2021-DSE
MATH CP
PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2021

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Candidate Number									

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

8:30 am – 10:45 am (2¼ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of THREE sections, A(1), A(2) and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

(8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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* A 0 3 0 E 0 0 1 *

SECTION A(1) (35 marks)

1. Simplify $(\alpha\beta^3)(\alpha^{-2}\beta^4)^5$ and express your answer with positive indices. (3 marks)

2. Make a the subject of the formula $\frac{4-3a}{b}=5$. (3 marks)

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3. Factorize

(a) $6x^2+xy-2y^2$,

(b) $8x-4y-6x^2-xy+2y^2$.

(3 marks)

4. (a) Find the range of values of x which satisfy both $\frac{7(x-2)}{5}+11>3(x-1)$ and $x+4\geq 0$.

(b) How many positive integers satisfy both inequalities in (a)?

(4 marks)

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5. The number of stickers owned by a boy is 3 times that owned by a girl. If the boy gives 20 of his stickers to the girl, then the number of stickers owned by the girl is 2 times that owned by the boy. Find the total number of stickers owned by the boy and the girl. (4 marks)

6. The marked price of a shirt is higher than its cost by $80. The shirt is sold at a discount of 10% on its marked price. After selling the shirt, the percentage profit is 30%. Find the marked price of the shirt. (4 marks)

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7. In a polar coordinate system, O is the pole. The polar coordinates of the points P and Q are $(r, 80°)$ and $(r, 140°)$ respectively, where r is a positive constant. It is given that the distance between P and Q is 21. Find

(a) $\angle POQ$,

(b) r,

(c) the perimeter of ΔOPQ.

(4 marks)

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8. In Figure 1, AB produced and CD produced meet at the point E. It is given that $\angle CAE = \angle BDE$.

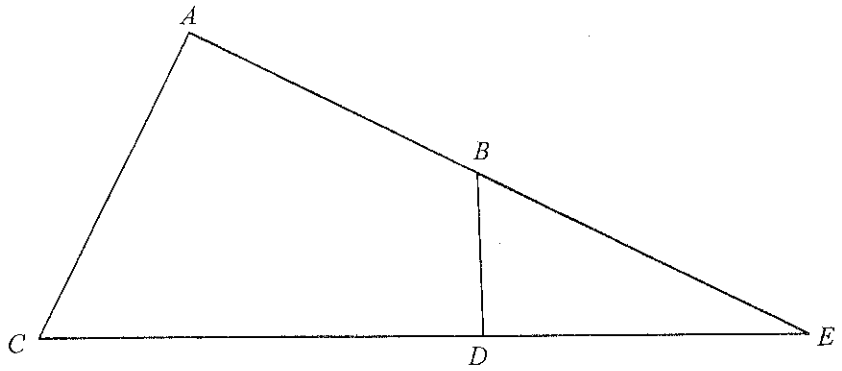

Figure 1

(a) Prove that $\Delta ACE \sim \Delta DBE$.

(b) It is given that $AC = 25$ cm, $AE = 60$ cm, $CE = 65$ cm and $BD = 15$ cm.

(i) Is ΔACE a right-angled triangle? Explain your answer.

(ii) Find the area of ΔBDE.

(5 marks)

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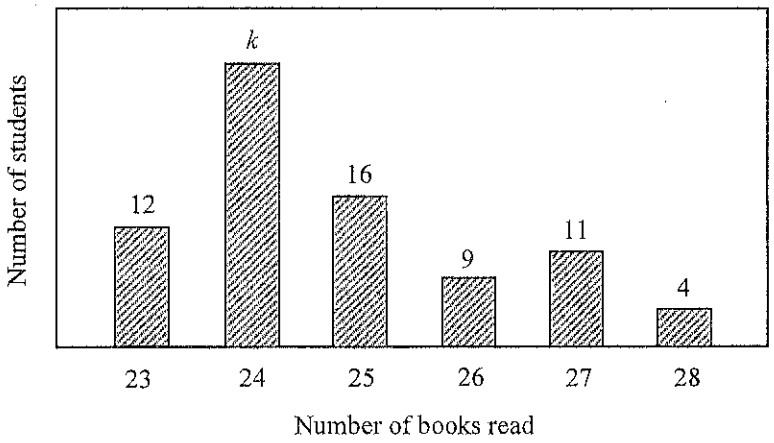
9. The bar chart below shows the distribution of the numbers of books read by a group of students in a year.

Distribution of the numbers of books read by the group of students in the year



If a student is randomly selected from the group, then the probability that the selected student reads fewer than 26 books in the year is $\frac{7}{10}$.

(a) Find k.

(b) Write down the range, the inter-quartile range and the standard deviation of the distribution.

(5 marks)

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SECTION A(2) (35 marks)

10. It is given that $f(x)$ is partly constant and partly varies as $(x+4)^2$. Suppose that $f(-3)=0$ and $f(2)=105$.

(a) Find $f(0)$. (3 marks)

(b) Denote the graph of $y=f(x)+3$ by G.

(i) Write down the y-intercept of G.

(ii) Find the x-intercept(s) of G. (3 marks)

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11. The table below shows the distribution of the numbers of tokens got by a group of children in a game.

