Quadratic Equations in One Unknown

一元二次方程

Exercise(練習)

1. Convert the following recurring decimals into fractions.

把下列各循環小數化為分數。

- (a) $0.\dot{7}\dot{2}$
- (b) $0.\dot{4}\dot{1}\dot{8}$
- 2. Solve the quadratic equation $6x^2 + 5x 6 = 0$ using the factor method.

利用因式法解二次方程 $6x^2 + 5x - 6 = 0$ 。

3. Solve the following quadratic equations using the factor method.

利用因式法解下列各二次方程。

- (a) $2x^2 = 5x$
- **(b)** $4x^2 25 = 0$
- 4. Convert the following quadratic equations into the general form $ax^2 + bx + c = 0$, where a > 0. Write down the values of a, b and c.

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把下列各二次方程寫成一般式 $ax^2 + bx + c = 0$, 其中 a > 0, 並找出 $a \cdot b$ 和 c 的值。

- (a) $5x^2 8 = 4x$
- **(b)** x(x-3) + 2 = 0
- (c) $(3x-1)^2 = 4x$
- 5. Form a quadratic equation in x whose roots are -3 and 4.

建立一個以 x 為變數,並以 -3 和 4 為根的二次方程。

6. Solve the quadratic equation $(4x - 3)^2 - (x + 2)(4x - 3) = 0$.

解二次方程 $(4x-3)^2 - (x+2)(4x-3) = 0$ 。

7. Find a quadratic equation in x whose roots are $-\frac{2}{3}$ and $-\frac{1}{4}$.

建立一個以 x 為變數,並以 $-\frac{2}{3}$ 和 $-\frac{1}{4}$ 為根的二次方程。

8. Find a quadratic equation in x whose roots are two-third of the roots of $3x^2 + 10x - 8 = 0$.

建立一個以 x 為變數的二次方程,而該方程的根是 $3x^2+10x-8=0$ 的根的 $\frac{2}{3}$ 。

9. Solve $\left(x + \frac{2}{3}\right)^2 = 7$ and give your answers correct to 3 decimal places.

解
$$\left(x+\frac{2}{3}\right)^2 = 7$$
。(答案須準確至三位小數。)

10. Add a number to the following expressions to form perfect squares.

在下列各數式中加上一個數,使之成為一個完全平方。

- (a) $x^2 8x$
- (b) $x^2 + 5x$

11. Solve the equation $\left(x + \frac{1}{3}\right)^2 = 9$.

解方程
$$\left(x+\frac{1}{3}\right)^2=9$$
。

12. Solve the equation $x^2 - 6x + 1 = 0$ by completing the square. (Leave your answers in surd form.) 利用配方法解方程 $x^2 - 6x + 1 = 0$ 。(答案須以根式表示。)

13. Solve the equation $(x - 1)^2 = 5$ and leave your answers in surd form.

解方程 $(x-1)^2=5$,並以根式表示答案。

14. Solve the equation $x^2 - 4x + 3 = 0$ by completing the square.

利用配方法解方程 $x^2 - 4x + 3 = 0$ 。

15. Find the value of k if $x^2 - 2x + k = 0$ has a double real root.

若
$$x^2 - 2x + k = 0$$
 有一個二重實根,求 k 的值。

16. Solve $9x^2 + 24x + 16 = 0$ using the quadratic formula.

利用二次公式解 $9x^2 + 24x + 16 = 0$ 。

17. Solve $3x^2 + 5x - 2 = 0$ using the quadratic formula.

利用二次公式解 $3x^2 + 5x - 2 = 0$ 。

18. Solve the equation $5x^2 + 7x - 6 = 0$ by completing the square.

利用配方法解方程 $5x^2 + 7x - 6 = 0$ 。

19. Solve $x^2 + 3x + 5 = 0$ using the quadratic formula.

利用二次公式解 $x^2 + 3x + 5 = 0$ 。

- 20. If the quadratic equation $(k-2)x^2 + 5x 3 = 0$ has no real roots, find the range of possible values of k. 若二次方程 $(k-2)x^2 + 5x 3 = 0$ 沒有實根,求 k 值的可能範圍。
- 21. Find the discriminant for the following quadratic equations and determine the nature of the roots. 求下列各二次方程的判別式的值,並由此判別它的根的性質。
- (a) $5x^2 2x 3 = 0$
- **(b)** $4x^2 20x + 25 = 0$
- (c) $3x^2 4x + 7 = 0$
- 22. If the quadratic equation $x^2 + 2x + 3k 2 = 0$ has real roots, find the range of possible values of k. 若二次方程 $x^2 + 2x + 3k 2 = 0$ 有實根,求 k 值的可能範圍。

