HKUST

MATH1014 Calculus II

Midterm Examination (White Version) Name:_____

7th Apr 2013 Student ID:_____

10:30-12:00 Tutorial Section:_____

Directions:

- This is a closed book examination. No Calculator is allowed in this examination.
- DO NOT open the exam until instructed to do so.
- Turn off all phones and pagers, and remove headphones. All electronic devices should be kept in a bag away from your body.
- Write your name, ID number, and Tutorial Section in the space provided above, and also in the **Multiple Choice Item Answer Sheet** provided.
- **At the top-right corner of the Multiple Choice Item Answer Sheet**, write the color version of your exam (Green/Orange/Yellow/White).
- DO NOT use any of your own scratch paper. Write your name on every scratch paper supplied by the examination, and do not take any scratch paper away after the examination.
- When instructed to open the exam, please check that you have **7** pages of questions in addition to the cover page.
- Answer all questions. Show an appropriate amount of work for each long problem. If you do not show enough work, you will get only partial credit.
- You may write on the backside of the pages, but if you use the backside, clearly indicate that you have done so.
- **Cheating is a serious violation of the HKUST Academic Code. Students caught cheating will receive a zero score for the examination, and will also be subjected to further penalties imposed by the University.**

Please read the following statement and sign your signature.

I have neither given nor received any unauthorized aid during this examination. The answers submitted are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University's regulations governing academic integrity.

Student's Signature :

Question No.	Points	Out of
Q. 1-12		72
Q. 13		14
Q. 14		14
Total Points		100

Part I: Answer all of the following multiple choice questions.

- **Do not forget to put your name, student ID number, and the color version of your exam paper on the Multiple Choice Item Answer Sheet.**
- **Use an HB pencil to mark your answers to the MC questions on the Multiple Choice Item Answer Sheet provided.**
- Enter also your MC answers to the following boxes **for back-up use only**. The marking will be completely based on the answers on the Multiple Choice Item Answer Sheet.

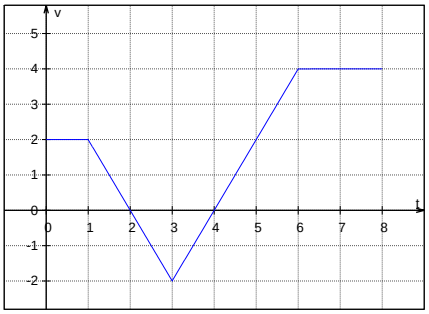
Question	1	2	3	4	5	6	7	8	9	10	11	12
Answer												

Each of the following MC questions is worth 6 points. No partial credit.

1. Use the Trapezoidal Rule on $n = 3$ subintervals of equal length to find an approximate value for the definite integral $\int_0^3 (4 - x^2)\,dx$.

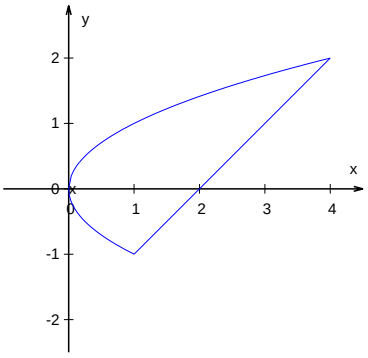
   (a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{9}{2}$ (d) $\frac{5}{4}$ (e) $\frac{7}{4}$

2. The graph of the velocity function $v = v(t)$ of a particle moving along a line is given. What is the distance traveled by the particle between $t = 0$ and $t = 5$? (Time in seconds, velocity in meters per second.)



   (a) 6 m (b) 8 m (c) 10 m (d) 13 m (e) 17 m

3. Find the area of the region enclosed by the curves $y^2 = x$, $y = x - 2$ and the y-axis as shown in the given figure.



(a) $\frac{3}{2}$ (b) $\frac{7}{2}$ (c) $\frac{9}{2}$ (d) $\frac{5}{3}$ (e) $\frac{8}{3}$

4. The region enclosed by the curves $y = \frac{1}{x}$, $x = 1$, $x = 4$ and the x-axis is rotated about the y-axis to generate a solid. Find the volume of the solid.

(a) $\frac{5}{2}\pi$ (b) $\frac{7}{2}\pi$ (c) 4π (d) 6π (e) 8π

5. The base of a solid sitting on the xy-plane is the region enclosed by the curve $y = x^2$ and $y = x$. If the cross sections of the solid perpendicular to the base and parallel to the x-axis are *isosceles right triangles* (i.e., two sides of the right triangle have equal length) with the right angle vertex lying on the curve $y = x^2$. Find the volume of the solid.

(a) $\frac{1}{60}$ (b) $\frac{7}{60}$ (c) $\frac{1}{30}$ (d) $\frac{7}{30}$ (e) $\frac{8}{15}$

6. Evaluate the integral $\int_0^{\frac{\pi}{4}} 3\sin^3(2x)\cos^2(2x)\,dx$.

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{6}{5}$ (e) $\frac{7}{5}$

7. Evaluate the integral $\int_1^3 \frac{x+7}{(x+1)(x+3)}\,dx$.

(a) $2\ln 2 - 3\ln 3$ (b) $2\ln 2 - 5\ln 3$ (c) $3\ln 2 - 2\ln 3$

(d) $5\ln 2 - 2\ln 3$ (e) $5\ln 2 - 3\ln 3$

8. Evaluate the integral $\int_0^2 \frac{1}{\sqrt{x^2+4}}\,dx$

(a) $\frac{1}{2}\ln(\sqrt{2}+1)$ (b) $\ln(\sqrt{2}+1)$ (c) $\frac{3}{2}\ln(\sqrt{2}+1)$

