Solution	Marks	Remarks
1. $\mathrm{f}(1+h)-\mathrm{f}(1)$		
$=\dfrac{10(1+h)}{7+3(1+h)^2}-\dfrac{10}{7+3}$		
$=\dfrac{10+10h}{10+6h+3h^2}-1$		
$=\dfrac{10+10h-10-6h-3h^2}{10+6h+3h^2}$		
$=\dfrac{4h-3h^2}{10+6h+3h^2}$	1	
$\mathrm{f}'(1)$		
$=\lim\limits_{h\to 0}\dfrac{\mathrm{f}(1+h)-\mathrm{f}(1)}{h}$	1M	
$=\lim\limits_{h\to 0}\dfrac{1}{h}\left(\dfrac{4h-3h^2}{10+6h+3h^2}\right)$		
$=\lim\limits_{h\to 0}\dfrac{4-3h}{10+6h+3h^2}$	1M	withhold 1M if the step is skipped
$=\dfrac{4-3(0)}{10+6(0)+3(0)^2}$		
$=\dfrac{2}{5}$	1A	0.4
	----------(4)	
2. (a) $\mathrm{P}(x)$		
$=(x+\lambda)(x+\lambda)^2(x+\lambda)^3-(3)(5)(x+\lambda)+(1)(3)(4)-(2)(4)(x+\lambda)^2$	1M	
$=(x+\lambda)^6-8(x+\lambda)^2-15(x+\lambda)+12$		
Note that the coefficient of x^3 in the expansion of $\mathrm{P}(x)$ is 160 .		
So, we have $C_3^6\lambda^3=160$.	1M	for $C_3^6\lambda^3$
Thus, we have $\lambda=2$.	1A	
(b) Note that $\mathrm{P}'(0)$ is the coefficient of x in the expansion of $\mathrm{P}(x)$.	1M	can be absorbed
Also note that the coefficient of x is $6\lambda^5-16\lambda-15$.		
By (a), we have $\mathrm{P}'(0)=145$.	1A	
	----------(5)	

Solution	Marks	Remarks
3. (a) V		
$= \int -2t \, \mathrm{d}t$	1M	
$= -t^2 + C$, where C is a constant		
Since $V = 580$ when $t = 0$, we have $C = 580$.	1M	
So, we have $V = 580 - t^2$.		
When $t = 24$, we have $V = 4 > 0$.		
Thus, the claim is correct.	1A	f.t.
(b) Let p cm be the depth of liquid X in the vessel when $t = 18$.		
Since $V\big\vert_{t=18} = 580 - 18^2 = 256$, we have $p^2 + 24p = 256$.	1M	
So, we have $p^2 + 24p - 256 = 0$.		
Solving, we have $p = 8$ or $p = -32$ (rejected) .		
Note that $\frac{\mathrm{d}V}{\mathrm{d}t} = (2h + 24)\frac{\mathrm{d}h}{\mathrm{d}t}$.	1M	
Since $\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right\vert_{t=18} = -36$, we have $-36 = (2(8) + 24)\left.\frac{\mathrm{d}h}{\mathrm{d}t}\right\vert_{t=18}$.		
Thus, we have $\left.\frac{\mathrm{d}h}{\mathrm{d}t}\right\vert_{t=18} = \frac{-9}{10}$.	1A	-0.9
	----------(6)	
4. (a) $g'(x)$		
$= \dfrac{\sqrt{x}\left(\frac{1}{x}\right) - (\ln x)\left(\frac{1}{2\sqrt{x}}\right)}{x}$	1M	for quotient rule
$= \dfrac{2 - \ln x}{2\sqrt{x^3}}$		
So, we have $g'(x) = 0 \Leftrightarrow x = e^2$.		
(table below)	1M	
Therefore, G attains its maximum value only at $x = e^2$.		
Thus, G has only one maximum point.	1	
(b) Note that $g(x) < 0$ for all $x \in (0, 1)$ and $g(x) > 0$ for all $x \in (1, 99)$.		
So, we have $g(x) = 0 \Leftrightarrow x = 1$.		
