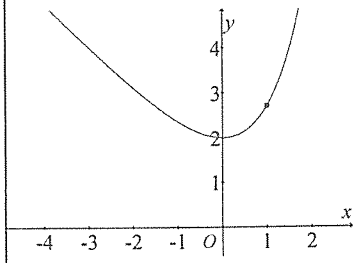
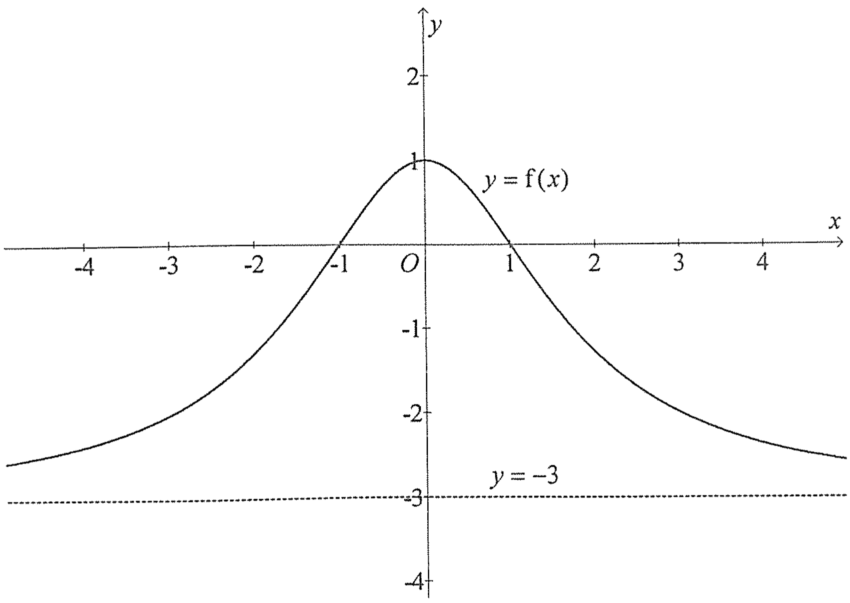
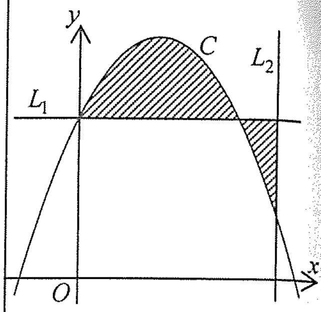
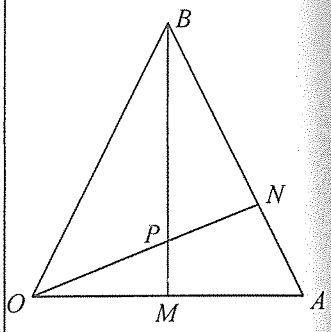
Solution	Marks	Remarks
1. $\dfrac{d}{dx}(\sin 2x)=\lim\limits_{h\to 0}\dfrac{\sin 2(x+h)-\sin 2x}{h}$	1M	
$=\lim\limits_{h\to 0}\left(\dfrac{2}{h}\cos\dfrac{2x+2h+2x}{2}\sin\dfrac{2x+2h-2x}{2}\right)$	1M	
$=\lim\limits_{h\to 0}\left[2\cos(2x+h)\dfrac{\sin h}{h}\right]$		
$=2\lim\limits_{h\to 0}\cos(2x+h)\cdot\lim\limits_{h\to 0}\dfrac{\sin h}{h}$	1M	
Alternative Solution		
$=\lim\limits_{h\to 0}\dfrac{\sin 2h\cos 2x+\cos 2h\sin 2x-\sin 2x}{h}$	1M	
$=\lim\limits_{h\to 0}\dfrac{\sin 2h\cos 2x-\sin 2x\cdot 2\sin^2 h}{h}$		
$=2\cos 2x\cdot\lim\limits_{h\to 0}\dfrac{\sin 2h}{2h}-2\sin 2x\cdot\lim\limits_{h\to 0}\sin h\cdot\lim\limits_{h\to 0}\dfrac{\sin h}{h}$	1M	
$=2\cos 2x$	1A	
	(4)	
2. $(1+ax)^n=1+C_1^n ax+C_2^n(ax)^2+\cdots$		OR general term $=C_r^n(ax)^r$
$\begin{cases} na=-20 & \text{-------------- (1)} \\ \dfrac{n(n-1)}{2}a^2=180 & \text{-------------- (2)} \end{cases}$	1M	
$(2)\div(1)^2$:		
$\dfrac{n-1}{2n}=\dfrac{180}{400}$	1M	
$n=10$	1A	
$\therefore\quad a=-2$	1A	
	(4)	
3. For $n=1$,		
L.H.S. $1+\dfrac{1}{1\times 4}=\dfrac{5}{4}$ and R.H.S. $=\dfrac{4(1)+1}{3(1)+1}=\dfrac{5}{4}$		
$\therefore$ L.H.S. = R.H.S. and the statement is true for $n=1$.	1	
