

# HKDSE MATH CORE 2022 Past Paper I

## 1. HKDSE MATH CORE 2022 Past Paper I Q1

Simplify  $\frac{(a^3b^{-2})^4}{a^{-5}b^6}$  and express your answer with positive indices.  
(3 marks)

## 2. HKDSE MATH CORE 2022 Past Paper I Q2

Let  $x$  and  $y$  be two numbers. The sum of  $x$  and  $y$  is 456 while the product of 7 and  $x$  is  $y$ . Find  $x$ .  
(3 marks)

## 3. HKDSE MATH CORE 2022 Past Paper I Q3

Simplify  $\frac{3}{k-9} + \frac{2}{5k+6}$ .  
(3 marks)

## 4. HKDSE MATH CORE 2022 Past Paper I Q4

Factorize

(a)  $9c^2 - 6x + 1$ ,

(b)  $(4c + d)^2 - 9c^2 + 6x - 1$ .

(4 marks)

## 5. HKDSE MATH CORE 2022 Past Paper I Q5

A fan is sold at a discount of 30% on its marked price. After selling the fan, the profit is \$78 and the percentage profit is 26%. Find the marked price of the fan.  
(4 marks)

## 6. HKDSE MATH CORE 2022 Past Paper I Q6

Consider the compound inequality

$$-2(3x + 2) > x + 10 \text{ or } 2x \leq -8 \dots\dots\dots (*).$$

(a) Solve (\*).

(b) Write down the greatest integer satisfying (\*).

(4 marks)

## 7. HKDSE MATH CORE 2022 Past Paper I Q7

The coordinates of the points  $S$  and  $T$  are  $(12, -5)$  and  $(-3, -7)$  respectively.  $S$  is rotated anticlockwise about  $O$  through  $90^\circ$  to  $S'$ , where  $O$  is the origin.  $T'$  is the reflection image of  $T$  with respect to the  $x$ -axis.

(a) Write down the coordinates of  $S'$  and  $T'$ .

(b) Find the slope of  $S'T'$ .

(4 marks)

**8. HKDSE MATH CORE 2022 Past Paper I Q8**

In Figure 1,  $A$  is a point lying inside the quadrilateral  $BCDE$  such that  $AC \parallel ED$  and  $AD \parallel BC$ . It is given that  $\angle ABD = \angle AED$  and  $AB = AE$ .

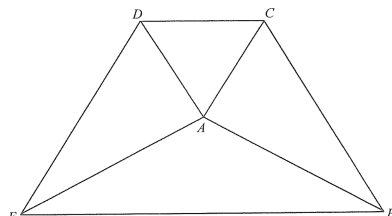


Figure 1

(a) Prove that  $\triangle ABC \cong \triangle AED$ .

(b) If  $\angle ABC = 39^\circ$  and  $\angle DAE = 87^\circ$ , find  $\angle ACD$ .

(5 marks)

**9. HKDSE MATH CORE 2022 Past Paper I Q9**

The frequency distribution table and the cumulative frequency distribution table below show the distribution of the times taken to complete a 3 km race by a group of students.

Time taken (minutes)	Frequency
10 – 14	$a$
15 – 19	9
20 – 24	$b$
25 – 29	3

Time taken less than (minutes)	Cumulative frequency
14.5	3
19.5	$x$
24.5	$y$
29.5	20

(a) Write down the value of  $x$ .

(b) Find the mean of the distribution.

(c) Find the probability that the time taken to complete the 3 km race by a randomly selected student from the group is less than 19.5 minutes.

(5 marks)

**10. HKDSE MATH CORE 2022 Past Paper I Q10**

It is given that  $f(x)$  partly varies as  $x^2$  and partly varies as  $x$ . Suppose that  $f(4) = 96$  and  $f(-5) = 15$ .

(a) Find  $f(x)$ .

(3 marks)

(b) Write down the  $x$ -intercept of the graph of  $y = 8f(x)$ .

(1 marks)

- (c) Let  $k$  be a real constant. Find the range of values of  $k$  such that the equation  $f(x) = k$  has two distinct real roots.  
(2 marks)

**11. HKDSE MATH CORE 2022 Past Paper I Q11**

The stem-and-leaf diagram below shows the distribution of the ages of the players of a football team. The inter-quartile range and the median of the distribution are 14 and 31 respectively.

Stem (tens)	Leaf (units)
1	7 8 9
2	0 $a$ $a$ 8 8 9
3	$b$ $b$ 5 5 6 6 6 6 7 8
4	3

- (a) Find  $a$  and  $b$ .  
(3 marks)
- (b) A player now leaves the football team.
- Is there any change in the mode of the distribution due to the leaving of the player? Explain your answer.
  - If the range of the distribution is decreased, find the greatest possible standard deviation of the distribution.
- (4 marks)

**12. HKDSE MATH CORE 2022 Past Paper I Q12**

The equation of the circle  $C$  is  $x^2 + y^2 - 154x - 128y + 224 = 0$ . Denote the centre of  $C$  by  $G$ . The coordinates of the point  $H$  are  $(65, 48)$ .

