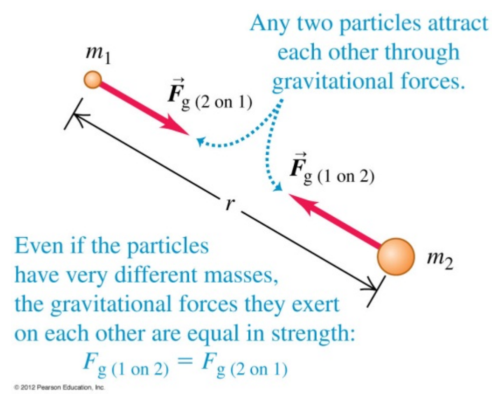
GRAVITATION I

Intended Learning Outcomes – after this lecture you will learn:
1. Newton's law of gravitation, a central inverse square law.
2. Cavendish experiment to measure gravitational constant.
3. gravitational force and potential energy.
4. escape speed from a planet.
5. satellites in circular obits.

Textbook Reference: Ch 13.1 – 13.4

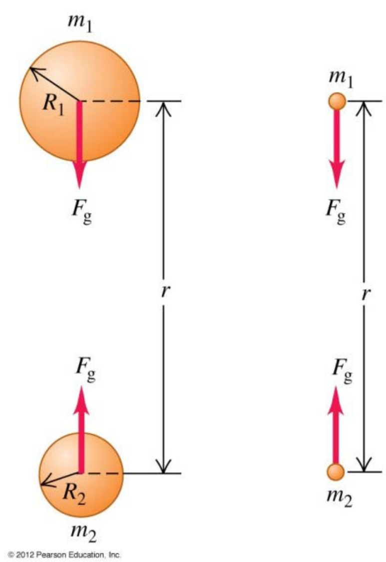
Newton's Law of Gravitation



$$F_g = \frac{Gm_1m_2}{r^2} \qquad \textbf{inverse square law}$$

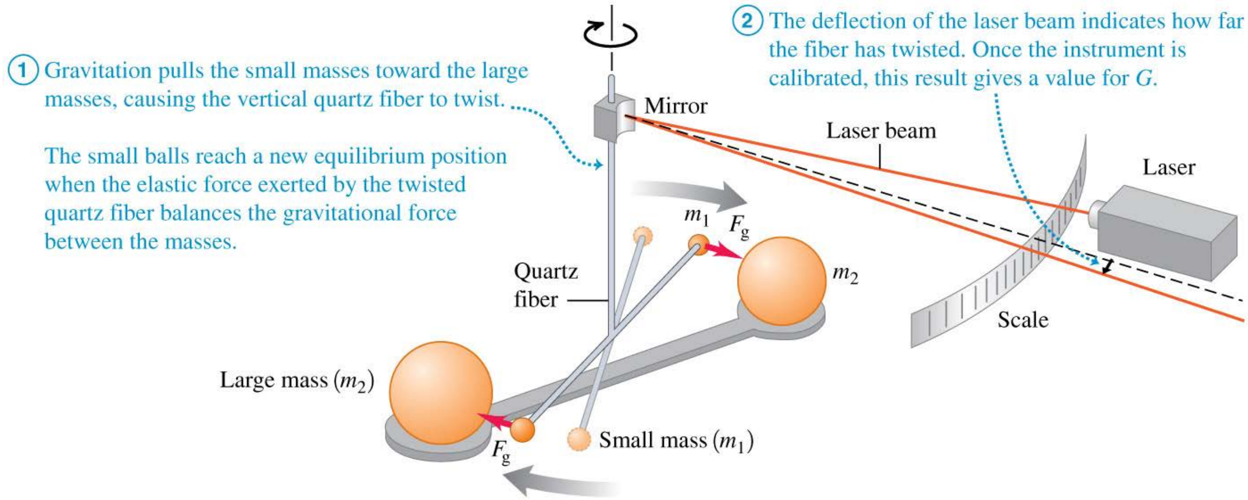
G: **gravitational constant** $6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$

