HKDSE
MATH EP
M2

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HONG KONG DIPLOMA OF
SECONDARY EDUCATION EXAMINATION

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)
MOCK EXAM 1
Question-Answer Book

Time allowed: 2½ hours
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
2. This paper consists of **TWO** sections, A and B.
3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
8. The diagrams in this paper are not necessarily drawn to scale.
9. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

Please stick the barcode label here.

Candidate Number									

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

Answers written in the margins will not be marked.

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SECTION A (50 marks)

2016

1. Expand $(4-x)^3$. Hence, find the constant term in the expansion of $(4-x)^3\left(1+\frac{6}{x}\right)^4$.

(5 marks)

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2013

2. Find $\dfrac{d}{dx}(\sin x)$ from first principles.

(4 marks)

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2016

3. Consider the curve C: $y = 4 \ln x$, where $x > 1$. Let O be the origin and P be a point lying on C. The vertical line which passes through P cuts the x-axis at the point Q. Denote the x-coordinate of Q by u.

(a) Express the area of ΔOPQ in terms of u.

(b) If P moves along C such that PQ decreases at a constant rate of 4 units per second, find the rate of change of the area of ΔOPQ when $u = e$.

(5 marks)

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2014

4. It is given that the curve $x\ln y^2 + y^2 = 9$ cuts the y-axis at two points. Find the equations of the tangents to the curve at these two points.

(7 marks)

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2016

5. (a) Using mathematical induction, prove that $\sum_{k=1}^{n}(-1)^k k = \frac{(-1)^n(2n+1)-1}{4}$ for all positive integers n.

(b) Using (a), evaluate $\sum_{k=3}^{2017}(-1)^k(k+1)$.

(6 marks)

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6. (a) Show that $\sin 3x = 3\sin x - 4\sin^3 x$.

(b) Using (a) and the fact that $\sin 108° = \sin 72°$, show that $\cos 36°$ is a root of the equation $4x^2 - 2x - 1 = 0$. Hence find the exact value of $\cos 36°$.

(7 marks)

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7. (a) Using the substitution $x = 3 + 3\sin\theta$, find $\int \sqrt{x(6-x)}\, dx$.

(b) Let R be the region bounded by the curve $x^2 + y^4 - 6x = 0$ in the first quadrant. Find the volume of the solid of revolution generated by revolving R about the x-axis.

(8 marks)

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2016

8. Let n be a positive integer.

(a) Define $M = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Evaluate

(i) M^2,

(ii) M^n,

(iii) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^n$.

(b) Evaluate

(i) $\sum_{k=1}^{n} 2^k$,

(ii) $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}^n$.

(8 marks)

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SECTION B (50 marks)

2016

9. Define $f(x) = x^3 + px^2 + qx + 1$ for all real numbers x, where p and q are constants. Denote the curve $y = f(x)$ by C. It is given that $P(-2, 9)$ is a stationary point of C.

(a) Find p and q. (3 marks)

(b) Is P a minimum point of C? Explain your answer. (2 marks)

(c) Find the relative extreme points(s) of C. (2 marks)

(d) Find the point(s) of inflexion of C. (2 marks)

(e) Let L be the tangent to C at P. Find the area of the region bounded by C and L. (4 marks)

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10. (a) Suppose $\frac{\pi}{2} < k < \pi$.

(i) Show that $\int_{\frac{\pi}{4}}^{k-\frac{\pi}{4}} \ln\sin(k-x)\,dx = \int_{\frac{\pi}{4}}^{k-\frac{\pi}{4}} \ln\sin x\,dx$.

(ii) Hence show that $\int_{\frac{\pi}{4}}^{k-\frac{\pi}{4}} \ln(\sin k\cot x - \cos k)\,dx = 0$.

(6 marks)

(b) (i) Show that $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x\csc^2 x}{\cot x + 1}\,dx = \frac{\pi\ln 2}{4} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \ln(\cot x + 1)\,dx$.

(ii) Using the results of (a)(ii) and (b)(i), evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x\csc^2 x}{\cot x + 1}\,dx$.

(6 marks)

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2016

11. (a) Consider the system of linear equations in real variables x, y, z

$$(E):\begin{cases} x - y + z = 2 \\ 2x - y + pz = q \\ 4x+(2-p)y+(4p-5)z = q-8 \end{cases},$$

where p and q are real numbers.

(i) Assume that (E) has a unique solution.

(1) Prove that $p \neq 1$ and $p \neq 3$.

(2) Solve (E).

(ii) Assume that $p = 3$ and (E) is consistent.

(1) Find q.

(2) Solve (E).

(9 marks)

(b) Is there a real solution of the system of linear equations

$$\begin{cases} x - y + z = 2 \\ 8x - 4y + 12z = -8 \\ 8x - 2y + 14z = -20 \end{cases}$$

satisfying $xy = 3z$? Explain your answer.

(3 marks)

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12.

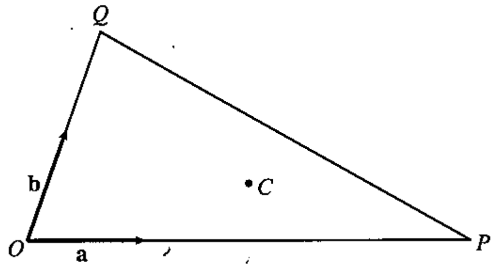

Figure 1

In Figure 1, **a** and **b** are unit vectors with $\mathbf{a} \cdot \mathbf{b} = \dfrac{1}{3}$. Let $\overrightarrow{OP} = 4\mathbf{a}$, $\overrightarrow{OQ} = 2\mathbf{b}$ and $\overrightarrow{OC} = r\mathbf{a} + s\mathbf{b}$, where C is the circumcentre of ΔOPQ, r and s are constants.

(a) By considering $\overrightarrow{OP} \cdot \overrightarrow{CM}$, where M is the mid-point of OP, show that $3r + s = 6$.

(3 marks)

(b) Find the values of r and s.

(3 marks)

(c)

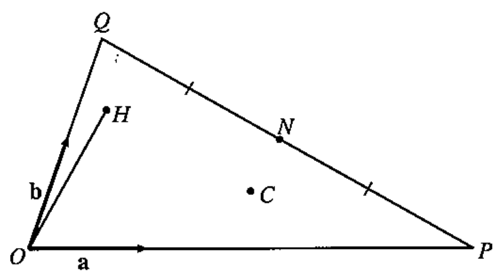

Figure 2

In Figure 2, H is the orthocentre of ΔOPQ and N is the mid-point of PQ. Let $OH : CN = k : 1$, where k is a constant. By expressing $\overrightarrow{OH}$ in terms of k, **a** and **b**, find $\overrightarrow{OH}$ in terms of **a** and **b**.

(7 marks)

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