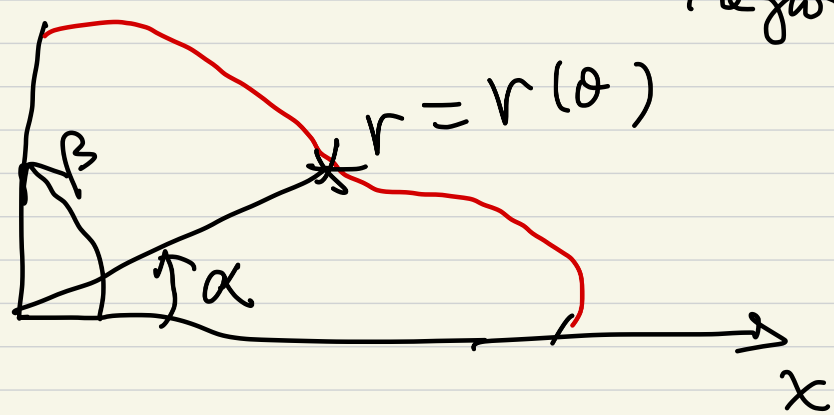
Area

$$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2(\theta)\, d\theta = \text{area of a "polar sector" region.}$$



Example

$$\boxed{r = 1 + 2\cos 3\theta}, \quad -1 \le r \le 3$$

$r = 0$, $\theta = \frac{2\pi}{9} = 20° \times 2$ ← correction!!

