HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2021

MATHEMATICS Compulsory Part
PAPER 2

11:30 am – 12:45 pm (1¼ hours)

INSTRUCTIONS

1. Read carefully the instructions on the Answer Sheet. After the announcement of the start of the examination, you should first stick a barcode label and insert the information required in the spaces provided. No extra time will be given for sticking on the barcode label after the 'Time is up' announcement.

2. When told to open this book, you should check that all the questions are there. Look for the words '**END OF PAPER**' after the last question.

3. All questions carry equal marks.

4. **ANSWER ALL QUESTIONS**. You are advised to use an HB pencil to mark all the answers on the Answer Sheet, so that wrong marks can be completely erased with a clean rubber. You must mark the answers clearly; otherwise you will lose marks if the answers cannot be captured.

5. You should mark only **ONE** answer for each question. If you mark more than one answer, you will receive **NO MARKS** for that question.

6. No marks will be deducted for wrong answers.

Not to be taken away before the end of the examination session

There are 30 questions in Section A and 15 questions in Section B.
The diagrams in this paper are not necessarily drawn to scale.
Choose the best answer for each question.

Section A

1. $\dfrac{(2^n)(8^{3n})}{64^n} =$

   A. 4^n .

   B. 4^{2n} .

   C. 4^{-3n} .

   D. 4^{-4n} .

2. If $m(m-a)=a(1-m)$, then $a=$

   A. m .

   B. $2m$.

   C. m^2 .

   D. $\dfrac{m^2+m}{2}$.

3. $(u+v)(u-v)(u-1) =$

   A. $u^3+u^2+uv^2+v^2$.

   B. $u^3+u^2-uv^2+v^2$.

   C. $u^3-u^2+uv^2+v^2$.

   D. $u^3-u^2-uv^2+v^2$.

4. $\frac{6}{n-6}-\frac{7}{n-7}=$

A. $\frac{n}{(n-6)(n-7)}$.

B. $\frac{n}{(n-6)(7-n)}$.

C. $\frac{n+84}{(n-6)(n-7)}$.

D. $\frac{n+84}{(n-6)(7-n)}$.

5. If $x=6.24$ (correct to 2 decimal places), find the range of values of x.

A. $6.23 < x \leq 6.25$

B. $6.23 \leq x < 6.25$

C. $6.235 < x \leq 6.245$

D. $6.235 \leq x < 6.245$

6. If a, b and c are non-zero constants such that $a(x+3)+b(3x+1) \equiv c(x+2)$, then $a:b=$

A. 1:3 .

B. 1:5 .

C. 3:1 .

D. 5:1 .

7. Let $f(x)=(x+h)(x-3)+k$, where h and k are constants. If $f(0)=f(8)=1$, find k.

A. -14

B. -5

C. 20

D. 31

Go on to the next page

8. Let $p(x)$ be a polynomial. When $p(x)$ is divided by $x+1$, the remainder is -2. If $p(x)$ is divisible by $x-1$, find the remainder when $p(x)$ is divided by x^2-1.

A. $x+1$

B. $x-1$

C. $-x+1$

D. $-x-1$

9. In a school, 33% of the students are overweight. It is given that 60% of the students in the school are girls and 45% of the girls are overweight. If $x\%$ of the boys in the school are overweight, then $x=$

A. 15.

B. 18.

C. 25.

D. 55.

10. The solution of $9x+8\leq 4(x-3)$ or $6-7x>20$ is

A. $x\leq -4$.

B. $x\geq -4$.

C. $x<-2$.

D. $x>-2$.

11. If α and β are non-zero numbers such that $\frac{2\alpha+3\beta}{3\alpha+2\beta}=\frac{7}{10}$, then $\frac{2\alpha+\beta}{\alpha+2\beta}=$

