HKDSE MATH M2 2016

1. **HKDSE Math M2 2016 Q1**

   Expand $(5+x)^4$. Hence, find the constant term in the expansion of $(5+x)^4\left(1-\frac{2}{x}\right)^3$.

   (5 marks)

2. **HKDSE Math M2 2016 Q2**

   Prove that $\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{x+h}}=\frac{h}{(x+h)\sqrt{x}+x\sqrt{x+h}}$. Hence, find $\frac{d}{dx}\sqrt{\frac{3}{x}}$ from first principles.

   (5 marks)

3. **HKDSE Math M2 2016 Q3**

   Consider the curve $C : y = 2e^x$, where $x > 0$. It is given that P is a point lying on C. The horizontal line which passes through P cuts the y-axis at the point Q. Let O be the origin. Denote the x-coordinate of P by u.

   (a) Express the area of $\triangle OPQ$ in terms of u.

   (b) If P moves along C such that OQ increases at a constant rate of 6 units per second, find the rate of change of the area of $\triangle OPQ$ when $u = 4$.

   (5 marks)

4. **HKDSE Math M2 2016 Q4**

   Define $f(x)=\frac{2x^2+x+1}{x-1}$ for all $x \neq 1$. Denote the graph of $y = f(x)$ by G. Find

   (a) the asymptote(s) of G,

   (b) The slope of the normal to G at the point $(2, 11)$.

   (7 marks)

5. **HKDSE Math M2 2016 Q5**

   (a) Using mathematical induction, prove that $\sum_{k=1}^{n}(-1)^k k^2=\frac{(-1)^n n(n+1)}{2}$ for all positive integers n.

   (b) Using (a), evaluate $\sum_{k=3}^{333}(-1)^{k+1}k^2$.

   (6 marks)

6. **HKDSE Math M2 2016 Q6**

   (a) Prove that $x + 1$ is a factor of $4x^3 + 2x^2 - 3x - 1$.

   (b) Express $\cos 3\theta$ in terms of $\cos\theta$.

(C) Using the results of (a) and (b), prove that $\cos\frac{3\pi}{5} = \frac{1-\sqrt{5}}{4}$.
(6 marks)

(6 marks)

7. **HKDSE Math M2 2016 Q7**

(a) Using integration by substitution, find $\int (1+\sqrt{t+1})^2\,dt$.

(b) Consider the curve $\Gamma : y = 4x^2 - 4x$, where $1 \le x \le 4$. Let R be the region bounded by Γ, the straight line $y = 48$ and the two axes. Find the volume of the solid of revolution generated by revolving R about the y-axis.

(8 marks)

8. **HKDSE Math M2 2016 Q8**
Let n be a positive integer.

(a) Define $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. Evaluate

(i) A^2,

(ii) A^n,

(iii) $(A^{-1})^n$.

(b) Evaluate

(i) $\sum_{k=0}^{n-1} 2^k$,

(ii) $\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}^n$.

(8 marks)

9. **HKDSE Math M2 2016 Q9**
Let a and b be constants. Define $f(x) = x^3 + ax^2 + bx + 5$ for all real numbers x. Denote the curve $y = f(x)$ by C. It is given that $P(-1, 10)$ is a turning point of C.

(a) Find a and b.
(3 marks)

(b) Is P a maximum point of C? Explain your answer.
(2 marks)

(c) Find the minimum value(s) of $f(x)$.
(2 marks)

(d) Find the point(s) of inflexion of C.
(2 marks)

(e) Let L be the tangent to C at P. Find the area of the region bounded by C and L.
(4 marks)

10. **HKDSE Math M2 2016 Q10**

(a) Let $f(x)$ be a continuous function defined on the interval $[0, a]$, where a is a positive constant. Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.
(3 marks)

(b) Prove that $\displaystyle\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx = \int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan x}\right) dx$.
(3 marks)

(c) Using (b), prove that $\displaystyle\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx = \frac{\pi \ln 2}{8}$.
(3 marks)

(d) Using integration by parts, evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{x\sec^2 x}{1+\tan x}\,dx$.
(3 marks)

11. **HKDSE Math M2 2016 Q11**

(a) Consider the system of linear equations in real variables x, y, z

$$(E):\begin{cases} x + y - z = 3 \\ 4x + 6y + az = b \\ 5x + (1-a)y + (3a-1)z = b-1 \end{cases}, \text{ where } a \text{ and } b \text{ are real numbers.}$$

(i) Assume that (E) has a unique solution.

(1) Prove that $a \neq -2$ and $a \neq -12$.

(2) Solve (E).

(ii) Assume that $a = -2$ and (E) is consistent.

(1) Find b.

(2) Solve (E).

(9 marks)

(b) Is there a real solution of the system of linear equations

$$\begin{cases} x + y - z = 3 \\ 2x + 3y - z = 7 \\ 5x + 3y - 7z = 13 \end{cases}$$

satisfying $x^2 + y^2 - 6z^2 > 14$? Explain your answer.
(3 marks)

12. **HKDSE Math M2 2016 Q12**
Let $\overrightarrow{OA} = 2\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\overrightarrow{OP} = \mathbf{i} + t\mathbf{j}$, where t is a constant and O is the origin. It is given that P is equidistant from A and B.

(a) Find t.

(3 marks)

(b) Let $\overrightarrow{OC} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{OD} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$. Denote the plane which contains A, B and C by Π.

(i) Find a unit vector which is perpendicular to Π.

(ii) Find the angle between CD and Π.

(iii) It is given that E is a point lying on Π such that $\overrightarrow{DE}$ is perpendicular to Π. Let F be a point such that $\overrightarrow{PF} = \overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$. Describe the geometric relationship between D, E and F. Explain your answer.

(10 marks)