FINA 1303

FOUNDATIONS OF INTEREST RATES Part II

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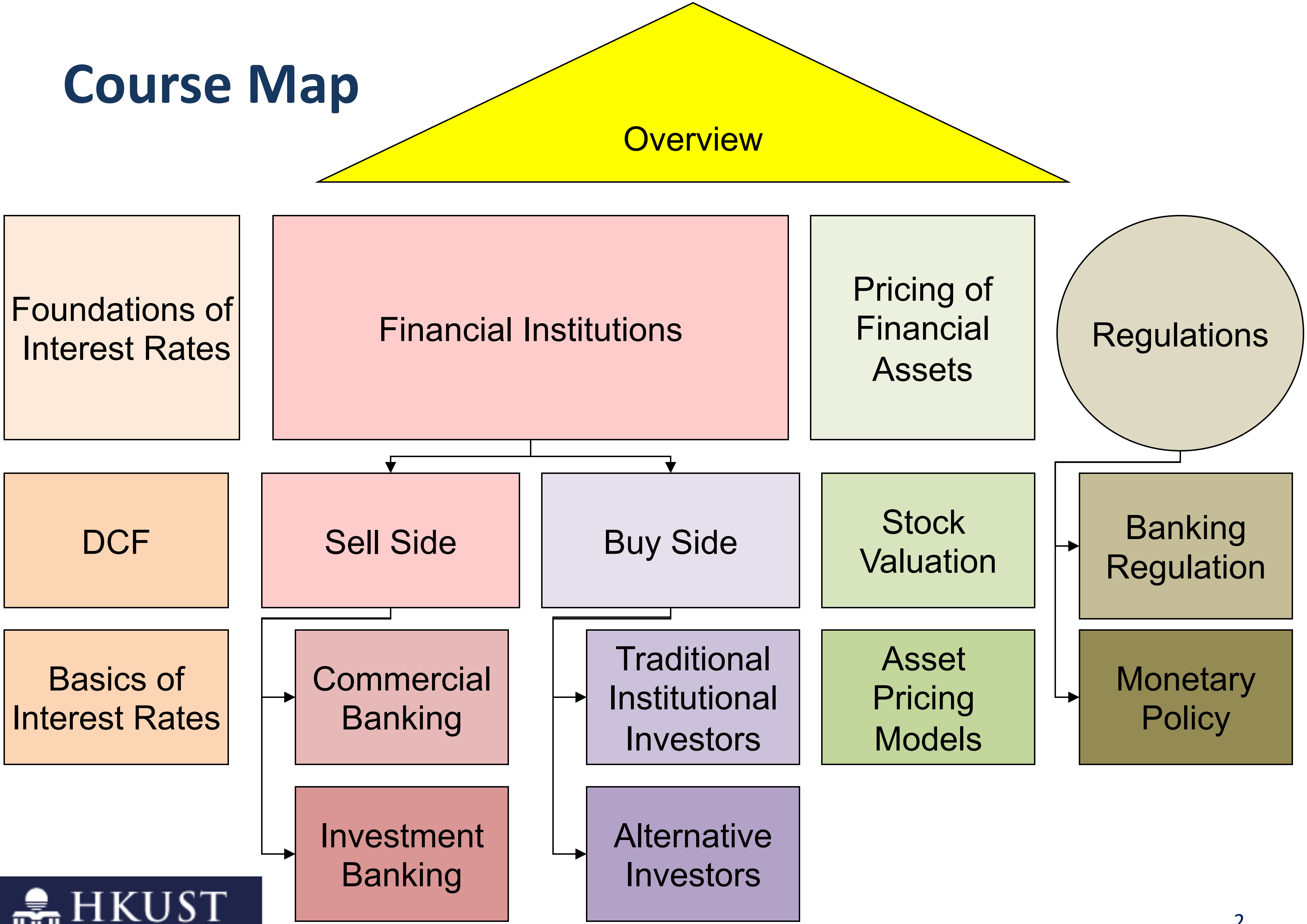
Course Map



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Overview

Foundations of Interest Rates

DCF

Basics of Interest Rates

Students will establish an understanding of

1. Bond Pricing Basics
    1. Zero Coupon Bond
    2. Coupon Bond
2. Computing interest for short term debt

Bond Basics

- A bond is a financial instrument promising to make a series of payments on specific dates. It is similar to a loan but unlike loans, bonds are securities (negotiable, transferable financial instruments)
- Bonds are legal contracts between an issuer and the investors (buyers) that:
  - Require the issuer to make payments to the buyer, and
  - Specify in great detail all the terms and conditions, including what happens in case the issuer fails to make a payment (called "event of default")

Bond Basics: Coupon Bond

- The most common type of bond is **a coupon bond**:
  - Issuer is required to make regular payments, called *coupon* payments.
  - The interest the issuer pays, and which is used to calculate the coupons, is the *coupon rate*.
  - The frequency of the coupon payments is annual or semi-annual (USA).
  - The date on which the principal of the bond is repaid is the *maturity date*.
  - The final payment includes (1) the principal, face value, or par value of the bond and (2) the final coupon payment.

Coupon Bond

Script coupon bond as buyer would receive a certificate with coupons attached.

Coupons not yet paid

Key Formulas: Bond Price

- That formula for fixed cash flows with a residual value on maturity date is also the general formula that covers many different cases of DCF calculations but is also the way (for you for now) to value bonds based on Coupon (*C*), Principal (*P*), Number of Years (*n*), Yield To Maturity (*i*):

$$Bond\ Price = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{P}{(1+i)^n}$$

$$\textbf{Bond Price} = \frac{C.[1-(1+i)^{-n}]}{i} + \frac{P}{(1+i)^n}$$

Key Formulas: Bond Price

- If P=0 that is our mortgage case (in part I),
- If P=0 and n=$\propto$ AND i > 0, then it is a consol or a perpetual bond:

$$\text{Bond Price} = \frac{C}{i}$$

- If C= 0 it is a Zero-Coupon Bond:

$$\text{Bond Price} = \frac{P}{(1+i)^n}$$

Zero Coupon Bonds

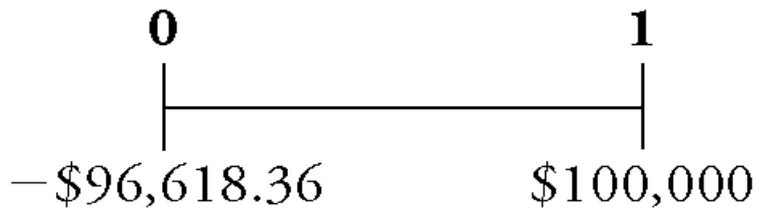
Zero-Coupon Bonds

- Zero-coupon bonds
  - Only two cash flows
    - The bond's market price at the time of purchase
    - The bond's face value (principal *P*) at maturity

$$\textbf{Bond Price} = \frac{P}{(1+i)^n}$$

Zero-Coupon Bonds - Example

- A one-year zero-coupon bond with a $100,000 face value has an initial price of $96,618.36
  - If you purchased this bond and held it to maturity, you would have the following cash flows:



