Updated to 04-05-2019

Mathematics
Module 1
HKDSE
2012–2019

Name: ________________

Class: ___ (   )

1 – Binomial

1. Let n be a positive integer.

   (a) Expand $(1+3x)^n$ in ascending powers of x up to the term x^2.

   (b) It is given that the coefficient of x^2 in the expansion of $e^{-2x}(1+3x)^n$ is 62. Find the value of n. (4 marks)

[HKDSE 2012' Section A#1]

2. (a) Expand $\left(u+\frac{1}{u}\right)^4$ in descending powers of u.

(b) Express $(e^{ax}+e^{-ax})^4$ in ascending powers of x up to the term in x^2.

(c) Suppose the coefficient of x^2 in the results of (b) is 2. Find all possible values of a. (5 marks)

[HKDSE 2013' Section A #1]

3. (a) Expand e^{-4x} in ascending powers of x as far as the term in x^2.

(b) Find the coefficient of x^2 in the expansion of $\dfrac{(2+x)^5}{e^{4x}}$. (5 marks)

[HKDSE 2015' Section A#5]

4. Let k be a constant.

(a) Expand e^{kx} in ascending powers of x as far as the term in x^2.

(b) If the coefficient of x in the expansion of $(1+2x)^7 e^{kx}$ is 8, find the coefficient of x^2. (5 marks)

[HKDSE 2016' Section A#5]

5. (a) Expand $(1+e^{3x})^2$ in ascending powers of x as far as the term in x^2.

(b) Find the coefficient of x^2 in the expansion of $(5-x)^4(1+e^{3x})^2$.

(6 marks)

[HKDSE 2017' Section A#5]

6. Let k be a constant.

(a) Expand $e^{kx}+e^{2x}$ in ascending powers of x as far as the term in x^2.

(b) If the coefficient of x and the coefficient of x^2 in the expansion of $(1-3x)^8(e^{kx}+e^{2x}-1)$ are equal, find k. (6 marks)

[HKDSE 2018' Section A#6]

7. (a) Expand e^{-18x} in ascending powers of x as far as the term in x^2.

(b) Let n be a positive integer. If the coefficient of x^2 in the expansion of $e^{-18x}(1+4x)^n$ is -38, find n. (6 marks)

[HKDSE 2019' Section A#6]

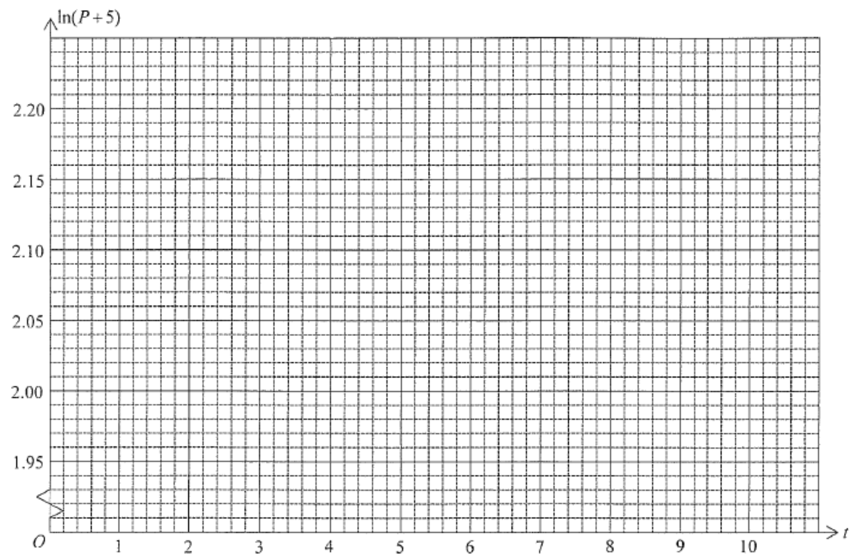
2 – Linear Relationship

1. The population P (in million) of a city can be modeled by $P = ae^{\frac{kt}{40}} - 5$, where a and k are constants and t is the number of years since the beginning of a certain year. The population of the city is recorded as follows.

t	2	4	6	8	10
P	2.36	2.81	3.23	3.55	4.01

(a) Express $\ln(P+5)$ as a linear function of t.

(b) Using the graph paper below, estimate the values of a and k. Correct your answers to the nearest integers. (5 marks)



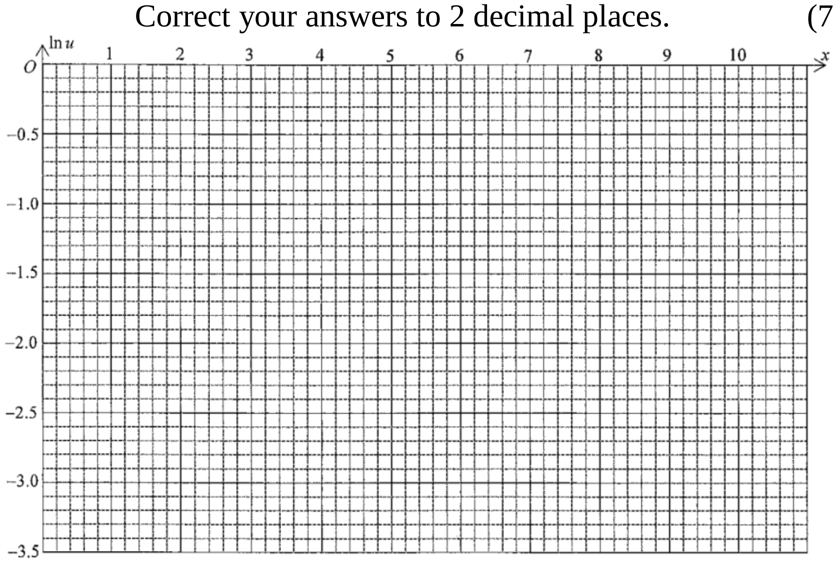
[HKDSE 2012' Section A#3]

2. After launching an advertisement for x weeks, the number y (in thousand) of members of a club can be modeled by $y = \dfrac{8(1 - ae^{-bx})}{1 + ae^{-bx}}$, where a and b are positive integers and $x \geq 0$.
The values of y when x = 2, 4, 6, 8, 10 were recorded as follows:

x	2	4	6	8	10
y	5.97	6.26	6.75	7.11	7.37

(a) Let $u = ae^{-bx}$.
  (i) Express $\ln u$ as a linear function of x.
  (ii) Find u in terms of y.

(b) It is known that one of the values of y in the above table is incorrect.
  (i) Using the graph paper to determine which value of y is incorrect.
  (ii) By removing the incorrect value of y, estimate the values of a and b. Correct your answers to 2 decimal places. (7 marks)



[HKDSE 2013' Section A#4]

3 –Application of Differentiation (A)

1. Let $y=\sqrt[3]{\dfrac{3x-1}{x-2}}$, where $x>2$.

   (a) Use logarithmic differentiation to express $\dfrac{1}{y}\cdot\dfrac{dy}{dx}$ in terms of *x*.

   (b) Using the result of (a), find $\dfrac{d^2y}{dx^2}$ when $x=3$. (6 marks)

[HKDSE 2012' Section A#4]

2. The population p (in million) of a city at time t (in years) can be modeled by

$$p = 8 - \frac{2.1}{\sqrt{t+4}} \text{ for } t \geq 0 .$$

An environment study indicates that, when the population is p million, the concentration of carbon dioxide in the air is given by

$$C = 2^p \text{ units.}$$

Find the rate of change of the concentration of carbon dioxide in the air at $t = 5$.

(4 marks)

[HKDSE 2013' Section A#2]

3. Air is leaking from a spherical balloon at a constant rate of 100 cm^3 per second. Find the rate of change of the radius of the balloon at the instant when the radius is 10 cm. (3 marks)

[HKDSE 2014' Section A#1]

4. Let $f(x)=\dfrac{x^x}{(2x+13)^6}$, where $x>1$.

(a) By considering $\ln f(x)$, find $f'(x)$.

(b) Show that $f(x)$ is increasing for $x>1$. (6 marks)

[HKDSE 2014' Section A#2]

5. Consider the curve $C : y = x\sqrt{2x^2 + 1}$.

(a) Find $\frac{dy}{dx}$.

