Math1013 Calculus I **Name:** Kin Nam KOO

Homework-2 : Due 10/03/2021 at 11:59pm HKT

This is a set of homework questions on the basic concepts and usages of functions. To solve the problems in this homework set, you need to know the following:

(1) meaning of the domain and range of a function;
(2) the basic operations of functions: $+$, $-$, $\times$, $\div$, and $\circ$ (composition of functions);
(3) graphs of functions and their symmetry;
(4) inverse functions;
(5) exponential and logarithmic functions, trigonometric functions and their inverses.

Give 4 or 5 significant digits for numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as 3^2 or 3**2 instead of 9, $\sin(3*pi/2)$ instead of -1, e^(ln(3)) instead of 3, $(1+\tan(3))*(4-\sin(5))$^$6-15/8$ instead of 12748.8657, etc.

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1. (6 points) Use **interval notation** to indicate the domain of

$$f(x) = \sqrt[4]{x^2 - 8x}$$

and

$$g(x) = \sqrt[9]{3x^2 - 8x}.$$

The domain of $f(x)$ is ____________
The domain of $g(x)$ is ____________

Correct Answers:

- `(-infinity,0] U [8,infinity)`
- `(-infinity,infinity)`

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2. (8 points) Suppose that

$$f(x) = \frac{1}{x-4} \quad \text{and} \quad g(x) = \frac{x-4}{x+7}.$$

For each function h given below, find a formula for $h(x)$ and the domain of h. Use **interval notation** for entering the domains.

(A) $h(x) = (f \circ g)(x)$.

$h(x)$ = ____________
Domain = ____________

(B) $h(x) = (g \circ f)(x)$.

$h(x)$ = ____________
Domain = ____________

(C) $h(x) = (f \circ f)(x)$.

$h(x)$ = ____________
Domain = ____________

(D) $h(x) = (g \circ g)(x)$.

$h(x)$ = ____________
Domain = ____________

Correct Answers:

- `(x+7)/(x-4-4*(x+7))`
- `(-infinity,-10.6666666666667) U (-10.6666666666667,-7) U (`
- `(1-4*(x-4))/(1+7*(x-4))`
- `(-infinity,3.85714285714286) U (3.85714285714286,4) U (4,i`
- `(x-4)/(1-4*(x-4))`
- `(-infinity,4) U (4,4.25) U (4.25,infinity)`
- `(x-4-4*(x+7))/(x-4+7*(x+7))`
- `(-infinity,-7) U (-7,-5.625) U (-5.625,infinity)`

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3. (2 points) Find the equations of the lines that pass through the point $(5,4)$ and are parallel to and perpendicular to the line with equation $y+6x=8$.
Parallel: $y =$ ________________
Perpendicular: $y =$ ________________

Solution:
SOLUTION
The line $y+6x=8$ has slope -6. Therefore, the parallel line has slope -6 and equation $y-4=-6(x-5)$ or $y=-6(x-5)+4$. The perpendicular line has slope $\frac{1}{6}$ and equation $y-4=\frac{1}{6}(x-5)$ or $y=\frac{1}{6}(x-5)+4$.

Correct Answers:

- `-6*(x-5)+4`
- `1/6*(x-5)+4`

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4. (3 points) The monthly charge for a waste collection service is 1080 dollars for 100 kg of waste and 1280 dollars for 120 kg of waste.

(a) Find a linear model for the cost, C, of waste collection as a function of the number of kilograms, w.
$C =$ ________________

(b) What is the slope of the line found in part (a)?
Slope = ________________

Think about the interpretation of the slope: are the units of the slope

- A. dollars
- B. dollars per kilogram
- C. kilograms per dollar
- D. kilograms

(c) What is the value of the vertical intercept of the line found in part (a)?
Value= ________________

Think about the interpretation of the intercept: are the units of the intercept

- A. kilograms
- B. kilograms per dollar
- C. dollars per kilogram
- D. dollars

Solution:

SOLUTION

(a) We find the slope m and intercept b in the linear equation $C = b + mw$. To find the slope m, we use

$m = \frac{\Delta C}{\Delta w} = \frac{1280-1080}{120-100} = 10.$

We substitute to find b:

$1280 = b + (10)(20)$

so that $b = 80$.

The linear formula is $C = 10w + 80$.

(b) The slope is 10 dollars per kilogram. Each additional kilogram of waste costs 10 dollars.

(c) The intercept is 80 dollars. The flat monthly fee to subscribe to the waste collection is 80 dollars, even if there is no waste.

Correct Answers:

- `10*w+80`
- `10`
- `B`
- `80`
- `D`

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5. (3 points) The point $P(3,15)$ lies on the curve $y = x^2 + x + 3$. Let Q be the point $(x, x^2 + x + 3)$.

a.) Compute the slope of the secant line PQ for the following values of x.