- (a) Find the distance between  $G$  and  $H$ .  
(3 marks)
- (b) Let  $P$  be a moving point on  $C$ . When the area of  $\triangle GHP$  is the greatest,
- describe the geometric relationship between  $GH$  and  $GP$ ;
  - find the perimeter of  $\triangle GHP$ .
- (4 marks)

**13. HKDSE MATH CORE 2022 Past Paper I Q13**

There are two solid metal spheres. The ration of the surface area of the smaller sphere to the surface area of the larger sphere is 4:9. The radius of the larger sphere 9 cm.

- (a) Express, in terms of  $\pi$ , the volume of the smaller sphere.  
(3 marks)

- (b) The two spheres are melted and recast into two solid right circular cones. Denote these two circular cones by  $A$  and  $B$ . It is given that the height and the base radius of  $A$  are 10 cm and 6 cm respectively. A student finds that the base radius of  $B$  is 12 cm. The student claims that  $A$  and  $B$  are similar. Is the claim correct? Explain your answer.

(4 marks)

**14. HKDSE MATH CORE 2022 Past Paper I Q14**

Let  $p(x) = 2x^3 + ax^2 + bx - 20$ , where  $a$  and  $b$  are constants. When  $p(x)$  is divided by  $x^2 - 2x + 3$ , the remainder is  $x + 13$ .

- (a) Find  $a$  and  $b$ .

(3 marks)

- (b) Is  $x - 5$  a factor of  $p(x)$ ? Explain your answer.

(2 marks)

- (c) Someone claims that the equation  $p(x) = 0$  has two irrational roots. Do you agree? Explain your answer.

(3 marks)

**15. HKDSE MATH CORE 2022 Past Paper I Q15**

There are 10 boys and 12 girls in a class. If 4 students are randomly selected from the class to form a committee.

- (a) find the probability that there are 2 boys and 2 girls in the committee.

(2 marks)

- (b) find the probability that the number of boys and the number of girls in the committee are different.

(2 marks)

**16. HKDSE MATH CORE 2022 Past Paper I Q16**

Let  $g(x) = 3x^2 + 12kx^2 + 16k^2 + 8$ , where  $k$  is a non-zero constant.

- (a) Using the method of completing the square, express, in terms of  $k$ , the coordinates of the vertex of the graph of  $y = g(x)$ .

(2 marks)

- (b) On the same rectangular coordinates system, denote the vertex of the graph of  $y = g(x)$  and the vertex of the graph of  $y = 2g(-x)$  by  $A$  and  $B$  respectively. Let  $M$  be a point lying on  $AB$  such that the area of  $\triangle OBM$  is the triple of the area of  $\triangle OAM$ , where  $O$  is the origin. Express, in terms of  $k$ , the coordinates of  $M$ .

(3 marks)

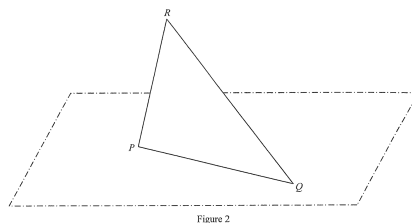
**17. HKDSE MATH CORE 2022 Past Paper I Q17**

Let  $c$  be a real constant. The roots of the equation  $x^2 + cx - 9 = 0$  are  $\alpha$  and  $\beta$ .

- (a) Express  $\alpha^2 + \beta^2$  in terms of  $c$ .  
(3 marks)
- (b) The 1st term, the 2nd term and the 3rd term of an arithmetic sequence are  $c^2$ ,  $\alpha^2 + \beta^2$  and 85 respectively. Find the least value of  $n$  such that the sum of the first  $n$  terms of the sequence is greater than  $2 \times 10^6$ .  
(4 marks)

**18. HKDSE MATH CORE 2022 Past Paper I Q18**

In Figure 2, the triangular paper and  $PQR$  is held such that  $PQ$  lies on the horizontal ground. It is given that  $PQ = 30$  cm,  $PR = 25$  cm and  $\angle QPR = 95^\circ$ .



- (a) Find
- (i) the length of  $QR$ ,
  - (ii)  $\angle PQR$ .
- (4 marks)
- (b) Let  $M$  be the mid-point of  $QR$ . A craftman finds that the angle between  $PR$  and the horizontal ground is  $70^\circ$ . The craftman claims that the angle between  $PM$  and the horizontal ground exceeds  $40^\circ$ . Is the claim correct? Explain your answer.  
(3 marks)

**19. HKDSE MATH CORE 2022 Past Paper I Q19**

The centre of the circle  $C$  is the point  $G(83, 112)$ . It is found that the point  $A(158, 12)$  lies outside  $C$ ,  $AP$  and  $AQ$  are the tangents to  $C$  at the points  $P$  and  $Q$  respectively. It is given that  $C$  passes through the point  $(23, 67)$ .

- (a) Find the equation of the straight line passing through  $A$  and  $G$ .  
(2 marks)
- (b) Find the coordinates of the point of intersection of  $AG$  and  $PQ$ .  
(3 marks)
- (c) Find the equation of the inscribed circle of  $\triangle APQ$ .  
(4 marks)
- (d) Someone claims that the ratio of the area of the inscribed circle to the area of the circumcircle of  $\triangle APQ$  is  $1 : 4$ . Do you agree? Explain your answer.  
(3 marks)