(b) Two of the tangents to C are perpendicular to the straight line $3x + 17y = 0$. Find the equations of the two tangents. (7 marks)

[HKDSE 2015' Section A#7]

6. Consider the curve $C: y=(2x+8)^{\frac{3}{2}}+3x^2$, where $x>-4$.

(a) Find $\dfrac{dy}{dx}$.

(b) Someone claims that two of the tangents to C are parallel to the straight line $6x+y+4=0$. Do you agree? Explain your answer. (7 marks)

[HKDSE 2016' Section A#7]

7. Let $f(x) = 4x^3 + mx^2 + nx + 615$, where m and n are constants. It is given that $(6, -33)$ is a turning point of the graph of $y = f(x)$. Find

(a) m and n,

(b) the minimum value(s) and the maximum value(s) of $f(x)$. (6 marks)

[HKDSE 2017' Section A#6]

8. Consider the curve C: $y = \dfrac{x}{\sqrt{x-2}}$, where $x > 2$.

(a) Find $\dfrac{dy}{dx}$.

(b) A tangent to C passes through the point (9, 0). Find the slope of this tangent. (7 marks)

[HKDSE 2017' Section A#7]

9. Let h be a constant. Consider the curve $C: y = x^2\sqrt{h-x}$, where $0 < x < h$.

It is given that $\frac{dy}{dx} = 30$ when $x = 4$.

(a) Prove that $h = 20$.

(b) Find the maximum point(s) of C.

(c) Write down the equation(s) of the horizontal tangent(s) to C.

(7 marks)

[HKDSE 2018' Section A#7]

10. Consider the curve $C : y = (x-2)\sqrt{3x+6} - 8x$, where $x > -2$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(b) Someone claims that two of the tangents to C are horizontal lines. Do you agree? Explain your answer. (6 marks)

[HKDSE 2019' Section A#7]

4 –Application of Differentiation (B)

1. Let y be the amount (in suitable units) of suspended particulate in a laboratory. It is given that

$$(E): y = \frac{340}{2 + e^{-t} - 2e^{-2t}} \qquad (t \geq 0),$$

where t is the time (in hours) which has elapsed since an experiment started.

(a) Will the value of y exceed 171 in the long run? Justify your answer. (2 marks)

(b) Find the greatest value and least value of y. (6 marks)

(c) (i) Rewrite (E) as a quadratic equation in e^{-t}.

(ii) It is known that the amounts of suspended particulate are the same at the time $t = \alpha$ and $t = 3 - \alpha$. Given that $0 \leq \alpha < 3 - \alpha$, find α. (4 marks)

[HKDSE 2014' Section B#11]

2. In an experiment, the temperature (in $°C$) of a certain liquid can be modelled by

$$S = \frac{200}{1 + a2^{bt}},$$

where a and b are constants and t is the number of hours elapsed since the start of the experiment.

(a) Express $\ln\left(\frac{200}{S} - 1\right)$ as a linear function of t. (2 marks)

(b) It is found that the intercepts on the vertical axis and the horizontal axis of the graph of the linear function obtained in (a) are ln 4 and 4 respectively.

(i) Find a and b.

(ii) Find $\frac{dS}{dt}$ and $\frac{d^2S}{dt^2}$.

(iii) Describe how S and $\frac{dS}{dt}$ vary during the first 48 hours after the start of the experiment. Explain your answer. (11 marks)

[HKDSE 2015' Section B#12]

3. The chickens in a farm are infected by a certain bird flu. The number of chickens (in thousand) in the farm is modelled by

$$N = \frac{27}{2 + \alpha t e^{\beta t}},$$

where t (≥ 0) is the number of days elapsed since the start of the spread of the bird flu and α and β are constants.

(a) Express $\ln\left(\frac{27-2N}{Nt}\right)$ as a linear function of t. (2 marks)

(b) It is given that the slope and the intercept on the horizontal axis of the graph of the linear function obtained in (a) are -0.1 and $10 \ln 0.03$ respectively.

(i) Find α and β.

(ii) Will the number of chickens in the farm be less than 12 thousand on a certain day after the start of the spread of the bird flu? Explain your answer.

(iii) Describe how the rate of change of the number of chickens in the farm varies during the first 20 days after the start of the spread of the bird flu. Explain your answer. (10 marks)

[HKDSE 2016' Section B#12]

4. A researcher, Peter, models the number of crocodiles in a lake by

$$x = 4 + \frac{3k}{2^{\lambda t} - k},$$

where λ and k are positive constants, x is the number in thousands of crocodiles in the lake and t (≥ 0) is the number of years elapsed since the start of the research.

(a) (i) Express $(x - 4)(x - 1)$ in terms of λ, k and t.

(ii) Peter claims that the number of crocodiles in the lake does not lie between 1 thousand and 4 thousand. Is the claim correct? Explain your answer. (3 marks)

(b) Peter finds that $\dfrac{dx}{dt} = \dfrac{-\ln 2}{24}(x-4)(x-1)$.

(i) Prove that $\lambda = \dfrac{1}{8}$.

(ii) For each of the following conditions (1) and (2), find t. Also determine whether the crocodiles in the lake will eventually become extinct or not. If your answer is 'yes', find the time it will take for the crocodiles to become extinct; if your answer is 'no', estimate the number of crocodiles in the lake after a very long time.

(1) When $t = 0$, $x = 0.8$.

(2) When $t = 0$, $x = 7$. (9 marks)

[HKDSE 2017' Section B#12]

5. In an experiment, the number of certain bacteria in a room under controlled conditions is recorded. The temperature Q (in$^{\circ}$C) in the room can be modelled by the following linear function

$$Q = \ln r + (s \ln 3)t$$

where r and s are constants, t ($0 \le t \le 20$) is the number of hours elapsed since the start of the experiment. It is given that the slope and the intercept on the vertical axis of the graph of this linear function of t are $-0.1\ln 9$ and $\ln 9$ respectively.

(a) Find r and s. (2 marks)

(b) It is given that

$$Q = \ln\left(\frac{120 - 3N}{N}\right),$$

where N is the number in millions of bacteria

(i) Prove that $N = \dfrac{40}{3^{1-0.2t} + 1}$.

(ii) Is it possible that there are 4 million bacteria in the room during the experiment? Explain your answer.

(iii) Find $\dfrac{dN}{dt}$ and $\dfrac{d^2N}{dt^2}$.

(iv) Describe how $\dfrac{dN}{dt}$ varies during the experiment. Explain your answer. (11 marks)

[HKDSE 2018' Section B#12]

6. A tank contains some water. Water is now leaking from the tank. Let V m^3 be the volume of water in the tank. It is given that

$$V = \frac{64}{he^{kt} + 4} \ ,$$

where $t\ (\geq 0)$ is the number of hours elapsed since the leaking begins and h and k are constants.

(a) Express $\ln\left(\frac{64}{V} - 4\right)$ as a linear function of t. (1 mark)

(b) It is given that the graph of the linear function obtained in (a) passes through the origin and the point (2, 1). Find

(i) h and k,

(ii) $\frac{dV}{dt}$,

(iii) the value of V when $\frac{dV}{dt}$ attains its least value. (8 marks)

(c) The owner of the tank finds that $S = V^{\frac{2}{3}}$, where S m^2 is the wet total surface area of the tank.

(i) Find the value of $\frac{dS}{dt}$ when $\frac{dV}{dt}$ attains its least value.

(ii) The owner claims that $\frac{dS}{dt}$ attains its least value when $\frac{dV}{dt}$ attains its least value. Is the claim correct? Explain your answer.

(4 marks)

[HKDSE 2019' Section B#12]

5 –Application of Integration (A)

1. The rate of change of the value V (in million dollars) of a flat is given by $\frac{dV}{dt}=\frac{t}{\sqrt{4t+1}}$, where t is the number of years since the beginning of 2012. The value of the flat is 3 million dollars at the beginning of 2012. Find the percentage change in the value of the flat from the beginning of 2012 to the beginning of 2014. (5 marks)

[HKDSE 2012' Section A#2]

2. The slope of the tangent to a curve S at any point (x, y) on S is given by $\frac{dy}{dx} = e^{2x}$.

Let L be the tangent to S at the point $A(0,1)$ on S.

(a) Find the equation of S.

(b) Find the equation of L.

(c) Find the area of the region bounded by S, L and the line $x = 1$.

(7 marks)

[HKDSE 2012' Section A#5]

3. Consider the curve $C: y = x(x-2)^{\frac{1}{3}}$ and the straight line L that passes through the origin and is parallel to the tangent to C at $x = 3$.

(a) Find the equation of L.

(b) Find the x-coordinates of the two intersecting points of C and L.

(c) Find the area bounded by C and L.

[Note: For definite integrals, answers obtained by using numerical integration functions in calculators are not accepted.] (8 marks)

[HKDSE 2013' Section A#3]

4. (a) Find $\dfrac{d}{dx}(x\ln x)$.

(b) Use (a) to evaluate $\int_1^e \ln x dx$. (4 marks)

[HKDSE 2013' Section A#5]

5. The slope of the tangent to a curve S at any point (x, y) on S is given by $\frac{dy}{dx} = \left(2x - \frac{1}{x}\right)^3$, where $x > 0$. A point $P(1, 5)$ lies on S.

(a) Find the equation of the tangent to S at P.

(b) (i) Expand $\left(2x - \frac{1}{x}\right)^3$.

(ii) Find the equation of S for $x > 0$. (7 marks)

[HKDSE 2014' Section A#3]

6. Evaluate the following definite integrals:

(a) $\displaystyle\int_1^3 \frac{t+2}{t^2+4t+11}\,dt$,

(b) $\displaystyle\int_1^3 \frac{t^2+3t+9}{t^2+4t+11}\,dt$.

[Note: For definite integrals, answers obtained by using numerical integration functions in calculators are not accepted.] (6 marks)

[HKDSE 2014' Section A#4]

7. The government of a country is going to announce a new immigration policy which will last for 3 years. At the moment of the announcement, the population of the country is 8 million. After the announcement, the rate of change of the population can be modeled by

$$\frac{dx}{dt}=\frac{t\sqrt{9-t^2}}{3} \qquad (0\le t\le 3),$$

where x is the population (in million) of the country and t is the time (in years) which has elapsed since the announcement. Find x in terms of t. (5 marks)

[HKDSE 2014' Section A#5]

8. Consider the curves $C_1 : y = e^{2x} + e^4$ and $C_2 : y = e^{x+3} + e^{x+1}$.

(a) Find the *x*-coordinates of the two points of intersection of C_1 and C_2.

(b) Express, in terms of *e*, the area of the region bounded by C_1 and C_2.

(6 marks)

[HKDSE 2015' Section A#6]

9. (a) Express $\frac{d}{dx}\left((x^6+1)\ln(x^2+1)\right)$ in the form $f(x)+g(x)\ln(x^2+1)$, where $f(x)$ and $g(x)$ are polynomials.

(b) Find $\int x^5 \ln(x^2+1)dx$. (7 marks)

[HKDSE 2015' Section A#8]

10. Let $f(x) = 3^{2x} - 10(3^x) + 9$.

(a) Find $\int f(x)dx$.

(b) The equation of the curve C is $y = f(x)$. Find

(i) the two x-intercepts of C.

(ii) the exact value of the area of the region bounded by C and the x-axis. (6 marks)

[HKDSE 2016' Section A#6]

11. Define $f(x)=\frac{(\ln x)^2}{x}$ for all $x > 0$. Let α and β be two roots of the equation $f'(x)=0$, where $\alpha > \beta$.

(a) Express α in terms of e. Also find β.

(b) Using the integration by substitution, evaluate $\int_{\beta}^{\alpha} f(x)dx$. (7 marks)

HKDSE 2016' Section A#8]

12. Define $g(x)=\dfrac{1}{x}\ln\left(\dfrac{e}{x}\right)$ for all $x>0$.

(a) Using integration by substitution, find $\int g(x)dx$.

(b) Denote the curve $y = g(x)$ by Γ.

(i) Write down the x-intercept(s) of Γ.

(ii) Find the area of the region bounded by Γ, the x-axis and the straight lines $x = 1$ and $x = e^2$. (7 marks)

[HKDSE 2017' Section A#8]

13. Let $f(x)$ be a continuous function such that $f'(x)=\dfrac{12x-48}{(3x^2-24x+49)^2}$ for all real numbers x .

(a) If $f(x)$ attains it minimum value at $x=\alpha$, find α .

(b) It is given that the extreme value of $f(x)$ is 5 . Find

(i) $f(x)$,

(ii) $\lim_{x\to\infty} f(x)$. (6 marks)

[HKDSE 2018' Section A#5]

14. (a) By considering $\frac{d}{dx}(x\ln x)$, find $\int \ln x dx$.

(b) Find $\int \frac{\ln x}{x} dx$.

(c) Let C be the curve $y = \frac{(x-1)(\ln x - 1)}{x}$ where $x > 0$. Express, in term of e, the area of the region bounded by C and the x-axis. (7 marks)

[HKDSE 2018' Section A#8]

15. Define $f(x)=\dfrac{6-x}{x+3}$ for all $x>-3$.

(a) Prove that $f(x)$ is decreasing.

(b) Find $\lim_{x\to\infty} \mathrm{f}(x)$.

(c) Find the exact value of the area of the region bounded by the graph of $y=f(x)$, the x-axis and the y-axis. (6 marks)

[HKDSE 2019' Section A#5]

16. (a) Express $7^{\frac{-1}{\ln 7}}$ in terms of e .

(b) By considering $\frac{d}{dx}(x7^{-x})$, find $\int x7^{-x}dx$.

(c) Define $h(x) = x7^{-x}$ for all real numbers x . It is given that the equation $h'(x) = 0$ has only one real root α . Find α . Also express $\int_0^{\alpha} h(x)dx$ in terms of e .

(7 marks)

[HKDSE 2019' Section A#8]

6 – Application of Integration (B)

1. Let $I = \int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}} dt$.

(a) (i) Use the trapezoidal rule with 6 sub-intervals to estimate I.

(ii) Is the estimate in (a)(i) an over-estimate or under-estimate? Justify your answer. (7 marks)

(b) Using a suitable substitution, show that $I = 2\int_1^2 e^{\frac{-x^2}{2}} dx$ (3 marks)

(c) Using the above results and Standard Normal Distribution Table, show that $\pi < 3.25$. (3 marks)

[HKDSE 2012' Section B#10]

2. In a research of the radiation intensity of a city, an expert modeled the rate of change of the radiation intensity R (in suitable units) by

$$\frac{dR}{dt}=\frac{a(30-t)+10}{(t-35)^2+b}$$

where t $(0\le t\le T)$ is the number of days elapsed since the start of the research, a, b and T are positive constants.

It is know that the intensity increased to the greatest value of 6 units at t = 35, and then decreased to the level as at the start of the research at t = T. Moreover, the decrease of the intensity from $t=40$ to $t=41$ is $\ln\frac{61}{50}$ units.

(a) Find the value of a. (2 marks)

(b) Find the value of T. (4 marks)

(c) Express R in terms of t. (4 marks)

(d) For $0\le t\le 35$, when would the rate of change of the radiation intensity attain its greatest value? (4 marks)

[HKDSE 2012' Section B#11]

3. (a) Consider the function $f(x)=\ln(x^2+16)-\ln(3x+20)$ for $x>\frac{-20}{3}$.

(i) Find the range of values of x such that $f(x)<0$.

(ii) Consider the integral $I=\int_0^4 f(x)dx$.

(1) Using the trapezoidal rule with 4 subintervals, find an estimate for I.

(2) Determine whether the estimate in (1) is an over-estimate or under-estimate. Justify your answer. (8 marks)

(b) A certain species of insects lives in a certain environment. Let $N(t)$ (in thousand) be the number of the insects at time t (in months). Assume that $N(t)$ can be treated as a differentiable function when $N(t)>0$. The birth rate and death rate of the insects at time t are equal to $10\ln(t^2+16)$ and $10\ln(3t+20)$ respectively when $N(t)>0$. It is given that $N(0)=8$.

