

MATH 2111: Tutorial 3

Matrix Equation and Solution Sets of Linear Systems

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- Relationship between matrix equation and vector equation
- Computation of matrix equation
- Homogeneous Linear Systems
- Parametric Vector Form

Write the matrix equation as a vector equation, or vice versa

$$\begin{bmatrix} 5 & 1 & -8 & 4 \\ -2 & -7 & 3 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

(1) Could a set of n vectors in \mathbb{R}^m span all of \mathbb{R}^m if $n < m$? Explain.

(2) Suppose A is a 3×3 matrix and \mathbf{b} is a vector in \mathbb{R}^3 with the property that $A\mathbf{x} = \mathbf{b}$ has a unique solution. Explain why the columns of A must span \mathbb{R}^3 .

Determine if the columns of the matrix span \mathbb{R}^4

$$\begin{bmatrix} 5 & -7 & -4 & 9 \\ 6 & -8 & -7 & 5 \\ 4 & -4 & -9 & -9 \\ -9 & 11 & 16 & 7 \end{bmatrix}$$

Determine if the system has a nontrivial solution

$$2x_1 - 5x_2 + 8x_3 = 0$$

$$-2x_1 - 7x_2 + x_3 = 0$$

$$4x_1 + 2x_2 + 7x_3 = 0$$

$$x_1 - 3x_2 + 7x_3 = 0$$

$$-2x_1 + x_2 - 4x_3 = 0$$

$$x_1 + 2x_2 + 9x_3 = 0$$

Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form

$$A = \begin{bmatrix} 1 & 5 & 2 & -6 & 9 & 0 \\ 0 & 0 & 1 & -7 & 4 & -8 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(1) Suppose \mathbf{w}, \mathbf{p} are two solutions of the equation $A\mathbf{x} = \mathbf{b}$ and define $\mathbf{v}_h = \mathbf{w} - \mathbf{p}$. Show that \mathbf{v}_h is a solution of $A\mathbf{x} = \mathbf{0}$.

(2) Suppose $A\mathbf{x} = \mathbf{b}$ has a solution. Explain why the solution is unique precisely when $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.