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TBAB01

# TABELL- OCH FORMELSAMLING

## SANNOLIKHETSFÖRDELNINGAR

- **Binomialfördelning**

$$X \sim Bin(n,p)$$
$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}, \qquad x = 0,1,\ldots,n$$
$$\mathbb{E}X = np, \qquad Var(X) = np(1-p).$$

- **Poissonfördelning**

$$X \sim Po(\mu)$$
$$P(x) = \frac{\mu^x}{x!} e^{-\mu}, \qquad x = 0,1,2,\ldots$$
$$\mathbb{E}X = \mu, \qquad Var(X) = \mu.$$

- **Geometrisk fördelning**

$$X \sim Ge(p)$$
$$P(x) = (1-p)^{x-1} p, \qquad x = 1,2,\ldots$$
$$\mathbb{E}X = \frac{1}{p}, \qquad Var(X) = \frac{1-p}{p^2}.$$

- **Multinomialfördelning**

$$(X_1,...,X_k) \sim Multinomial(n, p_1,...,p_k)$$
$$P(x_1,...,x_k) = \frac{n!}{x_1! \cdots x_k!} p_1^{x_1} \cdots p_k^{x_k}, \qquad x_i = 0,1,2,...,n \text{ och } \sum_{i=1}^{n} x_i = n.$$
$$\mathbb{E}X_i = np_i, \qquad Var(X_i) = np_i(1-p_i), \qquad Cov(X_i,X_j) = -np_ip_j \ (i \neq j).$$

- **Likformig (rektangulär) fördelning på intervallet (a,b)**

$$X \sim U(a,b)$$
$$f(x) = \frac{1}{b-a}, \qquad a \leq x \leq b$$
$$\mathbb{E}\mathbb{X} = \frac{a+b}{2}, \qquad Var(X) = \frac{(b-a)^2}{12}.$$

- **Exponentialfördelning**

$$X \sim Exp(\lambda),$$

där $\lambda$ betecknar intensiteten. Ibland används väntevärdet $\mu = \frac{1}{\lambda}$ som parameter.

$$f(x) = \lambda e^{-\lambda x}, \qquad x \geq 0$$

$$\mathbb{E}X = \frac{1}{\lambda}, \qquad Var(X) = \frac{1}{\lambda^2}.$$

- **Normalfördelning**

$$X \sim N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \qquad -\infty < x < +\infty$$

$$\mathbb{E}X = \mu, \qquad Var(X) = \sigma^2.$$

- **$\chi^2$-fördelning**

$$Y \sim \chi^2(\nu)$$

Uppkomst: Om $X_1, \ldots, X_n$ är oberoende, var och en $N(0,1)$, gäller att $Y = X_1^2 + \ldots + X_n^2$ får en $\chi^2$ fördelning med $\nu$ frihetsgrader.

$$f(x) = \frac{x^{(\nu/2)-1} e^{-x/2}}{2^{(\nu/2)}\Gamma(\nu/2)}, \qquad x \geq 0,$$

där $\Gamma(\cdot)$ är gammafunktionen

$$\Gamma(c) = \int_0^\infty x^{c-1} e^{-x} dx, \quad \text{där } c > 0.$$

$$\mathbb{E}Y = \nu, \qquad Var(Y) = 2\nu.$$

- **t-fördelning**

$$Z \sim t(\nu)$$

Uppkomst: Om $X \sim N(0,1)$ och $Y \sim \chi^2(\nu)$ samt $X$ och $Y$ är oberoende, så gäller att $Z = \dfrac{X}{\sqrt{Y/n}}$ får en t-fördelning med $\nu$ frihetsgrader.

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)\left(1+\frac{x^2}{\nu}\right)^{(\nu+1)/2}}, \qquad -\infty < x < +\infty$$

- **Gammafördelning**

$$Y \sim \Gamma(\alpha, \lambda)$$

Uppkomst: Om $X_1, \ldots, X_n$ är oberoende, var och en $Exp(\lambda)$, så blir $Y = X_1 + \ldots + X_n$ gammafördelad med parametrarna $n$ och $\lambda$.

$$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, \qquad x \geq 0$$

$$\mathbb{E}Y = \frac{\alpha}{\lambda}, \qquad Var(Y) = \frac{\alpha}{\lambda^2}.$$

- **Betafördelning**

$$X \sim Beta(\alpha, \beta)$$

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \qquad 0 < x < 1$$

$$\mathbb{E}X = \frac{\alpha}{\alpha+\beta}, \qquad Var(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}.$$

- **Dirichletfördelningen**

$$(X_1, ..., X_k) \sim Dirichlet(\alpha_1, ..., \alpha_k)$$

$$P(x_1, ..., x_k) = \frac{\Gamma(\sum_{i=1}^k \alpha_i)}{\prod_{i=1}^k \Gamma(\alpha_i)} x_1^{\alpha_1-1} \cdots x_k^{\alpha_k-1}, \qquad 0 < x_i < 1 \text{ och } \sum_{i=1}^n x_i = 1.$$

$$\mathbb{E}X_i = \frac{\alpha_i}{\alpha_0}, \text{ där } \alpha_0 = \sum_{i=1}^k \alpha_i \qquad Var(X_i) = \frac{\alpha_i(\alpha_0 - \alpha_i)}{\alpha_0^2(\alpha_0+1)}, \qquad Cov(X_i, X_j) = -\frac{\alpha_i\alpha_j}{\alpha_0^2(\alpha_0+1)} (i \neq j).$$