(i) Express $N'(t)$ in terms of t when $N(t)>0$

(ii) Jane claims that the species will not die out until $t=4$.
Do you agree? Justify your answer. (4 marks)

[HKDSE 2013' Section B#10]

4. Let $P(t)$ and $C(t)$ (in suitable units) be the electric energy produced and consumed respectively in a city during the time period [0, t], where t is in years and $t \geq 0$. It is known that $P'(t) = 4(4 - e^{\frac{-t}{5}})$ and $C'(t) = 9(2 - e^{\frac{-t}{10}})$. The redundant electric energy being generated during the time period [0, t] is $R(t)$, where $R(t) = P(t) - C(t)$ and $t \geq 0$.

(a) Find t such that $R'(t) = 0$. (3 marks)

(b) Show that $R'(t)$ decreases with t. (3 marks)

(c) Find the total redundant electric energy generated during the period when $R'(t) > 0$. (3 marks)

(d) The electric energy production is improved at $t = 5$. Let $Q(t)$ be the electric energy produced during the period [5, t], where $t \geq 5$, and

$$Q'(t) = \frac{(t+1)[\ln(t^2 + 2t + 3)]^3}{t^2 + 2t + 3} + 9.$$

Find the total electric energy produced for the first 3 years after the improvement. (5 marks)

[Note: For definite integrals, answers obtained by using numerical integration functions in calculators are not accepted.]

[HKDSE 2013' Section B#11]

5. (a) (i) Find $\frac{d}{dv}(ve^{-v})$.

(ii) Using (a)(i), or otherwise, show that $\int ve^{-v}dv = -e^{-v}(1+v)+C$, where C is a constant. (3 marks)

(b)

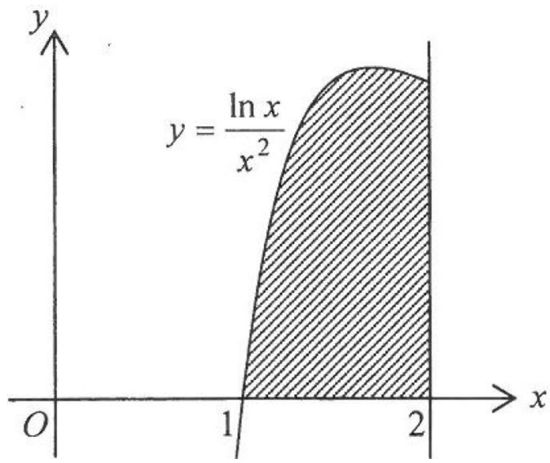


Figure 1

Figure 1 shows a shaded region bounded by the curve $y=\frac{\ln x}{x^2}$, the line $x=2$ and the x-axis. Using a suitable substitution and the result of (a), show that the area of the shaded region is $\frac{1-\ln 2}{2}$. (5 marks)

(c) (i) Find $\frac{d^2}{dx^2}\left(\frac{\ln x}{x^2}\right)$.

(ii) Using (b) and (c)(i), show that

$\frac{\ln 1.1}{1.1^2}+\frac{\ln 1.2}{1.2^2}+\frac{\ln 1.3}{1.3^2}+\cdots+\frac{\ln 1.9}{1.9^2}<5-\frac{41}{8}\ln 2$. (6 marks)

[HKDSE 2014' Section B#10]

6. An engineer models the rates of change of the amount of oil produced (in hundred barrels per day) by oil companies X and Y respectively by

$$f(t) = \ln(e^t - t) \text{ and } g(t) = \frac{8t}{1+t},$$

where $t (2 \le t \le 12)$ is the time measured in days.

(a) Using the trapezoidal rule with 5 sub-intervals, estimate the total amount of oil produced by oil company X from $t = 2$ to $t = 12$. (3 marks)

(b) Determine whether the estimate in (a) is an over-estimate or an under-estimate. Explain your answer. (3 marks)

(c) Find $\int \frac{t}{1+t} dt$. (3 marks)

(d) The engineer claims that the total amount of oil produced by oil company X from $t = 2$ to $t = 12$ is less than that of oil company Y. Do you agree? Explain your answer. (3 marks)

[HKDSE 2015' Section B#11]

7. An investment consultant, Albert, predicts the total profit made by a factory in the coming year. He models the rate of change of profit (in million dollars per month) made by the factory by

$A(t)=\ln(t^2-8t+95)$,

where $t(0\le t\le 12)$ is the number of months elapsed since the prediction begins. Let P_1 million dollars be the total profit made by the factory in the coming year under Albert's model.

(a) (i) Using the trapezoidal rule with 4 sub-intervals, estimate P_1.

(ii) Find $\frac{d^2A(t)}{dt^2}$. (4 marks)

(b) The factory manager, Christine, models the rate of change of profit (in million dollars per month) made by the factory in the coming year by

$B(t)=\frac{t+8}{\sqrt{t+3}}$,

where $t(0\le t\le 12)$ is the number of months elapsed since the prediction begins. Let P_2 million dollars be the total profit made by the factory in the coming year under Christine's model.

(i) Find P_2.

(ii) Albert claims that the difference between P_1 and P_2 does not exceed 2. Do you agree? Explain your answer. (9 marks)

[HKDSE 2016' Section B#11]

8. Let $f(x)=\dfrac{e^{0.1x}}{x}$. Define $I=\int_{0.5}^{1} f(x)dx$. In order to estimate the value of I, Ada suggests using trapezoidal rule with 5 sub-intervals while Billy suggests replacing $e^{0.1x}$ with $1 + 0.1x + 0.005x^2$ and then evaluating the integral.

(a) Find the estimates of I according to the suggestions of Ada and Billy respectively. (5 marks)

(b) Determine each of the two estimates in (a) is an over-estimate or an under-estimate. Explain your answer. (6 marks)

(c) Someone claims that the difference of I and 0.746 is less than 0.002. Do you agree? Explain your answer. (2 marks)

[HKDSE 2017' Section B#11]

9. In a research, the rate of change of the distance (in cm/s) travelled by a particle is given by

$$A(t) = 60(1+10t)e^{-2t}$$

where t is the number of seconds elapsed since the start of the research. Let D cm be the distance travelled by the particle from t = 0.1 to t = 0.5. Denote the estimate of D by using the trapezoidal rule with 4 sub-intervals by D_1.

(a) (i) Find D_1.

(ii) Is D_1 an over-estimate or an under-estimate? Explain your answer.

(6 marks)

(b) In order to estimate D, a researcher Mary, models the rate of change of the distance travelled by the particle by

$$B(t) = \frac{50(1+10t)}{1+2t}$$

where t is the number of seconds elapsed since the start of the research. Let D_2 cm be the distance travelled by the particle from t = 0.1 to t = 0.5 under this model.

(i) Find D_2.

(ii) Mary claims that in order to estimate D, D_2 is more accurate than D_1. Do you agree? Explain your answer. (6 marks)

[HKDSE 2018' Section B#11]

10. A steel factory has two machines, P and Q, for producing steel. The two machines start production at the same time. The manager of the factory models the rates of change of the amount of steel produced (in thousand tones per month) by P and Q respectively by

$$p(t) = 2t\ln(t^2+4) \text{ and } q(t) = \frac{4\ln(2e^t+1)}{e^{-t}+2} \qquad (0 \le t \le 4),$$

where t is the number of months elapsed since the steel production begins. Denote the total amount of steel produced by P in the first 4 months by α thousand tones. Let α_1 be the estimate of α by using the trapezoidal rule with 4 sub-intervals.

(a) (i) Find α_1.

(ii) Is α_1 an over-estimate or an under-estimate? Explain your answer.

(6 marks)

(b) Let β thousand tones be the total amount of steel produced by Q in the first 4 months.

(i) Using the substitution $u = \ln(2e^t+1)$, find β.

(ii) The manager claims that the total amount of steel produced by Q in the first 4 months exceeds 30% of the sum of the total amount of steel produced by P and Q in the first 4 months. Do you agree? Explain your answer. (6 marks)

[HKDSE 2019' Section B#11]

7 – Further Probability

1. Let A and B be two events such that $P(A\,|\,B)=0.4$, $P(A\cup B)=0.45$ and $P(B')=0.75$, where B' is the complementary event of B.
   (a) Find $P(A\cap B)$ and $P(A)$.
   (b) Are events A and B independent? Justify your answer. (6 marks)

[HKDSE 2014' Section A#7]

2. A and B are two events. Suppose that $P(A) = 0.3$, $P(B) = 0.28$ and $P(B'|A') = 0.6$, where A' and B' are the complementary events of A and B respectively.

