COMP 2711 Discrete Math Tools for Computer Science

2022 Fall Semester - Homework 6

Review: We learned about several special types of graphs: complete graphs K_n, cycles C_n, bipartite graphs (denoted as $G_{(b)}$ here), and complete bipartite graphs $K_{m,n}$. Recall the definitions:

- K_n For $V = \{v_1, v_2, \cdots, v_n\}$ $(n \geq 1)$, there is exactly one edge between every pair of vertices in V. K_1 is a single vertex and K_2 is two vertices connected by an edge.
- C_n For $V = \{v_1, v_2, \cdots, v_n\}$ $(n \geq 3)$, there is exactly one edge between v_i and v_{i+1} for all $1 \leq i \leq n$, plus exactly one edge from v_n to v_1.
- $G_{(b)}$ For $V = \{v_1, v_2, \cdots, v_n\}$ $(n \geq 2)$, it can be partitioned into two disjoint subsets V_1 and V_2 such that $(V_1 \cap V_2 = \varnothing) \wedge (V_1 \cup V_2 = V)$. Every edge connects u_i in V_1 and v_j in V_2.
- $K_{m,n}$ For every vertex u_i in $U = \{u_1, u_2, \cdots, u_m\}$, and v_j in $V = \{v_1, v_2, \cdots, v_n\}$ $(m \geq 1, n \geq 1)$, there is exactly one edge connecting u_i and v_j. There are no edges between two vertices in U, and no edges between two vertices in V.

Question 1:

(a) Draw a complete graph K_7.

(b) Draw a bipartite graph $G_{(b)}$ which is not a complete bipartite graph.

(c) Represent your bipartite graph in (b) by adjacency matrix. Please label the vertices in (b) and declare the order in (c).

(d) Can a complete graph K_n be bipartite? Explain what conditions n must satisfy if it is possible.

(e) Can a cycle C_n be bipartite? Explain what conditions n must satisfy if it is possible.

(f) Can a complete bipartite graph $K_{m,n}$ have an Euler path but not an Euler circuit? Explain what conditions m and n must satisfy if it is possible.

Question 2: Show that a directed multigraph (graphs that may have multiple edges connecting the same vertices) having no isolated vertices has an Euler circuit if and only if the graph is weakly connected and the in-degree and out-degree of each vertex are equal.