MATH1012 Calculus IA

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Topic of Calculus IA: one-variable calculus.

Things you need to notice:

(1). How to use "Canvas"? https://canvas.ust.hk/.

Login → find "Course" → choose "MATH 1012"

→ find "outline" in "Syllabus".

find notes in "File" (will upload in next days).

(2) Time:

Lecture time: Mon/Wed 9 a.m. — 10:20 a.m. LTE (Xu Zili).

Wed 16:30 p.m. — 17:20 p.m. LTC (Liang Shixin).

Tutorial: once a week. (start from next Wednesday, i.e. 8. Sep).

TA: Li jiayi and Liang Shixin.

(3). Credit points: 4.

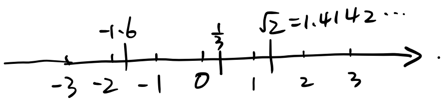
(4) grade
- assignments 25% ← given once a week, "web work" https://webwork.math.ust.hk/, deadlines
- final exam 75%

(5). Textbook: "Calculus Early Transcendentals". 7th edition.

Basic concepts and notations

1. Real numbers

A real number is a number that can be found on the number line.



Example: integers: $1, 2, 3, \cdots$.

$\frac{1}{3}$, $\sqrt{2}$, -1.6.

Notice: (1) Every number on this number is a real number.

(2) Every real number can be found on this number line.

"R": the set of all real numbers.

$$\text{real numbers} \begin{cases} \text{rational numbers} \\ \text{irrational numbers} \end{cases}$$

1.1 Rational numbers

A rational number is a real number that can be expressed as a fraction using integers.

$m = \frac{a}{b}$ → integers.

a rational number

Example: $5 = \frac{5}{1} = \frac{10}{2}$.

$\frac{1}{3}$ $\quad -1.6 = \frac{-8}{5}$.

1.2 Irrational numbers

An irrational number is a real number that can not be expressed as a fraction using integers.

Example: $\sqrt{2}$, $\pi = 3.1415926\cdots$

1.3 How to tell if a number is rational or irrational?

Notice: Every real number has a decimal representation.

m is rational $\Leftrightarrow$ its decimal is repeating.

Example: $\frac{1}{3} = 0.33\cdots = 0.\bar{3}$

$0.285714\,285714\cdots = 0.\overline{285714} = \frac{2}{7}$.

m is irrational $\Leftrightarrow$ its decimal is non-repeating.

Example: $\pi = 3.1415926\cdots$

$\sqrt{2} = 1.41421356\cdots$

2. basic notation

2.1 order

Assume a, b are two real numbers.

(1). $a < b$: "a is less than b"

or "$b-a$ is a positie number".

Equivalenty, we can write $b > a$.

(2). $a \le b$: "a is less than or equal to b"

or "$b-a$ is zero or a positive number".

Equivalently, we can write $b \ge a$.

Example: $1 < 2 < 3$. $4 \le 4$.

2.2 set

(1). A set is a collection of objects, and these objects are called "elements".

Example: $\underset{\sim}{A} = \{1 \underset{\text{comma}}{,} 2, 3\}$ is a set. → braces

"2" is an element of A.

(2). "belong to".

$\underset{\text{element}}{a} \in \underset{\text{set}}{S}$: "a" is an element of S.

or "a" belongs to S.

"not belong to":

$a \notin S$: "a" is not an element of S.

or "a" does not belong to S.

Example: $2 \in A$. $4 \notin A$.

(3). union

$\underbrace{S \cup T}_{\text{a new set}}$ (S: set, T: set) : the set that contains all elements in S or T.

(4). intersection.

$\underbrace{S \cap T}_{\text{a new set}}$ (S: set, T: set) : the set that contains all elements in both S and T.

Example : $A = \{1, 2, 3\}$, $B = \{2, 5\}$.

$A \cup B = \{1, 2, 3, 5\}$.

$A \cap B = \{2\}$.

(5). empty set ϕ : the set that contains no element.

Notice : $S \cup \phi = S$. $S \cap \phi = \phi$.

(6). How to represent a set ?

$A = \{1, 2, 3\}$. , list every element in A.

" set-builder notation ".

$A = \{x \mid x \text{ is an integer and } 0 < x < 4\}$. use words to describe the conditions on the element of A.

$\mid$: a vertical bar meaning " such that ".

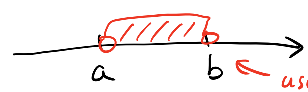
$\boxed{\text{2-3 intervals}}$

Assume a and b are two real numbers and $a < b$.

(1). $(a, b) = \{x \mid a < x < b\}$.

round brackets

called " the open interval from a to b ".

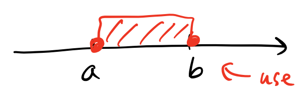


use open dots.

(2). $[a, b] = \{x \mid a \le x \le b\}$

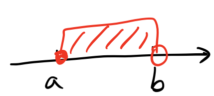
square brackets

called " the closed interval from a to b ".

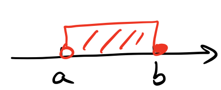


use solid dots.

(3). $[a, b) = \{x \mid a \le x < b\}$.



$(a, b] = \{x \mid a < x \le b\}$



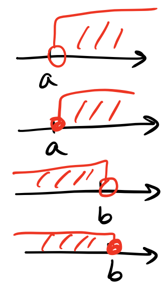
(4) infinite intervals.

$(a, +\infty) = \{x \mid x > a\}$

$[a, +\infty) = \{x \mid x \geq a\}$.

$(-\infty, b) = \{x \mid x < b\}$

$(-\infty, b] = \{x \mid x \leq b\}$.



Example: $1 \in (-3, 5)$.

2.4 Inequality

" $1+2=3$ " → an equality. " $1+2<4$ " → an inequality.

Assume that a, b, c, d are four real numbers.

Rules
(1). If $a<b$ then $a+c < b+c$.
(2). If $a<b$ and $c<d$, then $a+c < b+d$.
(3). If $a<b$ and $c>0$, then $ac < bc$.
(4). If $a<b$ and $c<0$, then $ac > bc$.
(5). If $0<a<b$, then $\frac{1}{a} > \frac{1}{b}$.

Example: Solve $x-3>5$ ← we need to find the values of x such that this inequality is true.

$$x-3+3 > 5+3.$$

$$x > 8.$$

Solution: $(8, +\infty)$.

2.5. Absolute values.

$|a|$: distance from a to zero.

real number

Notice: $|a| \geq 0$. for all real numbers a.

Example: $|-2.4| = 2.4$. $|5.6| = 5.6$.

In general, we have $|a| = \begin{cases} a & \text{if } a \geq 0. \\ -a & \text{if } a < 0. \end{cases}$

Notice: we have $\sqrt{a^2} = |a|$. for all real numbers a.

Example : Solve $|x-3| > 5$. ← we need to find the value of x such that this inequality is true.

1) The first case : $x-3 \geq 0$.

$\rightarrow |x-3| = x-3$.

Then we need to solve $x-3 > 5$.

$x > 8$.

Combining with $x-3 \geq 0$, we obtain $x > 8$.

2) The second case : $x-3 < 0$

$\rightarrow |x-3| = -(x-3) = -x+3$.

Then we need to solve $-x+3 > 5$.

$3 > 5+x$.

$x < -2$.

Combining with $x-3<0$, we obtain $x<-2$.

Final solution: $(8, +\infty) \cup (-\infty, -2)$.