TBAB01

#### TABELL- OCH FORMELSAMLING

#### SANNOLIKHETSFÖRDELNINGAR

• Binomialfördelning

$$X \sim Bin(n, p)$$
 
$$P(x) = \binom{n}{x} p^{x} (1-p)^{n-x}, \qquad x = 0, 1, \dots, n$$
 
$$\mathbb{E}X = np, \qquad Var(X) = np(1-p).$$

• Poissonfördelning

$$X \sim Po(\mu)$$
 
$$P(x) = \frac{\mu^x}{x!} e^{-\mu}, \qquad x = 0, 1, 2, \dots$$
 
$$\mathbb{E}X = \mu, \qquad Var(X) = \mu.$$

• Geometrisk fördelning

$$X \sim Ge(p)$$
 
$$P(x) = (1-p)^{x-1}p, \qquad x = 1, 2, \dots$$
 
$$\mathbb{E}X = \frac{1}{p}, \qquad Var(X) = \frac{1-p}{p^2}.$$

• Multinomialfördelning

$$(X_1, ..., X_k) \sim Multinomial(n, p_1, ..., p_k)$$
 
$$P(x_1, ..., x_k) = \frac{n!}{x_1! \cdots x_k!} p_1^{x_1} \cdots p_k^{x_k}, \qquad x_i = 0, 1, 2, ..., n \text{ och } \sum_{i=1}^n x_i = n.$$
 
$$\mathbb{E}X_i = np_i, \qquad Var(X_i) = np_i(1 - p_i), \qquad Cov(X_i, X_j) = -np_ip_j \ (i \neq j).$$

• Likformig (rektangulär) fördelning på intervallet (a,b)

$$X \sim U(a,b)$$
 
$$f(x) = \frac{1}{b-a}, \qquad a \le x \le b$$
 
$$\mathbb{EX} = \frac{a+b}{2}, \qquad Var(X) = \frac{(b-a)^2}{12}.$$

#### • Exponentialfördelning

$$X \sim Exp(\lambda),$$

där  $\lambda$  betecknar intensiteten. Ibland används väntevärdet  $\mu=\frac{1}{\lambda}$  som parameter.

$$f(x) = \lambda e^{-\lambda x}, \qquad x \ge 0$$

$$\mathbb{E}X = \frac{1}{\lambda}, \qquad Var(X) = \frac{1}{\lambda^2}.$$

#### • Normalfördelning

$$X \sim N(\mu, \sigma^2)$$
 
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \qquad -\infty < x < +\infty$$
 
$$\mathbb{E}X = \mu, \qquad Var(X) = \sigma^2.$$

### • $\chi^2$ -fördelning

$$Y \sim \chi^2(\nu)$$

Uppkomst: Om  $X_1, \ldots, X_n$  är oberoende, var och en N(0,1), gäller att  $Y = X_1^2 + \ldots + X_n^2$  får en  $\chi^2$  fördelning med  $\nu$  frihetsgrader.

$$f(x) = \frac{x^{(\nu/2)-1}e^{-x/2}}{2^{(\nu/2)}\Gamma(\nu/2)}, \qquad x \ge 0,$$

där  $\Gamma(\cdot)$  är gammafunktionen

$$\Gamma(c) = \int_0^\infty x^{c-1} e^{-x} dx, \quad \text{där } c > 0.$$
 
$$\mathbb{E}Y = \nu, \qquad Var(Y) = 2\nu.$$

### • t-fördelning

$$Z \sim t(\nu)$$

Uppkomst: Om  $X \sim N(0,1)$  och  $Y \sim \chi^2(\nu)$  samt X och Y är oberoende, så gäller att  $Z = \frac{X}{\sqrt{Y/n}}$  får en t-fördelning med  $\nu$  frihetsgrader.

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)\left(1 + \frac{x^2}{\nu}\right)^{(\nu+1)/2}}, \quad -\infty < x < +\infty$$

#### • Gammafördelning

$$Y \sim \Gamma(\alpha, \lambda)$$

Uppkomst: Om  $X_1, \ldots, X_n$  är oberoende, var och en  $Exp(\lambda)$ , så blir  $Y = X_1 + \ldots + X_n$  gammafördelad med parametrarna n och  $\lambda$ .

$$f(x) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha - 1} e^{-\lambda x}, \qquad x \ge 0$$

$$\mathbb{E}Y = \frac{\alpha}{\lambda}, \qquad Var(Y) = \frac{\alpha}{\lambda^2}.$$

#### • Betafördelning

$$X \sim Beta(\alpha, \beta)$$

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha - 1} (1 - x)^{\beta - 1}, \qquad 0 < x < 1$$

$$\mathbb{E}X = \frac{\alpha}{\alpha + \beta}, \qquad Var(X) = \frac{\alpha\beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)}.$$

