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HONG KONG EXAMINATION & ASSESSMENT AUTHORITY

Hong Kong Diploma of Secondary Education Examination 2024

Mathematics (Compulsory Part) Paper 1

Time: 2 hours, 15 minutes Maximum Marks: 105

2024 HKDSE Math Paper, 15 Apr 2024

- Documented on LaTeX, 16 Apr 2024
- Good luck!

Section A(1) (35 marks)

1. Simplify $\dfrac{2}{4h-7}-\dfrac{3}{6h-5}$. [3 marks]

2. Make x the subject of the formula $\dfrac{Ax+C}{B}=3x$. [3 marks]

3. Factorize

   (a) $6r^2-13rs-28s^2$,

   (b) $4r-14s+6r^2-13rs-28s^2$.

   [3 marks]

4. (a) Find the range of values of x which satisfy both $\dfrac{5x+7}{4}-1<2x$ and $3x+9\geq 0$.

   (b) Write down the least integer satisfying both inequalities in (a).

   [4 marks]

5. Let a, b and c be non-zero numbers such that $5a=6c$ and $\dfrac{2b+7c}{b+c}=4$. Find $\dfrac{5a+8b}{2b+3c}$ [4 marks]

6. The marked price of a calculator is 40% higher than its cost. The calculator is sold at a discount of 25% on its marked price and the profit is \$13. Find the marked price of the calculator. [4 marks]

7. In a polar coordinate system, O is the pole. The polar coordinates of the points P, Q and R are $(11, 59°)$, $(60, 149°)$ and $(144, 239°)$ respectively.

   (a) Find $\angle POQ$.

   (b) Are P, O and R collinear? Explain your answer.

   (c) Find the perimeter of ΔPQR.

[4 marks]

8. In Figure 1, E is the point of intersection of AC and BD.

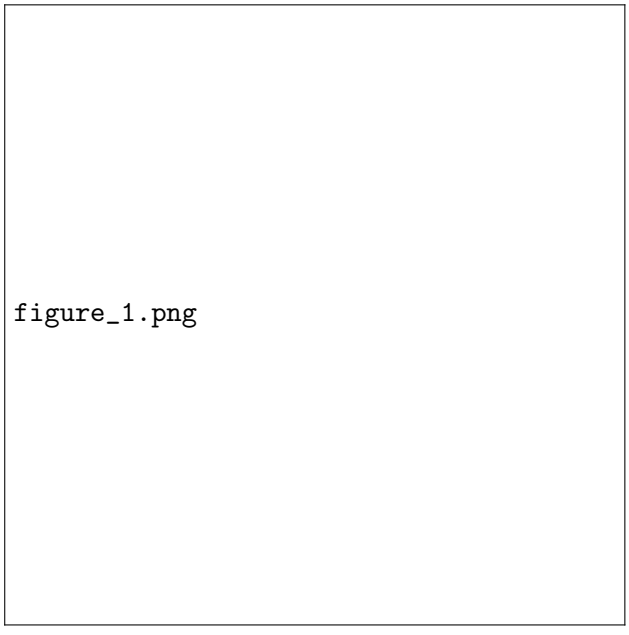

Figure 1

It is given that $\angle ACB = \angle ADB = 90°$ and $AD = BC$.

(a) Prove that $\Delta ABC \cong \Delta BAD$.

(b) If $AD = 12$ cm and $DE = 9$ cm, find the area of the pentagon $ABCED$.

[5 marks]

9. The table shows the distribution of the numbers of keys owned by a group of housewives.

Number of keys	3	4	5	6	7	8
Number of housewives	10	9	4	3	4	k

If a housewife is randomly selected from the group, then the probability that she owns more than 6 keys is $\frac{5}{18}$.

(a) Find k.

(b) Write down the mean, mode and median of the distribution.

[5 marks]

Section A(2) (35 marks)

10. It is given that $g(x)$ is partly constant and partly varies as x.

    Suppose that $g(-3) = -21$ and $g(7) = 9$.

    (a) Find $g(x)$. [3 marks]

    (b) Let $h(x) = xg(x) + k$, where k is a real constant.
    If all the roots of the equation $h(x) = 0$ are real numbers, find the range of values of k. [3 marks]

11. The stem-and-leaf diagram below shows the distribution of the numbers of hours spent on reading journals in a month by a group of researchers.

