# HKDSE MATH CORE 2019 Past Paper I

1. **HKDSE MATH CORE 2019 Past Paper I Q1**
   Make $h$ the subject of the formula $9(h + 6k) = 7h + 8$.
   (3 marks)

2. **HKDSE MATH CORE 2019 Past Paper I Q2**
   Simplify $\frac{3}{7x - 6} - \frac{2}{5x - 4}$.
   (3 marks)

3. **HKDSE MATH CORE 2019 Past Paper I Q3**
   The length and the breadth of a rectangle are 24 cm and $(13 + r)$ cm respectively. If the length of a diagonal of the rectangle is $(17 - 3r)$ cm, find $r$.
   (3 marks)

4. **HKDSE MATH CORE 2019 Past Paper I Q4** Factorize
   (a) $4m^2 - 9$,
   (b) $2m^2n + 7mn - 15n$.
   (c) $4m^2 - 9 - 2m^2n - 7mn + 15n$.
   (4 marks)

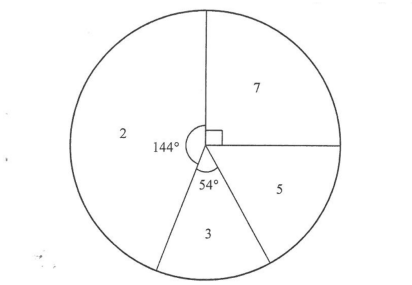
5. **HKDSE MATH CORE 2019 Past Paper I Q5**
   A wallet is sold at a discount of 25% on its marked price. The selling price of the wallet is $690.
   (a) Find the marked price of the wallet.
   (b) After selling the wallet, the percentage profit is 15%. Find the cost of the wallet.
   (4 marks)

6. **HKDSE MATH CORE 2019 Past Paper I Q6**
   (a) Solve the inequality $\frac{7x + 26}{4} \leq 2(3x - 1)$.
   (b) Find the number of integers satisfying both inequalities $\frac{7x + 26}{4} \leq 2(3x - 1)$ and $45 - 5x \geq 0$ .
   (4 marks)

7. **HKDSE MATH CORE 2019 Past Paper I Q7**
   In a playground, the ratio of the number of adults to the number of children is 13 : 6. If 9 adults and 24 children enter the playground, then the ratio of the number of adults to the number of children is 8 : 7. Find the original number of adults in the playground.
   (4 marks)

8. **HKDSE MATH CORE 2019 Past Paper I Q8**

The pie chart below shows the distribution of the numbers of rings owned by the girls in a group.




Distribution of the numbers of rings owned by the girls in the group

(a) Write down the mode of the distribution.

(b) Find the mean of the distribution.

(c) If a girl is randomly selected from the group, find the probability that the selected girl owns more than 3 rings.

(5 marks)

9. **HKDSE MATH CORE 2019 Past Paper I Q9**

The sum of the volumes of two spheres is $324\pi$ cm3. The radius of the larger sphere is equal to the diameter of the smaller sphere. Express, in terms of $\pi$,

(a) The volume of the larger sphere;

(b) The sum of the surface areas of the two spheres.

(5 marks)

10. **HKDSE MATH CORE 2019 Past Paper I Q10**

It is given that $h(x)$ is partly constant and partly varies as $x$. Suppose that $h(-2) = -96$ and $h(5) = 72$.

(a) Find $h(x)$.
(3 marks)

(b) Solve the equation $h(x) = 3x^2$.
(2 marks)

11. **HKDSE MATH CORE 2019 Past Paper I Q11**

Let $p(x)$ be a cubic polynomial. When $p(x)$ is divided by $x - 1$, the remainder is 50. When $p(x)$ is divided by $x + 2$, the remainder is $-52$. It is given that $p(x)$ is divisible by $2x^2 + 9x + 14$.

(a) Find the quotient when $p(x)$ is divided by $2x^2 + 9x + 14$.
(3 marks)

(b) How many rational roots does the equation have? Explain your answer.
(3 marks)

12. **HKDSE MATH CORE 2019 Past Paper I Q12**

The stem-and-leaf diagram below shows the distribution of the results (in seconds) of some boys in a 400 m race. It is given that the inter-quartile range of the distribution is 8 seconds.

| Stem (tens) | Leaf (units) |
|---|---|
| 5 | $a$ |
| 6 | 0 0 3 $c$ $c$ 8 9 9 9 |
| 7 | 0 1 1 1 2 2 5 6 9 |
| 8 | $b$ |

(a) Find $c$.

(2 marks)

(b) It is given that the range of the distribution exceeds 34 seconds and the mean of the distribution is 69 seconds. Find

(i) $a$ and $b$,

(ii) the least possible standard deviation of the distribution.

(6 marks)

13. **HKDSE MATH CORE 2019 Past Paper I Q13**

In Figure 1, $O$ is the centre of circle $ABCDE$. $AC$ is a diameter of the circle. $BD$ and $OC$ intersect at the point $F$. It is given that $\angle AED = 115°$.

