

MOMENTUM, IMPULSE, AND COLLISIONS I

PHYS1112

Lecture 6

Intended Learning Outcomes

- After this lecture you will learn:
 - 1) impulse as an indication of the effect of a force which is in effect for a short time.
 - 2) the relation between impulse and momentum change – the impulse momentum theorem.
 - 3) conservation of momentum.
 - 4) elastic, inelastic, and completely elastic collisions.

Define **momentum**

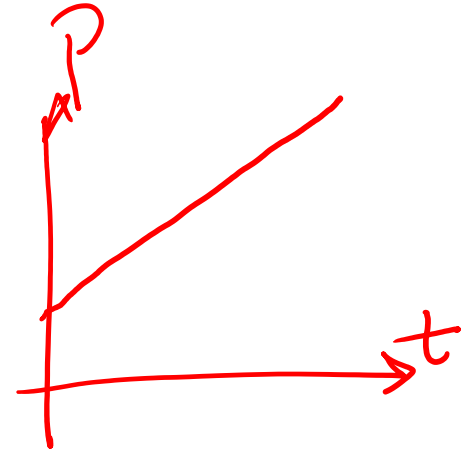
$$\vec{p} = m\vec{v},$$

SI unit: kg·m/s

Newton's second law in terms of momentum:

$$\sum \vec{F} = m \frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt} \approx C$$

Suppose net force $\sum \vec{F}$ is constant \rightarrow ???



Define **impulse**

$$\vec{J} = \sum \vec{F} (t_2 - t_1) = \sum \vec{F} \Delta t,$$

SI unit: N·s



Most useful if the force is in effect for a short time, i.e., when Δt is small

From Newton's second law

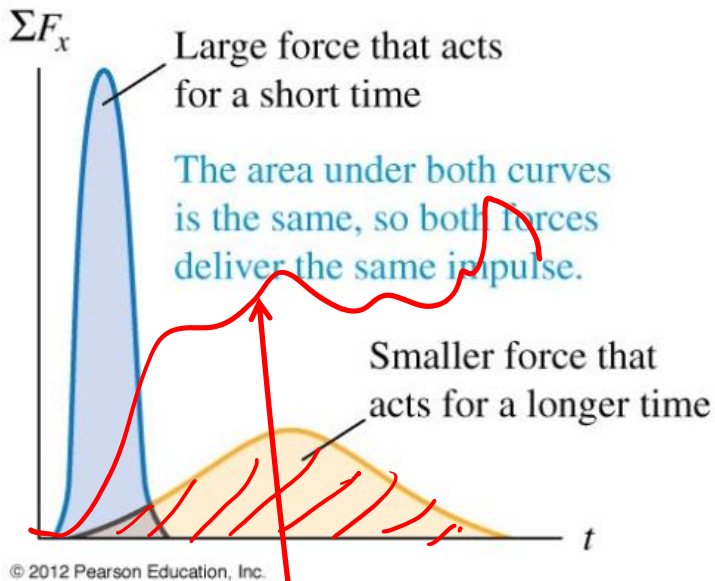
$$\Sigma \vec{F} (t_2 - t_1) = \vec{p}_2 - \vec{p}_1$$

i.e., $\vec{J} = \vec{p}_2 - \vec{p}_1$

Impulse-momentum theorem:

The change in momentum of a particle during a time interval equals the impulse of the net force acting on the particle during that interval

But in general, $\sum \vec{F}$ is not constant!



Define impulse as

$$\begin{aligned} & \sum \Sigma \vec{F}_i \cdot \Delta t_i \\ & \downarrow \quad \downarrow \\ & \text{area under graph} \quad dt \\ & \underbrace{\qquad\qquad\qquad} \\ & \vec{J} = \int_{t_1}^{t_2} \Sigma \vec{F} dt \\ & = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt} dt = \vec{p}_2 - \vec{p}_1 \end{aligned}$$

Handwritten red annotations: (t_c) and (t_i) are written above \vec{p}_2 and \vec{p}_1 respectively. Red lines are drawn under \vec{p}_2 and \vec{p}_1 . Red lines are also drawn under $\frac{d\vec{p}}{dt}$ and dt in the second line.

Impulse-momentum theorem again !!

⚠ Different forces can give the same impulse

Q8.2

You are testing a new car using crash test dummies. Consider two ways to slow the car from 90 km/h (56 mi/h) to a complete stop:

- (i) You let the car slam into a wall, bringing it to a sudden stop. $\vec{v} = 0$
- (ii) You let the car plow into a giant tub of gelatin so that it comes to a gradual halt. $\Delta t_{ii} > \Delta t_i$

In which case is there a greater impulse of the net force on the car?

A. In case (i).

B. In case (ii).

☒ C. The impulse is the same in both cases.

D. The answer depends on how rigid the front of the car is.

E. The answer depends on how rigid the front of the car is and on the mass of the car.

$$\vec{p}_2 - \vec{p}_1 = 0$$

A8.2


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Define average net force \vec{F}_{av} as the constant force that gives the same impulse

