HKDSE
MATH EP
M2

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HONG KONG DIPLOMA OF
SECONDARY EDUCATION EXAMINATION

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)
MOCK EXAM 3
Question-Answer Book

Time allowed: 2½ hours
This paper must be answered in English

Please stick the barcode label here.

Candidate Number									

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
2. This paper consists of **TWO** sections, A and B.
3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
8. The diagrams in this paper are not necessarily drawn to scale.
9. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Answers written in the margins will not be marked.

SECTION A (50 marks)

2014

1. In the expansion of $(1+2x)^2(1-x)^n$, the coefficient of x is -7.

   (a) Find the value of n.

   (b) Find the coefficient of x^2.

(4 marks)

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2. Find $\frac{d}{dx}(\sqrt{x})$ from first principles.

(4 marks)

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3. (a) Let $x = \cot\theta$. Show that $\dfrac{x^2 - 1}{x^2 + 1} = \cos 2\theta$.

(b) Using (a), find the least value of $\dfrac{2(x+1)(x-1)}{x^2 + 1}$, where x is real.

(5 marks)

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2015

4. Let $f(x) = xe^{kx}$, where k is a constant.

   (a) Find $f'(x)$ and $f''(x)$.

   (b) If $f''(x) - 2kf'(x) + 4f(x) = 0$ for all real values of x, find the value(s) of k.

(5 marks)

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2015

5. (a) Using integration by substitution, find $\int \frac{dx}{x^2+9}$.

(b) At any point (x, y) on the curve Γ, the slope of the tangent to Γ is $\frac{x^2}{x^2+9}$. It is given that Γ passes through the point $(0, 6)$. Find the equation of Γ.

(7 marks)

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6. Let $f(x)$ be a polynomial. Figure 1 shows a sketch of the curve $y = f'(x)$, where $a \le x \le c$. The curve cuts the x-axis at the origin and $(b, 0)$, where $0 < b < c$.

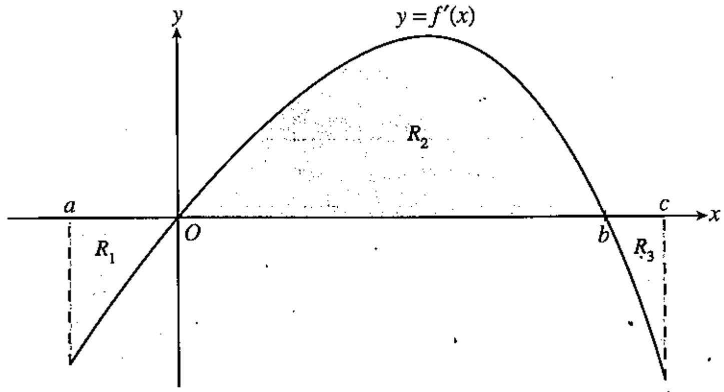

Figure 1

(a) Write down the x-coordinates of the maximum and minimum points of the curve $y = f(x)$ for $a < x < c$.

(b) It is known that $f(0) = 1$ and the area of the shaded region R_2 as shown in Figure 1 is 6.

(i) By considering $\int_0^b f'(x)\,dx$, find the value of $f(b)$.

(ii) If $f(a) = f(c)$ and the areas of the shaded regions R_1 and R_3 as shown in Figure 1 are equal, find the value of $f(a)$.

(7 marks)

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7. Consider the following system of linear equations in x, y, z:

$$\begin{cases} x + 3y + kz = 0 \\ x - ky - 4z = 0 \\ 3x + 5y - 9z = 0 \end{cases}, \text{ where } k \text{ is a real number.}$$

If the system has non-trivial solutions, find the possible values of k.

(4 marks)

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2015

8. (a) Let A be a 3×3 matrix such that $A^T = -A$, where A^T is the transpose of A. Prove that $|A| = 0$.

(b) Let $M = \begin{pmatrix} 1 & x & y \\ -x & 1 & z \\ -y & -z & 1 \end{pmatrix}$, where x, y and z are real numbers. It is given that M is a non-singular matrix. Denote the 3×3 identity matrix by I.

(i) Using (a), or otherwise, prove that $|I - M| = 0$.

(ii) Someone claims that $I - M^{-1}$ is a singular matrix. Do you agree? Explain your answer.

(6 marks)

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9. It is given that $\overrightarrow{OP} = 4\mathbf{i} + \mathbf{j}$ and $\overrightarrow{OQ} = 12\mathbf{i} - 3\mathbf{j}$. R is a point on PQ such that $\overrightarrow{PR} = 3\overrightarrow{RQ}$.

(a) Find the unit vector in the direction of $\overrightarrow{OR}$.

(b) Find $\angle ROP$ correct to the nearest degree.

(8 marks)

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SECTION B (50 marks)

10. Let $A = \begin{pmatrix} 0 & 0 & 1 \\ 4 & 4 & 4 \\ 5 & 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 4 & -4 \\ 0 & -5 & 4 \\ -4 & 0 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} -3 & -4 & -4 \\ 5 & 6 & 5 \\ 0 & 0 & 1 \end{pmatrix}$.

(a) (i) Find AB and BA.

(ii) Using the result of (i), or otherwise, find A^{-1}.

(4 marks)

(b) (i) Find ACA^{-1}.

(ii) Show that C is invertible.

