2020-DSE
MATH CP

PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2020

Please stick the barcode label here.

Candidate Number

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

8:30 am – 10:45 am (2¼ hours)

This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of THREE sections, A(1), A(2) and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

(8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

* A 0 3 0 E 0 0 1 *

SECTION A(1) (35 marks)

1. Simplify $\dfrac{(mn^{-2})^5}{m^{-4}}$ and express your answer with positive indices. (3 marks)

2. Factorize

(a) $\alpha^2+\alpha-6$,

(b) $\alpha^4+\alpha^3-6\alpha^2$.

(3 marks)

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3. (a) Round up 534.7698 to the nearest hundred.

(b) Round down 534.7698 to 2 decimal places.

(c) Round off 534.7698 to 2 significant figures.

(3 marks)

4. Let a, b and c be non-zero numbers such that $\frac{a}{b}=\frac{6}{7}$ and $3a=4c$. Find $\frac{b+2c}{a+2b}$. (3 marks)

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5. In a recruitment exercise, the number of male applicants is 28% more than the number of female applicants. The difference of the number of male applicants and the number of female applicants is 91. Find the number of male applicants in the recruitment exercise. (4 marks)

6. Consider the compound inequality

$$3 - x > \frac{7 - x}{2} \quad \text{or} \quad 5 + x > 4 \qquad \ldots\ldots\ldots\ldots (*).$$

(a) Solve (*).

(b) Write down the greatest negative integer satisfying (*).

(4 marks)

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7. Let $p(x) = 4x^2 + 12x + c$, where c is a constant. The equation $p(x) = 0$ has equal roots. Find

(a) c,

(b) the x-intercept(s) of the graph of $y = p(x) - 169$.

(5 marks)

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8. In Figure 1, B and D are points lying on AC and AE respectively. BE and CD intersect at the point F. It is given that $AB = BE$, $BD // CE$, $\angle CAE = 30°$ and $\angle ADB = 42°$.

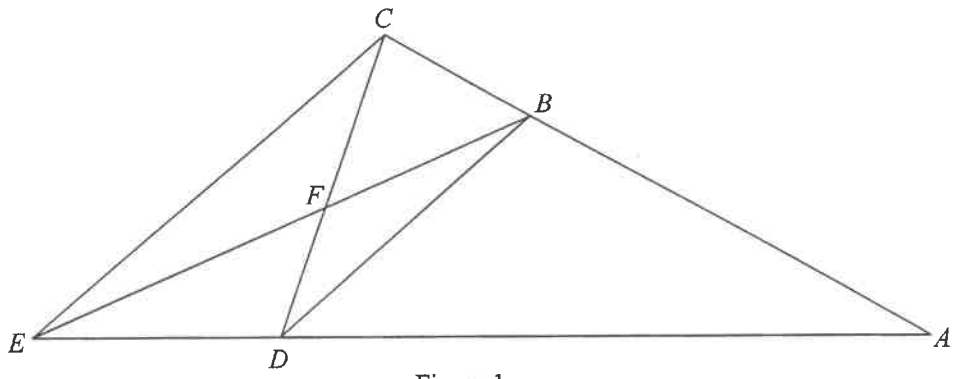

Figure 1

(a) Find $\angle BEC$.

(b) Let $\angle BDC = \theta$. Express $\angle CFE$ in terms of θ.

(5 marks)

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9. The table below shows the distribution of the numbers of subjects taken by a class of students.

Number of subjects taken	4	5	6	7
Number of students	8	12	16	4

(a) Write down the mean, the median and the standard deviation of the above distribution.

(b) A new student now joins the class. The number of subjects taken by the new student is 5. Find the change in the median of the distribution due to the joining of this student.

(5 marks)

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SECTION A(2) (35 marks)

10. The price of a brand X souvenir of height h cm is $\$P$. P is partly constant and partly varies as h^3. When $h=3$, $P=59$ and when $h=7$, $P=691$.

