732A96/TDDE15 Advanced Machine Learning

Reinforcement Learning

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Lectures 9: REINFORCE and Deep Q-Learning Algorithms

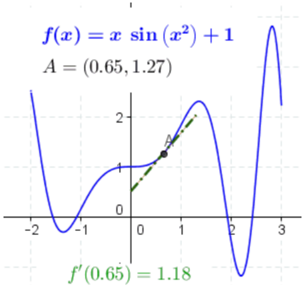
Contents

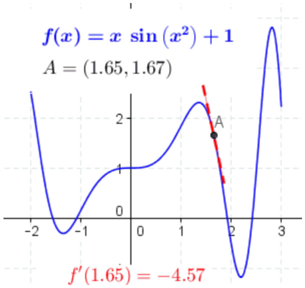
Literature

- Main source
  - Sutton, R. S. and Barto, A. G. *Reinforcement Learning: An Introduction*. The MIT Press, 2018. Chapters 13 and 16.
  - Mnih, V. et al. Playing Atari with Deep Reinforcement Learning. *NIPS Deep Learning Workshop*, 2013.
- Additional source
  - Russel, S. and Norvig, P. *Artifical Intelligence: A Modern Approach*. Pearson, 2010. Chapters 16, 17 and 21.
  - Mnih, V. et al. Human-level Control through Deep Reinforcement Learning. *Nature 518*, 529–533, 2015.

REINFORCE

- Consider learning a **parameterized policy** $\pi(a|s,\theta)$ to select actions without consulting state or action values.
- Advantages:
  - The policy function may be simpler to learn than the state and action value functions.
  - Possibility to inject prior knowledge through the parameterization chosen.
  - Possibility to model deterministic and stochastic policies. The latter may be interesting in problems with imperfect knowledge, e.g. bluffing in poker. Learning is also easier as the gradient is smoother.
- Choose the parameter values that maximize the following function for **episodic** tasks: $J(\theta) = v_{\pi_\theta}(S_0)$ where S_0 is the initial state. It can also be adapted to continuing tasks.
- Stochastic **gradient ascent**: $\theta^{i+1} \leftarrow \theta^i + \alpha \nabla_{\theta^i} J(\theta^i)$.





REINFORCE

- Theorem:

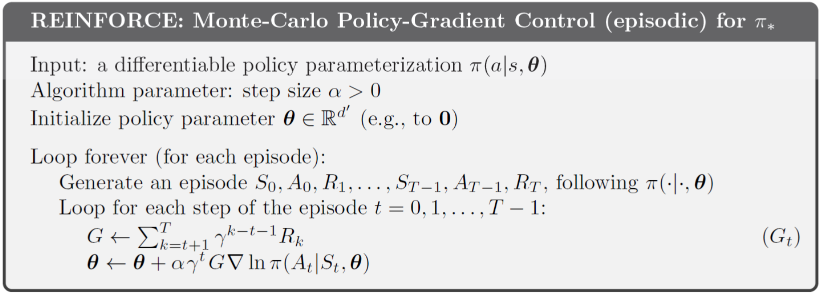
$$\nabla_\theta J(\theta) = \nabla_\theta v_{\pi_\theta}(S_0) \propto E_{\pi_\theta}\Big[\sum_a q_\pi(S_t, a)\nabla_\theta \pi(a|S_t, \theta)\Big]$$

  and thus

$$\nabla_\theta J(\theta) \propto E_{\pi_\theta}\Big[\sum_a \pi(a|S_t,\theta) q_\pi(S_t, a)\frac{\nabla_\theta \pi(a|S_t,\theta)}{\pi(a|S_t,\theta)}\Big] = E_{\pi_\theta}\Big[q_\pi(S_t, A_t)\frac{\nabla_\theta \pi(A_t|S_t,\theta)}{\pi(A_t|S_t,\theta)}\Big]$$

$$= E_{\pi_\theta}\Big[G_t \frac{\nabla_\theta \pi(A_t|S_t,\theta)}{\pi(A_t|S_t,\theta)}\Big] = E_{\pi_\theta}[G_t \nabla_\theta \ln \pi(A_t|S_t,\theta)].$$

- All this results in the REINFORCE algorithm, which asymptotically converges to a local optimum of $J(\theta)$.