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- 23. (a) Plot the graph of $y = 2x^2 + x 10$ from x = -3 to x = 3.
 - (a) 繪畫 $y = 2x^2 + x 10$ 在 x = -3 與 x = 3 之間的圖像。
 - (b) Hence, solve the equation $2x^2 + x 10 = 0$ graphically.
 - (b) 由此,利用圖解法解方程 $2x^2 + x 10 = 0$ 。
- 24. Plot the graph of $y = -x^2 + 2x + 4$ from x = -2 to x = 4.
- (a) 繪畫 $y = -x^2 + 2x + 4$ 在 x = -2 與 x = 4 之間的圖像。
- (b) Hence, solve the equation $x^2 2x 4 = 0$ graphically.
- (b) 由此,利用圖解法解方程 $x^2 2x 4 = 0$ 。
- 25. Plot the graph of $y = x^2 6x + 9$ from x = 0 to x = 6.
- (a) 繪畫 $y = x^2 6x + 9$ 在 x = 0 與 x = 6 之間的圖像。
- (b) Hence, solve the equation $x^2 6x + 9 = 0$ graphically.
- (b) 由此,利用圖解法解方程 $x^2 6x + 9 = 0$ 。

26. girl drops a beanbag from the top of a building. After x seconds, it reaches a height of $(30 - 5x^2)$ m above the ground. Find the time taken by the beanbag to reach the ground.

(Give your answer correct to 3 significant figures.)

一個豆袋從一座建築物的頂部跌下,並於 x 秒後到達離地面 $(30-5x^2)$ m 的高度。求該豆袋到達地面所需的時間。

(答案須準確至三位有效數字。)

27. The product of two consecutive positive even numbers is 16 greater than 16 times of the odd number between them. Find the two even numbers.

兩個連續正偶數的積較位於它們之間的正奇數的 16 倍大 16。求該兩個偶數。

28. Mabel pays \$54 for a number of stickers. If the price of a sticker is reduced by \$1, she can buy 14 more stickers by paying \$10 more. What is the original price of a sticker?

詠珊用 \$54 購買了一些貼紙。若每款貼紙的售價均降低 \$1,則詠珊多付 \$10 便可多購得 14 款貼紙。求每款貼紙的原價。

29. The height of a triangular label is 6 cm longer than half of its base. The area of the label is 27 cm². Find the base and the height of the label.

兩個連續正偶數的積較位於它們之間的正奇數的 16 倍大 16。求該兩個偶數。

30. Sarah and Cherie both travel 200 km in a journey. If Sarah's driving speed is 10 km/h less than that of Cherie, Sarah will take 1 hour more than Cherie to complete the journey. Find Cherie's driving speed. 佩君和慧玲均完成了 200 km 的車程,其中佩君較慧玲多需 1 小時。若佩君的行車速率較慧玲的慢 10 km/h,求慧玲的行車速率。

31. Given a quadratic equation $kx^2 + 5x + 4k = 0$, where $k \neq 0$.

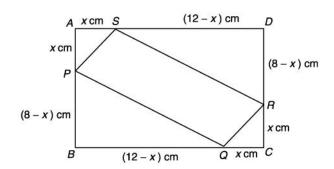
已知一個二次方程 $kx^2 + 5x + 4k = 0$, 其中 $k \neq 0$ 。

- (a) Solve the given equation and express your answer in terms of k. 求二次方程 $kx^2+5x+4k=0$ 的解,答案以 k 表示。
- (c) Hence, by using (b) or otherwise, determine whether the following equations have two distinct real roots.

由此,利用(b)或其他方法,判斷下列各二次方程是否具有兩個相異實根。

- (i) $3x^2 5x + 12 = 0$
- (ii) $2x^2 + 15x + 8 = 0$

32.



The figure shows a rectangular piece of paper ABCD. A triangle is cut away from each corner of the paper to form a quadrilateral PQRS as shown. The length and the width of ABCD are 12 cm and 8 cm respectively.

