Momentum, Impulse, and Collisions II

PHYS1112

Lecture 7

Intended Learning Outcomes

- After this lecture you will learn:
  1) characteristics of elastic collisions.
  2) center of mass and its relation to center of gravity.
  3) the dynamics of the center of mass of a system or a body.

Elastic collision

Conservation of energy: $$\frac{1}{2}m_A v_{A1x}^2 + \frac{1}{2}m_B v_{B1x}^2 = \frac{1}{2}m_A v_{A2x}^2 + \frac{1}{2}m_B v_{B2x}^2 \quad ①$$

Conservation of momentum: $$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x} \quad ②$$

$$v_{B2x} = \frac{m_A v_{A1x} + m_B v_{B1x} - m_A v_{A2x}}{m_B}$$

Want to solve for v_{A2x} and v_{B2x}.

Trick:

$$m_A\left(v_{A1x}^2 - v_{A2x}^2\right) = m_B\left(v_{B2x}^2 - v_{B1x}^2\right)$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$m_A(v_{A1x} + v_{A2x})(v_{A1x} - v_{A2x}) = m_B(v_{B2x} + v_{B1x})(v_{B2x} - v_{B1x}) \quad ③$$

From momentum conservation we have

$$m_A(v_{A1x} - v_{A2x}) = m_B(v_{B2x} - v_{B1x}) \quad ④$$

$$\Rightarrow \quad v_{A1x} + v_{A2x} = v_{B1x} + v_{B2x} \qquad ③/④$$

Physical meaning: $$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$$

relative velocity *after* collision = – (relative velocity *before* collision)

In an elastic collision we can write down three equations: 3

1. conservation of energy
2. conservation of momentum.
3. relative velocity *after* collision = – (relative velocity *before* collision)

But only two of them are independent. Usually 2 and 3 are preferred because they are linear.

$$\begin{cases} m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x} \\ v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x}) \end{cases}$$

$$V_{B2x} = V_{A2x}$$

$$-(V_{B1x} - V_{A1x})$$

$$\Rightarrow v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x} + \frac{2m_B}{m_A + m_B} v_{B1x}, \quad = 0$$

$$v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x} + \frac{m_B - m_A}{m_A + m_B} v_{B1x} \quad = 0$$

Special case: Elastic collision with one body initially at rest

$v_{B1x} = 0$

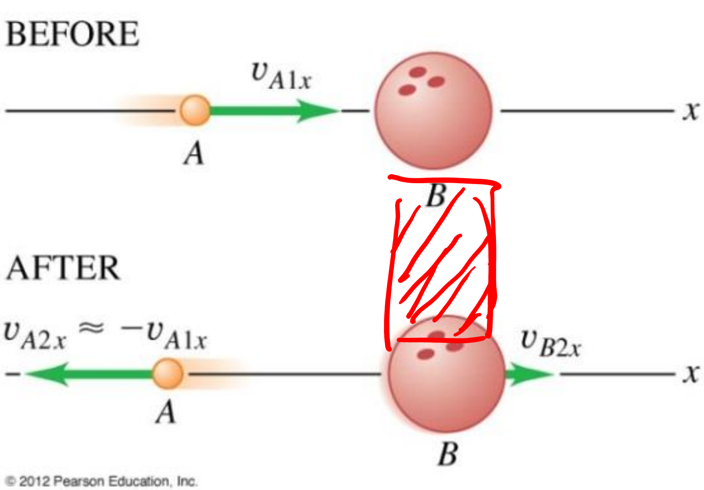
B initially at rest

$$\vec{v}_{A2x} = \frac{m_A - m_B}{m_A + m_B}\vec{v}_{A1x}, \qquad \vec{v}_{B2x} = \frac{2m_A}{m_A + m_B}\vec{v}_{A1x}$$

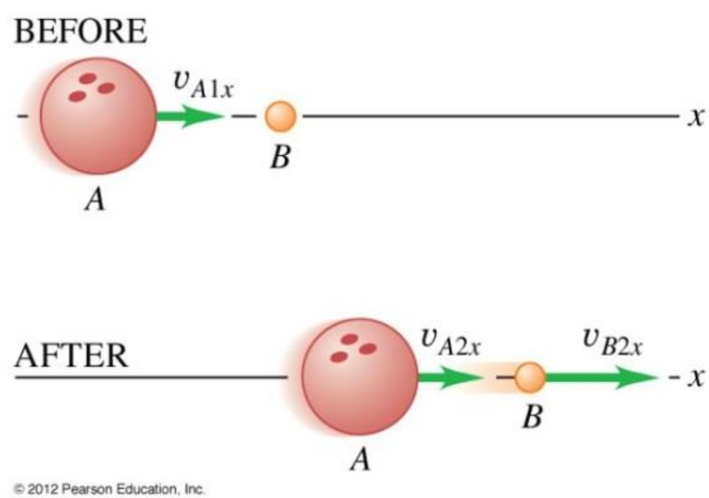
⚠ v_{B2x} same direction (same sign) as v_{A1x}, but direction of v_{A2x} depends on m_A-m_B

$\vec{v}_{A2x} \approx -\vec{v}_{A1x}$ $\qquad$ $\vec{v}_{B2x} = 0$

(a) Ping-Pong ball strikes bowling ball.