✓ dimension analysis

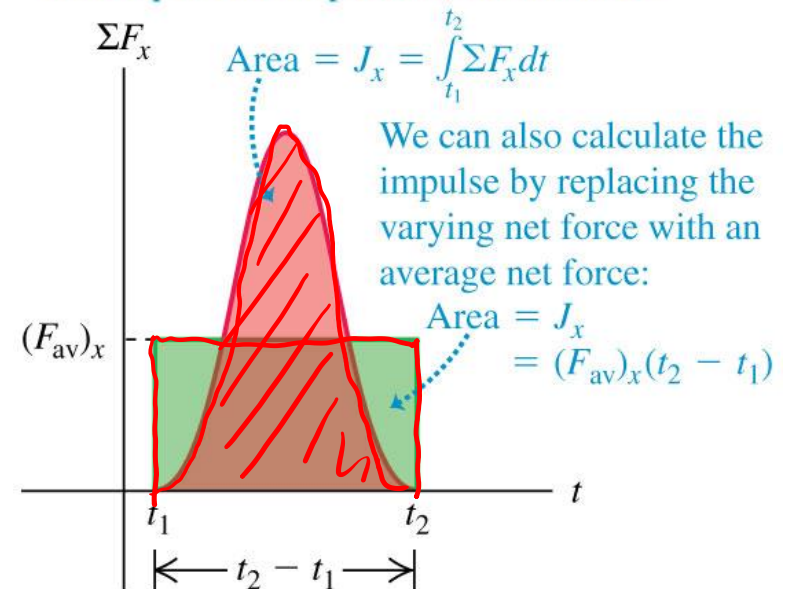
$$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F} dt = \vec{F}_{av}(t_2 - t_1)$$

$$\Rightarrow \vec{F}_{av} = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} \sum \vec{F} dt$$

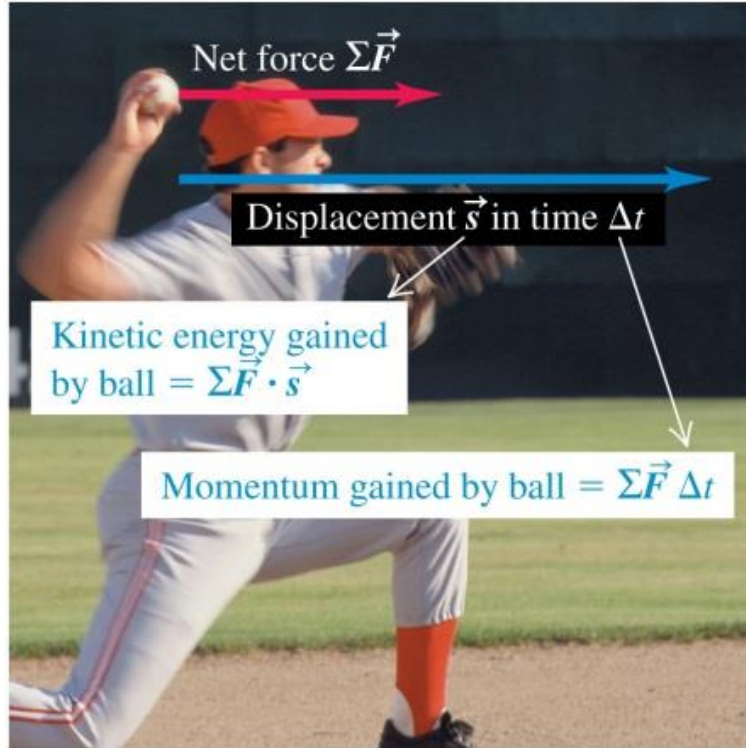
mean value theorem in definite integral ✓

Geometric interpretation: \vec{F}_{av} is a constant force that has the same area under it as the variable force

The area under the curve of net force versus time equals the impulse of the net force:



Example: Catching a ball



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Case 1: 0.50 kg ball moving at 4.0 m/s,
 $p = 2.0 \text{ N}\cdot\text{s}$, $K = 4.0 \text{ J}$ $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Case 2: 0.10 kg ball moving at 20 m/s,
 $p = 2.0 \text{ N}\cdot\text{s}$, $K = 20 \text{ J}$

Which one is easier to catch?

$$\vec{F} \cdot \Delta t = \Delta \vec{p}$$

Suppose your hand exerts the same force in both cases: F

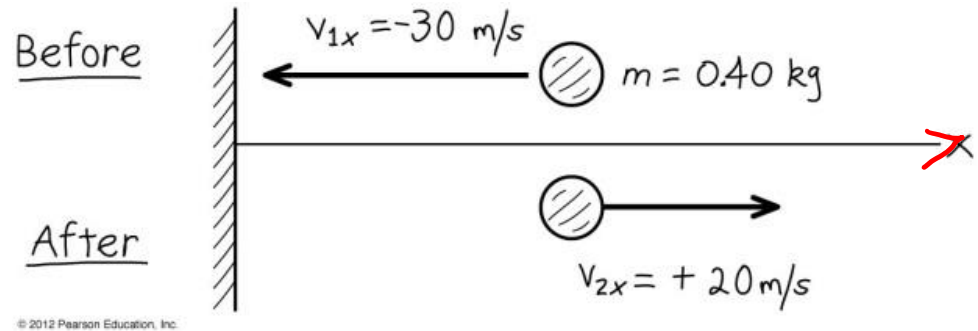
Both stop within the same time interval (\because same impulse)

But case 2 stops at 5 times the distance ($\because K$ is 5 times larger)

$$W = \Delta K$$
$$\vec{F} \cdot \vec{l} = \Delta K$$

Example

A ball hits a wall and bounced back. Assume the ball is in contact with the wall for 0.010 s



