More about Equations

續方程

Exercise(練習)

1. Solve $x^4 + x^2 - 20 = 0$.
解 $x^4 + x^2 - 20 = 0$。

2. Solve $x^4 - 20x^2 + 64 = 0$.
解 $x^4 - 20x^2 + 64 = 0$。

3. Solve $2x - 3\sqrt{x-1} - 4 = 0$.
解 $2x - 3\sqrt{x-1} - 4 = 0$。

4. Solve $x^6 + 28x^3 + 27 = 0$.
解 $x^6 + 28x^3 + 27 = 0$。

5. Solve $\sqrt{x-2} - x = -4$.
解 $\sqrt{x-2} - x = -4$。

6. Solve the following simultaneous equations graphically.
利用圖解法解以下的聯立方程。

$$\begin{cases} y = x^2 - 2x + 2 \\ y = 2x - 1 \end{cases}$$

7. Solve the following simultaneous equations graphically.
利用圖解法解以下的聯立方程。

$$\begin{cases} y = -x^2 + 2x - 3 \\ y = -4x + 6 \end{cases}$$

8. Plot the graph of $y = x^2 + 2x - 2$ for $-4 \le x \le 2$. Solve the following equations by drawing suitable straight lines on this graph.
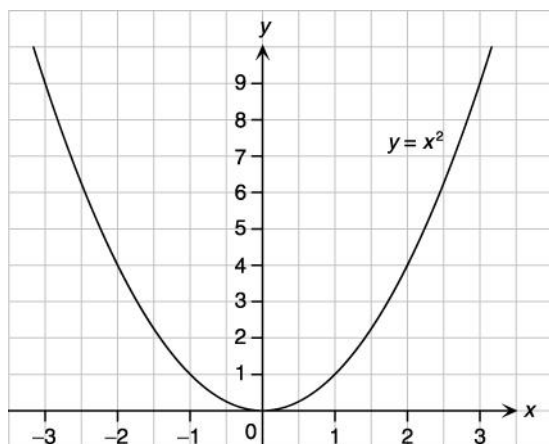
繪畫 $y = x^2 + 2x - 2$ 在 $-4 \le x \le 2$ 的圖像，並在所得的圖像中繪畫適當的直線，從而解下列各方程。

(a) $x^2 + x - 2 = 0$

(b) $x^2 + 2x - 3 = 0$

9. The figure shows the graph of $y = x^2$ for $-3 \le x \le 3$.

圖中所示為 $y = x^2$ 在 $-3 \le x \le 3$ 的圖像。

Solve the following quadratic equations graphically by drawing suitable straight lines on the graph.

在圖中繪畫適當的直線，從而解下列各二次方程。

(a) $x^2 + x - 2 = 0$

(b) $x^2 - 4x + 3 = 0$

10. Solve the following simultaneous equations graphically.

利用圖解法解以下的聯立方程。

$$\begin{cases} y = x^2 - 3x + 3 \\ y = 2x - 4 \end{cases}$$

11. Solve the following simultaneous equations.

解以下的聯立方程。

$$\begin{cases} y = x^2 + x - 2 & \text{...... (1)} \\ y = 3x + 1 & \text{...... (2)} \end{cases}$$

12. Solve the following simultaneous equations.

解以下的聯立方程。

$$\begin{cases} 2x^2 - xy = 2 & \text{...... (1)} \\ x + 2y = 8 & \text{...... (2)} \end{cases}$$

13. Plot the graph of $y = 2x^2 - x - 2$ for $-4 \le x \le 3$. Solve the following equations by drawing suitable straight lines on this graph.

繪畫 $y = 2x^2 - x - 2$ 在 $-4 \le x \le 3$ 的圖像，並在所得的圖像中繪畫適當的直線，從而解下列各方程。

(a) $2x^2 - x - 4 = 0$

(b) $x^2 + 3x - 2 = 0$

14. Suppose the graph of $y = x^2$ is given. Find the equations of straight lines needed to be drawn on the given graph so as to solve the following equations.

假設已給定 $y = x^2$ 的圖像。求在解下列各方程時，應在圖像中加上的直線的方程。

(a) $x^2 - 2x - 3 = 0$

(b) $3x^2 - 2x + 1 = 0$

15. Solve the following simultaneous equations.

解以下的聯立方程。

$$\begin{cases} y = 2x^2 - 3x - 4 \\ 2x - y - 1 = 0 \end{cases}$$

16. A two-digit positive number is increased by 45 when its digits are reversed. The sum of the squares of the two digits is 73. Find the number.

把一個二位正整數的十位數字與個位數字對調後，其數值會增加 45。若該兩個數字的平方和為 73，求該正整數。

17. It is known that the simultaneous equations $\begin{cases} y = 2x^2 + 3x + k \\ 4x - y = -2 \end{cases}$ have only one solution.

已知以下的聯立方程只有一個解。

$$\begin{cases} y = 2x^2 + 3x + k & \text{......(1)} \\ 4x - y = -2 & \text{......(2)} \end{cases}$$

(a) Find the value of k.

求 k 的值。

(b) Solve the simultaneous equations.

解該聯立方程。

18. Find the number of solutions of the following simultaneous equations.

求下列各聯立方程的解的數目。

(a) $\begin{cases} y = 2x^2 - x + 1 \\ 2x + y + 2 = 0 \end{cases}$

(b) $\begin{cases} x^2 + x = y + 2 \\ x + y = 3 \end{cases}$

19. The figure shows two concentric circles. The radius of the larger circle is 2 cm less than twice the radius of the smaller circle. The ratio of the area of the larger circle to that of the smaller circle is 9 : 4.
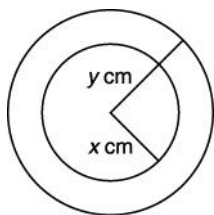
圖中所示為兩個同心圓，其中大圓的半徑較小圓的半徑的兩倍小 2 cm。已知大圓與小圓的面積的比為 9：4。

(a) Set up a pair of simultaneous equations in x and y.

建立一組以 x 和 y 為變數的聯立方程。

(b) Find x and y.

求 x 和 y。

20. Let $f(x) = x^3 - 3x^2 - 10x + 24$.

設 $f(x) = x^3 - 3x^2 - 10x + 24$。

(a) Show that $x - 2$ is a factor of $f(x)$.

證明 $x - 2$ 是 $f(x)$ 的因式。

(b) Factorize $f(x)$ completely.

因式分解 $f(x)$。

(c) Hence, solve $f(x) = 0$.

由此，解 $f(x) = 0$。

21. (a) Factorize $x^4 - 9x^2 + 20$.

因式分解 $x^4 - 9x^2 + 20$。

(b) Hence, solve the equation $x^4 + x^3 - 9x^2 - 4x + 20 = 0$.

由此，解方程 $x^4 + x^3 - 9x^2 - 4x + 20 = 0$。

22. Solve $\dfrac{2}{x-1} - \dfrac{1}{x+2} = \dfrac{1}{2}$.

解　$\dfrac{2}{x-1} - \dfrac{1}{x+2} = \dfrac{1}{2}$。

23. (a)　Factorize $x^3 - 7x - 6$ completely.
　　　　因式分解 $x^3 - 7x - 6$。

(b)　Hence, solve the equation $x^3 - 7x - 6 = 0$.
　　　由此，解方程 $x^3 - 7x - 6 = 0$。

24. (a)　Factorize $8a^3 - 27b^3$.
　　　　因式分解 $8a^3 - 27b^3$。

(b)　Hence, solve $8(x+2)^3 - 27(x-1)^3 = 0$.
　　　由此，解方程 $8(x+2)^3 - 27(x-1)^3 = 0$。

25. It is given that $f(x) = x^4 + 5x^3 + 3x^2 - 15x - 18$.
已知 $f(x) = x^4 + 5x^3 + 3x^2 - 15x - 18$。

(a)　Find two linear factors of $f(x)$.
　　　求 $f(x)$ 的兩個一次因式。

(b)　Hence, solve $f(x) = 0$.
　　　由此，解 $f(x) = 0$。

26. (a)　Factorize $x^4 - 5x^2 + 6$.
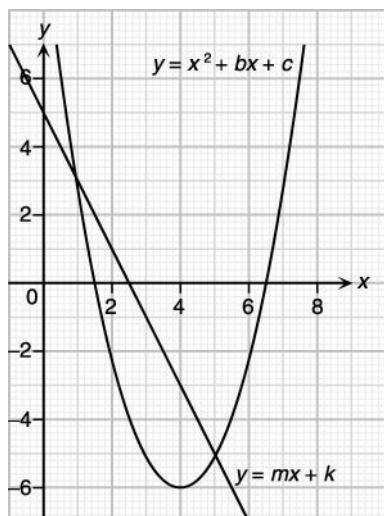　　　　因式分解 $x^4 - 5x^2 + 6$。

(b)　Hence, solve the equation $x^4 - 3x^3 - 5x^2 + 9x + 6 = 0$.
　　　(Leave your answers in surd form.)
　　　由此，解 $x^4 - 3x^3 - 5x^2 + 9x + 6 = 0$。（答案以根式表示。）

27. The figure shows the graphs of $y = x^2 + bx + c$ and $y = mx + k$.

圖中所示為 $y = x^2 + bx + c$ 及 $y = mx + k$ 的圖像。

(a) Solve the quadratic equation $x^2 + (b - m)x + (c - k) = 0$ graphically.

