

PHYSICAL QUANTITIES, VECTORS AND 2D MOTION

Intended Learning Outcomes – after this lecture you will learn:

1. meanings of theories and models in physics.
2. how to define units for fundamental physical quantities.
3. how to use significant figures.
4. vectors and its algebraic operations: addition and subtraction
5. displacement, velocity and acceleration in vector notation
6. to predict the trajectory of projectile motion

Textbook reference: Ch 1, 3.1 – 3.3

Physics is an *experimental natural* science.

Theory: an explanation of natural phenomena based on observation and accepted fundamental principles, e.g. theory of evolution in biology

Model: a simplified version of a physical system that would be too complicated to analyze in full detail

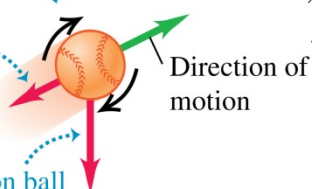
e.g. throwing a baseball

(a) A real baseball in flight

Baseball spins and has a complex shape.

Air resistance and wind exert forces on the ball.

Gravitational force on ball depends on altitude.



throw away
“unimportant
” parts

(b) An idealized model of the baseball

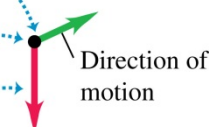
Baseball is treated as a point object (particle).

No air resistance.

Gravitational force on ball is constant.

use *theory* to make prediction

means calculation

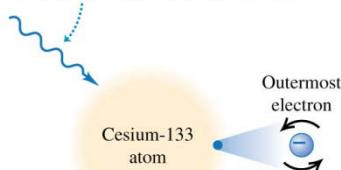


Standards and Units The *International Standard*, or SI (Système International) Units

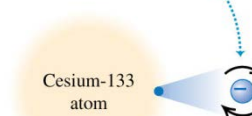
TIME: in second – starting 1967, defined using an atomic clock

(a) Measuring the second

Microwave radiation with a frequency of exactly 9,192,631,770 cycles per second ...



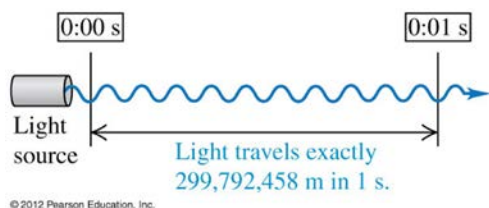
... causes the outermost electron of a cesium-133 atom to reverse its spin direction.



An atomic clock uses this phenomenon to tune microwaves to this exact frequency. It then counts 1 second for each 9,192,631,770 cycles.

LENGTH: in meter – starting 1983,
defined based on the **speed of light** in
vacuum, which is *defined* to be (exactly)
 $c = 299,792,458 \text{ m/s}$

(b) Measuring the meter



MASS: in kilogram – starting 2018,
kilogram is defined based on a
fundamental constant of nature called
Planck's constant, which is *defined* to
be (exactly)

$$h = 6.62607015 \times 10^{-34} \text{ kg}\cdot\text{m}^2/\text{s}$$

Uncertainty and Significant Figures

What is the difference among the following representations of π ?

1. 3.14 (means between 3.135 and 3.145, or 3.14 ± 0.005)
2. 3.1416 (means 3.1416 ± 0.00005)
3. $22/7$ (rational number usually means exact, ⚠ misleading here, not exact!)

What is the difference between 3 and 3.00?

⚠ Be careful about the number of significant figures.