圖中所示為一長方形紙張 ABCD。現從紙張的四個角分別切割出一個三角形,以得出一個四邊形 PQRS。長方形 ABCD 的長和闊分別是 12 cm 和 8 cm。

- (a) Express the area of PQRS in terms of x.
 - 以 x 表示 PQRS 的面積。
- (b) (i) If the area of △APS is 6.75 cm² less than half the area of △PBQ, find the value of x. 若 △APS 的面積比 △PBQ 的面積的一半小 6.75 cm², 求 x 的值。
 - (ii) Hence, find the area of PQRS. 由此,求 PQRS 的面積。
- 33. (a) Solve the quadratic equation (4x 3)(3x + 2) + 5 = 0 using the factor method.
- (a) 利用因式法解二次方程 (4x-3)(3x+2)+5=0。
- (b) Form a quadratic equation in x whose roots are one greater than twice the roots of that in (a).
- (b) 建立一個以 x 為變數的二次方程,而該方程的根是 (a) 中的二次方程的根的兩倍加 1。
- 34. Suppose the equation $(2k + 1)x^2 2x 2 = 0$ has real roots.

已知二次方程 $(2k+1)x^2-2x-2=0$ 有實根。

(a) Find the range of possible values of k.

求 k 值的可能範圍。

(b) For the minimum value of k obtained in (a),

對於在 (a) 中所得的最小值,

- (i) solve the equation $(2k+1)x^2-2x-2=0$, 對於在 (a) 中所得的最小值,
- (ii) plot the graph of $y = (2k+1)x^2 2x 2$ and find the x-intercept(s), 繪畫 $y = (2k+1)x^2 2x 2$ 的圖像,並找出圖像的 x 軸截距;
- (iii) describe the relation between the root(s) of the equation $(2k + 1)x^2 2x 2 = 0$ and the x-intercept(s) of the graph of $y = (2k + 1)x^2 2x 2$.

試描述方程 $(2k+1)x^2-2x-2=0$ 的根與 $y=(2k+1)x^2-2x-2$ 的圖像的 x 軸截距之間的關係。

Pre-requisite Questions

預備測驗

1. Factorize
$$r^2 - rs + ps - pr$$
.
因式分解 $r^2 - rs + ps - pr$ 。

- 2. Factorize ap + 2aq.因式分解 ap + 2aq。
- 3. Factorize 2a 2b bc + ac. 因式分解 2a - 2b - bc + ac。
- 4. Factorize −3ab − 6a²b³c. 因式分解 −3ab − 6a²b³c。
- 5. Factorize $4x^2 y^2$. 因式分解 $4x^2 y^2$ 。
- 6. Factorize $2m^2 + 20mn + 50n^2$. 因式分解 $2m^2 + 20mn + 50n^2$ 。
- 7. Factorize $2k^2 8$. 因式分解 $2k^2 - 8$ 。
- 8. Factorize (x + y)(x 1) (2 x)(x + y). 因式分解 (x + y)(x - 1) - (2 - x)(x + y)。
- 9. Factorize $(r + s)^2 4$. 因式分解 $(r + s)^2 - 4$ 。
- 10. Factorize $b 6b^2 + 9b^3$. 因式分解 $b - 6b^2 + 9b^3$ 。
- 11. Factorize $z^2 8z + 16$. 因式分解 $z^2 - 8z + 16$ 。
- 12. Factorize $k^2n 2k^2n^2 + k^3n^3$. 因式分解 $k^2n - 2k^2n^2 + k^3n^3$ 。
- 13. Factorize $a^2b^2 + 2ab + 1$. 因式分解 $a^2b^2 + 2ab + 1$ 。