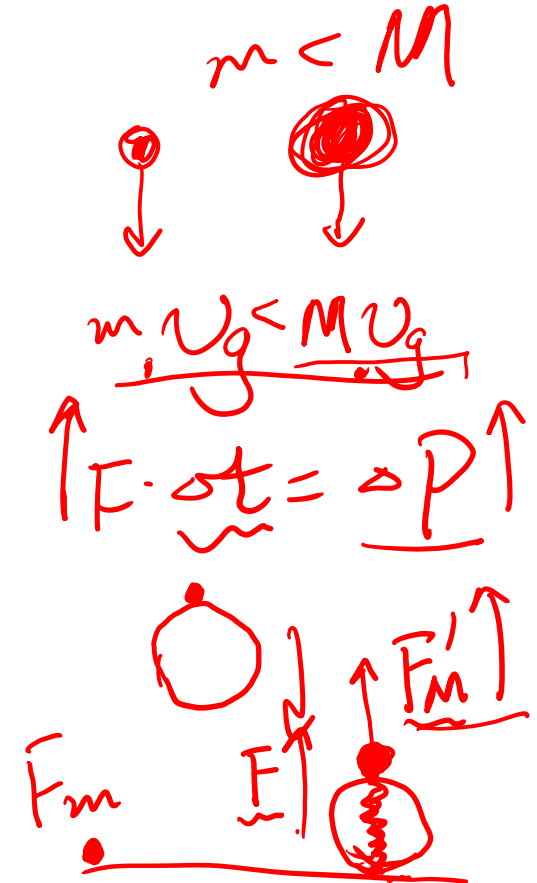
$$\begin{aligned} \text{Impulse } J &= m(\overrightarrow{v_{2x}} - \overrightarrow{v_{1x}}) \\ &= (0.40 \text{ kg})(20 - (-30) \text{ m/s}) = 20 \text{ N}\cdot\text{s} \end{aligned}$$

+ -

J is a vector, be careful about the direction

$$\text{average force } F_{av} = \frac{J}{\Delta t} = \frac{20 \text{ N}\cdot\text{s}}{0.010 \text{ s}} = 2000 \text{ N}$$

Demonstration: velocity amplification – impulse can be transmitted from one object to another: impulse due to the normal reaction of the ground on the larger ball is transmitted to the smaller one.



Question



- Arrange the following cases in decreasing order of the magnitude of the average net force. In each case a 1000 kg automobile is along a straight east-west road.
 - a) { It is initially moving east at 25 m/s and comes to a stop in 10 s. $F = \frac{m \Delta v}{\Delta t}$ 2500 N
 - b) { It is initially moving east at 25 m/s and comes to a stop in 5 s. 5000 N
 - c) { It is initially at rest, and a 2000 N net force toward the east is applied to it for 10 s.
 - d) { It is initially moving east at 25 m/s, and a 2000 N net force toward the west is applied to it for 10 s.
 - e) It is initially moving east at 25 m/s. Over a 30 s period, it reverses direction and ends up moving west at 25 m/s. $\leftarrow -25 \rightarrow$

$$1000 \times \frac{50}{30} \text{ N} = 1666.7 \text{ N}$$

Some terminologies:

A **system** means a collection of bodies, e.g. the 2 astronauts form a system.

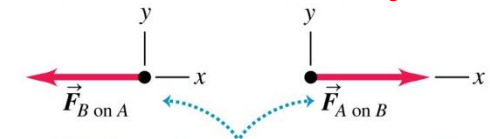
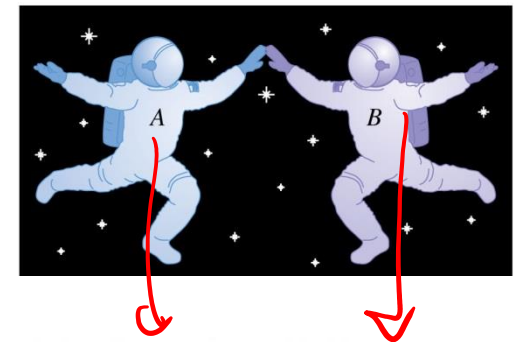


Internal forces are forces which individual bodies in the same system exert on others, e.g., the push between the astronauts.

⚠ **Internal forces** always exist as action and reaction pairs.

External forces are forces exerted on one or more bodies of the system by another object outside it, e.g., gravitational (if any) pull on the astronauts.

A system with no external forces is called an isolated system.

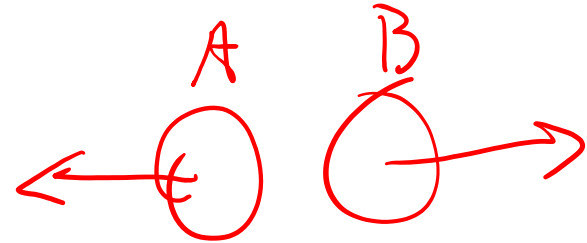


The forces the astronauts exert on each other form an action–reaction pair.

Consider a 2 body system,

Net force on A, $\vec{F}_A = \frac{d\vec{p}_A}{dt}$, ✓

net force on B, $\vec{F}_B = \frac{d\vec{p}_B}{dt}$ ✓



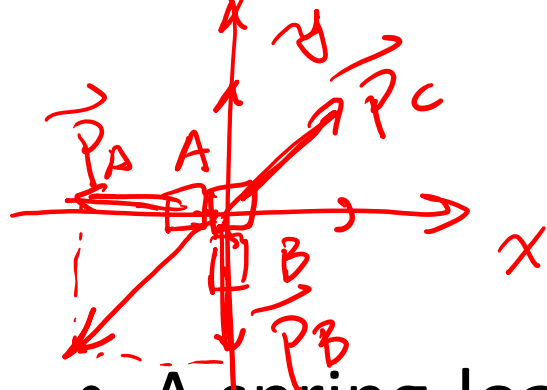
If it is an isolated system, \vec{F}_A and \vec{F}_B are action and reaction pair

$$\vec{F}_A = -\vec{F}_B \Rightarrow \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = 0$$

Define **total momentum** of the system $\vec{P} = \vec{p}_A + \vec{p}_B \Rightarrow$

$$\frac{d\vec{P}}{dt} = 0,$$

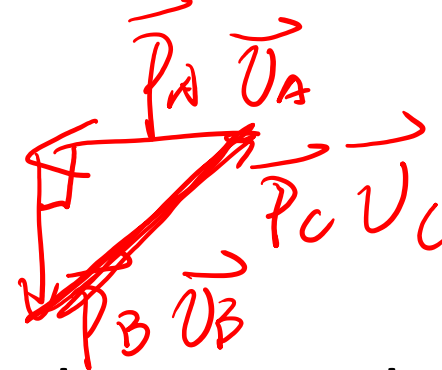
\vec{P} is constant or conserved



Question

$$\sum m_i \vec{v}_i = 0$$

$$\sum \vec{p}_i = 0$$



- A spring-loaded toy sits at rest on a horizontal, frictionless surface. When the spring releases, the toy breaks into three equal mass pieces, A, B, and C, which slide along the surface. A moves off in the negative x direction, while B moves off in the negative y direction.

