機密 (只限閱卷員使用) CONFIDENTIAL (FOR MARKER'S USE ONLY)

香港考試及評核局 HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2014 年香港中學文憑考試 HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2014

數學 延伸部分 單元二 (代數與微積分)

MATHEMATICS Extended Part Module 2 (Algebra and Calculus)

評卷參考 MARKING SCHEME

本評卷參考乃香港考試及評核局專為今年本科考試而編寫,供閱卷員參考之 用。本評卷參考之使用,均受制於閱卷員有關之委任條款及閱卷員指引。特 別是:

- 本局擁有並保留本評卷參考的所有財產權利(包括知識產權)。在未獲本局之書面批准下,閱卷員均不得複製、發表、透露、提供、使用或經營本評卷參考之全部或其部份。在遵守上述條款之情況下,本局有限地容許閱卷員可在應屆香港中學文憑考試的考試成績公布後,將本評卷參考提供任教本科的教師參閱。
- 在任何情況下,均不得容許本評卷參考之全部或其部份落入學生手中。本局籲請各閱卷員/教師通力合作,堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for the reference of markers. The use of this marking scheme is subject to the relevant appointment terms and Instructions to Markers. In particular:

- The Authority retains all proprietary rights (including intellectual property rights) in this marking scheme. This marking scheme, whether in whole or in part, must not be copied, published, disclosed, made available, used or dealt in without the prior written approval of the Authority. Subject to compliance with the foregoing, a limited permission is granted to markers to share this marking scheme, after release of examination results of the current HKDSE examination, with teachers who are teaching the same subject.
- Under no circumstances should students be given access to this marking scheme or any part of it. The Authority is counting on the co-operation of markers/teachers in this regard.

@香港考試及評核局 保留版權 Hong Kong Examinations and Assessment Authority All Rights Reserved 2014



機密 (只限閱卷員使用) CONFIDENTIAL (FOR MARKER'S USE ONLY)

General Marking Instructions

- It is very important that all markers should adhere as closely as possible to the marking scheme. In many
 cases, however, candidates will have obtained a correct answer by an alternative method not specified in the
 marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a
 particular method has been specified in the question. Markers should be patient in marking alternative
 solutions not specified in the marking scheme.
- 2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.
- 3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
- Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.
- 5. In the marking scheme, marks are classified into the following three categories:

'M' marks - awarded for applying correct methods
'A' marks - awarded for the accuracy of the answers

Marks without 'M' or 'A' - awarded for correctly completing a proof or arriving at an answer given in the question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.

- In the marking scheme, steps which can be skipped are enclosed by dotted rectangles; whereas alternative
 answers are enclosed by solid rectangles.
- (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.
 - (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted.

CONFIDENTIAL (FOR MARKER'S USE ONLY)

	CONFIDENTIAL (FOR MARKER 5 USE		D
	Solution $(1 + 4x)^2 (1 + x)^n$	Marks	Remarks
l. (a)	$(1-4x)^{2}(1+x)^{n}$ $= (1-8x+16x^{2})\left[1+nx+\frac{n(n-1)}{2}x^{2}+\cdots\right]$	1M €	
	Coefficient of $x = n - 8$ $\therefore n - 8 = 1$ i.e. $n = 9$	1A	For binomial expansion of $(1+x)^n$ up to the x^2 term
(b)	$\therefore (1-4x)^2 (1+x)^9 = (1-8x+16x^2)(1+9x+36x^2+\cdots)$ Coefficient of $x^2 = 36-8\cdot 9+16$	1M	
	Alternative Solution Coefficient of $x^2 = \frac{n(n-1)}{2} - 8n + 16$ by (*)	1M	
	= -20	1A (4)	
(a)	$y = x^{3} - 3x$ $\frac{dy}{dx} = \lim_{h \to 0} \frac{[(x+h)^{3} - 3(x+h)] - (x^{3} - 3x)}{h}$	1M	
	$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 3x - 3h - x^3 + 3x}{h}$	1M	OR $\frac{h[(x+h)^2 + (x+h)x + x^2] - 3}{h}$
	$= \lim_{h \to 0} (3x^2 + 3xh + h^2 - 3)$ = $3x^2 - 3$	1A	
(b)	When C is decreasing, $\frac{dy}{dx} \le 0$.		
	$3x^2 - 3 \le 0$ (x+1)(x-1) \le 0	1M	Accept $3x^2 - 3 < 0$
	$-1 \le x \le 1$	1A	Accept -1 < x < 1
		(5)	
	$y + y = 2$ $y + x \cdot \frac{1}{y} \frac{dy}{dx} + \frac{dy}{dx} = 0$	1M+1M	1M for product rule
	$y dx dx$ $= \frac{-y \ln y}{x + y}$		1M for chain rule
Alte	ernative Solution		
	$= \frac{\frac{2-y}{\ln y}}{\frac{(\ln y)^2}{(\ln y)^2}}$	1M	For quotient rule
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y(\ln y)^2}{y - 2 - y \ln y}$	1M	For $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$
	en the curve cuts the y-axis, $x = 0$. y = 2	1A	100

