MATH 2111: Tutorial 5 Linear Transformations and Matrix Operations

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Review

- Linear Transformation Definition
- Matrix of a linear transformation, Standard Matrix
- Onto Map & One-to-one Map
- Matrix Operation: sums, scalar multiples, matrix multiples, matrix transpose

Example 1

Linear Transformation

Given transformation $T(x_1, x_2, x_3) = (x_2 + 1, x_3 + 1)$.

(1) What is $T(1, 2, 1)$?

(2) Is $T(\cdot)$ a linear transformation?

Example 2

Standard Matrix

(1) Find the standard matrix of the following linear transformation

$$T(x_1, x_2, x_3, x_4) = (5x_1 - x_2, 5x_2 - x_3, 5x_3 - x_4, 5x_4 - x_1).$$

(2) Find the linear transformation of the following standard matrix

$$A = \begin{pmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{pmatrix}$$

Example 3

Onto Map & One-to-One Map

Given linear transformation
$T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 + x_3)$, determine whether

(1) T is a one-to-one map,

(2) T maps $\mathbb{R}^3$ onto $\mathbb{R}^3$.

Example 4

Onto Map & One-to-One Map

Suppose α is an angle. Given linear transformation $T(x_1, x_2) = (\cos\alpha \cdot x_1 + \sin\alpha \cdot x_2, -\sin\alpha \cdot x_1 + \cos\alpha \cdot x_2)$.
Determine whether

(1) T is a one-to-one map,

(2) T maps $\mathbb{R}^2$ onto $\mathbb{R}^2$.

Example 5

Matrix Operations

Given $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$.

(1) Compute AB.

(2) Compute A^2, A^3.

(3) Compute $A^\top B$.