Newton's Laws of Motion II

PHYS1112

Lecture 3

~~$c = 299{,}792{,}458\ m/s$~~

"static"

frame of reference :

$v_{t/A}$

c

A

c

"special relativity"

$\boxed{E = mc^2}$

$$\boxed{\vec{v}_{b/A} = \vec{v}_{b/t} + \vec{v}_{t/A}}$$

Galilean transformation

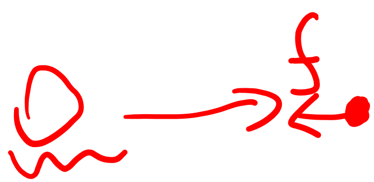
Intended Learning Outcomes

- After this lecture you will learn:
  1. to describe friction in a macroscopic picture and solve problems involving it.
  2. to contrast fluid resistance to friction.
  3. uniform circular motion and centripetal acceleration
  4. to solve problems involving uniform circular motion

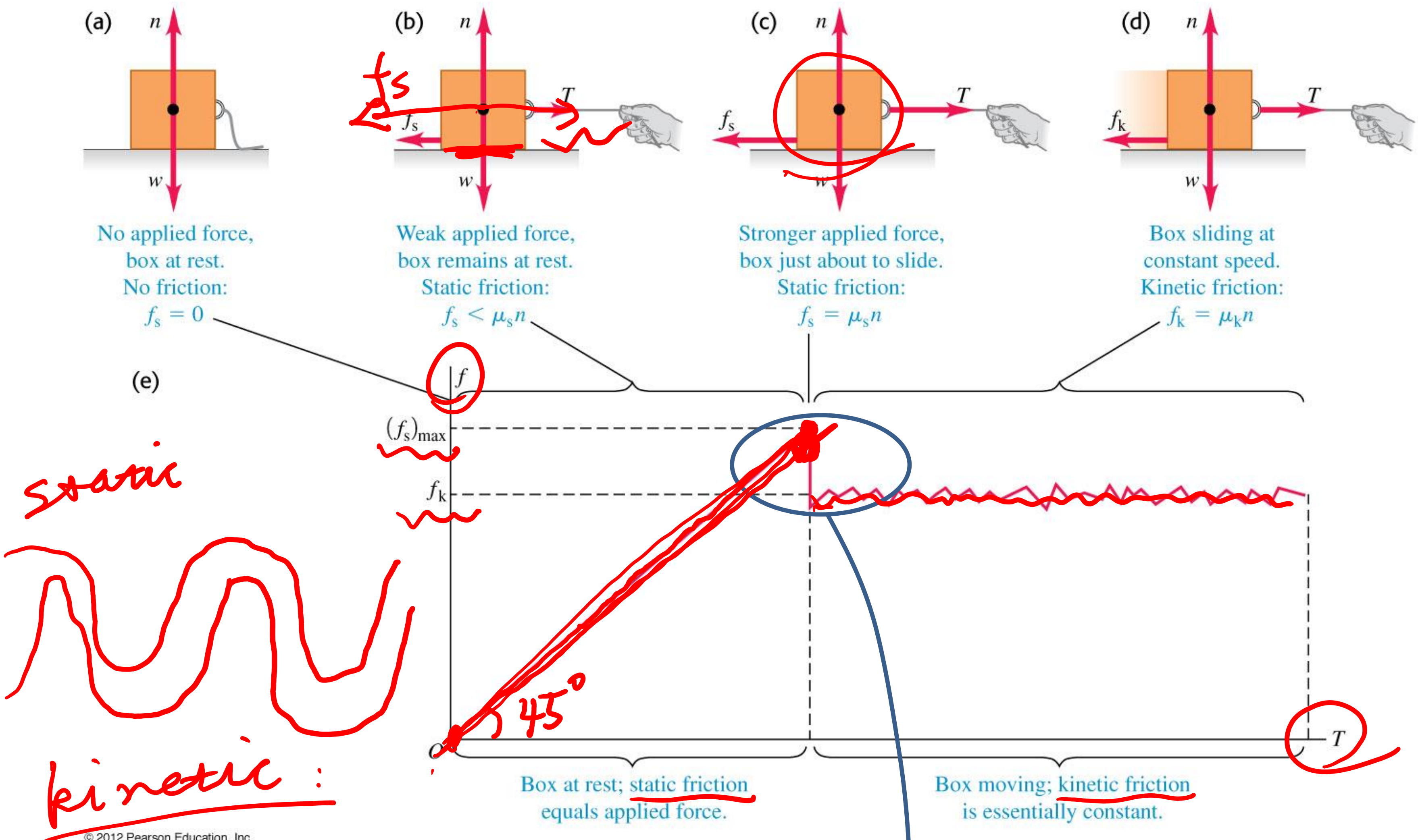
Frictional Forces

- Microscopic: due to interactions between molecules of surfaces in contact ? 10^{-9} m "nano"
- Macroscopic (phenomenological): ignore microscopic level and look at the outcome only "empirical"

"model"

