

## HKDSE MATH CORE 2021 Past Paper I

### 1. HKDSE MATH CORE 2021 Past Paper I Q1

Simplify  $(\alpha\beta^3)(\alpha^{-2}\beta^4)^5$  and express your answer with positive indices.

(3 marks)

### 2. HKDSE MATH CORE 2021 Past Paper I Q2

Make  $a$  the subject of the formula  $\frac{4-3a}{b} = 5$ .

(3 marks)

### 3. HKDSE MATH CORE 2021 Past Paper I Q3

Factorize

(a)  $6x^2 + xy - 2y^2$ ,

(b)  $8x - 4y - 6x^2 - xy + 2y^2$ .

(3 marks)

### 4. HKDSE MATH CORE 2021 Past Paper I Q4

(a) Find the range of values of  $x$  which satisfy both  $\frac{7(x-2)}{5} > 3(x-1)$  and  $x+4 \geq 0$ .

(b) How many positive integers satisfy both inequalities in (a)?

(4 marks)

### 5. HKDSE MATH CORE 2021 Past Paper I Q5

The number of stickers owned by a boy is 3 times that owned by a girl. If the boy gives 20 of his stickers to the girl, then the number of stickers owned by the girl is 2 times that owned by the boy. Find the total number of stickers owned by the boy and the girl.

(4 marks)

### 6. HKDSE MATH CORE 2021 Past Paper I Q6

The marked price of a shirt is higher than its cost by \$80. The shirt is sold at a discount of 10% on its marked price. After selling the shirt, the percentage profit is 30%. Find the marked price of the shirt.

(4 marks)

### 7. HKDSE MATH CORE 2021 Past Paper I Q7

In a polar coordinate system,  $O$  is the pole. The polar coordinates of the points  $P$  and  $Q$  are  $(r, 80^\circ)$  and  $(r, 140^\circ)$  respectively, where  $r$  is a positive constant. It is given that the distance between  $P$  and  $Q$  is 21. Find

(a)  $\angle POQ$ ,

(b)  $r$ ,

(c) the perimeter of  $\triangle OPQ$ .

(4 marks)

**8. HKDSE MATH CORE 2021 Past Paper I Q8**

In Figure 1,  $AB$  produced and  $CD$  produced meet at the point  $E$ . It is given that  $\angle CAE = \angle BDE$ .

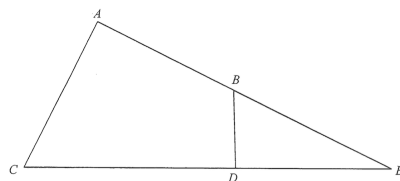


Figure 1

(a) Prove that  $\triangle ACE \sim \triangle DBE$ .

(b) It is given that  $AC = 25$  cm,  $AE = 60$  cm and  $BD = 15$  cm.

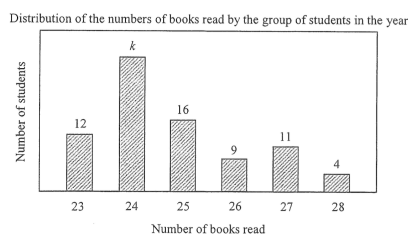
(i) Is  $\triangle ACE$  a right-angled triangle? Explain your answer.

(ii) Find the area of  $\triangle BDE$ .

(5 marks)

**9. HKDSE MATH CORE 2021 Past Paper I Q9**

The bar chart below shows the distribution of the numbers of books read by a group of students in a year.



If a student is randomly selected from the group, then the probability that the selected student reads fewer than 26 books in the year is  $\frac{7}{10}$ .

(a) Find  $k$ .

(b) Write down the range the inter-quartile range and the standard deviation of the distribution.

(5 marks)

**10. HKDSE MATH CORE 2021 Past Paper I Q10**

It is given that  $f(x)$  is partly constant and partly varies as  $(x+4)^2$ . Suppose that  $f(-3) = 0$  and  $f(2) = 105$ .

(a) Find  $f(0)$ .

(3 marks)

(b) Denote the graph of  $y = f(x) + 3$  by  $G$ .

(i) Write down the  $y$ -intercept of  $G$ .

(ii) Find the  $x$ -intercept of  $G$ .

(3 marks)

**11. HKDSE MATH CORE 2021 Past Paper I Q11**

The table below shows the distribution of the numbers of tokens got by the group of children in a game.

Number of tokens got	1	2	3	4	5	6	7
Number of children	15	9	2	5	4	2	5

(a) Find the mean of the distribution.

(2 marks)

(b) Are the median and the mode of the distribution equal? Explain your answer.

(2 marks)

(c) If  $n$  more children play the game and each of them gets 5 tokens, write down

(i) the value of  $n$  such that the mean of the distribution is increased by 1;

(ii) the least value of  $n$  such that the median of the distribution is increased by 2;

(iii) the greatest value of  $n$  such that the mode of the distribution remains unchanged.

(3 marks)

**12. HKDSE MATH CORE 2021 Past Paper I Q12**

The polynomial  $p(x)$  is divisible by  $x - 5$ . When  $p(x)$  is divided by  $x^2 + x + 1$ , the quotient and the remainder are  $2x^2 - 37$  and  $cx + c - 1$  respectively, where  $c$  is a constant.

(a) Find  $c$ .

(3 marks)

(b) Prove that  $x + 3$  is a factor of  $p(x)$ .

(1 marks)

(c) Someone claims that all the roots of the equation  $p(x) = 0$  are real numbers. Is the claim correct? Explain your answer.

(3 marks)

**13. HKDSE MATH CORE 2021 Past Paper I Q13**

The equation of the circle  $C$  is  $x^2 + y^2 - 12x - 16y - 69 = 0$ . Let  $G$  be the centre of  $C$ . Denote the origin by  $O$ .

(a) Find  $OG$ .

(2 marks)

(b) Does  $O$  lie inside  $C$ ? Explain your answer.

(1 marks)

- (c) Let  $P$  be a moving point in the rectangular coordinate plane such that  $OP = GP$ . Denote the locus of  $P$  by  $\Gamma$ . Suppose that  $\Gamma$  cuts  $C$  at the points  $M$  and  $N$ . Find the area of the quadrilateral  $OMGN$ .  
(4 marks)

**14. HKDSE MATH CORE 2021 Past Paper I Q14**

The base radius of the solid right circular cylinder  $X$  and the base radius of the solid right circular cone  $Y$  are equal. The heights of  $X$  and  $Y$  are 20 cm and 24 cm respectively. The volume of the solid right circular cone  $Z$  is equal to the sum of the volume of  $X$  and the volume of  $Y$ . The base radius of  $Z$  is equal to the base diameter of  $X$ . A craftsman finds that the volume of  $Y$  is  $800\pi$  cm<sup>3</sup>.

- (a) Find the base radius of  $Y$ .  
(2 marks)
- (b) Are  $Y$  and  $Z$  similar? Explain your answer.  
(3 marks)
- (c) The craftsman claims that the sum of the curved surface area of  $X$  and the curved surface area of  $Y$  is greater than the curved surface area of  $Z$ . Do you agree? Explain your answer.  
(3 marks)

**15. HKDSE MATH CORE 2021 Past Paper I Q15**

A queue is randomly formed by 7 teachers and 3 students.

- (a) How many different queues can be formed?  
(1 marks)
- (b) Find the probability that no students are next to each other in the queue.  
(3 marks)

**16. HKDSE MATH CORE 2021 Past Paper I Q16**

The straight lines  $L_1$  and  $L_2$  are perpendicular to each other, The  $y$ -intercept of  $L_1$  is 3. It is given that  $L_1$  and  $L_2$  intersect at the point  $(2, 6)$ . Let  $R$  be the region (including the boundary) bounded by  $L_1$ ,  $L_2$  and the  $x$ -axis.

- (a) It is given that  $R$  represents the solution of a system of inequalities. Find the system of inequalities.  
(3 marks)
- (b) Find the least value of  $8x - 5y$ , where  $(x, y)$  is a point lying in  $R$ .  
(2 marks)

**17. HKDSE MATH CORE 2021 Past Paper I Q17**

Let  $A(n)$  be the  $n$ th term of an arithmetic sequence. It is given that  $A(5) = 26$  and  $A(12) = 61$ .

- (a) Find  $A(1)$ .  
(2 marks)

- (b) Suppose that  $\log_2 G(n) = A(n)$  for any positive integer  $n$ . Find the greatest value of  $k$  such that  $\log_8 (G(1)G(2)G(3) \cdots G(k)) < 999$ .  
(5 marks)

18. HKDSE MATH CORE 2021 Past Paper I Q18

- (a) A thin metal sheet  $ABCD$  is in the shape of a trapezium, where  $AD \parallel BC$ . It is given that  $AB = 45$  cm,  $\angle ADC = 70^\circ$  and  $\angle BAD = 50^\circ$ . Find  $CD$ .  
(2 marks)
- (b) The metal sheet  $ABCD$  described in (a) is now given. Let  $E$  be a point lying on  $AD$  such that  $BE$  is perpendicular to  $AD$ . The metal sheet is folded along  $BE$  such that  $AE$  is perpendicular to the plane  $BCDE$ . Three thin triangular metal sheets are placed to this folded metal sheet to form a pyramid ( see Figure2 ). It is found that  $BC = 40$  cm.

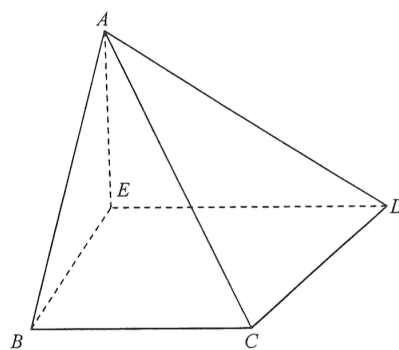


Figure 2

- (i) Find  $\angle CAD$ .
- (ii) Does the angle between the plane  $ACD$  and the plane  $BCDE$  exceed  $30^\circ$ ? Explain your answer.  
(5 marks)

19. HKDSE MATH CORE 2021 Past Paper I Q19

Let  $f(x) = x^2 - 12kx - 14x + 36k^2 + 89k + 53$ , where  $k$  is a positive constant. On the same rectangular coordinate system, denote the vertex of the graph of  $y = f(x)$  and the vertex of the graph of  $y = f(14 - x)$  by  $Q$  and  $R$  respectively.

- (a) Using the method of completing the square, express, in terms of  $k$ , the coordinates of  $Q$ .  
(2 marks)
- (b) Write down, in terms of  $k$ , the coordinates of  $R$ .  
(1 marks)
- (c) The coordinates of the point  $S$  are  $(7, 4 - 3k)$ . Denote the inscribed circle  $\triangle QRS$  by  $C$ .
- (i) Express, in terms of  $k$ , the equation of the straight line which passes through  $Q$  and  $S$ .
- (ii) Express, in terms of  $k$ , the equation of  $C$ .

- (iii) Suppose that  $QS$  is the tangent to  $C$  at the point  $T$ . Let  $U$  be the centre of  $C$ . It is given that the coordinates of the point  $V$  are  $(-29, -14)$ . Is it possible that  $STUV$  is a rectangle? Explain your answer.

(9 marks)