Math2001 Answer to Homework 3

Exercise 2.18

(1). $[0,1] \cup (2,4)$.
(2). $[0,1] \cup (2,5)$.
(3). $[0,4)$.
(4). $[-5,5)$.
(5). $[0,1] \cup (2,4)$.
(6). $(-1,1) \cup (1,2) \cup (2,3) \cup (3,4)$.
(7). $[-5,-4) \cup (-4,-2) \cup (-2,0) \cup (0,2) \cup (2,4) \cup (4,5)$.

Exercise 2.19

(1). $10\mathbb{Z}$.
(2). $\mathbb{R}$.
(3). $\{2,4\}$.

Exercise 2.21

(1). $X \cap (Y \cup Z)$.
(2). $(X \cap Y) \cup Z$.
(3). $(X \cap Z) - Y$.

Exercise 2.22

(1). If $x \in Z$, then $x \in X$ and $x \in Y$. Thus we have $x \in X \cap Y$. Therefore $x \in Z$ implies $x \in X \cap Y$. In other words, we have $Z \subset X \cap Y$.

Exercise 2.24

(1). T. (2). T. (3). F. (4). F. (5). T. (6). T. (7). T.

Exercise 2.25

If $x \in X - (X - Y)$, then $x \in X$ and $x \notin X - Y$. $x \notin X - Y$ implies $x \notin X$ or $x \in Y$. Equivalently, there is $x \in X$ and $x \notin X - Y$ imply $x \in Y$. Thus $x \in X - (X - Y)$ implies that $x \in X$ and $x \in Y$, which, in other words, is saying that $x \in X \cap Y$. Therefore, there is $X - (X - Y) \subset X \cap Y$.

If $x \in X \cap Y$, then $x \in X$ and $x \in Y$. $x \in Y$ implies that $x \notin X - Y$. $x \in X$ and $x \notin X - Y$ imply that $x \in X - (X - Y)$. Hence there is $X \cap Y \subset X - (X - Y)$.

Therefore, $X - (X - Y) = X \cap Y$.

Exercise 2.26

If $x \in X$, then $x \in Y$ or $x \in Z$. Since $X = Y \sqcup Z$, the proposition $x \in Y \cap Z$ is always false. In other words, the proposition '$x \in Y$ and $x \in Z$' is false.Thus $x \in Z$ implies $x \notin Y$.

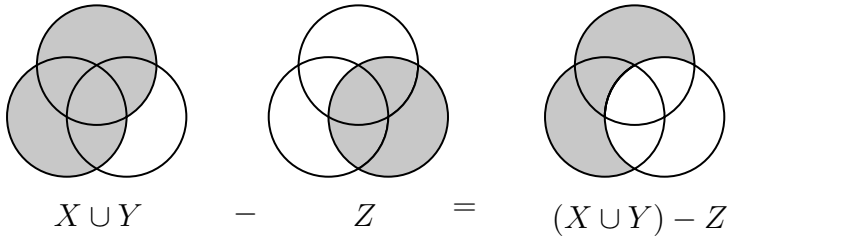
If $x \in Z$, then $x \in X$ and $x \notin Y$. Hence $x \in X - Y$.

If $x \in X - Y$, then $x \in X$ and $x \notin Y$. Since $x \in X$ implies $x \in Y$ or $x \in Z$, $x \in X$ and $x \notin Y$ imply that $x \in Z$. Therefore, $X - Y = Z$.

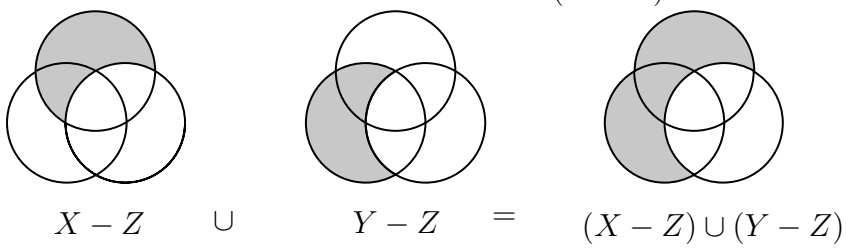
By Excercise 2.25, $X - Z = X - (X - Y) = Y$.

