MI Exercise

1. Prove, by mathematical induction, that

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

for all positive integer n.

2. Prove, by mathematical induction, that

$$1 + 2 + 3 + \ldots + (2n-1) + (2n) = n(2n+1)$$

for all positive integer n.

3. Prove, by mathematical induction, that

$$1 + 3 + 5 + \ldots + (2n-1) = n^2$$

for all positive integer n.

4. Prove, by mathematical induction, that

$$\sum_{k=1}^{n} kr^{k-1} = \frac{(r-1)(nr^n) - (r^n - 1)}{(r-1)^2}, \text{ for } r \neq 1$$

for all positive integer n.

5. Prove, by mathematical induction, that

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

for all positive integer n.

6. Prove, by mathematical induction, that

$$\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{n \cdot (n+1)} = 1 - \frac{1}{n+1}$$

for all positive integer n.

7. Prove, by mathematical induction, that

$$1 + r + r^2 + \ldots + r^{2n} = \frac{r^{2n+1} - 1}{r - 1}$$

for all positive integer n.