A. 1.

B. $\frac{3}{2}$.

C. $\frac{11}{6}$.

D. $\frac{13}{8}$.

12. If w varies directly as the square of x and inversely as the cube of y, which of the following must be constant?

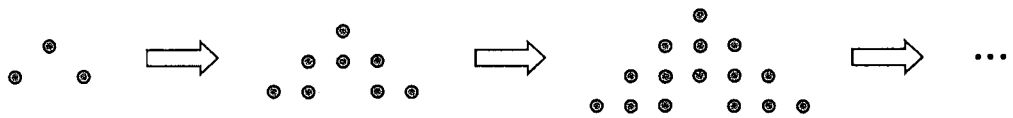
A. $\dfrac{x}{w^2y^6}$

B. $\dfrac{x^2}{wy^3}$

C. $\dfrac{w}{x^2y^3}$

D. $\dfrac{w^2}{xy^2}$

13. In the figure, the 1st pattern consists of 3 dots. For any positive integer n, the $(n+1)$th pattern is formed by adding $(2n+3)$ dots to the nth pattern. Find the number of dots in the 8th pattern.



A. 63

B. 75

C. 80

D. 99

14. Let m and n be real constants. Which of the following statements about the graph of $y=(m-x)^2+n$ must be true?

I. The graph opens upwards.
II. The y-intercept of the graph is positive.
III. The graph passes through the point (n, m).

A. I only

B. II only

C. I and III only

D. II and III only

Go on to the next page

15. The base of a solid right prism is a regular 6-sided polygon of side 8 cm. If the volume of the prism is 288 cm^3, find the total surface area of the prism correct to the nearest cm^2.

A. 166 cm^2

B. 249 cm^2

C. 416 cm^2

D. 748 cm^2

16. The sum of the total surface areas of two solid hemispheres is $351\pi\text{ cm}^2$. If the ratio of the radius of the smaller hemisphere to the radius of the larger hemisphere is $2:3$, then the difference of the volumes of the two hemispheres is

A. $342\pi\text{ cm}^3$.

B. $630\pi\text{ cm}^3$.

C. $684\pi\text{ cm}^3$.

D. $1260\pi\text{ cm}^3$.

17. The area of the sector OAB is $\pi\text{ cm}^2$, where O is the centre of the sector OAB. If $\angle AOB = 90°$, which of the following are true?

I. The radius of the sector OAB is 2 cm.
II. The perimeter of the sector OAB is $\pi\text{ cm}$.
III. The area of the circle passing through O, A and B is $2\pi\text{ cm}^2$.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

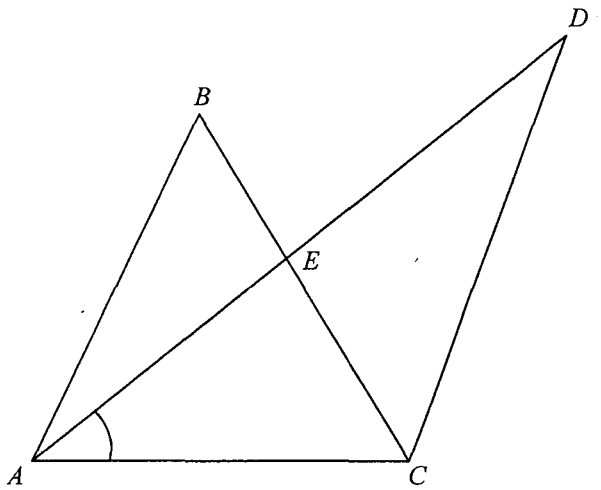
18. In the figure, $AB = BC$ and $AB // CD$. Let E be the point of intersection of AD and BC. If $\angle ADC = 28°$ and $\angle AEB = 94°$, then $\angle CAD =$

A. $30°$.

B. $33°$.

C. $36°$.

D. $39°$.



19. In the figure, $ABCD$ is a rectangle. Let E be a point lying on AC such that BE is perpendicular to AC. BE is produced to the point F such that $CF = AD$. Denote the point of intersection of BF and CD by G. Which of the following are true?

