Lecture 1

General Physics I with Calculus.

General → Broad

Physics → Study of natural phenomena in terms of matters and their interactions.

Calculus → Derivative & Integration

e.g. constant $a \longrightarrow \vec{a}(t)$

$V = u + at \longrightarrow \vec{v}(t) = \vec{v}(0) + \int_0^t \vec{a}(t')\,dt'$

phenomena → e.g. gravity: projectile motion, orbital motion

matters → mass

their interactions → gravitational attraction

Real world is complicated → model is needed.

Reality → model → theory → prediction.

model → keep important features/factors only.

SI unit

Fundamental Quantities (Dimension)

	Time (T)	Length (L)	Mass (M)	Temperature	Charge ...
units:	day, month...	foot, yard...	oz, pond....	°C, °F	...
SI unit:	second (s)	meter (m)	kilogram.	K.	...

Uncertainty and Significant Figure

Every value obtained from measurement has error / uncertainty.

Precision of the value is represented by its significant figure.

e.g. $\pi = 3.14 \longrightarrow$ range: 3.135 to 3.145

or $3.135 \leq x < 3.145$

$\pi = 3.1416 \longrightarrow 3.14155 \leq x < 3.14165$

Experiment

$2.017676 \pm \underbrace{0.0132}_{\text{keep 1 or 2 sig. fig}}$

$2.\underline{017}\underbrace{676}_{\text{not important.}} \pm 0.\underline{013}$ — 3 d.p.

$$\Rightarrow \boxed{2.018 \pm 0.013}$$

Multiplication: $\underset{3.885}{0.745 \times 2.2} = 0.42$ to smallest sig. fig.

Addition / Subtraction: $27.153 + 138.2 - 11.74$

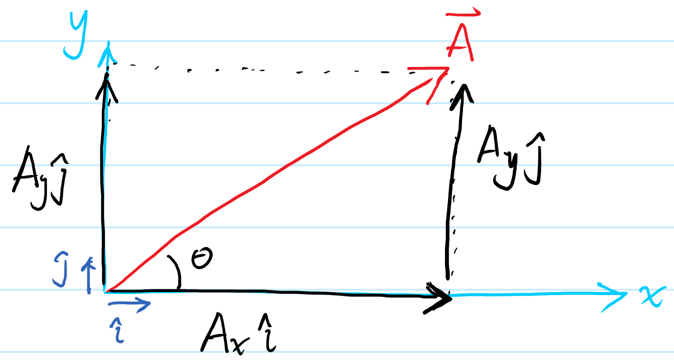
$= 153.6$ to the least d.p.

(the greatest place value.)

Vector

"arrow" — length / magnitude; direction

2D Cartesian Coordinate



$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

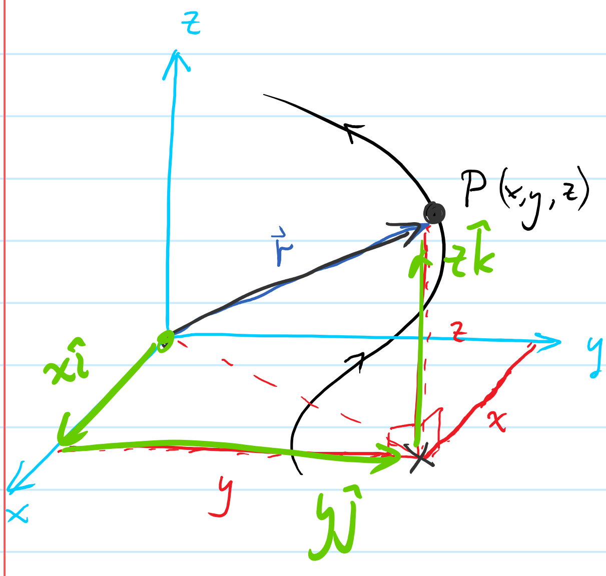
$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\Rightarrow \tan\theta = \frac{A_y}{A_x}$$