Assume $1+\dfrac{1}{1\times 4}+\dfrac{1}{4\times 7}+\dfrac{1}{7\times 10}+\cdots+\dfrac{1}{(3k-2)(3k+1)}=\dfrac{4k+1}{3k+1}$, where k is a positive integer.	1	
$1+\dfrac{1}{1\times 4}+\dfrac{1}{4\times 7}+\dfrac{1}{7\times 10}+\cdots+\dfrac{1}{(3k-2)(3k+1)}+\dfrac{1}{[3(k+1)-2][3(k+1)+1]}$		
$=\dfrac{4k+1}{3k+1}+\dfrac{1}{(3k+1)(3k+4)}$ by the assumption	1	
$=\dfrac{(12k^2+19k+4)+1}{(3k+1)(3k+4)}$		
$=\dfrac{(3k+1)(4k+5)}{(3k+1)(3k+4)}$		
$=\dfrac{4(k+1)+1}{3(k+1)+1}$	1	
Hence the statement is true for $n=k+1$.		
By the principle of mathematical induction, the statement is true for all positive integers n .	1	Follow through
	(5)	

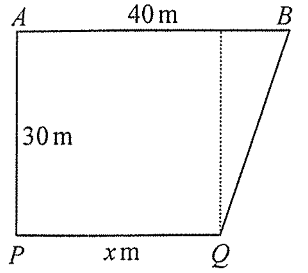
Solution	Marks	Remarks
4. (a) $\dfrac{dy}{dx} = e^x - 1$		
$y = \int (e^x - 1)\,dx$		
$= e^x - x + C$	1A	
Since the curve passes through the point $(1, e)$, $e = e^1 - 1 + C$.	1M	
i.e. $C = 1$		
$\therefore\ y = e^x - x + 1$	1A	
(b) The curve cuts the y-axis at $(0, 2)$.		
When $x = 0$, $\dfrac{dy}{dx} = 0$.	1M	
Hence the equation of tangent to the curve at $(0, 2)$ is		
$\boxed{y - 2 = 0(x - 0)}$		
$y = 2$	1A	
	(5)	
5. (a) $f(x) = \dfrac{3 - 3x^2}{3 + x^2}$		
$\therefore\ f(0) = 1,\ f(1) = 0$ and $f(-1) = 0$		
$\therefore$ maximum point is $(0, 1)$,	1A	
and points of inflexion are $(1, 0)$ and $(-1, 0)$.	1A	For both
(b) Since $x^2 + 3 > 0$, there is no vertical asymptote.		
$f(x) = -3 + \dfrac{12}{x^2 + 3}$	1M	OR $f(x) = \dfrac{\frac{3}{x^2} - 3}{\frac{3}{x^2} + 1}$
When $x \to \pm\infty$, $y \to -3$.		
