HKDSE MATH M2 2014

1. **HKDSE Math M2 2014 Q1**

   In the expansion of $(1-4x)^2(1+x)^n$, the coefficient of x is 1.

   (a) Find the value of n.

   (b) Find the coefficient of x^2.

   (4 marks)

2. **HKDSE Math M2 2014 Q2**

   Consider the curve $C : y = x^3 - 3x$.

   (a) Find $\dfrac{dy}{dx}$ from first principles.

   (b) Find the range of x where C is decreasing.

   (5 marks)

3. **HKDSE Math M2 2014 Q3**

   Find the equation of tangent to the curve $x \ln y + y = 2$ at the point where the curve cuts the y-axis.

   (5 marks)

4. **HKDSE Math M2 2014 Q4**

   Let $x = 2y + \sin y$. Find $\dfrac{d^2y}{dx^2}$ in terms of y.

   (3 marks)

5. **HKDSE Math M2 2014 Q5**

   (a) Find $\displaystyle\int \frac{dx}{\sqrt{9-x}}$, where $x < 9$.

   (b) Using integration by substitution, find $\displaystyle\int \frac{dx}{\sqrt{9-x^2}}$, where $-3 < x < 3$.

   (6 marks)

6. **HKDSE Math M2 2014 Q6**

   (a) Find $\displaystyle\int xe^{-x}\, dx$.

   (b) Figure 1 shows the shaded region bounded by the curve $y = xe^{-x}$ and the straight line $y = \dfrac{x}{e}$. Find the area of the shaded region.

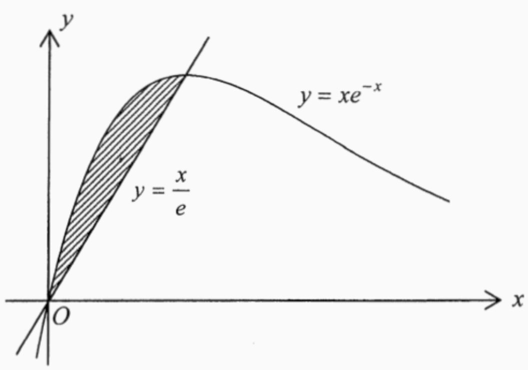

Figure 1

(6 marks)

7. **HKDSE Math M2 2014 Q7**
   Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$.

   (a) Prove, by mathematical induction, that for all positive integers n, $A^{n+1} = 2^n A$.

   (b) Using the result of (a), Willy proceeds in the following way:
   $A^2 = 2A$
   $A^2 A^{-1} = 2AA^{-1}$
   $A = 2I$
   Explain why Willy arrives at a wrong conclusion.

   (7 marks)

8. **HKDSE Math M2 2014 Q8**
   Let $\overrightarrow{OP} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OQ} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OR} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$.

   (a) Find $\overrightarrow{OP} \times \overrightarrow{OQ}$.
   Hence find the volume of tetrahedron $OPQR$.

   (b) Find the acute angle between the plane OPQ and the line OR, correct to the nearest $0.1°$.

   (8 marks)

9. **HKDSE Math M2 2014 Q9**

   (a) Solve the system of linear equations $\begin{cases} x + y + z = 100 \\ x + 6y + 10z = 200 \end{cases}$

   (b) In a store, the prices of each of small, medium and large marbles are \$0.5, \$3 and \$5 respectively. Aubrey plans to spend all \$100 for exactly 100 marbles, which include m small marbles, n medium marbles and k large marbles.
   Aubrey claims that there is only one set of combination of m, n and k. Do you agree? Explain your answer.

(6 marks)

10. **HKDSE Math M2 2014 Q10**

Thomas has a bookcase of dimensions 100 cm $\times$ 24 cm $\times$ 192 cm at the corner in his room. He wants to hang a decoration on the wall above the bookcase. Therefore, he finds a ladder to climb up. Initially, the ladder touches the wall, the edge of the top of the bookcase and the floor at the same time. Let rectangle $ABCD$ be the side-view of the bookcase and HK be the side-view of the ladder, so that $AB = 24$ cm and $BC = 192$ cm (see Figure 2). Let $\angle HKD = \theta$.

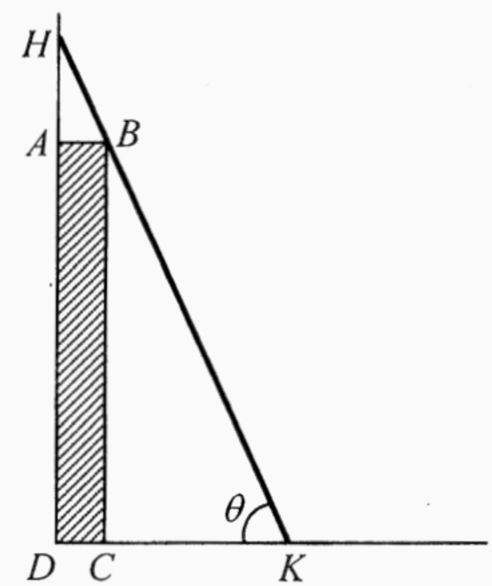

Figure 2

(a) Find the length of HK in terms of θ.
(1 marks)

(b) Prove that the shortest length of the ladder is $120\sqrt{5}$ cm.
(5 marks)

(c) Suppose the length of the ladder is 270 cm. Suddenly, the ladder slides down so that the end of the ladder, K, moves towards E (see Figure 3). The ladder touches the edge of the top of the bookcase and the floor at the same time. Let x cm and y cm be the horizontal distances from H and K respectively to the wall.