16. Factorize
$$1 - k^2 + 4kc - 4c^2$$
.
因式分解 $1 - k^2 + 4kc - 4c^2$ 。

17. Factorize
$$-x^2 - 9xy - 8y^2$$
.
因式分解 $-x^2 - 9xy - 8y^2$ 。

18. Factorize
$$3p^2 + 10pq + 7q^2$$
.
因式分解 $3p^2 + 10pq + 7q^2$ 。

21. Factorize
$$t^2 - t - 2$$
.
因式分解 $t^2 - t - 2$ 。

22. Factorize
$$3cd - 3d^2 + 6c^2$$
.
因式分解 $3cd - 3d^2 + 6c^2$ 。

23. Factorize
$$3h^2 - 14h + 15$$
.
因式分解 $3h^2 - 14h + 15$ 。

24. Factorize
$$-35c^2 - 3c + 2$$
.
因式分解 $-35c^2 - 3c + 2$ 。

25. Factorize
$$4n^4 - 180n^2 - 16n^3$$
.
因式分解 $4n^4 - 180n^2 - 16n^3$ 。

Level 1 Questions 程度 1 題目

1. Solve the equation
$$(2x-3)(3x+1) = 0$$
. 解二次方程 $(2x-3)(3x+1) = 0$ 。

2. Solve the equation
$$3x^2 - 12 = 0$$
. 解二次方程 $3x^2 - 12 = 0$ °

3. Solve the equation
$$5(3x-1)(2x+5) = 0$$
. 解二次方程 $5(3x-1)(2x+5) = 0$ 。

4. Solve the equation
$$(x-4)(x+5) = 0$$
. 解二次方程 $(x-4)(x+5) = 0$ 。

5. Solve the equation
$$98 - 2x^2 = 0$$
. 解二次方程 $98 - 2x^2 = 0$ °

6. Solve the equation
$$2x(x-1) = 0$$
. 解二次方程 $2x(x-1) = 0$ 。

7. Solve the equation
$$8x^2 - 2x = 0$$
. 解二次方程 $8x^2 - 2x = 0$ 。

8. Solve the equation
$$(2x-1)^2 - 1 = 0$$
. 解二次方程 $(2x-1)^2 - 1 = 0$ °

9. Solve the equation
$$(x + 1)^2 = 9$$
. 解二次方程 $(x + 1)^2 = 9$ °

10. Solve the equation
$$2\left(1-\frac{x}{2}\right)^2=\frac{25}{2}$$
, give your answer in surd form.
解二次方程 $2\left(1-\frac{x}{2}\right)^2=\frac{25}{2}$ \circ (答案以根式表示 \circ)

11. Solve the equation
$$5x - 15x^2 = 0$$
. 解二次方程 $5x - 15x^2 = 0$ °

12. Solve the equation
$$(3 + x)^2 = 3$$
, give your answer in surd form. 解二次方程 $(3 + x)^2 = 3$ 。(答案以根式表示。)

- 13. Solve the equation $x^2 4x + 3 = 0$ using the factor method. 利用因式法解二次方程 $x^2 4x + 3 = 0$ 。
- 14. Solve the equation $x^2 + 6x + 9 = 0$ using the factor method. 利用因式法解二次方程 $x^2 + 6x + 9 = 0$ 。
- 15. Solve the equation $-x^2 + 3x 2 = 0$ using the factor method. 利用因式法解二次方程 $-x^2 + 3x 2 = 0$ 。
- 16. Solve the equation $x^2 8x + 16 = 0$ using the factor method. 利用因式法解二次方程 $x^2 8x + 16 = 0$ 。
- 17. Solve the equation $x^2-11x+28=0$ using the factor method. 利用因式法解二次方程 $x^2-11x+28=0$ \circ
- 18. Set up a quadratic equation in x whose roots are 2 and -3. 建立一個以 x 為變數,並以 2 和 -3 為根的二次方程。
- 19. Solve the equation $x^2 7x + 12 = 0$ using the factor method. 利用因式法解二次方程 $x^2 7x + 12 = 0$ \circ
- 20. Solve the equation $4x^2 20x + 25 = 0$ using the factor method. 利用因式法解二次方程 $4x^2 20x + 25 = 0$ 。
- 21. Solve the equation $-x^2 + 2x + 8 = 0$ using the factor method. 利用因式法解二次方程 $-x^2 + 2x + 8 = 0$ 。
- 22. Set up a quadratic equation in x whose roots are $\frac{1}{2}$ and 0. 建立一個以 x 為變數,並以 $\frac{1}{2}$ 和 0 為根的二次方程。
- 23. Set up a quadratic equation in x whose roots are $-\frac{5}{2}$ and $-\frac{7}{3}$. 建立一個以 x 為變數,並以 $-\frac{5}{2}$ 和 $-\frac{7}{3}$ 為根的二次方程。