## DIVERSE DEFINITIONER OCH RESULTAT

- Kovarians: $Cov(X, Y) = \mathbb{E}[(X - \mu_X)(Y - \mu_Y)]$, där $\mu_X = \mathbb{E}X]$ och $\mu_Y = \mathbb{E}Y$
- Korrelation: $\rho(X, Y) = \dfrac{Cov(X, Y)}{\sigma_X \sigma_Y}$, där $\sigma_X^2 = Var(X)$ och $\sigma_Y^2 = Var(Y)$
- Generellt gäller att

$$\mathbb{E}(a_1 X_1 + \ldots + a_n X_n + b) = a_1 \mathbb{E}X_1 + \ldots + a_n \mathbb{E}X_n + b.$$

- För *oberoende* slumpvariabler $X_1, \ldots, X_n$ gäller att

$$Var(a_1 X_1 + \ldots + a_n X_n + b) = a_1^2 Var(X_1) + \ldots + a_n^2 Var(X_n).$$

- Generellt gäller att

$$Var(a_1 X_1 + \ldots + a_n X_n + b) = \sum_{j=1}^n a_j^2 Var(X_j) + 2 \sum_{1 \leq j < k \leq n} a_j a_k Cov(X_j, X_k).$$

- $X \sim Bin(n, p)$ och $n \geq 10$, $p \leq 0.1 \quad \Rightarrow \quad X \approx Po(np)$
- $X \sim Bin(n, p)$ och $np(1-p) \geq 10 \quad \Rightarrow \quad X \approx N(np, np(1-p))$,
- $X \sim Po(\mu)$ och $\mu \geq 15 \quad \Rightarrow \quad X \approx N(\mu, \mu)$.
- Om $X_1, ..., X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, gäller följande:

1. $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$,
2. $\dfrac{(n-1)S^2}{\sigma^2} = \dfrac{\sum_{j=1}^n (X_j - \bar{X}_n)^2}{\sigma^2} \sim \chi^2(n-1)$,

3. $\dfrac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$.

- Vid **enkel linjär regression** ges modellen av

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \qquad \text{för } i = 1, \ldots, n,$$

där $\varepsilon_i \sim N(0, \sigma)$ och oberoende.
**Minsta kvadrat-skattningar**

$$\begin{aligned} \hat{\beta}_1 &= \frac{SS_{xy}}{SS_{xx}}, \\ \hat{\beta}_0 &= \overline{y} - \hat{\beta}_1 \overline{x}, \\ \widehat{\sigma^2} &= \frac{1}{n-2} \sum_{i=1}^{n} \left( y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i \right)^2 \end{aligned}$$

där

$$\begin{aligned} S_{xx} &= \sum_{i=1}^{n} (x_i - \overline{x})^2 \\ S_{xy} &= \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y}) \end{aligned}$$

- $\chi^2$ **goodness of fit-test**.

  – $H_0$: Fördelningsfunktioner är $F_0(x)$ (inga okända parametrar).

  Låt $p_i = F_0(a_i) - F_0(a_{i-1})$ och $N_i$ antalet $x_i$ i intervallet $(a_{i-1}, a_i]$.

  Teststatistika: $T = \sum_{i=1}^{k} \dfrac{(N_i - np_i)^2}{np_i} \approx \chi^2(k-1)$-fördelad under $H_0$.

  – $H_0$: Given parametrisk fördelningsklass med fördelningsfunktion $F(x)$.

  Teststatistika: $T = \sum_{i=1}^{k} \dfrac{(N_i - np_i)^2}{np_i}$,

  där $p_i$ beräknas som enligt föregående punkt sedan parametrarna i $F(x)$ har skattats. $T$ är approximativt $\chi^2(k-1-r)$-fördelad under $H_0$ där $r$ = antalet skattade parametrar i $F(x)$.

  I båda fallen krävs att alla $np_i \geq 5$.

## BAYESIANSK INFERENS

## Bernoulli data - Beta prior

- Modell: $X_1, ..., X_n|\theta \sim Bernoulli(\theta)$
- Prior: $\theta \sim Beta(\alpha, \beta)$
- Posterior: $\theta|x_1, ..., x_n \sim Beta(\alpha + s, \beta + f)$, där $s = \sum_{i=1}^{n} x_i$ och $f = n - s$.

