

MATH 2111: Tutorial 9

Coordinate Systems, Dimension and Rank

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- The unique representation theorem, coordinate mapping
- The dimension of a vector space, the basis theorem
- The rank of a matrix, the rank theorem
- The invertible matrix theorem

Find the coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ of \mathbf{x} relative to the given basis $\mathcal{B} = \{\mathbf{b}_1, \dots, \mathbf{b}_n\}$

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 5 \\ -6 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}, \mathbf{b}_3 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$$

$$\mathcal{B} = \{b_1, b_2, b_3\} = \left\{ \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 8 \\ -2 \\ 7 \end{bmatrix} \right\}$$

1. Show that the set \mathcal{B} is a basis of \mathbb{R}^3
2. Find the change-of-coordinates matrix from \mathcal{B} to the standard basis.
3. Write the equation that relates x in \mathbb{R}^3 to $[x]_{\mathcal{B}}$

For each subspace, find a basis, and state the dimension

$$\left\{ \begin{bmatrix} 4s \\ -3s \\ -t \end{bmatrix} : s, t \text{ in } \mathbb{R} \right\}$$

$$\left\{ \begin{bmatrix} 3a + 6b - c \\ 6a - 2b - 2c \\ -9a + 5b + 3c \\ -3a + b + c \end{bmatrix} : a, b, c \text{ in } \mathbb{R} \right\}$$

Determine the dimensions of $\text{Nul } A$ and $\text{Col } A$ for the matrices

$$A = \begin{bmatrix} 1 & 0 & 9 & 5 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -4 & 2 & -1 & 6 \\ 0 & 0 & 1 & -3 & 7 & 0 \\ 0 & 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Assume that the matrix A is row equivalent to B . Without calculations, list rank A and $\dim \text{Nul } A$. Then find bases for $\text{Col } A$, $\text{Row } A$, and $\text{Nul } A$.

$$A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \\ 1 & -2 & 0 & 0 & -5 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 0 & 1 & -1 & 3 & 4 & -3 \\ 0 & 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$