

Poker Exercises 1

- A poker hand contains 5 out of 52 cards. The cards are 13 **kinds** (1-Ace, 2,...,10,J,Q,K) and 4 **suites** (spades, hearts, clubs, diamonds)
- What is the probability that a hand contains the Ace of Spades and the King of Spades?
- I need to compute the number of pokers hands that contain AoS and KoS. Since these 2 cards are fixed, I have $C(50,3)$ choices for the remaining 3 cards.
- Probability is: $C(50,3)/C(52,5)$
- What is the probability that a hand contains exactly 1 Ace?
- There are 4 ways to choose the Ace. Once the Ace is selected, there are $C(48,4)$ ways to choose the remaining four cards.
- Probability is: $4 \cdot C(48,4) / C(52,5)$
- What is the probability that a hand contains 5 different kinds?
- There are $C(13,5)$ ways to choose the 5 kinds. For each card there are 4 choices of suit.
- Probability is: $4^5 \cdot C(13,5) / C(52,5)$

Poker Exercises 2

- A poker hand contains 5 out of 52 cards. The cards are 13 **kinds** (1-Ace, 2,...,10,J,Q,K) and 4 **suites** (spades, hearts, clubs, diamonds)
- What is the probability that a hand is a **royal flush**? Royal flush is {10,J,Q,K,Ace} of the same suit. This is the strongest card in poker.
- There are 4 royal flushes/one per suit
- Probability is: $4 / C(52,5)$
- What is the probability that a hand is a **straight flush**? A straight flush is a hand with all five cards of the same suit in sequential order starting at 1,2,...,9. 10 is excluded because it would lead to a Royal flush.
- I have $C(9,1)=9$ options for the beginning of straight flush, and 4 options for the suit, yielding a total of $4 \cdot 9=36$. There is a single option for each of the remaining cards.
- Probability is: $36 / C(52,5)$
- What is the probability that a hand is a **straight**? A straight is a hand with all five cards in sequential order. At least two cards are of different suit; otherwise it would be a royal or straight flush.
- I will first count all straights including straight and royal flushes.
- I have $C(10,1)=10$ options for the beginning of the straight. For each of the five cards, there are 4 suite options. Therefore, total number of straights is: $10 \cdot 4^5$.
- If I exclude the straight and royal flushes, the number becomes: $10 \cdot 4^5 - 40$.
- Probability is: $(10 \cdot 4^5 - 40) / C(52,5)$

Poker Exercises 3

- A poker hand contains 5 out of 52 cards. The cards are 13 **kinds** (1-Ace, 2,...,10,J,Q,K) and 4 **suites** (spades, hearts, clubs, diamonds)
- What is the probability that a hand is **flush**, i.e., it consists of 5 card of the same suit, excluding the cases of royal and straight flush.
- The flush is a combination of 5 out of 13 cards of the same suit. Thus, there are $4 \cdot C(13,5)$ flushes. After excluding the straight and the royal flushed the number is $4 \cdot C(13,5) - 40$
- Probability is: $(4 \cdot C(13,5) - 40) / C(52,5)$
- What is the probability that a hand contains **3-of a kind** (e.g., 3 aces - the 4th and 5th card must be different because otherwise we would have stronger hands, to be discussed later):
- Number of ways to select the kind of the 3-of a kind $C(13,1)$.
- Number of ways to select the suites of the 3-of a kind $C(4,3)$.
- Number of ways to select the kind of the 4th and 5th card: $C(12,2)$.
- Number of ways to select the suit of the 4th and 5th card: $C(4,1) \cdot C(4,1)$
- Probability is: $C(13,1) \cdot C(4,3) \cdot C(12,2) \cdot C(4,1) \cdot C(4,1) / C(52,5)$
- What is the probability that a hand contains **2 pairs** (e.g., 2 aces and 2 kings - the 5th card be different):
- Number of ways to select the 2 pairs is $C(13,2)$.
- Number of ways to select the suites of the 2 pairs $C(4,2) \cdot C(4,2)$.
- Number of ways to select the fifth card: $52 - 8 = 44$.
- Probability is: $C(13,2) \cdot C(4,2) \cdot C(4,2) \cdot 44 / C(52,5)$

Poker

Exercises 4

Four of a kind	
Full house	

- What is the probability that a hand contains **4-of a kind**?
- Number of ways to select the kind of the 4-of a kind $C(13,1)$.
- Number of ways to select the 5th card: 48 (12 for the rank times 4 for the suit).
- Probability is: $13 \cdot 48 / C(52,5)$
- What is the probability that a hand is a **full house**?
- Number of ways to select the 2 kinds $C(13,2)$.
- Number of ways to select the kind for the 3-of a kind $C(2,1)$.
- Number of ways to select the suits for the 3-of a kind $C(4,3)$
- Number of ways to select the suits for the pair $C(4,2)$.
- Probability is: $C(13,2) \cdot C(2,1) \cdot C(4,3) \cdot C(2,2) / C(52,5)$
- **Poker Hands**: Royal Flush > Straight Flush > 4-of a kind > Full house > Flush > Straight > 3-of a kind > 2 pairs > 1 pair