COMP 2711 Discrete Mathematical Tools for Computer Science 2022 Fall Semester - Tutorial 1

Question 1: Let p, q, and r be the propositions

- p: You get an A in the final exam.
- q: You do every exercise in this book.
- r: You get an A in the class.

Write these propositions using p, q, and r and the Boolean connectives

- (a) You get an A in this class, but you do not do every exercise in the book
- (b) You get an A in the final, you do every exercise in this book, and you get an A in this class
- (c) Getting an A in the final and doing every exercise in this book is sufficient for getting an A in this class.
- (d) You get an A in this class if and only if you either do every exercise in this book or you get an A in the final.

Using p, q, r above express each of the following as an English sentence

- (e) $p \leftrightarrow r$
- (f) $\neg r \land q \rightarrow \neg p$

Question 2: (Distributive "Laws")

- (a) Is $w \wedge (w \oplus v)$ equivalent to $(w \wedge w) \oplus (w \wedge v)$?
- (b) Is $w \lor (u \oplus v)$ equivalent to $(w \lor u) \oplus (w \lor v)$? (Noted. $a \oplus b$ evaluates F if and only if a and b are the same.)

Question 3: Let p and q be statements, prove each of the following compound statement is always true. (such statement is called "Tautology")

- (a) $(q \land \neg q) \to p$
- (b) $(p \land q) \to p$

Question 4: For each of the following pairs of logic statements, either prove that the two statements are logically equivalent, or give a counterexample. In your proof, you may use either a truth table or logic laws. A counterexample should consist of a truth setting of the variables and the truth values of the statements under the setting.

- (a) $(p \land q) \to r$ and $\neg p \lor \neg q \lor r$
- (b) $(p \land q) \to r$ and $\neg r \to (p \to \neg q)$
- (c) $(p \to r) \land (q \to r)$ and $(p \land q) \to r$
- (d) $(p \land \neg q) \rightarrow (r \land \neg r)$ and $p \rightarrow q$

Question 5: (a) Given the statement $(a \lor b) \land (\neg b \lor c)$. Express its equivalent statement using only $NOT(\neg)$ and $Implication(\rightarrow)$.

- (b) Given $\neg a \lor (b \to \neg c)$. Express its equivalent statements using only:
 - (i) $NOT(\neg)$ and $Implication(\rightarrow)$.
 - (ii) $NOT(\neg)$ and $OR(\lor)$.
 - (iii) $NOT(\neg)$ and $AND(\wedge)$.