## 香港考試及評核局 HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

## 2 0 1 2 年 香港中 學文憑考試 HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2012

數學 延伸部分單元二(代數與微積分)

MATHEMATICS Extended Part Module 2 (Algebra and Calculus)

評 卷 參 考

MARKING SCHEME

本評卷參考乃香港考試及評核局專為今年本科考試而編寫,供閱卷員參考之用。閱卷員在完成閱卷工作後,若將本評卷參考提供其任教會考班的本科同事參閱,本局不表反對,但須切記,在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件,閱卷員/教師應嚴詞拒絕,因學生極可能將評卷參考視為標準答案,以致但知硬背死記,活剝生吞。這種落伍的學習態度,既不符現代教育原則,亦有違考試着重理解能力與運用技巧之旨。因此,本局籲請各閱卷員/教師通力合作,堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

	Solution	Marks	Remarks
1.	$f(x) = e^{2x}$		
	$f'(0) = \lim_{h \to 0} \frac{e^{2(0+h)} - e^{2(0)}}{h}$	1M	
	$h \rightarrow 0$ $h$		$a^{2h}-1$
	$=\lim_{h\to 0}\frac{e^{2h}-1}{2h}\cdot 2$	1M	Accept $\lim_{h \to 0} \frac{e^{2h} - 1}{h}$
	= 2	1A	
		(3)	
-			
2.	$(1+ax)^n = 1 + nax + \frac{n(n-1)}{2}(ax)^2 + \cdots$	1M+1A	1A for $\frac{n(n-1)}{2}$
	Hence $na = 6$ and $\frac{n(n-1)}{2}a^2 = 16$ .	1M	2
	•	1171	
	Solving, $\frac{n(n-1)}{2} \left(\frac{6}{n}\right)^2 = 16$		
	18(n-1) = 16n		,
	n = 9	1A	:
	Therefore, $a = \frac{2}{3}$ .	1A	:
		(5)	
3.	For $n=1$ , L.H.S. = $1 \times 2 = 2$ and R.H.S. = $1^2(1+1) = 2$		
	$\therefore$ L.H.S. = R.H.S. and the statement is true for $n=1$ .	1	
	Assume $1 \times 2 + 2 \times 5 + 3 \times 8 + \dots + k(3k-1) = k^2(k+1)$ , where k is a positive integer.	1	
	$1 \times 2 + 2 \times 5 + 3 \times 8 + \dots + k(3k-1) + (k+1)[3(k+1)-1]$		
	$= k^{2}(k+1) + (k+1)(3k+2)$ by the assumption	1	
	$= (k+1)(k^2+3k+2)$	1	
	$= (k+1)^{2}(k+1+1)$ Hence the statement is true for $n = k+1$ .	1	
	By the principle of mathematical induction, the statement is true for all positive integers $n$ .	1	Follow through
		(5)	
			:
4.	(a) $\int \frac{x+1}{x} dx = \int \left(1 + \frac{1}{x}\right) dx$	1M	
	$= x + \ln x  + C$	1A	
			4
	(b) Let $u = x^2 - 1$ . du = 2xdx		
	$\int \frac{x^3}{x^2 - 1} dx = \int \frac{u + 1}{u} \cdot \frac{du}{2}$	1A	
	· · · · ·	""	
	$=\frac{1}{2}u + \frac{1}{2}\ln u  + C$ by (a)	1M	
	$= \frac{1}{2}(x^2 - 1) + \frac{1}{2}\ln x^2 - 1  + C$	1A	OR $\frac{1}{2}x^2 + \frac{1}{2}\ln x^2 - 1  + C$
	2 2 1 1	(5)	

