**MATH 2111 Matrix Algebra and Applications**
**Homework-5 : Due 10/24/2022 at 11:59pm HKT**

---

**1.** (1 point) Find the inverse of $AB$ if
$A^{-1} = \begin{bmatrix} -3 & 4 \\ -4 & -3 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -5 & -5 \\ -2 & 0 \end{bmatrix}$.

$(AB)^{-1} = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$

*Correct Answers:*

- 35
- -5
- 6
- -8

---

**2.** (2 points) Solve for $X$.

$$\begin{bmatrix} -8 & -2 \\ -1 & 3 \end{bmatrix} X + \begin{bmatrix} -5 & 4 \\ -5 & -6 \end{bmatrix} = \begin{bmatrix} 9 & 9 \\ 8 & 1 \end{bmatrix} X.$$

$X = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$

**Solution:** Write $A = \begin{bmatrix} -8 & -2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -5 & 4 \\ -5 & -6 \end{bmatrix}$, and $C = \begin{bmatrix} 9 & 9 \\ 8 & 1 \end{bmatrix}$. Provided that $C - A$ is invertible,

$$\begin{aligned} AX + B &= CX \\ B &= CX - AX \\ B &= (C-A)X \\ (C-A)^{-1}B &= (C-A)^{-1}(C-A)X \\ (C-A)^{-1}B &= IX \\ (C-A)^{-1}B &= X \end{aligned}$$

and therefore

$$\begin{aligned} X &= \left( \begin{bmatrix} 9 & 9 \\ 8 & 1 \end{bmatrix} - \begin{bmatrix} -8 & -2 \\ -1 & 3 \end{bmatrix} \right)^{-1} \begin{bmatrix} -5 & 4 \\ -5 & -6 \end{bmatrix} \\ &= \begin{bmatrix} 17 & 11 \\ 9 & -2 \end{bmatrix}^{-1} \begin{bmatrix} -5 & 4 \\ -5 & -6 \end{bmatrix} \\ &= \frac{1}{-133} \begin{bmatrix} -2 & -11 \\ -9 & 17 \end{bmatrix} \begin{bmatrix} -5 & 4 \\ -5 & -6 \end{bmatrix} \\ &= \frac{1}{-133} \begin{bmatrix} 65 & 58 \\ -40 & -138 \end{bmatrix} \\ &\approx \begin{bmatrix} -0.488722 & -0.43609 \\ 0.300752 & 1.03759 \end{bmatrix}. \end{aligned}$$

*Correct Answers:*

- 1/-133*[[65,58],[-40,-138]]

---

**3.** (2 points) If

$$A = \begin{bmatrix} -4 & -12 & -31 \\ -5 & -14 & -37 \\ 1 & 3 & 8 \end{bmatrix},$$

then
$A^{-1} = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$.

Given $\vec{b} = \begin{bmatrix} -3 \\ 4 \\ 4 \end{bmatrix}$, solve $A\vec{x} = \vec{b}$.

$\vec{x} = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$.

*Correct Answers:*

- $\begin{bmatrix} 1 & -3 & -10 \\ -3 & 1 & -7 \\ 1 & 0 & 4 \end{bmatrix}$
- $\begin{bmatrix} -55 \\ -15 \\ 13 \end{bmatrix}$

---

**4.** (2 points) If

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ -7 & 2 & 1 & 0 \\ -1 & -6 & -1 & 1 \end{bmatrix},$$

then

$$A^{-1} = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}.$$

*Correct Answers:*

- $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 15 & -2 & 1 & 0 \\ -8 & 4 & 1 & 1 \end{bmatrix}$

**5.** (1 point) The $2 \times 2$ elementary matrix $E$ can be gotten from the identity matrix using the row operation $R_1 = r_1 + 2r_2$. Find $EA$ if
$A = \begin{bmatrix} 1 & 3 \\ -2 & -5 \end{bmatrix}$.

$EA = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

*Correct Answers:*

- -3
- -7
- -2
- -5

**6.** (4 points) a. Suppose that $E_1 \begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -12 & 6 \\ -1 & 1 \end{bmatrix}$. Find $E_1$ and $E_1^{-1}$.

$E_1 = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$, $E_1^{-1} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$.

b. Suppose that $E_2 \begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$. Find $E_2$ and $E_2^{-1}$.

$E_2 = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$, $E_2^{-1} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$.

c. Suppose that $E_3 \begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -8 & 7 \\ -1 & 1 \end{bmatrix}$. Find $E_3$ and $E_3^{-1}$.

$E_3 = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$, $E_3^{-1} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$.

d. Suppose that $E_4 \begin{bmatrix} 3 & -4 & 4 \\ 1 & -3 & -3 \\ -1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 4 \\ 3 & -9 & -9 \\ -1 & -1 & 2 \end{bmatrix}$.

Find $E_4$ and $E_4^{-1}$.

$E_4 = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$,

$E_4^{-1} = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$.

e. Suppose that $E_5 \begin{bmatrix} 3 & -4 & 4 \\ 1 & -3 & -3 \\ -1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 4 \\ -1 & -1 & 2 \\ 1 & -3 & -3 \end{bmatrix}$.

Find $E_5$ and $E_5^{-1}$.

$E_5 = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$,

$E_5^{-1} = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$.

f. Suppose that $E_6 \begin{bmatrix} 3 & -4 & 4 \\ 1 & -3 & -3 \\ -1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 4 \\ 1 & -3 & -3 \\ 20 & -29 & 30 \end{bmatrix}$.

Find $E_6$ and $E_6^{-1}$.

$E_6 = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$,

$E_6^{-1} = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$.

*Correct Answers:*

- $\begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 0.166667 & 0 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- $\begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & -6 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.333333 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

- $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$
- $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{bmatrix}$$
- $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & 0 & 1 \end{bmatrix}$$

---

**7.** (1 point) Find the determinant of the matrix

$$B = \begin{bmatrix} -3 & 1 & -2 \\ 4 & -3 & -2 \\ -5 & 0 & 4 \end{bmatrix}.$$

$\det(B) =$ _____.

*Correct Answers:*

- `60`

---

**8.** (1 point) If $A = \begin{bmatrix} -4 & -8 & -6 & -7 & -7 \\ 0 & -3 & 5 & 6 & -5 \\ 0 & 0 & -6 & -5 & -6 \\ 0 & 0 & 0 & 9 & -2 \\ 0 & 0 & 0 & 0 & -4 \end{bmatrix}$

then $\det(A) =$ _____

*Correct Answers:*

- `2592`

---

**9.** (2 points) Find the determinant of the matrix

$$M = \begin{bmatrix} -1 & 0 & 0 & -1 & 0 \\ 2 & 0 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 3 & -3 \\ 0 & -1 & -3 & 0 & 0 \end{bmatrix}.$$

$\det(M) =$ _______.

*Correct Answers:*

- `-1*-3*-3*3*-1+-1*2*-3*1*-3`

---

**10.** (2 points) Find $k$ such that the following matrix $M$ is singular.

$$M = \begin{bmatrix} 3 & 4 & -1 \\ -9 & -13 & 5 \\ -16+k & -34 & 12 \end{bmatrix}$$

$k =$ _____

*Correct Answers:*

- `-8`