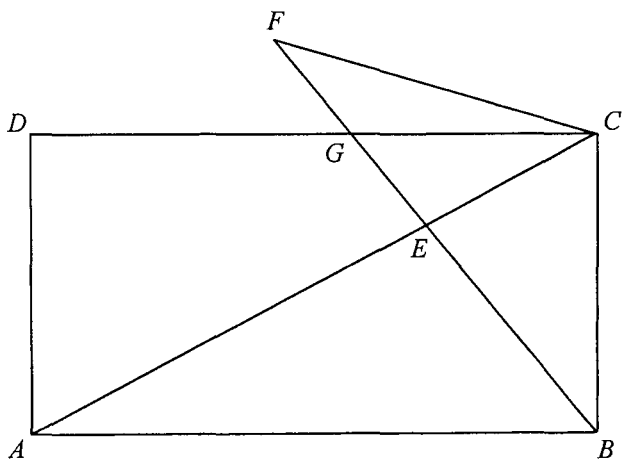
I. $\angle DAE = \angle DGF$
II. $\Delta BCE \sim \Delta CGE$
III. $\Delta BCE \cong \Delta FCE$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III



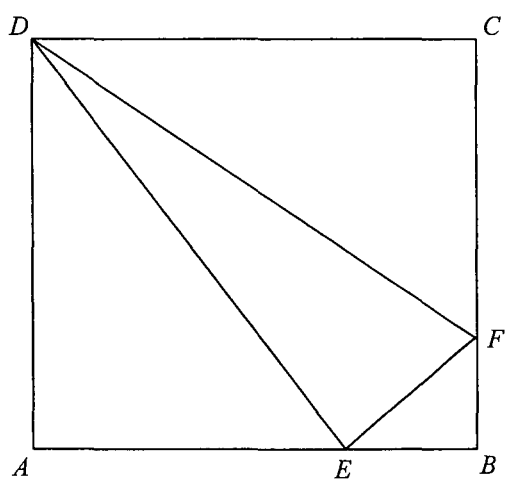
20. In the figure, $ABCD$ is a square. Let E and F be points lying on AB and BC respectively such that $AE = 3BE$ and $\angle DEF = 90°$. If the area of ΔDEF is $25\ \text{cm}^2$, then the area of ΔCDF is

A. $48\ \text{cm}^2$.

B. $50\ \text{cm}^2$.

C. $52\ \text{cm}^2$.

D. $75\ \text{cm}^2$.



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21. If $ABCDEFGH$ is a regular 8-sided polygon, which of the following are true?

I. $AG // BF$
II. $BD = EG$
III. $\angle CAG = 2\angle BDH$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

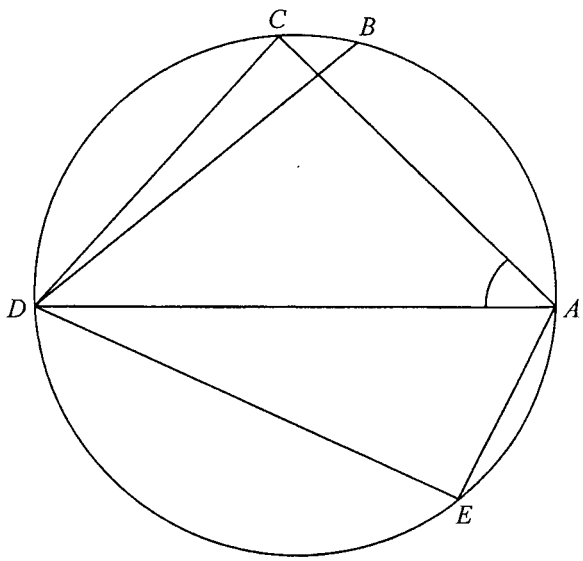
22. In the figure, $ABCDE$ is a circle. If $AC = BD$, $\angle AED = 96°$ and $\angle BDC = 14°$, then $\angle CAD =$

A. $41°$.

B. $44°$.

C. $49°$.

D. $55°$.



23. The coordinates of the point P are $(7, -5)$. P is reflected with respect to the y-axis to the point Q. Q is then rotated clockwise about the origin through $90°$ to the point R. Find the x-coordinate of R.

