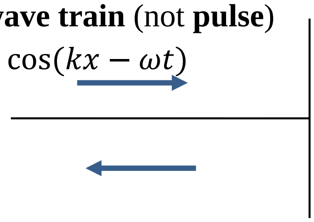
WAVE MOTION AND SOUND II

Intended Learning Outcomes – after this lecture you will learn:
1. standing wave as the result of superposition of incident and reflected wave trains
2. beats due to interference of two traveling waves with *slightly* different frequencies
3. Doppler effect in sound

Textbook Reference: 15.7-15.8, 16.7-16.8

Standing wave – result of superposition between incident and reflected waves

continuous incident **wave train** (not **pulse**)
$A\cos(kx-\omega t)$



For open boundary condition, reflected wave is $A\cos(kx+\omega t)$

Resulting wave:

$$y(x,t) = A\cos(kx-\omega t) + A\cos(kx+\omega t)$$
$$= \underbrace{2A\cos kx}_{\text{sinusoidal amplitude}}\ \underbrace{\cos\omega t}_{\text{time variation}}$$

⚠ not propagating because no $\cos(kx-\omega t)$ term

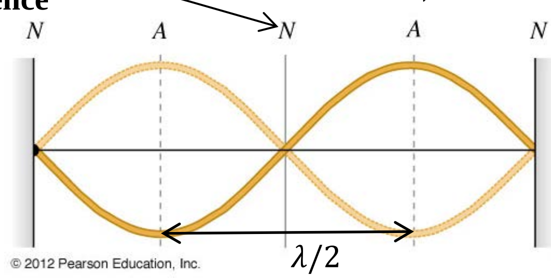
For fixed boundary condition, reflected wave is $-A\cos(kx+\omega t)$

resulting wave:

$$y(x,t) = A\cos(kx-\omega t) - A\cos(kx+\omega t)$$
$$= 2A\sin kx \sin\omega t$$

antinode – maximum amplitude, called **constructive interference**

node – zero amplitude, called **destructive interference**



Demonstration:
1. Standing wave applet
2. Standing waves on vibrating string

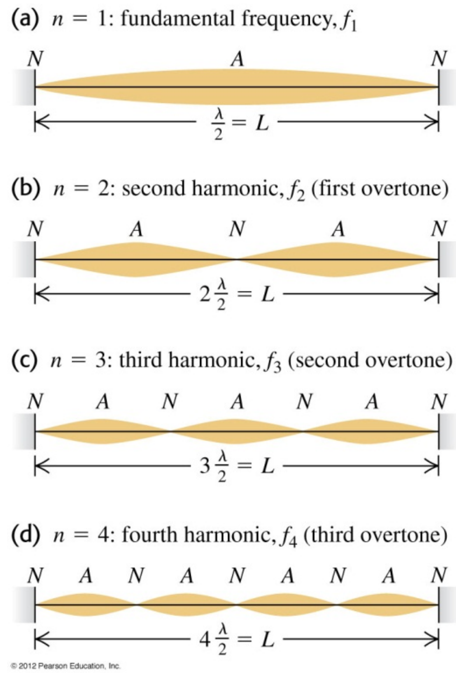
For a string of length L clamped on both ends, **normal modes** of vibration are those standing waves that can be fitted into the string

Normal mode frequencies are

$$L = n\frac{\lambda}{2} \quad \Rightarrow \quad \lambda_n = \frac{2L}{n}, \;\; n = 1, 2, \ldots$$

and frequencies are

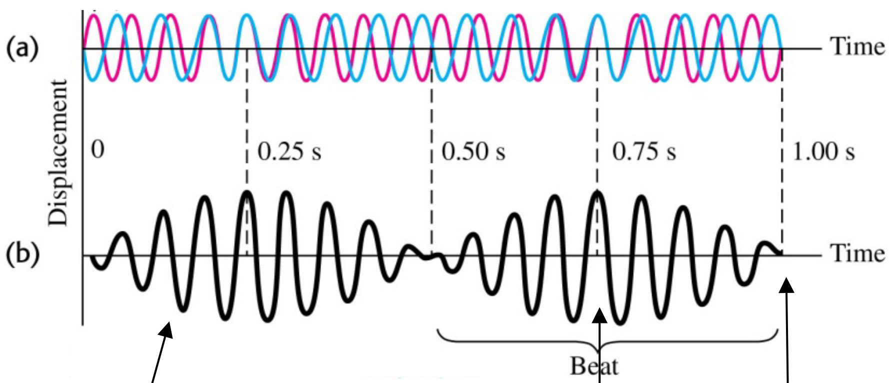
$$f_n = n\left(\frac{v}{2L}\right) = nf_1, \;\; n = 1, 2, \ldots$$



Beats – interference of two traveling waves with *slightly* different frequencies