Hence $y = -3$ is a horizontal asymptote.	1A	
(c) 	1A 1A	For shape of $y = f(x)$ For all correct
	(6)	

Solution	Marks	Remarks
6. (a) Area $=\int_0^4\left[\left(\frac{-x^2}{2}+2x+4\right)-4\right]dx+\int_4^5\left[4-\left(\frac{-x^2}{2}+2x+4\right)\right]dx$	1M	
$=\int_0^4\left(\frac{-x^2}{2}+2x\right)dx+\int_4^5\left(\frac{x^2}{2}-2x\right)dx$		
$=\left[\frac{-x^3}{6}+x^2\right]_0^4+\left[\frac{x^3}{6}-x^2\right]_4^5$	1M	
$=\frac{13}{2}$	1A	
(b) Volume $=\pi\int_0^5\left(\frac{-x^2}{2}+2x+4-4\right)^2 dx$	1M+1A	
$=\pi\int_0^5\left(\frac{x^4}{4}-2x^3+4x^2\right)dx$		
$=\pi\left[\frac{x^5}{20}-\frac{x^4}{2}+\frac{4x^3}{3}\right]_0^5$		
$=\frac{125\pi}{12}$	1A	
	(6)	
7. (a) R.H.S. $=\frac{\sin 2x}{1+\cos 2x}$		
$=\frac{2\sin x\cos x}{1+2\cos^2 x-1}$	1M	For either formula
$=\frac{\sin x}{\cos x}$		
$=\tan x$	1	
= L.H.S.		
(b) R.H.S. $=\frac{\sin 8y\cos 4y\cos 2y}{(1+\cos 8y)(1+\cos 4y)(1+\cos 2y)}$		
$=\tan 4y\cdot\frac{\cos 4y\cos 2y}{(1+\cos 4y)(1+\cos 2y)}$ by (a)	1M	
$=\frac{\sin 4y\cos 2y}{(1+\cos 4y)(1+\cos 2y)}$	1M	
$=\tan 2y\cdot\frac{\cos 2y}{1+\cos 2y}$ by (a)		
$=\frac{\sin 2y}{1+\cos 2y}$		
$=\tan y$ by (a)	1	
Alternative Solution		
$=\frac{\sin 8y\cos 4y\cos 2y}{\left(\frac{\sin 8y}{\tan 4y}\right)\left(\frac{\sin 4y}{\tan 2y}\right)\left(\frac{\sin 2y}{\tan y}\right)}$ by (a)	1M	
$=\frac{\sin 8y}{\sin 8y}\cdot\frac{\tan 4y\cos 4y}{\sin 4y}\cdot\frac{\tan 2y\cos 2y}{\sin 2y}\cdot\tan y$		
$=\tan y$	1M + 1	1M for $\tan x\cos x=\sin x$
= L.H.S.		
	(5)	

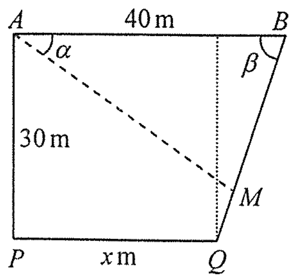
Solution	Marks	Remarks
8. (a) $\begin{pmatrix}1 & k & 0\\0 & 1 & 1\\k & 0 & 0\end{pmatrix}^{-1} = \dfrac{1}{\begin{vmatrix}1 & k & 0\\0 & 1 & 1\\k & 0 & 0\end{vmatrix}}\begin{pmatrix}0 & k & -k\\0 & 0 & k^2\\k & -1 & 1\end{pmatrix}^T$	1M+1A	1M for minors
$= \dfrac{1}{k^2}\begin{pmatrix}0 & 0 & k\\k & 0 & -1\\-k & k^2 & 1\end{pmatrix}$	1A	
(b) $\begin{pmatrix}1 & k & 0\\0 & 1 & 1\\k & 0 & 0\end{pmatrix}\begin{pmatrix}x\\1\\z\end{pmatrix} = \begin{pmatrix}2\\2\\1\end{pmatrix}$		
$\begin{pmatrix}x\\1\\z\end{pmatrix} = \dfrac{1}{k^2}\begin{pmatrix}0 & 0 & k\\k & 0 & -1\\-k & k^2 & 1\end{pmatrix}\begin{pmatrix}2\\2\\1\end{pmatrix}$ by (a)	1M	
$= \dfrac{1}{k^2}\begin{pmatrix}k\\2k-1\\2k^2-2k+1\end{pmatrix}$		
From the second row, we have $\dfrac{2k-1}{k^2} = 1$.		
