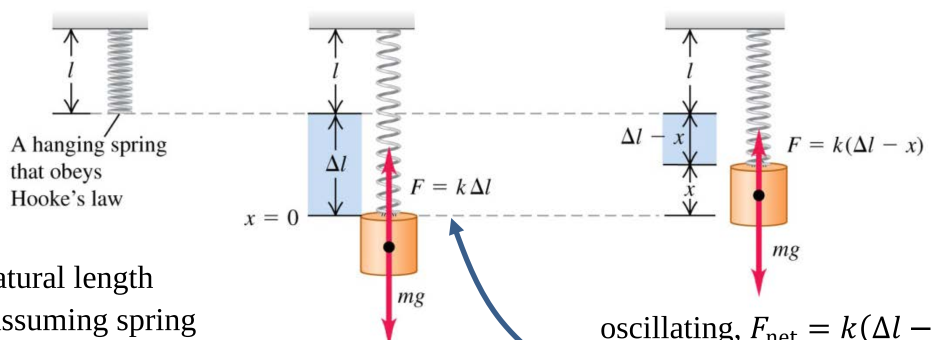
Periodic Motion II

Intended Learning Outcomes – after this lecture you will learn: 1. various types of harmonic oscillators 2. damping effect on a harmonic oscillator 3. forced oscillator and the phenomenon of resonance Textbook Reference: Ch 14.4 – 14.8

Recap from last lecture

A mechanical system with equation of motion $\boxed{d^2x/dt^2 = -\omega^2 x}$ is said to execute simple harmonic motion in the coordinate x, with $\boxed{\text{frequency } f = \omega/2\pi}$, and period $\boxed{T = 1/f = 2\pi/\omega}$.

Vertical Spring and Mass System



natural length (assuming spring massless)

in equilibrium, $mg = k\Delta l$

oscillating, $F_{\text{net}} = k(\Delta l - x) - mg = -kx$

x measured from equilibrium position

<u>Implication</u>: same as horizontal spring and mass system *if* x measured from equilibrium position, *not* from natural (unextended) position.

Example 14.6 P. 471

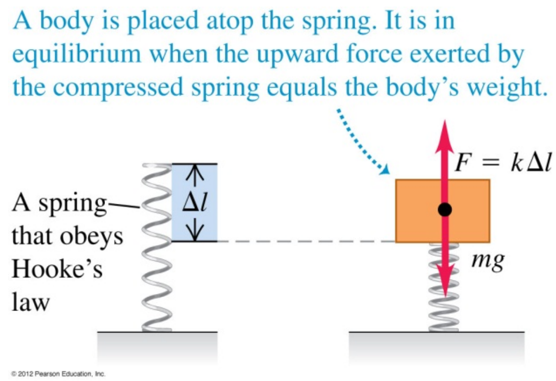
Suppose a car's shock absorbers are worn out so that it provides no damping to oscillations. Its mass is 1000 kg. A 100 kg person sits in it and its center of gravity lowers by 2.8 cm. It then hits a bump and start oscillating.

Spring constant

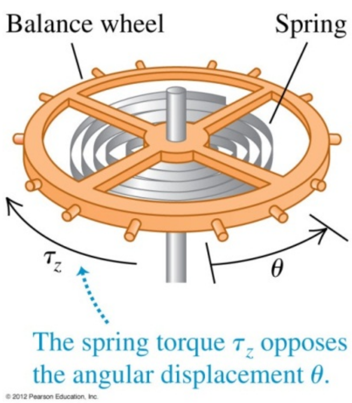
$$k = -\frac{F}{x} = -\frac{980\text{ N}}{-0.028\text{ m}} = 3.5 \times 10^4\text{ kg/s}^2$$

Period of the oscillation

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1100\text{ kg}}{3.5 \times 10^4\text{ kg/s}^2}} = 1.11\text{ s}$$



Angular Oscillation e.g. a clockwork spring



Assumption: restoring *torque* proportional to angular displacement

$$\tau = -\kappa\theta = I\alpha \quad \Rightarrow \quad \boxed{\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta}$$

$$\omega = \sqrt{\frac{\kappa}{I}}, \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}, \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{\kappa}}$$

