GRAVITATION II

Intended Learning Outcomes – after this lecture you will learn:
1. Kepler's laws of planetary motion
2. gravitation effect of a spherical mass distribution is the same as a point mass
3. the apparent weight due to rotation of the earth
4. the idea of a black hole starting from the concept of escape speed

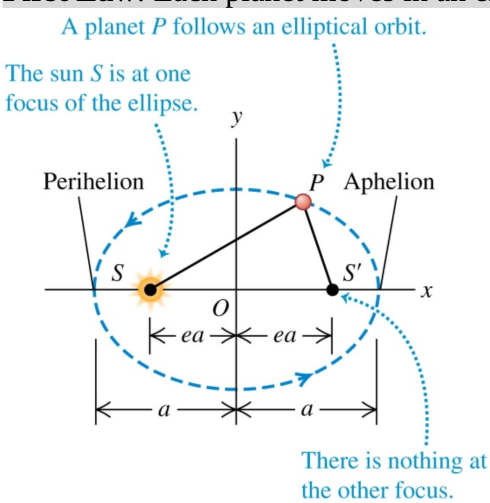
Textbook Reference: Ch 13.5 – 13.8

Kepler's Laws of Planetary Motion

Purely phenomenological – Kepler didn't know why

Later derived by Newton using his laws of motion and gravitation – significance: heavenly objects obey the same physical laws as terrestrial objects, don't need, e.g., Greek myths!

First Law: Each planet moves in an elliptical orbit, with the sun at one focus of the ellipse.



An **ellipse** is defined by the locus of a point P such that $|PS'| + |SP| = \text{constant}$

S and S' are the two **foci** of the ellipse

Semi-major axis a (⚠ a length, not an axis)

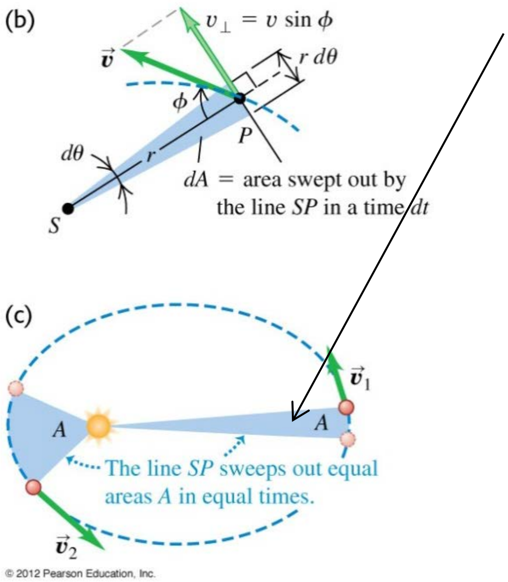
Eccentricity e ($e = 0$ for circle, $0 < e < 1$ for ellipse)

Aphelion – farthest $[(1+e)a]$ point from sun

Perihelion – closest $[(1-e)a]$ point to sun

Note: aphelion distance + perihelion distance = $2a$

Second Law: A line from the sun to a given planet sweeps out equal areas in equal times.
See http://en.wikipedia.org/wiki/File:Kepler-second-law.gif



$dA \approx$ area of blue triangle $= \frac{1}{2}(r d\theta) r$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

$$v_\perp = v \sin\phi = r\frac{d\theta}{dt}$$

$$\therefore \quad \frac{dA}{dt} = \frac{1}{2} r v \sin\phi = \frac{1}{2m}|\vec{r} \times m\vec{v}| = \frac{L}{2m}$$

i.e., Kepler's second law ⇔ conservation of angular momentum

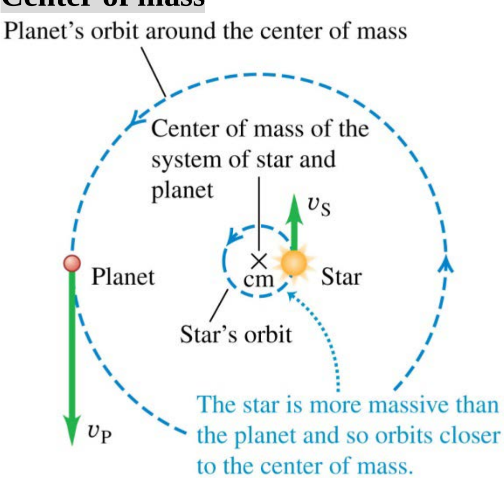
⚠ Angular momentum is conserved because gravitational force (a central force) produces no torque

