HKDSE
MATH EP
M2

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HONG KONG DIPLOMA OF
SECONDARY EDUCATION EXAMINATION

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)
MOCK EXAM 6
Question-Answer Book

Time allowed: 2½ hours
This paper must be answered in English

Please stick the barcode label here.

Candidate Number									

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
2. This paper consists of **TWO** sections, A and B.
3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
8. The diagrams in this paper are not necessarily drawn to scale.
9. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

Answers written in the margins will not be marked.

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SECTION A (50 marks)

2015

1. (a) Prove that $\tan^2 x = \dfrac{1-\cos 2x}{1+\cos 2x}$.

   (b) Let $f(x) = \sec^4 x - 2\tan^4 x$. Express $f(x)$ in the form $\dfrac{A + B\cos 2x + C\cos 4x}{D + E\cos 2x + F\cos 4x}$, where A, B, C, D, E and F are constants.

(5 marks)

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2. Find the coefficient of x^2 in the expansion of $\left(3x - \frac{2}{x^2}\right)^8$. (3 marks)

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2016

3. Let $f(x) = \dfrac{1}{\sqrt{2x-1}}$. Find $f'(1)$ from first principles. (4 marks)

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4. If $y^2 = \sin x$, show that $\frac{d^2y}{dx^2} = -\frac{1+y^4}{4y^3}$. (4 marks)

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5. Using integration by parts, find $\int \frac{\ln x}{x^3}\,dx$. (4 marks)

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2013

6. Prove, by mathematical induction, that for all positive integers n,

$$1-\frac{1}{2!}-\frac{2}{3!}-\frac{3}{4!}-\cdots-\frac{n}{(n+1)!}=\frac{1}{(n+1)!}.$$

(5 marks)

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7. The graph of $y = Ax + \dfrac{1}{x+B}$ intersects the x-axis at $(1, 0)$ and has the vertical asymptote $x = 2$, where A and B are constants. Find the local extreme point(s) and the other asymptote(s) of the graph. (7 marks)

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8. (a) Using integration by substitution, show that $\int \sqrt{4-x^2}\,dx = 2\sin^{-1}\frac{x}{2} + \frac{1}{2}x\sqrt{4-x^2} + C$, where $-2 \le x \le 2$ and C is a constant.

(b)

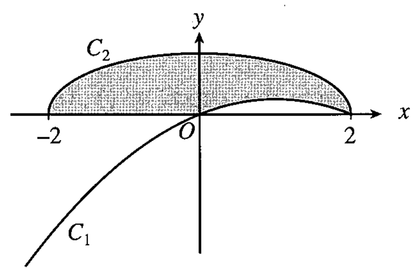

Figure 1

Figure 1 shows the shaded region bounded by the curves $C_1 : y = 2x - x^2$, $C_2 : y = 2\sqrt{4-x^2}$ and the x-axis. Find the area of the shaded region.

(6 marks)

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9. Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors in a plane such that $|\mathbf{a}| = 13$ and $|\mathbf{b}| = 10$. The angle between $\mathbf{a}$ and $\mathbf{b}$ is θ, where $\cos\theta = \dfrac{16}{65}$.

(a) Find $\mathbf{a} \cdot \mathbf{b}$.

(b) Find the value of k if the projection of $(5\mathbf{a} + k\mathbf{b})$ on $\mathbf{b}$ is a unit vector.

(5 marks)

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10. The following table shows the volume and selling price of each bottle of each type of detergent.

	Volume (L)	Selling price ($)
Detergent A	1	100
Detergent B	2	150
Detergent C	5	300

Tom wants to buy x bottles of detergent A, y bottles of detergent B and z bottles of detergent C. Suppose he buys a total of 8 bottles of detergent with a total volume of 14 L.

(a) Tom claims that there is only one set of combination of x, y and z. Do you agree? Explain your answer.

(b) Moreover, suppose Tom spends exactly $1100 to buy the detergent. Do you think that there is only one set of x, y and z? Explain your answer.

(7 marks)

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SECTION B (50 marks)

11. Figure 2 shows a prism $ABCDEFGH$ with a trapezoidal base $ABCD$, where $AB \parallel DC$. The position vectors of A, B and C are $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ and $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ respectively. The area of the trapezoidal base $ABCD$ is 6.

