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MATH EP
M2

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MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)
MOCK EXAM 2
Question-Answer Book

Time allowed: 2½ hours
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
2. This paper consists of **TWO** sections, A and B.
3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
8. The diagrams in this paper are not necessarily drawn to scale.
9. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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Candidate Number								

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

* *

Answers written in the margins will not be marked.

SECTION A (50 marks)

2013

1. Consider the expansion of $(1 + ax)^n$, where a is a constant and n is a positive integer. The coefficient of x in the expansion is -16. The sum of the coefficients of x^2 and x^3 is -336. Find the values of a and n.

(4 marks)

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2016

2. Prove that $\dfrac{1}{x^{\frac{1}{3}}} - \dfrac{1}{(x+h)^{\frac{1}{3}}} = \dfrac{h}{x^{\frac{1}{3}}(x+h) + x^{\frac{2}{3}}(x+h)^{\frac{2}{3}} + x(x+h)^{\frac{1}{3}}}$.

Hence, find $\dfrac{d}{dx}\left(\dfrac{6}{x^{\frac{1}{3}}}\right)$ from first principles.

(5 marks)

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3. (a) Express $\frac{x^7}{x^2+1}$ in the form of $f(x) + \frac{kx}{x^2+1}$, where $f(x)$ is a polynomial and k is a constant.

(b) Using integration by parts, find $\int x^5 \ln(x^2+1)\,dx$.

(6 marks)

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4. (a) Let $f(x)$ be a continuous function defined on the interval $[0, \pi]$. Prove that

$$\int_0^{\pi} f(x)\,dx = \int_0^{\pi} f(\pi - x)\,dx .$$

(b) Evaluate $\displaystyle\int_0^{\pi}\left(1+\frac{\cos^3 x}{1+\sin x}\right)dx$.

(6 marks)

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5. (a) Let A be a 3×3 non-singular matrix. Show that $\det A \times \det(A^{-1} - xI) = -x^3 \det(A - x^{-1}I)$.

(b) Let $A = \begin{pmatrix} 4 & -11 & 12 \\ 0 & 0 & 4 \\ -2 & 7 & -2 \end{pmatrix}$.

(i) Prove that 2 is a root of $\det(A - xI) = 0$ and hence find the other roots in surd form.

(ii) Solve $\det(A^{-1} - xI) = 0$.

(7 marks)

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2016

6. (a) Solve the equation $\cos 3\theta = \sin 2\theta$ for $0° < \theta < 45°$.

   (b) Prove that $\cos 3\theta = \sin 2\theta$ can be expressed as $16 \cos^4 \theta - 20 \cos^2 \theta + 5 = 0$ for $0° < \theta < 45°$.

   (c) Using the results of (a) and (b), find the value of $\sin 54°$.

(7 marks)

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2016

7. Define $f(x) = \dfrac{x^2 + 5x + 1}{x + 2}$ for all $x \neq -2$. Denote the graph of $y = f(x)$ by C.

   (a) Find the asymptote(s) of C.

   (b) Find the equation of the normal to C at the point where C cuts the y-axis.

(7 marks)

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2015

8. (a) Using mathematical induction, prove that $\sin x \sum_{k=1}^{n} \cos 2kx = \sin nx \cos(n+1)x$ for all positive integers n.

(b) Using (a), evaluate $\sum_{k=1}^{2025} \cos \frac{k\pi}{9}$. (8 marks)

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SECTION B (50 marks)

2016

9. Let a and b be constants. Define $f(x) = e^x + \frac{a}{e^x} + bx$ for all real numbers x. Denote the curve $y = f(x)$ by C. It is given that $P(0, -1)$ is a stationary point of C. Another point Q is a minimum point of C.

