GRAVITATION I

PHYS1112

Lecture 13

Intended Learning Outcomes

- After this lecture you will learn:
  1) Newton's law of gravitation, a central inverse square law.
  2) Cavendish experiment to measure gravitational constant.
  3) gravitational force and potential energy.
  4) escape speed from a planet.
  5) satellites in circular obits.

Newton's Law of Gravitation

$$\boxed{F_g = \frac{G m_1 m_2}{r^2}}$$

inverse square law

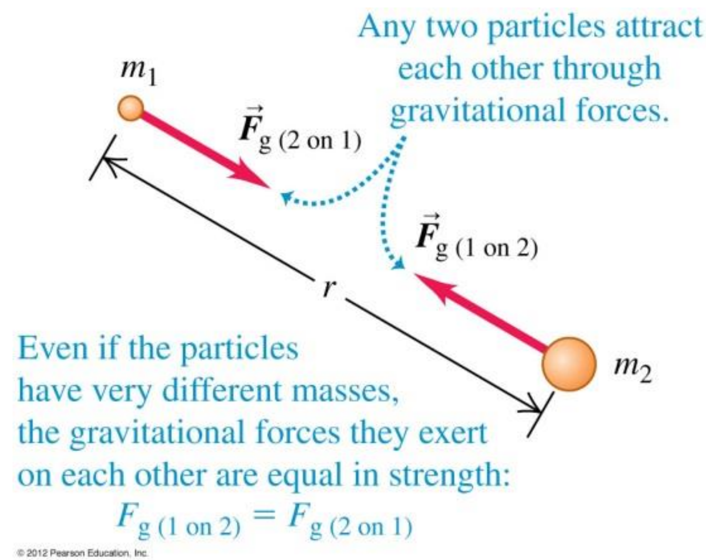
$r^2 + \delta$

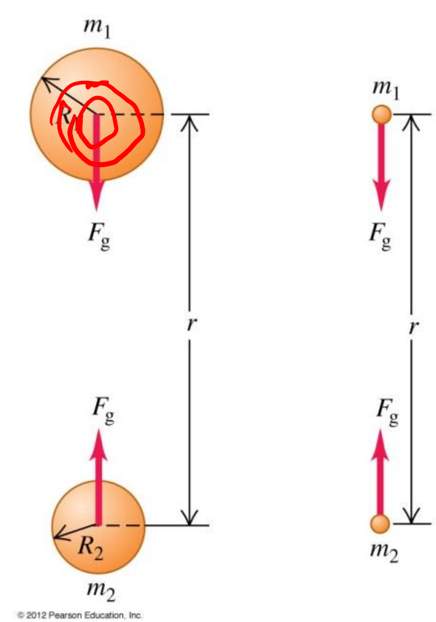
$\delta \to 0$

G: **gravitational constant** $6.67 \times 10^{-11}\ \mathrm{Nm}^2/\mathrm{kg}^2$

Gravitational attraction between two point masses is always along the line joining them (called **central force**), and forms an action-reaction pair

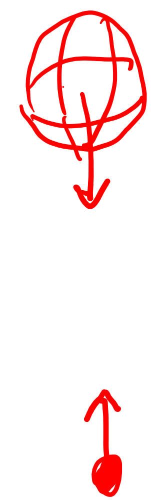
When outside a spherically symmetric body (i.e., density $\rho(r)$ depends on radial distance r only, not on direction), the gravitational effect is the same as if all of the mass were concentrated at its center





Q13.1

The mass of the moon is 1/81 of the mass of the earth. Compared to the gravitational force that the earth exerts on the moon, the gravitational force that the moon exerts on the earth is



