COMP 2711 Discrete Mathematical Tools for Computer Science
2022 Fall Semester - Tutorial 1

Question 1: Let p, q, and r be the propositions

p: You get an A in the final exam.
q: You do every exercise in this book.
r: You get an A in the class.

Write these propositions using p, q, and r and the Boolean connectives

(a) You get an A in this class, but you do not do every exercise in the book

(b) You get an A in the final, you do every exercise in this book, and you get an A in this class

(c) Getting an A in the final and doing every exercise in this book is sufficient for getting an A in this class.

(d) You get an A in this class if and only if you either do every exercise in this book or you get an A in the final.

Using p, q, r above express each of the following as an English sentence

(e) $p \leftrightarrow r$

(f) $\neg r \wedge q \rightarrow \neg p$

Solution : (a) $r \wedge \neg q$

(b) $p \wedge q \wedge r$

(c) $(p \wedge q) \rightarrow r$

(d) $r \leftrightarrow (q \vee p)$

(e) You get an A in the course if and only if you get an A in the final.

(f) If you don't get an A in the course but you have done all the exercises, then you must not get an A in the final.

Question 2: (Distributive "Laws")

(a) Is $w \wedge (w \oplus v)$ equivalent to $(w \wedge w) \oplus (w \wedge v)$?

(b) Is $w \vee (u \oplus v)$ equivalent to $(w \vee u) \oplus (w \vee v)$? (Noted. $a \oplus b$ evaluates F if and only if a and b are the same.)

Solution : (a) $\wedge$ distributes over $\oplus$. Compare the truth tables of $w \wedge (w \oplus v)$ and $(w \wedge w) \oplus (w \wedge v)$.

w	v	$w \wedge w$	$w \wedge v$	$(w \wedge w) \oplus (w \wedge v)$	$(w \oplus v)$	$w \wedge (w \oplus v)$
T	T	T	T	F	F	F
T	F	T	F	T	T	T
F	T	F	F	F	T	F
F	F	F	F	F	F	F

(b) $\vee$ doesn't distribute over $\oplus$. Try $w = T, u = T$, and $v = T$, where T stands for a statement that is always true, to get the value of $w \vee (u \oplus v)$ and $(w \vee u) \oplus (w \vee v)$.
$T \vee (T \oplus T) = T \neq F = (T \vee T) \oplus (T \vee T)$.

Question 3: Let p and q be statements, prove each of the following compound statement is always true. (such statement is called "Tautology")

(a) $(q \wedge \neg q) \rightarrow p$

(b) $(p \wedge q) \rightarrow p$

Solution : (a) Compute the truth tables of $(q \wedge \neg q) \rightarrow p$.

q	p	$q \wedge \neg q$	$(q \wedge \neg q) \rightarrow p$
T	T	F	T
T	F	F	T
F	T	F	T
F	F	F	T

(b) Compute the truth table of $(p \wedge q) \rightarrow p$.

q	p	$q \wedge p$	$(q \wedge p) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Question 4: For each of the following pairs of logic statements, either prove that the two statements are logically equivalent, or give a counterexample. In your proof, you may use either a truth table or logic laws. A counterexample should consist of a truth setting of the variables and the truth values of the statements under the setting.

(a) $(p \wedge q) \rightarrow r$ and $\neg p \vee \neg q \vee r$

(b) $(p \wedge q) \rightarrow r$ and $\neg r \rightarrow (p \rightarrow \neg q)$

(c) $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$

(d) $(p \wedge \neg q) \rightarrow (r \wedge \neg r)$ and $p \rightarrow q$

Solution : (a) Equivalent.

$$\begin{aligned}(p \wedge q) \rightarrow r &\equiv \neg(p \wedge q) \vee r \quad (s \rightarrow t \equiv \neg s \vee t)\\ &\equiv \neg p \vee \neg q \vee r \quad \text{(by DeMorgan's law)}\end{aligned}$$

(b) Equivalent.

$$\begin{aligned}\neg r \rightarrow (p \rightarrow \neg q) &\equiv r \vee (\neg p \vee \neg q) \quad (s \rightarrow t \equiv \neg s \vee t)\\ &\equiv (p \wedge q) \rightarrow r \quad \text{(by part (a))}\end{aligned}$$

(c) Not equivalent. Counter example: $p = T, q = F, r = F$. The first statement is false, while the second statement is true.

(d) Equivalent.

$$\begin{aligned}(p \wedge \neg q) \rightarrow (r \wedge \neg r) &\equiv \neg(p \wedge \neg q) \vee (r \wedge \neg r)\\ &\equiv \neg p \vee q \vee F\\ &\equiv \neg p \vee q\\ &\equiv p \rightarrow q\end{aligned}$$

Question 5: (a) Given the statement $(a \vee b) \wedge (\neg b \vee c)$. Express its equivalent statement using only *NOT* $(\neg)$ and *Implication* $(\rightarrow)$.

(b) Given $\neg a \vee (b \rightarrow \neg c)$. Express its equivalent statements using only:

(i) *NOT* $(\neg)$ and *Implication* $(\rightarrow)$.

(ii) *NOT* $(\neg)$ and *OR* $(\vee)$.

(iii) *NOT* $(\neg)$ and *AND* $(\wedge)$.

Solution : (a)

$$\begin{aligned}&(a \vee b) \wedge (\neg b \vee c)\\ \equiv\ &(\neg a \rightarrow b) \wedge (b \rightarrow c)\\ \equiv\ &\neg((\neg a \rightarrow b) \rightarrow \neg(b \rightarrow c))\end{aligned}$$

(b) (i) $a \rightarrow (b \rightarrow \neg c)$

(ii) $\neg a \vee \neg b \vee \neg c$

(iii) $\neg(a \wedge b \wedge c)$