Alternative Solution $\begin{pmatrix}x+k\\1+z\\kx\end{pmatrix} = \begin{pmatrix}2\\2\\1\end{pmatrix}$ From the first and third rows, we have $x+k=2$ and $x=\dfrac{1}{k}$. $\therefore\ \dfrac{1}{k}+k=2$.	1M	
i.e. $k^2-2k+1=0$ $k=1$	1A	
	(5)	
9. (a) The augmented matrix is $\left(\begin{array}{ccc\|c}1 & -a & 1 & 2\\2 & 1-2a & 2-b & a+4\\3 & 1-3a & 3-ab & 4\end{array}\right)$		
$\sim\left(\begin{array}{ccc\|c}1 & -a & 1 & 2\\0 & 1 & -b & a\\0 & 1 & -ab & -2\end{array}\right)$	1M	
$\sim\left(\begin{array}{ccc\|c}1 & -a & 1 & 2\\0 & 1 & -b & a\\0 & 0 & ab-b & a+2\end{array}\right)$		
Hence the system has infinitely many solutions when $\begin{cases}b(a-1)=0\\a+2=0\end{cases}$	1M	
i.e. $a=-2$ and $b=0$	1A	For both

Solution	Marks	Remarks
(b) The system becomes $\begin{cases} x + 2y + z = 2 \\ 2x + 5y + 2z = 2 \\ 3x + 7y + 3z = 4 \end{cases}$	1M	
i.e. $\begin{cases} x + z = 6 \\ y = -2 \end{cases}$		
$(x, y, z) = (6-t, -2, t)$ for any real number t	1A	OR $(t, -2, 6-t)$
	(5)	
10. (a) $\overrightarrow{ON} = \dfrac{k\overrightarrow{OA} + \overrightarrow{OB}}{k+1}$	1M	
$= \dfrac{k(2\mathbf{i}) + (\mathbf{i} + 2\mathbf{j})}{k+1}$		
$= \dfrac{(2k+1)\mathbf{i} + 2\mathbf{j}}{k+1}$	1A	
(b) $\because$ $\overrightarrow{MB} = 2\mathbf{j}$, $\therefore$ $BM \perp OA$	1M	
Since A, N, P and M are concyclic, $ON \perp AB$.		
$\therefore$ $\overrightarrow{ON} \cdot \overrightarrow{AB} = 0$	1M	
$\dfrac{(2k+1)\mathbf{i} + 2\mathbf{j}}{k+1} \cdot (\mathbf{i} + 2\mathbf{j} - 2\mathbf{i}) = 0$		
$-(2k+1) + 2 \cdot 2 = 0$		
$k = \dfrac{3}{2}$	1A	
	(5)	
11. (a) $\dfrac{\mathrm{d}}{\mathrm{d}\theta} \ln(\sec\theta + \tan\theta) = \dfrac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta + \tan\theta}$	1M	
$= \sec\theta$		
Hence $\int \sec\theta \, \mathrm{d}\theta = \int \dfrac{\mathrm{d}}{\mathrm{d}\theta} \ln(\sec\theta + \tan\theta) \, \mathrm{d}\theta$		
$= \ln(\sec\theta + \tan\theta) + C$	1	
Alternative Solution		
$\int \sec\theta \, \mathrm{d}\theta = \int \dfrac{\sec\theta(\sec\theta + \tan\theta)}{\sec\theta + \tan\theta} \, \mathrm{d}\theta$	1M	
Let $u = \sec\theta + \tan\theta$ which gives $\mathrm{d}u = (\sec\theta\tan\theta + \sec^2\theta)\mathrm{d}\theta$.		