⚠ Another consequence of conservation of angular momentum – orbit lies in a plane

Third Law: The periods of the planets are proportional to the $\frac{3}{2}$ powers of the major axis lengths of their orbits.

$$T = \frac{2\pi a^{3/2}}{\sqrt{G m_S}}$$

Center of mass



Both sun and planet orbit around their center of mass

Mass of sun ~ 750 times the total mass of planets → sun effectively at rest

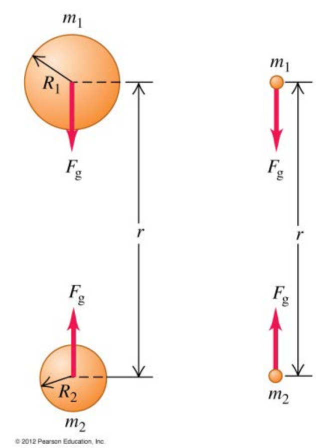
A **binary star** consists of two stars orbiting about their common CM, one called primary (the brighter one) and the other secondary. Can detect the secondary based on wobbling of the primary around their CM

Spherical Mass Distribution

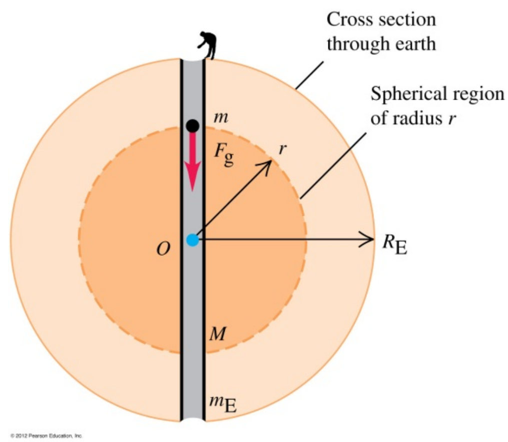
Means density $\rho(r)$ depends on distance from the center only, not on the direction. Can be a shell or solid.

Major results: (see textbook for proofs)

1. The gravitational effect *outside* a spherical mass distribution is the same as if all the mass is concentrated at the center of the sphere.



2. The gravitational effect *inside* a spherical mass distribution is the same as if all the mass *interior* to that point is concentrated at the center of the sphere.



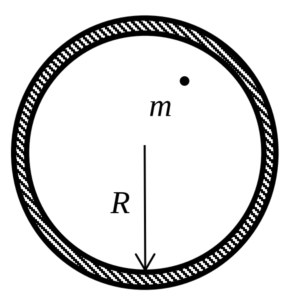
Example: when passing through a tunnel through the earth (assume constant density ρ), only the spherical region of radius r contributes to the gravitational force at r

$$F_g = \frac{GMm}{r^2} = \frac{Gm}{r^2}\left[\left(\frac{4}{3}\pi r^3\right)\rho\right] = \frac{Gm_E m}{R_E^3} r$$