A. $81^2 = 6561$ times greater.

B. 81 times greater.

C. equally strong.

D. 1/81 as great.

E. $(1/81)^2 = 1/6561$ as great.

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A13.1

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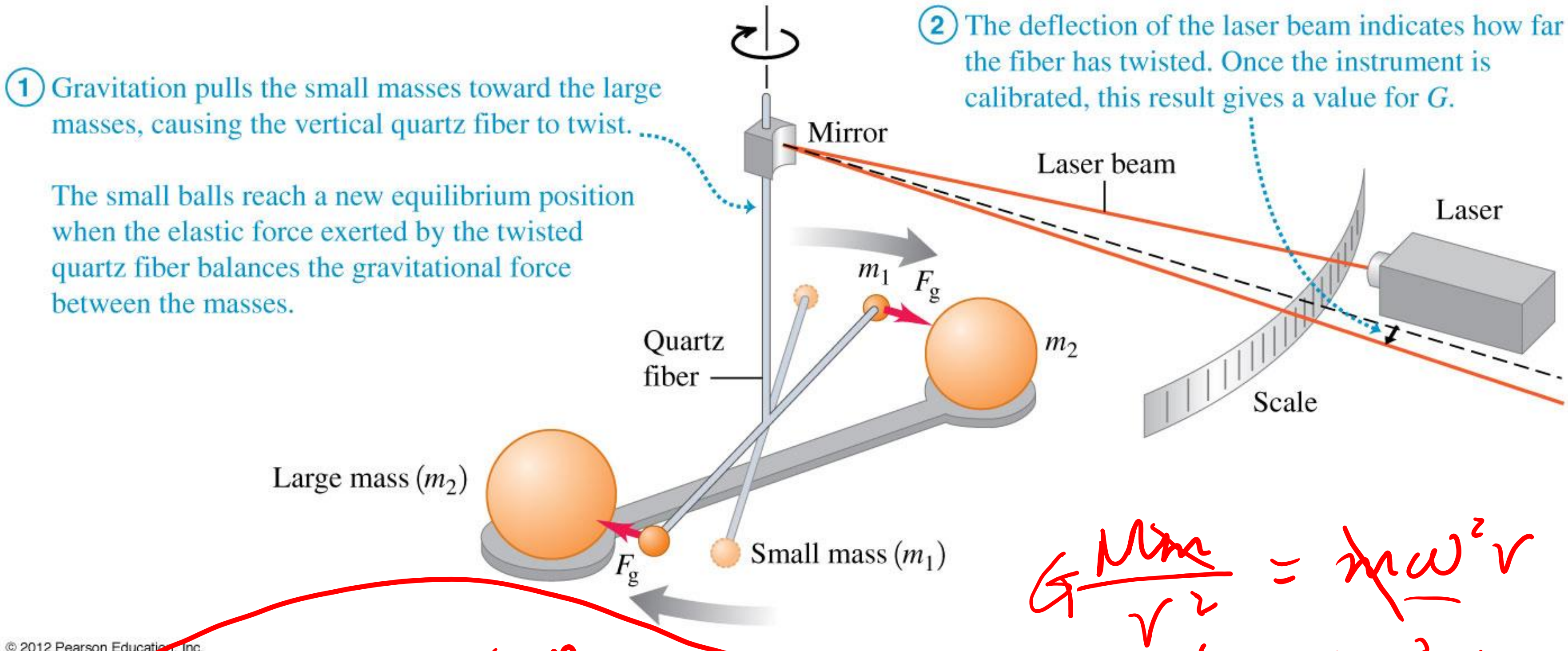
Cavendish Experiment – to measure G (or to "weight the earth")



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$$G\frac{M\cancel{m}}{r^2} = \cancel{m}\omega^2 r$$

$$M = \frac{\omega^2 r^3}{G}$$

$$\omega = \frac{2\pi}{T}$$

Question

Saturn is about 100 times the mass of the earth and about 10 times farther from the sun than the earth. Compared to the acceleration of the earth caused by the sun's gravitational pull, the acceleration of Saturn due to the sun's gravitation is (100 times greater / 10 times greater / the same / $\frac{1}{10}$ as great / $\frac{1}{100}$ as great).

