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Adjusting a Levelling Network using the Parametric Case of Least Squares

The parametric case of the least squares is the special case of the combined case of least squares. This is the technique that was used to optimised h_A and h_B in Figure 1. h_1 and h_2

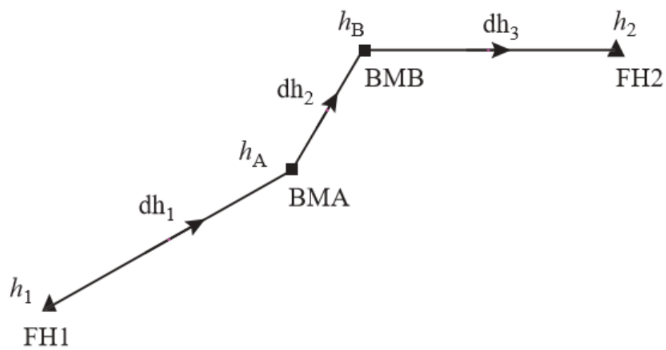

Figure 1: Levelling network.

are known elevations, meaning that these parameters would not contribute in the adjustment process.

Without derivations, in matrix notation, the parametric case takes the form of:

$$V = AX - L; \tag{1}$$

where V is the residual matrix, A is the design matrix, X is the unknown matrix, and L is the misclosure matrix. Note that sometimes L can also be denoted as $-W$, which would then alter Equation 2. Moreover, to solve for X, the formula is given as:

$$X = (A^T PA)^{-1} A^T PL; \tag{2}$$

where P is the weight matrix. This was the main formula used to find the adjusted elevations.

The design matrix, A is populated by the jacobian of the observation equations. For this problem, the observation equations can be derived as follows:

$$dh_1 = h_A^o - h_1; \tag{3a}$$

$$dh_2 = h_B^o - h_A^o; \tag{3b}$$

$$dh_3 = h_2 - h_B^o; \tag{3c}$$

where dh_1, dh_2, and dh_3 are the change in elevation from FH1 to BMA, BMA to BMB, and BMB to FH2, respectively. h_A^o and h_B^o are the the parameters or unknowns that must be adjusted.

Therefore, to solve Equation 2, we first established the A, L, and P matrices. These matrices are further exploded below.

Each row of the design matrix, A represents the observation equations and each columns refers to the respective partial derivatives of the unknowns (i.e, h_A^o and h_B^o). Hence a 3×2 matrix.

$$A = \begin{bmatrix} \frac{\partial dh_1}{\partial h_A^o} & \frac{\partial dh_1}{\partial h_B^o} \\ \\ \frac{\partial dh_2}{\partial h_A^o} & \frac{\partial dh_2}{\partial h_B^o} \\ \\ \frac{\partial dh_3}{\partial h_A^o} & \frac{\partial dh_3}{\partial h_B^o} \end{bmatrix}_{3\times 2}$$

The misclosure matrix, L is simply making Equations 3a, 3b, and 3c equal to 0. Ideally, the equations should equal to 0, however, it won't be most of the time, due to the random and systematic errors inherent during observations. The size of this matrix is 3×1.

$$L = \begin{bmatrix} dh_1 - (h_A^o - h_1) \\ \\ dh_2 - (h_B^o - h_A^o) \\ \\ dh_3 - (h_2 - h_B^o) \end{bmatrix}_{3\times 1}$$

The weight matrix, P is always a square matrix provided that there exists no correlations between the observations, hence the off-diagonal 0s. Here, P is a 3×3 matrix. σ_o is called the standard deviation of an observation of unit weight introduced a priori, and assumed to be 1.

$$P = \begin{bmatrix} \frac{\sigma_o^2}{\sigma_{dh_1}^2} & 0 & 0 \\ \\ 0 & \frac{\sigma_o^2}{\sigma_{dh_2}^2} & 0 \\ \\ 0 & 0 & \frac{\sigma_o^2}{\sigma_{dh_3}^2} \end{bmatrix}_{3\times 3}$$

Once the above matrices were evaluated, Equation 2 was applied to get the corrections for the unknowns in the matrix X. These unknown values were still added to the initialised h_A^o and h_B^o for the adjusted elevations.

Reference

Ogundare, J.O., 2018. Understanding least squares estimation and geomatics data analysis. John Wiley & Sons.