1. Consider the function $f(x) = \ln(x+1) - \sqrt{4-x} + 2\sin x$

   (a) Find the domain of $f(x)$.

   (b) Show that there is a root $c \in (0, \frac{\pi}{2})$ such that $f(c) = 0$

2. Car A and car B meet somewhere at time $t = 0$. After that, car A goes east with a time-dependent velocity: it covers $30m$ during the period $t \in [0, 4\sec]$, and the instantaneous velocity at $t = 4\sec$ is $15m/\sec$; Car B goes south with a constant velocity $10m/\sec$. Calculate the rate of change of the distance between the two cars at $t = 4\sec$?

3. Evaluate the following limits if they exist.

   (a)
   $$\lim_{x \to -\infty} x + \sqrt{x^2 + 5x + 6}$$

   (b)
   $$\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$$

4. Let $f(x) = \frac{x^2 - 4}{x^3}$

   (a) Find the domain of $f(x)$.

   (b) Find all the local maxima and local minima (relative maxima and relatice minima) of $f(x)$. Present your reasoning for full credit.

5. A function $y = g(x)$ is implicitly defined through the relation
   $$x^2 y^3 + e^{2xy} + \tan 2x = 2$$
   in the domain $(-\frac{\pi}{4}, 0) \cup (0, \frac{\pi}{4})$

   (a) Find the derivative $\frac{dy}{dx}$ in terms of x and y.

   (b) Find the equation of the line tangent to the graph of $y = g(x)$ at the point where this graph crosses the x-axis.

6. Consider the functino $f(x) = \ln(1 + \frac{1}{4}e^x - \frac{1}{8}e^{2x})$.

   (a) Find the domain of $f(x)$.

   (b) Find the x-intercept(s) and y-intercept(s) of the graph of $y = f(x)$ (if any).

   (c) Find the vertical and horizontal asymptotes of the graph of $y = f(x)$ (if any).

   (d) Locate the local maximum and minimum (relative maximum and minimum) of f (if any).

(e) Sketch the graph of $y = f(x)$.

7. (a) Evaluate

$$\lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{k^7}{n^8} + \frac{k^3}{n^4} \right)$$

if it exists. (Definite integrals may be used.)

(b) Find the dericative $g'(x)$ of

$$g(x) = \int_0^{\sin^2 x} e^{s^2} \, ds.$$

8. (a) Find the derivative of $f(x) = xe^x$.

(b) Use the result found in (a) to derive the indefinite integral $\int xe^x \, dx$.

(c) Use the result found in (b) to derive the indefinite integral $\int_0^2 xe^x \, dx$.

9. Evaluate the following definite integrals:

(a)

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin x} \, dx,$$

(b)

$$\int_{-3}^{3} (9x^3 + 6)\sqrt{9 - x^2} \, dx.$$

10. Evaluate the definite integral

$$\int_1^2 \frac{1}{x^2 + 5x + 6} \, dx$$

using a technique called the partial fraction decomposition, which can be illustrated by the following example

$$\frac{4}{x^2 + 2x - 3} = \frac{4}{(x+3)(x-1)} = \frac{1}{x-1} - \frac{1}{x+3}$$

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