$$G\frac{M\cancel{m}}{r^2} = \cancel{m}a$$

$r^2 \rightarrow 10^2$

Four fundamental forces of nature:

Force	Example	Range
Gravitation force	Hold planets together	∞
Electromagnetic force	Hold molecules together	∞
Strong force	Hold nucleons (protons and neutrons in an atomic nucleus) together	10^{-15} m
Weak force	Beta decay of nuclei	10^{-18} m

$$\vec{F} = k\frac{q_1 q_2}{r^2}$$

Weight – defined as the total gravitational forces exert on the body by all other bodies in the universe
On earth's surface, gravitational attraction by the earth dominates over others

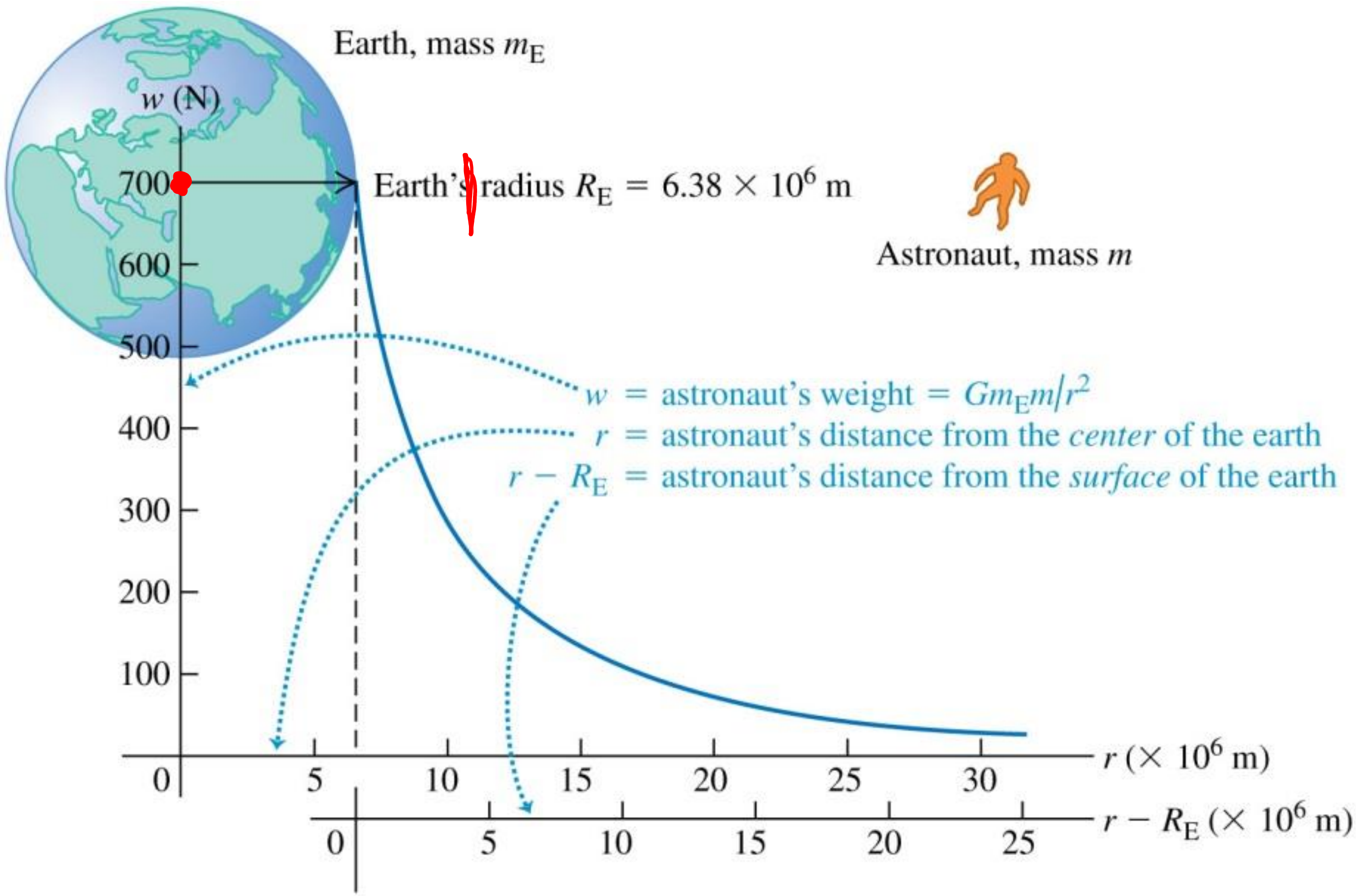
$$W = F_g = \frac{G m_E m}{R_E^2} = mg \quad \Longrightarrow \quad g = \frac{G m_E}{R_E^2}$$