2014-DSE-MATH-EP(M2)-3

CONFIDENTIAL (FOR MARKER 3 03		
Solution	Marks	Remarks
$\left. \frac{\mathrm{d}y}{\mathrm{d}x} \right _{(0,2)} = \frac{-2\ln 2}{0+2}$	1M	OR $\frac{2(\ln 2)^2}{2-2-2\ln 2}$
= $-\ln 2$ Hence the equation of the tangent is $y = -x \ln 2 + 2$.	1A	
	(5)	
$4. \qquad x = 2y + \sin y$		
$\frac{\mathrm{d}x}{\mathrm{d}y} = 2 + \cos y$	1M	
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2 + \cos y}$		
$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -1 \cdot (2 + \cos y)^{-2} \left(-\sin y\right) \frac{\mathrm{d}y}{\mathrm{d}x}$	1M	OR $\frac{0 - 1(-\sin y) \frac{dy}{dx}}{(2 + \cos y)^2}$
Alternative Solution		
$1 = 2\frac{dy}{dx} + \cos y \cdot \frac{dy}{dx}$	1M	
$0 = 2\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left[\cos y \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + (-\sin y) \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]$	1M	
$\sin y \left(\frac{1}{2 + \cos y}\right)^2 = (2 + \cos y) \frac{d^2 y}{dx^2}$		
$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\sin y}{\left(2 + \cos y\right)^3}$	1A	
$dx = (2 + \cos y)$	(3)	
5. (a) $\int \frac{dx}{\sqrt{9-x}} = \int -(9-x)^{\frac{-1}{2}} d(9-x)$	1M+1A	
Alternative Solution Let $u = 9 - x$. $du = - dx$	1M	
$\int \frac{\mathrm{d}x}{\sqrt{9-x}} = \int -u^{\frac{-1}{2}} \mathrm{d}u$	1A	
$=-2u^{\frac{1}{2}}+C$		
$=-2\sqrt{9-x}+C$	1A	
(b) Let $x = 3 \sin \theta$. $dx = 3 \cos \theta d\theta$	1M	
$\int \frac{\mathrm{d}x}{\sqrt{9-x^2}} = \int \frac{3\cos\theta \mathrm{d}\theta}{\sqrt{9-9\sin^2\theta}}$		
$= \int d\theta$ $= \theta + C$	1A	
$=\sin^{-1}\frac{x}{3}+C$	1A	
	(6)	

-		Solution Solution	Marks	Remarks
6	(a)	$\int xe^{-x} dx = x(-e^{-x}) - \int (-e^{-x}) dx$	1M	Lonning
0.	(a)	ž ž		
		$=-xe^{-x}-e^{-x}+C$	1A	
				, y
	(6)	$\begin{cases} y = xe^{-x} \\ y = \frac{x}{a} \end{cases}$		$y = xe^{-x}$
	(0)	$y = \frac{x}{e}$		
		$xe^{-x} = \frac{x}{e}$		$y = \frac{x}{e}$
				2
		$x\left(e^{-x} - \frac{1}{e}\right) = 0$		No -
		x = 0 or 1	1A	For $x=1$
		$\therefore \text{ the area } = \int_0^1 \left(x e^{-x} - \frac{x}{e} \right) dx$	1M	For $\int_a^b (y_1 - y_2) dx$
		$= \left[-xe^{-x} - e^{-x} - \frac{x^2}{2e} \right]_0^1$	1M	For using (a)
			11/1	ror using (a)
		$=\left(-e^{-1}-e^{-1}-\frac{1}{2e}\right)-(-1)$		
		$=1-\frac{5}{2e}$	1A	
		- · 2e	(6)	
			(0)	
		(1 0 1)(1 0 1)		
7.	(a)	$A^2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$		
		(1 0 1)(1 0 1)		
		(2 0 2)		
		$= \begin{bmatrix} 0 & 4 & 0 \\ 2 & 0 & 2 \end{bmatrix}$		
		=2A	1	
		Hence the statement is true for $n=1$.		Ÿ
		Assume the statement is true for $n = k$, i.e. $A^{k+1} = 2^k A$.	1	
		$A^{k+2} = A^{k+1}A$		
		$=(2^k A)A$ by assumption	1	
		$=2^kA^2$		
		= $2^k \cdot 2A$ by the statement for $n = 1$ = $2^{k+1}A$	1.0	
		Hence the statement is also true for $n = k + 1$.	1	
		By the principle of mathematical induction, the statement is true for all positive		
		integers n.	1	
	(b)	A = 0	1A	
		Hence A^{-1} does not exist and so Willy arrives at a wrong conclusion by using A^{-1}	¹ . 1	
			(7)	7
		,		3

	CONFIDENTIAL (FOR MARKER'S USE		Pamarka
	Solution	Marks	Remarks
. (a)	$\overrightarrow{OP} \times \overrightarrow{OQ} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 2 \\ 1 & -1 & 2 \end{vmatrix}$		
	$= 6\mathbf{i} + 4\mathbf{j} - \mathbf{k}$	1A	
	The volume of tetrahedron $OPQR = \frac{1}{6} \overrightarrow{OP} \times \overrightarrow{OQ}) \cdot \overrightarrow{OR} $		
	$= \frac{1}{6} \left (6\mathbf{i} + 4\mathbf{j} - \mathbf{k}) \cdot (2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) \right $	1M	
	=1	1A	
(b)	$OR = \sqrt{2^2 + (-3)^2 + 6^2}$		
(0)	=7	1A	
	The area of $\triangle OPQ = \frac{1}{2} \overrightarrow{OP} \times \overrightarrow{OQ} $		
			R
	$=\frac{1}{2}\sqrt{6^2+4^2+(-1)^2}$		h
	$=\frac{\sqrt{53}}{2}$	1A	0
	Let h be the height of the tetrahedron with OPQ as base.		Q
	$\therefore \frac{1}{3} \cdot \frac{\sqrt{53}}{2} h = 1$	1M	2
	$h = \frac{6}{\sqrt{53}}$		
	$\sqrt{53}$ Let θ be the angle between the plane OPQ and the line OR .		
	6		D
	$\therefore \sin \theta = \frac{\sqrt{53}}{7}$	1M	R
		-	0.0
	Alternative Solution $ \overrightarrow{OP} \times \overrightarrow{OQ} = \sqrt{6^2 + 4^2 + (-1)^2}$		
	La L		Ω
	$= \sqrt{53}$ Let θ be the angle between the plane OPQ and the line OR .	1A	$\overrightarrow{OP} \times \overrightarrow{OQ}$
	$\overrightarrow{OR} \cdot (\overrightarrow{OP} \times \overrightarrow{OQ}) = \overrightarrow{OR} \cdot \overrightarrow{OP} \times \overrightarrow{OQ} \cos(\theta + 90^{\circ})$	1M+1M	1M for dot product form 1M for $\theta + 90^{\circ}$
			1141101 0 1 30
	$\cos(\theta + 90^{\circ}) = \frac{2 \cdot 6 - 3 \cdot 4 + 6(-1)}{7\sqrt{53}}$		
	$\theta \approx 6.8^{\circ}$	1A	
		(8)	
		IE.	1

