Part I: Answer all of the following multiple choice questions.

Each of the following MC questions is worth 3 points. No partial credit.

1. What is the colour version of your exam paper? (Read the top left corner of the cover page!) **Make sure that you have written your ID number correctly in the I.D. No. Box in the MC answer sheet. If you do not do both correctly, you lose the points of this question.**

   (a) Green (b) Orange (c) White (d) Yellow (e) None of the previous

2. Evaluate the integral $\int_0^3 4sin^2(\pi x)\cos(2\pi x)\,dx$.

   (a) -2 (b) -1 (c) 1 (d) 2 (e) 4

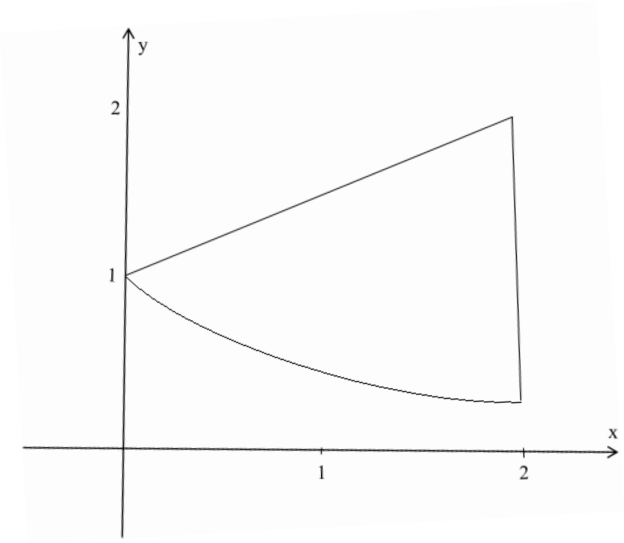
3. Evaluate the improper integral $\int_0^3 \frac{2x}{\sqrt{9-x^2}}\,dx$.

   (a) 2 (b) 4 (c) 6 (d) 8 (e) 10

4. Evaluate the improper integral $\int_0^\infty \frac{8x}{(x^2+2)^2}\,dx$.

   (a) 1 (b) 2 (c) 3 (d) 4 (e) divergent

5. Find the area enclosed by the curves $y = \frac{1}{2}x + 1$, $y = \frac{6}{(x+2)(x+3)}$ and $x = 2$.

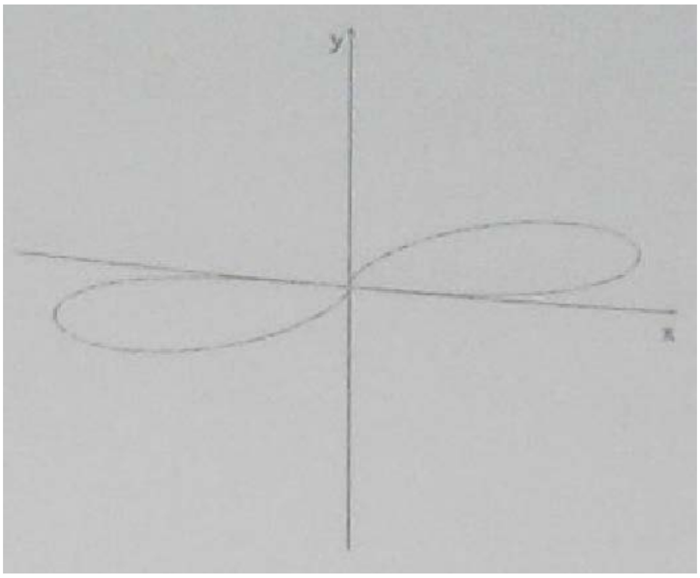


(a) $3\text{-}4 \ln\frac{6}{5}$ (b) $3\text{-}2 \ln\frac{3}{5}$ (c) $4\text{-}4 \ln\frac{3}{5}$ (d) $4\text{-}\ln\frac{3}{5}$ (e) $3\text{-}6 \ln\frac{6}{5}$

6. The graph of the function $y = 4 + 2x^2$ over the interval $0 \le x \le \sqrt{5}$ is rotated about the y-axis to generate a surface of revolution. Find the *area* of the surface.