(a) Find $P(A' \cap B')$ and $P(A' \cap B)$.

(b) Are A and B mutually exclusive? Explain your answer. (6 marks)

[HKDSE 2015' Section A#2]

3. Let X and Y be two events such that $P(X) = 0.4$, $P(Y) = 0.7$ and $P(Y \mid X) = 0.5$.

(a) Are X and Y independent? Explain your answer.

(b) Find $P(X \cup Y)$. (5 marks)

[HKDSE 2016' Section A#1]

4. Let A and B be two events. Suppose that $\mathrm{P}(A) = 0.2$, $\mathrm{P}(B') = 0.7$ and $\mathrm{P}(A|B) = 0.6$, where B' Is the complementary event of B.

(a) Find $\mathrm{P}(B|A)$.

(b) Are A and B mutually exclusive? Explain your answer.

(c) Are A and B independent? Explain your answer. (6 marks)

[HKDSE 2017' Section A#2]

5. Let A and B be two events. Suppose that $P(A) = 0.8$, $P(B|A) = 0.45$ and $P(B|A') = 0.6$, where A' is the complementary event of A. Find

(a) $P(B)$,

(b) $P(A|B)$,

(c) $P(A \cup B)$.

(5 marks)

[HKDSE 2018' Section A#1]

6. Let A and B be two events. Denote the complementary event of A and B by A' and B' respectively. Suppose that $\mathrm{P}(A' \cap B) = 0.12$ and $\mathrm{P}(B' | A') = 2\mathrm{P}(A)$.

(a) By considering $\mathrm{P}(A' \cap B')$, or otherwise, find $\mathrm{P}(A)$.

(b) If A and B are independent, find $\mathrm{P}(B)$. (6 marks)

[HKDSE 2019' Section A#2]

8 – Expectation and Variance

1. Let X be a discrete random variable with probability function shown below:

x	1	3	4	6	9	13
$P(X = x)$	0.1	a	0.25	0.15	b	0.05

where a and b are constants. It is known that $E(X) = 5.5$.

(a) Find the values of a and b.

(b) Let F be the event that $X \geq 4$ and G be the event that $X < 8$.

(i) Find $P(F \cap G)$.

(ii) Are F and G independent events? Justify your answer.

(6 marks)

[HKDSE 2012' Section A#8]

2. Let X and Y be two independent discrete random variables with their respective probability distributions shown as follows:

x	0	1	3	5	7
$P(X = x)$	0.2	0.3	0.3	0.1	0.1

x	1	2	4	m
$P(Y = y)$	0.4	0.3	0.2	0.1

Suppose that $E(Y) = 2.4$.

(a) Find the value of m.

(b) Let A be the event that $X + Y \leq 2$ and B be the event that $X = 0$.

(i) Find $P(A)$.

(ii) Are events A and B independent? Justify your answer.

(5 marks)

[HKDSE 2013' Section A#7]

3. Let X be a discrete random variable with probability function as shown in the following table.

x	k	0	4	6
$P(X = x)$	0.1	0.2	0.3	0.4

It is given that $E(X) = 3.4$.

(a) Find the value of k.

(b) Find $Var(3-4X)$.

(c) Let G be the event that $X < 4$ and H be the event that $X \geq -1$. Find $P(G \cap H)$. (5 marks)

[HKDSE 2014' Section A#6]

4. The table below shows the probability distribution of a discrete random variable X, where a and b are constants:

x	2	3	5	7	9
$P(X = x)$	0.08	0.15	a	0.45	b

It is given that $E(X) = 5.64$. Find

(a) a and b,

(b) $E((6-5X)^2)$ and $Var(6-5X)$. (6 marks)

[HKDSE 2015' Section A#1]

5. The table below shows the probability distribution of a discrete random variable X, where k is a constant:

x	0	2	4	5	8	9
$P(X = x)$	k^2	0.16	0.18	0.3	k	0.12

Find

(a) k.

(b) $E(X)$,

(c) $\text{Var}(2 - 3X)$. (6 marks)

[HKDSE 2017' Section A#1]

6. The table below show the probably distribution of a discrete random variable Y, where m and p are constant:

y	-2	2	m
$P(Y = y)$	p	0.25	0.5

(a) Prove that $\text{Var}(Y) = 0.25m^2 + 2$,

(b) If $\text{Var}(2Y - 1) = 8E(2Y - 1)$, find m. (7 marks)

[HKDSE 2018' Section A#4]

7. The table below shows the probability distribution of a discrete random variable X, where k is a constant:

x	8	11	k	27	32
$P(X = x)$	0.2	0.1	0.3	0.3	0.1

It is given that $\mathrm{Var}(X) = 66$. Find k, $E(3X + 5)$ and $\mathrm{Var}(3X + 5)$. (6 marks)

[HKDSE 2019' Section A#1]

9 – Discrete Probability Distribution (A)

1. The number of goals scored in a randomly selected match by a football team follows Poisson distribution with mean λ. The probability that the team scores no goals in a match is 0.1653.
   (a) Find the value of λ correct to 1 decimal place.
   (b) Find the probability that the team scores less than 3 goals in a match.
   (c) It is known that the numbers of goals scored by the team in any two matches are independent. Find the probability that the team totally scores less than 3 goals in two randomly selected matches. (5 marks)

   [HKDSE 2012' Section A#7]

2. In a shooting game, one member from each team will be selected to shoot a target three times. The team will get a prize if the target is hit at least once. Team A consists of Mabel and Owen, with the probability that Mabel is selected to shoot being 0.7. Suppose that the probabilities of Mabel and Owen to hit the target in each shot are 0.6 and 0.5 respectively.

(a) Find the probability that Team A will get a prize if Mabel is selected.

(b) Find the probability that Team A will get a prize.

(c) Given that Team A does not get a prize, find the probability that Owen is selected. (6 marks)

[HKDSE 2013' Section A#8]

3. A company produces microwave ovens by production lines A and B. It is known that 4% of all microwave ovens fail to function properly and that 2% of microwave ovens produced by line A fail to function properly. Among the microwave ovens which function properly, $\frac{2}{3}$ of them are produced by line B. Suppose a microwave oven is randomly selected.

(a) What is the probability that the microwave oven is produced by line B and functions properly?

(b) What is the probability that the microwave oven is produced by line A?

(c) If the microwave oven is produced by line B, what is the probability that it functions properly? (5 marks)

[HKDSE 2014' Section A#8]

4. A bag contains 2 white balls and 5 yellow balls. In a survey, each interviewee draws a ball randomly from the bag. If a white ball is drawn, then the interviewee consider the question ‘Are you a smoker?’. If a yellow ball is drawn, then the interviewee considers the question ‘Are you a non-smoker?’. Finally, the interviewee answers either ‘Yes’ or ‘No’. Let p be the probability that a randomly selected interviewee is a smoker.

(a) Express, in terms of p, the probability that a randomly selected interviewee answers ‘Yes’.

(b) In this survey, 50 out of 91 interviewee answer ‘Yes’.

(i) Find p.

(ii) Given that an interviewee answers ‘No’, find the probability that the interviewee is a non-smoker. (6 marks)

[HKDSE 2015’ Section A#3]

5. A manufacturer of brand *B* biscuits starts a promotion plan by giving one reward points card in each packet of biscuits. It is found that 75% of the packets of brand *B* biscuits contain 3-point cards and the rest contain 7-point cards. A total of 20 points or more can be exchanged for a gift coupon. John buys 4 packets of brand *B* biscuits and he opens them one by one.

(a) Find the probability that John gets the first 7-point card when the 4th packet of brand B biscuits has been opened.

(b) Find the probability that John can exchange for a gift coupon.

