MATH 2111: Tutorial 7 Determinants, Vector Spaces and Subspaces

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Review

- Cramer's Rule
- Inverse formula
- Area and volume (using determinant)
- Vector spaces and subspaces

Example 1

Use Cramer's rule to solve the following linear system.

$$\begin{cases} x_1 + x_2 = 3 \\ -3x_1 + 2x_3 = 0 \\ x_2 - 2x_3 = 2 \end{cases}$$

Example 2

Compute the adjugate of the given matrix, and then use the inverse formula to give A^{-1}.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

Example 3

Let S be the parallelogram determined by the vectors $\mathbf{b}_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$, and let $A = \begin{bmatrix} 6 & -2 \\ -3 & 2 \end{bmatrix}$. Compute the area of the image of S under the mapping $\mathbf{x} \mapsto A\mathbf{x}$

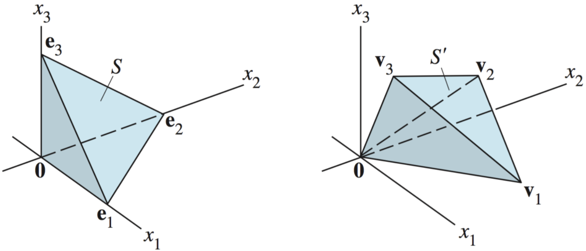
Example 4

Let R be the triangle with vertices at (x_1, y_1), (x_2, y_2), and (x_3, y_3). Show that

$$\{ \text{ area of triangle } \} = \frac{1}{2} \det \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

Example 5

Let S be the tetrahedron in $\mathbb{R}^3$ with vertices at the vectors $\mathbf{0}, \mathbf{e}_1, \mathbf{e}_2$, and $\mathbf{e}_3$, and let S' be the tetrahedron with vertices at vectors $\mathbf{0}, \mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$.



a. Describe a linear transformation that maps S onto S'.

b. Find a formula for the volume of the tetrahedron S' using the fact that $\{$ volume of $S\} = (1/3)\{$ area of base $\} \cdot \{$ height $\}$

Example 6

Let S be a set of 2×2 matrices, whose sum of all diagonal entries is zero. Verify S is a subspace of the vector space of all 2×2 matrices.