Differentiation Note

1 Definition of Differentiation

The slope two points in a function:
$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Differentiation(First principle):

$$\frac{dy}{dx} = \frac{d}{dx}f(x) = f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$\left.\frac{dy}{dx}\right|_{x=a} = f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

2 Exercise

1. **HKDSE Math M2 Sample Paper Q1**
   Find $\dfrac{d}{dx}(\sqrt{2x})$ from the first principles.
   (4 marks)

2. **HKDSE Math M2 Practice Paper Q6**
   Find $\dfrac{d}{dx}\left(\dfrac{1}{x}\right)$ from the first principles.
   (4 marks)

3. **HKDSE Math M2 2012 Q1**
   Let $f(x) = e^{2x}$. Find $f'(0)$ from first principles.
   (3 marks)

4. **HKDSE Math M2 2013 Q1**
   Find $\dfrac{d}{dx}(\sin 2x)$ from first principles.
   (4 marks)

5. **HKDSE Math M2 2014 Q2 (a)**
   Consider the curve $C : y = x^3 - 3x$.

   (a) Find $\dfrac{dy}{dx}$ from first principles.

6. **HKDSE Math M2 2015 Q1**
   Find $\dfrac{d}{dx}(x^5 + 4)$ from first principles.
   (4 marks)

7. **HKDSE Math M2 2016 Q2**
   Prove that $\dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt{x+h}} = \dfrac{h}{(x+h)\sqrt{x} + x\sqrt{x+h}}$. Hence, find $\dfrac{d}{dx}\sqrt{\dfrac{3}{x}}$ from first principles.
   (5 marks)

8. **HKDSE Math M2 2017 Q1**
   Let $\dfrac{d}{d\theta}\sec 6\theta$ from first principles.
   (5 marks)

9. **HKDSE Math M2 2018 Q1**
   Let $f(x) = (x^2 - 1)e^x$. Express $f(1 + h)$ in terms of h. Hence, find $f'(1)$ from first principles.
   (4 marks)

10. **HKDSE Math M2 2019 Q1**
    Let $f(x) = \dfrac{10x}{7 + 3x^2}$. Prove that $f(1 + h) - f(1) = \dfrac{4h - 3h^2}{10 + 6h + 3h^2}$. Hence, find $f'(1)$ from first principles.
    (4 marks)

11. **HKDSE Math M2 2020 Q2**
    Define $f(x) = \dfrac{x}{\sqrt{2+x}}$, for all $x > -2$. Find $f'(2)$ from first principles.
    (4 marks)

12. **HKDSE Math M2 2021 Q1**
    Let $f(x) = \dfrac{1}{3x^2 + 4}$. Find $f'(x)$ from first principles.
    (4 marks)

13. **HKDSE Math M2 2022 Q1**
    Let $g(x) = \dfrac{1}{\sqrt{5x+4}}$, where $x > 0$. Prove that $g(1+h) - g(1) = \dfrac{-5h}{3\sqrt{5h+9}(3 + \sqrt{5h+9})}$. Hence, find $g'(1)$ from first principles.
    (4 marks)

3 Rules of Differentiation

1. $\dfrac{d}{dx}C = 0$, where C is a constant

2. $\dfrac{d}{dx}x^n = nx^{n-1}$, where n is a constant

3. $\dfrac{d}{dx}a^x = a^x \ln a$ where a is a positive constant and $a \neq 1$

4. $\dfrac{d}{dx}e^x = e^x$

5. $\dfrac{d}{dx}\ln x = \dfrac{1}{x}$

6. $\dfrac{d}{dx}\sin x = \cos x$

7. $\dfrac{d}{dx}\cos x = -\sin x$

8. $\dfrac{d}{dx}\tan x = \sec^2 x$

4 Quiz

(a) Given $f(x) = x^2$, find $f'(x)$

(b) Prove (a)

5 Sum and Difference Rule

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$$

6 Product Rule

$$\frac{d}{dx}[f(x) \times g(x)] = g(x)\frac{d}{dx}f(x) + f(x)\frac{d}{dx}g(x)$$

7 Quotient Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$$

8 Chain Rule

$$\frac{d}{dx}f(g(x)) = \frac{d\,f(g(x))}{d(g(x))} \cdot \frac{d(g(x))}{dx} = f'(g(x)) \cdot g'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$