Potential Energy & Energy Conservation

PHYS1112

Lecture 5

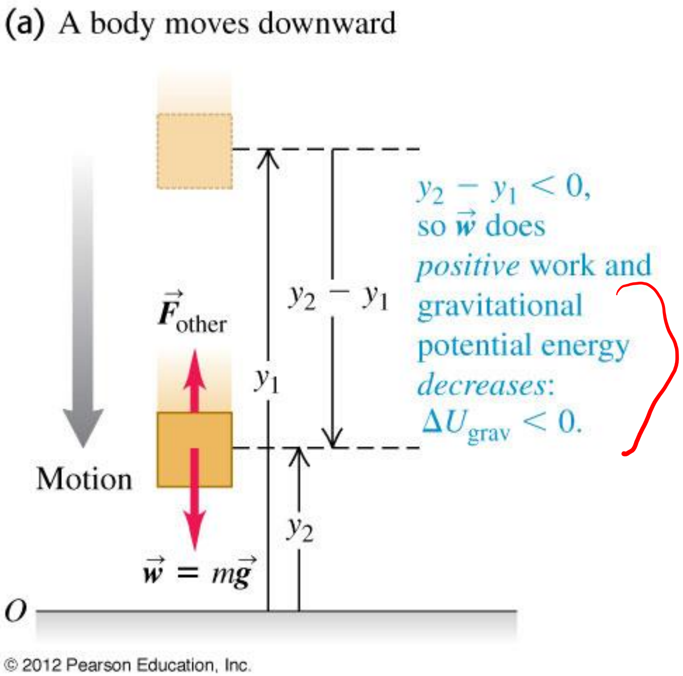
Intended Learning Outcomes

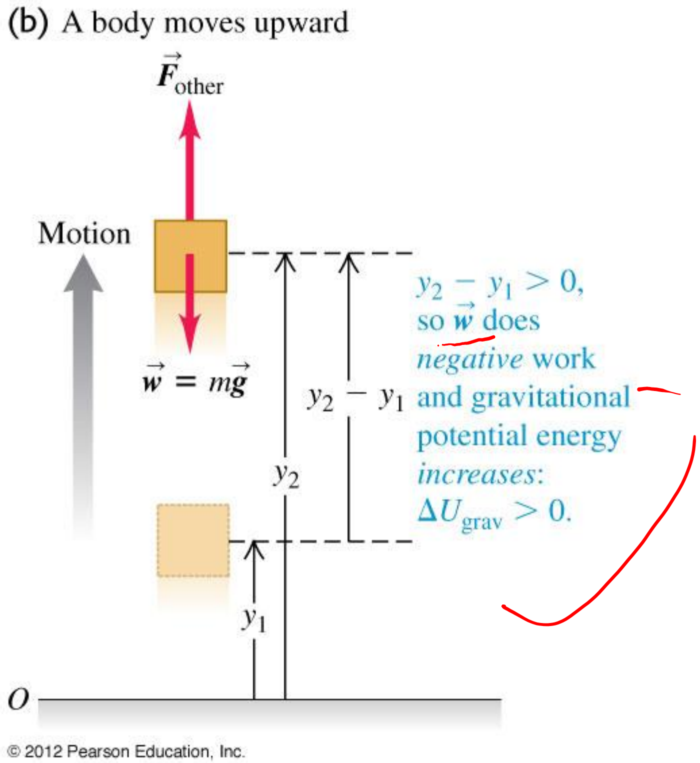
- After this lecture you will learn:
  1) gravitational and elastic forces as examples of conservative force
  2) properties of the potential energy function of a conservative force
  3) to derive the force from the potential energy function

Potential energy –
energy associated with the position of bodies in a system

Gravitational PE

Defined by $U_{grav} = mgy$





Work done by the weight of the body

$W_{\text{grav}} = mg(y_1 - y_2) > 0,$

$\vec{\boldsymbol{w}}$ does +ve work

$\Delta U_{\text{grav}} = mg(y_2 - y_1) = -W_{\text{grav}} < 0$

gravitational PE decreases

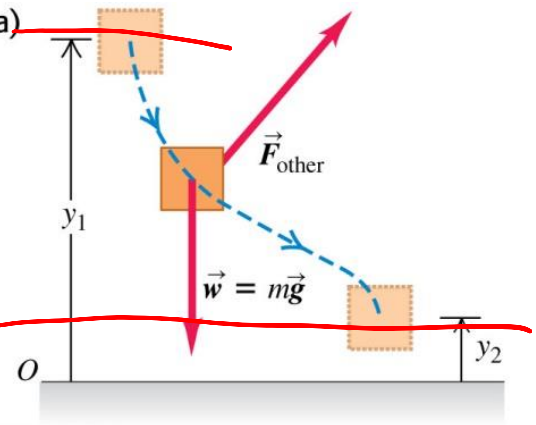
$W_{\text{grav}} = -mg(y_2 - y_1) < 0,$

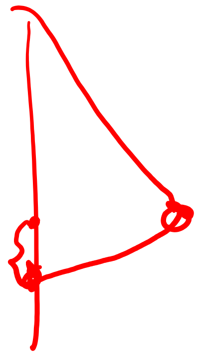
$\vec{\boldsymbol{w}}$ does –ve work

$\Delta U_{\text{grav}} = mg(y_2 - y_1) = -W_{\text{grav}} > 0$

gravitational PE increases

Along a curved path

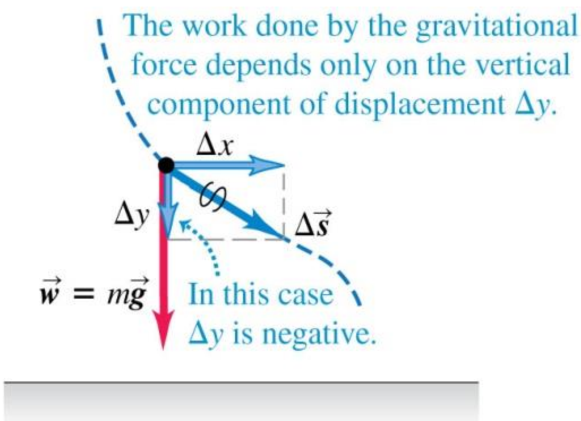
(a)




work done by the weight

$$W_{\text{grav}} = \vec{\boldsymbol{w}} \cdot \overrightarrow{\Delta \boldsymbol{s}} = -mg\Delta y$$
$$= -\Delta U_{\text{grav}}$$

same as vertical motion!

(b)


<u>Conclusion</u>: $\boxed{W_{\text{grav}} = -\Delta U_{\text{grav}}}$

Note: gravitational PE acts like a bank to store workdone for later use

energy

if $W_{\text{grav}} < 0$, $\Delta U_{\text{grav}} > 0$, U_{grav} increases, *c.f.* deposit money into a bank

if $W_{\text{grav}} > 0$, $\Delta U_{\text{grav}} < 0$, U_{grav} decreases *c.f.* draw money from the bank and spend it

⚠ Gravitational PE does not belong to the body only, it belongs to both the body and the earth

Q7.1

A piece of fruit falls straight down. As it falls,

A. the gravitational force does positive work on it and the gravitational potential energy increases.

B. the gravitational force does positive work on it and the gravitational potential energy decreases.

C. the gravitational force does negative work on it and the gravitational potential energy increases.

D. the gravitational force does negative work on it and the gravitational potential energy decreases.

A7.1

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By **work-energy theorem**

$\Delta W = \Delta K$

$= -\Delta U$ $\quad \Delta K = -\Delta U_{\text{grav}} \quad \Rightarrow \boxed{\Delta K + \Delta U_{\text{grav}} = 0,}$

