

Part 1

Incorrect

Question 1

0 / 1 pts

Evaluate $\tan\left(\frac{\pi}{12}\right)$.

- ☒ $1 - 1/\sqrt{3}$
- ☐ $2 + \sqrt{3}$
- ☐ $1 + \sqrt{3}$
- ☐ $2 - \sqrt{3}$
- ☐ $\sqrt{3} - 1$

Incorrect

Question 2

0 / 1 pts

The population of germs in a container is known to grow exponentially. A single germ is placed in the container initially, and 10 germs were observed one hour later. How many more hours are needed to see 1000 germs in this container?

- ☐ 2
- ☐ $\ln 1000 - \ln 10$
- ☐ $\ln 1000$
- ☐ $\ln 1000 - 1$
- ☒ 3

Incorrect

Question 3

0 / 1 pts

Let f be an odd function. Which of the following is always an even function?

- (I) $|f|$
- (II) $-f$
- (III) f^2

- ☐ (I), (II) and (III) are always even
- ☐ (I) and (II) only
- ☐ (I) and (III) only
- ☐ (II) and (III) only
- ☒ none of them is always even

Incorrect

Question 4

0 / 1 pts

Let f be the function defined by

$$f(x) = \frac{x}{x+1} \quad \text{for all } x \neq -1,$$

and f^{-1} is its inverse function. Compute $f^{-1} \circ f^{-1}(2)$.

- ☐ 2
- ☐ 1
- ☒ -2
- ☐ 2/3
- ☐ -2/3

Part 2

Incorrect

Question 1

0 / 1 pts

Evaluate

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 + 1} - 1}{\sqrt{x^4 + 1} + 1}$$

if it exists.

☐ -4☐ 0☐ 4☒ -1☐ 1

Incorrect

Question 2

0 / 2 pts

Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{\sin^2 x}$$

if it exists.

☐ 1☐ 0☒ does not exist☐ 2☐ 1/2

Question 3

1 / 1 pts

Evaluate the limit

$$\lim_{x \rightarrow +\infty} e^{-x} \sin(e^x)$$

if it exists.

☐ -1☐ 1☐ $+\infty$ ☒ 0☐ $-\infty$

Part 3

Incorrect

Question 1

0 / 1 pts

Evaluate $f'(\frac{\pi}{2})$ if

$$f(x) = \ln\left(\frac{x}{\sin x}\right) \quad \text{for all } 0 < x < \pi.$$

- ☐ $1 + 2/\pi$
- ☐ $2/\pi$
- ☐ f is not differentiable at $\pi/2$
- ☒ $-2/\pi$
- ☐ $1 - 2/\pi$

Incorrect

Question 2

0 / 1 pts

Evaluate $f'(0)$ if

$$f(x) = (\sqrt{x^2 + 1} + 1)^2 \quad \text{for all } x.$$

- ☒ 8
- ☐ 4
- ☐ 0
- ☐ 1
- ☐ 2

Incorrect

Question 3

0 / 1 pts

Evaluate the second derivative $f''(0)$ if

$$f(x) = xe^{-x} \quad \text{for all } x.$$

- ☐ 2
- ☐ -2
- ☐ 1
- ☐ -1
- ☒ 0

Question 1

2 / 2 pts

Let the lengths of the minute arm and the hour arm of a clock be a meters and b meters respectively. Evaluate the rate at which the distance between the tips of these arms are separating at three o'clock (in meter per minute).

- ☐ $-\frac{\pi}{360} \frac{ab}{\sqrt{a^2 + b^2}}$
- ☐ $-\frac{\pi}{60} \frac{ab}{\sqrt{a^2 + b^2}}$
- ☐ $-\frac{\pi}{30} \frac{ab}{\sqrt{a^2 + b^2}}$
- ☒ $-\frac{11\pi}{360} \frac{ab}{\sqrt{a^2 + b^2}}$

Incorrect

Question 2

0 / 2 pts

Let $a, b > 0$, and E is the ellipse defined by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Evaluate the slope of the line tangent to E , which also passes through the point $(2a, 0)$. (Note: the point $(2a, 0)$ does not lie on E .)

- ☐ $-\frac{2b}{a}$
- ☐ $-\frac{\sqrt{3}b}{a}$
- ☐ $-\frac{b}{\sqrt{3}a}$
- ☐ $-\frac{b}{a}$

Part 5

Incorrect

Question 1

0 / 1 pts

Find the number of points at which f attains local maximum if

$$f(x) = \int_0^x e^{y^2} (y^4 - 1) dy \quad \text{for all } x.$$

- ☐ 4
- ☐ 0
- ☐ 1
- ☐ 3
- ☒ 2

Question 2

1 / 1 pts

Let

$$f(x) = x^2(1 - x) \quad \text{for all } x.$$

Find the interval on which f is increasing.

- ☒ $[0, \frac{2}{3}]$
- ☐ f is a decreasing function
- ☐ $(-\infty, \frac{2}{3})$
- ☐ $[\frac{2}{3}, +\infty)$
- ☐ $[0, +\infty)$

Incorrect

Question 3

0 / 2 pts

A cardboard of dimension 1 meter \times 2 meter is given. Four equal squares are cut from the corners of this cardboard, then fold up to create a (half-opened) rectangular box. Compute the width of the squares cut at the corners (in meter) so that the volume of the box created is as large as possible.

- ☐ $(3 + \sqrt{3})/6$
- ☒ $1/2\sqrt{3}$
- ☐ $(3 - \sqrt{3})/6$
- ☐ $1/3\sqrt{3}$
- ☐ $(1 + \sqrt{3})/6$

Part 6

Incorrect

Question 1

0 / 1 pts

Evaluate the integral

$$\int_{-1}^1 |e^x - 1| dx.$$

☐ $e+1/e-2$

☐ $e-1/e+2$

☒ $e-1/e$

☐ $e-1/e-2$

☐ $e+1/e+2$

Incorrect

Question 2

0 / 1 pts

Evaluate the integral

$$\int_1^e \frac{\ln x}{x} dx.$$

☐ 1

☒ $1/e^2$

☐ $\frac{1}{2}$

☐ $1 - 1/e^2$

☐ $\frac{1}{2}(1 - 1/e^2)$

Incorrect

Question 3

0 / 1 pts

Evaluate the integral

$$\int_0^{\pi/2} \sin(2x) \cos^2(2x) dx.$$

☐ π

☐ $2\pi/3$

☐ $1/3$

☒ $\pi/3$

☐ $2/3$