Fast Bayesian Whole-Brain fMRI Analysis with Spatial 3D Priors

Per Sidén, Division of Statistics and Machine Learning, Linköping University, Sweden.

Jointly with Anders Eklund, David Bolin and Mattias Villani. Mail: per.siden@liu.se

LINKÖPING UNIVERSITY

Summary

Spatial Bayesian modeling of task-related fMRI is a serious computational challenge. Therefore many methods do (i) approximate inference using e.g. Variational Bayes (VB) and (ii) slice-wise processing of data. One example is the Bayesian single subject analysis in SPM [1]. Using modern techniques for efficient high-dimensional Gaussian sampling, we develop a fast and practical Markov Chain Monte Carlo (MCMC) algorithm to be used both for slice-wise 2D and whole-brain 3D inference. Using MCMC, we are for the first time able to evaluate the approximate VB posterior against the exact MCMC posterior and show that VB can lead to spurious activation. We also develop an improved, even faster VB algorithm which shows negligible error compared to the MCMC posterior. A preprint of this paper is available at `http://arxiv.org/abs/1606.00980`.

GLM with Spatial Prior

Following [1], we define the General Linear Model (GLM) as

$$\underset{T\times N}{\mathbf{Y}} = \underset{T\times K}{\mathbf{X}}\,\underset{K\times N}{\mathbf{W}} + \underset{T\times N}{\mathbf{E}},$$

with T volumes, N voxels and K regressors in the design matrix $\mathbf{X}$. $\mathbf{E}$ is Gaussian order P AR noise with voxel-specific precision $\boldsymbol{\lambda}$ and AR parameters $\mathbf{A}$. There is a spatial unweighted graph Laplacian prior on the regression coefficients according to

$$\mathbf{W}'_{k,.}|\alpha_k \sim \mathcal{N}\left(0, \alpha_k^{-1}\mathbf{D}_w^{-1}\right),$$

where $\mathbf{D}_w$ is a sparse precision matrix and similar for $\mathbf{A}$. We use gamma priors for α_k and λ_n.

IVB, SVB and MCMC

SPM's VB makes two simplifying independence assumptions about the posterior

$$(1)\;\; q\left(\mathbf{W},\boldsymbol{\alpha},\boldsymbol{\lambda}|\mathbf{Y}\right) = q\left(\mathbf{W}|\mathbf{Y}\right)q\left(\boldsymbol{\alpha}|\mathbf{Y}\right)q\left(\boldsymbol{\lambda}|\mathbf{Y}\right)$$

$$(2)\;\; q\left(\mathbf{W}|\mathbf{Y}\right) = \prod_{n=1}^{N} q\left(\mathbf{W}_{\cdot,n}|\mathbf{Y}\right).$$

We construct an algorithm that drops the second assumption, Spatial VB (SVB), in contrast to SPM's Independent VB (IVB). We also develop a fast MCMC algorithm that drops both assumptions.

References

[1] Penny, W. D., Trujillo-Barreto, N. J. and Friston, K. J.: *Bayesian fMRI time series analysis with spatial priors*, NeuroImage (2005)

[2] Papandreou, G. and Yuille, A.: *Gaussian sampling by local perturbations*, Advances in Neural Information Processing Systems 23 (2010)

[3] Rue, H. and Held, L.: *Gaussian Markov Random Fields: Theory and Applications*, CRC Press (2005)

[4] Bishop, C. M.: *Pattern Recognition and Machine Learning*, Springer (2006)

Acknowledgements

This work was funded by Swedish Research Council (Vetenskapsrådet) grant no 2013-5229.

Inference Algorithms

A key to fast inference for both SVB and MCMC is efficient sampling from Gaussian Markov Random Fields (GMRFs) using the pre-conditioned conjugate gradient (PCG) techniques in [2]. The full conditional posterior for activity parameters $\mathbf{w}_r = vec\left(\mathbf{W}'\right)$ is a GMRF on the form