24. Set up a quadratic equation in x whose roots are
$$\frac{1}{2}$$
 and $-\frac{3}{4}$.

建立一個以
$$x$$
 為變數,並以 $\frac{1}{2}$ 和 $-\frac{3}{4}$ 為根的二次方程。

25. Solve the equation
$$3x^2 + 2x - 3 = 0$$
 using the quadratic formula. 利用二次公式解 $3x^2 + 2x - 3 = 0$ °

26. Solve the equation
$$x^2 + 7x + 1 = 0$$
 using the quadratic formula. 利用二次公式解 $x^2 + 7x + 1 = 0$ 。

27. Solve the equation
$$8x^2 - 14x - 9 = 0$$
 using the factor method. 利用因式法解二次方程 $8x^2 - 14x - 9 = 0$ 。

28. Solve the equation
$$5x^2 - x - 2 = 0$$
 using the quadratic formula. 利用二次公式解 $5x^2 - x - 2 = 0$ 。

29. Solve the equation
$$6x^2 - 7x - 3 = 0$$
 using the factor method. 利用因式法解二次方程 $6x^2 - 7x - 3 = 0$ 。

30. Solve the equation
$$x^2 + 2x + 1 = 0$$
 using the quadratic formula. 利用二次公式解 $x^2 + 2x + 1 = 0$ \circ

31. Solve the equation
$$2x^2 + 5x - 3 = 0$$
 using the factor method. 利用因式法解二次方程 $2x^2 + 5x - 3 = 0$ \circ

32. Solve the equation
$$3x^2 + 5x - 12 = 0$$
 using the factor method. 利用因式法解二次方程 $3x^2 + 5x - 12 = 0$ 。

33. Find the discriminant for the equation
$$5x^2 - 7x + 6 = 0$$
 and determine its nature of roots. 求二次方程 $5x^2 - 7x + 6 = 0$ 的判別式的值,並由此判別它的根的性質。

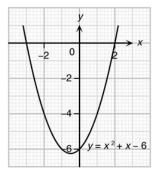
34. Find the discriminant for the equation
$$-3x^2+x+1=0$$
 and determine its nature of roots. 求二次方程 $-3x^2+x+1=0$ 的判別式的值,並由此判別它的根的性質。

35. If the roots of the equation
$$-2x^2+5x+k=0$$
 are the same, find the value(s) of k. 若 $-2x^2+5x+k=0$ 有一個二重實根,求 k 的值。

- 36. Find the discriminant for the equation $2x^2 + 5x + 1 = 0$ and determine its nature of roots. 求二次方程 $2x^2 + 5x + 1 = 0$ 的判別式的值,並由此判別它的根的性質。
- 37. Find the range of possible values of k if the equation $x^2+x+k=0$ has no real roots. 若二次方程 $x^2+x+k=0$ 沒有實根,求 k 值的可能範圍。
- 38. Find the discriminant for the equation $9x^2 + 42x + 49 = 0$ and determine its nature of roots. 求二次方程 $9x^2 + 42x + 49 = 0$ 的判別式的值,並由此判別它的根的性質。
- 39. Find the range of possible values of k if the equation $3x^2 2x + k + 2 = 0$ has two distinct real roots. 若二次方程 $3x^2 2x + k + 2 = 0$ 有兩個相異實根,求 k 值的可能範圍。
- 40. Find the values of k if the equation $3x^2 + (k+1)x + 12 = 0$ has a double real root. 若 $3x^2 + (k+1)x + 12 = 0$ 有一個二重實根,求 k 的值。
- 41. Solve the equation $x^2 + x 6 = 0$ using the graph below.

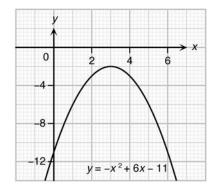
(Give your answer correct to 1 decimal place.)

根據以下的圖像,利用圖解法解方程 $x^2 + x - 6 = 0$ 。(答案須準確至一位小數。)



42. Solve the equation $x^2 - 6x + 11 = 0$ using the graph below.

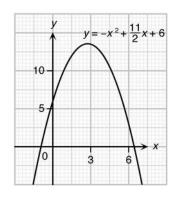
根據以下的圖像,利用圖解法解方程 $x^2 - 6x + 11 = 0$ 。



43. Solve the equation $-2x^2 + 11x + 12 = 0$ using the graph below.

(Give your answer correct to 1 decimal place.)

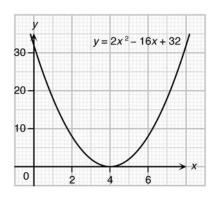
根據以下的圖像,利用圖解法解方程 $-2x^2 + 11x + 12 = 0$ 。(答案須準確至一位小數。)



44. Solve the equation $x^2 - 8x + 16 = 0$ using the graph below.

(Give your answer correct to 1 decimal place.)

根據以下的圖像,利用圖解法解方程 $x^2 - 8x + 16 = 0$ 。(答案須準確至一位小數。)



45. The figure shows the graph of $y = 2x^2 + (m + 1)x + 2$.

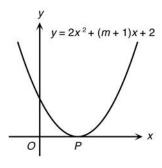
圖中所示為 $y = 2x^2 + (m+1)x + 2$ 的圖像。

(a) Find the value(s) of m.

求 m 的值。

(b) Find the coordinates of P.

求 P 的坐標。



46. \$4000 was shared equally among a certain number of people. If 3 more people share the amount, each will get \$300 less. Find the original number of people.