	Calvein	Moules	Domonico
	Solution	Marks	Remarks
5.	$y = \frac{x^2 + x + 1}{x + 1}$		
	$=x+\frac{1}{x+1}$	1M	
			2 . 2
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \frac{1}{(x+1)^2}$	1A	$OR \frac{x^2 + 2x}{(x+1)^2}$
			(3/1)
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0  \text{when}  x = -2  \text{or}  0$		
	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}$ > 0 0 < 0 undefined < 0 0 > 0	1M	
	Alternative Solution	_	
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{2}{(x+1)^3}$		
		} 1M	
	When $x = 0$ , $\frac{d^2 y}{dx^2} = 2 > 0$ , when $x = -2$ , $\frac{d^2 y}{dx^2} = -2 < 0$ .	J	
			,
	Hence $(0,1)$ is a minimum point. The vertical asymptote is $x = -1$ .	1A 1A	
	, ,	171	Accept lim 1 -0
	$\lim_{x \to \pm \infty} \frac{1}{x+1} = 0$		Accept $\lim_{x \to \infty} \frac{1}{x+1} = 0$
	$\therefore$ the oblique asymptote is $y = x$	1A (6)	
		(0)	
	i		
•	(a) Let the radius of the water surface be $a \text{ cm}$ . $a-3  h$	13.4	€ 4
	By considering similar triangles, $\frac{a-3}{4-3} = \frac{h}{10}$ .	1M	<b>\</b> \
	i.e. $a = \frac{h+30}{10}$		
			10
	$V = \frac{\pi}{3} h \left[ 3^2 + 3 \left( \frac{h+30}{10} \right) + \left( \frac{h+30}{10} \right)^2 \right]$	1M	
	$= \frac{\pi}{300}h[900 + 30(h + 30) + (h^2 + 60h + 900)]$		$\longleftrightarrow$ 3
	$=\frac{\pi}{300}(h^3+90h^2+2700h)$	1	
	$dV = \pi (2)^2 \cdot 100I \cdot 2700 \cdot dh$	13.6 / 1.4	
	(b) $\frac{dV}{dt} = \frac{\pi}{300} (3h^2 + 180h + 2700) \frac{dh}{dt}$	1M+1A	
	$\therefore 7\pi = \frac{\pi}{300} [3(5)^2 + 180(5) + 2700] \frac{dh}{dt}$		
	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{4}{7}$	1A	
	u ,	IA	
	i.e. the rate of increase of depth of water is $\frac{4}{7}$ cm s <sup>-1</sup> .		
	·	(6)	

		Solution	Marks	Remarks
7.		The area of the parallelogram <i>OADB</i>		
		$= \left  (6\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) \right $	1M	E   G
		$=  \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} $		C/F/
		$= \sqrt{1^2 + 2^2 + 2^2}$		17 / / /
		= 3	1A	$\int_{R}$
	(b)	The volume of the parallelepiped <i>OADBECFG</i>		
	(0)	$= (6\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) \cdot (5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$	1M	O A
		$= (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \cdot (5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$		
		=1.5+(-2)(-1)+2.2		
		=11		
		Hence, the distance between point $C$ and plane $OADB$ is $\frac{11}{3}$ .	1M+1A	1M for height = $\frac{\text{volume}}{1}$
		3	(5)	base
		<i>f</i>	(3)	· · · · · · · · · · · · · · · · · · ·
-				
		$(1 \ 1 \ 1 \ 0)$		
8.	(a)	The augmented matrix is $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & -1 & 5 & 6 \end{pmatrix}$		
		$       \sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & 3 & 6 \end{pmatrix}   $	1M	
		$\sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & -2 \end{pmatrix}$		
		Let $z = t$ , where t is a real number. Then $y = t - 2$ and $x = 2 - 2t$ .	1A	OR Solution Set =
				$\{(2-2t, t-2, t): t \in \mathbf{R}\}$
	(b)	Substitute $(x, y, z) = (2-2t, t-2, t)$ into the last equation:	1M	
		$(2-2t)-(t-2)+\lambda(t)=4$		
		$(\lambda - 3)t = 0$		
		When $\lambda \neq 3$ , $t = 0$ (x, y, z) = (2, -2, 0)	1A	
		When $\lambda = 3$ , t can be any real number.	""	•
		$\therefore (x, y, z) = (2 - 2t, t - 2, t)$	1A	
			+	
		Alternative Solution		
		The augmented matrix is $ \begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & -1 & 5 & 6 \\ 1 & -1 & \lambda & 4 \end{pmatrix} $		
		$\sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & 3 & 6 \\ 0 & -2 & \lambda - 1 & 4 \end{pmatrix}$	1M	
		$\begin{bmatrix} -1 & -3 & 3 & 0 \\ 0 & 2 & 1 & 1 \end{bmatrix}$	11/1	
		$\begin{pmatrix} 0 & -2 & \lambda - 1 \mid 4 \end{pmatrix}$		
		$\sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & \lambda - 3 & 0 \end{pmatrix}$		
		$\sim \begin{vmatrix} 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 3 & 0 \end{vmatrix}$		
		When $\lambda \neq 3$ , $z=0$ .		
		when $\lambda \neq 3$ , $z = 0$ . $\therefore (x, y, z) = (2, -2, 0)$	1A	
		When $\lambda = 3$ , z can be any real number.		
		(x, y, z) = (2-2t, t-2, t), where t is a real number.	1A	
			7.5	
			(5)	

