WAVE MOTION AND SOUND I

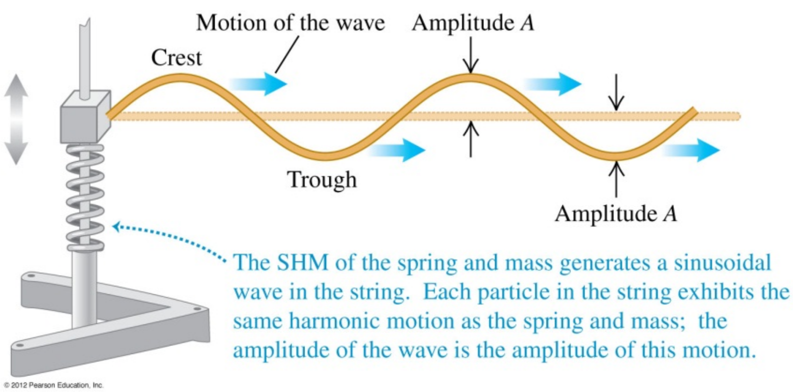
Intended Learning Outcomes – after this lecture you will learn:
1. types of waves – longitudinal and transverse
2. mathematical description of waves – wave equation and wavefunction
3. power propagation in waves
4. reflection of traveling wave under open and fixed boundary conditions

Textbook Reference: Ch 15.1 – 15.6

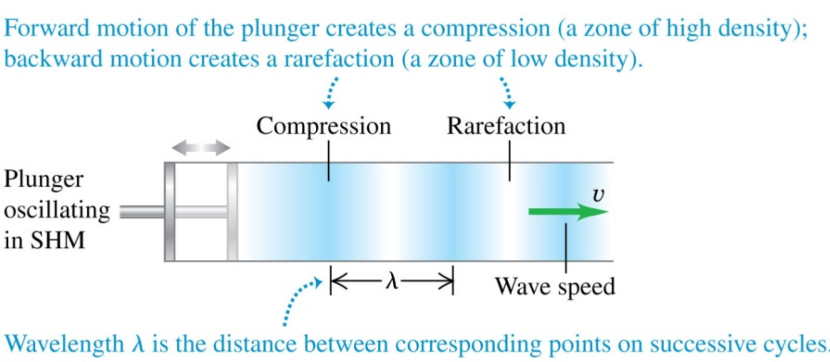
Types of waves:

Mechanical waves – need a medium for propagation

Demonstration: Slinky (helical spring)



transverse,
e.g. wave in a string



longitudinal,
e.g. sound wave

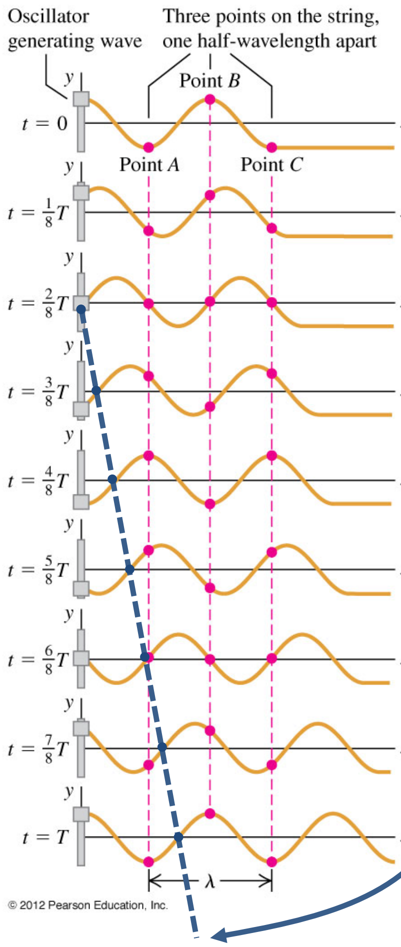
$$v = \lambda f$$

wave speed (v), **wavelength** (λ), **frequency** (f)

period $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$

⚠ a traveling wave transports energy, but not mass of the medium

Mathematical description



Sinusoidal wave on a string as an example:

