Problem Set - 2 (Discrete Random Variables)

ECO 104 (Section 11)
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In-Class Practice Problems and Due Problems (Due Date: 5th May, in class hard copy submission)

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Please Note: Please submit individually in class.

§. Problems from Discrete Random Variables

1. From Anderson et al. (2020) Chapter 5, #1

   Consider the experiment of tossing a coin twice.

   (a) List the experimental outcomes.

   (b) Define a random variable that represents the number of heads occurring on the two tosses.

   (c) Show what value the random variable would assume for each of the experimental outcomes.

   (d) Is this random variable discrete or continuous?

2. From Anderson et al. (2020) Chapter 5, #2

   Consider the experiment of a worker assembling a product.

   (a) Define a random variable X that represents the time in minutes required to assemble the product.

   (b) What values may X take?

   (c) Is X a discrete or continuous random variable?

3. From Anderson et al. (2020) Chapter 5, #7.

   The probability mass function or PMF for the random variable X follows.

x	$f(x)$
20	0.20
25	0.15
30	0.25
35	0.40

   (a) Is this a valid PMF? Explain.

   (b) What is $\mathbb{P}(X = 30)$?

   (c) What is $\mathbb{P}(X \leq 25)$?

   (d) What is $\mathbb{P}(X \geq 30)$?

   (e) What is $\mathbb{E}(X)$ and $\mathbb{V}\text{ar}(X)$?

   (f) What is $\mathbb{E}(X^2)$ and $\mathbb{V}\text{ar}(X^3)$?

   (g) What is $\mathbb{E}(2X + 3)$ and $\mathbb{V}\text{ar}(2X + 3)$?

   (h) What is $\mathbb{E}(2X^2 + 3)$ and $\mathbb{V}\text{ar}(2X^2 + 3)$?

4. From Anderson et al. (2020) Chapter 5, #15.

   The following table provides a PMF for the random variable X.

x	$f(x)$
3	0.25
6	0.50
9	0.25

   (a) Compute $\mathbb{E}(X)$ and $\mathbb{V}\text{ar}(X)$.

   (b) Compute the standard deviation of X.

(c) What will happen to variance if we change the distribution as follows,

x	$f(x)$
3	0.3333
6	0.3333
9	0.3333

5. From Anderson et al. (2020) Chapter 5, #16 (slightly modified)

The following table provides a probability distribution for the random variable Y,

y	$f(y)$
2	0.20
4	0.30
7	0.40
8	0.10

(a) Compute $\mathbb{E}(Y)$ and $\mathbb{V}\text{ar}(Y)$.

(b) Compute $\mathbb{E}(Y^2)$.

(c) Calculate $\mathbb{E}(Y^2) - [\mathbb{E}(Y)]^2$. Check whether this is equal to $\mathbb{V}\text{ar}(Y)$.

6. From Anderson et al. (2020) Chapter 5, #31.

Consider a Binomial experiment with 2 trials and $p = .4$. This means we have a random variable X such that $X \sim \text{Bin}(2, 0.4)$

(a) Compute $f(0), f(1), f(2)$ and interpret the value.

(b) Compute the probability of at least one success, this means $\mathbb{P}(X \geq 1)$

(c) Compute $\mathbb{E}(X)$ and $\mathbb{V}\text{ar}(X)$.

7. From Anderson et al. (2020) Chapter 5, #32.

Consider a Binomial experiment with $n = 10$ and $p = .10$. This means we have a random variable $X \sim \text{Bin}(10, 0.10)$

(a) Compute $f(0)$.

(b) Compute $f(2)$.

(c) Compute $\mathbb{P}(X \leq 2)$.

(d) Compute $\mathbb{P}(X \geq 1)$.

(e) Compute $\mathbb{E}(X)$ and $\mathbb{V}\text{ar}(X)$.

8. From Anderson et al. (2020) Chapter 5, #33.

Consider a binomial experiment with $n = 20$ and $p = .70$. This means we have a random variable $X \sim \text{Bin}(20, 0.70)$

(a) Compute $f(12)$.

(b) Compute $f(16)$.

(c) Compute $\mathbb{P}(X \geq 16)$.

(d) Compute $\mathbb{P}(X \leq 15)$.

(e) Compute $\mathbb{E}(X)$ and $\mathbb{V}\text{ar}(X)$.

§. **Applied Problems** Solve following applied problems from Anderson et al. (2020)

9. From Anderson et al. (2020) Chapter 5, #36.

Number of Defective Parts. When a new machine is functioning properly, only 3% of the items produced are defective. Assume that we will randomly select two parts produced on the machine and that we are interested in the number of defective parts found.

(a) How can you think about a Bernoulli random variable here?

(b) How can you think about a Binomial random variable here? And what is the key condition under which we can think about a Binomial random variable?

(c) What are the possible values of the Binomial random variable?

(d) What is the mean and variance of the Binomial random variable?

10. From Anderson et al. (2020) Chapter 5, #41.

Introductory Statistics Course Withdrawals. A university found that 20% of its students withdraw without completing the introductory statistics course. Assume that 20 students registered for the course.

(a) Compute the probability that 2 or fewer will withdraw.

(b) Compute the probability that exactly 4 will withdraw.

(c) Compute the probability that more than 3 will withdraw.

(d) Compute the expected number of withdrawals.

11. From Anderson et al. (2020) Chapter 5, #42.

State of the Nation Survey. Suppose a sample of 20 Americans is selected as part of a study of the state of the nation. The Americans in the sample are asked whether or not they are satisfied with the way things are going in the United States.

(a) Compute the probability that exactly 4 of the 20 Americans surveyed are satisfied with the way things are going in the United States.

(b) Compute the probability that at least 2 of the Americans surveyed are satisfied with the way things are going in the United States.

(c) For the sample of 20 Americans, compute the expected number of Americans who are satisfied with the way things are going in the United States.

(d) For the sample of 20 Americans, compute the variance and standard deviation of the number of Americans who are satisfied with the way things are going in the United States.

Remarks: All problems are taken from Anderson et al. (2020). If possible you should do more problems from there.

References:

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., Cochran, J. J., Fry, M. J. and Ohlmann, J. W. (2020), *Statistics for Business & Economics*, 14th edn, Cengage, Boston, MA.