$\therefore$ $\int \sec\theta \, \mathrm{d}\theta = \int \dfrac{\mathrm{d}u}{u}$		
$= \ln\lvert u\rvert + C$		
$= \ln(\sec\theta + \tan\theta) + C$ since $\sec\theta + \tan\theta > 0$ for $0 < \theta < \dfrac{\pi}{2}$	1	
	(2)	

Solution	Marks	Remarks
(b) (i) Let $u=\sec\theta$, where $0<\theta<\frac{\pi}{2}$.	1M	
$\therefore\ \mathrm{d}u=\sec\theta\tan\theta\,\mathrm{d}\theta$		
$\int\frac{\mathrm{d}u}{\sqrt{u^2-1}}=\int\frac{\sec\theta\tan\theta\,\mathrm{d}\theta}{\sqrt{\sec^2\theta-1}}$		
$=\int\sec\theta\,\mathrm{d}\theta$ since $\tan\theta>0$ for $0<\theta<\frac{\pi}{2}$		
$=\ln(\sec\theta+\tan\theta)+C$ by (a)		
$=\ln(\sec\theta+\sqrt{\sec^2\theta-1})+C$ since $\tan\theta>0$ for $0<\theta<\frac{\pi}{2}$		
$=\ln(u+\sqrt{u^2-1})+C$	1	
(ii) $\int_0^1\frac{2x}{\sqrt{x^4+4x^2+3}}\,\mathrm{d}x=\int_0^1\frac{2x}{\sqrt{(x^2+2)^2-1}}\,\mathrm{d}x$		
Let $u=x^2+2$ which gives $\mathrm{d}u=2x\,\mathrm{d}x$.	1M	
When $x=0$, $u=2$; when $x=1$, $u=3$.		
$\therefore\ \int_0^1\frac{2x}{\sqrt{x^4+4x^2+3}}\,\mathrm{d}x=\int_2^3\frac{\mathrm{d}u}{\sqrt{u^2-1}}$		
$=\left[\ln(u+\sqrt{u^2-1})\right]_2^3$ by (i)	1M	For primitive function
$=\ln(3+\sqrt{8})-\ln(2+\sqrt{3})$		
$=\ln\left(\frac{3+2\sqrt{2}}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)$		
$=\ln(6+4\sqrt{2}-3\sqrt{3}-2\sqrt{6})$	1	
	(5)	
(c) $t=\tan\phi$		
$\frac{\mathrm{d}t}{\mathrm{d}\phi}=\sec^2\phi$		
$=1+\tan^2\phi$		
$\therefore\ \frac{\mathrm{d}\phi}{\mathrm{d}t}=\frac{1}{1+t^2}$	1	
$\cos^2\phi=\frac{1}{\sec^2\phi}$		
$=\frac{1}{1+t^2}$	1A	
$\int_0^{\frac{\pi}{4}}\frac{\tan\phi}{\sqrt{1+2\cos^2\phi}}\,\mathrm{d}\phi=\int_0^1\frac{t}{\sqrt{1+\frac{2}{1+t^2}}}\cdot\frac{1}{1+t^2}\,\mathrm{d}t$ where $t=\tan\phi$	1M	
$=\int_0^1\frac{t}{\sqrt{(3+t^2)(1+t^2)}}\,\mathrm{d}t$	1A	
$=\frac{1}{2}\int_0^1\frac{2t}{\sqrt{t^4+4t^2+3}}\,\mathrm{d}t$		
$=\frac{1}{2}\ln(6+4\sqrt{2}-3\sqrt{3}-2\sqrt{6})$	1A	OR $\ln\sqrt{6+4\sqrt{2}-3\sqrt{3}-2\sqrt{6}}$
	(5)	

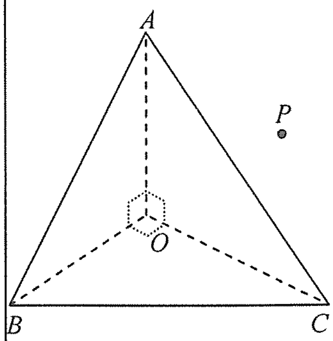
Solution	Marks	Remarks
12. (a) (i) $T=\dfrac{PQ}{7}+\dfrac{QB}{1.4}$	1M	
$=\dfrac{x}{7}+\dfrac{5\sqrt{30^2+(40-x)^2}}{7}$		
