This homework set covers the basics of definite integrals.

1. Interpreting definite integrals as signed areas.
2. Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$ if $F'(x) = f(x)$, where f is continuous on $[a,b]$. Also
$$\frac{d}{dx}\int_a^x f(t)dt = f(x).$$
It relates the two kinds of integrals: definite and indefinite.
3. Substitution rule: turning a complicated integral $\int f(x)dx$ into an easier one $\int g(u)du$ by an appropriate choice of substitution: $u = g(x), du = g'(x)dx$.

Give 4 or 5 significant digits for numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as 3^2 or 3**2 instead of 9, $\sin(3*pi/2)$ instead of -1, e^(ln(3)) instead of 3, $(1+\tan(3))*(4-\sin(5))$^6 $-15/8$ instead of 12748.8657, etc.

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1. (3 points)

Evaluate the following integral by interpreting it in terms of areas:
$$\int_0^{10} |x-5|\,dx$$
Value of integral = ____________

Correct Answers:

- `25`

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2. (3 points) Let
$$f(x) = \begin{cases} 0 & \text{if } x < -4 \\ 3 & \text{if } -4 \le x < -1 \\ -4 & \text{if } -1 \le x < 5 \\ 0 & \text{if } x \ge 5 \end{cases}$$
and
$$g(x) = \int_{-4}^x f(t)dt$$
Determine the value of each of the following:
(a) $g(-5) =$ ___
(b) $g(-3) =$ ___
(c) $g(0) =$ ___
(d) $g(6) =$ ___
(e) The absolute maximum of $g(x)$ occurs when $x =$ ___ and is the value ___

It may be helpful to make a graph of $f(x)$ when answering these questions.

Correct Answers:

- `0`
- `3`
- `5`
- `-15`
- `-1`
- `9`

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3. (3 points) The velocity function is $v(t) = -t^2 + 4t - 3$ for a particle moving along a line. Find the displacement (net distance covered) of the particle during the time interval $[-3,6]$.

displacement = ____________

Correct Answers:

- `-1 * (18 - -36)`

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4. (4 points) Evaluate the definite integral
$$\int_4^7 \frac{4x^2+2}{\sqrt{x}}dx$$

6. (4 points) Suppose that $\int_0^1 f(t)\,dt = 15$. Calculate each of the following.

A. $\int_0^{0.1} f(10t)\,dt =$ ______________

B. $\int_0^{0.5} f(1-2t)\,dt =$ ______________

C. $\int_{0.1}^{0.2} f(2-10t)\,dt =$ ______________

Solution:

SOLUTION

A. We substitute $w = 10t$, so that $w(0) = 0$, $w(0.1) = 1$, and $dw = 10\,dt$. Thus

$$\int_0^{0.1} f(10t)\,dt = \frac{1}{10}\int_0^1 f(w)\,dw = \frac{15}{10}.$$

B. We substitute $w = 1-2t$, so that $w(0) = 1$, $w(0.5) = 0$, and $dw = -2\,dt$. Thus

$$\int_0^{0.5} f(1-2t)\,dt = -\frac{1}{2}\int_1^0 f(w)\,dw = \frac{1}{2}\int_0^1 f(w)\,dw = \frac{15}{2}.$$

C. We substitute $w = 2-10t$, so that $w(0.1) = 1$, $w(0.2) = 0$, and $dw = -10\,dt$. Thus

$$\int_{0.1}^{0.2} f(2-10t)\,dt = -\frac{1}{10}\int_1^0 f(w)\,dw = \frac{15}{10}.$$

Correct Answers:

- `15/10`
- `15/2`
- `15/10`

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7. (4 points)

Evaluate

$$\int_{-2}^{2} (x+6)\sqrt{4-x^2}\,dx$$

by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.

$\int_{-2}^{2} (x+6)\sqrt{4-x^2}\,dx =$ ______________

Correct Answers:

- `2*6*pi`

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8. (4 points)

Evaluate the indefinite integral

$$\int e^x\sqrt{2+e^x}\,dx$$

Note: Any arbitrary constants used must be an upper-case "C".

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Correct Answers:

- `2/3*(2+exp(x))^(3/2)+C+c`

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9. (5 points)

Evaluate the indefinite integral

$$\int \frac{-5\sin(x)}{1+\cos^2(x)}\,dx$$

Note: Any arbitrary constants used must be an upper-case "C".

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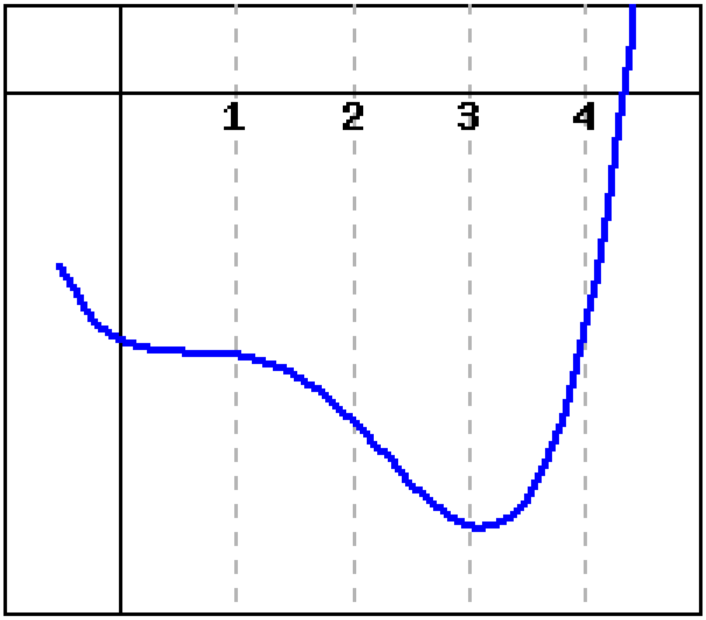
Correct Answers:

- `-1*-5*arctan(cos(x))+C+c`

10. (5 points) Consider the graph of f' given below.



Note that the graph shows f', not f.

Which is largest, $f(0)$ or $f(4)$?

- A. $f(0)$
- B. $f(4)$

List the following in increasing order:

A. $\frac{f(3)-f(1)}{2}$
B. $f(2)-f(1)$
C. $f(3)-f(2)$
(Enter the letter, **A**, **B** *or* **C***, in each of the following answer blanks.)*

$$\_\_\_ < \_\_\_ < \_\_\_$$

Solution:
SOLUTION
By the Fundamental Theorem,

$$f(4)-f(0)=\int_0^4 f'(x)dx.$$

Since $f'(x)$ is negative for $0 \le x \le 4$, this integral must be negative and so $f(4) < f(0)$.

For the second part, we first rewrite each of the quantities in terms of f', since we have the graph of f'. If A_1 and A_2 are the positive areas between the x-axis and the graph of f' for $1 \le x \le 2$ and $2 \le x \le 3$, respectively, then the Fundamental Theorem tells us

$$f(2)-f(1)=\int_1^2 f'(x)dx=-A_1, \quad \text{and} \quad f(3)-f(2)=\int_2^3 f'(x)dx=-A_2.$$

In addition, we know that $\frac{f(3)-f(1)}{2}$ is just

$$\frac{f(3)-f(1)}{2}=\frac{1}{2}\int_1^3 f'(x)dx=\frac{1}{2}\left(\int_1^2 f'(x)dx+\int_2^3 f'(x)dx\right)=\frac{1}{2}(-A_1-A_2).$$

Since $A_1 < A_2$ and this is just the the average of these, we have

$$f(3)-f(2) < \frac{f(3)-f(1)}{2} < f(2)-f(1).$$

Correct Answers: