2019-ME
MATH EP
M2

2019 HKDSE MOCK EXAMINATION

(Canotta Production)

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)
Question-Answer Book

Time allowed: 2 hours 30 minutes

This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number, Candidate Name, Class and Class Number in the spaces provided on Page 1 and stick thebarcode label in the space provided on Page 1.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Supplementary answer sheets are attached at the end of this booklet. Mark the question number box for the question you answer on the supplementary answer sheet.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answersmust be exact.

7. No extra time will be given to candidates for sticking on the barcode label or filling in the question number boxes after the 'Time is up' announcement.

Please stick the barcode label here.

Candidate Number					
Name of Candidate					
Class					
Class Number					

	Marker's Use Only	Examiner's Use Only
	Marker No.	Examiner No.
Question No.	Marks	Marks
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FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Answers written in the margins will not be marked.

Section A (50 marks)

1. If the coefficients of x and x^2 in the expansion of $(1 + ax)^n$ are $4\sqrt{3}$ and 18 respectively, find the values of a and n. (4 marks)

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2. Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{1+x^3}\right)$ from first principles. (5 marks)

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3. (a) Prove that, for any positive integer n ,

$$\tan^{-1}\frac{1}{n}-\tan^{-1}\frac{1}{n+1}=\tan^{-1}\frac{1}{n^2+n+1} .$$

(b) Using(a), prove that, for any positive integer m ,

$$\frac{\pi}{4}-\tan^{-1}\frac{1}{m+1}=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}+\cdots+\tan^{-1}\frac{1}{m^2+m+1} .$$

(6 marks)

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4. Consider the curve $3x^2 + 5xy + 8y^2 = 148$. Find the equations of the two tangents to the curve which are parallel to the line $x + 60y = 0$. (6 marks)

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5. (a) Using mathematical induction, prove that

$$\sum_{k=1}^{n} \frac{1}{(5k-1)(5k+4)(5k+9)} = \frac{1}{10}\left(\frac{1}{36} - \frac{1}{(5n+4)(5n+9)}\right)$$

for all positive integers n .

(b) Using (a), evaluate $\displaystyle\sum_{k=13}^{36} \frac{5520}{(5k-1)(5k+4)(5k+9)}$.

(7 marks)

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6. Define $f(x) = \sqrt{x}e^{\sqrt{x}}$ for all $x \geq 0$.

(a) Using a suitable substitution and integration by parts, find $\int \sqrt{x}e^{\sqrt{x}}\,dx$.

(b) Find the area of the region bounded by the graph of $y = f(x)$, the y-axis and the straight line $y = 2e^2$.

(7 marks)

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7. Let $A = \begin{pmatrix} 26 & -18 \\ 36 & -25 \end{pmatrix}$ and $P = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$. Denote the 2×2 identity matrix by I.

(a) Evaluate $P^{-1}AP$.

(b) Let n be a positive integer.

(i) Find A^n.

(ii) Someone claims that for any positive integer n, there exists an odd integer m such that $A^n - (-1)^n I = m \begin{pmatrix} 9 & -6 \\ 12 & -8 \end{pmatrix}$. Do you agree? Explain your answer.

(7 marks)

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8. Define $f(x)=\dfrac{x^2+ax+10}{x+6}$ for all $x \neq -6$, where a is a constant. Denote the graph of $y=f(x)$ by G.

It is given that G has an extreme point at $x=-10$.

(a) Find $f'(x)$.

(b) Find the asymptote(s) of G.

(c) Someone claims that G has no points of inflexion. Do you agree? Explain your answer.

(8 marks)

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Section B (50 marks)

9. (a) (i) Find $\int \frac{1}{\sqrt{a^2-x^2}}\,\mathrm{d}x$.

(ii) Prove that

$$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{a^2}{2}\sin^{-1}\frac{x}{a}+\frac{x}{2}\sqrt{a^2-x^2}+C \text{ , where } C \text{ is a constant.}$$

(4 marks)

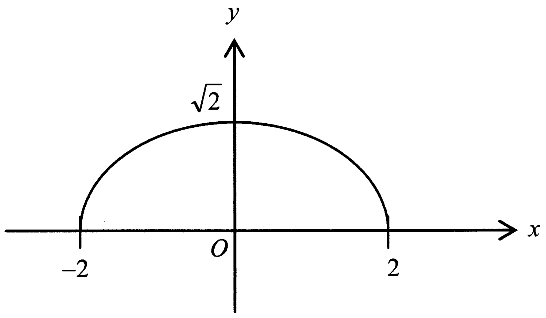
(b) Let $\mathrm{f}(x)$ be a continuous function on $[-a, a]$, where a is a positive constant.

