HKDSE MATH CORE 2022 Past Paper I

1. **HKDSE MATH CORE 2022 Past Paper I Q1**
   Simplify $\frac{(a^3b^{-2})^4}{a^{-5}b^6}$ and express your answer with positive indices.
   (3 marks)

2. **HKDSE MATH CORE 2022 Past Paper I Q2**
   Let x and y be two numbers. The sum of x and y is 456 while the product of 7 and x is y. Find x.
   (3 marks)

3. **HKDSE MATH CORE 2022 Past Paper I Q3**
   Simplify $\frac{3}{k-9} + \frac{2}{5k+6}$.
   (3 marks)

4. **HKDSE MATH CORE 2022 Past Paper I Q4**
   Factorize

   (a) $9c^2 - 6c + 1$,

   (b) $(4c + d)^2 - 9c^2 + 6c - 1$.

   (4 marks)

5. **HKDSE MATH CORE 2022 Past Paper I Q5**
   A fan is sold at a discount of 30% on its marked price. After selling the fan, the profit is \$78 and the percentage profit is 26%. Find the marked price of the fan.
   (4 marks)

6. **HKDSE MATH CORE 2022 Past Paper I Q6**
   Consider the compound inequality

   $$-2(3x + 2) > x + 10 \text{ or } 2x \leq -8 \ldots\ldots\ldots\ldots\ldots (*).$$

   (a) Solve $(*)$.

   (b) Write down the greatest integer satisfying $(*)$.

   (4 marks)

7. **HKDSE MATH CORE 2022 Past Paper I Q7**
   The coordinates of the points S and T are $(12, -5)$ and $(-3, -7)$ respectively. S is rotated anticlockwise about O through $90°$ to S', where O is the origin. T' is the reflection image of T with respect to the x-axis.

   (a) Write down the coordinates of S' and T'.

(b) Find the slope of $S'T'$.

(4 marks)

8. **HKDSE MATH CORE 2022 Past Paper I Q8**

In Figure 1, A is a poinjt lying inside the quadrilateral $BCDE$ such that $AC//ED$ and $AD//BC$. It is given that $\angle ABD = \angle AED$ and $AB = AE$.

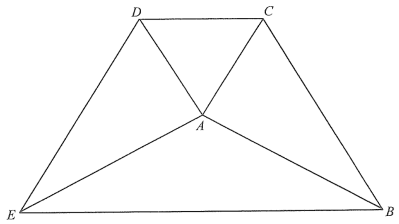

Figure 1

(a) Prove that $\triangle ABC \cong \triangle AED$.

(b) If $\angle ABC = 39°$ and $\angle DAE = 87°$, find $\angle ACD$.

(5 marks)

9. **HKDSE MATH CORE 2022 Past Paper I Q9**

The frequency distribution table and the cumulative frequency distribution table below show the distribution of the times taken to complete a 3 km race by a group of students.

Time taken (minutes)	Frequency
$10 - 14$	a
$15 - 19$	9
$20 - 24$	b
$25 - 29$	3

Time taken less than(minutes)	Cumulative frequency
14.5	3
19.5	x
24.5	y
29.5	20

(a) Write down the value of x.

(b) Find the mean of the distribution.

(c) Find the probability that the time taken to complete the 3 km race by a randomly selected student from the group is less than 19.5 minutes.

(5 marks)

10. **HKDSE MATH CORE 2022 Past Paper I Q10**

It is given that $f(x)$ partly varies as x^2 and partly varies as x. Suppose that $f(4) = 96$ and $f(-5) = 15$.

(a) Find $f(x)$.
(3 marks)

(b) Wrtie down the x-intercept of the graph of $y = 8f(x)$.
(1 marks)

(c) Let k be a real constant. Find the range of values of k such that the equation $f(x) = k$ has two distinct real roots.

(2 marks)

11. **HKDSE MATH CORE 2022 Past Paper I Q11**

The stem-and-leaf diagram below shows the distribution of the ages of the players of a football team. The inter-quartile range and the median of the distribution are 14 and 31 respectively.

Stem (tens)	Leaf (units)
1	7 8 9
2	0 a a 8 8 9
3	b b 5 5 6 6 6 6 7 8
4	3

(a) Find a and b.

(3 marks)

(b) A player now leaves the football team.

(i) Is there any change in the mode of the distribution due to the leaving of the player? Explain your answer.

(ii) If the range of the distribution is decreased, find the greatest possible standard deviation of the distribution.

(4 marks)

12. **HKDSE MATH CORE 2022 Past Paper I Q12**

The equation of the circle C is $x^2 + y^2 - 154x - 128y + 224 = 0$. Denote the centre of C by G. The coordinates of the point H are $(65, 48)$.