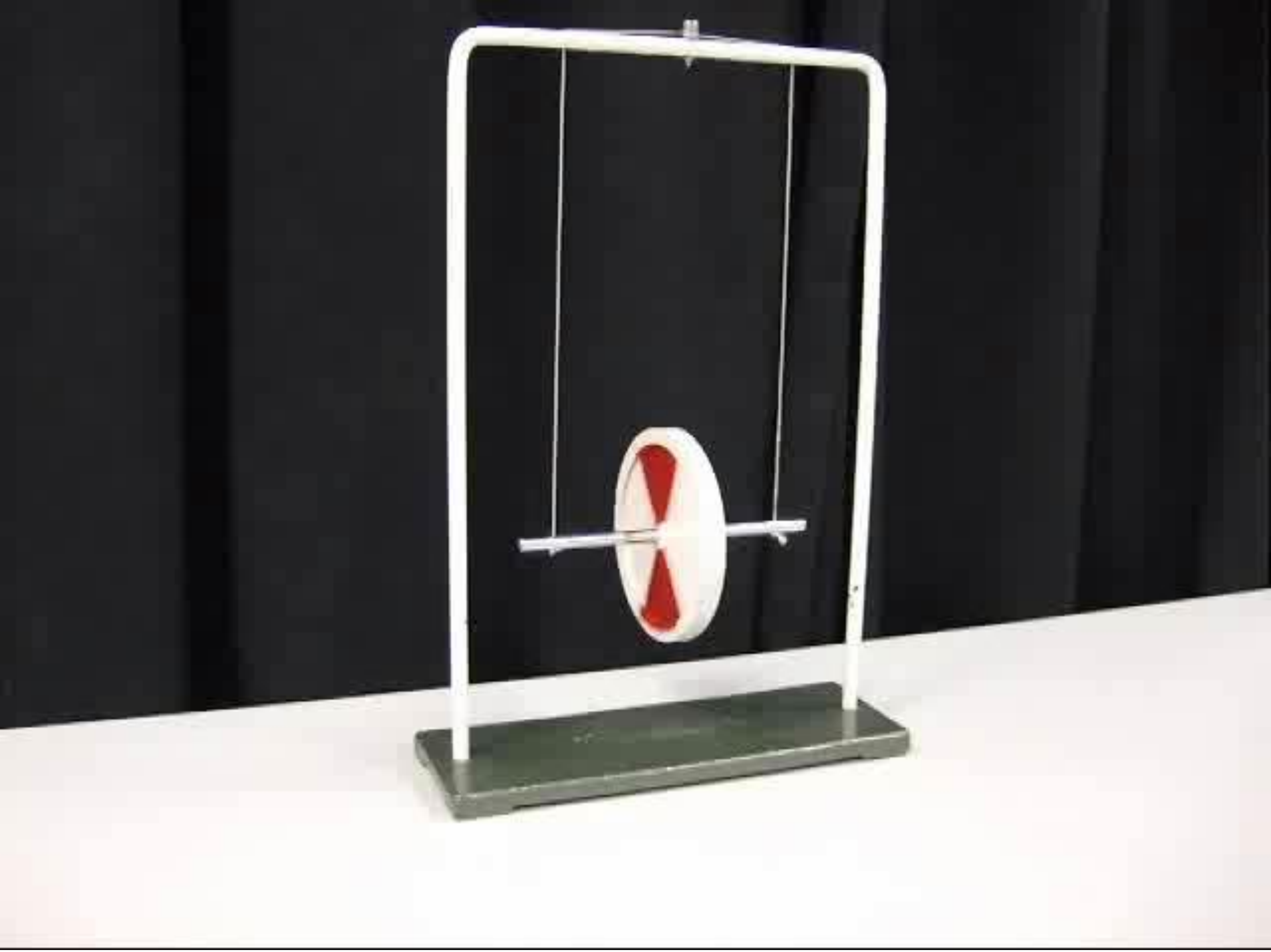
or $\quad K_{\text{initial}} + U_{\text{grav,initial}} = K_{\text{final}} + U_{\text{grav,final}}$

Conservation of mechanical energy

What if other forces also do work?

Work-energy theorem $\quad \Rightarrow W_{other} + W_{grav} = \Delta K$

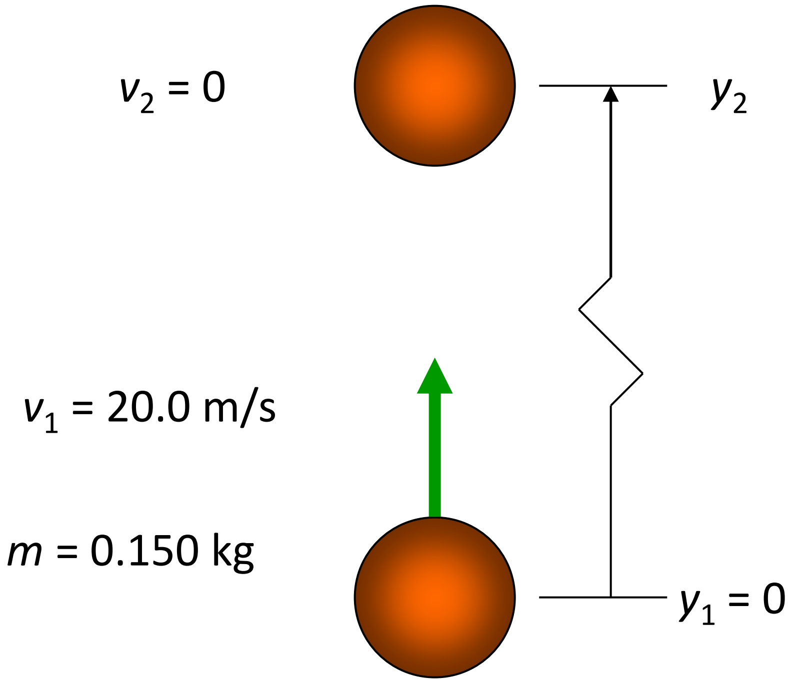
$\Rightarrow \boxed{W_{other} = \Delta K + \Delta U_{grav}}$



Q7.2

You toss a 0.150-kg baseball straight upward so that it leaves your hand moving at 20.0 m/s. The ball reaches a maximum height y_2.

What is the speed of the ball when it is at a height of $y_2/2$? Ignore air resistance.



A. 10.0 m/s

B. less than 10.0 m/s but greater than zero

C. greater than 10.0 m/s

D. not enough information given to decide

A7.2

You toss a 0.150-kg baseball straight upward so that it leaves your hand moving at 20.0 m/s. The ball reaches a maximum height y_2.

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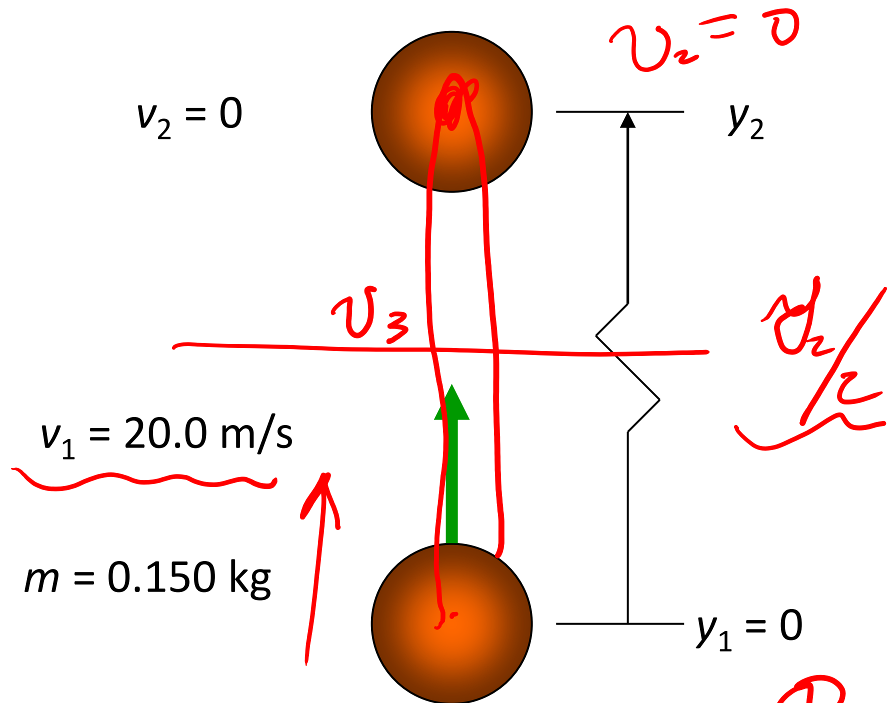


