University of the West Indies, Cave Hill Campus
Department of Computer Science, Mathematics and Physics
COMP1180: Mathematics for Computer Science I
Semester I, 2016/2017

ASSIGNMENT
Thursday 15th September, 2016

TO BE SUBMITTED) by hand, (signature required) no later than 5:00PM on Thursday 10th November, 2016. SHOW ALL WORKING CLEARLY!!

1. (a) Show that $[\sim r \wedge (\sim t \rightarrow r)] \rightarrow r$ is a tautology using a truth table. **[3 mks]**

ii) Contruct a truth table for $(p \leftrightarrow q) \oplus (\sim p \leftrightarrow \sim r)$ **[3 mks]**

(b) Show, without the use of truth tables, that $r \leftrightarrow s$ is logically equivalent to $\sim (r \vee s) \vee (r \wedge s)$ **[4 mks]**

(c) Show that $(\sim p \wedge \sim q) \vee \sim(p \vee \sim q)$ is logically equivalent to $\sim p$ **[2 mks]**

(d) Determine the truth value of each of the following: **[1 mk each]**

i) Pigs can fly iff $0 > 1$
ii) If $3 + 1 = 5$ then $2 + 2 = 4$
iii) $1 + 1 = 0$ if and only if $3 + 7 = 9$
iv) If 5 is a factor of 20 then 3 is a factor of 23

(e) Let **k**: Kite flying; and let **r**: The rain is falling. Express the following sentence as a logical expression:

There is no kite flying when the rain is falling. **[2 mk]**

(f) Let p, q and r be propositions: **[2 mk each]**

p: Rattle snakes have been seen in the area
q: Riding is safe on the trail
r: Tamarinds are ripe along the trail

i. Whenever rattle snakes have been seen in the area and tamarinds are ripe along the trail, riding is not safe
ii. If Tamarinds are ripe along the trail and rattle snakes have not been seen in the area then riding is safe
iii. Tamarinds are ripe along the trail, iff riding is safe and rattle snakes have not been seen in the area.

2. (a) Given the hypotheses $\neg q \rightarrow (r \rightarrow \neg s)$, $\neg p \rightarrow r$, $\neg q \vee w$, $\neg(p \vee w)$, show that $\neg s$ is a valid conclusion. **[4 mks]**

(b) Given the hypotheses $(p \wedge s) \vee t$, and $t \to q$. Show that $s \vee q$ is a valid conclusion. **[3 mks]**

(c) Given the hypotheses $(t \vee q) \to (r \vee \neg p)$ and $\neg r \wedge p$. Show that $\neg \mathbf{t}$ is a valid conclusion. **[3 mks]**

iv. Construct an argument using the rules of inference to show that the hypotheses "If it does not snow or if it is not misty, then the boat race will be held," "If the boat race is held, then the cup will be awarded," and "The cup was not awarded" imply the conclusion, "**It snowed**". [**5 mks**]

- Let c be the proposition "It snowed"
- Let b be the proposition "It is misty"
- Let h be the proposition "The boat race will be held"
- Let s be the proposition "The cup will be awarded"

3. (a) Prove by mathematical induction, when *n* is a positive integer that

(i) $2 + 7 + 12 + 17 + \ldots. + (5n - 3) = n(5n - 1)/2$ [**5 mks**]

(ii) $\frac{1}{1\times 5} + \frac{1}{5\times 9} + \frac{1}{9\times 13} + \cdots + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1}$ [**5 mks**]

(iii) $1^3 + 2^3 + 3^3 + \ldots + n^3 = [n(n + 1)/2]^2$ **[5 mks]**

(b) Use mathematical induction to prove that

$n^3 + 2n$ is divisible by 3 $\forall n \geq 1$ [**4 mks**]

4. a) If a set R has *p* elements, in terms of *p*, how many elements does *P*(R) has? [**2 mks**]

b) Find the power set of each of the following:
   i) $\{x\}$
   ii) $\{\emptyset, \{\emptyset\}\}$ [**1, 3 mks**]

5. a) What is the Cartesian product $B \times A$ if $A = \{1, 2\}$ and $B = \{x, y, z\}$ [**2 mks**]

b) How many different elements does $B \times A$ have if A has *n* elements and B has *m* elements. [**2 mks**]

6. a) Let *U* be the set of the first 12 nonnegative integers that are greater than zero, and the ordering of the elements in *U* are in increasing order; i.e $a_i = i$. What **bit strings** represents the following subsets in *U* :

i) the factors of 12

ii) all odd numbers
iii) all even numbers less than 12 **[1,1,1 mks]**

Show all steps clearly:
b) What is the bit string representation for the complement of a(i) from above?
c) What is the bit string representation for the union of a(i) and a(ii)?
d) What is the bit string representation for the intersection of a(i) and a(ii)?
[2,2,2 mks]

7. Give the definition of each of the following functions: **[1,1,1 mks]**
   a. Injection
   b. Surjection
   c. Bijection

8. Determine, giving reasons, whether the function, f: Z × Z → Z is surjective:
   a. $f(x, y) = x + y$ **[2 mks]**
   b. $f(x, y) = x^2 + y^2$ **[2 mks]**

9. By showing all working, determine whether each of these functions is a bijection from **R** to **R**:
   a) $f(x) = 2x + 1$ **[3 mks]**

   b) $f(x) = x^2 + 1$ **[3 mks]**

10. Give the definition of each of the following relations: **[1,1,1 mks]**
   a) Reflexive
   b) Symmetric
   c) Transitive

11. Let A be a set {1, 2, 3, 4,5}. List the ordered pairs in the relation R = { (a,b) | a divides b } **[2,2,2 mks]**
   a) State, with reasons, whether of not, R is reflexive
   b) State, with reasons, whether or not, if R is symmetric
   c) State, with reasons, whether or not, if R is transitive.

12. If A = {1, 2, 3, 4, 5, 6, 7, 8, 9} be the given set and the relation R is defined on A As R = { a, b) | a and b are even }. Show that R is an equivalence relation.
[6 mks]

TOTAL MARKS: 104