•		Solution	Marks	Remarks
9. (	(a)	$\int x \sin x dx = x(-\cos x) - \int (-\cos x) dx$	1M	
		$= -x\cos x + \sin x + C$	1A	** -
(	(b)	The volume $= \pi \int_0^{\pi} x \sin x dx$	1M	$ \begin{array}{ccc} 1M & \text{for } V = \pi \int y^2 dx \\ y & y = \sqrt{x \sin x} \end{array} $
		$= \pi[-x\cos x + \sin x]_0^{\pi}$ $= \pi^2$	1A	$y = \sqrt{x \sin x}$
			(4)	
10. (	(a)	Let $F$ be a point on $AB$ such that $OF \perp AB$ . Let $OA$ be $x$ .		
		$\therefore AF = \frac{1}{2} \text{ and } \angle AOF = 2\theta \text{ (properties of isos. } \Delta\text{)}$		x 0/30
	_	In $\triangle OAF$ , $\sin 2\theta = \frac{1}{x}$	1M	A $y$ $Y$ $F$ $B$
		Alternative Solution In $\triangle OAB$ , $\frac{1}{\sin 4\theta} = \frac{x}{\sin(90^\circ - 2\theta)}$	1M	OR $x^2 = x^2 + 1^2 - 2x\cos(90^\circ - 2\theta)$
		$x = \frac{1}{2\sin 2\theta} $ (1)		
		In $\triangle OAY$ , $\frac{y}{\sin \theta} = \frac{x}{\sin(90^\circ + \theta)}$ (2)	1M	
		Substitute (1) into (2): $\frac{y}{\sin \theta} = \frac{1}{2\sin 2\theta} \cdot \frac{1}{\cos \theta}$	1M	
		Alternative Solution In $\triangle OAY$ , $\frac{y}{\sin \theta} = \frac{OY}{\sin \angle OAY}$	1M	0
		In $\triangle OBY$ , $\frac{\sin \theta}{\sin 3\theta} = \frac{OY}{\sin \angle OBY}$	1M	$\theta$ 3 $\theta$
		$Sin 3\theta Sin ∠OBT$ ∴ $\angle OAY = \angle OBY$ (base ∠s, isos. Δs) ∴ $\frac{y}{\sin \theta} = \frac{1 - y}{3\sin \theta - 4\sin^3 \theta}$	1M	$A  y  Y \qquad 1-y \qquad B$
		$3y - 4y\sin^2\theta = 1 - y$ $4y(1 - \sin^2\theta) = 1$		
		$y = \frac{1}{4}\sec^2\theta$	1	
(	(b)	$0^{\circ} < 4\theta < 180^{\circ}$ $0^{\circ} < \theta < 45^{\circ}$	1M	
		$\frac{1}{4}\sec^2 0^\circ < y < \frac{1}{4}\sec^2 45^\circ \left[ \text{since } \sec^2 \theta \text{ is an increasing function for } 0^\circ < \theta < 45^\circ \right]$		Accept using "≤" sign
		i.e. $\frac{1}{4} < y < \frac{1}{2}$	1A	
			(6)	- -