Stem (tens)	Leaf (units)									
2	0	0	1	a	a	a	8	8	9	9
3	0	0	2	3	4	4	7	9		
4	0	b								

    (a) Find a and b. [3 marks]

    (b) Write down the least possible range of the distribution. [1 mark]

    (c) Find the greatest possible inter-quartile range of the distribution. [2 marks]

12. Denote the origin by O.

    (a) A and B are points lying on the positive x-axis such that the x-coordinate of A is greater than the x-coordinate of B. A vertical line which passes through B cuts the straight line $y = mx$ at the point C such that $AB = BC$, where m is a positive constant.

    Let D be a point such that $ABCD$ is a square. Express the slope of OD in terms of m. [3 marks]

    (b) The coordinates of the points M and N are (6, 5) and (10, 0) respectively. Let P and Q be points lying on OM and MN respectively while R and S be points lyings on the x-axis.

    If the quadrilateral $PQRS$ is a square, find the x-coordinate of P. [4 marks]

13. The base of a solid right pyramid is a square of side 64 cm. The height of a pyramid is 24 cm. The pyramid is divided into a frustum X and a pyramid Y by a plane parallel to its base.

    It is given that the height of Y is 18 cm.

    (a) Find the volume of X. [3 marks]

    (b) The base of another solid right pyramid is a square. The pyramid is divided into a frustum Z and a pyramid by a plane parallel to its base. The height and the total surface area of Z are 3 cm and 960 cm^2 respectively.

    Are X and Z similar? Explain your answer. [4 marks]

14. Let $F(x) = (6x^2 + x + p)(qx^2 + rx - 10)$, where p, q and r are constants.

    The constant term of $F(x)$ is 40.

    (a) Write down the value of p. [1 mark]

    (b) When $F(x)$ is divided by $x + 1$, the remainder is -12. Given that $x - 2$ is a factor of $F(x)$.

    (i) Find q and r.

    (ii) How many irrational roots does the equation $F(x) = 0$ have? Explain your answer.

    [7 marks]

Section B (35 marks)

15. It is given that $\log_9 y$ is a linear function of $\log_3 x$. Denote the graph of the linear function by L. The slope of L is 4 and L passes through the point (5, 22).

    Express y in terms of x. [3 marks]

16. In a bag, there are 16 red cups and 4 white cups. If 5 cups are randomly drawn from the bag at the same time, find

    (a) the probability that exactly 1 white cup is drawn, [2 marks]

    (b) the probability that at most 3 red cups are drawn. [2 marks]

17. The coordinates of the points Q and R are $(10, -1)$ and $(-4, -9)$ respectively.

    (a) Let P be a moving point in the rectangular coordinate plane such that $PQ = PR$. Denote the locus of P by Γ.

    (i) Describe the geometric relationship between Γ and QR.

    (ii) Find the equation of Γ.

    [3 marks]

    (b) Let C be the circle which passes through Q, R and the point $(4, 3)$.

    (i) Find the equation of C.

    (ii) The coordinates of the point U is (10, 4). It is found that U lies outside C. UV and UW are the tangents to C at the points V and W respectively.

    Is the area of the circumcircle of ΔUVW greater than 100? Explain your answer.

    [5 marks]

18. (a) $PQRS$ is a thin quadrilateral metal sheet where $PQ = 12$cm, $PS = 10$cm, $QR = 13$cm, $\angle QPS = 82°$ and $\angle QRS = 65°$. Find

(i) the length of QS,

(ii) $\angle RQS$.

[4 marks]

(b) The metal sheet $PQRS$ described in (a) is now folded along QS (see Figure 2). It is given that the angle between the plane PQS and the plane QRS is 80°.

figure_2.png

(i) Find the shortest distance from R to the plane PQS.

(ii) Let X be any point lying on the plane QRS. Someone claims that the distance between P and X exceeds 8 cm. Is the claim correct? Explain your answer.

[4 marks]

19. Let $f(x) = 2x^2 + 4mx + 8x + 2m^2 + 8m + n$, where m and n are real constants such that $mn < 0$. Denote the vertex of the graph of $y = f(x)$ by P.

(a) Using the method of completing the square, express the coordinates of P in terms of m and n. [2 marks]

(b) Describe the geometric meaning represented by transforming $f(x)$ to $f(\frac{x}{5}) + 7$. [2 marks]

(c) Denote the vertex of the graph of $f(\frac{x}{5})+7$ by Q. Let (a_1, b_1) and (a_2, b_2) be the coordinates of P and Q respectively. Given that $a_1, 1+n, a_2$ is an arithmetic sequence, and $b_1, 4-m, b_2$ is a geometric sequence.

(i) Find the coordinates of P and Q.

(ii) The coordinates of the points R and S are $(3t+27, t)$ and $(3t+3, 2t-3)$ respectively, where t is a real number.

Is it possible that $PQRS$ is a rhombus? Explain your answer.

[8 marks]