Discrete Distributions

Followingis a list of discrete distributions, abbreviations, their probability functions, means, variances, and characteristic functions. An asterisk (*) indicates that the expression is too complicated to present here; in some cases a closed formula does not even exist.

Distribution, notation	Probability function	EX	$\text{Var}\,X$	$\varphi_X(t)$
One point $\delta(a)$	$p(a) = 1$	a	0	e^{ita}
Symmetric Bernoulli	$p(-1) = p(1) = \frac{1}{2}$	0	1	$\cos t$
Bernoulli $\text{Be}(p),\ 0 \le p \le 1$	$p(0) = q,\ p(1) = p;\ q = 1 - p$	p	pq	$q + pe^{it}$
Binomial $\text{Bin}(n,p),\ n = 1,2,\dots,\ 0 \le p \le 1$	$p(k) = \binom{n}{k} p^k q^{n-k},\ k = 0,1,\dots,n;\ q = 1 - p$	np	npq	$(q + pe^{it})^n$
Geometric $\text{Ge}(p),\ 0 \le p \le 1$	$p(k) = pq^k,\ k = 0,1,2,\dots;\ q = 1 - p$	$\frac{q}{p}$	$\frac{q}{p^2}$	$\frac{p}{1 - qe^{it}}$
First success $\text{Fs}(p),\ 0 \le p \le 1$	$p(k) = pq^{k-1},\ k = 1,2,\dots;\ q = 1 - p$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^{it}}{1 - qe^{it}}$
Negative binomial $\text{NBin}(n,p),\ n = 1,2,3,\dots,\ 0 \le p \le 1$	$p(k) = \binom{n+k-1}{k} p^n q^k,\ k = 0,1,2,\dots;\ q = 1 - p$	$n\frac{q}{p}$	$n\frac{q}{p^2}$	$(\frac{p}{1-qe^{it}})^n$
Poisson $\text{Po}(m),\ m > 0$	$p(k) = e^{-m}\frac{m^k}{k!},\ k = 0,1,2,\dots$	m	m	$e^{m(e^{it}-1)}$
Hypergeometric $H(N,n,p),\ n = 0,1,\dots,N,\ N = 1,2,\dots,\ p = 0, \frac{1}{N}, \frac{2}{N}, \dots, 1$	$p(k) = \frac{\binom{Np}{k}\binom{Nq}{n-k}}{\binom{N}{n}},\ k = 0,1,\dots,Np;\ q = 1 - p;\ n - k = 0,\dots,Nq$	np	$npq\frac{N-n}{N-1}$	*

Continuous Distributions

Following is a list of some continuous distributions, abbreviations, their densities, means, variances, and characteristic functions. An asterisk (*) indicates that the expression is too complicated to present here; in some cases a closed formula does not even exist.

Distribution, notation	Density	EX	$\text{Var}\,X$	$\varphi_X(t)$
Uniform/Rectangular				
$U(a,b)$	$f(x) = \frac{1}{b-a},\ a < x < b$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$	$\frac{e^{itb}-e^{ita}}{it(b-a)}$
$U(0,1)$	$f(x) = 1,\ 0 < x < 1$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{e^{it}-1}{it}$
$U(-1,1)$	$f(x) = \frac{1}{2},\ \lvert x\rvert < 1$	0	$\frac{1}{3}$	$\frac{\sin t}{t}$
Triangular				
$\text{Tri}(a,b)$	$f(x) = \frac{2}{b-a}\left(1 - \frac{2}{b-a}\left\lvert x - \frac{a+b}{2}\right\rvert\right),\ a < x < b$	$\frac{1}{2}(a+b)$	$\frac{1}{24}(b-a)^2$	$\left(\frac{e^{itb/2}-e^{ita/2}}{\frac{1}{2}it(b-a)}\right)^2$
$\text{Tri}(-1,1)$	$f(x) = 1 - \lvert x\rvert,\ \lvert x\rvert < 1$	0	$\frac{1}{6}$	$\left(\frac{\sin\frac{t}{2}}{\frac{t}{2}}\right)^2$
Exponential $\text{Exp}(a),\ a > 0$	$f(x) = \frac{1}{a}e^{-x/a},\ x > 0$	a	a^2	$\frac{1}{1 - ait}$
Gamma $\Gamma(p,a),\ a > 0,\ p > 0$	$f(x) = \frac{1}{\Gamma(p)}x^{p-1}\frac{1}{a^p}e^{-x/a},\ x > 0$	pa	pa^2	$\frac{1}{(1-ait)^p}$
Chi-square $\chi^2(n),\ n = 1,2,3,\dots$	$f(x) = \frac{1}{\Gamma(\frac{n}{2})}x^{\frac{1}{2}n-1}\left(\frac{1}{2}\right)^{n/2}e^{-x/2},\ x > 0$	n	$2n$	$\frac{1}{(1-2it)^{n/2}}$
Laplace $L(a),\ a > 0$	$f(x) = \frac{1}{2a}e^{-\lvert x\rvert/a},\ -\infty < x < \infty$	0	$2a^2$	$\frac{1}{1 + a^2t^2}$
Beta $\beta(r,s),\ r,s > 0$	$f(x) = \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)}x^{r-1}(1-x)^{s-1},\ 0 < x < 1$	$\frac{r}{r+s}$	$\frac{rs}{(r+s)^2(r+s+1)}$	*