What are the problems with the following representation?

$$2.017676 \pm 0.0132$$

smaller than
error,
meaningless!

error estimation cannot be so
accurate! Usually take 1, at
most 2 sig. fig.

keep the same decimal places

→ should be ____ \pm ____

Note: uncertainty propagates in calculations:

Multiplication or division:

Result may have no more significant figures
than **the starting number with the fewest
significant figures:**

$$\frac{0.745 \times 2.2}{3.885} = 0.42$$

$$1.32578 \times 10^7 \times 4.11 \times 10^{-3} = 5.45 \times 10^4$$

Addition or subtraction:

Number of significant figures is determined by
**the starting number with the largest uncertainty
(i.e., fewest digits to the right of the decimal
point):**

$$27.153 + 138.2 - 11.74 = 153.6$$

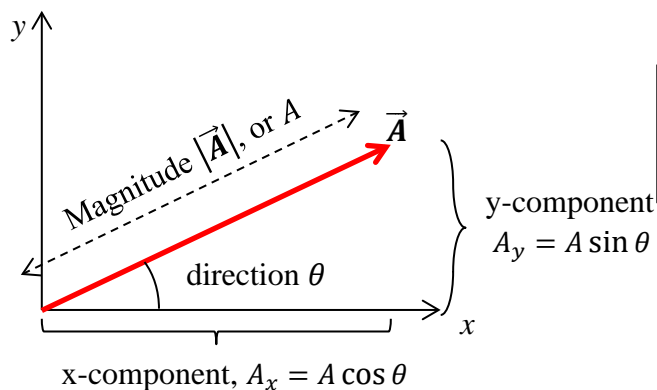
Question: What is the density (in kg/m^3) of a rock of mass 1.80 kg and volume $6.0 \times 10^{-4} \text{ m}^3$?

(a) $3 \times 10^3 \text{ kg/m}^3$, (b) $3.0 \times 10^3 \text{ kg/m}^3$, (c) $3.00 \times 10^3 \text{ kg/m}^3$, (d) $3.000 \times 10^3 \text{ kg/m}^3$

Answer: see inverted text on P. 37 of textbook

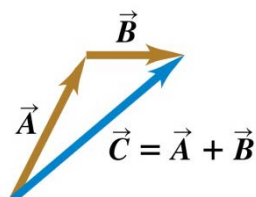
Vector

An “arrow” in space, has magnitude (length) and direction
e.g. in 2D Cartesian coordinates (due to René Descartes)

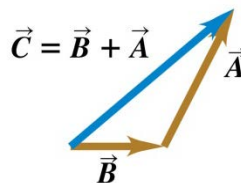


Note: $A = \sqrt{A_x^2 + A_y^2}$ (Pythagoras thm)
 $\tan \theta = \frac{A_y}{A_x}$

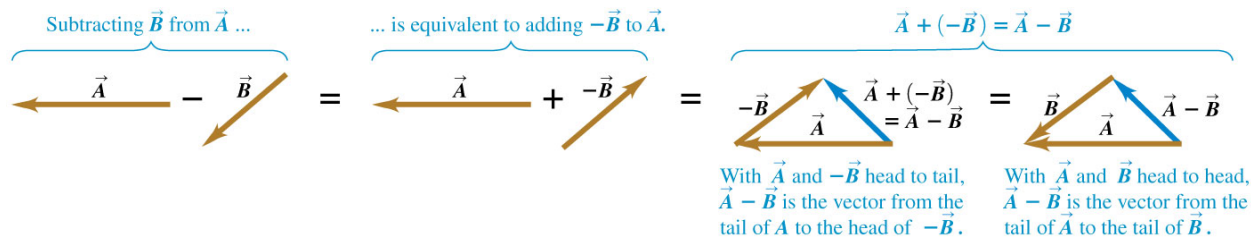
Addition:



or

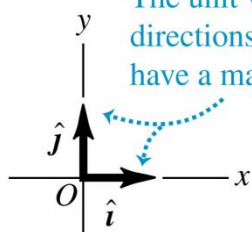


Subtraction:



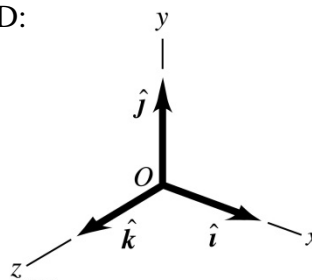
Vectors of unit magnitude are called unit vectors. Most commonly used unit vectors are \hat{i} , \hat{j} , and \hat{k} , along x , y , and z directions in Cartesian coordinates

2D:



The unit vectors \hat{i} and \hat{j} point in the directions of the x - and y -axes and have a magnitude of 1.

