Paper 1

Solution	Marks	Remarks
1. $\dfrac{(xy)^2}{x^{-5}y^6}$		
$=\dfrac{x^2y^2}{x^{-5}y^6}$	1M	for $(ab)^m = a^m b^m$
$=\dfrac{x^{2-(-5)}}{y^{6-2}}$	1M	for $\dfrac{a^m}{a^n} = a^{m-n}$
$=\dfrac{x^7}{y^4}$	1A	
	----------(3)	
2. $a(b+7)=a+b$		
$ab+7a=a+b$		
$7a-a=b-ab$	1M	for putting b on one side
$6a=b(1-a)$	1M	for factorization
$b=\dfrac{6a}{1-a}$	1A	
$b+7=\dfrac{a+b}{a}$		
$b-\dfrac{b}{a}=1-7$	1M	for putting b on one side
$b(1-\dfrac{1}{a})=-6$	1M	for factorization
$b(\dfrac{a-1}{a})=-6$		
$b=\dfrac{-6a}{a-1}$	1A	
	----------(3)	
3. (a) $3m^2-mn-2n^2$		
$=(3m+2n)(m-n)$	1A	
(b) $3m^2-mn-2n^2-m+n$		
$=(3m+2n)(m-n)-m+n$	1M	for using (a)
$=(3m+2n)(m-n)-(m-n)$		
$=(m-n)(3m+2n-1)$	1A	
	----------(3)	
4. (a) Let $\$x$ be the cost of the handbag.		
$x(1+40\%)=560$	1M	
$x=\dfrac{560}{1+40\%}$		
$x=400$	1A	
Thus, the cost of the handbag is $\$400$.		u–1 for missing unit
(b) The percentage profit		
$=\left(\dfrac{460-400}{400}\right)100\%$	1M	accept without 100%
$=15\%$	1A	
	----------(4)	

Solution	Marks	Remarks
5. Let x be the number of games that the champion wins.		
Then, the number of games drawn will be $36-x$.	1A	
Now, we have $3x+(36-x)=84$.	1M + 1A	1M for $3x+(a-x)$
Solving, we have $x=24$.	1A	
Thus, the required number of games is 24 .		
Let x be the number of games that the champion wins and y be the number of games that the champion draws.		
$\begin{cases} x+y=36 \\ 3x+y=84 \end{cases}$	1A 1M + 1A	 1M for $3x+y$
Solving, we have $x=24$.	1A	
Thus, the required number of games is 24 .		
	----------(4)	
6. (a) $\frac{1}{3}\pi r^2(12)=2\left[\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)\right]$	1M	for $V_1=2V_2$
$r=3$	1A	u−1 for having unit
(b) The required volume		
$=\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)+\frac{1}{3}\pi r^2(12)$	1M	
$=\frac{2}{3}\pi(3^3)+4\pi(3^2)$		
$=54\pi \text{ cm}^3$	1A	u−1 for missing unit
The required volume		
$=\frac{3}{2}\left(\frac{4}{3}\pi r^3\right)$	1M	
$=2\pi(3^3)$		
$=54\pi \text{ cm}^3$	1A	u−1 for missing unit
	----------(4)	
7. Note that $\angle ABD=90°$.	1A	
Also note that $\angle COD=\angle BAD=38°$.	1M	
Further note that $\angle ADB=180°-90°-38°=52°$.		
Since $OC=OD$, we have $\angle ODC=\angle OCD$.		
So, we have $\angle ODC=\frac{180°-38°}{2}$.	1M	
Therefore, we have $\angle ODC=71°$.		
$\angle BDC$ $=\angle ODC-\angle ADB$ $=71°-52°$ $=19°$	1A	u−1 for missing unit
Join O and B .		
Since $OA=OB$, we have $\angle OBA=\angle OAB=38°$.	1A	
So, we have $\angle BOC=\angle OBA=38°$.	1M	
Thus, $\angle BDC=\frac{1}{2}\angle BOC$	1M	
$=\frac{1}{2}(38°)$ $=19°$	1A	u−1 for missing unit
	----------(4)	

