HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION

MATHEMATICS Extended Part Module 2 (Algebra and Calculus)

(Sample Paper)

Time allowed: 2 hours 30 minutes

This paper must be answered in English

INSTRUCTIONS

- 1. This paper consists of Section A and Section B. Each section carries 50 marks.
- 2. Answer **ALL** questions in this paper.
- 3. All working must be clearly shown.
- 4. Unless otherwise specified, numerical answers must be exact.

Not to be taken away before the end of the examination session

FORMULAS FOR REFERENCE

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

 $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$2\sin A\cos B = \sin (A+B) + \sin (A-B)$$

$$2\cos A\cos B = \cos (A+B) + \cos (A-B)$$

$$2 \sin A \sin B = \cos (A - B) - \cos (A + B)$$

$$\sin A + \sin B = 2\sin \frac{A+B}{2}\cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$2 \sin A \cos B = \sin (A+B) + \sin (A-B)$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Section A (50 marks)

1. Find $\frac{d}{dx}(\sqrt{2x})$ from first principles.

(4 marks)

A snowball in a shape of sphere is melting with its volume decreasing at a constant rate of $4\,\mathrm{cm}^3\mathrm{s}^{-1}$. When its radius is 5 cm, find the rate of change of its radius.

(4 marks)

The slope at any point (x, y) of a curve is given by $\frac{dy}{dx} = 2x \ln(x^2 + 1)$. It is given that the curve passes through the point (0,1).

Find the equation of the curve.

(4 marks)

4. Find
$$\int \left(x^2 - \frac{1}{x}\right)^4 dx$$
.

(4 marks)

5. By considering $\sin \frac{\pi}{7} \cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{3\pi}{7}$, find the value of $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{3\pi}{7}$. (4 marks)

Let C be the curve $3e^{x-y} = x^2 + y^2 + 1$. Find the equation of the tangent to C at the point (1,1).

(5 marks)

7. Solve the system of linear equations

$$\begin{cases} x + 7y - 6z = -4 \\ 3x - 4y + 7z = 13 \\ 4x + 3y + z = 9 \end{cases}$$

(5 marks)

8. (a) Using integration by parts, find $\int x \cos x \, dx$.

(b)

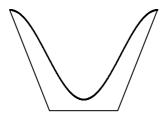
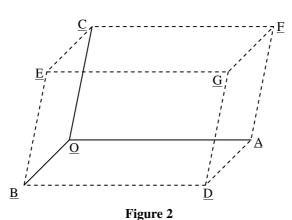


Figure 1

The **inner surface** of a container is formed by revolving the curve $y = -\cos x$ (for $0 \le x \le \pi$) about the y-axis (see Figure 1). Find the capacity of the container.

(6 marks)

9.



Let $\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j}$, $\overrightarrow{OB} = 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 3\mathbf{i} + \mathbf{j} + 5\mathbf{k}$. Figure 2 shows the parallelepiped OADBECFG formed by \overrightarrow{OA} , \overrightarrow{OB} and \overrightarrow{OC} .

- (a) Find the area of the parallelogram *OADB*.
- (b) Find the volume of the parallelepiped *OADBECFG* .
- (c) If C' is a point different from C such that the volume of the parallelepiped formed by \overrightarrow{OA} , \overrightarrow{OB} and $\overrightarrow{OC'}$ is the same as that of OADBECFG, find a possible vector of $\overrightarrow{OC'}$.

(6 marks)

- 10. Let $0^{\circ} < \theta < 180^{\circ}$ and define $A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$.
 - (a) Prove, by mathematical induction, that

$$A^{n} = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$$

for all positive integers n.

- (b) Solve $\sin 3\theta + \sin 2\theta + \sin \theta = 0$.
- (c) It is given that $A^3 + A^2 + A = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$. Find the value(s) of a.

(8 marks)

Section B (50 marks)

11. Let
$$A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$
, $P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

- (a) Let I and O be the 3×3 identity matrix and zero matrix respectively.
 - (i) Prove that $P^3 2P^2 P + I = O$.
 - (ii) Using the result of (i), or otherwise, find P^{-1} .

(5 marks)

- (b) (i) Prove that $D = P^{-1}AP$.
 - (ii) Prove that D and A are non-singular.
 - (iii) Find $(D^{-1})^{100}$. Hence, or otherwise, find $(A^{-1})^{100}$.

(7 marks)

- 12. Let $f(x) = \frac{4}{x-1} \frac{4}{x+1} 1$, where $x \neq \pm 1$.
 - (a) (i) Find the x- and y-intercept(s) of the graph of y = f(x).
 - (ii) Find f'(x) and prove that

$$f''(x) = \frac{16(3x^2 + 1)}{(x-1)^3(x+1)^3}$$

for $x \neq \pm 1$.

(iii) For the graph of y = f(x), find all the extreme points and show that there are no points of inflexion.

(6 marks)

(b) Find all the asymptote(s) of the graph of y = f(x).

(2 marks)

(c) Sketch the graph of y = f(x).

(3 marks)

(d) Let S be the area bounded by the graph of y = f(x), the straight lines x = 3, x = a (a > 3) and y = -1.

Find S in terms of a.

Deduce that $S < 4 \ln 2$.

(3 marks)

13. (a) Let a > 0 and f(x) be a continuous function.

Prove that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$.

Hence, prove that $\int_0^a f(x) dx = \frac{1}{2} \int_0^a [f(x) + f(a - x)] dx$.

(3 marks)

(b) Show that $\int_0^1 \frac{dx}{x^2 - x + 1} = \frac{2\sqrt{3}\pi}{9}$.

(5 marks)

(c) Using (a) and (b), or otherwise, evaluate $\int_0^1 \frac{dx}{(x^2 - x + 1)(e^{2x - 1} + 1)}$.

(6 marks)

14.

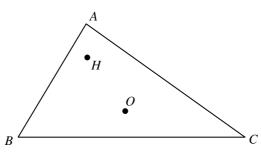


Figure 3

In Figure 3, $\triangle ABC$ is an acute-angled triangle, where O and H are the circumcentre and orthocentre respectively. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OH} = \mathbf{h}$

(a) Show that $(\mathbf{h} - \mathbf{a}) / (\mathbf{b} + \mathbf{c})$.

(3 marks)

- (b) Let $\mathbf{h} \mathbf{a} = t(\mathbf{b} + \mathbf{c})$, where t is a non-zero constant. Show that
 - (i) $t(\mathbf{b} + \mathbf{c}) + \mathbf{a} \mathbf{b} = s(\mathbf{c} + \mathbf{a})$ for some scalar s,
 - (ii) $(t-1)(\mathbf{b}-\mathbf{a})\cdot(\mathbf{c}-\mathbf{a})=0$.

(5 marks)

(c) Express \mathbf{h} in terms of \mathbf{a} , \mathbf{b} and \mathbf{c} .

(2 marks)

END OF PAPER