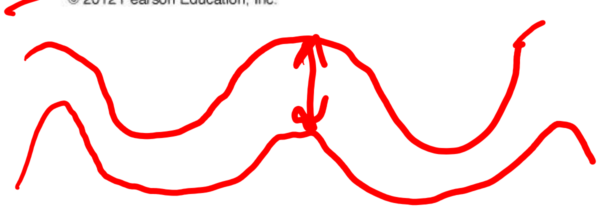


Can be classified into two types: *static* friction, and *dynamic* (or *kinetic*) friction



static

kinetic :

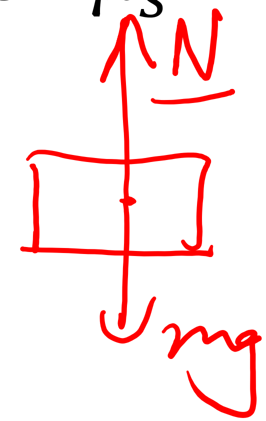


Interpretation: easier to keep the block moving than to start it moving

Note

- the coefficients of static and kinetic friction μ_s and μ_k depends on the two surfaces in contact

$f_s^{max} = \mu_s N$ $f_k = \mu_k N$

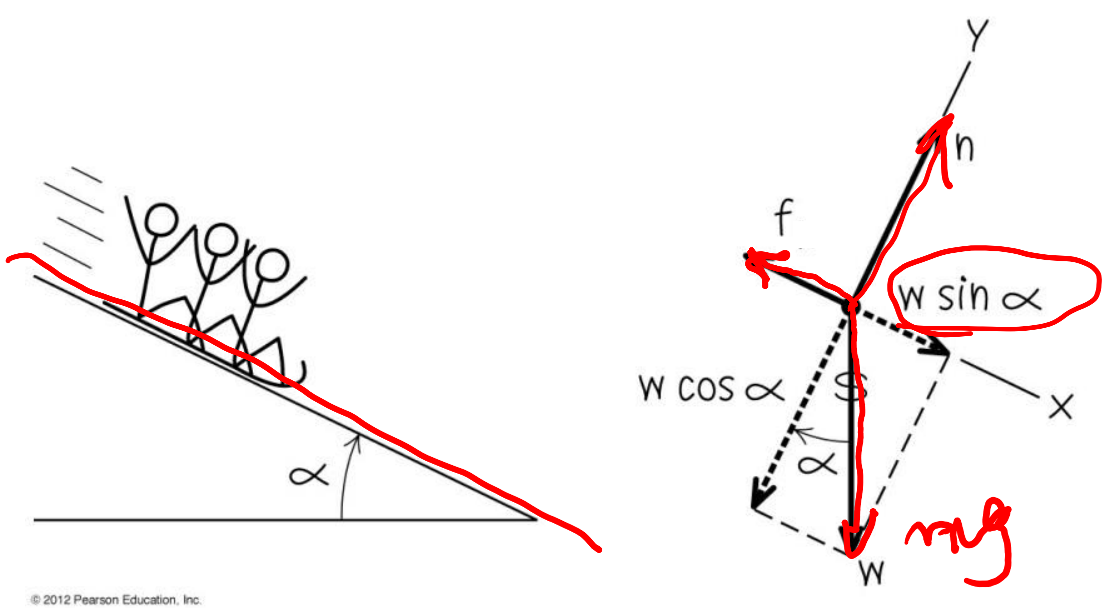


- friction always along contact surface and therefore $\perp$ to normal force
- static friction can be less than the maximum value

Example: A block (or toboggan) sliding down an inclined plane

"level"

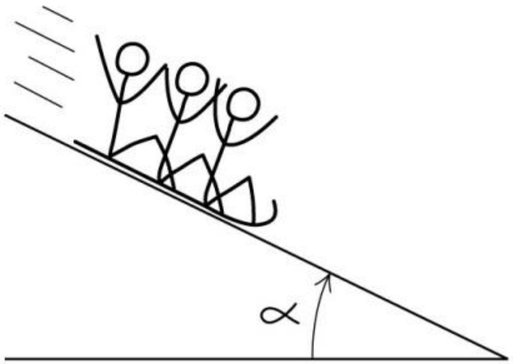
(a) The situation   (b) Free-body diagram for toboggan



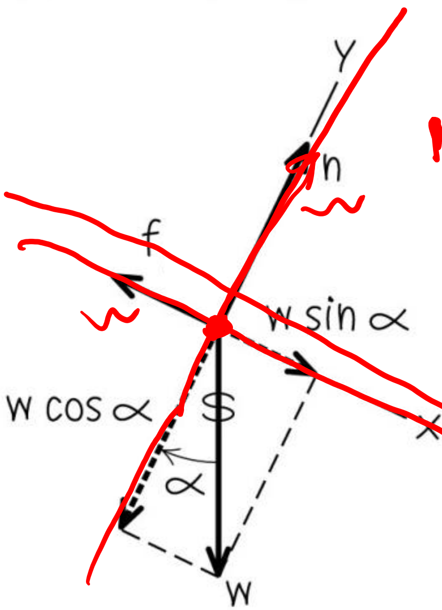
Given: μ_s and μ_k, angle α increases from zero

Before the block starts to slide, friction is (static / kinetic), and equals w sin α

(a) The situation



(b) Free-body diagram for toboggan



If at a particular α, the block just begins to slide, right before the block begins to slide, friction is (static / kinetic):

