HKDSE
MATH EP
M2

HONG KONG EDUCATIONAL PUBLISHING COMPANY

HONG KONG DIPLOMA OF
SECONDARY EDUCATION EXAMINATION

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)
MOCK EXAM 8
Question-Answer Book

Time allowed: 2½ hours
This paper must be answered in English

Please stick the barcode label here.

Candidate Number									

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
2. This paper consists of **TWO** sections, A and B.
3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. In this paper, vectors may be represented by bold-type letters such as $\mathbf{u}$, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
8. The diagrams in this paper are not necessarily drawn to scale.
9. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

2014

2. In the expansion of $(1 + 2x)^n(1 - x)^2$, where n is a positive integer, the coefficient of x^2 is 9.

(a) Find the value of n.

(b) Find the coefficient of x.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

2015

3. (a) Find $\int \tan x \, dx$.

(b) Using integration by substitution, evaluate $\int_{\frac{\pi^2}{16}}^{\frac{\pi^2}{9}} \frac{\tan\sqrt{x}}{\sqrt{x}} \, dx$.

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

2015

3. (a) Find $\int \tan x \, dx$.

(b) Using integration by substitution, evaluate $\int_{\frac{\pi^2}{16}}^{\frac{\pi^2}{9}} \frac{\tan\sqrt{x}}{\sqrt{x}} \, dx$.

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

4. Find the equations of the two tangents to the curve C: $x^2 + 2xy - y^2 = 1$ which pass through the point $A(0, -1)$.

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

5. Consider the following system of linear equations in real variables x, y, z:

$$(E): \begin{cases} x - y + z = 0 \\ 2x + 3y + z = 0 \\ kx + 5y - z = 0 \end{cases}, \text{ where } k \text{ is a real number.}$$

It is given that (E) has non-trivial solutions.

(a) Find the value(s) of k and solve (E).

(b) If some solution (x, y, z) of (E) satisfies $25x^2 - 175y^2 + (z - p)^2 = 10$, find the range of values of p.

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

2013

6. Let $M = \begin{pmatrix} 1+x & -x \\ x & 1-x \end{pmatrix}$ for any real number x.

(a) Show that M is invertible.

Hence find M^{-1}.

(b) If $M^T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 28 \\ -22 \end{pmatrix}$, where M^T is the transpose of M, find the values of x and y.

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

2014

7. Let $\overrightarrow{OA} = 3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{i} + \mathbf{k}$.

(a) Find the volume of the tetrahedron $OABC$.

(b) Suppose L is a straight line passing through C and perpendicular to the plane OAB. Let θ be the acute angle between L and the straight line OC. Find the value of $\cos\theta$.

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

8. (a) Prove, by mathematical induction, that
$1 \times 2 + 2 \times 2^2 + 3 \times 2^3 + ... + n \times 2^n = (n - 1) \times 2^{n+1} + 2$ for all positive integers n.

(b) Using the result of (a), simplify $\sum_{r=1}^{n} (r + 1) \times 2^r$.

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

SECTION B (50 marks)

9. (a) (i) Using integration by parts, find $\int x \ln x \, dx$.

(ii) Figure 3 shows a shaded region enclosed by the curve $y = \sqrt{x \ln x}$, the line $x = h$ $(h > 1)$ and the x-axis.

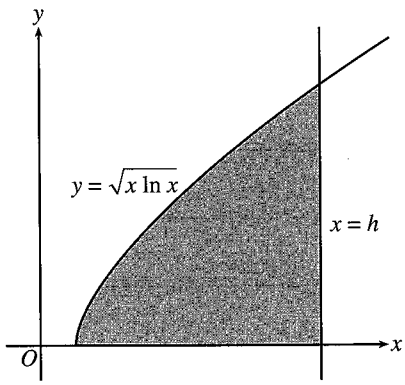

Figure 3

A solid is formed by revolving the shaded region about the x-axis. Show that the volume of the solid is $\frac{\pi}{4}(2h^2 \ln h - h^2 + 1)$ cubic units.

(6 marks)

(b) By revolving the shaded region about the x-axis, solids X and Y are formed with heights $(e^2 - 1)$ units and $(e - 1)$ units respectively (see Figure 4 and Figure 5 respectively). Find the volumes of solids X and Y.

