Math1013 Calculus I **Name:** Kin Nam KOO

Homework-8 : Due 12/05/2021 at 11:59pm HKT

This homework set covers the basics of antiderivatives (indefinite integrals) and initial value problems, Riemann sums and definite integrals, Fundamental Theorem of Calculus, and the Substitution Rule.

1. Antiderivatives $\int f(x)dx = F(x)+C \longleftrightarrow \frac{dF}{dx} = f$
2. Riemann sums are just sum of +ve/-ve rectangular area over subintervals, which lead to definite integrals by taking limit.

Give 4 or 5 significant digits for numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as 3^2 or 3**2 instead of 9, $\sin(3*pi/2)$ instead of -1, $e^{\wedge}(\ln(3))$ instead of 3, $(1+\tan(3))*(4-\sin(5))^{\wedge}6-15/8$ instead of 12748.8657, etc.

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1. (5 points)

Find the derivative of $f(x) = -\frac{\sqrt{x^2+25}}{25x}+C$ to complete the following integration formula:

$\int$ _____ $dx = -\frac{\sqrt{x^2+25}}{25x}+C$

Correct Answers:

- `1/(x^2*sqrt(x^2+25))`

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2. (5 points) Calculate the following antiderivatives:

(a) $\int 8t-6t^7+10\,dt =$ _____ $+C$.

(b) $\int \frac{1}{u^{1/4}}+6.5\sqrt{u}\,du =$ _____ $+C$.

(c) $\int \frac{1}{5x^6}\,dx =$ _____ $+C$.

Correct Answers:

- `-6/8*t^8+8/2*t^2+10*t`
- `4/3*u^(3/4)+6.5*2/3*u^(3/2)`
- `1/5*1/(-5)*x^(-5)`

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3. (5 points) Calculate the following antiderivatives:

(a) $\int \frac{3}{x}\,dx =$ _____ $+C$.

(b) $\int -4\sin x+8\cos x\,dx =$ _____ $+C$.

(c) $\int 3e^x\,dx =$ _____ $+C$.

Correct Answers:

- `3*ln(|x|)`
- `4*cos(x)+8*sin(x)`
- `3*e^x`

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4. (5 points) A particle is moving with acceleration $a(t) = 30t+12$. its position at time $t=0$ is $s(0)=10$ and its velocity at time $t=0$ is $v(0)=8$. What is its position at time $t=13$?

_____

Solution:

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To solve this problem, one must remember the relationship between position at time t ($s(t)$), instantaneous velocity at time t ($v(t)$), and acceleration at time t ($a(t)$). This relationship is given by:

$$v(t) = s'(t)$$
$$a(t) = v'(t) = s''(t)$$

With this in mind, this problem becomes an exercise in finding antideriviatives. We are told that the acceleration is given by $a(t) = 30t+12$. The antiderivative (velocity) of this is therefore given by $v(t) = \frac{30}{2}t^2+12t+C_1$. To find out what the constant C_1 is, we use the fact that the velocity of the partical at time $t=0$ is given by $v(0)=8$. Plugging this into our general antiderivative and solving for C_1 yeilds the following.

$$v(0) = 8$$
$$\frac{30}{2}(0)^2+12(0)+C_1 = 8$$
$$C_1 = 8$$

So, the expression for the velocity of the partical at time t is given by $v(t) = 15t^2+12t+8$. Now, we need to find the formula for the partical's position, which is just the antiderivative of this. The general antiderivative of this is $s(t) = \frac{15}{3}t^3+\frac{12}{2}t^2+8t+C_2$ where C_2 is again an unknown constant. However, as before, we can make use of the inforamation given, namely that the position of the partical at time $t=0$ is $s(0)=10$, to find the value of C_2.

$$s(0) = 10$$
$$5(0)^3+6(0)^2+8(0)+C_2 = 10$$
$$C_2 = 10$$

Therefore, the formula for the partical's position is:

$$s(t) = 5t^3+6t^2+8t+10$$

So, the partical's position at time $t=13$ is found by simply plugging 13 into the function $s(t)$. This is $s(13) = 12113$.

Correct Answers:

- `12113`

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5. (5 points) You are testing your brand new Ferrari Testarossa. To see how well the brakes work you accelerate to 100 miles per hour, slam on the brakes, and determine that you brought the car to a stop over a distance of 541 feet. Assuming a constant deceleration you figure that that deceleration is _____ feet per second squared. (Enter a positive number.)

I trust that you don't have the courage to try this, but that night you wonder how long it would take you to stop (with the same constant deceleration) if you were moving at 200 miles per hour. Your stopping distance would be _____ feet. (Enter a number, not an arithmetic expression.)

Correct Answers:

- `19.8808790306018`
- `2164`