(a) Find the price of a brand X souvenir of height 4 cm. (4 marks)

(b) Someone claims that the price of a brand X souvenir of height 5 cm is higher than the total price of two brand X souvenirs of height 4 cm. Is the claim correct? Explain your answer. (2 marks)

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11. The stem-and-leaf diagram below shows the distribution of the weights (in grams) of the letters in a bag.

Stem (tens)	Leaf (units)						
1	1	2	3	3			
2	3	3	4	5	6	9	9
3	1	6	7	8	8	8	
4	2						
5	0	w					

It is given that the range of the above distribution is the triple of its inter-quartile range.

(a) Find w. (4 marks)

(b) If a letter is randomly chosen from the bag, find the probability that the weight of the chosen letter is not less than the mode of the distribution. (2 marks)

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12. The height and the base radius of a solid right circular cone are 36 cm and 15 cm respectively. The circular cone is divided into three parts by two planes which are parallel to its base. The heights of the three parts are equal. Express, in terms of π,

(a) the volume of the middle part of the circular cone; (3 marks)

(b) the curved surface area of the middle part of the circular cone. (3 marks)

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13. The cubic polynomial $f(x)$ is divisible by $x-1$. When $f(x)$ is divided by x^2-1, the remainder is $kx+8$, where k is a constant.

(a) Find k. (3 marks)

(b) It is given that $x+3$ is a factor of $f(x)$. When $f(x)$ is divided by x, the remainder is 24. Someone claims that all the roots of the equation $f(x)=0$ are integers. Is the claim correct? Explain your answer. (5 marks)

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14. The coordinates of the points A and B are $(-10, 0)$ and $(30, 0)$ respectively. The circle C passes through A and B. Denote the centre of C by G. It is given that the y-coordinate of G is -15.

(a) Find the equation of C. (3 marks)

(b) The straight line L passes through B and G. Another straight line ℓ is parallel to L. Let P be a moving point in the rectangular coordinate plane such that the perpendicular distance from P to L is equal to the perpendicular distance from P to ℓ. Denote the locus of P by Γ. It is given that Γ passes through A.

(i) Describe the geometric relationship between Γ and L.

(ii) Find the equation of Γ.

(iii) Suppose that Γ cuts C at another point H. Someone claims that $\angle GAH < 70°$. Do you agree? Explain your answer.

(6 marks)

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SECTION B (35 marks)

15. In a box, there are 3 blue plates, 7 green plates and 9 purple plates. If 4 plates are randomly selected from the box at the same time, find

(a) the probability that 4 plates of the same colour are selected; (3 marks)

(b) the probability that at least 2 plates of different colours are selected. (2 marks)

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16. The 3rd term and the 6th term of a geometric sequence are 144 and 486 respectively.

(a) Find the 1st term of the sequence. (2 marks)

(b) Find the least value of n such that the sum of the first n terms of the sequence is greater than 8×10^{18}. (3 marks)

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17. Let $g(x) = x^2 - 2kx + 2k^2 + 4$, where k is a real constant.

(a) Using the method of completing the square, express, in terms of k, the coordinates of the vertex of the graph of $y = g(x)$. (2 marks)

(b) On the same rectangular coordinate system, let D and E be the vertex of the graph of $y = g(x+2)$ and the vertex of the graph of $y = -g(x-2)$ respectively. Is there a point F on this rectangular coordinate system such that the coordinates of the circumcentre of ΔDEF are $(0, 3)$? Explain your answer. (4 marks)

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18. In Figure 2, U, V and W are points lying on a circle. Denote the circle by C. TU is the tangent to C at U such that TVW is a straight line.