A. 10.0 m/s

B. less than 10.0 m/s but greater than zero

✔ C. greater than 10.0 m/s $\quad \frac{1}{2}v_1$

D. not enough information given to decide

① $\frac{1}{2}mv_1^2 = mgy_2$

② $\frac{1}{2}mv_1^2 - \frac{1}{2}mv_3^2 = \frac{1}{2}mgy_2$

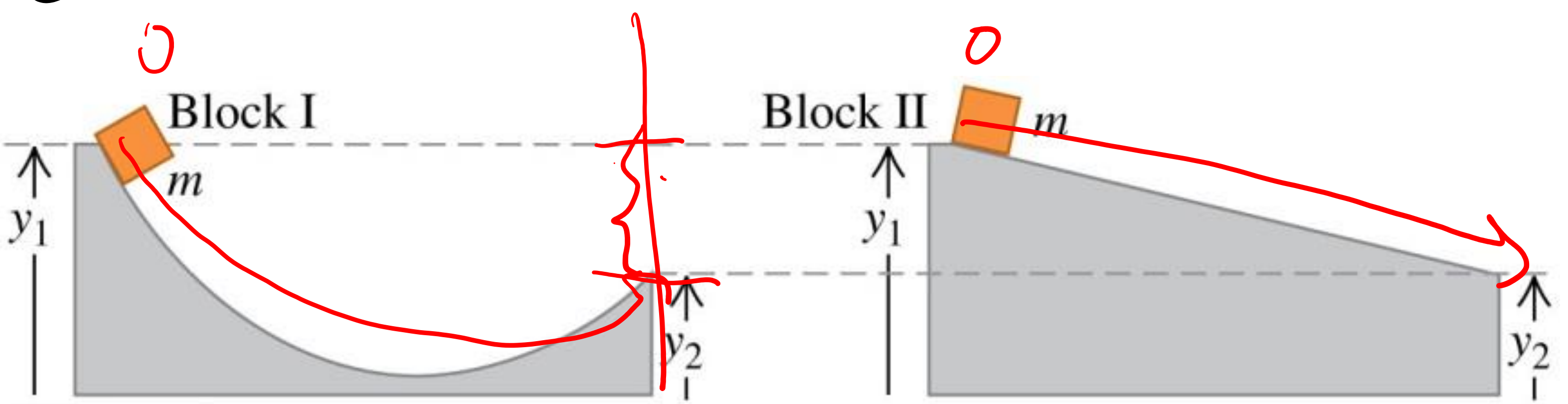
$\frac{1}{2}\cancel{m}(v_1^2 - v_3^2) = \frac{1}{2}\cdot\frac{1}{2}\cancel{m}v_1^2$

$\frac{1}{2}v_1^2 = v_3^2 \qquad v_3 = \frac{v_1}{\sqrt{2}} > \frac{v_1}{2}$

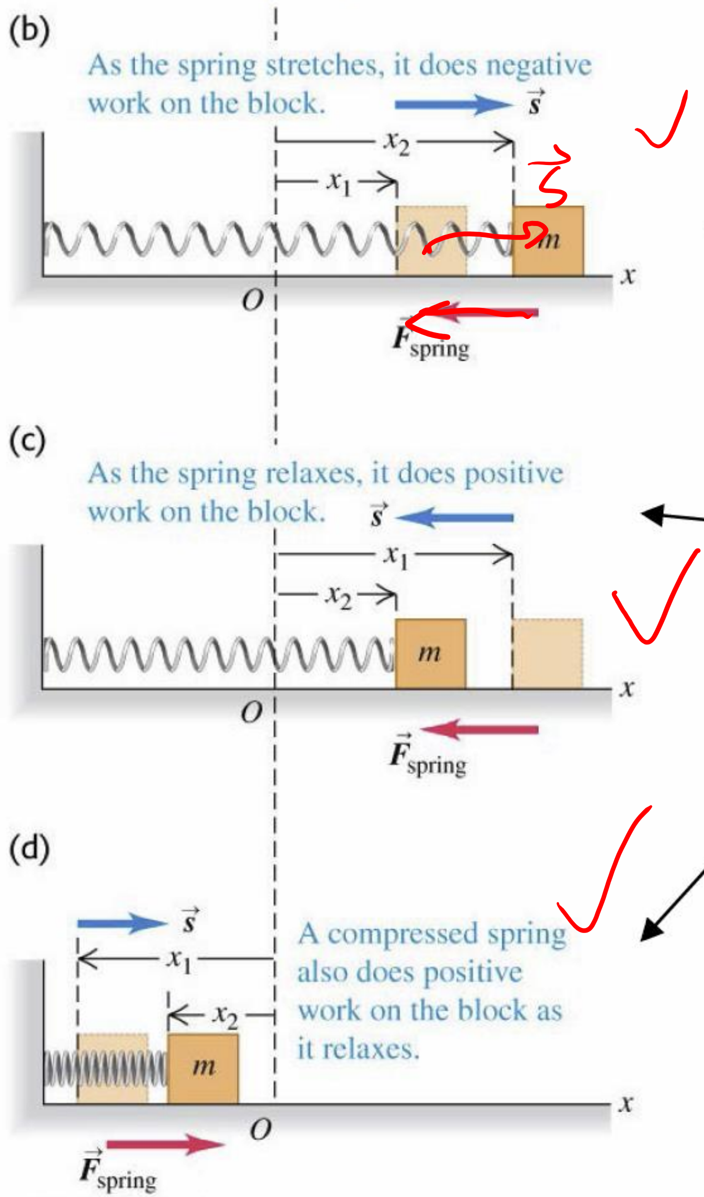
$= \frac{v_1}{1.414}$

Question

- The figure shows two different frictionless ramps. The heights y_1 and y_2 are the same for both ramps. If a block of mass m is released from rest at the left-hand end of each ramp, which block arrives at the right-hand end with the greater speed?
    1) block I;
    2) block II;
    3) the speed is the same for both blocks.



Elastic PE – spring



Check that it works in all 3 cases

Work done by restoring force in spring

$$W_{el} = \int_{x_1}^{x_2} \underbrace{(-kx)\,dx}_{\vec{F}\cdot\vec{s}} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

Define elastic PE of spring $\boxed{U_{el} = \frac{1}{2}kx^2}$

$$W_{el} = -\Delta U_{el}$$

c.f. gravitational PE

⚠ U_{grav} free to choose zero level position, but for U_{el}, zero level position must correspond to unstretched position.

In the presence of gravitational, elastic, and other forces

Work-energy theorem $\Rightarrow$ $\boxed{\overset{-\Delta U_{grav}}{W_{\text{grav}}} + \overset{-\Delta U_{el}}{W_{\text{el}}} + W_{\text{other}}} = \Delta K$

$$\Rightarrow \quad W_{\text{other}} = \Delta K + \Delta\left(U_{\text{grav}} + U_{\text{el}}\right)$$

$$= \Delta K + \Delta PE$$

If $\boxed{W_{\text{other}} = 0}$, $\boxed{\Delta K + \Delta PE = 0}$,

or $K_{\text{initial}} + PE_{\text{initial}} = K_{\text{final}} + PE_{\text{final}}$

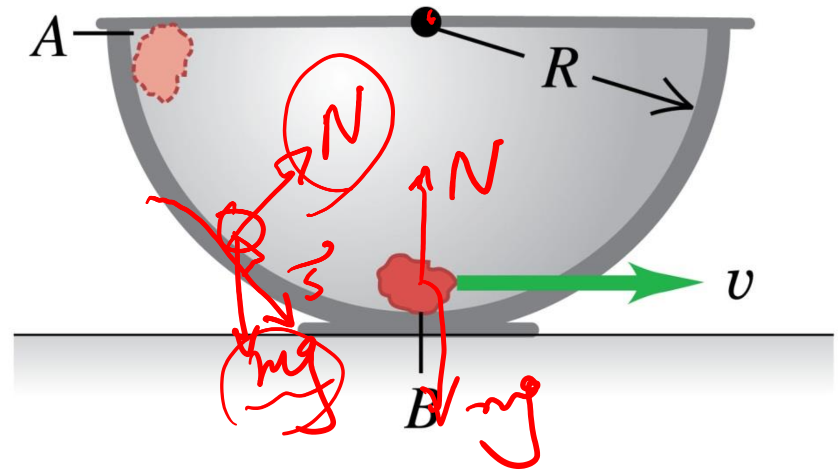
Conservation of mechanical energy

Demonstration: Maxwell's wheel

Q7.3

As a rock slides from *A* to *B* along the inside of this frictionless hemispherical bowl, mechanical energy is conserved. Why?