When $x = 3.1$, the slope of PQ is: ____________

When $x = 3.01$, the slope of PQ is: ____________

When $x = 2.9$, the slope of PQ is: ____________

When $x = 2.99$, the slope of PQ is: ____________

b.) Based on the above results, guess the slope of the tangent line to the curve at $P(3,15)$.

Answer: ____________

Correct Answers:

- `3 +3.1 + 1`
- `3 +3.01 + 1`
- `3 +2.9 + 1`
- `3 +2.99 + 1`
- `2*3 + 1`

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6. (4 points)

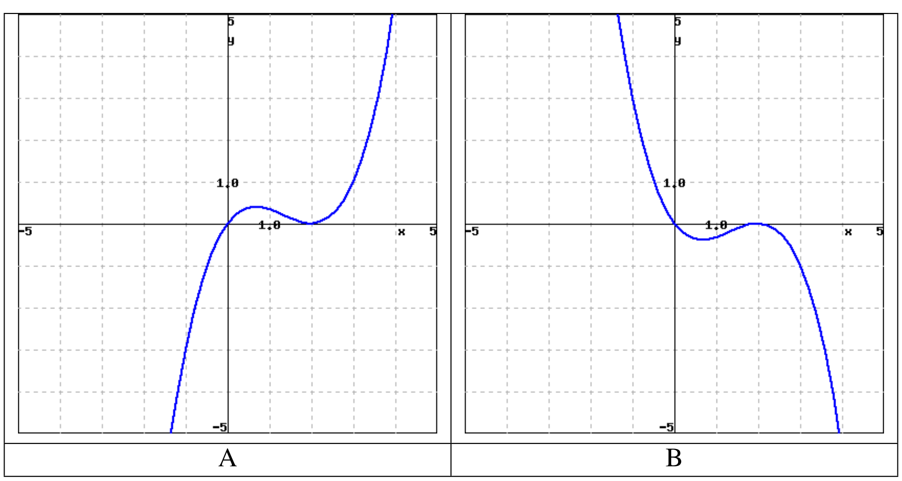
Use properties of functions to match each of the following functions with its graph. *Do not use your calculator.* Clicking on a graph will give you an enlarged view.

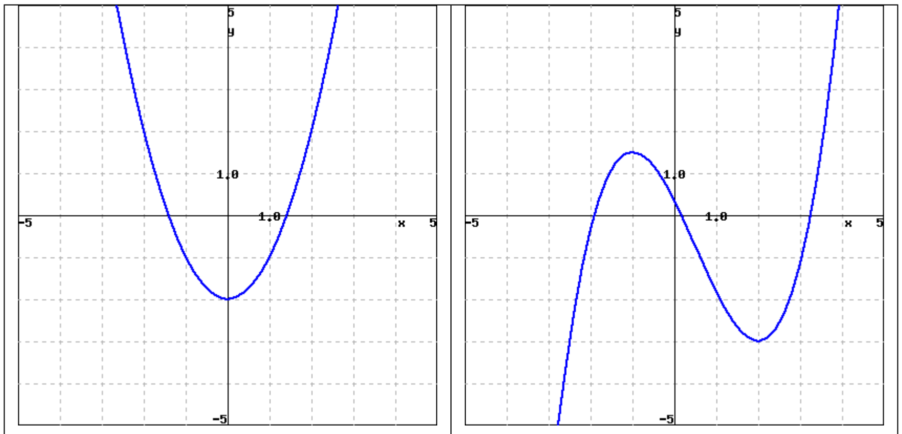
$\boxed{?}$ 1. $f(x) = -x(2-x)^2/3$

$\boxed{?}$ 2. $f(x) = x(2-x)^2/3$

$\boxed{?}$ 3. $f(x) = x^2 - 2$

$\boxed{?}$ 4. $f(x) = x^3/3 - x^2/2 - 2x + 1/3$





C | D

Correct Answers:

- B
- A
- C
- D

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7. (4 points) Relative to the graph of

$$y = x^3$$

the graphs of the following equations have been changed in what way?

___1. $y = (x+19)^3$
___2. $y = 19^3 x^3$
___3. $y = x^3 - 19$
___4. $y = x^3 + 19$

A. shifted 19 units left
B. compressed horizontally by the factor 19
C. shifted 19 units up
D. shifted 19 units down

Correct Answers:

- A
- B
- D
- C

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8. (3 points)
Find a formula for the inverse of the function.

$f(x) = \frac{4x-1}{2x+3}$.
$f^{-1}(x) =$ ______________________
Correct Answers:

- `(3x+1)/(4-2x)`

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9. (3 points)
Find (a) the domain of f, (b) f^{-1}, and (c) the domain of f^{-1}.

