

COMP 2711 Discrete Mathematical Tools for Computer Science
2022 Fall Semester – Tutorial 11

- Question 1:**
- (a) Find a recurrence relation for the number of bit strings of length n that contain a pair of consecutive 0s.
 - (b) What are the initial conditions?
 - (c) How many bit strings of length seven contain two consecutive 0s?

- Question 2:** You are given a real number a and a positive integer n that is a power of 2, i.e. $n = 2^k$ for some integer $k \geq 0$.
- (a) Devise a recursive algorithm to find a^n . Your algorithm should use as few multiplications as possible.
 - (b) Give a recurrence equation of the number of multiplications used in your algorithm in (a).
 - (c) Solve your recurrence equation in (b).

Note that in this question, we assume

- (i) b^m uses $m-1$ multiplications for any real number b and positive integer m .
- (ii) b/m is not counted as a multiplication if m is a positive integer.
- (iii) In the computation of $(f(n))^m$, $f(n)$ is evaluated only once. But, in the computation of $f(n) \cdot f(n)$, $f(n)$ is evaluated twice.

- Question 3:** The **reversal** of a string is the string consisting of the symbols of the string in reverse order. The reversal of the string w is denoted by w^R .
- (a) Give a recursive definition of the reversal of a string. [*Hint:* First define the reversal of the empty string. Then write a string w of length $n+1$ as xy , where x is a string of length n , and express the reversal of w in terms of x^R and y .]
 - (b) Use structural induction to prove that $(w_1w_2)^R = w_2^Rw_1^R$.

- Question 4:** Show that a simple graph G with n vertices is connected if it has more than $(n-1)(n-2)/2$ edges.