⚠ Gravitational attraction between two point masses is always along the line joining them (called **central force**), and forms an action-reaction pair



When outside a spherically symmetric body (i.e., density $\rho(r)$ depends on radial distance r only, not on direction), the gravitational effect is the same as if all of the mass were concentrated at its center

Cavendish Experiment – to measure G (or to "weight the earth")



Question
Saturn is about 100 times the mass of the earth and about 10 times farther from the sun than the earth. Compared to the acceleration of the earth caused by the sun's gravitational pull, the acceleration of Saturn due to the sun's gravitation is (100 times greater / 10 times greater / the same / $\frac{1}{10}$ as great / $\frac{1}{100}$ as great).
Answer: see inverted text on P. 427 of textbook

Four fundamental forces of nature:

Gravitation force	Hold planets together	Range	∞
Electromagnetic force	Hold molecules together		∞
Strong force	Hold nucleons (protons and neutrons in an atomic nucleus) together		10^{-15} m
Weak force	Beta decay of nuclei		10^{-18} m

Weight – defined as the total gravitational forces exert on the body by all other bodies in the universe
On earth's surface, gravitational attraction by the earth dominates over others

$$W = F_g = \frac{G m_E m}{R_E^2} = mg \quad \Rightarrow \quad g = \frac{G m_E}{R_E^2}$$

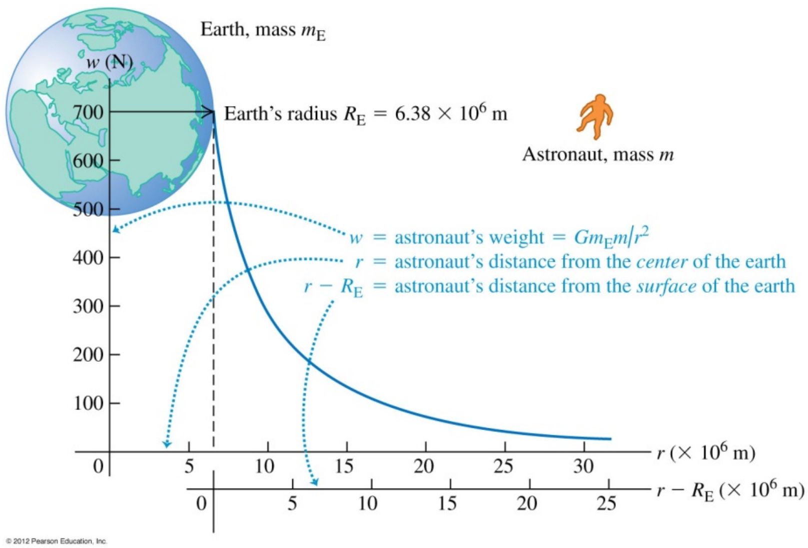
measure $g = 9.8\ \mathrm{m/s^2}$

$R_E = 6.38 \times 10^6$ m

$\Rightarrow \quad m_E = 5.974 \times 10^{24}$ kg

assuming earth is a uniform sphere with radius R_E and mass m_E

Digression: R_E was first measured by an ancient Greek called Eratosthenes in ~200 BC. See https://en.wikipedia.org/wiki/Eratosthenes



when $r > R_E$, weight decreases as $1/r^2$, $W = Gm_E m/r^2$

Question
Rank the hypothetical planets in order from highest to lowest value of g at the surface:
a) mass $2m_E$, radius $2R_E$;
b) mass $4m_E$, radius $4R_E$;
c) mass $4m_E$, radius $2R_E$;
d) mass $2m_E$, radius $4R_E$.
Answer: see inverted text on P. 430 of textbook

Gravitational Potential Energy – beyond $U = mgy$ (near earth surface only)
Recall: gravitation is a conservative force
Reminder: revisit the properties of conservative forces in Lecture 5