Solution	Marks	Remarks
8. (a) The coordinates of A'		
$= (5, 2)$	1A	pp–1 for missing '(' or ')'
The coordinates of A''		
$= (2, 5)$	1A	pp–1 for missing '(' or ')'
(b) The slope of AA'		
$= \dfrac{5-2}{-2-5}$	1M	either one
$= \dfrac{-3}{7}$		
The slope of OA''		
$= \dfrac{5-0}{2-0}$		
$= \dfrac{5}{2}$		
Note that the product of the slope of AA' and the slope of OA'' is not equal to -1 .	1M	
Thus, AA' is not perpendicular to OA'' .	1A	f.t.
	----------(5)	
9. (a) $x(1+20\%) = 72$	1M	
$x = \dfrac{72}{1+20\%}$		
$x = 60$	1A	u–1 for having unit
(b) Let y be the angle subtended at the centre for the sector representing District C .		
y		
$= 360° - 72° - 120° - 30° - 60°$	1M	
$= 78°$	1A	
Since the angle subtended at the centre for the sector representing District C is greater than that for District A , the number of traffic accidents occurred in District A is not greater than that in District C .	1M	
	----------(5)	

Solution	Marks	Remarks
10. (a) The quotient		
$= 5x + 2$	1M + 1A	1M for division process
	----------(2)	
(b) (i) $a = 2$ and $b = -1$	1A	for both correct
(ii) $g(x) = 0$		
$(5x+2)(x^2+2x-3) = 0$	1M	for using (a)
$(5x+2)(x+3)(x-1) = 0$	1A	
Thus, we have $x = \frac{-2}{5}$, $x = -3$ or $x = 1$.	1M	
	----------(4)	
11. (a) Let $C = as + bs^2$, where a and b are non-zero constants.	1A	
$\begin{cases} a(2) + b(2^2) = 356 \\ a(5) + b(5^2) = 1\,250 \end{cases}$	1M	for substitution
Solving, we have $a = 130$ and $b = 24$.	1A	for both correct
The required cost		
$= 130(6) + 24(6^2)$		
$= \$1\,644$	1A	u–1 for missing unit
	----------(4)	
(b) $130s + 24s^2 = 539$		
$24s^2 + 130s - 539 = 0$	1M	
$(4s-11)(6s+49) = 0$		
$s = \frac{11}{4}$ or $s = \frac{-49}{6}$ (rejected)	1A	
Thus, the perimeter of the carpet is 2.75 metres.		u–1 for missing unit
	----------(2)	

Solution	Marks	Remarks
12. (a) Note that the slope of a line segment in the graph represents the average speed of that part of the journey.	1M	can be absorbed
Since the slope of the line segment for Part I is the least, John drives at the lowest speed for Part I of the journey.	1A	
	----------(2)	
(b) Let x h be the time required to drive from B to C.		
$\frac{18-4}{x}=56$	1M	
$x=\frac{1}{4}$		
Thus, John will reach C at 8:26 .	1A	
	----------(2)	
(c) The average speed		
$=\frac{27}{\frac{30}{60}}$	1M	for $\frac{27}{30}$
$=54 \text{ km/h}$		
$=\frac{(54)(1\,000)}{3600}$	1M	
$=15 \text{ m/s}$	1A	
	----------(3)	

Solution	Marks	Remarks
13. (a) Note that the equation of L_1 is $4x-3y+12=0$.		
So, the slope of L_1 is $\frac{4}{3}$.		
Therefore, the slope of L_2 is $\frac{-3}{4}$.	1M	
The equation of L_2 is		
$y-9=\frac{-3}{4}(x-4)$	1M	
$3x+4y-48=0$	1A	
	----------(3)	
(b) (i) Note that Γ is the perpendicular bisector of AB . Also, AB is perpendicular to L_2 .	1M	accept Γ is a straight line
Thus, Γ is parallel to L_2 .	1A	
(ii) The slope of Γ is $\frac{-3}{4}$.	1M	
Putting $x=0$ in $4x-3y+12=0$, we have $y=4$.		
So, we have $B=(0,4)$.	1A	
Note that L_2 cuts the y-axis at $C(0,12)$.		
The mid-point of BC is $(0,8)$.	1M	
The required equation is		
$y=\frac{-3}{4}x+8$	1A	
$3x+4y-32=0$		
By solving $\begin{cases}4x-3y+12=0\\3x+4y-48=0\end{cases}$, we have $A=\left(\frac{96}{25},\frac{228}{25}\right)$.	1M	either one
Putting $x=0$ in $4x-3y+12=0$, we have $y=4$.		
So, we have $B=(0,4)$.	1A	
The required equation is		
$\sqrt{(x-0)^2+(y-4)^2}=\sqrt{\left(x-\frac{96}{25}\right)^2+\left(y-\frac{228}{25}\right)^2}$	1M	
$3x+4y-32=0$	1A	
The slope of Γ is $\frac{-3}{4}$.	1M	either one
By solving $\begin{cases}4x-3y+12=0\\3x+4y-48=0\end{cases}$, we have $A=\left(\frac{96}{25},\frac{228}{25}\right)$.		
Putting $x=0$ in $4x-3y+12=0$, we have $y=4$.		
So, we have $B=(0,4)$.	1A	
The mid-point of AB is $\left(\frac{48}{25},\frac{164}{25}\right)$.	1M	
The required equation is		
$y-\frac{164}{25}=\frac{-3}{4}\left(x-\frac{48}{25}\right)$	1A	
$3x+4y-32=0$		
	----------(6)	