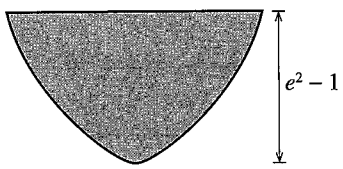

Figure 4

Figure 5

(2 marks)

(c)

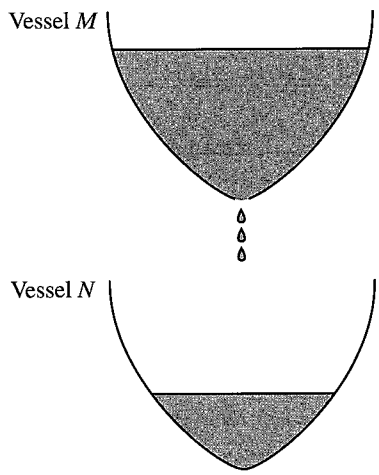

Figure 6

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

In Figure 6, vessels M and N are formed by revolving the curve $y = \sqrt{x \ln x}$ about the x-axis. Initially, vessel N is empty and vessel M contains some water. There is a small hole at the bottom of vessel M such that water is leaking through the hole into vessel N. When the volumes of water in vessels M and N are $\frac{\pi}{4}(3e^4 + 1)$ cubic units and $\frac{\pi}{4}(e^2 + 1)$ cubic units respectively, the water level in vessel M drops at a rate of 1 unit per second, find the rate of increase of water level in vessel N at this moment.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

10. In Figure 7, $OACB$ is a rectangle. $OA = 2OB$. AC is produced to D. OD intersects AB and BC at E and F respectively. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\angle OEA = \theta$ and $AD : AC = k : 1$, where k is a constant.

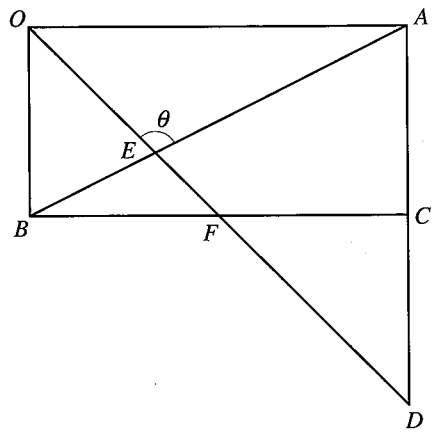

Figure 7

(a) (i) Show that $|\overrightarrow{OD}| = \sqrt{k^2 + 4}\,|\mathbf{b}|$.

(ii) Show that $\cos\theta = \dfrac{k-4}{\sqrt{5(k^2+4)}}$.

(5 marks)

(b) It is given that $AB \perp OD$ and G is a point on OD such that $CG \,//\, AE$.

(i) Find $\overrightarrow{CG}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Someone claims that E is the mid-point of OG. Do you agree? Explain your answer.

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

2015

11. (a) Let α and β be real numbers such that $\beta - \alpha \neq 4$. Denote the 2×2 identity matrix by I. Define

$$P = \frac{1}{\alpha - \beta + 4}(M - \beta I + 2I) \text{ and}$$

$$Q = \frac{1}{\alpha - \beta + 4}(M - \alpha I - 2I),$$

where $M = \begin{pmatrix} \alpha & 2 \\ \alpha - \beta + 2 & \beta \end{pmatrix}$.

(i) Evaluate PQ, QP and $P - Q$.

(ii) Prove that $P^2 = P$ and $Q^2 = -Q$.

(iii) Prove that $M^n = (\alpha + 2)^n P - (\beta - 2)^n Q$ for all positive integers n.

(8 marks)

(b) Using (a), or otherwise, evaluate $\begin{pmatrix} 4 & 1 \\ 5 & 0 \end{pmatrix}^{2017}$. (4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

12. Let $f(x) = x - \dfrac{x}{x+1}$, where $x \neq -1$.

(a) Find $f'(x)$ and $f''(x)$, where $x \neq -1$.

(2 marks)

(b) (i) Find the relative extreme point(s) of the graph of $y = f(x)$.

(ii) Show that the graph of $y = f(x)$ does not have any point of inflexion.

(6 marks)

(c) Find the asymptote(s) of the graph of $y = f(x)$.

(2 marks)

(d) Sketch the graph of $y = f(x)$.

(3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

END OF PAPER

Answers written in the margins will not be marked.