CONFIDENTIAL (FOR MARKER'S USE	ONLY)	
Solution	Marks	Remarks
The augmented matrix is $\begin{pmatrix} 1 & 1 & 1 & 100 \\ 1 & 1 & 1 & 100 \end{pmatrix}$		
$\sim \begin{pmatrix} 1 & 1 & 1 & 100 \\ 2 & 2 & 2 & 100 \end{pmatrix}$	1M	
	AND UP SERVICEN	
$\therefore y = 20 - \frac{1}{5}$ and $x = 80 + \frac{1}{5}$	1A+1A	
Alternative Solution		OP 1
		OR let $x = t$ and so
$\therefore z = \frac{100 - 3t}{9} \text{ and } x = \frac{800 - 4t}{9}$	1A+1A	$y = 200 - \frac{9t}{4}, \ z = \frac{5t}{4} - 100$
$\int m + n + k = 100$	1.4	
0.5m + 3n + 5k = 100	IA	
$\lim_{n \to \infty} \int m + n + k = 100$		
By (a), if both $20 - \frac{9t}{5}$ and $80 + \frac{4t}{5}$ are integers, then t is a multiple of 5.		
$m \ge 0$ gives $t \ge -100$		
$n \ge 0$ gives $t \le \frac{100}{9}$	> 1M	
Combining all the conditions above, we have $t = 0$, 5 or 10.	J	
Alternative Solution (1)		
	} 1M	
negative when t = 0, 5 or 10 (OK any two of these).		
Alternative Solution (2)	5	
500 to 1000 km A	} 1M	
(80, 20, 0), (84, 11, 3) or (88, 2, 10) (OR any two of these).		
Hence there are more than one set of combination of m , n and k and so Aubrey cannot be agreed with.	1	
,		
	(0)	
		P
HK - HR + RK		
$=\left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)$ cm	1A	OR $(24\sec\theta + 192\csc\theta)$ cm
	(1)	
	The augmented matrix is	The augmented matrix is $ \begin{bmatrix} 1 & 1 & 1 & 100 \\ 1 & 6 & 10 & 200 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 100 \\ 0 & 5 & 9 & 100 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 100 \\ 0 & 5 & 9 & 100 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 100 \\ 0 & 5 & 9 & 100 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 100 \\ 0 & 5 & 9 & 100 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 100 \\ 0 & 5 & 9 & 100 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 100 \\ 0 & 5 & 1 & 100 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 100 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 100 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 \end{bmatrix} $ $ - \begin{bmatrix} 1 & 1 & $