## Normal data - Normal prior

- Modell: $X_1, ..., X_n|\theta, \sigma^2 \sim N(\theta, \sigma^2)$, $\sigma^2$ känd.
- Prior: $\theta \sim N(\mu, \tau^2)$
- Posterior: $\theta|x_1, ..., x_n \sim N\left(\mu_x, \tau_x^2\right)$, där $\frac{1}{\tau_x^2} = \frac{n}{\sigma^2} + \frac{1}{\tau^2}$, $\mu_x = w\bar{x} + (1 - w)\mu$ och $w = \frac{\frac{n}{\sigma^2}}{\frac{n}{\sigma^2} + \frac{1}{\tau^2}}$.

## Multinomial data - Dirichlet prior

- Modell: $X_1, ..., X_K|\theta_1, ..., \theta_K \sim Multinomial(n, \theta_1, ..., \theta_K)$.
- Prior: $(\theta_1, ..., \theta_K) \sim Dirichlet(\alpha_1, ..., \alpha_K)$
- Posterior: $(\theta_1, ..., \theta_K)\,|x_1, ..., x_k \sim Dirichlet(\alpha_1 + x_1, ..., \alpha_K + x_K)$.

## Normalfördelning

Tabell för $\Phi(x) = P(X \leq x)$, där $X \sim N(0,1)$. För $x < 0$, använd att $\Phi(x) = 1 - \Phi(-x)$.

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.7 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.8 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 4.0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

## t-fördelning

Tabell för $F(x) = P(X \leq x)$, där $X \sim t(\nu)$. För $F(x) < 0.5$, använd att $F(x) = 1 - F(-x)$.

| | $F(x)$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\nu$ | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.9995 |
| 1 | 1.00 | 3.08 | 6.31 | 12.71 | 31.82 | 63.66 | 127.32 | 636.62 |
| 2 | 0.82 | 1.89 | 2.92 | 4.30 | 6.96 | 9.92 | 14.09 | 31.60 |
| 3 | 0.76 | 1.64 | 2.35 | 3.18 | 4.54 | 5.84 | 7.45 | 12.92 |
| 4 | 0.74 | 1.53 | 2.13 | 2.78 | 3.75 | 4.60 | 5.60 | 8.61 |
| 5 | 0.73 | 1.48 | 2.02 | 2.57 | 3.36 | 4.03 | 4.77 | 6.87 |
| 6 | 0.72 | 1.44 | 1.94 | 2.45 | 3.14 | 3.71 | 4.32 | 5.96 |
| 7 | 0.71 | 1.41 | 1.89 | 2.36 | 3.00 | 3.50 | 4.03 | 5.41 |
| 8 | 0.71 | 1.40 | 1.86 | 2.31 | 2.90 | 3.36 | 3.83 | 5.04 |
| 9 | 0.70 | 1.38 | 1.83 | 2.26 | 2.82 | 3.25 | 3.69 | 4.78 |
| 10 | 0.70 | 1.37 | 1.81 | 2.23 | 2.76 | 3.17 | 3.58 | 4.59 |
| 11 | 0.70 | 1.36 | 1.80 | 2.20 | 2.72 | 3.11 | 3.50 | 4.44 |
| 12 | 0.70 | 1.36 | 1.78 | 2.18 | 2.68 | 3.05 | 3.43 | 4.32 |
| 13 | 0.69 | 1.35 | 1.77 | 2.16 | 2.65 | 3.01 | 3.37 | 4.22 |
| 14 | 0.69 | 1.35 | 1.76 | 2.14 | 2.62 | 2.98 | 3.33 | 4.14 |
| 15 | 0.69 | 1.34 | 1.75 | 2.13 | 2.60 | 2.95 | 3.29 | 4.07 |
| 16 | 0.69 | 1.34 | 1.75 | 2.12 | 2.58 | 2.92 | 3.25 | 4.01 |
| 17 | 0.69 | 1.33 | 1.74 | 2.11 | 2.57 | 2.90 | 3.22 | 3.97 |
| 18 | 0.69 | 1.33 | 1.73 | 2.10 | 2.55 | 2.88 | 3.20 | 3.92 |
| 19 | 0.69 | 1.33 | 1.73 | 2.09 | 2.54 | 2.86 | 3.17 | 3.88 |
| 20 | 0.69 | 1.33 | 1.72 | 2.09 | 2.53 | 2.85 | 3.15 | 3.85 |
| 21 | 0.69 | 1.32 | 1.72 | 2.08 | 2.52 | 2.83 | 3.14 | 3.82 |
| 22 | 0.69 | 1.32 | 1.72 | 2.07 | 2.51 | 2.82 | 3.12 | 3.79 |
| 23 | 0.69 | 1.32 | 1.71 | 2.07 | 2.50 | 2.81 | 3.10 | 3.77 |
| 24 | 0.68 | 1.32 | 1.71 | 2.06 | 2.49 | 2.80 | 3.09 | 3.75 |
| 25 | 0.68 | 1.32 | 1.71 | 2.06 | 2.49 | 2.79 | 3.08 | 3.73 |
| 26 | 0.68 | 1.31 | 1.71 | 2.06 | 2.48 | 2.78 | 3.07 | 3.71 |
| 27 | 0.68 | 1.31 | 1.70 | 2.05 | 2.47 | 2.77 | 3.06 | 3.69 |
| 28 | 0.68 | 1.31 | 1.70 | 2.05 | 2.47 | 2.76 | 3.05 | 3.67 |
| 29 | 0.68 | 1.31 | 1.70 | 2.05 | 2.46 | 2.76 | 3.04 | 3.66 |
| 30 | 0.68 | 1.31 | 1.70 | 2.04 | 2.46 | 2.75 | 3.03 | 3.65 |
| 40 | 0.68 | 1.30 | 1.68 | 2.02 | 2.42 | 2.70 | 2.97 | 3.55 |
| 50 | 0.68 | 1.30 | 1.68 | 2.01 | 2.40 | 2.68 | 2.94 | 3.50 |
| 60 | 0.68 | 1.30 | 1.67 | 2.00 | 2.39 | 2.66 | 2.91 | 3.46 |
| 100 | 0.68 | 1.29 | 1.66 | 1.98 | 2.36 | 2.63 | 2.87 | 3.39 |
| $\infty$ | 0.67 | 1.28 | 1.65 | 1.96 | 2.33 | 2.58 | 2.81 | 3.29 |