利用圖解法，解二次方程 $x^2 + (b - m)x + (c - k) = 0$。

**(b)** Find the values of b, m, c and k of the equation in **(a)**.

求 **(a)** 中 b、m、c 和 k 的值。

(c) If the straight line $3x + 2y = 4$ is added to the graph of $y = x^2 + bx + c$, which quadratic equation can be solved graphically?

若在同一個直角坐標平面上繪畫直線 $3x + 2y = 4$ 的圖像，問可求得哪一個二次方程的根。

28. A tent in the shape of a right triangular prism (without base) is fixed on a horizontal ground. If the height and the slant height of the tent are x m and $2x$ m respectively, and its length is $4\sqrt{3}$ m.

如圖所示，將一個三角棱柱形的帳篷（沒有底部）固定在地面上。已知該帳篷的高和斜高分別是 x m 和 $2x$ m，而長度則為 $4\sqrt{3}$ m。
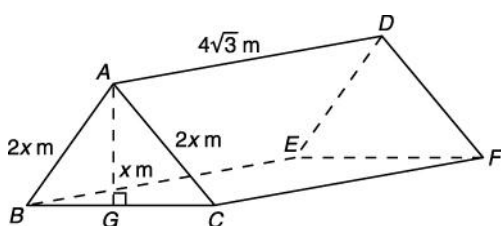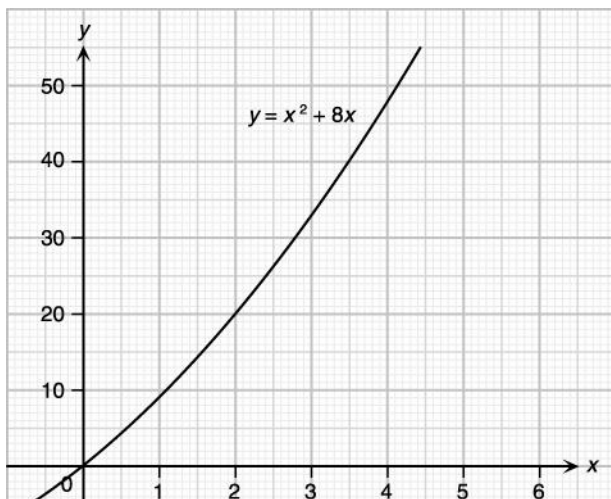

(a) Show that the surface area A (in m²) of the tent is $A = 2\sqrt{3}\,(x^2 + 8x)$.

試證明帳篷的表面面積 A (m²) 為 $A = 2\sqrt{3}\,(x^2 + 8x)$。

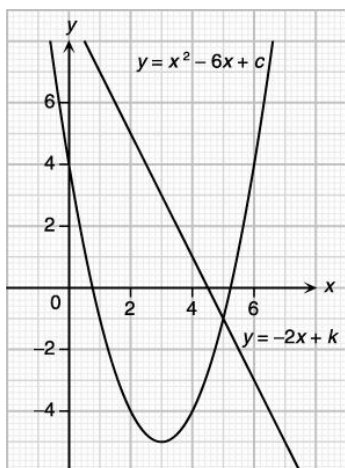**(b)** Given the graph of $y = x^2 + 8x$,

以下所示為 $y = x^2 + 8x$ 的圖像。

if the surface area of the tent is $40\sqrt{3}$ m², find the height of the tent by adding a suitable straight line on the graph.

若帳篷的表面面積為 $40\sqrt{3}$ m²，試在圖像中繪畫適當的直線，從而求帳篷的高度。

29. The figure shows the graphs of $y = x^2 - 6x + c$ and $y = -2x + k$.

圖中所示為 $y = x^2 - 6x + c$ 及 $y = -2x + k$ 的圖像。

(a) Find a solution of the simultaneous equations $\begin{cases} y = x^2 - 6x + c \\ y = -2x + k \end{cases}$ graphically.

利用圖解法，求聯立方程 $\begin{cases} y = x^2 - 6x + c \\ y = -2x + k \end{cases}$ 的一個解。

(b) Find the values of c and k.

求 c 和 k 的值。

(c) Find the other solution of the simultaneous equations by algebraic method.

利用代數方法，求 **(a)** 中聯立方程的另一個解。

Pre-requisite Questions
預備測驗

1. Solve $3x^2 - 2x = x + 3$ using the quadratic formula. (Leave your answers in surd form.)
   利用二次公式解 $3x^2 - 2x = x + 3$。(答案以根式表示。)

2. Solve $8x^2 + 10x + 8 = 5$ using the factor method.
   利用因式法解 $8x^2 + 10x + 8 = 5$。

3. Solve $3x^2 + 7x + 2 = 0$ using the factor method.
   利用因式法解 $3x^2 + 7x + 2 = 0$。

4. Solve $x^2 + x - 2 = 0$ using the quadratic formula.
   利用二次公式解 $x^2 + x - 2 = 0$。

5. **(a)** Complete the following tables according to the given equations.
   根據已給定的方程，完成下表。

   (i) $y = x - 2$

x	0	1	2
y			

   (ii) $3y - x = -4$

x			
y	-2	-1	0

   (b) Plot the graphs of $y = x - 2$ and $3y - x = -4$.
   繪畫 $y = x - 2$ 及 $3y - x = -4$ 的圖像。

   (c) Using the graphs in **(b)**, solve the simultaneous equations $\begin{cases} y = x - 2 \\ 3y - x = -4 \end{cases}$.

   利用 **(b)** 所得的圖像，解聯立方程 $\begin{cases} y = x - 2 \\ 3y - x = -4 \end{cases}$。

6. Solve the following simultaneous equations graphically.
   利用圖解法解以下的聯立方程。
   $\begin{cases} 2x - y = 1 \\ x + 2y = 8 \end{cases}$

7. Solve the following simultaneous equations using the algebraic method.
   利用圖解法解以下的聯立方程。
   $$\begin{cases} x - 3y = 7 \\ 2x - y = 5 \end{cases}$$

8. Solve the following simultaneous equations using the algebraic method.
   利用圖解法解以下的聯立方程。
   $$\begin{cases} 2x + y = 3 \\ y = x - 1 \end{cases}$$

9. Factorize the following expressions.
   因式分解下列各數式。
   (a) $54a^3 + 16b^3$ **(b)** $(x + 1)^3 - 64$

10. Factorize the following expressions.
    因式分解下列各數式。
    (a) $3x^2 - 27$ **(b)** $x - x^5$

11. Find the quotient and the remainder of $(x^3 - 3x^2 - 7x + 2) \div (x - 6)$.
    求 $(x^3 - 3x^2 - 7x + 2) \div (x - 6)$ 的商式和餘式。

12. Find the quotient and the remainder of $(2x^2 + 11x - 20) \div (x + 8)$.
    求 $(2x^2 + 11x - 20) \div (x + 8)$ 的商式和餘式。

Level 1 Questions
程度 1 題目

1. Solve $x^4 - 37x^2 + 36 = 0$.
   解 $x^4 - 37x^2 + 36 = 0$。

2. Solve $x^6 - 28x^3 + 27 = 0$.
   解 $x^6 - 28x^3 + 27 = 0$。

3. Solve $x^4 + 3x^3 - 40x^2 = 0$.
   解 $x^4 + 3x^3 - 40x^2 = 0$。

4. Solve $\sqrt{x^2 + x - 4} = 4$.

   解 $\sqrt{x^2 + x - 4} = 4$。

5. Solve $x^4 - 6x^2 + 5 = 0$. (Leave your answers in surd form if necessary.)
   解 $x^4 - 6x^2 + 5 = 0$。(如有需要，答案以根式表示。)

6. Solve $x^5 - 4x^3 - 21x = 0$. (Leave your answers in surd form if necessary.)
   解 $x^5 - 4x^3 - 21x = 0$。(如有需要，答案以根式表示。)

7. Solve $x - 2\sqrt{x} - 3 = 0$.
   解 $x - 2\sqrt{x} - 3 = 0$。

8. Solve $x - \sqrt{3x - 2} = 4$.
   解 $x - \sqrt{3x - 2} = 4$。

9. Solve $x + \sqrt{6 + x} + 4 = 0$.
   解 $x + \sqrt{6 + x} + 4 = 0$。

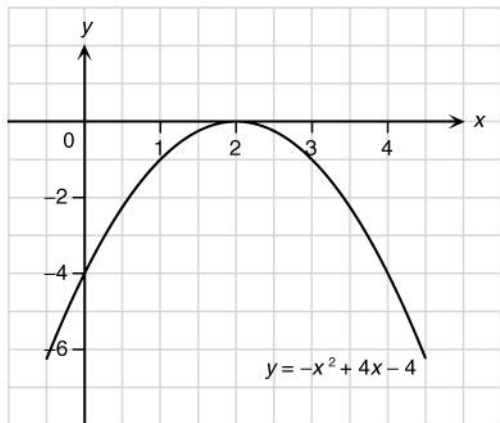**10.** Solve $7\sqrt{x + 1} - (x + 1) - 10 = 0$.
   解 $7\sqrt{x + 1} - (x + 1) - 10 = 0$。

11. Solve $x - \sqrt{2x} = 4$.
   解 $x - \sqrt{2x} = 4$。

12. The figure shows the graph of $y = -x^2 + 4x - 4$ for $-0.5 \leq x \leq 4.5$. Solve the following pairs of simultaneous equations by drawing suitable straight lines on the graph.

下圖所示為 $y = -x^2 + 4x - 4$ 在 $-0.5 \leq x \leq 4.5$ 的圖像。試在圖像中繪畫適當的直線，從而解下列各聯立方程。

$y = -x^2 + 4x - 4$

(a) $\begin{cases} y = -x^2 + 4x - 4 \\ y = x - 4 \end{cases}$ **(b)** $\begin{cases} y = -x^2 + 4x - 4 \\ y = -2x + 4 \end{cases}$

13. The figure shows the graph of $y = x^2 - 3x + 1$ for $-0.5 \leq x \leq 3.5$. Solve the following pairs of simultaneous equations by drawing suitable straight lines on the graph.
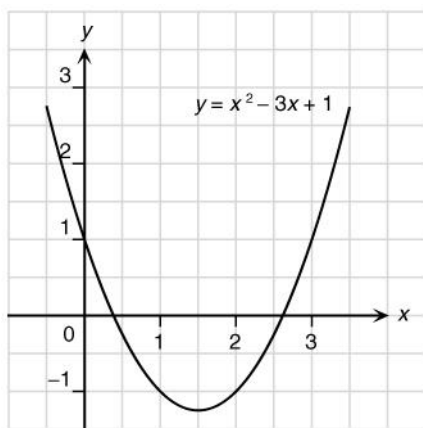
下圖所示為 $y = x^2 - 3x + 1$ 在 $-0.5 \leq x \leq 3.5$ 的圖像。試在圖像中繪畫適當的直線，從而解下列各聯立方程。

$y = x^2 - 3x + 1$

(a) $\begin{cases} y = x^2 - 3x + 1 \\ y = 1 \end{cases}$ **(b)** $\begin{cases} y = x^2 - 3x + 1 \\ y = x - 2 \end{cases}$

14. The figure shows the graph of $y = 2x^2 - 3x + 1$ for $-1.5 \le x \le 3$. Solve the following pairs of simultaneous equations by drawing suitable straight lines on the graph.
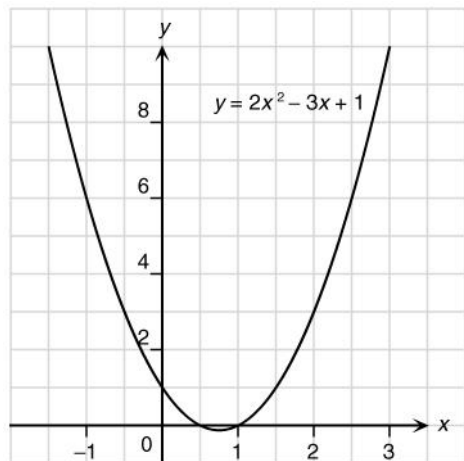
下圖所示為 $y = 2x^2 - 3x + 1$ 在 $-1.5 \le x \le 3$ 的圖像。試在圖像中繪畫適當的直線，從而解下列各聯立方程。

(a) $\begin{cases} y = 2x^2 - 3x + 1 \\ y = x + 1 \end{cases}$ **(b)** $\begin{cases} y = 2x^2 - 3x + 1 \\ y = -x + 5 \end{cases}$

15. Write a quadratic equation which can be solved by drawing the straight line $2y = x + 3$ on the graph of $y = x^2 + 7x + 4$.

假設已給定 $y = x^2 + 7x + 4$ 的圖像。若在同一個直角坐標平面上繪畫直線 $2y = x + 3$，問可求得哪一個二次方程的根？

16. Write a quadratic equation which can be solved by drawing the straight line $y = 5x - 6$ on the graph of $y = x^2 + 3$.

假設已給定 $y = x^2 + 3$ 的圖像。若在同一個直角坐標平面上繪畫直線 $y = 5x - 6$，問可求得哪一個二次方程的根？

17. Suppose the graph of $y = x^2 + 3$ is given. Find the equation of the straight line needed to be drawn on the given graph so as to solve $2x^2 - 4x + 5 = 0$.

假設已給定 $y = x^2 + 3$ 的圖像。求在解 $2x^2 - 4x + 5 = 0$ 時，應在該圖像中加上的直線的方程。

18. Suppose the graph of $y = x^2 - 5x + 9$ is given. Find the equation of the straight line needed to be drawn on the given graph so as to solve $x^2 + 3x + 2 = 0$.

假設已給定 $y = x^2 - 5x + 9$ 的圖像。求在解 $x^2 + 3x + 2 = 0$ 時，應在該圖像中加上的直線的方程。

19. Write a quadratic equation which can be solved by drawing the straight line $y = -3x + 1$ on the graph of $y = 2x^2 + x + 3$.

假設已給定 $y = 2x^2 + x + 3$ 的圖像。若在同一個直角坐標平面上繪畫直線 $y = -3x + 1$，問可求得哪一個二次方程的根？

20. Suppose the graph of $y = x^2 + x - 2$ is given. Find the equation of the straight line needed to be drawn on the given graph so as to solve $x^2 + 5x - 7 = 0$.

假設已給定 $y = x^2 + x - 2$ 的圖像。求在解 $x^2 + 5x - 7 = 0$ 時，應在該圖像中加上的直線的方程。

21. Solve the simultaneous equations $\begin{cases} y = 3x^2 + 4x + 5 \\ y = -2x + 2 \end{cases}$ using the algebraic method.

利用代數方法解聯立方程 $\begin{cases} y = 3x^2 + 4x + 5 \\ y = -2x + 2 \end{cases}$。

22. Find the number of solutions of the simultaneous equations $\begin{cases} y = 4x^2 + x \\ y = 3x - 7 \end{cases}$.

求聯立方程 $\begin{cases} y = 4x^2 + x \\ y = 3x - 7 \end{cases}$ 的解的數目。

23. Solve the simultaneous equations $\begin{cases} x^2 + y^2 = 5 \\ y = x - 1 \end{cases}$ using the algebraic method.

利用代數方法解聯立方程 $\begin{cases} x^2 + y^2 = 5 \\ y = x - 1 \end{cases}$。

24. Find the number of solutions of the simultaneous equations $\begin{cases} x^2 - y^2 = 1 \\ y = -2x + 3 \end{cases}$.

求聯立方程 $\begin{cases} x^2 - y^2 = 1 \\ y = -2x + 3 \end{cases}$ 的解的數目。

25. Given that the simultaneous equations $\begin{cases} x^2 + y^2 = k \\ y = 2x + 1 \end{cases}$ have only one solution, find the value of k.

已知聯立方程 $\begin{cases} x^2 + y^2 = k \\ y = 2x + 1 \end{cases}$ 只有一個解，求 k 的值。

26. Find the number of solutions of the simultaneous equations $\begin{cases} 9x^2 + 2x = 4 - y \\ 4x - y + 5 = 0 \end{cases}$.

求聯立方程 $\begin{cases} 9x^2 + 2x = 4 - y \\ 4x - y + 5 = 0 \end{cases}$ 的解的數目。

27. Solve the simultaneous equations $\begin{cases} y - 4x = 2x^2 - 1 \\ 3x - y = 4 \end{cases}$ using the algebraic method.

利用代數方法解聯立方程 $\begin{cases} y - 4x = 2x^2 - 1 \\ 3x - y = 4 \end{cases}$。

28. Given that the simultaneous equations $\begin{cases} y = 9x^2 - 6x + 2 \\ y = 6x + k \end{cases}$ have only one solution, find the value of k.

已知聯立方程 $\begin{cases} y = 9x^2 - 6x + 2 \\ y = 6x + k \end{cases}$ 只有一個解，求 k 的值。

29. A two-digit positive number is decreased by 27 when its digits are reversed. The product of the two digits is 10. Find the number.

把一個二位正整數的十位數字與個位數字對調後，其數值會減少 27。若該兩個數字的積為 10，求該二位正整數。

30. In the figure, a fence 18 m long is used to form three sides of the rectangle. If the enclosed area is 40 m^2, find the values of x and y.

如圖所示，長 18 m 的籬笆圍成一個長方形的三邊。若被圍起的面積是 40 m^2，求 x 和 y 的值。
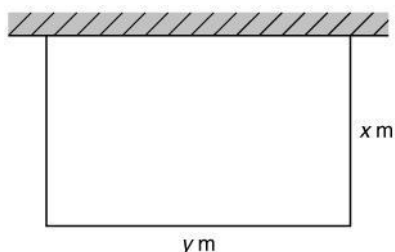
31. Let $f(x) = 2x^3 + 9x^2 + x - 12$.

設 $f(x) = 2x^3 + 9x^2 + x - 12$。

   (a) Show that $x + 4$ is a factor of $f(x)$.

   證明 $x + 4$ 是 $f(x)$ 的因式。

   (b) Factorize $f(x)$ completely.

   因式分解 $f(x)$。

   (c) Hence, solve $f(x) = 0$.

   由此，解 $f(x) = 0$。

32. The height of a right-angled triangle is longer than its base by 7 cm and the area of the triangle is 30 cm^2. Suppose the base and the height of the triangle are x cm and y cm respectively.

一個直角三角形的高度比它的底邊長 7 cm，而它的面積是 30 cm^2。設 x cm 和 y cm 分別是該三角形的底邊和高度。

   (a) Set up a pair of simultaneous equations in x and y.

   建立一組以 x 和 y 為變數的聯立方程。

   (b) Find the perimeter of the triangle.

   求該三角形的周界。

33. Let $f(x) = x^3 - 3x^2 - 9x - 5$.

    設 $f(x) = x^3 - 3x^2 - 9x - 5$。

    (a) Show that $x + 1$ is a factor of $f(x)$.

    證明 $x + 1$ 是 $f(x)$ 的因式。

    (b) Factorize $f(x)$ completely.

    因式分解 $f(x)$。

    (c) Hence, solve $f(x) = 0$.

    由此，解 $f(x) = 0$。

34. Let $f(x) = x^3 - 6x^2 + 11x - 6$.

    設 $f(x) = x^3 - 6x^2 + 11x - 6$。

    (a) Show that $x - 2$ is a factor of $f(x)$.

    證明 $x - 2$ 是 $f(x)$ 的因式。

    (b) Factorize $f(x)$ completely.

    因式分解 $f(x)$。

    (c) Hence, solve $f(x) = 0$.

    由此，解 $f(x) = 0$。

35. Let $f(x) = 2x^4 + x^3 - 9x^2 - 4x + 4$.

    設 $f(x) = 2x^4 + x^3 - 9x^2 - 4x + 4$。

    (a) Show that $x^2 - 4$ is a factor of $f(x)$.

    證明 $x^2 - 4$ 是 $f(x)$ 的因式。

    (b) Factorize $f(x)$ completely.

    因式分解 $f(x)$。

    (c) Hence, solve $f(x) = 0$.

    由此，解 $f(x) = 0$。

36. Let $f(x) = 3x^4 + 13x^3 - 16x^2 - 68x + 48$.

    設 $f(x) = 3x^4 + 13x^3 - 16x^2 - 68x + 48$。

    (a) Show that $x - 2$ and $x + 3$ are factors of $f(x)$.

    證明 $x - 2$ 和 $x + 3$ 是 $f(x)$ 的因式。

    (b) Factorize $f(x)$ completely.

    因式分解 $f(x)$。

    (c) Hence, solve $f(x) = 0$.

    由此，解 $f(x) = 0$。

37. Let $f(x) = x^4 + 2x^3 - 16x^2 - 2x + 15$.
    設 $f(x) = x^4 + 2x^3 - 16x^2 - 2x + 15$。

    (a) Show that $x + 1$ and $x - 3$ are factors of $f(x)$.
    證明 $x + 1$ 和 $x - 3$ 是 $f(x)$ 的因式。

    (b) Factorize $f(x)$ completely.
    因式分解 $f(x)$。

    (c) Hence, solve $f(x) = 0$.
    由此，解 $f(x) = 0$。

38. Let $f(x) = 10x^4 - 9x^3 - x^2 + x - 1$.
    設 $f(x) = 10x^4 - 9x^3 - x^2 + x - 1$。

    (a) Show that $2x + 1$ and $x - 1$ are factors of $f(x)$.
    證明 $2x + 1$ 和 $x - 1$ 是 $f(x)$ 的因式。

    (b) Factorize $f(x)$ completely.
    因式分解 $f(x)$。

    (c) Hence, solve $f(x) = 0$.
    由此，解 $f(x) = 0$。

39. Solve $x^3 - 8x - 7 = 0$. (Leave your answers in surd form if necessary.)
    解 $x^3 - 8x - 7 = 0$。(如有需要，答案以根式表示。)

40. Solve $2(2 - x)^3 + 128 = 0$.
    解 $2(2 - x)^3 + 128 = 0$。

41. Solve $(x + 1)^3 - 8 = 0$.
    解 $(x + 1)^3 - 8 = 0$。

42. Solve $(3x + 2)^3 - 8x^3 = 0$.
    解 $(3x + 2)^3 - 8x^3 = 0$。

Level 2　Questions
程度 2　題目

1. Solve $8x^6 + 19x^3 - 27 = 0$.
   解 $8x^6 + 19x^3 - 27 = 0$。

2. Solve $2x^4 - 19x^2 + 9 = 0$. (Leave your answers in surd form if necessary.)
   解 $2x^4 - 19x^2 + 9 = 0$。(如有需要，答案以根式表示。)

3. Solve $27x^6 + 26x^3 - 1 = 0$.
   解 $27x^6 + 26x^3 - 1 = 0$。

4. Solve $5\sqrt{3x + 2} = 3(x + 2)$.
   解 $5\sqrt{3x + 2} = 3(x + 2)$。

5. Solve $27x^4 - 51x^2 + 20 = 0$. (Leave your answers in surd form if necessary.)
   解 $27x^4 - 51x^2 + 20 = 0$。(如有需要，答案以根式表示。)

6. Solve $\dfrac{3}{(x^2 - 2x)^2} = 1 - \dfrac{2}{x^2 - 2x}$.
   解 $\dfrac{3}{(x^2 - 2x)^2} = 1 - \dfrac{2}{x^2 - 2x}$。

7. Solve $3x^4 - 11x^2 - 4 = 0$.
   解 $3x^4 - 11x^2 - 4 = 0$。

8. Solve $\sqrt{6 - 5x} - 3 = 3x - 1$.
   解 $\sqrt{6 - 5x} - 3 = 3x - 1$。

9. Solve $\dfrac{1}{x - 4} - \dfrac{1}{x - 3} = \dfrac{1}{2}$.
   解 $\dfrac{1}{x - 4} - \dfrac{1}{x - 3} = \dfrac{1}{2}$。

10. Solve $(x^2 + x)^2 + 3(x^2 + x) - 10 = 0$.
    解 $(x^2 + x)^2 + 3(x^2 + x) - 10 = 0$。

11. Solve $3x(x - 4) - (x^2 - 4x)^2 = -18$. (Leave your answers in surd form if necessary.)
    解 $3x(x - 4) - (x^2 - 4x)^2 = -18$。(如有需要，答案以根式表示。)

12. **(a)** By substituting $\sqrt{(x+6)^2+9}=u$, show that the expression

$x^2+12x+60-8\sqrt{x^2+12x+45}$ can be transformed into $u^2-8u+15$.

設 $\sqrt{(x+6)^2+9}=u$，證明 $x^2+12x+60-8\sqrt{x^2+12x+45}$ 可變換成 $u^2-8u+15$。

   (b) Hence, solve $8\sqrt{x^2+12x+45}-x^2=12(x+5)$.

由此，解 $8\sqrt{x^2+12x+45}-x^2=12(x+5)$。

13. Solve $\dfrac{1}{x+2}=2\left(\dfrac{1}{3}-\dfrac{1}{x}\right)$.

解 $\dfrac{1}{x+2}=2\left(\dfrac{1}{3}-\dfrac{1}{x}\right)$。

14. Solve $\dfrac{2}{4x+1}-\dfrac{1}{4x-1}=3$.

解 $\dfrac{2}{4x+1}-\dfrac{1}{4x-1}=3$。

15. Solve the simultaneous equations $\begin{cases} y=x^2-x-6 \\ y=x-7 \end{cases}$ graphically.

利用圖解法解聯立方程 $\begin{cases} y=x^2-x-6 \\ y=x-7 \end{cases}$。

16. Solve the simultaneous equations $\begin{cases} y=x^2+x \\ 4y-x-7=0 \end{cases}$ graphically.

(Give your answers correct to the nearest 0.2.)

利用圖解法解聯立方程 $\begin{cases} y=x^2+x \\ 4y-x-7=0 \end{cases}$。

(答案須準確至最接近的 0.2。)

17. Solve the simultaneous equations $\begin{cases} y=x^2+1 \\ 2y-x=1 \end{cases}$ graphically.

利用圖解法解聯立方程 $\begin{cases} y=x^2+1 \\ 2y-x=1 \end{cases}$。

18. Suppose the graph of $y=4x^2-x-1$ is given. Find the equation of the straight line needed to be drawn on the given graph so as to solve $x^2+3x-2=0$.

假設已給定 $y=4x^2-x-1$ 的圖像。求在解 $x^2+3x-2=0$ 時，應在該圖像中加上的直線的方程。

19. Suppose the graph of $y = 2x^2 - 3x + 5$ is given. Find the equation of the straight line needed to be drawn on the given graph so as to solve $4x^2 + 6x + 13 = 0$.

    假設已給定 $y = 2x^2 - 3x + 5$ 的圖像。求在解 $4x^2 + 6x + 13 = 0$ 時，應在該圖像中加上的直線的方程。

20. Suppose the graph of $y = 3x^2 + 2x - 6$ is given. Find the equation of the straight line needed to be drawn on the given graph so as to solve $2x^2 - x + 1 = 0$.

    假設已給定 $y = 3x^2 + 2x - 6$ 的圖像。求在解 $2x^2 - x + 1 = 0$ 時，應在該圖像中加上的直線的方程。

21. **(a)** Given $y = x^2 - 3x + 2$, complete the table for the corresponding values of y.

    已知 $y = x^2 - 3x + 2$，在下表中寫出對應的 y 值。

x	–2	–1	0	1	2	3	4
y							

    (b) Draw the graph of $y = x^2 - 3x + 2$ for $-2 \le x \le 4$.

    繪畫 $y = x^2 - 3x + 2$ 在 $-2 \le x \le 4$ 的圖像。

    (c) Solve the following quadratic equations by drawing suitable straight lines on the graph obtained in **(b)**.

    在 **(b)** 所得的圖像中繪畫適當的直線，從而解下列各二次方程。

    (i) $x^2 - 3x - 2 = 0$

    (ii) $x^2 - 2x - 3 = 0$

    (iii) $2x^2 + 3x - 2 = 0$

    (Give your answers correct to 1 decimal place.) （答案須準確至一位小數。）

22. Solve the simultaneous equations $\begin{cases} 3x - 2y + 1 = 0 \\ 4y^2 + 6x = -3 \end{cases}$ using the algebraic method.

    利用代數方法解聯立方程 $\begin{cases} 3x - 2y + 1 = 0 \\ 4y^2 + 6x = -3 \end{cases}$。

23. **(a)** Plot the graph of $y = x^2 + 4x - 3$ for $-5 \le x \le 1$.

    繪畫 $y = x^2 + 4x - 3$ 在 $-5 \le x \le 1$ 的圖像。

    (b) Solve $2x^2 + 7x - 8 = 0$ graphically. (Give your answers correct to 1 decimal place.)

    利用圖解法解 $2x^2 + 7x - 8 = 0$。(答案須準確至一位小數。)

24. Given that the simultaneous equations $\begin{cases} 3y^2 + 2x - k = 0 \\ y = x - 5 \end{cases}$ have no real solutions, find the range of possible values of k.

    已知聯立方程 $\begin{cases} 3y^2 + 2x - k = 0 \\ y = x - 5 \end{cases}$ 沒有實數解，求 k 值的可能範圍。

25. Solve the simultaneous equations $\begin{cases} x + y - 2 = 0 \\ 5x^2 - 2(y^2 + 3x) = 0 \end{cases}$ using the algebraic method.

    利用代數方法解聯立方程 $\begin{cases} x + y - 2 = 0 \\ 5x^2 - 2(y^2 + 3x) = 0 \end{cases}$。

26. Given that the simultaneous equations $\begin{cases} y = 4x(x - 2) \\ y + 9 = 2kx \end{cases}$ have only one solution, find the value(s) of k.

    已知聯立方程 $\begin{cases} y = 4x(x - 2) \\ y + 9 = 2kx \end{cases}$ 只有一個解，求 k 的值。

27. The graphs of $y = mx - 11$ and $y = x^2 + 2x - 7$ intersect at only one point P.

    $y = mx - 11$ 的圖像與 $y = x^2 + 2x - 7$ 的圖像只相交於一點 P。

    (a) Find the values of m.

    求 m 的值。

    (b) For each of the values of m obtained in **(a)**, find the coordinates of P.

    利用從 **(a)** 所得的 m 值，求 P 的坐標。

28. Given that the simultaneous equations $\begin{cases} 5x + k = y^2 \\ x + y - 1 = 0 \end{cases}$ have two distinct real solutions, find the range

    of possible values of k.

    已知聯立方程 $\begin{cases} 5x + k = y^2 \\ x + y - 1 = 0 \end{cases}$ 有兩個相異的實數解，求 k 值的可能範圍。

29. It is given that the following simultaneous equations have real solutions.

    已知以下的聯立方程有實數解。

    $\begin{cases} y = kx^2 + 6 \\ 7x - y - 8 = 0 \end{cases}$

    (a) Find the range of possible values of k.

    求 k 值的可能範圍。

    (b) For the maximum value of k, solve the simultaneous equations.

    當 k 為最大值時，求該聯立方程的解。

30. Find the minimum value of k so that the following simultaneous equations have real solutions.

    求 k 的最小值，使以下的聯立方程有實數解。

    $\begin{cases} y = 3x - 8 \\ y = x^2 + 7x - k \end{cases}$

31. A wire 40 cm long is bent to form two squares. If the sum of their areas is 52 cm^2, find the lengths of the sides of the two squares.

把一條長 40 cm 的鐵絲屈曲，造成兩個正方形。若該兩個正方形的面積之和是 52 cm^2，求它們的邊長。

32. Given a two-digit positive number, its units digit is greater than its tens digit by 4, and the sum of squares of its digits is 58. Find the number.

已知一個二位正整數的個位數字比十位數字大 4，而兩個數字的平方和為 58，求該二位正整數。

33. It is given that $f(x) = x^3 + 8x^2 + 5x - 14$.

已知 $f(x) = x^3 + 8x^2 + 5x - 14$。

 (a) Factorize $f(x)$ completely.

   因式分解 $f(x)$。

 (b) Hence, solve $f(x) = 0$.

   由此，解 $f(x) = 0$。

34. The perimeter of a rectangle is 42 cm. The product of the lengths of its two diagonals is 225 cm^2. Find the length and the width of the rectangle.

一個長方形的周界是 42 cm，其兩條對角線的長度的積是 225 cm^2。求該長方形的長度和闊度。

35. It is given that $f(x) = x^4 + 2x^3 - 14x^2 + 7x + 10$.

已知 $f(x) = x^4 + 2x^3 - 14x^2 + 7x + 10$。

 (a) Factorize $f(x)$ completely.

   因式分解 $f(x)$。

 (b) Hence, solve $f(x) = 0$.

   (Leave your answers in surd form if necessary.)

   由此，解 $f(x) = 0$。(如有需要，答案以根式表示。)

36. It is given that $f(x) = x^4 + 3x^3 - 2x^2 - 10x - 12$.

已知 $f(x) = x^4 + 3x^3 - 2x^2 - 10x - 12$。

 (a) Factorize $f(x)$ completely.

   因式分解 $f(x)$。

 (b) Hence, solve $f(x) = 0$.

   由此，解 $f(x) = 0$。

37. Solve the equation $x^4 + 4x^3 - x^2 - 16x - 12 = 0$ by regrouping terms.

    (Hint: $x^4 + 4x^3 - x^2 - 16x - 12 = (x^4 - x^2 - 12) + (4x^3 - 16x)$)

    透過適當的併項，解方程 $x^4 + 4x^3 - x^2 - 16x - 12 = 0$。

    (提示：$x^4 + 4x^3 - x^2 - 16x - 12 = (x^4 - x^2 - 12) + (4x^3 - 16x)$)

38. Solve $(3x - 1)^3 + 8(x + 6)^3 = 0$.

    解 $(3x - 1)^3 + 8(x + 6)^3 = 0$。

39. Solve the equation $x^4 - 4x^3 + x^2 + 8x - 6 = 0$ by regrouping terms.

    (Leave your answers in surd form if necessary.)

    (Hint: $x^4 - 4x^3 + x^2 + 8x - 6 = (x^4 + x^2 - 6) - (4x^3 - 8x)$)

    透過適當的併項，解方程 $x^4 - 4x^3 + x^2 + 8x - 6 = 0$。(如有需要，答案以根式表示。)

    (提示：$x^4 - 4x^3 + x^2 + 8x - 6 = (x^4 + x^2 - 6) - (4x^3 - 8x)$)

40. Solve $81(x + 1)^3 - 3(x - 3)^3 = 0$.

    解 $81(x + 1)^3 - 3(x - 3)^3 = 0$。

Level 2+ Questions
程度 2+ 題目

1. Mary wants to wrap a gift for her mother. She has a rectangular wrapping paper that its length and width are in the ratio 4 : 3. She cuts two adjacent sides by k cm, and the area of the rectangular paper used is half of the original paper. Let x cm and y cm be the length and the width of the original wrapping paper respectively.

    明慧正在包裝一份送給媽媽的禮物，她有一張長度和闊度比例為 4：3 的長方形包裝紙。她從包裝紙的兩條鄰邊各剪下 k cm，所用去的長方形包裝紙的面積是原來的一半。設原來的包裝紙的長度和闊度分別是 x cm 和 y cm。
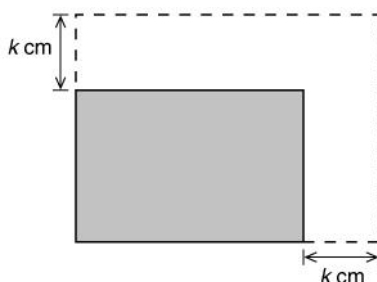
    (a) Set up a pair of simultaneous equations in x and y.
    建立一組以 x 和 y 為變數的聯立方程。

    (b) Express x and y in terms of k.
    試以 k 表示 x 和 y。

    (c) **(i)** If the perimeter of the original wrapping paper is 112 cm, find the value of k.
    若原來的包裝紙的周界是 112 cm，求 k 的值。

    (ii) Hence, find the area of the original wrapping paper.
    由此，求原來的包裝紙的面積。

2. May is Mike's daughter. The product of their ages is 210. After k years, Mike will be twice as old as May. Let x and y be the ages of May and Mike respectively.

    美儀是偉強的女兒，他們二人的年齡的積是 210。k 年後，偉強的年齡是美儀的兩倍。設美儀和偉強的歲數分別是 x 和 y。

    (a) Set up a pair of simultaneous equations in x and y.
    建立一組以 x 和 y 為變數的聯立方程。

    (b) Will Mike be twice as old as May 10 years later? Briefly explain your answer.
    10 年後，偉強的年齡會是美儀的兩倍嗎？試說明理由。

    (c) If Mike is 29 years older than May, how many years later will Mike be twice as old as May?
    若偉強比美儀大 29 歲，問多少年後偉強的年齡是美儀的兩倍？

3. There are k candies shared among x children, each of them gets y candies and one candy is left. If one more child comes, they can share all candies, and the number of candies each child gets is equal to the number of children.

    有 x 位小朋友分享 k 顆糖果，每人可分得 y 顆糖果，還餘下一顆。若多一位小朋友到來，他們便可以將糖果平分，而每人得到的糖果的數目等於小朋友的人數。

    (a) Set up a pair of simultaneous equations in x and y.

    建立一組以 x 和 y 為變數的聯立方程。

    (b) Express y in terms of x.

    試以 x 表示 y。

    (c) If there are 3 more candies shared among x children now, each child can get one more candy without any candies left. Find the values of k, x and y.

    若現在有多 3 顆糖果給 x 位小朋友分享，每人可多分一顆，且不會餘下糖果。求 k、x 和 y 的值。

4. Solve $\sqrt{x+2} + \sqrt{2x-3} = 3$.

    解 $\sqrt{x+2} + \sqrt{2x-3} = 3$。

5. Given is an identity $ax^4 - (3a+2b)x^2 + 4 \equiv (x^2 - b)[(b+1)x^2 - 3] - 2b$.

    已知恆等式 $ax^4 - (3a+2b)x^2 + 4 \equiv (x^2 - b)[(b+1)x^2 - 3] - 2b$。

    (a) Find the values of a and b.

    求 a 和 b 的值。

    (b) Hence, solve the simultaneous equations $\begin{cases} y = 5x^4 - 22x^2 + 6 \\ y = x^2 - 6 \end{cases}$.

    (Leave your answers in surd form if necessary.)

    由此，解聯立方程 $\begin{cases} y = 5x^4 - 22x^2 + 6 \\ y = x^2 - 6 \end{cases}$。 (如有需要，答案以根式表示。)

6. **(a)** It is given that the graphs of $y = x^2$ and $y - k^2 = m(x - k)$, where $m \neq 0$, intersect at only one point.

已知 $y = x^2$ 的圖像與 $y - k^2 = m(x - k)$ 的圖像 (其中 $m \neq 0$) 只相交於一點。

    (i) Express m in terms of k.

    試以 k 表示 m。

    (ii) If the two graphs intersect at $P(3, 9)$, write down the equation of the straight line in the form $y = ax + b$.

    若該兩個圖像相交於 $P(3, 9)$，試以 $y = ax + b$ 的形式寫出直線的方程。

(b) **(i)** By translating the straight line obtained in **(a)(ii)** in the direction of the y-axis, suggest a straight line such that it has no intersections with the graph of $y = x^2$.

    透過把 **(a)(ii)** 中所得的直線沿 y 軸平移，試舉出一條與 $y = x^2$ 的圖像不相交的直線。

    (ii) By translating the straight line obtained in **(a)(ii)** in the direction of the x-axis, suggest a straight line such that it has two intersections with the graph of $y = x^2$.

    透過把 **(a)(ii)** 中所得的直線沿 x 軸平移，試舉出一條與 $y = x^2$ 的圖像相交於兩點的直線。

7. Let $f(x) = 3x^2 - (a+3)x + a$ and $g(x) = \dfrac{x}{b-x}$. It is given that $f(-3) = 32$ and

$g\left(\dfrac{a}{1-a}\right) = \dfrac{a}{1-2a}$.

設 $f(x) = 3x^2 - (a+3)x + a$ 及 $g(x) = \dfrac{x}{b-x}$。已知 $f(-3) = 32$ 及 $g\left(\dfrac{a}{1-a}\right) = \dfrac{a}{1-2a}$。

(a) Find the values of a and b.

求 a 和 b 的值。

(b) Prove that $g\left(\dfrac{x}{1-kx}\right) = \dfrac{x}{1-(k+1)x}$, where k is a constant.

證明 $g\left(\dfrac{x}{1-kx}\right) = \dfrac{x}{1-(k+1)x}$，其中 k 是一個常數。

(c) Solve the equation $f(x) - 8(1-x)g\left(\dfrac{x}{1-2x}\right) = 0$.

解方程 $f(x) - 8(1-x)g\left(\dfrac{x}{1-2x}\right) = 0$。

8. **(a)** Show that the simultaneous equations $\begin{cases} 2x - \sqrt{x+k} = y - 1 \\ y = x + 2 \end{cases}$ have two distinct real solutions for
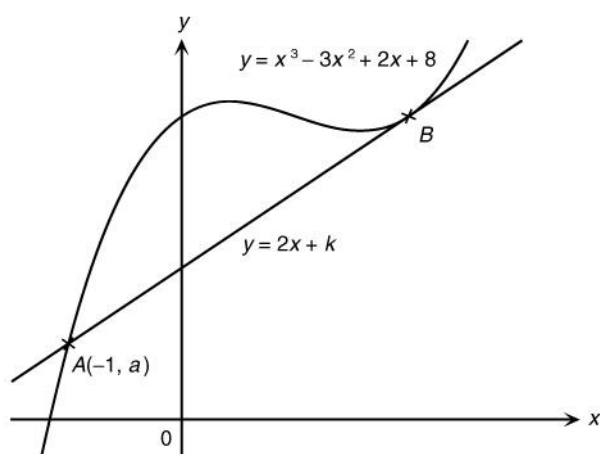
   $-\dfrac{5}{4} < k \le -1$.

   對於 $-\dfrac{5}{4} < k \le -1$，證明聯立方程 $\begin{cases} 2x - \sqrt{x+k} = y - 1 \\ y = x + 2 \end{cases}$ 有兩個相異的實數解。

   (b) Suggest a value of k such that the simultaneous equations have no real solutions.
   試舉出一個 k 值，使該聯立方程沒有實數解。

9. The graph of $y = x^3 - 3x^2 + 2x + 8$ intersects the straight line $y = 2x + k$ at $A(-1, a)$ and B.
   $y = x^3 - 3x^2 + 2x + 8$ 的圖像與直線 $y = 2x + k$ 相交於 $A(-1, a)$ 和 B。

   (a) Find the values of a and k.
   求 a 和 k 的值。

   (b) Find the coordinates of B.
   求 B 的坐標。

   (c) **(i)** Suggest a straight line such that it has only one intersection with the graph of
   $y = x^3 - 3x^2 + 2x + 8$.
   試舉出一條與 $y = x^3 - 3x^2 + 2x + 8$ 的圖像只相交於一點的直線。

   (ii) Suggest a straight line such that it has three intersections with the graph of
   $y = x^3 - 3x^2 + 2x + 8$.
   試舉出一條與 $y = x^3 - 3x^2 + 2x + 8$ 的圖像相交於三點的直線。

Multiple Choice Questions
多項選擇題

1. Solve $x^4 - 3x^2 - 54 = 0$.

解 $x^4 - 3x^2 - 54 = 0$。

A. $x = -6$ or 9

B. $x = -3$ or 3

C. $x = -\sqrt{6}$ or $\sqrt{6}$

D. $x = -3, -\sqrt{6}, \sqrt{6}$ or 3

2. Solve $\dfrac{1}{x-1} - \dfrac{1}{x} = \dfrac{1}{6}$.

解 $\dfrac{1}{x-1} - \dfrac{1}{x} = \dfrac{1}{6}$。

A. $x = 2$

B. $x = 3$

C. $x = -2$ or 3

D. $x = -3$ or 2

3. Solve $x - 2 = 5\sqrt{x-2} - 6$.

解 $x - 2 = 5\sqrt{x-2} - 6$。

A. $x = 6$

B. $x = 11$

C. $x = 6$ or 11

D. No real roots　沒有實根

4. Solve $x(x+1)^2 - 25x = 0$.

解 $x(x+1)^2 - 25x = 0$。

A. $x = -4$ or 6

B. $x = -6$ or 4

C. $x = -4, 0$ or 6

D. $x = -6, 0$ or 4

5. Find the range of possible values of k such that the equation $x^4 + 5x^2 + k = 0$ has real roots.

求 k 值的可能範圍，使方程 $x^4 + 5x^2 + k = 0$ 有實根。

A. $k \le \dfrac{25}{4}$

B. $k > \dfrac{25}{4}$

C. $k \le 0$

D. $k > 0$

6. Solve $\dfrac{4}{x^4} - \dfrac{65}{x^2} + 16 = 0$.

解 $\dfrac{4}{x^4} - \dfrac{65}{x^2} + 16 = 0$。

A. $x = -4, -\dfrac{1}{2}, \dfrac{1}{2}$ or 4

B. $x = -2, -\dfrac{1}{4}, \dfrac{1}{4}$ or 2

C. $x = \dfrac{1}{4}$ or 16

D. $x = \dfrac{1}{16}$ or 4

7. Solve $8x^6 + 63x^3 - 8 = 0$.

解 $8x^6 + 63x^3 - 8 = 0$。

A. $x = -\dfrac{1}{2}$ or $\dfrac{1}{2}$

B. $x = -2$ or 2

C. $x = -\dfrac{1}{2}$ or 2

D. $x = -2$ or $\dfrac{1}{2}$

8. If –2 is a root of $a(x + 1)^6 - b(x + 1)^3 + c = 0$, which of the following is true?

若 –2 是 $a(x + 1)^6 - b(x + 1)^3 + c = 0$ 的一個根，下列哪一項正確?

A. $a = b + c$

B. $c = a + b$
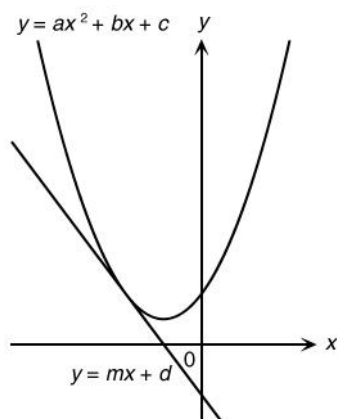
C. $a - b + c = 0$

D. $a + b + c = 0$

9. Find the quadratic equation which can be solved by drawing the straight line $2x + y = 9$ on the graph of $y = x^2 - 7x + 3$.

若在 $y = x^2 - 7x + 3$ 的圖像中繪畫直線 $2x + y = 9$，問可解哪一個二次方程?

A. $x^2 - 9x + 12 = 0$

B. $x^2 - 9x - 6 = 0$

C. $x^2 - 5x + 12 = 0$

D. $x^2 - 5x - 6 = 0$

10. The figure shows the graphs of $y = ax^2 + bx + c$ and $y = mx + d$. Which of the following MUST be true?

下圖所示為 $y = ax^2 + bx + c$ 和 $y = mx + d$ 的圖像。下列哪項必為正確?

I. $b^2 > 4ac$

II. $b^2 = 4ac$

III. $(b - m)^2 > 4a(c - d)$

IV. $(b - m)^2 = 4a(c - d)$

A. II only　只有 II

B. IV only　只有 IV

C. I and III only　只有 I 及 III

D. II and IV only　只有 II 及 IV

11. Find the quadratic equation which can be solved by drawing the straight line $3x - 2y = 4$ on the graph of $y = 2x^2 - 5x + 1$.

若在 $y = 2x^2 - 5x + 1$ 的圖像中繪畫直線 $3x - 2y = 4$，問可解哪一個二次方程?

A. $4x^2 - 13x + 2 = 0$

B. $4x^2 + 13x + 2 = 0$

C. $4x^2 - 13x + 6 = 0$

D. $4x^2 + 13x + 6 = 0$

12. Find the equation of the straight line that should be drawn on the graph of $y = 3x^2 + x - 8$ to solve $3x^2 - x - 5 = 0$.
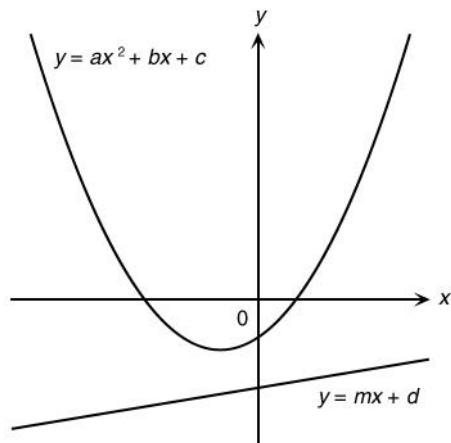
假設已給定 $y = 3x^2 + x - 8$ 的圖像。求在解方程 $3x^2 - x - 5 = 0$ 時，應在該圖像中加上的直線的方程。

A. $y = 2x + 3$

B. $y = 2x - 3$

C. $y = 3x + 2$

D. $y = 3x - 2$

13. The figure shows the graphs of
$y = ax^2 + bx + c$ and $y = mx + d$. Which of the following MUST be true?

下圖所示為 $y = ax^2 + bx + c$ 和 $y = mx + d$ 的圖像。下列哪項必為正確?

I. $(b - m)^2 > 4a(c - d)$

II. $b^2 > 4ac$

III. The equation $ax^2 + (b - m)x + (c - d) = 0$ has no real roots.

A. I only　只有 I

B. II only　只有 II

C. I and II only　只有 I 及 II

D. II and III only 只有 II 及 III

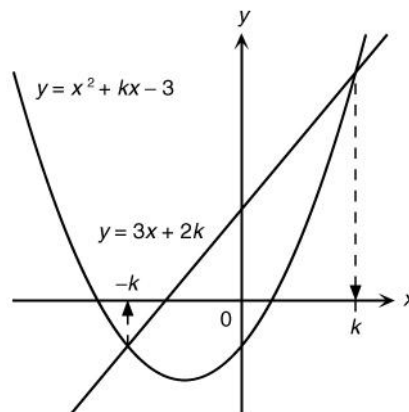**14.** Find the equation of the straight line that should be drawn on the graph of
$y = 2x^2 - 6x + 5$ to solve $3x^2 + 9x - 5 = 0$.

假設已給定 $y = 2x^2 - 6x + 5$ 的圖像。求在解方程 $3x^2 + 9x - 5 = 0$ 時,應在該圖像中加上的直線的方程。

A. $y = 12x - \dfrac{25}{3}$

B. $y = 6x - \dfrac{10}{3}$

C. $y = -12x + \dfrac{25}{3}$

D. $y = -6x + \dfrac{10}{3}$

15. The figure shows the graphs of $y = x^2 + kx - 3$ and $y = 3x + 2k$. Find the value(s) of k.

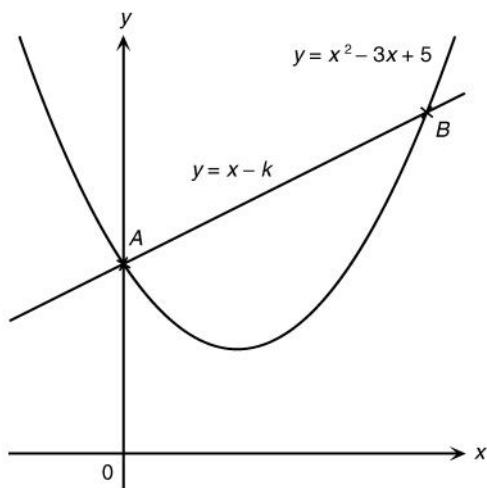下圖所示為 $y = x^2 + kx - 3$ 和 $y = 3x + 2k$ 的圖像。求 k 的值。

A. -3

B. 3

C. $-3, \dfrac{1}{2}$

D. $-\dfrac{1}{2}, 3$

16 Suppose the graphs of $y = ax^2 + bx + k$ and $y = mx + k$ are given. Find the x-coordinate(s) of the point(s) of intersection(s) of the graphs.

假設已給定 $y = ax^2 + bx + k$ 和 $y = mx + k$ 的圖像。求該兩個圖像的交點的 x 坐標。

A. $\dfrac{m - b}{a}$

B. $\dfrac{b - m}{a}$

C. $0, \dfrac{m - b}{a}$

D. $0, \dfrac{b - m}{a}$

17. In the figure, the graphs of $y = x^2 - 3x + 5$ and $y = x - k$ intersect at A and B. Find the value of k.
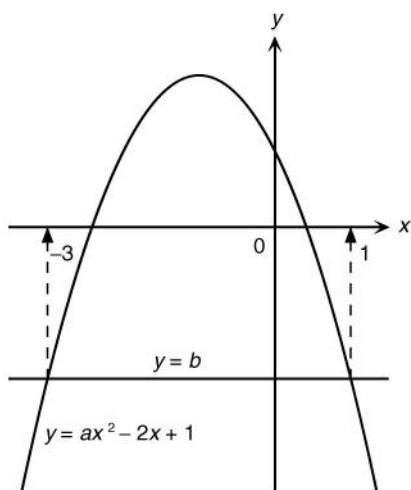
在圖中，$y = x^2 - 3x + 5$ 的圖像與 $y = x - k$ 的圖像相交於 A 和 B。求 k 的值。

 A. -5

 B. -1

 C. 1

 D. 5

18. The figure shows the graphs of $y = ax^2 - 2x + 1$ and $y = b$. Find the values of a and b.
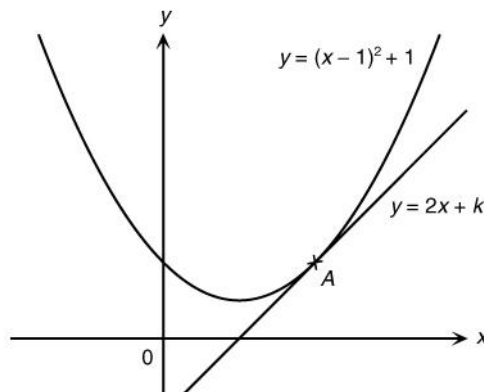
下圖所示為 $y = ax^2 - 2x + 1$ 和 $y = b$ 的圖像。求 a 和 b 的值。

 A. $a = -1, b = -1$

 B. $a = -1, b = -2$

 C. $a = -2, b = -1$

 D. $a = -2, b = -2$

19. In the figure, the graphs of $y = (x - 1)^2 + 1$ and $y = 2x + k$ intersect at only one point A. Find the value of k.

在圖中，$y = (x - 1)^2 + 1$ 的圖像與 $y = 2x + k$ 的圖像只相交於一點 A。求 k 的值。

 A. $-\dfrac{1}{2}$

 B. $\dfrac{1}{2}$

 C. -2

 D. 2

20. Which of the following simultaneous equations have two real solutions?

下列哪個聯立方程有兩個實數解?

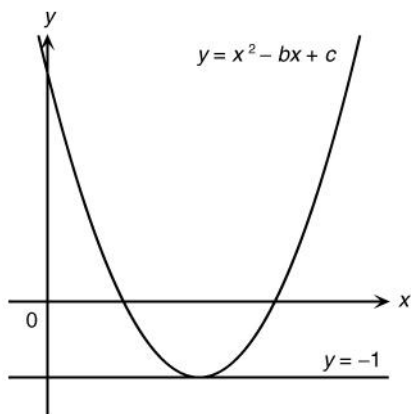 **A.** $\begin{cases} \dfrac{y}{3} = x^2 + x \\ 4x + y + 8 = 0 \end{cases}$

 B. $\begin{cases} y = 3x^2 - 3x + 2 \\ y = 3x - 1 \end{cases}$

 C. $\begin{cases} y = 3x^2 + 2x \\ y + 4x = 8 \end{cases}$

 D. $\begin{cases} y = x^2 - x \\ 3x - 3y - 8 = 0 \end{cases}$

21. The figure shows the graphs of $y = x^2 - bx + c$ and $y = -1$. By considering the two graphs, which of the following simultaneous equations have two real solutions?

下圖所示為 $y = x^2 - bx + c$ 和 $y = -1$ 的圖像。透過觀察該兩個圖像，下列哪個聯立方程有兩個實數解?

A. $\begin{cases} y = (x+1)^2 - b(x+1) + c \\ y = -1 \end{cases}$

B. $\begin{cases} y = (x-1)^2 - b(x-1) + c \\ y = -1 \end{cases}$

C. $\begin{cases} y + 1 = x^2 - bx + c \\ y = -1 \end{cases}$

D. $\begin{cases} y - 1 = x^2 - bx + c \\ y = -1 \end{cases}$

22. Solve the simultaneous equations
$\begin{cases} x^2 - 4xy = 4 \\ x - 2y + 2 = 0 \end{cases}$.

解聯立方程 $\begin{cases} x^2 - 4xy = 4 \\ x - 2y + 2 = 0 \end{cases}$。

A. $(-2, 0)$

B. $(2, 2)$

C. $(-2, 0), (2, 2)$

D. No real solutions   沒有實數解

23. Find the minimum value of k such that the simultaneous equations $\begin{cases} y^2 - 5x^2 = k \\ y = 3x - 2 \end{cases}$ have real solutions.

求 k 的最小值，使聯立方程 $\begin{cases} y^2 - 5x^2 = k \\ y = 3x - 2 \end{cases}$ 有實數解。

A. -10

B. 10

C. -5

D. 5

24. Solve the simultaneous equations
$\begin{cases} y = x^2 + 3x + 15 \\ y = 3 - 4x \end{cases}$.

解聯立方程 $\begin{cases} y = x^2 + 3x + 15 \\ y = 3 - 4x \end{cases}$。

A. $(2, -5), (6, -21)$

B. $(-6, 27), (-2, 11)$

C. $(-4, 19), (-3, 15)$

D. $(3, -9), (4, -13)$

25. Given that the simultaneous equations
$\begin{cases} y = x^2 + 5x + a \\ y = bx + 5 \end{cases}$ have two real solutions $(-4, y_1)$ and $(3, y_2)$. Find the values of a and b.

已知聯立方程 $\begin{cases} y = x^2 + 5x + a \\ y = bx + 5 \end{cases}$ 有兩個實數解 $(-4, y_1)$ 和 $(3, y_2)$。求 a 和 b 的值。

A. $a = -4, b = 7$

B. $a = 4, b = -7$

C. $a = -7, b = 4$

D. $a = 7, b = -4$

26. Given that the simultaneous equations
$\begin{cases} kx^2 - 3y^2 = 5 \\ y = x - 1 \end{cases}$ have two real solutions, find
the range of possible values of k.

已知聯立方程 $\begin{cases} kx^2 - 3y^2 = 5 \\ y = x - 1 \end{cases}$ 有兩個實數

解，求 k 值的可能範圍。

A. $k < -\dfrac{15}{8}$

B. $k > -\dfrac{15}{8}$

C. $k < \dfrac{15}{8}$

D. $k > \dfrac{15}{8}$

27. Solve $x^4 - 2x^3 + x - 2 = 0$.
解 $x^4 - 2x^3 + x - 2 = 0$。

A. $x = -2$

B. $x = 2$

C. $x = -1$ or 2

D. $x = -2$ or 1

28. Given that the simultaneous equations
$\begin{cases} 2y^2 - kx - 18 = 0 \\ y = x - 3 \end{cases}$ have only one solution,
find the value of k.

已知聯立方程 $\begin{cases} 2y^2 - kx - 18 = 0 \\ y = x - 3 \end{cases}$ 只有一

個解，求 k 的值。

A. 0

B. −12

C. 12

D. Cannot be determined　不可能求得

29. Solve $64(x - 1)^3 + 27 = 0$.
解 $64(x - 1)^3 + 27 = 0$。

A. $x = -\dfrac{3}{4}$

B. $x = \dfrac{3}{4}$

C. $x = -\dfrac{1}{4}$

D. $x = \dfrac{1}{4}$

30. Solve $x^4 - 4x^3 + 4x^2 - 4x + 3 = 0$.
解 $x^4 - 4x^3 + 4x^2 - 4x + 3 = 0$。

A. $x = -3, -1$ or 1

B. $x = -1, 1$ or 3

C. $x = -3$ or -1

D. $x = 1$ or 3

31. Let $f(x) = x^4 + 8x^3 + 7x^2 - 36x - 36$. If $x - 2$ is
a factor of $f(x)$, solve $f(x) = 0$.
設 $f(x) = x^4 + 8x^3 + 7x^2 - 36x - 36$。若 $x - 2$
是 $f(x)$ 的因式，解 $f(x) = 0$。

A. $x = -6, -3, -1$ or 2

B. $x = -1, 2, 3$ or 6

C. $x = -2, 1, 3$ or 6

D. $x = -6, -3, -2$ or 1

32. Solve $x^3 + 3x^2 - 4x - 12 = 0$.
解 $x^3 + 3x^2 - 4x - 12 = 0$。

A. $x = -3, -2$ or 3

B. $x = -3, 2$ or 3

C. $x = -3, -2$ or 2

D. $x = -2, 2$ or 3