Continuous Distributions (continued)

Distribution, notation	Density	$E\,X$	$\mathrm{Var}\,X$	$\varphi_X(t)$
Weibull $W(\alpha,\beta),\ \alpha,\beta>0$	$f(x)=\frac{1}{\alpha\beta}x^{(1/\beta)-1}\,e^{-x^{1/\beta}/\alpha},\ x>0$	$\alpha^{\beta}\,\Gamma(\beta+1)$	$a^{2\beta}\big(\Gamma(2\beta+1)$ $-\Gamma(\beta+1)^2\big)$	*
Rayleigh $\mathrm{Ra}\,(\alpha),\ \alpha>0$	$f(x)=\frac{2}{\alpha}\,x\,e^{-x^2/\alpha},\ x>0$	$\frac{1}{2}\sqrt{\pi\alpha}$	$\alpha(1-\frac{1}{4}\pi)$	*
Normal $N(\mu,\sigma^2)$, $-\infty<\mu<\infty,\sigma>0$	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{1}{2}(x-\mu)^2/\sigma^2}$, $-\infty<x<\infty$	μ	σ^2	$e^{i\mu t-\frac{1}{2}t^2\sigma^2}$
$N(0,1)$	$f(x)=\dfrac{1}{\sqrt{2\pi}}\,e^{-x^2/2},\ -\infty<x<\infty$	0	1	$e^{-t^2/2}$
Log-normal $LN(\mu,\sigma^2)$, $-\infty<\mu<\infty,\ \sigma>0$	$f(x)=\dfrac{1}{\sigma x\sqrt{2\pi}}\,e^{-\frac{1}{2}(\log x-\mu)^2/\sigma^2},\ x>0$	$e^{\mu+\frac{1}{2}\sigma^2}$	$e^{2\mu}\big(e^{2\sigma^2}-e^{\sigma^2}\big)$	*
(Student's) t $t(n),\ n=1,2,\ldots$	$f(x)=\frac{\Gamma(\frac{n+1}{2})}{\sqrt{\pi n}\Gamma(\frac{n}{2})}\cdot d\frac{1}{(1+\frac{x^2}{n})^{(n+1)/2}}$, $-\infty<x<\infty$	0	$\dfrac{n}{n-2},\ n>2$	*
(Fisher's) F $F(m,n),\ m,n=1,2,\ldots$	$f(x)=\frac{\Gamma(\frac{m+n}{2})(\frac{m}{n})^{m/2}}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})}\cdot\frac{x^{m/2-1}}{(1+\frac{mx}{n})^{(m+n)/2}}$, $x>0$	$\frac{n}{n-2}$, $n>2$	$\frac{n^2(m+2)}{m(n-2)(n-4)}-\left(\frac{n}{n-2}\right)^2$, $n>4$	*

Continuous Distributions (continued)

Distribution, notation	Density	$E\,X$	$\mathrm{Var}\,X$	$\varphi_X(t)$
Cauchy $C(m,a)$	$f(x)=\frac{1}{\pi}\cdot\frac{a}{a^2+(x-m)^2},\ -\infty<x<\infty$	$\nexists$	$\nexists$	$e^{imt-a\lvert t\rvert}$
$C(0,1)$	$f(x)=\dfrac{1}{\pi}\cdot\dfrac{1}{1+x^2},\ -\infty<x<\infty$	$\nexists$	$\nexists$	$e^{-\lvert t\rvert}$
Pareto $\mathrm{Pa}\,(k,\alpha),\ k>0,\ \alpha>0$	$f(x)=\dfrac{\alpha k^{\alpha}}{x^{\alpha+1}},\ x>k$	$\dfrac{\alpha k}{\alpha-1},\ \alpha>1$	$\dfrac{\alpha k^2}{(\alpha-2)(\alpha-1)^2},\ \alpha>2,$	*