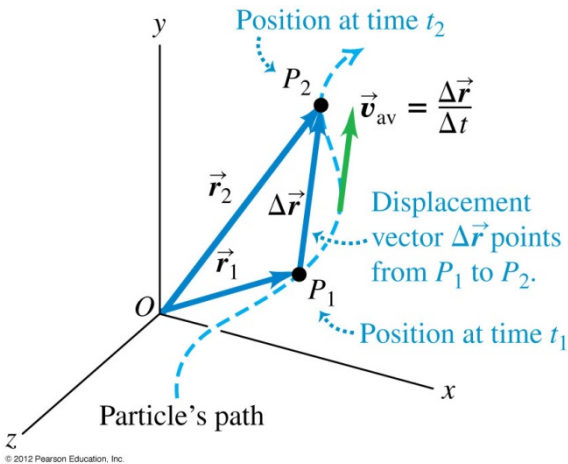
3D:



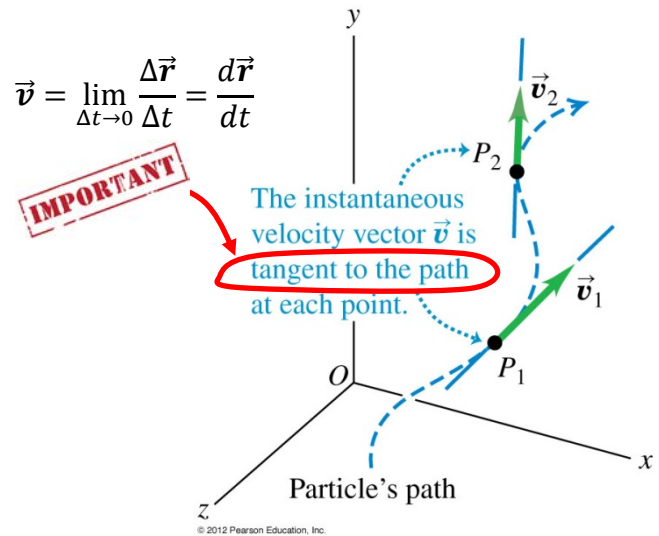
Analytical representation of a vector: $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

Displacement and velocity vectors

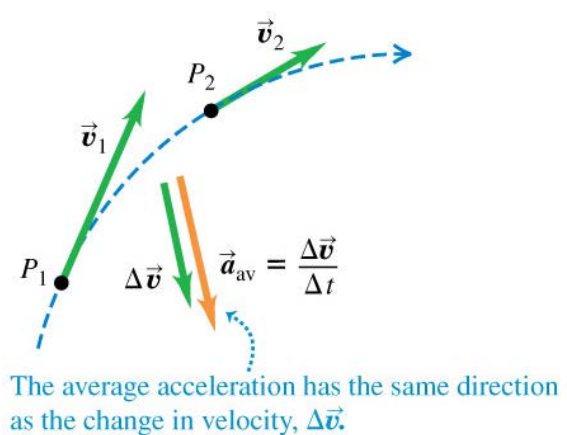
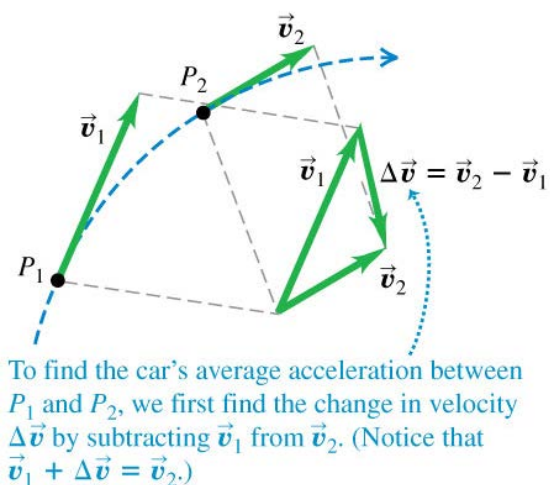
Distance and speed – scalars