Zero-Coupon Bonds

- Yield to Maturity of a Zero-Coupon Bond
  - The **discount rate** that sets the **present value** of the promised bond payments **equal to** the current **market price** of the bond
  - This is the same problem as when we solved for the rate of return
  - Yield to Maturity of an n-Year Zero-Coupon Bond:

$$1 + YTM_n = \left(\frac{Face\,Value}{Price} \right)^{1/n}$$

Example: Yields for Different Maturities

- Suppose the following zero-coupon bonds are trading at the prices shown below per $100 face value.
- **Determine the corresponding yield to maturity (YTM) for each bond.**

Maturity	1 year	2 years	3 years	4 years
Price	$96.62	$92.45	$87.63	$83.06

Example: Yields for Different Maturities

- We can use our equation to solve for the YTM of the bonds.

$$1 + YTM_n = \left(\frac{Face\,Value}{Price} \right)^{1/n}$$

- The table gives the prices and number of years to maturity.

Example: Yields for Different Maturities

- We have

$$YTM_1 = (100/96.62)^{1/1} - 1 = 3.50\%$$

$$YTM_2 = (100/92.45)^{1/2} - 1 = 4.00\%$$

$$YTM_3 = (100/87.63)^{1/3} - 1 = 4.50\%$$

$$YTM_4 = (100/83.06)^{1/4} - 1 = 4.75\%$$

Example: Yields for Different Maturities

- Solving for the YTM of a zero-coupon bond is **the same process we used to solve for the rate of return.**
- Indeed, the YTM is the rate of return of buying the bond.
- NB: we can do it quickly in our calculator, where we solve for *i*, with *n* = tenor (number of periods) *PMT* is zero (no coupons), *FV* is $100 (par value) and *PV* is the price (with negative cash flow)

Your Turn!

- Suppose the following zero-coupon bonds are trading at the prices shown below per $100 face value.
- Determine the corresponding yield to maturity for each bond.

Maturity	1 year	2 years	3 years	4 years
Price	$98.52	$96.59	$94.23	$91.48

Your Turn (PRS please)

- The YTMs are
  - 1 year bond:
    - 1.2%
    - 1.5%
    - 1.7%
  - 2 year bond:
    - 1.6%
    - 1.75%
    - 1.8%
  - 3 year bond:
    - 1.9%
    - 2.0%
    - 2.5%
  - 4 year bond:
    - 2.0%
    - 2.25%
    - 3%

Solution: Yields for Different Maturities

- We can solve for the YTM of the bonds. The table gives the prices and number of years to maturity.
- NB: we can do it quickly in our calculator, where we solve for *i*, with *n*= tenor (number of periods) *PMT* is zero (no coupons), *FV* is $100 (par value) and *PV* is the price (with negative cash flow)

Solution: Yields for Different Maturities

- We have

$$\text{YTM}_1 = (100/98.52)^{1/1} - 1 = 1.50\%$$

$$\text{YTM}_2 = (100/96.59)^{1/2} - 1 = 1.75\%$$

$$\text{YTM}_3 = (100/94.23)^{1/3} - 1 = 2.00\%$$

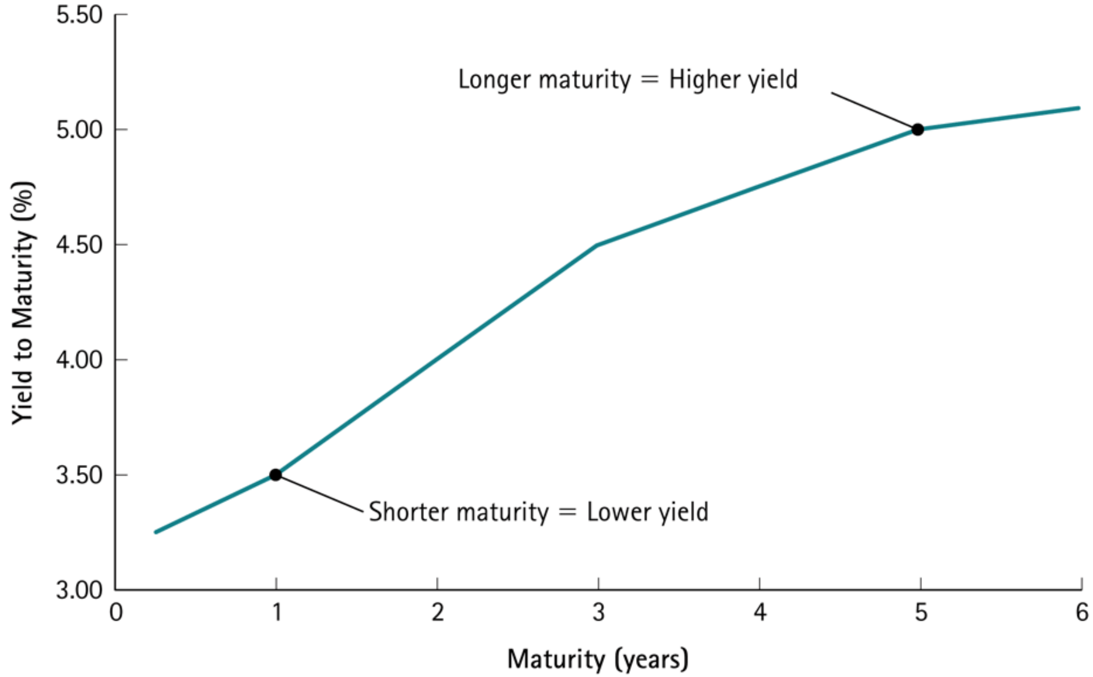
$$\text{YTM}_4 = (100/91.48)^{1/4} - 1 = 2.25\%$$

Solution: Yields for Different Maturities

- Solving for the YTM of a zero-coupon bond is the same process we used to solve for the rate of return.
- Indeed, the YTM is the rate of return of buying the bond.