(c) Given that John can exchange for a gift coupon, find the probability that he gets a 7-point card when the 4^{th} packet of brand B biscuits has been opened. (7 marks)

[HKDSE 2015' Section A#4]

6. A box contains six cards numbered 1, 2, 3, 4, 5 and 6 respectively.

(a) Three cards are drawn randomly from the box one by one with replacement. Given that the sum of the numbers drawn is 7, find the probability that the number 1 is drawn exactly two times.

(b) If the card numbered 6 is taken away before three cards are drawn, will the probability described in (a) change? Explain your answer. (6 marks)

[HKDSE 2016' Section A#2]

7. A museum opens at 10:00. The number of visitors entering the museum in a minute follows a Poisson distribution with a mean of 1.8.

(a) Write down the variance of the number of visitors entering the museum in a minute.

(b) Find the probability that 3 visitors entered the museum in the first two minutes after the museum opens.

(c) At 10:00, only one gate at the entrance of the museum is opened. If in any two consecutive minutes, there are at least 4 visitors entering the museum in each minute, then a second gate will be opened. Find the probability that the second gate is opened three minutes after the museum opens.

(7 marks)

[HKDSE 2016' Section A#3]

8. Susan plays a game. In each trial of the game, her probability of winning a doll is 0.6. Susan plays the game until she wins a doll.

(a) Find the probability that Susan wins a doll at the 4th trial in the game.

(b) If Susan cannot win a doll in k trials, then the probability that she wins a doll within 10 trials in the game is greater than 0.95. Find the greatest value of k.

(c) In each trial of the game, Susan has to pay $15. Find the expected amount of money she has to pay to win a doll in the game. (7 marks)

[HKDSE 2017' Section A#4]

9. A Lucky draw is held in a shop. In each day, there is a big prize in this lucky draw. When the big prize is won, the lucky draw on that day stops. For each draw, the probability of winning the big prize is 0.2 .

(a) Write down the mean and the variance of the number of draws for winning the big prize in a day.

(b) Within the first 4 draws in a day, are winning the big prize and not winning the big prize of equal chance? Explain your answer.

(c) Find the probability of not winning the big prize within the first 4 draws in each day for 5 days. (7 marks)

[HKDSE 2019' Section A#3]

10. In each month, the probability that a store offers a discount to its products is 0.35 . If a discount is offered in a certain month, then the probability that the store makes a profit in that month is 0.7 ; otherwise, the probability of making a profit in that month is 0.28 .

(a) Find the probability that the store makes a profit in a certain month.

(b) Given that the store makes a profit in a certain month, find the probability that the store offers a discount in that month.

(c) Find the probability that the store makes a profit in at least 2 months out of 12 months. (6 marks)

[HKDSE 2019' Section A#4]

10 – Discrete Probability Distributions (B)

1. Drunk driving is against the law in a city. The police set up an inspection block at the entrance of a certain highway at night in order to arrest drunk drivers. From the past experience, the number of drunk drivers arrest follows a Poisson distribution with mean 2.3 per hour.

   (a) Find the probability that at least 2 drunk drivers are arrested in a certain hour. (2 marks)

   (b) Given that at least 2 drunk drivers are arrested in a certain hour, find the probability that not more than 4 drunk drivers are arrested. (3 marks)

   (c) In a certain week, the police sets up an inspection block for three nights, all at the same period from 1: 00 am to 2: 00 am. It if known that the numbers of drunk drivers arrested in different nights are independent.

      (i) Find the probability that the third night is the first night to have at least 2 drunk drivers arrested.

      (ii) Find the probability that at least 2 drunk drivers are arrested in each of the 3 nights and there are totally 10 drunk drivers arrested.

   (5 marks)

   [HKDSE 2012'Section B#13]

2. A lift company provides a regular maintenance service for every lift in an estate at the beginning of each month. Assume that the number of breakdowns of a lift in a month follows the Poisson distribution with mean 1.9. Suppose there are totally 15 lifts in the estate, and the regular maintenance service of a lift in a month is regarded as unacceptable if there are more than 2 breakdowns in that month after the regular maintenance. Assume that the monthly numbers of breakdowns of lifts are independent.

(a) Find the probability that the regular maintenance service of a randomly selected lift in a certain month in the estate is unacceptable. (2 marks)

(b) For a certain lift, find the probability that June of 2014 is the 3rd month in 2014 such that the regular maintenance service of that lift is unacceptable. (2 marks)

(c) Find the expected total number of unacceptable regular maintenance services of all lifts in the estate for one year. (2 marks)

(d) In order to assure the quality of the maintenance service provided by the lift company, the estate management office introduces the following term in the new maintenance contract for the 15 lifts, which will be effective on 1st January 2015.

For each lift in the estate, if the regular maintenance services is unacceptable for 3 consecutive months in the new contract period, one warning letter will be immediately issued to the lift company, provided that no warning letter has been issued for that lift before.

(i) For a randomly selected lift, find the probability that a warning letter will be issued to the lift company on or before 30th April 2015.

(ii) Find the probability that 3 or more warning letters will be issued to the lift company on or before 30th April 2015. (6 marks)

[HKDSE 2013'Section B#13]

3. The number of delays in a day of railway system follows the Poisson distribution with mean 4.8. Assume that the daily numbers of delays are independent.

(a) Find the probability that there are not more than 3 delays in a day. (2 marks)

(b) Find the probability that, in 3 consecutive days, there are at most 2 days with not more than 3 delays in each day. (2 marks)

(c) A day is called a *bad day* if there are more than 5 delays in that day; otherwise it is called a *good day*.

(i) Suppose today is a *bad day*. Find the mean number of *good days* between today and next *bad day*.

(ii) Find the probability that the last day of a week is the third *bad day* in that week.

(iii) Find the probability that there are at least 4 consecutive *bad days* in a week. (7 marks)

[HKDSE 2014'Section B#13]

4. The number of customers buying tickets at cinema A in a minute can be modelled by a Poisson distribution with a mean of 3.2. The probability distribution of the number of tickets bought by a customer at cinema A is shown in the following table:

Number of tickets bought	1	2	3	4	5	6	≥ 7
Probability	0.12	0.7	0.08	0.04	0.03	0.02	0.01

(a) Find the probability that fewer than 4 customers buy tickets at cinema A in a certain minute. (3 marks)

(b) Find the probability that the 8th customer buying tickets at cinema A is the 3rd customer who buys 2 tickets. (2 marks)

(c) Find the probability that exactly 3 customers buy tickets at cinema A in a certain minute and each of them buys 2 tickets. (2 marks)

(d) Find the probability that exactly 3 customers buy tickets at cinema A in a certain minute and they buy a total of 6 tickets. (3 marks)

(e) Given that fewer than 4 customers buy tickets at cinema A in a certain minute, find the probability that they buy a total of 6 tickets.

(3 marks)

[HKDSE 2015'Section B#10]

5. Tom arrives at the bus stop at 7:10. A bus arrives at 7:20 and another bus arrives at 7:30. The probability that Tom can take the bus is 0.9 each time. If Tom takes the bus at 7:20, the probability for him to be late is 0.1. If Tom takes the bus at 7:30, the probability for him to be late is 0.4. Tom will be late if he cannot take these two buses.

(a) Find the probability that Tom takes a bus on or before 7:30 on a certain day. (2 marks)

(b) Find the probability that Tom is late on a certain day. (2 marks)

(c) Find the probability that Tom is late 2 times in 6 days. (2 marks)

(d) There are 7 persons, including Tom, waiting for a lift at the lobby. If Tom is late, he will go to the second floor; otherwise he will go to the third floor. The probabilities for each of the other 6 persons to go to the second and third floor are 0.7 and 0.3 respectively. When an empty lift arrives, the 7 persons enter the lift. No person enters the lift afterwards.

(i) Find the probability that the 7 persons are going to the same floor.

(ii) Find the probability that exactly 3 persons are going to the third floor.

(iii) Given that exactly 3 persons are going to the third floor, find the probability that Tom is late. (7 marks)

[HKDSE 2016'Section B#10]

6. A department store issues a cash coupon to a customer spending at least $500 in a transaction. The details are given in the following table:

Transaction amount($$x$)	Cash coupon
$500 \le x < 1000$	$50
$1000 \le x < 2000$	$100
$x \ge 2000$	$200

At the department store, 45%, 20% and 10% of the customers each gets one cash coupon of $50, $100 and $200 respectively in a transaction. Assume that the number of transactions per minute follows a Poisson distribution with a mean of 2.