Resolving force ⊥ the plane:

$$y: \sum F_y = n - mg\cos\alpha = 0$$

$mg\cos\alpha = n$ ①

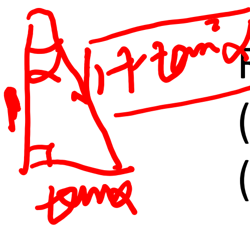
along the plane:

$mg\sin\alpha = \mu_s n$ ②

$$x: \sum F_x = mg\sin\alpha - \underbrace{\mu_s n}_{f} = 0 \implies \alpha = \tan^{-1}\mu_s$$

$\boxed{\tan\alpha = \mu_s}$ ③

$\alpha = \arctan\mu_s$



Right after the block begins to slide, friction is (static / kinetic) and the block slides with (constant speed / an acceleration):

$$x: \sum F_x = mg\sin\alpha - \mu_k n = ma$$

$$\implies a = g(\sin\alpha - \mu_k\cos\alpha) = g\frac{\mu_s - \mu_k}{\sqrt{1+\mu_s^2}}$$

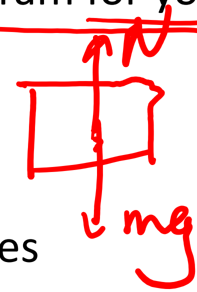
$\cancel{m}a = \cancel{m}g\sin\alpha - \mu_k \cancel{m}g\cos\alpha$

$a = g\cos\alpha(\tan\alpha - \mu_k) = g\frac{1}{\sqrt{1+\tan^2\alpha}}(\tan\alpha - \mu_k)$

Q5.10

You are walking on a level floor. You are getting good traction, so the soles of your shoes don't slip on the floor.

Which of the following forces *should* be included in a free-body diagram for your body?



A. the force of ~~kinetic~~ friction that the floor exerts on your shoes

B. the force of static friction that the floor exerts on your shoes

C. the force of kinetic friction that your shoes exert on the floor

D. the force of static friction that your shoes exert on the floor

E. more than one of these

A5.10

You are walking on a level floor. You are getting good traction, so the soles of your shoes don’t slip on the floor.

Which of the following forces *should* be included in a free-body diagram for your body?

A. the force of kinetic friction that the floor exerts on your shoes

B. the force of static friction that the floor exerts on your shoes

C. the force of kinetic friction that your shoes exert on the floor

D. the force of static friction that your shoes exert on the floor

E. more than one of these

Fluid Resistance

⚠ Fluid resistance depends on speed
At high speed (or non-viscous fluid),
$f \propto v^2$, or $f = Dv^2$
e.g. air resistance

