HKUST

MATH1014 Calculus II

Midterm Examination (White Version) Name:_____

30th Mar 2014 Student ID:_____

10:15-11:45 Lecture Section:_____

Directions:

- This is a closed book examination. **No Calculator is allowed in this examination**.
- DO NOT open the exam until instructed to do so.
- Turn off all phones and pagers, and remove headphones. All electronic devices should be kept in a bag away from your body.
- Write your name, ID number, and Tutorial Section in the space provided above, and also in the **Multiple Choice Item Answer Sheet** provided.
- Check that the version of your **Multiple Choice Item Answer Sheet** matches the color version of your exam paper: Green, Orange, Yellow, or White (if no color version printed on the MC answer sheet).
- Answer all questions. Show an appropriate amount of work for each long problem. If you do not show enough work, you will get only partial credit.
- DO NOT use any of your own scratch paper. Write your name on every scratch paper supplied by the examination, and do not take any scratch paper away after the examination.
- When instructed to open the exam, please check that you have **7** pages of questions in addition to the cover page. There are two blank pages and a **formula sheet** attached which can be used as scratch paper.
- You may write on the backside of the pages, but if you use the backside, clearly indicate that you have done so.
- **Cheating is a serious violation of the HKUST Academic Code. Students caught cheating will receive a zero score for the examination, and will also be subjected to further penalties imposed by the University.**

Please read the following statement and sign your signature.

I have neither given nor received any unauthorized aid during this examination. The answers submitted are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University's regulations governing academic integrity.

Student's Signature :

Question No.	Points	Out of
Q. 1-10		60
Q. 11		10
Q. 12		15
Q. 13		15
Total Points		100

Part I: Answer all of the following multiple choice questions.

- **Do not forget to write your name and student ID number on your Multiple Choice Item Answer Sheet. Mark also your student ID number in the I. D. No. box there.**
- **Use an HB pencil to mark your answers to the MC questions on the Multiple Choice Item Answer Sheet provided.**
- Enter also your MC answers to the following boxes **for back-up use only**. The marking will be completely based on the answers on the Multiple Choice Item Answer Sheet.

Question	1	2	3	4	5	6	7	8	9	10
Answer										

Each of the following MC questions is worth 6 points. No partial credit.

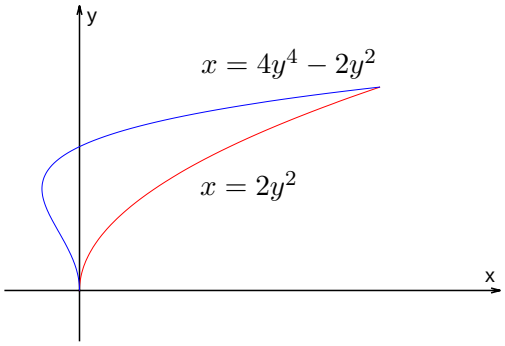
1. The oil in an oil reserve is being extracted at a rate of

$$r(t) = \frac{t^2(30-t)}{100} \quad \text{millions of barrels/year}$$

for $0 \le t \le 30$ (in years). How much oil (in millions of barrels) is extracted from $t = 0$ to $t = 20$ year?

(a) 200 (b) 250 (c) 300 (d) 350 (e) 400

2. Find the area of the region enclosed by the curves $x = 4y^4 - 2y^2$ and $x = 2y^2$.



(a) $\frac{9}{5}$ (b) $\frac{12}{5}$ (c) $\frac{14}{5}$ (d) $\frac{8}{15}$ (e) $\frac{16}{15}$

3. The base of a solid sitting on the xy-plane is enclosed by the ellipse defined by the equation $x^2 + \frac{y^2}{4} = 1$. Cross sections of the solid perpendicular to the base and parallel to the x-axis are *equilateral triangles* (i.e., triangles with equal side length). Find the volume of the solid.