(a) Find the probability that there are at most 4 transactions at the department store in a certain minute. (3 marks)

(b) Find the probability that there are exactly 3 transactions at the department store in a certain minute and cash coupons of total value $200 are issued. (3 marks)

(c) If there are exactly 4 transactions at the department store in a certain minute, find the probability that cash coupons of total value $200 are issued by the department store in this minute. (3 marks)

(d) Given that there are at most 4 transactions at the department store in a certain minute, find the probability that cash coupons of total value $200 are issued by the department store in this minute. (3 marks)

[HKDSE 2017'Section B#10]

7. A company records the number of lateness of its staff monthly. The performance of a staff member in a month is regarded as *good* if the staff member is late for fewer than 2 times in that month. Albert is a staff member of the company. The number of lateness of Albert in a month follows a Poisson distribution with a mean of 1.8.

(a) Find the probability that Albert's performance in a certain month is *good*.
(2 marks)

(b) To improve the performance of the staff, the company launches a bonus scheme on staff performance in the coming four months. Two suggestions for the bonus scheme are listed below.

Suggestion I

Number of month with good performance	4	3	2	1	0
Bonus	\$5000	\$2500	\$1500	\$600	\$0

Suggestion II

Total number of lateness in these four month	Fewer than 5	otherwise
Bonus	\$8000	\$0

Which one of the above suggestion is more favorable to Albert? Explain your answer. (6 marks)

(c) The company also records the numbers of early leaves of its staff monthly. The number of early leaves of Albert in a month follows a Poisson distribution with a mean of λ. It is assumed that whether Albert is late and whether he leaves early are independent events.

(i) Express, in term of e and λ, the probability that Albert is late for 2 times and does not leave early in a certain month.

(ii) Given that the sum of the number of lateness and the number of early leaves of Albert in a certain month is 2, the probability that Albert is late for 2 times and does not leave early in that month is 0.36. Find λ. (5 marks)

[HKDSE 2018'Section B#10]

9. The number of matches won by a basketball team in a season follows a Poisson distribution with a mean of 3 matches per season. The points scored by the team in a match follows a normal distribution with a mean of 66 points and a standard deviation of 10 points.

(a) Find the probability that the team wins fewer than 6 matches in a certain season. (3 marks)

(b) Find the probability that the team scores higher than 70 points in a certain match. (2 marks)

(c) The team receives a certificate if the team wins a match and scores more than 70 points in that match. The team is awarded a bonus in a certain season if the team receives more than 2 certificates in that season.

(i) Find the probability that the team wins exactly 3 matches in a certain season and is awarded a bonus in that season.

(ii) If the team wins exactly 4 matches in a certain season, find the probability that the team is awarded a bonus in that season.

(iii) Given that the team wins fewer than 6 matches in a certain season, find the probability that the team is awarded a bonus in that season.

(7 marks)

[HKDSE 2019'Section B#9]

11 – Normal Distribution (A)

1. Among the students sitting for a Mathematics test, 73% of them had revised before the test. For those who had revised, their scores are real numbers which can be modeled by $N(59, 10^2)$; and for those who had not revised, their scores are real numbers which can be modeled by $N(35.2, 12^2)$. Students who scored at least 43 passed the test.
   (a) Find the probability that a randomly selected student passed the test.
   (b) Given that a randomly selected student passed the test, find the probability that he had not revised before the test.
   (c) Ten students are randomly selected among those who passed the test. Find the probability that exactly four of them had not revised before the test.

(7 marks)

[HKDSE 2012'Section A#9]

2. The lifetime of a randomly selected LED bulb produced by a manufacturer is assumed to be normally distributed with mean μ hours and standard deviation 5000 hours. It is known that 96.41% of the bulbs will have a lifetime shorter than 39000 hours.

(a) Find the value of μ.

(b) Suppose a random sample of 100 bulbs is drawn. Find the probability that the mean lifetime of the sample lies between 30200 hours and 30800 hours.

(c) The manufacturer wants to select another random sample of n bulbs such that the probability that the mean lifetime of the sample exceeding 28500 hours is at least 0.985. Find the least value of n. (7 marks)

[HKDSE 2013'Section A#9]

3. In a large farm, the weights of chickens follow a normal distribution with a mean of μ kg and a standard deviation of σ kg. It is given that the percentage of chickens being lighter than 1.83 kg is the same as the percentage of those being heavier than 3.43 kg. Moreover, 89.04 % chickens weigh between 1.83 kg and 3.43 kg.

(a) Find μ and σ.

(b) If 9 chickens are selected randomly from the farm, find the probability that the mean of their weights lies between 2.5 kg and 3.1kg (5 marks)

[HKDSE 2017'Section A#3]

4. A factory manufactures a batch of marbles. The diameters of the marbles follow a normal distribution with a mean of 9mm and a standard deviation of 0.125mm. A marble is classified as *oversized* if its diameter is more than 9.16mm.

(a) Find probability that a randomly selected marble from the batch is *oversized*.

(b) The diameter of the marbles is measured one by one. Let X be the random variable representing the number of measurements taken then the first *oversized* marble is found. Find

(i) $P(X \leq 3)$,

(ii) E(X). (6 marks)

[HKDSE 2018'Section A#3]

12 – Sampling and Confidence Interval

1. The weights (in kg) of the students in a school can be modeled by the normal distribution with mean 67 and standard deviation 15. A random sample of 36 students is taken.
   (a) Find the probability that the mean weight of the 36 students is over 70 kg.
   (b) It is found that 9 students in the sample like French fries. Find an approximate 95% confidence interval for the proportion of students in the school who like French fries. (5 marks)

[HKDSE 2012'Section A#6]

2. In a random sample of 120 swimmers in a certain beach, 75 of them are not satisfied with the water quality of the beach. Let p be the population proportion of the swimmers in this beach who are not satisfied with water quality of the beach. Find an approximate 90% confidence interval for p. (4 marks)

[HKDSE 2013'Section A#6]

3. The manager of a fitness centre wants to promote aerobic classes.

(a) The manager randomly selected 200 Hong Kong residents and found out that 80 of them had taken part in aerobic classes. Let p be the proportion of Hong Kong residents who had taken part in aerobic classes. Find an approximate 95% confidence interval for p.

(b) The manager wants to randomly select n Hong Kong residents and invite them to take part in a free aerobic class. The probability that an invited resident will show up is 0.85. Let X be the proportion of the n invited residents who will show up. Assume that X can be modeled by a normal distribution with mean 0.85 and variance $\frac{0.85(1-0.85)}{n}$. Find the maximum number of n such that the probability that more than 100 invited residents will show up is less than 0.05. (7 marks)

[HKDSE 2014'Section A#9]

4. There are many packs of seeds and each pack contains 100 seeds. Let p be the population proportion of seeds that germinate in a pack.

(a) A pack of seeds is randomly selected, 64 seeds germinate. Find an approximate 95% confidence interval for p.

(b) It is given that the proportion of seeds that germinate in these packs of seeds follows a normal distribution with mean of p and a standard deviation of 0.05. Find the least sample size to be taken such that the width of a 90% confidence interval for p is less than 0.04.

(7 marks)

[HKDSE 2016'Section A#4]

5. In an estate, Peter wants to study the proportion p of households who keep pets. He conducts a survey of a random sample of 64 households and finds an approximate β% confidence interval for p is (0.0915, 0.3085).

(a) Find

(i) the sample proportion of households who keep pets,

(ii) β.

(b) Using the sample proportion obtained in (a)(i), find the least number of household such that the probability of at least 1 of these households who keeps pets is greater than 0.99. (6 marks)

[HKDSE 2018'Section A#2]

13 – Normal Distribution (B)

1. A company provides cable-car service for tourists. Tourists complain that the waiting time for the cable-car is too long. From the past experience, the waiting time (in minutes) of a randomly selected tourists follows a normal distribution with mean μ and standard deviation 9.

   (a) The customer service manager of the company conducts a survey on the waiting time to estimate μ.

   (i) A random sample of 16 tourists is taken and their waiting times are recorded as below:

56	36	48	63	57	41	50	43
56	55	62	46	55	69	38	50

   Construct a 90% confidence interval for μ.