- What are the signs of the velocity components of C along the x and y directions? $+$ $+$
- Which of the three pieces is moving the fastest?

C



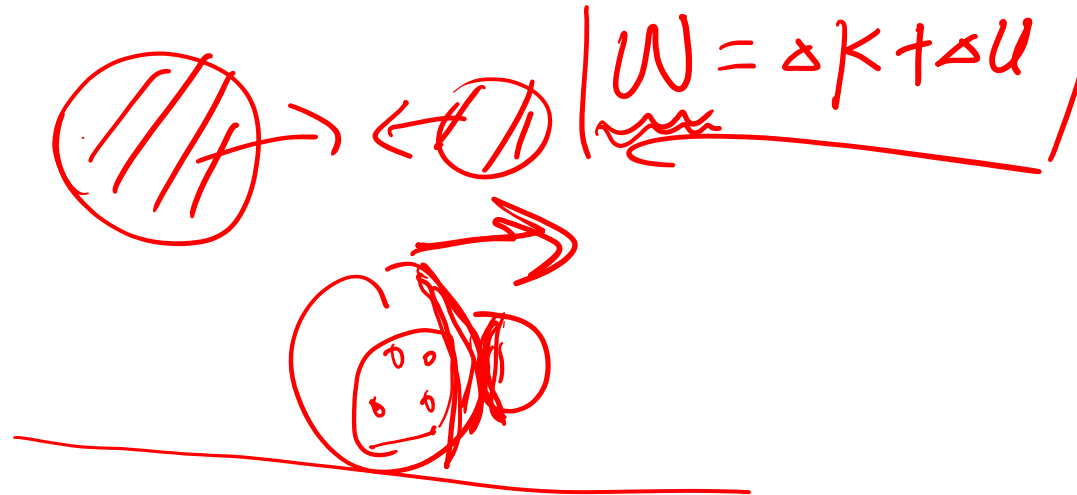
Under no net external force, momentum always conserved, but not mechanical energy.



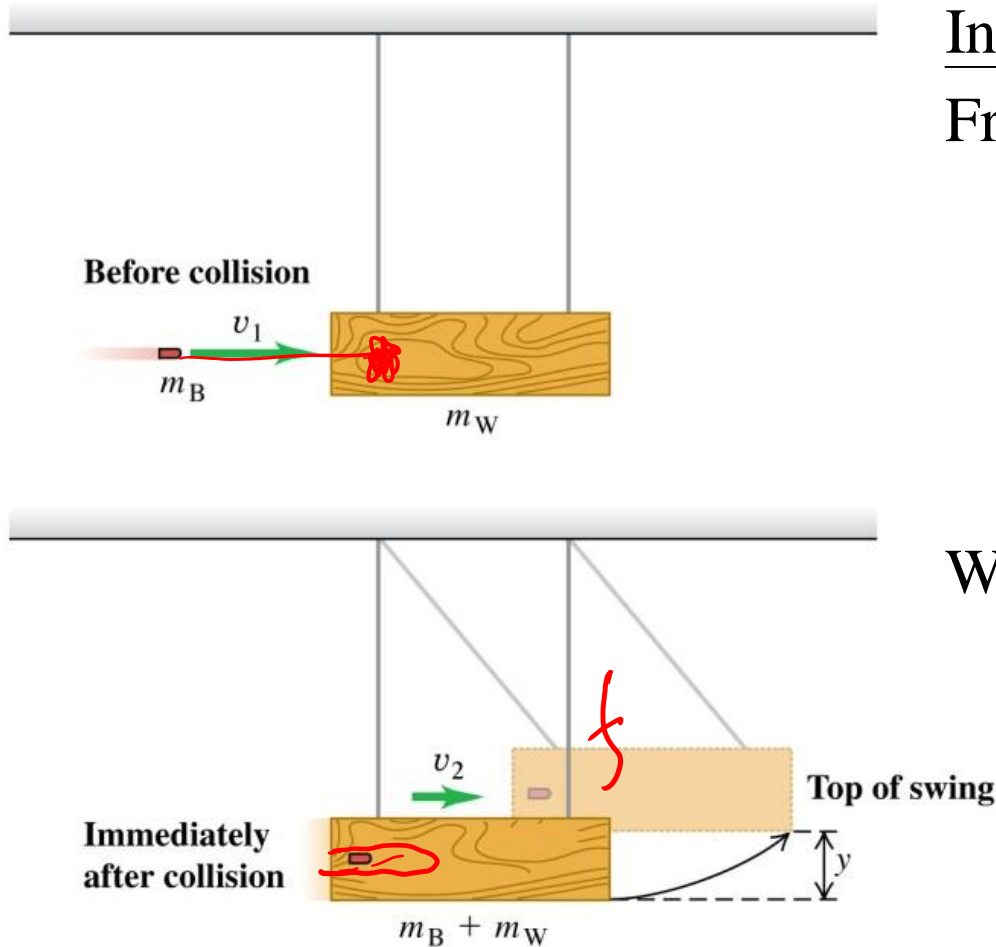
In an elastic collision, the KE is the same before and after the collision. (No change in PE during the impact.) $\Delta K + \Delta U = 0$

In an inelastic collision, the KE before the collision is larger.

In a completely inelastic collision, the bodies stick together after collision.



