Math1013 Calculus I **Name:** Kin Nam KOO

Homework-3 : Due 10/17/2021 at 11:59pm HKT

The problems in this homework set cover the basic concept of limits of functions and limit calculation. You need to know:

1. the idea of limits: trending behaviour of function values from tables and/or graphs;
2. basic limit laws;
3. algebraic techniques in limit computation, and the Squeeze Theorem;
4. infinite limits and limits at infinity, vertical and horizontal asymptotes;
5. The concept of continuity of functions.

Give 4 or 5 significant digits for numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as 3^2 or 3**2 instead of 9, sin(3 * pi/2) instead of -1, e^(ln(3)) instead of 3, (1+tan(3)) * (4 − sin(5))^6 − 15/8 instead of 12748.8657, etc.

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1. (3 points) Consider the function $f(x) = \frac{x^2-36}{x-6}$.

(a) Fill in the following table of values for $f(x)$:

$x =$	5.9	5.99	5.999	5.9999	6.0001	6.001	6.01	6.1
$f(x) =$	_____	_____	_____	_____	_____	_____	_____	_____

(b) Based on your table of values, what would you expect the limit of $f(x)$ as x approaches 6 to be?

$\lim_{x\to 6} \frac{x^2-36}{x-6} =$ _____

(c) Graph the function to see if it is consistent with your answers to parts (a) and (b). By graphing, find an interval for $x-6$ near zero such that the difference between your conjectured limit and the value of the function is less than 0.01. In other words, find a window of height 0.02 such that the graph exits the sides of the window and not the top or bottom. What is the window?

_____ $\le x \le$ _____,

_____ $\le y \le$ _____.

Solution:

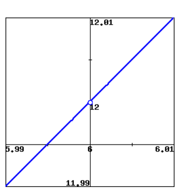
SOLUTION

(a) The values of $f(x)$ are

$x =$	5.9	5.99	5.999	5.9999	6.0001	6.001	6.01	6.1
$f(x) =$	11.9	11.99	11.999	11.9999	12.0001	12.001	12.01	12.1

(b) From the values in this table, it appears that the limit is 12.

(c) When we graph the function $f(x)$ we see the graph:



This confirms our estimate of the limit. A reasonable range for x so that the graph of $f(x)$ enters and leaves a window of height 0.02 around $y = 12$ is $-5.99 \le x \le 6.01$ and $11.99 \le y \le 12.01$.

Correct Answers:

- 11.9
- 11.99
- 11.999
- 11.9999
- 12.0001
- 12.001
- 12.01
- 12.1
- 2*6
- 5.99
- 6.01
- 11.99
- 12.01

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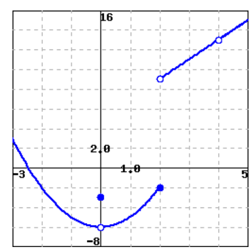
2. (3 points) Use the figure below, which gives a graph of the function $f(x)$, to give values for the indicated limits.



(If any of the limits does not exist, enter the word **none** *in the answer blank for that limit.)*

(a) $\lim_{x\to -2} f(x) =$ ________

(b) $\lim_{x\to 0} f(x) =$ ________

(c) $\lim_{x\to 2} f(x) =$ ________

(d) $\lim_{x\to 4} f(x) =$ ________

Solution:

SOLUTION

(a) As x approaches -2 from either side, the values of $f(x)$ get closer and closer to -2, so the limit appears to be about -2.

(b) As x approaches 0 from either side, the values of $f(x)$ get closer and closer to -6. (Recall that to find a limit, we are interested in what happens to the function near x but not at x.) The limit appears to be about -6.

(c) As x approaches 2 from either side, the values of $f(x)$ get closer and closer to -2 on one side of $x = 2$ and get closer and closer to 9 on the other side of $x = 2$. Thus the limit does not exist.

(d) As x approaches 4 from either side, the values of $f(x)$ get closer and closer to 13. (Again, recall that we don't care what happens right at $x = 4$.) The limit appears to be about 13.