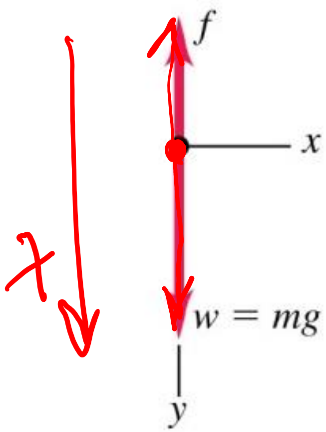
$$\sum F_y = mg - Dv^2 = ma = 0$$

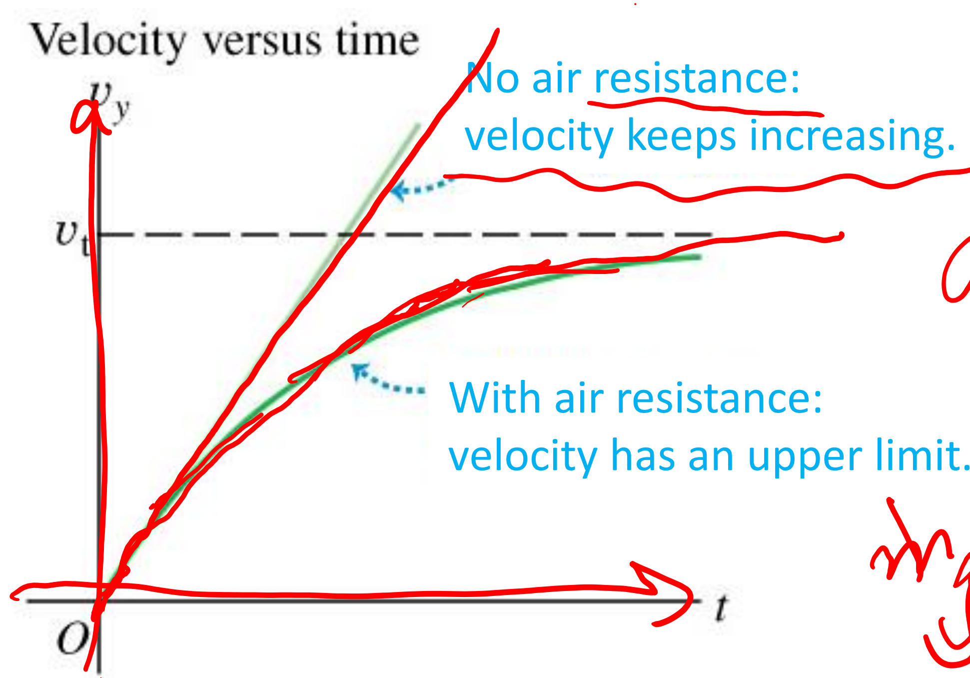
Note: $v = \sqrt{\frac{mg}{D}}$

1) a decreases as v increases

2) there exists a terminal speed

$$v_t = \sqrt{mg/D} \text{ when } a = 0$$

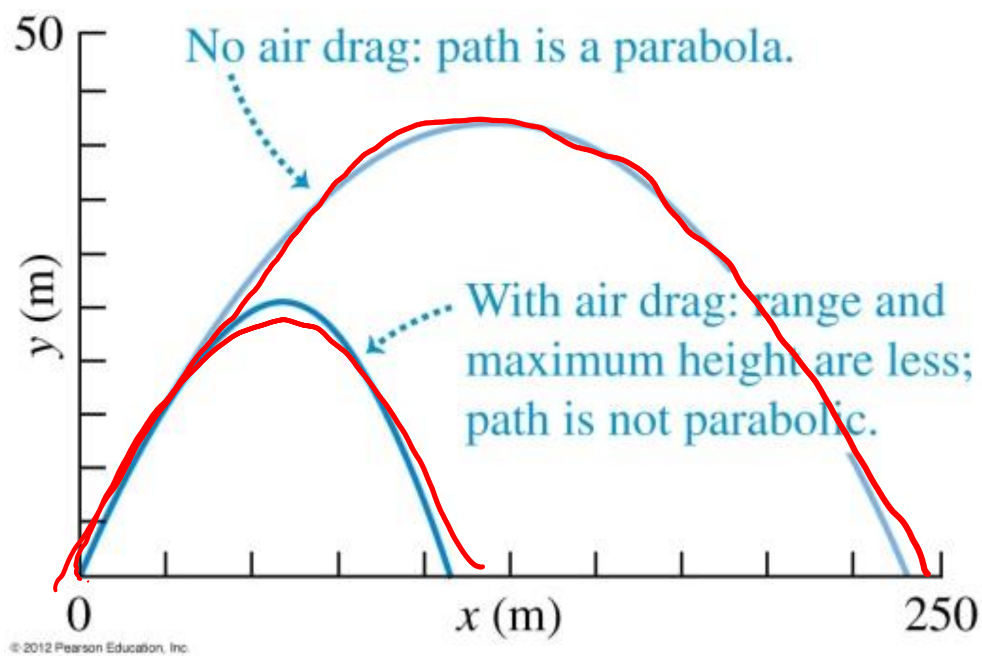




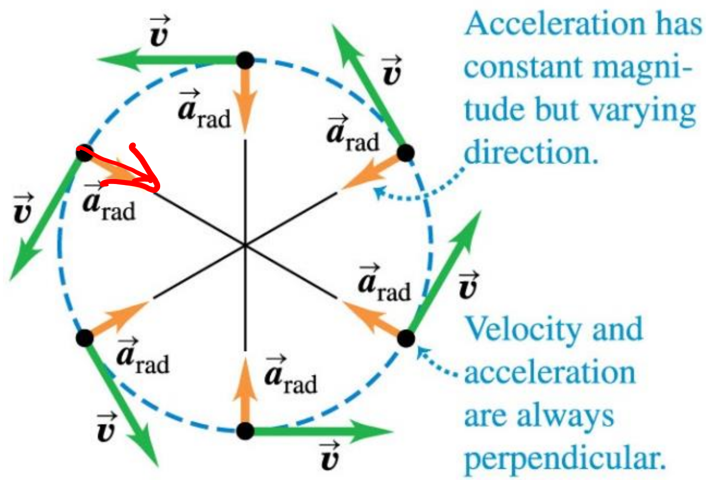
$$a = \frac{d\vec{v}}{dt}$$

$$\cancel{m}g = \cancel{m}a$$

- heavy bodies fall faster
  ∵ larger m
- a sheet of paper falls faster if crumpled into a ball
  ∵ D smaller
- with air resistance, a projectile is no longer a parabola



Dynamics of Uniform Circular Motion

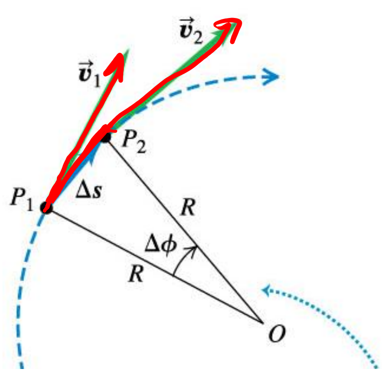


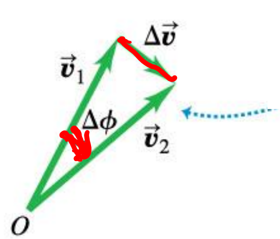
Speed (NOT velocity) constant

$\Rightarrow a_{||} = 0$

$\Rightarrow \vec{a}$ along radial direction (inward / outward)

called centripetal acceleration





same $\Delta\phi$ $\therefore$ $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ are $\perp$ to OP_1 and OP_2

$$\boxed{\Delta\phi = \frac{\Delta s}{R} = \frac{|\Delta\vec{\mathbf{v}}|}{v}}$$

$$a_{\text{rad}} = \frac{|\Delta\vec{\mathbf{v}}|}{\Delta t} = \frac{v}{R}\frac{\Delta s}{\Delta t}$$