Solution	Marks	Remarks
14. (a) The median		
$= 62\%$	1A	
The mean		
$= \dfrac{55\% + 58\% + 62\% + 62\% + 63\%}{5}$		
$= 60\%$	1A	
	----------(2)	
(b) (i) 58%	1A	
(ii) $a = 63$	1M	1M for either one of the following conditions satisfied:
$b = 57$	1M	(1) $a + b = 120$ (2) $\begin{cases} a \ge 62 \\ 0 \le b < 62 \end{cases}$ or $\begin{cases} 0 \le a < 62 \\ b \ge 62 \end{cases}$
	----------(3)	
(c) Note that the data are collected from only one magazine stall. Also note that the week may not be randomly selected.	1M	accept any suitable reason
Thus, the claim is disagreed.	1M	
Note that the number of magazine stall in city H may be small. Also note that the stall may be randomly selected. Further note that the week may be randomly selected.	1M	accept any suitable reason
Thus, the claim is agreed.	1M	
	----------(2)	

Solution	Marks	Remarks
15. Let n be the number of rows of seats.		
$\frac{n}{2}\left(2(12)+(n-1)3\right)\le 930$	1M	for sum of arithmetic sequence
$\frac{3n^2}{2}+\frac{21n}{2}-930\le 0$	1M	for quadratic inequality in n
$n^2+7n-620\le 0$		
$\frac{-7-\sqrt{2\,529}}{2}\le n\le\frac{-7+\sqrt{2\,529}}{2}$	1A	accept $-28.6\le n\le 21.6$
Therefore, the greatest value of n is 21 . Thus, the greatest number of rows of seats is 21 .	1A	
	----------(4)	
16. (a) The required probability		
$=\frac{C_2^5C_2^4}{C_4^9}$	1M + 1A	1M for numerator using C_2^n , $n=4,5$ 1A for denominator
$=\frac{10}{21}$	1A	r.t. 0.476
The required probability $=6\left(\frac{5}{9}\right)\left(\frac{4}{8}\right)\left(\frac{4}{7}\right)\left(\frac{3}{6}\right)$	1M + 1A	1M for $\left(\frac{r}{n}\right)\left(\frac{r-1}{n-1}\right)\left(\frac{r-1}{n-2}\right)\left(\frac{r-2}{n-3}\right)$ 1A for $6p$, $0<p<1$
$=\frac{10}{21}$	1A	r.t. 0.476
	----------(3)	
(b) The required probability		
$=1-\frac{10}{21}$	1M	for 1 – (a)
$=\frac{11}{21}$	1A	r.t. 0.524
The required probability $=\frac{C_4^5+C_4^4+C_1^5C_3^4+C_3^5C_1^4}{C_4^9}$	1M	for considering 4 cases
$=\frac{11}{21}$	1A	r.t. 0.524
The required probability $=\left(\frac{5}{9}\right)\left(\frac{4}{8}\right)\left(\frac{3}{7}\right)\left(\frac{2}{6}\right)+\left(\frac{4}{9}\right)\left(\frac{3}{8}\right)\left(\frac{2}{7}\right)\left(\frac{1}{6}\right)+4\left(\frac{5}{9}\right)\left(\frac{4}{8}\right)\left(\frac{3}{7}\right)\left(\frac{2}{6}\right)+4\left(\frac{5}{9}\right)\left(\frac{4}{8}\right)\left(\frac{3}{7}\right)\left(\frac{4}{6}\right)$	1M	for considering 10 cases
$=\frac{11}{21}$	1A	r.t. 0.524
	----------(2)	