assuming earth is a uniform sphere with radius R_E and mass m_E

$$\text{measure } g = 9.8 \text{ m}/s^2$$

$$R_E = 6.38 \times 10^6 \text{ m}$$

$$\Longrightarrow \quad m_E = 5.974 \times 10^{24} \text{ kg}$$



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when $r > R_E$, weight decreases as $1/r^2$,

$$W = Gm_E m/r^2$$

Question

Rank the hypothetical planets in order from highest to lowest value of g at the surface:

mass $2m_E$, radius $2R_E$; ② $\frac{2}{2^2}$

mass $4m_E$, radius $4R_E$; ③ $\frac{4}{4^2}$

mass $4m_E$, radius $2R_E$; ① $\frac{4}{2^2}$ ✓

mass $2m_E$, radius $4R_E$. ④ $\frac{2}{4^2}$

$$g = G\frac{M}{r^2}$$

$$= G\frac{4 m_E}{2^2 R_E^2}$$

Gravitational Potential Energy – beyond $U = mgy$ (near earth surface only)
Recall: gravitation is a conservative force
Reminder: revisit the properties of conservative forces in Lecture 5

$$\boxed{\int_{r_1}^{r_2} r^{-2}\,dr}$$

work done by gravitational attraction from 1 to 2

path independent, just choose radial path (straight line)

$$-\Delta U = W_{\text{grav}} = \int_1^2 \vec{\boldsymbol{F}}_g \cdot d\vec{\boldsymbol{r}}$$

$$= \int_{r_1}^{r_2} (-F_g\,dr) = \int_{r_1}^{r_2} \left(-G\frac{m_E M}{r^2}\right) dr$$

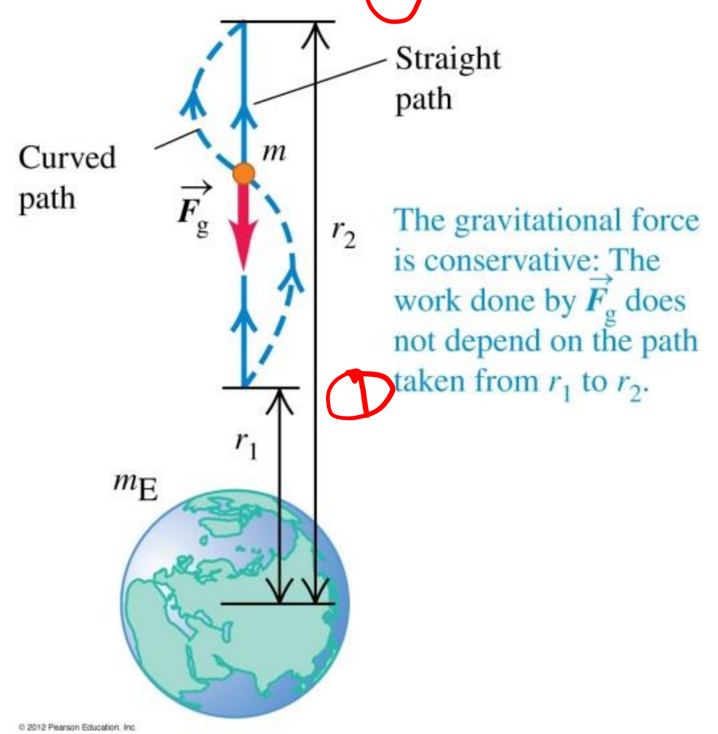
$$= -Gm_E m(-1)\,r^{-1}\Big|_{r_1}^{r_2}$$