(Ignore air resistance.)



A. The bowl is hemispherical.

B. The normal force is balanced by centrifugal force.

C. The normal force is balanced by centripetal force.

D. The normal force acts perpendicular to the bowl's surface.

E. The rock's acceleration is perpendicular to the bowl's surface.

$\boxed{W_{other} = 0}$

$\vec{N} \cdot \vec{s} = 0$

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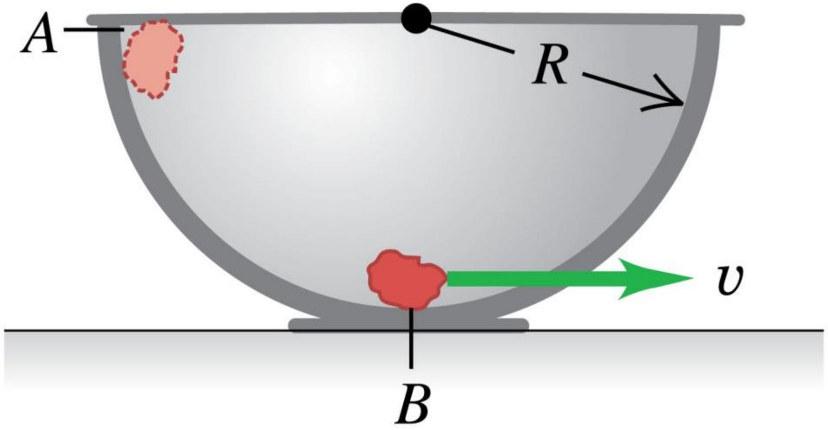

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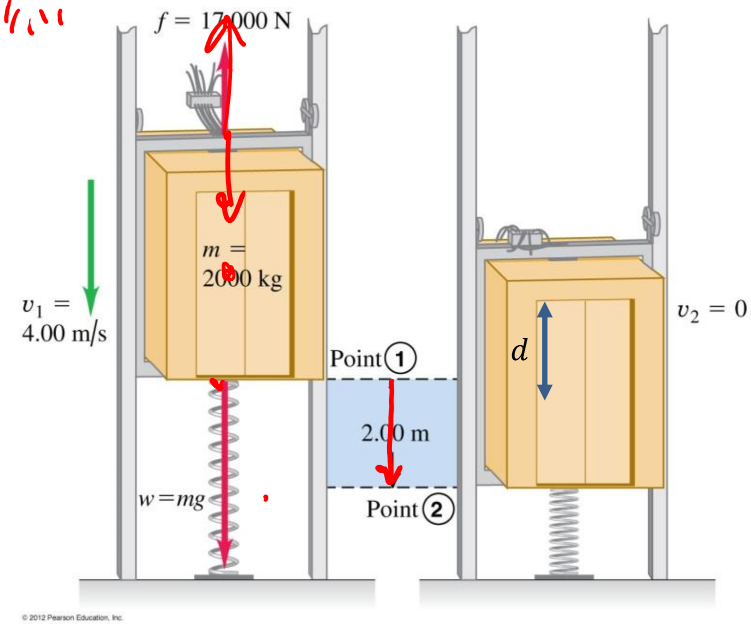
Example

An elevator with a broken cable. Friction between the rail and the elevator is f. What is the spring constant k if the elevator has initial speed v_1 when it just touches the spring, and comes to rest at a distance $d = 2.00$ m?

$v=0$



work done by friction $W_{\text{other}} = -fd$

$$\Delta K = 0 - \frac{1}{2} m v_1^2$$

$$\Delta PE = -mgd + \frac{1}{2} k d^2 - \frac{1}{2} k 0^2$$

$$W_{\text{other}} = \Delta K + \Delta PE$$

$$\Rightarrow \quad -fd = -\tfrac{1}{2} m v_1^2 - mgd + \tfrac{1}{2} k d^2$$

$$\Rightarrow \quad k = \frac{2\left(mgd + \frac{1}{2} m v_1^2 - fd\right)}{d^2}$$