(b) Bowling ball strikes Ping-Pong ball.

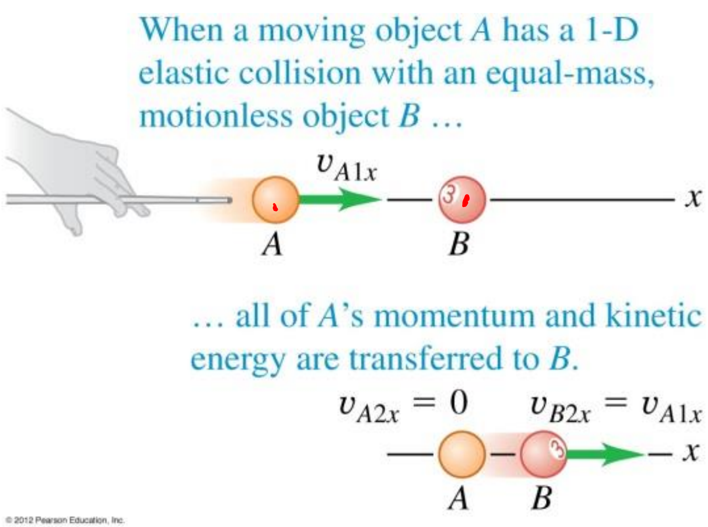


$m_B > m_A$, A reflected back

$m_B >> m_A$

$m_B < m_A$, A continue forward and $v_{A2x} < v_{B2x}$

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x}, \qquad v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x}$$



$$m_A = m_B, v_{A2x} = 0, v_{B2x} = v_{A1x}$$

$m_A v_{A1} = m_B v_{B2}$

Demonstration: Newton’s Cradle

Demonstration: Gaussian gun – where comes the extra energy when there is a magnet?

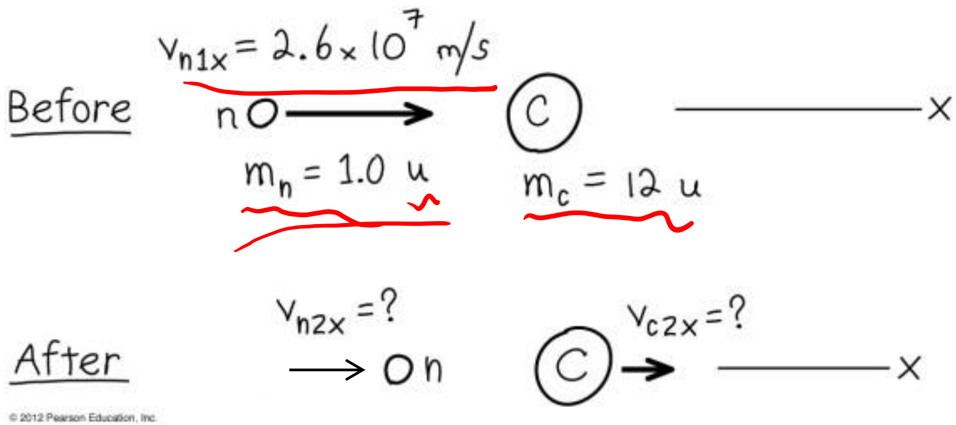
potential

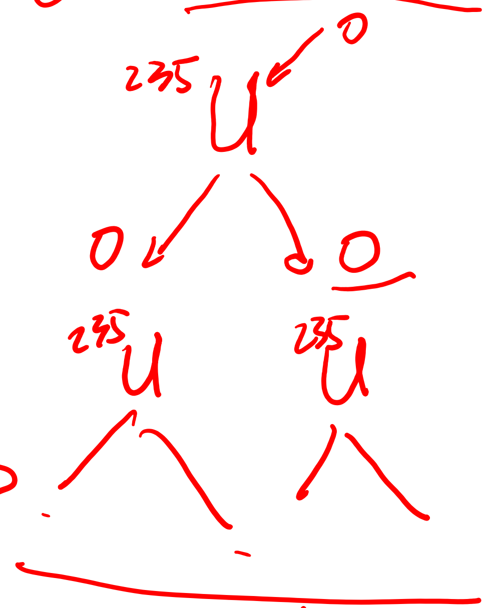
Example Moderating fission neutrons in a nuclear reactor

