Digital Tool For Finance

Final Assignment

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1 Risk Measurement

1.1 Standard Deviation, Semi Variance, Lower Partial Moment Index

Standard deviation is the most commonly used risk measurement indicator in not only industry but also academic research study. In the 1950s, Markowitz used the expected rate of return, variance and covariance to determine the efficient frontier of investment. Each portfolio on the boundary either maximizes its expected return given the variance, or has a minimized variance given a target rate of return. In later studies, many scholars found that investors' views on risk are not symmetrical, instead, they pay more attention to the downside risk of financial assets. Later, Markowitz (1959) further proposed two indicators to measure the downside risk: mean downside semi variance SV_m and target downside semi variance SV_t:

$$SV_m = \frac{1}{T}\sum_{t=1}^{T} \max\left[0, E(R_t) - R_t\right]^2$$
$$SV_m = \frac{1}{T}\sum_{t=1}^{T} \max\left[0, R^* - R_t\right]^2$$

where $E(R_t)$ is the average rate of return, and R^* is the target rate of return. The above indicators assigned a great impact on asset portfolio theory, but they have also caused great controversy on the description of the skewness of the financial asset distribution. Bawa (1975) first proposed the concept of lower partial moment, LPM, which defined as follow:

$$LPM(a,t) = \frac{1}{T}\sum_{t=1}^{T} \max\left[0, (R^* - R_t)\right]^a$$

where a is the order of the lower partial moment, and R^* is the target rate of return. (1) When $a = 0$, LPM represents the probability lower than the target rate of return; (2) When $a = 1$, LPM is the average value that is lower than the target rate of return. This indicator can then be used to represent risk-neutral investors; (3) When $a = 2$, LPM is the risk below the target rate of return, which can be used to describe risk-averse investors; (4) As a increases, investors become more risk averse. Following a long period of time, the method of determining the order a of LPM has attracted great attention from the academic community, and many related algorithms and research on the properties of LPM have been fast developed.

1.2 Value At Risk Indicators

Besides the risk metrics mentioned above, J. P. Morgan (1994) introduced the concept of VaR in response to the financial disasters of the 1990s. It is defined as follows

$$\mathrm{VaR}_\alpha(X) = -q_\alpha(X) = -\inf\{x : P[X \leq x] \geq \alpha\}$$

where X is a given random variable, and α is the confidence level. This indicator was widely noticed by researchers and practitioners at that time. However, with the in-depth of research, this indicator has been questioned a lot. It does not meet the consistency condition of risk measurement indicators proposed by Artzner. P (1999), so it cannot measure the tail risk and the risk of occurrence of the extreme events. Artzner (1999) proposed a new risk measurement indicator called conditional value at risk CVaR, which is defined as the following

$$\mathrm{CVaR}_\alpha(X) = -E\left[X \leq q_\alpha(X)\right]$$

where X is required to be a continuous random variable. This indicator can make up for the disadvantage of VaR, because it is consistent and compatible, and can better measure the tail risk. Rockafellar and Uryasevb (2002) believed that another defect of VaR index is that it cannot describe the degree of loss after the loss exceeds the critical value of the index. It only provides a minimum limit for the loss at the tail of the loss distribution with optimism rather than following the conservatism that prevails in risk management. Additionally, they believe that compared with VaR, the advantages of CVaR indicators include its subadditivity, measurability of risks exceeded VaR, and ability to solve the optimization problem of large-scale asset portfolios. However, CVaR is no longer a consistent measure of risk whenever the density of loss function is not continuous. Therefore, Dirk Tasche (2002) brought up the concept of Expected Shortfall ES:

$$ES(\alpha) = \frac{\int_0^\alpha \mathrm{VaR}(u)du}{\alpha}$$

This indicator overcomes the disadvantages of $CVaR_\alpha(X)$, and has been widely used in practice.