## $\chi^2$-fördelning

Tabell för $F(x) = P(X \leq x)$, där $X \sim \chi^2(\nu)$.

| | $F(x)$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu$ | 0.0005 | 0.001 | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 |
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.06 | 0.15 | 0.27 | 0.45 |
| 2 | 0.00 | 0.00 | 0.01 | 0.02 | 0.05 | 0.10 | 0.21 | 0.45 | 0.71 | 1.02 | 1.39 |
| 3 | 0.02 | 0.02 | 0.07 | 0.11 | 0.22 | 0.35 | 0.58 | 1.01 | 1.42 | 1.87 | 2.37 |
| 4 | 0.06 | 0.09 | 0.21 | 0.30 | 0.48 | 0.71 | 1.06 | 1.65 | 2.19 | 2.75 | 3.36 |
| 5 | 0.16 | 0.21 | 0.41 | 0.55 | 0.83 | 1.15 | 1.61 | 2.34 | 3.00 | 3.66 | 4.35 |
| 6 | 0.30 | 0.38 | 0.68 | 0.87 | 1.24 | 1.64 | 2.20 | 3.07 | 3.83 | 4.57 | 5.35 |
| 7 | 0.48 | 0.60 | 0.99 | 1.24 | 1.69 | 2.17 | 2.83 | 3.82 | 4.67 | 5.49 | 6.35 |
| 8 | 0.71 | 0.86 | 1.34 | 1.65 | 2.18 | 2.73 | 3.49 | 4.59 | 5.53 | 6.42 | 7.34 |
| 9 | 0.97 | 1.15 | 1.73 | 2.09 | 2.70 | 3.33 | 4.17 | 5.38 | 6.39 | 7.36 | 8.34 |
| 10 | 1.26 | 1.48 | 2.16 | 2.56 | 3.25 | 3.94 | 4.87 | 6.18 | 7.27 | 8.30 | 9.34 |
| 11 | 1.59 | 1.83 | 2.60 | 3.05 | 3.82 | 4.57 | 5.58 | 6.99 | 8.15 | 9.24 | 10.34 |
| 12 | 1.93 | 2.21 | 3.07 | 3.57 | 4.40 | 5.23 | 6.30 | 7.81 | 9.03 | 10.18 | 11.34 |
| 13 | 2.31 | 2.62 | 3.57 | 4.11 | 5.01 | 5.89 | 7.04 | 8.63 | 9.93 | 11.13 | 12.34 |
| 14 | 2.70 | 3.04 | 4.07 | 4.66 | 5.63 | 6.57 | 7.79 | 9.47 | 10.82 | 12.08 | 13.34 |
| 15 | 3.11 | 3.48 | 4.60 | 5.23 | 6.26 | 7.26 | 8.55 | 10.31 | 11.72 | 13.03 | 14.34 |
| 16 | 3.54 | 3.94 | 5.14 | 5.81 | 6.91 | 7.96 | 9.31 | 11.15 | 12.62 | 13.98 | 15.34 |
| 17 | 3.98 | 4.42 | 5.70 | 6.41 | 7.56 | 8.67 | 10.09 | 12.00 | 13.53 | 14.94 | 16.34 |
| 18 | 4.44 | 4.90 | 6.26 | 7.01 | 8.23 | 9.39 | 10.86 | 12.86 | 14.44 | 15.89 | 17.34 |
| 19 | 4.91 | 5.41 | 6.84 | 7.63 | 8.91 | 10.12 | 11.65 | 13.72 | 15.35 | 16.85 | 18.34 |
| 20 | 5.40 | 5.92 | 7.43 | 8.26 | 9.59 | 10.85 | 12.44 | 14.58 | 16.27 | 17.81 | 19.34 |
| 21 | 5.90 | 6.45 | 8.03 | 8.90 | 10.28 | 11.59 | 13.24 | 15.44 | 17.18 | 18.77 | 20.34 |
| 22 | 6.40 | 6.98 | 8.64 | 9.54 | 10.98 | 12.34 | 14.04 | 16.31 | 18.10 | 19.73 | 21.34 |
| 23 | 6.92 | 7.53 | 9.26 | 10.20 | 11.69 | 13.09 | 14.85 | 17.19 | 19.02 | 20.69 | 22.34 |
| 24 | 7.45 | 8.08 | 9.89 | 10.86 | 12.40 | 13.85 | 15.66 | 18.06 | 19.94 | 21.65 | 23.34 |
| 25 | 7.99 | 8.65 | 10.52 | 11.52 | 13.12 | 14.61 | 16.47 | 18.94 | 20.87 | 22.62 | 24.34 |
| 26 | 8.54 | 9.22 | 11.16 | 12.20 | 13.84 | 15.38 | 17.29 | 19.82 | 21.79 | 23.58 | 25.34 |
| 27 | 9.09 | 9.80 | 11.81 | 12.88 | 14.57 | 16.15 | 18.11 | 20.70 | 22.72 | 24.54 | 26.34 |
| 28 | 9.66 | 10.39 | 12.46 | 13.56 | 15.31 | 16.93 | 18.94 | 21.59 | 23.65 | 25.51 | 27.34 |
| 29 | 10.23 | 10.99 | 13.12 | 14.26 | 16.05 | 17.71 | 19.77 | 22.48 | 24.58 | 26.48 | 28.34 |
| 30 | 10.80 | 11.59 | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 23.36 | 25.51 | 27.44 | 29.34 |
| 40 | 16.91 | 17.92 | 20.71 | 22.16 | 24.43 | 26.51 | 29.05 | 32.34 | 34.87 | 37.13 | 39.34 |
| 50 | 23.46 | 24.67 | 27.99 | 29.71 | 32.36 | 34.76 | 37.69 | 41.45 | 44.31 | 46.86 | 49.33 |
| 60 | 30.34 | 31.74 | 35.53 | 37.48 | 40.48 | 43.19 | 46.46 | 50.64 | 53.81 | 56.62 | 59.33 |
| 100 | 59.90 | 61.92 | 67.33 | 70.06 | 74.22 | 77.93 | 82.36 | 87.95 | 92.13 | 95.81 | 99.33 |

