HKDSE
MATH EP
M2

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HONG KONG DIPLOMA OF
SECONDARY EDUCATION EXAMINATION

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)
MOCK EXAM 4
Question-Answer Book

Time allowed: 2½ hours
This paper must be answered in English

Please stick the barcode label here.

Candidate Number									

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
2. This paper consists of **TWO** sections, A and B.
3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
8. The diagrams in this paper are not necessarily drawn to scale.
9. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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SECTION A (50 marks)

2015

1. Find $\frac{d}{dx}(x^3 + 2x)$ from first principles. (4 marks)

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2. Let $e^x + \ln y + 7xy = 1$. Find $\dfrac{dy}{dx}$ when $x = 0$. (4 marks)

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3. The slope at any point (x, y) of a curve is given by $\frac{dy}{dx} = \frac{-4x}{(x^2+1)^2}$. If the straight line $y = 6$ is a tangent to the curve, find the equation of the curve.

(4 marks)

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4. If n is a positive integer and the coefficient of x^2 in the expansion of

$$(1 - x^2)^n + (1 + 2x)^n$$

is 104, find the value(s) of n.

(4 marks)

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5. Consider the curve C: $y = x^3 - 4x^2 + 8x - 7$. Find the equations of the two tangents to the curve C which are parallel to the line L: $6x - 2y + 7 = 0$.

(4 marks)

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6. (a) Show that $\sin^2 x \cos^2 x = \dfrac{1-\cos 4x}{8}$.

(b) Hence find $\displaystyle\int (\sin^4 x + \cos^4 x)\,dx$.

(5 marks)

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7. Solve the system of linear equations $\begin{cases} x - 4y + 3z = 9 \\ 5x + y - 6z = 3 \\ 3x + 2y - 5z = -1 \end{cases}$. (5 marks)

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2013

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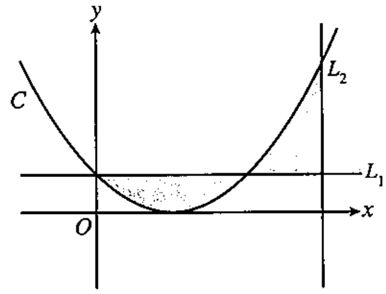

Figure 1

Figure 1 shows the shaded region with boundaries C: $y = (x - 1)^2$, L_1: $y = 1$ and L_2: $x = 3$. It is given that C intersects L_1 at $(0, 1)$ and $(2, 1)$.

(a) Find the area of the shaded region.

(b) Find the volume of solid of revolution when the shaded region is revolved about L_1.

(6 marks)

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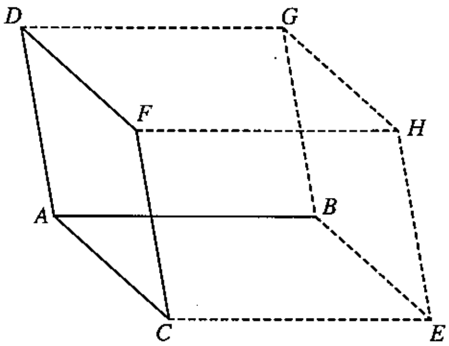

Figure 2

Let $\overrightarrow{AB} = \mathbf{i} + 3\mathbf{j}$, $\overrightarrow{AC} = 3\mathbf{i} - 4\mathbf{k}$ and $\overrightarrow{AD} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$. Figure 2 shows the parallelepiped $ABECFDGH$ formed by $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$.

(a) Find the area of the parallelogram $ABEC$.

(b) Find the volume of the tetrahedron $ABCD$.

(c) If D' is a point different from D such that the volume of the tetrahedron formed by $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD'}$ is the same as that of $ABCD$, find a possible vector $\overrightarrow{AD'}$.

(6 marks)

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10. (a) Let $M = \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$. Prove, by mathematical induction, that

$$M^n = \begin{pmatrix} \cos nx & -\sin nx \\ \sin nx & \cos nx \end{pmatrix}$$

for all positive integers n.

(b) Let $A = \begin{pmatrix} -\sqrt{3} & 1 \\ -1 & -\sqrt{3} \end{pmatrix}$.

(i) Suppose that $A = r\begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$, where $r > 0$ and $0° < x < 360°$. Find r and x.

(ii) A matrix of the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ is called a 2×2 diagonal matrix. Find the least positive integer n such that A^n is a 2×2 diagonal matrix, and evaluate A^n for this value of n.

(8 marks)

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SECTION B (50 marks)

11. Let $M = \begin{pmatrix} 3+x & -y \\ -x & 3+y \end{pmatrix}$ and $P = \begin{pmatrix} y & 1 \\ x & -1 \end{pmatrix}$, where x and y are real numbers such that $xy > 0$.

    (a) Show that P is an invertible matrix. (2 marks)

    (b) Evaluate $P^{-1}MP$.
    Hence find $P^{-1}M^nP$ in terms of x, y and n, where n is a positive integer.
    (5 marks)

    (c) Suppose that $A = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$.
    Using the result of (b), find A^{2n} in terms of n, where n is a positive integer.
    (5 marks)

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12. Let C_1 be the curve $y = \dfrac{px+q}{2x+r}$, where p, q and r are constants with $p \neq 0$ and $x \neq -\dfrac{r}{2}$. It is given that C_1 passes through $(0, 0)$ and it has a vertical asymptote $x = -\dfrac{3}{2}$ and a horizontal asymptote $y + 1 = 0$. Let C_2 be the curve $y = \dfrac{2x+r}{px+q}$, where $x \neq -\dfrac{q}{p}$.

(a) Find the equation of C_1. (4 marks)

(b) Find the coordinates of the point(s) of intersection of C_1 and C_2. (2 marks)

(c) Show that $\dfrac{d}{dx}\left(\dfrac{px+q}{2x+r}\right) < 0$ for $x \neq -\dfrac{r}{2}$ and $\dfrac{d}{dx}\left(\dfrac{2x+r}{px+q}\right) > 0$ for $x \neq -\dfrac{q}{p}$. (3 marks)

(d) Write down all the asymptotes of C_2. (2 marks)

(e) Sketch the curves C_1 and C_2 on the same diagram, indicating their asymptotes, intercepts and their point(s) of intersection. (3 marks)

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13. (a) Using the substitution $\tan x = 2\tan\theta$, find $\displaystyle\int \frac{dx}{1+3\cos^2 x}$. (5 marks)

(b) Let $f(x)$ be a continuous function such that $f(2\pi - x) = -f(x)$ for all real numbers x. Using the substitution $y = 2\pi - x$, prove that

$$\int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} f(x)\ln(1+e^{\sin x})dx = \frac{1}{2}\int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} f(x)\sin x dx .$$

(3 marks)

(c) Hence evaluate $\displaystyle\int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} \frac{\sin x \ln(1+e^{\sin x})}{1+3\cos^2 x}dx$. (Give the answer correct to 3 significant figures.)

(4 marks)

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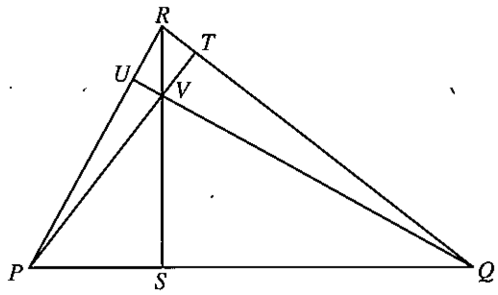

Figure 3

In Figure 3, S is a point on PQ such that RS is an altitude of ΔPQR. V divides RS in the ratio $1:2$. PV and QV are produced to meet QR and PR at T and U respectively. Suppose $QT:TR = r:1$, where r is a constant.

Let $\mathbf{a}$, $k\mathbf{a}$ $(k > 1)$ and $\mathbf{b}$ be $\overrightarrow{PS}$, $\overrightarrow{PQ}$ and $\overrightarrow{SV}$ respectively.

(a) Find $\overrightarrow{PV}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (1 mark)

(b) Express $\overrightarrow{PT}$ in terms of r, k, $\mathbf{a}$ and $\mathbf{b}$.
Hence find r in terms of k.

(4 marks)

(c) It is given that $|\mathbf{a}| = |\mathbf{b}|$ and V is the orthocentre of ΔPQR.

(i) Find $\overrightarrow{PT}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Find $\dfrac{RU}{UP}$.

(7 marks)

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