MATH 2111: Tutorial 10

T1A&T1B QUAN Xueyang
T1C&T2A SHEN Yinan
T2B&T2C ZHANG Fa

Department of Mathematics, HKUST

Review

- Eigenvectors and eigenvalues
- The characteristic equation
- Similarity

Example 1

Let λ be an eigenvalue of A. Find an eigenvalue of the following matrices.
(1) A^2
(2) $A^3 + A^2$
(3) $A^3 + 2I$
(4) If A is invertible, A^{-1}
(5) If $p(t) = c_0 + c_1 t + c_2 t^2 + \cdots + c_n t^n$, define $p(A)$ to be the matrix formed by replacing each power of t in $p(t)$ by the corresponding power of A (with $A^0 = I$). That is,

$$p(A) = c_0 I + c_1 A + c_2 A^2 + \cdots + c_n A^n.$$

Example 2

Let

$$A = \begin{bmatrix} -1 & 4 & 6 \\ -3 & 7 & 9 \\ 1 & -2 & -2 \end{bmatrix}$$

Determine whether the following vectors are eigenvectors of A.

(1) $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ (2) $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

Example 3

For the given matrix A and the given eigenvalue λ, find the corresponding collection of eigenvectors.

$$A = \begin{bmatrix} 5 & 9 & 7 \\ 4 & 10 & 7 \\ -8 & -18 & -13 \end{bmatrix}, \ \lambda = 1$$

Example 4

Suppose that λ and ρ are two different eigenvalues of the square matrix A. Prove that the intersection of the eigenspaces for these two eigenvalues is trivial. That is, $\mathcal{E}_A(\lambda) \cap \mathcal{E}_A(\rho) = \{\mathbf{0}\}$

Example 5

Find the eigenvalues eigenvalues, eigenspaces, algebraic and geometric multiplicities of the following matrices.

(1) $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

(2) $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$