**MATH 2111 Matrix Algebra and Applications**
**Homework-1 : Due 09/22/2022 at 11:59pm HKT**

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**1.** (2 points) For each system, determine whether it has a unique solution (in this case, find the solution), infinitely many solutions, or no solutions.

(1) $\begin{cases} 2x+2y = 2 \\ -6x-6y = -5 \end{cases}$

- A. Unique solution: $x = 2,\ y = -5$
- B. Unique solution: $x = 0,\ y = 0$
- C. No solutions
- D. Unique solution: $x = -5,\ y = 2$
- E. Infinitely many solutions
- F. None of the above

(2) $\begin{cases} 5x-3y = 0 \\ -3x+7y = 0 \end{cases}$

- A. Unique solution: $x = 0,\ y = 0$
- B. Unique solution: $x = -7,\ y = 5$
- C. Unique solution: $x = 2,\ y = 4$
- D. No solutions
- E. Infinitely many solutions
- F. None of the above

(3) $\begin{cases} -2x+4y = 26 \\ 6x-12y = -78 \end{cases}$

- A. Infinitely many solutions
- B. Unique solution: $x = 26,\ y = -78$
- C. Unique solution: $x = 0,\ y = 0$
- D. No solutions
- E. Unique solution: $x = -13,\ y = 0$
- F. None of the above

(4) $\begin{cases} 9x-10y = 73 \\ 8x-3y = 59 \end{cases}$

- A. No solutions
- B. Unique solution: $x = 7,\ y = -1$
- C. Unique solution: $x = 0,\ y = 0$
- D. Unique solution: $x = -1,\ y = 7$
- E. Infinitely many solutions
- F. None of the above

*Correct Answers:*

- C
- A
- A
- B

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**2.** (1 point)

$$\begin{cases} x - 3y + 9z = -2 \\ -3x + 4y - 17z = 21 \\ -5x + 10y - 35z = k \end{cases}$$

In order for the system of equations above to be a consistent system, $k$ must be equal to _____.

*Correct Answers:*

- 25

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**3.** (2 points) Write the augmented matrix of the system

$$\begin{cases} 7x-29y+50z = 95 \\ \quad -85y-1z = -7 \\ 64x \quad +47z = -15 \end{cases}$$

$$\left[\begin{array}{ccc|c} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{array}\right]$$

*Correct Answers:*

- 7
- -29
- 50
- 95
- 0
- -85
- -1
- -7
- 64
- 0
- 47
- -15

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**4.** (2 points) Determine how many pivot positions each of the following matrices have.

1. $\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 9 \\ 0 & 0 & 1 & 0 & -9 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$
   - A. One Pivot
   - B. Two Pivots
   - C. Three Pivots
   - D. Four Pivots
2. $\begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -8 \end{bmatrix}$
   - A. One Pivot
   - B. Two Pivots
   - C. Three Pivots
   - D. Four Pivots
3. $\begin{bmatrix} 1 & 0 & -8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
   - A. One Pivot
   - B. Two Pivots

- C. Three Pivots
- D. Four Pivots

4. $\begin{bmatrix} 1 & 0 & 6 & -7 \\ 0 & 1 & 6 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

- A. One Pivot
- B. Two Pivots
- C. Three Pivots
- D. Four Pivots

*Correct Answers:*

- D
- C
- A
- B

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**5.** (2 points) The reduced row-echelon forms of the augmented matrices of four systems are given below. How many solutions does each system have?

(1) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

- A. Infinitely many solutions
- B. No solutions
- C. Unique solution
- D. None of the above

(2) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & -7 \end{array}\right]$

- A. Unique solution
- B. No solutions
- C. Infinitely many solutions
- D. None of the above

(3) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & 2 \end{array}\right]$

- A. Infinitely many solutions
- B. No solutions
- C. Unique solution
- D. None of the above

(4) $\left[\begin{array}{cc|c} 1 & 0 & 19 \\ 0 & 1 & -5 \end{array}\right]$

- A. No solutions
- B. Infinitely many solutions
- C. Unique solution
- D. None of the above

*Correct Answers:*

- B
- C
- A
- C

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**6.** (3 points) Solve the system by finding the reduced row-echelon form of the augmented matrix.

$$\begin{cases} x+2y+2z & = -4 \\ -3x-5y-4z & = 14 \\ -3x-7y-8z & = 10 \end{cases}$$

reduced row-echelon form:

$$\left[\begin{array}{ccc|c} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{array}\right]$$

How many solutions are there to this system?