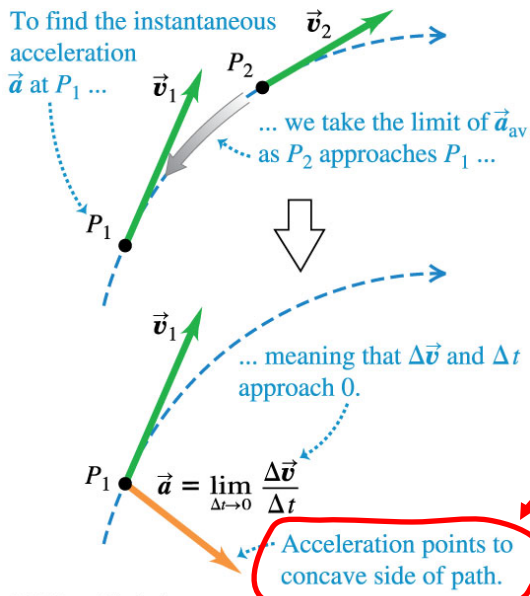
Displacement and velocity – vectors



Acceleration vector



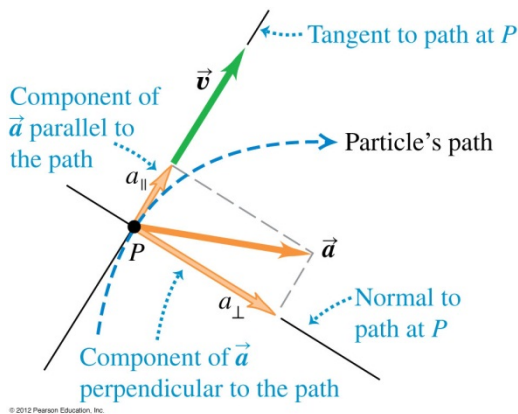
(a) Acceleration: curved trajectory



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$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

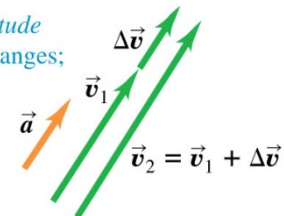
Resolve into parallel (or tangential) a_{\parallel} , and perpendicular (or radial) a_{\perp} components



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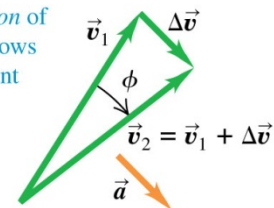
(a) Acceleration parallel to velocity

Changes only *magnitude* of velocity: speed changes; direction doesn't.



(b) Acceleration perpendicular to velocity

Changes only *direction* of velocity: particle follows curved path at constant speed.



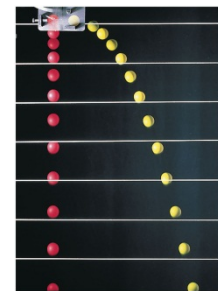
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Projectile

Principle: x and y motions are independent

Vertical motion of red and yellow balls are identical – at the same height at any time

$$\vec{a} = a_x \hat{i} + a_y \hat{j} = -g \hat{j}, \text{ i.e., } a_x = 0, a_y = -g$$