Fission of uranium produces high speed neutrons which must be slowed down before it can initiate another fission process. Suppose graphite (carbon) is used as moderator to slow down neutrons.

chain reaction

^{235}U





bomb

assuming elastic collision

relative velocities: $v_{C2\,x} - v_{n2x} = -(0 - v_{n1x})$

conservation of momentum:

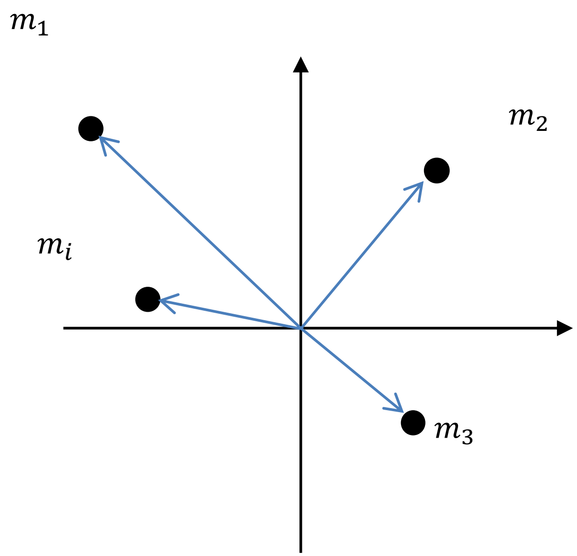
$$m_{\mathrm{n}} v_{\mathrm{n1}\,x} = m_{\mathrm{C}} v_{\mathrm{C2}\,x} + m_{\mathrm{n}} v_{\mathrm{n2}\,x}$$

get: $v_{\mathrm{n2}\,x} = -2.2 \times 10^7$ m/s, $v_{\mathrm{C2}\,x} = 0.4 \times 10^7$ m/s

Don’t worry about the direction (forward or backward) of neutron after collision. Assume all v are +ve, *c.f.*, figure in the textbook.

Center of Mass

The center of mass (CM) of a system of point particles is defined as



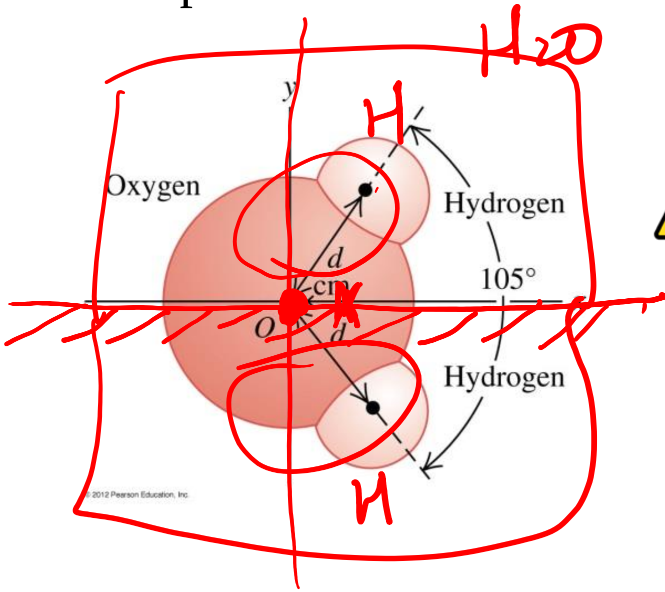
$$\vec{\boldsymbol{r}}_{\text{cm}} = \frac{m_1\vec{\boldsymbol{r}}_1 + m_2\vec{\boldsymbol{r}}_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum m_i \vec{\boldsymbol{r}}_i}{\sum m_i}$$

$$\boxed{\vec{r} = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}}$$

i.e.

$$x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i}, \qquad y_{\text{cm}} = \frac{\sum m_i y_i}{\sum m_i}, \qquad z_{\text{cm}} = \frac{\sum m_i z_i}{\sum m_i}$$

Example



$m_e \ll 1\,u$

Need to worry about mass of nuclei only (why not e^-?)

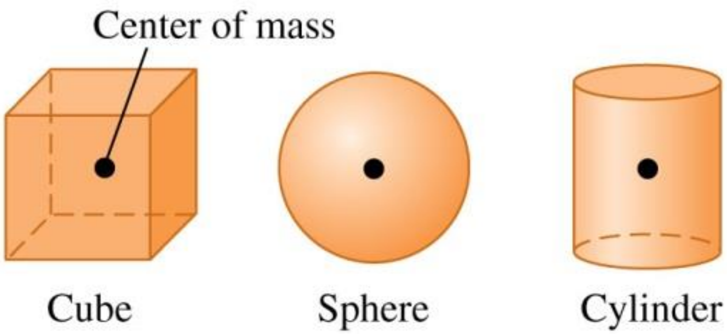
⚠ choose symmetry axis of the molecule as the *x*, *y*, and *z* directions

meaning under rotation about that axis, the molecule looks the same

$$x_{cm} = \frac{(1.0\text{ u})(d\cos 52.5°) + (1.0\text{ u})(d\cos 52.5°) + (16.0\text{ u})(0)}{1.0\text{ u} + 1.0\text{ u} + 16.0\text{ u}}$$
$$= 0.068d = 6.5 \times 10^{-12}\text{ m}$$