Example The ballistic pendulum – one way to measure the speed of a bullet

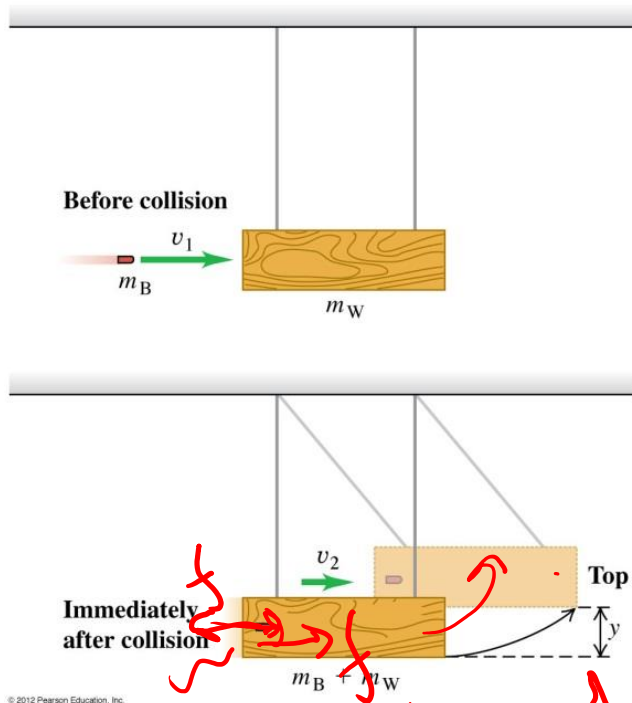


Incorrect solution:

From conservation of energy

$$\frac{1}{2}m_B v_1^2 = (m_B + m_W)gy$$
$$\Rightarrow v_1 = \sqrt{\frac{2(m_B + m_W)gy}{m_B}}$$

What is wrong?



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Correct solution:

Conservation of momentum:

$$m_B v_1 = (m_B + m_W) v_2 \Rightarrow v_2 = \frac{m_B v_1}{m_B + m_W}$$

mechanics

Conservation of energy after collision:

$$\frac{1}{2} (m_B + m_W) v_2^2 = (m_B + m_W) g y \quad \checkmark$$

$$\Rightarrow \frac{1}{2} \left(\frac{m_B v_1}{m_B + m_W} \right)^2 = g y$$

$$\Rightarrow v_1 = \frac{m_B + m_W}{m_B} \sqrt{2 g y}$$

Put in realistic numbers,

$m_B = 5.00 \text{ g}$, $m_W = 2.00 \text{ kg}$, $y = 3.00 \text{ cm}$, then

$v_1 = 307 \text{ m/s}$

KE before impact is

$$\frac{1}{2} (0.00500 \text{ kg}) (307 \text{ m/s})^2 = \underline{236 \text{ J}}$$

KE after impact is

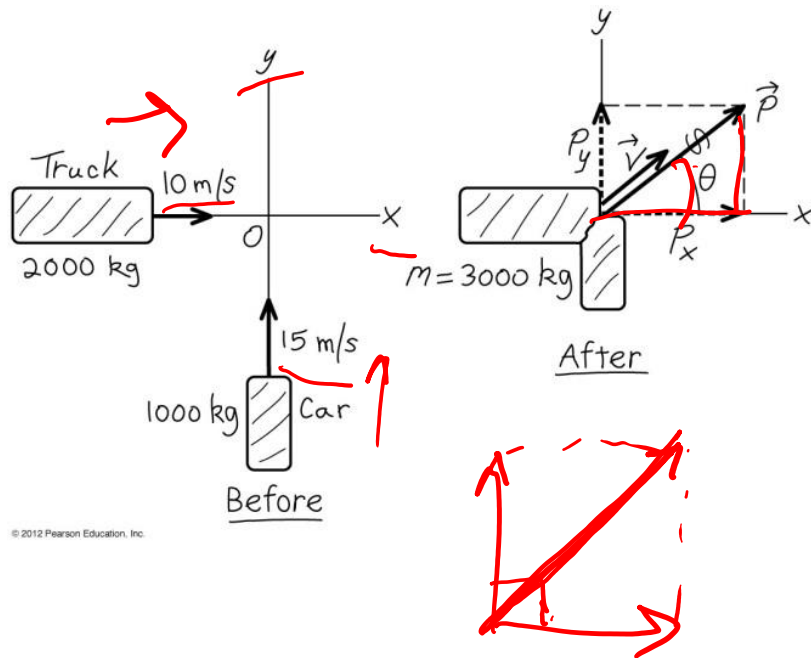
$$(0.00500 + 2.00 \text{ kg}) (9.80 \text{ m/s}^2) (0.0300 \text{ m}) = \underline{0.590 \text{ J}}$$

Most of the original KE is lost! What happens to this amount of energy?

heat

Example An automobile collision

A 1000-kg car traveling north collides with a 2000-kg truck traveling east. Just before the collision, the speed of the car is 15 m/s and that of the truck is 10 m/s. The two vehicles move away from the impact point as one. Find the velocity just after the collision.



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By conservation of momentum:

$$(m_C + m_T)V_x = m_T v_{Tx} + m_C v_{Cx}$$

$$\Rightarrow V_x = \frac{m_T v_{Tx}}{(m_C + m_T)} = 6.7 \text{ m/s}$$

$$(m_C + m_T)V_y = m_T v_{Ty} + m_C v_{Cy}$$

$$\Rightarrow V_y = \frac{m_C v_{Cy}}{(m_C + m_T)} = 5.0 \text{ m/s}$$

$$\therefore V = \sqrt{V_x^2 + V_y^2} = 8.3 \text{ m/s}$$

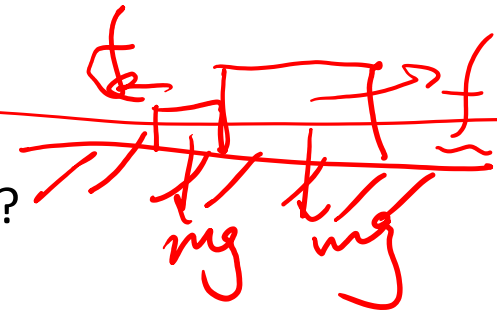
$$\tan \theta = \frac{V_y}{V_x} = 0.75 \Rightarrow \theta = 37^\circ$$



Are there external forces acting on the vehicles?

