MATH 2111 Matrix Algebra and Applications
Homework-6 : Due 11/02/2022 at 11:59pm HKT

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1. (2 points) Which of the following sets are subspaces of $\mathbb{R}^3$?

- A. $\{(x,y,z) \mid x,y,z > 0\}$
- B. $\{(x,y,z) \mid -9x-8y+3z=0\}$
- C. $\{(-7x,-5x,-2x) \mid x \text{ arbitrary number }\}$
- D. $\{(x,y,z) \mid x+y+z=7\}$
- E. $\{(x,x-9,x-8) \mid x \text{ arbitrary number }\}$
- F. $\{(x,y,z) \mid 9x+8y=0,-3x-7z=0\}$

Correct Answers:

- `BCF`

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2. (2 points) Determine if each of the following sets is a subspace of $\mathbb{P}$ (the vector space of polynomials). Type "yes" or "no" for each answer.

Let W_1 be the set of all polynomials of the form $p(t) = at^2$, where a is in $\mathbb{R}$. ____

Let W_2 be the set of all polynomials of the form $p(t) = t^2 + a$, where a is in $\mathbb{R}$. ____

Let W_3 be the set of all polynomials of the form $p(t) = at^2 + at$, where a is in $\mathbb{R}$. ____

Correct Answers:

- `YES`
- `NO`
- `YES`

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3. (3 points) Determine whether the given set S is a subspace of the vector space V.

- A. $V = \mathbb{P}_3$, and S is the subset of $\mathbb{P}_3$ consisting of all polynomials of the form $p(x) = x^2 + c$.
- B. V is the vector space of all real-valued functions defined on the interval $(-\infty,\infty)$, and S is the subset of V consisting of those functions satisfying $f(0) = 0$.
- C. $V = \mathbb{P}_n$, and S is the subset of $\mathbb{P}_n$ consisting of those polynomials satisfying $p(0) = 0$.
- D. $V = C^1(\mathbb{R})$ (continuously differentiable functions), and S is the subset of V consisting of those functions satisfying $f'(0) \geq 0$.
- E. $V = \mathbb{R}^n$, and S is the set of solutions to the homogeneous linear system $Ax = 0$ where A is a fixed $m \times n$ matrix.
- F. $V = \mathbb{R}^3$, and S is the set of vectors (x_1,x_2,x_3) in V satisfying $x_1 - 5x_2 + x_3 = 4$.
- G. $V = M_{n\times n}(\mathbb{R})$, and S is the subset of all upper triangular matrices.

Correct Answers:

- `BCEG`

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4. (3 points) Let x,y,z be (non-zero) vectors and suppose $w = 6x+2y-3z$.

If $z = 3x+y$, then $w =$ ___ $x+$ ___ y.

Using the calculation above, mark the statements below that must be true.

- A. Span(w, x, z) = Span(w, z)
- B. Span(w, x) = Span(x, y)
- C. Span(w, y, z) = Span(x, y, z)
- D. Span(x, z) = Span(w, x, y)
- E. Span(w, z) = Span(x, y)

Correct Answers:

- `-3`
- `-1`
- `BCD`

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5. (2 points) Let $A = \begin{bmatrix} 4 & -8 \\ 1 & -2 \\ -1 & 2 \\ -3 & 6 \end{bmatrix}$.

Find k such that $Nul(A)$ is a subspace of $\mathbb{R}^k$ ____

Find k such that $Col(A)$ is a subspace of $\mathbb{R}^k$ ____

Let $B = \begin{bmatrix} -6 & 5 & -9 & 6 & 4 \end{bmatrix}$.

Find k such that $Row(B)$ is a subspace of $\mathbb{R}^k$ ____

Find k such that $Nul(B)$ is a subspace of $\mathbb{R}^k$ ____

Let $C = \begin{bmatrix} 7 & 5 & 0 \\ 5 & 0 & 2 \\ 0 & 2 & 7 \\ 2 & 7 & 5 \end{bmatrix}$.

Find k such that $Col(C)$ is a subspace of $\mathbb{R}^k$ ____

Find k such that $Row(C)$ is a subspace of $\mathbb{R}^k$ ____

Correct Answers:

- `2`
- `4`
- `5`
- `5`
- `4`
- `3`

6. (2 points) Let $A = \begin{bmatrix} 5 & 7 & 1 \\ -2 & -4 & 2 \\ 3 & 7 & -5 \end{bmatrix}$, $v = \begin{bmatrix} -2 \\ -5 \\ 5 \end{bmatrix}$, $w = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$ and $x = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

Is v in Nul(A)? Type "yes" or "no". ____

Is w in Nul(A)? Type "yes" or "no". ____

Is x in Nul(A)? Type "yes" or "no". ____

Correct Answers:

- NO
- YES
- NO

7. (2 points) Let W be the set of all vectors of the form: $\begin{bmatrix} \text{-3r-4s-3t} \\ \text{5r-4s+5t} \\ \text{r+2s-t} \\ \text{4r+4s-4t} \end{bmatrix}$, where r,s,t are arbitrary real numbers. Find A such that $W = ColA$.

$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$.

Correct Answers:

- -3
- -4
- -3
- 5
- -4
- 5
- 1
- 2
- -1
- 4
- 4
- -4

8. (2 points) Let u, v, w be three linearly independent vectors in $\mathbb{R}^7$. Determine whether the following sets of vectors are linearly independent or dependent.

$\boxed{?}$ 1. The set $\{u - v, v - w, w - u\}$

$\boxed{?}$ 2. The set $\{u + v, v + w, w + u\}$

Correct Answers:

- LINEARLY_DEPENDENT
- LINEARLY_INDEPENDENT

9. (1 point) Let u, v, w be three linearly independent vectors in $\mathbb{R}^7$. Determine a value of k,

$k =$ ___, so that the set $S = \{u - 5v,\ v - 3w,\ w - ku\}$ is linearly dependent.

Correct Answers:

- 0.0666666666666667

10. (2 points) Determine which of the following pairs of functions are linearly independent.

$\boxed{?}$ 1. $f(\theta) = \cos(3\theta)$, $g(\theta) = 2\cos^3(\theta) - 2\cos(\theta)$

$\boxed{?}$ 2. $f(t) = 2t^2 + 14t$, $g(t) = 2t^2 - 14t$

$\boxed{?}$ 3. $f(t) = 3t$, $g(t) = |t|$

$\boxed{?}$ 4. $f(x) = e^{2x}$, $g(x) = e^{2(x-3)}$

Correct Answers:

- LINEARLY INDEPENDENT
- LINEARLY INDEPENDENT
- LINEARLY INDEPENDENT
- LINEARLY DEPENDENT

11. (2 points) Find a basis of the subspace of $\mathbb{R}^4$ consisting of all vectors of the form

$$\begin{bmatrix} x_1 \\ 5x_1 + x_2 \\ 3x_1 + 2x_2 \\ 2x_1 + 3x_2 \end{bmatrix}$$

Your answer should be a list of row vectors separated by commas. (Click **vector** to learn about entering vectors.)

Basis = { ________________ }.

Correct Answers:

- <1,5,3,2>, <0,1,2,3>