HKUST

MATH1014 Calculus II

Midterm Examination (Makeup Version) Name:_____

24th Mar 2019 Student ID:_____

10:15-11:45 Lecture Section:_____

Directions:

- This is a closed book examination. **No Calculator is allowed in this examination**.
- DO NOT open the exam until instructed to do so.
- Turn off all phones and pagers, and remove headphones. All electronic devices should be kept in a bag away from your body.
- Write your name, ID number, and Lecture Section in the space provided above, and also in the Multiple Choice Item Answer Sheet.
- **Make sure you also write and mark your ID number correctly in the I.D. No. Box, in the Multiple Choice Item Answer Sheet**.
- DO NOT use any of your own scratch paper. Write your name on every scratch paper supplied by the examination, and do not take any scratch paper away after the examination.
- Answer all questions. Show an appropriate amount of work for each long problem. If you do not show enough work, you will get only partial credit.
- When instructed to open the exam, please check that you have **7** pages of questions in addition to the cover page. There are two blank pages and a **formula sheet** attached which can be used as scratch paper.
- You may write on the backside of the pages, but if you use the backside, clearly indicate that you have done so.
- **Cheating is a serious violation of the HKUST Academic Code. Students caught cheating will receive a zero score for the examination, and will also be subjected to further penalties imposed by the University.**

Please read the following statement and sign your signature.

I have neither given nor received any unauthorized aid during this examination. The answers submitted are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University's regulations governing academic integrity.

Student's Signature :

Question No.	Points	Out of
Q. 1-11		55
Q. 12		15
Q. 13		15
Q. 14		15
Total Points		100

Part I: Answer all of the following multiple choice questions.

- Mark your MC answers to the boxes in the Multiple Choice Item Answer Sheet provided.
- Write your name and mark your student ID number on the Multiple Choice Item Answer Sheet.
- Mark only one answer for each MC question. Multiple answers entered for each single MC question will result in a 3 point deduction.

Write also your MC question answers in the following boxes for back up use only. The grading will be based on the answers you mark on the MC item answer sheet.

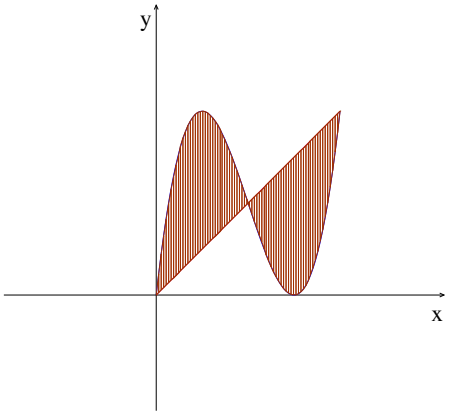
Question	1	2	3	4	5	6	7	8	9	10	11	Total
Answer												

Each of the following MC questions is worth 5 points. No partial credit.

1. What is the color version of your examination paper? (Read the top left corner of the cover page of your exam!) Make sure your ID number has also been **written and marked** correctly in the I.D. No. Box in the MC answer form. **If you do not do both correctly, you lose the points of this question.**

   (a) Green (b) Orange (c) White (d) Yellow (e) None of the previous

2. Find the area or the region bounded between the two curves given by equations $y = x$ and $y = x(x-3)^2$.



   (a) 6 (b) 7 (c) 8 (d) 9 (e) 10

3. Evaluate the integral $\int_0^{\frac{\pi}{4}} \sin(2x)\sin(4x)\,dx$.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$ (e) $\frac{7}{2}$

4. Evaluate the integral $\int_0^{\frac{\pi}{4}} 3\tan^3(x)\sec^2(x)\,dx$.

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$ (e) $\frac{3}{4}$

5. Find the average value of the function $2x^2\sqrt{4-x^2}$ over the interval $[0,2]$.

(a) π (b) 2π (c) $\frac{\pi}{2}$

(d) $\frac{3\pi}{2}$ (e) $\frac{2\pi}{3}$

6. Some function values of a function f with continuous second derivative f'' are given below.

x	0	1	2
$f(x)$	1	-1	1
$f'(x)$	2	1	2

Evaluate the integral $\int_0^2 \left[x^2 f''(x) - 2f(x)\right] dx$.