	Solution	Marks	Remarks
I. (a	$\begin{vmatrix} 1-x & 4 \\ 2 & 3-x \end{vmatrix} = 0$		
	$(1-x)(3-x)-2\cdot 4 = 0$ $x^2 - 4x - 5 = 0$	1M	
	$ \begin{array}{ccc} x & -4x - 5 = 0 \\ x = -1 & \text{or } 5 \end{array} $	1A	
		(2)	
(b	(i) $ \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = -1 \cdot \begin{pmatrix} a \\ b \end{pmatrix} $		
	$\begin{cases} a+4b=-a\\ 2a+3b=-b \end{cases}$	1M €	
	a + 2b = 0(1)	1A <	
	$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} c \\ 1 \end{pmatrix} = 5 \begin{pmatrix} c \\ 1 \end{pmatrix}$		Either one
	$\begin{cases} c+4=5c\\ 2c+3=5 \end{cases}$	<	Either one
	c=1	<	
	$\begin{vmatrix} a & c \\ b & 1 \end{vmatrix} = 1$	·	
	By (2), $a-b=1$ (3) Solving (1) and (3), we have $a=\frac{2}{3}$ and $b=\frac{-1}{3}$ .	1M	For $a-bc=1$
	$\therefore P = \begin{pmatrix} \frac{2}{3} & 1 \\ \frac{-1}{3} & 1 \end{pmatrix}$	1A	
	(3)		
	$\left(1,\frac{1}{2}\right)^t$		
	(ii) $P^{-1} = \frac{1}{\frac{2}{3} + \frac{1}{3}} \begin{pmatrix} 1 & \frac{1}{3} \\ -1 & \frac{2}{3} \end{pmatrix}^{t}$		
	$= \begin{pmatrix} 1 & -1 \\ \frac{1}{2} & \frac{2}{3} \end{pmatrix}$	126	
	(3 3)	1M	
	$P^{-1} \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} P = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} \frac{2}{3} & 1 \\ -\frac{1}{3} & 1 \end{pmatrix}$		
	\ - '		
	$= \begin{pmatrix} -1 & 1 \\ \frac{5}{3} & \frac{10}{3} \end{pmatrix} \begin{pmatrix} \frac{2}{3} & 1 \\ \frac{-1}{3} & 1 \end{pmatrix}$	1M	$ \begin{array}{c cccc}  & 1 & -1 & \frac{-2}{3} & 5 \\  & 0 & 1 & 2 & \frac{3}{3} & 5 \end{array} $
	$\left(\frac{3}{3}  \frac{3}{3}\right)\left(\frac{-1}{3}  1\right)$		OR $ \begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} \frac{-2}{3} & 5 \\ \frac{1}{3} & 5 \end{pmatrix} $
	$= \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix}$	1A	

Solution	Marks	Remarks
(iii) $P^{-1} \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} P = \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix}$		
$ \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} = P \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} P^{-1} $	1M	
$ \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}^{12} = P \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} P^{-1} P \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} P^{-1} \cdots P \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} P^{-1} $		
$\begin{pmatrix} -1 & 0 \end{pmatrix}^{12} = 0$		
$=P\begin{pmatrix} -1 & 0\\ 0 & 5 \end{pmatrix}^{12}P^{-1}$	1M	
$= \begin{pmatrix} \frac{2}{3} & 1\\ -\frac{1}{3} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0\\ 0 & 5^{12} \end{pmatrix} \begin{pmatrix} \frac{1}{1} & \frac{-1}{2}\\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	1M	For $\begin{pmatrix} 1 & 0 \\ 0 & 5^{12} \end{pmatrix}$
$= \begin{pmatrix} \frac{2}{3} & 5^{12} \\ \frac{-1}{2} & 5^{12} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$		For $\begin{pmatrix} 1 & 0 \\ 0 & 5^{12} \end{pmatrix}$ $OR \begin{pmatrix} \frac{2}{3} & 1 \\ \frac{-1}{3} & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{5^{12}} & \frac{-1}{3} \\ \frac{5^{12}}{3} & \frac{2 \cdot 5^{12}}{3} \end{pmatrix}$
(3)		
$= \begin{pmatrix} \frac{5^{12} + 2}{3} & \frac{2 \cdot 5^{12} - 2}{3} \\ \frac{5^{12} - 1}{3} & \frac{2 \cdot 5^{12} + 1}{3} \end{pmatrix}$	1A	OR (81380209 162760416) 81380208 162760417)
	(11)	
12. (a) Let $\overrightarrow{AG} = \frac{\overrightarrow{AC} + \lambda \overrightarrow{AD}}{1 + \lambda}$ .		
Since $\overrightarrow{AG}$ lies on a median, $\overrightarrow{AG} = k \frac{\overrightarrow{AC} + \overrightarrow{AB}}{2} = k \frac{\overrightarrow{AC} + 2\overrightarrow{AD}}{2}$ for some $k$ .		
Comparing the two expressions of $\overrightarrow{AG}$ , we get $\lambda = 2$ .		C A
$\overrightarrow{AG} = \frac{\overrightarrow{AC} + 2\overrightarrow{AD}}{\overrightarrow{AD}}$	.	F $M$
3	1A	Go
$=\frac{\overrightarrow{AC}+\overrightarrow{AB}}{3}$		A E D B
$=\frac{(\mathbf{c}-\mathbf{a})+(\mathbf{b}-\mathbf{a})}{3}$	1M	For tip-to-tail method
Alternative Solution 1 Let $M$ be the mid-point of $BC$ .		
$\overrightarrow{AG} = \frac{2}{3}\overrightarrow{AM}$	1A	
$=\frac{2}{3}\cdot\frac{\overrightarrow{AC}+\overrightarrow{AB}}{2}$		`
$=\frac{3}{(\mathbf{c}-\mathbf{a})+(\mathbf{b}-\mathbf{a})}$	1 <sub>M</sub>	For tip-to-tail method
		To up to tall motion
$\frac{\text{Alternative Solution 2}}{\overrightarrow{AG} = \overrightarrow{OG} - \overrightarrow{OA}}$	1M	
$=\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}-\mathbf{a}$	1A	For $\frac{a+b+c}{3}$
$=\frac{\mathbf{b}+\mathbf{c}-2\mathbf{a}}{3}$	1A	3
3	(3)	