Example: Computing the Price of a Zero-Coupon Bond

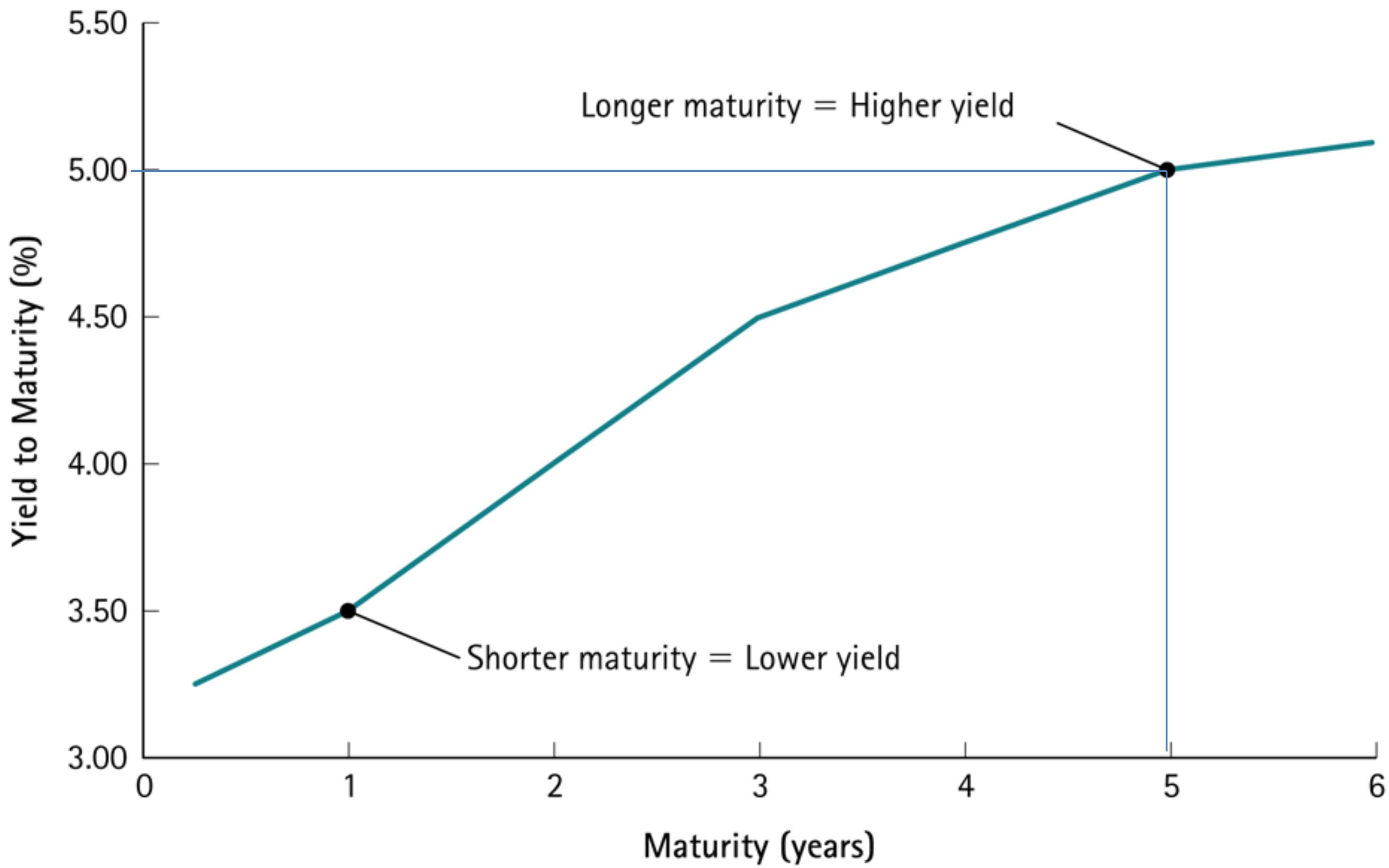
- Given the yield curve below, what is the price of a 5-year "risk-free" zero-coupon bond with a face value of $100?



Example: Computing the Price of a Zero-Coupon Bond

- We can use the bond's yield to maturity to compute the bond's price as the present value of its face amount, where the discount rate is the bond's yield to maturity.
- From the yield curve, the yield to maturity for **5-year** "risk-free" zero-coupon bonds is **5.0%**.

Zero-Coupon Yield Curve Consistent with the Bond Prices in Example



Example: Computing the Price of a Zero-Coupon Bond

- Execute:

$$\text{Bond Price} = \frac{P}{(1+i)^n}$$

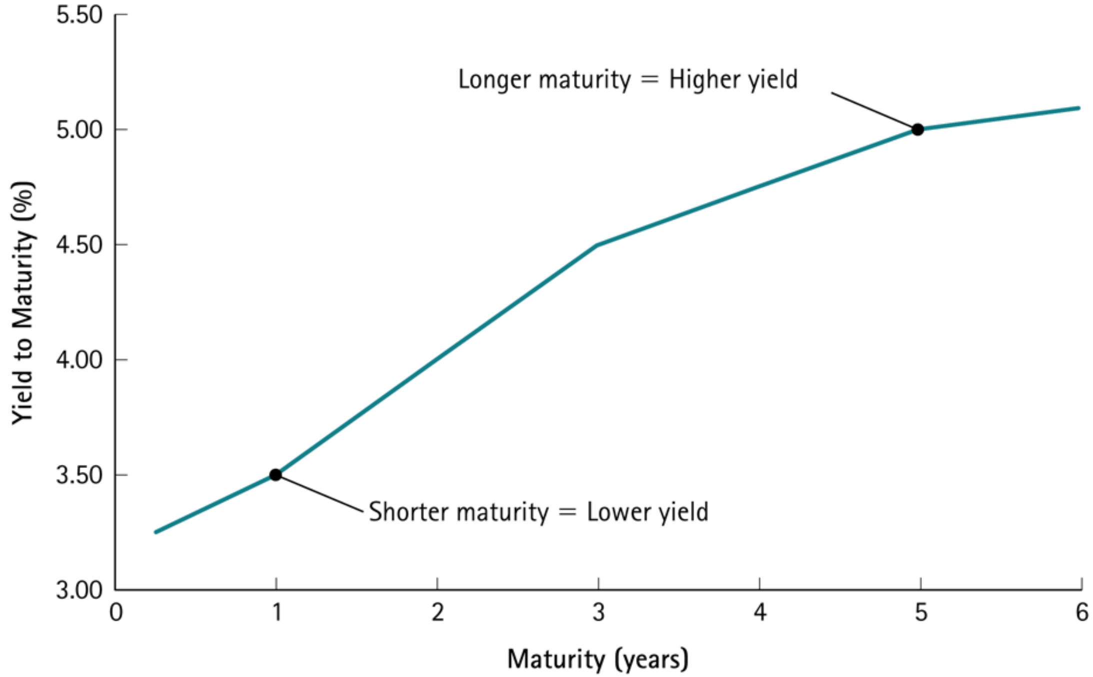
$$P = 100/(1.05)^5 = 78.35$$

Example: Computing the Price of a Zero-Coupon Bond

- We can compute the price of a zero-coupon bond simply by computing the **present value** of the face amount using the bond's yield to maturity.
- Note that the price of the 5-year zero-coupon bond is even lower than the price of the other zero-coupon bonds in Example 6.1, because the face amount is the same but we must wait longer to receive it.

Your Turn!

- Given the yield curve below, what is the price of a **3-year** risk-free zero-coupon bond with a face value of **$900**?