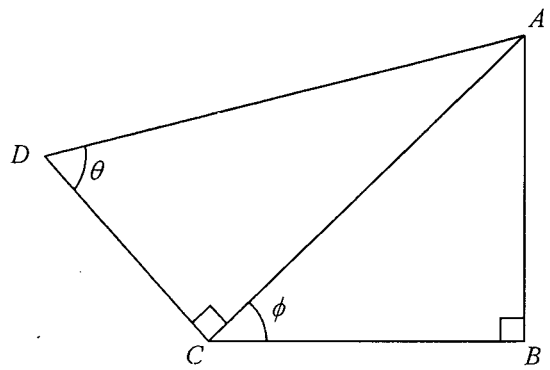
A. -7

B. -5

C. 5

D. 7

24. In the figure, $\frac{AB}{CD} =$



A. $\cos\theta \sin\phi$.

B. $\sin\theta \cos\phi$.

C. $\tan\theta \cos\phi$.

D. $\tan\theta \sin\phi$.

25. The coordinates of the points M and N are $(5, 7)$ and $(6, 8)$ respectively. Let P be a moving point in the rectangular coordinate plane such that $PM = MN$. Find the equation of the locus of P .

A. $x - y + 2 = 0$

B. $x + y - 13 = 0$

C. $x^2 + y^2 - 10x - 14y + 72 = 0$

D. $x^2 + y^2 - 12x - 16y + 98 = 0$

26. The coordinates of the points A , B and C are $(3, 3)$, $(5, 8)$ and $(9, 2)$ respectively. Let P be a point such that AP is a median of ΔABC . Find the equation of the straight line which passes through A and P .

A. $x - 2y + 3 = 0$

B. $2x - 3y + 1 = 0$

C. $2x - 3y + 3 = 0$

D. $3x + 2y - 15 = 0$

27. The slope of the straight line L is 4 . It is given that L and the circle $x^2 + y^2 - 18x - 20y + 96 = 0$ intersect at the points P and Q . If the coordinates of the mid-point of PQ are (s, t) , which of the following must be true?

A. $s - 4t - 49 = 0$

B. $s - 4t + 31 = 0$

C. $s + 4t - 49 = 0$

D. $s + 4t + 31 = 0$

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28. The stem-and-leaf diagram below shows the distribution of the weights (in kg) of a group of workers.

Stem (tens)	Leaf (units)
5	3 6 7
6	1 2 2 2 6 8
7	2 3 4 5 7 7 9 9
8	3 4 5 6 6 7 8

If a worker is randomly selected from the group, find the probability that the weight of the selected worker is not less than the lower quartile of the distribution.

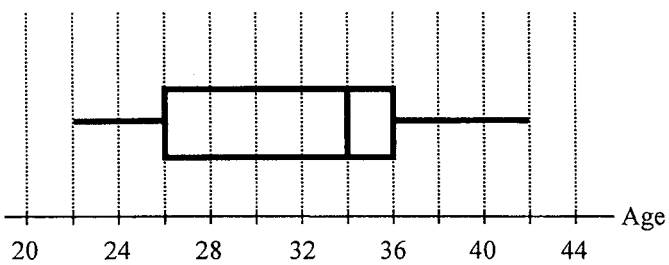
A. $\frac{1}{4}$

B. $\frac{1}{5}$

C. $\frac{1}{6}$

D. $\frac{5}{6}$

29. The box-and-whisker diagram below shows the distribution of the ages of a group of researchers. Find the inter-quartile range of the distribution.