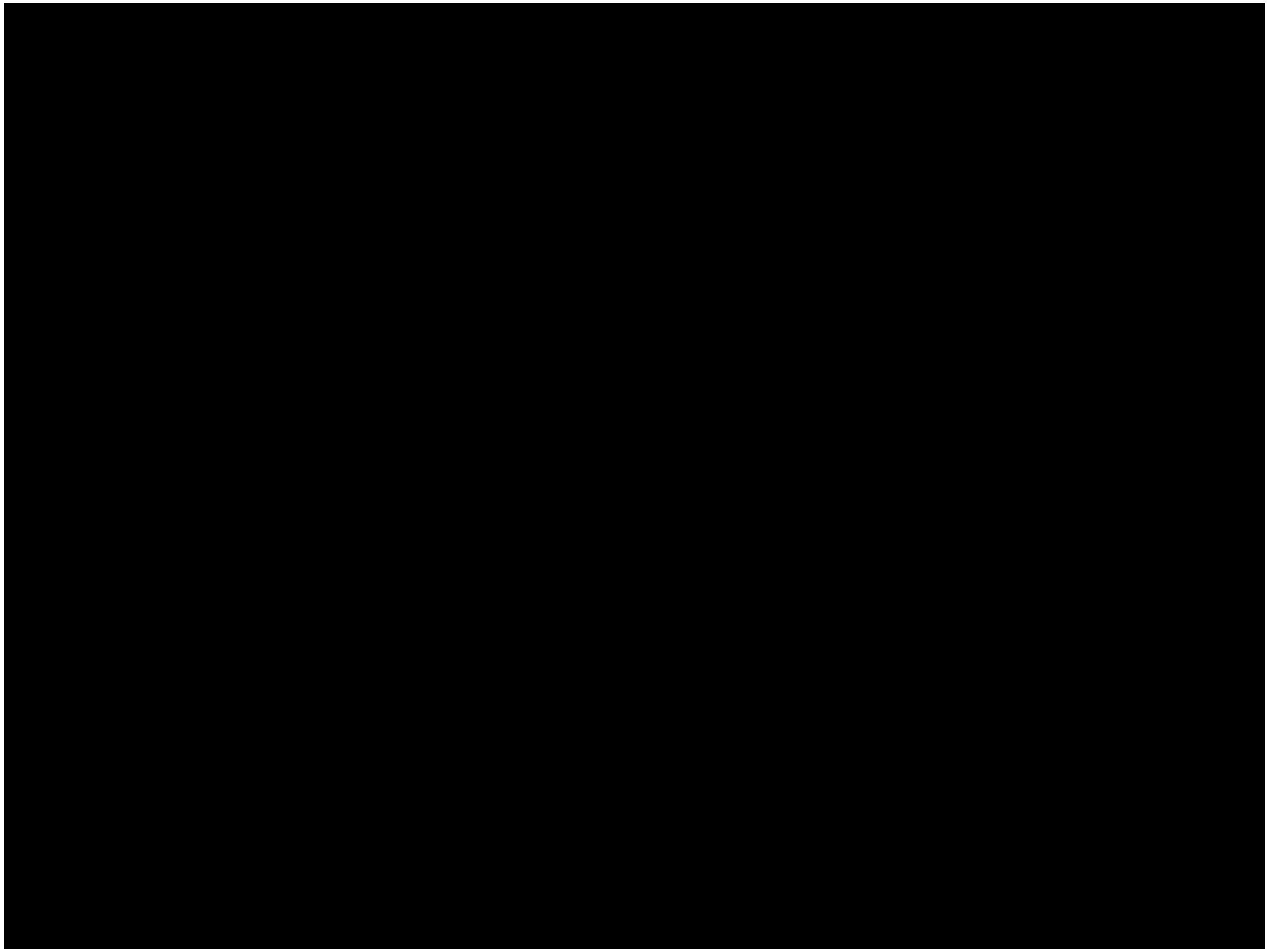
Conservative Forces

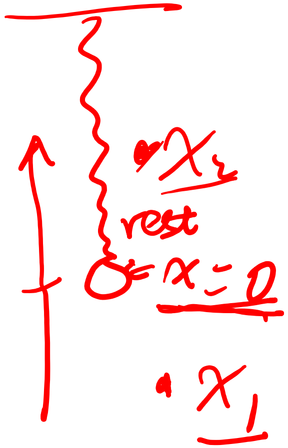
The work done on a system can be "reclaimed" as KE, e.g. gravitation, spring

These are called **conservative forces**

Demonstration: energy stored in a spring

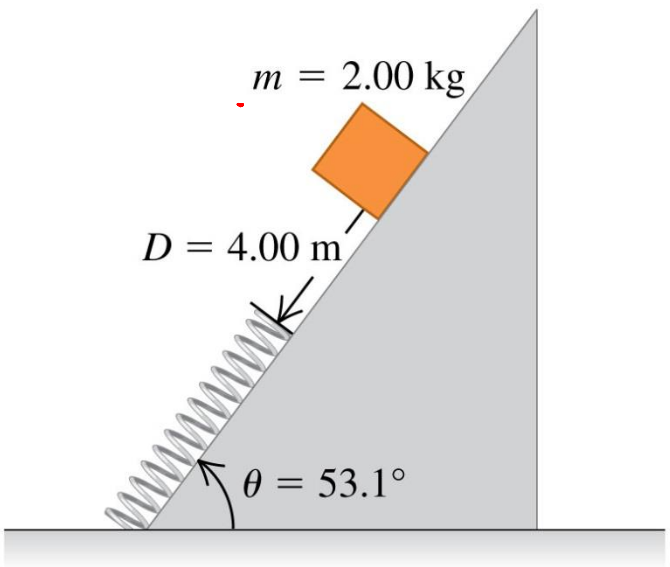
$$\boxed{\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2}$$





Q7.5

A block is released from rest on a frictionless incline as shown. When the moving block is in contact with the spring and compressing it, what is happening to the gravitational potential energy U_{grav} and the elastic potential energy U_{el}?



A. U_{grav} and U_{el} are both increasing.

B. U_{grav} and U_{el} are both decreasing.

C. U_{grav} is increasing; U_{el} is decreasing.

D. U_{grav} is decreasing; U_{el} is increasing.

E. The answer depends on how the block's speed is changing.

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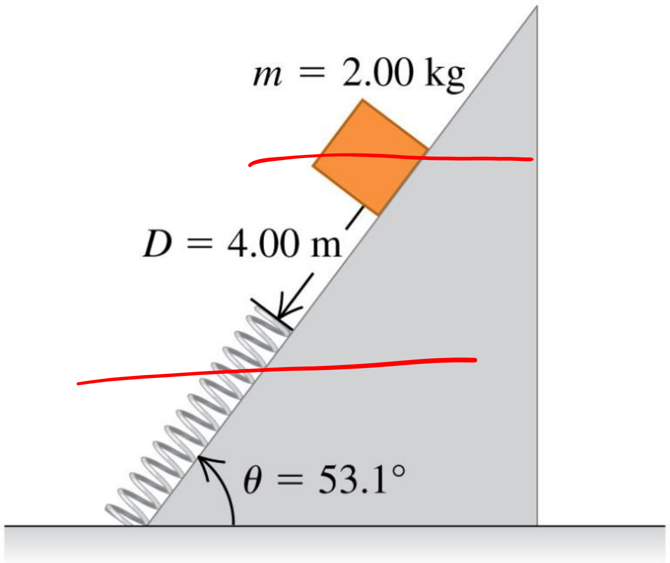

A. U_{grav} and U_{el} are both increasing.

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C. U_{grav} is increasing; U_{el} is decreasing.

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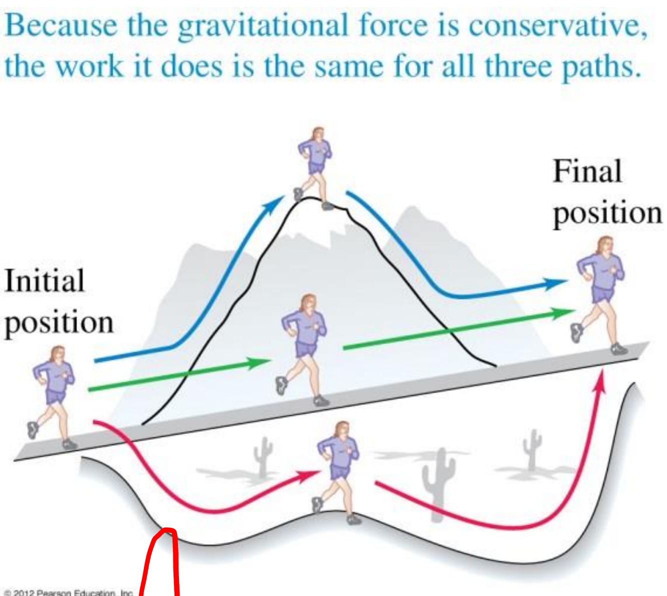
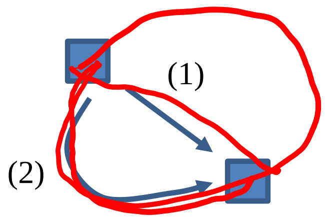


Properties of the work done by conservative forces:

1. It can be expressed as the difference between the initial and final values of a potential energy function.
2. It is reversible, i.e., if path is reversed, workdone changes sign.
3. It depends on the starting and ending point only, not on the path.
4. When the starting and ending points are the same (path forms a close loop), the total work is zero.

c.f. The work done by friction cannot be "reclaimed", called **non-conservative forces**.

Work done by non-conservative force is path dependent $W_f = \Delta K$



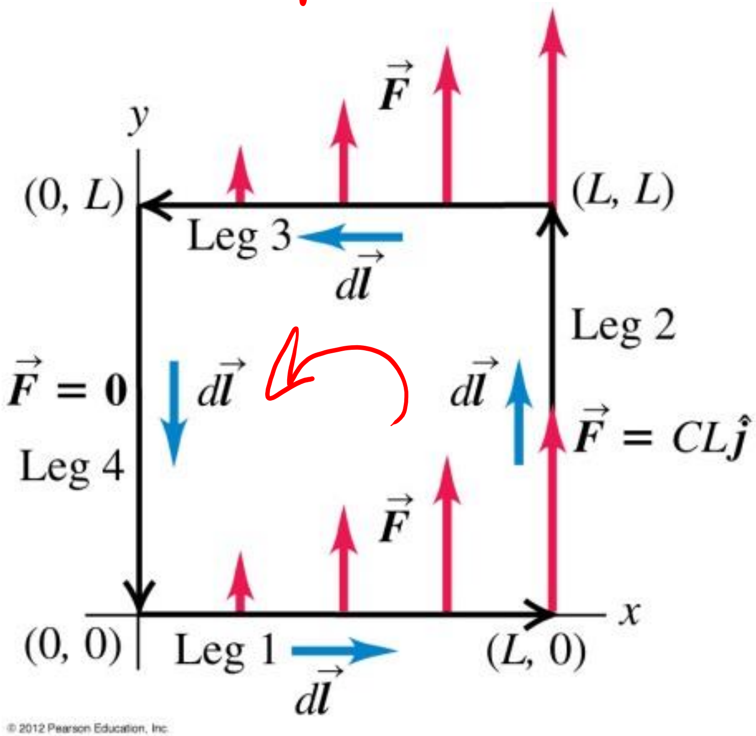
work done by friction in path (2) is more negative than in path (1).

