Trigonometry Exercise

1. Let x be a real number such that $\sec x - \tan x = 2$. Find the value of $\sec x + \tan x$.
   Ans : $1/2$

2. Find the maximum value of $y = 5 + \dfrac{4}{2\sec^2 x - 1}$.
Ans : 9

3. Find the minimum value of $y = 3 - \dfrac{1}{4\csc^2 2x - 3}$.

   Ans : 2

4. (a) Find the maximum and minimum value of $1 - \dfrac{3y^2}{4}$ where $0 \leq y \leq 1$.

(b) i. Express $\sin^4 x + \cos^4 x$ in terms of $\sin 2x$.

ii. Hence express $\sin^6 x + \cos^6 x$ in terms of $\sin 2x$.

(c) Using (a) and (b), or otherwise, find the maximum and minimum value of $\sin^6 x + \cos^6 x$.

Ans : (a) Min $= 1/4$, Max $= 1$ (b)(i) $1 - \dfrac{\sin^2 2x}{2}$ (ii) $1 - \dfrac{3\sin^2 2x}{4}$ (c) Min $= 1/4$, Max $= 1$

5. Prove that

$$\frac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$$

6. Let $y = \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$, by considering $y\sin\frac{\pi}{7}$, find the value of y.
   Ans : $-1/8$

7. Let $y = \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$, by considering $y\sin\frac{\pi}{15}$, find the value of y.
   Ans : $-1/16$

8. Let $y = \cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7}$.

   (a) Prove that $2y\sin\frac{\pi}{7} = \sin\frac{6\pi}{7}$.

   (b) Using (a), find the value of y.

   (c) Using (b), find the value of $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$. [Hint: $\cos(\pi - x) = -\cos x$]

   Ans : (b) $y = 1/2$ (c) $-1/2$

9. Solve the equation

$$\sin 2x + \sin 4x = \cos x$$

for $0 \leq x \leq \pi$.

Ans : $\pi/18$, $5\pi/18$, $\pi/2$, $13\pi/18$, $17\pi/18$

10. **HKALE Pure Math 2007 Paper 2 Q11(b)(i)(ii)(Modified)**

(a) Prove that $\tan\dfrac{3\pi}{8} = 1+\sqrt{2}$.

(b) Using (a), prove that $\tan\dfrac{\pi}{24} = \dfrac{1+\sqrt{2}-\sqrt{3}}{1+\sqrt{3}+\sqrt{6}}$.

11. Prove by mathematical induction that

$$\sin x + \sin 3x + \cdots + \sin(2n-1)x = \frac{\sin^2 nx}{\sin x}$$

where $\sin x \neq 0$, for all positive integers n.