Yes!

Then how to justify using conservation of momentum?



Weight and normal reaction: cancel each other, does not contribute to the net external force.

Friction: contribute to the net external force, but can we neglect it?

The friction f between the vehicles and the road has finite magnitude.

Suppose the collision is ideal and takes time $\Delta t \rightarrow 0$, then the impulse is $f\Delta t \rightarrow 0$. Hence friction can be neglected.

In general, any external forces with bounded magnitude can be neglected in ideal collisions.

Of course no collision is ideal in the real world. From the given speeds, it is reasonable to assume that the collision takes a time $\Delta t \sim 0.1\text{ s}$. Suppose $\mu_k = 0.5$. Then the frictions are of the order $\mu_k mg \sim (0.5)(2000\text{ kg})(10\text{ m/s}^2) = 10^4\text{ N}$. The impulses are of the order $\sim 10^4\text{ N} \times 0.1\text{ s} = 10^3\text{ N} \cdot \text{s}$. The initial momenta of the vehicles are of the order of $2 \times 10^4\text{ N} \cdot \text{s}$. Therefore momentum is conserved approximately and we can simplify the question by neglecting friction.

Question

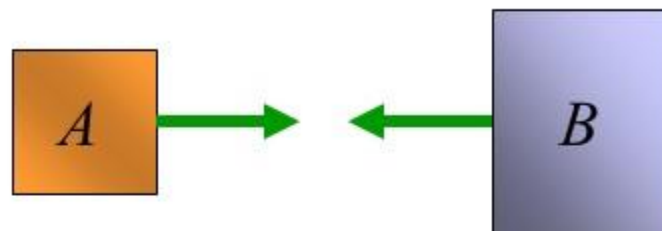
- For each situation, state whether the collision is elastic, inelastic, or completely inelastic.
 - a) You drop a ball from your hand. It collides with the floor and bounces back up so that it just reaches your hand.
 - b) You drop a different ball from your hand and let it collide with the ground. This ball bounces back up to half the height from which it was dropped.
 - c) You drop a ball of clay from your hand. When it collides with the ground, it stops.

elastic



Q8.5

Two objects with different masses collide with and *stick* to each other. Compared to *before* the collision, the system of two objects *after* the collision has



- A. the same amount of total momentum and the same total kinetic energy.
- B. the same amount of total momentum but less total kinetic energy.
- C. less total momentum but the same amount of total kinetic energy.
- D. less total momentum and less total kinetic energy.
- E. Not enough information is given to decide.

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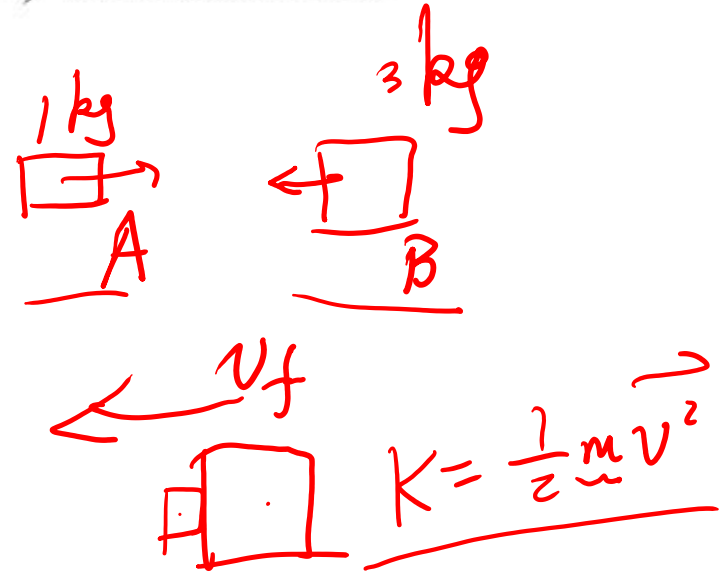
D. less total momentum and less total kinetic energy.

E. Not enough information is given to decide.

Q8.7


Block A has mass 1.00 kg and block B has mass 3.00 kg . The blocks collide and stick together on a level, frictionless surface. After the collision, the kinetic energy (KE) of block A is

- A. one-ninth the KE of block B .
- B. one-third the KE of block B .
- C. three times the KE of block B .
- D. nine times the KE of block B .
- E. the same as the KE of block B .



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- D. nine times the KE of block B .
- E. the same as the KE of block B .

Q8.9

Block A on the left has mass 1.00 kg . Block B on the right has mass 3.00 kg . The blocks are forced together, compressing the spring. Then the system is released from rest on a level, frictionless surface. After the blocks are released, how does K_A (the kinetic energy of block A) compare to K_B (the kinetic energy of block B)?