$\Delta\phi \to 0$

$$\frac{ds}{dt} = v$$

$$\therefore \boxed{a_{\text{rad}} = \frac{v^2}{R}}$$

$$a_{rad} = \frac{|\Delta \vec{v}|}{\Delta t}$$

$$= \frac{v\,\Delta\phi}{\Delta t} = \frac{v\,\Delta s}{\Delta t\,R}$$

$$\frac{\Delta s}{\Delta t} = v$$

Q3.11

You drive a race car around a circular track of radius 100 m at a constant speed of 100 km/h. If you then drive the same car around a different circular track of radius 200 m at a constant speed of 200 km/h, your acceleration will be

A. 8 times greater.

B. 4 times greater.

C. twice as great.

D. the same.

E. half as great.

Q3.11

R

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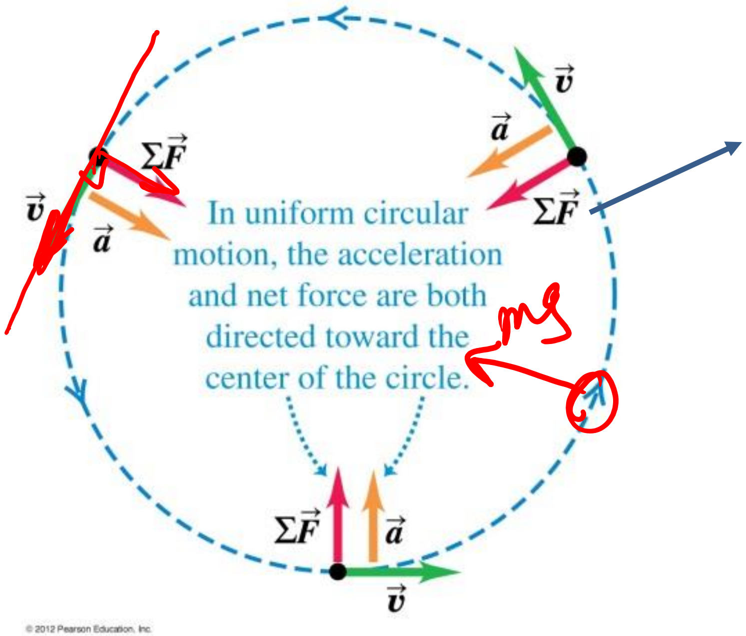
C. twice as great.

D. the same.

E. half as great.

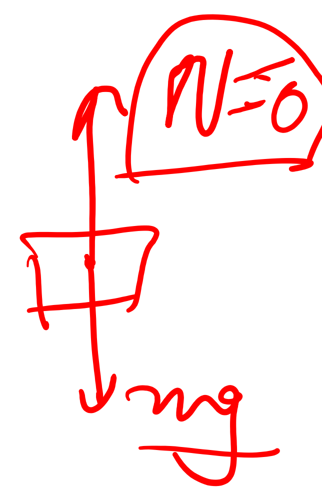
$$a_{rad} = \frac{v^2}{R} \quad \frac{2 \times \cancel{2}}{\cancel{2}}$$



force providing the centripetal acceleration, sometimes called the "centripetal force".

$$F_{net} = ma = \boxed{m\frac{v^2}{R}}$$

$$\boxed{mg = ma}$$



Demonstration: vertical circular motion

Q5.13

A pendulum of length L with a bob of mass m swings back and forth. At the low point of its motion (point Q), the tension in the string is $(3/2)mg$. What is the speed of the bob at this point?