A. 5

B. 10

C. 20

D. 34

30. The mean of 70 integers is 32 . If the mean of 30 of these 70 integers is 24 , then the mean of the remaining 40 integers is

A. 38 .

B. 40 .

C. 43 .

D. 74 .

Section B

31. The H.C.F. and the L.C.M. of three expressions are x^2y^2z and $x^3y^4z^5$ respectively. If the first expression and the second expression are $x^3y^2z^2$ and $x^3y^3z^5$ respectively, then the third expression is

A. x^2y^4z .

B. $x^2y^4z^2$.

C. x^3y^3z .

D. $x^3y^3z^2$.

32. $14\times16^{15}+17\times16^{14}+16^2+17=$

A. $\text{E10100000000021}_{16}$.

B. $\text{F10000000000111}_{16}$.

C. $\text{E110000000000021}_{16}$.

D. $\text{F100000000000111}_{16}$.

33. Let a , b and c be positive constants. On the same rectangular coordinate system, the graph of $y=a+\log_b x$ and the graph of $y=\log_c x$ cut the x-axis at the points S and T respectively. Denote the origin by O. Find $OT:OS$.

A. $1:b^a$

B. $1:c^a$

C. $b^a:1$

D. $c^a:1$

Go on to the next page

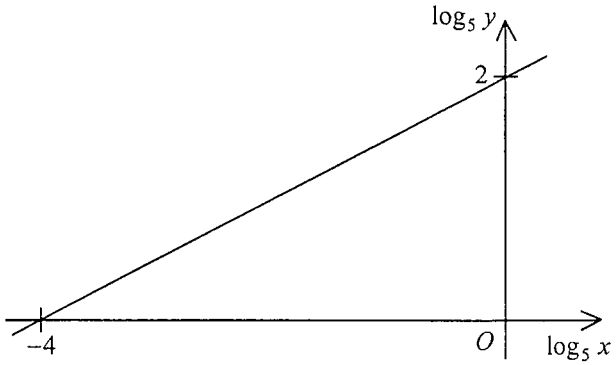
34. The graph in the figure shows the linear relation between $\log_5 x$ and $\log_5 y$. Which of the following must be true?

A. $xy^2 = 625$

B. $x^2y = 625$

C. $\frac{y^2}{x} = 625$

D. $\frac{y}{x^2} = 625$



35. Let α be a real number. Define $u = w + \frac{1}{w}$ and $v = w - \frac{1}{w}$, where $w = \frac{\alpha + i}{\alpha - i}$. Which of the following must be true?

I. u is a real number.
II. The real part of v is equal to 0.
III. The imaginary part of w is equal to the imaginary part of $2w$.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

36. If p, q, r, s is a geometric sequence, which of the following must be true?

I. $ps = qr$
II. $p + s = q + r$
III. $p < q < r < s$

A. I only

B. II only

C. I and III only

D. II and III only

37. Let k be a constant. Find the range of values of k such that $x^2+kx+k+8\geq 0$ for any real number x.

A. $-8\leq k\leq 4$

B. $-4\leq k\leq 8$

C. $k\leq -8$ or $k\geq 4$

D. $k\leq -4$ or $k\geq 8$

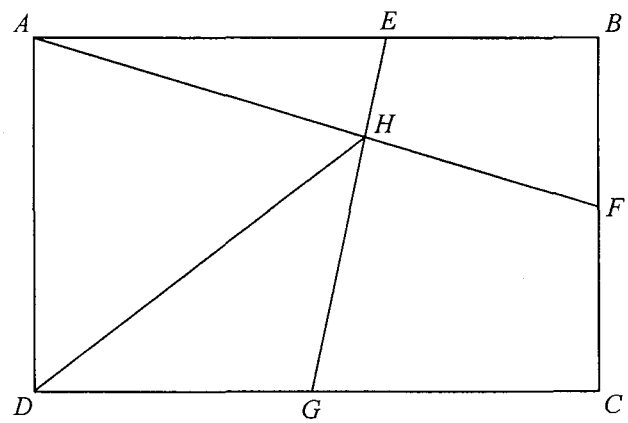
38. The figure shows the rectangle $ABCD$, where $AB=960$ cm and $BC=597$ cm. Let E, F and G be points lying on AB, BC and CD respectively such that $AE=638$ cm, $BF=280$ cm and $CG=480$ cm. Denote the point of intersection of AF and EG by H. Find DH correct to the nearest cm.