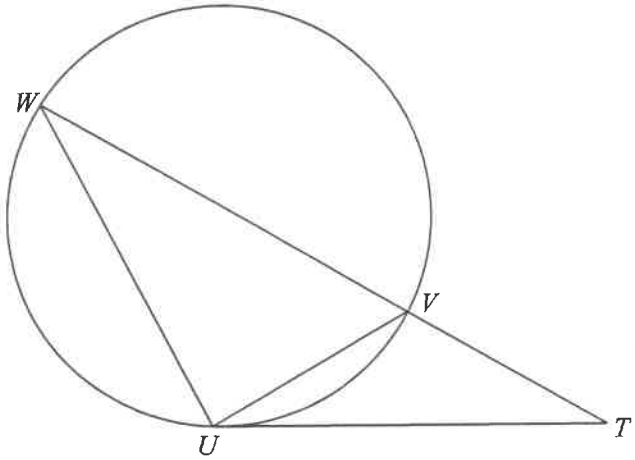

Figure 2

(a) Prove that $\Delta UTV \sim \Delta WTU$. (2 marks)

(b) It is given that VW is a diameter of C. Suppose that $TU = 780$ cm and $TV = 325$ cm.

(i) Express the circumference of C in terms of π.

(ii) Someone claims that the perimeter of ΔUVW exceeds 35 m. Do you agree? Explain your answer.

(5 marks)

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19. $PQRS$ is a quadrilateral paper card, where $PQ = 60$ cm , $PS = 40$ cm , $\angle PQR = 30°$, $\angle PRQ = 55°$ and $\angle QPS = 120°$. The paper card is held with QR lying on the horizontal ground as shown in Figure 3.

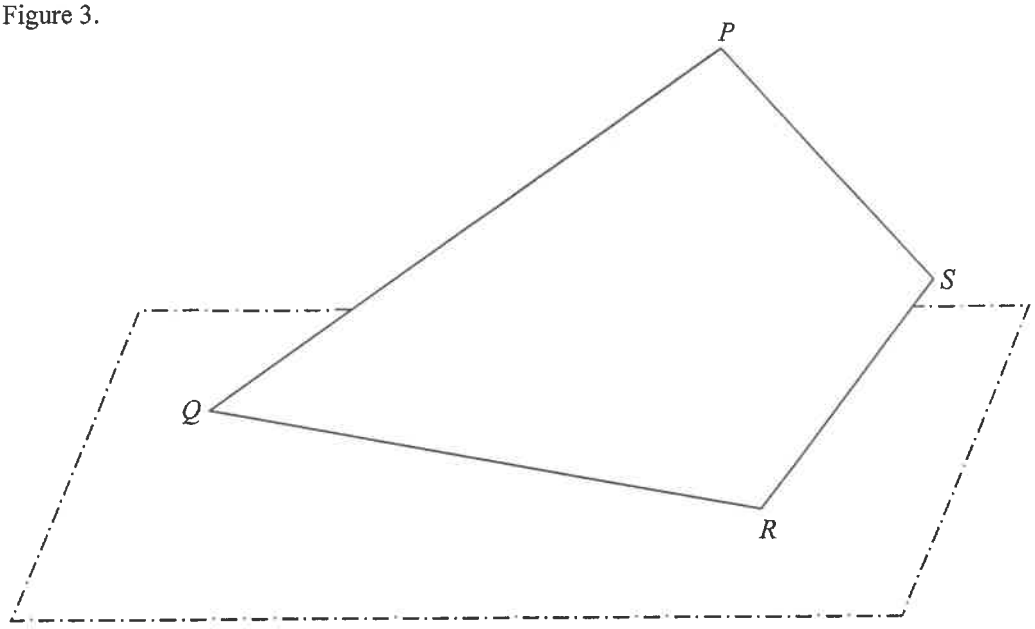

Figure 3

(a) Find the length of RS. (3 marks)

(b) Find the area of the paper card. (2 marks)

(c) It is given that the angle between the paper card and the horizontal ground is $32°$.

(i) Find the shortest distance from P to the horizontal ground.

(ii) A student claims that the angle between RS and the horizontal ground is at most $20°$. Is the claim correct? Explain your answer.

(7 marks)

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