An imaginary case – A Tunnel through the Earth

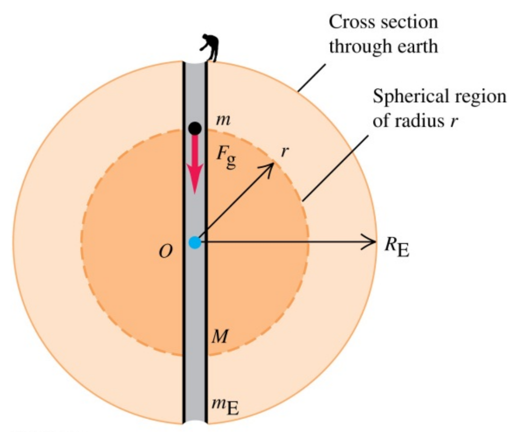
From lecture 13, a mass in the tunnel experience a force

$$F_g = -\frac{Gm_E m}{R_E^3}r \quad \Rightarrow \quad \frac{d^2r}{dt^2} = \frac{F_g}{m} = -\underbrace{\frac{Gm_E}{R_E^3}}_{\omega^2}r$$

⚠ F_g opposite to r

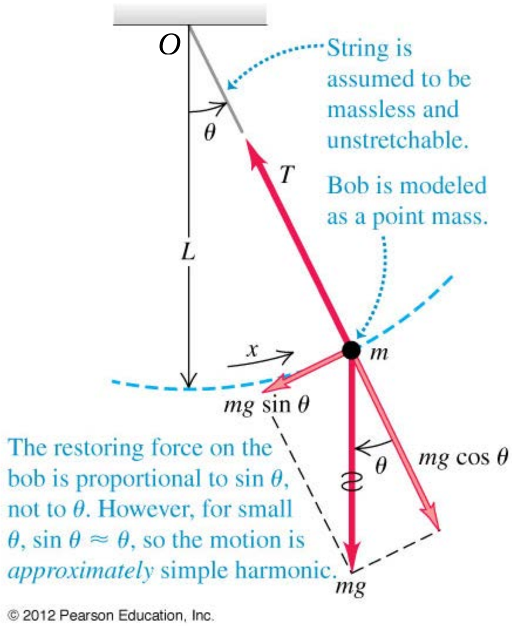
Execute SHM with period with

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R_E^3}{Gm_E}}$$



Simple Pendulum

Galilei observed that a lamp hung from the ceiling of a church swung with constant period



x is the arc length

If θ small enough, $\sin\theta \approx \theta = x/L$

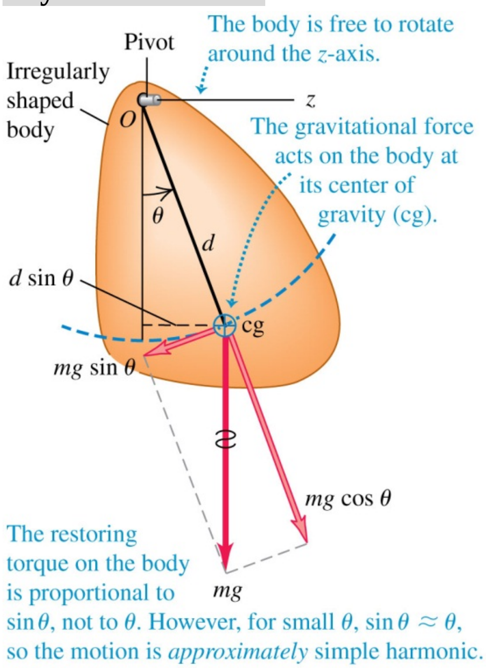
In the tangential direction, the restoring force is

$$-mg\sin\theta \approx -mg\frac{x}{L} = ma_{\tan}$$

$$\Rightarrow \quad \boxed{\frac{d^2x}{dt^2} = -\frac{g}{L}x}$$

$$\omega = \sqrt{\frac{g}{L}}, \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}, \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

Physical Pendulum



In small θ limit:

$$\tau = -(mg)d\sin\theta \approx -(mgd)\theta = I\alpha$$

$$\Rightarrow \quad \boxed{\frac{d^2\theta}{dt^2} = -\frac{mgd}{I}\theta}$$

$$\omega = \sqrt{\frac{mgd}{I}}, \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}},$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$$

Demonstration: Simple Pendulum vs. Meter Stick (same center of mass, different periods)

Question:
The CG of a simple pendulum of mass *m* and length *L* is at a distance *L* from its pivot. The CG of a uniform rod of mass *m* and length 2*L*, pivoted at one end, is also at a distance *L* from its pivot. The period of the rod is (longer / shorter / the same) as the period of the pendulum.

