

MATH 2111: Tutorial 8

T1A&T1B QUAN Xueyang
T1C&T2A SHEN Yinan
T2B&T2C ZHANG Fa

Department of Mathematics, HKUST

- Null Space
- Column Space
- Kernel
- Range
- Basis

Example One

subspace

Determine whether the following is a subspace or not.

(1) $\{(1 + a, b, a + b) \mid a, b \in \mathbb{R}\},$

(2) $\{(1 + a, b, 1 + a + b) \mid a, b \in \mathbb{R}\},$

(3) $\{(a, 3b, a + 2b, 2b - a) \mid a, b \in \mathbb{R}\}$

Example Two

Null Space

Determine the null space of the following matrix:

$$A = \begin{pmatrix} 1 & 3 & 2 & 8 \\ 2 & 7 & 2 & 3 \end{pmatrix}, \quad (1)$$

if $\text{col}(A)$ is subspace of \mathbb{R}^k , what is k ?

Example Three

Range

What is the base of the range for the above given matrix?

$$A = \begin{pmatrix} 1 & 3 & 2 & 8 \\ 2 & 7 & 2 & 3 \end{pmatrix}, \quad (2)$$

Example Four

Basis

(1) Is $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right\}$ basis for \mathbb{R}^3 ?

(2) $S_1 = \{1, x, x^2\}$ is a basis of \mathbb{P}_2 . Is $S_2 = \{1, x + 1, (x + 1)^2\}$ also a basis of \mathbb{P}_2 ?

Example Five

- (1) Is $\left\{ \begin{pmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ linearly independent?
- (2) Suppose nonzero vectors v_1, v_2, \dots, v_n are orthogonal to each other, namely, $v_i^\top v_j = 0$ holds for any $i \neq j, i, j = 1, \dots, n$. Prove v_1, v_2, \dots, v_n are linearly independent.