$\theta = \frac{\pi}{3} + \frac{\pi}{9} = \frac{4\pi}{9} = 40° \times 2$

$\theta = \frac{2\pi}{9} + \frac{2\pi}{3} = \frac{8\pi}{9}$

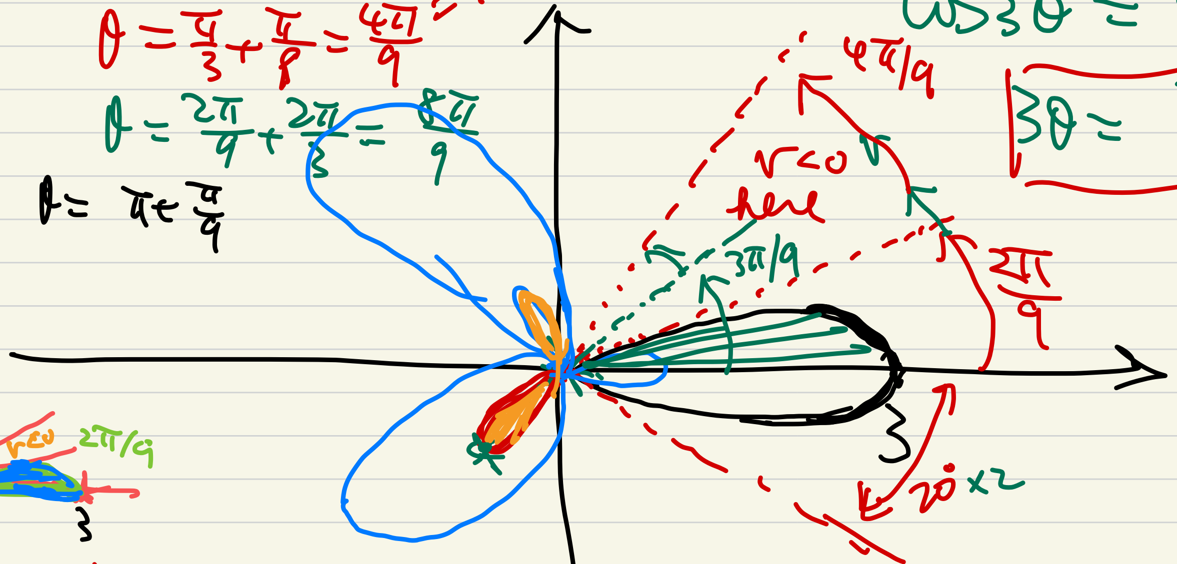
$\theta = \pi + \frac{\pi}{9}$

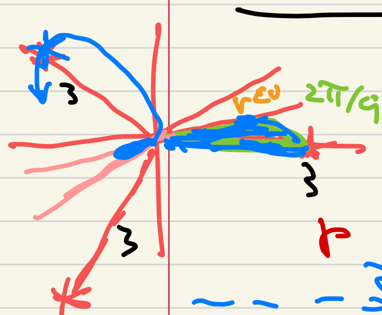
$r = 0$

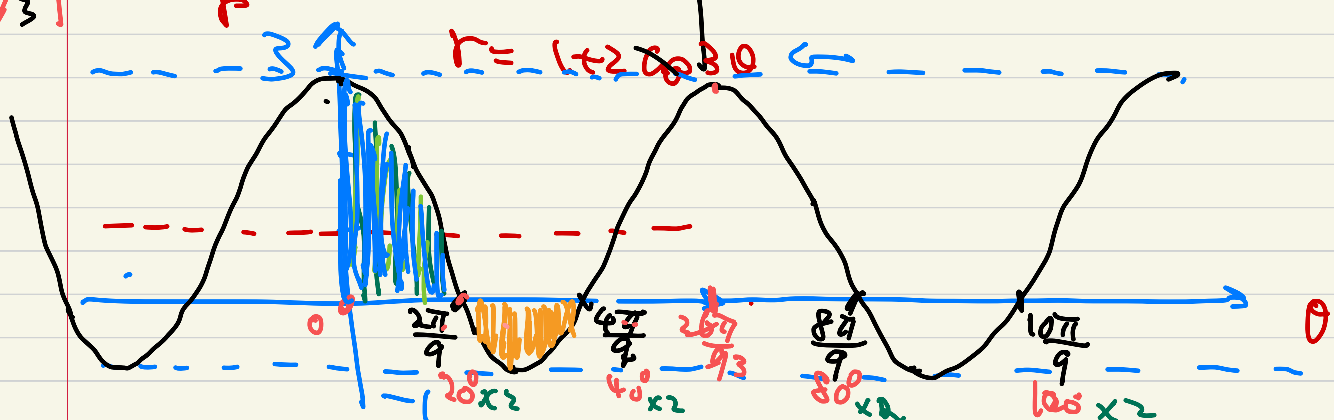
$\cos 3\theta = -\frac{1}{2}$

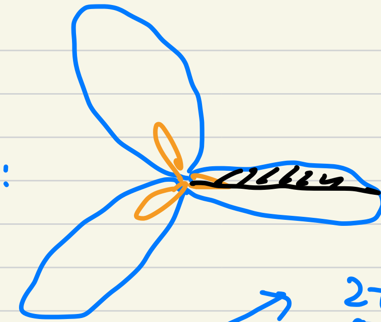
$\boxed{3\theta = \frac{2\pi}{3}}$, $\pi + \frac{\pi}{3}$, $\boxed{\frac{2\pi}{3} + 2\pi}$, $3\pi + \frac{\pi}{3}$, ...

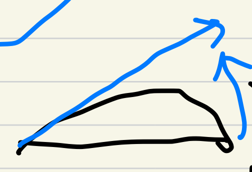
$\frac{2\pi}{3}$







Find the area of the three large leaves:  $r = 1 + 2\cos 3\theta$

$A = 6 \text{ area}\Big($  $\Big)$ $\frac{2\pi}{9}$

$$= 6 \cdot \int_0^{2\pi/9} \frac{1}{2}(1 + 2\cos 3\theta)^2 \, d\theta$$

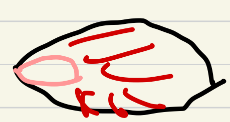
$$= 3 \int_0^{\frac{2\pi}{9}} (1 + 2\cos 3\theta)^2 \, d\theta$$

Area() $= 3 \times \underbrace{\int_{\frac{2\pi}{9}}^{\frac{4\pi}{9}} \frac{1}{2}(1 + 2\cos 3\theta)^2 \, d\theta}_{\text{area}(\ )}$ 