$$\vec{\boldsymbol{F}}_g \cdot d\vec{\boldsymbol{r}} = -F_g\,dr$$

inwards outwards

$$= -\left[\left(-\frac{Gm_E m}{r_2}\right) - \left(-\frac{Gm_E m}{r_1}\right)\right] \checkmark$$

$$= -[U(2) - U(1)]$$



Define

$$\boxed{U(r) = -\frac{Gm_E m}{r}}$$

$U(\infty) = 0$, i.e. zero level of PE at ∞

$U(r) < 0$, decreases (more negative) as r decreases

When close to earth surface, $r_1, r_2 \approx R_E$ $\quad -(U(r_2) - U(r_1))$

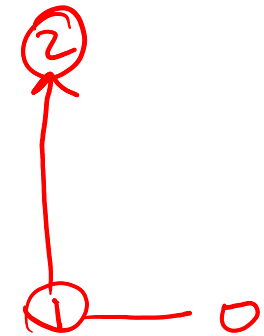
$$W_{\text{grav}} = Gm_E m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

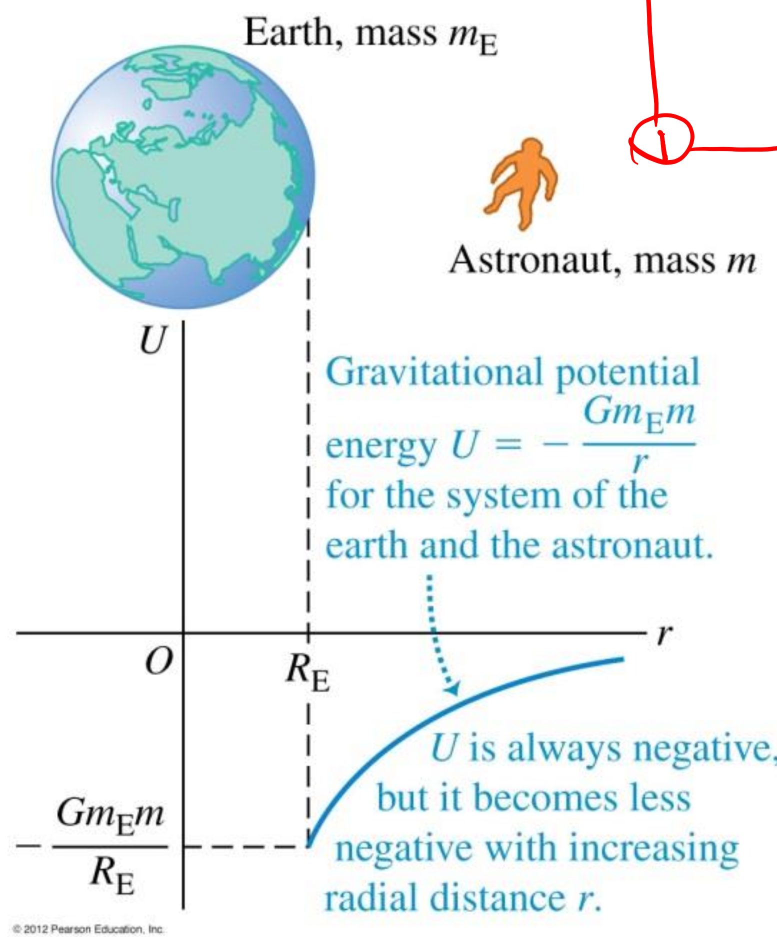
$$= Gm_E m\frac{r_1 - r_2}{r_1 r_2} \approx m\frac{Gm_E}{R_E^2}(r_1 - r_2)$$

