COMP 2711 Discrete Mathematical Tools for Computer Science
2022 Fall Semester – Tutorial 7

Question 1: Find the integer a such that

(a) $a \equiv -15 \pmod{27}$ and $-26 \leq a \leq 0$.

(b) $a \equiv 24 \pmod{31}$ and $-15 \leq a \leq 15$.

(c) $a \equiv 99 \pmod{41}$ and $100 \leq a \leq 140$.

Question 2: Use the extended Euclidean algorithm to express $\gcd(26, 91)$ as a linear combination of 26 and 91.

Question 3: Prove that if $a - c \mid ab + cd$ then $a - c \mid ad + bc$

Question 4: assume a, b are non-zero integers. Prove that:

(a) $gcd(a, b) = gcd(a, b + ka)$ for any $k \in \mathbb{Z}$.

(b) $gcd(na, nb) = n \cdot gcd(a, b)$ for any $n \in \mathbb{N}$.

Question 5: Prove that the following fraction can not be simplified for any $n \in N$.

$$\frac{21n + 4}{14n + 3}$$

Question 6: (a) Prove that $gcd(n, n + 1) = 1$ for any natural number n

(b) Prove that there are infinitely many prime numbers. (Hint: Use part a)