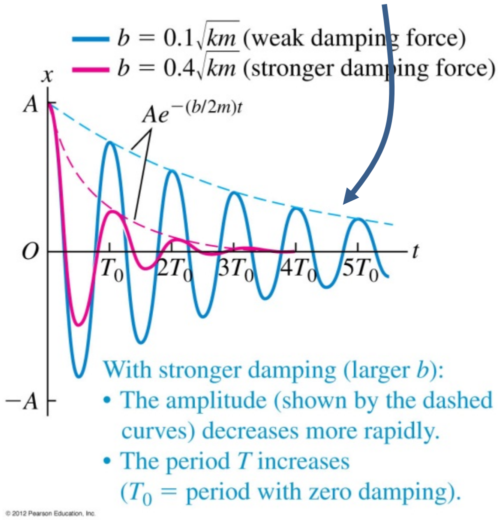
Answer: see inverted text on P. 477 of textbook

Damped Harmonic Oscillator

Suppose oscillator experience fluid resistance at low speed, $f = -bv$ (oppose to v)

$$ma = -kx - bv, \text{ or } \quad m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \quad 2^{\text{nd}} \text{ order } \textit{differential equation}$$

Solution: $$x(t) = \underbrace{Ae^{-\left(\frac{b}{2m}\right)t}}_{\substack{\text{exponentially decaying}\\ \text{amplitude, or envelop,}\\ \text{damp out the oscillation}}} \underbrace{\cos(\omega' t + \phi)}_{\substack{\text{periodic oscillation with}\\ \text{angular frequency } \omega'}} \qquad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$



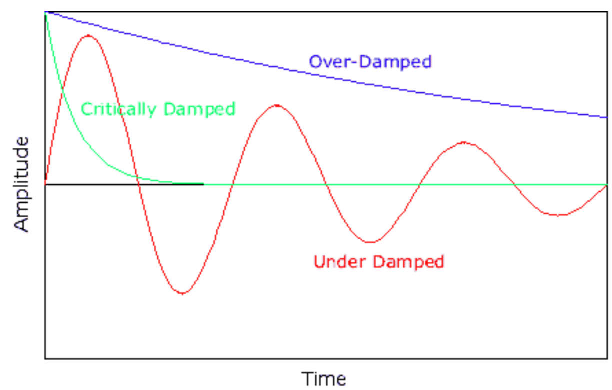
When $b < 2\sqrt{km}$, $\omega' > 0$, called **underdamping**. Stronger damping (larger b), oscillation dies off faster

When $b > 2\sqrt{km}$, ω' imaginary (only if you know complex numbers), no oscillation, solution becomes

$$x(t) = C_1 e^{-a_1 t} + C_2 e^{-a_2 t}$$

When $b = 2\sqrt{km}$, $\omega' = 0$, no oscillation, return to equilibrium position in shortest time, called **critical damping**

Demonstration
[Under, critical, and overdamping]



Rate of energy dissipation

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = mv\underbrace{\frac{dv}{dt}}_{a} + kx\underbrace{\frac{dx}{dt}}_{v} = v\underbrace{(ma + kx)}_{-bv} = -bv^2$$

