Math1014 Calculus II
Brief Summary of Some Trigonometric Identities

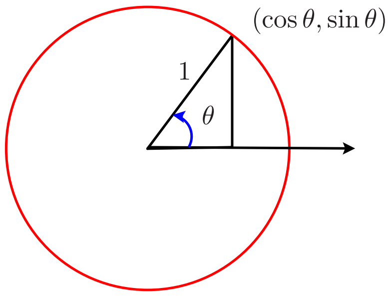
Here are some useful trigonometric identities for Math1014.

"Pythagoras Theorem"

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$\longleftrightarrow$



Angle Sum/Difference Formulas vs Product To Sum/Difference Formula

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$
$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$\longleftrightarrow$

$$\sin A\cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$$
$$\cos A\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$
$$\sin A\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

Double-Angle Formulas vs Half-Angle Formula

Taking $A = B$, the follow formulas follow:

$$\sin 2A = 2\sin A\cos B$$
$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

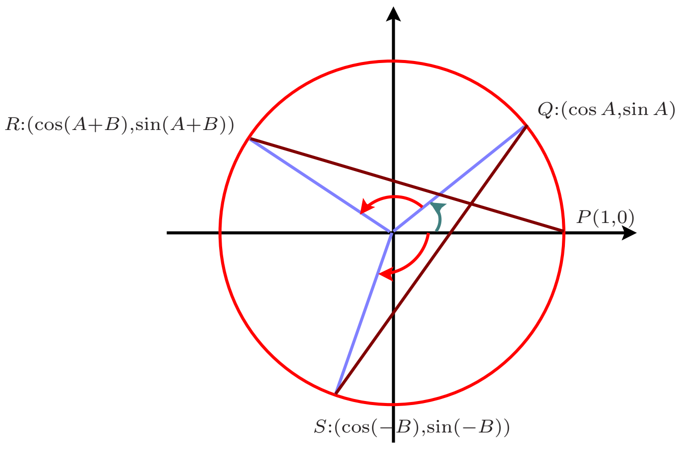
$\longleftrightarrow$

$$\cos^2 A = \frac{1+\cos 2A}{2}$$
$$\sin^2 A = \frac{1-\cos 2A}{2}$$

Exercise

1. Check that $\cos(A+B) = \cos A\cos B - \sin A\sin B$ is the same as the distance identity $PR^2 = QS^2$:

$$[\cos A - \cos(-B)]^2 + [\sin A - \sin(-B)]^2 = [\cos(A+B) - 1)^2 + (\sin(A+B) - 0]^2$$



The other angle sum/difference formulas can be derived from this one, either by replacing B by $-B$ or by using $\sin(\theta) = \cos(\frac{\pi}{2} - \theta)$, or just by differentiation.)