$$f(x)=\sum_{n=0}^{\infty} c_n(x-a)^n \quad |x-a|<R$$

$$c_n=\frac{f^{(n)}(a)}{n!} \quad R=\lim_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right|$$

26.2 Power Series

(& where they converge)

Video: https://youtu.be/LKhvdkUdLtE

FOR ALL CALC2 STUDENTS!

blackpenredpen

Nov. 4th, 2019

$$\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n$$

$$|x|<1 \quad R=1 \quad I=(-1,1)$$

(Q1.) $\dfrac{x}{1-4x}$ at $a = 0$

(Q2.) $\dfrac{x^4}{9+x^2}$ at $a = 0$

(Q3.) $\dfrac{1+2x}{1-x}$ at $a = 0$

(Q4.) $\dfrac{1}{x^2-5x-6}$ at $a = 0$

(Q5.) $\dfrac{1}{(1-x)^2}$ at $a = 0$

(Q6.) $\ln(1+x)$ at $a = 0$

(Q7.) $\tan^{-1}x$ at $a = 0$

(Q8.) $\dfrac{1}{1-x}$ at $a = 3$

(Q9.) $\dfrac{1}{x^2}$ at $a = -2$

(Q10.) $\dfrac{1}{x^2+6x+10}$ at $a = -3$

(Q11.) e^x at $a = 0$

(Q12.) $\sin x$ at $a = 0$

(Q13.) $\cos x$ at $a = 0$

(Q14.) e^{3x} at $a = 2$

(Q15.) $\sin x$ at $a = \frac{\pi}{2}$

(Q16.) $\sin x$ at $a = -\pi$

(Q17.) $\sin^2 x$ at $a = 0$

(Q18.) $\cos x$ at a = pi/4

(Q19.) $\sinh x = \dfrac{e^x - e^{-x}}{2}$ at $a = 0$

(Q20.) $\cosh x = \dfrac{e^x + e^{-x}}{2}$ at $a = 0$

(Q21.) $\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$ at $a = 0$

(Q22.) $\ln(x)$ at $a = 2$

(Q23.) $2x^3 - 5x^2 + 1$ at $a = 1$

(Q24.) $(1+x)^r$ at $a = 0$

(Q25.) $\sqrt{4+x}$ at $a = 0$

(Q26.) $\sin^{-1}x$ at $a = 0$

(Q26.2) $x^{0.2}$ at $a = 26$

(Q1.) $\dfrac{x}{1-4x}$ at $a = 0$ $\qquad \sum_{n=0}^{\infty} 4^n x^{n+1}$, $R=\frac{1}{4}$, $I=\left(\frac{-1}{4},\frac{1}{4}\right)$

(Q2.) $\dfrac{x^4}{9+x^2}$ at $a = 0$ $\qquad \sum_{n=0}^{\infty}\frac{(-1)^n}{9^{n+1}}x^{2n+4}$, $R=3$, $I=\left(\frac{-1}{3},\frac{1}{3}\right)$

(Q3.) $\dfrac{1+2x}{1-x}$ at $a = 0$ $\qquad 1+\sum_{n=1}^{\infty}3x^n$, $R=1$, $I=(-1,1)$

(Q4.) $\dfrac{1}{x^2-5x-6}$ at $a = 0$ $\qquad \sum_{n=0}^{\infty}\left(\frac{-1+(-6)^{n+1}}{7\cdot 6^{n+1}}\right)x^n$, $R=1$, $I=(-1,1)$

(Q5.) $\dfrac{1}{(1-x)^2}$ at $a = 0$ $\qquad \sum_{n=1}^{\infty}nx^{n-1}$, $R=1$, $I=(-1,1)$

(Q6.) $\ln(1+x)$ at $a = 0$ $\qquad \sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}x^{n+1}$, $R=1$, $I=(-1,1]$

(Q7.) $\tan^{-1}x$ at $a = 0$ $\qquad \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1}$, $R=1$, $I=[-1,1]$

(Q8.) $\dfrac{1}{1-x}$ at $a = 3$ $\qquad \sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{2^{n+1}}(x-3)^n$, $R=2$, $I=(1,5)$

(Q9.) $\dfrac{1}{x^2}$ at $a = -2$ $\qquad \sum_{n=1}^{\infty}\frac{n}{2^{n+1}}(x+2)^{n-1}$ or $\sum_{n=0}^{\infty}\frac{n+1}{2^{n+2}}(x+2)^n$, $R=2$, $I=(-4,0)$

