Math1013 Calculus I **Name:** Kin Nam KOO

Homework-6 : Due 11/14/2021 at 11:59pm HKT

This homework set covers the basic usages of derivatives, including:

1. using linear approximations;
2. using Newton's Method to find approximation solutions of equations;
3. locating local or absolute maxima and minima;
4. determining the intervals of increase and decrease;
5. determining concavity and inflection points;
6. graphing functions;
7. using the Mean Value Theorem.

Give 4 or 5 significant digits for numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as 3^2 or 3**2 instead of 9, $\sin(3*pi/2)$ instead of -1, $e^\wedge(\ln(3))$ instead of 3, $(1+\tan(3))*(4-\sin(5))^\wedge 6-15/8$ instead of 12748.8657, etc.

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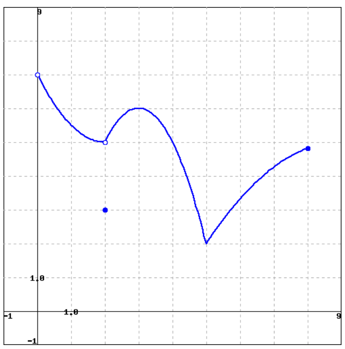
1. (2 points) Use the given graph of the function on the interval $(0,8]$ to answer the following questions.



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1. Where does the function f have a local maximum?
Answer (separate by commas): $x =$ ______

2. Where does the function f have a local minimum?
Answer (separate by commas): $x =$ ______

3. What is the global maximum of f?
Answer (write 'none' if there is none): ______

4. What is the global minimum of f?
Answer (write 'none' if there is none): ______

Note: *You can click on the graph to enlarge the image.*

Correct Answers:

- `3, 8`
- `2, 5`
- `none`
- `2-0`

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2. (2 points) Find the exact global maximum and minimum values of the function $g(t) = 9te^{-3t}$ if $t > 0$.

global maximum at $t =$ ______
global minimum at $t =$ ______

(Enter **none** *if there is no global maximum or global minimum for this function.)*

Solution:

SOLUTION

Differentiating using the product rule gives

$$g'(t) = 9\cdot e^{-3t} - 27te^{-3t} = 9(1-3t)e^{-3t},$$

so the critical point is $t = 1/3$.

Since $g'(t) > 0$ for $0 < t < 1/3$ and $g'(t) < 0$ for $t > 1/3$, the critical point is a local maximum.

As $t \to \infty$, the value of $g(t) \to 0$, and as $t \to 0^+$, the value of $g(t) \to 0$. Thus, the local maximum at $t = 1/3$ is a global maximum of $g(1/3) = \frac{9}{3}e^{-1} = \frac{9}{3e}$. In addition, the value of $g(t)$ is positive for all $t > 0$; it tends to 0 but never reaches 0. Thus, there is no global minimum.

Correct Answers:

- `1/3`
- `none`

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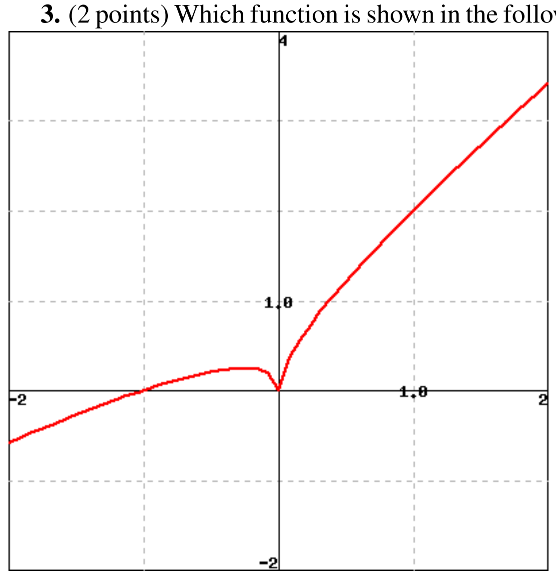
3. (2 points) Which function is shown in the following graph?



a.) $f(x) = x + 3x^{2/3}$
b.) $f(x) = x + \sqrt{|x|}$
Answer: (input a or b) ___

Note: You can click on the graph to make it larger.

Correct Answers:

- `B`

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4. (4 points) For the function $f(x) = -6x + \sin(x)$, find all intervals where the function is increasing.

f is increasing on ______

(Give your answer as an interval or a list of intervals, e.g., **(-infinity,8]** *or* **(1,5),(7,10)** *. Enter* **none** *if there are no such intervals.)*

Similarly, find all intervals where the function is decreasing:

f is decreasing on ______

(Give your answer as an interval or a list of intervals, e.g., **(-infinity,8]** *or* **(1,5),(7,10)** *. Enter* **none** *if there are no such intervals.)*

Finally, find all critical points in the graph of $f(x)$ (enter x values as a comma-separated list, or **none** if there are no critical points): ______________

Solution:

SOLUTION

The function is increasing and decreasing where $f'(x) > 0$ and $f'(x) < 0$; critical points are where $f'(x) = 0$. Here $f'(x) = -6 + \cos(x)$. But $-6 > \cos(x)$ for all x, so f is decreasing on $(-\infty, \infty)$ and decreasing nowhere. There are no critical points, because $f'(x)$ is never zero.

Correct Answers:

- `none`
- `(-infinity,infinity)`
- `none`

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5. (4 points) Let $f(x) = \dfrac{6x^2}{x-2}$. Find the open intervals on which f is increasing (decreasing). Then determine the x-coordinates of all relative maxima (minima).

1. f is increasing on the intervals ______________
2. f is decreasing on the intervals ______________
3. The relative maxima of f occur at x = ______________
4. The relative minima of f occur at x = ______________

Notes: In the first two, your answer should either be a single interval, such as (0,1), a comma separated list of intervals, such as (-inf, 2), (3,4), or the word "none".

In the last two, your answer should be a comma separated list of x values or the word "none".

Correct Answers:

- `(-infinity,0), (4,infinity)`
- `(0, 2), (2, 4)`
- `0`
- `4`

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6. (3 points) Find the absolute maximum and absolute minimum values of the function

$$f(x) = (x-3)(x-6)^3 + 5$$

on each of the indicated intervals.

(a) Interval = $[1,4]$.

1. Absolute maximum = __________
2. Absolute minimum = __________

(b) Interval = $[1,8]$.

1. Absolute maximum = __________
2. Absolute minimum = __________

(c) Interval = $[4,9]$.

1. Absolute maximum = __________
2. Absolute minimum = __________

Correct Answers:

- `255`
- `-3.54297`
- `255`
- `-3.54297`
- `167`
- `-3`