(b) $\frac{dHK}{d\theta} = -24(\cos\theta)^{-2}(-\sin\theta) - 192(\sin\theta)^{-2}\cos\theta$ $\frac{dHK}{d\theta} = 0 \text{ when } \frac{24\sin\theta}{\cos^2\theta} = \frac{192\cos\theta}{\sin^2\theta}$ $\tan^3\theta = 8$ $\tan\theta = 8$ $\tan\theta = 8$ $\theta = \tan^{-1}2$ $\theta = \tan^{-1}2, HK \text{ is minimum.}$ By (a), the shortest length of the ladder $= 24 \cdot \frac{\sqrt{5}}{1} + 192 \cdot \frac{\sqrt{5}}{2}$ $= 120\sqrt{5} \text{ cm}$ $\frac{dx}{dx} = 270 \cos\theta + 24 + 192 \cos\theta$ $\frac{dx}{dx} = 270 \sin\theta \cdot \frac{dx}{dx} - 192 \cos\theta^2\theta \cdot \frac{d\theta}{dx}$ $\frac{dx}{dx} = 270 \sin\theta \cdot \frac{d\theta}{dx} - 192 \cos\theta^2\theta \cdot \frac{d\theta}{dx}$ $\frac{dx}{dx} = 270 \sin\theta \cdot \frac{d\theta}{dx} - 192 \cos\theta^2\theta \cdot \frac{d\theta}{dx}$ $\frac{dx}{dx} = 270 \left(\frac{6}{\sqrt{61}} \right) (-0.1) - 192 \left(\frac{\sqrt{61}}{6} \right)^2 (-0.1)$ $= 11.79$ i.e. the rate of change of x is 11.79 cm s^{-1} . (ii) $y - x = 270 \cos\theta$ $\frac{dy}{dx} = -270 \sin\theta \cdot \frac{d\theta}{dx}$ $\frac{dy}{dx} = -192 \cos^2\theta \cdot \frac{d\theta}{dx}$ $\frac{d\theta}{dx} = -192 $		Solution Solution	Marks	Remarks
$\frac{dHK}{d\theta} = 0 \text{when} \frac{24 \sin \theta}{\cos^2 \theta} = \frac{192 \cos \theta}{\sin^2 \theta}$ $\tan^3 \theta = 8$ $\tan \theta = 2$ $\theta = \tan^{-1} 2$ $\theta = 0 < \theta < \tan^{-1} 2 \theta = \tan^{-1} 2 \tan^{-1} 2 < \theta < \frac{\pi}{2}$ $\frac{dHK}{d\theta} -ve 0 +ve$ $When \theta = \tan^{-1} 2, HK \text{ is minimum.} By (a), the shortest length of the ladder = 24 \cdot \frac{\sqrt{5}}{1} + 192 \cdot \frac{\sqrt{5}}{2} = 120\sqrt{5} \text{ cm} (c) (i) x + HK \cos \theta = AB + CK x = -270 \cos \theta + 24 + 192 \cot \theta \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt} - 192 \csc^2 \theta \cdot \frac{d\theta}{dt} When CK = 160 \text{ cm}, \tan \theta = \frac{192}{160} = \frac{6}{5} \therefore \sin \theta = \frac{6}{\sqrt{61}} (-0.1) - 192 \left(\frac{\sqrt{61}}{6} \right)^2 (-0.1) = 11.79 i.e. the rate of change of x is 11.79 \text{ cm s}^{-1}. (ii) y - x = 270 \cos \theta \frac{dy}{dt} = \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt} \frac{dy}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt} \frac{d\theta}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt$	(b)			
$\tan^3\theta = 8$ $\tan\theta = 2$ $\theta = \tan^{-1}2$ $\theta = 0 < \theta < \tan^{-1}2$ $\theta = \tan^{-1}2$ $\frac{dH}{d\theta} - ve$ $0 + ve$ $When \theta = \tan^{-1}2, HK is minimum. By (a), the shortest length of the ladder = 24 \cdot \frac{\sqrt{5}}{1} + 192 \cdot \frac{\sqrt{5}}{2} = 120\sqrt{5} \text{ cm} (c) (i) x + HK \cos\theta = AB + CK x = -270 \cos\theta + 274 + 192 \cot\theta \frac{dx}{dx} = 270 \sin\theta \cdot \frac{d\theta}{dt} - 192 \csc^2\theta \cdot \frac{d\theta}{dt} \frac{dx}{dt} = 270 \cos\theta - \frac{1}{2}(-0.1) - 192\left(\frac{\sqrt{6}}{6}\right)^2(-0.1) = 11.79 i.e. the rate of change of x is 11.79 \text{ cm s}^{-1}. (ii) y - x = 270 \cos\theta \frac{dy}{dt} \frac{dx}{dt} = -270 \sin\theta \cdot \frac{d\theta}{dt} \frac{dy}{dt} = -270 \sin\theta \cdot \frac{d\theta}{dt} \frac{dy}{dt} = -192 \csc^2\theta \cdot \frac{d\theta}{dt} \frac{d\theta}{dt} = -192 \cot^2\theta \cdot \frac{d\theta}{dt} \frac{d\theta}{dt} = -192 \cot^2$	(0)		1M	H
$\theta = \tan^{-1} 2$ $\theta = 0 < \theta < \tan^{-1} 2 \theta = \tan^{-1} 2 \theta = \tan^{-1} 2 \tan^{-1} 2 < \theta < \frac{\pi}{2}$ $\frac{dHK}{d\theta} -ve 0 +ve$ $When \theta = \tan^{-1} 2, HK \text{ is minimum.} By (a), the shortest length of the ladder = 24, \frac{\sqrt{5}}{1} + 192, \frac{\sqrt{5}}{2} = 120\sqrt{5} \text{ cm} 1 (c) (i) x + HK \cos \theta = AB + CK x = -270 \cos \theta + 24 + 192 \cot \theta \frac{dx}{dt} = 270 \sin \theta, \frac{d\theta}{dt} - 192 \cos^2 \theta, \frac{d\theta}{dt} \frac{dx}{dt} = 270 \sin \theta, \frac{d\theta}{dt} - 192 \cos^2 \theta, \frac{d\theta}{dt} \frac{dx}{dt} = 270 \left(\frac{6}{\sqrt{61}} \right) (-0.1) - 192 \left(\frac{\sqrt{61}}{6} \right)^2 (-0.1) \approx 11.79 i.e. the rate of change of x is 11.79 \text{ cm s}^{-1}. (ii) y - x = 270 \cos \theta \frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta, \frac{d\theta}{dt} \frac{dy}{dt} = -192 \cos^2 \theta, \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta, \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta, \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta, \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta, \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta, \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta, \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta, \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} = 270 \sin \theta + \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} - \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} \frac{dy}{dt} - \frac{dy}{dt} - \frac{dy}{dt} $				$A \longrightarrow B$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				
$\frac{dHK}{d\theta} - ve \qquad 0 \qquad + ve$ $When \theta = \tan^{-1} 2, HK is minimum. By (a), the shortest length of the ladder = 24 \cdot \frac{\sqrt{5}}{1} + 192 \cdot \frac{\sqrt{5}}{2} = 120\sqrt{5} cm 1 \qquad (5) (c) (i) x + HK \cos \theta = AB + CK \\ x = -270 \cos \theta + 24 + 192 \cot \theta \\ \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt} - 192 \csc^2 \theta \cdot \frac{d\theta}{dt} When CK = 160 \text{ cm}, \tan \theta = \frac{192}{160} = \frac{6}{5} \therefore \sin \theta = \frac{6}{\sqrt{61}} \frac{dx}{dt} = 270 \left(\frac{6}{\sqrt{61}} \right) (-0.1) - 192 \left(\frac{\sqrt{61}}{6} \right)^2 (-0.1) = 11.79 i.e. the rate of change of x is 11.79 \text{ cm s}^{-1}. (ii) y - x = 270 \cos \theta \frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt} \frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt} \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \frac{dy}{dt} > \frac{dx}{dt} \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \frac{dy}{dt} > \frac{dx}{dt} \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \frac{dy}{dt} > \frac{dx}{dt} \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \frac{dy}{dt} > \frac{dx}{dt} \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \cos \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \cos \theta > 0 \text{ and } \frac{d\theta}{dt} < 0 \therefore \cos \theta > 0 \text{ and } d\theta$				
When $\theta = \tan^{-1} 2$, HK is minimum. By (a), the shortest length of the ladder $= 24 \cdot \frac{\sqrt{5}}{1} + 192 \cdot \frac{\sqrt{5}}{2}$ $= 120\sqrt{5} \text{ cm}$ IM $\frac{dx}{dx} = 270 \cos \theta + 24 + 192 \cot \theta$ $\frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt} - 192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta = \frac{6}{\sqrt{61}}$ $\frac{dx}{dt} = 270 \left(\frac{6}{\sqrt{61}} \left(-0.1 \right) - 192 \left(\frac{\sqrt{61}}{6} \right)^2 \left(-0.1 \right) \right)$ ≈ 11.79 i.e. the rate of change of x is 11.79 cm s^{-1} . (ii) $y - x = 270 \cos \theta$ $\frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt}$ Alternative Solution $y = 24 + 192 \cot \theta$ $\frac{dy}{dt} - 192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = \frac{1}{2} \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = \frac{1}{2} \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dx}{dt} = \frac{1}{2} \cos^2 \theta \cdot \frac{d\theta}{dt}$ $\frac{dx}{dt} = \frac{1}{2} \cos^2$		d <i>HK</i>	1M	
By (a), the shortest length of the ladder $= 24 \cdot \frac{\sqrt{5}}{1} + 192 \cdot \frac{\sqrt{5}}{2}$ $= 120\sqrt{5} \text{ cm}$ (c) (i) $x + HK \cos \theta = AB + CK$ $x = -270 \cos \theta + 24 + 192 \cot \theta$ $\frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt} = 192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta = \frac{6}{\sqrt{61}}$ $\frac{dx}{dt} = 270 \left(\frac{6}{\sqrt{61}}\right)(-0.1) - 192 \left(\frac{\sqrt{61}}{6}\right)^2(-0.1)$ ≈ 11.79 i.e. the rate of change of x is 11.79 cm s^{-1} . (ii) $y - x = 270 \cos \theta$ $\frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt}$ Alternative Solution $y = 24 + 192 \cot \theta$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} - \frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dx}{dt} = 192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ $\frac{dx}{dt} = 192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ $\frac{dx}{dt} = 192 \cos^2 \theta \cdot \frac{d\theta}$		$d\theta$	1A	θ