REINFORCE: Monte-Carlo Policy-Gradient Control (episodic) for π_*

```
Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta})$
Algorithm parameter: step size $\alpha > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
    Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot,\boldsymbol{\theta})$
    Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
        $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$                    $(G_t)$
        $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha\gamma^t G \nabla \ln \pi(A_t|S_t,\boldsymbol{\theta})$
```

REINFORCE

- REINFORCE has one major disadvantage: It updates the policy only **at the end of each episode**, which makes it not suitable for incremental online learning and, typically, implies slow convergence and high variance.
- A particularly interesting advantage of REINFORCE is that updating the policy parameters may change the policy for every action-state pair, and not only for the pairs visited, i.e. **generalization**.
- Another interesting feature of REINFORCE is that it can be applied to **continuous state and action spaces** by choosing an appropriate parametric policy model, e.g. $\mathcal{N}(\mu s, \sigma)$:

  $$\pi(a|s,\theta) = \pi(a|s,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(- \frac{(a-\mu s)^2}{2\sigma^2} \right)$$

  which implies that

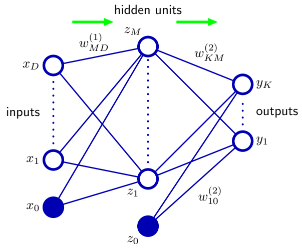
  $$\nabla_\mu \ln \pi(a|s,\mu,\sigma) = \frac{a-\mu s}{\sigma^2} s$$

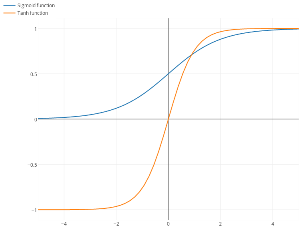
  and

  $$\nabla_\sigma \ln \pi(a|s,\mu,\sigma) = \left(\frac{(a-\mu s)^2}{\sigma^2} - 1 \right) s$$

  and, thus, the REINFORCE update has a simple form.
- However, we can easily do better than this.

Neural Networks





- Consider a multiclass classification problem, i.e. $C \in \{1, 2, \dots, K\}$.
- Activations: $a_j = \sum_{i=1}^{D} w_{ji}^{(1)} x_i + w_{j0}^{(1)}$ for all $j = 1, \dots, M$.
- Hidden units and activation function: $z_j = h(a_j)$ for all $j = 1, \dots, M$.
- Output activations: $a_k = \sum_{j=1}^{M} w_{kj}^{(2)} z_j + w_{k0}^{(2)}$ for all $k = 1, \dots, K$.
- Output : $y_k(\boldsymbol{x}) = a_k$ for all $k = 1, \dots, K$.
- Two-layer NN: $y_k(\boldsymbol{x}) = \sum_{j=1}^{M} w_{kj}^{(2)} h\left(\sum_{i=1}^{D} w_{ji}^{(1)} x_i + w_{j0}^{(1)} \right) + w_{k0}^{(2)}$.
- Class probabilities: $p(C = k|\boldsymbol{x}) = \frac{\exp(y_k(\boldsymbol{x}))}{\sum_{k=1}^{K} \exp(y_k(\boldsymbol{x}))}$.
- All the previous is generalizable to several hidden layers.

Neural Networks

- Consider some training data $\{(\boldsymbol{x}_n, c_n)\}$ of size N. Consider minimizing the cross-entropy or negative log-likelihood loss function:

$$L(\boldsymbol{w}) = \sum_{n=1}^{N} L_n(\boldsymbol{w}) = -\sum_{n=1}^{N} \log p(c_n|\boldsymbol{x}_n, \boldsymbol{w}).$$

- The weight space is highly multimodal and, thus, we have to resort to approximate iterative methods to minimize the previous expression.
- Stochastic gradient descent:

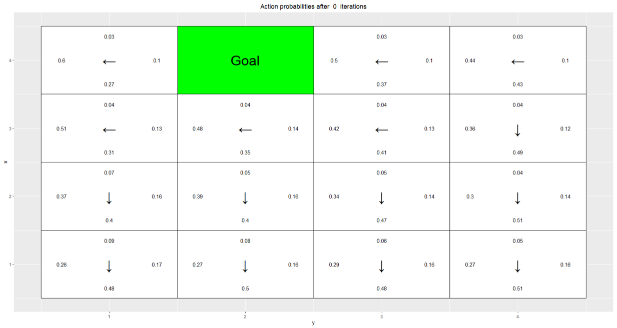
$$\boldsymbol{w}^{i+1} = \boldsymbol{w}^i - \alpha \nabla_{\boldsymbol{w}^i} L_n(\boldsymbol{w}^i) = \boldsymbol{w}^i + \alpha \nabla_{\boldsymbol{w}^i} \log p(c_n|\boldsymbol{x}_n, \boldsymbol{w}^i)$$

