# HKDSE MATH M2 Sample Paper

1. **HKDSE Math M2 Sample Paper Q1**

   Find $\dfrac{d}{dx}(\sqrt{2x})$ from first principles.

   (4 marks)

2. **HKDSE Math M2 Sample Paper Q2**

   A snowball in a shape of sphere is melting with its volume decreasing at a constant rate of 4 $\text{cm}^3\text{s}^{-1}$. When its radius is 5 cm, find the rate of change of its radius.

   (4 marks)

3. **HKDSE Math M2 Sample Paper Q3**

   The slope at any point $(x, y)$ of a curve is given by $\dfrac{dy}{dx} = 2x \ln\left(x^2 + 1\right)$. It is given that the curve passes through the point $(0, 1)$.

   Find the equation of the curve.

   (4 marks)

4. **HKDSE Math M2 Sample Paper Q4**

   Find $\displaystyle\int \left(x^2 - \dfrac{1}{x}\right)^4 dx$.

   (4 marks)

5. **HKDSE Math M2 Sample Paper Q5**

   By considering $\sin\dfrac{\pi}{7}\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{3\pi}{7}$, find the value of $\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{3\pi}{7}$.

   (4 marks)

6. **HKDSE Math M2 Sample Paper Q6**

   Let $C$ be the curve $3e^{x-y} = x^2 + y^2 + 1$.

   Find the equation of the tangent to $C$ at the point $(1, 1)$.

   (5 marks)

7. **HKDSE Math M2 Sample Paper Q7**

   Solve the system of linear equations

   $$\begin{cases} x & + & 7y & - & 6z & = & -4 \\ 3x & - & 4y & + & 7z & = & 13 \\ 4x & + & 3y & + & z & = & 9 \end{cases}.$$

   (5 marks)

8. **HKDSE Math M2 Sample Paper Q8**

   (a) Using integration by parts, find $\displaystyle\int x \cos x \, dx$.

(b) The **inner surface** of a container is formed by revolving the curve $y = -\cos x$ (for $0 \le x \le \pi$) about the $y$-axis (see Figure 1). Find the capacity of the container.
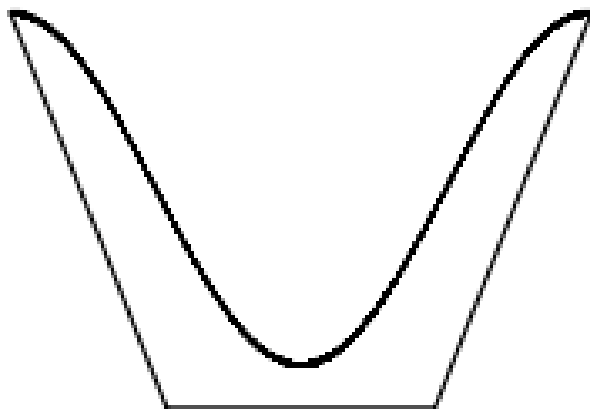


Figure 1

(6 marks)

9. **HKDSE Math M2 Sample Paper Q9**
Let $\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j}$, $\overrightarrow{OB} = 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 3\mathbf{i} + \mathbf{j} + 5\mathbf{k}$. Figure 2 shows the parallelepiped $OADBECFG$ formed by $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$.
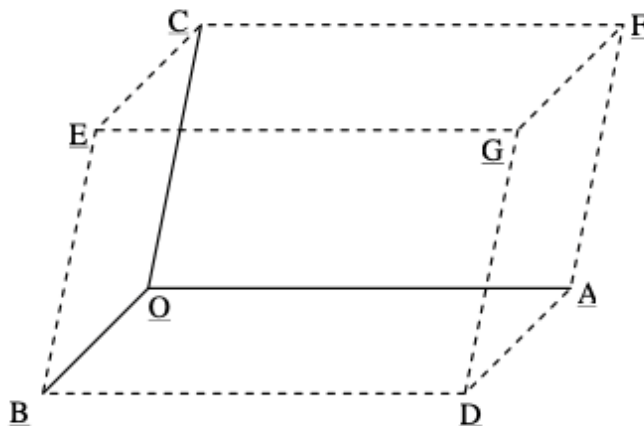


Figure 2

(a) Find the area of the parallelogram $OADB$.

(b) Find the volume of the parallelepiped $OADBECFG$.

(c) If $C'$ is a point different from $C$ such that the volume of the parallelepiped formed by $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC'}$ is the same as that of $OADBECFG$, find a possible vector of $\overrightarrow{OC'}$.