$$\rho = \frac{m_E}{\frac{4}{3}\pi R_E^3}$$

⚠ $\propto r$, not $1/r^2$

3. The gravitational PE of a mass *m anywhere inside* a hollow sphere is a constant



$$U = -\frac{GMm}{R}$$

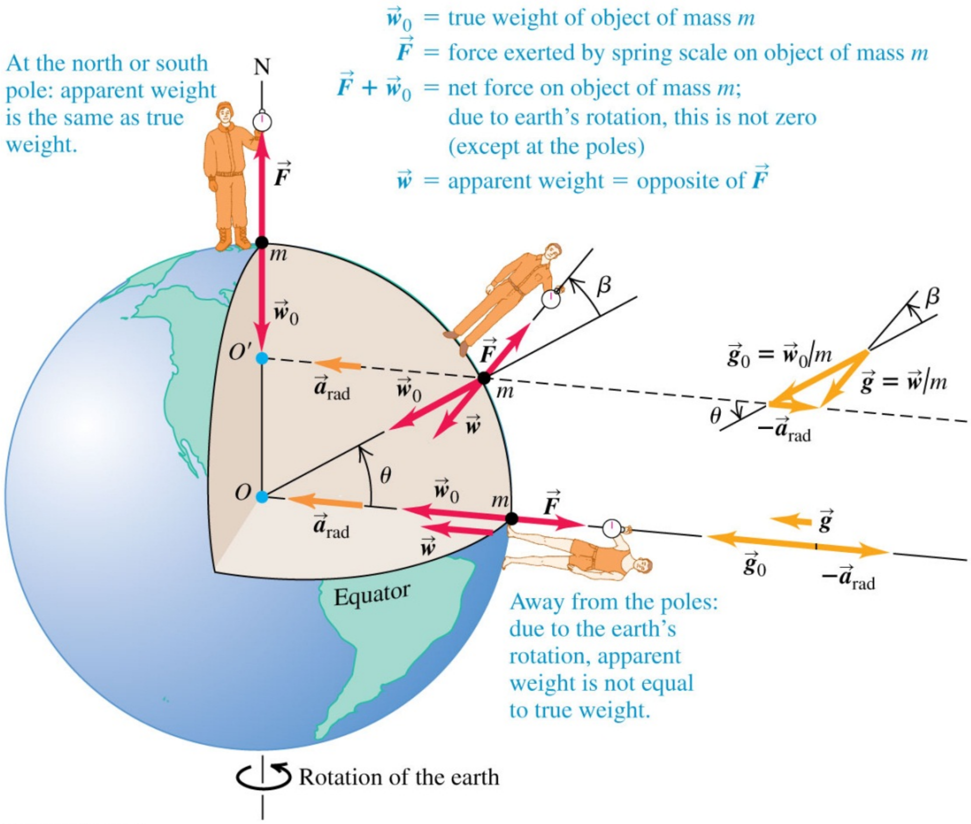
Gravitational force $\vec{\boldsymbol{F}}_g = -\left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right) = 0$

Question

If the earth were a hollow sphere and were not rotating, would it be possible to stand and walk on its inner surface?

Answer: see inverted text on P. 441 of textbook

Apparent Weight and the Earth's Rotation



$\vec{\boldsymbol{w}}_0$ true weight (due to earth's gravitational attraction)

If earth not rotating, body in equilibrium, $\vec{\boldsymbol{F}} = -\vec{\boldsymbol{w}}_0$ (this is true at the north/south poles of the rotating earth)

At equator

$$w_0 - F = \frac{mv^2}{R_E}$$

acceleration due to gravity of a rotating earth

Apparent weight

$$w = F = \underbrace{\frac{Gm_E}{R_E^2}}_{g_0}m - \frac{mv^2}{R_E} = mg$$

g_0, acceleration due to gravity of a non-rotating earth

$$\Rightarrow \quad g = g_0 - \frac{v^2}{R_E}$$

$\sim 0.0339\ \mathrm{m/s^2}$

Question

A planet has the same mass and radius as the earth, but rotates 10 times faster. The difference between the acceleration due to gravity at its equator and poles is (0.00339 / 0.0339 / 0.339 / 3.39) m/s^2.