## $\chi^2$-fördelning, forts.

Tabell för $F(x) = P(X \leq x)$, där $X \sim \chi^2(\nu)$.

| | $F(x)$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\nu$ | 0.60 | 0.70 | 0.80 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.999 | 0.9995 |
| 1 | 0.71 | 1.07 | 1.64 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 | 10.83 | 12.12 |
| 2 | 1.83 | 2.41 | 3.22 | 4.61 | 5.99 | 7.38 | 9.21 | 10.60 | 13.82 | 15.20 |
| 3 | 2.95 | 3.66 | 4.64 | 6.25 | 7.81 | 9.35 | 11.34 | 12.84 | 16.27 | 17.73 |
| 4 | 4.04 | 4.88 | 5.99 | 7.78 | 9.49 | 11.14 | 13.28 | 14.86 | 18.47 | 20.00 |
| 5 | 5.13 | 6.06 | 7.29 | 9.24 | 11.07 | 12.83 | 15.09 | 16.75 | 20.52 | 22.11 |
| 6 | 6.21 | 7.23 | 8.56 | 10.64 | 12.59 | 14.45 | 16.81 | 18.55 | 22.46 | 24.10 |
| 7 | 7.28 | 8.38 | 9.80 | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 | 24.32 | 26.02 |
| 8 | 8.35 | 9.52 | 11.03 | 13.36 | 15.51 | 17.53 | 20.09 | 21.95 | 26.12 | 27.87 |
| 9 | 9.41 | 10.66 | 12.24 | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 | 27.88 | 29.67 |
| 10 | 10.47 | 11.78 | 13.44 | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 | 29.59 | 31.42 |
| 11 | 11.53 | 12.90 | 14.63 | 17.28 | 19.68 | 21.92 | 24.72 | 26.76 | 31.26 | 33.14 |
| 12 | 12.58 | 14.01 | 15.81 | 18.55 | 21.03 | 23.34 | 26.22 | 28.30 | 32.91 | 34.82 |
| 13 | 13.64 | 15.12 | 16.98 | 19.81 | 22.36 | 24.74 | 27.69 | 29.82 | 34.53 | 36.48 |
| 14 | 14.69 | 16.22 | 18.15 | 21.06 | 23.68 | 26.12 | 29.14 | 31.32 | 36.12 | 38.11 |
| 15 | 15.73 | 17.32 | 19.31 | 22.31 | 25.00 | 27.49 | 30.58 | 32.80 | 37.70 | 39.72 |
| 16 | 16.78 | 18.42 | 20.47 | 23.54 | 26.30 | 28.85 | 32.00 | 34.27 | 39.25 | 41.31 |
| 17 | 17.82 | 19.51 | 21.61 | 24.77 | 27.59 | 30.19 | 33.41 | 35.72 | 40.79 | 42.88 |
| 18 | 18.87 | 20.60 | 22.76 | 25.99 | 28.87 | 31.53 | 34.81 | 37.16 | 42.31 | 44.43 |
| 19 | 19.91 | 21.69 | 23.90 | 27.20 | 30.14 | 32.85 | 36.19 | 38.58 | 43.82 | 45.97 |
| 20 | 20.95 | 22.77 | 25.04 | 28.41 | 31.41 | 34.17 | 37.57 | 40.00 | 45.31 | 47.50 |
| 21 | 21.99 | 23.86 | 26.17 | 29.62 | 32.67 | 35.48 | 38.93 | 41.40 | 46.80 | 49.01 |
| 22 | 23.03 | 24.94 | 27.30 | 30.81 | 33.92 | 36.78 | 40.29 | 42.80 | 48.27 | 50.51 |
| 23 | 24.07 | 26.02 | 28.43 | 32.01 | 35.17 | 38.08 | 41.64 | 44.18 | 49.73 | 52.00 |
| 24 | 25.11 | 27.10 | 29.55 | 33.20 | 36.42 | 39.36 | 42.98 | 45.56 | 51.18 | 53.48 |
| 25 | 26.14 | 28.17 | 30.68 | 34.38 | 37.65 | 40.65 | 44.31 | 46.93 | 52.62 | 54.95 |
| 26 | 27.18 | 29.25 | 31.79 | 35.56 | 38.89 | 41.92 | 45.64 | 48.29 | 54.05 | 56.41 |
| 27 | 28.21 | 30.32 | 32.91 | 36.74 | 40.11 | 43.19 | 46.96 | 49.64 | 55.48 | 57.86 |
| 28 | 29.25 | 31.39 | 34.03 | 37.92 | 41.34 | 44.46 | 48.28 | 50.99 | 56.89 | 59.30 |
| 29 | 30.28 | 32.46 | 35.14 | 39.09 | 42.56 | 45.72 | 49.59 | 52.34 | 58.30 | 60.73 |
| 30 | 31.32 | 33.53 | 36.25 | 40.26 | 43.77 | 46.98 | 50.89 | 53.67 | 59.70 | 62.16 |
| 40 | 41.62 | 44.16 | 47.27 | 51.81 | 55.76 | 59.34 | 63.69 | 66.77 | 73.40 | 76.09 |
| 50 | 51.89 | 54.72 | 58.16 | 63.17 | 67.50 | 71.42 | 76.15 | 79.49 | 86.66 | 89.56 |
| 60 | 62.13 | 65.23 | 68.97 | 74.40 | 79.08 | 83.30 | 88.38 | 91.95 | 99.61 | 102.69 |
| 100 | 102.95 | 106.91 | 111.67 | 118.50 | 124.34 | 129.56 | 135.81 | 140.17 | 149.45 | 153.17 |

## Binomialfördelning

Tabell för $P(X \leq k)$ där $X \sim Bin(n, p)$.
För $p > 0.5$, använd att $P(X \leq k) = P(Y \geq n - k)$ där $Y \sim Bin(n, 1-p)$.

