HKDSE MATH M2 2015

1. **HKDSE Math M2 2015 Q1**
   Find $\frac{d}{dx}(x^5+4)$ from first principles.
   (4 marks)

2. **HKDSE Math M2 2015 Q2**
   Let $y = x\sin x + \cos x$.
   (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
   (b) Let k be a constant such that $x\frac{d^2y}{dx^2} + k\frac{dy}{dx} + xy = 0$ for all real values of x. Find the value of k.

   (5 marks)

3. **HKDSE Math M2 2015 Q3**
   (a) Find $\int \frac{1}{e^{2u}}\,du$.
   (b) Using integration by substitution, evaluate $\int_1^9 \frac{1}{\sqrt{x}e^{2\sqrt{x}}}\,dx$.

   (7 marks)

4. **HKDSE Math M2 2015 Q4**
   (a) Using integration by parts, find $\int x^2 \ln x\,dx$.
   (b) At any point (x, y) on the curve Γ, the slope of the tangent to Γ is $9x^2\ln x$. It is given that Γ passes through the point $(1, 4)$. Find the equation of Γ.

   (7 marks)

5. **HKDSE Math M2 2015 Q5**
   Solve the following systems of linear equations in real variables x, y, z:
   (a) $\begin{cases} x + y + z = 2 \\ 2x + 3y - 3z = 4 \end{cases}$;
   (b) $\begin{cases} x + y + z = 2 \\ 2x + 3y - 3z = 4 \\ 3x + 2y + kz = 6 \end{cases}$, where k is a real constant.

   (6 marks)

6. **HKDSE Math M2 2015 Q6**
   (a) Let M be a 3×3 real matrix such that $M^T = -M$, where M^T is the transpose of M. Prove that $|M| = 0$.

(b) Let $A = \begin{pmatrix} -1 & a & b \\ -a & -1 & -8 \\ -b & 8 & -1 \end{pmatrix}$, where a and b are real numbers. Denote the 3×3 identity matrix by I.

(i) Using (a), or otherwise, prove that $|A + I| = 0$.

(ii) Someone claims that $A^3 + I$ is a singular matrix. Do you agree? Explain your answer.

(6 marks)

7. **HKDSE Math M2 2015 Q7**

(a) Prove that $\sin^2 x \cos^2 x = \dfrac{1 - \cos 4x}{8}$.

(b) Let $f(x) = \sin^4 x + \cos^4 x$.

(i) Express $f(x)$ in the form $A \cos Bx + C$, where A, B and C are constants.

(ii) Solve the equation $8f(x) = 7$, where $0 \le x \le \dfrac{\pi}{2}$.

(7 marks)

8. **HKDSE Math M2 2015 Q8**

(a) Using mathematical induction, prove that $\sin \dfrac{x}{2} \displaystyle\sum_{k=1}^{n} \cos kx = \sin \dfrac{nx}{2} \cos \dfrac{(n+1)x}{2}$ for all positive integers n.

(b) Using (a), evaluate $\displaystyle\sum_{k=1}^{567} \cos \frac{k\pi}{7}$.

(8 marks)

9. **HKDSE Math M2 2015 Q9**

Define $f(x) = \dfrac{x^2 + 12}{x - 2}$ for all $x \ne 2$.

(a) Find $f'(x)$.
(2 marks)

(b) Prove that the maximum value and the minimum value of $f(x)$ are -4 and 12 respectively.
(4 marks)

(c) Find the asymptote(s) of the graph of $y = f(x)$.
(3 marks)

(d) Find the area of the region bounded by the graph of $y = f(x)$ and the horizontal line $y = 14$.
(4 marks)

10. **HKDSE Math M2 2015 Q10**

OAB is a triangle. P is the mid-point of OA. Q is a point lying on AB such that $AQ : QB = 1 : 2$ while R is a point lying on OB such that $OR : RB = 3 : 1$. PR and OQ intersect at C.

(a) (i) Let t be a constant such that $PC : CR = t : (1-t)$.
By expressing $\overrightarrow{OQ}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$, find the value of t.

(ii) Find $CQ : OQ$.

(7 marks)

(b) Suppose that $\overrightarrow{OA} = 20\mathbf{i} - 6\mathbf{j} - 12\mathbf{k}$, $\overrightarrow{OB} = 16\mathbf{i} - 16\mathbf{j}$ and $\overrightarrow{OD} = \mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$, where O is the origin. Find

(i) the area of $\triangle OAB$,

(ii) the volume of tetrahedron $ABCD$.

(5 marks)

11. **HKDSE Math M2 2015 Q11**

(a) Let λ and μ be real numbers such that $\mu - \lambda \neq 2$. Denote the 2×2 identity matrix by I.
Define $A = \dfrac{1}{\lambda - \mu + 2}(I - \mu I + M)$ and $B = \dfrac{1}{\lambda - \mu + 2}(I + \lambda I - M)$,
where $M = \begin{pmatrix} \lambda & 1 \\ \lambda - \mu + 1 & \mu \end{pmatrix}$.

(i) Evaluate AB, BA and $A + B$.

(ii) Prove that $A^2 = A$ and $B^2 = B$.

(iii) Prove that $M^n = (\lambda + 1)^n A + (\mu - 1)^n B$ for all positive integers n.

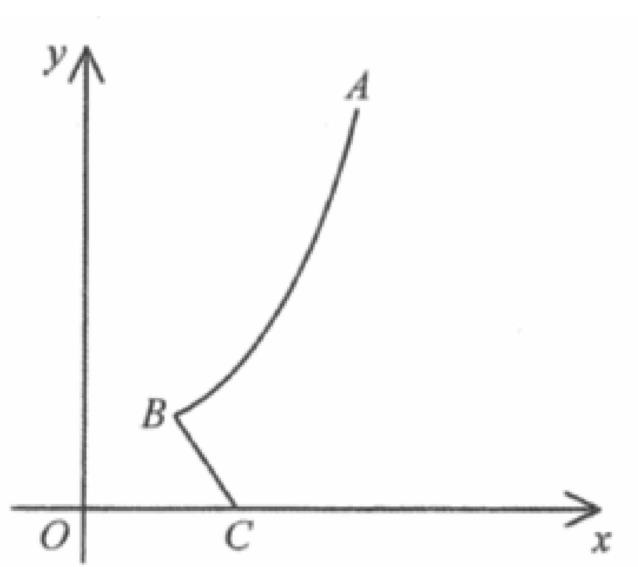
(8 marks)

(b) Using (a), or otherwise, evaluate $\begin{pmatrix} 4 & 2 \\ 0 & 6 \end{pmatrix}^{315}$.

(4 marks)

12. **HKDSE Math M2 2015 Q12**

(a) In the figure, the curve Γ consists of curve AB, the line segments BC and CO, where O is the origin, B lies in the first quadrat and C lies on the x-axis. The equations of AB and BC are $x^2 - 4y + 8 = 0$ and $3x + y - 9 = 0$ respectively.



(i) Find the coordinates of B.

(ii) Let h be the y-coordinate of A, where $h > 3$. A cup is formed by revolving Γ about the y-axis. Prove that the capacity of the cup is $\pi(2h^2 - 8h + 25)$.

(7 marks)

(b) A cup described in (a)(ii) is placed on a horizontal table. The radii of the base and the lip of the cup are 3 cm and 6 cm respectively.

(i) Find the capacity of the cup.

(ii) Water is poured into the cup at a constant rate of 24π cm^3/s. Find the rate of change of the depth of water when the volume of water in the cup is 35π cm^3.

(6 marks)