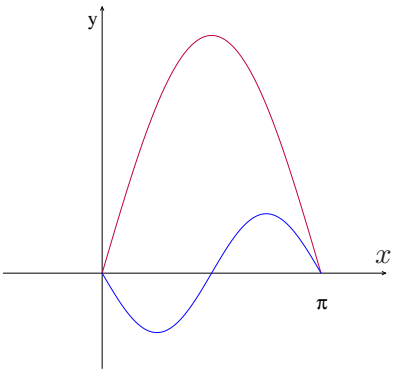
(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

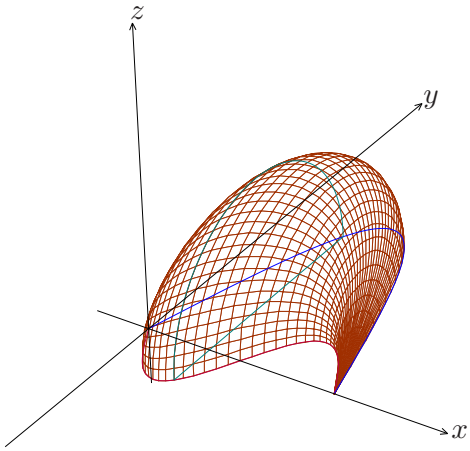
7. Evaluate the integral $\int_0^{\ln 2} \frac{2}{(e^x+1)(e^x+2)}\, dx$.

(a) $3\ln 2 - \ln 3$ (b) $4\ln 2 - 2\ln 3$ (c) $4\ln 2 - 3\ln 3$

(d) $5\ln 2 - 2\ln 3$ (e) $5\ln 2 - 3\ln 3$

8. The base of a solid sitting on the xy-plane is the region enclosed by the graphs of $y = 4\sin x$ and the $y = -\sin(2x)$, for $0 \le x \le \pi$. Suppose that each cross section of the solid perpendicular to the x-axis is a semi-disc. Find the volume of the solid.





(a) $\frac{11\pi^2}{8}$ (b) $\frac{15\pi^2}{8}$ (c) $\frac{21\pi^2}{8}$ (d) $\frac{17\pi^2}{16}$ (e) $\frac{19\pi^2}{16}$

9. The graph of the function $y = x^3/3$ for $0 \le x \le 1$ is rotated about the x-axis to generate a surface of revolution. Find the area of this surface.

(a) $\dfrac{\pi(\sqrt{2}-1)}{3}$ (b) $\dfrac{\pi(\sqrt{2}+1)}{3}$ (c) $\dfrac{\pi(\sqrt{2}+2)}{9}$

(d) $\dfrac{\pi(2\sqrt{2}-1)}{9}$ (e) $\dfrac{\pi(2\sqrt{2}-2)}{9}$

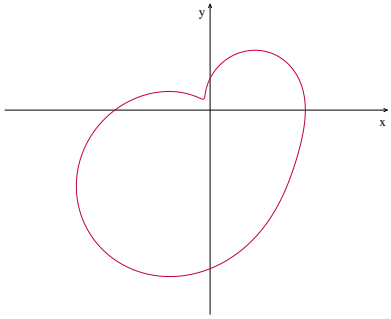
10. Find the length of the graph of the function $f(x) = \displaystyle\int_0^x \tan t\, dt$, where $0 \le x \le \frac{\pi}{4}$.

(a) $\frac{1}{2}\ln 2 - 1$ (b) $\ln 2 - 1$ (c) $\ln(\sqrt{2}-1)$ (d) $\ln(2\sqrt{2}-1)$ (e) $\ln(\sqrt{2}+1)$

11. Find the slope of the tangent line to the curve defined by the **polar equation**

$$r = 3 - 2\sin\theta + \sin(2\theta)$$

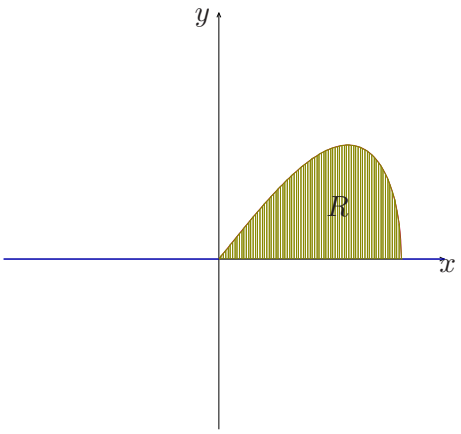
at the point on the curve with angular coordinate $\theta = -\frac{\pi}{4}$.