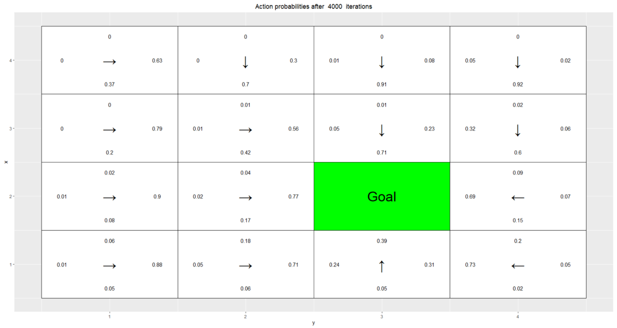
  where n is chosen randomly, and $\nabla_{\boldsymbol{w}^i} L_n(\boldsymbol{w}^i)$ can be computed efficiently via **backpropagation**.
- Note the similarities between the update above and the REINFORCE update:

$$\theta^{i+1} = \theta^i + \alpha \gamma^t G \nabla_{\theta^i} \ln \pi(A_t|S_t, \theta^i).$$

- Therefore, REINFORCE is straightforward to implement when a NN is used to represent the parameterized policy. We only need to set the sample weights $\gamma^t G$. In other words, the policy can be seen as a classifier whose training involves certain peculiarities: Incremental online, and **delayed labels** via sample weights.

Example: Grid Worlds





Neural Networks

- Now, consider a K-dimensional regression problem, i.e. $\boldsymbol{C} = (C_1, \dots, C_K)$. Consider also some training data $\{(\boldsymbol{x}_n, \boldsymbol{c}_n)\}$ of size N. Consider minimizing the mean squared-error loss function:

$$L(\boldsymbol{w}) = \sum_{n=1}^{N} L_n(\boldsymbol{w}) = \sum_{n=1}^{N} \frac{1}{2} \|\boldsymbol{c}_n - \boldsymbol{y}(\boldsymbol{x}_n)\|^2 = \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{1}{2} (c_{nk} - y_k(\boldsymbol{x}_n))^2.$$

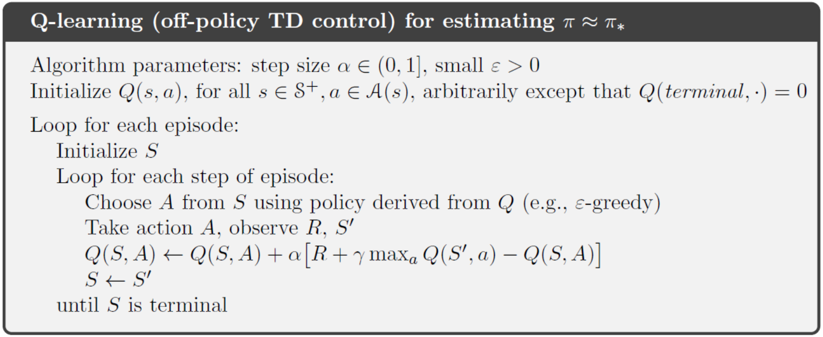
- The above implies the following stochastic gradient descent update:

$$\boldsymbol{w}^{i+1} = \boldsymbol{w}^{i} - \alpha \nabla_{\boldsymbol{w}^i} L_n(\boldsymbol{w}^i) = \boldsymbol{w}^{i} - \alpha \sum_{k=1}^{K} (c_{nk} - y_k(\boldsymbol{x}_n)) \nabla_{\boldsymbol{w}^i} \boldsymbol{y}(\boldsymbol{x}_n)$$

  where n is chosen randomly, and $\nabla_{\boldsymbol{w}^i} \boldsymbol{y}(\boldsymbol{x}_n)$ can be computed efficiently via **backpropagation**.

Deep Q-Learning

- If the transition model were so that s and a are always followed by s' and r, then $q_*(s,a) = r + \gamma \max_{a'} q_*(s',a')$ by the Bellman optimality equation and, thus, $0 = r + \gamma \max_{a'} q_*(s',a') - q_*(s,a)$. We can try to enforce this constraint by executing π one step from s and a and, then, updating the estimate of $q_*(s,a)$ as
$$q_*(s,a) \leftarrow q_*(s,a) + \alpha(r + \gamma \max_{a'} q_*(s',a') - q_*(s,a)).$$
where $\alpha > 0$ is the learning rate.
- The reasoning above also applies to stochastic transition models, since the number of times that s and a are followed by s' and r in the sampled episodes is proportional to the transition probability.



Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

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Algorithm parameters: step size $\alpha \in (0,1]$, small $\varepsilon > 0$
Initialize $Q(s,a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
    Initialize $S$
    Loop for each step of episode:
        Choose $A$ from $S$ using policy derived from $Q$ (e.g., $\varepsilon$-greedy)
        Take action $A$, observe $R$, $S'$
        $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
        $S \leftarrow S'$
    until $S$ is terminal
```

Deep Q-Learning

- Q-learning has a major disadvantage: Representing the action value function $q(s, a)$ as a look-up table is not feasible for **large state and/or action spaces**, due to storage space and time to convergence.
- A solution is to represent it as a **parameterized function** $q(s, a|\theta)$. This brings advantages such as fewer parameters than table entries and thus easier to learn and reach convergence, less storage space, **generalization to unvisited state-action pairs**, etc.
- It also comes with the challenge of selecting an appropriate parameterized function: It has to be expressive and allow for incremental online training. A NN is typically used.

Deep Q-Learning

- For the i-th iteration (i.e., episode step), the NN is trained by minimizing the mean squared-error loss function:

$$L(\theta^i) = E_{s,a\sim\rho(s,a)}[(y^i - q(s,a|\theta^i))^2]$$

where $\rho(s,a)$ is the behaviour distribution induced by ϵ-greedy, and

$$y^i = E_{s'\sim p(s'|s,a)}[r + \gamma \max_{a'} q(s',a'|\theta^{i-1})|s,a].$$

- The gradient is given by

$$\nabla_{\theta^i} L(\theta^i) = E_{s,a\sim\rho(s,a);s'\sim p(s'|s,a)}[(r+\gamma \max_{a'} q(s',a'|\theta^{i-1}) - q(s,a|\theta^i))\nabla_{\theta^i} q(s,a|\theta^i)]$$

and stochastic gradient descent is typically considered:

$$\theta^{i+1} = \theta^i - \alpha(r + \gamma \max_{a'} q(s',a'|\theta^{i-1}) - q(s,a|\theta^i))\nabla_{\theta^i} q(s,a|\theta^i).$$

- Note the similarities between the update above and the NN update:

$$\boldsymbol{w}^{i+1} = \boldsymbol{w}^i - \alpha\nabla_{\boldsymbol{w}^i} L_n(\boldsymbol{w}^i) = \boldsymbol{w}^i + \alpha\sum_{k=1}^{K}(c_{nk} - y_k(\boldsymbol{x}_n))\nabla_{\boldsymbol{w}^i}\boldsymbol{y}(\boldsymbol{x}_n)$$

which implies that deep Q-learning is straightforward to implement when a NN is used to represent the parameterized state value function. In other words, the action value function can be seen as a regressor whose training involves certain peculiarities: Incremental online, and **delayed targets** via bootstrapping on the model from the previous iteration.

Deep Q-Learning

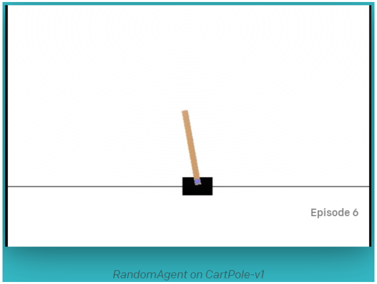
```
Deep Q-learning with experience replay for estimating $\pi \approx \pi_*$

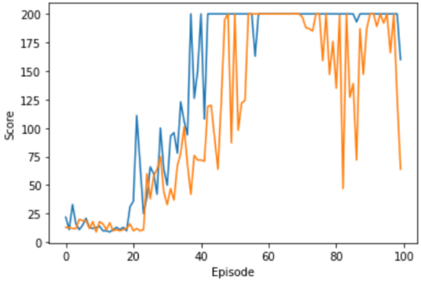
Algorithm parameters: small $\epsilon > 0$
Initialize $Q(S, A|\theta)$ with random weights, and initialize $\mathcal{D}$ with $N$ random experiences

Loop for each episode:
    Initialize $S$
    Loop for each step of episode:
        Choose $A$ from $S$ using policy derived from $Q$ (e.g., $\epsilon$-greedy)
        Take action $A$, observed $R, S'$
        Store transition $(S, A, R, S')$ in $\mathcal{D}$
        Sample random minibatch of transitions $(S_j, A_j, R_j, S'_j)$ from $\mathcal{D}$
        $Y_j \leftarrow \begin{cases} R_j & \text{for terminal } S'_j \\ R_j + \gamma \max_a Q(S'_j, a|\theta) & \text{for non-terminal } S'_j \end{cases}$
        Perform a gradient descent step on $(Y_j - Q(S_j, A_j|\theta))^2$
        $S \leftarrow S'$
    until $S'$ is terminal
```

- Learning a parameterized state value function may resemble supervised learning but the learning data is **not i.i.d.** due to highly correlated states and changing agent behaviour. Experience replay alleviates these problems by smoothing the training distribution over many past behaviours.
- All action values at once: $S \rightarrow NN \rightarrow (Q(S, A_1), \ldots, Q(S, A_k))$.

Example: CartPole





- Open AI's gym
- PilcoLearner

Summary

- REINFORCE
- Neural Networks
- Example: Grid Worlds
- Deep Q-Learning
- Example: CartPole
- Interested in more ? Check out AlphaGo - The Movie.

Thank you