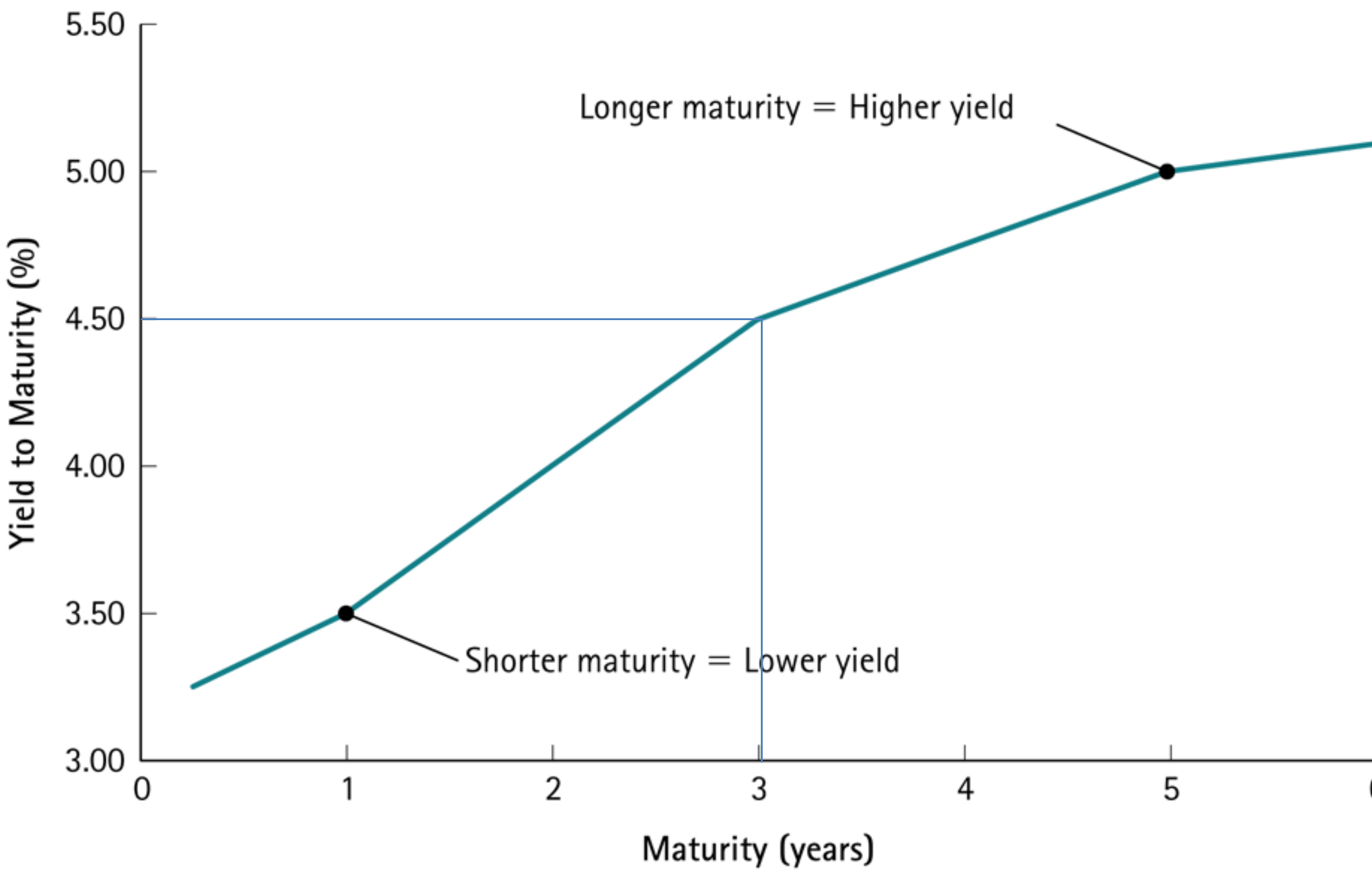
Your Turn (PRS please)

- The price is:
- $900
- $879
- $788.67

Solution: Computing the Price of a Zero-Coupon Bond

- We can use the bond's yield to maturity to compute the bond's price as the present value of its face amount, where the discount rate is the bond's yield to maturity.
- **From the yield curve**, the yield to maturity for **3-year** risk-free zero-coupon bonds is **4.50%**.

Zero-Coupon Yield Curve Consistent with the Bond Prices in Example



Solution: Computing the Price of a Zero-Coupon Bond

Execute:

$$\textbf{Bond Price} = \frac{P}{(1+i)^n}$$

$$P = 900 \,/\, (1.045)^3 = \$788.67$$

U.S. Government Securities "Treasuries"

Video on Tbills: https://www.youtube.com/watch?v=aVvDy9gVe90

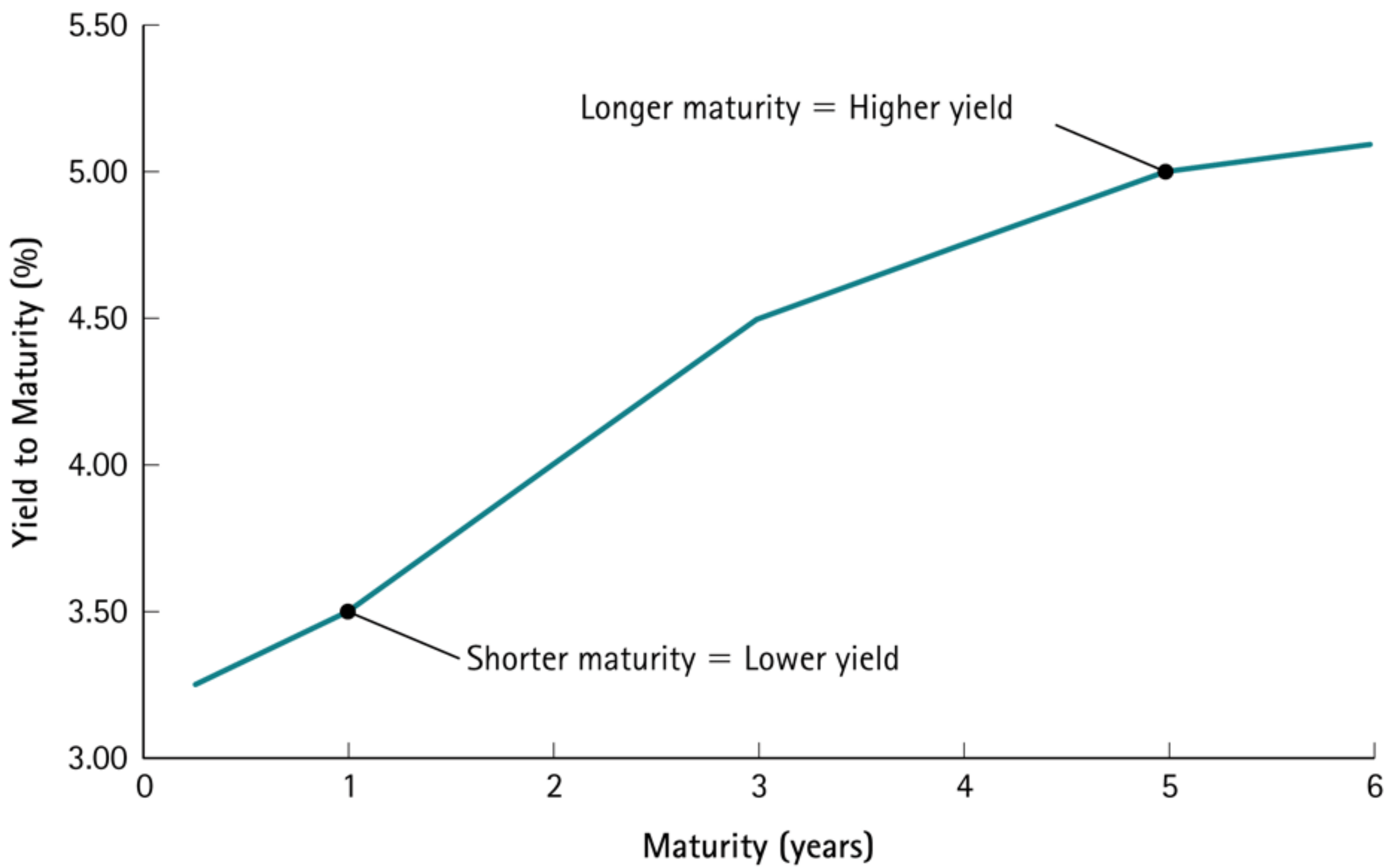
https://www.treasury.gov/resource-center/data-chart-center/quarterly-refunding/Documents/auctions.pdf