| | | $p$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $k$ | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
| 2 | 0 | 0.9025 | 0.8100 | 0.7225 | 0.6400 | 0.5625 | 0.4900 | 0.4225 | 0.3600 | 0.3025 | 0.2500 |
| | 1 | 0.9975 | 0.9900 | 0.9775 | 0.9600 | 0.9375 | 0.9100 | 0.8775 | 0.8400 | 0.7975 | 0.7500 |
| 3 | 0 | 0.8574 | 0.7290 | 0.6141 | 0.5120 | 0.4219 | 0.3430 | 0.2746 | 0.2160 | 0.1664 | 0.1250 |
| | 1 | 0.9928 | 0.9720 | 0.9392 | 0.8960 | 0.8438 | 0.7840 | 0.7183 | 0.6480 | 0.5747 | 0.5000 |
| | 2 | 0.9999 | 0.9990 | 0.9966 | 0.9920 | 0.9844 | 0.9730 | 0.9571 | 0.9360 | 0.9089 | 0.8750 |
| 4 | 0 | 0.8145 | 0.6561 | 0.5220 | 0.4096 | 0.3164 | 0.2401 | 0.1785 | 0.1296 | 0.0915 | 0.0625 |
| | 1 | 0.9860 | 0.9477 | 0.8905 | 0.8192 | 0.7383 | 0.6517 | 0.5630 | 0.4752 | 0.3910 | 0.3125 |
| | 2 | 0.9995 | 0.9963 | 0.9880 | 0.9728 | 0.9492 | 0.9163 | 0.8735 | 0.8208 | 0.7585 | 0.6875 |
| | 3 | 1.0000 | 0.9999 | 0.9995 | 0.9984 | 0.9961 | 0.9919 | 0.9850 | 0.9744 | 0.9590 | 0.9375 |
| 5 | 0 | 0.7738 | 0.5905 | 0.4437 | 0.3277 | 0.2373 | 0.1681 | 0.1160 | 0.0778 | 0.0503 | 0.0313 |
| | 1 | 0.9774 | 0.9185 | 0.8352 | 0.7373 | 0.6328 | 0.5282 | 0.4284 | 0.3370 | 0.2562 | 0.1875 |
| | 2 | 0.9988 | 0.9914 | 0.9734 | 0.9421 | 0.8965 | 0.8369 | 0.7648 | 0.6826 | 0.5931 | 0.5000 |
| | 3 | 1.0000 | 0.9995 | 0.9978 | 0.9933 | 0.9844 | 0.9692 | 0.9460 | 0.9130 | 0.8688 | 0.8125 |
| | 4 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9976 | 0.9947 | 0.9898 | 0.9815 | 0.9688 |
| 6 | 0 | 0.7351 | 0.5314 | 0.3771 | 0.2621 | 0.1780 | 0.1176 | 0.0754 | 0.0467 | 0.0277 | 0.0156 |
| | 1 | 0.9672 | 0.8857 | 0.7765 | 0.6554 | 0.5339 | 0.4202 | 0.3191 | 0.2333 | 0.1636 | 0.1094 |
| | 2 | 0.9978 | 0.9842 | 0.9527 | 0.9011 | 0.8306 | 0.7443 | 0.6471 | 0.5443 | 0.4415 | 0.3438 |
| | 3 | 0.9999 | 0.9987 | 0.9941 | 0.9830 | 0.9624 | 0.9295 | 0.8826 | 0.8208 | 0.7447 | 0.6563 |
| | 4 | 1.0000 | 0.9999 | 0.9996 | 0.9984 | 0.9954 | 0.9891 | 0.9777 | 0.9590 | 0.9308 | 0.8906 |
| | 5 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9982 | 0.9959 | 0.9917 | 0.9844 |
| 7 | 0 | 0.6983 | 0.4783 | 0.3206 | 0.2097 | 0.1335 | 0.0824 | 0.0490 | 0.0280 | 0.0152 | 0.0078 |