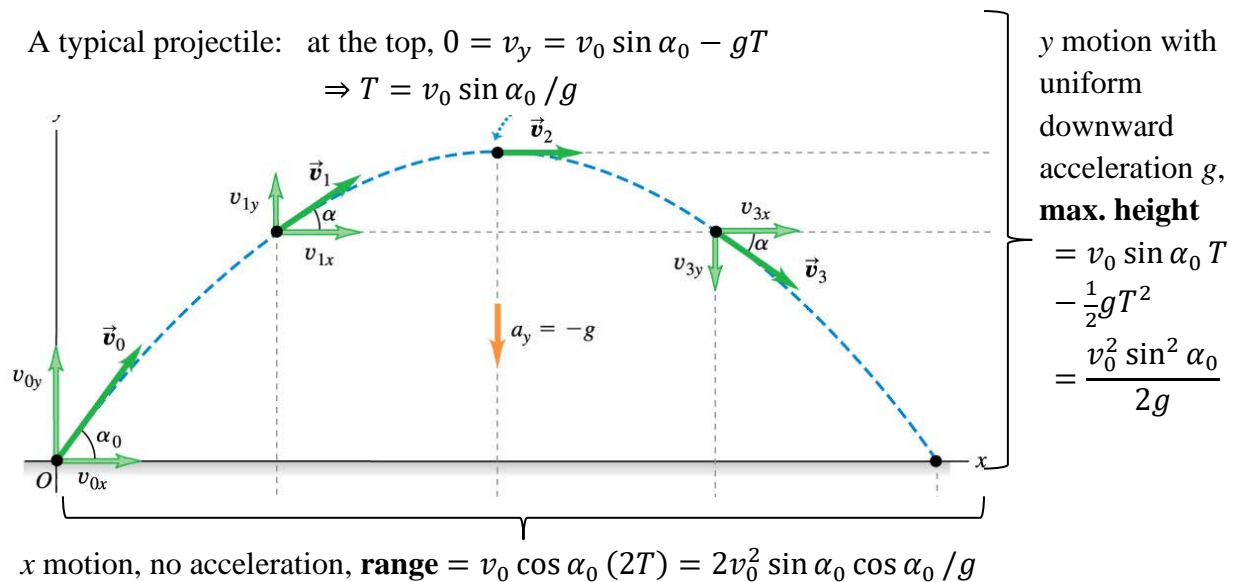


Recall from high school: rectilinear motion with uniform acceleration a

$$\begin{aligned}v &= v_0 + at \\x - x_0 &= v_0 t + \frac{1}{2}at^2 \\v^2 &= v_0^2 + 2a(x - x_0)\end{aligned}$$

Trajectory: $x(t) = v_0 \cos \alpha_0 t$, $y(t) = v_0 \sin \alpha_0 t - \frac{1}{2}gt^2$

Eliminate $t \Rightarrow y = (\tan \alpha_0)x - \frac{g}{2v_0^2 \cos^2 \alpha_0}x^2$
i.e. $y = bx - cx^2$ a parabola 拋物線



⚠ As derived in the general case, i) \vec{v} is tangential to path, ii) \vec{a} points to concave side

Demonstration: see Example 3.10 on P. 108

A ball fired at the same instant when the monkey is dropped. Ball *always* hit the monkey, **AMAZING!!**

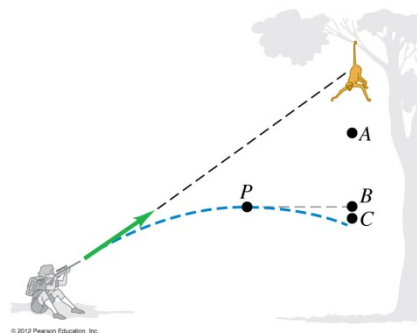
See the textbook for a proof.