Treasury Security	Type	Original Maturity
Bills	Discount	4, 13, 26, and 52 weeks
Notes	Coupon	2, 3, 5, and 10 year
Bonds	Coupon	20 and 30 year

Because of their short tenor, US T-Bills are zero coupon. They trade at a discount to yield.

The 30 year T-Bond is called the "long bond"

Zero-Coupon Yield Curve



Coupon Bonds

Coupon Bonds

- Coupon bonds
  - Pay face value at maturity
  - Also make regular coupon interest payments
- Return on a coupon bond comes from:
  - The difference between the purchase price and the principal value
  - Periodic coupon payments
- To compute the yield to maturity of a coupon bond, we need to know the coupon interest payments, and when they are paid

Key Formulas: Bond Price

- That formula for fixed cash flows with a residual value on maturity date is also the general formula that covers many different cases of DCF calculations but is also the way (for you for now) to value bonds based on Coupon (*C*), Principal (*P*), Number of Years (*n*), Yield To Maturity (*i*):

$$Bond\ Price = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{P}{(1+i)^n}$$

$$\textbf{Bond Price} = \frac{C.[1-(1+i)^{-n}]}{i} + \frac{P}{(1+i)^n}$$

Example: Computing the Yield to Maturity of a Coupon Bond

- Consider a five-year, $1,000 bond with a 2.2% coupon rate and semiannual coupons .
- If this bond is currently trading for a price of $963.11, **what is the bond's yield to maturity?**

Example: Computing the Yield to Maturity of a Coupon Bond

- We can solve it by financial calculator, or a spreadsheet. To use a financial calculator, we enter the price we pay as a negative number for the PV (it is a cash outflow), the coupon payments as the PMT, and the bond's par value as its FV. Finally, we enter the number of coupon payments remaining (10) as N.

	N	I/Y	PV	PMT	FV
Given:	10		-963.11	11	1,000
Solve for:		**1.50**			
Excel Formula: =RATE(NPER,PMT,PV,FV)= RATE(10,11,-963.11,1000)					

Example: Computing the Yield to Maturity of a Coupon Bond

- Therefore, $y = 1.50\%$.
- Because the bond pays coupons semiannually, this yield is for a **six-month** period.
- We convert it to an APR (annual percentage rate) by multiplying by the number of coupon payments per year.
- Thus the bond has a yield to maturity equal to a **3.0% APR** with semiannual compounding.

Example: Computing the Yield to Maturity of a Coupon Bond

- As the equation shows, the yield to maturity is the discount rate that equates the present value of the bond's cash flows with its price.
- Note that the YTM is higher than the coupon rate and the price is lower than the par value.

Example: Computing a Bond Price from Its Yield to Maturity

- Consider a five-year, $1,000 bond with a 2.2% coupon rate and semiannual coupons.
- Suppose interest rates drop and the bond's **yield to maturity** decreases to **2%** (expressed as an APR with semiannual compounding).
- What **price** is the bond trading for now?

Example: Computing a Bond Price from Its Yield to Maturity

- Given the yield, we can compute the price.
- First, note that a 2.0% APR is equivalent to a semiannual rate of 1.0%.
- Also, recall that the cash flows of this bond are 10 payments of $11, paid every 6 months, and one lump-sum cash flow of $1,000 (the face value), paid in 5 years (ten 6-month periods).
- We can compute the effective annual yield from the bond's yield to maturity expressed as an APR.

Example: Computing a Bond Price from Its Yield to Maturity

- We can use a financial calculator:

	N	I/Y	PV	PMT	FV
Given:	10	1.0		11	1,000
Solve for:			**-1,009.47**		
Excel Formula: = PV(RATE,NPER,PMT,FV)=PV(.01,10,11,1000)					

Your Turn!

- Consider a **nine-year**, $1,000 note with a **3%** coupon rate and **semiannual** coupons.
- If this bond is currently trading for a price of **$1,038.32**, **what is the bond's yield to maturity**?

Your Turn (PRS please)

- The bond's YTM (on an APR basis) will be:
- 1.26%
- 2.52%
- 3.16%

Solution: Computing the Yield to Maturity of a Coupon Bond

- From the cash flow timeline, we can see that the bond consists of 18 payments of $15, paid every 6 months, and one lump-sum payment of $1,000 in 9 years (eighteen 6-month periods).
- We can solve for the yield to maturity.
- However, we must use 6-month intervals consistently.

Solution: Computing the Yield to Maturity of a Coupon Bond

- We can solve it by financial calculator, or a spreadsheet. To use a financial calculator, we enter the price we pay as a negative number for the PV (it is a cash outflow), the coupon payments as the PMT, and the bond's par value as its FV. Finally, we enter the number of coupon payments remaining (10) as N.

	N	I/Y	PV	PMT	FV
Given:	18		-1038.32	15	1,000
Solve for:		**1.26**			
Excel Formula: =RATE(NPER,PMT,PV,FV)= RATE(18,15,-1038.32,1000)					

Solution: Computing the Yield to Maturity of a Coupon Bond

- Therefore, $\boldsymbol{y}$ = 1.26%.
- Because the bond pays coupons semiannually, this yield is for a six-month period.
- We convert it to an APR by multiplying by the number of coupon payments per year.
- Thus the bond has a yield to maturity equal to a **2.52%** APR with semiannual compounding.
- As the equation shows, the yield to maturity is the discount rate that equates the present value of the bond's cash flows with its price.