$=120\sqrt{5} \text{ cm}$ (c) (i) $x+HK\cos\theta=AB+CK$ $x=-270\cos\theta+24+192\cot\theta$ $\frac{dx}{dt}=270\sin\theta\cdot\frac{d\theta}{dt}-192\csc^2\theta\cdot\frac{d\theta}{dt}$ When $CK=160$ cm, $\tan\theta=\frac{192}{160}-\frac{6}{5}$ $\therefore \sin\theta=\frac{6}{\sqrt{61}}$ $\frac{dx}{dt}=270\left(\frac{6}{\sqrt{61}}\right)(-0.1)-192\left(\frac{\sqrt{61}}{6}\right)^2(-0.1)$ ≈ 11.79 i.e. the rate of change of x is 11.79 cm s ⁻¹ . (ii) $y-x=270\cos\theta$ $\frac{dy}{dt}-\frac{dx}{dt}=270\sin\theta\cdot\frac{d\theta}{dt}$ Alternative Solution $y=24+192\cot\theta$ $\frac{dy}{dt}=-192\csc^2\theta\cdot\frac{d\theta}{dt}$ By (*), $\frac{dx}{dt}-\frac{dy}{dt}=270\sin\theta\cdot\frac{d\theta}{dt}$ $\therefore \sin\theta>0 \text{ and } \frac{d\theta}{dt}<0$ $\therefore \sin\theta>0 \text{ and } \frac{d\theta}{dt}<0$ $\therefore \sin\theta>0 \text{ and } \frac{d\theta}{dt}<0$ $\therefore \sin\theta>0 \text{ and } \frac{d\theta}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.				D C K
(c) (i) $x + HK \cos\theta = AB + CK$ $x = -270 \cos\theta + 24 + 192 \cot\theta$ $\frac{dx}{dt} = 270 \sin\theta \cdot \frac{d\theta}{dt} - 192 \csc^2\theta \cdot \frac{d\theta}{dt}$ When $CK = 160$ cm, $\tan\theta = \frac{192}{160} = \frac{6}{5}$ $\therefore \sin\theta = \frac{6}{\sqrt{61}}$ OR $\theta = 0.87605805$			1	
(c) (i) $x + HK \cos \theta = AB + CK$ $x = -270 \cos \theta + 24 + 192 \cot \theta$ $\frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt} - 192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ When $CK = 160$ cm, $\tan \theta = \frac{192}{160} = \frac{6}{5}$ $\therefore \sin \theta = \frac{6}{\sqrt{61}}$ OR $\theta = 0.87605805$ OR		1000 to 1 2000	(5)	
$x = -270\cos\theta + 24 + 192\cot\theta$ $\frac{dx}{dt} = 270\sin\theta \cdot \frac{d\theta}{dt} - 192\csc^2\theta \cdot \frac{d\theta}{dt}$ When $CK = 160$ cm, $\tan\theta = \frac{192}{160} = \frac{6}{5}$ $\therefore \sin\theta = \frac{6}{\sqrt{61}}$ $\frac{dx}{dt} = 270\left(\frac{6}{\sqrt{61}}\right)(-0.1) - 192\left(\frac{\sqrt{61}}{6}\right)^2(-0.1)$ ≈ 11.79 i.e. the rate of change of x is 11.79 cm s ⁻¹ . (ii) $y - x = 270\cos\theta$ $\frac{dy}{dt} - \frac{dx}{dt} = -270\sin\theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} = -192\csc^2\theta \cdot \frac{d\theta}{dt}$ By (*), $\frac{dx}{dt} - \frac{dy}{dt} = 270\sin\theta \cdot \frac{d\theta}{dt}$ $\therefore \sin\theta > 0 \text{ and } \frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.			(-)	1
$\frac{dx}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt} - 192 \csc^2 \theta \cdot \frac{d\theta}{dt} \qquad (*)$ When $CK = 160 \text{ cm}$, $\tan \theta = \frac{192}{160} = \frac{6}{5}$ $\therefore \sin \theta = \frac{6}{\sqrt{61}}$ $\frac{dx}{dt} = 270 \left(\frac{6}{\sqrt{61}}\right)(-0.1) - 192 \left(\frac{\sqrt{61}}{6}\right)^2 (-0.1)$ ≈ 11.79 i.e. the rate of change of x is 11.79 cm s^{-1} . (ii) $y - x = 270 \cos \theta$ $\frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ $\frac{dy}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.	(c)		1M	
When $CK = 160 \text{cm}$, $\tan \theta = \frac{192}{160} = \frac{6}{5}$ $\therefore \sin \theta = \frac{6}{\sqrt{61}}$ $\frac{dx}{dt} = 270 \left(\frac{6}{\sqrt{61}}\right) (-0.1) - 192 \left(\frac{\sqrt{61}}{6}\right)^2 (-0.1)$ ≈ 11.79 i.e. the rate of change of x is 11.79cm s^{-1} . (ii) $y - x = 270 \text{cos} \theta$ $\frac{dy}{dt} - \frac{dx}{dt} = -270 \text{sin} \theta \cdot \frac{d\theta}{dt}$ Alternative Solution $y = 24 + 192 \text{cot} \theta$ $\frac{dy}{dt} = -192 \text{csc}^2 \theta \cdot \frac{d\theta}{dt}$ By (*), $\frac{dx}{dt} - \frac{dy}{dt} = 270 \text{sin} \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta > 0 \text{and} \frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.				
		di di		
$\frac{dx}{dt} = 270 \left(\frac{6}{\sqrt{61}}\right) (-0.1) - 192 \left(\frac{\sqrt{61}}{6}\right)^2 (-0.1)$ ≈ 11.79 i.e. the rate of change of x is 11.79 cm s^{-1} . (ii) $y - x = 270 \cos \theta$ $\frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt}$ Alternative Solution $y = 24 + 192 \cot \theta$ $\frac{dy}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ By (*), $\frac{dx}{dt} - \frac{dy}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.				
$\frac{dx}{dt} = 270 \left(\frac{dy}{\sqrt{61}} \right) (-0.1) - 192 \left(\frac{401}{6} \right) (-0.1)$ ≈ 11.79 i.e. the rate of change of x is 11.79 cm s^{-1} . (ii) $y - x = 270 \cos \theta$ $\frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt}$ Alternative Solution $y = 24 + 192 \cot \theta$ $\frac{dy}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ By (*), $\frac{dx}{dt} - \frac{dy}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed E is leaving the wall and Thomas is agreed with.		$\therefore \sin \theta = \frac{6}{\sqrt{61}}$		OR $\theta = 0.87605805$
i.e. the rate of change of x is 11.79 cm s^{-1} . (ii) $y - x = 270 \cos \theta$ $\frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt}$ Alternative Solution $y = 24 + 192 \cot \theta$ $y = 24 + 192 \cot \theta$ $By (*), \frac{dx}{dt} - \frac{dy}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.		$\frac{\mathrm{d}x}{\mathrm{d}t} = 270 \left(\frac{6}{\sqrt{61}} \right) (-0.1) - 192 \left(\frac{\sqrt{61}}{6} \right)^2 (-0.1)$		100
(ii) $y - x = 270 \cos \theta$ $\frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt}$ Alternative Solution $y = 24 + 192 \cot \theta$ $\frac{dy}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ By (*), $\frac{dx}{dt} - \frac{dy}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.			1A	U
$\frac{dy}{dt} - \frac{dx}{dt} = -270 \sin \theta \cdot \frac{d\theta}{dt}$ Alternative Solution $y = 24 + 192 \cot \theta$ $\frac{dy}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ By (*), $\frac{dx}{dt} - \frac{dy}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence <i>K</i> is moving towards <i>E</i> at a speed faster than the horizontal speed <i>H</i> is leaving the wall and Thomas is agreed with.		i.e. the rate of change of x is 11.79 cm s $^{\circ}$.		$A \stackrel{H}{\longrightarrow} B$
Alternative Solution $y = 24 + 192 \cot \theta$ $\frac{dy}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ By (*), $\frac{dx}{dt} - \frac{dy}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.		(ii) $y - x = 270 \cos \theta$	1M	
Alternative Solution $y = 24 + 192 \cot \theta$ $\frac{dy}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ By (*), $\frac{dx}{dt} - \frac{dy}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.		$\frac{\mathrm{d}y}{\mathrm{d}t} - \frac{\mathrm{d}x}{\mathrm{d}t} = -270\sin\theta \cdot \frac{\mathrm{d}\theta}{\mathrm{d}t}$		
$\frac{dy}{dt} = -192 \csc^2 \theta \cdot \frac{d\theta}{dt}$ By (*), $\frac{dx}{dt} - \frac{dy}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.		Alternative Solution		
By (*), $\frac{dx}{dt} - \frac{dy}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$ $\therefore \sin \theta > 0$ and $\frac{d\theta}{dt} < 0$ $\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed E is leaving the wall and Thomas is agreed with.			IM	θ
		1 17 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		L A
$\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.		By (*), $\frac{dx}{dt} - \frac{dy}{dt} = 270 \sin \theta \cdot \frac{d\theta}{dt}$		
$\therefore \frac{dy}{dt} > \frac{dx}{dt}$ Hence <i>K</i> is moving towards <i>E</i> at a speed faster than the horizontal speed <i>H</i> is leaving the wall and Thomas is agreed with.		$\therefore \sin \theta > 0 \text{ and } \frac{d\theta}{dt} < 0$		
Hence K is moving towards E at a speed faster than the horizontal speed H is leaving the wall and Thomas is agreed with.			1A	
II to rearing the front and research to the		Hence K is moving towards E at a speed faster than the horizontal speed		
(6)		H is leaving the wall and Thomas is agreed with.		
			(6)	-