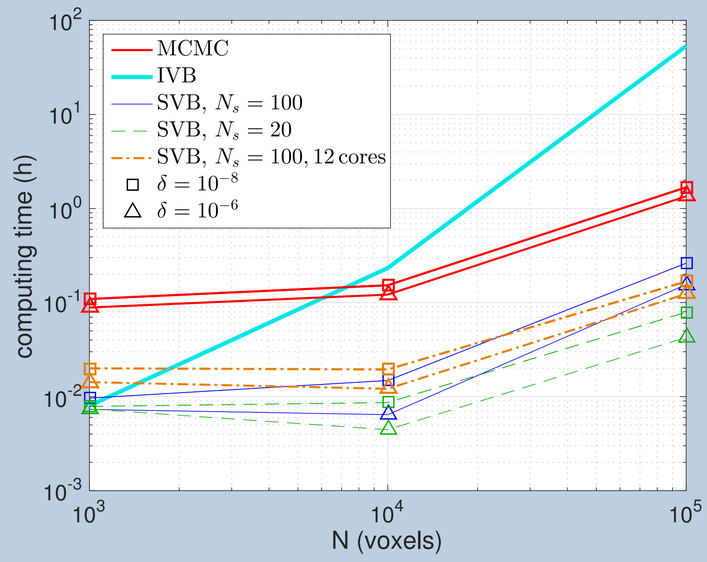
$$p\left(\mathbf{w}_r|\mathbf{Y},\boldsymbol{\alpha},\boldsymbol{\lambda}\right) \propto \exp\left(-\frac{1}{2}\mathbf{w}'_r\tilde{\mathbf{B}}\mathbf{w}_r + \mathbf{b}'_w\mathbf{w}_r\right)$$

$$\mathbf{b}_w = vec\left(diag\left(\boldsymbol{\lambda}\right)\mathbf{Y}'\mathbf{X}\right)$$

$$\tilde{\mathbf{B}} = \mathbf{X}'\mathbf{X}\otimes diag\left(\boldsymbol{\lambda}\right) + diag\left(\boldsymbol{\alpha}\right)\otimes\mathbf{D}_w$$

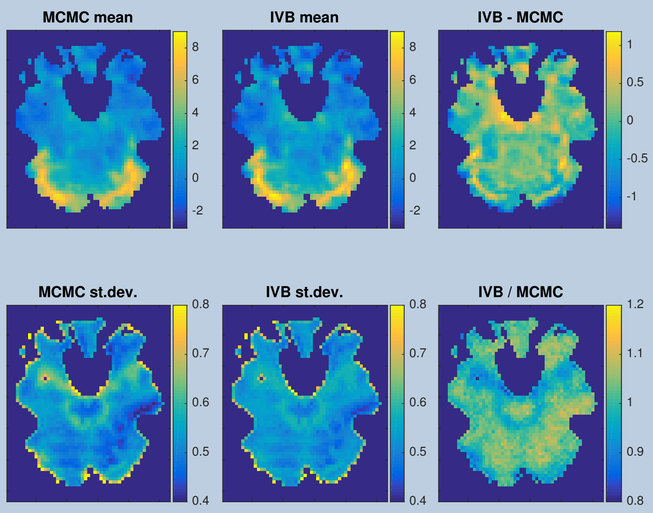
As the number of voxels increase, PCG sampling can give speed-ups greater than a factor 100, compared to the efficient, exact sparse Cholesky based sampling methods in [3].

The resulting MCMC and SVB algorithms scale much better with number of voxels than the IVB method implemented in SPM12.