$$= \frac{3}{2} \int_{2\pi/9}^{4\pi/9} (1 + 2\cos 3\theta)^2 \, d\theta$$

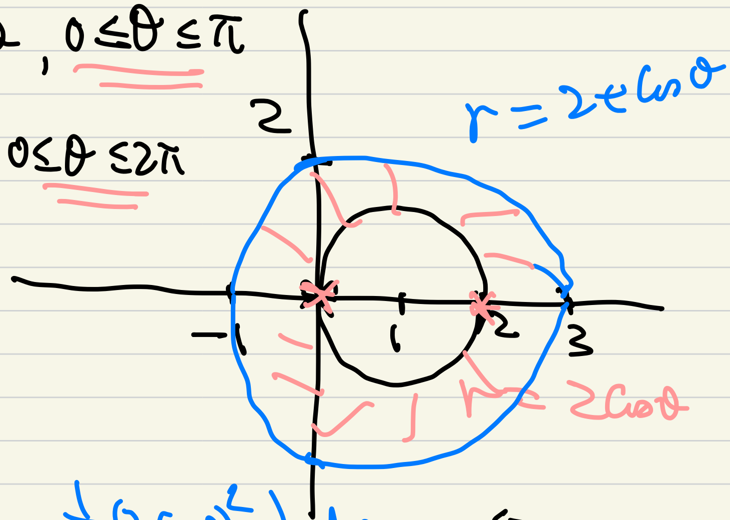
area() $= \left(2 \cdot \int_0^{2\pi/9} \frac{1}{2}(1 + 2\cos 3\theta)^2 \, d\theta - \int_{\frac{2\pi}{9}}^{4\pi/9} \frac{1}{2}(1 + 2\cos 3\theta)^2 \, d\theta \right) \times 3$

area inside the larger loops but outside the smaller loops



Example $r = 2\cos\theta$, $0 \le \theta \le \pi$

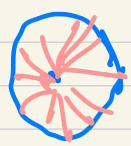
$r = 2 + \cos\theta$, $0 \le \theta \le 2\pi$



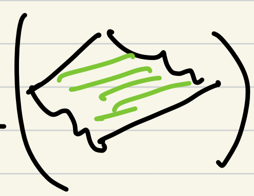
area between the two curves

$$\neq \int_0^{2\pi} \frac{1}{2}\left((2+\cos\theta)^2 - \frac{1}{2}(2\cos\theta)^2 \right) d\theta$$

$$= \underbrace{\int_0^{2\pi} \frac{1}{2}(2+\cos\theta)^2\, d\theta}_{\text{area}(\ )} - \underbrace{\int_0^{2\pi} \frac{1}{2}(2\cos\theta)^2\, d\theta}_{2 \times \text{area of the circle}}$$

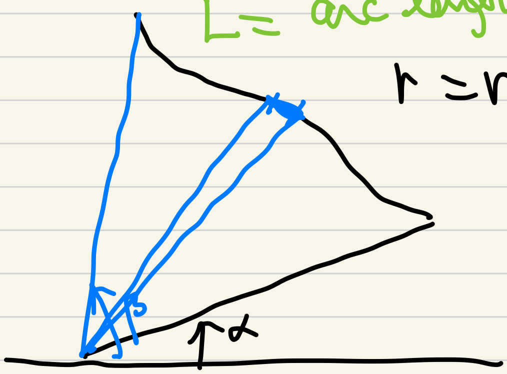


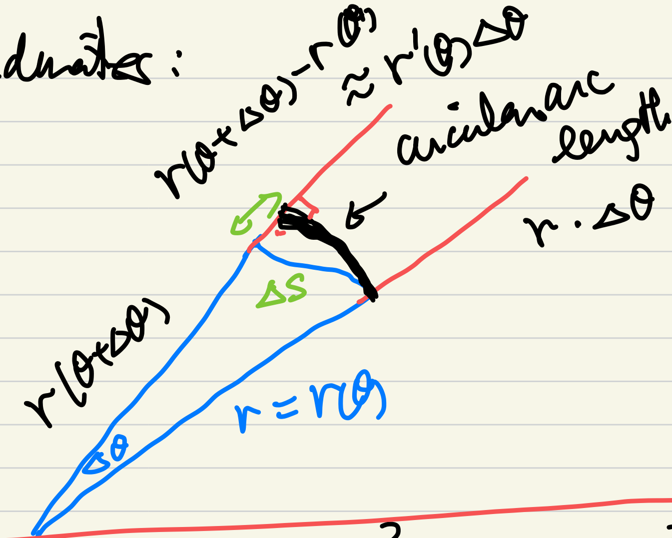
area between the curves $= \int_0^{2\pi} \frac{1}{2}(2+\cos\theta)^2\, d\theta - \int_0^{\pi} \frac{1}{2}(2\cos\theta)^2\, d\theta$

area 

$$= \frac{1}{2}\int_\alpha^\beta r_1^2(\theta)\, d\theta - \frac{1}{2}\int_\alpha^\beta r_2^2(\theta)\, d\theta$$