The term PE is reserved for conservative forces only

To test whether a force is conservative – check if the work done is zero around a close loop.

Example

An electron goes counter clockwise around a square loop under a force $\vec{\boldsymbol{F}} = Cx\hat{j}$, C constant



Leg 1, $W_1 = \int \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = 0$

Leg 2, $W_2 = CL^2 = \int cL \cdot dl = cL \cdot L$

Leg 3, $W_3 = 0$

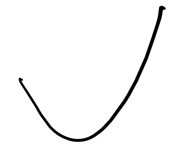
Leg 4, $W_4 = 0$

$\vec{\boldsymbol{F}}$ is (conservative / non-conservative)

To derive a conservative force $\vec{F}$ from its potential energy function U:

$$\text{Work done by a conservative force } \underset{F\Delta x}{W} = -\Delta U(x) \quad \text{in 1D}$$

$$\Rightarrow \quad F = -\frac{\Delta U}{\Delta x} \xrightarrow{\Delta x \to 0} \boxed{F = -\frac{dU}{dx}}$$



Free to add a constant to $U(x)$ without changing the force

Check: $U_{\text{grav}} = mgh$, $F = \underline{-mg} = -\frac{dmgh}{dh}$

$U_{\text{el}} = \frac{1}{2}kx^2$, $F = -kx$

In 3D, $F_x = -\frac{\partial U}{\partial x}$, $F_y = -\frac{\partial U}{\partial y}$, $F_z = -\frac{\partial U}{\partial z}$

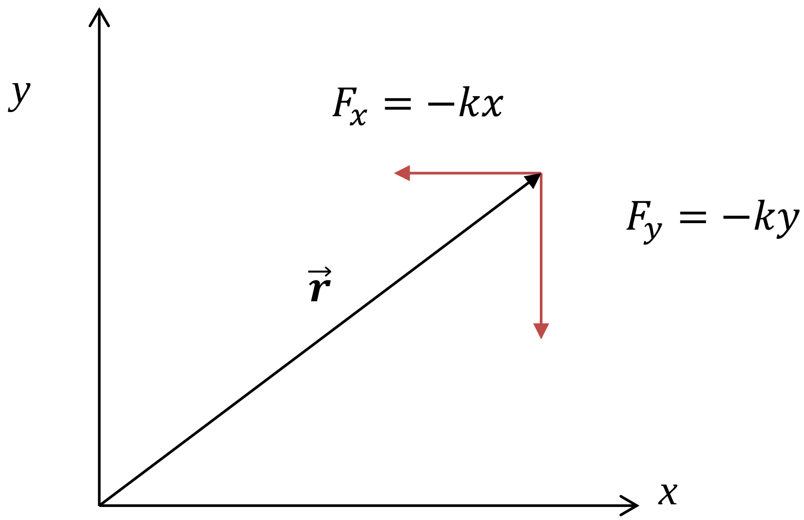
$$\boxed{\vec{F} = -\underline{\nabla U}}$$

Example

$$U(x,y) = \frac{1}{2}k\left(x^2 + y^2\right)$$

$\vec{F} = -\nabla U$

$$F_x = -\frac{\partial U}{\partial x} = -kx, F_y = -\frac{\partial U}{\partial y} = -ky$$



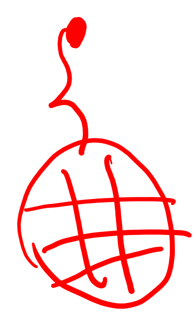
Question

- A particle moving along the x-axis is acted on by a conservative force F_x.
- At a certain point, the force is zero.
- At that point the value of the potential energy function $U(x)$ is
  1) $= 0$
  2) > 0
  3) < 0
  4) not enough information to decide