$=\dfrac{x+5\sqrt{x^2-80x+2500}}{7}$	1A	OR $\dfrac{x}{7}+\dfrac{\sqrt{x^2-80x+2500}}{1.4}$
(ii) When T is minimum, $\dfrac{\mathrm{d}T}{\mathrm{d}x}=0$.		
$\dfrac{1}{7}\left[1+\dfrac{5(2x-80)}{2\sqrt{x^2-80x+2500}}\right]=0$	1M	
$5(x-40)=-\sqrt{x^2-80x+2500}$		
$25x^2-2000x+40000=x^2-80x+2500$		
$2x^2-160x+3125=0$	1	
$\therefore\quad x=40-\dfrac{5\sqrt{6}}{2}$ or $40+\dfrac{5\sqrt{6}}{2}$ (rejected by checking)		

x	$0<x<40-\dfrac{5\sqrt{6}}{2}$	$x=40-\dfrac{5\sqrt{6}}{2}$	$x>40-\dfrac{5\sqrt{6}}{2}$
$\dfrac{\mathrm{d}T}{\mathrm{d}x}$	$-$	0	$+$

Solution	Marks	Remarks
So, when T is minimum, $x=40-\dfrac{5\sqrt{6}}{2}$.	1M	
$QB=\sqrt{30^2+\left[40-\left(40-\dfrac{5\sqrt{6}}{2}\right)\right]^2}$		
$=\dfrac{25\sqrt{6}}{2}$ m	1	
	(6)	
(b) (i) $\sin\beta=\dfrac{30}{\dfrac{25\sqrt{6}}{2}}=\dfrac{2\sqrt{6}}{5}$	1A	
$\cos\beta=\dfrac{40-\left(40-\dfrac{5\sqrt{6}}{2}\right)}{\dfrac{25\sqrt{6}}{2}}=\dfrac{1}{5}$	1A	
In ΔMAB , $\dfrac{MB}{\sin\alpha}=\dfrac{AB}{\sin(\pi-\alpha-\beta)}$.	1M	
$MB=\dfrac{40\sin\alpha}{\sin(\alpha+\beta)}$		
$=\dfrac{40\sin\alpha}{\sin\alpha\cos\beta+\cos\alpha\sin\beta}$	1M	
$=\dfrac{40\sin\alpha}{\dfrac{1}{5}\sin\alpha+\dfrac{2\sqrt{6}}{5}\cos\alpha}$		
$=\dfrac{200\tan\alpha}{\tan\alpha+2\sqrt{6}}$	1	

Solution	Marks	Remarks
(ii) $\dfrac{\mathrm{d}MB}{\mathrm{d}t}=200\cdot\dfrac{(\tan\alpha+2\sqrt{6})\sec^2\alpha-\tan\alpha\sec^2\alpha}{(\tan\alpha+2\sqrt{6})^2}\cdot\dfrac{\mathrm{d}\alpha}{\mathrm{d}t}$	1M	
$=\dfrac{400\sqrt{6}\sec^2\alpha}{(\tan\alpha+2\sqrt{6})^2}\cdot\dfrac{\mathrm{d}\alpha}{\mathrm{d}t}$		
$\therefore\ -1.4=\dfrac{400\sqrt{6}\sec^2 0.2}{(\tan 0.2+2\sqrt{6})^2}\cdot\dfrac{\mathrm{d}\alpha}{\mathrm{d}t}$		
$\dfrac{\mathrm{d}\alpha}{\mathrm{d}t}\approx-0.0357\ \text{rad s}^{-1}$	1A	
	(7)	
13. (a) (i) $MN=\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}e & f\\ g & h\end{pmatrix}$		
$=\begin{pmatrix}ae+bg & af+bh\\ ce+dg & cf+dh\end{pmatrix}$		
$\operatorname{tr}(MN)=ae+bg+cf+dh$	1A	Either one
$NM=\begin{pmatrix}e & f\\ g & h\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}$		