$y(x,t)$ is the vertical displacement of the string, called a **wave function**

<u>Assume</u> a sinusoidal generator $y(0,t) = A\cos\omega t$

Follow the time evolution of an arbitrary point, it propagates with speed v, called **phase velocity**

At time t, vertical displacement at location x is the same as that at $x = 0$ but at earlier time $t - x/v$
i.e.

$$y(x,t) = y\left(0, t - \frac{x}{v}\right) = A\cos\omega\left(t - \frac{x}{v}\right)$$

Define **wave number** (not a pure number, but has dimension 1/length)

$$\boxed{k = \frac{2\pi}{\lambda}} = \frac{2\pi f}{\lambda f} = \frac{\omega}{v}$$

Wave function becomes

$$y(x,t) = A\cos(kx - \omega t)$$

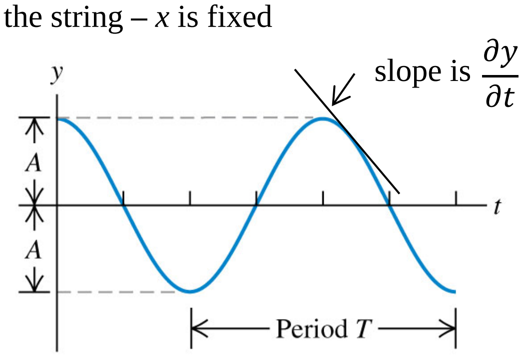
If wave traveling to the left, $v \to -v$

$$y(x,t) = A\cos\omega\left(t + \frac{x}{v}\right) = A\ \cos(kx + \omega t)$$

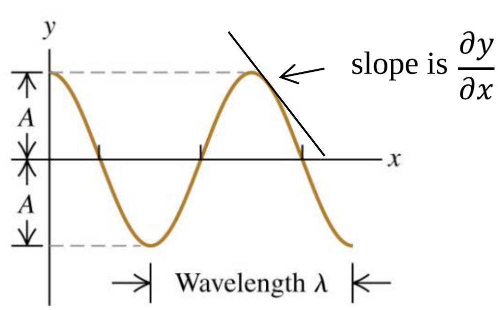
v is the magnitude, i.e., $v > 0$. The direction is shown in the phase angle $(kx \pm \omega t)$

$y(x,t)$ is a function of two variables:

displacement of a particular point on the string – x is fixed

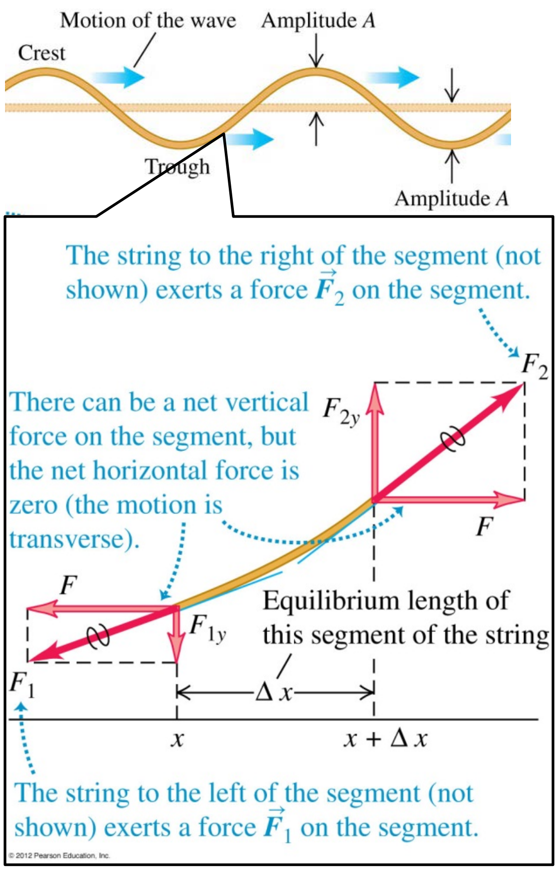


a snapshot (or photo) of the wave motion – t is fixed



Question
In the previous diagram that shows a traveling wave at $t = T/8, 2T/8, \ldots T$, at which time will the point A have (a) maximum upward speed, (b) greatest upward acceleration, (c) downward acceleration but an upward velocity?
Answer: see inverted text on P. 502 of textbook