$$\begin{cases} A = \sqrt{A_x^2 + A_y^2} \\ \tan\theta = A_y / A_x \end{cases} \Longleftrightarrow \begin{cases} A_x = A\cos\theta \\ A_y = A\sin\theta \end{cases}$$

$$\vec{A} = (A_x, A_y)$$

Kinematics in 3D — trajectory of a particle in vector notation



position Vector : $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

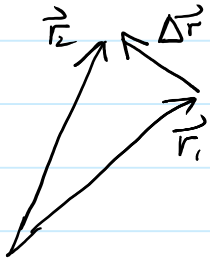
x, y, z are the coordinates of the particle

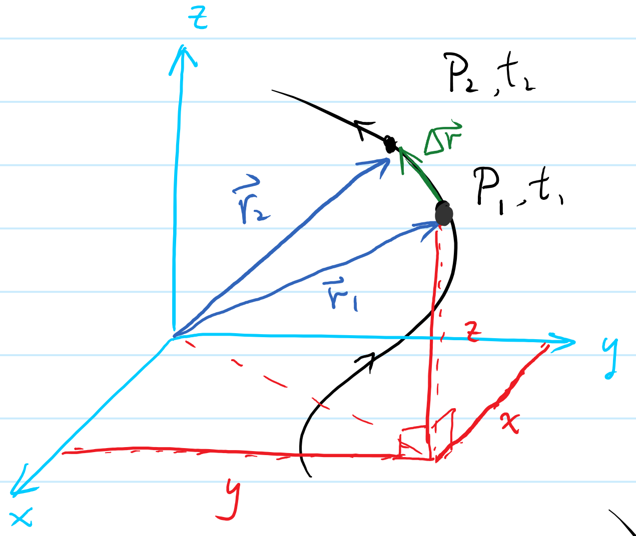
They can be a function of time.

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}.$$

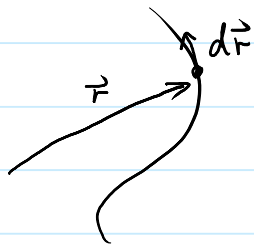
Displacement:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$





Infinitesimal displacement:



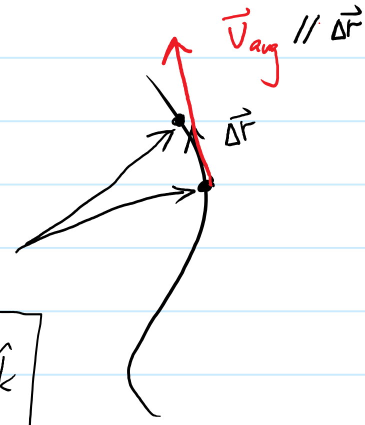
$d\vec{r}$ is always tangential to the trajectory.

Velocity

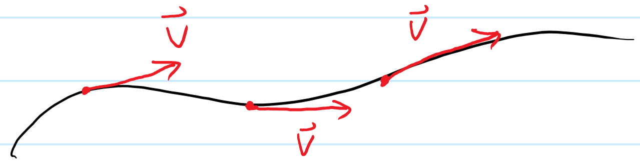
average velocity $= \vec{v}_{avg} = \dfrac{\Delta\vec{r}}{\Delta t}$

instantaneous velocity $= \vec{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{r}}{\Delta t}$

or $\boxed{\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j} + \dfrac{dz}{dt}\hat{k}}$

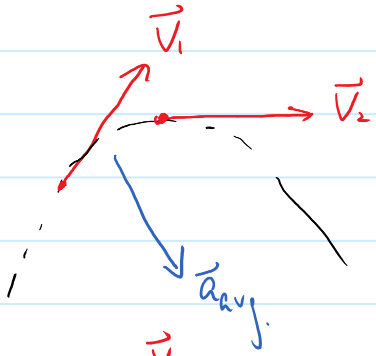


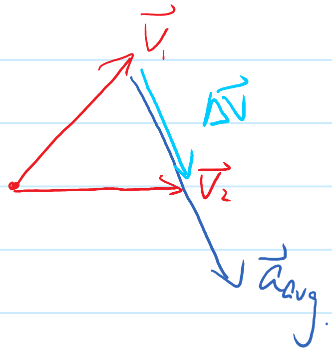
$\vec{v}$ is tangential to trajectory at any point.