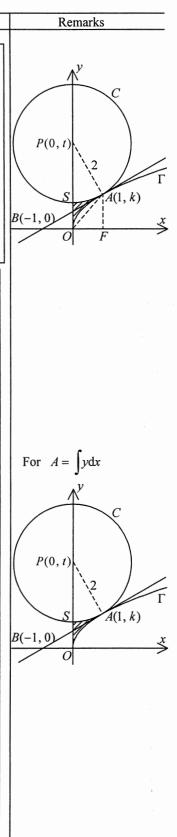
	Solution	Marks	Remarks
(i)	Since $O$ is the circumcentre of the $\triangle ABC$ , $OD \perp AB$ .		C
	∴ OD//CE	1	
	$\angle DOG = \angle CFG$ (alt. $\angle s$ , $OD//CF$ )		
	$\angle ODG = \angle FCG$ (alt. $\angle s$ , $OD//CF$ )		F
	$\angle OGD = \angle FGC$ (vert. opp. $\angle s$ )		
	$\therefore  \Delta DOG \sim \Delta CFG  (A.A.A.)$	1	/ 0
	$FG:GO=CG:GD$ (corr. sides, $\sim \Delta s$ )		A   E   D
	= 2:1	1A	
(ii)	$\overrightarrow{AG} = \frac{\overrightarrow{AF} + 2\overrightarrow{AO}}{3}$	1M	For using (b)(i)
(11)		1111	Tor using (b)(i)
	$\overrightarrow{AF} = 3\overrightarrow{AG} - 2\overrightarrow{AO}$		
	$=3\cdot\frac{\mathbf{b}+\mathbf{c}-2\mathbf{a}}{3}-2(-\mathbf{a})$	1M	For using (a)
	$=3\cdot{3}-2(-\mathbf{a})$	IIVI	For using (a)
	Alternative Solution 1		1 · · · · · · · · · · · · · · · · · · ·
	$\overrightarrow{AF} = \overrightarrow{AG} + \overrightarrow{GF}$		:
	$=\overrightarrow{AG}+2\overrightarrow{OG}$	1M	For using (b)(i)
	$= \frac{\mathbf{b} + \mathbf{c} - 2\mathbf{a}}{3} + 2 \cdot \frac{\mathbf{a} + \mathbf{b} + \mathbf{c}}{3}$	1M	For using (a)
	=	1141	Tor using (a)
	Alternative Solution 2		:
	$\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF}$		
	$=\overrightarrow{AC}+2\overrightarrow{OD}$	1M	
	$=\overrightarrow{AC}+\overrightarrow{OA}+\overrightarrow{OB}$	1M	
	$= (\mathbf{c} - \mathbf{a}) + \mathbf{a} + \mathbf{b}$		
	Alternative Solution 3		
	$\overrightarrow{AF} = \overrightarrow{OF} - \overrightarrow{OA}$		'
	$=3\overrightarrow{OG}-\overrightarrow{OA}$	1M	For using (b)(i)
			1 31 31 31 31 31 31
	$=3\cdot\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}-\mathbf{a}$	1M	
	$= \mathbf{b} + \mathbf{c}$	1	
	$\overrightarrow{AF} \cdot \overrightarrow{BC} = (\mathbf{b} + \mathbf{c}) \cdot (\mathbf{c} - \mathbf{b})$	1M	
		1	
	$= \mathbf{c} ^2- \mathbf{b} ^2$		
	= 0 (: O is the circumcentre)	1A	
	$\therefore AF \perp BC$		
	$\therefore$ AF is another altitude of $\triangle ABC$ .		
	Alternative Solution		
	$\overrightarrow{BF} \cdot \overrightarrow{AC} = (\overrightarrow{BA} + \overrightarrow{AF}) \cdot \overrightarrow{AC}$	1M	
	$= (\mathbf{a} - \mathbf{b} + \mathbf{b} + \mathbf{c}) \cdot (\mathbf{c} - \mathbf{a})$		
	$= \mathbf{c} ^2- \mathbf{a} ^2$		
	$ = 0 \qquad (\because O \text{ is the circumcentre}) $	1A	
	$\therefore$ BF is another altitude of $\triangle ABC$ .		
	$\therefore$ F is the orthocentre of $\triangle ABC$ .	1	
		(9)	1

	Solution	Marks	Remarks
13. (a) (i)	$\tan u = \frac{-1 + \cos\frac{2\pi}{5}}{\sin\frac{2\pi}{5}}$		
	$= \frac{-1 + 1 - 2\sin^2\frac{\pi}{5}}{2\sin\frac{\pi}{5}\cos\frac{\pi}{5}}$	1M	
	$= -\tan \frac{\pi}{5}$ $= \tan \frac{-\pi}{5}$ $\therefore u = \frac{-\pi}{5}  \text{for } \frac{-\pi}{2} < u < \frac{\pi}{2}$	1	
(ii)	$\tan v = \frac{1 + \cos \frac{2\pi}{5}}{\sin \frac{2\pi}{5}}$		
	$ \frac{\sin\frac{2\pi}{5}}{2\sin\frac{\pi}{5}\cos\frac{\pi}{5}} = \frac{1+2\cos^2\frac{\pi}{5}-1}{2\sin\frac{\pi}{5}\cos\frac{\pi}{5}} $	1M	
	$= \cot \frac{\pi}{5}$ $= \tan \left(\frac{\pi}{2} - \frac{\pi}{5}\right)$ $\therefore v = \frac{3\pi}{10} \text{ for } \frac{-\pi}{2} < v < \frac{\pi}{2}$	1A	
		(4)	
(b) (i)	$x^{2} + 2x\cos\frac{2\pi}{5} + 1$ $x^{2} + 2x\cos\frac{2\pi}{5} + \cos^{2}\frac{2\pi}{5} + \sin^{2}\frac{2\pi}{5}$		
	$= (x + \cos\frac{2\pi}{5})^2 + \sin^2\frac{2\pi}{5}$ $\sin^2 2\pi$ $\sin^2 2\pi$	1A	
(ii)	$\int_{-1}^{1} \frac{\sin\frac{2\pi}{5}}{x^2 + 2x\cos\frac{2\pi}{5} + 1} dx = \int_{-1}^{1} \frac{\sin\frac{2\pi}{5}}{(x + \cos\frac{2\pi}{5})^2 + \sin^2\frac{2\pi}{5}} dx$		
	Let $x + \cos \frac{2\pi}{5} = \sin \frac{2\pi}{5} \tan \theta$	1M	
	$\therefore dx = \sin\frac{2\pi}{5}\sec^2\theta d\theta$	1A	
	When $x = -1$ , $\tan \theta = \frac{-1 + \cos \frac{2\pi}{5}}{\sin \frac{2\pi}{5}}$ which gives $\theta = \frac{-\pi}{5}$ (by (a)(i))	1M	For using (a)
	When $x = 1$ , $\tan \theta = \frac{1 + \cos \frac{2\pi}{5}}{\sin \frac{2\pi}{5}}$ which gives $\theta = \frac{3\pi}{10}$ (by (a)(ii))		

