COMP 2711 Discrete Mathematical Tools for Computer Science
2022 Fall Semester – Tutorial 10

Question 1: What is the largest n for which one can solve within one second a problem using an algorithm that requires $f(n)$ bit operations, where each bit operation is carried out in 10^{-9} seconds, with these functions $f(n)$?

(a) $\log n$

(b) n

(c) n^2

(d) 2^n

(e) $n!$

Question 2: Describe the worst-case time complexity, measured in terms of comparisons, of the following algorithm.

procedure *ternary search*(x : integer, $a_1, a_2, \ldots, a_n$: increasing integers)

$i := 1$

$j := n$

while $i < j - 1$

  $l := \lfloor (i + j)/3 \rfloor$

  $u := \lfloor 2(i + j)/3 \rfloor$

  if $x > a_u$ **then** $i := u + 1$

  else if $x > a_l$ **then**

    $i := l + 1$

    $j := u$

  else $j := l$

if $x = a_i$ **then** *location* $:= i$

else if $x = a_j$ **then** *location* $:= j$

else *location* $:= 0$

return *location*

Question 3: Prove that $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer.

Question 4: You are given an infinite supply of 3-cent and 7-cent postage stamps.

Note that some integer postage values can be formed using these stamps and some cannot. For example, it is impossible to form 8 cents of postage

value using these stamps but 27 cents can be formed, e.g., by using three 7-cent stamps and two 3-cent stamps.

Which integer postage values can and which cannot be formed using just 3-cent and 7-cent postage stamps? You must prove the correctness of your answer.

Hint: Use induction

Question 5: Consider the following statement.

Given a positive integer n. For any non-negative integer m, there exist integers q and r such that

$$m = qn + r, \quad 0 \leq r < n$$

(a) Prove the statement by weak induction.

(b) Prove the statement by strong induction.