MI Note

1 Mathematical induction

Stage 0 Let $P(n)$ be the desired statement.
Stage 1 (Initial Kick)

$$\text{For } n = 1, \text{ show that } P(1) \text{ is true.}$$

Stage 2 (Induction Hypothesis)

$$\text{Assume } P(k) \text{ is true for some positive integer } k.$$

Stage 3 (Inductive Step - From n to $n+1$)

$$\text{For } n = k+1, \text{ show that } P(k+1) \text{ is true.}$$

Stage 4 (End of Story)

$$\text{Therefore } P(n) \text{is true for ALL positive integers } n \text{ by M.I.}$$

2 Exercise

Prove, by mathematical induction, that

$$1 + 3 + 5 + \ldots + (2n-1) = n^2$$

for all positive integer n.

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Let $P(n)$ be the desired statement.
For $n = 1$, we have LHS $= 1 = 1^2 =$ RHS. Hence $P(1)$ is true.
Assume $P(k)$ is true for some positive k.
For $n = k + 1$

$$\begin{aligned} 1 + 3 + \ldots + (2k-1) + (2k+1) &= k^2 + (2k+1) \\ &= (k+1)^2 \end{aligned}$$

Hence $P(k+1)$ is true assuming $P(k)$ is true.
$\therefore P(n)$ is true for all positive integer $n \geq 1$ by induction.

1. **HKCEE A.Maths 1994 Past Paper II Q5**
   Prove by induction that

   $$\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \ldots + \frac{2n-1}{2^n} = 3 - \frac{2n+3}{2^n}$$

   for all positive integers n.

2. **HKCEE A.Maths 1994 Past Paper II Q5**

   Prove by induction that

$$\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\ldots+\frac{2n-1}{2^n}=3-\frac{2n+3}{2^n}$$

   for all positive integers n.

3. **HKCEE A.Maths 2005 Past Paper Q8**

   Prove by induction that

$$\frac{1\ldots 2}{2\ldots 3}+\frac{2\ldots 2^2}{3\ldots 4}+\frac{3\ldots 2^3}{4\ldots 5}+\ldots+\frac{n\ldots 2^n}{(n+1)(n+2)}=\frac{2^{n+1}-(n+2)}{n+2}$$

   for all positive integers n.

4. **HKCEE A.Maths 2007 Past Paper Q5**

   Let $a \neq 0$ and $a \neq 1$. Prove by mathematical induction, that

$$\frac{1}{a-1}-\frac{1}{a}-\frac{1}{a^2}-\cdots-\frac{1}{a^n}=\frac{1}{a^{n+1}-a^n}$$

   for all positive integers n.