Solution	Marks	Remarks
17. Note that $6.4 = \log_8 E$.		
Therefore, we have $E = 8^{6.4}$.	1A	
Also note that $M = \log_4 E$.		
$M = \log_4 (8^{6.4})$	1A	
$4^M = 8^{6.4}$		
$(2^2)^M = (2^3)^{6.4}$	1M	
$2^{2M} = 2^{19.2}$		
$2M = 19.2$	1M	
$M = 9.6$	1A	
Thus, the magnitude of the explosion on Scale A is 9.6 .		
M		
$= \log_4 E$		
$= \dfrac{\log_8 E}{\log_8 4}$	1A	
$= \dfrac{6.4}{\log_8 (8^{\frac{2}{3}})}$	1M + 1A	
$= \dfrac{6.4}{\frac{2}{3}}$	1M	for denominator
$= 9.6$	1A	
Thus, the magnitude of the explosion on Scale A is 9.6 .		
	----------(5)	

Solution	Marks	Remarks
8. (a) By sine formula, $\dfrac{BC}{\sin 45^\circ}=\dfrac{20}{\sin 30^\circ}$	1M	accept finding CD and using sine twice
$BC=20\sqrt{2}$ cm	1A	
BD $=20\sqrt{2}\cos 30^\circ$ $=10\sqrt{6}$ cm	1A	
	----------(3)	
(b) (i) Note that the required angle is $\angle ADB$.	1A	
Also note that $AD=20\cos 45^\circ=10\sqrt{2}$ cm . By cosine formula, $\cos\angle ADB=\dfrac{(10\sqrt{6})^2+(10\sqrt{2})^2-18^2}{2(10\sqrt{6})(10\sqrt{2})}$	1M	accept using Pythagoras' theorem twice
$\angle ADB\approx 46.60320866^\circ$ $\angle ADB\approx 46.6^\circ$	1A	
(ii) Note that $AD=CD=10\sqrt{2}$ cm . Also note that $\angle ADC=90^\circ$. The volume of the tetrahedron $ABCD$ $=\dfrac{1}{3}\left(\dfrac{1}{2}(AD)(BD)\sin\angle ADB\right)(CD)$ $=\dfrac{1}{3}\left(\dfrac{1}{2}(10\sqrt{2})(10\sqrt{6})(\sin\angle ADB)\right)\left(10\sqrt{2}\right)$ $=\dfrac{1\,000\sqrt{6}\sin\angle ADB}{3}$ So, the volume of the tetrahedron varies directly as $\sin\angle ADB$.	1M	either one
When $\angle ADB$ increases from 40° to 90° , the volume of the tetrahedron $ABCD$ increases. When $\angle ADB$ increases from 90° to 140° , the volume of the tetrahedron $ABCD$ decreases.	1A	
The volume of the tetrahedron $ABCD$ $=\dfrac{1}{3}(\text{The area of }\Delta ACD)(BD\sin\angle ADB)$ Since the area of ΔACD and the length of BD are constants, the volume of the tetrahedron varies directly as $\sin\angle ADB$.	1M	either one
When $\angle ADB$ increases from 40° to 90° , the volume of the tetrahedron $ABCD$ increases. When $\angle ADB$ increases from 90° to 140° , the volume of the tetrahedron $ABCD$ decreases.	1A	
	----------(5)	

Solution	Marks	Remarks
19. (a) (i) Join C and D.		
$\angle CDB = \angle CAB$ ($\angle$s in the same segment)		
$\angle QCD = \angle CAD$ ($\angle$ in alt segment)		
$\angle QCD = \angle CDB$ (alt. $\angle$s, PQ // BD)		
So, $\angle CAB = \angle CAD$		
$AE = AE$ (common side)		
$AB = AD$ (given)		
$\Delta ABE \cong \Delta ABD$ (SAS)		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	3	
Case 2 Any correct proof without reasons.	2	
Case 3 Incomplete proof with any one correct step and one correct reason.	1	
(ii) By (a)(i), $\angle CAB = \angle CAD$. Therefore, AC is the angle bisector of $\angle BAD$. So, AC must pass through the in-centre of ΔABD. By (a)(i), $\angle AED = \angle AEB$. Note that $\angle AED + \angle AEB = 180°$. Therefore, $\angle AED = \angle AEB = 90°$. So, AE is an altitude of ΔABD. Hence, the orthocentre of ΔABD lies on the straight line passing through A and E. Also, $BE = DE$. Therefore, AE is a median of ΔABD. So, AC passes through the centroid of ΔABD. Also note that AC is the perpendicular bisector of BD. So, AC passes through the circumcentre of ΔABD.	1A + 1A	1A for any one correct 1A for any two correct
Therefore, the in-centre, the orthocentre, the centroid and the circumcentre lie on the straight line passing through A and C. Thus, the in-centre, the orthocentre, the centroid and the circumcentre are collinear.	1A ----------(6)	f.t.
(b) The x-coordinate of the centre of the circle is $\dfrac{14+4}{2} = 9$.	1A	
Let $(9, t)$ be the coordinates of the centre of the circle.		
$\sqrt{(9-4)^2 + (t-4)^2} = \sqrt{(9-8)^2 + (t-12)^2}$	1M	
$t = \dfrac{13}{2}$		
The slope of the tangent PQ = the slope of BD $= \dfrac{12-4}{8-4}$ $= 2$	1M	for slope formula
Let (a, b) be the coordinates of C.		
Note that AC passes through the centre of the circle.	1M	can be absorbed
Then, we have $\dfrac{a+14}{2} = 9$ and $\dfrac{b+4}{2} = \dfrac{13}{2}$.	1M	for mid-point formula
So, we have $a = 4$ and $b = 9$.		
The required equation is		
$y - 9 = 2(x-4)$	1M	
$2x - y + 1 = 0$	1A	