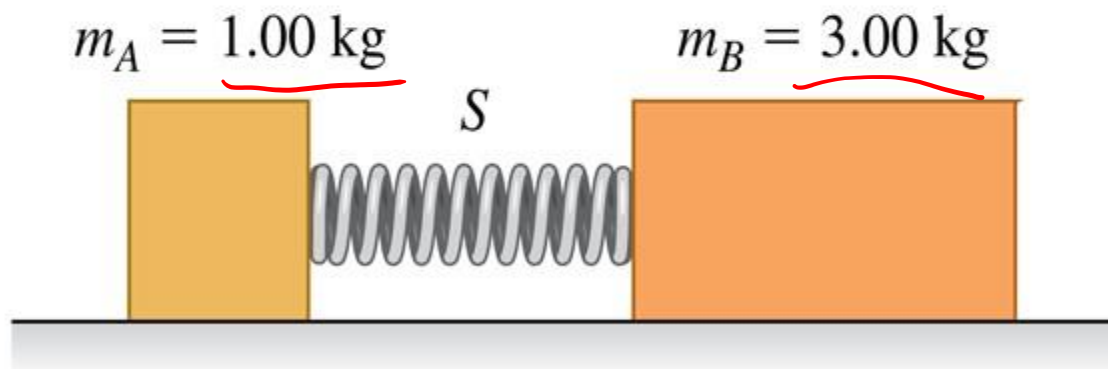
A. $K_A = K_B/9$

B. $K_A = K_B/3$

C. $K_A = K_B$

D. $K_A = 3K_B$

E. $K_A = 9K_B$



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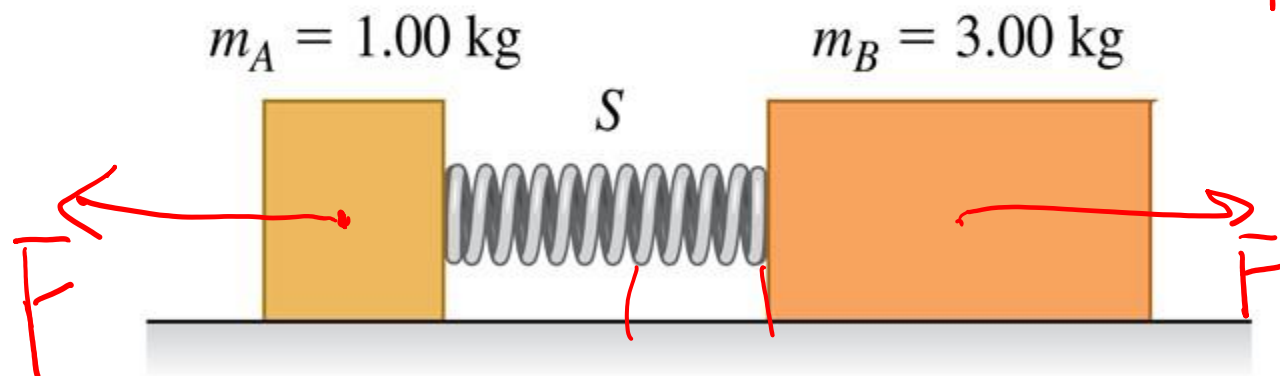
C. $K_A = K_B$

✓ D. $K_A = 3K_B$

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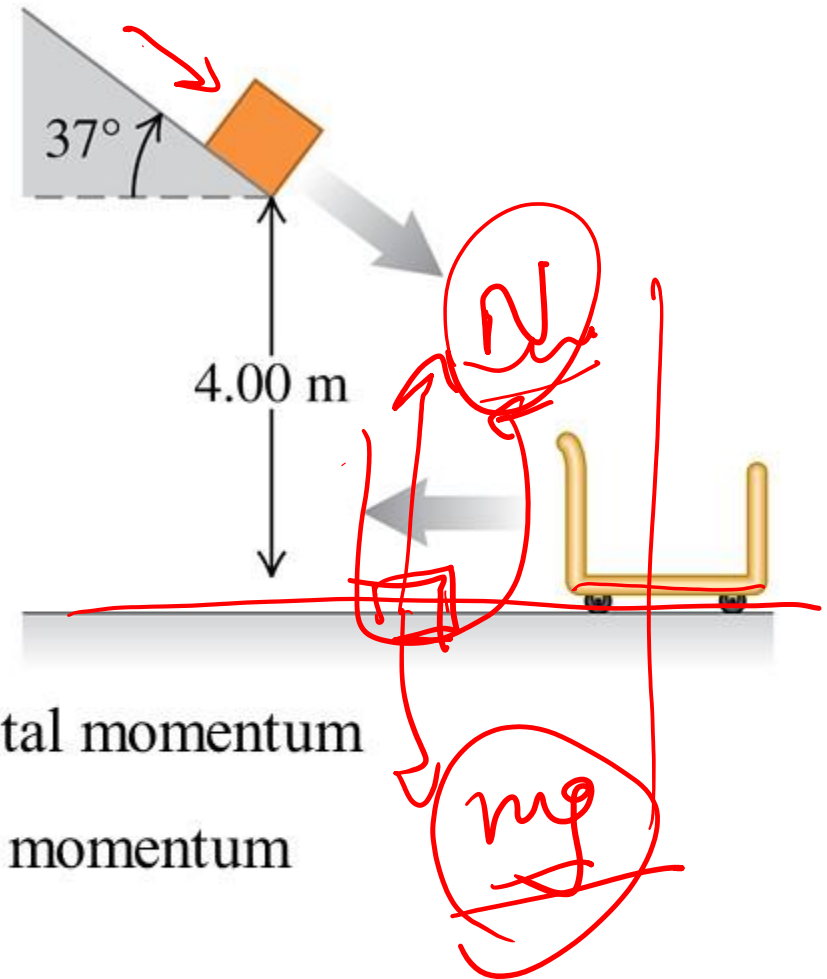
$$F \cdot \Delta t = m \Delta v \quad \Delta v = \frac{F \Delta t}{m}$$

$$K = \frac{1}{2} m \left(\frac{F \Delta t}{m} \right)^2 = \frac{(F \Delta t)^2}{2m}$$



