COMP 2711 Discrete Mathematical Tools for Computer Science
2022 Fall Semester – Tutorial 3

Question 1: Prove these statements by contrapositive.

(a) Suppose $x, y \in \mathbb{R}$. Show that if $y^3 + yx^2 \leq x^3 + xy^2$ then $y \leq x$.

(b) For any integer a, b, $a + b \geq 15$ implies $a \geq 8$ or $b \geq 8$.

(c) Assume A, B, C are three sets. Prove $A \cap B \subseteq C \implies (A - C) \cap B = \emptyset$

Question 2: Determine whether each of these functions is a bijection from $\mathbf{R}$ to $\mathbf{R}$.

(a) $f(x) = 2x + 1$

(b) $f(x) = x^2 + 1$

(c) $f(x) = x^3$

(d) $f(x) = (x + 1)/(x + 2)$

Question 3: Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

(a) the even integers

(b) the real numbers between 0 and $\frac{1}{2}$

(c) the integers that are multiples of 7

Question 4: Show that the set of irrational numbers is an uncountable set.

Question 5: Answer these questions.

(a) Is it true that every uncountable set has the same cardinality?

(b) Suppose you have two line segments X, Y of different lengths. the set of the points on each segment (both endings are included) is uncountable. we call these sets S_X, S_Y accordingly. Prove that the $|S_X| = |S_Y|$.