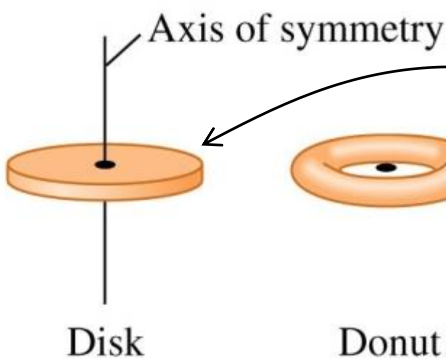
$$y_{cm} = \frac{(1.0\text{ u})(d\sin 52.5°) + (1.0\text{ u})(-d\sin 52.5°) + (16.0\text{ u})(0)}{1.0\text{ u} + 1.0\text{ u} + 16.0\text{ u}}$$
$$= 0$$

If not point particles, need integration … but not in this course



Make use of *symmetry* rules to find CM of homogeneous (constant density) solids:

1) at the geometric center of regular solids
2) lies on symmetry axis
3) CM can lie outside a body



c.f. center of gravity (CG) which you have learned in high school.

⚠ If g is the same at all points on a body, its CG is identical to its CM.

⚠ You know how to determine the CG of a rigid body experimentally.

From definition of $\vec{r}_{cm}$, (by differentiation)

ΣP:

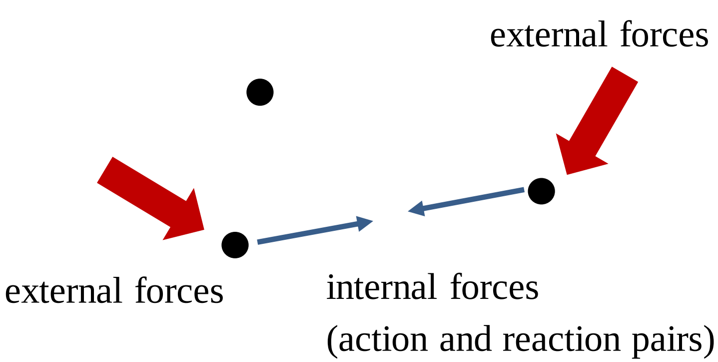
$$\frac{d}{dt} \qquad \frac{d}{dt}$$

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + \cdots}{m_1 + \cdots} \quad \Longrightarrow \quad M\vec{v}_{cm} = m_1\vec{v}_1 + \cdots = \vec{P}$$

total linear momentum

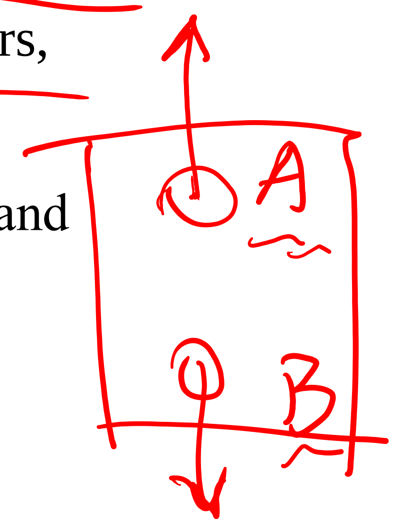
$$M\vec{a}_{cm} = m_1\vec{a}_1 + \cdots = \sum \vec{F}_{ext} + \underbrace{\sum \vec{F}_{int}}_{} = \sum \vec{F}_{ext}$$

$= 0$



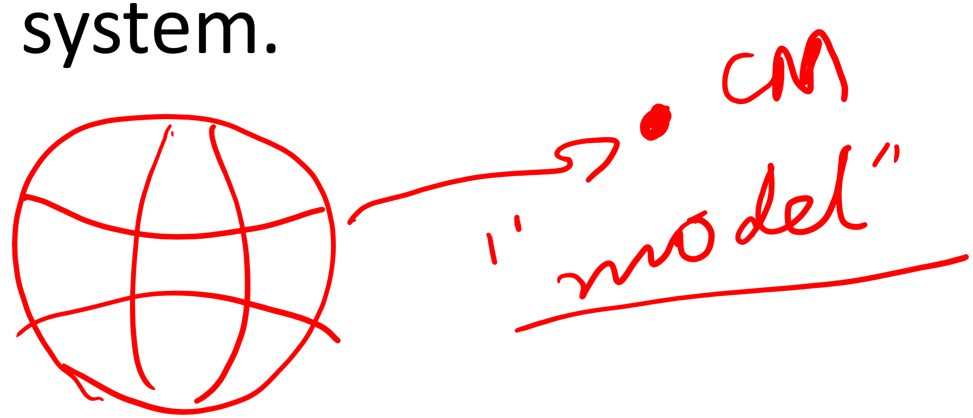
in equal and opposite pairs, add up to zero