$$= -mg(r_2 - r_1)$$

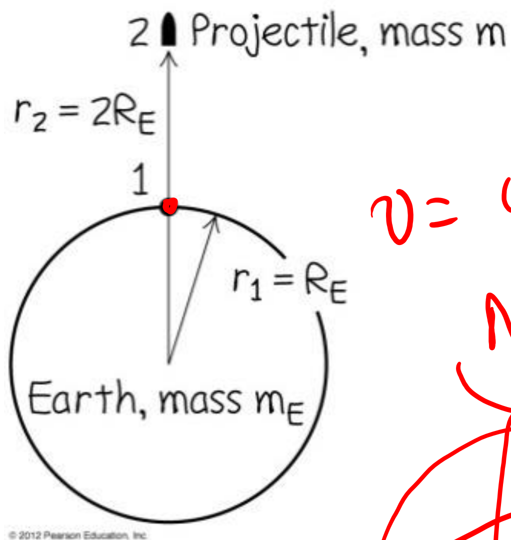
g

same as defining $U = mgy$ with zero level of PE arbitrary



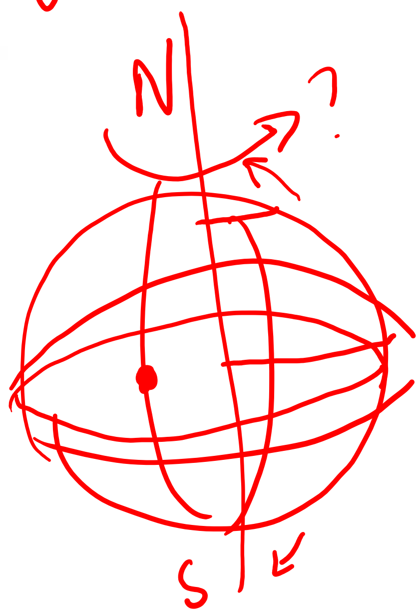


Example Escape speed (2nd cosmic speed)



Shoot a projectile vertically with speed v_1, can it escape from earth's gravitational attraction?

$v = \omega \cdot r$



①

$$\frac{1}{2}\cancel{m}v_1^2 + \left(-\frac{Gm_E\cancel{m}}{R_E}\right) = 0 + 0$$

zero KE, i.e., correspond to minimum v_1

PE at ∞

"black hole"

$$\boxed{c = \sqrt{\frac{2Gm}{R}}}$$

$$\Longrightarrow \boxed{v_1 = \sqrt{\frac{2Gm_E}{R_E}}}$$

escape speed, ⚠ independent of *m*

On substitution

$$v_1 = \sqrt{\frac{2(6.67\times10^{-11}\ \mathrm{Nm^2/kg^2})(5.97\times10^{24}\ \mathrm{kg})}{6.38\times10^{6}\ \mathrm{m}}} = 1.12\times10^{4}\ \mathrm{m/s}$$

⚠ better to launch a spacecraft towards to east ∵ before launching, its already moving to the east at 410 m/s due to earth's rotation

"$v_t = v_E + v_r$"