Wave equation



F – equilibrium tension of the string
μ – mass per unit length of the string

$$\frac{F_{1y}}{F} = -(\text{slope at } x) = -\left(\frac{\partial y}{\partial x}\right)_x,$$

$$\frac{F_{2y}}{F} = \left(\frac{\partial y}{\partial x}\right)_{x+\Delta x}$$

Newton's 2nd law

$$F_y = F_{1y} + F_{2y} = F\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right]$$

$$= ma_y = (\mu\Delta x)\frac{\partial^2 y}{\partial t^2}$$

$$\Rightarrow \frac{1}{\Delta x}\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right] = \frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$$

From wave function $y(x,t) = A\cos(kx - \omega t)$

$$\left.\begin{aligned}\frac{\partial^2 y}{\partial t^2} &= -\omega^2 A\cos(kx - \omega t)\\ \frac{\partial^2 y}{\partial x^2} &= -k^2 A\cos(kx - \omega t)\end{aligned}\right\} \Rightarrow \frac{\mu}{F}\omega^2 = k^2$$

$$vk = \omega$$

$$\Rightarrow \boxed{v = \sqrt{\frac{F}{\mu}}}$$ wave speed on a string

∴ wavefunctions are solutions of the wave equation

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

D'Alembert's equation, hold for different kinds of waves

Different kinds of waves have different speed. For mechanical waves

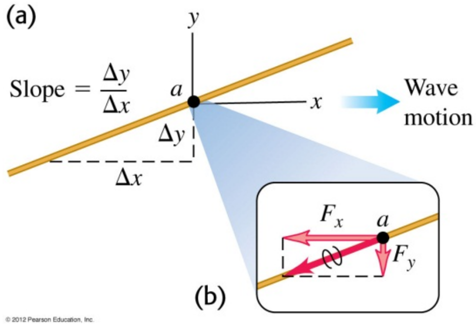
$$v = \sqrt{\frac{\text{restoring force returning the system to equilibrium}}{\text{inertia resisting the return to equilibrium}}}$$

⚠ v does not depend on λ nor f

Power propagation in wave motion

Que: Is there anything propagating, besides the shape? Ans: yes, energy.

To see why, consider a vibrating string



F_y y-component of force acting on point a

as point a moves, F_y does work. The power is

$$P(x,t) = F_y v_y = \left(-F\frac{\partial y}{\partial x}\right)\frac{\partial y}{\partial t}$$
$$= [FkA\sin(kx-\omega t)][\omega A\sin(kx-\omega t)]$$
$$= Fk\omega A^2\sin^2(kx-\omega t)$$

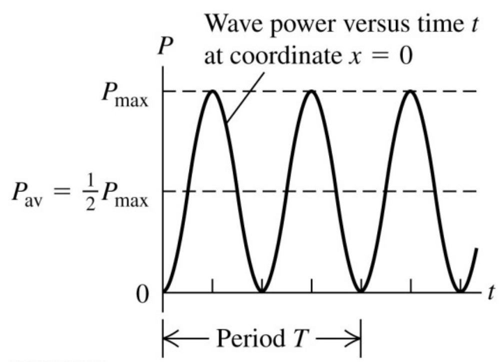
$$k = \frac{\omega}{v} = \omega\sqrt{\frac{\mu}{F}}$$

$$\boxed{P(x,t) = \sqrt{\mu F}\,\omega^2 A^2 \underbrace{\sin^2(kx-\omega t)}_{\text{propagating, like a wave}}}$$