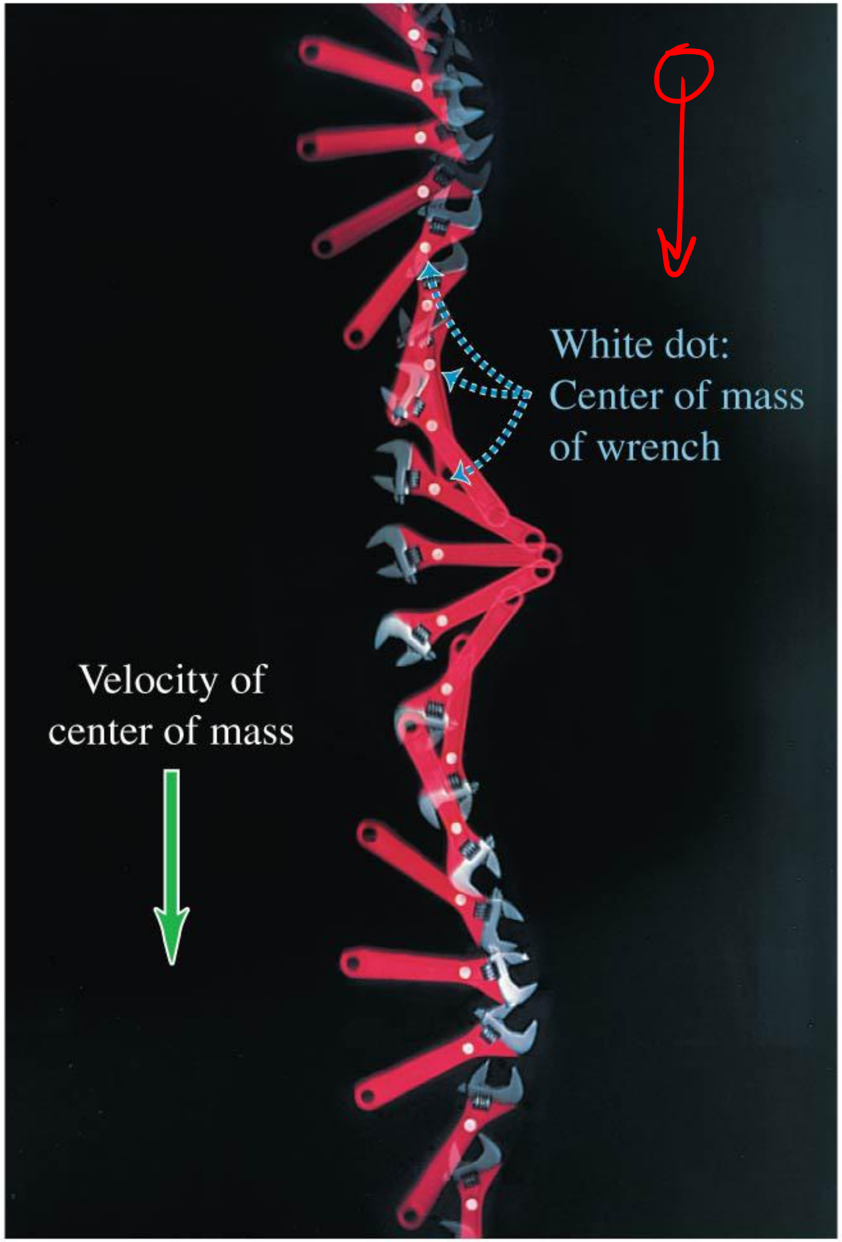
(⚠ didn't we say action and reaction do not cancel?)