- A. None
- B. Exactly 1
- C. Exactly 2
- D. Exactly 3
- E. Infinitely many
- F. None of the above

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If there is one solution, give its coordinates in the answer spaces below.

If there are infinitely many solutions, enter $z$ in the answer blank for $z$, enter a formula for $y$ in terms of $z$ in the answer blank for $y$ and enter a formula for $x$ in terms of $z$ in the answer blank for $x$.

If there are no solutions, leave the answer blanks for $x$, $y$ and $z$ empty.

$x =$ ____________

$y =$ ____________

$z =$ ____________

*Correct Answers:*

- 1
- 0
- -2
- -8
- 0
- 1
- 2
- 2
- 0
- 0
- 0
- 0
- E
- -8--2*z
- 2 - 2*z
- z

**7.** (1 point) Write a vector equation

$$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} x + \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} y + \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} z = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

that is equivalent to the system of equations:

$$\begin{cases} 2y - 5x + 6z &= 8, \\ 9y - 2x + 5z &= -6, \\ -2x - 5y - 5z &= -9. \end{cases}$$

*Correct Answers:*

- $\begin{bmatrix} -5 \\ -2 \\ -2 \end{bmatrix}$
- $\begin{bmatrix} 2 \\ 9 \\ -5 \end{bmatrix}$
- $\begin{bmatrix} 6 \\ 5 \\ -5 \end{bmatrix}$
- $\begin{bmatrix} 8 \\ -6 \\ -9 \end{bmatrix}$

**8.** (1 point) Write the system of equations

$$-1x + 2y - 2z = -4$$

$$2x + 2y - 5z = -5$$

$$1x + 3y - 2z = -3$$

as a matrix equation, that is, rewrite it in the form $A$ $v$= $b$. Find the matrix $A$ and vector $b$

$$\begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

*Correct Answers:*

- -1
- 2
- -2
- 2
- 2
- -5
- 1
- 3
- -2
- -4
- -5
- -3

**9.** (2 points) Determine whether the product $Ax$ is defined or undefined.

$\boxed{?}$ 1. $A = \begin{bmatrix} 1 & 1 \\ 8 & -10 \\ -5 & 0 \end{bmatrix}, x = \begin{bmatrix} -3 \\ -5 \\ -1 \end{bmatrix}$

$\boxed{?}$ 2. $A = \begin{bmatrix} 0 & -1 & -4 \\ -4 & -9 & 1 \\ 0 & 5 & -5 \\ -2 & 3 & -10 \end{bmatrix}, x = \begin{bmatrix} 8 \\ 4 \\ -8 \end{bmatrix}$

$\boxed{?}$ 3. $A = \begin{bmatrix} -6 & -2 & -8 \\ -7 & -2 & -5 \end{bmatrix}, x = \begin{bmatrix} -9 \\ -2 \\ 9 \\ -7 \end{bmatrix}$

$\boxed{?}$ 4. $A = \begin{bmatrix} -7 & 7 & 1 & -7 & 4 \end{bmatrix}, x = \begin{bmatrix} 6 \\ 3 \\ 10 \\ -6 \\ 9 \end{bmatrix}$

$\boxed{?}$ 5. $A = \begin{bmatrix} -4 & 3 & -4 & 6 \\ 8 & -5 & 4 & -5 \\ -8 & -8 & 2 & -3 \end{bmatrix}, x = \begin{bmatrix} 8 \\ 3 \\ -5 \\ -5 \end{bmatrix}$

*Correct Answers:*

- UNDEFINED
- DEFINED
- UNDEFINED
- DEFINED
- DEFINED

**10.** (1 point) Compute the following matrix-vector product.

$$\begin{bmatrix} -7 & 6 & 7 \\ 10 & 2 & 10 \\ -5 & 3 & -7 \end{bmatrix} \begin{bmatrix} -7 \\ 9 \\ -5 \end{bmatrix} = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

*Correct Answers:*

- 68
- -102
- 97

**11.** (1 point) Use the substitution method to solve the system

$$-x + y = -1,$$
$$4x - 3y = 0.$$

Your answer is
$x =$ ________
$y =$ ________

*Correct Answers:*

- -3
- -4

**12.** (1 point)

The solution of the linear system

$$\begin{array}{rrcl} x & +y & = & 1 \\ 3x & -y & = & 2 \end{array}$$

is

$x =$ _____ and $y =$ _____.

