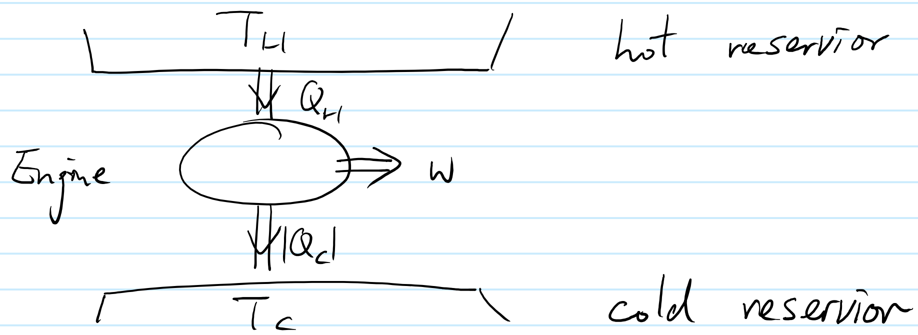
Heat Engine

- operates between two reservoirs : one hot & one cold.
  - large bath with fixed temperature.
  - allows heat extracting from or absorbing into it without changing its temperature.
- cyclic process.
- Absorb heat & Do work — e.g. steam engine
  or
  Receive work & Release heat — e.g. refrigerator.
- It is the working sustance doing work & exchanging heat
  e.g. coolant in refrigerator & air-conditioner

