COMP 2711 Discrete Math Tools for Computer Science

2022 Fall Semester - Homework 1

Question 1: Let $Z(x)$, $D(x)$, $F(x)$ and $C(x)$ be the following predicates:

$Z(x)$: "x attended every COMP2711 tutorial classes".

$D(x)$: "x gets F in COMP2711".

$F(x)$: "x cheated in the exams".

$C(x)$: "x has not done any tutorial question".

$K(x)$: "x asked some questions in the telegram group".

Express the following statements using quantifiers, logical connectives, and the predicates above, where the domain consists of all students in COMP2711.

(a) A student gets F in COMP2711 if and only if he/she hasn't done any tutorial question and cheated in the exams.

(b) Some students did some tutorial questions but he/she either absent from some of the tutorial classes or cheated in the exams.

(c) If a student attended every tutorial classes but gets F, then he/she must have cheated in the exams.

(d) Any student who asked some questions in the telegram group and didn't cheat in the exams won't get F.

Question 2: Show that the following two propositions are logically equivalent by developing series of logical equivalences.

(i) $(((p \to q) \leftrightarrow (\neg q \vee r)) \wedge (p \to \neg r)) \to \neg((s \vee r) \leftarrow (\neg r \wedge p))$,

(ii) $(r \vee (\neg q \wedge (s \vee \neg p))) \to (p \wedge (\neg q \vee r))$

Question 3: Determine the truth value of each of these statements if the domain for all variables consists of all real numbers.

(a) $\forall x \exists y (y > 2711x)$

(b) $\exists x \forall y (x \leq y^2)$

(c) $\exists x \exists y \forall z (x^2 + y^2 = z^3)$

(d) $\forall x ((x > 2) \rightarrow (\log_2 x < x - 1) \leftrightarrow \neg \exists x ((x > 2) \wedge (\log_2 x \geq x - 1))$

Question 4: Prove the following statement by contradiction for any integers a, b, c.

"If $a^2 + b^2 = c^2$, then a or b is even"