#### • Dirichletfördelningen

$$(X_1,...,X_k) \sim Dirichlet(\alpha_1,...,\alpha_k)$$
 
$$P(x_1,...,x_k) = \frac{\Gamma(\sum_{i=1}^k \alpha_i)}{\prod_{i=1}^k \Gamma(\alpha_i)} x_1^{\alpha_1-1} \cdots x_k^{\alpha_k-1}, \qquad 0 < x_i < 1 \text{ och } \sum_{i=1}^n x_i = 1.$$
 
$$\mathbb{E} X_i = \frac{\alpha_i}{\alpha_0}, \text{ där } \alpha_0 = \sum_{i=1}^k \alpha_i \qquad Var(X_i) = \frac{\alpha_i(\alpha_0 - \alpha_i)}{\alpha_0^2 (\alpha_0 + 1)}, \qquad Cov(X_i, X_j) = -\frac{\alpha_i \alpha_j}{\alpha_0^2 (\alpha_0 + 1)} (i \neq j).$$

#### DIVERSE DEFINITIONER OCH RESULTAT

- Kovarians:  $Cov(X,Y) = \mathbb{E}[(X-\mu_X)(Y-\mu_Y)]$ , där  $\mu_X = \mathbb{E}X$ ] och  $\mu_Y = \mathbb{E}Y$
- Korrelation:  $\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$ , där  $\sigma_X^2 = Var(X)$  och  $\sigma_Y^2 = Var(Y)$
- Generellt gäller att

$$\mathbb{E}(a_1X_1 + \ldots + a_nX_n + b) = a_1\mathbb{E}X_1 + \ldots + a_n\mathbb{E}X_n + b.$$

• För *oberoende* slumpvariabler  $X_1, \ldots, X_n$  gäller att

$$Var(a_1X_1 + \ldots + a_nX_n + b) = a_1^2Var(X_1) + \ldots + a_n^2Var(X_n).$$

• Generellt gäller att

$$Var(a_1X_1 + \ldots + a_nX_n + b) = \sum_{j=1}^{n} a_j^2 Var(X_j) + 2 \sum_{1 \le j < k \le n} a_j a_k Cov(X_j, X_k).$$

- $X \sim Bin(n, p)$  och  $n \ge 10, p \le 0.1$   $\Rightarrow$   $X \approx Po(np)$
- $X \sim Bin(n,p)$  och  $np(1-p) \ge 10 \implies X \approx N(np, np(1-p))$
- $X \sim Po(\mu)$  och  $\mu \ge 15 \implies X \approx N(\mu, \mu)$ .
- Om  $X_1, ..., X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$ , gäller följande:

1. 
$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1),$$
  
2.  $\frac{(n-1)S^2}{\sigma^2} = \frac{\sum_{j=1}^n (X_j - \bar{X}_n)^2}{\sigma^2} \sim \chi^2(n-1),$ 

3. 
$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1).$$

• Vid enkel linjär regression ges modellen av

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i,$$

där  $\varepsilon_i \sim N(0, \sigma)$   $i = 1, \dots, n$  och oberoende.

Minsta kvadrat-skattningar

$$\hat{\beta}_1 = \frac{\sum_i (x_i - \overline{x}) Y_i}{\sum_i (x_i - \overline{x})^2},$$

$$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{x},$$

$$\widehat{\sigma}^2 = S^2 = \frac{1}{n-2} \sum_j (Y_j - \hat{\beta}_0 - \hat{\beta}_1 x_j)^2.$$

- $\chi^2$  goodness of fit-test.
  - $-H_0$ : Fördelningsfunktioner är  $F_0(x)$  (inga okända parametrar).

Låt  $p_i = F_0(a_i) - F_0(a_{i-1})$  och  $N_i$  antalet  $x_i$  i intervallet  $(a_{i-1}, a_i]$ .

Teststatistika:  $T = \sum_{i=1}^k \frac{(N_i - np_i)^2}{np_i} \approx \chi^2(k-1)$ -fördelad under  $H_0$ .

-  $H_0$ : Given parametrisk fördelningsklass med fördelningsfunktion F(x).

Teststatistika:  $T = \sum_{i=1}^{k} \frac{(N_i - np_i)^2}{np_i}$ ,

där  $p_i$  beräknas som enligt föregående punkt sedan parametrarna i F(x) har skattats. T är approximativt  $\chi^2(k-1-r)$ -fördelad under  $H_0$  där r= antalet skattade parametrar i F(x).

I båda fallen krävs att alla  $np_i \geq 5$ .

#### **BAYESIANSK INFERENS**

## Bernoulli data - Beta prior

- Modell:  $X_1, ..., X_n | \theta \sim Bernoulli(\theta)$
- Prior:  $\theta \sim Beta(\alpha, \beta)$
- Posterior:  $\theta|x_1,...,x_n \sim Beta(\alpha+s,\beta+f)$ , där  $s=\sum_{i=1}^n x_i$  och f=n-s.

## Normal data - Normal prior

• Modell:  $X_1, ..., X_n | \theta, \sigma^2 \sim N(\theta, \sigma^2), \sigma^2$  känd.

• Prior:  $\theta \sim N(\mu, \tau^2)$ 

• Posterior:  $\theta|x_1,...,x_n \sim N\left(\mu_x,\tau_x^2\right)$ , där  $\frac{1}{\tau_x^2} = \frac{n}{\sigma^2} + \frac{1}{\tau^2}$ ,  $\mu_x = w\bar{x} + (1-w)\mu$  och  $w = \frac{\frac{n}{\sigma^2}}{\frac{n}{\sigma^2} + \frac{1}{\tau^2}}$ .

