Math1014 Calculus II
Brief Summary of Some Basic Geometry

Here is some basic geometry you should know.

Triangles

Similar triangles: $\triangle ABC \sim \triangle ADE$

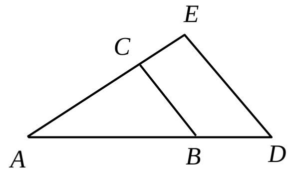
$$\iff \frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$$

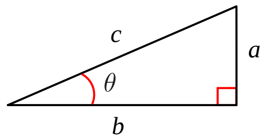
Pythagoras Theorem: $a^2 + b^2 = c^2$

Trigonometry: $\sin\theta = \dfrac{a}{c}$, $\cos\theta = \dfrac{b}{c}$, $\tan\theta = \dfrac{a}{b}$

Cosine Law: $BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cos\angle A$

Area: $\text{area}(\triangle ABC) = \frac{1}{2} AB \cdot AC \sin\angle A$





Circles

Circle: Area $= \pi r^2$

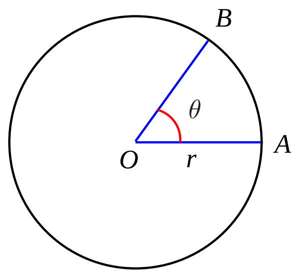
Arc length $= 2\pi r$

Circular sector: Area $= \frac{1}{2} r^2 \theta$

Arc length $= r\theta$

where θ is measured in radians.

$$\text{from} \quad \frac{\text{circular sector area}}{\text{circle area}} = \frac{\theta}{2\pi} = \frac{\text{circular arc length}}{\text{circle length}}$$



Volumes and Surface Areas

Cylinder: Volume $= \pi r^2 h$

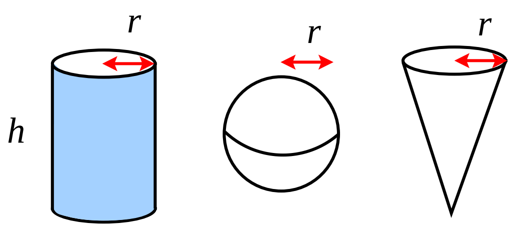
Lateral area $= 2\pi r h$

Sphere: Volume $= \frac{4}{3}\pi r^3$

Area $= 4\pi r^2$

Cone: Volume $= \frac{1}{3}\pi r^2 h$

Area $= \pi r \sqrt{r^2 + h^2}$



In Math1014, using the concept of definite integral, you will see how some volume formulas can be found from considering basic area formulas!

A Geometry⊗Calculus Exercise

Let $V(r) = \frac{4}{3}\pi r^3$ be the volume of a sphere of radius r. Then, miraculously, $V'(r) = 4\pi r^2 = S(r)$ is the area of the sphere! In case you haven't done this before, try to see if you could figure out an intuitive argument for this from the *limit definition of derivative*, without using the exact volume formula $\frac{4}{3}\pi r^3$:

$$V'(r) = \lim_{h \to 0} \frac{V(r+h) - V(r)}{h} \quad = \quad S(r).$$

(Hint: How is $V(r+h) - V(r)$ controlled by spherical areas and h? Use the Squeeze Theorem!)