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HONG KONG EXAMINATION & ASSESSMENT AUTHORITY

Hong Kong Diploma of Secondary Education Examination 2024
Mathematics (Compulsory Part) Paper 1

Time: 2 hours, 15 minutes

Maximum Marks: 105

2024 HKDSE Math Paper, 15 Apr 2024

- Documented on L^AT_EX, 16 Apr 2024
- Good luck!

Section A(1) (35 marks)

1. Simplify $\frac{2}{4h-7} - \frac{3}{6h-5}$. [3 marks]

2. Make x the subject of the formula $\frac{Ax+C}{B} = 3x$. [3 marks]

3. Factorize

(a) $6r^2 - 13rs - 28s^2$,

(b) $4r - 14s + 6r^2 - 13rs - 28s^2$.

[3 marks]

4. (a) Find the range of values of x which satisfy both $\frac{5x+7}{4} - 1 < 2x$ and $3x+9 \geq 0$.

(b) Write down the least integer satisfying both inequalities in (a).

[4 marks]

5. Let a, b and c be non-zero numbers such that $5a = 6c$ and $\frac{2b+7c}{b+c} = 4$. Find $\frac{5a+8b}{2b+3c}$ [4 marks]

6. The marked price of a calculator is 40% higher than its cost. The calculator is sold at a discount of 25% on its marked price and the profit is \$13. Find the marked price of the calculator. [4 marks]

7. In a polar coordinate system, O is the pole. The polar coordinates of the points P, Q and R are $(11, 59^\circ)$, $(60, 149^\circ)$ and $(144, 239^\circ)$ respectively.

- Find $\angle POQ$.
- Are P, O and R collinear? Explain your answer.
- Find the perimeter of $\triangle PQR$.

[4 marks]

8. In Figure 1, E is the point of intersection of AC and BD .

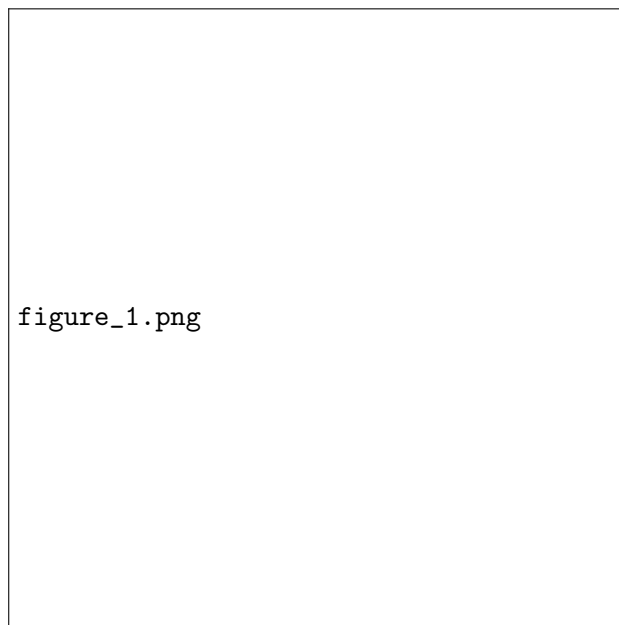


Figure 1

It is given that $\angle ACB = \angle ADB = 90^\circ$ and $AD = BC$.

- Prove that $\triangle ABC \cong \triangle BAD$.
- If $AD = 12$ cm and $DE = 9$ cm, find the area of the pentagon $ABCED$.

[5 marks]

9. The table shows the distribution of the numbers of keys owned by a group of housewives.

Number of keys	3	4	5	6	7	8
Number of housewives	10	9	4	3	4	k

If a housewife is randomly selected from the group, then the probability that she owns more than 6 keys is $\frac{5}{18}$.

- (a) Find k .
- (b) Write down the mean, mode and median of the distribution.

[5 marks]

Section A(2) (35 marks)

10. It is given that $g(x)$ is partly constant and partly varies as x .

Suppose that $g(-3) = -21$ and $g(7) = 9$.

(a) Find $g(x)$. [3 marks]

(b) Let $h(x) = xg(x) + k$, where k is a real constant.

If all the roots of the equation $h(x) = 0$ are real numbers, find the range of values of k . [3 marks]

11. The stem-and-leaf diagram below shows the distribution of the numbers of hours spent on reading journals in a month by a group of researchers.

Stem (tens)	Leaf (units)
2	0 0 1 a a a 8 8 9 9
3	0 0 2 3 4 4 7 9
4	0 b

(a) Find a and b . [3 marks]

(b) Write down the least possible range of the distribution. [1 mark]

(c) Find the greatest possible inter-quartile range of the distribution. [2 marks]

12. Denote the origin by O .

(a) A and B are points lying on the positive x -axis such that the x -coordinate of A is greater than the x -coordinate of B . A vertical line which passes through B cuts the straight line $y = mx$ at the point C such that $AB = BC$, where m is a positive constant.

Let D be a point such that $ABCD$ is a square. Express the slope of OD in terms of m . [3 marks]

(b) The coordinates of the points M and N are $(6, 5)$ and $(10, 0)$ respectively. Let P and Q be points lying on OM and MN respectively while R and S be points lying on the x -axis.