Answer: see inverted text on P. 443 of textbook

Black Holes

Recall *escape speed* from a star of mass M and radius R

$$v = \sqrt{\frac{2GM}{R}}$$

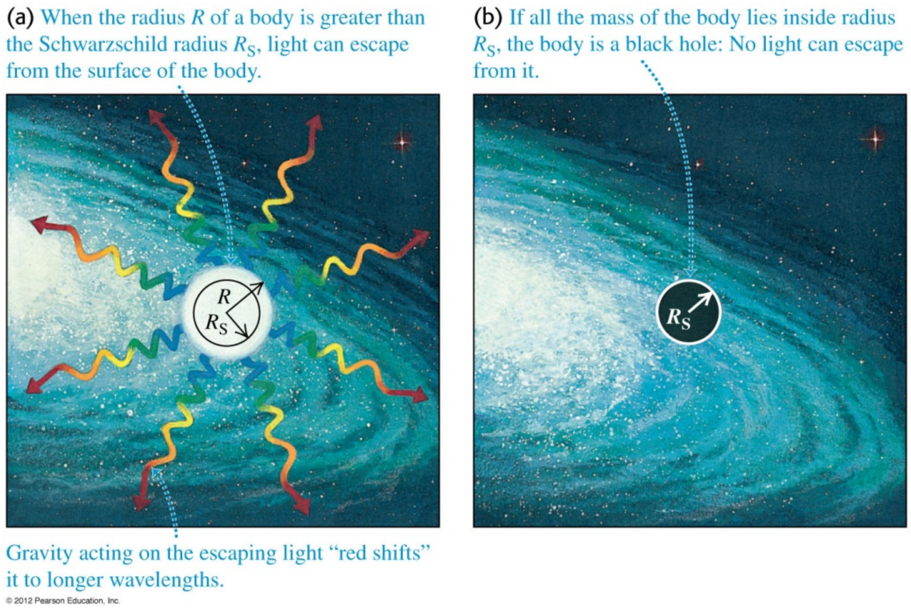
What if a star collapses, keeping the same M but R decreases? v increases.
When R small enough (reaches a critical value R_S), $v \to c$, no light can escape?

$$c = \sqrt{\frac{2GM}{R_S}} \quad \rightarrow \quad \boxed{R_S = \frac{2GM}{c^2}} \quad \textbf{Schwarzschild radius}$$

Problems: 1) KE of light (photon) is not $\frac{1}{2}mc^2$

2) gravitational PE near black hole is not $-GMm/r$

Schwarzschild (1916) derived exactly the same critical radius using *General Relativity* (a relativistic theory of gravitation)

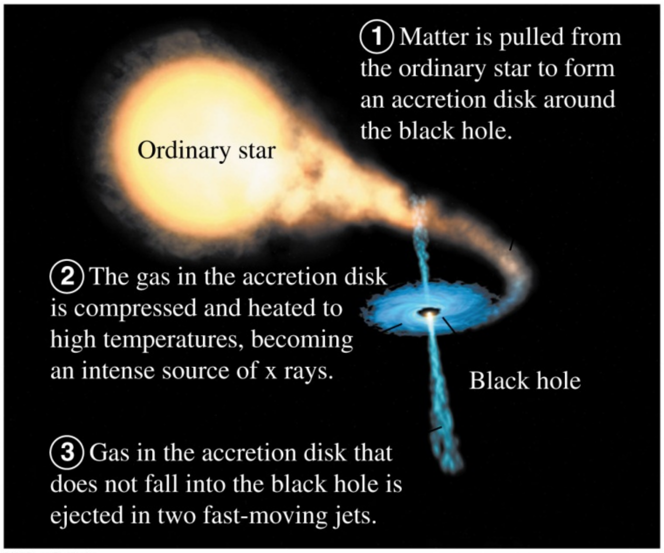


Surface of the sphere with radius R_S surrounding a black hole is called the **event horizon**: light inside cannot escape ∴ cannot know what happens inside a black hole

We know a black hole's

1) mass – through its gravitational force on others
2) electric charge – through its electric force on other charged bodies
3) angular momentum – through its rotating gravitational field that drags space and everything in that space around it

Can't see light from a black hole, how to detect it?



