Think Bayes

Chapter 1 - Bayes Theorem

→ probability - chance that something will happen
↳ 0 - impossible
1 - certain.
0.5 - outcome is as likely as not.

→ conditional probability
↳ background information is given

$$p(\text{first heart attack}) = \frac{7.85 \times 10^5}{3.11 \times 10^8} = 0.3\%$$

↳ each person has attributes that make event A more or less likely
- high cholesterol vs. - low blood press.
- age - smoker

$P(A|B)$ = probability of A given B

→ conjoint probability

↳ How likely is it that two events happen together?

$$p(A \cap B) = p(A)\,p(B)$$

↳ given A & B are independent.

or $P(B|A) = P(B)$

& $P(A|B) = P(A)$

∴ it is really

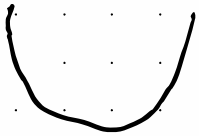
these are equal

→ $P(A \cap B) = P(A)\,P(B|A)$

→ $P(B \cap A) = P(B)\,P(A|B)$

$$\Rightarrow P(A|B) = \frac{P(B|A)\,P(A)}{P(A|B)}$$

→ eg



cocoa-flow	A	B
vanilla	30	20
chocolate	10	20

If you draw vanilla which bowl is most likely?

$$P(V|A) = \frac{30}{30+10} \qquad P(V|B) = \frac{20}{20+20}$$

$$\therefore P(V|A) > P(V|B)$$

now think of the problem in terms of bayes theorem.

$$P(A|V) = \frac{P(V|A)\,P(A)}{P(V)}$$

$$\begin{cases} P(V|A) = \frac{3}{4} \\ P(V) = \frac{30+20}{2(40)} = \frac{5}{8} \\ P(A) = \frac{1}{2} \end{cases}$$

$$= \frac{(1/2)(3/4)}{(5/8)}$$

$$= 3/5$$

Possible to use B.T. to get $P(A|B)$ from $P(B|A)$

→ Diachronic Interpretation

update hypothesis given data.

$$P(H|X) = \frac{P(X|H)\,P(H)}{P(X)}$$

$P(H)$ ← prior

$P(X)$ ← normalizing

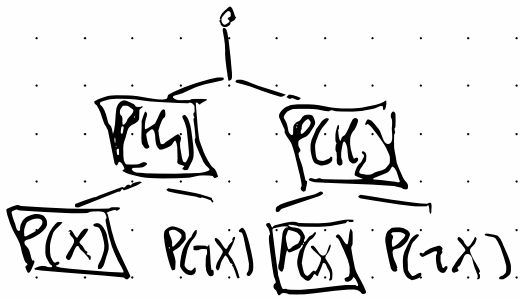
$P(H|X)$ ← posterior

$P(X|H)$ ← likelihood.

- likelihood can be difficult to calculate re prob. of data given any hypothesis solution:
  - → mutual exclusive hypothesis ? suite
  - → collective exhaustion !

If these cases are true then

$$P(X) = P(X|H_1)P(H_1) + P(X|H_2)P(H_2)$$



→ The M&M problem

M&Ms change over time

	Brown	yellow	red	green	tan	blue	orange
<1994	0.3	0.2	0.2	0.1	0.1	0	0.1
>1995	0.13	0.14	0.13	0.2	0	0.24	0.16

↳ friend gives two bags, one of each kind

↳ take 1 m&m from each bag

we get: yellow, green. what is $P(y|<1995)$?

$$P(y|<1995) = \frac{P(<1995|y)\,P(y)}{P(<1995)}$$

$$= \frac{\frac{0.2}{0.34}\ \frac{0.34}{2}}{\frac{1}{2}} = 0.2?$$

you can also make tables showing the outcomes

hypothesis	prior $P(H)$	likelihood	$P(H)P(D\|H)$	Posterior
A	1/2	(20)(20)	200	20/27
B	1/2	(14)(10)	70	7/27

Σ of these is norm. = 270

→ The monty hall problem

↳ 3 doors → 1 car, 2 goats (🐐)

you pick door A, monty shows you a goat in B

Should you change to C.

hypothesis	prior	likelihood	$P(H)P(X\|H)$	Posterior
A	1/3	1/2	1/6	1/3
B	1/3	0	0	0
C	1/3	1	1/3	2/3

car is in A, B, or C

$P(\text{goat} \mid B)$

$\Sigma = \frac{1}{6} + \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$

Better at switching