Exercise 2.27

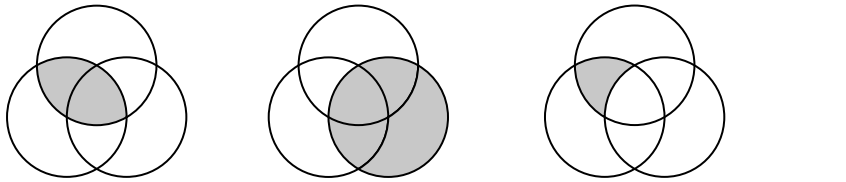
(4).



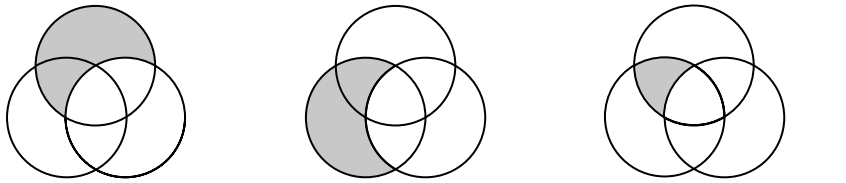
$X \cup Y \quad - \quad Z \quad = \quad (X \cup Y) - Z$



$X - Z \quad \cup \quad Y - Z \quad = \quad (X - Z) \cup (Y - Z)$

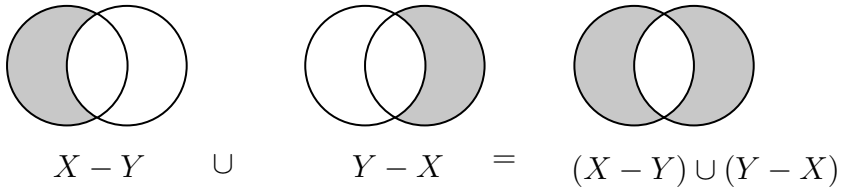


$X \cap Y \quad - \quad Z \quad = \quad (X \cup Y) - Z$

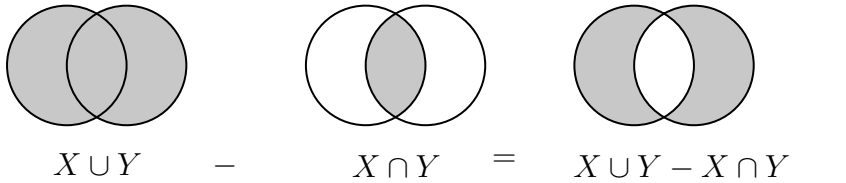


$X - Z \quad \cap \quad Y - Z \quad = \quad (X - Z) \cap (Y - Z)$

(5).