## Multinomial data - Dirichlet prior

• Modell:  $X_1, ..., X_K | \theta_1, ..., \theta_K \sim Multinomial(n, \theta_1, ..., \theta_K)$ .

• Prior:  $(\theta_1, ..., \theta_K) \sim Dirichlet(\alpha_1, ..., \alpha_K)$ 

• Posterior:  $(\theta_1, ..., \theta_K) | x_1, ..., x_k \sim Dirichlet(\alpha_1 + x_1, ..., \alpha_K + x_K)$ .

## TABELLER

# Normalfördelning

Tabell för  $\Phi(x) = P(X \le x)$ , där  $X \sim N(0,1)$ . För x < 0, använd att  $\Phi(x) = 1 - \Phi(-x)$ .

| x   | 0      | 1      | 2      | 3      | 4      | 5      | 6      | 7               | 8      | 9      |
|-----|--------|--------|--------|--------|--------|--------|--------|-----------------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279          | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675          | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064          | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443          | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808          | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157          | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486          | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794          | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078          | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340          | 0.8365 | 0.8389 |
|     |        |        |        |        |        |        |        |                 |        |        |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577          | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790          | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980          | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147          | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292          | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418          | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525          | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616          | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693          | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756          | 0.9761 | 0.9767 |
|     |        |        |        |        |        |        |        |                 |        |        |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808          | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850          | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884          | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911          | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932          | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949          | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962          | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972          | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979          | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985          | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989          | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9999 | 0.9992          | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9992 $0.9995$ | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996          | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9995 | 0.9995 | 0.9997 | 0.9990 | 0.9990 | 0.9990 | 0.9990          | 0.9990 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998          | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999          | 0.9999 | 0.9999 |
| 3.7 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999          | 0.9999 | 0.9999 |
| 3.8 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999          | 0.9999 | 0.9999 |
| 3.9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000          | 1.0000 | 1.0000 |
| 4.0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000          | 1.0000 | 1.0000 |
|     | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000          | 1.0000 | 1.0000 |

t-fördelning Tabell för  $F(x)=P(X\leq x)$ , där  $X\sim t(\nu)$ . För F(x)<0.5, använd att F(x)=1-F(-x).

|           |      |      |      |       | F(x)  |       |        |        |
|-----------|------|------|------|-------|-------|-------|--------|--------|
| $\nu$     | 0.75 | 0.90 | 0.95 | 0.975 | 0.99  | 0.995 | 0.9975 | 0.9995 |
| 1         | 1.00 | 3.08 | 6.31 | 12.71 | 31.82 | 63.66 | 127.32 | 636.62 |
| 2         | 0.82 | 1.89 | 2.92 | 4.30  | 6.96  | 9.92  | 14.09  | 31.60  |
| 3         | 0.76 | 1.64 | 2.35 | 3.18  | 4.54  | 5.84  | 7.45   | 12.92  |
| 4         | 0.74 | 1.53 | 2.13 | 2.78  | 3.75  | 4.60  | 5.60   | 8.61   |
| 5         | 0.73 | 1.48 | 2.02 | 2.57  | 3.36  | 4.03  | 4.77   | 6.87   |
| 6         | 0.72 | 1.44 | 1.94 | 2.45  | 3.14  | 3.71  | 4.32   | 5.96   |
| 7         | 0.71 | 1.41 | 1.89 | 2.36  | 3.00  | 3.50  | 4.03   | 5.41   |
| 8         | 0.71 | 1.40 | 1.86 | 2.31  | 2.90  | 3.36  | 3.83   | 5.04   |
| 9         | 0.70 | 1.38 | 1.83 | 2.26  | 2.82  | 3.25  | 3.69   | 4.78   |
|           |      |      |      |       |       |       |        |        |
| 10        | 0.70 | 1.37 | 1.81 | 2.23  | 2.76  | 3.17  | 3.58   | 4.59   |
| 11        | 0.70 | 1.36 | 1.80 | 2.20  | 2.72  | 3.11  | 3.50   | 4.44   |
| 12        | 0.70 | 1.36 | 1.78 | 2.18  | 2.68  | 3.05  | 3.43   | 4.32   |
| 13        | 0.69 | 1.35 | 1.77 | 2.16  | 2.65  | 3.01  | 3.37   | 4.22   |
| 14        | 0.69 | 1.35 | 1.76 | 2.14  | 2.62  | 2.98  | 3.33   | 4.14   |
| 15        | 0.69 | 1.34 | 1.75 | 2.13  | 2.60  | 2.95  | 3.29   | 4.07   |
| 16        | 0.69 | 1.34 | 1.75 | 2.12  | 2.58  | 2.92  | 3.25   | 4.01   |
| 17        | 0.69 | 1.33 | 1.74 | 2.11  | 2.57  | 2.90  | 3.22   | 3.97   |
| 18        | 0.69 | 1.33 | 1.73 | 2.10  | 2.55  | 2.88  | 3.20   | 3.92   |
| 19        | 0.69 | 1.33 | 1.73 | 2.09  | 2.54  | 2.86  | 3.17   | 3.88   |
|           |      |      |      |       |       |       |        |        |
| 20        | 0.69 | 1.33 | 1.72 | 2.09  | 2.53  | 2.85  | 3.15   | 3.85   |
| 21        | 0.69 | 1.32 | 1.72 | 2.08  | 2.52  | 2.83  | 3.14   | 3.82   |
| 22        | 0.69 | 1.32 | 1.72 | 2.07  | 2.51  | 2.82  | 3.12   | 3.79   |
| 23        | 0.69 | 1.32 | 1.71 | 2.07  | 2.50  | 2.81  | 3.10   | 3.77   |
| 24        | 0.68 | 1.32 | 1.71 | 2.06  | 2.49  | 2.80  | 3.09   | 3.75   |
| 25        | 0.68 | 1.32 | 1.71 | 2.06  | 2.49  | 2.79  | 3.08   | 3.73   |
| 26        | 0.68 | 1.31 | 1.71 | 2.06  | 2.48  | 2.78  | 3.07   | 3.71   |
| 27        | 0.68 | 1.31 | 1.70 | 2.05  | 2.47  | 2.77  | 3.06   | 3.69   |
| 28        | 0.68 | 1.31 | 1.70 | 2.05  | 2.47  | 2.76  | 3.05   | 3.67   |
| 29        | 0.68 | 1.31 | 1.70 | 2.05  | 2.46  | 2.76  | 3.04   | 3.66   |
|           |      |      |      |       |       |       |        |        |
| 30        | 0.68 | 1.31 | 1.70 | 2.04  | 2.46  | 2.75  | 3.03   | 3.65   |
| 40        | 0.68 | 1.30 | 1.68 | 2.02  | 2.42  | 2.70  | 2.97   | 3.55   |
| 50        | 0.68 | 1.30 | 1.68 | 2.01  | 2.40  | 2.68  | 2.94   | 3.50   |
| 60        | 0.68 | 1.30 | 1.67 | 2.00  | 2.39  | 2.66  | 2.91   | 3.46   |
| 100       | 0.68 | 1.29 | 1.66 | 1.98  | 2.36  | 2.63  | 2.87   | 3.39   |
| $-\infty$ | 0.67 | 1.28 | 1.65 | 1.96  | 2.33  | 2.58  | 2.81   | 3.29   |