$=\begin{pmatrix}ea+fc & eb+fd\\ ga+hc & gb+hd\end{pmatrix}$		
$\operatorname{tr}(NM)=ea+fc+gb+hd$		
$\therefore\ \operatorname{tr}(MN)=\operatorname{tr}(NM)$	1	
(ii) $\operatorname{tr}(BAB^{-1})=1+3$		
$\operatorname{tr}(AB^{-1}B)=4$ by (a)(i)	1M	OR $\operatorname{tr}(B^{-1}BA)=4$
$\operatorname{tr}(A)=4$	1	
(iii) $BAB^{-1}=\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$		
$\lvert BAB^{-1}\rvert=\begin{vmatrix}1 & 0\\ 0 & 3\end{vmatrix}$		
$\lvert B\rvert\cdot\lvert A\rvert\cdot\lvert B^{-1}\rvert=1\cdot3-0\cdot0$	1M	
$\lvert B\rvert\cdot\lvert A\rvert\cdot\lvert B\rvert^{-1}=3$		
$\lvert A\rvert=3$	1A	
	(6)	

Solution	Marks	Remarks
(b) (i) $C\begin{pmatrix}x\\y\end{pmatrix}=\lambda_1\begin{pmatrix}x\\y\end{pmatrix}$		
$\begin{pmatrix}px+qy\\rx+sy\end{pmatrix}=\begin{pmatrix}\lambda_1 x\\\lambda_1 y\end{pmatrix}$		
$\begin{cases}(p-\lambda_1)x+qy=0\\rx+(s-\lambda_1)y=0\end{cases}$	1A	
Since this system of equations has non-zero solutions $\begin{pmatrix}x\\y\end{pmatrix}$, $\begin{vmatrix}p-\lambda_1 & q\\r & s-\lambda_1\end{vmatrix}=0$.	1	
Similarly, $C\begin{pmatrix}x\\y\end{pmatrix}=\lambda_2\begin{pmatrix}x\\y\end{pmatrix}$ gives $\begin{vmatrix}p-\lambda_2 & q\\r & s-\lambda_2\end{vmatrix}=0$.	1	
(ii) By (b)(i), λ_1 and λ_2 are the roots of the equation $\begin{vmatrix}p-\lambda & q\\r & s-\lambda\end{vmatrix}=0$	1M	
$(p-\lambda)(s-\lambda)-qr=0$		
$\lambda^2-(p+s)\lambda+ps-qr=0$		
$\lambda^2-\operatorname{tr}(C)\cdot\lambda+\lvert C\rvert=0$	1	
	(5)	
(c) $A\begin{pmatrix}x\\y\end{pmatrix}=\lambda\begin{pmatrix}x\\y\end{pmatrix}$ for some non-zero matrices $\begin{pmatrix}x\\y\end{pmatrix}$		
$\lambda^2-\operatorname{tr}(A)\cdot\lambda+\lvert A\rvert=0$ by (b)(ii) $\lambda^2-4\lambda+3=0$ by (a)(ii) & (a)(iii)	1M	For either one
$\lambda=1$ or 3	1A	
	(2)	

Solution	Marks	Remarks
14. (a) (i) $\overrightarrow{AP}\cdot\overrightarrow{BP} = (\mathbf{p}-\mathbf{a})\cdot(\mathbf{p}-\mathbf{b})$	1M	For $\mathbf{p}-\mathbf{a}$ or $\mathbf{p}-\mathbf{b}$
$= \mathbf{p}\cdot\mathbf{p}-\mathbf{a}\cdot\mathbf{p}-\mathbf{b}\cdot\mathbf{p}+\mathbf{a}\cdot\mathbf{b}$		