In a binary star system (ordinary star + black hole), look for x-ray source

Demonstration: gravity well

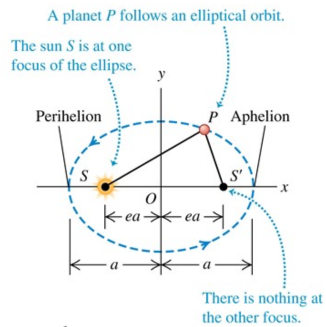
Or study orbits of surrounding stars in other cases

Question
If the sun collapses to form a black hole, the orbit of the earth would (shrink / expand / remain the same).
Answer: see inverted text on P. 447

Clicker Questions:

Q13.12

A planet (P) is moving around the sun (S) in an elliptical orbit. As the planet moves from aphelion to perihelion, the planet's angular momentum



A. increases at all times.

B. decreases at all times.

C. decreases during part of the motion and increases during the other part.

D. increases, decreases, or remains the same during various parts of the motion.

E. remains the same at all points between aphelion and perihelion.

Q13.8

Star X has twice the mass of the sun. One of star X's planets moves in a circular orbit around star X. This orbit has the same radius as the earth's orbit around the sun. The orbital *speed* of this planet of star X

A. is faster than the earth's orbital speed.

B. is the same as the earth's orbital speed.

C. is slower than the earth's orbital speed.

D. depends on the mass of the planet.

E. depends on the mass and radius of the planet.

Ans: Q13.12) E, Q13.8) A

Johannes Kepler

From Wikipedia, the free encyclopedia
(Redirected from Kepler)

Johannes Kepler (German: [ˈkʰɛplɐ]; December 27, 1571 – November 15, 1630) was a German mathematician, astronomer and astrologer. A key figure in the 17th century scientific revolution, he is best known for his eponymous laws of planetary motion, codified by later astronomers, based on his works *Astronomia nova*, *Harmonices Mundi*, and *Epitome of Copernican Astronomy*. These works also provided one of the foundations for Isaac Newton's theory of universal gravitation.

During his career, Kepler was a mathematics teacher at a seminary school in Graz, Austria, where he became an associate of Prince Hans Ulrich von Eggenberg. Later he became an assistant to astronomer Tycho Brahe, and eventually the imperial mathematician to Emperor Rudolf II and his two successors Matthias and Ferdinand II. He was also a mathematics teacher in Linz, Austria, and an adviser to General Wallenstein. Additionally, he did fundamental work in the field of optics, invented an improved version of the refracting telescope (the Keplerian Telescope), and mentioned the telescopic discoveries of his contemporary Galileo Galilei.

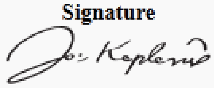
Kepler lived in an era when there was no clear distinction between astronomy and astrology, but there was a strong division between astronomy (a branch of mathematics within the liberal arts) and physics (a branch of natural philosophy). Kepler also incorporated religious arguments and reasoning into his work, motivated by the religious conviction and belief that God had created the world according to an intelligible plan that is accessible through the natural light of reason.[1] Kepler described his new astronomy as "celestial physics",[2] as "an excursion into Aristotle's *Metaphysics*",[3] and as "a supplement to Aristotle's *On the Heavens*",[4] transforming the ancient tradition of physical cosmology by treating astronomy as part of a universal mathematical physics.[5]

Johannes Kepler

A 1610 portrait of Johannes Kepler by an unknown artist

Born	December 27, 1571 Free Imperial City of Weil der Stadt near Stuttgart, HRE (now part of the Stuttgart Region of Baden-Württemberg, Germany)
Died	November 15, 1630 (aged 58) Regensburg, Electorate of Bavaria, HRE (now Germany)
Residence	Germany
Nationality	German
Fields	Astronomy, astrology, mathematics and natural philosophy
Institutions	University of Linz
Alma mater	University of Tübingen
Known for	Kepler's laws of planetary motion Kepler conjecture

