HKUST

Name:_____

MATH1014 Calculus II

Student ID:_____

Midterm Examination (Sample Version)

Lecture Section:_____

Directions:

- This is a closed book examination. **No Calculator is allowed in this examination**.
- DO NOT open the exam until instructed to do so.
- Turn off all phones and pagers, and remove headphones. All electronic devices should be kept in a bag away from your body.
- Write your name, ID number, and Lecture Section in the space provided above, and also in the Multiple Choice Item Answer Sheet.
- **Make sure you also write and mark your ID number correctly in the** I.D. No. Box**, in the Multiple Choice Item Answer Sheet**.
- DO NOT use any of your own scratch paper. Write your name on every scratch paper supplied by the examination, and do not take any scratch paper away after the examination.
- Answer all questions. Show an appropriate amount of work for each long problem. If you do not show enough work, you will get only partial credit.
- When instructed to open the exam, please check that you have **7** pages of questions in addition to the cover page. There are two blank pages and a **formula sheet** attached which can be used as scratch paper.
- You may write on the backside of the pages, but if you use the backside, clearly indicate that you have done so.
- **Cheating is a serious violation of the HKUST Academic Code. Students caught cheating will receive a zero score for the examination, and will also be subjected to further penalties imposed by the University.**

Please read the following statement and sign your signature.

I have neither given nor received any unauthorized aid during this examination. The answers submitted are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University's regulations governing academic integrity.

Student's Signature :

_____

Question No.	Points	Out of
Q. 1-11		55
Q. 12		15
Q. 13		15
Q. 14		15
Total Points		100

Part I: Answer all of the following multiple choice questions.

- Mark your MC answers to the boxes in the Multiple Choice Item Answer Sheet provided.
- Write your name and mark your student ID number on the Multiple Choice Item Answer Sheet.
- Mark only one answer for each MC question. Multiple answers entered for each single MC question will result in a 3 point deduction.

Write also your MC question answers in the following boxes for back up use only. The grading will be based on the answers you mark on the MC item answer sheet.

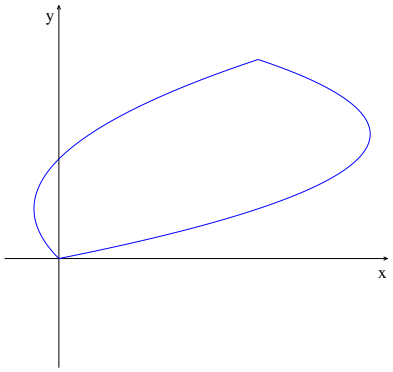
Question	1	2	3	4	5	6	7	8	9	10	11	**Total**
Answer												

Each of the following MC questions is worth 5 points. No partial credit.

1. What is the color version of your examination paper? Make sure your ID number has also been marked correctly in the I.D. No. Box in the MC answer form. **If you do not do both correctly, you lose the points of this question.**

   (a) Green (b) Orange (c) White (d) Yellow (e) Makeup

2. Find the area enclosed by the curves defined by the equations $x = 5y - 2y^2$ and $x = y^2 - y$.



   (a) 4 (b) 5 (c) 6 (d) 7 (e) 8

3. Evaluate the integral $\int_0^{\frac{\pi}{4}} 30\sin(2x)\cos^2(4x)\,dx$.

(a) 5 (b) 6 (c) 7 (d) 8 (e) 9

4. Evaluate the integral $\int_{-4}^{4} x^2\sqrt{16-x^2}\,dx$.

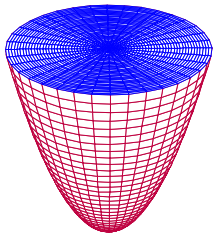
(a) 12π (b) 16π (c) 20π (d) 24π (e) 32π

5. Evaluate the integral $\int_0^2 \frac{x+3}{(x+1)(x+2)}dx$.

(a) $2\ln 3 - \ln 2$ (b) $3\ln 2 - 2\ln 3$ (c) $3\ln 2 - \ln 3$

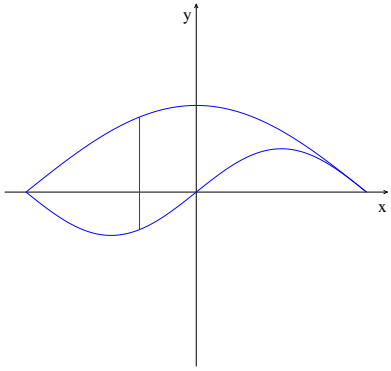
(d) $4\ln 3 + 2\ln 2$ (e) $5\ln 2 - 4\ln 3$