 $\chi^2\text{-}\mbox{f\"{o}}\mbox{rdelning}$  Tabell för  $F(x)=P(X\leq x),$  där  $X\sim \chi^2(\nu).$ 

|          |              |               |               |               |               | F(x)          |               |               |               |               |               |
|----------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| ν        | 0.0005       | 0.001         | 0.005         | 0.01          | 0.025         | 0.05          | 0.10          | 0.20          | 0.30          | 0.40          | 0.50          |
| 1        | 0.00         | 0.00          | 0.00          | 0.00          | 0.00          | 0.00          | 0.02          | 0.06          | 0.15          | 0.27          | 0.45          |
| 2        | 0.00         | 0.00          | 0.01          | 0.02          | 0.05          | 0.10          | 0.21          | 0.45          | 0.71          | 1.02          | 1.39          |
| 3        | 0.02         | 0.02          | 0.07          | 0.11          | 0.22          | 0.35          | 0.58          | 1.01          | 1.42          | 1.87          | 2.37          |
| 4        | 0.06         | 0.09          | 0.21          | 0.30          | 0.48          | 0.71          | 1.06          | 1.65          | 2.19          | 2.75          | 3.36          |
| 5        | 0.16         | 0.21          | 0.41          | 0.55          | 0.83          | 1.15          | 1.61          | 2.34          | 3.00          | 3.66          | 4.35          |
| 6        | 0.30         | 0.38          | 0.68          | 0.87          | 1.24          | 1.64          | 2.20          | 3.07          | 3.83          | 4.57          | 5.35          |
| 7        | 0.48         | 0.60          | 0.99          | 1.24          | 1.69          | 2.17          | 2.83          | 3.82          | 4.67          | 5.49          | 6.35          |
| 8        | 0.71         | 0.86          | 1.34          | 1.65          | 2.18          | 2.73          | 3.49          | 4.59          | 5.53          | 6.42          | 7.34          |
| 9        | 0.97         | 1.15          | 1.73          | 2.09          | 2.70          | 3.33          | 4.17          | 5.38          | 6.39          | 7.36          | 8.34          |
|          |              |               |               |               |               |               |               |               |               |               |               |
| 10       | 1.26         | 1.48          | 2.16          | 2.56          | 3.25          | 3.94          | 4.87          | 6.18          | 7.27          | 8.30          | 9.34          |
| 11       | 1.59         | 1.83          | 2.60          | 3.05          | 3.82          | 4.57          | 5.58          | 6.99          | 8.15          | 9.24          | 10.34         |
| 12       | 1.93         | 2.21          | 3.07          | 3.57          | 4.40          | 5.23          | 6.30          | 7.81          | 9.03          | 10.18         | 11.34         |
| 13       | 2.31         | 2.62          | 3.57          | 4.11          | 5.01          | 5.89          | 7.04          | 8.63          | 9.93          | 11.13         | 12.34         |
| 14       | 2.70         | 3.04          | 4.07          | 4.66          | 5.63          | 6.57          | 7.79          | 9.47          | 10.82         | 12.08         | 13.34         |
| 15       | 3.11         | 3.48          | 4.60          | 5.23          | 6.26          | 7.26          | 8.55          | 10.31         | 11.72         | 13.03         | 14.34         |
| 16       | 3.54         | 3.94          | 5.14          | 5.81          | 6.91          | 7.96          | 9.31          | 11.15         | 12.62         | 13.98         | 15.34         |
| 17       | 3.98         | 4.42          | 5.70          | 6.41          | 7.56          | 8.67          | 10.09         | 12.00         | 13.53         | 14.94         | 16.34         |
| 18       | 4.44         | 4.90          | 6.26          | 7.01          | 8.23          | 9.39          | 10.86         | 12.86         | 14.44         | 15.89         | 17.34         |
| 19       | 4.91         | 5.41          | 6.84          | 7.63          | 8.91          | 10.12         | 11.65         | 13.72         | 15.35         | 16.85         | 18.34         |
| 90       | F 40         | F 00          | 7.49          | 0.00          | 0.50          | 10.05         | 10.44         | 14 50         | 16.07         | 17.01         | 10.24         |
| 20       | 5.40         | 5.92          | 7.43          | 8.26          | 9.59          | 10.85         | 12.44         | 14.58         | 16.27         | 17.81         | 19.34         |
| 21       | 5.90         | 6.45          | 8.03          | 8.90          | 10.28         | 11.59         | 13.24         | 15.44         | 17.18         | 18.77         | 20.34         |
| 22       | 6.40         | 6.98          | 8.64          | 9.54          | 10.98         | 12.34         | 14.04         | 16.31         | 18.10         | 19.73         | 21.34         |
| 23       | 6.92         | 7.53          | 9.26          | 10.20         | 11.69         | 13.09         | 14.85         | 17.19         | 19.02         | 20.69         | 22.34         |
| 24       | 7.45         | 8.08          | 9.89          | 10.86         | 12.40         | 13.85         | 15.66         | 18.06         | 19.94         | 21.65         | 23.34         |
| 25       | 7.99         | 8.65          | 10.52         | 11.52         | 13.12         | 14.61         | 16.47         | 18.94         | 20.87         | 22.62         | 24.34         |
| 26       | 8.54         | 9.22          | 11.16         | 12.20         | 13.84         | 15.38         | 17.29         | 19.82         | 21.79         | 23.58         | 25.34         |
| 27<br>28 | 9.09<br>9.66 | 9.80          | 11.81 $12.46$ | 12.88 $13.56$ | 14.57 $15.31$ | 16.15         | 18.11         | 20.70         | 22.72 $23.65$ | 24.54 $25.51$ | 26.34 $27.34$ |
| 29       | 10.23        | 10.39 $10.99$ | 12.40 $13.12$ | 13.36 $14.26$ | 16.05         | 16.93 $17.71$ | 18.94 $19.77$ | 21.59 $22.48$ | 23.03 $24.58$ | 26.48         | 28.34         |
| 29       | 10.25        | 10.99         | 15.12         | 14.20         | 10.03         | 17.71         | 19.77         | 22.46         | 24.36         | 20.46         | 26.34         |
| 30       | 10.80        | 11.59         | 13.79         | 14.95         | 16.79         | 18.49         | 20.60         | 23.36         | 25.51         | 27.44         | 29.34         |
| 40       | 16.91        | 17.92         | 20.71         | 22.16         | 24.43         | 26.51         | 29.05         | 32.34         | 34.87         | 37.13         | 39.34         |
| 50       | 23.46        | 24.67         | 27.99         | 29.71         | 32.36         | 34.76         | 37.69         | 41.45         | 44.31         | 46.86         | 49.33         |
| 60       | 30.34        | 31.74         | 35.53         | 37.48         | 40.48         | 43.19         | 46.46         | 50.64         | 53.81         | 56.62         | 59.33         |
| 100      | 59.90        | 61.92         | 67.33         | 70.06         | 74.22         | 77.93         | 82.36         | 87.95         | 92.13         | 95.81         | 99.33         |
|          | 1 33.00      | 01.02         | 000           |               | ,             |               | 02.00         | 000           | 02.10         | 55.01         |               |

 $\chi^2$ -fördelning, forts. Tabell för  $F(x) = P(X \le x)$ , där  $X \sim \chi^2(\nu)$ .