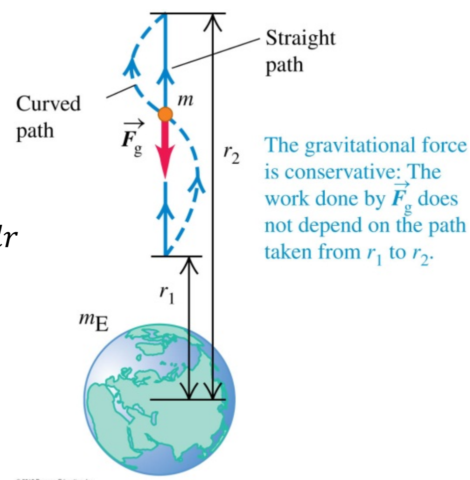
work done by gravitational attraction from 1 to 2

$$-\Delta U = W_{\text{grav}} = \int_1^2 \vec{\boldsymbol{F}}_g \cdot d\vec{\boldsymbol{r}} = \int_{r_1}^{r_2} (-F_g dr)$$

path independent, just choose radial path (straight line)

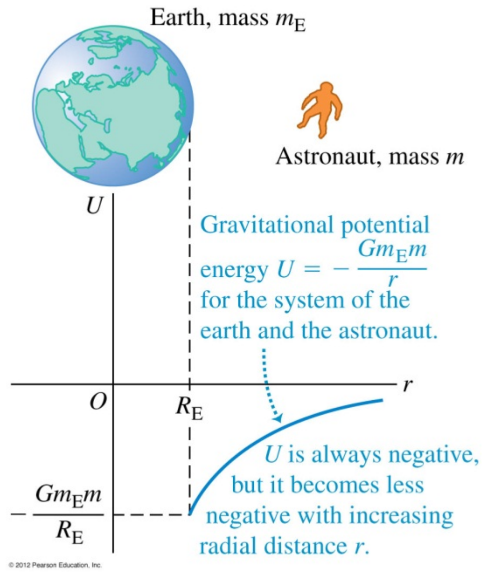
$$\underbrace{\vec{\boldsymbol{F}}_g}_{\text{inwards}} \cdot \underbrace{d\vec{\boldsymbol{r}}}_{\text{outwards}} = -F_g dr$$

$$= -\left[\left(-\frac{Gm_E m}{r_2}\right) - \left(-\frac{Gm_E m}{r_1}\right)\right]$$
$$= -[U(2) - U(1)]$$



Define

$$\boxed{U(r) = -\frac{Gm_Em}{r}}$$



⚠ $U(\infty) = 0$, i.e. zero level of PE at ∞

⚠ $U(r) < 0$, decreases (more negative) as r decreases

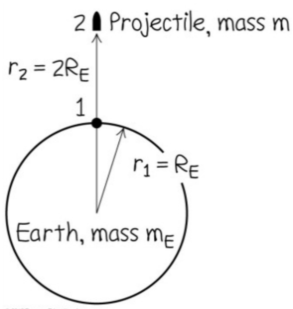
When close to earth surface, $r_1, r_2 \approx R_E$

$$W_{\text{grav}} = Gm_Em\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

$$= Gm_Em\frac{r_1 - r_2}{r_1 r_2} \approx m\underbrace{\frac{Gm_E}{R_E^2}}_{g}(r_1 - r_2) = -mg(r_2 - r_1)$$

same as defining $U = mgy$
zero level of PE arbitrary

Example 13.5, P. 431 Escape speed



Shoot a projectile vertically with speed v_1, can it escape from earth's gravitational attraction?