Solution	Marks	Remarks
The x-coordinate of the centre of the circle is $\frac{14+4}{2}=9$.	1A	
Let $(9,t)$ be the coordinates of the centre of the circle.		
$\sqrt{(9-4)^2+(t-4)^2}=\sqrt{(9-8)^2+(t-12)^2}$	1M	
$t=\frac{13}{2}$		
So, the equation of the circle is $x^2+y^2-18x-13y+92=0$.	1M	
The slope of the tangent PQ		
= the slope of BD		
$=\frac{12-4}{8-4}$	1M	for slope formula
$=2$		
Let the equation of the tangent be $y=2x+k$, where k is a constant.		
Putting $y=2x+k$ in $x^2+y^2-18x-13y+92=0$, we have		
$x^2+(2x+k)^2-18x-13(2x+k)+92=0$.	1M	
Therefore, we have $5x^2-(4k-44)x+(k^2-13k+92)=0$.		
For tangency, we have $\Delta=0$.		
$(4k-44)^2-4(5)(k^2-13k+92)=0$	1M	
$k^2+23k-24=0$		
$k=1$ or $k=-24$ (rejected)		
Thus, the required equation is $2x-y+1=0$.	1A	
Let the equation of the circle be		
$x^2+y^2+k_1x+k_2y+k_3=0$, where k_1 , k_2 and k_3 are constants.		
$\begin{cases}14^2+4^2+k_1(14)+k_2(4)+k_3=0\\8^2+12^2+k_1(8)+k_2(12)+k_3=0\\4^2+4^2+k_1(4)+k_2(4)+k_3=0\end{cases}$	1A	
Solving, we have $k_1=-18$, $k_2=-13$ and $k_3=92$.	1M	for solving
So, the equation of the circle is $x^2+y^2-18x-13y+92=0$.	1M	
The slope of the tangent PQ		
= the slope of BD		
$=\frac{12-4}{8-4}$	1M	for slope formula
$=2$		
Let the equation of the tangent be $y=2x+k$, where k is a constant.		
Putting $y=2x+k$ in $x^2+y^2-18x-13y+92=0$, we have		
$x^2+(2x+k)^2-18x-13(2x+k)+92=0$.	1M	
Therefore, we have $5x^2-(4k-44)x+(k^2-13k+92)=0$.		
For tangency, we have $\Delta=0$.		
$(4k-44)^2-4(5)(k^2-13k+92)=0$	1M	
$k^2+23k-24=0$		
$k=1$ or $k=-24$ (rejected)		
Thus, the required equation is $2x-y+1=0$.	1A	
	----------(7)	

Paper 2

Question No.	Key	Question No.	Key
1.	C	31.	B
2.	C	32.	D
3.	C	33.	A
4.	A	34.	A
5.	C	35.	D
6.	D	36.	C
7.	D	37.	C
8.	A	38.	A
9.	C	39.	A
10.	A	40.	C
11.	C	41.	B
12.	B	42.	B
13.	D	43.	D
14.	C	44.	B
15.	B	45.	D
16.	B		
17.	A		
18.	B		
19.	A		
20.	A		
21.	B		
22.	C		
23.	B		
24.	D		
25.	A		
26.	D		
27.	D		
28.	B		
29.	D		
30.	B		