60

100

62.13

102.95

65.23

106.91

68.97

111.67

74.40

118.50

79.08

124.34

83.30

129.56

88.38

135.81

91.95

140.17

99.61

149.45

102.69

153.17

F(x) $\nu$ 0.60 0.700.80 0.90 0.950.9750.99 0.995 0.999 0.99950.711.07 1.642.713.845.026.637.8810.83 12.122 1.83 2.413.22 4.615.997.38 9.2110.60 13.8215.203 2.953.664.646.257.81 9.3511.3412.8416.2717.734 4.044.88 5.99 7.78 9.4911.1413.28 14.8618.4720.00 5 5.136.067.299.2411.0712.8315.0916.7520.5222.116 6.217.238.5610.64 12.5914.4516.8118.55 22.4624.107 7.28 8.38 9.80 12.02 14.07 16.01 18.48 20.28 24.3226.028 11.0313.36 17.5320.09 21.9526.1227.878.359.5215.519 9.4110.66 12.2414.68 16.92 19.02 21.6723.59 27.8829.67 10 10.47 11.78 13.44 18.31 20.48 23.21 29.59 31.42 15.99 25.1933.14 1111.5312.9014.6317.2819.68 21.9224.7226.7631.2612 12.58 14.01 15.81 18.55 21.03 23.34 26.22 28.30 32.91 34.82 13 13.6415.1216.9819.81 22.36 24.7427.6929.8234.5336.4814 14.69 16.2218.1521.06 23.68 26.1229.14 31.3236.1238.11 27.4915 15.7317.3219.3122.3125.00 30.5832.8037.7039.7216 16.78 18.4220.4723.5426.30 28.8532.00 34.2739.2541.3117 17.8219.5121.6124.7727.5930.1933.4135.7240.7942.8818.87 20.60 22.76 25.99 28.87 31.5334.81 37.1642.31 44.43 18 19 19.9121.6923.9027.2030.1432.8536.1938.5843.8245.9720 20.9522.7725.0428.4131.4134.1737.5740.00 45.3147.5021 21.99 26.17 29.62 46.8049.01 23.8632.6735.4838.9341.4022 23.03 24.94 27.30 30.81 33.92 36.78 40.29 42.80 48.2750.51 2324.07 26.0228.4332.01 35.1741.6449.7352.0038.0844.1824 25.1127.10 29.5533.20 36.4239.36 42.98 45.5651.1853.4825 26.14 28.17 44.31 52.6230.6834.3837.6540.6546.9354.9526 27.18 29.25 45.64 31.79 35.5638.89 41.92 48.29 54.0556.4127 28.2130.32 32.9136.7440.1143.1946.9649.6455.4857.8628 29.25 31.39 34.03 37.92 44.4648.28 56.8941.3450.9959.3029 39.09 30.28 32.4635.1442.5645.7249.5952.3458.3060.7330 33.53 36.2546.98 59.7031.3240.2643.7750.8953.6762.1640 41.6244.1647.2751.8155.7659.34 63.69 66.7773.4076.09 50 51.8954.7258.1663.1767.50 71.4276.1579.4986.6689.56

Binomialfördelning

Tabell för  $P(X \le k)$  där  $X \sim Bin(n,p)$ . För p>0.5, använd att  $P(X \le k)=P(Y \ge n-k)$  där  $Y \sim Bin(n,1-p)$ .