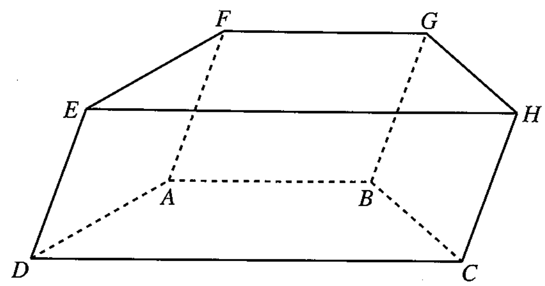

Figure 2

(a) (i) By considering the areas of ΔABC and ΔACD, find $AB : DC$.

(ii) Find the position vector of D.

(6 marks)

(b) Suppose the position vector of F is $4\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

(i) By considering the volume of the prism $ABCHFG$, find the volume of the prism $ABCDEFGH$.

(ii) Find the acute angle between the line FC and the plane $ABCD$. (Give the answer correct to the nearest 0.1°.)

(6 marks)

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12. (a) Let $t = \tan x$. Show that $\cos 2x = \dfrac{1-t^2}{1+t^2}$. (2 marks)

(b) (i) Using (a) and the substitution $t = \tan x$, show that $\displaystyle\int \frac{dx}{3+\cos 2x} = \int \frac{dt}{2t^2+4}$.

(ii) Using integration by substitution, show that $\displaystyle\int \frac{dt}{t^2+2} = \frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + C$, where C is a constant.

(5 marks)

(c) (i) Express $\dfrac{d}{dx}\left(\dfrac{\sin 2x}{3+\cos 2x}\right)$ in the form $\dfrac{P}{3+\cos 2x} + \dfrac{Q}{(3+\cos 2x)^2}$, where P and Q are constants.

(ii) Hence or otherwise, show that $\displaystyle\int_0^{\frac{\pi}{4}} \frac{dx}{(3+\cos 2x)^2} = \frac{3\sqrt{2}}{32}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{48}$.

(5 marks)

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13. Let I be the 3×3 identity matrix, $P = \begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix}$ and $Q = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, where a, b and c are real numbers with $a^2 + b^2 + c^2 = 1$.

(a) (i) Show that QQ^TP is the 3×3 zero matrix.

(ii) Show that $(I + P)^{-1} = \frac{1}{2}(I - P + QQ^T)$

Hence show that $I + P^2 = QQ^T$.

(10 marks)

(b) Let $M = \begin{pmatrix} 0 & -\sqrt{2} & \sqrt{3} \\ \sqrt{2} & 0 & -1 \\ -\sqrt{3} & 1 & 0 \end{pmatrix}$. Evaluate $M + \frac{1}{6}M^3 + \frac{1}{6^2}M^5 + \cdots + \frac{1}{6^{1008}}M^{2017}$.

(4 marks)

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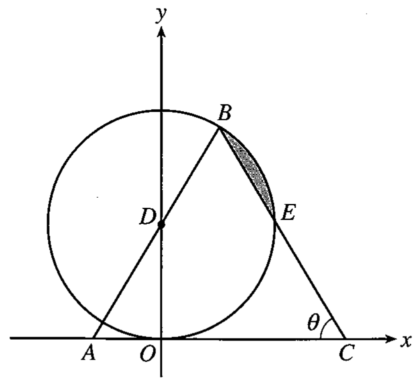

Figure 3

Figure 3 shows a circle with centre D and radius 1 unit, where O is the origin and D lies on the y-axis. The circle touches the x-axis at O. A is a moving point on the negative x-axis. AD is produced to intersect the circle at B. C is a point on the positive x-axis such that $BC = AB$ and BC intersects the circle at E. It is given that $\angle ACB = \theta$, where $\frac{\pi}{4} < \theta < \frac{\pi}{2}$. Let S square units be the area of the shaded region.

(a) Show that $S = 2\theta - \frac{\pi}{2} + \frac{1}{2}\sin 4\theta$.

(4 marks)

(b) Let x units be the horizontal distance between B and C. C is moving towards O such that the rate of change of x is 1 unit per second.

(i) Show that $x = \cos\theta + \cot\theta$.

(ii) Hence find the rate of increase of S when ΔABC is equilateral.

(8 marks)

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