6. The region enclosed by the graph of the function $y = 2x^4$ and the line $y = 32$ is rotated about the y-axis to generate a solid of revolution. Find the average value of the cross section areas of the solid which are perpendicular to the y-axis.



(a) $\frac{8\pi}{3}$ (b) $\frac{7\pi}{3}$ (c) $\frac{6\pi}{3}$ (d) $\frac{5\pi}{3}$ (e) $\frac{4\pi}{3}$

7. The base of a solid sitting on the xy-plane is the region enclosed by the curves $y = 2\sin 2x$ and $y = 4\cos x$ over the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Suppose that each cross section of the solid perpendicular to the x-axis is an equilateral triangle (i.e. all sides have equal length). Find the volume of the solid.



(a) $\frac{9\sqrt{3}\pi}{2}$ (b) $\frac{7\sqrt{3}\pi}{2}$ (c) $\frac{5\sqrt{3}\pi}{2}$ (d) $\frac{3\sqrt{3}\pi}{2}$ (e) $\frac{\sqrt{3}\pi}{2}$

8. Let f be a positive differentiable function satisfying

$$f(0) = 2, \quad f'(0) = 2, \quad f(1) = 4, \quad f'(1) = 3\ .$$

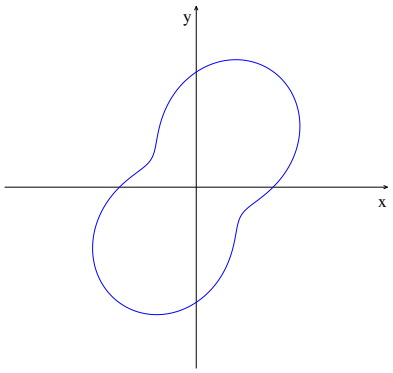
Evaluate the integral $\int_0^1 f'(x)\ln(f(x))dx$.

(a) $2\ln 2 - 1$ (b) $2\ln 3 - 1$ (c) $4\ln 2 - 2$ (d) $4\ln 3 - \ln 2$ (e) $6\ln 2 - 2$

9. The curve defined by $y = \sqrt{2x}$, where $0 \le y \le 4$, is rotated about the x-axis to generate a surface of revolution. Find the area of the surface.

(a) $\frac{49\pi}{3}$ (b) $\frac{52\pi}{3}$ (c) $\frac{68\pi}{3}$ (d) $\frac{92\pi}{3}$ (e) $\frac{124\pi}{3}$

10. Find the slope of the tangent line to the curve defined by the polar equation $r = 3 + \sin 2\theta - \cos^2\theta$ at the point on the curve with angular coordinate $\theta = \frac{\pi}{4}$.



(a) $-\frac{3}{2}$ (b) $-\frac{5}{3}$ (c) $-\frac{7}{4}$ (d) $-\frac{9}{5}$ (e) -1

11. A curve in the xy-plane is defined by the polar equation $r = \sqrt{\sec 2\theta}$, where $0 \le \theta \le \frac{\pi}{6}$. Which of the following definite integrals is the arc length of the curve?