|   |   |        |        |        |        | 1      | 9      |        |        |        |        |
|---|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| n | k | 0.05   | 0.10   | 0.15   | 0.20   | 0.25   | 0.30   | 0.35   | 0.40   | 0.45   | 0.50   |
| 2 | 0 | 0.9025 | 0.8100 | 0.7225 | 0.6400 | 0.5625 | 0.4900 | 0.4225 | 0.3600 | 0.3025 | 0.2500 |
|   | 1 | 0.9975 | 0.9900 | 0.9775 | 0.9600 | 0.9375 | 0.9100 | 0.8775 | 0.8400 | 0.7975 | 0.7500 |
| 3 | 0 | 0.8574 | 0.7290 | 0.6141 | 0.5120 | 0.4219 | 0.3430 | 0.2746 | 0.2160 | 0.1664 | 0.1250 |
|   | 1 | 0.9928 | 0.9720 | 0.9392 | 0.8960 | 0.8438 | 0.7840 | 0.7183 | 0.6480 | 0.5747 | 0.5000 |
|   | 2 | 0.9999 | 0.9990 | 0.9966 | 0.9920 | 0.9844 | 0.9730 | 0.9571 | 0.9360 | 0.9089 | 0.8750 |
| 4 | 0 | 0.8145 | 0.6561 | 0.5220 | 0.4096 | 0.3164 | 0.2401 | 0.1785 | 0.1296 | 0.0915 | 0.0625 |
|   | 1 | 0.9860 | 0.9477 | 0.8905 | 0.8192 | 0.7383 | 0.6517 | 0.5630 | 0.4752 | 0.3910 | 0.3125 |
|   | 2 | 0.9995 | 0.9963 | 0.9880 | 0.9728 | 0.9492 | 0.9163 | 0.8735 | 0.8208 | 0.7585 | 0.6875 |
|   | 3 | 1.0000 | 0.9999 | 0.9995 | 0.9984 | 0.9961 | 0.9919 | 0.9850 | 0.9744 | 0.9590 | 0.9375 |
| 5 | 0 | 0.7738 | 0.5905 | 0.4437 | 0.3277 | 0.2373 | 0.1681 | 0.1160 | 0.0778 | 0.0503 | 0.0313 |
|   | 1 | 0.9774 | 0.9185 | 0.8352 | 0.7373 | 0.6328 | 0.5282 | 0.4284 | 0.3370 | 0.2562 | 0.1875 |
|   | 2 | 0.9988 | 0.9914 | 0.9734 | 0.9421 | 0.8965 | 0.8369 | 0.7648 | 0.6826 | 0.5931 | 0.5000 |
|   | 3 | 1.0000 | 0.9995 | 0.9978 | 0.9933 | 0.9844 | 0.9692 | 0.9460 | 0.9130 | 0.8688 | 0.8125 |
|   | 4 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9976 | 0.9947 | 0.9898 | 0.9815 | 0.9688 |
| 6 | 0 | 0.7351 | 0.5314 | 0.3771 | 0.2621 | 0.1780 | 0.1176 | 0.0754 | 0.0467 | 0.0277 | 0.0156 |
|   | 1 | 0.9672 | 0.8857 | 0.7765 | 0.6554 | 0.5339 | 0.4202 | 0.3191 | 0.2333 | 0.1636 | 0.1094 |
|   | 2 | 0.9978 | 0.9842 | 0.9527 | 0.9011 | 0.8306 | 0.7443 | 0.6471 | 0.5443 | 0.4415 | 0.3438 |
|   | 3 | 0.9999 | 0.9987 | 0.9941 | 0.9830 | 0.9624 | 0.9295 | 0.8826 | 0.8208 | 0.7447 | 0.6563 |
|   | 4 | 1.0000 | 0.9999 | 0.9996 | 0.9984 | 0.9954 | 0.9891 | 0.9777 | 0.9590 | 0.9308 | 0.8906 |
|   | 5 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9982 | 0.9959 | 0.9917 | 0.9844 |
| 7 | 0 | 0.6983 | 0.4783 | 0.3206 | 0.2097 | 0.1335 | 0.0824 | 0.0490 | 0.0280 | 0.0152 | 0.0078 |