⚠ $dE/dt < 0$ means energy is decreasing, i.e., energy dissipation

⚠ consistent with $dE/dt = Fv = (-bv)v$

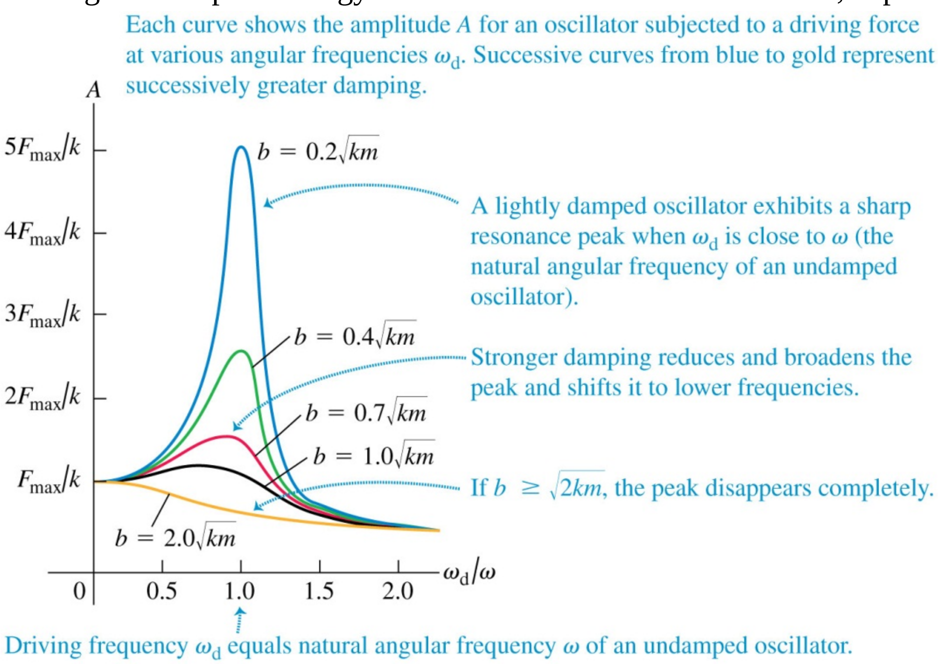
Forced Oscillations

Suppose an external periodic driving force $F(t) = F_{\max}\cos\omega_d t$

$$ma = -kx - bv + F(t) \quad \Rightarrow \quad \boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_{\max}\cos\omega_d t}$$

Oscillator vibrates with ω_d. What if $\omega_d \to \omega = \sqrt{k/m}$, the **natural frequency** of the free (undamped) oscillator?

Driving force deposit energy into the "natural mode" of vibration, expect amplitude to increase



$$A = \frac{F_{\max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

A reaches maximum when $\omega_d \approx \sqrt{k/m}$, called **resonance**

- ⚠ when damping b small, peak is higher, sharper, and closer to the **natural frequency** of the free (undamped) oscillator
- ⚠ resonance vibration can be strong enough to make bridges and buildings collapse

Demonstration: [forced oscillation and resonance of a wine glass]

Galileo Galilei

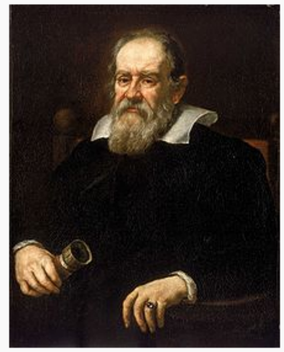
From Wikipedia, the free encyclopedia

Galileo Galilei (Italian pronunciation: [ɡaliˈlɛːo ɡaliˈlɛi]; 15 February 1564[4] – 8 January 1642),[5] was an Italian physicist, mathematician, astronomer, and philosopher who played a major role in the Scientific Revolution. His achievements include improvements to the telescope and consequent astronomical observations and support for Copernicanism. Galileo has been called the "father of modern observational astronomy",[6] the "father of modern physics",[7] the "father of science",[7] and "the Father of Modern Science".[8]

His contributions to observational astronomy include the telescopic confirmation of the phases of Venus, the discovery of the four largest satellites of Jupiter (named the Galilean moons in his honour), and the observation and analysis of sunspots. Galileo also worked in applied science and technology, inventing an improved military compass and other instruments.