Conclusion: $\boxed{M\vec{\boldsymbol{a}}_{\text{cm}} = \sum \vec{\boldsymbol{F}}_{\text{ext}} = d\vec{\boldsymbol{P}}/dt}$

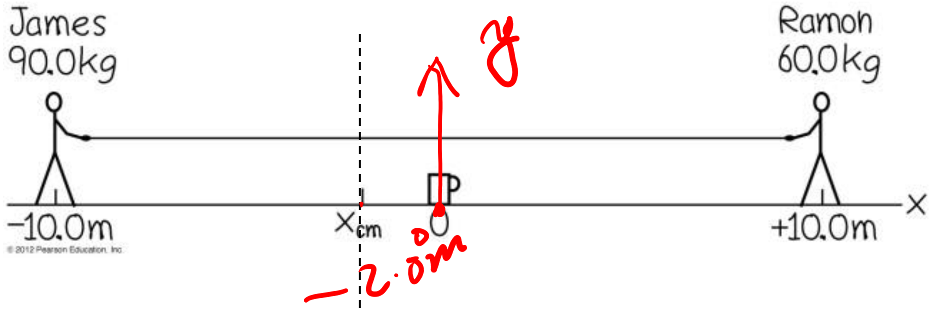
When a body or a collection of particles is acted on by external forces, the CM moves just as though all the mass were concentrated at that point and it were acted on by a net force equal to the sum of the external forces on the system.

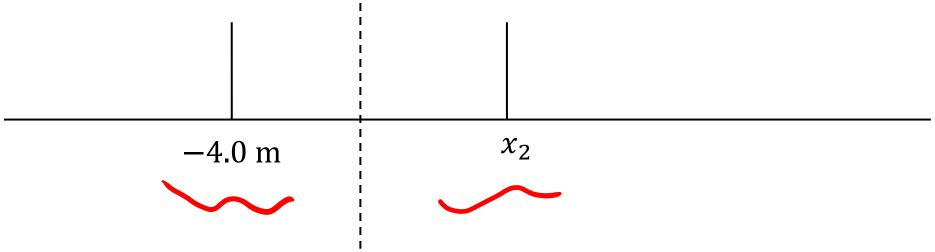




e.g. a falling rotating wrench (spanner)

An example with no external force – tug of war on ice





$$x_{\mathrm{cm}} = \frac{(90.0\ \mathrm{kg})(-10.0\ \mathrm{m}) + (60.0\ \mathrm{kg})(10.0\ \mathrm{m})}{90.0\ \mathrm{kg} + 60.0\ \mathrm{kg}}$$

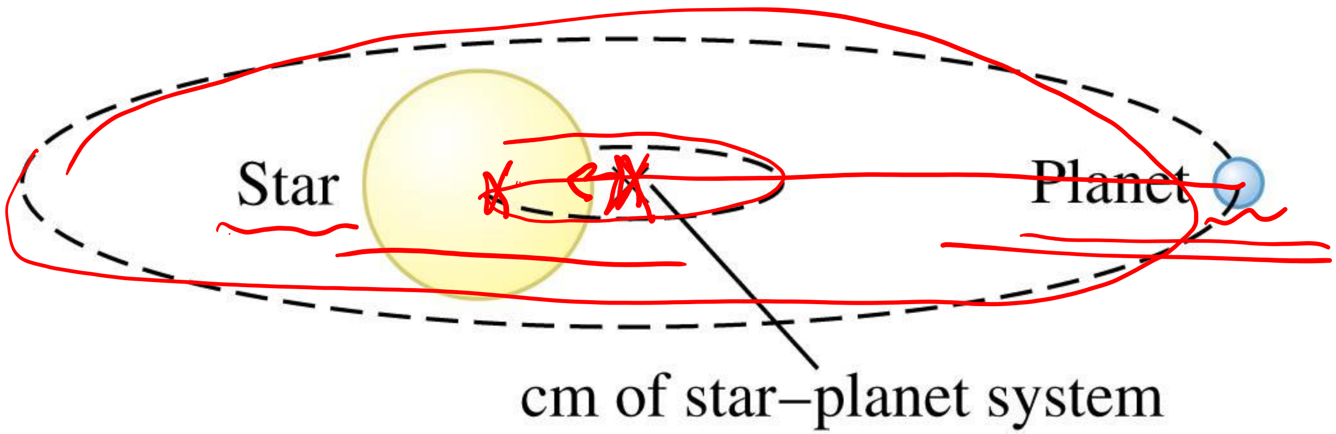
$$= -\frac{300}{150}\ m = -2.00\ m$$

$$x_{\mathrm{cm}} = \frac{(90.0\ \mathrm{kg})(-4.0\ \mathrm{m}) + (60.0\ \mathrm{kg})x_2}{90.0\ \mathrm{kg} + 60.0\ \mathrm{kg}}$$