|   | 1 | 0.9556 | 0.8503 | 0.7166 | 0.5767 | 0.4449 | 0.3294 | 0.2338 | 0.1586 | 0.1024 | 0.0625 |
|   | 2 | 0.9962 | 0.9743 | 0.9262 | 0.8520 | 0.7564 | 0.6471 | 0.5323 | 0.4199 | 0.3164 | 0.2266 |
|   | 3 | 0.9998 | 0.9973 | 0.9879 | 0.9667 | 0.9294 | 0.8740 | 0.8002 | 0.7102 | 0.6083 | 0.5000 |
|   | 4 | 1.0000 | 0.9998 | 0.9988 | 0.9953 | 0.9871 | 0.9712 | 0.9444 | 0.9037 | 0.8471 | 0.7734 |
|   | 5 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9987 | 0.9962 | 0.9910 | 0.9812 | 0.9643 | 0.9375 |
|   | 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9984 | 0.9963 | 0.9922 |
| 8 | 0 | 0.6634 | 0.4305 | 0.2725 | 0.1678 | 0.1001 | 0.0576 | 0.0319 | 0.0168 | 0.0084 | 0.0039 |
|   | 1 | 0.9428 | 0.8131 | 0.6572 | 0.5033 | 0.3671 | 0.2553 | 0.1691 | 0.1064 | 0.0632 | 0.0352 |
|   | 2 | 0.9942 | 0.9619 | 0.8948 | 0.7969 | 0.6785 | 0.5518 | 0.4278 | 0.3154 | 0.2201 | 0.1445 |
|   | 3 | 0.9996 | 0.9950 | 0.9786 | 0.9437 | 0.8862 | 0.8059 | 0.7064 | 0.5941 | 0.4770 | 0.3633 |
|   | 4 | 1.0000 | 0.9996 | 0.9971 | 0.9896 | 0.9727 | 0.9420 | 0.8939 | 0.8263 | 0.7396 | 0.6367 |
|   | 5 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9958 | 0.9887 | 0.9747 | 0.9502 | 0.9115 | 0.8555 |
|   | 6 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9987 | 0.9964 | 0.9915 | 0.9819 | 0.9648 |
|   | 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9983 | 0.9961 |
| 9 | 0 | 0.6302 | 0.3874 | 0.2316 | 0.1342 | 0.0751 | 0.0404 | 0.0207 | 0.0101 | 0.0046 | 0.0020 |
|   | 1 | 0.9288 | 0.7748 | 0.5995 | 0.4362 | 0.3003 | 0.1960 | 0.1211 | 0.0705 | 0.0385 | 0.0195 |
|   | 2 | 0.9916 | 0.9470 | 0.8591 | 0.7382 | 0.6007 | 0.4628 | 0.3373 | 0.2318 | 0.1495 | 0.0898 |
|   | 3 | 0.9994 | 0.9917 | 0.9661 | 0.9144 | 0.8343 | 0.7297 | 0.6089 | 0.4826 | 0.3614 | 0.2539 |
|   | 4 | 1.0000 | 0.9991 | 0.9944 | 0.9804 | 0.9511 | 0.9012 | 0.8283 | 0.7334 | 0.6214 | 0.5000 |
|   | 5 | 1.0000 | 0.9999 | 0.9994 | 0.9969 | 0.9900 | 0.9747 | 0.9464 | 0.9006 | 0.8342 | 0.7461 |
|   | 6 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 | 0.9957 | 0.9888 | 0.9750 | 0.9502 | 0.9102 |
|   | 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9986 | 0.9962 | 0.9909 | 0.9805 |
|   | 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9980 |

Poissonfördelning

Tabell för  $P(X \leq k)$  där  $X \sim Po(\mu)$ .