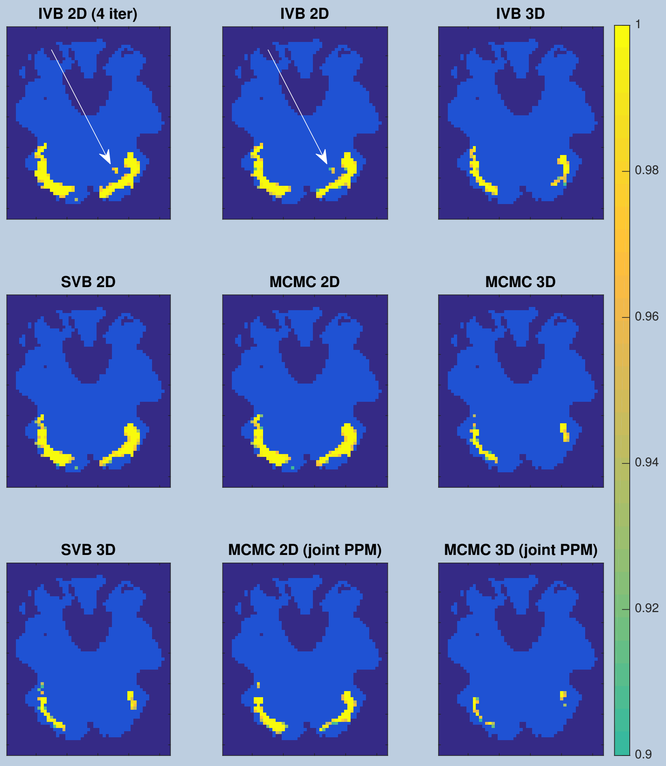


Results

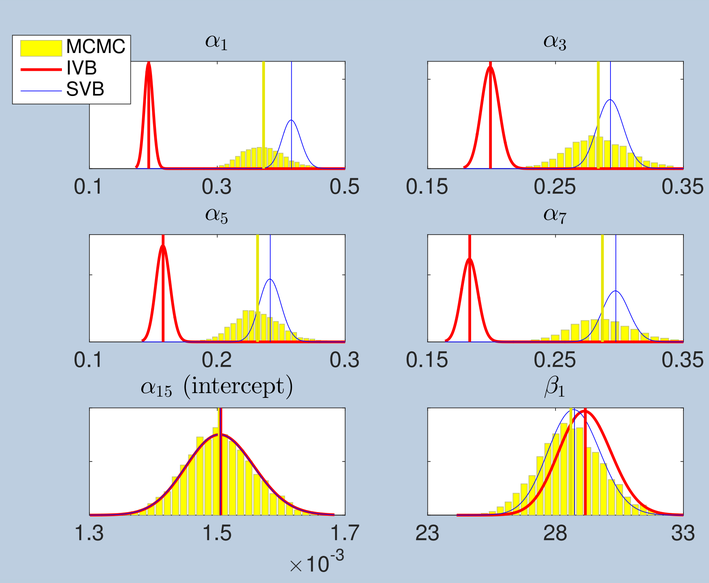
For the face repetition data in [1] we observe some differences between the IVB and the exact MCMC posterior mean and standard deviation:



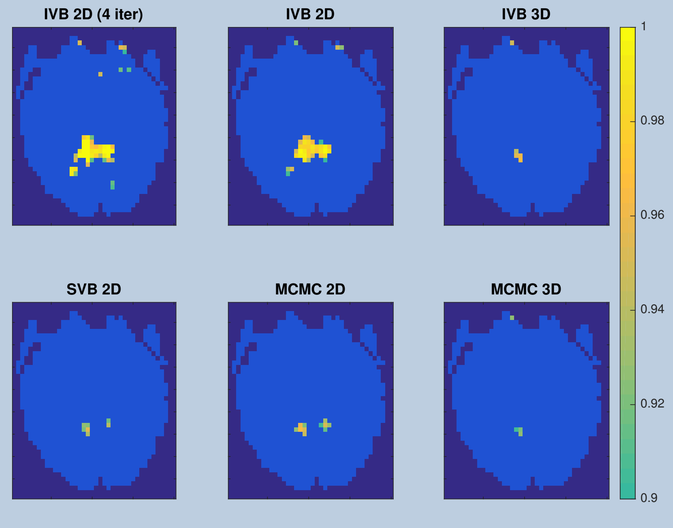
and small but existing differences in 90% thresholded PPMs, while the SVB PPMs show negligible error:



It is well-known that factorized VBs in general tend to underestimate posterior standard deviations [4]. Here, however, much of the differences in the activity posterior can be explained by IVB underestimating also the posterior mean of the hyperparameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ that control the spatial smoothing of activity coefficients $\mathbf{W}$ and AR coefficients $\mathbf{A}$:



A different data set from OpenfMRI gives evidence than IVB can lead to large errors also for thresholded PPMs:



Conclusions and Future Work

We present two fast methods for whole-brain 3D single-subject task-fMRI analysis, SVB and MCMC, and show that they outperform the IVB method in SPM in both speed and accuracy. Future work includes adopting model improvements into this framework, e.g. non-stationary spatial activity priors and spatial measurement noise, and the computation of model selection criteria such as the marginal likelihood to motivate these changes. We also plan to take this to the group level analysis.