If the quadrilateral $PQRS$ is a square, find the x -coordinate of P . [4 marks]

13. The base of a solid right pyramid is a square of side 64 cm. The height of a pyramid is 24 cm. The pyramid is divided into a frustum X and a pyramid Y by a plane parallel to its base.

It is given that the height of Y is 18 cm.

(a) Find the volume of X . [3 marks]

- (b) The base of another solid right pyramid is a square. The pyramid is divided into a frustum Z and a pyramid by a plane parallel to its base. The height and the total surface area of Z are 3 cm and 960 cm^2 respectively.

Are X and Z similar? Explain your answer. [4 marks]

14. Let $F(x) = (6x^2 + x + p)(qx^2 + rx - 10)$, where p, q and r are constants.

The constant term of $F(x)$ is 40.

(a) Write down the value of p . [1 mark]

- (b) When $F(x)$ is divided by $x + 1$, the remainder is -12 . Given that $x - 2$ is a factor of $F(x)$.

(i) Find q and r .

(ii) How many irrational roots does the equation $F(x) = 0$ have? Explain your answer.

[7 marks]

Section B (35 marks)

15. It is given that $\log_9 y$ is a linear function of $\log_3 x$. Denote the graph of the linear function by L . The slope of L is 4 and L passes through the point $(5, 22)$.

Express y in terms of x .

[3 marks]

16. In a bag, there are 16 red cups and 4 white cups. If 5 cups are randomly drawn from the bag at the same time, find

(a) the probability that exactly 1 white cup is drawn,

[2 marks]

(b) the probability that at most 3 red cups are drawn.

[2 marks]

17. The coordinates of the points Q and R are $(10, -1)$ and $(-4, -9)$ respectively.

(a) Let P be a moving point in the rectangular coordinate plane such that $PQ = PR$. Denote the locus of P by Γ .

(i) Describe the geometric relationship between Γ and QR .

(ii) Find the equation of Γ .

[3 marks]

(b) Let C be the circle which passes through Q , R and the point $(4, 3)$.

(i) Find the equation of C .

(ii) The coordinates of the point U is $(10, 4)$. It is found that U lies outside C . UV and UW are the tangents to C at the points V and W respectively.

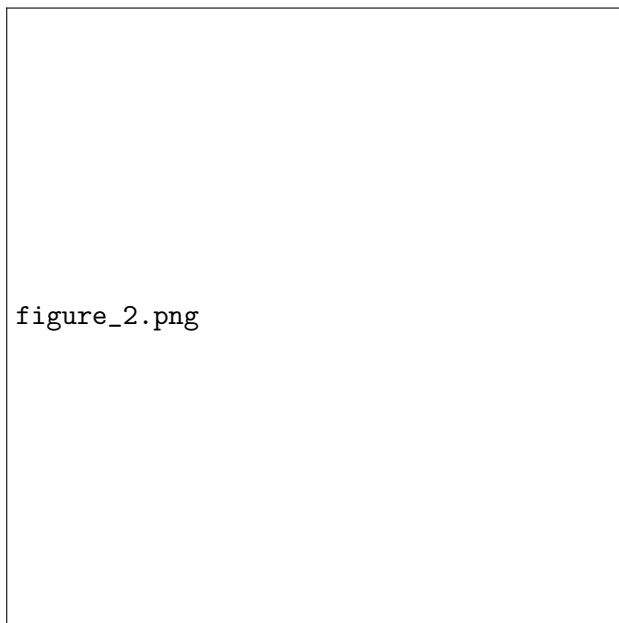
Is the area of the circumcircle of $\triangle UVW$ greater than 100? Explain your answer.

[5 marks]

18. (a) $PQRS$ is a thin quadrilateral metal sheet where $PQ = 12\text{cm}$, $PS = 10\text{cm}$, $QR = 13\text{cm}$, $\angle QPS = 82^\circ$ and $\angle QRS = 65^\circ$. Find
- (i) the length of QS ,
 - (ii) $\angle RQS$.

[4 marks]

- (b) The metal sheet $PQRS$ described in (a) is now folded along QS (see Figure 2). It is given that the angle between the plane PQS and the plane QRS is 80° .



- (i) Find the shortest distance from R to the plane PQS .
- (ii) Let X be any point lying on the plane QRS . Someone claims that the distance between P and X exceeds 8 cm. Is the claim correct? Explain your answer.

[4 marks]

19. Let $f(x) = 2x^2 + 4mx + 8x + 2m^2 + 8m + n$, where m and n are real constants such that $mn < 0$. Denote the vertex of the graph of $y = f(x)$ by P .

- (a) Using the method of completing the square, express the coordinates of P in terms of m and n . [2 marks]
- (b) Describe the geometric meaning represented by transforming $f(x)$ to $f(\frac{x}{5}) + 7$. [2 marks]
- (c) Denote the vertex of the graph of $f(\frac{x}{5}) + 7$ by Q . Let (a_1, b_1) and (a_2, b_2) be the coordinates of P and Q respectively. Given that $a_1, 1+n, a_2$ is an arithmetic sequence, and $b_1, 4-m, b_2$ is a geometric sequence.
 - (i) Find the coordinates of P and Q .

- (ii) The coordinates of the points R and S are $(3t + 27, t)$ and $(3t + 3, 2t - 3)$ respectively, where t is a real number.

Is it possible that $PQRS$ is a rhombus? Explain your answer.

[8 marks]