Consider two waves at a fixed spatial point $x_0 = 0$ for simplicity

$$y_a(t) = A\cos(-2\pi f_a t + \phi_a)$$
$$y_b(t) = A\cos(-2\pi f_b t + \phi_b)$$



rise ⟷ *fall*

only half of the corresponding period

Resulting note at the same point $x_0 = 0$:

$$y_a(t) + y_b(t)$$
$$= \underbrace{2A\cos\left(-2\pi\frac{f_a + f_b}{2}t + \frac{\phi_a + \phi_b}{2}\right)}\underbrace{\cos\left(-2\pi\frac{f_a - f_b}{2}t + \frac{\phi_a - \phi_b}{2}\right)}$$

fast varying with frequency $\frac{1}{2}(f_a + f_b) \approx f_a \approx f_b$

slow varying with frequency $\frac{1}{2}|f_a - f_b|$, hear *__rise__* *__and fall__* in intensity with period

$$T = \frac{1}{2}\frac{1}{\frac{1}{2}|f_a - f_b|} = \frac{1}{|f_a - f_b|}$$

Beat frequency $\boxed{f_{\text{beat}} = |f_a - f_b|}$

Demonstration			
1. Beats		2. Beats animation	

Question

A tuning fork vibrates at 440 Hz, while a second tuning fork vibrates at an unknown frequency. They produce a tone that rises and falls in intensity three times per second. The frequency of the second tuning fork is (434 Hz / 437 Hz / 443 Hz / 446 Hz / either 434 or 446 Hz / either 437 or 443 Hz).

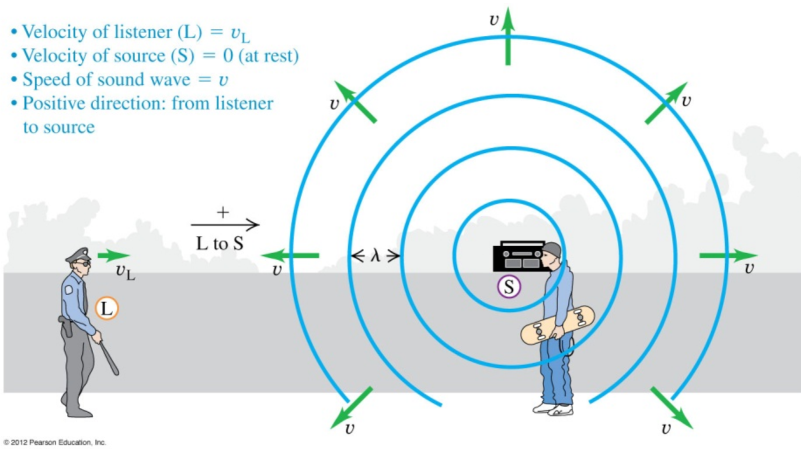
Answer: see inverted text on P. 552

Doppler effect – frequency changes when source and/or observer are “moving”

Demonstration – [a passing-by fire truck]()

Consider mechanical wave (sound as an example) only, *all speeds relative to the medium* (air), which is assumed to be stationary.

Case I: Source not moving (relative to the medium)



⚠ f_S fixed and λ does not change

Listener “hits” wave fronts with speed $v + v_L$

Time to hit 2 consecutive wave fronts

$$T = \frac{\lambda}{v + v_L}$$

$$\therefore\ f_L = \frac{1}{T} = \frac{v + v_L}{\lambda}$$

v/f_S

$$= \left(\frac{v + v_L}{v}\right) f_S$$

If listener *approaching* source, $v_L > 0$ and $f_L > f_S$, hear a higher pitch

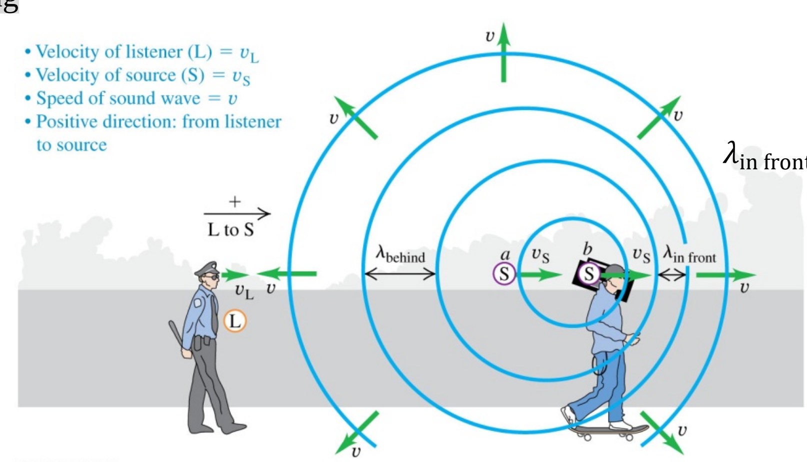
If listener *leaving* source, $v_L < 0$ and $f_L < f_S$, hear a lower pitch

Case II: Source moving

⚠ f_S remains fixed but λ changes.

