HKDSE MATH CORE 2019 Past Paper I

1. **HKDSE MATH CORE 2019 Past Paper I Q1**
   Make h the subject of the formula $9(h + 6k) = 7h + 8$.
   (3 marks)

2. **HKDSE MATH CORE 2019 Past Paper I Q2**
   Simplify $\frac{3}{7x - 6} - \frac{2}{5x - 4}$.
   (3 marks)

3. **HKDSE MATH CORE 2019 Past Paper I Q3**
   The length and the breadth of a rectangle are 24 cm and $(13 + r)$ cm respectively. If the length of a diagonal of the rectangle is $(17 - 3r)$ cm, find r.
   (3 marks)

4. **HKDSE MATH CORE 2019 Past Paper I Q4** Factorize
   (a) $4m^2 - 9$,
   (b) $2m^2n + 7mn - 15n$.
   (c) $4m^2 - 9 - 2m^2n - 7mn + 15n$.
   (4 marks)

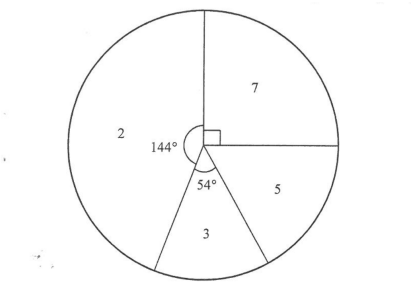
5. **HKDSE MATH CORE 2019 Past Paper I Q5**
   A wallet is sold at a discount of 25% on its marked price. The selling price of the wallet is $690.
   (a) Find the marked price of the wallet.
   (b) After selling the wallet, the percentage profit is 15%. Find the cost of the wallet.
   (4 marks)

6. **HKDSE MATH CORE 2019 Past Paper I Q6**
   (a) Solve the inequality $\frac{7x + 26}{4} \leq 2(3x - 1)$.
   (b) Find the number of integers satisfying both inequalities $\frac{7x + 26}{4} \leq 2(3x - 1)$ and $45 - 5x \geq 0$.
   (4 marks)

7. **HKDSE MATH CORE 2019 Past Paper I Q7**
   In a playground, the ratio of the number of adults to the number of children is 13 : 6. If 9 adults and 24 children enter the playground, then the ratio of the number of adults to the number of children is 8 : 7. Find the original number of adults in the playground.
   (4 marks)

8. **HKDSE MATH CORE 2019 Past Paper I Q8**

The pie chart below shows the distribution of the numbers of rings owned by the girls in a group.



Distribution of the numbers of rings owned by the girls in the group

(a) Write down the mode of the distribution.

(b) Find the mean of the distribution.

(c) If a girl is randomly selected from the group, find the probability that the selected girl owns more than 3 rings.

(5 marks)

9. **HKDSE MATH CORE 2019 Past Paper I Q9**

The sum of the volumes of two spheres is 324π cm3. The radius of the larger sphere is equal to the diameter of the smaller sphere. Express, in terms of π,

(a) The volume of the larger sphere;

(b) The sum of the surface areas of the two spheres.

(5 marks)

10. **HKDSE MATH CORE 2019 Past Paper I Q10**

It is given that $h(x)$ is partly constant and partly varies as x. Suppose that $h(-2) = -96$ and $h(5) = 72$.

(a) Find $h(x)$.
(3 marks)

(b) Solve the equation $h(x) = 3x^2$.
(2 marks)

11. **HKDSE MATH CORE 2019 Past Paper I Q11**

Let $p(X)$ be a cubic polynomial. When $p(x)$ is divided by $x - 1$, the remainder is 50. When $p(X)$ is divided by $x + 2$, the remainder is -52. It is given that $p(x)$ is divisible by $2x^2 + 9x + 14$.

(a) Find the quotient when $p(X)$ is divided by $2x^2 + 9x + 14$.
(3 marks)

(b) How many rational roots does the equation have? Explain your answer.
(3 marks)

12. **HKDSE MATH CORE 2019 Past Paper I Q12**

The stem-and-leaf diagram below shows the distribution of the results (in seconds) of some boys in a 400 m race. It is given that the inter-quartile range of the distribution is 8 seconds.

Stem (tens)	Leaf (units)
5	a
6	0 0 3 c c 8 9 9 9
7	0 1 1 1 2 2 5 6 9
8	b

(a) Find c.

(2 marks)

(b) It is given that the range of the distribution exceeds 34 seconds and the mean of the distribution is 69 seconds. Find

(i) a and b,

(ii) the least possible standard deviation of the distribution.