   (a) Find a and b. (3 marks)

   (b) Someone claims that P is a maximum point of C. Do you agree? Explain your answer. (2 marks)

   (c) Find the coordinates of Q. (3 marks)

   (d) Find the point(s) of inflexion of C. (2 marks)

   (e) Let L be the tangent to C at Q. Find the area of the region bounded by C, L and the y-axis. (3 marks)

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10. Consider two points $A(-4, 0)$ and $B(4, 0)$. $P(x, y)$ is a moving point such that $PA + PB = 10$.

(a) Show that the equation of the locus of P is $\frac{x^2}{25}+\frac{y^2}{9}=1$.

(3 marks)

(b)

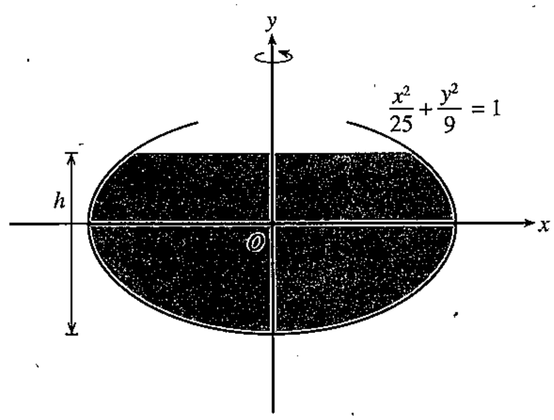

Figure 1

A container is generated by revolving part of the locus of P about the y-axis (see Figure 1). Initially, the container is empty and water is poured into the container at a constant rate of 30 cubic units per minute. At the same time, water evaporates at a constant rate of 5 cubic units per minute.

(i) When the depth of water is h units, find the volume of water in terms of h.

(ii) When the rate of change of the depth of water is minimum, find the depth of water.

(iii) When the rate of change of the depth of water is minimum, water is stopped to pour into the container. At this moment, the container is cracked and water leaks out. At time t minutes after cracking, the volume of water decreases at a rate of $\frac{\pi}{800}(t+100)$ cubic units per minute. Find the time required to dry up the water in the container.

(11 marks)

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11. Consider a system of linear equations in real variables x, y and z:

$$(E):\begin{cases} x+y=a \\ x+z=b \\ 3x+2y+z=c \end{cases}\text{, where } a, b \text{ and } c \text{ are real numbers.}$$

(a) If $c = 2a + b$, show that (E) is consistent and solve (E) in terms of a and b.

(4 marks)

(b) Consider a system of linear equations in real variables x, y and z:

$$(F):\begin{cases} x+y=2 \\ x+z=2 \\ 3x+2y+z=\alpha \\ 2x+3y-z=\beta \end{cases}\text{, where } \alpha \text{ and } \beta \text{ are real numbers.}$$

Find the values of α and β for which (F) is consistent.

(3 marks)

(c) Consider a system of linear equations in real variables x, y and z:

$$(G):\begin{cases} x+y=p \\ x+z=q \\ 3x+2y+z=7 \end{cases}\text{, where } p \text{ and } q \text{ are real numbers.}$$

If (G) is consistent and $x^3 + y^2 + z$ attains a local minimum when $z = 2$, find the values of p and q.

(5 marks)

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[2016]

12. Let $\overrightarrow{OA} = 3\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OB} = \frac{32}{5}\mathbf{i} - \frac{24}{5}\mathbf{j} + 6\mathbf{k}$ and $\overrightarrow{OC} = m\mathbf{i} + n\mathbf{j} + 15\mathbf{k}$, where m and n are constants. It is given that OC is the angle bisector of $\angle AOB$.

(a) Find m and n.

(4 marks)

(b) Let $\overrightarrow{OD} = \frac{15}{8}\mathbf{j} + \frac{17}{3}\mathbf{k}$. E is a point on the plane OAB such that DE is the altitude of the tetrahedron $OABD$ with base OAB.

(i) Find $\overrightarrow{DE}$.

(ii) Someone claims that E is the incentre of ΔOAB. Do you agree? Explain your answer.

(iii) Someone claims that E is the circumcentre of ΔOAB. Do you agree? Explain your answer.

(7 marks)

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END OF PAPER

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