當把 \$4000 平均分給一組人士時,若人數較原來的多 3 人,則每人分得的金額將較原來的少 \$300。求該組人士的數目。

- 47. If the graph of $y = kx^2 + 3x + 1$ touches the x-axis, find the value of k. 若 $y = kx^2 + 3x + 1$ 的圖像與 x 軸只相交於一點,求 k 的值。
- 48. The product of two consecutive positive integers is 342. Find the smaller integer. 兩個連續正整數的積是 342。求較小的數。
- 49. If a ball is thrown vertically upwards, its height above the ground after x seconds is given by $(20 + 6x 3x^2)$ m.

把一個球垂直向上抛起,它在 x 秒後離地面的高度是 $(20 + 6x - 3x^2)$ m \circ

- (a) Find the height of the ball after 3 seconds. 求該球在 3 秒後離地面的高度。
- (b) When will the ball be at 5 m above the ground?
 (Give your answer correct to 3 decimal places.)
 問在甚麼時候,球會離地面 5 m?(答案須準確至三位小數。)
- 50. The height of a solid cylinder is 4 cm. If its total surface area is 24π cm², find its radius.
 - 一個圓柱體的高度是 4 cm。若該圓柱體的總表面面積是 24π cm², 求它的半徑。

Level 2 Questions 程度 2 題目

1. Solve the equation
$$(x + 2)^2 = (2x + 1)^2$$
. 解二次方程 $(x + 2)^2 = (2x + 1)^2$ ∘

2. Solve the equation
$$x^2 + 17 = (2x + 7)^2$$
.
 解二次方程 $x^2 + 17 = (2x + 7)^2$ \circ

3. Solve the equation
$$(3x - 1)^2 = 3x - 1$$
. 解二次方程 $(3x - 1)^2 = 3x - 1$ °

4. Solve the equation
$$2(2x + 3)^2 + 6x + 7 = 0$$
. 解二次方程 $2(2x + 3)^2 + 6x + 7 = 0$ 。

5. Solve the equation
$$x^2 + 2x + 5 = (3x + 5)^2$$
.
 解二次方程 $x^2 + 2x + 5 = (3x + 5)^2$ \circ

6. Solve the equation
$$3x(2x + 5) = 2(x - 1)$$
. 解二次方程 $3x(2x + 5) = 2(x - 1)$ 。

7. Solve the equation
$$(3x-2)(2x+1) = 5$$
. 解二次方程 $(3x-2)(2x+1) = 5$ 。

8. Solve the equation
$$(5x + 2)^2 + (3x - 1)^2 = 40$$
, give your answer correct to 3 significant figures. 解二次方程 $(5x + 2)^2 + (3x - 1)^2 = 40$,並取答案準確至三位有效數字。

9. Solve the equation
$$(x + 1)(x - 3) = (x + 1)(2x + 3)$$
. 解二次方程 $(x + 1)(x - 3) = (x + 1)(2x + 3)$ \circ

10. Solve the equation
$$(4x + 3)^2 - (5x + 1)^2 = 0$$
.
解二次方程 $(4x + 3)^2 - (5x + 1)^2 = 0$ °

11. (a) Solve the equation
$$y^2 + 4y + 3 = 0$$
. 解二次方程 $y^2 + 4y + 3 = 0$ °

(b) Hence solve the equation
$$(2x + 1)^2 + 4(2x + 1) + 3 = 0$$
.
由此,解二次方程 $(2x + 1)^2 + 4(2x + 1) + 3 = 0$ 。