Number of tokens got	1	2	3	4	5	6	7
Number of children	15	9	2	5	4	2	5

(a) Find the mean of the distribution. (2 marks)

(b) Are the median and the mode of the distribution equal? Explain your answer. (2 marks)

(c) If n more children play the game and each of them gets 5 tokens, write down

(i) the value of n such that the mean of the distribution is increased by 1;

(ii) the least value of n such that the median of the distribution is increased by 2;

(iii) the greatest value of n such that the mode of the distribution remains unchanged. (3 marks)

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12. The polynomial $p(x)$ is divisible by $x-5$. When $p(x)$ is divided by x^2+x+1, the quotient and the remainder are $2x^2-37$ and $cx+c-1$ respectively, where c is a constant.

(a) Find c. (3 marks)

(b) Prove that $x+3$ is a factor of $p(x)$. (1 mark)

(c) Someone claims that all the roots of the equation $p(x)=0$ are real numbers. Is the claim correct? Explain your answer. (3 marks)

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13. The equation of the circle C is $x^2+y^2-12x-16y-69=0$. Let G be the centre of C. Denote the origin by O.

(a) Find OG. (2 marks)

(b) Does O lie inside C? Explain your answer. (1 mark)

(c) Let P be a moving point in the rectangular coordinate plane such that $OP=GP$. Denote the locus of P by Γ. Suppose that Γ cuts C at the points M and N. Find the area of the quadrilateral $OMGN$. (4 marks)

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14. The base radius of the solid right circular cylinder X and the base radius of the solid right circular cone Y are equal. The heights of X and Y are 20 cm and 24 cm respectively. The volume of the solid right circular cone Z is equal to the sum of the volume of X and the volume of Y. The base radius of Z is equal to the base diameter of X. A craftsman finds that the volume of Y is 800π cm^3.

(a) Find the base radius of Y. (2 marks)

(b) Are Y and Z similar? Explain your answer. (3 marks)

(c) The craftsman claims that the sum of the curved surface area of X and the curved surface area of Y is greater than the curved surface area of Z. Do you agree? Explain your answer. (3 marks)

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SECTION B (35 marks)

15. A queue is randomly formed by 7 teachers and 3 students.

(a) How many different queues can be formed? (1 mark)

(b) Find the probability that no students are next to each other in the queue. (3 marks)

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16. The straight lines L_1 and L_2 are perpendicular to each other. The y-intercept of L_1 is 3. It is given that L_1 and L_2 intersect at the point $(2, 6)$. Let R be the region (including the boundary) bounded by L_1, L_2 and the x-axis.

(a) It is given that R represents the solution of a system of inequalities. Find the system of inequalities. (3 marks)

(b) Find the least value of $8x - 5y$, where (x, y) is a point lying in R. (2 marks)

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17. Let $A(n)$ be the nth term of an arithmetic sequence. It is given that $A(5) = 26$ and $A(12) = 61$.

(a) Find $A(1)$. (2 marks)

(b) Suppose that $\log_2 G(n) = A(n)$ for any positive integer n.
Find the greatest value of k such that $\log_8\big(G(1)G(2)G(3)\cdots G(k)\big) < 999$. (5 marks)

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18. (a) A thin metal sheet $ABCD$ is in the shape of a trapezium, where $AD//BC$. It is given that $AB = 45$ cm, $\angle ADC = 70°$ and $\angle BAD = 50°$. Find CD. (2 marks)

(b) The metal sheet $ABCD$ described in (a) is now given. Let E be a point lying on AD such that BE is perpendicular to AD. The metal sheet is folded along BE such that AE is perpendicular to the plane $BCDE$. Three thin triangular metal sheets are placed to this folded metal sheet to form a pyramid (see Figure 2). It is found that $BC = 40$ cm.

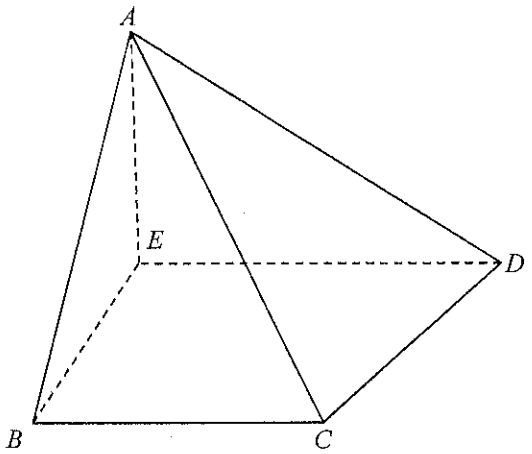

Figure 2

(i) Find $\angle CAD$.

(ii) Does the angle between the plane ACD and the plane $BCDE$ exceed $30°$? Explain your answer.

(5 marks)

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19. Let $f(x)=x^2-12kx-14x+36k^2+89k+53$, where k is a positive constant. On the same rectangular coordinate system, denote the vertex of the graph of $y=f(x)$ and the vertex of the graph of $y=f(14-x)$ by Q and R respectively.

(a) Using the method of completing the square, express, in terms of k, the coordinates of Q. (2 marks)

(b) Write down, in terms of k, the coordinates of R. (1 mark)

(c) The coordinates of the point S are $(7, 4-3k)$. Denote the inscribed circle of ΔQRS by C.

(i) Express, in terms of k, the equation of the straight line which passes through Q and S.

(ii) Express, in terms of k, the equation of C.

(iii) Suppose that QS is the tangent to C at the point T. Let U be the centre of C. It is given that the coordinates of the point V are $(-29, -14)$. Is it possible that $STUV$ is a rectangle? Explain your answer.

(9 marks)

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