(d) $\ln(\sqrt{2}-1)$ (e) $2\ln(\sqrt{2}-1)$

9. Find the arc length of the curve defined by $y = \frac{4}{3}x^{3/2} - 4$, where $0 \leq x \leq 2$.

(a) $\frac{7}{3}$ (b) $\frac{10}{3}$ (c) $\frac{13}{3}$ (d) $\frac{16}{3}$ (e) $\frac{19}{3}$

10. Evaluate the integral $\int_1^e (\ln x)^2 dx$.

(a) $e - 1$ (b) $e - 2$ (c) $2e - 1$ (d) $2e + 1$ (e) $2e - 3$

11. A thin rod lying on the interval $0 \leq x \leq 2$ (length in meters) has a density function

$$\rho(x) = (1+x)e^{2x}, \quad \text{(kg/m)}.$$

Find the mass of the thin rod.

(a) $\frac{1}{4}(e^4 - 1)$ (b) $\frac{1}{4}(2e^4 - 1)$ (c) $\frac{1}{4}(3e^4 - 3)$

(d) $\frac{1}{4}(4e^4 - 2)$ (e) $\frac{1}{4}(5e^4 - 1)$

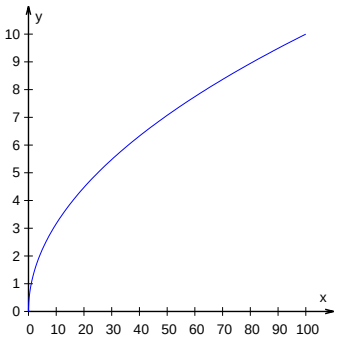
12. The curve on the xy-plane is defined by the polar equation $r = \cos\theta - \sin(2\theta)$. Find the slope of the tangent line to the curve at the point when $\theta = \frac{\pi}{4}$.

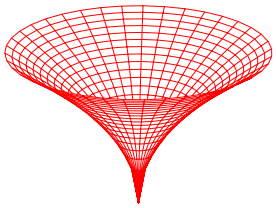
(a) $\frac{1}{-\sqrt{2}+1}$ (b) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (c) $\frac{1+\sqrt{2}}{\sqrt{2}-1}$ (d) $-\frac{\sqrt{2}}{\sqrt{2}-1}$ (e) $\frac{1}{\sqrt{2}-1}$

Part II: Answer each of the following questions.

13. [*14 pts*] The shape of a container is the same as the surface of revolution obtained by rotating the curve $x = y^2$ about the y-axis, where $0 \le x \le 100$ (in meters). Suppose water flows into the container and stops flowing in just when 50% of the volume is filled. (Water density =1000 kg/m^3, and $g = 9.8$ m/s^2.)

(a) Find the minimum work required to pump the water back to the top of the container. [9 pts]





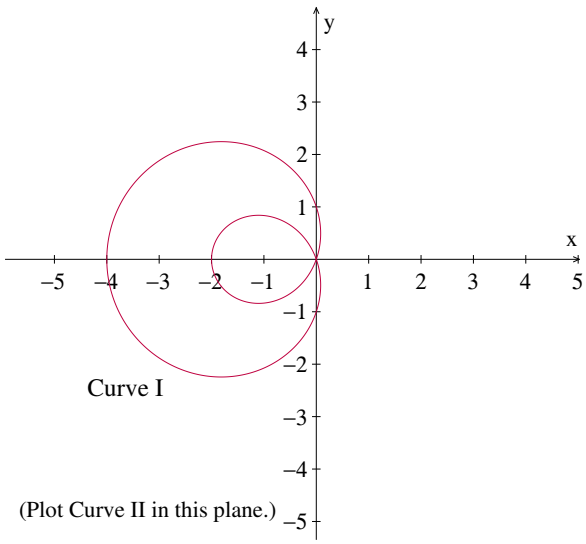
(b) Express by a definite integral the hydrostatic force on the inside surface of the container when 50% of its volume is filled. [5 pts]
You do not need to evaluate the integral.

14. [*14 pts*] Curve I on the xy-plane is defined by the polar equation $r = 1 - 3\cos\theta$ whose graph is given below.

(a) Consider polar Curve II defined by the polar equation $r = 3 + \cos(2\theta)$. Fill in the exact radial coordinates of some points on this curve in the following table, and then sketch Curve II together with Curve I in the given figure. [6 pts]

$r = 3 + \cos(2\theta)$

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
r					
θ	$-\pi/4$	$-\pi/2$	$-3\pi/4$	$-\pi$	
r					



(b) Find the area of the region which lies inside Curve II, but does not overlap with any part of the region enclosed by Curve I. [8 pts]

Math1014 Midterm Exam Formula Sheet

Trigonometric Identities

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$
$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\sin A\cos B = \tfrac{1}{2}\Big(\sin(A+B) + \sin(A-B)\Big)$$
$$\cos A\cos B = \tfrac{1}{2}\Big(\cos(A+B) + \cos(A-B)\Big)$$
$$\sin A\sin B = \tfrac{1}{2}\Big(\cos(A-B) - \cos(A+B)\Big)$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$$

$$\int \cos^n x dx = \frac{1}{n}\sin x\cos^{n-1}x + \frac{n-1}{n}\int \cos^{n-2}x dx$$

$$\int \sin^n x dx = -\frac{1}{n}\cos x\sin^{n-1}x + \frac{n-1}{n}\int \sin^{n-2}x dx$$