- 12. (a) Solve the equation $4y^2 + 7y 2 = 0$. 解二次方程 $4y^2 + 7y 2 = 0$ \circ
 - (b) Hence solve the equation $\frac{4}{x^2} + \frac{7}{x} 2 = 0$. 由此,解二次方程
- 13. Set up a quadratic equation in x whose roots are a and 1-a. 建立一個以 x 為變數,並以 a 和 1-a 為根的二次方程。
- 14. Set up a quadratic equation in x whose roots are $\sqrt{3}$ and $-\sqrt{3}$. 建立一個以 x 為變數,並以 $\sqrt{3}$ 和 $-\sqrt{3}$ 為根的二次方程。
- 15. Set up a quadratic equation in x whose roots are the reciprocals of the roots of $x^2-5x+4=0$. 建立一個以 x 為變數的二次方程,而該方程的根是 $x^2-5x+4=0$ 的根的倒數。
- 16. Solve the equation $2x^2 11x 5 = 0$ by completing the square. 利用配方法解方程 $2x^2 11x 5 = 0$ \circ
- 17. Set up a quadratic equation in x whose roots are twice of the roots of $3x^2 + 4x 15 = 0$. 建立一個以 x 為變數的二次方程,而該方程的根是 $3x^2 + 4x 15 = 0$ 的根的兩倍。
- 18. If the equation 2x(1-2kx) = 3x + 2k has no real roots, set up an inequality in k. 若二次方程 2x(1-2kx) = 3x + 2k 沒有實根,試建立一個以 k 為變數的不等式。
- 19. Set up a quadratic equation in x whose roots are one-third of the roots of $5x^2-x-6=0$. 建立一個以 x 為變數的二次方程,而該方程的根是 $5x^2-x-6=0$ 的根的 $\frac{1}{3}$ 。
- 20. Solve the equation $x^2 + 6x + 1 = 0$ by completing the square. 利用配方法解方程 $x^2 + 6x + 1 = 0$ \circ
- 21. Solve the equation $x^2 9x + 9 = 0$ by completing the square. 利用配方法解方程 $x^2 9x + 9 = 0$ 。
- 22. Solve the equation $x^2 + 4x + 2 = 0$ by completing the square. 利用配方法解方程 $x^2 + 4x + 2 = 0$ °

23. Solve the equation $3x^2 - 10x - 1 = 0$ by completing the square.

利用配方法解方程 $3x^2 - 10x - 1 = 0$ 。

- 24. If the equation $kx^2 5x k = 3(x^2 5)$ has two distinct real roots, set up an inequality in k. 若二次方程 $kx^2 5x k = 3(x^2 5)$ 有兩個相異實根,試建立一個以 k 為變數的不等式。
- 25. Given that the equation $5(x^2 3x + 4) = 6(x^2 x + k)$ has no real roots.

已知方程 $5(x^2-3x+4)=6(x^2-x+k)$ 沒有實根,

 $\textbf{(a)} \quad \text{Find the range of possible values of } k.$

求 k 值的可能範圍;

(b) Find the minimum value of k if k is an integer.

若 k 是一個整數, 求 k 的最小值。

26. Given that the equation (5x - 4)(3x + 1) = 6(x - k) has two distinct real roots.

已知方程 (5x-4)(3x+1) = 6(x-k) 有兩個相異實根,

(a) Find the range of possible values of k.

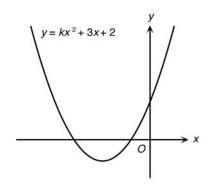
求 k 值的可能範圍;

(b) Find the maximum value of k if k is an integer.

若 k 是一個整數, 求 k 的最大值。

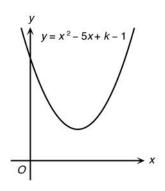
27. By using the graph below, state the number of roots of the equation $kx^2 + 3x + 2 = 0$ and hence find the range of possible values of k.

根據以下的圖像,寫出方程 $kx^2 + 3x + 2 = 0$ 的實根的數目。由此,求 k 值的可能範圍。



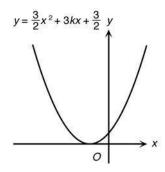
28. By using the graph below, state the number of roots of the equation $x^2 - 5x + k - 1 = 0$ and hence find the range of possible values of k.

根據以下的圖像,寫出方程 $x^2 - 5x + k - 1 = 0$ 的實根的數目。由此,求 k 值的可能範圍。



29. By using the graph below, state the number of roots of the equation $\frac{1}{2}x^2 + kx + \frac{1}{2} = 0$ and hence find the values of k.

根據以下的圖像,寫出方程 $\frac{1}{2}x^2 + kx + \frac{1}{2} = 0$ 的實根的數目。由此,求 k 值的可能範圍。



- 30. (a) Find the discriminant of the equation $2x^2 + kx + k^2 = 0$. 求二次方程 $2x^2 + kx + k^2 = 0$ 的判別式的值。
 - (b) Hence, find the number of x-intercept(s) of the graph of $y = 2x^2 + kx + k^2$. 由此,求 $y = 2x^2 + kx + k^2$ 的圖像中 x 軸截距的數目。
- 31. If the graph of $y = -3x^2 5x + 4k 3$ cuts the x-axis at two different points, find the range of possible values of k.