*Correct Answers:*

- `0.75`
- `0.25`

**13.** (1 point)

Linear systems of equations arise frequently in applications. They are usually solved by Gaussian Elimination and Backward Substitution. When working by hand it is crucial to organize the computation in a clear way that lets you check your calculations so far. A linear system may have none, one, or infinitely many solutions.

Solve the linear system

$$\begin{array}{rrcl} 4x & -y & = & 3 \\ 2x & +y & = & 5 \end{array}$$

$x =$_________ and $y =$ _________.

**Solution:**

**Solution:**

Adding the equations eliminates $y$ and gives $6x = 8$, i.e., $x = \frac{4}{3}$ Substituting $x$ in the first equation gives $\frac{16}{3} - y = 3$, i.e., $y = \frac{7}{3}$.

*Correct Answers:*

- `1.33333333333333`
- `2.33333333333333`

**14.** (1 point)

A linear system may have a unique solution, no solution, or infinitely many solutions. Indicate the type of the system for the following examples by $U$ , $N$ , or $I$ , respectively.

___1. $\begin{array}{rcr} -2x+y & = & 5 \\ 2x-y & = & -5 \end{array}$

___2. $\begin{array}{rcr} 2x+3y & = & 5 \\ x+6y & = & 7 \end{array}$

___3. $\begin{array}{rcr} 2x+3y & = & 1 \\ 4x+6y & = & 1 \end{array}$

___4. $\begin{array}{rcr} x-y & = & 15 \\ y-x & = & 15 \end{array}$

___5. $\begin{array}{rcr} 2x+3y & = & 0 \\ 2x+4y & = & 0 \end{array}$

**Solution:**

**Solution:** The following list contains more examples than are given in your problem. It does include your particular problems, however.

$\begin{array}{rcr} -2x+y & = & 5 \\ 2x-y & = & -5 \end{array}$

There are infinitely many solutions since the second equation is equivalent to the first, from which it can be obtained by multiplying with $-1$.

$\begin{array}{rcr} 2x+3y & = & 0 \\ 2x+4y & = & 0 \end{array}$

The unique solution of this system is $x = y = 0$.

$\begin{array}{rcr} 2x-3y & = & 5 \\ 4x-6y & = & 10 \end{array}$

There are infinitely many solutions since the second equation is equivalent to the first, from which it can be obtained by multiplying with 2.

$\begin{array}{rcr} x-3y & = & 5 \\ x+3y & = & 5 \end{array}$

The unique solution of this system is $x = 5$ and $y = 0$.

$\begin{array}{rcr} x-y & = & 15 \\ y-x & = & 15 \end{array}$

There is no solution since the left hand sides are negatives of each other, whereas the right hand sides are not.

$\begin{array}{rcr} 2x+3y & = & 0 \\ 4x+6y & = & 0 \end{array}$

There are infinitely many solutions since the second equation can be obtained form the first by multiplying with 2.

$\begin{array}{rcr} 2x+3y & = & 1 \\ 4x+6y & = & 1 \end{array}$

There is no solution since the left hand sides of the second equation is twice the left hand side of the first, but the corresponding right hand sides are equal.

$\begin{array}{rcr} 2x+3y & = & 5 \\ x+6y & = & 7 \end{array}$

The unique solution of this system is $x = y = 1$.

$\begin{array}{rcr} x+y & = & 5 \\ x+2y & = & 10 \end{array}$

The unique solution o f this system is $x = 0$ and $y = 5$.

$\begin{array}{rcr} 7x+3y & = & \pi \\ 4x-6y & = & \pi^2 \end{array}$

This system has a unique solution. This follows from the fact that the left hand sides are not multiples of each other. However you can also see this by actually solving the system which gives

$$x = \frac{\pi^2 + 2\pi}{18} \qquad \text{and} \qquad y = \frac{-7\pi^2 + 4\pi}{54}.$$

*Correct Answers:*

- `I`
- `U`
- `N`
- `N`
- `U`

**15.** (1 point) Solve the system:

$$\begin{cases} 2x - 3y = a \\ -3x + 5y = b \end{cases}$$

$x =$ __________
$y =$ __________

*Correct Answers:*

- `(5*a--3*b)*1`
- `(2*b--3*a)*1`

**16.** (1 point) Determine all values of $h$ and $k$ for which the system

$$\begin{cases} x - 2y = h \\ 6x + ky = -9 \end{cases}$$

has no solution.
$k =$ _____
$h \neq$ _____

*Correct Answers:*

- `-12`
- `-1.5`