Correct Answers:

- -2
- -6
- none
- 13

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3. (3 points) For the function

$$f(x) = \begin{cases} x^2-5, & 0 \le x < 2 \\ 5, & x = 2 \\ 4x-9, & \\ 2 < x & \end{cases}$$

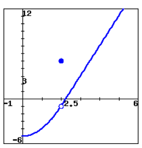
use algebra to find each of the following limits:
$\lim_{x\to 2^+} f(x) =$ ________
$\lim_{x\to 2^-} f(x) =$ ________
$\lim_{x\to 2} f(x) =$ ________
(For each, enter **dne** *if the limit does not exist.)*

Sketch a graph of $f(x)$ to confirm your answers.

Solution:

SOLUTION

The graph of $f(x)$ is shown below.



From this, and from the given formula for $f(x)$, we can see that
$\lim_{x\to 2^+} f(x) = -1$, and
$\lim_{x\to 2^-} f(x) = -1$.
These are the same value, so we know that the limit must exist: $\lim_{x\to 2} f(x) = -1$.

Note, however, that $f(x)$ is not continuous at $x = 2$ since $f(2) = 5 \neq -1$.

Correct Answers:

- 4*2+-9
- 2*2-5
- 2*2-5

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4. (3 points) Let $\lim_{x\to a} h(x) = -4$, $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = -4$.
Find following limits if they exist. If not, enter DNE ('does not exist') as your answer.

___1. $\lim_{x\to a} h(x) + f(x)$
___2. $\lim_{x\to a} h(x) - f(x)$
___3. $\lim_{x\to a} h(x) * g(x)$
___4. $\lim_{x\to a} \frac{h(x)}{f(x)}$
___5. $\lim_{x\to a} \frac{h(x)}{g(x)}$
___6. $\lim_{x\to a} \frac{g(x)}{h(x)}$
___7. $\lim_{x\to a} \sqrt{f(x)}$
___8. $\lim_{x\to a} f(x)^{-1}$
___9. $\lim_{x\to a} \frac{1}{f(x)-g(x)}$

Correct Answers:

- -4
- -4
- 16
- DNE
- 1
- 1
- 0
- DNE
- 0.25

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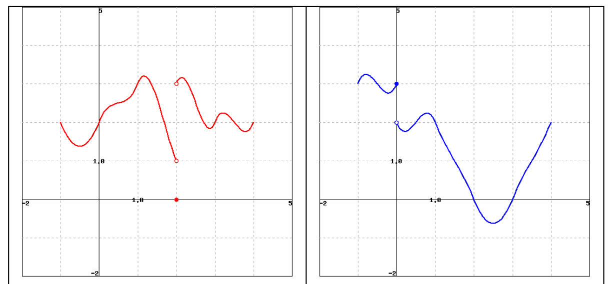
5. (4 points)



$f(x)$	$g(x)$

The graphs of f and g are given above. You may click on the graphs to get larger images of them. Use the graphs to evaluate each quantity below. Write *DNE* if the limit or value does not exist (or if it's infinity).

___1. $\lim_{x\to 2^+} [f(x)+g(x)]$
___2. $f(g(0))$
___3. $\lim_{x\to 2^-} [f(x)g(x)]$
___4. $\lim_{x\to 2^-} [f(x)+g(x)]$

Correct Answers:

- 3
- 2
- 0
- 1

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6. (2 points) Consider the function

$$g(x) = \frac{6x+15|x|}{5x-8|x|}$$

Evaluate the following expressions. Write DNE if the expression is undefined.
$\lim_{x\to 0^-} g(x) =$__________
$g(0) =$__________
$\lim_{x\to 0^+} g(x) =$__________
$\lim_{x\to 0} g(x) =$__________

Correct Answers:

- -0.692307692307692
- DNE
- -7

- DNE

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7. (2 points) Find (in terms of the constant a)

$$\lim_{h\to 0} \frac{\frac{6}{a+h} - \frac{6}{a}}{h}.$$

Limit = ________

Correct Answers:

- `-6/(a**2)`

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8. (2 points) Find the one-sided limit

$$\lim_{t\to 3^+} \frac{|9-t^2|}{3-t}$$

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Use INF to denote ∞ and MINF to denote $-\infty$.