$$T - mg = m\frac{v^2}{R}$$

$$\frac{1}{2}mg = m\frac{v^2}{R} \qquad v = \sqrt{\frac{gR}{2}}$$

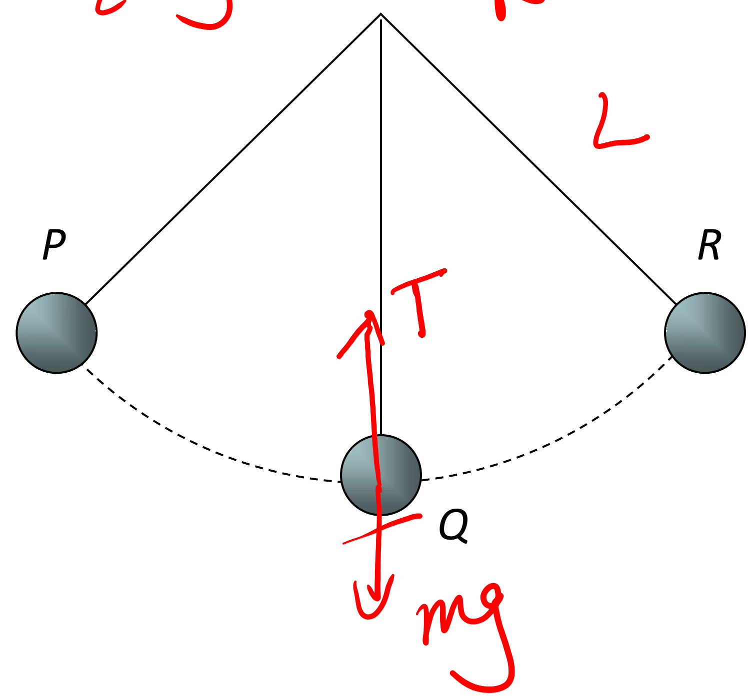
A. $2\sqrt{gL}$

B. $\sqrt{2gL}$

C. $\sqrt{gL}$

D. $\sqrt{\dfrac{gL}{2}}$

E. $\dfrac{\sqrt{gL}}{2}$



A5.13

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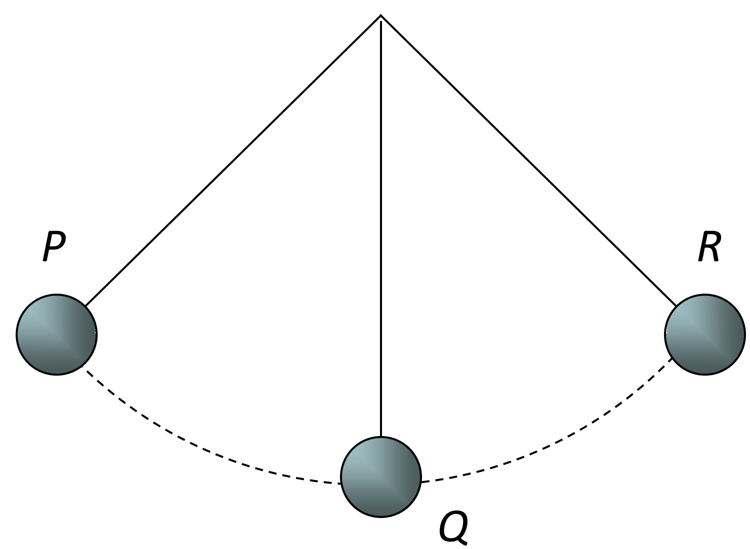
B. $\sqrt{2gL}$

C. $\sqrt{gL}$

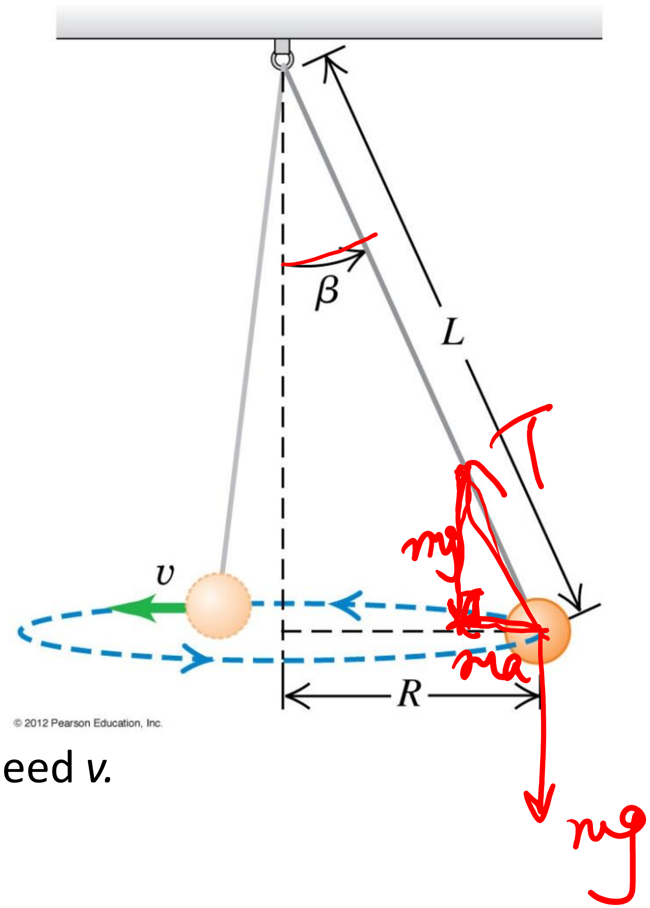
D. $\sqrt{\frac{gL}{2}}$

E. $\frac{\sqrt{gL}}{2}$



Q5.12

A pendulum bob of mass *m* is attached to the ceiling by a thin wire of length *L*. The bob moves at constant speed in a horizontal circle of radius *R*, with the wire making a constant angle β with the vertical. The tension in the wire