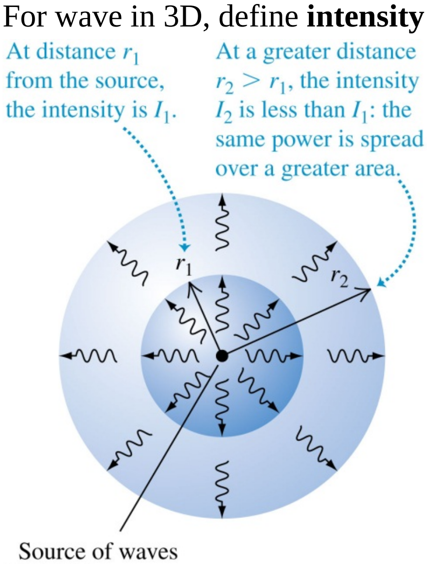
$$P_{\max} = \sqrt{\mu F}\,\omega^2 A^2$$

$$P_{\text{av}} = \frac{1}{T}\int_0^T P(x,t)\,dt$$
$$= \sqrt{\mu F}\,\omega^2 A^2\left(\frac{\omega}{2\pi}\right)\underbrace{\int_0^{\frac{2\pi}{\omega}}\sin^2(kx-\omega t)\,dt}_{\pi/\omega}$$

$$P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \frac{1}{2}P_{\max}$$



For wave in 3D, define **intensity** = average power per unit area, SI unit: W/m^2



Suppose power of source is P, intensity at distance r is

$$I = \frac{P}{4\pi r^2}$$

an *inverse square law!* Just like the Newton's law of gravitation and the Coulomb's law, although in a different context

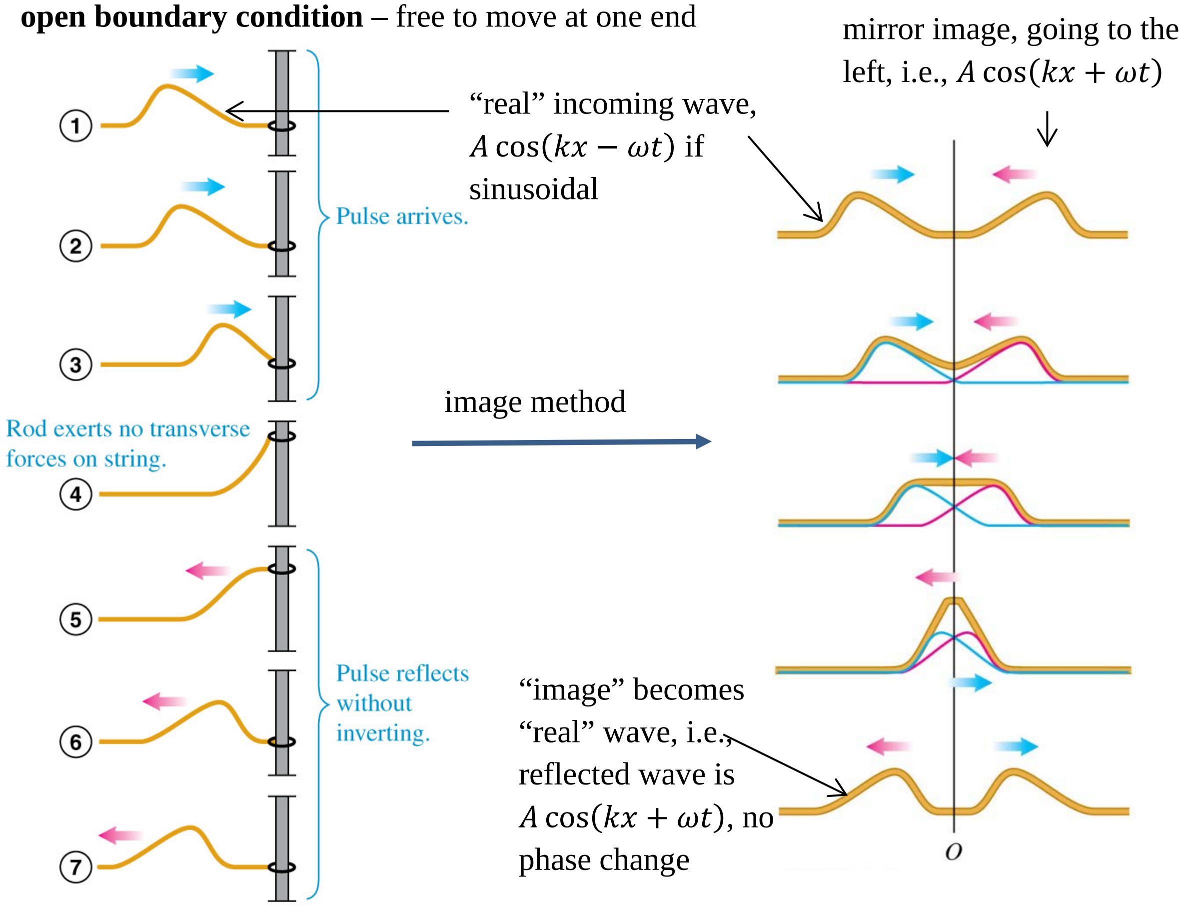
⚠ In the case of intensity it is clear that the inverse square law results from the surface area of a sphere, i.e., the dimensionality of space. The Newton's law of gravitation and Coulomb's law can also be formulated in a similar way to show that the inverse square laws are results of the dimensionality of space. This more general formulation is known as the Gauss Law.