Galileo's championing of heliocentrism was controversial within his lifetime, when most subscribed to either geocentrism or the Tychonic system.[9] He met with opposition from astronomers, who doubted heliocentrism due to the absence of an observed stellar parallax.[9] The matter was investigated by the Roman Inquisition in 1615, and they concluded that it could be supported as only a possibility, not an established fact.[9][10] Galileo later defended his views in *Dialogue Concerning the Two Chief World Systems*, which appeared to attack Pope Urban VIII and thus alienated him and the Jesuits, who had both supported Galileo up until this point.[9] He was tried by the Inquisition, found "vehemently suspect of heresy", forced to recant, and spent the rest of his life under house arrest.[11][12] It was while Galileo was under house arrest that he wrote one of his finest works, *Two New Sciences*, in which he summarised the work he had done some forty years earlier, on the two sciences now called kinematics and strength of materials. [13][14]

Galileo Galilei



Portrait of Galileo Galilei by Giusto Sustermans

Born	15 February 1564[1] Pisa,[1] Duchy of Florence, Italy
Died	8 January 1642 (aged 77)[1] Arcetri,[1] Grand Duchy of Tuscany, Italy
Residence	Grand Duchy of Tuscany, Italy
Nationality	Italian (Tuscan)
Fields	Astronomy, physics and mathematics
Institutions	University of Pisa University of Padua
Alma mater	University of Pisa
Academic advisors	Ostilio Ricci[2]
Notable students	Benedetto Castelli Mario Guiducci Vincenzo Viviani[3]
Known for	Kinematics Dynamics Telescopic observational astronomy Heliocentrism

Signature

Notes

His father was the musician Vincenzo Galilei. Galileo Galilei's mistress Marina Gamba (1570 – 21 August 1612?) bore him two daughters (Maria Celeste (Virginia, 1600–1634) and Livia (1601–1659), both of whom became nuns) and a son Vincenzo (1606–1649), a lutenist.

Early life

Galileo was born in Pisa (then part of the Duchy of Florence), Italy, the first of six children of Vincenzo Galilei, a famous lutenist, composer, and music theorist; and Giulia Ammannati. Galileo became an accomplished lutenist himself and would have learned early from his father a healthy scepticism for established authority,[15] the value of well-measured or quantified experimentation, an appreciation for a periodic or musical measure of time or rhythm, as well as the illuminative progeny to expect from a marriage of mathematics and experiment. Three of Galileo's five siblings survived infancy, and the youngest Michelangelo (or Michelagnolo) also became a noted lutenist and composer, although he contributed to financial burdens during Galileo's young adulthood. Michelangelo was incapable of contributing his fair share for their father's promised dowries to their brothers-in-law, who would later attempt to seek legal remedies for payments due. Michelangelo would also occasionally have to borrow funds from Galileo for support of his musical endeavours and excursions. These financial burdens may have contributed to Galileo's early fire to develop inventions that would bring him additional income.

Galileo was named after an ancestor, Galileo Bonaiuti, a physician, university teacher and politician who lived in Florence from 1370 to 1450; at that time in the late 14th century, the family's surname shifted from Bonaiuti (or Buonaiuti) to Galilei. Galileo Bonaiuti was buried in the same church, the Basilica of Santa Croce in Florence, where about 200 years later his more famous descendant Galileo Galilei was buried too. When Galileo Galilei was 8, his family moved to Florence, but he was left with Jacopo Borghini for two years.[1] He then was educated in the Camaldolese Monastery at Vallombrosa, 35 km southeast of Florence.[1]

Although a genuinely pious Roman Catholic,[16] Galileo fathered three children out of wedlock with Marina Gamba. They had two daughters, Virginia in 1600 and Livia in 1601, and one son, Vincenzo, in 1606. Because of their illegitimate birth, their father considered the girls unmarriageable, if not posing problems of prohibitively expensive support or dowries, which would have been similar to Galileo's previous extensive financial problems with two of his sisters.[17] Their only worthy alternative was the religious life. Both girls were sent to the convent of San Matteo in Arcetri and remained there for the rest of their lives.[18] Virginia took the name Maria Celeste upon entering the convent. She died on 2 April 1634, and is buried with Galileo at the Basilica of Santa Croce, Florence. Livia took the name Sister Arcangela and was ill for most of her life. Vincenzo was later legitimised as the legal heir of Galileo, and married Sestilia Bocchineri.[19]

For more information see http://en.wikipedia.org/wiki/Galileo_Galilei