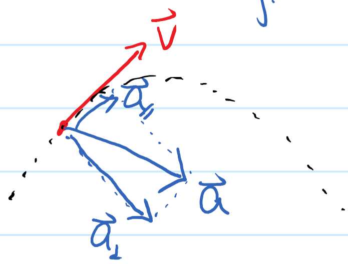
Acceleration

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$





$$\rightarrow \quad \vec{a} = \frac{d\vec{v}}{dt}$$



For every $\vec{a}$, we decompose it into 2 vectors

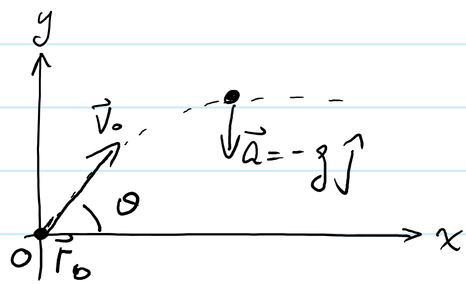
① $\vec{a}_{\parallel} \parallel \vec{v}$, $\vec{a}_{\parallel} = a_{\parallel}\hat{v}$, $a_{\parallel} = \dfrac{d|\vec{v}|}{dt}$ rate of change of speed.

② $\vec{a}_{\perp} \perp \vec{v}$, $|\vec{a}_{\perp}| = \dfrac{V^2}{R}$ centripetal acc.

change direction only

Projectile motion

Given initial position $\vec{r}_0$ and
initial velocity $\vec{v}_0$



$$\vec{a} = -g\hat{j}$$

find $\vec{V}(t)$ and $r(t)$.

$$\vec{V}_0 = V_0 \cos\theta \,\hat{i} + V_0 \sin\theta \,\hat{j}$$

$$\vec{r}_0 = \vec{0}$$

$$\vec{V}(t) = \vec{V}(0) + \int_0^t \vec{a}(t')\,dt'$$

$$= V_0\cos\theta\,\hat{i} + V_0\sin\theta\,\hat{j} + \int_0^t -g\,dt'\,\hat{j}$$

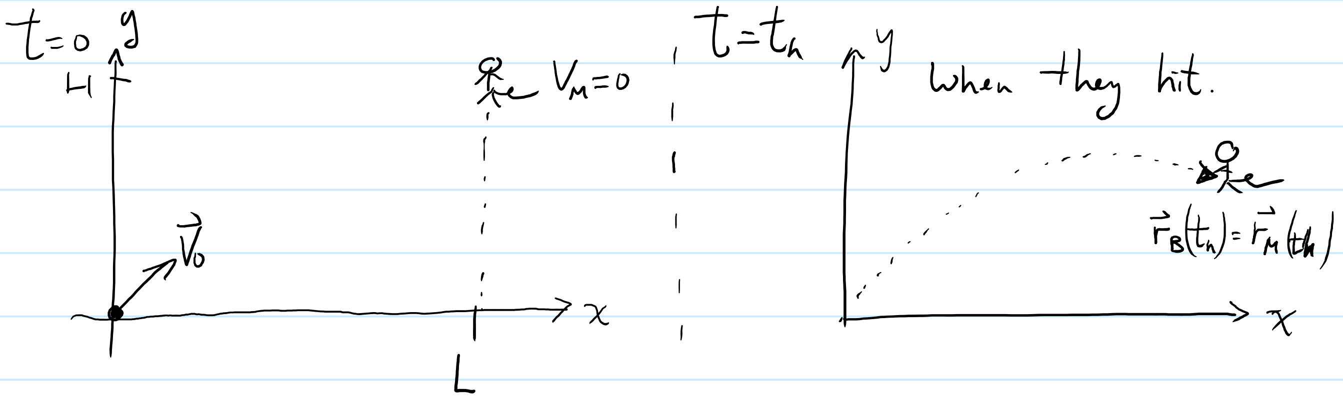
$$\boxed{\vec{V}(t) = V_0\cos\theta\,\hat{i} + (V_0\sin\theta - gt)\hat{j}}$$