Wave reflection

Demonstration: [wave motion demonstrator]

Image method –proof involves solving the d'Alembert equation
Mirror image of the wave coming in opposite direction, reflected wave results from the superposition of the "real" wave and its "image". ⚠ must observe the **boundary condition**

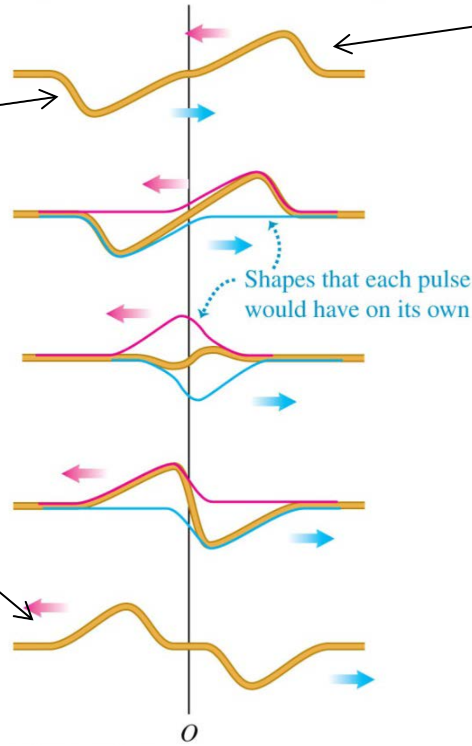
open boundary condition – free to move at one end



fixed boundary condition – string clamped at one end

mirror image – must be inverted to preserve boundary condition, i.e. $-A\cos(kx+\omega t)$

"real" in coming wave, $A\cos(kx-\omega t)$ if sinusoidal



"image" becomes "real" wave, i.e., reflected wave is $-A\cos(kx+\omega t) =$ $A\ \cos(kx+\omega t+\pi)$

means a phase change of π

Clicker Questions:

Q15.1

If you double the wavelength λ of a wave on a string, what happens to the wave speed v and the wave frequency f?

A. v is doubled and f is doubled.

B. v is doubled and f is unchanged.

C. v is unchanged and f is halved.

D. v is unchanged and f is doubled.

E. v is halved and f is unchanged.

Q15.2

Which of the following wave functions describe(s) a wave that moves in the $-x$-direction?

A. $y(x,t) = A \sin(-kx - \omega t)$

B. $y(x,t) = A \sin(kx + \omega t)$

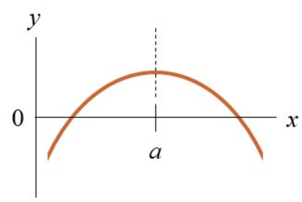
C. $y(x,t) = A \cos(kx + \omega t)$

D. both B and C

E. all of A, B, and C

Q15.3

A wave on a string is moving to the right. This graph of $y(x, t)$ versus coordinate x for a specific time t shows the shape of part of the string at that time. At this time, what is the *velocity* of a particle of the string at $x = a$?