(a) Find $\angle CBF$.

(3 marks)

(b) Suppose that $BC//OD$ and $OB = 18$ cm. Is the perimeter of the sector $OBC$ less than 60 cm? Explain your answer.

(5 marks)

14. **HKDSE MATH CORE 2019 Past Paper I Q14**

In Figure 2, $ABCD$ is a square. It is given that $E$ is a point lying on $AD$. $BD$ and $CE$ intersect at the point $F$. Let $G$ be a point such that $BG//EC$ and $CG//DB$.

(a) Prove that

(i) $\triangle BCG \cong \triangle CBF$,

(ii) $\triangle BCF \sim \triangle DEF$.

(4 marks)

(b) Suppose that $\angle BCF = \angle BGC$.

(i) Let $BC = l$. Express $DF$ in terms of $l$.

(ii) Someone claims that $AE > DF$. Do you agree? Explain your answer.

(4 marks)

15. **HKDSE MATH CORE 2019 Past Paper I Q15**

There are 21 boys and 11 girls in a class. If 5 students are selected from the class to form a committee consisting of at least 1 boy, how many different committees can be formed?
(3 marks)

16. **HKDSE MATH CORE 2019 Past Paper I Q16**

Let $\alpha$ and $\beta$ be real numbers such that $\begin{cases} \beta = 5\alpha - 18 \\ \beta = \alpha^2 - 13\alpha + 63 \end{cases}$.

(a) Find $\alpha$ and $\beta$.
(2 marks)

(b) The 1st term and the 2nd term of an arithmetic sequence are $\log \alpha$ and $\log \beta$ respectively. Find the least value of $n$ such that the sum of the first $n$ terms of the sequence is greater than 888.
(4 mark)

17. **HKDSE MATH CORE 2019 Past Paper I Q17**

(a) Let $a$ and $p$ be the area and the perimeter of $\triangle CDE$ respectively. Denote the radius of the inscribed circle of $\triangle CDE$ by $r$. Prove that $pr = 2a$.
(2 marks)

(b) The coordinates of the points $H$ and $K$ are $(9, 12)$ and $(14, 0)$ respectively. Let $P$ be a moving point in the rectangular coordinate plane such that the perpendicular distance from $P$ to $OH$ is equal to the perpendicular distance from $P$ to $HK$, where $O$ is the origin. Denote the locus of $P$ by $\Gamma$.

(i) Describe the geometric relationship between $\Gamma$ and $\triangle OHK$.

(ii) Using (a), find the equation of $\Gamma$.

(5 marks)

18. **HKDSE MATH CORE 2019 Past Paper I Q18**

Figure 3 shows a tetrahedron $ABCD$. Let $P$ be a point lying on $AD$ such that $BP$ is perpendicular to $AD$. A craftsman finds that $AC = AD = CD = 13$ cm, $BC = 8$ cm, $BD = 12$ cm and $\angle ABD = 72°$.

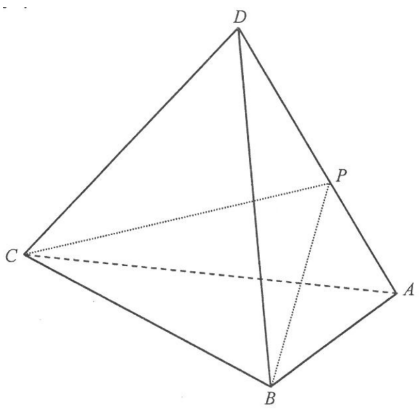


Figure 3

(a) Find

(i) $\angle BAD$,

(ii) $CP$.

(5 marks)

(b) TThe craftsman claims that $\angle BPC$ is the angle between the face $ABD$ and the face $ACD$. Is the claim correct? Explain your answer.
(2 marks)

19. **HKDSE MATH CORE 2019 Past Paper I Q19**

Let $f(x) = \dfrac{1}{1+k}(x^2 + (6k-2)x + (9k+25))$, where $k$ is a positive constant. Denote the point $(4, 33)$ by $F$.

(a) Prove that the graph of $y = f(x)$ passes through $F$.
(1 mark)

(b) The graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ with respect to the $y$-axis and then translating the resulting graph upwards by 4 units. Let $U$ be the vertex of the graph of $y = g(x)$. Denote the origin by $O$.

(i) Using the method of completing the square, express the coordinates of $U$ in terms of $k$.

(ii) Find $k$ such that the area of the circle passing through $F$, $O$ and $U$ is the least.

(iii) For any positive constant $k$, the graph of $y = g(x)$ passes through the same point $G$. Let $V$ be the vertex of the graph of $y = g(x)$ such that the area of the circle passing through $F$, $O$ and $V$ is the least. Are $F$, $G$, $O$ and $V$ concyclic? Explain your answer.

(11 marks)