		Solution Solution	Marks	Remarks
11 (-)	(i)		iviarks	Kemarks
11. (a)	(1)			
		$\overrightarrow{OE} = \frac{\mathbf{a} + mt\mathbf{b}}{1 + m}$	1M+1A	Å
	(ii)	$\overrightarrow{OD} = (1-t)\mathbf{a}$		E
		$\overrightarrow{OF} = n(1-t)\mathbf{a} + \mathbf{b}$		
		$\therefore \overrightarrow{OE} = \frac{n(1-t)\mathbf{a} + \mathbf{b}}{1+n}$	1A	
	(111)	Commented (D and (D) we have		0 C
	(111)	Comparing (i) and (ii), we have	1	
		$\begin{cases} \frac{1}{1+m} = \frac{n(1-t)}{1+n}$		
		$\begin{bmatrix} mt & 1 \\ mt & 1 \end{bmatrix}$	1M	
		(1+m-1+n)		
		(2)÷(1):		
		$mt = \frac{1}{n(1-t)}$	1M	
		By (1), $\frac{1}{1 + \frac{1}{nt(1-t)}} = \frac{n(1-t)}{1+n}$		
		$1+\frac{1}{nt(1-t)}$		
		t(1+n) = nt(1-t) + 1		
		$t = -nt^2 + 1$		
		$n = \frac{1-t}{t^2}$	1	
		•	1	
		By (3), $mt = \frac{1}{1}$		
		By (3), $mt = \frac{1}{\frac{1-t}{t^2}(1-t)}$		
		$m = \frac{t}{(1-t)^2}$	1	
	(iv)	If $m = n$, then $\frac{t}{(1-t)^2} = \frac{1-t}{t^2}$.	1A	
		$t^3 = (1-t)^3$		
		$t = \frac{1}{2}$		
		Hence C and D are the mid-points of OB and OA respectively.		
		Therefore, E is the centroid of $\triangle OAB$ and Chris is agreed with.	1A	
			(0)	
			(9)	
(b)		= fb - a		
	AC	$\overrightarrow{OB} = (t\mathbf{b} - \mathbf{a}) \cdot \mathbf{b}$		
	172546	$= 4t - a \cdot b$	1A	
	-	en $AC \perp OB$, $AC \cdot OB = 0$ which gives $\mathbf{a} \cdot \mathbf{b} = 4t$ (4)	1M	
		$=(1-t)\mathbf{a}-\mathbf{b}$		
	BD	$\overrightarrow{OA} = [(1-t)\mathbf{a} - \mathbf{b}] \cdot \mathbf{a}$		
		$= (1-t) - \mathbf{a} \cdot \mathbf{b}$	1A	
	Ву	$(4), \ \overrightarrow{BD} \cdot \overrightarrow{OA} = 1 - 5t .$		
		$\overrightarrow{BD} \cdot \overrightarrow{OA} \neq 0$ in general.		
		BD is not always perpendicular to OA and Francis is not agreed with.	1A	
			745	
			(4)	