|                | $\mu$  |        |        |        |        |        |        |        |        |        |  |  |  |
|----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--|--|--|
| $\underline{}$ | 0.1    | 0.2    | 0.3    | 0.4    | 0.5    | 0.6    | 0.7    | 0.8    | 0.9    | 1.0    |  |  |  |
| 0              | 0.9048 | 0.8187 | 0.7408 | 0.6703 | 0.6065 | 0.5488 | 0.4966 | 0.4493 | 0.4066 | 0.3679 |  |  |  |
| 1              | 0.9953 | 0.9825 | 0.9631 | 0.9384 | 0.9098 | 0.8781 | 0.8442 | 0.8088 | 0.7725 | 0.7358 |  |  |  |
| 2              | 0.9998 | 0.9989 | 0.9964 | 0.9921 | 0.9856 | 0.9769 | 0.9659 | 0.9526 | 0.9371 | 0.9197 |  |  |  |
| 3              | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9982 | 0.9966 | 0.9942 | 0.9909 | 0.9865 | 0.9810 |  |  |  |
| 4              | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9986 | 0.9977 | 0.9963 |  |  |  |
| 5              | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9997 | 0.9994 |  |  |  |
| 6              | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |  |  |  |
| 7              | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |  |  |  |
|                | $\mu$  |        |        |        |        |        |        |        |        |        |  |  |  |
| k              | 1.1    | 1.2    | 1.3    | 1.4    | 1.5    | 1.6    | 1.7    | 1.8    | 1.9    | 2.0    |  |  |  |
| 0              | 0.3329 | 0.3012 | 0.2725 | 0.2466 | 0.2231 | 0.2019 | 0.1827 | 0.1653 | 0.1496 | 0.1353 |  |  |  |
| 1              | 0.6990 | 0.6626 | 0.6268 | 0.5918 | 0.5578 | 0.5249 | 0.4932 | 0.4628 | 0.4337 | 0.4060 |  |  |  |
| 2              | 0.9004 | 0.8795 | 0.8571 | 0.8335 | 0.8088 | 0.7834 | 0.7572 | 0.7306 | 0.7037 | 0.6767 |  |  |  |
| 3              | 0.9743 | 0.9662 | 0.9569 | 0.9463 | 0.9344 | 0.9212 | 0.9068 | 0.8913 | 0.8747 | 0.8571 |  |  |  |
| 4              | 0.9946 | 0.9923 | 0.9893 | 0.9857 | 0.9814 | 0.9763 | 0.9704 | 0.9636 | 0.9559 | 0.9473 |  |  |  |
| 5              | 0.9990 | 0.9985 | 0.9978 | 0.9968 | 0.9955 | 0.9940 | 0.9920 | 0.9896 | 0.9868 | 0.9834 |  |  |  |
| 6              | 0.9999 | 0.9997 | 0.9996 | 0.9994 | 0.9991 | 0.9987 | 0.9981 | 0.9974 | 0.9966 | 0.9955 |  |  |  |
| 7              | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9994 | 0.9992 | 0.9989 |  |  |  |
| 8              | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9998 |  |  |  |
| 9              | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |  |  |  |
|                |        |        |        |        | ŀ      | ı      |        |        |        |        |  |  |  |
| k              | 2.1    | 2.2    | 2.3    | 2.4    | 2.5    | 2.6    | 2.7    | 2.8    | 2.9    | 3.0    |  |  |  |
| 0              | 0.1225 | 0.1108 | 0.1003 | 0.0907 | 0.0821 | 0.0743 | 0.0672 | 0.0608 | 0.0550 | 0.0498 |  |  |  |
| 1              | 0.3796 | 0.3546 | 0.3309 | 0.3084 | 0.2873 | 0.2674 | 0.2487 | 0.2311 | 0.2146 | 0.1991 |  |  |  |
| 2              | 0.6496 | 0.6227 | 0.5960 | 0.5697 | 0.5438 | 0.5184 | 0.4936 | 0.4695 | 0.4460 | 0.4232 |  |  |  |
| 3              | 0.8386 | 0.8194 | 0.7993 | 0.7787 | 0.7576 | 0.7360 | 0.7141 | 0.6919 | 0.6696 | 0.6472 |  |  |  |
| 4              | 0.9379 | 0.9275 | 0.9162 | 0.9041 | 0.8912 | 0.8774 | 0.8629 | 0.8477 | 0.8318 | 0.8153 |  |  |  |
| 5              | 0.9796 | 0.9751 | 0.9700 | 0.9643 | 0.9580 | 0.9510 | 0.9433 | 0.9349 | 0.9258 | 0.9161 |  |  |  |
| 6              | 0.9941 | 0.9925 | 0.9906 | 0.9884 | 0.9858 | 0.9828 | 0.9794 | 0.9756 | 0.9713 | 0.9665 |  |  |  |
| 7              | 0.9985 | 0.9980 | 0.9974 | 0.9967 | 0.9958 | 0.9947 | 0.9934 | 0.9919 | 0.9901 | 0.9881 |  |  |  |
| 8              | 0.9997 | 0.9995 | 0.9994 | 0.9991 | 0.9989 | 0.9985 | 0.9981 | 0.9976 | 0.9969 | 0.9962 |  |  |  |
| 9              | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9995 | 0.9993 | 0.9991 | 0.9989 |  |  |  |
| 10             | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 |  |  |  |
| 11             | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 |  |  |  |
| 12             | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |  |  |  |