(6 marks)

10. **HKDSE Math M2 Sample Paper Q10**

Let $0° < \theta < 180°$ and define $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

(a) Prove, by mathematical induction, that
$$A^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$$
for all positive integers $n$.

(b) Solve $\sin 3\theta + \sin 2\theta + \sin \theta = 0$.

(c) It is given that $A^3 + A^2 + A = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$.

Find the value(s) of $a$.

(8 marks)

11. **HKDSE Math M2 Sample Paper Q11**

Let $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, $P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

(a) Let $I$ and $O$ be the $3 \times 3$ identity matrix and zero matrix respectively.

(i) Prove that $P^3 - 2P^2 - P + I = O$.

(ii) Using the result of (i), or otherwise, find $P^{-1}$.

(5 marks)

(b) (i) Prove that $D = P^{-1}AP$.

(ii) Prove that $D$ and $A$ are non-singular.

(iii) Find $(D^{-1})^{100}$.

Hence, or otherwise, find $(A^{-1})^{100}$.

(7 marks)

12. **HKDSE Math M2 Sample Paper Q12**

Let $f(x) = \dfrac{4}{x-1} - \dfrac{4}{x+1} - 1$, where $x \neq \pm 1$.

(a) (i) Find the $x$- and $y$-intercept(s) of the graph of $y = f(x)$.

(ii) Find $f'(x)$ and prove that
$$f''(x) = \frac{16(3x^2 + 1)}{(x-1)^3(x+1)^3}$$
for $x \neq \pm 1$.

(iii) For the graph of $y = f(x)$, find all the extreme points and show that there are no points of inflexion.

(6 marks)

(b) Find all the asymptote(s) of the graph of $y = f(x)$.
(2 marks)

(c) Sketch the graph of $y = f(x)$.

(3 marks)

(d) Let $S$ be the area bounded by the graph of $y = f(x)$, the straight lines $x = 3$, $x = a$ $(a > 3)$ and $y = -1$.

Find $S$ in terms of $a$.

Deduce that $S < 4\ln 2$.

(3 marks)

13. **HKDSE Math M2 Sample Paper Q13**

(a) Let $a > 0$ and $f(x)$ be a continuous function.

Prove that $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

Hence, prove that $\displaystyle\int_0^a f(x)\,dx = \frac{1}{2}\int_0^a [f(x) + f(a-x)]\,dx$.

(3 marks)

(b) Show that $\displaystyle\int_0^1 \frac{dx}{x^2 - x + 1} = \frac{2\sqrt{3}\pi}{9}$.

(5 marks)

(c) Using (a) and (b), or otherwise, evaluate $\displaystyle\int_0^1 \frac{dx}{(x^2 - x + 1)(e^{2x-1} + 1)}$.

(6 marks)

14. **HKDSE Math M2 Sample Paper Q14**

In Figure 3, $\triangle ABC$ is an acute-angled triangle, where $O$ and $H$ are the circumcentre and orthocentre respectively. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OH} = \mathbf{h}$.
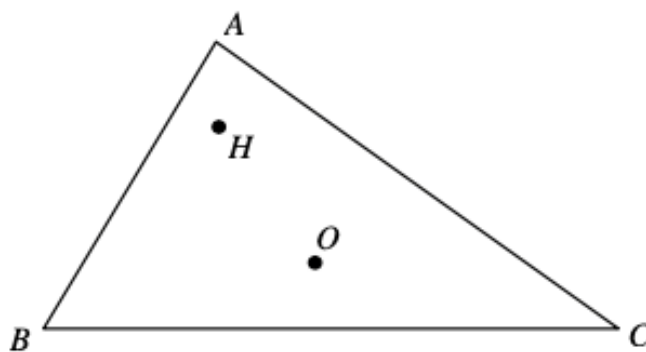


**Figure 3**

(a) Show that $(\mathbf{h} - \mathbf{a})//(\mathbf{b} + \mathbf{c})$.

(3 marks)

(b) Let $\mathbf{h} - \mathbf{a} = t(\mathbf{b} + \mathbf{c})$, where $t$ is a non-zero constant.

Show that

(i) $t(\mathbf{b} + \mathbf{c}) + \mathbf{a} - \mathbf{b} = s(\mathbf{c} + \mathbf{a})$ for some scalar $s$,

(ii) $(t-1)(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{c} - \mathbf{a}) = 0$.

(5 marks)

(c) Express **h** in terms of **a**, **b** and **c**.

(2 marks)