Distance between consecutive wave fronts:

$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S}$$



$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S}$$

$$f_L = \frac{v + v_L}{\lambda_{\text{behind}}} \quad \Longrightarrow \quad \boxed{f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S}$$

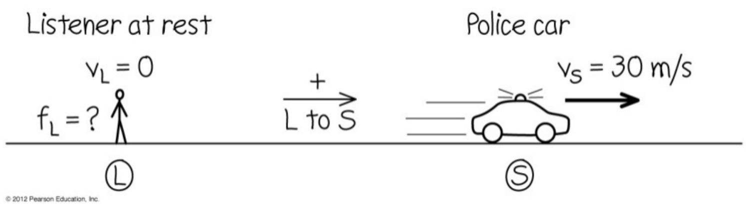
- Sign convention for v_L and v_S – the direction pointing *from listener to source* is taken to be +ve – check that the formula works in all possible cases
- If listener at rest ($v_L = 0$), source approaching listener, then v_S(> / <) 0, and f_L (> / <) f_S
- What if $v_S > v$? A condition called **supersonic**, leads to **shock wave**. Read textbook if you are interested.

Question In an outdoor concert with wind blowing steadily at 10 m/s from the performer towards you, is the sound you hear Doppler-shifted? Answer: see inverted text on P. 557 of textbook

Example 16.15 – 16.17 P. 555

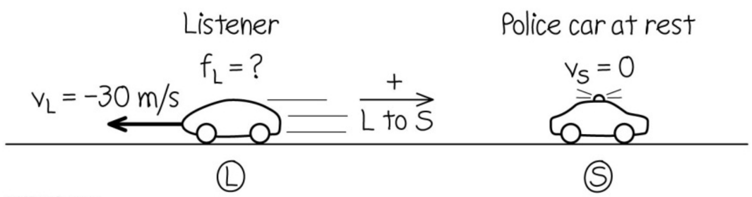
A police car's siren has frequency $f_S = 300$ Hz. Take speed of sound in still air, v, to be 340 m/s

Case I:



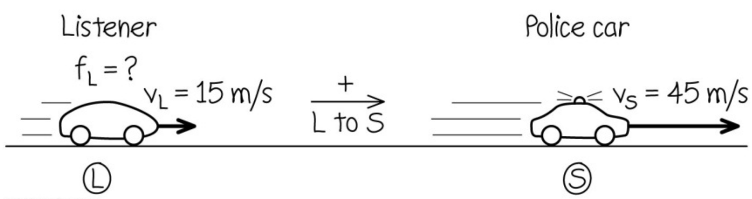
$$f_L = \frac{340\ \mathrm{m/s}}{340\ \mathrm{m/s} + 30\ \mathrm{m/s}}(300\ \mathrm{Hz}) = 276\ \mathrm{Hz}$$

Case II:



$$f_L = \frac{340\ \mathrm{m/s} - 30\ \mathrm{m/s}}{340\ \mathrm{m/s}}(300\ \mathrm{Hz}) = 274\ \mathrm{Hz}$$

Case III:



$$f_L = \frac{340\ \mathrm{m/s} + 15\ \mathrm{m/s}}{340\ \mathrm{m/s} + 45\ \mathrm{m/s}}(300\ \mathrm{Hz}) = 277\ \mathrm{Hz}$$

- In all 3 cases, the source and listener have the same relative velocity, but different f_L, i.e., cannot use either source or listener as frame of reference because there exist an absolute frame of reference – the medium.
- How about waves without medium, such as light? All inertia frame of references are equivalent and Doppler effect can depend on the relative motion of the source and receiver only.

$$\boxed{f_R = \sqrt{\frac{c - v}{c + v}} f_S}$$

v is the relative velocity between source and receiver, +ve if moving away from each other.

Question
If remote star moving away from us, see (red / blue) shift in the light it emits.

Clicker Questions

Q15.9

While a guitar string is vibrating, you gently touch the midpoint of the string to ensure that the string does not vibrate at that point. The lowest-frequency standing wave that could be present on the string vibrates at

A. the fundamental frequency.

B. twice the fundamental frequency.

C. three times the fundamental frequency.

D. four times the fundamental frequency.

E. There is not enough information given to decide.

Q16.8

You hear a sound with a frequency of 256 Hz. The amplitude of the sound increases and decreases periodically: It takes 2 seconds for the sound to go from loud to soft and back to loud. This sound can be thought of as a sum of two waves with frequencies

A. 256 Hz and 2 Hz.

B. 254 Hz and 258 Hz.

C. 255 Hz and 257 Hz.

D. 255.5 Hz and 256.5 Hz.

E. 255.75 Hz and 256.25 Hz.

Q16.9

On a day when there is no wind, you are moving toward a stationary source of sound waves. Compared to what you would hear if you were not moving, the sound that you hear has

A. a higher frequency and a shorter wavelength.

B. the same frequency and a shorter wavelength.

C. a higher frequency and the same wavelength.

D. the same frequency and the same wavelength.

E. none of the above.

Q16.10

On a day when there is no wind, you are at rest and a source of sound waves is moving toward you. Compared to what you would hear if the source were not moving, the sound that you hear has

A. a higher frequency and a shorter wavelength.

B. the same frequency and a shorter wavelength.

C. a higher frequency and the same wavelength.

D. the same frequency and the same wavelength.

E. none of the above.

Ans: Q15.9) B, Q16.8) E, Q16.9) C, Q16.10) A