(a) $1+\sqrt{2}$ (b) $2+2\sqrt{2}$ (c) $2-\sqrt{2}$ (d) $2+\sqrt{2}$ (e) $1+2\sqrt{2}$

Part II: Answer each of the following questions.

12. [*15 pts*] Consider the shaded region R bounded between the graph of $y = x\sqrt{25 - x^2}$ and the x-axis for $0 \leq x \leq 5$.



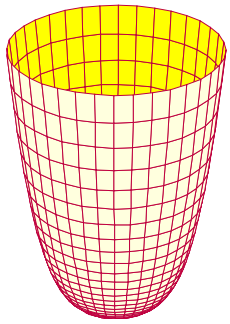
(a) Express the volume of the solid of revolution obtained by rotating the region R about the x-axis by a definite integral. **No partial credit.** [5 pts]

(b) Express the volume of the solid of revolution obtained by rotating the region R about the y-axis by a definite integral. **No partial credit.** [5 pts]

(c) Express the volume of the solid of revolution obtained by rotating the region R about the line $y = -2$ by a definite integral. **No partial credit.** [5 pts]

13. [*15 pts*] A tank is in the shape of a surface of revolution given by rotating the curve $y = x^4$, for $0 \leq x \leq 2$, about the y-axis. Denote the density of water by ρ kg/m^3, and gravity acceleration by g m/s^2.

    (a) Suppose that 60% of the volume of the tank is filled with water. Find the depth of the water in the tank. [7 pts]

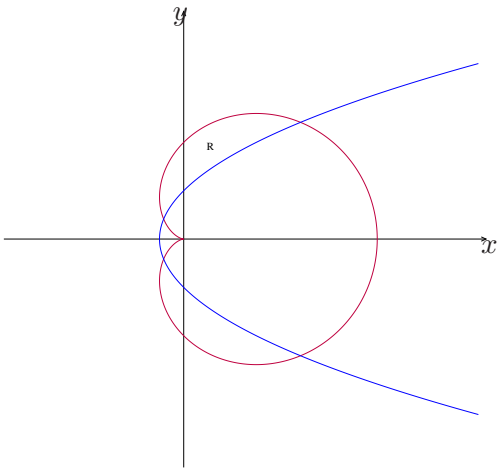


    (b) Find the work required to pump the water in the tank to an outlet at the top of the tank. [8 pts]

14. [*15 pts*] Consider the curves defined by the following polar equations

$$\text{Curve I: } r = 2 + 2\cos\theta, \qquad \text{Curve II: } r = \frac{1}{1-\cos\theta}.$$

(a) Find the rectangular coordinates ((x, y) coordinates) of the four intersection points of the two curves. [5 pts]



(b) Find the xy-equation of Curve II. [4 pts]

(c) Express the area of the shaded region R as a definite integral. **Do not need to evaluate the integral.** [6 pts]

Math1014 Midterm Exam Formula Sheet

Trigonometric Identities

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$
$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\sin A\cos B = \tfrac{1}{2}\Big(\sin(A+B) + \sin(A-B)\Big)$$
$$\cos A\cos B = \tfrac{1}{2}\Big(\cos(A+B) + \cos(A-B)\Big)$$
$$\sin A\sin B = \tfrac{1}{2}\Big(\cos(A-B) - \cos(A+B)\Big)$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x dx$$

$$\int \sin^n x dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x dx$$

$$\int \sec^n x dx = \frac{1}{n-1}\sec^{n-2} x\tan x + \frac{n-2}{n-1}\int \sec^{n-2} x dx$$