

COMP 2711 Discrete Math Tools for Computer Science
2022 Fall Semester - Homework 2

Question 1: Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set, or give a counting method. For those that are uncountable, give a simple proof.

- (a) the set $A \times B \times \mathbb{N}$ where $A = \{1, 2\}$, $B = \{3, 4\}$.
- (b) all bit strings not containing the bit 1.
- (c) all bit strings that is finite.
- (d) all bit strings that is infinite
- (e) the real numbers containing a finite number of 1s in their decimal representation (reminder: the real numbers can contain a infinite number of 2s or 3s)
- (f) the real numbers containing only 1s in their decimal representation

Question 2: In COMP2711 T5, there are totally 50 students. For HW1, each grade (A, B, C, D, F(didn't submit)) has 10 students. TA wants to select students from COMP2711 T5 based on the grade of HW1.

- (a) There are 4 different tasks, each needs one student for help. A student might be assigned to multiple tasks. Task 1 and Task 2 only accept students with A grade or B grade. Students with F grade can only be assigned to Task 4 and cannot be assigned to any other tasks. Among these 4 tasks, at least one task should be done by student with C grade. How many different ways to assign students in these 4 tasks?
- (b) TA wants to randomly select students from COMP2711 T5, how many students must he select to be sure of having at least four students with A grade?
- (c) TA wants to randomly select students from COMP2711 T5, how many students must he select to be sure of having at least four students with the same grade?

Question 3: In how many ways can a dozen books be placed on four distinguishable shelves

- (a) if the books are indistinguishable copies of the same title?
- (b) if no two books are the same, and the positions of the books on the shelves matter? [Hint: Place the books on the shelves one by one. How many ways can you put each book?]

Question 4: This semester for COMP2711, there are 81 students in L1, 86 students in L2, 74 students in L3. Assume that

- (a) in L3, students may also take COMP2011, MATH1012, or COMP1021. Notice that COMP2011 and COMP1021 cannot take together. There are exactly 16 students that take none of these three courses. There are 26 students take COMP2011, 38 take MATH1012, 24 take COMP1021. And there are 10 students take both COMP2011 and MATH1012. How many students take both MATH1012 and COMP1021?
- (b) the instructors want to recruit student helpers. There are 15 different positions, and they need to hire 3 students from L1, 8 students from L2, 4 students from L3. How many different ways for instructors to assign students to these 15 positions? (No calculation required)
- (c) the instructors want to recruit a team of 90 student helpers. How many different student helper team can be formed if they require that a student helper team must contain at least one student from each lectures? (No calculation required)
- (d) we already have 6 student helpers working on different tasks. After midterm, instructors want to rearrange the student helpers so that only two of them keep working on their original tasks, other student helpers will be swapped to do different tasks. How many ways can the instructors make the rearrangement?

Question 5: Prove the identity $\binom{n}{r}\binom{r}{a} = \binom{n}{a}\binom{n-a}{r-a}$, whenever n and r are nonnegative integers with $n \geq r \geq a$,

- (a) using a combinatorial argument.
- (b) using an argument based on the formula for the number of r -combinations of a set with n elements.