Schematic Picture



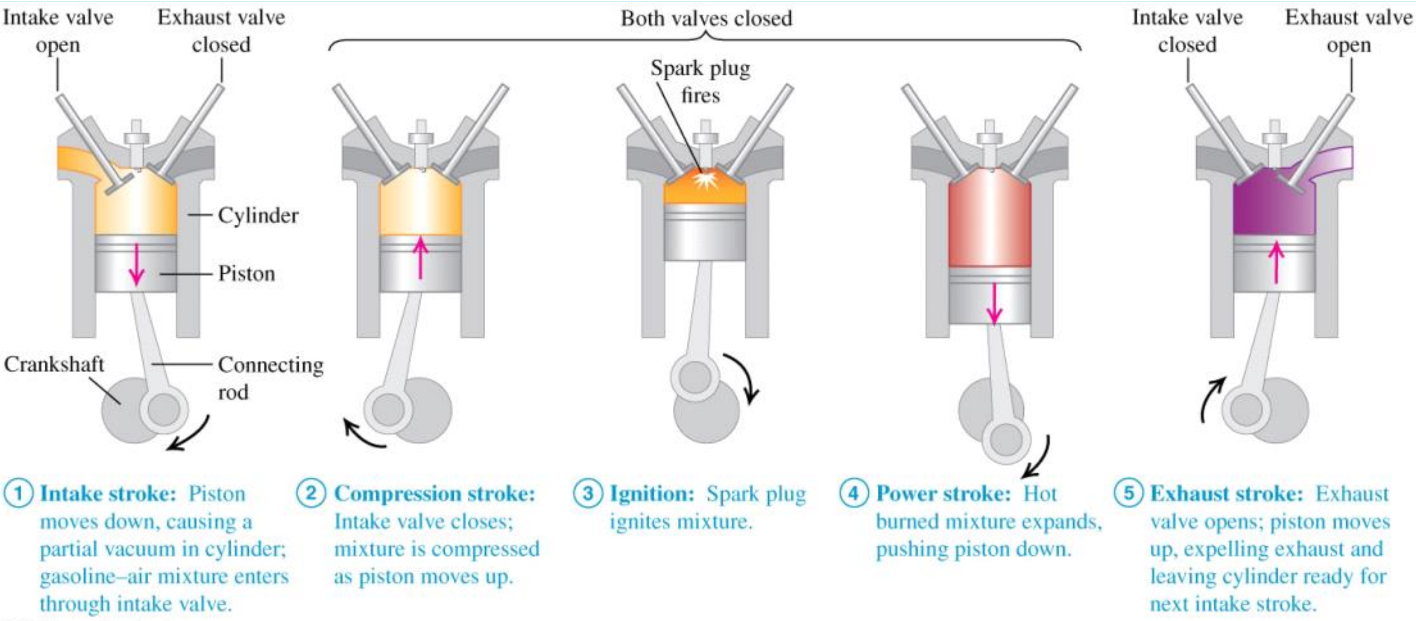
$\Delta U = 0$, $Q = |Q_H| - |Q_c|$

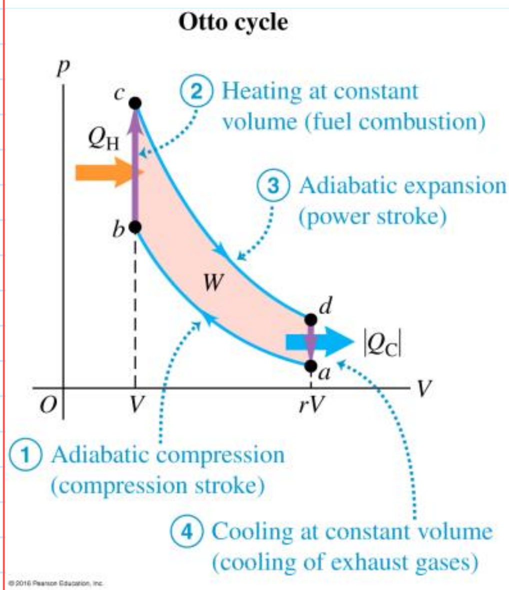
$\Rightarrow \quad Q = W + \overset{\nearrow 0}{\Delta U}$

$W = |Q_H| - |Q_c|$

Efficiency : $\boxed{e = \dfrac{\text{useful work}}{\text{energy input}} = \dfrac{W}{|Q_H|} = 1 - \dfrac{|Q_c|}{|Q_H|}}$

Practical Example: Otto Engine





$$Q_H = nC_v(T_c - T_b) > 0$$

$$Q_c = nC_v(T_a - T_d) < 0$$

$$e = 1 - \frac{|Q_c|}{|Q_H|}$$

$$= 1 - \frac{nC_v(T_d - T_a)}{nC_v(T_c - T_b)}$$

$$= 1 - \frac{T_d - T_a}{T_c - T_b}$$

for the 2 adiabatic processes.

$$T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1} \quad , \quad T_c V_c^{\gamma-1} = T_d V_d^{\gamma-1}$$

$$V_a = V_d = rV$$
$$V_b = V_c = V$$

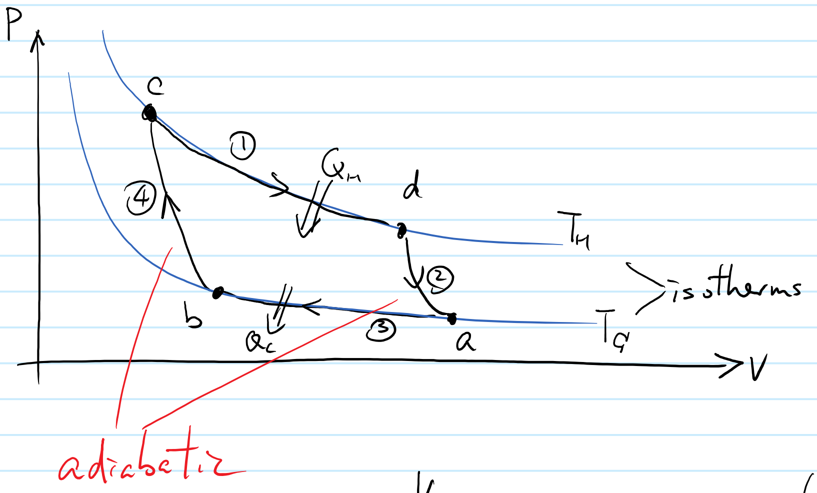
$$T_a (rV)^{\gamma-1} = T_b V^{\gamma-1} \quad , \quad T_c V^{\gamma-1} = T_d (rV)^{\gamma-1}$$

$$T_b = r^{\gamma-1} T_a \quad , \quad T_c = r^{\gamma-1} T_d$$

$$e_{otto} = 1 - \frac{T_d - T_a}{r^{\gamma-1}(T_d - T_a)} = 1 - \frac{1}{r^{\gamma-1}} \quad \text{or} \quad 1 - \frac{T_a}{T_b}$$

Carnot Engine (Carnot cycle)

2 isothermal + 2 adiabatic processes.