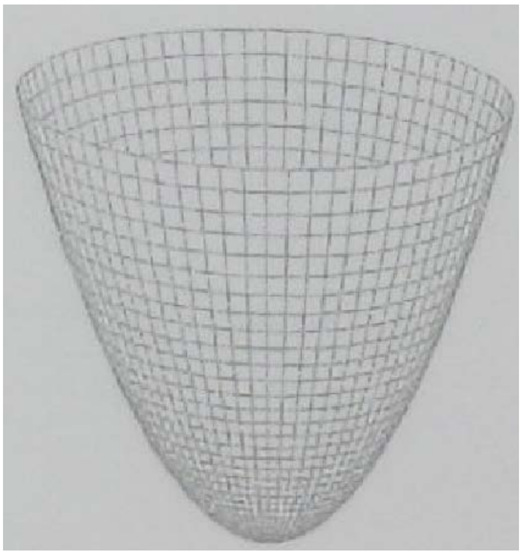
(a) $\frac{91\pi}{3}$ (b) $\frac{73\pi}{3}$ (c) $\frac{67\pi}{3}$ (d) $\frac{56\pi}{3}$ (e) $\frac{47\pi}{3}$

7. Find the area of the region enclosed by the curve defined by the *polar equation* $\mathrm{r} = 2\sqrt{\sin(2t)}e^{2\cos(2t)}$.



(a) $e^4 - e^{-4}$ (b) $\frac{e^4-1}{2}$ (c) $e^4 - 1$ (d) $\frac{e^4-e^{-4}}{2}$ (e) $e^4 - e^{-2}$

8.The shape of a container is obtained by rotating the curve $y = 2x^2$ over the interval $0 \leq x \leq 2$ about the y-axis. If the tank is full of water, the work required for pumping all water to the top of the tank can be expressed as an integral $\mathrm{W} = \int_a^b f(y)dy$. Use trapezoidal rule on four subintervals of equal length to estimate the work. (Assume that water density is ρ kg/m^3, and gravity acceleration g m/s^2.)



(a) $20\pi\rho g$ (b) $30\pi\rho g$ (c) $40\pi\rho g$ (d) $50\pi\rho g$ (e) $60\pi\rho g$

9. Which of the following improper integrals is divergent?

(i) $\int_1^\infty xe^{-\sqrt{x}}dx$ (ii) $\int_1^\infty \frac{\tan^{-1}x}{x^2}dx$ (iii) $\int_1^\infty \frac{\ln x}{\ln x+2x^2}dx$ (iv) $\int_1^\infty \frac{x}{e^{-2x}+x^2}dx$

(Recall that another notation for the inverse trigonometric function $\tan^{-1}x$ is $arctan\,x$.)

(a) (i) (b) (ii) (c) (iii) (d) (iv) (e) None

10. Find the sum of the series $\sum_{n=1}^{\infty}\left(\frac{3^{n+3}-2^n}{4^{n+3}}\right)$.

(a) 5 (b) 15 (c) 20 (d) 25 (e) divergent

11. Find the sum of the series: $\sum_{n=1}^{\infty}\left(4^{1/n}-4^{1/(n+2)}\right)$.

(a) 10 (b) 8 (c) 6 (d) 4 (e) divergent

12. Find all convergent infinite series from the following:

(i) $\sum_{n=1}^{\infty}(-1)^{n+1}\pi^{n/(n+1)}$ (ii) $\sum_{n=1}^{\infty}\frac{2^n n!}{(2n)!}$ (iii) $\sum_{n=1}^{\infty}\frac{\cos(n!)}{n^2}$ (iv) $\sum_{n=1}^{\infty}\frac{\ln\sqrt{n}}{\ln(n+4)}$

(a) Only (i) and (iv) are convergent.

(b) Only (ii) and (iii) are convergent.

(c) Only (ii) and (iv) are convergent.

(d) Only (i), (iii) and (iv) are convergent.

