

Math1014 Calculus II, Spring 2019

Midterm Exam Solution

Part I: MC Questions.

	1	2	3	4	5	6	7	8	9	10	11	12	13
Green	A	D	A	C	E	D	E	B	D	B	A	A	D
Orange	B	E	C	D	C	B	D	B	D	B	E	A	E
White	C	C	A	E	A	D	E	D	E	D	C	B	C
Yellow	D	E	B	A	C	E	C	D	E	D	B	B	E

Part II: Long Questions

14.

(a)

[5 pts]

Green-Yellow Version

The volume of the solid is

$$V = \int_0^4 \pi x^2 (16 - x^2) dx$$

White-Orange Version

The volume of the solid is

$$V = \int_0^5 \pi x^2 (25 - x^2) dx$$

(b)

[6 pts]

Green-Yellow Version

The volume of the solid is

$$\int_0^4 2\pi x^2 \sqrt{16 - x^2} dx$$

White-Orange Version

The volume of the solid is

$$\int_0^5 2\pi x^2 \sqrt{25 - x^2} dx$$

(c)

[6 pts]

Green-Yellow Version

The volume of the solid is

$$\int_0^4 [\pi(x\sqrt{16 - x^2} + 1)^2 - \pi \cdot 1^2] dx$$

White-Orange Version

The volume of the solid is

$$\int_0^5 [\pi(x\sqrt{25 - x^2} + 2)^2 - \pi \cdot 2^2] dx$$

15. [18 pts]

(a)

[9 pts]

Green-Yellow VersionThe depth of water h in the tank is given by

$$\begin{aligned}\int_0^h \pi \sqrt{y} dy &= \frac{1}{2} \cdot \int_0^{16} \pi \sqrt{y} dy \\ \left[\frac{2}{3} y^{3/2} \right]_0^h &= \frac{1}{2} \left[\frac{2}{3} y^{3/2} \right]_0^{16} \\ h^{\frac{3}{2}} &= 32, \quad h = 32^{2/3} \quad (\text{m})\end{aligned}$$

White-Orange VersionThe depth of water h in the tank is given by

$$\begin{aligned}\int_0^h \pi \sqrt{y} dy &= \frac{3}{4} \cdot \int_0^{16} \pi \sqrt{y} dy \\ \left[\frac{2}{3} y^{3/2} \right]_0^h &= \frac{3}{4} \left[\frac{2}{3} y^{3/2} \right]_0^{16} \\ h^{\frac{3}{2}} &= 48, \quad h = 48^{2/3} \quad (\text{m})\end{aligned}$$

(b)

[9 pts]

Green-Yellow Version

The work required is

$$\begin{aligned}W &= \int_0^{32^{2/3}} \pi \rho g \sqrt{y} (16 - y) dy \\ &= \pi \rho g \left[\frac{32}{3} y^{3/2} - \frac{2}{5} y^{5/2} \right]_0^{32^{2/3}} \\ &= \pi \rho g \left[\frac{32^2}{3} - \frac{2}{5} (32)^{5/3} \right] \quad (J)\end{aligned}$$

White-Orange Version

The work required is

$$\begin{aligned}W &= \int_0^{48^{2/3}} \pi \rho g \sqrt{y} (16 - y) dy \\ &= \pi \rho g \left[\frac{32}{3} y^{3/2} - \frac{2}{5} y^{5/2} \right]_0^{48^{2/3}} \\ &= \pi \rho g \left[\frac{32 \cdot 48}{3} - \frac{2}{5} (48)^{5/3} \right] \quad (J)\end{aligned}$$