Math1014 Calculus II

Week 8-9: Brief Review and Some Practice Problems

Improper Integrals

- Evaluating *improper integrals* by taking suitable limits of ordinary integrals.
- Determining *convergence or divergence of improper integrals* by comparison.

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1. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

   (i) $\displaystyle\int_{-\infty}^{0} \frac{1}{2x-5}dx$ (ii) $\displaystyle\int_{0}^{\infty} \frac{x}{(x^2+2)^2}dx$ (iii) $\displaystyle\int_{-\infty}^{1} e^{-2t}dt$

   (iv) $\displaystyle\int_{0}^{\infty} \frac{1}{z^2+3z+2}dz$ (v) $\displaystyle\int_{-\infty}^{6} re^{r/3}dr$ (vi) $\displaystyle\int_{2}^{3} \frac{1}{\sqrt{3-x}}dx$

   (vii) $\displaystyle\int_{6}^{8} \frac{1}{(x-6)^3}dx$ (viii) $\displaystyle\int_{0}^{2} \frac{e^{1/x}}{x^3}dx$ (ix) $\displaystyle\int_{0}^{1} \frac{\ln x}{\sqrt{x}}dx$

2. The average speed of molecules in an ideal gas is

   $$\bar{v} = \frac{4}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{3/2}\int_{0}^{\infty} v^3 e^{-Mv^2/(2RT)}dv$$

   where M is the molecular weight of the gas, R is the gas constant, T is the temperature, and v is the molecular speed. Show that $\bar{v} = \sqrt{\dfrac{8RT}{\pi M}}$. (*Hint: Make a substitution* $u = Mv^2/(2RT)$ *to simplify the calculation.*)

3. Find the value of the constant C for which the integral

   $$\int_{0}^{\infty}\left(\frac{x}{x^2+1} - \frac{C}{3x+1}\right)dx$$

   converges. Evaluate the integral for this value of C.

4. Show that if $a > -1$, and $b > a+1$, then the following integral is convergent.

   $$\int_{0}^{\infty} \frac{x^a}{1+x^b}dx$$