Question:

When the ball is at its highest point P , the monkey will be at

- (i) point A (higher than P),
- (ii) point B (at the same height as P)
- (iii) C (lower than P)



Answer: see inverted text on P. 109 of textbook

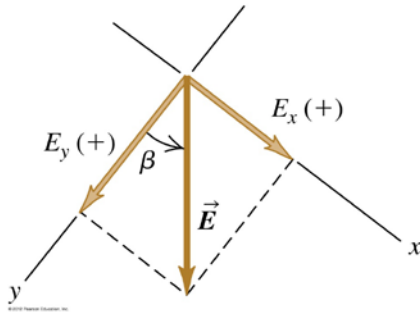
Clicker Questions

Q1.1

What are the x- and y-components of the vector \vec{E} ?



(b)



- A. $E_x = E \cos \beta$, $E_y = E \sin \beta$
- B. $E_x = E \sin \beta$, $E_y = E \cos \beta$
- C. $E_x = -E \cos \beta$, $E_y = -E \sin \beta$
- D. $E_x = -E \sin \beta$, $E_y = -E \cos \beta$
- E. $E_x = -E \cos \beta$, $E_y = E \sin \beta$

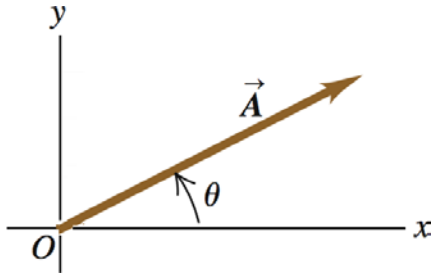
Q1.2



Which of the following statements is correct for *any* two vectors \vec{A} and \vec{B} ?

- A. The magnitude of $\vec{A} + \vec{B}$ is $A + B$
- B. The magnitude of $\vec{A} + \vec{B}$ is $A - B$
- C. The magnitude of $\vec{A} + \vec{B}$ is greater than or equal to $|A - B|$
- D. The magnitude of $\vec{A} + \vec{B}$ is greater than the magnitude of $\vec{A} - \vec{B}$
- E. The magnitude of $\vec{A} + \vec{B}$ is $\sqrt{A^2 + B^2}$

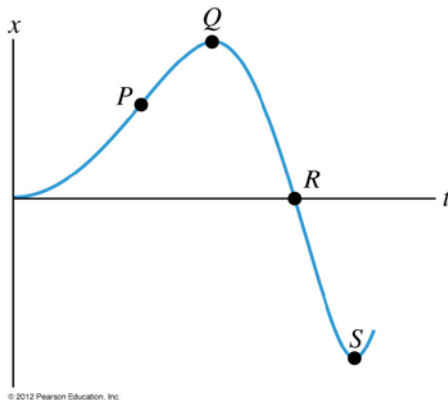
Q1.3



The angle θ is measured counterclockwise from the positive x -axis as shown. For which of these vectors is θ greatest?

- A. $24\hat{i} + 18\hat{j}$
- B. $-24\hat{i} - 18\hat{j}$
- C. $-18\hat{i} + 24\hat{j}$
- D. $-18\hat{i} - 24\hat{j}$

Q2.3



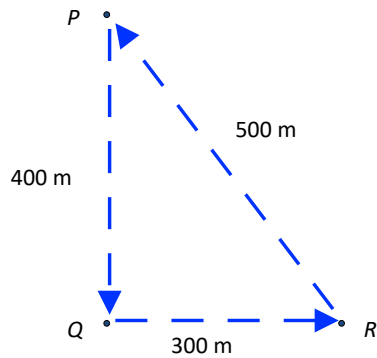
This is the x - t graph of the motion of a particle. Of the four points P , Q , R , and S , the acceleration a_x is greatest (most positive) at

- A. point P
- B. point Q
- C. point R
- D. point S
- E. not enough information in the graph to decide

Q3.1



A bicyclist starts at point P and travels around a triangular path that takes her through points Q and R before returning to point P . What is the magnitude of her net displacement for the entire round trip?

