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Covariance Matrix of Observations

The network of observations below depicts the observations propagated from a known point, P1. The observations are mainly distances and directions typically observed from an instrument called the Total Station. A technique that would normally be used to calculate

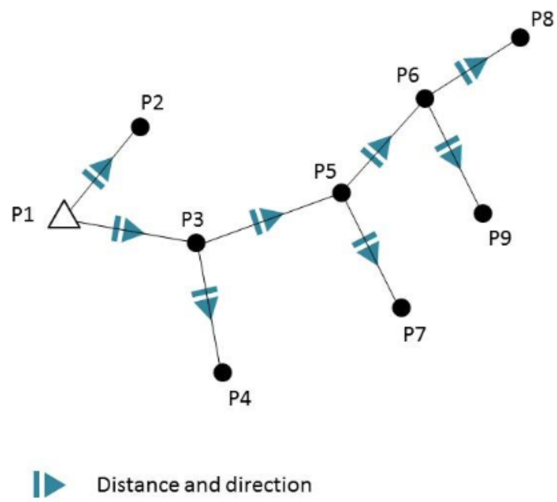

Figure 1: Sequential polars

the coordinates of a second point based on an initial point is called **polar**. In this case, sequential polars would be calculated to obtain the coordinates of all the points since P1 is the only known point. The general formulae to calculate the coordinates are:

$$y_j = y_i + d_{ij} sin\theta_{ij} \tag{1}$$

$$x_j = x_i + d_{ij} cos\theta_{ij} \tag{2}$$

where i is the initial point, j is the observed point, d_{ij} is the distance from i to j, and θ_{ij} is the direction from i to j.

Note that the simulated data (refer to uploaded `Sequential polar obs.xlsx` file) asked in the problem was just an educated guess. In reality, the observations from the total station would be exported from the instrument as a `.csv` file. Nevertheless, a more formal way of simulating the data would be to use a statistical distribution.

To compute the covariance matrix of the observations, the observation equations must first be derived for each measurement in the form of Equations 1 and 2. Here, we have 8 pairs of observation (d_{ij} and θ_{ij} for each measurement), which means that 16 observation equations are expected. Applying the laws of variance-covariance propagation, the formula for the estimated variance-covariance matrix of the parameters, C_x, is given as (Ogundare 2018):

$$C_x = JC_lJ^T \tag{3}$$

where J is the jacobian matrix of the observation equations with respect to their respective d_{ij} and θ_{ij}, and C_l is the covariance matrix of observations. Note that for this problem, since the parameters are assumed to be uncorrelated, it really is only a variance matrix of observations.

Let us further explode Equation 3 based on the network configuration in Figure 1:

Let the 16 observation equations be denoted as y_1, x_1, y_2, x_2, ..., y_8, x_8, with their respective parameters denoted as (d_1, θ_1) for both y_1 and x_1, (d_2, θ_2) for both y_2 and x_2,..., (d_8, θ_8) for both y_8 and x_8.

The Jacobian matrix will contain these partial derivatives with respect to the parameters:

$$J = \begin{bmatrix} \frac{\partial y_1}{\partial d_1} & \frac{\partial y_1}{\partial \theta_1} & \frac{\partial y_1}{\partial d_2} & \frac{\partial y_1}{\partial \theta_2} & \cdots & \frac{\partial y_1}{\partial d_8} & \frac{\partial y_1}{\partial \theta_8} \\ \frac{\partial x_1}{\partial d_1} & \frac{\partial x_1}{\partial \theta_1} & \frac{\partial x_1}{\partial d_2} & \frac{\partial x_1}{\partial \theta_2} & \cdots & \frac{\partial x_1}{\partial d_8} & \frac{\partial x_1}{\partial \theta_8} \\ \frac{\partial y_2}{\partial d_1} & \frac{\partial y_2}{\partial \theta_1} & \frac{\partial y_2}{\partial d_2} & \frac{\partial y_2}{\partial \theta_2} & \cdots & \frac{\partial y_2}{\partial d_8} & \frac{\partial y_2}{\partial \theta_8} \\ \frac{\partial x_2}{\partial d_1} & \frac{\partial x_2}{\partial \theta_1} & \frac{\partial x_2}{\partial d_2} & \frac{\partial x_2}{\partial \theta_2} & \cdots & \frac{\partial x_2}{\partial d_8} & \frac{\partial x_2}{\partial \theta_8} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial y_8}{\partial d_1} & \frac{\partial y_8}{\partial \theta_1} & \frac{\partial y_8}{\partial d_2} & \frac{\partial y_8}{\partial \theta_2} & \cdots & \frac{\partial y_8}{\partial d_8} & \frac{\partial y_8}{\partial \theta_8} \\ \frac{\partial x_8}{\partial d_1} & \frac{\partial x_8}{\partial \theta_1} & \frac{\partial x_8}{\partial d_2} & \frac{\partial x_8}{\partial \theta_2} & \cdots & \frac{\partial x_8}{\partial d_8} & \frac{\partial x_8}{\partial \theta_8} \end{bmatrix}_{16\times16}$$

The number of rows in the Jacobian matrix J, is defined by each observation equation, and the number of columns is defined by the total unique number of parameters contained in all the observation equations.

Moreover, for the matrix computations to be computable, the estimated variance matrix of the observations C_l, must be diagonalised with zero elements populated in the off-diagonal as such:

$$C_l = \begin{bmatrix} \sigma^2_{d_1} & & & & & & \\ & \sigma^2_{\theta_1} & & & & & \\ & & \sigma^2_{d_2} & & & & \\ & & & \sigma^2_{\theta_2} & & & \\ & & & & \ddots & & \\ & & & & & \sigma^2_{d_8} & \\ & & & & & & \sigma^2_{\theta_8} \end{bmatrix}_{16\times16}$$

The reason for C_l to be a diagonal matrix is due to the fact that the distances and directions are assumed to be uncorrelated. Eventually, when the relevant matrices have been computed and organised appropriately, Equation 3 can be applied to obtain the desired output.

Reference

Ogundare, J.O., 2018. Understanding least squares estimation and geomatics data analysis. John Wiley & Sons.