(6 marks)

13. **HKDSE MATH CORE 2019 Past Paper I Q13**

In Figure 1, O is the centre of circle $ABCDE$. AC is a diameter of the circle. BD and OC intersect at the point F. It is given that $\angle AED = 115°$.

(a) Find $\angle CBF$.

(3 marks)

(b) Suppose that $BC//OD$ and $OB = 18$ cm. Is the perimeter of the sector OBC less than 60 cm? Explain your answer.

(5 marks)

14. **HKDSE MATH CORE 2019 Past Paper I Q14**

In Figure 2, $ABCD$ is a square. It is given that E is a point lying on AD. BD and CE intersect at the point F. Let G be a point such that $BG//EC$ and $CG//DB$.

(a) Prove that

(i) $\triangle BCG \cong \triangle CBF$,

(ii) $\triangle BCF \sim \triangle DEF$.

(4 marks)

(b) Suppose that $\angle BCF = \angle BGC$.

(i) Let $BC = l$. Express DF in terms of l.

(ii) Someone claims that $AE > DF$. Do you agree? Explain your answer.

(4 marks)

15. **HKDSE MATH CORE 2019 Past Paper I Q15**

There are 21 boys and 11 girls in a class. If 5 students are selected from the class to form a committee consisting of at least 1 boy, how many different committees can be formed?
(3 marks)

16. **HKDSE MATH CORE 2019 Past Paper I Q16**

Let α and β be real numbers such that $\begin{cases} \beta = 5\alpha - 18 \\ \beta = \alpha^2 - 13\alpha + 63 \end{cases}$.

(a) Find α and β.
(2 marks)

(b) The 1st term and the 2nd term of an arithmetic sequence are $\log \alpha$ and $\log \beta$ respectively. Find the least value of n such that the sum of the first n terms of the sequence is greater than 888.
(4 mark)

17. **HKDSE MATH CORE 2019 Past Paper I Q17**

(a) Let a and p be the area and the perimeter of $\triangle CDE$ respectively. Denote the radius of the inscribed circle of $\triangle CDE$ by r. Prove that $pr = 2a$.
(2 marks)

(b) The coordinates of the points H and K are $(9, 12)$ and $(14, 0)$ respectively. Let P be a moving point in the rectangular coordinate plane such that the perpendicular distance from P to OH is equal to the perpendicular distance from P to HK, where O is the origin. Denote the locus of P by Γ.

(i) Describe the geometric relationship between Γ and $\triangle OHK$.

(ii) Using (a), find the equation of Γ.

(5 marks)

18. **HKDSE MATH CORE 2019 Past Paper I Q18**

Figure 3 shows a tetrahedron $ABCD$. Let P be a point lying on AD such that BP is perpendicular to AD. A craftsman finds that $AC = AD = CD = 13$ cm, $BC = 8$ cm, $BD = 12$ cm and $\angle ABD = 72°$.

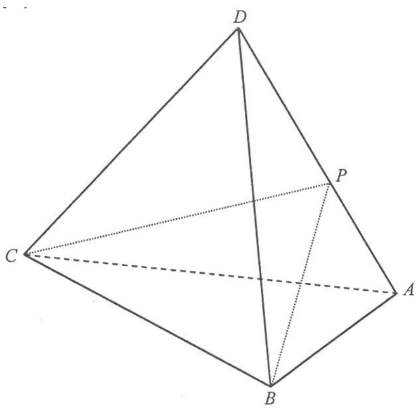


Figure 3

(a) Find

(i) $\angle BAD$,

(ii) CP.

(5 marks)

(b) TThe craftsman claims that $\angle BPC$ is the angle between the face ABD and the face ACD. Is the claim correct? Explain your answer.
(2 marks)

19. **HKDSE MATH CORE 2019 Past Paper I Q19**

Let $f(x) = \dfrac{1}{1+k}(x^2 + (6k-2)x + (9k+25))$, where k is a positive constant. Denote the point $(4, 33)$ by F.

(a) Prove that the graph of $y = f(x)$ passes through F.
(1 mark)

(b) The graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ with respect to the y-axis and then translating the resulting graph upwards by 4 units. Let U be the vertex of the graph of $y = g(x)$. Denote the origin by O.

(i) Using the method of completing the square, express the coordinates of U in terms of k.

(ii) Find k such that the area of the circle passing through F, O and U is the least.

(iii) For any positive constant k, the graph of $y = g(x)$ passes through the same point G. Let V be the vertex of the graph of $y = g(x)$ such that the area of the circle passing through F, O and V is the least. Are F, G, O and V concyclic? Explain your answer.

(11 marks)