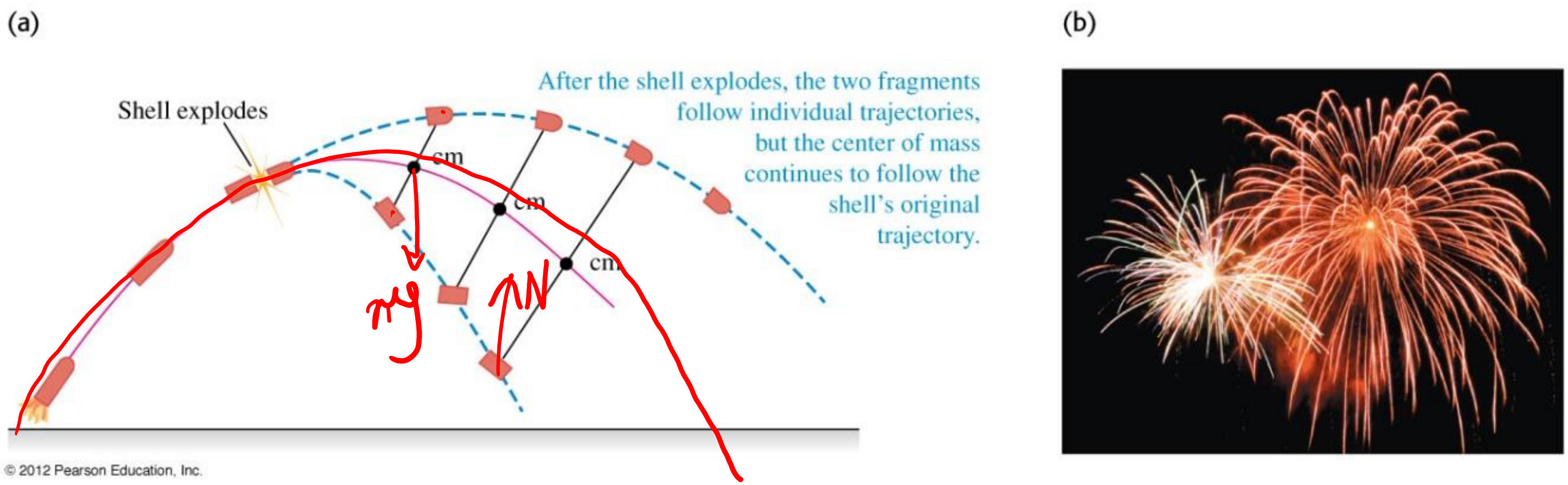
$$\Rightarrow \quad x_2 = 1.0\ \mathrm{m}$$

An example of a wobbling star

A wobbling star shows the presence of an accompanying planet which is too dim to be seen



An example with external force – A shell explodes into two fragments in flight.



Question: Will the CM in the above problem continue on the same parabolic trajectory even after one of the fragments hits the ground?

Example Walking on a canoe in still water

Canoe is 5.00 m long. CM of canoe is at its center

CM of the system (canoe + girl) doesn't move. Girl's mass 45.0 kg, canoe's mass 60.0 kg

$$x_{cm} = \frac{(2.50\text{ m})(60.0\text{ kg}) + (1.00\text{ m})(45.0\text{ kg})}{45.0\text{ kg} + 60.0\text{ kg}}$$

