HKDSE
MATH EP
M2

HONG KONG EDUCATIONAL PUBLISHING COMPANY

HONG KONG DIPLOMA OF
SECONDARY EDUCATION EXAMINATION

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)
MOCK EXAM 7
Question-Answer Book

Time allowed: 2½ hours
This paper must be answered in English

Please stick the barcode label here.

Candidate Number									

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
2. This paper consists of **TWO** sections, A and B.
3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. In this paper, vectors may be represented by bold-type letters such as $\mathbf{u}$, but candidates are expected to use appropriate symbols such as $\bar{u}$ in their working.
8. The diagrams in this paper are not necessarily drawn to scale.
9. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

* *

SECTION A (50 marks)

2012

1. It is given that $(1 + kx)^n = 1 - 24x + 252x^2 +$ terms involving higher powers of x, where n is a positive integer and k is a constant.

   (a) Find the values of k and n.

   (b) Find the coefficient of x^3.

   (6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

2. Prove, by mathematical induction, that $\sum_{k=1}^{n} k \times k! = (n+1)! - 1$ for all positive integers n.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

2013

3. Show that $\frac{d}{dx}(\csc x) = -\csc x \cot x$ from first principles.

(4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

4. It is given that $A - B = \frac{\pi}{4}$.

(a) Show that $\tan A = \frac{1 + \tan B}{1 - \tan B}$.

(b) Using the result of (a), find the value of $\tan \frac{3\pi}{8}$.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

2012

5. (a) Using integration by parts, find $\int (\ln x)^2 \, dx$.

(b) In Figure 1, the shaded region is bounded by the curve $y = e^x$ for $0 \leq x \leq 2$ and the x-axis. Find the volume of the solid of revolution when the shaded region is revolved about the y-axis.

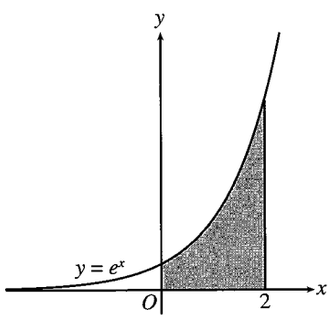

Figure 1

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

2013

6. The slope at any point (x, y) of a curve is given by $\frac{dy}{dx} = \frac{1}{x} + 1$. It is given that the line $2x - y - 3 = 0$ touches the curve at the point P.

(a) Find the coordinates of P.

(b) Find the equation of the curve.

(c) Find the equation of normal to the curve at the point P.

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

7. Let $f(x) = \dfrac{\ln x}{x}$, where $x > 0$.

(a) Find the extreme value of $f(x)$.

(b) Using the result of (a), show that $e^x \geq x^e$ for $x > 0$.

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

2012

8.

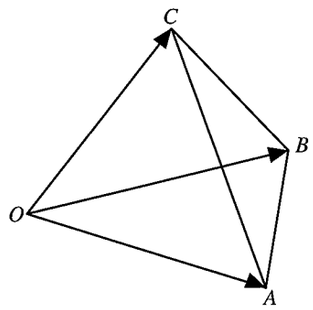

Figure 2

Figure 2 shows a tetrahedron $OABC$. Let $\overrightarrow{OA} = 5\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} + \mathbf{j}$ and $\overrightarrow{OC} = 4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

(a) Find the area of ΔOAB.

(b) Find the distance between point C and the plane OAB.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

9. Let $A = \begin{pmatrix} 7 & 3 & 1 \\ k & 1 & -4 \\ -5 & -4 & 3 \end{pmatrix}$, where k is a positive constant.

(a) Show that the inverse of A is $\frac{1}{13k+26}\begin{pmatrix} 13 & 13 & 13 \\ 3k-20 & -26 & -28-k \\ 4k-5 & -13 & 3k-7 \end{pmatrix}$.

(b) Using the result of (a), solve the system of linear equations $\begin{cases} x+y+z=5 \\ x-2y-3z=-3 \\ 3x-y+2z=0 \end{cases}$.

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

SECTION B (50 marks)

10.