A. The velocity is upward.

B. The velocity is downward.

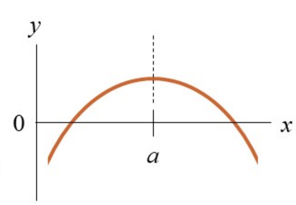
C. The velocity is zero.

D. Either A or B is possible.

E. Any of A, B, or C is possible.

Q15.4

A wave on a string is moving to the right. This graph of $y(x, t)$ versus coordinate x for a specific time t shows the shape of part of the string at that time. At this time, what is the *acceleration* of a particle of the string at $x = a$?



A. The acceleration is upward.

B. The acceleration is downward.

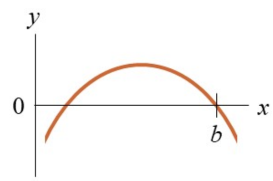
C. The acceleration is zero.

D. Either A or B is possible.

E. Any of A, B, or C is possible.

Q15.5

A wave on a string is moving to the right. This graph of $y(x, t)$ versus coordinate x for a specific time t shows the shape of part of the string at that time. At this time, what is the *velocity* of a particle of the string at $x = b$?



A. The velocity is upward.

B. The velocity is downward.

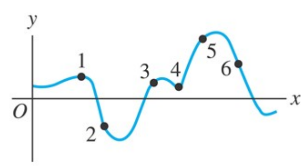
C. The velocity is zero.

D. Either A or B is possible.

E. Any of A, B, or C is possible.

Q15.6

A wave on a string is moving to the right. This graph of $y(x, t)$ versus coordinate x for a specific time t shows the shape of part of the string at that time. At this time, the velocity of a particle on the string is *upward* at



A. only one of points 1, 2, 3, 4, 5, and 6.

B. point 1 and point 4 only.

C. point 2 and point 6 only.

D. point 3 and point 5 only.

E. three or more of points 1, 2, 3, 4, 5, and 6.

Q15.7

Two identical strings are each under the same tension. Each string has a sinusoidal wave with the same average power P_{av}. If the wave on string #2 has twice the amplitude of the wave on string #1, the *wavelength* of the wave on string #2 must be

A. four times the wavelength of the wave on string #1.

B. twice the wavelength of the wave on string #1.

C. the same as the wavelength of the wave on string #1.

D. half of the wavelength of the wave on string #1.

E. one-quarter of the wavelength of the wave on string #1.

Q15.8

The four strings of a musical instrument are all made of the same material and are under the same tension, but have different thicknesses. Waves travel

A. fastest on the thickest string.

B. fastest on the thinnest string.

C. at the same speed on all strings.

D. Either A or B is possible.

E. Any of A, B, or C is possible.

Ans: Q15.1) C, Q15.2) E, Q15.3) C, Q15.4) B, Q15.5) A, Q15.6) C, Q15.7) B, Q15.8) B

Jean le Rond d'Alembert

From Wikipedia, the free encyclopedia
(Redirected from D'Alembert)

Jean-Baptiste le Rond d'Alembert (French pronunciation: [ʒɑ̃ batist lə ʁɔ̃ dalɑ̃bɛːʁ]) (16 November 1717 – 29 October 1783) was a French mathematician, mechanician, physicist, philosopher, and music theorist. He was also co-editor with Denis Diderot of the *Encyclopédie*. D'Alembert's formula for obtaining solutions to the wave equation is named after him.[1]

Jean-Baptiste le Rond d'Alembert

Jean-Baptiste le Rond d'Alembert, pastel by Maurice Quentin de La Tour

Born	16 November 1717 Paris
Died	29 October 1783 (aged 65) Paris
Nationality	French
Fields	Mathematics Mechanics Physics Philosophy
Known for	Fluid mechanics Encyclopédie

Early years

Born in Paris, d'Alembert was the illegitimate child of the writer Claudine Guérin de Tencin and the chevalier Louis-Camus Destouches, an artillery officer. Destouches was abroad at the time of d'Alembert's birth, and a couple of days after birth his mother left him on the steps of the Saint-Jean-le-Rond de Paris church. According to custom, he was named after the patron saint of the church. D'Alembert was placed in an orphanage for found children, but was soon adopted by the wife of a glazier. Destouches secretly paid for the education of Jean le Rond, but did not want his paternity officially recognized.