(a) Find the distance between G and H.

(3 marks)

(b) Let P be a moving point on C. When the area of $\triangle GHP$ is the greatest,

(i) describe the geometric relationship between GH and GP;

(ii) find the perimeter of $\triangle GHP$.

(4 marks)

13. **HKDSE MATH CORE 2022 Past Paper I Q13**

There are two solid metal spheres. The ration of the surface area of the smaller sphere to the surface area of the larger sphere is 4:9. The radius of the larger sphere 9 cm.

(a) Express, in terms of π, the volume of the smaller sphere.

(3 marks)

(b) The two spheres are melted and recast into two solid right circular cones. Denote these two circular cones by A and B. It is given tha tht eheight and the base radius A are 10 cm and 6 cm respectively. A student finds thatthe base radius of B is 12 cm. The student claims that A and B are similar. Is the claim correct? Explain your answer.
(4 marks)

14. **HKDSE MATH CORE 2022 Past Paper I Q14**
Let $p(x) = 2x^3 + ax^2 + bx - 20$, where a and b are constants. When $p(x)$ is divided by $x^2 - 2x + 3$, the remainder is $x + 13$.

(a) Find a and b.
(3 marks)

(b) Is $x - 5$ a factor of $p(x)$? Explain your answer.
(2 marks)

(c) Someone claims that the equation $p(x) = 0$ has two irrational roots. Do you agree? Explain your answer.
(3 marks)

15. **HKDSE MATH CORE 2022 Past Paper I Q15**
There are 10 boys and 12 girls in a class. If 4 students are randomly selected from the class to form a committee.

(a) find the probability that there are 2 boys and 2 girls in the committee.
(2 marks)

(b) find the probability that the number of boys and the number of girls in the committee are different.
(2 marks)

16. **HKDSE MATH CORE 2022 Past Paper I Q16**
Let $g(x) = 3x^2 + 12kx^2 + 16k^2 + 8$, where k is a non-zero constant.

(a) Using the method of completing the square, express, in terms of k, the coordinates of the vertex of the graph of $y = g(x)$.
(2 marks)

(b) On the same rectangular coordinates system, denote the vertex of the graph of $y = g(x)$ and the vertex of the graph of $y = 2g(-x)$ by A and B respectively. Let M be a point lying on AB such that the area of $\triangle OBM$ is the triple of the area of $\triangle OAM$, where O is the origin. Express, in terms of k, the coordinates of M.
(3 marks)

17. **HKDSE MATH CORE 2022 Past Paper I Q17**
Let c be a real constant. The roots of the equation $x^2 + cx - 9 = 0$ are α and β.

(a) Express $\alpha^2 + \beta^2$ in terms of c.
(3 marks)

(b) The 1st term, the 2nd term and the 3rd term of an arithmetic sequence are c^2, $\alpha^2 + \beta^2$ and 85 respectively. Find the least value of n such that the sum of the first n terms of the sequence is greater than 2×10^6.
(4 marks)

18. **HKDSE MATH CORE 2022 Past Paper I Q18**

In Figure 2, the triangular paper and PQR is held such that PQ lies on the horizontal ground. It is given that $PQ = 30$ cm, $PR = 25$ cm and $\angle QPR = 95°$.

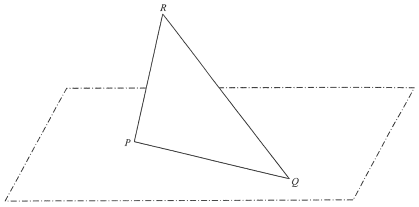

Figure 2

(a) Find

(i) the length of QR,

(ii) $\angle PQR$.

(4 marks)

(b) Let M be the mid-point of QR. A craftman finds that the angle between PR and the horizontal ground is 70°. The craftman claims that the angle between PM and the horizontal ground exceeds 40°. Is the claim correct? Explain your answer.
(3 marks)

19. **HKDSE MATH CORE 2022 Past Paper I Q19**

The centre of the circle C is the point $G(83, 112)$. It is found that the point $A(158, 12)$ lies outside C, AP and AQ are the tangents to C at the points P and Q respectively. It is given that C passes through the point $(23, 67)$.

(a) Find the equation of the straight line passing through A and G.
(2 marks)

(b) Find the coordinates of the point of intersection of AG and PQ.
(3 marks)

(c) Find the equation of the inscribed circle of $\triangle APQ$.
(4 marks)

(d) Someone claims that the ratio of the area of the inscribed circle to the area of the circumcircle of $\triangle APQ$ is $1 : 4$. Do you agree? Explain your answer.
(3 marks)