(e) All are convergent.

13. Which of the following values of *p* is the *largest* to keep the infinite series $\sum_{n=1}^{\infty} \frac{2^n n!}{(2n)!}$ *divergent*?

(a) p = 1 (b) p = 2 (c) p = -4 (d) p = -2 (e) p = -1

14. Find the coefficient of the x^4 term in the Maclaurin series (i.e. Taylor series centered at 0) of the function $f(x) = (1 + 4x^2)^{\frac{3}{2}}$.

(a) 4 (b) 6 (c) 8 (d) 10 (e) 12

15. Find the vector projection (orthogonal projection) of the vector $\langle -2,12,4\rangle$ onto the vector $\langle 2,3,1\rangle$.

(a) $\langle 4,6,2\rangle$ (b) $\langle 8,12,4\rangle$ (c) $\langle 2,3,1\rangle$ (d) $\langle \frac{8}{3}, 4, \frac{4}{3}\rangle$ (e) $\langle 6,9,3\rangle$

16. Find the perpendicular distance from the point given by the vector $\langle 6,0,0\rangle$ to the plane generated by the vectors $\langle 1,2,0\rangle$ and $\langle 1,0,-1\rangle$.

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

Part II: Answer each of the following questions.

17. [*12 pts*] Let $I_n = \int \frac{x^n}{\sqrt{x+3}} dx$, where *n* is a positive integer.

(a) Find constants A_n, B_n, which depend on n, such that [7 pts]

$$I_n = A_n x^n \cdot \sqrt{x+3} + B_n I_{n-1}.$$

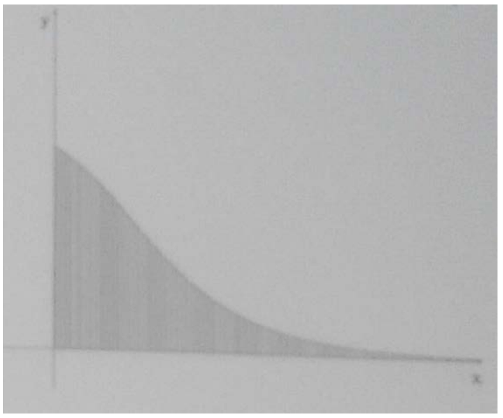
$A_n =$ ______________________________ $B_n =$ ______________________________

(Both are some expression in *n*.)

(b) Using part (a) or otherwise, evaluate the definite integral $\int_1^6 \frac{5x^2}{\sqrt{x+3}} dx$. [5 pts]

18. [*14 pts*] Consider the area under the graph of the function $y = \frac{4}{\sqrt{e^{2x}+8}}$ over the interval $0 < x < \infty$.

(a) Rotate the area about the *x-axis* to generate a solid of revolution. Find the volume of the solid thus obtained. [8 pts]



(b) If the area is rotated about the *y-axis*, does the solid of revolution thus obtained have a finite volume? *Show your reasoning for full credit.* [6 pts]

19. [*12 pts*] Determine whether the given series is convergent or divergent. Given brief reason to justify your answer.

(a) $\sum_{n=0}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$ [3 pts]

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n n}{(\ln n)^2}$ [3 pts]

(c) $\sum_{n=1}^{\infty} \left(\frac{n}{n+2}\right)^{n^3}$ [3 pts]

(d) $\sum_{n=1}^{\infty} e^{-n} \cos n$ [3 pts]

20. [*14 pts*] Consider a power series $f(x) = \sum_{n=0}^{\infty} \frac{2n+3}{6^{2n}} (x-1)^{2n+1}$.

(a) Find the largest open interval (open interval of convergence) in which the given power series converges absolutely. Show your work for full credit. [7 pts]

(b) Determine if the given power series is convergent at the endpoints of the open interval of convergence of *f(x)*. Justify your answer for full credit. [3 pts]

(c) Suppose that H(x) is a differentiable function in the open interval of convergence of *f(x)* such that $\frac{dH}{dx} = (x-1)f(x)$, and H(1) = -2. Find the function value H(2). [4 pts]