(a) $\frac{8\sqrt{3}}{3}$ (b) $\frac{12\sqrt{3}}{5}$ (c) $\frac{16\sqrt{3}}{5}$ (d) $\frac{5\sqrt{3}}{3}$ (e) $\frac{16\sqrt{3}}{3}$

4. Evaluate the integral $\int_0^{\frac{\pi}{8}} 4\tan^3(2x)\sec^2(2x)\,dx$.

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{5}{2}$ (e) $\frac{5}{4}$

5. Evaluate the integral $\int_0^{\frac{\pi}{3}} 3\cos^2(3x)\cos(6x)\,dx$.

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$ (e) $\frac{5\pi}{6}$

6. Evaluate the integral $\displaystyle\int_0^{\ln 3} \frac{e^x - 3}{(e^x + 1)(e^x + 3)}\, dx$.

(a) $2\ln 2 - 3\ln 3$ (b) $3\ln 2 - 2\ln 3$ (c) $3\ln 2 + 2\ln 3$

(d) $3\ln 2 - 5\ln 3$ (e) $3\ln 2 + 5\ln 3$

7. Evaluate the integral $\displaystyle\int_{-1}^{2} \frac{1}{\sqrt{x^2 + 2x + 5}}\, dx$

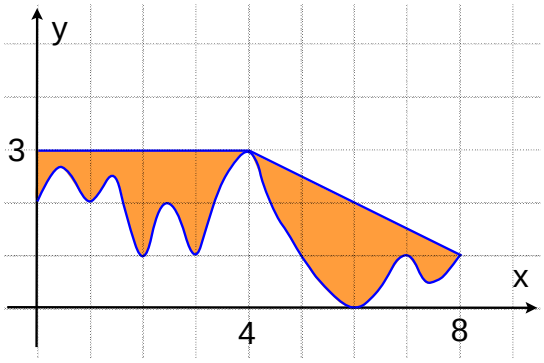
(a) $\ln(\sqrt{13} + 3)$ (b) $2\ln(\sqrt{13} + 3)$ (c) $\ln \frac{\sqrt{13}+3}{2}$

(d) $2\ln \frac{\sqrt{13}+3}{2}$ (e) $\frac{3}{2}\ln \frac{\sqrt{13}+3}{2}$

8. Find the arc length of the graph of the function defined by $y = \displaystyle\int_0^x \sqrt{e^t - 1}\, dt$, where $0 \le x \le 1$.

(a) $e - 1$ (b) e (c) $2\sqrt{e-1}$ (d) $\sqrt{e} - 1$ (e) $2\sqrt{e} - 2$

9. The area in the given figure is rotated about the x-axis to generate a solid. Use Simpson's Rule on 4 subintervals to find an approximate value of the volume of the solid.



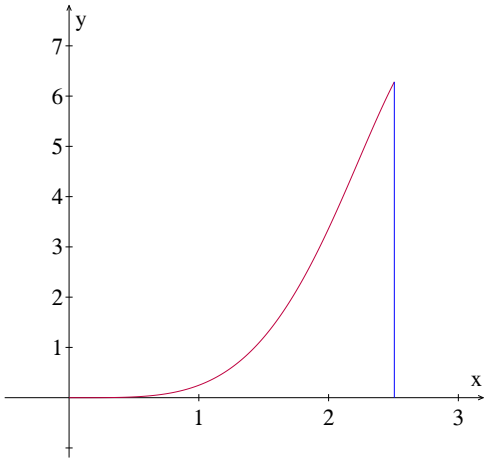
(a) $\frac{106\pi}{3}$ (b) $\frac{134\pi}{3}$ (c) $\frac{168\pi}{3}$ (d) $\frac{152\pi}{6}$ (e) $\frac{215\pi}{6}$

10. A curve in the xy-plane is defined by the polar equation $r = \theta \sin\theta$. Find the slope of the tangent line to the curve at the point with angular coordinate $\theta = \frac{\pi}{4}$.