$$\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{v}(t')\,dt'$$

$$= \vec{0} + \int_0^t V_0\cos\theta\,dt'\,\hat{i} + \int_0^t (V_0\sin\theta - gt')\,dt'\,\hat{j}$$

$$\boxed{\vec{r}(t) = V_0\cos\theta\, t\,\hat{i} + \left(V_0\sin\theta\, t - \frac{1}{2}gt^2\right)\hat{j}}$$

Example Monkey and Hunter



Position Vector of bullet's trajectory.

$$\vec{r}_B(t) = V_0\cos\theta\, t\,\hat{i} + \left(V_0\sin\theta\, t - \frac{1}{2}gt^2\right)\hat{j}$$

Position vector of monkey's trajectory.

$$\vec{r}_M(t) = L\,\hat{i} + \left(H - \frac{1}{2}gt^2\right)\hat{j}$$

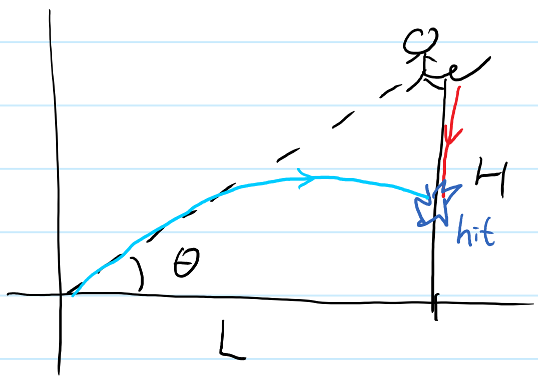
To hit, $\exists\, t_h$ s.t. $\underline{\vec{r}_M(t_h) = \vec{r}_B(t_h)}$

$$\Rightarrow \quad L\,\hat{i} + \left(H - \tfrac{1}{2} g t_h^2\right)\hat{j} = V_0 \cos\theta\, t_h\, \hat{i} + \left(V_0 \sin\theta\, t_h - \tfrac{1}{2} g t_h^2\right)\hat{j}$$

$$\Rightarrow \begin{cases} L = V_0 \cos\theta\, t_h \\ H - \cancel{\tfrac{1}{2} g t_h^2} = V_0 \sin\theta\, t_h - \cancel{\tfrac{1}{2} g t_h^2} \end{cases}$$

$$\Rightarrow \begin{cases} L = V_0 \cos\theta\, t_h \\ H = V_0 \sin\theta\, t_h \end{cases}$$

$$\Rightarrow \quad \tan\theta = \frac{H}{L}$$



Dimension

<u>in equation</u> Dimension on both sides of a equality must be the same.

e.g. $A = \pi r^2$ $\qquad\qquad$ $A = \pi r$

$L^2 = 1 \times L^2$ $\qquad\qquad$ $L^2 \neq L$

Dimensionless

$$[A] = [\pi][r^2]$$

$$\Rightarrow L^2 = 1 \times L^2$$

<u>in elementary function</u>

$[\sin x] = 1$ dimensionless

so as the argument : $[x] = 1$

e.g. Suppose in oscillating motion

$x(t) = A\sin(\omega t) \Rightarrow [\omega t] = 1$

$$[\omega] = \frac{1}{[t]} = \frac{1}{T} = \text{frequency.}$$