Binomial Theorem Exercise

1. **Question 1.**

   It is given that

   $$\left(x^2+\frac{1}{x}\right)^5+\left(x^2-\frac{1}{x}\right)^5=2x^{10}+hx^4+\frac{k}{x^2}.$$

   (a) Find the values of h and k.

   (b) Using the result of (a), evaluate

   $$\left(3+\frac{1}{\sqrt{3}}\right)^5+\left(3-\frac{1}{\sqrt{3}}\right)^5.$$

   Ans:

   (a) $h=20, k=10$

   (b) $\dfrac{2008}{3}$

2. **Question 2.**

   It is given that

   $$\left(x+\frac{1}{x^2}\right)^4+\left(x-\frac{1}{x^2}\right)^4=ax^4+\frac{b}{x^2}+\frac{c}{x^8}.$$

   Find the values of a, b and c.

   Ans: $(a,b,c)=(2,12,2)$

3. **Question 3.**

   Determine whether the expansion of $\left(2x+\dfrac{3}{x^2}\right)^7$ consists of

   (a) a constant term,

   (b) an x term.

   Find each term if it exists.

   Ans:

   (a) No

   (b) Yes, $6048x$

4. **Question 4.**

   (a) If k is a positive integerm expand $(1-3x)^k$ in ascending powers of x up to powers of 2.

   (b) It is given that the coefficient of x^2 in the expansion of $(1-3x)^k(1+x+2x^2)$ is 77. Find the value of k.

   Ans:

(a) $1 - 3kx + \frac{9}{2}k(k-1)x^2 + \ldots$

(b) $k = 5$

5. **Question 5.**

   (a) Expand $(1-3x)^4$ and $\left(1+\frac{2}{x}\right)^3$.

   (b) In the expansion of $(1-3x)^4\left(1+\frac{2}{x}\right)^3$, find

   (i) the constant term,

   (ii) the coefficient of x.

   Ans:

   (a) $(1-3x)^4 = 1 - 12x + 54x^2 - 108x^3 + 81x^4$,
   $\left(1+\frac{2}{x}\right)^3 = 1 + \frac{6}{x} + \frac{12}{x^2} + \frac{8}{x^3}$

   (b) (i) -287

   (ii) -336

6. **Question 6.**

   (a) Given that n is a positive integer, expand $\left(ax+\frac{b}{x}\right)^n$ in descending powers of x up to the 5th term, where $a \neq 0$ and $b \neq 0$.

   (b) If the 4th term in the expansion is the constant term, find the value of n.

   Ans:

   (a) $a^n x^n + \binom{n}{1}a^{n-1}bx^{n-2} + \binom{n}{2}a^{n-2}b^2x^{n-4} + \binom{n}{3}a^{n-3}b^3x^{n-6} + \binom{n}{4}a^{n-4}b^4x^{n-8} + \cdots$

   (b) $n = 6$

7. **Question 7.**
   It is given that n is a positive integer where $n > 3$, the coefficients of x^5 and x^6 in the expansion of $(1+3x)^n$ are the same. Find the value of n.
   Ans: $n = 7$

8. **Question 8.**
   It is given that n is a positive integer, the 5th term in the expansion of $\left(2x^2+\frac{1}{2x}\right)^n$ in descending powers of x is the constant. Find the value of n and the 5th term.
   Ans: $n = 6$, 5th term $= \frac{15}{4}$

9. **Question 9.**
   Let T_r be the coefficient of x^r in the expansion of $\left(x^2+\frac{a}{2x}\right)^7$, where $a \neq 0$. If $T_2 = 2T_5$, find the value of a.
   Ans: $a = 4$

10. **Question 10.**

In the expansion of $\left(ax+\frac{2}{x^2}\right)^n$, the 3rd term in descending powers of x is $\frac{20}{27}$, where n is a positive integer and $a<0$. Find the values of n and a.

Ans: $n=6, a=-\frac{1}{3}$

11. **Question 11.**

It is given that the coefficient of x^3 in the expansion of $\left(1+\frac{x}{2n}\right)^n$ is $\frac{1}{100}$, where n is a positive integer. Find the value of n and the coefficient of x^4.

Ans: $n=5$, coefficient of $x^4=\frac{1}{2000}$

12. **Question 12.**

(a) Given that n is a positive integer, expand $(1-kx)^6-(1+x)^n$ in ascending powers of x up to the term in x^2.

(b) If the coefficients of x and x^2 in the expansion are -23 and 125 respectively, find the values of n and k.

Ans:

(a) $-(6k+n)x+\frac{1}{2}(30k^2-n^2+n)x^2+\cdots$

(b) $n=5, k=3$

13. **Question 13.**

It is givne that $\left(2+\frac{x}{10}\right)^n=1024+px+qx^2+\cdots$.

(a) Find the value of n.

(b) Find the values of p and q.

Ans:

(a) $n=10$

(b) $p=512, q=\frac{576}{5}$

14. **Question 14.**

It is given that $(hx-1)^k=-1+10x-10h^2x^2+\cdots$, where k is a positive integer.

(a) Find the values of h and k.

(b) Hence, find the coefficient of x^3 in the expansion.

Ans:

(a) $h=2, k=5$

(b) 80

15. **Question 15.**

It is given that $(hx-2)^k = 64 - 576x + 240h^2x^2 + \cdots$, where k is a positive integer.

(a) Find the values of h and k.

(b) Hence, find the coefficient of x^3 in the expansion.

Ans:

(a) $h = 3, k = 6$

(b) -4320