A. is greater than *mg.*

B. is equal to *mg.*

C. is less than *mg.*

D. is any of the above, depending on the bob's speed *v.*

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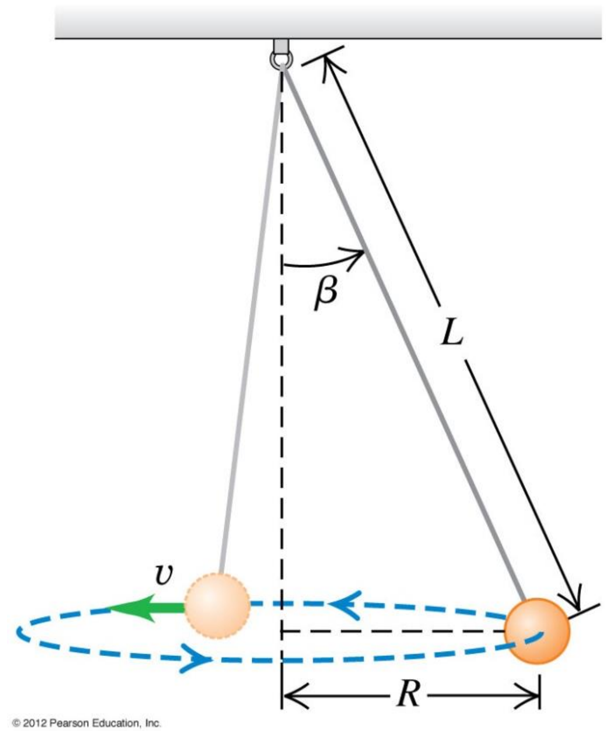


A. is greater than *mg.*

B. is equal to *mg.*

C. is less than *mg.*

D. is any of the above, depending on the bob's speed *v.*

Example: A conical pendulum

horizontal uniform circular motion

$$\sum F_x = F\sin\beta = ma$$
$$\sum F_y = F\cos\beta - mg = 0$$

$$\Rightarrow \quad a = g\tan\beta = \frac{v^2}{R}$$

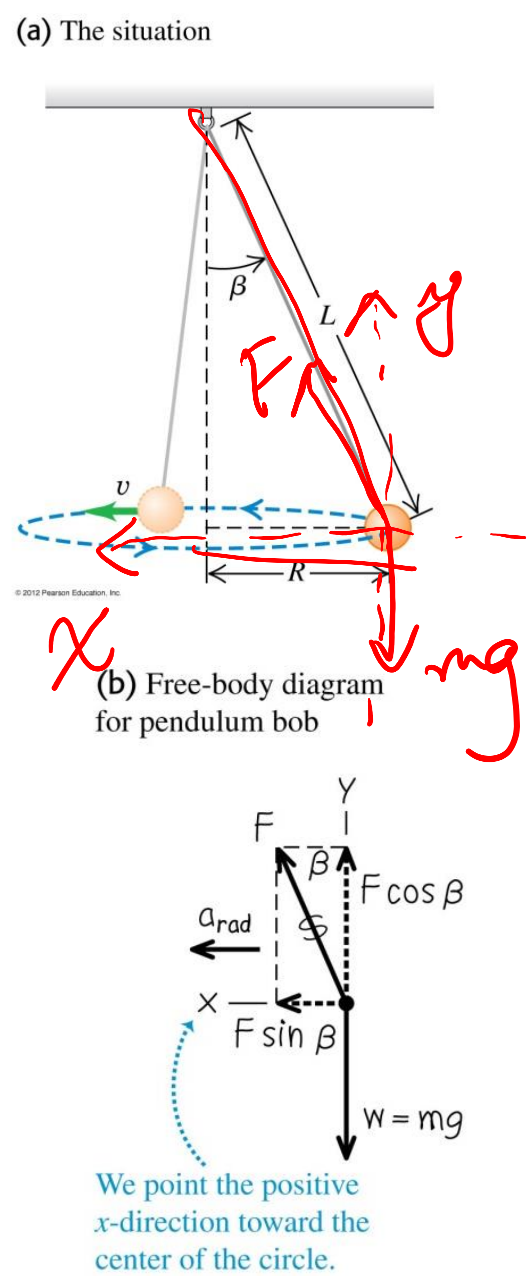
$v^2 = aR$

Period of the pendulum:

$$T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{R}{a}} = 2\pi\sqrt{\frac{L\cos\beta}{g}}$$

c.f. a planar pendulum

$$T = 2\pi\frac{R}{\sqrt{aR}} = 2\pi\sqrt{\frac{R}{a}} = 2\pi\sqrt{\frac{L\sin\beta}{g\tan\beta}}$$



Observation: Why banked curves in a racing track help?

On a flat curve

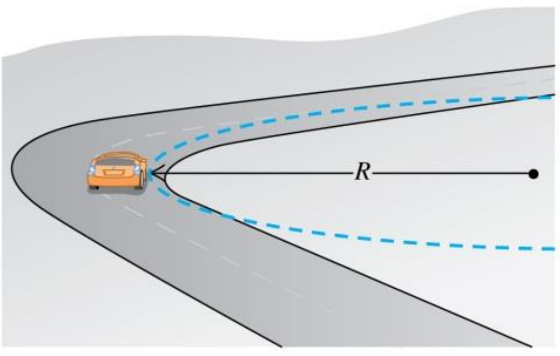
Assume no skidding, what supplies the centripetal force? (Static / Kinetic) friction!
Max. speed without skidding:

$$\underline{f = f_{max} = m\frac{v_{max}^2}{R}} \Rightarrow \boxed{v_{max} = \sqrt{\mu_s g R}}$$

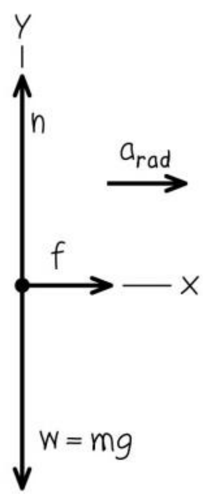
$$\mu_s n = \mu_s mg$$

$$\mu_s \cancel{m} g = \cancel{m}\frac{v^2}{R}$$

(a) Car rounding flat curve



(b) Free-body diagram for car



If banked at angle β

What supplies the centripetal force? n and f!