(iii) Find $(D^{-1})^{2016}$, where $D = ACA^{-1}$.

Hence, or otherwise, find $(C^{-1})^{2016}$.

(7 marks)

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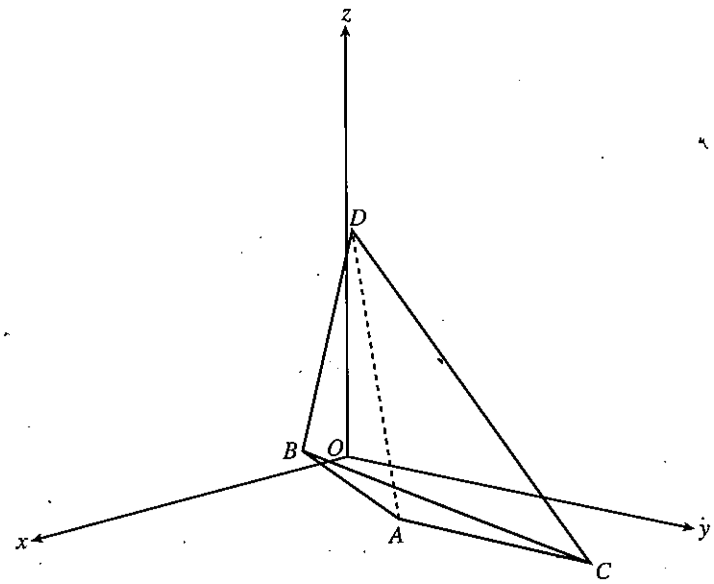

Figure 2

Let $\overrightarrow{OA} = -\mathbf{i} - \mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\overrightarrow{OC} = -\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\overrightarrow{OD} = \mathbf{i} + \mathbf{j} + 3\mathbf{k}$ (see Figure 2). Let P and Q be points on the straight lines AB and CD respectively such that $AP : PB = r : (1 - r)$ and $CQ : QD = s : (1 - s)$, where $0 < r < 1$ and $0 < s < 1$. Suppose that PQ is perpendicular to both AB and CD.

(a) (i) Express $\overrightarrow{PQ}$ in terms of r, s, $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.

(ii) Hence find the shortest distance between the straight lines AB and CD.

(8 marks)

(b) (i) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

(ii) Let R be the projection of D on the plane ABC. Find the coordinates of R by considering $\overrightarrow{AR} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})$.

(5 marks)

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12. Let $f(x) = \dfrac{x^4}{x^3+2}$, where $x \neq -2^{\frac{1}{3}}$.

(a) Find the x- and y-intercept(s) of the graph of $y = f(x)$.

(1 mark)

(b) Find $f'(x)$ and prove that $f''(x) = \dfrac{12x^2(4-x^3)}{(x^3+2)^3}$.

(4 marks)

(c) (i) For the graph of $y = f(x)$, find all the extreme point(s) and point(s) of inflexion.

(ii) Find all the asymptote(s) of the graph of $y = f(x)$.

(iii) Sketch the graph of $y = f(x)$.

(8 marks)

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13. (a)

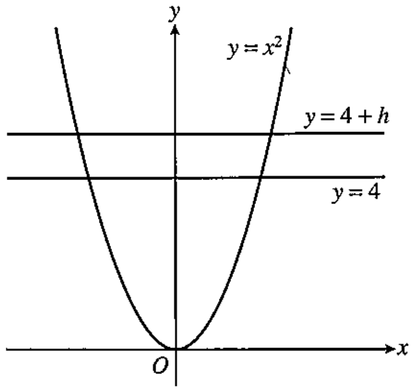

Figure 3

In Figure 3, the region enclosed by the curve $y = x^2$, the straight lines $y = 4$ and $y = 4 + h$ (where $h \geq 0$) is revolved about the y-axis. Show the the volume of the solid of revolution is $\frac{\pi}{2}(h^2 + 8h)$.

(2 marks)

(b)

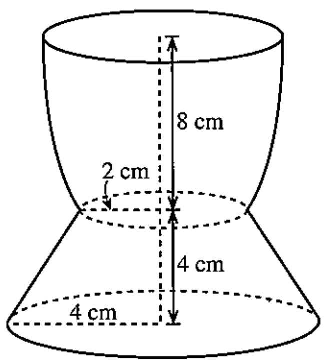

Figure 4

In Figure 4, an empty cup consists of two portions. The upper portion is in the shape of the solid described in (a) with height 8 cm. The lower portion is a frustum of a right circular cone. The height of the frustum is 4 cm and the radii of the bases of the frustum are 4 cm and 2 cm respectively. Milk is poured into the cup to a depth H cm at a rate of 8 cm^3s^{-1}, where $0 \leq H \leq 12$.

Let V cm^3 be the volume of milk in the cup.

(i) Find the rate of increase of the depth of milk when the depth is 7 cm.

(ii) Show that $V = \frac{128\pi}{3}\left[1 - \left(1 - \frac{H}{8}\right)^3\right]$ for $0 \leq H \leq 4$.

(iii) After the cup is fully filled, suddenly it cracks at the bottom and the milk is leaking out. It is given that the rate of leaking is $\frac{\pi}{2} \cdot \text{cm}^3\text{s}^{-1}$. Find the rate of decrease of the depth of milk after 130 seconds of leaking. (Give the answer correct to 3 significant figures.)

(11 marks)

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