   (ii) Find the least sample size to be taken such that the width of the 90% confidence interval for μ is less than 6 minutes.

   (7 marks)

   (b) Suppose that $\mu = 51.5$. The customer service manager of the company interviews tourists and will give a coupon to a tourists whose waiting time is more than 65 minutes.

   (i) Find the probability that he gives less than 2 coupons to the first 10 tourists interviewed.

   (ii) Find the probability that the 5th coupon is given to the 20th tourists interviewed. (6 marks)

   [HKDSE 2012'Section B#12]

2. The cholesterol levels (in suitable units) of the adults in a city are assumed to be normally distributed with mean μ and variance σ^2. From a random sample of 49 adults, a 95% confidence interval for μ is found to be (4.596, 5.044).

(a) (i) Find the value of σ.

(ii) Find the mean of the sample. (3 marks)

(b) Another sample of 15 adults is randomly selected and their cholesterol levels are recorded as follows:

3.6 3.8 3.9 4.3 4.3 4.5 4.8 5.0

5.1 5.2 5.3 5.5 5.8 6.0 6.4

The two samples are then combined. Construct a 99% confidence interval for μ using the combined sample. (4 marks)

(c) A health organization classifies the cholesterol level of an adult to be low, medium and high if his/her cholesterol value is respectively at most 5.2, between 5.2 and 6.2, and at least 6.2. Suppose $\mu = 4.8$.

(i) Find the probability that the cholesterol level of a randomly selected adult in the city is low.

(ii) A sample of 20 adults is randomly selected in the city. Find the probability that there are more than 17 adults with low cholesterol level and at least 1 adult with medium cholesterol level in this sample. (5 marks)

[HKDSE 2013'Section B#12]

3. The delivery time X (in minutes) of an order received by a pizza restaurant follows a normal distribution with mean μ and standard deviation σ. It is known that 27.43% of the delivery times are longer than 25 minutes and 51.60% of the delivery times fall within 3.5 minutes of μ.

(a) Find μ and σ. (4 marks)

(b) If the delivery time of an order is longer than k minutes, then a coupon will be given as a compensation to the customer who has made the order. Suppose that a total of 200 orders are received in a day. Assuming independence among delivery times of different orders, find the minimum integral value of k such that the expected number of coupons given out is at most 5 in that day. (3 marks)

(c) The employees of the pizza restaurant recently received training to improve their efficiency. After training, the delivery time Y (in minutes) of an order follows a normal distribution with mean θ and standard deviation 4.7.

(i) Manager A draws a random sample of 12 orders and the delivery times (in minutes) are recorded as follows:

22 15 18 21 22 31
20 16 21 19 23 24

Construct a 90% confidence interval for θ.

(ii) Manager B is going to draw another random sample of n orders. He requires that the probability that the mean delivery time of the n orders falls within 3 minutes of θ be greater than 0.99. Find the minimum value of n to meet his requirement. (6 marks)

[HKDSE 2014'Section B#12]

4. The speeds of cars passing a checkpoint on a highway follow a normal distribution with a mean of μ km/h and a standard deviation of 16km/h.

(a) A survey on the speeds of cars to estimate μ is conducted.

(i) A random sample of 25 cars is taken and the stem-and-leaf diagram below shows the distribution of their speeds (in km/h):

Stem (tens)	Leaf (units)
6	0 0 1 1 1 2 2 3 4 4 5 5 6 6 7
7	1 1 2 3 5 5 6
8	3 6 7

Find a 95% confidence interval for μ.

(ii) Find the least sample size to be taken such that the width of a 97.5% confidence interval for μ is less than 9. (7 marks)

(b) Suppose that $\mu = 66$. If the speed of a car passing the checkpoint exceeds 90 km/h, a penalty ticket will be issued.

(i) If a car passes the checkpoint, find the probability that a penalty ticket will be issued.

(ii) If 12 cars pass the checkpoint, find the probability that more than 2 penalty tickets will be issued. (5 marks)

[HKDSE 2015'Section B#9]

5. X and Y are two schools with the same number of students. The daily reading times (in minutes) of the students in each school are assumed to be normally distributed. In school X, 0.6% of the students read less than 40 minutes daily while 1.5% read more than 70 minutes. In school Y, 1.5% of the students read less than 48 minutes daily while 1.7% read more than 72 minutes.

(a) Which school has less students reading more than 60 minutes daily? Explain your answer. (6 marks)

(b) For the school that has less students reading more than 60 minutes daily, find the probability that the 4th randomly selected student is the 2nd one who reads more than 60 minutes daily. (2 marks)

(c) Students reading T minutes or more daily will be awarded. What should the least value of T be so that no more than 10% of students are awarded in each school? Give your answer in integral minutes. (4 marks)

[HKDSE 2016'Section B#9]

6. The daily times spent on homework of the students in a school follow a normal distribution with a mean of μ hours and a standard deviation of 0.4 hour.

(a) A survey is conducted in the school to estimate μ.

(i) A sample of 40 students in the school is randomly selected and their daily times spent on homework are recorded below:

Daily time spent (x hours)	Number of students
$0.5 < x \le 1.0$	11
$1.0 < x \le 1.5$	13
$1.5 < x \le 2.0$	8
$2.0 < x \le 2.5$	5
$2.5 < x \le 3.0$	3

Find a 90% confidence interval for μ.

(ii) Find the least sample size to be taken such that the width of a 97% confidence interval for μ is at most 0.3. (7 marks)

(b) Suppose that μ = 1.48. If the daily time spent on homework of a student exceeds 2 hours, then the student has to attend homework guidance class.

(i) If a student is randomly selected from the school, find the probability that the student has to attend homework guidance class.

(ii) A sample of 15 students 1s now randomly drawn from the school and their daily times spent on homework are examined one by one. Given that more than 1 student in the sample have to attend homework guidance class, find the probability that the 10th student is the 2nd student who has to attend homework guidance class.

(6 marks)

[HKDSE 2017'Section B#9]

7. A fruit wholesaler, John, grades a batch of apples according to their weights. The following table shows the classification of the apples, where a is a constant.

Weight of an apple (W g)	$W \le a$	$a < W \le 260$	$260 < W$
Classification	*small*	*medium*	*large*

The weights of the apple follow a normal distribution with a mean of μ g and a standard deviation of 16g. It is known that 10.56% and 73.57% of the apples are *large* and *medium* respectively. Every 8 of apples are packed in a box. A box of apples is regarded as *regular* if there are at least 6 *medium* apples in the box.

(a) Find μ and a. (3 marks)

(b) Find the probability that a randomly chosen box of apples is *regular*. (2 marks)

(c) John randomly chooses 3 boxes of apples.

(i) Find the probability that these 3 boxes of apples are *regular* and there are totally 21 *medium* apples and 3 *small* apples.

(ii) Given that these 3 boxes of apple are *regular*, find the probability that there are totally 21 *medium* and 3 *small* apples.

(iii) Given that there are totally 21 *medium* apples and 3 *small* apples in these 3 boxes of apple, find the probability that these 3 boxes of apples are *regular*. (7 marks)

[HKDSE 2018'Section B#9]

10. In city H, the water consumption (in m^3) of each family in a certain month follows a normal distribution with a mean of μ m^3 and a standard deviation of 4 m^3.

(a) A survey is conducted to estimate μ.

(i) A random sample of 16 families is selected and their water consumptions (in m^3) in that month are recorded as follows:

17	17	18	19	19	20	20	21	21
21	22	23	23	23	24	24		

Find a 95% confidence interval for μ.

(ii) Find the least sample size to be taken such that the width of a 99.5% confidence interval for μ is less than 3. (7 marks)

(b) Suppose that $\mu = 20$. If the water consumption of a family in that month lies between 18 m^3 and 23 m^3, the family is regarded as *ordinary*.

(i) Find the percentage of *ordinary* families in city H.

(ii) The families in city H are randomly selected one by one and their water consumptions in that month are recorded. The recording stops when 3 ordinary families are found. Given that more than 6 families are selected in this recording process, find the probability that the water consumptions of exactly 9 families are recorded. (6 marks)