MATH 2111 Matrix Algebra and Applications
Homework-8 : Due 11/17/2022 at 11:59pm HKT

---

1. (1 point) Determine if λ is an eigenvalue of the matrix A.

? 1. $A = \begin{bmatrix} -6 & -1 \\ 2 & -9 \end{bmatrix}$ and $\lambda = -8$

? 2. $A = \begin{bmatrix} 9 & -14 \\ 7 & -12 \end{bmatrix}$ and $\lambda = -5$

? 3. $A = \begin{bmatrix} 0 & -1 \\ 6 & -5 \end{bmatrix}$ and $\lambda = -8$

Correct Answers:

- `YES`
- `YES`
- `NO`

---

2. (1 point) Determine if v is an eigenvector of the matrix A.

? 1. $A = \begin{bmatrix} -7 & 9 \\ -12 & 14 \end{bmatrix}$, $v = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

? 2. $A = \begin{bmatrix} 15 & -14 \\ 21 & -20 \end{bmatrix}$, $v = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$

? 3. $A = \begin{bmatrix} 26 & -12 \\ 40 & -18 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$

Correct Answers:

- `YES`
- `NO`
- `NO`

---

3. (1 point) If $\vec{v}_1 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ are eigenvectors of a matrix A corresponding to the eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -5$, respectively,

then $A(\vec{v}_1 + \vec{v}_2) = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

and $A(3\vec{v}_1) = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} -19 \\ -5 \end{bmatrix}$
- $\begin{bmatrix} 3 \\ -15 \end{bmatrix}$

---

4. (1 point) Supppose A is an invertible $n \times n$ matrix and $\vec{v}$ is an eigenvector of A with associated eigenvalue 8. Convince yourself that $\vec{v}$ is an eigenvector of the following matrices, and find the associated eigenvalues.

(1) The matrix A^7 has an eigenvalue _____.

(2) The matrix A^{-1} has an eigenvalue _____.

(3) The matrix $A + 7I_n$ has an eigenvalue _____.

(4) The matrix $4A$ has an eigenvalue _____.

Correct Answers:

- `8^7`
- `1/8`
- `8+7`
- `4*8`

---

5. (1 point) Let $A = \begin{bmatrix} -3 & 7 \\ 3 & k \end{bmatrix}$.

For A to have 0 as an eigenvalue, k must be ___

Correct Answers:

- `-7`

---

6. (1 point) Find the eigenvalues of A, given that $A = \begin{bmatrix} 7 & 3 & 12 \\ 6 & 4 & 12 \\ -3 & -3 & -8 \end{bmatrix}$

and its eigenvectors are $v_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$ and $v_3 = \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$.

The corresponding eigenvalues are $\lambda_1 =$_____, $\lambda_2 =$_____, and $\lambda_2 =$_____.

Correct Answers:

- `-2`
- `1`
- `4`

---

7. (2 points) Find the eigenvalues $\lambda_1 < \lambda_2 < \lambda_3$ and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} -4 & 20 & 132 \\ 0 & 0 & -20 \\ 0 & 0 & 4 \end{bmatrix}.$$

The eigenvalue $\lambda_1 =$___ corresponds to the eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

The eigenvalue $\lambda_2 =$___ corresponds to the eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

The eigenvalue $\lambda_3 =$___ corresponds to the eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- `-4`
- $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$
- `0`
- $\begin{bmatrix} -5 \\ -1 \\ 0 \end{bmatrix}$
- `4`
- $\begin{bmatrix} 4 \\ -5 \\ 1 \end{bmatrix}$

---

8. (2 points) Given that the matrix A has eigenvalues $\lambda_1 = 3$ with corresponding eigenvector $\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\lambda_2 = 1$ with corresponding eigenvector $\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, find A.

$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} -3 & 2 \\ -12 & 7 \end{bmatrix}$

---

9. (2 points) The matrix

$$A = \begin{bmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ -5 & -5 & -5 \end{bmatrix}$$

has two real eigenvalues, one of multiplicity 1 and one of multiplicity 2. Find the eigenvalues and a basis for each eigenspace.

The eigenvalue λ_1 is ____ and a basis for its associated eigenspace is $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

The eigenvalue λ_2 is ____ and a basis for its associated eigenspace is $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

Correct Answers:

- `5`
- $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$
- `0`
- `[[1],[0],[-1]],` $\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$

---

10. (2 points) A, P and D are $n \times n$ matrices.

Check the true statements below:

- A. If A is invertible, then A is diagonalizable.
- B. If A is diagonalizable, then A has n distinct eigenvalues.
- C. If $AP = PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A.
- D. A is diagonalizable if A has n distinct eigenvectors.

Correct Answers:

- `C`

---

11. (3 points) Let

$$A = \begin{bmatrix} -3 & -1 & -4 \\ 0 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix}.$$

If possible, find an invertible matrix P so that $D = P^{-1}AP$ is a diagonal matrix. If it is not possible, enter the identity matrix for P and the matrix A for D. You must enter a number in every answer blank for the answer evaluator to work properly.

$P = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$.

$D = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$.

Is A diagonalizable over $\mathbb{R}$?

- choose
- diagonalizable
- not diagonalizable

Be sure you can explain why or why not.

Correct Answers:

- $\begin{bmatrix} -1 & -1 & -2 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$
- $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

- diagonalizable

12. (3 points) Let

$$M = \begin{bmatrix} 4 & 2 \\ -4 & -2 \end{bmatrix}.$$

Find formulas for the entries of M^n, where n is a positive integer.

$$M^n = \begin{bmatrix} \underline{\hspace{6em}} & \underline{\hspace{1em}} \\ \underline{\hspace{4em}} & \underline{\hspace{1em}} \end{bmatrix}$$

Correct Answers:

- $$\begin{bmatrix} 2*2^n & 1*2^n \\ (-2)*2^n & -2^n \end{bmatrix}$$