Math1013 Calculus I **Name:** Kin Nam KOO

Homework-4 : Due 10/31/2021 at 11:59pm HKT

The problems in this homework set cover the basic concepts of limits of functions and their calculation. You need to know:

1. the limit definition of derivative;
2. derivatives as rates of change, or slopes of tangent lines;
3. derivative formulas of elementary functions;
4. basic rules of differentiation: power rule, product and quotient rules, chain rule.

Give 4 or 5 significant digits for numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as 3^2 or 3**2 instead of 9, sin(3 * pi/2) instead of -1, e^(ln(3)) instead of 3, (1+tan(3)) * (4−sin(5))^6−15/8 instead of 12748.8657, etc.

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1. (2 points) Let $f(x)$ be the function $\frac{1}{x+7}$. Then the quotient $\frac{f(4+h)-f(4)}{h}$ can be simplified to $\frac{-1}{ah+b}$ for:

$a =$_____

and

$b =$ _____

Correct Answers:

- 11
- 121

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2. (3 points) Let

$$f(x) = \begin{cases} 4x - 2x^2 & \text{for } x < 0\,, \\ 5x^2 + 4x & \text{for } x \geq 0\,. \end{cases}$$

According to the definition of the derivative, to compute $f'(0)$, we need to compute the left-hand limit

$\lim_{x\to 0^-}$ _____, which is _____,

and the right-hand limit

$\lim_{x\to 0^+}$ _____, which is _____.

We conclude that $f'(0)$ is _____

Note: If a limit or derivative is undefined, enter 'undefined' as your answer.

Correct Answers:

- (4*x-2*x^2)/x
- 4
- (5*x^2+4*x)/x
- 4
- 4

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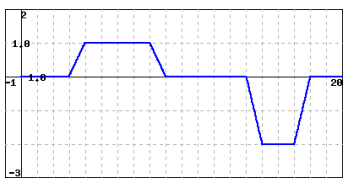
3. (3 points) A child fills a pail by using a water hose. After finishing, the child plays in a sandbox for a while before tipping the pail over to empty it. If $V(t)$ gives the volume of the water in the pail at time t, then the figure below shows $V'(t)$ as a function of t.



At what time does the child:

A. Begin to fill the pail? $t =$ ___

B. Finish filling the pail? $t =$ ___

C. Tip the pail over? $t =$ ___

(What would the graph of $V'(t)$ look like if the child filled the pail by using a play shovel to repeatedly scoop water from a larger bucket and dump it in the pail instead of using a hose?)

Solution:

SOLUTION

The child begins filling the pail when $V'(t)$ becomes positive, which is when the volume of water in the pail starts increasing. This is at $t = 3$. The child finishes filling the pail when the volume stops increasing, which is at $t = 9$. The child tips the pail over when the volume of water in the pail start decreasing, which is where $V'(t)$ first becomes negative, which is at $t = 14$.

Correct Answers:

- 3
- 9
- 14

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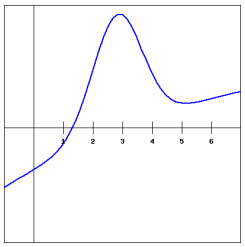
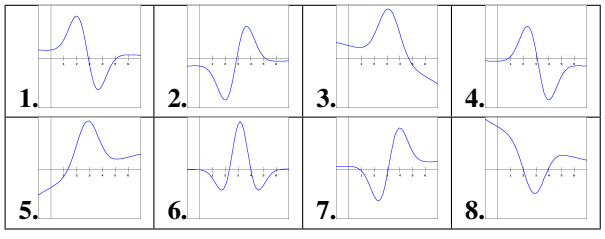
4. (2 points) For the function $f(x)$ shown in the graph below, sketch a graph of the derivative. You will then be picking which of the following is the correct derivative graph, but should be sure to first sketch the derivative yourself.



Which of the following graphs is the derivative of $f(x)$? [?/1/2/3/4/5/6/7/8]

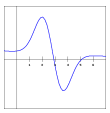
(Click on a graph to enlarge it.)



