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HKUST

MATH1014 Calculus II

Sample Final Exam

Name:____________________

Student ID:____________________

Lecture Section:____________________

Directions:

- This is a closed book examination. **No Calculator is allowed in this examination**.
- DO NOT open the exam until instructed to do so.
- Turn off all phones and pagers, and remove headphones. All electronic devices should be kept in a bag away from your body.
- Write your name, ID number, and Lecture Section in the space provided above, and also in the **Multiple Choice Item Answer Sheet** provided.
- **Mark your ID numbers correctly in the I.D. No. box in the Multiple Choice Item Answer Sheet.**
- DO NOT use any of your own scratch paper. Use only the scratch papers provided by the examination. Write also your name on every scratch paper you use, and do not take any scratch paper away from the examination venue.
- When instructed to open the exam, please check that you have **10** pages of questions in addition to the cover page.
- Answer all questions. Show an appropriate amount of work for each long problem. If you do not show enough work, you will get only partial credit.
- You may write on the backside of the pages, but if you use the backside, clearly indicate that you have done so.
- **Cheating is a serious violation of the HKUST Academic Code. Students caught cheating will receive a zero score for the examination, and will also be subjected to further penalties imposed by the University.**

Please read the following statement and sign your signature.

I have neither given nor received any unauthorized aid during this examination. The answers submitted are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University's regulations governing academic integrity.

Student's Signature :

____

Question No.	Points	Out of
Q. 1-16		48
Q. 17		12
Q. 18		14
Q. 19		12
Q. 20		14
Total Points		100

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Part I: Answer all of the following multiple choice questions.

- Mark you answers to the multiple choice questions in the Multiple Choice Item Answer Sheet.
- **3 points will be deducted for each attempt to enter more than one answer to any MC question**.
- **For backup purpose, put your MC answers in the following boxes**. Grading will be based on the answers marked in the MC answer sheet.

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Answer																

Each of the following MC questions is worth 3 points. No partial credit.

1. What is the colour version of your exam paper? (Read the top left corner of the cover page!) **Make sure that you have also written and marked your ID number correctly in the I.D. No. Box in the MC answer sheet. If you do not do both correctly, you lose the points of this question.**

   (a) Green (b) Orange (c) White (d) Yellow (e) Sample

2. Evaluate the integral $\int_0^1 4\cos(2\pi x)\sin^2(\pi x)dx$.

   (a) -2 (b) -1 (c) 1 (d) 2 (e) 4

3. Evaluate the improper integral $\int_0^2 \frac{4x}{\sqrt{4-x^2}}dx$.

   (a) 2 (b) 4 (c) 6 (d) 8 (e) 10

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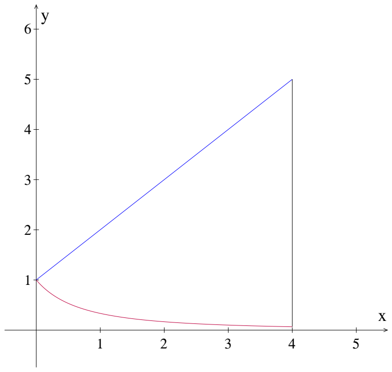
4. Evaluate the improper integral $\int_0^{\infty} \frac{2x}{(x^2+1)^2} dx$.

(a) 1 (b) 2 (c) 3 (d) 4 (e) divergent

5. Find the area enclosed by the curves $y = x + 1$, $y = \frac{2}{(x+1)(x+2)}$ and $x = 4$.



(a) $\ln \frac{3}{5} + 12$ (b) $\ln \frac{2}{5} + 12$ (c) $2\ln \frac{3}{5} + 12$ (d) $2\ln \frac{2}{5} + 12$ (e) $3\ln \frac{3}{5} + 6$

6. The graph of the function $y = 3 + x^2$ over the interval $0 \le x \le \sqrt{12}$ is rotated about the y-axis to generate a surface of revolution. Find the *area* of the surface.

