MATH 2111 Matrix Algebra and Applications
Homework-3 : Due 10/08/2022 at 11:59pm HKT

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1. (2 points) Determine which of the following transformations are linear transformations.

- A. The transformation T defined by $T(x_1,x_2) = (4x_1 - 2x_2, 3|x_2|)$.
- B. The transformation T defined by $T(x_1,x_2,x_3) = (x_1,x_2,-x_3)$
- C. The transformation T defined by $T(x_1,x_2) = (2x_1 - 3x_2, x_1+4, 5x_2)$.
- D. The transformation T defined by $T(x_1,x_2,x_3) = (1,x_2,x_3)$
- E. The transformation T defined by $T(x_1,x_2,x_3) = (x_1,0,x_3)$

Correct Answers:

- BE

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2. (2 points) Let

$$A = \begin{bmatrix} -6 & 6 \\ 9 & -8 \\ -8 & 9 \end{bmatrix}.$$

Define the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T(\vec{x}) = A\vec{x}$. Find the images of $\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} a \\ b \end{bmatrix}$ under T.

$T(\vec{u}) = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

$T(\vec{v}) = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} 18 \\ -25 \\ 26 \end{bmatrix}$
- $\begin{bmatrix} 6*b-6*a \\ 9*a-8*b \\ 9*b-8*a \end{bmatrix}$

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3. (1 point) If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation such that

$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$, $T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, $T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix}$

then $T\left(\begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} 19 \\ -2 \\ 14 \end{bmatrix}$

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4. (1 point) Let

$$A = \begin{bmatrix} 5 & -5 \\ 4 & -6 \\ 1 & 1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} 35 \\ 38 \\ -3 \end{bmatrix}.$$

A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by $T(x) = Ax$. Find an $\vec{x}$ in $\mathbb{R}^2$ whose image under T is $\vec{b}$.

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$

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5. (1 point) Find the standard matrix A of the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ given by

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \\ 5 \end{bmatrix} x_1 + \begin{bmatrix} -3 \\ 6 \\ 5 \end{bmatrix} x_2.$$

$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$.

Correct Answers:

- 9
- -3
- 7
- 6
- 5
- 5

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6. (3 points) To every linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$, there is an associated 2×2 matrix. Match the following linear transformations with their associated matrix.

___1. Reflection about the y-axis
___2. Reflection about the x-axis
___3. Clockwise rotation by $\pi/2$ radians
___4. The projection onto the x-axis given by T(x,y)=(x,0)
___5. Reflection about the line y=x
___6. Counter-clockwise rotation by $\pi/2$ radians

A. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
B. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
C. $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

D. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

E. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

F. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

G. None of the above

Correct Answers:

- B
- F
- D
- C
- E
- A

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7. (2 points)
Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflects points through the x-axis and then reflects points through the line $y = -x$.

Find the standard matrix A for T.
$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$.

Correct Answers:

- 0
- 1
- -1
- 0

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8. (2 points) Let T be a linear transformation from $\mathbb{R}^r$ to $\mathbb{R}^s$.
Determine whether or not T is one-to-one in each of the following situations:

___1. $r < s$
___2. $r = s$
___3. $r > s$

A. T is a one-to-one transformation
B. T is not a one-to-one transformation
C. There is not enough information to tell

Correct Answers:

- C
- C
- B

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9. (2 points) Let T be an linear transformation from $\mathbb{R}^r$ to $\mathbb{R}^s$. Let A be the standard matrix of T.

Fill in the correct answer for each of the following situations.

___1. The row-echelon form of A has a pivot in every column.
___2. Two rows in the row-echelon form of A do not have pivots.
___3. The row-echelon form of A has a row of zeros.
___4. Every row in the row-echelon form of A has a pivot.

A. T is onto
B. T is not onto
C. There is not enough information to tell.

Correct Answers:

- C
- B
- B
- A

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10. (1 point) A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ whose standard matrix is
$\begin{bmatrix} 1 & 5 & 3 \\ 1 & 5 & \text{-6.5+k} \end{bmatrix}$
is onto if and only if $k \neq$ ___.

Correct Answers:

- 9.5

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11. (2 points) Let

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \vec{v}_2 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}.$$

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation satisfying

$$T(\vec{v}_1) = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \text{ and } T(\vec{v}_2) = \begin{bmatrix} -10 \\ 8 \end{bmatrix}.$$

Find the image of an arbitrary vector $\begin{bmatrix} x \\ y \end{bmatrix}$.

$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} 9*x+(-8)*y \\ -4*x+0*y \end{bmatrix}$