$f(x) = \sqrt{3 - e^{2x}}$
(a) $x \leq$ ________
(b) $f^{-1}(x) =$ ________
(c) ________ $\leq x <$ ________

Correct Answers:

- `ln(3)/2`
- `(.5)*ln(3-x^2)`
- `0`
- `3^(1/2)`

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10. (3 points)
Find (a) the domain of f and (b) f^{-1}.

$f(x) = \ln(2 + \ln x)$
(a) $x >$ ________
(b) $f^{-1}(x) =$ ________
Correct Answers:

- `e^(-2)`
- `e^(e^x-2)`

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11. (2 points)
How can you tell from the graph of a function whether it is one-to-one?

(a) Use the Vertical Line Test.
(b) Use the Horizontal Line Test.
(c) None of the above.

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Correct Answers:

- b

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12. (3 points) The population of a region is growing exponentially. There were 10 million people in 1980 (when $t = 0$) and 55 million people in 1990. Find an exponential model for the population (in millions of people) at any time t, in years after 1980.

$P(t)$=__________
What population do you predict for the year 2000?
Predicted population in the year 2000 =__________ million people.
What is the doubling time?
Doubling time =__________ years.

Solution:
SOLUTION
The population has increased by a factor of $\frac{55}{10} = \frac{11}{2}$ in 10 years. Thus we have the formula

$$P(t) = 10(\frac{11}{2})^{t/10},$$

and $t/10$ gives the number of 10 year periods that have passed since 1980.
In 1980, $t/10 = 0$, so we have $P = 10$.
In 1990, $t/10 = 1$, so we have $P = 10(\frac{11}{2}) = 55$.
In 2000, $t/10 = 2$, so we have $P = 10(\frac{11}{2})^2 = \frac{605}{2}$.
To find the doubling time, solve $20 = 10(\frac{11}{2})^{t/10}$ by dividing by 10 and taking the natural logarithm of both sides, to get $t = 10 \cdot \frac{\ln(2)}{\ln(\frac{11}{2})}$.
Correct Answers:

- `10*(55/10)^(t/10)`
- `10*(55/10)^2`
- `10*ln(2)/[ln(55/10)]`

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13. (2 points) A mass is oscillating on the end of a spring. The distance, y, of the mass from its equilibrium point is given by the formula

$$y = 4z\cos(6\pi wt)$$

where y is in centimeters, t is time in seconds, and z and w are positive constants.
(a) What is the furthest distance of the mass from its equilibrium point?
distance = ________________ cm
(b) How many oscillations are completed in 1 second?
number of oscillations = ________________
Solution:
SOLUTION
(a) The furthest distance the mass can travel from its equilibrium point is the amplitude $4z$ of the formula representing its motion.
(b) One complete cyclic is executed when

$$6\pi wt = 2\pi, \quad \text{so} \quad t = \frac{2}{6w}.$$

Therefore, the period is $\frac{2}{6w} = \frac{1}{3w}$ seconds and the number of complete oscillations that take place in 1 second are $3w$.
Correct Answers:

- `4*z`
- `6*w/2`

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14. (2 points)
Solve each equation for x.

(a) $7^{x-5} = 2$
(b) $\ln x + \ln(x-1) = 1$
(a) __________
(b) __________
Correct Answers:

- `5.35620718710802`
- `.5*(1+sqrt(1+4*e))`

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15. (2 points)
Find the domain and the range of $g(x) = \sin^{-1}(3x+1)$.
Domain: _____ $\le x \le$ _____
Range: _____ $\le y \le$ _____
Correct Answers:

- `-0.666666666666667`
- `0`
- `-pi/2`
- `pi/2`

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16. (1 point) Use an addition or subtraction formula to find the exact value of $\tan 75^\circ = \frac{\sqrt{A}+1}{\sqrt{B}-1}$.
$A =$ _____;
$B =$ _____.

Correct Answers:

- `3`
- `3`

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17. (1 point) Sometimes, however, things are simpler than they appear. Using the results of the preceding problem, simplify as much as possible:
$(1-\sin x)(1+\sin x)(1+\tan^2(x)) =$ ______
This is a little more tricky: Simplify as much as possible:
$\sin^4 x - \cos^4 x - \sin^2 x + \cos^2 x =$ ______
Correct Answers:

- `1`
- `0`

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18. (1 point)
Suppose you inscribe a regular octagon into a circle of radius r. Then the area of that octagon is ____________.
Solution:
Solution: Connecting the center of the circle to the vertices of the octagon divides the octagon into 8 congruent triangles with an angle of 45° on the top and a side length of r. According to the result of the preceding question, the area of one such triangle is $r^2 \cos\frac{45^\circ}{2} \sin\frac{45^\circ}{2}$ The area A of the octagon is 8 times that value:

$$A = 8r^2 \cos\frac{45^\circ}{2} \sin\frac{45^\circ}{2}$$

Correct Answers:

- `8*r**2*sin(0.785398163397448/2)*cos(0.785398163397448/2)`