(i) If $\mathrm{f}(-x)=-\mathrm{f}(x)$ where $-a \le x \le a$, prove that $\int_{-a}^{a}\mathrm{f}(x)\,\mathrm{d}x = 0$.

(ii) If $\mathrm{f}(-x)=\mathrm{f}(x)$ where $-a \le x \le a$, prove that $\int_{-a}^{a}\mathrm{f}(x)\,\mathrm{d}x = 2\int_{0}^{a}\mathrm{f}(x)\,\mathrm{d}x$.

(3 marks)

(c)



The figure shows the curve $y=(4-x^2)^{\frac{1}{4}}$, where $-2 \le x \le 2$. Let R be the region bounded by the curve, the two vertical lines $x=-\sqrt{3}$, $x=\sqrt{3}$ and the x-axis. Find the volume of the solid generated by revolving R about the x-axis. (5 marks)

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10. A curve passing through the point $P(0, 1)$ has an equation $y = \mathrm{f}(x)$ which satisfies the condition

$$a\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2} \quad ,$$

where a is a constant.

(a) Show that the function $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$ satisfies the above condition, and find the value of a.

(3 marks)

(b) A variable point $M(x, y)$ initially at P moves on the curve in (a), where $x \geq 0$. The normal at M to the curve cuts the y-axis at N. The circle with centre N and radius NM touches the curve and has an area A square units.

(i) Let R be the radius of the circle. Show that

$$R = \frac{x(e^{2x}+1)}{e^{2x}-1} \quad .$$

(ii) It is given that M moves so that its y-coordinate increases at a rate of 2 units per second.

(1) Show that

$$\frac{\mathrm{d}A}{\mathrm{d}t} = 8\pi\left(\frac{xe^{x}(e^{2x}+1)(e^{4x}-4xe^{2x}-1)}{(e^{2x}-1)^4}\right) \quad .$$

(2) Find the rate of change of A after 0.125 seconds.

(10 marks)

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11. Consider the system of simultaneous linear equations in real variables x , y , z

$$(E):\begin{cases} 4x + ay + 3z = k \\ x - y + 2z = 2 \\ 2x + 3y - az = k-4 \end{cases}, \text{ where } a, k \in \mathbf{R}.$$

(a) Find the values of a for which (E) has a unique solution. (3 marks)

(b) Assume that $a = 1$.

(i) Show that (E) is consistent for all values of k .

(ii) If $k = 3$, solve (E) where x , y and z are integers satisfying $x^2 + y^2 + z^2 \leq 9$.

(7 marks)

(c) Assume that $a = -9$ and (E) is consistent.

(i) Find k .

(ii) Solve (E) .

(3 marks)

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12. $ABCD$ is a parallelogram. Let $\overrightarrow{OA} = a\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$, $\overrightarrow{OB} = 16\mathbf{i} + b\mathbf{j} - 9\mathbf{k}$, $\overrightarrow{OC} = 17\mathbf{i} + 2\mathbf{j} + c\mathbf{k}$ and $\overrightarrow{OD} = 7\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, where O is the origin and a , b and c are real constants.

(a) Find the values of a , b and c. (2 marks)

(b) Find $\overrightarrow{OA} \cdot \overrightarrow{AD}$. (1 mark)

(c) Find $\overrightarrow{AB} \times \overrightarrow{AD}$. (1 mark)

(d) Let E be the projection of O on the plane $ABCD$.

(i) Find the volume of the pyramid $OABCD$, where O is the vertex.

(ii) Find the shortest distance from O to the plane $ABCD$. Hence, find the angle between OA and the plane $ABCD$.

(iii) Someone claims that the angle between ΔOAD and the plane $ABCD$ is $\angle OAE$. Do you agree? Explain your answer.

(8 marks)

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