Q8.10

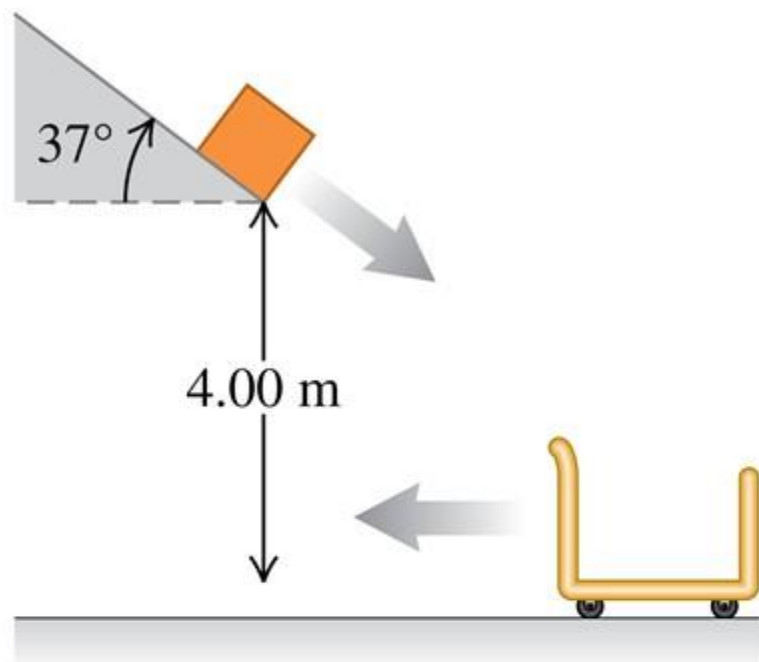
An open cart is rolling to the left on a horizontal surface. A package slides down a chute and lands in the cart. Which quantity or quantities have the same value just before and just after the package lands in the cart?



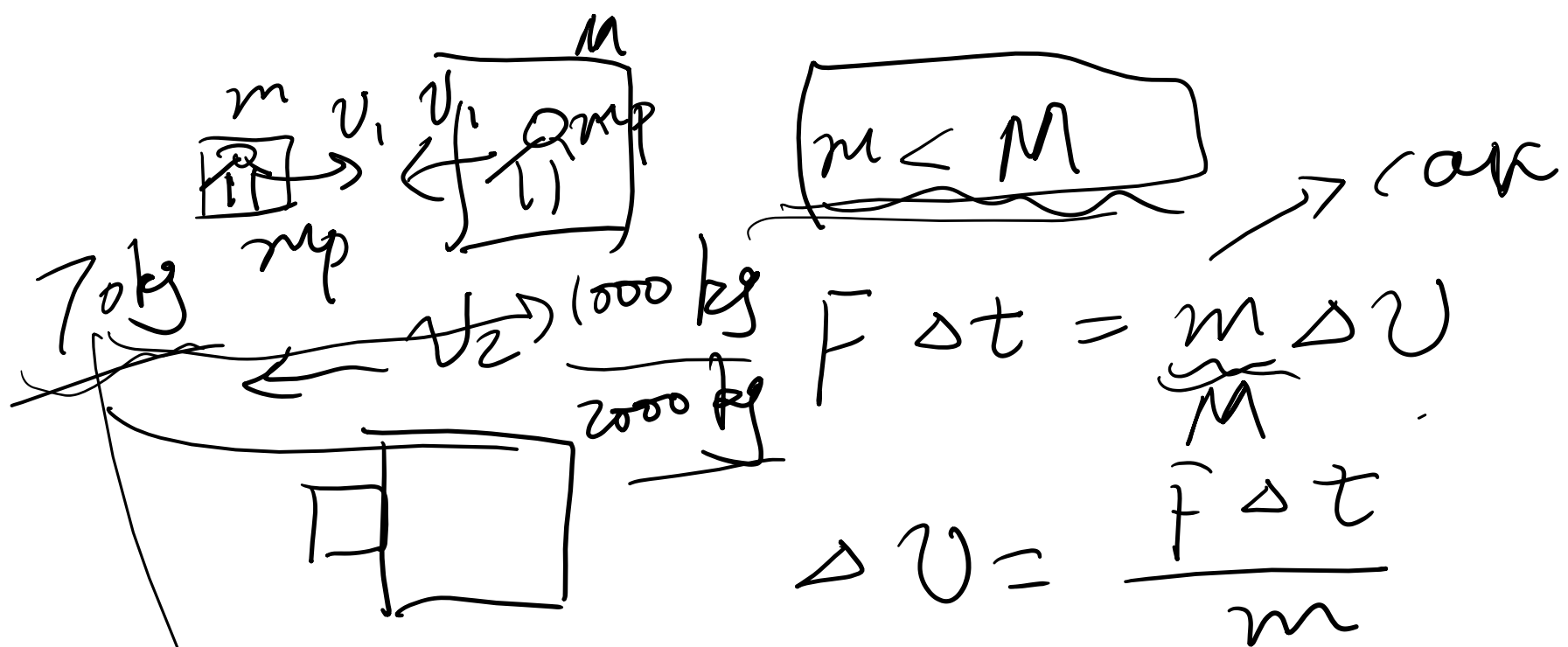
- ☒ A. the horizontal component of total momentum
- ☐ B. the vertical component of total momentum
- ☐ C. the total kinetic energy
- ☐ D. two of A, B, and C
- ☐ E. all of A, B, and C

A8.10

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- ✓ A. the horizontal component of total momentum
- B. the vertical component of total momentum
- C. the total kinetic energy
- D. two of A, B, and C
- E. all of A, B, and C



$$F_p \Delta t = m_p \Delta v$$

The diagram shows a car with a person inside. The car is labeled with F_p and $F \Delta t$. The person is labeled with m . The car is shown with a leftward arrow labeled v_1 and a rightward arrow labeled v_2 . The person is shown with a rightward arrow labeled v_1 and a leftward arrow labeled v_2 . The car is labeled with 70 kg and 1000 kg . The person is labeled with 70 kg .