(a) $\frac{1+\pi}{2}$ (b) $\frac{2}{1-\pi}$ (c) $\frac{2}{2+\pi}$ (d) $\frac{2+\pi}{2}$ (e) $\frac{1-\pi}{2}$

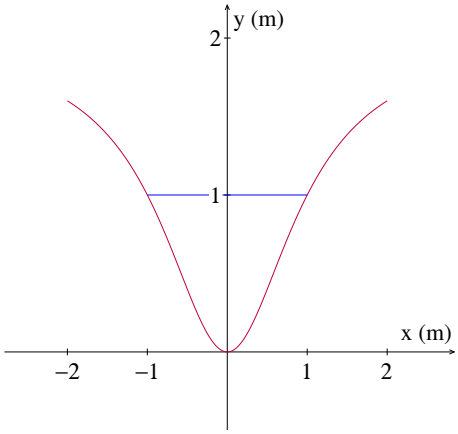
Part II: Answer each of the following questions.

11. [*10 pts*] The region enclosed by the curve defined by $y = x^2 \sin(\frac{x^2}{4})$, the line defined by $x = \sqrt{2\pi}$ and the x-axis is revolved about the y-axis to generate a solid. Find the volume of the solid.



12. [*15 pts*] The shape of a container is given by rotating the curve $y = \frac{2x^2}{1+x^2}$ about the y-axis, where $-2 \le x \le 2$. Suppose the container is filled with water up to a depth of 1 m.

(a) Express the volume of the water as a definite integral. [5 pts]
Do not need to calculate the integral.



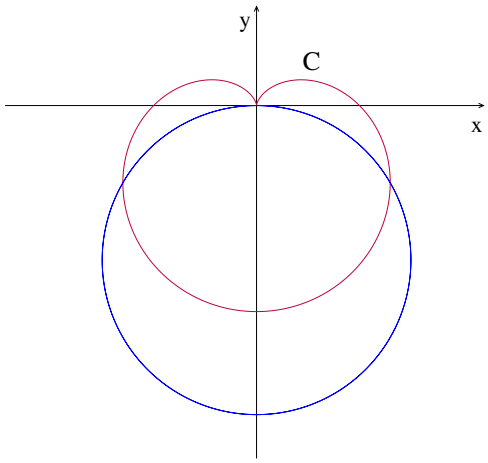
(b) Express the work required to pump all water to the top of the container as a definite integral. (Water density: 1000 kg/m^3, gravity acceleration: $g = 9.8$ m/s^2) [5 pts]
Do not need to calculate the integral.

(c) Express the surface area of the whole container as a definite integral. [5 pts]
Do not need to calculate the integral.

13. [*15 pts*] A curve C defined by the polar equation $r = 1 - \sin\theta$ is plotted in the given figure together with the circle defined by the polar equation $r = -3\sin\theta$.

(a) Find the rectangular coordinates of the points on curve C with the following angular coordinates: [2 pts]

	$\theta = \pi + \frac{\pi}{3}$	$\theta = -\frac{\pi}{4}$
x coordinate		
y coordinate		



(b) Find the rectangular coordinates of the intersection points of the two curves. [4 pts]

(c) Find the area of the region lying inside curve C but outside the circle. [9 pts]

Math1014 Midterm Exam Formula Sheet

Trigonometric Identities

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$
$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\sin A\cos B = \tfrac{1}{2}\Big(\sin(A+B) + \sin(A-B)\Big)$$
$$\cos A\cos B = \tfrac{1}{2}\Big(\cos(A+B) + \cos(A-B)\Big)$$
$$\sin A\sin B = \tfrac{1}{2}\Big(\cos(A-B) - \cos(A+B)\Big)$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$$

$$\int \cos^n x dx = \frac{1}{n}\sin x\cos^{n-1}x + \frac{n-1}{n}\int \cos^{n-2}x dx$$

$$\int \sin^n x dx = -\frac{1}{n}\cos x\sin^{n-1}x + \frac{n-1}{n}\int \sin^{n-2}x dx$$