Signature



Early years

Johannes Kepler was born on December 27, 1571, at the Free Imperial City of Weil der Stadt (now part of the Stuttgart Region in the German state of Baden-Württemberg, 30 km west of Stuttgart's center). His grandfather, Sebald Kepler, had been Lord Mayor of that town but, by the time Johannes was born, he had two brothers and one sister and the Kepler family fortune was in decline. His father, Heinrich Kepler, earned a precarious living as a mercenary, and he left the family when Johannes was five years old. He was believed to have died in the Eighty Years' War in the Netherlands. His mother Katharina Guldenmann, an inn-keeper's daughter, was a healer and herbalist who was later tried for witchcraft. Born prematurely, Johannes claimed to have been weak and sickly as a child. Nevertheless, he often impressed travelers at his grandfather's inn with his phenomenal mathematical faculty.[6]

For more information see http://en.wikipedia.org/wiki/Kepler

Karl Schwarzschild

From Wikipedia, the free encyclopedia

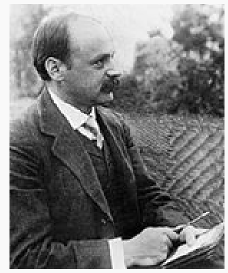
Karl Schwarzschild (/ˈshvarts shīld/) (October 9, 1873 – May 11, 1916) was a German physicist. He is also the father of astrophysicist Martin Schwarzschild.

He is best known for providing the first exact solution to the Einstein field equations of general relativity, for the limited case of a single spherical non-rotating mass, which he accomplished in 1915, the same year that Einstein first introduced general relativity. The Schwarzschild solution, which makes use of Schwarzschild coordinates and the Schwarzschild metric, leads to the well-known Schwarzschild radius, which is the size of the event horizon of a non-rotating black hole.

Schwarzschild accomplished this triumph while serving in the German army during World War I. He died the following year from pemphigus, a painful autoimmune disease which he developed while at the Russian front.

Asteroid 837 Schwarzschilda is named in his honor.

Karl Schwarzschild



Karl Schwarzschild (1873-1916)

Born	October 9, 1873 Frankfurt am Main
Died	May 11, 1916 (aged 42) Potsdam
Nationality	German
Fields	Physics Astronomy
Alma mater	Ludwig Maximilian University of Munich
Doctoral advisor	Hugo von Seeliger
Influenced	Martin Schwarzschild

Life

Schwarzschild was born in Frankfurt am Main to Jewish parents. His father was active in the business community of the city, and the family had ancestors in the city dating back to the sixteenth century.[1] Karl attended a Jewish primary school until 11 years of age.[2] He was something of a child prodigy, having two papers on binary orbits (celestial mechanics) published before he was sixteen.[3][4] He studied at Straßburg and Munich, obtaining his doctorate in 1896 for a work on Jules Henri Poincaré's theories.

From 1897, he worked as assistant at the Kuffner observatory in Vienna.

From 1901 until 1909 he was a professor at the prestigious institute at Göttingen, where he had the opportunity to work with some significant figures including David Hilbert and Hermann Minkowski. Schwarzschild became the director of the observatory in Göttingen. He married Else Posenbach, the daughter of a professor of surgery at Göttingen, in 1909, and later that year moved to Potsdam, where he took up the post of director of the Astrophysical Observatory. This was then the most prestigious post available for an astronomer in Germany. He and Else had three children, Agathe, Martin (who went on to become a professor of astronomy at Princeton University), and Alfred.

From 1912, Schwarzschild was a member of the Prussian Academy of Sciences.

At the outbreak of World War I in 1914 he joined the German army despite being over 40 years old. He served on both the western and eastern fronts, rising to the rank of lieutenant in the artillery.

While serving on the front in Russia in 1915, he began to suffer from a rare and painful skin disease called pemphigus. Nevertheless, he managed to write three outstanding papers, two on relativity theory and one on quantum theory. His papers on relativity produced the first exact solutions to the Einstein field equations, and a minor modification of these results gives the well-known solution that now bears his name: the Schwarzschild metric.

Schwarzschild's struggle with pemphigus may have eventually led to his death. He died on May 11, 1916.

For more information see http://en.wikipedia.org/wiki/Karl_Schwarzschild