F_x

- dU/dx is
  1) $= 0$
  2) > 0
  3) < 0
  4) not enough information to decide

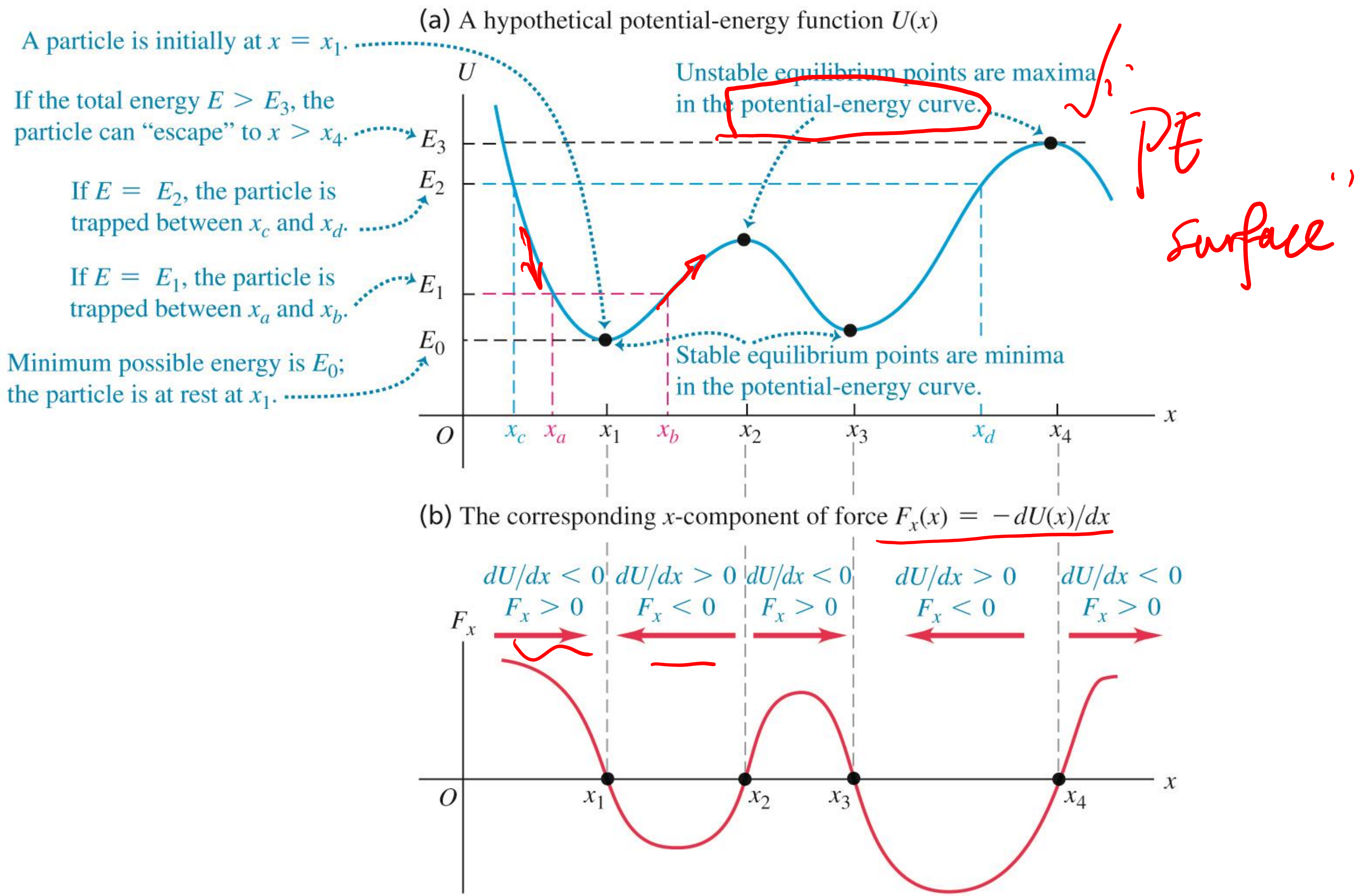
$F = -\frac{dU}{dx}$



$d \to +\infty$

$U(d) = 0$

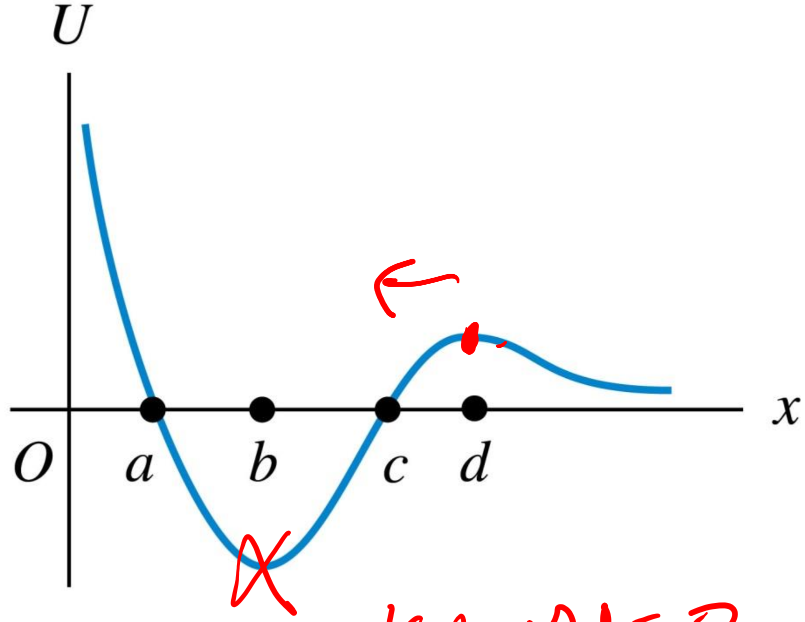
Interpretation of an **energy diagram**:
Note the meanings of stable and unstable equilibrium.



Q7.6

The graph shows the potential energy *U* for a particle that moves along the *x*-axis.

The particle is initially at $x = d$ and moves in the negative *x*-direction. At which of the labeled *x*-coordinates does the particle have the greatest *speed*?



$\Delta K + \Delta U = 0$

A. at $x = a$

B. at $x = b$

C. at $x = c$

D. at $x = d$

E. more than one of the above

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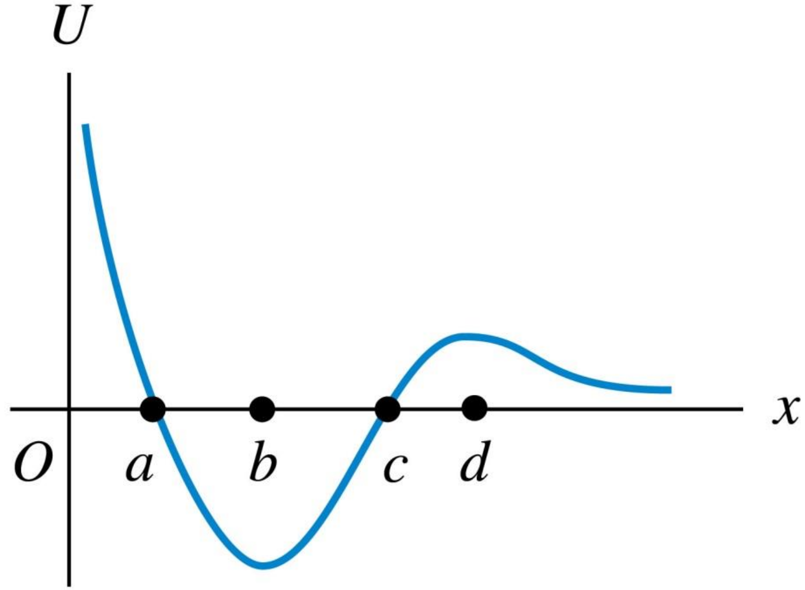

A. at $x = a$

B. at $x = b$

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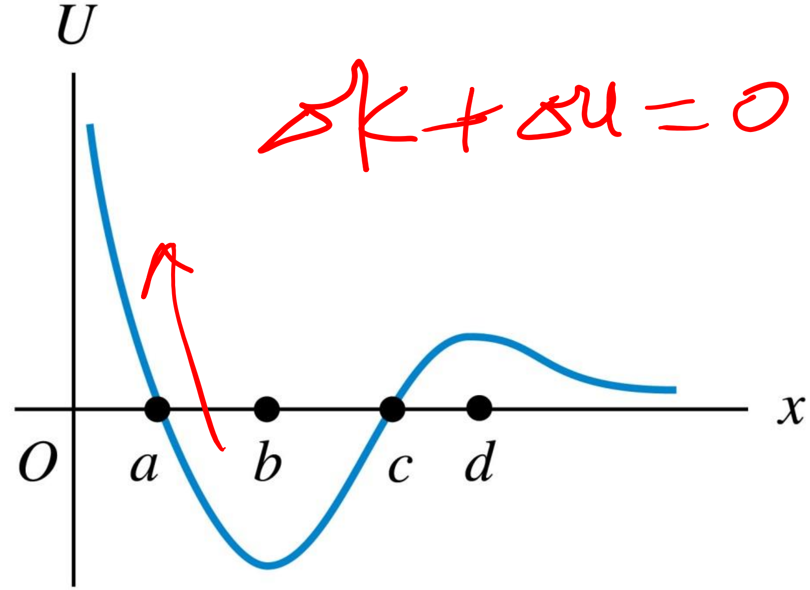
D. at $x = d$

E. more than one of the above

Q7.7

The graph shows the potential energy *U* for a particle that moves along the *x*-axis.

The particle is initially at *x* = *d* and moves in the negative *x*-direction. At which of the labeled *x*-coordinates is the particle *slowing down*?



A. at *x* = *a*

B. at *x* = *b*

C. at *x* = *c*

D. at *x* = *d*

E. more than one of the above

A7.7

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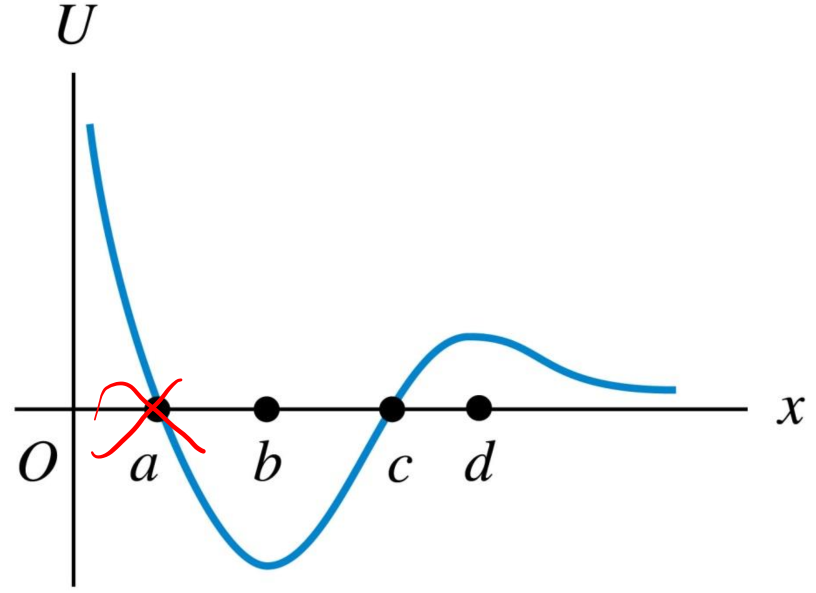
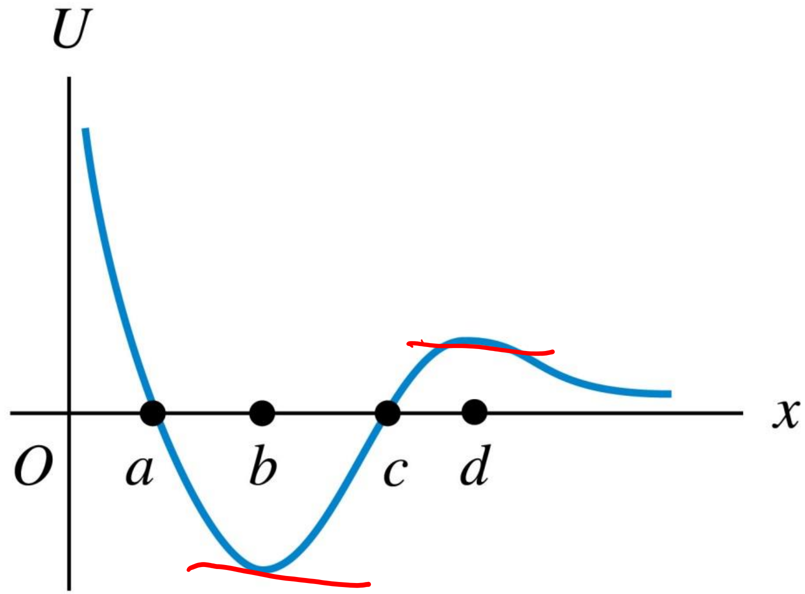