⚠ air molecules at room temperature ~ 500 m/s , → atmosphere exists

Question

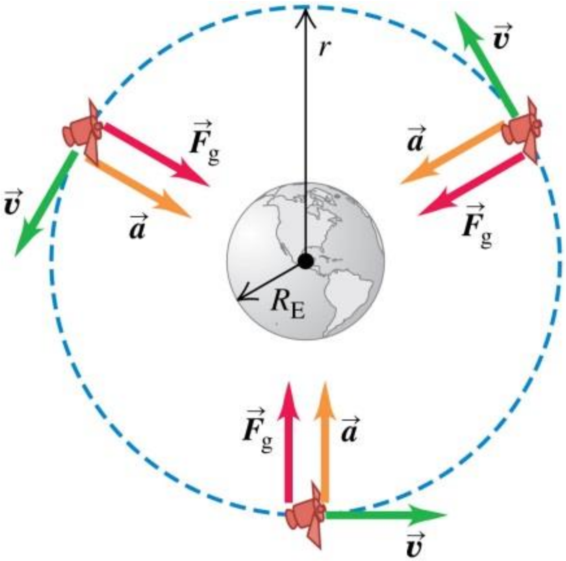
Is it possible for a planet to have the same surface gravity as the earth (i.e., same g) and yet have a greater escape speed?

$\checkmark$

$$v = \sqrt{\frac{2Gm}{r}} \qquad g = \frac{Gm}{r^2}$$

$$= \sqrt{2gr}$$

Satellites – Assuming circular orbit



⚠ No tangential force, v must be constant

$$\frac{Gm_E m}{r^2} = m\left(\frac{v^2}{r}\right)$$

centripetal acceleration

$$\Longrightarrow \quad v = \sqrt{\frac{Gm_E}{r}}$$

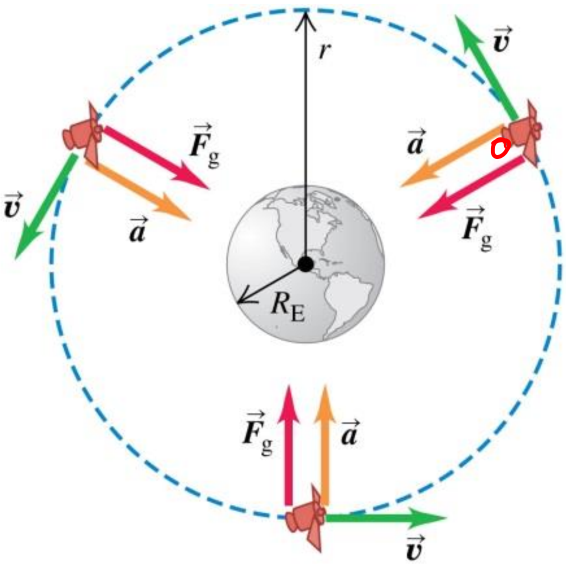
Do not confuse with escape velocity

orbital speed (1st cosmic speed)

⚠ v independent of mass, astronauts orbit about the earth together with spacecraft – <u>apparent weightlessness</u>

<u>True weightlessness</u> only if object is infinitely far from other masses

Satellites – Assuming circular orbit



$$v = \sqrt{\frac{Gm_E}{r}}$$

Period of orbit

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = \frac{2\pi}{\sqrt{Gm_E}}r^{3/2}$$

⚠ larger orbit ⟶ longer period

Total energy in an orbit

$\frac{1}{2} m \frac{G m_E}{r}$

$$E = K + U = \tfrac{1}{2}mv^2 + \left(-\frac{Gm_Em}{r}\right) = \frac{1}{2}\left(-\frac{Gm_Em}{r}\right) = \boxed{\frac{U}{2}}$$

✓

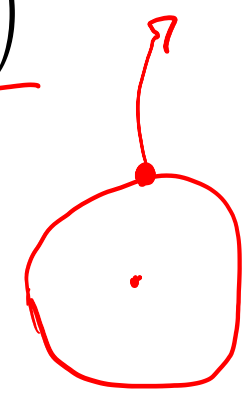
⚠ larger orbit, larger (less –ve) *E*. If spacecraft loses energy (due to air resistance when it is too close to the earth's atmosphere), *r* decreases and eventually falls to the earth