A. 728 cm

B. 729 cm

C. 741 cm

D. 742 cm



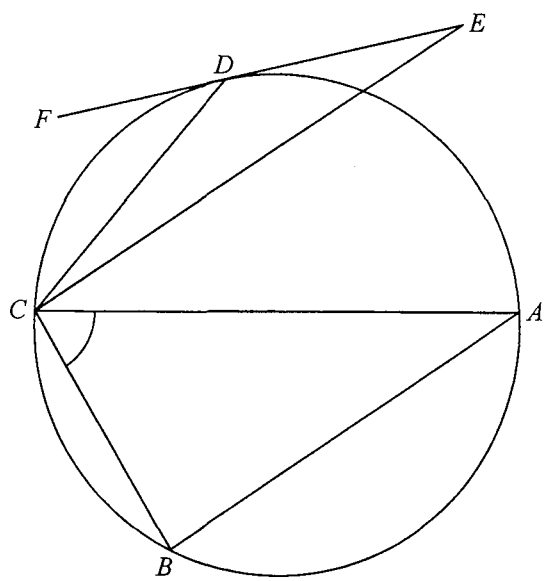
39. In the figure, AC is a diameter of the circle $ABCD$. EF is the tangent to the circle at D such that $AB//EC$. If $\angle CDF=49°$ and $\angle CED=31°$, then $\angle ACB=$

A. 49°.

B. 57°.

C. 59°.

D. 67°.



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40. If the straight line $4x=3y$ and the circle $x^2+y^2-4x-22y+75=0$ intersect at the points M and N, then the equation of the circle with MN as a diameter is

A. $(x-6)^2+(y-8)^2=25$.

B. $(x-8)^2+(y-6)^2=25$.

C. $(x-6)^2+(y-8)^2=100$.

D. $(x-8)^2+(y-6)^2=100$.

41. Let O be the origin. The coordinates of the point P are $(26,-18)$. If the coordinates of the orthocentre of ΔOPQ are $(21,-3)$, then the y-coordinate of Q is

A. -30.

B. -10.

C. 10.

D. 30.

42. A committee is formed by 20 students and 10 teachers. If 7 members are selected from the committee to form a choir consisting of at least 4 students, how many different choirs can be formed?

A. 581 400

B. 873 120

C. 1 162 800

D. 1 744 200

43. A bag contains 7 red balls, 3 yellow balls and 5 black balls. A child repeats drawing one ball at a time randomly from the bag without replacement until a black ball is drawn. Find the probability that the child needs at most three draws.

A. $\frac{4}{7}$

B. $\frac{5}{9}$

C. $\frac{19}{27}$

D. $\frac{67}{91}$

44. In an examination, the mean of the examination scores is 45 marks. A boy gets 25 marks in the examination and his standard score is -5. If the standard score of a girl in the examination is 7, then her examination score is

A. 4 marks.

B. 53 marks.

C. 73 marks.

D. 80 marks.

45. It is given that $T(n)$ is the nth term of an arithmetic sequence. Let x_1, y_1 and z_1 be the median, the range and the variance of the group of numbers $\{T(1), T(2), T(3), \ldots, T(49)\}$ respectively while x_2, y_2 and z_2 be the median, the range and the variance of the group of numbers $\{T(51), T(52), T(53), \ldots, T(99)\}$ respectively. Which of the following must be true?

I. $x_1 < x_2$

II. $y_1 = y_2$

III. $z_1 > z_2$

A. I only

B. II only

C. I and III only

D. II and III only

END OF PAPER