Bond Pricing Recap

Bond Pricing

- The relationship between the bond price and interest rates is very important.
  - Bonds promise fixed payments on future dates, so the higher the interest rate, the lower their present value.
- The value of a bond varies ***inversely*** with the interest rate used to calculate the present value of the promised payment.

Bond Pricing Example

- Bond terms and conditions:
  - Par Value / Face Value: US$1000
  - Coupon Rate: 3.15%
  - Coupon Frequency: Annual
  - Tenor: 7 years
  - Else: N/A
- Key terms for calculations:
  - Annual Coupon Amount (PMT): US$1000 x 3.15% = US$31.50
  - Principal Payment (FV): US$1000
  - Nbr Payments (NPER): 7

Bond Pricing Example

- Then depending on what is available, Bond Price (*PV*) or interest rate, Yield To Maturity "*YTM*" (*i*) it's possible to calculate the other:

$$Bond\ Price = \frac{\$31.5}{(1+i)^1} + \frac{\$31.5}{(1+i)^2} + \cdots + \frac{\$31.5}{(1+i)^7} + \frac{\$1000}{(1+i)^7}$$

- If YTM = *i* = 2%,

$$Bond\ Price = \frac{\$31.5}{(1+2\%)^1} + \frac{\$31.5}{(1+2\%)^2} + \cdots + \frac{\$31.5}{(1+2\%)^7} + \frac{\$1000}{(1+2\%)^7}$$

$$Bond\ Price = \$1074.43 = 107.44\% \times Par\ Value$$

- If YTM = *i* = 4%,

$$Bond\ Price = \frac{\$31.5}{(1+4\%)^1} + \frac{\$31.5}{(1+4\%)^2} + \cdots + \frac{\$31.5}{(1+4\%)^7} + \frac{\$1000}{(1+4\%)^7}$$

$$Bond\ Price = \$948.98 = 94.90\% \times Par\ Value$$

Bond Pricing

- Par, Premium, Discount
  - If YTM is equal to Coupon Rate then Bond Price = 100% of Par Value (the bond trades "at par")
  - If YTM is higher than Coupon Rate then Bond Price < 100% of Par Value (the bond trades "at a discount")
  - If YTM is lower than Coupon Rate then Bond Price > 100% of Par Value
  - (the bond trades "at a premium")
  - The Tenor further amplifies those variations

Interest for short term debt instruments

Money Market ($\leq$1 Year) Rates

- For debt instruments within a year, interest calculations as well as names are different!
- Money market instruments include: Time Deposits, T Bills, Certificate of Deposits, Commercial Paper, Bankers' Acceptances, Forward Rate Agreements (FRA) including those listed in future markets.
- The interest are calculated as follow:

$$Interest = Notional\,.\,Interest\ Rate.\frac{Number\ of\ Days}{Number\ of\ Days\ in\ Year}$$

Simple Interest on Short Term Loans

Simple interest applied to short term loans:

$$\boldsymbol{FV = PV\,[\,1 + r \times (d/y)]}$$

Where

- *FV* = Future Value
- *PV* = Present Value
- *r* = interest rate
- *d* = number of days to term
- *y* = year basis used in calculation

The DIY Dilemma

- On 1st January 2010, Delta Investments Yield has HK$10 million excess cash to invest for 2 months; the treasurer considers the following options:
- Placing the money in 2 month fixed deposit in Sunny Bank Ltd that pays **3%** on a money market (actual/365) day basis
- Buying a Lucky Gold Company bond with a remaining maturity of 2 months that yields **3%** on a bond basis (30/360)
- What should DIY do?

The DIY Dilemma (Continued)

- *Ceteris paribus*, what is the only difference between the two options?
- What **other** factors should DIY consider?
- How do we compare the two options?
  - Interest calculation: Principal * Interest rate * DTM/year
  - Why is that important?

The DIY Dilemma (Continued)

To compare the two options, we need to answer the following questions:

- How do we calculate the number of days to maturity?
- How is the number of days in the year defined?

Day Count Conventions

- **Day count convention** (year basis) is the market convention used in calculating interest expressed as a ratio of number of days in a month (d) to number of days in a year (y)
- Most common conventions:

	Actual	30
Actual	X	
365	X	
360	X	X

Market	Calculation Convention	Coupon Structures	Coupon Payment	Day Count Convention
Brunei Darussalam	N/A	N/A	N/A	N/A
Cambodia	N/A	N/A	N/A	N/A
China	Yield to Maturity	Fixed coupon mostly; two listed float –rate treasury bonds	Annual	Actual/365
Hong Kong	Yield to Maturity	EFBNs are issued on a discount basis	EFBNs are issued on a discount basis	Government: Actual/365
Japan	Yield to Maturity	Fixed coupon; float-rate for 15yr maturities	Semi-annual coupon	Government: Actual/Actual
Indonesia	Yield to Maturity	Fixed and variable coupons for government bonds; Variable rate bonds for some recapitalization bond issues	Quarterly or semi annual depending on the terms of the bonds	Government: Actual/Actual
Korea, Republic of	Yield to Maturity	All coupon-bearing bonds. Some Municipal bond (Seoul Sub) issues are deferred amortized and some MSB are discounted	Annual or semi-annual	Government: Actual/Actual
Lao PDR	N/A	Fixed	Annual	N/A
Malaysia	Yield to Maturity; Internal rate of return of cash flows	Fixed for government bonds	Annual or semi-annual	Government: Actual/Actual Corporate: Actual/365
Myanmar	N/A	Fixed	N/A	N/A
Philippines	Yield to Maturity	FXTNs/RTBs, fixed coupon; T-bills/CMBs, zero coupon	Semi-annual except for RTBs	Government: Actual/Actual Corporate: Actual/365
Singapore	Yield to Maturity	Varies depending on instrument	SGS Bond: Semi-annual coupon	SGS Bond: Actual/Actual
Thailand	Yield to Maturity	Fixed for government bonds; Floating rate notes are also issued	Semi-annual	Government: Actual/365
Vietnam	Varies depending on the instrument	Varies depending on instrument	Varies depending on the instrument	Varies depending on the instrument

The DIY Dilemma (Continued)

Day Count Conventions

Sunny Bank

- Actual/365
- Interest calculation:

  10,000,000*3%*actual number of days in 2 months/365

Lucky Gold

- 30/360
- Interest calculation:

  10,000,000*3%*(30 days/month for 2 months)/360

Can we do the calculation?
When does the deposit start and when does it mature?

The DIY Dilemma (Continued)

After the treasurer agrees a trade with the banker, it takes time for the transaction to be executed: the trade date is generally not the same as the settlement date

- What do we need to know?