A. at $x = a$ B. at $x = b$ C. at $x = c$ D. at $x = d$

E. more than one of the above

Q7.8

The graph shows the potential energy *U* for a particle that moves along the *x*-axis. At which of the labeled *x*-coordinates is there *zero* force on the particle?



A. at $x = a$ and $x = c$

B. at $x = b$ only

C. at $x = d$ only

D. at $x = b$ and d

E. misleading question—there is a force at all values of x

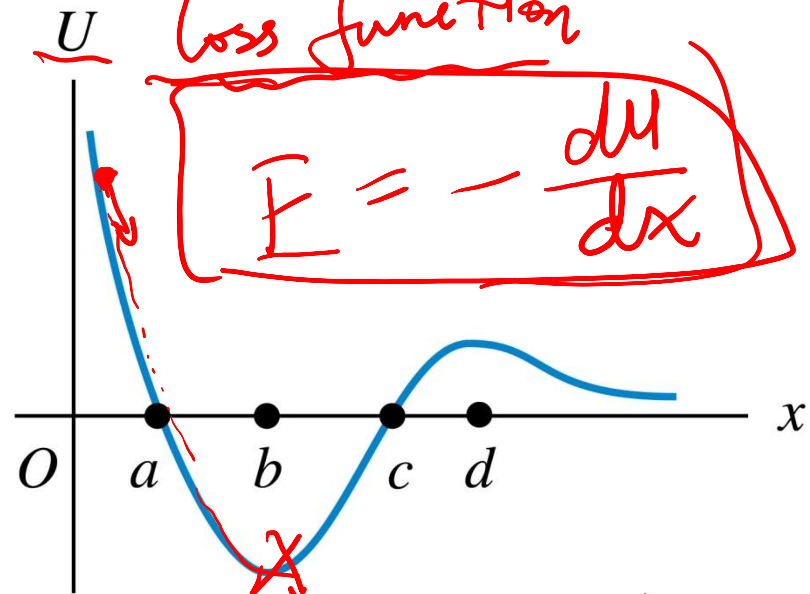
A7.8

"machine learning"

The graph shows the potential energy U for a particle that moves along the x-axis. At which of the labeled x-coordinates is there *zero* force on the particle?

"loss function"

$$\boxed{F = -\frac{dU}{dx}}$$



"gradient descent"

A. at $x = a$ and $x = c$

B. at $x = b$ only

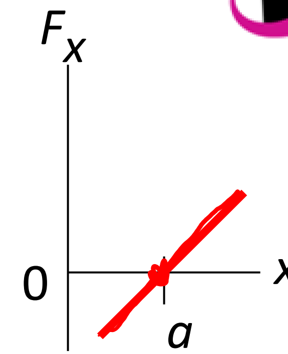
C. at $x = d$ only

D. at $x = b$ and d

E. misleading question—there is a force at all values of x

Q7.9

The graph shows a conservative force F_x as a function of x in the vicinity of $x = a$. As the graph shows, $F_x = 0$ at $x = a$. Which statement about the associated *potential energy* function U at $x = a$ is correct?



A. $U = 0$ at $x = a$

B. U is a maximum at $x = a$.

C. U is a minimum at $x = a$.

D. U is neither a minimum or a maximum at $x = a$, and its value at $x = a$ need not be zero.

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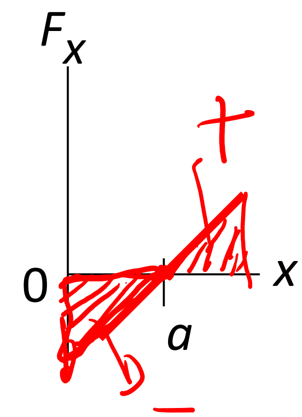

? $x=0$

$$F = -\frac{du}{dx}$$

$$\int_{x=0}^{x} du = -\int_0^x F\,dx$$

$$\boxed{u(x) = -\int_0^x F\,dx}$$

A. $U = 0$ at $x = a$

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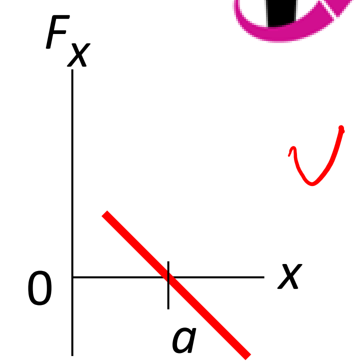

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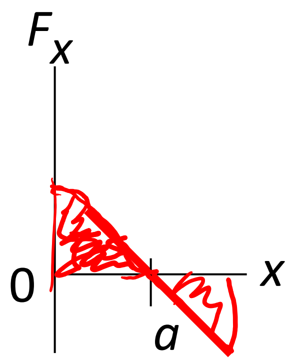
B. U is a maximum at $x = a$.

C. U is a minimum at $x = a$.

D. U is neither a minimum or a maximum at $x = a$, and its value at $x = a$ need not be zero.

A7.10

The graph shows a conservative force F_x as a function of x in the vicinity of $x = a$. As the graph shows, $F_x = 0$ at $x = a$. Which statement about the associated *potential energy* function U at $x = a$ is correct?



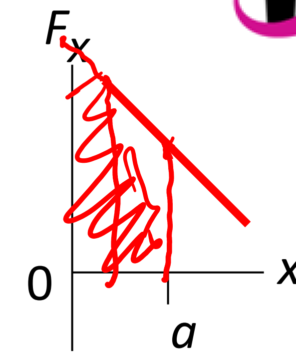
A. $U = 0$ at $x = a$

B. U is a maximum at $x = a$.

C. U is a minimum at $x = a$.

D. U is neither a minimum or a maximum at $x = a$, and its value at $x = a$ need not be zero.

Q7.11



The graph shows a conservative force F_x as a function of x in the vicinity of $x = a$. As the graph shows, $F_x > 0$ and $dF_x/dx < 0$ at $x = a$. Which statement about the associated *potential energy* function U at $x = a$ is correct?

$$F = -\frac{dU}{dx}$$

A. $dU/dx > 0$ at $x = a$

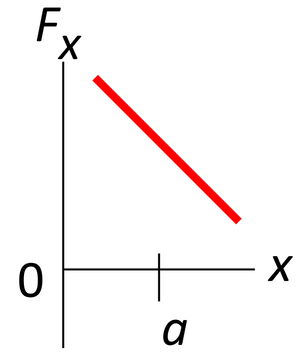
B. $dU/dx < 0$ at $x = a$

C. $dU/dx = 0$ at $x = a$

D. Any of the above could be correct.

A7.11

The graph shows a conservative force F_x as a function of x in the vicinity of $x = a$. As the graph shows, $F_x > 0$ and $dF_x/dx < 0$ at $x = a$. Which statement about the associated *potential energy* function U at $x = a$ is correct?



A. $dU/dx > 0$ at $x = a$

B. $dU/dx < 0$ at $x = a$

C. $dU/dx = 0$ at $x = a$

D. Any of the above could be correct.