$$\frac{1}{2}mv_1^2 + \left(-\frac{Gm_Em}{R_E}\right) = 0 + 0$$

PE at ∞

zero KE, i.e., correspond to minimum v_1

$$\Rightarrow \quad \boxed{v_1 = \sqrt{\frac{2Gm_E}{R_E}}}$$

escape speed, ⚠ independent of m

On substitution

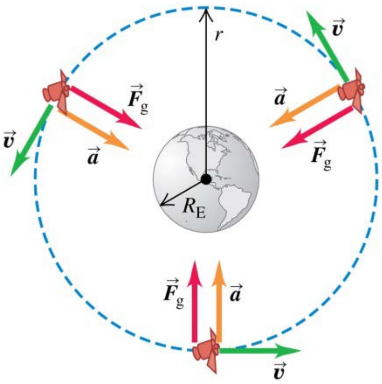
$$v_1 = \sqrt{\frac{2(6.67\times10^{-11}\ \text{Nm}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{6.38\times10^{6}\ \text{m}}} = 1.12\times10^{4}\ \text{m/s}$$

⚠ better to launch a spacecraft towards to east ∵ before launching, its already moving to the east at 410 m/s due to earth's rotation (but this effect is not significant, *c.f.* Example 13.6)

⚠ air molecules at room temperature $\sim 500\ \text{m/s}$, → atmosphere exists

Question
Is it possible for a planet to have the same surface gravity as the earth (i.e., same g) and yet have a greater escape speed?
Answer: see inverted text on P. 432 of textbool

Satellites – Assuming circular orbit



⚠ No tangential force, v must be constant

$$\frac{Gm_E m}{r^2} = m\underbrace{\left(\frac{v^2}{r}\right)}_{\text{centripetal acceleration}} \quad \Longrightarrow \quad v = \sqrt{\frac{Gm_E}{r}}$$

⚠ do not confuse with escape speed

⚠ v independent of mass, astronauts orbit about the earth together with spacecraft – <u>apparent weightlessness</u>

<u>True weightlessness</u> only if object is infinitely far from other masses

Period of orbit $T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = \frac{2\pi}{\sqrt{Gm_E}} r^{3/2}$ ⚠ larger orbit ⟶ longer period

Total energy in an orbit $E = K + U = \frac{1}{2}mv^2 + \left(-\frac{Gm_E m}{r}\right) = \frac{1}{2}\left(-\frac{Gm_E m}{r}\right) = \frac{U}{2}$

⚠ larger orbit, larger (less –ve) E. If spacecraft loses energy (due to air resistance when it is too close to the earth's atmosphere), r decreases and eventually falls to the earth

Example 13.6 P. 434

In order to launch a 1000-kg satellite into a circular orbit 300 km above the earth

$$\underbrace{\frac{1}{2}\left(-\frac{Gm_E m}{R_E + 300\text{ km}}\right)}_{\text{total energy in orbit}} = W_{\text{required}} + \underset{\uparrow\ \text{assume no initial KE}}{0} + \left(-\frac{Gm_E m}{R_E}\right) \quad \Longrightarrow \quad W_{\text{required}} = 3.26 \times 10^{10}\text{ J}$$

⚠ ignore rotation of the earth so that no KE before launching. Its contribution is about $\frac{1}{2}(1000\text{ kg})(410\ \text{m/s})^2 = 8.41 \times 10^7$ J, insignificant compared to W_{required}.

Question A spacecraft is in a low-altitude circular orbit around the earth. Air resistance from the outer regions of the atmosphere does negative work on the spacecraft, causing the orbital radius to decrease slightly. The speed of the spacecraft (remains the same / increases / decreases). Answer: see inverted text on P. 435 of textbook.

Clicker Questions:

Q13.1

The mass of the moon is 1/81 of the mass of the earth. Compared to the gravitational force that the earth exerts on the moon, the gravitational force that the moon exerts on the earth is

A. $81^2 = 6561$ times greater.

B. 81 times greater.

C. equally strong.

D. 1/81 as great.

E. $(1/81)^2 = 1/6561$ as great.

Q13.8

Star X has twice the mass of the sun. One of star X's planets moves in a circular orbit around star X. This orbit has the same radius as the earth's orbit around the sun. The orbital *speed* of this planet of star X

A. is faster than the earth's orbital speed.

B. is the same as the earth's orbital speed.

C. is slower than the earth's orbital speed.

D. depends on the mass of the planet.

E. depends on the mass and radius of the planet.

Q13.9

Suppose the sun were to shrink to half of its present radius while maintaining the same mass. What effect would this have on the radius r and the period T of earth's orbit around the sun?

