Little Summary

1 Theorem (Uniqueness of solutions to linear systems)

For a matrix A, the following are equivalent:
(a) $A\mathbf{x} = \mathbf{0}$ has no non-trivial solution (i.e. $\mathbf{x} = \mathbf{0}$ is the only solution).
(b) If $A\mathbf{x} = \mathbf{b}$ is consistent, then it has a unique solution.
(c) The columns of A are linearly independent.
(d) A has a pivot position in every column (i.e. all variables are basic).
(e) The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.
(f) The kernel of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is $\{\mathbf{0}\}$.

2 Theorem (Existence of solutions to linear systems)

For an $m \times n$ matrix A, the following statements are logically equivalent:
(a) For each $\mathbf{b}$ in $\mathbb{R}^m$, the equation $A\mathbf{x} = \mathbf{b}$ has a solution.
(b) Each $\mathbf{b}$ in $\mathbb{R}^m$ is a linear combination of the columns of A.
(c) The columns of A span $\mathbb{R}^m$.
(d) A has a pivot position in every row.
(e) The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto.
(f) The range of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is $\mathbb{R}^m$.

3 Geometric Linear Transformations of $\mathbb{R}^2$

(Pick from textbook)

TABLE 1 **Reflections**

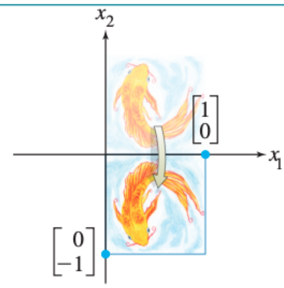
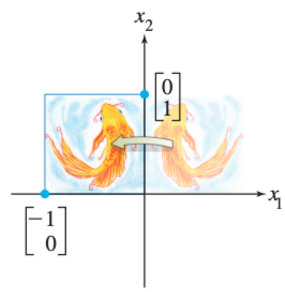
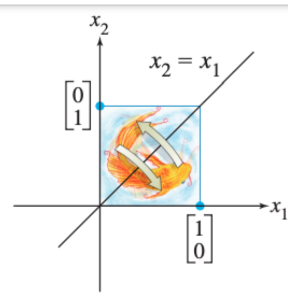
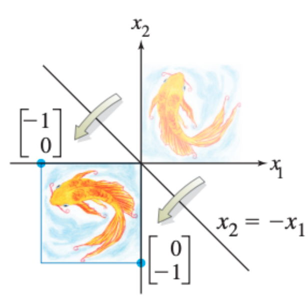
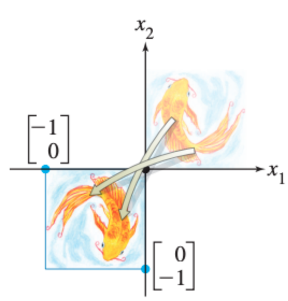
Transformation	Image of the Unit Square	Standard Matrix
Reflection through the x_1-axis		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Reflection through the x_2-axis		$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
Reflection through the line $x_2 = x_1$		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Reflection through the line $x_2 = -x_1$		$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
Reflection through the origin		$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

TABLE 2 **Contractions and Expansions**

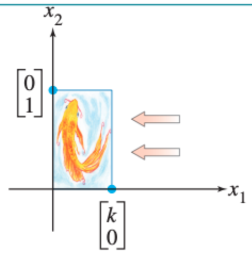
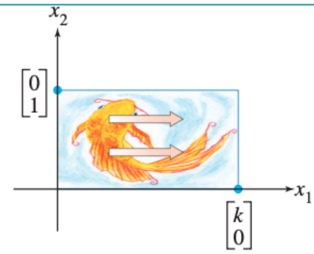
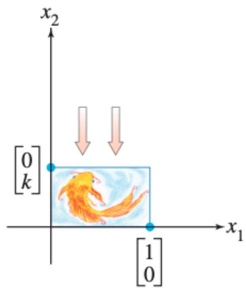
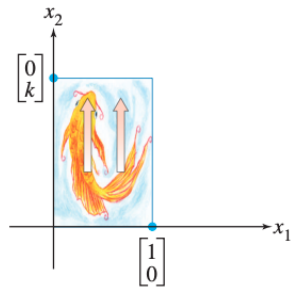
Transformation	Image of the Unit Square		Standard Matrix
Horizontal contraction and expansion	 $0 < k < 1$	 $k > 1$	$\begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$
Vertical contraction and expansion	 $0 < k < 1$	 $k > 1$	$\begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$

TABLE 3 **Shears**

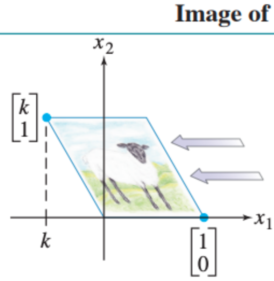
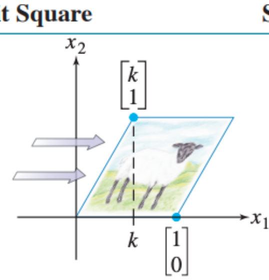
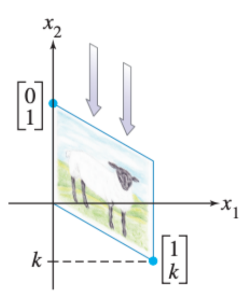
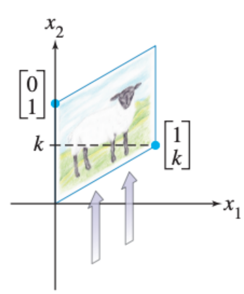
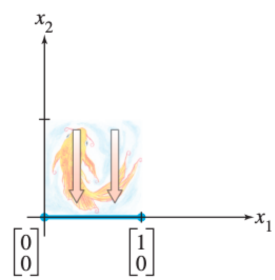
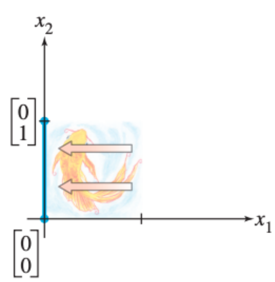
Transformation	Image of the Unit Square		Standard Matrix
Horizontal shear	 $k < 0$	 $k > 0$	$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$
Vertical shear	 $k < 0$	 $k > 0$	$\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$

TABLE 4 **Projections**

Transformation	Image of the Unit Square	Standard Matrix
Projection onto the x_1-axis		$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Projection onto the x_2-axis		$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$