MATH 2111 Matrix Algebra and Applications
Homework-7 : Due 11/08/2022 at 11:59pm HKT

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1. (2 points) Let

$$A = \begin{bmatrix} 0 & 2 & -2 & -4 \\ 0 & -3 & 3 & 6 \end{bmatrix}.$$

Find a basis for the null space of A.

$$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}.$$

Correct Answers:

- `[[1],[0],[0],[0]],`
$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$
,
$\begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

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2. (1 point) Find a basis of the subspace of $\mathbb{R}^3$ defined by the equation $5x_1 + 5x_2 - 4x_3 = 0$.

Basis: $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}.$

Correct Answers:

- `[[-4],[0],[-5]],`
$\begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix}$

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3. (2 points) Find a basis of the row space of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 \\ -4 & -12 & -1 & 0 & 5 \\ 0 & 4 & -3 & -4 & -4 \\ -2 & -12 & 4 & 6 & 9 \end{bmatrix}.$$

Basis:

$\{ [\ \_\_ \ \ \_\_ \ \ \_\_ \ \ \_\_ \ \ \_\_ \ ],$
$[\ \_\_ \ \ \_\_ \ \ \_\_ \ \ \_\_ \ \ \_\_ \ ],$
$[\ \_\_ \ \ \_\_ \ \ \_\_ \ \ \_\_ \ \ \_\_ \ ] \}.$

Correct Answers:

- `[1,2,1,1,0],`
$\begin{bmatrix} 0 & 4 & -3 & -4 & -4 \end{bmatrix}$
,
$\begin{bmatrix} -2 & -12 & 4 & 6 & 9 \end{bmatrix}$

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4. (2 points) Find a basis for the column space of

$$A = \begin{bmatrix} -4 & 0 & 2 & 3 & 2 \\ 4 & 1 & 0 & 2 & -4 \\ 4 & 1 & 0 & 2 & -3 \\ -8 & -2 & 0 & -4 & 7 \end{bmatrix}.$$

Basis = $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}.$

Correct Answers:

- `[[0],[1],[1],[-2]],`
$\begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
,
$\begin{bmatrix} 2 \\ -4 \\ -3 \\ 7 \end{bmatrix}$

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5. (2 points) Find a linearly independent set of vectors that spans the same subspace of $\mathbb{R}^4$ as that spanned by the vectors

$$\begin{bmatrix} 1 \\ -6 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -3 \\ 1 \end{bmatrix}.$$

A linearly independent spanning set for the subspace is:

$$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}.$$

Correct Answers:

- `[[-1],[2],[-3],[1]],`
$\begin{bmatrix} 0 \\ 3 \\ -2 \\ 1 \end{bmatrix}$
,
$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$

6. (1 point) The set $B = \left\{ \begin{bmatrix} -4 \\ -8 \end{bmatrix}, \begin{bmatrix} -12 \\ -21 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$.

Find the coordinate vector of $x = \begin{bmatrix} 28 \\ 47 \end{bmatrix}$ relative to the basis B:

$[x]_B = \begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$

Correct Answers:

- 2
- -3

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7. (1 point) Find the coordinate vector of $x = \begin{bmatrix} -5 \\ -4 \\ -3 \end{bmatrix}$ relative to the basis $B = \left\{ \begin{bmatrix} 1 \\ 7 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ of $\mathbb{R}^3$.

$[x]_B = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$

Correct Answers:

- -5
- 31
- 110

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8. (1 point) The set $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ is a basis of the vector space of 2×2 matrices.

Find the coordinate vector of $M = \begin{bmatrix} 2 & -2 \\ -9 & 9 \end{bmatrix}$ relative to this basis B.

$[M]_B = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$.

Correct Answers:

- 2
- -2
- -9
- 9

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9. (2 points) Determine whether or not the following sets S of 2×2 matrices are linearly independent.

[?] 1. $S = \left\{ \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 3 & 0 \end{pmatrix} \right\}$

[?] 2. $S = \left\{ \begin{pmatrix} -3 & 7 \\ -7 & -2 \end{pmatrix}, \begin{pmatrix} 9 & 0 \\ 42 & 6 \end{pmatrix} \right\}$

[?] 3. $S = \left\{ \begin{pmatrix} -3 & 7 \\ -7 & -2 \end{pmatrix}, \begin{pmatrix} 9 & -21 \\ 21 & 6 \end{pmatrix} \right\}$

[?] 4. $S = \left\{ \begin{pmatrix} -3 & 7 \\ -7 & -2 \end{pmatrix}, \begin{pmatrix} 9 & 0 \\ 42 & 6 \end{pmatrix}, \begin{pmatrix} 1 & -3 \\ 9 & 10 \end{pmatrix}, \begin{pmatrix} 7 & -3 \\ -21 & -3 \end{pmatrix}, \begin{pmatrix} 17 \\ \pi \end{pmatrix}\right.$

Correct Answers:

- LINEARLY_INDEPENDENT
- LINEARLY_INDEPENDENT
- LINEARLY_DEPENDENT
- LINEARLY_DEPENDENT

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10. (2 points) The set $B = \{-1 - 3x^2, -3 - 1x - 9x^2, -7 - 2x - 24x^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinates of $p(x) = 10 + 3x + 39x^2$ relative to this basis:

$[p(x)]_B = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$

Correct Answers:

- 2
- 3
- -3

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11. (1 point) Find the ranks of the following matrices.

$\text{rank} \begin{bmatrix} 0 & -7 \\ 0 & -1 \\ 0 & 4 \end{bmatrix} = \_\_\_$

$\text{rank} \begin{bmatrix} 0 & 0 & 2 & 0 \\ -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 \end{bmatrix} = \_\_\_$

$\text{rank} \begin{bmatrix} 9 & 1 & -9 \\ 0 & -7 & 0 \\ -2 & 0 & 2 \end{bmatrix} = \_\_\_$

Correct Answers:

- 1
- 4
- 2

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12. (2 points) Find the value of k for which the matrix

$$A = \begin{bmatrix} -6 & 1 & 4 \\ 3 & -5 & 7 \\ 7 & 5 & k \end{bmatrix}$$

has rank 2.

$k =$ ____

Correct Answers:

- -17