(a) $\displaystyle\int_0^{\frac{\pi}{6}} \sqrt{1 + \frac{1}{\sec 2\theta}}\, d\theta$

(b) $\displaystyle\int_0^{\frac{\pi}{6}} \sqrt{2\tan 2\theta + \frac{1}{\sec 2\theta}}\, d\theta$

(c) $\displaystyle\int_0^{\frac{\pi}{6}} \sqrt{\sec^3 2\theta}\, d\theta$

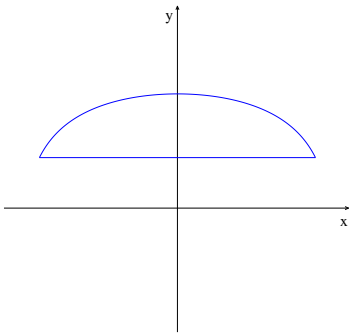
(d) $\displaystyle\int_0^{\frac{\pi}{6}} \sqrt{\sec 2\theta + \frac{1}{\tan 2\theta}}\, d\theta$

(e) $\displaystyle\int_0^{\frac{\pi}{6}} \sqrt{\tan 2\theta + \sec^2 2\theta}\, d\theta$

Part II: Answer each of the following questions.

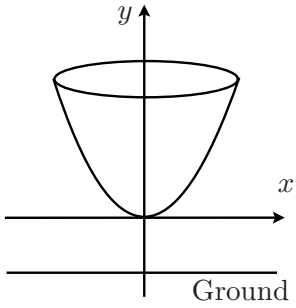
12. [*15 pts*] The region enclosed by the curves $y = \ln(12 - x^2)$ and $y = \ln 3$ is rotated about the y-axis to generate a solid of revolution.

(a) Express the volume of the solid as a definite integral. [5 pts]



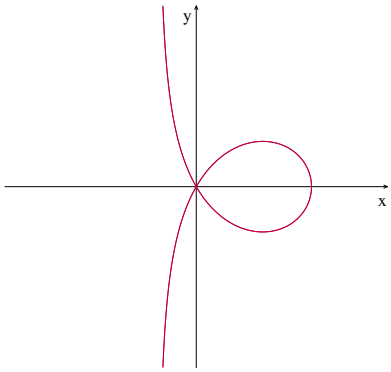
(b) Find the volume of the solid. [10 pts]

13. [*15 pts*] A container has the shape of a bowl obtained by rotating the curve $y = 8x^3$, $0 \le x \le 1$, around the y-axis. The lowest point of the container is 5 m above the ground. Find the work required to fill the tank by pumping water from the ground through a thin tube connecting the ground to the bottom of the tank. (Ignore the water in the thin tube.) Recall the density of water is $\rho = 1000$ kg/m^3, and $g = 9.8$ m/s^2 is gravity acceleration. [15 pts]
(*You may use the symbols ρ and g to represent your answer.*)



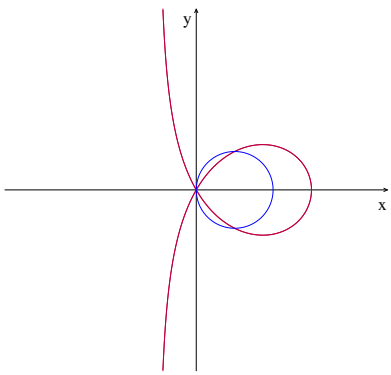
14. [*15 pts*] Consider the curve defined by the equation $y^2 = x^2\left(\dfrac{3-x}{1+x}\right)$.

(a) Show that the polar equation of the curve can be written as $r = (3\cos^2\theta - \sin^2\theta)\sec\theta$. [4 pts]



(b) Find the area inside the loop of the given curve. [7 pts]
(Hint: At what θ will the curve hit the origin?)

(c) Use integral(s) to express the area inside the loop of the given curve but outside the circle given by the polar equation $r = 2\cos\theta$. **Do not evaluate the integral.** [4 pts]
(Hint: At what θ will the curves intersect each other?)



Math1014 Midterm Exam Formula Sheet

Trigonometric Identities

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$
$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\sin A\cos B = \tfrac{1}{2}\Big(\sin(A+B) + \sin(A-B)\Big)$$
$$\cos A\cos B = \tfrac{1}{2}\Big(\cos(A+B) + \cos(A-B)\Big)$$
$$\sin A\sin B = \tfrac{1}{2}\Big(\cos(A-B) - \cos(A+B)\Big)$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x dx$$

$$\int \sin^n x dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x dx$$

$$\int \sec^n x dx = \frac{1}{n-1}\sec^{n-2} x\tan x + \frac{n-2}{n-1}\int \sec^{n-2} x dx$$