The DIY Dilemma (Continued)

Trade and Value Dates

- When does the DIY Treasurer call his banker?
  - The trade date
- When can the money be invested?
  - The value/settlement date
- When can the money be returned?
  - The maturity date

Trade, Value & Maturity Dates

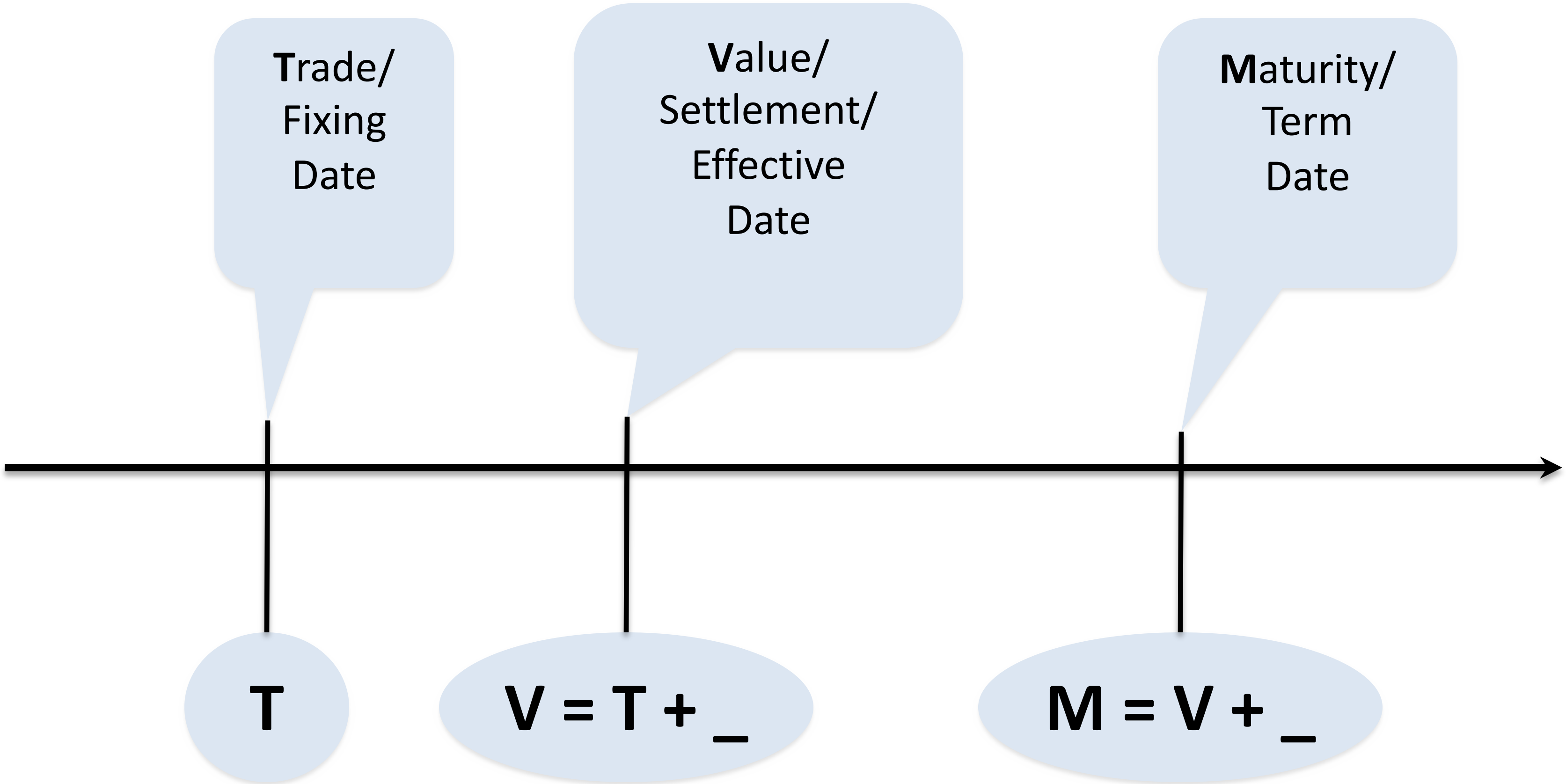
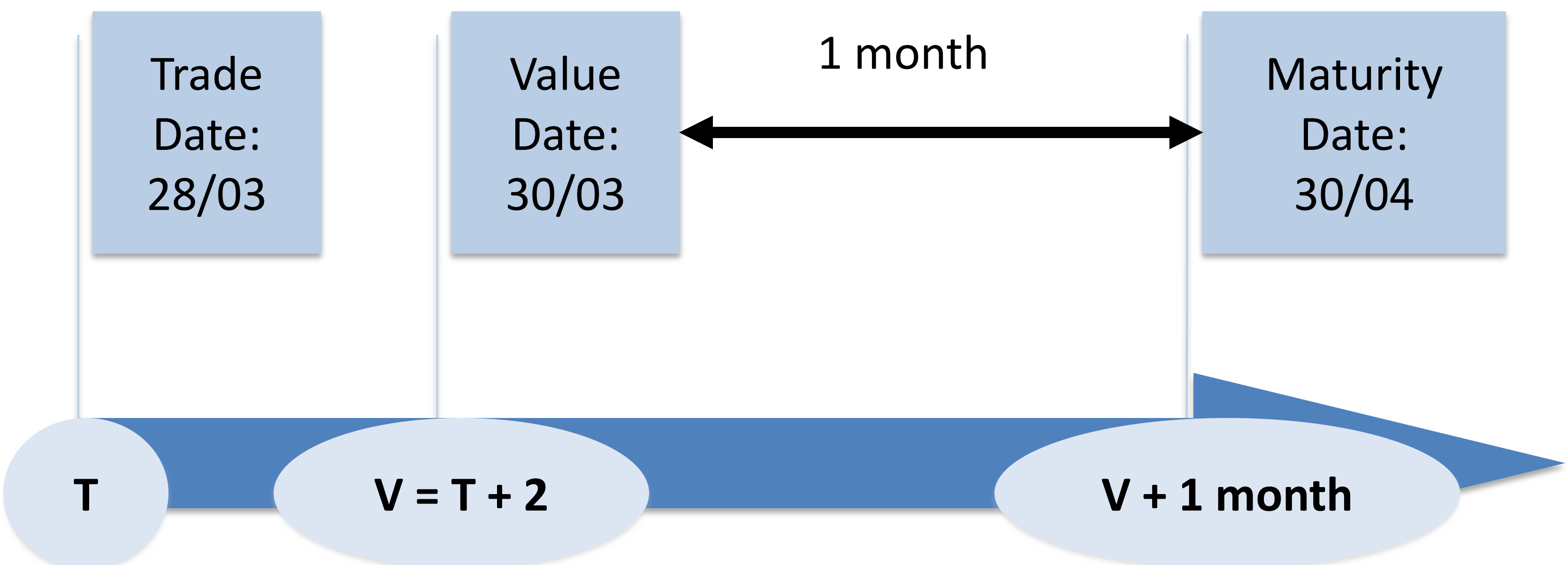


Illustration: 1 Month Deposit



The DIY Dilemma (Continued)
Trade and Value Dates

Sunny Bank

- Trade date : ?
- Value date: (spot) T+ 2 business days
- Maturity date : V+ 2 months

Lucky Gold

- Trade date : ?
- Value date: (spot) T+ 2 business days
- Maturity date : V+ 2 months

But, wait a minute… 1st January is a public holiday!