Correct Answers:

- `-6`

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9. (2 points)

$$\lim_{x\longrightarrow \frac{16}{10.5}} \frac{10.5x-16}{\sqrt{10.5x}-4} = \_\_\_\_\_\_\_\_\_.$$

Correct Answers:

- `8`

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10. (2 points)

Evaluate the limit, if it exists. If not, enter "n" below.

$$\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+8t}} - \frac{1}{t}\right)$$

Correct Answers:

- `-4`

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11. (3 points)

Evaluate the limit, if it exists. If not, enter "n" below.

$$\lim_{x\to 4} \frac{8\sqrt{x}-x^2}{2-\sqrt{x}}$$

Correct Answers:

- `24`

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12. (2 points) If

$$10x-25 \le f(x) \le x^2+6x-21$$

determine $\lim_{x\to 2} f(x) =$ ________

What theorem did you use to arrive at your answer?

____

Correct Answers:

- `-5`
- `The Squeeze theorem`

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13. (3 points) Determine the infinite limit of the following functions. Enter infinity for ∞ and -infinity for $-\infty$.

___1. $\lim_{x\to 0} \frac{1}{x^2(x+7)}$

___2. $\lim_{x\to 5} \frac{2}{(x-5)^6}$

___3. $\lim_{x\to 5^-} \frac{2}{(x-5)^3}$

___4. $\lim_{x\to 3^-} \frac{2}{x-3}$

___5. $\lim_{x\to 3^+} \frac{2}{x-3}$

___6. $\lim_{x\to -7^-} \frac{1}{x^2(x+7)}$

Correct Answers:

- `infinity`
- `infinity`
- `-infinity`
- `-infinity`
- `infinity`
- `-infinity`

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14. (3 points)

A function is said to have a **vertical asymptote** wherever the limit on the left or right (or both) is either positive or negative infinity.

For example, the function $f(x) = \frac{-3(x+2)}{x^2+4x+4}$ has a vertical asymptote at $x=-2$.

For each of the following limits:

enter either 'infinity' for positive infinity,

'-infinity' for negative infinity,

or 'DNE' when the limit simply does not exist.

$\lim_{x\to -2^-} \frac{-3(x+2)}{x^2+4x+4} =$ ________

$\lim_{x\to -2^+} \frac{-3(x+2)}{x^2+4x+4} =$ ________

$\lim_{x\to -2} \frac{-3(x+2)}{x^2+4x+4} =$ ________

Correct Answers:

- `infinity`
- `-infinity`
- `DNE`

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15. (5 points)

A function is said to have a **horizontal asymptote** if either the limit at infinity exists or the limit at negative infinity exists.

Show that each of the following functions has a horizontal asymptote by calculating the given limit.

$\lim_{x\to\infty} 14 + \frac{7x}{x^2-12x+4} =$ _____

$\lim_{x\to-\infty} \frac{10-11x}{15+x} + \frac{3x^2+2}{(9x-9)^2} =$ _____

$\lim_{x\to-\infty} \frac{11x+15}{x-11} \cdot \frac{6x-14}{-x-2} =$ _____

$\lim_{x\to\infty} \sqrt{x^2+14x-7} - x =$ _____
$\lim_{x\to-\infty} \sqrt{x^2+14x-7} + x =$ _____

Correct Answers:

- `14`
- `-10.962962962963`
- `-66`
- `7`
- `-7`

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16. (2 points) Evaluate

$$\lim_{x\to\infty} \frac{\sqrt{x^4+4x^3-9}}{8x^2-5}$$

____

Correct Answers:

- `0.125`

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17. (4 points)
Evaluate the following limits. If needed, enter INF for ∞ and MINF for $-\infty$.

(a)

$$\lim_{x\to\infty} \left(\sqrt{x^2-10x+1} - x\right) =$$

_____

(b)

$$\lim_{x\to-\infty} \left(\sqrt{x^2-10x+1} - x\right) =$$

_____

Correct Answers:

- `-5`
- `INF`

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18. (2 points) Find the values of c and d that make the following function

$$f(x) = \begin{cases} 8x & \text{if } x < 1 \\ cx^2+d & \text{if } 1 \le x < 2 \\ 4x & \text{if } x \ge 2 \end{cases}$$

continuous for all x.
$c =$ __________
$d =$ __________

Correct Answers:

- `0`
- `8`