Solution:

SOLUTION

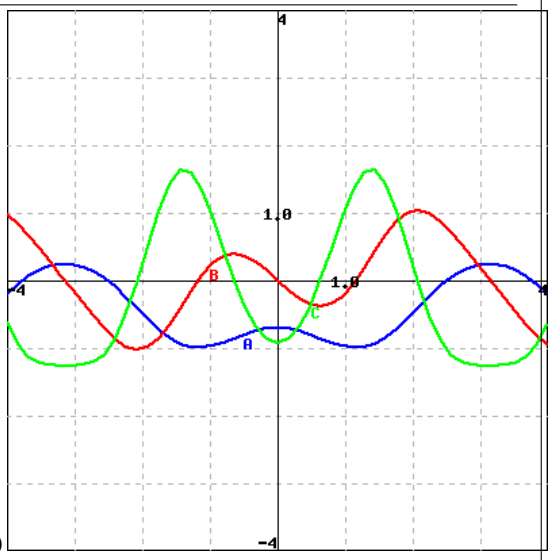
Because the derivative gives the slope of the original function at each point x, we know that the derivative is negative where $f(x)$ is decreasing and positive where it is increasing. Applying this to $f(x)$, we see that the derivative must be



which is answer 1.

Correct Answers:

- 1



5. (2 points)

Identify the graphs A (blue), B(red) and C (green) as the graphs of a function and its derivatives:

___ is the graph of the function

___ is the graph of the function's first derivative

___ is the graph of the function's second derivative

Correct Answers:

- A
- B
- C

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6. (2 points) Given that

$f(-3) = 3,\ f'(-3) = -1,$
$g(-3) = -6,\ g'(-3) = 2,$

calculate the following:

$(f+g)'(-3) =$ _____
$(f-g)'(-3) =$ _____
$(f \cdot g)'(-3) =$ _____
$(f/g)'(-3) =$ _____

Correct Answers:

- 1
- -3
- 12
- 0/36

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7. (2 points) Find the derivative of the function $f(z)$, below. It may be to your advantage to simplify first.

$$f(z) = \frac{z^9+11}{\sqrt{z}}$$

$f'(z) =$ _____

Solution:

SOLUTION

Note that $f(z) = \frac{z^9+11}{\sqrt{z}} = z^{17/2} + 11z^{-1/2}$. Thus

$$f'(z) = \frac{17}{2}z^{15/2} - \frac{11}{2}z^{-3/2}.$$

Correct Answers:

- (9-1/2)*z^(9-3/2)-11/[2*z^(3/2)]

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8. (2 points) Let

$$f(x) = \frac{3\sin(x) - 11}{\cos(x)}.$$

Find $f'(x) =$ _____

Correct Answers:

- 3*[sec(x)]^2+-11*sec(x)*tan(x)

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9. (2 points) Find the derivative of

$w(r) = \sqrt{r^7+7}$

$\frac{dw}{dr} =$ _____

Solution: $\frac{dw}{dr} = (7r^6)(1/2)(r^7+7))^{-1/2} = \frac{7}{2}r^6(r^7+7))^{-1/2}$

Correct Answers:

- 7*r^(7-1)*1/2*1/[sqrt(r^7+7)]

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10. (2 points) Find the derivative of

$y = \frac{e^{2x}}{x^2+1}$

$\frac{dy}{dx} =$ _____

Solution:

SOLUTION

$$\frac{dy}{dx} = \frac{2e^{2x}(x^2+1) - 2xe^{2x}}{(x^2+1)^2}$$

Correct Answers:

- [2*e^(2*x)*(x^2+1)-2*x^(2-1)*e^(2*x)]/[(x^2+1)^2]

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11. (2 points) Find the derivative of

$w = (t^2+2t)(1-e^{-2t})$

$\frac{dw}{dt} =$ _____

Solution:

SOLUTION

$$\frac{dw}{dt} = (2t+2)(1-e^{-2t}) + (t^2+2t)(2e^{-2t})$$

Correct Answers:

- [2*t^(2-1)+2]*[1-e^(-1*2*t)]+(t^2+2*t)*2*e^(-1*2*t)

12. (2 points)
Let

$$f(x) = \sqrt{2+\sin^2 x}$$

Then
$f'(x) =$ _____.
Correct Answers:

- `(1/2)*(2+sin(x)**2)^(-1/2)*2*sin(x)*cos(x)`

13. (2 points) If $\frac{d}{dx}(f(3x^2)) = 5x^3$, calculate $f'(x)$.
$f'(x) =$ _____

Correct Answers:

- `5/(2*3^[(3+1)/2])*x^[(3-2+1)/2]`