MATH 2111 Matrix Algebra and Applications
Homework-10 : Due 12/05/2022 at 11:59pm HKT

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1. (1 point) Find the projection of $\vec{v} = \begin{bmatrix} -4 \\ -1 \\ 4 \end{bmatrix}$ onto the line ℓ of $\mathbb{R}^3$ given by the parametric equation $\ell = t\vec{u}$, where $\vec{u} = \begin{bmatrix} 5 \\ 5 \\ 3 \end{bmatrix}$.

$\text{proj}_\ell(\vec{v}) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} -1.10169 \\ -1.10169 \\ -0.661017 \end{bmatrix}$

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2. (2 points) Perform the Gram-Schmidt process on the following sequence of vectors.

$x = \begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix}, y = \begin{bmatrix} -2 \\ -4 \\ -5 \end{bmatrix}, z = \begin{bmatrix} -9 \\ 0 \\ 0 \end{bmatrix}$.

[Note: You need to normalize your vectors.]

$\begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}, \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}, \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.

Correct Answers:

- `0.666666666666667`
- `0.333333333333333`
- `0.666666666666667`
- `0.666666666666667`
- `-0.666666666666667`
- `-0.333333333333333`
- `-0.333333333333333`
- `-0.666666666666667`
- `0.666666666666667`

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3. (2 points) Find the projection of $\vec{v} = \begin{bmatrix} -10 \\ 12 \\ 14 \end{bmatrix}$ onto the subspace W of $\mathbb{R}^3$ spanned by $\begin{bmatrix} -5 \\ 6 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 5 \\ -1 \end{bmatrix}$.

$\text{proj}_W(\vec{v}) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} -10.254 \\ 9.20635 \\ -1.49206 \end{bmatrix}$

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4. (2 points) Let

$$\vec{v}_1 = \begin{bmatrix} 0.5 \\ -0.5 \\ 0.5 \\ -0.5 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{bmatrix}.$$

Find a vector $\vec{v}_4$ in $\mathbb{R}^4$ such that the vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, and $\vec{v}_4$ are orthonormal.

$\vec{v}_4 = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$

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5. (3 points) Let

$$\vec{x} = \begin{bmatrix} 4 \\ -1 \\ -1 \\ 0 \end{bmatrix}, \vec{y} = \begin{bmatrix} 1 \\ -0.5 \\ -4.5 \\ 1 \end{bmatrix}, \vec{z} = \begin{bmatrix} -6.5 \\ -17.5 \\ 0.5 \\ 4.5 \end{bmatrix}.$$

Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by $\vec{x}$, $\vec{y}$, and $\vec{z}$.

$$\left\{ \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}, \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}, \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} \right\}.$$

Correct Answers:

- $\begin{bmatrix} 0.942809 \\ -0.235702 \\ -0.235702 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} -0.235702 \\ 0 \\ -0.942809 \\ 0.235702 \end{bmatrix}$
- $\begin{bmatrix} -0.210819 \\ -0.948683 \\ 0.105409 \\ 0.210819 \end{bmatrix}$

6. (2 points) Let $y = \begin{bmatrix} -4 \\ 6 \\ 0 \\ 2 \end{bmatrix}$, $u_1 = \begin{bmatrix} 6 \\ 4 \\ 2 \\ -1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -6 \\ -4 \\ -1 \\ -54 \end{bmatrix}$.

Compute the distance d from y to the subspace of $\mathbb{R}^4$ spanned by u_1 and u_2.

$d =$ ____

Correct Answers:

- `7.2111877714221`

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7. (2 points) Given $\vec{v} = \begin{bmatrix} 5 \\ 4 \\ 5 \\ -9 \end{bmatrix}$, find the closest point to $\vec{v}$ in the subspace W spanned by $\begin{bmatrix} -3 \\ 4 \\ 4 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ 4 \\ 4 \\ 44 \end{bmatrix}$.

$\begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} -1.37673 \\ 2.09101 \\ 2.09101 \\ -9.14171 \end{bmatrix}$

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8. (2 points) All vectors and subspaces are in $\mathbb{R}^n$.

Check the true statements below:

- A. If an $n \times p$ matrix U has orthonormal columns, then $UU^T x = x$ for all x in $\mathbb{R}^n$.
- B. In the Orthogonal Decomposition Theorem, each term $\hat{y} = \frac{y \cdot u_1}{u_1 \cdot u_1} u_1 + \ldots + \frac{y \cdot u_p}{u_p \cdot u_p} u_p$ is itself an orthogonal projection of y onto a subspace of W.
- C. The best approximation to y by elements of a subspace W is given by the vector $y - \text{proj}_W(y)$.
- D. If $y = z_1 + z_2$, where z_1 is in a subspace W and z_2 is in $W^\perp$, then z_1 must be the orthogonal projection of y onto W.
- E. If W is a subspace of $\mathbb{R}^n$ and if v is in both W and $W^\perp$, then v must be the zero vector.

Correct Answers:

- `BDE`

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9. (3 points) Let W be the subspace of $\mathbb{R}^3$ spanned by the vectors

$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \text{ and } \begin{bmatrix} 6 \\ -10 \\ 2 \end{bmatrix}.$$

Find the matrix A of the orthogonal projection onto W.

$A = \begin{bmatrix} \_\_\_\_ & \_ & \_ \\ \_\_\_\_ & \_ & \_ \\ \_\_\_\_ & \_ & \_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} 0.833333 & -0.166667 & -0.333333 \\ -0.166667 & 0.833333 & -0.333333 \\ -0.333333 & -0.333333 & 0.333333 \end{bmatrix}$

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10. (2 points) Find the least-squares solution $\vec{x}^*$ of the system

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 3 & 4 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}.$$

$\vec{x}^* = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

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11. (3 points) Fit a quadratic function of the form $f(t) = c_0 + c_1 t + c_2 t^2$ to the data points $(0, -6)$, $(1, 0)$, $(2, -12)$, $(3, -2)$, using least squares.

$f(t) =$ ________

Correct Answers:

- `t^2-(4+3*t)`

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12. (3 points) Fit a trigonometric function of the form $f(t) = c_0 + c_1 \sin(t) + c_2 \cos(t)$ to the data points $(0, 13.5)$, $(\frac{\pi}{2}, 5.5)$, $(\pi, -0.5)$, $(\frac{3\pi}{2}, 13.5)$, using least squares.

$f(t) =$ ________

Correct Answers:

- `8-4*sin(t)+7*cos(t)`