$$= 1.857\text{ m} \qquad \frac{195}{105}\text{ m}$$

⚠ may keep one extra significant figure for intermediate result

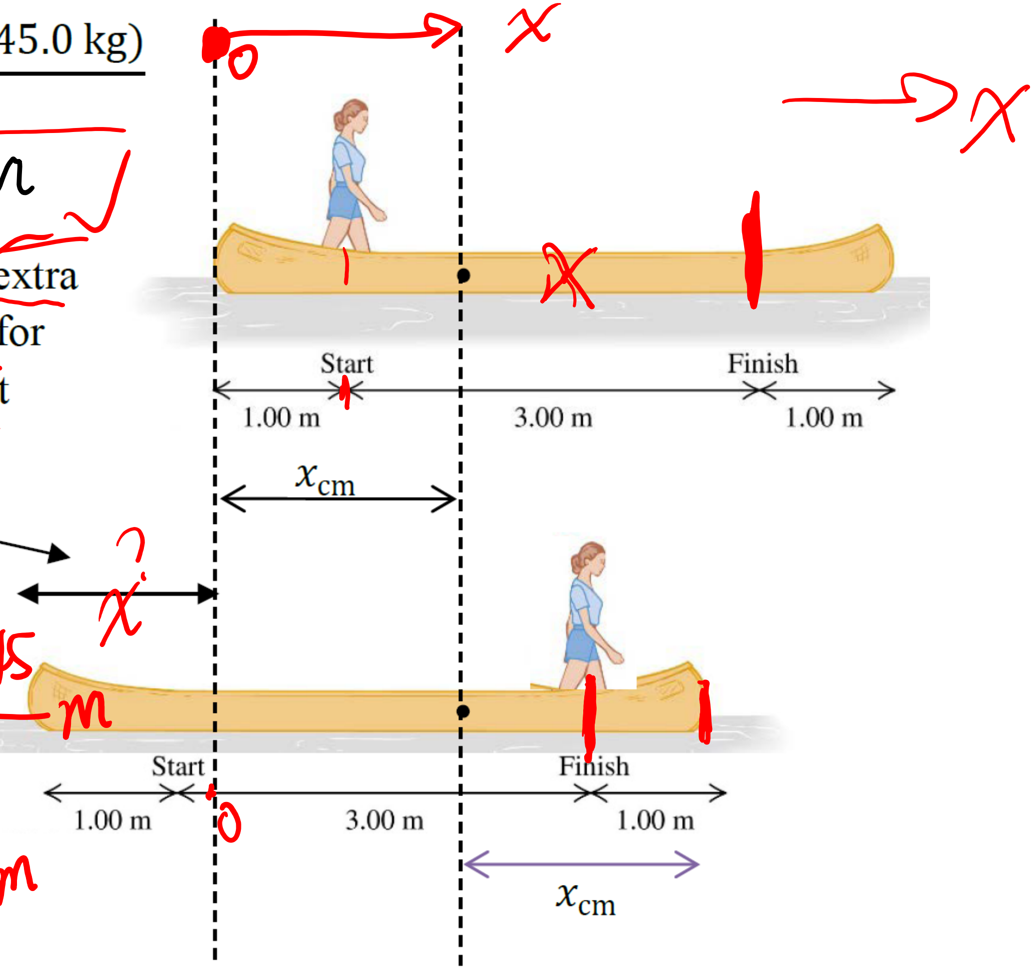
Canoe moves to the left by

$$\boxed{5.00\text{ m} - 2x_{cm} = 1.29\text{ m}}$$

$$x_{cm} = \frac{(2.5-x)\times 60 + (4-x)\times 45}{105}\text{ m}$$

$$\frac{330}{105}\text{ m} - x = \frac{195}{105}\text{ m}$$

$$x = \frac{135}{105}\text{ m} \approx 1.29\text{ m}$$



CM does not move

Q8.11

A yellow block and a red rod are joined together. Each object is of uniform density. The center of mass of the *combined* object is at the position shown by the black "X." Which has the *greater mass*, the yellow block or the red rod?

A. The yellow block has the greater mass.

B. The red rod has the greater mass.

C. They both have the same mass.

D. Either A or B is possible.

E. A, B, or C is possible.

A8.11

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$$\frac{m_1x_1 + m_2x_2}{\sum m}$$

A. The yellow block has the greater mass.

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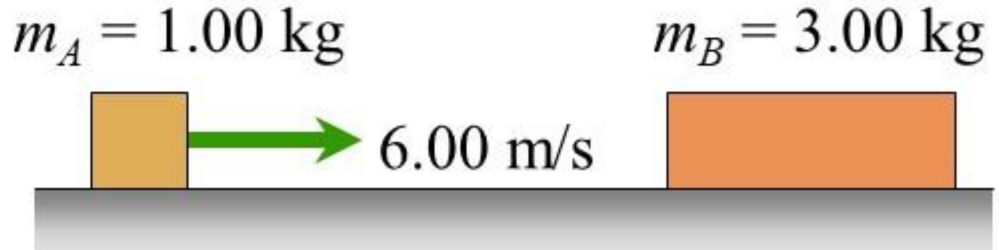
Q8.13

Block A on the left has mass 1.00 kg. Block B on the right has mass 3.00 kg.



Block A is initially moving to the right at 6.00 m/s, while block B is initially at rest. The surface they move on is level and frictionless. What is the velocity of the center of mass of the two blocks *after* the blocks collide?

A. 6.00 m/s, to the right

B. 3.00 m/s, to the right

C. 1.50 m/s, to the right

D. zero

E. Not enough information is given to decide.

A8.13

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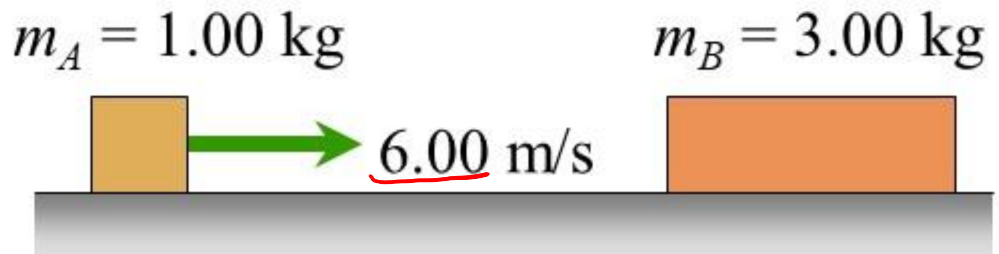

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A. 6.00 m/s, to the right

B. 3.00 m/s, to the right

✔ C. 1.50 m/s, to the right

D. zero

E. Not enough information is given to decide.

$$\frac{m_A v_A + m_B v_B}{m_A + m_B} \rightarrow$$

$$= 1.50 \text{ m/s}$$