Example

In order to launch a 1000-kg satellite into a circular orbit 300 km above the earth

$$\underbrace{\frac{1}{2}\left(-\frac{Gm_E m}{R_E + 300\text{ km}}\right)}_{\text{total energy in orbit}} = W_{\text{required}} + 0 + \left(-\frac{Gm_E m}{R_E}\right)$$

assume no initial KE



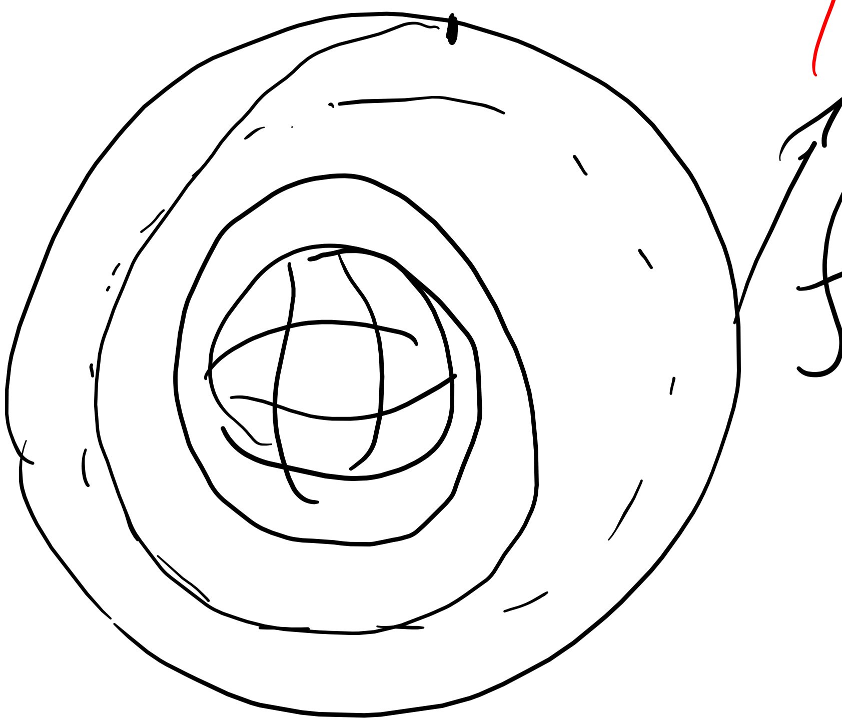
$$\Longrightarrow \quad W_{\text{required}} = 3.26 \times 10^{10}\text{ J}$$

Ignore rotation of the earth so that no KE before launching.
Its contribution is about $\frac{1}{2}(1000\text{ kg})(410\ \text{m/s})^2 = 8.41 \times 10^7$ J,
insignificant compared to W_{required}.

Question

A spacecraft is in a low-altitude circular orbit around the earth. Air resistance from the outer regions of the atmosphere does negative work on the spacecraft, causing the orbital radius to decrease slightly. The speed of the spacecraft (remains the same / increases / decreases).

$$v = \sqrt{\frac{GM}{r}}$$



$$f \propto v^2$$

Q13.8

Star X has twice the mass of the sun. One of star X's planets moves in a circular orbit around star X. This orbit has the same radius as the earth's orbit around the sun. The orbital *speed* of this planet of star X

A. is faster than the earth's orbital speed.

B. is the same as the earth's orbital speed.

C. is slower than the earth's orbital speed.

D. depends on the mass of the planet.

E. depends on the mass and radius of the planet.

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Q13.9

Suppose the sun were to shrink to half of its present radius while maintaining the same mass. What effect would this have on the radius r and the period T of earth's orbit around the sun?

A. r would decrease and T would decrease.

B. r would increase and T would increase.

C. r would decrease and T would increase.

D. r would increase and T would decrease.

E. r and T would both be unchanged.

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C. *r* would decrease and *T* would increase.

D. *r* would increase and *T* would decrease.

E. *r* and *T* would both be unchanged.

$$v = \sqrt{\frac{GM}{r}}$$

$$F = G\frac{Mm}{r^2}$$

$$T = \frac{2\pi r}{v}$$

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