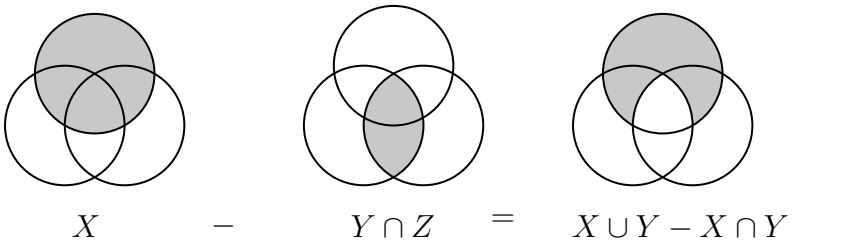


$X - Y \quad \cup \quad Y - X \quad = \quad (X - Y) \cup (Y - X)$

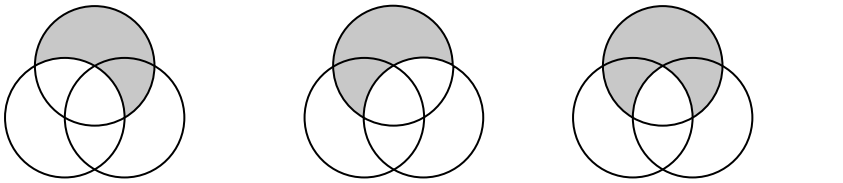


$X \cup Y \quad - \quad X \cap Y \quad = \quad X \cup Y - X \cap Y$

Exercise 2.29



$X \quad - \quad Y \cap Z \quad = \quad X \cup Y - X \cap Y$



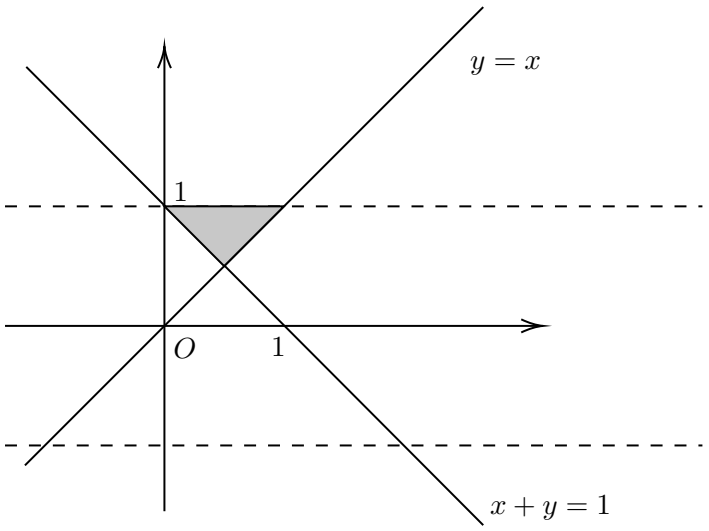
$X - Y \quad \cup \quad X - Z \quad = \quad (X - Y) \cup (X - Z)$

Exercise 2.30

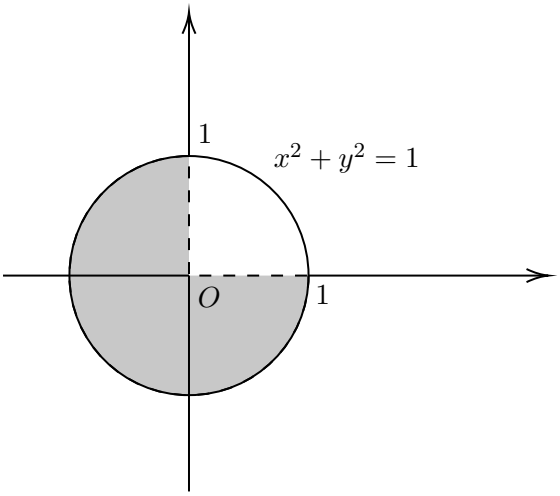
(1). F. (2). T. (3). T. (4). T. (5). F.

Exercise 2.32

(1).



(2).



Exercise 2.35

(1). $(1, 3) \times [2, 4]$.

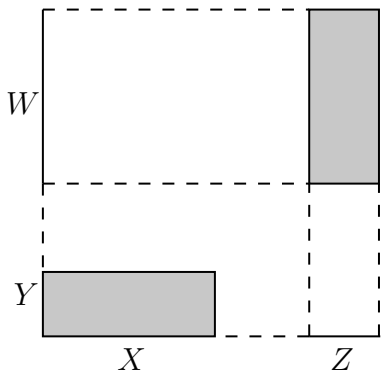
(2). $(1, 3) \times [2, 4] \sqcup (2, 3) \times (4, 5] \sqcup [3, 4) \times [3, 5]$.

(3). $(2, 3) \times [3, 4]$.

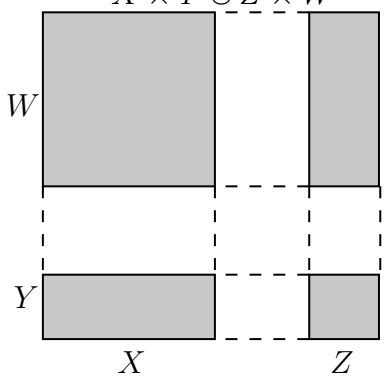
(4). $(1, 2] \times [2, 4] \sqcup (2, 3) \times [2, 3)$.

Exercise 2.36

(3). The equality doesn't hold.

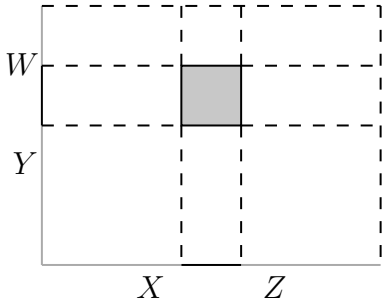


$X \times Y \cup Z \times W$

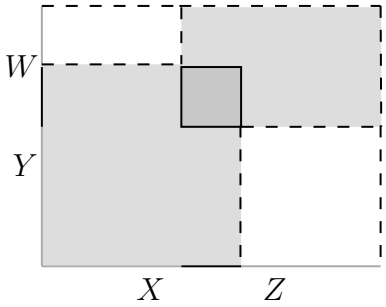


$(X \cup Z) \times (Y \cup W)$

(4). The equality holds.



$(X \cap Z) \times (Y \cap W)$



$(X \times Y) \cap (Z \times W)$