(Q10.) $\dfrac{1}{x^2+6x+10}$ at $a = -3$ $\qquad \sum_{n=0}^{\infty}(-1)^n(x+3)^{2n}$, $R=1$, $I=(-4,-2)$

(Q11.) e^x at $a = 0$ $\qquad \sum_{n=0}^{\infty}\frac{1}{n!}x^n$, $R=\infty$, $I=(-\infty,\infty)$

(Q12.) $\sin x$ at $a = 0$ $\qquad \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}$, $R=\infty$, $I=(-\infty,\infty)$

(Q13.) $\cos x$ at $a = 0$ $\qquad \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}$, $R=\infty$, $I=(-\infty,\infty)$

(Q14.) e^{3x} at $a = 2$ $\qquad \sum_{n=0}^{\infty}\frac{e^6 3^n}{n!}(x-2)^n$, $R=\infty$, $I=(-\infty,\infty)$

(Q15.) $\sin x$ at $a = \frac{\pi}{2}$ $\qquad \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(x-\frac{\pi}{2})^{2n}$, $R=\infty$, $I=(-\infty,\infty)$

(Q16.) $\sin x$ at $a = -\pi$ $\qquad \sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{(2n+1)!}(x+\pi)^{2n+1}$, $R=\infty$, $I=(-\infty,\infty)$

(Q17.) $\sin^2 x$ at $a = 0$ $\qquad \sum_{n=1}^{\infty}\frac{(-1)^{n+1}2^{2n-1}}{(2n)!}x^{2n}$, $R=\infty$, $I=(-\infty,\infty)$

(Q18.) $\cos x$ at $a = \frac{\pi}{4}$ $\qquad \sum_{n=1}^{\infty} \frac{\cos(\frac{\pi}{4}+\frac{\pi}{2}n)}{n!}(x-\frac{\pi}{4})^n$, $R=\infty$, $I=(-\infty,\infty)$

(Q19.) $\sinh x = \dfrac{e^x - e^{-x}}{2}$ at $a = 0$ $\qquad \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} x^{2n+1}$, $R=\infty$, $I=(-\infty,\infty)$

(Q20.) $\cosh x = \dfrac{e^x + e^{-x}}{2}$ at $a = 0$ $\qquad \sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n}$, $R=\infty$, $I=(-\infty,\infty)$

(Q21.) $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ at $a = 0$ $\qquad \sum_{n=0}^{\infty} \frac{1}{2n+1} x^{2n+1}$, $R=1$, $I=(-1,1)$

(Q22.) $\ln(x)$ at $a = 2$ $\qquad \ln(2)+\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)2^{n+1}}(x-2)^{n+1}$, $R=2$, $I=(0,4]$

(Q23.) $2x^3-5x^2+1$ at $a=1$ $\qquad -2-4(x-1)+(x-1)^2+2(x-1)^3$, $R=\infty$, $I=(-\infty,\infty)$

(Q24.) $(1+x)^r$ at $a = 0$ $\qquad \sum_{n=0}^{\infty} \binom{r}{n} x^n$, $R=1$, $I=-1,1$

For any real number r $\qquad \binom{r}{n} = \frac{r(r-1)(r-2)\cdots(r-n+1)}{n!}$, $\binom{r}{0}=1$

If $r \le -1$ then $I=(-1,1)$. If $-1<r<0$, then $I=(-1,1]$. If If $r \ge 0$, then $I=[-1,1]$

(Q25.) $\sqrt{4+x}$ at $a = 0$ $\qquad \sum_{n=0}^{\infty} \frac{1}{2^{2n-1}} \binom{\frac{1}{2}}{n} x^n$, $R=4$, $I=[-4,4]$

(Q26.) $\sin^{-1} x$ at $a = 0$ $\qquad \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \binom{\frac{-1}{2}}{n} x^{2n+1}$, $R=1$, $I=[-1,1]$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \binom{\frac{-1}{2}}{n} x^{2n+1} = \sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n+1)(2n)!!} x^{2n+1} = \sum_{n=0}^{\infty} \frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{2^n(2n+1)n!} x^{2n+1}$$

$$(2n)!!=2^n n!,\ (2n-1)!!=\frac{(2n)!}{2^n n!},\ 0!!=1,\ (-1)!!=1$$

(Q26.2) $x^{0.2}$ at $a = 26$ $\qquad \sum_{n=0}^{\infty} \frac{1}{26^{n-0.2}} \binom{\frac{1}{5}}{n} (x-26)^n$, $R=26$, $I=[0,52]$

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