Solution	Marks	Remarks
$\therefore \int_{-1}^{1} \frac{\sin\frac{2\pi}{5}}{x^2 + 2x\cos\frac{2\pi}{5} + 1} dx = \int_{-\frac{\pi}{5}}^{\frac{3\pi}{10}} \frac{\sin^2\frac{2\pi}{5}\sec^2\theta}{\sin^2\frac{2\pi}{5}(\tan^2\theta + 1)} d\theta$	1A	For integrand
$= [\theta] \frac{\frac{3\pi}{10}}{\frac{-\pi}{5}}$ $= \frac{\pi}{2}$	1A	
2	(6)	
(c) $\int_{-1}^{1} \frac{\sin \frac{7\pi}{5}}{x^2 + 2x \cos \frac{7\pi}{5} + 1} dx = \int_{-1}^{1} \frac{-\sin \frac{2\pi}{5}}{x^2 - 2x \cos \frac{2\pi}{5} + 1} dx$	1A	
Let $y = -x$ .	1M	
dy = -dx When $x = -1$ , $y = 1$ ; when $x = 1$ , $y = -1$ .		
$\therefore \int_{-1}^{1} \frac{\sin\frac{7\pi}{5}}{x^2 + 2x\cos\frac{7\pi}{5} + 1} dx = \int_{1}^{-1} \frac{-\sin\frac{2\pi}{5}}{y^2 + 2y\cos\frac{2\pi}{5} + 1} \cdot -dy$		
Alternative Solution		
$\int_{-1}^{1} \frac{\sin\frac{7\pi}{5}}{x^2 + 2x\cos\frac{7\pi}{5} + 1} dx = \int_{-1}^{1} \frac{\sin\frac{7\pi}{5}}{(x + \cos\frac{7\pi}{5})^2 + \sin^2\frac{7\pi}{5}} dx$		
Let $x + \cos \frac{7\pi}{5} = \sin \frac{7\pi}{5} \tan \theta$	1M	
$\therefore dx = \sin \frac{7\pi}{5} \sec^2 \theta d\theta$		
When $x = -1$ , $\theta = \frac{3\pi}{10}$ ; when $x = 1$ , $\theta = \frac{-\pi}{5}$ .	1A	* .
$\therefore \int_{-1}^{1} \frac{\sin\frac{7\pi}{5}}{x^2 + 2x\cos\frac{7\pi}{5} + 1} dx = \int_{\frac{3\pi}{10}}^{\frac{-\pi}{5}} \frac{\sin^2\frac{7\pi}{5}\sec^2\theta}{\sin^2\frac{7\pi}{5}(\tan^2\theta + 1)} d\theta$		
$=\frac{-\pi}{2}$ by (b)(ii)	1A	
2	(3)	
14. (a) $y = kx^p$		, i
$\frac{\mathrm{d}y}{\mathrm{d}x} = kpx^{p-1}$	1A	
The slope of the tangent to $\Gamma$ at $A$ is $kpa^{p-1}$ .		:
$\therefore \frac{ka^p - 0}{a - (-a)} = kpa^{p-1}$	1M	
$p = \frac{1}{2}$	1	
. 2	(3)	