| | 1 | 0.9556 | 0.8503 | 0.7166 | 0.5767 | 0.4449 | 0.3294 | 0.2338 | 0.1586 | 0.1024 | 0.0625 |
| | 2 | 0.9962 | 0.9743 | 0.9262 | 0.8520 | 0.7564 | 0.6471 | 0.5323 | 0.4199 | 0.3164 | 0.2266 |
| | 3 | 0.9998 | 0.9973 | 0.9879 | 0.9667 | 0.9294 | 0.8740 | 0.8002 | 0.7102 | 0.6083 | 0.5000 |
| | 4 | 1.0000 | 0.9998 | 0.9988 | 0.9953 | 0.9871 | 0.9712 | 0.9444 | 0.9037 | 0.8471 | 0.7734 |
| | 5 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9987 | 0.9962 | 0.9910 | 0.9812 | 0.9643 | 0.9375 |
| | 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9984 | 0.9963 | 0.9922 |
| 8 | 0 | 0.6634 | 0.4305 | 0.2725 | 0.1678 | 0.1001 | 0.0576 | 0.0319 | 0.0168 | 0.0084 | 0.0039 |
| | 1 | 0.9428 | 0.8131 | 0.6572 | 0.5033 | 0.3671 | 0.2553 | 0.1691 | 0.1064 | 0.0632 | 0.0352 |
| | 2 | 0.9942 | 0.9619 | 0.8948 | 0.7969 | 0.6785 | 0.5518 | 0.4278 | 0.3154 | 0.2201 | 0.1445 |
| | 3 | 0.9996 | 0.9950 | 0.9786 | 0.9437 | 0.8862 | 0.8059 | 0.7064 | 0.5941 | 0.4770 | 0.3633 |
| | 4 | 1.0000 | 0.9996 | 0.9971 | 0.9896 | 0.9727 | 0.9420 | 0.8939 | 0.8263 | 0.7396 | 0.6367 |
| | 5 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9958 | 0.9887 | 0.9747 | 0.9502 | 0.9115 | 0.8555 |
| | 6 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9987 | 0.9964 | 0.9915 | 0.9819 | 0.9648 |
| | 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9983 | 0.9961 |
| 9 | 0 | 0.6302 | 0.3874 | 0.2316 | 0.1342 | 0.0751 | 0.0404 | 0.0207 | 0.0101 | 0.0046 | 0.0020 |
| | 1 | 0.9288 | 0.7748 | 0.5995 | 0.4362 | 0.3003 | 0.1960 | 0.1211 | 0.0705 | 0.0385 | 0.0195 |
| | 2 | 0.9916 | 0.9470 | 0.8591 | 0.7382 | 0.6007 | 0.4628 | 0.3373 | 0.2318 | 0.1495 | 0.0898 |
| | 3 | 0.9994 | 0.9917 | 0.9661 | 0.9144 | 0.8343 | 0.7297 | 0.6089 | 0.4826 | 0.3614 | 0.2539 |
| | 4 | 1.0000 | 0.9991 | 0.9944 | 0.9804 | 0.9511 | 0.9012 | 0.8283 | 0.7334 | 0.6214 | 0.5000 |
| | 5 | 1.0000 | 0.9999 | 0.9994 | 0.9969 | 0.9900 | 0.9747 | 0.9464 | 0.9006 | 0.8342 | 0.7461 |
| | 6 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 | 0.9957 | 0.9888 | 0.9750 | 0.9502 | 0.9102 |
| | 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9986 | 0.9962 | 0.9909 | 0.9805 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9980 |