Consider ② & ④

$$T_C V_b^{\gamma-1} = T_H V_c^{\gamma-1}$$

$$T_C V_a^{\gamma-1} = T_H d^{\gamma-1}$$

$$\Rightarrow \left(\frac{V_b}{V_a}\right)^{\gamma-1} = \left(\frac{V_c}{V_d}\right)^{\gamma-1}$$

$$\Rightarrow \frac{V_b}{V_a} = \frac{V_c}{V_d}$$

$$Q_C = W_{ab} = \int_{V_a}^{V_b} p\,dV = nRT_C \ln\left(\frac{V_b}{V_a}\right) < 0$$

$$Q_H = W_{cd} = \int_{V_c}^{V_d} p\,dV = nRT_H \ln\left(\frac{V_d}{V_c}\right) > 0$$

$$e_{carnot} = 1 - \frac{-T_C \ln\left(\frac{V_b}{V_a}\right)}{T_H \ln\left(\frac{V_d}{V_c}\right)} = 1 - \frac{T_c \ln\left(\frac{V_a}{V_b}\right)}{T_H \ln\left(\frac{V_d}{V_c}\right)}$$

$$\boxed{e_{carnot} = 1 - \frac{T_c}{T_H}}$$

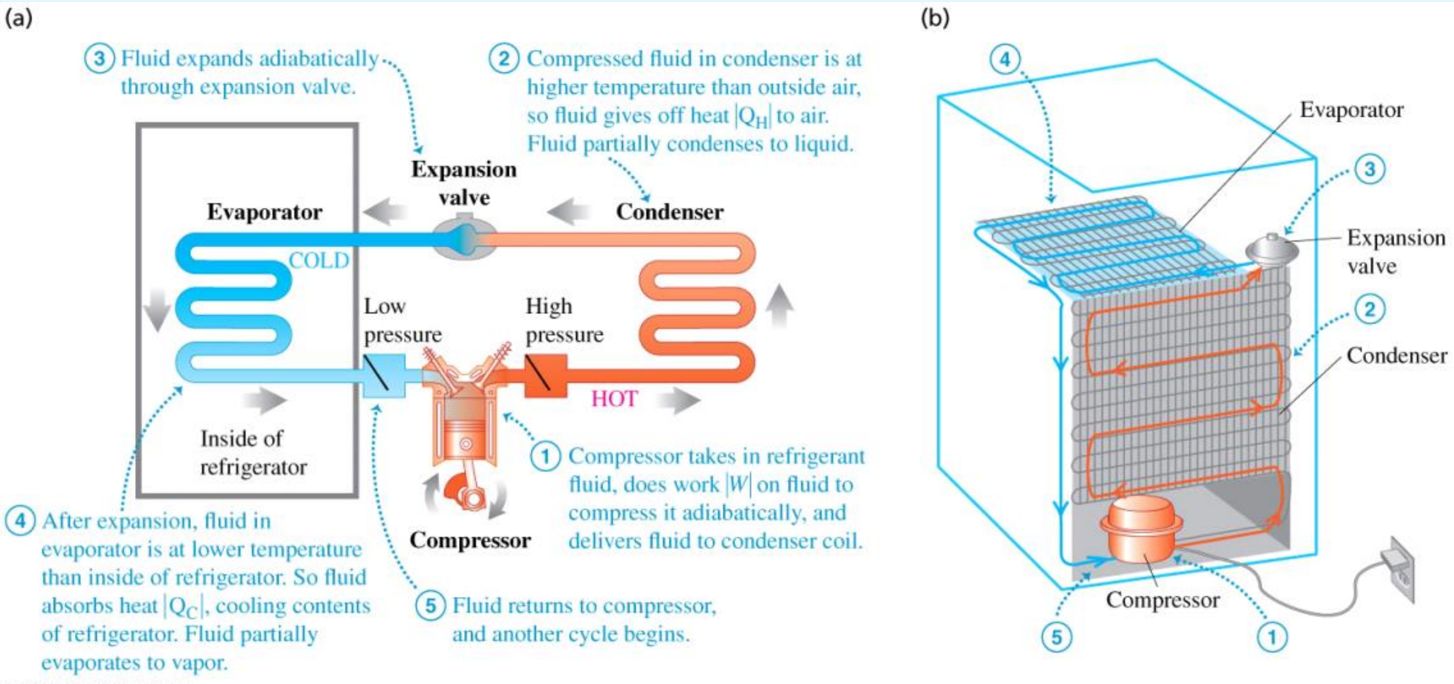
Since : $e = 1 - \dfrac{|Q_c|}{|Q_H|}$

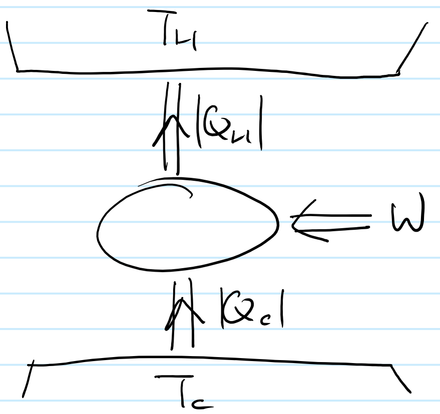
$\Rightarrow$ in Carnot cycle

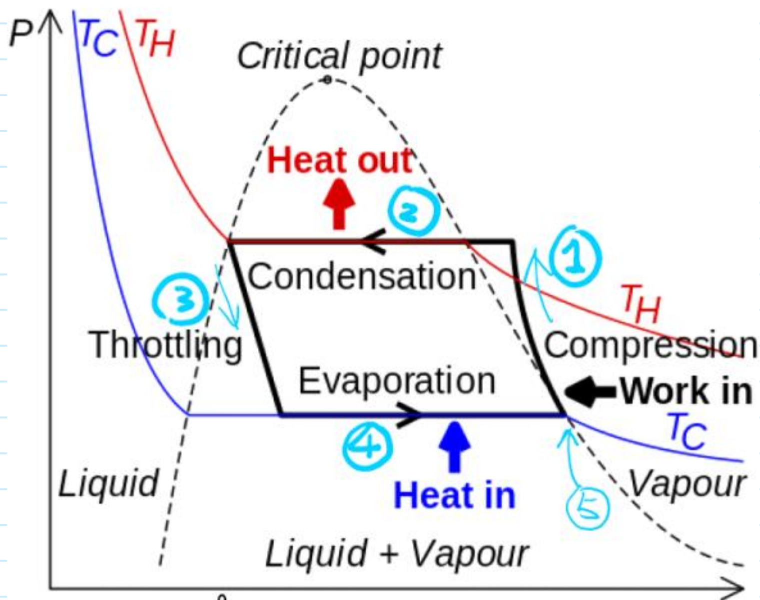
$$\frac{|Q_c|}{|Q_H|} = \frac{T_c}{T_H}$$

$$\Rightarrow \frac{|Q_c|}{T_c} = \frac{|Q_H|}{T_H}$$

Refrigerator







from wikipedia.

Coefficient of Performance

$$K = \frac{|Q_c|}{W} = \frac{|Q_c|}{|Q_H| - |Q_c|}$$

Carnot cycle can be reversed and it runs as a refrigerator. ∵ every steps in Carnot cycle are reversible.

in Carnot cycle: $\dfrac{|Q_c|}{|Q_H|} = \dfrac{T_c}{T_H}$

$$\Rightarrow \quad K_{Carnot} = \frac{T_c}{T_H - T_c}$$