Arc Length in Polar Coordinates:





$$\boxed{\Delta s^2 = (r\Delta\theta)^2 + (r'(\theta)\Delta\theta)^2}$$

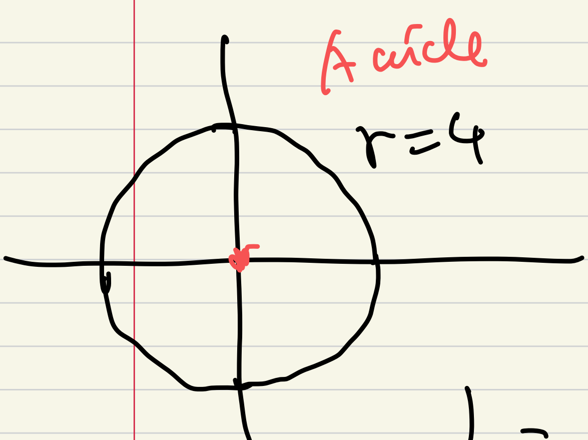
$$\Delta s \approx \sqrt{r^2 + (r'(\theta))^2}\ \Delta\theta$$

Σ

$\downarrow$ Integration

$$L = \int_\alpha^\beta \sqrt{r^2 + (r')^2}\, d\theta$$

arc length formula in polar coordinates

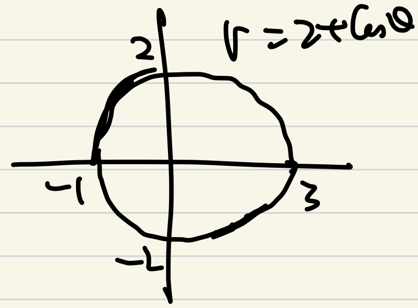


arc length $= (2\cdot 4)\pi = 8\pi$

$r = 4, \quad \frac{dr}{d\theta} = 0$ ||

$$L = \int_0^{2\pi} \sqrt{4^2 + 0^2}\, d\theta = 2\pi \cdot 4 = 8\pi$$

Example. $r = 2 + \cos\theta$



$\frac{dr}{d\theta} = -\sin\theta$

arc length $= \int_0^{2\pi} \sqrt{(2+\cos\theta)^2 + (-\sin\theta)^2}\, d\theta$

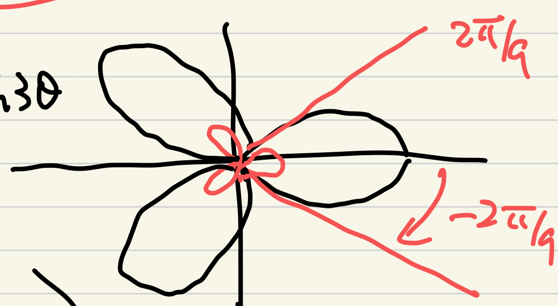
$(r)^2 + (r')^2$

$= \int_0^{2\pi} \sqrt{4 + 4\cos\theta + \cos^2\theta + \sin^2\theta}\, d\theta$

$= \int_0^{2\pi} \sqrt{6 + \underset{8\cos^2\theta/2 - 4}{4\cos\theta}}\, d\theta$

Can find approximate values by some numerical method.

$r = 1+2\cos 3\theta, \ \frac{dr}{d\theta} = -6\sin 3\theta$



arc length

$= (\text{arc length} \quad + \text{arc length} \quad) \times 3$

$= 3 \times \left(\int_{-\frac{2\pi}{9}}^{\frac{2\pi}{9}} \sqrt{(1+2\cos 3\theta)^2 + 36\sin^2\theta}\, d\theta + \int_{2\pi/9}^{\frac{4\pi}{9}} \sqrt{(1+2\cos 3\theta)^2 + 36\sin^2\theta}\, d\theta \right)$