

COMP 2711 Discrete Mathematical Tools for Computer Science
2022 Fall Semester – Tutorial 5

Question 1: How many permutations of the 10 digits (0 through 9) either begin with the 3 digits 987, contain the digits 45 in the fifth and sixth positions, or end with the 3 digits 123?

Question 2: There are ten groups and each group has two people. They sit down in a row of twenty seats. How many ways are there that at least one group sits together? Your answer could be a summation of at most ten terms.

Question 3: This problem was posed by the Chevalier de Méré and was solved by Blaise Pascal and Pierre de Fermat.

- (a) Find the probability of rolling at least one six when a fair die is rolled four times.
- (b) Find the probability that a double six comes up at least once when a pair of dice is rolled 24 times. Answer the query the Chevalier de Méré made to Pascal asking whether this probability was greater than $1/2$.
- (c) Is it more likely that a six comes up at least once when a fair die is rolled four times or that a double six comes up at least once when a pair of dice is rolled 24 times?

Question 4: There is a type of cereal that contains a toy in each box. There are 10 types of toys, T_1, T_2, \dots, T_{10} and every box has probability $\frac{1}{10}$ of containing each possible toy, independently of every other box.

Your little brother buys a box of cereal each week for 20 weeks and keeps all of the toys that he finds.

- (a) What is the probability that your brother has at least one copy of toy T_1 after 20 weeks?
- (b) What is the probability that after 20 weeks your brother has collected *all* of the 10 different toys?