The DIY Dilemma (Continued)

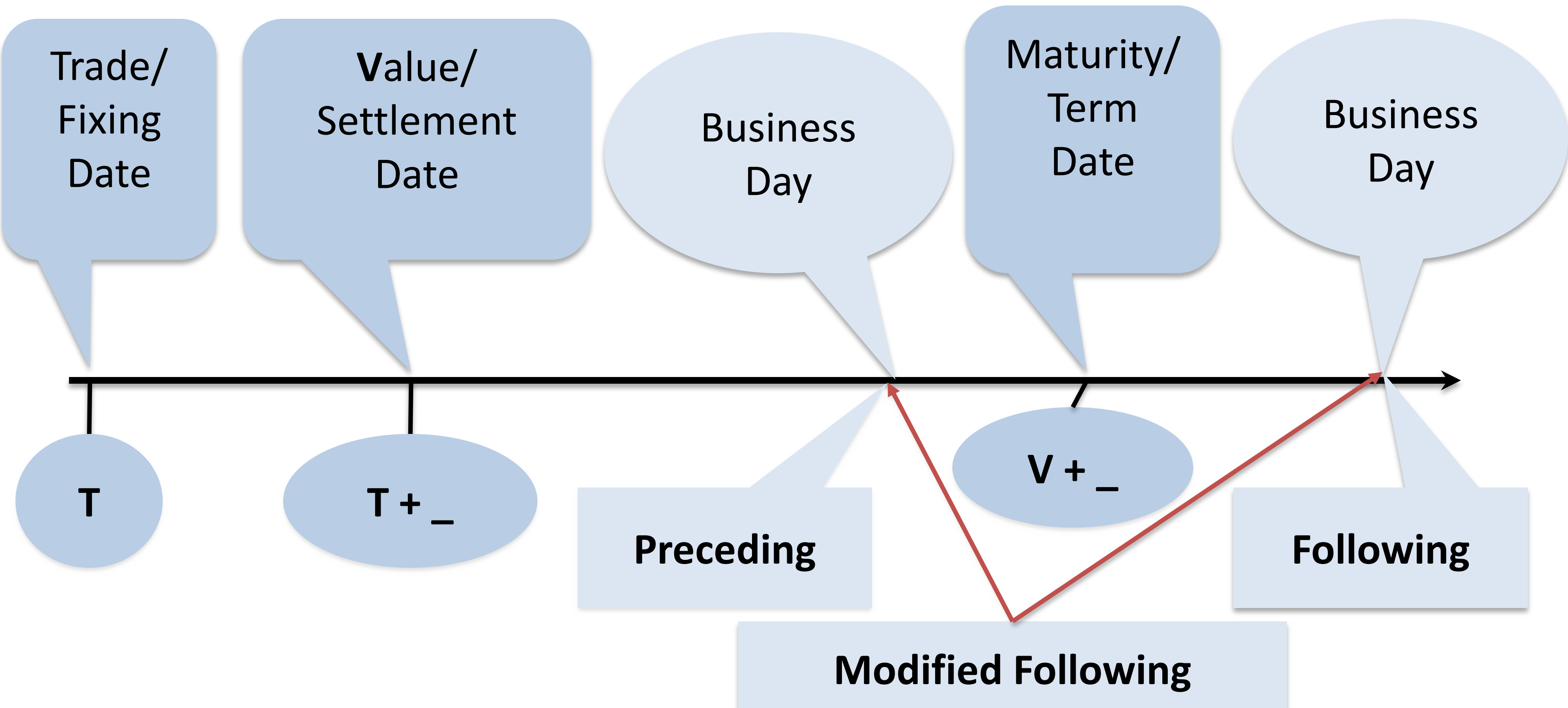
Business Day Definitions

- 1st January 2010 (Friday) is a public holiday in HK => DIY's treasurer cannot reach his banker and has to wait for the next **banking day** to agree a trade
  - The next day when banks are open is Monday 4th January, which is the Trade Date
- Let's assume market convention is **spot** (T+2) for deposits and bond purchases
  - The Value Date will be T + 2 : Wednesday 6th January
- To determine the Maturity Date we need to compute 2 (calendar) months from the Value Date
  - that is to say Saturday 6th March
- But Saturday is not a banking day, what do we do?

Business Day Conventions

- Market convention for adjusting payment dates in response to days that are **not business days**
- Definition of "business day"
- Most common business day conventions:
  - Following
  - Preceding
  - Modified following

Business Day Conventions



The DIY Dilemma (Continued)
Trade and Value Dates

Sunny Bank

- Trade date : Monday 4th January
- Value date: Wednesday 6th January
- Maturity date : (applying modified business day): Monday 8th March

Lucky Gold

- Trade date : Monday 4th January
- Value date: Wednesday 6th January
- Maturity date : (applying modified business day): Monday 8th March

Now we can calculate DTM using the relevant convention

The DIY Dilemma (Continued)
DTM applying day count convention

Sunny Bank

- Day count convention: actual/365
- Means we count the actual number of days in the relevant months and 365 days in a year
- January = 25 days (31-6) + February 28 days + March 8 days
- DTM = 61 days

Lucky Gold

- Day count convention: 30/360
- Means each month is assumed to have 30 days and the year 360 days
- January = 24 days (30-6)+ February 30 days+ March 8 days
- DTM = 62 days

The DIY Dilemma Solution

Sunny Bank

- Trade date : 4 January 2010
- Value date: 6 January
- Maturity date : 8 March
- DTM = 61
- Interest:= 10,000,000*3%*61/365

= 50,136.99

Lucky Gold

- Trade date : 4 January 2010
- Value date: 6 January
- Maturity date : 8 March
- DTM = 62
- Interest = 10,000,000*3%*62/360

= 51,666.67

Ceteris paribus, DIY should invest in Lucky Gold bond

Your turn! Supreme Bank

- Trade date: Friday 29 January 2010
- Supreme Bank (HK) Ltd borrows HK$ 50 million for 1 month from KS Lee Bankers (HK) Co at 5 % p.a.
- What amount of interest will Supreme Bank pay?

Supreme Bank: What Do We Need to Know?

- What is the market convention for value date?
  - Deposits: spot (T + 2)
- What is the day count convention?
  - Actual/365
- What is the business day definition?
  - Banking days in HK
  - Modified Following applies to maturity date

Supreme Bank Solution Steps

- First calculate the Value Date
  - V = T + 2
  - V =
- Then determine the Maturity Date
  - M = V + (calendar) month adjusted by the business day convention
  - M =
- Calculate DTM
  - DTM =
- Calculate interest applying day count convention
  - Interest =

Supreme Bank solution

- Trade date: 29 January (Friday)
- Value date: 2 February (Tuesday)
- Maturity date: 2 March (Tuesday)
- DTM: 28
- Convention: actual/365
- Interest: 191,780.82