$= \mathbf{p}\cdot\mathbf{p}-(\mathbf{a}+\mathbf{b})\cdot\mathbf{p}$ $(\because\ \mathbf{a}\cdot\mathbf{b}=0)$	1	
(ii) $\overrightarrow{AP}\cdot\overrightarrow{BP}+\overrightarrow{BP}\cdot\overrightarrow{CP}+\overrightarrow{CP}\cdot\overrightarrow{AP}=0$ By (i) and some similar results, we have		
$\mathbf{p}\cdot\mathbf{p}-(\mathbf{a}+\mathbf{b})\cdot\mathbf{p}+\mathbf{p}\cdot\mathbf{p}-(\mathbf{b}+\mathbf{c})\cdot\mathbf{p}+\mathbf{p}\cdot\mathbf{p}-(\mathbf{c}+\mathbf{a})\cdot\mathbf{p}=0$	1M	
$3\mathbf{p}\cdot\mathbf{p}-2(\mathbf{a}+\mathbf{b}+\mathbf{c})\cdot\mathbf{p}=0$		
$3\mathbf{p}\cdot\mathbf{p}-2(3\mathbf{d})\cdot\mathbf{p}=0$	1M	
$\mathbf{p}\cdot\mathbf{p}=2\mathbf{p}\cdot\mathbf{d}$ ---------------------------------- (*)	1	
(iii) $\lvert\mathbf{p}-\mathbf{d}\rvert^2=(\mathbf{p}-\mathbf{d})\cdot(\mathbf{p}-\mathbf{d})$	1M	
$=\mathbf{p}\cdot\mathbf{p}-2\mathbf{p}\cdot\mathbf{d}+\mathbf{d}\cdot\mathbf{d}$		
$=\mathbf{d}\cdot\mathbf{d}$ by (*)		
$=\lvert\mathbf{d}\rvert^2$		
Hence $\lvert\mathbf{p}-\mathbf{d}\rvert=\lvert\mathbf{d}\rvert$.	1	
$\therefore\ \lvert\overrightarrow{DP}\rvert=\lvert\overrightarrow{OD}\rvert$		
$\therefore\ PD=OD$ ------------------------------- (**)		
Thus, the distance between P and D is a constant and therefore P lies on the sphere centred at D with fixed radius.	1	
	(8)	
(b) (i) Yes. Since O satisfies (**), O lies on the sphere mentioned in (i).	1A	OR "Since OD is equal to the radius of the sphere, ..."
(ii) Yes. Since $\overrightarrow{DP_1}\times\overrightarrow{DP_2}=\overrightarrow{DP_2}\times\overrightarrow{DP_3}$, the normal vector of the plane containing D, P_1 and P_2 equals the normal vector of the plane containing D, P_2 and P_3.	1M	
Thus the plane containing D, P_1 and P_2 is parallel to the plane containing D, P_2 and P_3. Since D and P_2 are common points of the planes, D, P_1, P_2 and P_3 are on the same plane.	1M	
Since D is the centre of the sphere and P_1, P_2 and P_3 lie on the largest circle on the sphere, the radius of the circle equals the radius of the sphere, which is OD.	1A	Follow through
	(4)	