$$\sum F_x = n \sin\beta + f \cos\beta = mv^2/R$$

$$\sum F_y = n \cos\beta - f \sin\beta - mg = 0$$

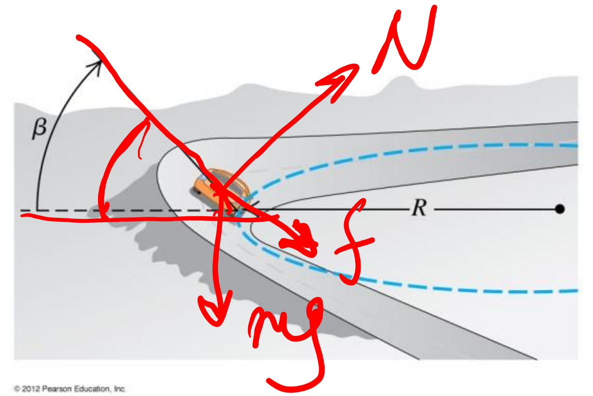
$$\Rightarrow f = \frac{m \cos\beta}{R}\left(v^2 - gR \tan\beta\right),$$

$$n = \frac{m \cos\beta}{R}\left(v^2 \tan\beta + gR\right)$$

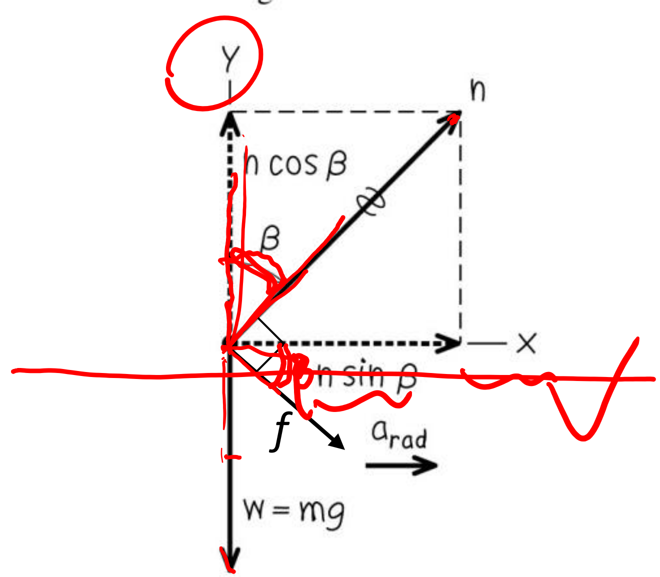
$$f \le \mu_s n \Rightarrow v \le v_{max} = \sqrt{\frac{\tan\beta + \mu_s}{1 - \mu_s \tan\beta} gR}$$

$$\ge \sqrt{\mu_s g R}$$

(a) Car rounding banked curve



(b) Free-body diagram for car



$$x: \quad n\sin\beta + f\cos\beta = m\frac{v^2}{R} \quad ①$$
$$y: \quad n\cos\beta - f\sin\beta = mg \quad ②$$

$f = 0 \Rightarrow \quad v = \boxed{\sqrt{gR\tan\beta}}$

$①\times\cos\beta - ②\times\sin\beta$

$$③ \quad f = m\frac{v^2}{R}\cos\beta - mg\sin\beta = \frac{m\cos\beta}{R}(v^2 - gR\tan\beta)$$

$①\times\sin\beta + ②\times\cos\beta$

$$n = m\frac{v^2}{R}\sin\beta + mg\cos\beta = \frac{m\cos\beta}{R}(v^2\tan\beta + gR)$$

$f \le f_{max} = \mu_s n$:

$$v^2 - gR\tan\beta \le \mu_s(v^2\tan\beta + gR)$$
$$(1 - \mu_s\tan\beta)v^2 \le \mu_s gR + gR\tan\beta$$
$$v \le \sqrt{\frac{\mu_s + \tan\beta}{1 - \mu_s\tan\beta}gR} \ge \sqrt{\mu_s gR}$$

$\beta = 0$

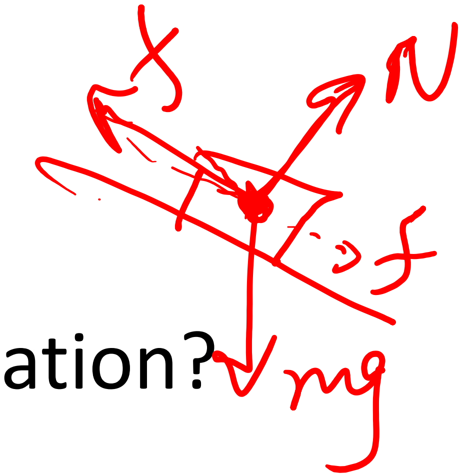
if $f = 0$

Challenging Question

- What happen to the friction f if

$$\boxed{v < \sqrt{gR\tan\beta}\ ?}$$

$v = 0$



- How would you interpret this situation?