# MATH 2111: Tutorial 11: Eigenvalue, Eigenspace, Similarity and DIagona

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# Review

- Eigenspace
- Characteristic Function
- Similarities & Diagonalization

Suppose $A \in \mathbb{R}^{n\times n}$ is a symmetric matrix. Given $\lambda$ and $\rho$ are two distinct eigenvalues of $A$. Show that eigenspaces of $\lambda$ and $\rho$ are orthogonal. Namely, for any vectors $x_1 \in \mathcal{E}_\rho(A)$, $x_2 \in \mathcal{E}_\lambda(A)$, it has $x_1^\top x_2 = 0$.

Given $A \in \mathbb{R}^{n\times n}$ and its characteristic function $f(\lambda) = \lambda^2(\lambda+1)(\lambda-1)(3-\lambda)^{n-4}$.

(1) Write down eigenvalues and their multiplicities.

(2) What is characteristics function of matrix $A + 2I$?

Suppose $A = \begin{pmatrix} 4 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

(1) Find out characteristics function of $A$.

(2) Determine whether $A$ is diagonalizable.

Diagonalize the following matrix, if possible,

$$A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 4 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

Determine range of $\alpha$ such that the following matrix is similar to some real diagonal matrix,

$$A = \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix}.$$

# Remark

Given $\lambda$ and $\rho$ are two distinct eigenvalues of matrix $A \in \mathbb{R}^{n \times n}$. Suppose $x_1$ is an eigenvector corresponding to $\lambda$ and $x_2$ is an eigenvector corresponding to $\rho$, namely,

$$Ax_1 = \lambda x_1, \quad Ax_2 = \rho x_2.$$

Then $x_1 + x_2$ is not eigenvector of $A$.