MATH 2111 Matrix Algebra and Applications
Homework-9 : Due 11/25/2022 at 11:59pm HKT

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1. (1 point) Find the dot product of

$$\vec{x} = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix} \text{ and } \vec{y} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}.$$

$\vec{x} \cdot \vec{y} =$ ______.

Correct Answers:

- `7`

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2. (1 point) Let $x = \begin{bmatrix} 4 \\ 2 \\ -5 \\ -5 \end{bmatrix}$.

Find the length of x and the unit vector u in the direction of x.

$||x|| =$ ______,

$u = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.

Correct Answers:

- `8.36660026534076`
- `0.478091443733757`
- `0.239045721866879`
- `-0.597614304667197`
- `-0.597614304667197`

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3. (1 point) Find the value of k for which the vectors

$$\begin{bmatrix} 5 \\ -3 \\ -4 \\ -2 \end{bmatrix} \text{ and } \begin{bmatrix} -3 \\ 3 \\ -4 \\ k \end{bmatrix}$$

are orthogonal.

$k =$ ____.

Correct Answers:

- `-(-8)/-2`

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4. (1 point) Let $\{\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4, \vec{e}_5, \vec{e}_6\}$ be the standard basis in $\mathbb{R}^6$. Find the length of the vector $\vec{x} = -2\vec{e}_1 - 4\vec{e}_2 + 3\vec{e}_3 + 4\vec{e}_4 - 3\vec{e}_5 - 4\vec{e}_6$.

$||\vec{x}|| =$ ______.

Correct Answers:

- `sqrt(70)`

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5. (2 points) All vectors are in $\mathbb{R}^n$.

Check the true statements below:

- A. If $||u||^2 + ||v||^2 = ||u+v||^2$, then u and v are orthogonal.
- B. $u \cdot v - v \cdot u = 0$.
- C. For an $m \times n$ matrix A, vectors in the null space of A are orthogonal to vectors in the row space of A.
- D. For any scalar c, $||cv|| = c||v||$.
- E. If x is orthogonal to every vector in a subspace W, then x is in $W^\perp$.

Correct Answers:

- `ABCE`

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6. (2 points) Let W be the set of all vectors $\begin{bmatrix} \text{x} \\ \text{y} \\ \text{x + y} \end{bmatrix}$ with x and y real.

Determine whether each of the following vectors is in $W^\perp$.

$\boxed{?}$ 1. $v = \begin{bmatrix} 9 \\ 0 \\ -1 \end{bmatrix}$

$\boxed{?}$ 2. $v = \begin{bmatrix} -2 \\ -2 \\ 2 \end{bmatrix}$

$\boxed{?}$ 3. $v = \begin{bmatrix} 5 \\ 5 \\ -5 \end{bmatrix}$

Correct Answers:

- `NO`
- `YES`
- `YES`

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7. (1 point) Given $v = \begin{bmatrix} 18 \\ 28 \\ -4 \\ 62 \end{bmatrix}$, find the linear combination for v in the subspace W spanned by $u_1 = \begin{bmatrix} 4 \\ -2 \\ -2 \\ 0 \end{bmatrix}$, $u_2 = \begin{bmatrix} -2 \\ 0 \\ -4 \\ 1 \end{bmatrix}$ and $u_3 = \begin{bmatrix} -6 \\ -10 \\ -2 \\ -20 \end{bmatrix}$. Note that u_1, u_2 and u_3 are orthogonal.

$v =$ ____ $u_1 +$ ____ $u_2 +$ ____ u_3

Correct Answers:

- 1
- 2
- -3

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8. (1 point) Given $v = \begin{bmatrix} -9 \\ 3 \\ -9 \\ -2 \end{bmatrix}$, find the linear combination for v in the subspace W spanned by $u_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 4 \end{bmatrix}$, $u_3 = \begin{bmatrix} 11 \\ -1 \\ 4 \\ -7 \end{bmatrix}$ and $u_4 = \begin{bmatrix} 0 \\ -3 \\ 1 \\ 1 \end{bmatrix}$. Note that u_1, u_2, u_3 and u_4 are orthogonal.

$v =$ ____ u_1+ ____ u_2+ ____ u_3+ ____ u_4

Correct Answers:

- -0.666666666666667
- -1.03921568627451
- -0.663101604278075
- -1.81818181818182

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9. (1 point) Suppose $v_1,\ v_2,\ v_3$ is an orthogonal set of vectors in $\mathbb{R}^5$ with
$v_1 \cdot v_1 = 26, v_2 \cdot v_2 = 30, v_3 \cdot v_3 = 1$.

Let w be a vector in $\mathrm{Span}(v_1, v_2, v_3)$ such that
$w \cdot v_1 = -26, w \cdot v_2 = -60, w \cdot v_3 = 2$.

Then $w =$____v_1+____v_2+____v_3.

Correct Answers:

- -1
- -2
- 2