(a) 63π (b) 57π (c) 42π (d) 35π (e) 26π

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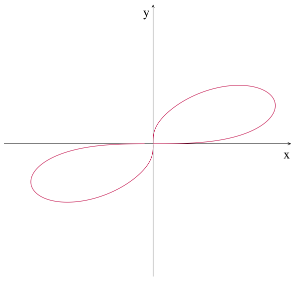
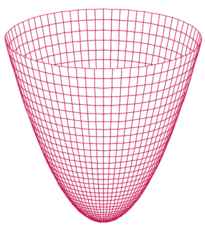
7. Find the area of the region enclosed by the curve defined by the *polar equation* $r = 2\sqrt{\sin(2t)}\, e^{\cos(2t)}$.



(a) $\frac{e^2-e^{-2}}{2}$ (b) $e^2 - e^{-2}$ (c) $\frac{e^2-1}{2}$ (d) $e^2 - 1$ (e) $e^2 - e$

8. The shape of a container is obtained by rotating the curve $y = x^2$ over the interval $0 \le x \le 2$ about the y-axis. If the tank is full of water, the work required for pumping all water to the top of the tank can be expressed as an integral $W = \displaystyle\int_a^b f(y)dy$. **Use trapezoidal rule on four subintervals of equal length** to estimate the work. (Assume that water density is ρ kg/m^3, and gravity acceleration g m/s^2.)



(a) $6\pi\rho g$ (b) $8\pi\rho g$ (c) $10\pi\rho g$ (d) $12\pi\rho g$ (e) $14\pi\rho g$

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9. Which of the following improper integrals is divergent?

(i) $\displaystyle\int_1^\infty xe^{-\sqrt{x}}\,dx$ (ii) $\displaystyle\int_1^\infty \frac{\ln x}{\ln x + x^2}\,dx$ (iii) $\displaystyle\int_1^\infty \frac{\tan^{-1} x}{x^2}\,dx$ (iv) $\displaystyle\int_1^\infty \frac{x}{e^{-x} + x^2}\,dx$

(Recall that another notation for the inverse trigonometric function $\tan^{-1} x$ is $\arctan x$.)

(a) None (b) (i) (c) (ii) (d) (iii) (e) (iv)

10. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3^{n+3} - 2^n}{4^{n+1}}\right)$.

(a) divergent (b) 8 (c) 12 (d) 16 (e) 20

11. Find the sum of the series: $\displaystyle\sum_{n=1}^{\infty} \left(9^{1/n} - 9^{1/(n+2)}\right)$.

(a) 1 (b) 6 (c) 9 (d) 10 (e) divergent

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12. Find **all** convergent infinite series from the following:

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{(2n)!}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln\sqrt{n}}{\ln(n+2)}$ (iii) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} e^{n/(n+1)}$ (iv) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n!)}{n^2}$

(a) Only (i) and (iv) are convergent.

(b) Only (ii) and (iv) are convergent.

(c) Only (i) and (iii) are convergent.

(d) Only (i), (iii), and (iv) are convergent.

(e) All are convergent.

13. Which of the following values of p is the *smallest* to keep the infinite series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^{p+2}}(\ln n)^2}$ *convergent*?

(a) $p = -4$ (b) $p = -2$ (c) $p = -1$ (d) $p = 1$ (e) $p = 2$

14. Find the coefficient of the x^4 term in the Maclaurin series (i.e., Taylor series centered at 0) of the function $f(x) = (1 + 8x^2)^{\frac{3}{2}}$.

(a) 16 (b) 12 (c) 24 (d) 32 (e) 20

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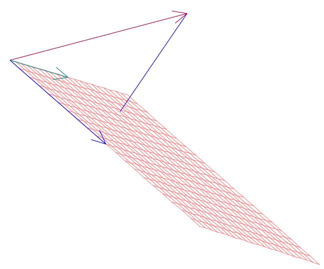
15. Find the vector projection (orthogonal projection) of the vector $\langle -4, 12, -2\rangle$ onto the vector $\langle 1, 3, 2\rangle$.