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(b) (i)		Marks	Remarks
	: $AP = 2$ : $(k-t)^2 + (1-0)^2 = 2^2$	11/4	$OR  t = k + \sqrt{4 - 1}$
	$k - t = -\sqrt{3}  \text{or } \sqrt{3}  \text{(rejected)}(1)$	1M	$\int_{y}^{y} \sqrt{y}$
	Slope of $AP = \frac{k-t}{1-0}$		C
	1 0		
	Slope of $AB = \frac{k}{2}$ (by (a))	1M	P(0,t)
	$\therefore (k-t)\frac{k}{2} = -1 \qquad (2)$	1A	ν 2 Γ
	Substitute (1) into (2): $(-\sqrt{3})\frac{k}{2} = -1$		$B(-1,0) \qquad \qquad x$
	Alternative Solution		0
	According to the figure, $\angle ROB = \angle RAP = \frac{\pi}{2}$ .		, v.
	$\angle ORB = \angle ARP$ (vert. opp. $\angle$ s) $\therefore \angle RBO = \angle RPA$ and let the angles be $\theta$ .	1M	C
	Since $PA = 2$ and $QA = 1$ , $\theta = \frac{\pi}{6}$	1A	P
	Slope of $AB = \frac{k}{2}$ (by (a))	1M	$\theta$ 2
	$\therefore \tan \frac{\pi}{6} = \frac{k}{2}$		B(-1,0) $A(1,k)$
-	$k = \frac{2\sqrt{3}}{2}$	1	$\frac{B(-1,0)}{O}$
	3		
(;;)	The sheded area		, v
(11)	The shaded area = area of $\triangle PQA$ + area on the left of $\Gamma$ from $O$ to $A$ – area of sector $PAS$	1M	C
	$= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_0^{2\sqrt{3}} \left( \frac{y}{2\sqrt{3}} \right)^2 dy - \frac{1}{2} (2)^2 \frac{\pi}{6}$	1M+1A	
	· · ·	·	
	$=\frac{\sqrt{3}}{2}+\frac{3}{4}\left[\frac{y^3}{3}\right]_0^{\frac{2\sqrt{3}}{3}}-\frac{\pi}{3}$	1M	B(-1,0) $A(1,k)$
	Alternative Solution 1		/ 0
	$t = k + \sqrt{3}$		
	$=\frac{5\sqrt{3}}{2}$		C
	The shaded area		
	= area of trapezium $OFAP$ – area of sector $PAS$ – area under $\Gamma$ from $O$ to $A$	1M	P(0,t)
	$= \frac{1}{2} \left( \frac{2\sqrt{3}}{3} + \frac{5\sqrt{3}}{3} \right) (1) - \frac{1}{2} (2)^2 \frac{\pi}{6} - \int_0^1 \frac{2\sqrt{3}}{3} x^{\frac{1}{2}} dx$	1M+1A	$\frac{1}{A(1,k)}$
	$ = \frac{7\sqrt{3}}{6} - \frac{\pi}{3} - \frac{4\sqrt{3}}{9} \left[ x^{\frac{3}{2}} \right]_{0}^{1} $	1M	$B(-1,0) \xrightarrow{A(1,k)} x$
		<b> </b>	'

_	Solution	Marks
	Alternative Solution 2 $t = k + \sqrt{3}$ $= \frac{5\sqrt{3}}{3}$ The shaded area	
	= area of $\triangle OAP$ + area of $\triangle OAF$ - area of sector $PAS$ - area under $\Gamma$ from $O$ to $A$	1 1M
	$= \frac{1}{2} \left( \frac{5\sqrt{3}}{3} \right) (1) + \frac{1}{2} (1) \left( \frac{2\sqrt{3}}{3} \right) - \frac{1}{2} (2)^2 \frac{\pi}{6} - \int_0^1 \frac{2\sqrt{3}}{3} x^{\frac{1}{2}} dx$	1M+1A
	$=\frac{5\sqrt{3}}{6} + \frac{\sqrt{3}}{3} - \frac{\pi}{3} - \frac{4\sqrt{3}}{9} \left[ x^{\frac{3}{2}} \right]_0^1$	1M
	Alternative Solution 3 $t = k + \sqrt{3}$ $= \frac{5\sqrt{3}}{3}$	
	The equation of $C$ is $x^2 + \left(y - \frac{5\sqrt{3}}{3}\right)^2 = 4$ . Hence, the equation of $\widehat{AS}$ is $y = \frac{5\sqrt{3}}{3} - \sqrt{4 - x^2}$ .	1A
	The shaded area $= \int_0^1 \left( \frac{5\sqrt{3}}{3} - \sqrt{4 - x^2} - \frac{2\sqrt{3}}{3} x^{\frac{1}{2}} \right) dx$	1M
	For $\int_0^1 \sqrt{4-x^2} dx$ , let $x = 2\sin \phi$ .	
	$\therefore dx = 2\cos\phi d\phi$	
	When $x=1$ , $\phi = \frac{\pi}{6}$ ; when $x=0$ , $\phi = 0$ .	
	$\int_0^1 \sqrt{4 - x^2}  dx = \int_0^{\frac{\pi}{6}} \sqrt{4 - 4\sin^2 \phi}  2\cos\phi  d\phi$	1M
	$= \int_0^{\frac{\pi}{6}} 2(1+\cos 2\phi) \mathrm{d}\phi$	



 $= \left[2\phi + \sin 2\phi\right]^{\frac{\pi}{6}}$ 

 $= \frac{\pi}{3} + \frac{\sqrt{3}}{2}$ Hence, the shaded area  $= \left[\frac{5\sqrt{3}}{3}x - \frac{4\sqrt{3}}{9}x^{\frac{3}{2}}\right]_{0}^{1} - \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$ 

1M