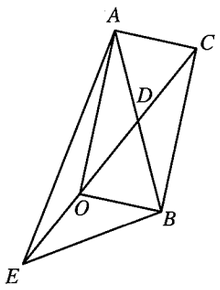

Figure 3

In Figure 3, $OACB$ is a parallelogram, where O is the origin. D is the point of intersection of the diagonals AB and OC. CO is produced to E such that $CD : DE = 1 : 2$. Suppose AB is a diameter of the circle passing through A, C and B, EB is the tangent to the circle at B. It is given that $\overrightarrow{OB} = m\mathbf{i} + n\mathbf{j}$ and $\overrightarrow{OC} = 6\mathbf{i} + 8\mathbf{j}$, where $m > 0$ and $n < 0$.

(a) Find $\overrightarrow{OE}$. (1 mark)

(b) Find the values of m and n. (7 marks)

(c) Is O the incentre of ΔABE? Explain your answer. (4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

11. (a) Show that $\begin{vmatrix} p & q & 1 \\ p^2 & q^2 & 1 \\ p^3 & q^3 & 1 \end{vmatrix} = pq(1-p)(1-q)(q-p)$.

(3 marks)

(b) Consider the system of linear equations in x, y, z:

$$(E): \begin{cases} hx + 3y + z = a \\ h^2x + 9y + z = b \\ h^3x + 27y + z = c \end{cases}, \text{ where } a, b, c \text{ and } h \text{ are real numbers.}$$

(i) Find the values of h such that (E) does not have unique solution.

(ii) Assume $h = 3$ and (E) is consistent. Show that $c - 4b + 3a = 0$ when (E) has infinitely many solutions.

(5 marks)

(c) Solve the system of equations $\begin{cases} 3x + 3y + z = 2 \\ 9x + 9y + z = 1 \\ 27x + 27y + z = -2 \\ 3x + 6y - 2z = 0 \end{cases}$.

(3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

12.

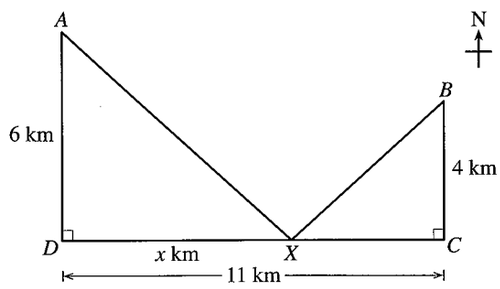

Figure 4

In Figure 4, the cities C and D are 6 km and 4 km due south of the cities A and B respectively. City C is 11 km due east of city D. The government plans to build a road connecting cities A and B via point X, where X is a point on CD and x km due east from city D. Let y km be the total length of the road.

(a) (i) Express y in terms of x.

(ii) Find the minimum length of the road.

(6 marks)

(b)

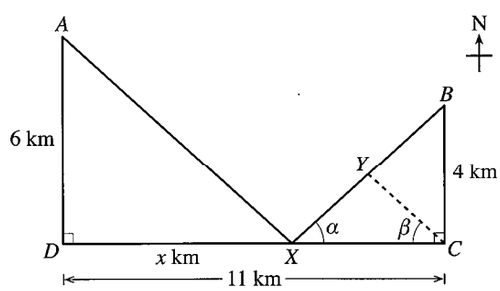

Figure 5

The road is built with the minimum length. In Figure 5, Sam is driving on the road from X to B at a constant speed of 50 km/h. Let $\angle YXC = \alpha$ and $\angle YCX = \beta$, where Y is the position of Sam's car.

(i) By finding $\sin\alpha$ and $\cos\alpha$, show that $XY = \dfrac{22\sqrt{221}}{50\cot\beta + 55}$ km.

(ii) Find the rate of change of β when CY is the shortest distance between city C and the road.

(8 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

13. (a) Let T be a positive number such that $f(x) = f(T - x)$ for all real values of x. By using the substitution $u = T - x$, show that $\int_0^T xf(x)\,dx = \frac{T}{2}\int_0^T f(x)\,dx$.

(4 marks)

(b) (i) Using a suitable substitution, or otherwise, show that $\int_0^{\pi} \frac{\sin x \cos^2 x}{1 + \cos^2 x}\,dx = 2 - \int_{-1}^{1} \frac{1}{1 + x^2}\,dx$.

(ii) Hence, or otherwise, evaluate $\int_0^{\pi} \frac{\sin x \cos^2 x}{1 + \cos^2 x}\,dx$.

(6 marks)

(c) Using the results of (a) and (b), evaluate $\int_0^{\pi} \frac{x \sin x \cos^2 x}{1 + \cos^2 x}\,dx$.

(3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

END OF PAPER

Answers written in the margins will not be marked.