機密 (只限閱卷員使用)
CONFIDENTIAL (FOR MARKER'S USE ONLY)

	CONFIDENTIAL (FOR MARKER'S USE Solution	Marks	Remarks
2. (a) (i)	$A^{-1} = \begin{pmatrix} 1 & p \\ -1 & 1 \end{pmatrix}^{-1}$		
	$=\frac{1}{\begin{vmatrix} 1 & p \\ -1 & 1 \end{vmatrix}} \begin{pmatrix} 1 & 1 \\ -p & 1 \end{pmatrix}^T$	1M	
	$=\frac{1}{1+p} \begin{pmatrix} 1 & -p \\ 1 & 1 \end{pmatrix}$	1A	
(ii)	$A^{-1}MA = \frac{1}{1+p} \begin{pmatrix} 1 & -p \\ 1 & 1 \end{pmatrix} \begin{pmatrix} k-1 & k \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & p \\ -1 & 1 \end{pmatrix}$ $= \frac{1}{1+p} \begin{pmatrix} k-p-1 & k \\ k & k \end{pmatrix} \begin{pmatrix} 1 & p \\ -1 & 1 \end{pmatrix}$ $= \frac{1}{1+p} \begin{pmatrix} -1-p & k+kp-p-p^2 \\ 0 & k+kp \end{pmatrix}$	1M+1A	OR $\frac{1}{1+p} \begin{pmatrix} 1 & -p \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & k+kp-1 \\ 1 & p \end{pmatrix}$
	$= \frac{1}{1+p} \begin{pmatrix} 0 & k+kp \end{pmatrix}$ $= \begin{pmatrix} -1 & k-p \\ 0 & k \end{pmatrix}$	1	
(iii)) By (ii), $(A^{-1}MA)^n = \begin{pmatrix} -1 & 0 \\ 0 & k \end{pmatrix}^n$ for $p = k$		
	$A^{-1}M^nA = \begin{pmatrix} (-1)^n & 0 \\ 0 & k^n \end{pmatrix}$	1M	For either side
	$M^{n} = \begin{pmatrix} 1 & k \\ -1 & 1 \end{pmatrix} \begin{pmatrix} (-1)^{n} & 0 \\ 0 & k^{n} \end{pmatrix} \cdot \frac{1}{1+k} \begin{pmatrix} 1 & -k \\ 1 & 1 \end{pmatrix}$	1M	
	$= \frac{1}{1+k} \begin{pmatrix} (-1)^n & k^{n+1} \\ (-1)^{n+1} & k^n \end{pmatrix} \begin{pmatrix} 1 & -k \\ 1 & 1 \end{pmatrix}$		$OR \frac{1}{1+k} \begin{pmatrix} 1 & k \\ -1 & 1 \end{pmatrix} \begin{pmatrix} (-1)^n & (-1)^{n+1} \\ k^n & k^n \end{pmatrix}$
	$= \frac{1}{1+k} \begin{pmatrix} k^{n+1} + (-1)^n & k^{n+1} + (-1)^{n+1} k \\ k^n + (-1)^{n+1} & k^n + (-1)^n k \end{pmatrix}$	1A (8)	_
(b) (x	$\begin{pmatrix} x_n \\ x_{n-1} \end{pmatrix} = M \begin{pmatrix} x_{n-1} \\ x_{n-2} \end{pmatrix}$ where $M = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$ after substituting $k = 2$		
	$= M^2 \begin{pmatrix} x_{n-2} \\ x_{n-3} \end{pmatrix}$ $= \cdots$	1M	
	$=M^{n-2} \binom{x_2}{x_1}$	1A	
	$= \frac{1}{1+2} \begin{pmatrix} 2^{n-1} + (-1)^{n-2} & 2^{n-1} + (-1)^{n-1} 2 \\ 2^{n-2} + (-1)^{n-1} & 2^{n-2} + (-1)^{n-2} 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} $ by (a)(iii)		20-1
.:	$x_n = \frac{2^{n-1} + (-1)^{n-2}}{3}$	1A (3)	OR $\frac{2^{n-1}+(-1)^n}{3}$
.4.	$= \frac{1}{1+2} \begin{pmatrix} 2^{n-1} + (-1)^{n-2} & 2^{n-1} + (-1)^{n-1} 2 \\ 2^{n-2} + (-1)^{n-1} & 2^{n-2} + (-1)^{n-2} 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} $ by (a)(iii) $x_n = \frac{2^{n-1} + (-1)^{n-2}}{3}$	1A (3)	OR $\frac{2^{n-1} + (-1)^n}{3}$

