COMP 2711 Discrete Mathematical Tools for Computer Science
2022 Spring Semester – Tutorial 4

Question 1: How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

Question 2: Give a combinatorial proof for the following statement for $n \geq 2$,

$$\binom{n}{2}^2 \leq \binom{2n}{4}.$$

Note: An algebraic proof of this statement will not be accepted as a solution.

Question 3:

(a) Let n and r be positive integers. Explain why the number of solutions of the equation $x_1 + x_2 + \cdots + x_n = r$, where x_i is a nonnegative integer for $i = 1, 2, 3, \cdots, n$, equals the number of r-combinations with repetition of a set with n elements.

(b) How many solutions in nonnegative integers are there to the equation $x_1 + x_2 + x_3 + x_4 = 17$?

(c) How many solutions in positive integers are there to the equation in part (b)?

Question 4: In how many ways can a dozen books be placed on four distinguishable shelves

(a) if the books are indistinguishable copies of the same title?

(b) if no two books are the same, and the positions of the books on the shelves matter? [*Hint:* Break this into 12 tasks, placing each book separately. Start with the sequence $1, 2, 3, 4$ to represent the shelves. Represent the books by b_i, $i = 1, 2, \ldots, 12$. Place b_1 to the right of one of the terms in $1, 2, 3, 4$. Then successively place $b_2, b_3, \ldots,$ and b_{12}.]

Question 5: Give a formula for the coefficient of x^k in the expansion of $(x^2 - 1/x)^{100}$, where k is an integer.

Question 6: Prove that if n and k are integers with $1 \leq k \leq n$, then $k\binom{n}{k} = n\binom{n-1}{k-1}$,

(a) using a combinatorial proof. [*Hint:* Show that the two sides of the identity count the number of ways to select a subset with k elements from a set with n elements and then an element of this subset.]

(b) using an algebraic proof based on the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.

Question 7: Use combinatorial proof to show that

$$\binom{2n-5}{10} = \sum_{k=0}^{10} \binom{n-3}{k} \binom{n-2}{10-k}$$