## Poissonfördelning

Tabell för $P(X \leq k)$ där $X \sim Po(\mu)$.

| | $\mu$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $k$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| 0 | 0.9048 | 0.8187 | 0.7408 | 0.6703 | 0.6065 | 0.5488 | 0.4966 | 0.4493 | 0.4066 | 0.3679 |
| 1 | 0.9953 | 0.9825 | 0.9631 | 0.9384 | 0.9098 | 0.8781 | 0.8442 | 0.8088 | 0.7725 | 0.7358 |
| 2 | 0.9998 | 0.9989 | 0.9964 | 0.9921 | 0.9856 | 0.9769 | 0.9659 | 0.9526 | 0.9371 | 0.9197 |
| 3 | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9982 | 0.9966 | 0.9942 | 0.9909 | 0.9865 | 0.9810 |
| 4 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9986 | 0.9977 | 0.9963 |
| 5 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9997 | 0.9994 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | $\mu$ | | | | | | | | | |
| $k$ | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
| 0 | 0.3329 | 0.3012 | 0.2725 | 0.2466 | 0.2231 | 0.2019 | 0.1827 | 0.1653 | 0.1496 | 0.1353 |
| 1 | 0.6990 | 0.6626 | 0.6268 | 0.5918 | 0.5578 | 0.5249 | 0.4932 | 0.4628 | 0.4337 | 0.4060 |
| 2 | 0.9004 | 0.8795 | 0.8571 | 0.8335 | 0.8088 | 0.7834 | 0.7572 | 0.7306 | 0.7037 | 0.6767 |
| 3 | 0.9743 | 0.9662 | 0.9569 | 0.9463 | 0.9344 | 0.9212 | 0.9068 | 0.8913 | 0.8747 | 0.8571 |
| 4 | 0.9946 | 0.9923 | 0.9893 | 0.9857 | 0.9814 | 0.9763 | 0.9704 | 0.9636 | 0.9559 | 0.9473 |
| 5 | 0.9990 | 0.9985 | 0.9978 | 0.9968 | 0.9955 | 0.9940 | 0.9920 | 0.9896 | 0.9868 | 0.9834 |
| 6 | 0.9999 | 0.9997 | 0.9996 | 0.9994 | 0.9991 | 0.9987 | 0.9981 | 0.9974 | 0.9966 | 0.9955 |
| 7 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9994 | 0.9992 | 0.9989 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9998 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | $\mu$ | | | | | | | | | |
| $k$ | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | 2.7 | 2.8 | 2.9 | 3.0 |
| 0 | 0.1225 | 0.1108 | 0.1003 | 0.0907 | 0.0821 | 0.0743 | 0.0672 | 0.0608 | 0.0550 | 0.0498 |
| 1 | 0.3796 | 0.3546 | 0.3309 | 0.3084 | 0.2873 | 0.2674 | 0.2487 | 0.2311 | 0.2146 | 0.1991 |
| 2 | 0.6496 | 0.6227 | 0.5960 | 0.5697 | 0.5438 | 0.5184 | 0.4936 | 0.4695 | 0.4460 | 0.4232 |
| 3 | 0.8386 | 0.8194 | 0.7993 | 0.7787 | 0.7576 | 0.7360 | 0.7141 | 0.6919 | 0.6696 | 0.6472 |
| 4 | 0.9379 | 0.9275 | 0.9162 | 0.9041 | 0.8912 | 0.8774 | 0.8629 | 0.8477 | 0.8318 | 0.8153 |
| 5 | 0.9796 | 0.9751 | 0.9700 | 0.9643 | 0.9580 | 0.9510 | 0.9433 | 0.9349 | 0.9258 | 0.9161 |
| 6 | 0.9941 | 0.9925 | 0.9906 | 0.9884 | 0.9858 | 0.9828 | 0.9794 | 0.9756 | 0.9713 | 0.9665 |
| 7 | 0.9985 | 0.9980 | 0.9974 | 0.9967 | 0.9958 | 0.9947 | 0.9934 | 0.9919 | 0.9901 | 0.9881 |
| 8 | 0.9997 | 0.9995 | 0.9994 | 0.9991 | 0.9989 | 0.9985 | 0.9981 | 0.9976 | 0.9969 | 0.9962 |
| 9 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9995 | 0.9993 | 0.9991 | 0.9989 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |