HKDSE
MATH EP
M2

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HONG KONG DIPLOMA OF
SECONDARY EDUCATION EXAMINATION

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)
MOCK EXAM 5
Question-Answer Book

Time allowed: 2½ hours
This paper must be answered in English

Please stick the barcode label here.

Candidate Number									

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
2. This paper consists of **TWO** sections, A and B.
3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
8. The diagrams in this paper are not necessarily drawn to scale.
9. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

* *

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SECTION A (50 marks)

2014

1. Let $x = 4y - \cos y$, where $0 \le y \le \pi$. Find the values of x and y when $\frac{d^2y}{dx^2} = 0$.

(3 marks)

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2. (a) Using integration by parts, show that $\int e^x \cos 2x \, dx = \frac{e^x(2\sin 2x + \cos 2x)}{5} + C$, where C is a constant.

(b) Evaluate $\int_0^{\frac{\pi}{4}} 5e^x \cos^2 x \, dx$.

(5 marks)

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3. At any point (x, y) on the curve Γ, the slope of the tangent to Γ is $\sin x \cos 3x$. It is given that the y-intercept of Γ is 1. Find the equation of Γ.

(4 marks)

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2014

4. Consider a curve C: $y = \frac{1}{x} + x$.

   (a) Find $\frac{dy}{dx}$ from first principles.

   (b) Find the range of values of x when the slope of tangent to C is positive.

(4 marks)

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5. The line L: $y = \frac{1}{\ln 10}x - \frac{1}{\ln 10}$ touches the curve C: $x = 10^y$ at the point P.

(a) Find the coordinates of P.

(b) Find the area of the region bounded by the curve C, the line L and the line $y = 1$.

(6 marks)

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2013

6. Consider a continuous function $f(x) = \dfrac{x^3 + 9x}{x^2 + 1}$. It is given that

x	$x < -\sqrt{3}$	$x = -\sqrt{3}$	$-\sqrt{3} < x < 0$	$x = 0$	$0 < x < \sqrt{3}$	$x = \sqrt{3}$	$x > \sqrt{3}$
$f'(x)$	+	0	+	+	+	0	+
$f''(x)$	−	0	+	0	−	0	+

('+' and '−' denote 'positive value' and 'negative value' respectively.)

(a) Find all the maximum and/or minimum point(s) and point(s) of inflexion.

(b) Find the asymptote(s) of the graph of $y = f(x)$.

(c) Sketch the graph of $y = f(x)$.

(6 marks)

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7. Consider the following system of linear equations in real variables x, y and z:

$$(E):\begin{cases} mx+y+z=1 \\ x+my+z=2 \\ x+y+mz=4 \end{cases}\text{, where } m \text{ is a real number.}$$

(a) Find the range of values of m for which (E) has a unique solution.

(b) Someone claims that (E) cannot have infinitely many solutions. Do you agree? Explain your answer.

(5 marks)

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8. (a) Let $-\frac{\pi}{2} < k < \frac{\pi}{2}$. It is given that $\cos(x+k) - \cos(x-k) = \sin x$ for any real x. Find the value of k.

(b) Without using a calculator, find the value of $\begin{vmatrix} \cos\frac{-\pi}{6} & 0 & \cos\frac{\pi}{6} \\ \cos\frac{\pi}{6} & 2\sqrt{3} & \cos\frac{\pi}{2} \\ \cos\frac{\pi}{12} & 2\sqrt{2} & \cos\frac{5\pi}{12} \end{vmatrix}$.

(5 marks)

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9. It is given that $ABCD$ is a parallelogram. $\overrightarrow{BA} = 5\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = 2x\mathbf{i} + x\mathbf{j} + 14\mathbf{k}$, where x is a constant.

(a) Find the area of $ABCD$ in terms of x.

(b) If the area of $ABCD$ is minimized, is $ABCD$ a rectangle? Explain your answer.

(5 marks)

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10. Let $X = \begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix}$ and $Y = \begin{pmatrix} 1 & 0 \\ 13 & 27 \end{pmatrix}$.

(a) Prove, by mathematical induction, that $X^n = \begin{pmatrix} 1 & 0 \\ \dfrac{3^n - 1}{2} & 3^n \end{pmatrix}$ for all positive integers n.

(b) Hence find $\left| XY^3 \right|$.

(7 marks)

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SECTION B (50 marks)

2013

11. (a) (i) Let $a > 0$. Using a suitable substitution, show that $\int \frac{du}{u^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$, where C is a constant.

(ii) Using (a)(i), show that $\int_0^{\frac{1}{\sqrt{3}}} \frac{dx}{x^2 - \sqrt{3}x + 1} = \frac{\pi}{3}$.

(6 marks)

(b) Evaluate $\int_0^{\frac{\pi}{6}} \frac{(\tan^2\alpha + \sqrt{3}\tan\alpha + 1)(\tan^2\alpha + 1)}{\tan^4\alpha - \tan^2\alpha + 1}\,d\alpha$.

(6 marks)

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12. Let $M = \begin{pmatrix} k+1 & -1 \\ k & 0 \end{pmatrix}$ and $A = \begin{pmatrix} 1 & 1 \\ p & 1 \end{pmatrix}$, where k and p are real numbers and $p \neq 1$.

(a) (i) Express $A^{-1}MA$ in terms of k and p.

(ii) Suppose $p = k$. Using (i), show that $M^n = \dfrac{1}{1-p}\begin{pmatrix} 1-p^{n+1} & p^n - 1 \\ p - p^{n+1} & p^n - p \end{pmatrix}$, where n is a positive integer.

(8 marks)

(b) A sequence is defined by

$x_1 = 0$, $x_2 = 3$ and $x_n = 3x_{n-1} - 2x_{n-2}$ for $n = 3, 4, 5, \ldots$.

It is known that this sequence can be expressed in the matrix form

$$\begin{pmatrix} x_n \\ 2x_{n-1} \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} x_{n-1} \\ 2x_{n-2} \end{pmatrix}.$$

Using the result of (a)(ii), express x_n in terms of n.

(4 marks)

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13. In Figure 1, $ABCD$ is an isosceles trapezium with $AD \parallel BC$ and $AB = DC$. O is a point lying on AD. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. It is given that $|\mathbf{a}| = 7$, $|\mathbf{b}| = 12$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{3}$.

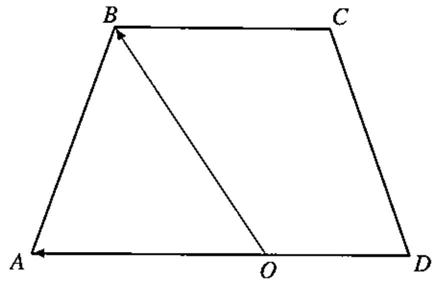

Figure 1

(a) Express $\overrightarrow{DC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(5 marks)

(b) P and Q are points lying on DC and CB respectively such that $DP : PC = CQ : QB = r : 1$ and $PQ \parallel DB$, where $r > 0$. It is given that AD is a diameter of the circle $ABCD$.

(i) Find the value of r.

(ii) Find the acute angle between AP and OB. Give the answer correct to the nearest degree.

(8 marks)

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14. Figure 2 shows a rail AOB with $\angle AOB = \frac{2\pi}{3}$ and AO lying on the horizontal plane. A rod MN of length 14 m is connected to the centres of two identical spheres of radius $\sqrt{3}$ m. The two spheres are free to slide on the rail and they touch OA and OB at P and Q respectively. It is given that the points A, B, O, P, Q, M and N lie on the same vertical plane. Let $OP = x$ m and $OQ = y$ m.

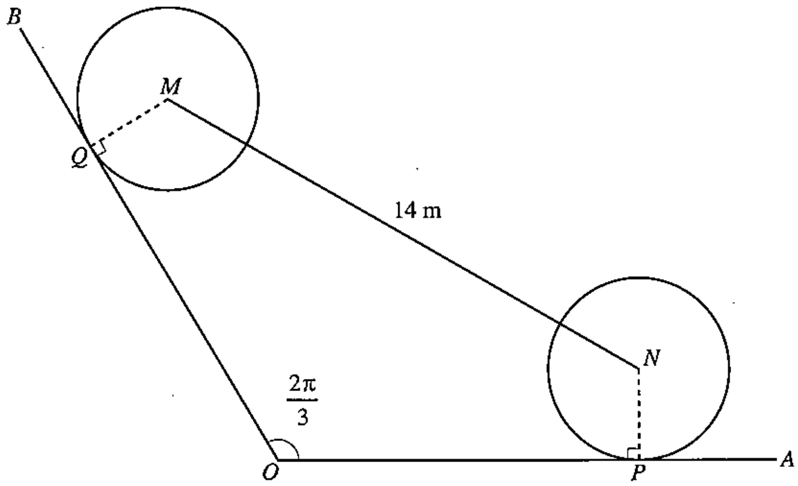

Figure 2

(a) Show that $\frac{dy}{dx} = -\frac{2x + y - 3}{x + 2y - 3}$.

(4 marks)

(b) The sphere with centre N is moving towards O at a constant speed of 2 m s^{-1} and the angle between the rod MN and the horizontal plane is θ (in radians). When $x = 11$, find the rate of increase of θ.

(9 marks)

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