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Application of Law of Propagation of Variances

Given the formula derived from the area of triangle ABC:

$$b = c \ \frac{\sin\beta}{\sin(180 - (\alpha+\beta))}; \tag{1}$$

the standard deviation of side b is to be calculated using the law of propagation of variances.

Let the right-hand side of Equation 1 be the function $f(\alpha, \beta, c) = F$. When applying the law of propagation of variances to Equation 1, the derivation is as follows:

$$\delta_b^2 = \underbrace{\left(\frac{\partial F}{\partial \alpha}\right)^2 \delta_\alpha^2}_{\text{Component 1}} + \underbrace{\left(\frac{\partial F}{\partial \beta}\right)^2 \delta_\beta^2}_{\text{Component 2}} + \underbrace{\left(\frac{\partial F}{\partial c}\right)^2 \delta_c^2}_{\text{Component 3}}. \tag{2}$$

The partial derivatives of each variable pertaining to function F (*without the squared*) are also known as the jacobians or the determinants of the jacobian matrix. Note that the components 1, 2, and 3 of Equation 2 relate to the error contributors over the total error. For example, if component 1 has the largest magnitude, it means that the angle α yields the largest error effect, therefore must be re-observed carefully. Finally, to solve for δ_b in Equation 2, the matrix notation would look as follows:

$$\delta_b = \sqrt{\begin{bmatrix} \left(\frac{\partial F}{\partial \alpha}\right)^2 & \left(\frac{\partial F}{\partial \beta}\right)^2 & \left(\frac{\partial F}{\partial c}\right)^2 \end{bmatrix}_{1\times 3} \cdot \begin{bmatrix} \delta_\alpha^2 \\ \delta_\beta^2 \\ \delta_c^2 \end{bmatrix}_{3\times 1}}. \tag{3}$$