Introduction to inverse problem resolution

Claude Goubet - Juan JFP Abascal

February 17, 2017

Chapter 1

Inverse problem and conditioning

1.1 Inverse problem

An inverse problem is a situation when you wish to describe a physical model from the results of measurements. As apposed to modelization problem where you estimate the outcome of measurements using the physical model.
Basically, a modelization problem is of the form:

$$A \times x = Y \tag{1.1}$$

Where you estimate a measurement Y of the estimated measurement of element x with a transformation matrix A. Hence the known values are x and A and compute Y.
The inverse problem is posed as in equation (1.1) although the known values are the measurement Y and the transformation matrix A and we want to infer from them the physical model x.
A naive solution to (1.1) in case of inverse problem would be:

$$x = A^{-1} \times Y \tag{1.2}$$

Thus this solution requires A^{-1} to be invertable such transformation requires a the problem to be well posed. In the contrary case, other methods must be used in order to avoid using A^{-1}.

1.2 well- and ill-posed problems

For the following, let us consider A a squared matrix of size $n \times n$; x of the size $n \times 1$ and Y of the size $n \times 1$.

In order for a matrix to be invertable, its rank must be equal to its number of columns. The rank of a matrix being its number of independent rows.