	Solution	Marks	Remarks
13. (a)	$1 - \cos 4\theta - 2\cos 2\theta \sin^2 2\theta$ $= 2\sin^2 2\theta - 2\cos 2\theta \sin^2 2\theta$ $= 2\sin^2 2\theta (1 - \cos 2\theta)$	1M	For $1 - \cos 4\theta = 2\sin^2 2\theta$ OR $1 - \cos 2\theta = 2\sin^2 \theta$
	$=2(2\sin\theta\cos\theta)^2(2\sin^2\theta)$		
	$=16\cos^2\theta\sin^4\theta$	1	
		(2)	
(b)	$\int_0^{n\pi} \cos^2 x \sin^4 x \mathrm{d}x$		
	$= \int_0^{n\pi} \frac{1 - \cos 4x - 2\cos 2x \sin^2 2x}{16} dx \qquad \text{by (a)}$		
	$= \frac{1}{16} \int_0^{n\pi} (1 - \cos 4x) dx - \frac{1}{16} \int_0^{n\pi} \sin^2 2x \cdot 2 \cos 2x dx$	1M	
	$= \frac{1}{16} \left[x - \frac{\sin 4x}{4} \right]_0^{n\pi} - \frac{1}{16} \int_{x=0}^{n\pi} \sin^2 2x \mathrm{d}\sin 2x$	1M	For $dsin2x$
	$= \frac{1}{16} \left[\left(n\pi - \frac{\sin 4n\pi}{4} \right) - 0 \right] - \frac{1}{16} \left[\frac{\sin^3 2x}{3} \right]_0^{n\pi}$	1A	For $\frac{\sin^3 2x}{3}$
	Alternative Solution		
	$= \frac{1}{16} \int_0^{n\pi} (1 - \cos 4x) dx - \frac{1}{16} \int_0^{n\pi} \sin 4x \sin 2x dx$		
	$= \frac{1}{16} \left[x - \frac{\sin 4x}{4} \right]_0^{n\pi} - \frac{1}{16} \int_0^{n\pi} \frac{\cos 2x - \cos 6x}{2} \mathrm{d}x$	1M	For $\frac{\cos 2x - \cos 6x}{2}$ For $\frac{\sin 2x}{2} - \frac{\sin 6x}{6}$
	$= \frac{1}{16} \left[\left(n\pi - \frac{\sin 4n\pi}{4} \right) - 0 \right] - \frac{1}{32} \left[\frac{\sin 2x}{2} - \frac{\sin 6x}{6} \right]_0^{n\pi}$	1A	For $\frac{\sin 2x}{2} - \frac{\sin 6x}{6}$
	$=\frac{n\pi}{16}$	1	
		(4)	
(c)	Let $x = k - u$.	1M	
	$\therefore dx = -du$ When $x = 0$, $u = k$; when $x = k$, $u = 0$.		
	$\int_{0}^{k} xf(x) dx = \int_{k}^{0} (k-u)f(k-u)(-du)$		
	$= \int_0^k (k-u) f(u) \mathrm{d}u$	1M+1M	1M for reversing the limits 1M for $f(k-u) = f(u)$
	$=k\int_0^k f(u) du - \int_0^k u f(u) du$		
	$=k\int_0^k f(x) dx - \int_0^k x f(x) dx$		
	$\therefore 2\int_0^k xf(x) dx = k \int_0^k f(x) dx$		
	i.e. $\int_0^k x f(x) dx = \frac{k}{2} \int_0^k f(x) dx$	1	
2014 Det	S-MATH-FP(M2)-11	(4)	

機密 (只限閲卷員使用)
CONFIDENTIAL (FOR MARKER'S USE ONLY)

CONFIDENTIAL (FOR MARKER'S US		
Solution	Marks	Remarks
(d) Let $f(x) = \cos^2 x \sin^4 x$		
$f(\pi - x) = \cos^2(\pi - x)\sin^4(\pi - x)$		
$=(-\cos x)^2(\sin x)^4$		
$=\cos^2 x \sin^4 x$) IM	
= f(x)	IM IM	
$f(2\pi - x) = \cos^2(2\pi - x)\sin^4(2\pi - x)$		
$=(\cos x)^2(-\sin x)^4$		
= f(x)	,	
The volume of the solid of revolution		v
$=2\pi\int_{-\infty}^{2\pi}x\cos^2x\sin^4x\mathrm{d}x$	1M	$y = \cos^2 x \sin^4 x$
$=2\pi\int_{\pi}x\cos x\sin xdx$	1.11	
$=2\pi \left(\int_{0}^{2\pi} x \cos^{2} x \sin^{4} x dx - \int_{0}^{\pi} x \cos^{2} x \sin^{4} x dx\right)$	i	
$=2\pi \left(\int_0^1 x\cos^2x\sin^2xdx - \int_0^1 x\cos^2x\sin^2xdx\right)$		O π 2π
$= 2\pi \cdot \frac{2\pi}{2} \int_0^{2\pi} \cos^2 x \sin^4 x dx - 2\pi \cdot \frac{\pi}{2} \int_0^{\pi} \cos^2 x \sin^4 x dx \qquad \text{by (c)}$	1M	
$= 2\pi \cdot \frac{1}{2} \int_0^{2\pi} \cos^2 x \sin^2 x dx - 2\pi \cdot \frac{1}{2} \int_0^{2\pi} \cos^2 x \sin^2 x dx$	1141	
$=2\pi^2 \left(\frac{2\pi}{16}\right) - \pi^2 \left(\frac{\pi}{16}\right)$ by (b)		
$=2\pi \left(\frac{1}{16}\right)^{-\chi} \left(\frac{1}{16}\right)$ by (b)		
$=\frac{3\pi^3}{16}$	1A	
16	200000	24
	(4)	