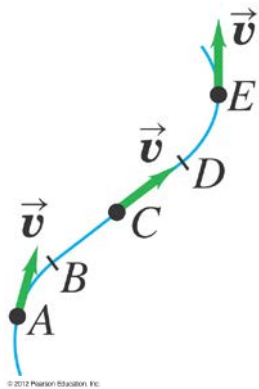


- A. 100 m
- B. 200 m
- C. 600 m
- D. 1200 m
- E. zero

Q3.3



The motion diagram shows an object moving along a curved path at constant speed. At which of the points A , C , and E does the object have zero acceleration?



- A. point A only
- B. point C only
- C. point E only
- D. points A and C only
- E. points A , C , and E

Ans. Q1.1) B, Q1.2) C, Q1.3) D, Q2.3) D, Q3.1) E, Q3.3) B

René Descartes

From Wikipedia, the free encyclopedia

"Descartes" redirects here. For other uses, see [Descartes \(disambiguation\)](#).

René Descartes (March 31, 1596 – February 11, 1650), also known as *Renatus Cartesius* (latinized form), was a highly influential French philosopher, mathematician, scientist, and writer. Dubbed the "Founder of Modern Philosophy" and the "Father of Modern Mathematics", much of subsequent western philosophy is a reaction to his writings, which have been closely studied from his time down to the present day. His influence in mathematics is also apparent, the [Cartesian coordinate system](#) used in plane geometry and algebra being named after him, and he was one of the key figures in the [Scientific Revolution](#).

Descartes frequently contrasted his views with those of his predecessors. In the opening section of the *Passions of the Soul*, he goes so far as to assert that he will write on his topic "as if no one had written on these matters before". Nevertheless many elements of his philosophy have precedents in late [Aristotelianism](#), the revived [Stoicism](#) of the 16th century, or in earlier philosophers like [Augustine](#). In his natural philosophy, he differs from the [Schools](#) on two major points: first, he rejects the analysis of [corporeal substance](#) into matter and form; second, he rejects any appeal to [ends](#)—divine or natural—in explaining natural phenomena. In his theology, he insists on the absolute freedom of God's act of creation.

Descartes was a major figure in 17th century continental [rationalism](#), later advocated by [Baruch Spinoza](#) and [Gottfried Leibniz](#), and opposed by the [empiricist](#) school of thought, consisting of [Hobbes](#), [Locke](#), [Berkeley](#), and [Hume](#). Leibniz, Spinoza and Descartes were all versed in mathematics as well as philosophy, and Descartes and Leibniz contributed greatly to science as well. As the inventor of the [Cartesian coordinate system](#), Descartes founded [analytic geometry](#), that bridge between algebra and geometry crucial to the invention of

[calculus](#) and [analysis](#). Descartes's reflections on mind and mechanism began the strain of western thought that much later, impelled by the invention of the [electronic computer](#) and by the possibility of [machine intelligence](#), blossomed into, e.g., the [Turing test](#). His most famous statement is: *Cogito ergo sum* (French: *Je pense, donc je suis*; English: *I think, therefore I am*), found in §7 of *Principles of Philosophy* (Latin) and part IV of *Discourse on Method* (French).

Western Philosophy 17th-century philosophy



René Descartes

Name: René Descartes

Birth: March 31, 1596 (La Haye en Touraine (now Descartes), Indre-et-Loire, France)

Death: February 11, 1650 (Stockholm, Sweden)

School/tradition: Cartesianism, Rationalism, Foundationalism

Main interests: Metaphysics, Epistemology, Science, Mathematics

Notable ideas: *Cogito ergo sum*, method of doubt, *Mathesis Universalis*, Cartesian coordinate system, Cartesian dualism, ontological argument for God's existence; regarded as a founder of Modern philosophy

Influences: Plato, Aristotle, Anselm, Aquinas, Ockham, Suarez, Mersenne, Sextus Empiricus, Michel de Montaigne

Influenced: Spinoza, Hobbes, Arnauld, Malebranche, Pascal, Locke, Leibniz, More, Kant, Husserl

For more information see <http://en.wikipedia.org/wiki/Descartes>