A. r would decrease and T would decrease.

B. r would increase and T would increase.

C. r would decrease and T would increase.

D. r would increase and T would decrease.

E. r and T would both be unchanged.

Ans: Q13.1) C, Q13.8) A, Q13.9) E

Henry Cavendish

From Wikipedia, the free encyclopedia

Henry Cavendish FRS (10 October 1731 – 24 February 1810) was a British natural philosopher, scientist, and an important experimental and theoretical chemist and physicist. Cavendish is noted for his discovery of hydrogen or what he called "inflammable air".[1] He described the density of inflammable air, which formed water on combustion, in a 1766 paper "On Factitious Airs". Antoine Lavoisier later reproduced Cavendish's experiment and gave the element its name. Cavendish was distinguished for great accuracy and precision in researches into the composition of atmospheric air, the properties of different gases, the synthesis of water, the law governing electrical attraction and repulsion, a mechanical theory of heat, and calculations of the density (and hence the weight) of the Earth. His experiment to weigh the Earth has come to be known as the Cavendish experiment.

Henry Cavendish

Henry Cavendish

Born	10 October 1731 Nice, France
Died	24 February 1810 (aged 78) London, England
Nationality	British
Fields	Chemistry, physics
Alma mater	University of Cambridge
Known for	Discovery of hydrogen Measured the Earth's density

Biography

Early life

Henry Cavendish was born on 10 October 1731 in Nice, France, where his family was living at the time. His mother was Lady Anne Grey, fourth daughter of Henry Grey, 1st Duke of Kent and his father was Lord Charles Cavendish, third son of William Cavendish, 2nd Duke of Devonshire.The family traces its lineage across eight centuries to Norman times and was closely connected to many aristocratic families of Great Britain. His mother died in 1733, three months after the birth of her second son, Frederick, and shortly before Henry's second birthday, leaving Lord Charles Cavendish to bring up his two sons. At age 11, Henry attended Hackney Academy, a private school near London, and at age 18 (on 24 November 1748) he entered the University of Cambridge in St Peter's College, now known as Peterhouse, but left three years later on 23 February 1751 without taking a degree (a common practice).[2][3] He then lived with his father in London, where he soon had his own laboratory.

Lord Charles Cavendish lived a life of service, first in politics and then increasingly in science, especially in the Royal Society of London. In 1758 he took Henry to meetings of the Royal Society and also to dinners of the Royal Society Club. In 1760 Henry Cavendish was elected to both these groups, and he was assiduous in his attendance thereafter. He took virtually no part in politics, but, like his father, he lived a life of service to science, both through his researches and through his participation in scientific organizations. He was active in the Council of the Royal Society of London (to which he was elected in 1765); his interest and expertise in the use of scientific instruments led him to head a committee to review the Royal Society's meteorological instruments and to help assess the instruments of the Royal Greenwich Observatory. His first paper, "Factitious Airs" appeared in 1766. Other committees on which he served included the committee of papers, which chose the papers for publication in the Philosophical Transactions, and the committees for the transit of Venus (1769), for the gravitational attraction of mountains (1774), and for the scientific instructions for Constantine Phipps's expedition (1773) in search of the North Pole and the Northwest Passage. In 1773 Henry joined his father as an elected trustee of the British Museum, to which he devoted a good deal of time and effort. Soon after the Royal Institution of Great Britain was established, Cavendish became a manager (1800) and took an active interest, especially in the laboratory, where he observed and helped in Humphry Davy's chemical experiments.

For more information see http://en.wikipedia.org/wiki/Henry_Cavendish and http://en.wikipedia.org/wiki/Cavendish_experiment