Chapter 4

Testing

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Outline

1. Basic Ideas and Steps

- We have already learned about *point estimation* and *interval estimation*. Estimation is an idea to make a guess about the unknown quantity, e.g., μ.
- Where Point estimation gives us a fixed number, the interval estimation gives us an interval with some confidence.
- Now we will learn another major theme of statistical inference, this is known as *Hypothesis Testing*, in short *Testing*.
- Hypothesis Testing or Testing is slightly different than estimation. Here
  - ✓ We initially start with two (competing) hypotheses about the population parameter (This is called "Null" and "Alternative" Hypothesis).
  - ✓ Then we use the sample data to reject or accept one of two hypotheses.

Question is - What is a hypothesis Ans: It's simply a conjecture about the population parameter. Let's see an example.

We always start with two hypotheses, here is one example. Suppose somehow we have an information that the true population mean of income is less than 24, this means we know $\mu < 24$. Then we can form two hypotheses

$$H_0: \mu \geq 24$$
$$H_a : \mu < 24$$

Where H_0 is what we call a *Null Hypothesis* and H_a is what we call an *Alternative Hypothesis*.

After we are done with constructing the hypotheses, we use the random sample (data) to either *reject Null Hypothesis* or *accept the Null Hypothesis* (sometimes it is written fail to reject). Notice! Everything is happening around Null (Why? We will come back to the answer later!)

Note that we are making conclusion about the Population using a sample. This seems to be a very difficult task, right? So obviously there is a chance of making error. Here are the possible errors.

		Population Reality	
		H_0 True	H_a True
Conclusion	Accept H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

Before interpreting the table, first of all note that, we do not know what is the truth. Now the table says, if the hypothesized Null is actually true and after the testing we accept Null, then there is no error and we have reached the correct conclusion. But if the hypothesized Null is true and we reject the Null then we will make an error and the error is called Type-I error. Can you interpret other cells of the table?

Let's not write 24, below we will write μ_0 for any fixed value, and μ is our unknown Population mean that we don't know.

There are other ways we can form Hypotheses, for example one possibility is

$$H_{0:} : \mu \leq \mu_0$$
$$H_a : \mu > \mu_0$$

Another possibility is

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

The first two formations are called *one tail test*, and the last one is called *two tail test*, Later you will understand why this naming.

So we have following three formations,

- *It can be*

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

- *or, it can be*

$$H_{0:} : \mu \leq \mu_0$$
$$H_a : \mu > \mu_0$$

- *or, it can be*

$$H_{0:} : \mu \geq \mu_0$$
$$H_a : \mu < \mu_0$$

In the hypothesis testing again we will use the *sampling distribution of* $\bar{X}_n$.

Recall before we called $\bar{X}_n$ an estimator of μ.

In the Hypothesis testing we won't call this an estimator, rather we will call it a *Test Statistic*. In general often a *Test Statistic* is same or very similar to a point estimator. But note, the goal is different, here the goal is NOT estimation, rather testing.

Recall in general, *any* function of the random sample is called a "*Statistic*". When a *Statistic* is used for estimation we call it an *Estimator*. Similarly, when a *Statistic* is used for *Testing* we call it a *Test Statistic*. These are just some naming conventions that you need to know.

For example $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ is a function of random sample, so this is a statistic. When we use it for point estimation, we call this point estimator, but if we use it for hypothesis testing we call it a *Test Statistic*.

The distribution of a *Statistic* is called *Sampling Distribution*. So the distribution of $\bar{X}_n$ is also called a sampling distribution.

Let's see the basic steps of Hypothesis testing. Depending upon the formation, the steps might be slightly different but the basic procedure is more or less similar.

1. Write down the Null and Alternative hypotheses. Again this step might be different depending upon the question. For example we can start with

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

   This is called *Two-Tail* test (But we can also start with one-tail test, we will see more details on this later!)

2. Using the random sample calculate $\bar{x}$.
3. Fix Type - I error. Conventionally Type - I error is denoted by α, where $0 < \alpha < 1$. This will be essentially a probability, e.g., $.05$, 0.025, etc. There are different names of *Type I Error*, sometimes we call it also *rejection region*, or *level of significance* or *significance level*. But all are same thing. Divide it equally with an area of $\alpha/2$ in two-tails of the distribution. Essentially this is the possible mistake we can make.

4. In this step we need to use the *sampling distribution* of the test statistic under the Null (this means the sampling distribution of the test statistic assuming the Null Hypothesis is true). For example, if the test statistic is $\bar{X}_n$, then we will assume $\bar{X}_n \sim \mathcal{N}(\mu_0, \sigma^2/n)$ and use it for testing, where μ_0 is coming from the Null Hypothesis, and we will talk about σ^2 later. Then at this step we need to use the sampling distribution to find *critical values* $\bar{x}_{\frac{\alpha}{2}}$ and $\bar{x}_{1-\frac{\alpha}{2}}$. Critical values are two values such that

$$P(\bar{X}_n \leq \bar{x}_{\frac{\alpha}{2}}) = \alpha/2 \quad \text{and} \quad P(\bar{X}_n \leq \bar{x}_{1-\frac{\alpha}{2}}) = a - \alpha/2$$

   Here we can assume we know σ^2 (but this is an unrealistic assumption, and later we will see what to do when we relax it). If you use MS-Excel to find critical values, the function is "`NORM.INV`(cumulative probability, mean, variance)"

5. Reject the Null if $\bar{x} > \bar{x}_{1-\frac{\alpha}{2}}$ or $\bar{x} < \bar{x}_{\frac{\alpha}{2}}$ (just reject the Null if any one of the two happens). Why? We will see the intuition later.

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