MATH 2111: Tutorial 2 Echelon Form and Linear Combinations

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Review

- Existence and Uniqueness Theorem (important!!!!!)
- Geometric visualization of linear equation
- Vector equation (sum & scalar multiple & some other algebraic properties) $--->$ Linear combinations
- The subset spanned by vector $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_p\}$.

Suppose $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & 2 & a & 3 \\ 1 & 1 & b & 4 \end{pmatrix}$ is an augumented matrix. Determine a and b such that the linear system

(1) is inconsistent,

(2) has a unique solution,

(3) has infinite many solutions.

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(2) has a unique solution,

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Plot the following linear systems:

(1) Two variables: $\begin{cases} x + y = 0, \\ 2x - 6y = 2. \end{cases}$

(2) Two variables: $\begin{cases} x + y = 0, \\ 2x - 2y = 2. \end{cases}$

(3) Three variables: $\begin{cases} x + y = 0, \\ y + z = 2. \end{cases}$

Let $\boldsymbol{u} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\boldsymbol{v} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$.

(1) Write down the subset of $\mathbb{R}^3$ spanned by $\boldsymbol{u}$ and $\boldsymbol{v}$.

(2) Determine whether vector $\boldsymbol{w} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ could be spanned by $\boldsymbol{u}$ and $\boldsymbol{v}$.

Let $\boldsymbol{u} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$, $\boldsymbol{v} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$.

(1) Write down the subset of $\mathbb{R}^3$ spanned by $\boldsymbol{u}$ and $\boldsymbol{v}$.

(2) Determine h such that vector $\boldsymbol{w} = \begin{pmatrix} 2 \\ 2 \\ h \end{pmatrix}$ could be spanned by $\boldsymbol{u}$ and $\boldsymbol{v}$.

Let $\boldsymbol{u} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$, $\boldsymbol{v} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$, $\boldsymbol{w} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$.

(1) Write down the subset of $\mathbb{R}^3$ spanned by $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$.

(2) Determine h such that vector $\boldsymbol{x} = \begin{pmatrix} 1 \\ 3 \\ h \end{pmatrix}$ could be spanned by $\boldsymbol{u}$ and $\boldsymbol{v}$.

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