MATH 2111 Matrix Algebra and Applications
Homework-2 : Due 09/30/2022 at 11:59pm HKT

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1. (2 points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 3 & 6 & 9 & -3 \end{bmatrix}.$$

Describe all solutions of $A\vec{x} = \vec{0}$.

$$\vec{x} = x_2 \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} + x_3 \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} + x_4 \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

Correct Answers:

- `[[-2],[1],[0],[0]],` $\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

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2. (1 point) Express the vector $\vec{v} = \begin{bmatrix} 23 \\ 11 \end{bmatrix}$ as a linear combination of $\vec{x} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$.

$\vec{v} = \_\_\_ \vec{x} + \_\_\_ \vec{y}$.

Correct Answers:

- `6`
- `1`

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3. (2 points)
Find the value of a for which

$$v = \begin{bmatrix} -3 \\ a \\ -3 \\ -4 \end{bmatrix}$$

is in the set

$$H = span\left\{ \begin{bmatrix} -3 \\ -3 \\ -5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4 \\ -5 \end{bmatrix} \right\}.$$

$a =$ _____

Correct Answers:

- `7`

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4. (3 points) Which of the following statements are true? (May contain one or more true statements, or none.)

- A. If the augmented matrix $[\ A\ b\ ]$ has a pivot position in every row, then the equation $Ax = b$ is inconsistent.
- B. If the equation $Ax = b$ is inconsistent, then b is not in the set spanned by the columns of A.
- C. Every matrix equation $Ax = b$ corresponds to a vector equation with the same solution set.
- D. Any linear combination of vectors can always be written in the form Ax for a suitable matrix A and vector x.
- E. The equation $Ax = b$ is consistent if the augmented matrix $[\ A\ b\ ]$ has a pivot position in every row.
- F. A vector b is a linear combination of the columns of a matrix A if and only if the equation $Ax = b$ has at least one solution.

Correct Answers:

- `BCDF`

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5. (2 points) How many basic variables does each augmented matrix have?

1. $\left[\begin{array}{cccc|c} 1 & 0 & 0 & -9 & 3 \\ 0 & 1 & 2 & 0 & -6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$
   - A. None
   - B. One
   - C. Two
   - D. Three
2. $\left[\begin{array}{cc|c} 1 & 5 & 9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$
   - A. None
   - B. One
   - C. Two
   - D. Three
3. $\left[\begin{array}{cccc|c} 1 & 0 & 0 & -3 & 7 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 2 \end{array}\right]$
   - A. None
   - B. One
   - C. Two
   - D. Three
4. $\left[\begin{array}{ccc|c} 1 & 7 & 2 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right]$
   - A. None
   - B. One
   - C. Two
   - D. Three

Correct Answers:

- C
- B
- D
- B

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6. (2 points) Do the columns of the matrix span $\mathbb{R}^3$?

? 1. $A = \begin{bmatrix} 1 & 5 & 7 \\ 4 & 21 & 26 \\ 4 & 20 & 24 \end{bmatrix}$

? 2. $A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -5 & 4 & 1 & 3 \\ 2 & -2 & 0 & 0 \end{bmatrix}$

? 3. $A = \begin{bmatrix} 4 & -5 \\ 6 & 9 \\ -2 & 1 \end{bmatrix}$

? 4. $A = \begin{bmatrix} -5 & -15 & -10 \\ -8 & -24 & -16 \\ 1 & 3 & 2 \end{bmatrix}$

Correct Answers:

- YES
- NO
- NO
- NO

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7. (2 points) Let $A = \begin{bmatrix} -2 & 2 & 0 \\ -3 & 5 & 2 \\ -5 & 3 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} -8 \\ -16 \\ -11 \end{bmatrix}$.
Is b is a linear combination of a_1, a_2 and a_3, the columns of the matrix A?

- No
- Yes

.
If b is a linear combination of the columns of A, determine a non-trivial linear relation between a_1, a_2, a_3 and b. Otherwise, enter 0's for the coefficients.
____ $a_1 +$ ____ $a_2 +$ ____ $a_3 = b$.

Correct Answers:

- Yes
- 1
- -3
- 1

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8. (2 points) Which of the following sets of vectors are linearly independent? (Check the boxes for linearly independent sets.)

- A. $\left\{ \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 9 \\ 2 \end{bmatrix} \right\}$
- B. $\left\{ \begin{bmatrix} -5 \\ -9 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 11 \\ 14 \\ 4 \end{bmatrix} \right\}$
- C. $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -8 \end{bmatrix} \right\}$
- D. $\left\{ \begin{bmatrix} 1 \\ 2 \\ -8 \end{bmatrix}, \begin{bmatrix} -6 \\ -9 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix} \right\}$
- E. $\left\{ \begin{bmatrix} -9 \\ -2 \end{bmatrix}, \begin{bmatrix} -9 \\ -2 \end{bmatrix} \right\}$
- F. $\left\{ \begin{bmatrix} -3 \\ 7 \end{bmatrix} \right\}$

Correct Answers:

- DF

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9. (1 point) The vectors

$$\vec{u} = \begin{bmatrix} -2 \\ -2 \\ -6 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} -2 \\ -4 \\ -22 + k \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} -2 \\ -5 \\ -9 \end{bmatrix}$$

are linearly independent if and only if $k \neq$ ____.

Correct Answers:

- 14