The required volume		
$= \int_1^{e^2} \pi \left(\frac{\ln x}{\sqrt{x}}\right)^2 \mathrm{d}x$	1M	
$= \pi \int_0^2 u^2 \, \mathrm{d}u$ (by letting $u = \ln x$)	1M	
$= \pi \left[\frac{u^3}{3}\right]_0^2$		
$= \frac{8\pi}{3}$	1A	
	----------(6)	

x	$(0, e^2)$	e^2	$(e^2, 99)$
$g'(x)$	$+$	0	$-$

Solution	Marks	Remarks
5. (a) Note that $\frac{1}{(1)(2)}+\frac{1}{(2)(3)}=\frac{2}{3}=\frac{1+1}{(1)(2+1)}$.		
So, the statement is true for $n=1$.	1	
Assume that $\sum_{k=m}^{2m}\frac{1}{k(k+1)}=\frac{m+1}{m(2m+1)}$, where m is a positive integer.	1M	
$\sum_{k=m+1}^{2m+2}\frac{1}{k(k+1)}$		
$=\sum_{k=m}^{2m}\frac{1}{k(k+1)}-\frac{1}{m(m+1)}+\frac{1}{(2m+1)(2m+2)}+\frac{1}{(2m+2)(2m+3)}$	1M	can be absorbed
$=\frac{m+1}{m(2m+1)}-\frac{1}{m(m+1)}+\frac{1}{(2m+1)(2m+2)}+\frac{1}{(2m+2)(2m+3)}$	1M	for using induction assumption
$=\frac{(m+1)^2-(2m+1)}{m(m+1)(2m+1)}+\frac{(2m+3)+(2m+1)}{(2m+1)(2m+2)(2m+3)}$		
$=\frac{m}{(m+1)(2m+1)}+\frac{2}{(2m+1)(2m+3)}$		
$=\frac{(2m+1)(m+2)}{(m+1)(2m+1)(2m+3)}$		
$=\frac{m+2}{(m+1)(2m+3)}$		
So, the statement is true for $n=m+1$ if it is true for $n=m$.		
By mathematical induction, the statement is true for all positive integers n .	1	
(b) Putting $n=50$ in (a), we have $\sum_{k=50}^{100}\frac{1}{k(k+1)}=\frac{51}{(50)(101)}=\frac{51}{5\,050}$.	1M	either one
Putting $n=100$ in (a), we have $\sum_{k=100}^{200}\frac{1}{k(k+1)}=\frac{101}{(100)(201)}=\frac{101}{20\,100}$.		
So, we have $\sum_{k=50}^{200}\frac{1}{k(k+1)}=\frac{51}{5\,050}+\frac{101}{20\,100}-\frac{1}{(100)(101)}$.		
Thus, we have $\sum_{k=50}^{200}\frac{1}{k(k+1)}=\frac{151}{10\,050}$.	1A	
	----------(7)	

Solution	Marks	Remarks
6. (a) (i) Note that $\begin{vmatrix} 1 & -2 & -2 \\ 5 & \alpha & \alpha \\ 7 & \alpha-3 & 2\alpha+1 \end{vmatrix}$		
$= (\alpha)(2\alpha+1)+(-2)(\alpha)(7)+(-2)(5)(\alpha-3)-(\alpha)(\alpha-3)-(-2)(5)(2\alpha+1)-(-2)(\alpha)(7)$	1A	
$= (\alpha+4)(\alpha+10)$		
Since (E) has a unique solution, we have $\begin{vmatrix} 1 & -2 & -2 \\ 5 & \alpha & \alpha \\ 7 & \alpha-3 & 2\alpha+1 \end{vmatrix} \neq 0$.	1M	
Hence, we have $(\alpha+4)(\alpha+10) \neq 0$.		
So, we have $\alpha \neq -4$ and $\alpha \neq -10$.	1A	
Thus, we have $\alpha < -10$, $-10 < \alpha < -4$ or $\alpha > -4$.		
The augmented matrix of (E) is		
$\left(\begin{array}{ccc\|c} 1 & -2 & -2 & \beta \\ 5 & \alpha & \alpha & 5\beta \\ 7 & \alpha-3 & 2\alpha+1 & 8\beta \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -2 & -2 & \beta \\ 0 & \alpha+10 & \alpha+10 & 0 \\ 0 & \alpha+11 & 2\alpha+15 & \beta \end{array}\right)$	1M	
$\sim \left(\begin{array}{ccc\|c} 1 & -2 & -2 & \beta \\ 0 & 1 & \alpha+5 & \beta \\ 0 & 0 & (\alpha+4)(\alpha+10) & (\alpha+10)\beta \end{array}\right)$	1A	
Since (E) has a unique solution, we have $(\alpha+4)(\alpha+10) \neq 0$.		
So, we have $\alpha \neq -4$ and $\alpha \neq -10$.	1A	
Thus, we have $\alpha < -10$, $-10 < \alpha < -4$ or $\alpha > -4$.		
(ii) Since (E) has a unique solution, we have		
$y = \dfrac{\begin{vmatrix} 1 & \beta & -2 \\ 5 & 5\beta & \alpha \\ 7 & 8\beta & 2\alpha+1 \end{vmatrix}}{(\alpha+4)(\alpha+10)}$	1M	for Cramer's Rule
$= \dfrac{-\beta}{\alpha+4}$	1A	
(b) When $\alpha = -4$, the augmented matrix of (E) is		
$\left(\begin{array}{ccc\|c} 1 & -2 & -2 & \beta \\ 5 & -4 & -4 & 5\beta \\ 7 & -7 & -7 & 8\beta \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -2 & -2 & \beta \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & \beta \end{array}\right)$	1M	
Since (E) is inconsistent, we have $\beta \neq 0$.	1A	
Thus, we have $\beta < 0$ or $\beta > 0$.		
	----------(7)	

Solution	Marks	Remarks
7. (a) $\int e^x \sin \pi x \, \mathrm{d}x$		
$= \int \sin \pi x \, \mathrm{d}e^x$		
$= e^x \sin \pi x - \pi \int e^x \cos \pi x \, \mathrm{d}x$	1M	
$= e^x \sin \pi x - \pi \int \cos \pi x \, \mathrm{d}e^x$		
$= e^x \sin \pi x - \pi \left(e^x \cos \pi x - \pi \int -e^x \sin \pi x \, \mathrm{d}x \right)$	1M	
$\int e^x \sin \pi x \, \mathrm{d}x = e^x \sin \pi x - \pi e^x \cos \pi x - \pi^2 \int e^x \sin \pi x \, \mathrm{d}x$		
$\pi^2 \int e^x \sin \pi x \, \mathrm{d}x + \int e^x \sin \pi x \, \mathrm{d}x = e^x \sin \pi x - \pi e^x \cos \pi x + \text{constant}$	1M	
$(\pi^2+1) \int e^x \sin \pi x \, \mathrm{d}x = e^x (\sin \pi x - \pi \cos \pi x) + \text{constant}$		
$\int e^x \sin \pi x \, \mathrm{d}x = \frac{1}{\pi^2+1} (e^x (\sin \pi x - \pi \cos \pi x)) + \text{constant}$	1A	
(b) $\int_0^3 e^{3-x} \sin \pi x \, \mathrm{d}x$		
$= \int_3^0 -e^u \sin \pi (3-u) \, \mathrm{d}u$ (by letting $u = 3 - x$)	1M	
$= \int_0^3 e^u \sin \pi u \, \mathrm{d}u$		
$= \int_0^3 e^x \sin \pi x \, \mathrm{d}x$		
$= \frac{1}{\pi^2+1} \left[e^x (\sin \pi x - \pi \cos \pi x) \right]_0^3$ (by (a))	1M	for using the result of (a)
$= \frac{\pi(e^3+1)}{\pi^2+1}$	1A	
	----------(7)	

Solution	Marks	Remarks
8. (a) For all $x>0$,		
$\mathrm{h}'(x)$		
$=\frac{2}{x}\left(\left(x^2-\frac{7}{2}x+\left(\frac{7}{4}\right)^2-\left(\frac{7}{4}\right)^2\right)+4\right)$	1M	
$=\frac{2}{x}\left(\left(x-\frac{7}{4}\right)^2+\frac{15}{16}\right)$		
>0		
Thus, $\mathrm{h}(x)$ is an increasing function.	1A	f.t.
(b) (i) y		
$=\int\left(2x-7+\frac{8}{x}\right)\mathrm{d}x$	1M	
$=x^2-7x+8\ln x+C$, where C is a constant		
Note that $y=3$ when $x=1$.		
So, we have $1^2-7(1)+8\ln 1+C=3$.	1M	
Therefore, we have $C=9$.		
Thus, the equation of H is $y=x^2-7x+8\ln x+9$.	1A	
(ii) $\mathrm{h}''(x)$		
$=\frac{x(4x-7)-(2x^2-7x+8)}{x^2}$	1M	
$=\frac{2x^2-8}{x^2}$		
$=\frac{2(x-2)(x+2)}{x^2}$		
Since $x>0$, we have $\mathrm{h}''(x)=0 \Leftrightarrow x=2$.		
(table below)	1M	
Thus, the point of inflexion of H is $(2, 8\ln 2-1)$.	1A	
	----------(8)	

x	$(0,2)$	2	$(2,\infty)$
$\mathrm{h}''(x)$	$-$	0	$+$

Solution	Marks	Remarks
9. (a) $\frac{dy}{dx}$		
$=\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)(12-x^2)^{\frac{-1}{2}}(-2x)$	1M	for chain rule
$=\frac{-x}{3\sqrt{12-x^2}}$		
So, we have $\left.\frac{dy}{dx}\right\|_{x=3}=\frac{-1}{\sqrt{3}}$.		
The equation of L is		
$y-\frac{\sqrt{3}}{3}=\frac{-1}{\sqrt{3}}(x-3)$	1M	
$x+\sqrt{3}y-4=0$	1A	
	----------(3)	
(b) (i) Putting $y=\frac{1}{\sqrt{3}}(4-x)$ in $y=\sqrt{4-x^2}$, we have	1M	
$\frac{1}{\sqrt{3}}(4-x)=\sqrt{4-x^2}$		
$x^2-2x+1=0$		
So, we have $x=1$ and $y=\sqrt{3}$.		
Thus, the point of contact of L and C is $(1,\sqrt{3})$.	1A	
(ii) When $\sqrt{4-x^2}=\frac{1}{3}\sqrt{12-x^2}$, we have $36-9x^2=12-x^2$.	1M	
So, we have $8x^2-24=0$.		
Since $0<x<2$, we have $x=\sqrt{3}$.		
Thus, the point of intersection of C and Γ is $(\sqrt{3},1)$.	1A	
(iii) The required area		
$=\int_1^{\sqrt{3}}\left(\frac{1}{\sqrt{3}}(4-x)-\sqrt{4-x^2}\right)dx+\int_{\sqrt{3}}^{3}\left(\frac{1}{\sqrt{3}}(4-x)-\frac{1}{3}\sqrt{12-x^2}\right)dx$	1M+1A	1M for either integral
$=\int_1^3\frac{1}{\sqrt{3}}(4-x)\,dx-\int_1^{\sqrt{3}}\sqrt{4-u^2}\,du-\int_{\sqrt{3}}^{3}\frac{1}{3}\sqrt{12-v^2}\,dv$		
$=\frac{4\sqrt{3}}{3}-\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}4\cos^2\alpha\,d\alpha-\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{12\cos^2\beta}{3}d\beta$ (by letting $u=2\sin\alpha$ and $v=\sqrt{12}\sin\beta$)	1M	for either substitution
$=\frac{4\sqrt{3}}{3}-8\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\cos 2\theta+1}{2}d\theta$	1M	
$=\frac{4\sqrt{3}}{3}-8\left[\frac{\sin 2\theta}{4}+\frac{\theta}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$		
$=\frac{4\sqrt{3}}{3}-\frac{2\pi}{3}$	1A	
	----------(9)	

Solution	Marks	Remarks
10. (a) $\dfrac{1}{2+\cos 2x}$		
$=\dfrac{1}{2+2\cos^2 x-1}$		
$=\dfrac{1}{2\cos^2 x+1}$		
$=\dfrac{\sec^2 x}{2+\sec^2 x}$	1	
	----------(1)	
(b) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{2+\cos 2x}\,dx$		
$\displaystyle=\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{2+\sec^2 x}\,dx$ (by (a))	1M	for using (a)
$\displaystyle=\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{3+\tan^2 x}\,dx$		
$\displaystyle=\int_0^{1} \frac{1}{3+t^2}\,dt$ (by letting $t=\tan x$)	1M	
$\displaystyle=\left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right]_0^1$		
$=\dfrac{\sqrt{3}\pi}{18}$	1A	
	----------(3)	
(c) By letting $u=-x$, we have		
$\displaystyle\int_{-a}^{0} f(x)\ln(1+e^x)\,dx=\int_a^0 -f(-u)\ln(1+e^{-u})\,du$.	1M	
So, we have $\displaystyle\int_{-a}^{0} f(x)\ln(1+e^x)\,dx=\int_0^a f(-x)\ln(1+e^{-x})\,dx$.		
$\displaystyle\int_{-a}^{a} f(x)\ln(1+e^x)\,dx$		
$\displaystyle=\int_{-a}^{0} f(x)\ln(1+e^x)\,dx+\int_0^a f(x)\ln(1+e^x)\,dx$	1M	
$\displaystyle=\int_0^a f(-x)\ln(1+e^{-x})\,dx+\int_0^a f(x)\ln(1+e^x)\,dx$		
$\displaystyle=-\int_0^a f(x)\ln(1+e^{-x})\,dx+\int_0^a f(x)\ln(1+e^x)\,dx$	1M	
$\displaystyle=-\int_0^a f(x)\ln\left(\frac{e^x+1}{e^x}\right)dx+\int_0^a f(x)\ln(1+e^x)\,dx$		
$\displaystyle=-\int_0^a f(x)\ln(e^x+1)\,dx+\int_0^a x\,f(x)\,dx+\int_0^a f(x)\ln(1+e^x)\,dx$		
$\displaystyle=\int_0^a x\,f(x)\,dx$	1	
	----------(4)	

Solution	Marks	Remarks
(d) Note that $\dfrac{\sin 2(-x)}{(2+\cos 2(-x))^2}=\dfrac{-\sin 2x}{(2+\cos 2x)^2}$ for all $x\in\mathbf{R}$.	1M	withhold 1M if checking is omitted
By (c), we have $\displaystyle\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\frac{\sin 2x}{(2+\cos 2x)^2}\ln(1+e^x)\,\mathrm{d}x=\int_0^{\frac{\pi}{4}}\frac{x\sin 2x}{(2+\cos 2x)^2}\,\mathrm{d}x$.	1M	for using (c)
Also note that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{2(2+\cos 2x)}\right)=\dfrac{\sin 2x}{(2+\cos 2x)^2}$.		
$\displaystyle\int_0^{\frac{\pi}{4}}\frac{x\sin 2x}{(2+\cos 2x)^2}\,\mathrm{d}x$		
$\displaystyle=\left[\frac{x}{2(2+\cos 2x)}\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}}\frac{1}{2(2+\cos 2x)}\,\mathrm{d}x$	1M	
$=\dfrac{1}{2}\left(\dfrac{\pi}{4}\right)\left(\dfrac{1}{2+0}\right)-\dfrac{1}{2}\left(\dfrac{\sqrt{3}\pi}{18}\right)$ (by (b))	1M	for using the result of (b)
$=\dfrac{(9-4\sqrt{3})\pi}{144}$		
Thus, we have $\displaystyle\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\frac{\sin 2x}{(2+\cos 2x)^2}\ln(1+e^x)\,\mathrm{d}x=\frac{(9-4\sqrt{3})\pi}{144}$.	1A	
	----------(5)	

Solution	Marks	Remarks
11. (a) M^2		
$=\begin{pmatrix}2 & 7\\ -1 & -6\end{pmatrix}\begin{pmatrix}2 & 7\\ -1 & -6\end{pmatrix}$		
$=\begin{pmatrix}-3 & -28\\ 4 & 29\end{pmatrix}$	1M	
$M^2=aM+bI$		
$\begin{pmatrix}-3 & -28\\ 4 & 29\end{pmatrix}=\begin{pmatrix}2a & 7a\\ -a & -6a\end{pmatrix}+\begin{pmatrix}b & 0\\ 0 & b\end{pmatrix}$		
$\begin{cases}-3=2a+b\\ -28=7a\\ 4=-a\\ 29=-6a+b\end{cases}$	1M	
Thus, we have $a=-4$ and $b=5$.	1A	for both correct
	----------(3)	
(b) Note that $(1-(-5))M+(5+(-5))I=6M$.	1M	
So, the statement is true for $n=1$.		
Assume that $6M^k=(1-(-5)^k)M+(5+(-5)^k)I$, where k is a positive integer.		
$6M^{k+1}$		
$=M(6M^k)$		
$=M((1-(-5)^k)M+(5+(-5)^k)I)$	1M	
$=(1-(-5)^k)M^2+(5+(-5)^k)M$		
$=(1-(-5)^k)((1+(-5))M+5I)+(5+(-5)^k)M$	1M	for using the result of (a)
$=(1+(-5)-(-5)^k-(-5)^{k+1})M+(5+(-5)^{k+1})I+(5+(-5)^k)M$		
$=(1-(-5)^{k+1})M+(5+(-5)^{k+1})I$		
So, the statement is true for $n=k+1$ if it is true for $n=k$.		
By mathematical induction, we have $6M^n=(1-(-5)^n)M+(5+(-5)^n)I$.	1	
$6M$		
$=(1-(-5))M+(5+(-5))I$	1M	
$6M^2$		
$=(1-(-5))M^2$		
$=(1-(-5))((1+(-5))M+5I)$	1M	for using the result of (a)
$=(1-(-5)^2)M+(5+(-5)^2)I$		
$6M^3$		
$=M(6M^2)$		
$=M((1-(-5)^2)M+(5+(-5)^2)I)$	1M	
$=(1-(-5)^2)M^2+(5+(-5)^2)M$		
$=(1-(-5)^2)((1+(-5))M+5I)+(5+(-5)^2)M$		
$=(1+(-5)-(-5)^2-(-5)^3)M+(5+(-5)^3)I+(5+(-5)^2)M$		
$=(1-(-5)^3)M+(5+(-5)^3)I$		
Thus, we have $6M^n=(1-(-5)^n)M+(5+(-5)^n)I$.	1	
	----------(4)	

Solution	Marks	Remarks
(c) By (b), we have $6M^n = (1-(-5)^n)M + (5+(-5)^n)I$ and		
$6M^{n+1} = (1-(-5)^{n+1})M + (5+(-5)^{n+1})I$.	1M	
$6(1-(-5)^n)M^{n+1} - 6(1-(-5)^{n+1})M^n = ((1-(-5)^n)(5+(-5)^{n+1}) - (1-(-5)^{n+1})(5+(-5)^n))I$	1M	
$M^n\left(\dfrac{(1-(-5)^n)M-(1-(-5)^{n+1})I}{-6(-5)^n}\right) = \left(\dfrac{(1-(-5)^n)M-(1-(-5)^{n+1})I}{-6(-5)^n}\right)M^n = I$		
So, we have $(M^n)^{-1} = \dfrac{(1-(-5)^n)M-(1-(-5)^{n+1})I}{-6(-5)^n}$.	1M	
$(M^n)^{-1}$ $= \dfrac{(-5)^n-1}{6(-5)^n}M + \dfrac{-(-5)^{n+1}+1}{6(-5)^n}I$		
$= \left(\dfrac{1}{6}M + \dfrac{5}{6}I\right) + \dfrac{1}{(-5)^n}\left(\dfrac{-1}{6}M + \dfrac{1}{6}I\right)$	1M	
Letting $A = \dfrac{1}{6}M + \dfrac{5}{6}I = \dfrac{1}{6}\begin{pmatrix}7 & 7\\ -1 & -1\end{pmatrix}$ and $B = \dfrac{-1}{6}M + \dfrac{1}{6}I = \dfrac{1}{6}\begin{pmatrix}-1 & -7\\ 1 & 7\end{pmatrix}$,		
we have $(M^n)^{-1} = A + \dfrac{1}{(-5)^n}B$.		
Thus, there exists a pair of 2×2 real matrices A and B such that $(M^n)^{-1} = A + \dfrac{1}{(-5)^n}B$ for all positive integers n .	1A	f.t.
$6M^n = (1-(-5)^n)M + (5+(-5)^n)I$ (by (b))		
$M^n = \dfrac{1}{6}(1-(-5)^n)\begin{pmatrix}2 & 7\\ -1 & -6\end{pmatrix} + \dfrac{1}{6}(5+(-5)^n)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$		
$= \begin{pmatrix}\dfrac{-(-5)^n+7}{6} & \dfrac{-7(-5)^n+7}{6}\\ \dfrac{(-5)^n-1}{6} & \dfrac{7(-5)^n-1}{6}\end{pmatrix}$	1M	
$\det(M^n) = \left(\dfrac{-(-5)^n+7}{6}\right)\left(\dfrac{7(-5)^n-1}{6}\right) - \left(\dfrac{-7(-5)^n+7}{6}\right)\left(\dfrac{(-5)^n-1}{6}\right)$	1M	
$= (-5)^n$		
$(M^n)^{-1} = \dfrac{1}{\det(M^n)}\begin{pmatrix}\dfrac{7(-5)^n-1}{6} & \dfrac{7(-5)^n-7}{6}\\ \dfrac{-(-5)^n+1}{6} & \dfrac{-(-5)^n+7}{6}\end{pmatrix}$	1M	
$= \begin{pmatrix}\dfrac{7}{6} & \dfrac{7}{6}\\ \dfrac{-1}{6} & \dfrac{-1}{6}\end{pmatrix} + \dfrac{1}{(-5)^n}\begin{pmatrix}\dfrac{-1}{6} & \dfrac{-7}{6}\\ \dfrac{1}{6} & \dfrac{7}{6}\end{pmatrix}$	1M	
Letting $A = \begin{pmatrix}\frac{7}{6} & \frac{7}{6}\\ \frac{-1}{6} & \frac{-1}{6}\end{pmatrix}$ and $B = \begin{pmatrix}\frac{-1}{6} & \frac{-7}{6}\\ \frac{1}{6} & \frac{7}{6}\end{pmatrix}$, we have $(M^n)^{-1} = A + \dfrac{1}{(-5)^n}B$.		
Thus, there exists a pair of 2×2 real matrices A and B such that $(M^n)^{-1} = A + \dfrac{1}{(-5)^n}B$ for all positive integers n .	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
12. (a) $\left\|\overrightarrow{AC}\right\|=\left\|\overrightarrow{BC}\right\|$		
$\left\|\overrightarrow{OC}-\overrightarrow{OA}\right\|=\left\|\overrightarrow{OC}-\overrightarrow{OB}\right\|$	1M	
$\left\|-6\mathbf{i}-8\mathbf{j}+(t-2)\mathbf{k}\right\|=\left\|-8\mathbf{j}+(t-8)\mathbf{k}\right\|$		
$\sqrt{(-6)^2+(-8)^2+(t-2)^2}=\sqrt{(-8)^2+(t-8)^2}$	1M	
$t^2-4t+104=t^2-16t+128$		
$12t=24$		
$t=2$	1A	
	----------(3)	
(b) $\overrightarrow{AB}\times\overrightarrow{AC}$		
$=(-6\mathbf{i}+6\mathbf{k})\times(-6\mathbf{i}-8\mathbf{j})$		
$=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -6 & 0 & 6\\ -6 & -8 & 0\end{vmatrix}$		
$=((0)(0)-(6)(-8))\mathbf{i}+((6)(-6)-(6)(0))\mathbf{j}+((-6)(-8)-(0)(-6))\mathbf{k}$	1M	
$=48\mathbf{i}-36\mathbf{j}+48\mathbf{k}$	1A	
	----------(2)	
(c) The volume of the pyramid $OABC$		
$=\frac{1}{6}\left\|\overrightarrow{OA}\cdot(\overrightarrow{AB}\times\overrightarrow{AC})\right\|$	1M	
$=\frac{1}{6}\left\|(\mathbf{i}-4\mathbf{j}+2\mathbf{k})\cdot(48\mathbf{i}-36\mathbf{j}+48\mathbf{k})\right\|$		
$=\frac{1}{6}\left\|(1)(48)+(-4)(-36)+(2)(48)\right\|$		
$=48$	1A	
The volume of the pyramid $OABC$		
$=\frac{1}{6}\left\|\overrightarrow{OA}\cdot(\overrightarrow{OB}\times\overrightarrow{OC})\right\|$	1M	
$=\frac{1}{6}\left\|\begin{vmatrix}1 & -4 & 2\\ -5 & -4 & 8\\ -5 & -12 & 2\end{vmatrix}\right\|$		
$=\frac{1}{6}\left\|(1)(-4)(2)+(-4)(8)(-5)+(2)(-5)(-12)-(1)(8)(-12)-(-4)(-5)(2)-(2)(-4)(-5)\right\|$		
$=48$	1A	
	----------(2)	
(d) (i) By (c), the volume of the pyramid $OABC$ is not equal to 0.		
So, O does not lie on Π.		
Therefore, $\overrightarrow{OP}$, $\overrightarrow{OQ}$ and $\overrightarrow{OR}$ are non-zero vectors.		
Hence, we have $p\neq 0$, $q\neq 0$ and $r\neq 0$.		
Thus, we have $pqr\neq 0$.	1	

Solution	Marks	Remarks
(ii) $\overrightarrow{OD}$		
$=\overrightarrow{OA}\cdot\dfrac{\overrightarrow{AB}\times\overrightarrow{AC}}{\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|^2}(\overrightarrow{AB}\times\overrightarrow{AC})$	1M	
$=\dfrac{(6)(48)}{48^2+(-36)^2+48^2}(48\mathbf{i}-36\mathbf{j}+48\mathbf{k})$		
$=\dfrac{288}{5\,904}(48\mathbf{i}-36\mathbf{j}+48\mathbf{k})$		
$=\dfrac{96}{41}\mathbf{i}-\dfrac{72}{41}\mathbf{j}+\dfrac{96}{41}\mathbf{k}$	1A	
(iii) $\overrightarrow{AP}\cdot(\overrightarrow{AB}\times\overrightarrow{AC})=0$	1M	
$((p-1)\mathbf{i}+4\mathbf{j}-2\mathbf{k})\cdot(48\mathbf{i}-36\mathbf{j}+48\mathbf{k})=0$		
$48p-48-144-96=0$		
$p=6$		
$\overrightarrow{AQ}\cdot(\overrightarrow{AB}\times\overrightarrow{AC})=0$		any one
$(-\mathbf{i}+(q+4)\mathbf{j}-2\mathbf{k})\cdot(48\mathbf{i}-36\mathbf{j}+48\mathbf{k})=0$		
$-48-36q-144-96=0$		
$q=-8$		for all
$\overrightarrow{AR}\cdot(\overrightarrow{AB}\times\overrightarrow{AC})=0$		
$(-\mathbf{i}+4\mathbf{j}+(r-2)\mathbf{k})\cdot(48\mathbf{i}-36\mathbf{j}+48\mathbf{k})=0$		
$-48-144+48r-96=0$		
$r=6$	1A	
So, we have $\overrightarrow{OE}=\dfrac{1}{6}\mathbf{i}-\dfrac{1}{8}\mathbf{j}+\dfrac{1}{6}\mathbf{k}$.		
By (b)(ii), we have $\overrightarrow{OE}=\dfrac{41}{576}\overrightarrow{OD}$.		
Thus, D, E and O are collinear.	1A	f.t.
	----------(6)	