Math1014 Calculus II
Definite Integrals: Net Changes, Areas, Volumes

- Review the limit definition of definite integrals, as a summing process through subdivisions of the interval and limit taking:
$$\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=1}^{n} f(x_k^*)$$
where $a = x_0 < x_1 < \cdots < x_n = b$ is a subdivision of the interval $[a, b]$ into subintervals of equal length $\Delta x_k = x_k - x_{k-1} = \frac{b-a}{n}$, and $x_{k-1} \le x_k^* \le x_k$.
- Recall the use of Fundamental Theorem of Calculus in evaluating definite integrals.
$$\int_a^b f(x)dx = F(b) - F(a), \quad \text{if } F'(x) = f(x)$$
(Most of the integrals in the early weeks of the semester should be quite easy to evaluate, up to some simple substitutions.)
- Area between curves: integrate over certain x-interval, or y-interval, whichever is more appropriate or easier to work with.
- Express volumes in terms of definite integrals by the slicing (cross-section) method or cylindrical shell method - cutting the solid into well known pieces. (Often need to consider the area of shapes like : rectangle, triangle, disk, washer, cylinder, etc.)

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1. If the birth rate of a population is $b(t) = 2200e^{0.024t}$ people per year, and the death rate is $d(t) = 1460e^{0.018t}$ people per year, find the area between these curves for $0 \le t \le 10$. What does this area represent?

2. If the amount of capital that a company has at time t is $f(t)$, then the derivative $f'(t)$ is called the *net investment flow*. Suppose that the net investment flow is $\sqrt{t}$ million dollars per year (where t is measured in year). Find the *increase in capital* (also called the *capital formation*) from the fourth year to the eighth year.

3. Evaluate the integral $\displaystyle\int_0^4 \left|\sqrt{x+2} - x\right| dx$ and interpret it as the area of a region. Sketch the region.

4. Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

   (i) $4x + y^2 = 12$, $x = y$. (ii) $y = 3x^2$, $y = 8x^2$, $4x + y = 4$, $x \ge 0$.

5. Consider the area under the curve $y = \dfrac{1}{x^2}$, $1 \le x \le 4$.

   (i) Find the number a such that $x = a$ bisects the given area.

   (ii) Find the number b such that $y = b$ bisects the given area.

6. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid and a typical disk or washer formed by the revolution.

   (i) $y = e^{-x}$, $y = 1$, $x = 2$; about $y = 2$. (ii) $y = x$, $y = \sqrt{x}$; about $x = 2$.

7. Find the volume of the described solid S.

(i) The base of S is a circular disk with radius r. Parallel cross-sections perpendicular to the base are squares.

(ii) The base of S is a circular disk with radius r. Parallel cross-sections perpendicular to the base are isosceles triangles with height h and unequal side in the base.

(iii) The common region S of two cylinders with the same radius r, if the axes of the cylinders intersect at right angles.

8. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis. Sketch the region and a typical shell.

   (i) $y = x^2$, $y = 2 - x^2$; about $x = 1$. (ii) $y = x^2$, $x = y^2$; about $y = -1$.