2 Risk Parity Model

Markowitz model represents a new stage of the development of modern portfolio theory. The model has been improved by scholars with respect to strong parameter sensitivity and insufficient risk measurement. At the same time, many scholars and practitioners began to focus on other asset portfolio models.

Bridgewater Associates first proposed the idea of risk parity in 1996, and created the All Weather(AW) strategy. The core idea of the model is to equates the risk of each asset by determining their weights in the asset portfolio. The total risk $\boldsymbol{\mathcal{R}}_{\mathbf{p}}(\vec{w})$ of a portfolio can be written as:

$$\boldsymbol{\mathcal{R}}_{\mathbf{p}} = \sqrt{\vec{w}^T \Sigma \vec{w}}$$

The marginal risk $\boldsymbol{M\mathcal{R}}(X_i)$ and the risk contribution $\boldsymbol{\mathcal{R}C}(X_i)$ of each asset i to the portfolio can be written as:

$$\boldsymbol{M\mathcal{R}}\left(X_i\right) = \frac{\partial \boldsymbol{\mathcal{R}}_{\mathbf{p}}}{\partial w_i}$$

$$\boldsymbol{\mathcal{R}C}\left(X_i\right) = w_i \frac{\partial \boldsymbol{R}_{\mathbf{p}}}{\partial w_i} = w_i \frac{\partial \sqrt{\vec{w}^T \sum \vec{w}}}{\partial w_i} = w_i \frac{(\Sigma \vec{w})_i}{\sqrt{\vec{w}^T \sum \vec{w}}}$$

Later, Q.Edward (2005) conducted an empirical analysis using the Russell 1000 Index and the Exchange Traded Funds, and found that the performance of the investment portfolio based on the risk parity model was significantly better than that of a single financial asset. At the same time, the strategy obtained by the risk parity model has a higher average rate of return and Sharpe ratio, which proves that the risk parity model can disperse risks and improve the robustness of returns.

3 Calculation of Risk Metrics based on MVT distribution

3.1 Data Description and Normality Test

3.1.1 Data Selection

In order to conduct empirical analysis, we use the daily trading data of CSI 300, CSI Aggregate Bond Index and Exchange Traded Fund to represent the performance of stock, marketable Securities, and commodity markets, respectively. Through the data of the daily closing price from 2015 to 2019, we can calculate the respective daily average return according to $r_t = \frac{P_t - P_{t-1}}{P_t}$ and draw the following figure.

3.1.2 Statistics Description and Normality Test

Through calculation, various statistics of the logarithmic return rate of CSI 300, CSI Aggregate Bond Index(ABI) and Gold Exchange Traded Fund(ETF) can be obtained, which are listed as follows:

Assets	Average	SD	Median	Max	Min	Kurtosis	Skewness	JB Test
CSI 300	0.01%	1.55%	0.05%	6.50%	-9.15%	9.46	-1.02	0.001
CSI ABI	0.02%	0.07%	0.02%	0.67%	-0.73%	16.89	-0.41	0.001
Gold ETF	0.03%	0.75%	0.04%	4.52%	-3.80%	7.56	0.36	0.001

Table 1: CSI 300, CSI ABI, and Gold ETF Logarithmic Statistics Description

According to the above results, we can draw the following conclusions:
(1) Volatility: The standard deviation(SD) of the CSI 300 Index is higher than that of the Gold ETF and much higher than that of the CSI ABI. It indicates that the volatility of the stock market is high, while that of the bond market is low, and the volatility of the gold market is in the middle.
(2) Leptokurtic: We found that the three types of assets all have high kurtosis, i.e., greater than 3, which shows that their logarithmic returns have a "fat tail" phenomenon. At the same time, the CSI 300 Index and the CSI ABI have a certain "negative bias",

while the Gold ETF has a certain "positive bias". Therefore, we found that the logarithmic return series of the three types of assets all have "peak and fat tail".

(3) Normality Test: According to the fact that the p-value of the Jarque–Bera test(JB test) of the three assets is less than 0.05, it can be seen that at the 95% confidence level, the original hypothesis of "H0: the logarithmic return follows a normal distribution" is rejected, which shows that the logarithmic return of the three assets does not follow the normal distribution. In addition, the corresponding P-P Plot can intuitively show this characteristic.