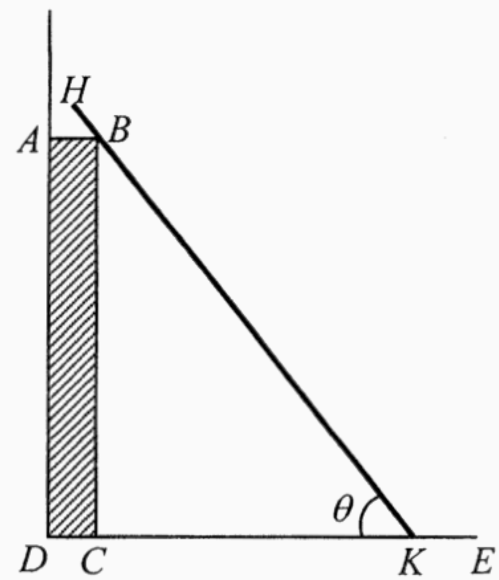

Figure 3

(i) When $CK = 160$ cm, the rate of change of θ is -0.1 rad s^{-1}. Find the rate of change of x at this moment, correct to 4 significant figures.

(ii) Thomas claims that K is moving towards E at a speed faster than the horizontal speed H is leaving the wall. Do you agree? Explain your answer.

(6 marks)

11. **HKDSE Math M2 2014 Q11**

In Figure 4, C and D are points on OB and OA respectively such that $AD : DO = OC : CB = t : 1 - t$, where $0 < t < 1$. BD and AC intersect at E such that $AE : EC = m : 1$ and $BE : ED = n : 1$, where m and n are positive. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

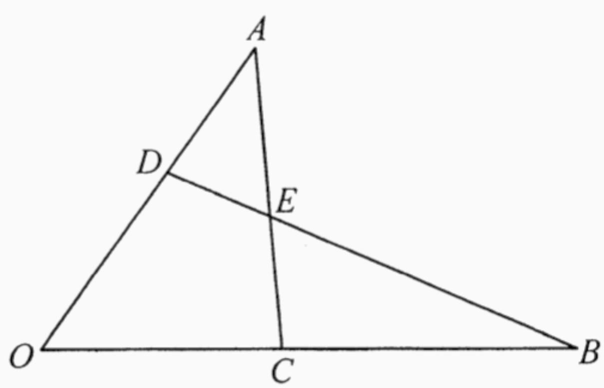

Figure 4

(a) (i) By considering $\triangle OAC$, express $\overrightarrow{OE}$ in terms of m, t, $\mathbf{a}$ and $\mathbf{b}$.

(ii) By considering $\triangle OBD$, express $\overrightarrow{OE}$ in terms of n, t, $\mathbf{a}$ and $\mathbf{b}$.

(iii) Show that $m = \dfrac{t}{(1-t)^2}$ and $n = \dfrac{1-t}{t^2}$.

(iv) Chris claims that

"if $m = n$, then E is the centroid of $\triangle OAB$ ".

Do you agree? Explain your answer.

(9 marks)

(b) It is given that $OA = 1$ and $OB = 2$. Francis claims that

"if AC is perpendicular to OB, then BD is always perpendicular to OA".

Do you agree? Explain your answer.

(4 marks)

12. **HKDSE Math M2 2014 Q12**

Let $M = \begin{pmatrix} k-1 & k \\ 1 & 0 \end{pmatrix}$ and $A = \begin{pmatrix} 1 & p \\ -1 & 1 \end{pmatrix}$, where k and p are real numbers and $p \neq -1$.

(a) (i) Find A^{-1} in terms of p.

(ii) Show that $A^{-1}MA = \begin{pmatrix} -1 & k-p \\ 0 & k \end{pmatrix}$.

(iii) Suppose $p = k$. Using (ii), find M^n in terms of k and n, where n is a positive integer.

(8 marks)

(b) A sequence is defined by $x_1 = 0$, $x_2 = 1$ and $x_n = x_{n-1} + 2x_{n-2}$ for $n = 3, 4, 5, \cdots$.

It is known that this sequence can be expressed in the matrix form $\begin{pmatrix} x_n \\ x_{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_{n-1} \\ x_{n-2} \end{pmatrix}$

Using the result of (a)(iii), express x_n in terms of n.

(3 marks)

13. **HKDSE Math M2 2014 Q13**

(a) Prove that $1 - \cos 4\theta - 2\cos 2\theta \sin^2 2\theta = 16\cos^2\theta \sin^4\theta$.

(2 marks)

(b) Show that $\displaystyle\int_0^{n\pi} \cos^2 x \sin^4 x \, dx = \frac{n\pi}{16}$, where n is a positive integer.

(4 marks)

(c) Let $f(x)$ be a continuous function such that $f(k-x) = f(x)$, where k is a constant.

Show that $\displaystyle\int_0^k xf(x)\,dx = \frac{k}{2}\int_0^k f(x)\,dx$.

(4 marks)

(d) Figure 5 shows the shaded region bounded by curve $y = cos^2 x \sin^4 x$ and the x-axis, where $\pi \leq x \leq 2\pi$. Find the volume of the solid of revolution when the shaded region is revolved about the y-axis.

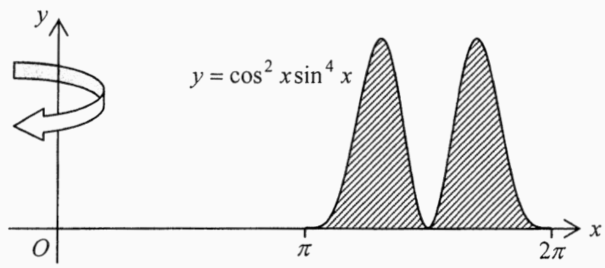

Figure 5

(4 marks)