Function By Topic

1. **HKDSE MATH CORE 2012 Past Paper I Q13**

   (a) Find the value of k such that $x - 2$ is a factor of $kx^3 - 21x^2 + 24x - 4$.

   (2 marks)

   (b) Figure 4 shows the graph of $y = 15x^2 - 63x + 72$. Q is a variable point on the graph in the first quadrant. P and R are the feet of the perpendiculars from Q to the x-axis and the y-axis respectively.
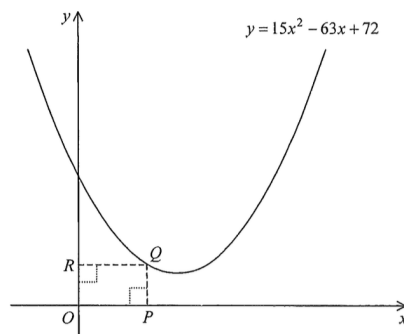

   Figure 4

   (i) Let $(m, 0)$ be the coordinates of P. Express the area of the rectangle $OPQR$ in terms of m.

   (ii) Are there three different positions of Q such that the area of the rectangle $OPQR$ is 12? Explain your answer.

   (5 marks)

2. **HKDSE MATH CORE 2013 Past Paper I Q12**

   Let $f(x) = 3x^3 - 7x^2 + kx - 8$, where k is a constant. It is given that $f(x) \equiv (x - 2)(ax^2 + bx + c)$, where a, b and c are constants.

   (a) Find a, b and c.

   (4 marks)

   (b) Someone claims that all the roots of the equation $f(x) = 0$ are real numbers. Do you agree? Explain your answer.

   (3 marks)

3. **HKDSE MATH CORE 2014 Past Paper I Q13**

   It is given that $f(x)$ is the sum of two parts, one part varies as x^2 and the other part is a constant. Suppose that $f(2) = 59$ and $f(7) = -121$.

   (a) Find $f(6)$.

   (4 marks)

   (b) $A(6, a)$ and $B(-6, b)$ are points lying on the graph of $y = f(x)$. Find the area of $\triangle ABC$, where C is a point lying on the x-axis.

   (4 marks)

4. **HKDSE MATH CORE 2015 Past Paper I Q11**

   Let $f(x) = (x-2)^2(x+h) + k$, where h and k are constants. When $f(x)$ is divided by $x - 2$, the remainder is 5. It is given that $f(x)$ is divisible by $x - 3$.

   (a) Find h and k.

   (3 marks)

   (b) Someone claims that all the roots of the equation $f(x) = 0$ are integers. Do you agree? Explain your answer.

   (3 marks)

5. **HKDSE MATH CORE 2016 Past Paper I Q14**

   Let $p(x) = 6x^4 + 7x^3 + ax^2 + bx + c$, where a, b and c are constants. When $p(x)$ is divided by $x + 2$ and when $p(x)$ is divided by $x - 2$, the two remainders are equal. It is given that $p(x) \equiv (lx^2 + 5x + 8)(2x^2 + mx + n)$, where l, m and n are constants.

   (a) Find l, m and n.

   (5 marks)

   (b) How many real roots does the equation $p(x) = 0$ have? Explain your answer.

   (5 marks)

6. **HKDSE MATH CORE 2017 Past Paper I Q14**

   Let $f(x) = 6x^3 - 13x^2 - 46x + 34$. When $f(x)$ is divided by $2x^2 + ax + 4$, the quotient and the remainder are $3x + 7$ and $bx + c$ respectively, where a, b and c are constants.

   (a) Find a.

   (3 marks)

   (b) Let $g(x)$ be a quadratic polynomial such that when $g(x)$ is divided by $2x^2 + ax + 4$, the remainder is $bx + c$.

   (i) Prove that $f(x) - g(x)$ is divisible by $2x^2 + ax + 4$.

   (ii) Someone claims that all the roots of the equation $f(x) - g(x) = 0$ are integers. Do you agree? Explain your answer.

   (5 marks)

7. **HKDSE MATH CORE 2018 Past Paper I Q12**

   Let $f(x) = 4x(x+1)^2 + ax + b$, where a and b are constants. It is given that $x - 3$ is a factor of $f(x)$. When $f(x)$ is divided by $x + 2$, the remainder is $2b + 165$.

   (a) Find a and b.

   (3 marks)

   (b) Someone claims that the equation $f(x) = 0$ has at least one irrational root. Do you agree? Explain your answer.

   (4 marks)

8. **HKDSE MATH CORE 2019 Past Paper I Q11**

   Let $p(X)$ be a cubic polynomial. When $p(x)$ is divided by $x - 1$, the remainder is 50. When $p(X)$ is divided by $x + 2$, the remainder is -52. It is given that $p(x)$ is divisible by $2x^2 + 9x + 14$.

   (a) Find the quotient when $p(X)$ is divided by $2x^2 + 9x + 14$.

   (3 marks)

   (b) How many rational roots does the equation have? Explain your answer.

   (3 marks)

9. **HKDSE MATH CORE 2020 Past Paper I Q13**

   The cubic polynomial $f(x)$ is divisible by $x - 1$. When $f(x)$ is divided by $x^2 - 1$, the remainder is $kx + 8$, where k is a constant.

   (a) Find k.

   (3 marks)

   (b) It is given that $x + 3$ is a factor of $f(x)$. When $f(x)$ is divided by x, the remainder is 24. Someone claims that all the roots of the equation $f(x) = 0$ are integers. Is the claim correct? Explain your answer.

   (5 marks)

10. **HKDSE MATH CORE 2021 Past Paper I Q12**

    The polynomial $p(x)$ is divisible by $x - 5$. When $p(x)$ is divided by $x^2 + x + 1$, the quotient and the remainder are $2x^2 - 37$ and $cx + c - 1$ respectively, where c is a constant.

    (a) Find c.

    (3 marks)

    (b) Prove that $x + 3$ is a factor of $p(x)$.

    (1 marks)

    (c) Someone claims that all the roots of the equation $p(x) = 0$ are real numbers. Is the claim correct? Explain your answer.

    (3 marks)

11. **HKDSE MATH CORE 2022 Past Paper I Q14**

    Let $p(x) = 2x^3 + ax^2 + bx - 20$, where a and b are constants. When $p(x)$ is divided by $x^2 - 2x + 3$, the remainder is $x + 13$.

    (a) Find a and b.

    (3 marks)

    (b) Is $x - 5$ a factor of $p(x)$? Explain your answer.

    (2 marks)

    (c) Someone claims that the equation $p(x) = 0$ has two irrational roots. Do you agree? Explain your answer.

    (3 marks)

12. **HKDSE MATH CORE 2023 Past Paper I Q13**

Define $g(x) = x^3 + 5x^2 - 12x - 1$. Let $h(x) = 3x^4 + ax^3 - 16x^2 + bx + c$, where a, b and c are constants. When $h(x)$ is divided by $g(x)$, the quotient and the remainder are equal.

(a) Find the quotient when $h(x)$ is divided by $g(x)$.

(3 marks)

(b) How many rational roots does the equation $h(x) = 0$ have? Explain your answer.

(4 marks)