Studies and adult life

D'Alembert first attended a private school. The chevalier Destouches left d'Alembert an annuity of 1200 livres on his death in 1726. Under the influence of the Destouches family, at the age of twelve d'Alembert entered the Jansenist Collège des Quatre-Nations (the institution was also known under the name "Collège Mazarin"). Here he studied philosophy, law, and the arts, graduating as *bachelier* in 1735. In his later life, D'Alembert scorned the Cartesian principles he had been taught by the Jansenists: "physical promotion, innate ideas and the vortices".

The Jansenists steered D'Alembert toward an ecclesiastical career, attempting to deter him from pursuits such as poetry and mathematics. Theology was, however, "rather unsubstantial fodder" for d'Alembert. He entered law school for two years, and was nominated *avocat* in 1738.

He was also interested in medicine and mathematics. Jean was first registered under the name *Daremberg*, but later changed it to *d'Alembert*. The name "d'Alembert" was proposed by Johann Heinrich Lambert for a suspected (but non-existent) moon of Venus.[*citation needed*]

Career

In July 1739 he made his first contribution to the field of mathematics, pointing out the errors he had detected in *L'analyse démontrée* (published 1708 by Charles René Reynaud) in a communication addressed to the Académie des Sciences. At the time *L'analyse démontrée* was a standard work, which d'Alembert himself had used to study the foundations of mathematics. D'Alembert was also a Latin scholar of some note and worked in the latter part of his life on a superb translation of Tacitus, for which he received wide praise including that of Denis Diderot.

In 1740, he submitted his second scientific work from the field of fluid mechanics *Mémoire sur la réfraction des corps solides*, which was recognized by Clairaut. In this work d'Alembert theoretically explained refraction.

In 1741, after several failed attempts, d'Alembert was elected into the Académie des Sciences. He was later elected to the Berlin Academy in 1746[2] and a Fellow of the Royal Society in 1748[3]

In 1743 he published his most famous work, *Traité de dynamique*, in which he developed his own laws of motion.[4]

When the *Encyclopédie* was organized in the late 1740s, d'Alembert was engaged as co-editor (for mathematics and science) with Diderot, and served until a series of crises temporarily interrupted the publication in 1757. He authored over a thousand articles for it, including the famous *Preliminary Discourse*. D'Alembert "abandoned the foundation of Materialism"[5] when he "doubted whether there exists outside us anything corresponding to what we suppose we see."[5] In this way, D'Alembert agreed with the Idealist Berkeley and anticipated the Transcendental idealism of Kant.

In 1752, he wrote about what is now called D'Alembert's paradox: that the drag on a body immersed in an inviscid, incompressible fluid is zero.

In 1754, d'Alembert was elected a member of the Académie française, of which he became Permanent Secretary on 9 April 1772.[6]

In 1757, an article by d'Alembert in the seventh volume of the Encyclopedia suggested that the Geneva clergymen had moved from Calvinism to pure Socinianism, basing this on information provided by Voltaire. The Pastors of Geneva were indignant, and appointed a committee to answer these charges. Under pressure from Jacob Vernes, Jean-Jacques Rousseau and others, d'Alembert eventually made the excuse that he considered anyone who did not accept the Church of Rome to be a Socinianist, and that was all he meant.[7] He was elected a Foreign Honorary Member of the American Academy of Arts and Sciences in 1781.[8]

For more information see http://en.wikipedia.org/wiki/D%27Alembert