



# Master of Science in Informatics at Grenoble Master Mathématiques Informatique - spécialité Informatique option Graphics Vision and Robotics

# Scalable image reconstruction methods for large data: Application to Synchrotron CT of biological samples Claude Goubet

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Research project performed at CREATIS

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Defended before a jury composed of: Head of the jury Jury member 1 Jury member 2

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#### **Abstract**

Your abstract goes here...

#### Acknowledgement

#### Résumé

Your abstract in French goes here...

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# Introduction

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#### **— 2** —

# State-of-the-Art

intro about CT and importance for osteoporosis diagnosis + use of SR + low dose problem CS ([?, ?, ?])

#### 2.1 Dose reduction in SR Micro-CT

Multiple CS algorithm were developed for Micro-CT allowing to generate different spacial resolutions. Alternative methods then FBP necessary to recover missing projections. Iterative algorithms are used.

#### 2.1.1 No SR

SART-L1 [?, ?] ASD-POCS TV [?]

#### 2.1.2 CS on SR micro-CT

multiple iterative methods using CGTV ([?]) ART with multiple denoising (TV [?]; L1 minimisation [?]; Discrete packet shrinkage [?]) SART ([?] with TV [?]) OS-SART [?]) EST [?, ?] PCCT [?] define resolution for each solution (maybe more details?)

#### 2.2 SR Nano-CT

Nano-CT general ref: [?] (I can have other references but are mostly about the hardware side, new materials and acquisition methodology, or image post-processing without having used low dose)

less CS reconstruction experimented Low dose nano OS-SART L1 norm TV [?]

## 2.3 Wrap-up

A lot of research these past few years of CSCT going toward a improvement of spacial resolution and dose reduction. Yet not so much has been done on Nano scale. In the context of osteoporosis nano scale is mandatory for a accurate diagnosis. Present our objective.

## **Problem Statement**

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# Proposed approach

We propose to use iterative reconstruction with the algorithm described by [?]. Split Bregman algorithm gives a solution to an L1-L2 constrained problem. We will here describe the bregman iteration and it's application to L2 minimisation reconstructions which will be used in our algorithm.

## 4.1 Split Bregman iterative reconstruction

#### 4.1.1 Split Bregman iteration

Using Split Bregman we wish to solve the constrained reconstruction optimization problem described in the section ??:

$$\min_{u} ||\nabla_{u}||_{1} \text{ such that } Fu = f \tag{4.1}$$

Such constrained problem problems are difficult to solve directly. For this reason we need to define a new unconstrained problem. Luckily it is possible to approximate (4.1) as:

$$\min_{u} ||\nabla_{u}||_{1} + \frac{\lambda}{2} ||Fu - f||_{2}^{2}$$
(4.2)

The Bregman iteration allows us to reduce 4.1 in even shorter unconstrained problems using Bregman distances. These constrained problem can be resolved iteratively as follows:

$$u^{k+1} = \min_{u} ||\nabla_{u}||_{1} + \frac{\lambda}{2} ||Fu - f^{k}||_{2}^{2}$$

$$f^{k+1} = f^{k} + f - Fu^{k}$$
(4.3)

#### 4.1.2 L1 regularization problem

Our compressed sensing reconstruction method is based on L1 regulation. A more faithful reconstruction problem must be formulated and we will see how to solve it iteratively with split Bregman.

The idea is to "de-couple" the L1 and L2 parts of our original problem. We wish to minimize the Total Variation  $\nabla_u$  of the image and a weight function H(). We write the problem as follows:

$$\min_{u,d} ||d||_1 + H(u) \text{ such that } d = \nabla_u$$
(4.4)

Which can be computed iteratively using Split Bregman iteration as:

$$(u^{k+1}, d^{k+1}) = \min_{u, d} ||d||_1 + H(u) + \frac{\lambda}{2} ||d - \nabla_u - b^k||_2^2$$

$$b^{k+1} = b^k + \nabla_{u^{k+1}} - d^{k+1}$$
(4.5)

#### 4.2 SB-TV-2D reconstruction

isotropic TV denoising pbl:

$$\min_{u} ||\nabla_{u}||_{1} \text{ such that } ||Fu - f||_{2}^{2} < \sigma^{2}$$

$$\tag{4.6}$$

where  $\nabla_u = (\nabla_x, \nabla_y)u$ , f represents the projection space, F the projection operator, u the image domain and  $\sigma$  represents the variance of the signal noise.

$$u^{k+1} = \min_{u} ||\nabla_{u}||_{1} + \frac{\lambda}{2} ||Fu - f^{k}||_{2}^{2}$$

$$f^{k+1} = f^{k} + f - F_{u}^{k}$$
(4.7)

We fall here into an unconstrained problem which is not steight forwardly solved. In order to get a constrained problem we will insert a variable d such that  $d = \nabla_u$ . We can now use the Split Bregman iteration in order to solve our new problem:

$$u^{k+1} = \min_{u,d} ||d||_1 + \frac{\lambda}{2} ||Fu - f^k||_2^2 \text{ such that } d = \nabla_u$$

$$f^{k+1} = f^k + f - F_u^k$$
(4.8)

And get to a solution where L1 and L2 elements of our original problem are spleted into two equations:

$$u^{k+1} = \min_{u} \frac{\mu}{2} ||Fu - f^{k}||_{2}^{2} + \frac{\lambda}{2} ||d^{k} - \nabla_{u} - b^{k}||_{2}^{2}$$

$$d^{k+1} = \min_{u} ||d||_{1} + \frac{\lambda}{2} ||d - \nabla_{u} - b^{k}||_{2}^{2}$$

$$b^{k+1} = b^{k} + \nabla_{u}^{k+1} - d^{k+1}$$

$$f^{k+1} = f^{k} + f - F_{u}^{k}$$

$$(4.9)$$

Now is left to sole the minimization on the  $u^{k+1}$  and  $d^{k+1}$  operations. Both operation are differentiable. We are hence get the minimum using the derivative.

Solution for u

$$u^{k+1} = (4.10)$$

Solution for d

# 4.3 SB-TV-3D

pbl: 
$$\alpha ||(\nabla_x, \nabla_y, \nabla_z)u||_1$$
 such that  $||Fu - f||_2^2 < \delta^2$  
$$u = d = d = d$$
 (4.11)

# **Method evaluation**

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# Conclusion

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# A —Appendix

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