(a) $\langle \frac{4}{3}, 4, \frac{8}{3}\rangle$ (b) $\langle 4, 12, 8\rangle$ (c) $\langle 1, 3, 2\rangle$ (d) $\langle 3, 9, 6\rangle$ (e) $\langle 2, 6, 4\rangle$

16. Find the perpendicular distance from the point given by the vector $\langle 3, 0, 0\rangle$ to the plane generated by the vectors $\langle 1, 2, 0\rangle$ and $\langle 1, 0, -1\rangle$.



(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

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Part II: Answer each of the following questions.

17. [*12 pts*] Let $I_n = \displaystyle\int \frac{x^n}{\sqrt{x+1}}\,dx$, where n is a positive integer.

(a) Find constants A_n, B_n, which depend on n, such that [7 pts]

$$I_n = A_n x^n \cdot \sqrt{x+1} + B_n I_{n-1}\ .$$

$A_n =$ ____________________ $B_n =$ ____________________

(Both are some expression in n.)

(b) Using part (a) or otherwise, evaluate the definite integral $\displaystyle\int_0^3 \frac{15x^2}{\sqrt{x+1}}\,dx$. [5 pts]

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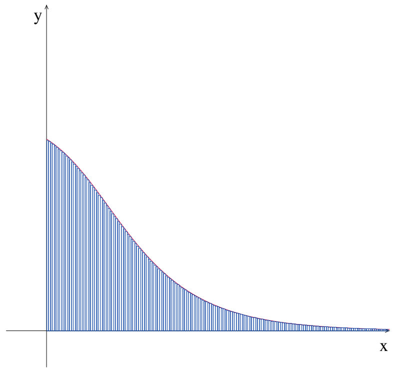
18. [*14 pts*] Consider the area under the graph of the function $y = \dfrac{8}{\sqrt{e^{2x}+4}}$ over the interval $0 < x < \infty$.

(a) Rotate the area about the x-axis to generate a solid of revolution. Find the volume of the solid thus obtained. [8 pts]



(b) If the area is rotated about the y-axis, does the solid of revolution thus obtained have a finite volume? *Show your reasoning for full credit.* [6 pts]

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19. [*12 pts*] Determine whether the given series is convergent or divergent. Given brief reason to justify your answer.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n}$ [3 pts]

(b) $\displaystyle\sum_{n=0}^{\infty} \sqrt{\frac{n+2}{n^2+1}}$ [3 pts]

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^3}$ [3 pts]

(d) $\displaystyle\sum_{n=1}^{\infty} e^{-n} \sin n$ [3 pts]

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20. [*14 pts*] Consider a power series $f(x) = \sum_{n=0}^{\infty} \frac{2n+3}{4^{2n}}(x-1)^{2n+1}$.

(a) Find the largest open interval (open interval of convergence) in which the given power series converges absolutely. Show your work for full credit. [7 pts]

(b) Determine if the given power series is convergent at the endpoints of the open interval of convergence of $f(x)$. Justify your answer for full credit. [3 pts]

(c) Suppose that $H(x)$ is a differentiable function in the open interval of convergence of $f(x)$ such that $\frac{dH}{dx} = (x-1)f(x)$, and $H(1) = -1$. Find the function value $H(2)$. [4 pts]

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Math1014 Final Exam Formula Sheet

Trigonometric Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$

$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\sin A\cos B = \tfrac{1}{2}\Big(\sin(A+B) + \sin(A-B)\Big)$$

$$\cos A\cos B = \tfrac{1}{2}\Big(\cos(A+B) + \cos(A-B)\Big)$$

$$\sin A\sin B = \tfrac{1}{2}\Big(\cos(A-B) - \cos(A+B)\Big)$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x dx$$

$$\int \sin^n x dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x dx$$

$$\int \sec^n x dx = \frac{1}{n-1}\sec^{n-2} x\tan x + \frac{n-2}{n-1}\int \sec^{n-2} x dx$$