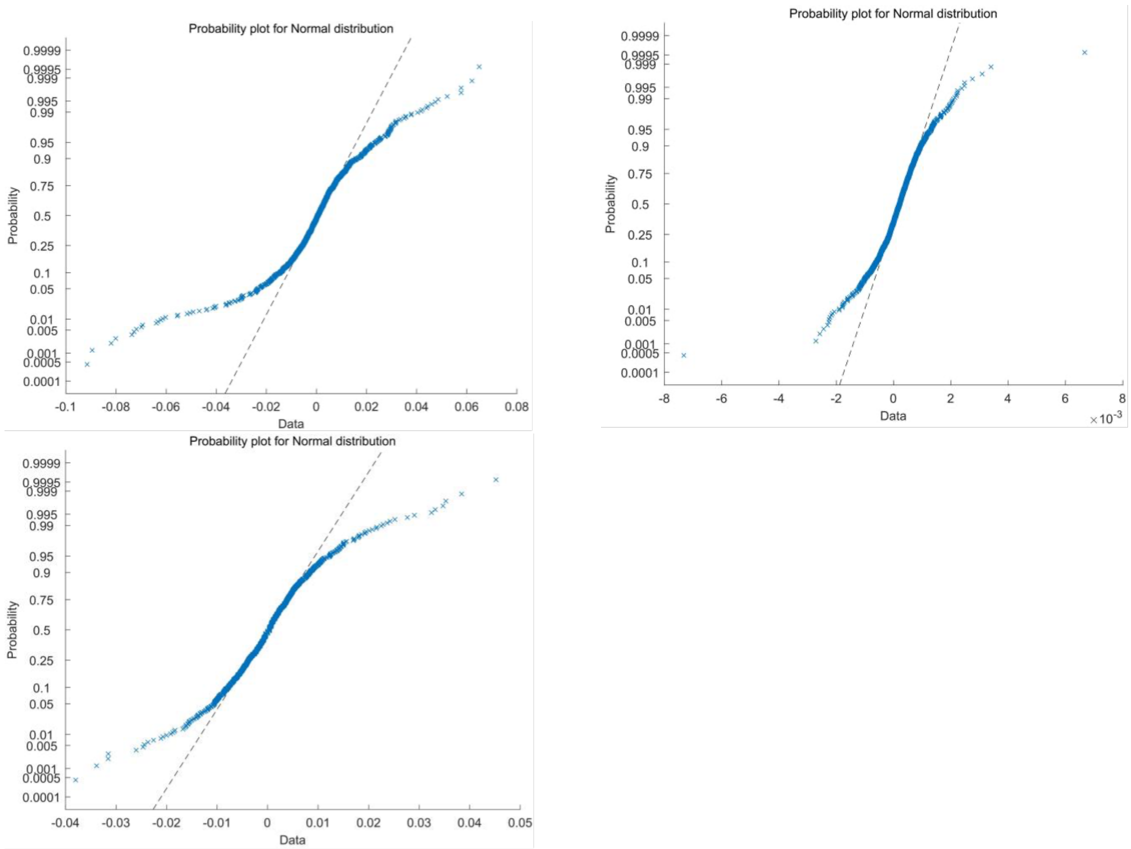

Figure 1: CSI-300 (upper left), CSI-ABI (upper right), and Gold-ETF (lower left) daily logarithmic return normal distribution P-P Plot of Year 2015-2020

3.2 Parameter Estimation of MVT Distribution

According to the relevant statistical analysis of the logarithmic return of the three types of assets in **Figure 1**, we know that using the normal distribution to describe the return on financial assets is not effective and there is correlation between the returns on each asset, so the MVT distribution is used to estimate The rate of return of the three major assets is a more reasonable choice. Through the MMF method, we can obtain the following results: