HKDSE MATH CORE 2023 Past Paper I

1. **HKDSE MATH CORE 2023 Past Paper I Q1**
   Make h the subject of the formula $\frac{5}{h+k} = \frac{k}{h-3}$.
   (3 marks)

2. **HKDSE MATH CORE 2023 Past Paper I Q2**
   Simplify $\frac{x^{-8}y}{(x^7y^9)^{-6}}$ and express your answer with positive indices.
   (3 marks)

3. **HKDSE MATH CORE 2023 Past Paper I Q3**
   A packet of cheese is termed regular if its weight is measured as 220 g correct to the nearest 10 g. Someone claims that the total weight of 250 regular packets of cheese can be measured as 53.6 kg correct to the nearest 0.1 kg. Is the claim correct? Explain your answer.
   (3 marks)

4. **HKDSE MATH CORE 2023 Past Paper I Q4**
   Consider the compound inequality

   $$3x + 2 > \frac{4x - 5}{2} \text{ and } 3x - 2 < 7 \ldots\ldots\ldots\ldots\ldots (*).$$

   (a) Solve $(*)$.

   (b) How many negative integers satisfy $(*)$?

   (4 marks)

5. **HKDSE MATH CORE 2023 Past Paper I Q5**
   On a ferry, the number of female passengers is 40% more than the number of male passengers. If 24 female passengers leave the ferry, then the number of male passengers is 40% more than the number of female passengers. Find the number of male passengers on the ferry.
   (4 marks)

6. **HKDSE MATH CORE 2023 Past Paper I Q6**
   Let a, b and c be non-zero numbers such that $7a = 6b$ and $\frac{4a - 3c}{2b - c} = 9$. Find

   (a) $a : b : c$,

   (b) $\frac{5a + 8b}{7b + 3c}$.

   (4 marks)

7. **HKDSE MATH CORE 2023 Past Paper I Q7**
   In Figure 1, PR is a diameter of the circle $PQRS$. Denote the point of intersection of PR and QS by T.

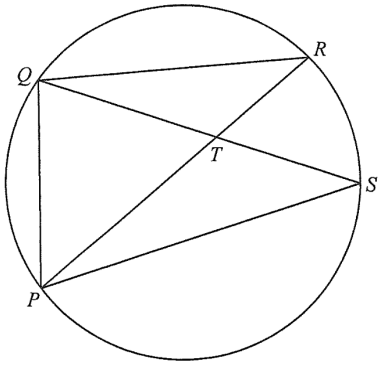

Figure 1

If $\angle PSQ = 41°$ and $\angle PTQ = 68°$, find $\angle RQS$ and $\angle PQS$.
(4 marks)

8. **HKDSE MATH CORE 2023 Past Paper I Q8**
In Figure 2, AB and CD intersect at the point E. It is given that $AC//DB$.

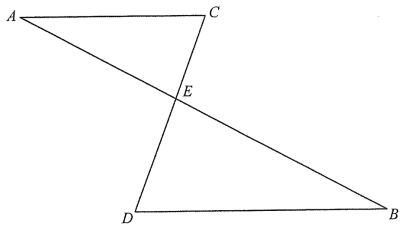

Figure 2

(a) Prove that $\triangle ACE \sim \triangle BDE$.

(b) Suppose that $AB = 20$ cm, $AC = 10$ cm and $CE = 7$ cm. Is $\triangle BDE$ is a right-angled triangle? Explain your answer.

(5 marks)

9. **HKDSE MATH CORE 2023 Past Paper I Q9**
The stem-and-leaf diagram below shoes the distribution of the numbers of working hours of a group of workers in a week. The range of the distribution is 27.

Stem (tens)	Leaf (units)
2	a 5 5 6 6 8 8
3	3 3 3 4 5 5 9 9
4	0 1 4 4 5 6 7 7 9

(a) Find the mean and the mode of the distribution.

(b) If a worker is randomly selected from the group, find the probability that the number of working hours of the selected worker in the week exceeds the mode of the distribution.

(5 marks)

10. **HKDSE MATH CORE 2023 Past Paper I Q10**

It is given that A and B are two distinct points in a rectangular coordinate plane. Let P be a moving point in the rectangular coordinate plane such that P is equidistant from A and B. Denote the locus of P by Γ.

(a) Describe the grometric relationship between Γ and AB.
(1 marks)

(b) Suppose that the coordinates of A are $(2, -4)$ and the equation of Γ is $3x + y - 12 = 0$. Find

(i) the equation of the straight line which passes through A and B,

(ii) the equation of the circle with AB as a diameter.

(5 marks)

11. **HKDSE MATH CORE 2023 Past Paper I Q11**

The table below shoes the distribution of the numbers of calculators owned by a class of students.

Number of calculators owned	1	2	3	4
Number of student	8	5	n	1

The mean of the distribution is 2.

(a) Find the median, inter-quatile range and the variance of the above distribution.
(5 marks)

(b) Two students now withdraw from the class. It is found that the mean of the distribution remains unchanged. Is there any change in the range of the distribution due to the withdrawal of the two students? Explain your answer.
(2 marks)

12. **HKDSE MATH CORE 2023 Past Paper I Q12**

It is given that $f(x)$ is partly constant and partly varies as x^2. Suppose that $f(10) = 62$ and $f(15) = 122$.

(a) Find $f(5)$.
(3 marks)

(b) Suppose that $U(0, u)$ and $V(5, v)$ are points lying on the graph of $y = f(x)$. The horizontal line passing through V cuts the y-axis at the point W. Denote the circle which passes through U, V and W by C. Express the circumference of C in terms of π.
(4 marks)

13. **HKDSE MATH CORE 2023 Past Paper I Q13**

Define $g(x) = x^3 + 5x^2 - 12x - 1$. Let $h(x) = 3x^4 + ax^3 - 16x + bx + c$, where a, b and c are constants. When $h(x)$ is divided by $g(x)$, the quotient and the remainder are equal.

(a) Find the quotient when $h(x)$ is divided by $g(x)$.
(3 marks)

(b) How many rational roots does the equation $h(x) = 0$ have? Explain your answer.
(4 marks)

14. **HKDSE MATH CORE 2023 Past Paper I Q14**
The base radius and the curved area of a solid metal reight circular cone are 14 cm and 700π cm^2 respectively.

(a) Find the height of the circular cone.
(3 marks)

(b) The circular cone is divided into a right circular cone X anda frustum Y by a plane which is parallel to its base. The curve surface area of Y is 15 times the curved surface area of X.

(i) Express the volume of Y in terms of π.

(ii) If Y is melted and recast into 2 identical solid spheres, find the diameter of each sphere.

(5 marks)

15. **HKDSE MATH CORE 2023 Past Paper I Q15**
In a box, there are 4 red balls and 4 black balls. From the box, 2 balls are randomly chosen at the same time.

(a) Find the probability that the 2 balls chosen are red.
(2 marks)

(b) In a bag, there are 8 red balls. The 2 balls form the box are put into the bag and then 3 balls are randomly chosen at the smae time from the bag. Find the probability tha tthe 3 balls chosen are of the same colour.
(2 marks)

16. **HKDSE MATH CORE 2023 Past Paper I Q16**

(a) Let a and b be real constants. If the roots of the equation $x^2 + ax + b = 0$ are p and $5p$, prove that $5a^2 = 36b$.
(2 marks)

(b) Denote the circle $x^2 + y^2 - 6x - 12y + 20 = 0$ by C. Find the constant m such that the straight line $y = mx$ cuts C at the points Q and R with $OQ : QR = 1 : 4$, where O is the origin.
(3 marks)

17. **HKDSE MATH CORE 2023 Past Paper I Q17**

(a) It is given that WXY is a triangle, where $WX = 6$ cm , $XY = 5$ cm and $\angle WYX = 70°$. Find $\angle XWY$.
(2 marks)

(b) Figure 3 shows the pyramid $WXYZ$, where $WZ = XZ = YZ$. The base of this pyramid is the triangle WXY described in (a).

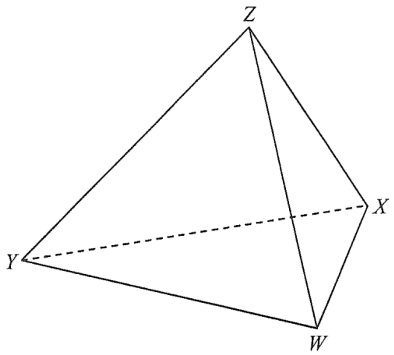

Figure 3

It is given that the angle between WZ and the triangle WXY is $30°$. Does the angle between the triangles WXY and XYZ exceeed $45°$? Explain your answer.
(4 marks)

18. **HKDSE MATH CORE 2023 Past Paper I Q18**
Suppose that $\alpha, 7, \beta$ is a geometric sequence, where $q < \alpha < \beta$.

(a) Express $\log_7 \alpha$ in terms of $\log_7 \beta$.
(3 marks)

(b) If $\log_\beta \alpha, \log_7 \beta, \log_\alpha \beta$ is an arithmetic sequence, find the common difference of the arithmetic sequence.
(5 marks)

19. **HKDSE MATH CORE 2023 Past Paper I Q19**
The coordinates of the points P and Q are $(50, 0)$ and $(32, t)$ respectively, where $t > 0$. Denote the origin by O. Let R be a point such that OQ is a median of $\triangle OPR$. Suppose that G and H are the circumcentre and the orthocentre of $\triangle OPR$ respectively.

(a) Express the coordinates of G and H in terms of t.
(5 marks)

(b) Let S be a point lying on OP such that QS is perpendicular to OP. It is given that $\angle PQS = \angle POQ$.

(i) By considering $\tan \angle PQS$, prove that $t = 24$.

(ii) Are O, G and Q collinear? Explain your answer.

(iii) Denote the in-centre of $\triangle OPR$ by I. Find the ratio of the area of $\triangle GHR$ to the area of $\triangle IPQ$.

(7 marks)