若 $y = -3x^2 - 5x + 4k - 3$ 的圖像與 x 軸相交於兩點,求 k 值的可能範圍。

32. If the graph of $y = 4x^2 + 3x + k$ cuts the x-axis at two different points, find the range of possible values of k.

若 $y = 4x^2 + 3x + k$ 的圖像與 x 軸相交於兩點,求 k 值的可能範圍。

33. If the graph of $y=-\frac{3}{4}x^2+x+k-1$ does not intersect the x-axis, find the range of possible values of k. 若 $y=-\frac{3}{4}x^2+x+k-1$ 的圖像與 x 軸並不相交,求 k 值的可能範圍。

34. Given that the graph of $y = 2x^2 - (m+2)x + 2$ touches the x-axis. 已知 $y = 2x^2 - (m+2)x + 2$ 的圖像與 x 輔只相交於一點。

- (a) Find the two values of m. 求 *m* 的兩個可能值。
- (b) For each of the values of m, solve the equation $2x^2 (m+2)x + 2 = 0$. 對於每個 m 值,解方程 $2x^2 (m+2)x + 2 = 0$ 。
- 35. If the graph of $y=3x^2+8x-\frac{k}{5}$ does not intersect the x-axis, find the range of possible values of k. 若 $y=3x^2+8x-\frac{k}{5}$ 的圖像與 x 軸並不相交,求 k 值的可能範圍。
- 36. Given that the graph of $y = 16px^2 8(3p + 2)x 1$ touches the x-axis. 已知 $y = 16px^2 8(3p + 2)x 1$ 的圖像與 x 軸只相交於一點。
 - **(a)** Find the two values of p. 求 p 的兩個可能值。
 - (b) For each of the values of p, solve the equation $16px^2 8(3p + 2)x 1 = 0$. 對於每個 p 值,解方程 $16px^2 8(3p + 2)x 1 = 0$ 。
- 37. In a journey, a car travels a distance of 300 km at a uniform speed. If the speed of the car is increased by 10 km/h, it takes one hour less to complete the journey. Find the original speed of the car.
 - 一輛汽車須完成 300 km 的路程。若駕駛者把速率增加 10 km/h,則可節省 1 小時的行車時間。 求原來的速率。
- 38. John and Peter are cycling from town A to town B, where the two towns are 20 km apart. Given that John is 4 km/h faster than Peter, and Peter takes 50 minutes longer than John to finish the journey. Find their speeds.

俊華及志達由城市 A 踏單車到城市 B,而兩個城市的距離是 20 km。已知俊華的速率較志達快 4 km/h,而志達完成旅程的時間較俊華多需 50 分鐘。求他們的速率。

39. If the sum of two numbers is 36 and the sum of their reciprocals is $\frac{1}{5}$, find the two numbers.

已知兩個數之和是 36,而它們的倒數之和是 $\frac{1}{5}$ 。求該兩個數。

40. Mr. Lee takes 8 days to finish a job. If Mr. Chan works alone, he needs 18 days more than they work together. Find the time taken for Mr. Chan to finish the job alone.

李先生需要 8 天時間完成一件工程。若陳先生單獨完成該工程,則所需時間較李、陳二人合力 完成的時間多 18 天。求陳先生單獨完成該工程所需的時間。

41. Miss Lam bought p goldfish for \$120, but 4 of them died shortly afterwards. Then she sold all the goldfish left and each of them was sold at a price \$1 more than she paid for it.

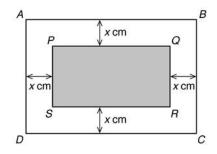
林小姐以 \$120 買了 p 尾金魚。回家後,她發現其中 4 尾死了。林小姐現把餘下的金魚轉售,而每尾的價格較購入時高 \$1。

- (a) Express, in terms of p, the price at which Miss Lam sold the goldfish. 以 p 表示每尾金魚的轉售價格。
- (b) If Miss Lam gains \$24 totally, find the value of p. 若林小姐總共賺得 \$24,求 p 的值。
- 42. Amy bought a dog at \$x and then sold it at \$1500 later. If her profit per cent is x%, find the value of x. (Given your answer correct to 3 significant figures.)

小美以 \$x 購買了一隻小狗,再以 \$1500 將小狗出售。若她的盈利百分率是 x%,求 x 的值。 (答案須準確至三位有效數字。)

43. In the figure, the dimensions of a rectangular park ABCD are 30 m × 10 m. In the park, a rectangular garden PQRS is surrounded by a path. Given that the path has a uniform width of x m, and its area is 50% of that of the garden PQRS.

圖中所示為一個 $30 \text{ m} \times 10 \text{ m}$ 的長方形公園。在公園內,有一條 小徑圍著長方形花園 PQRS。已知小徑的闊度為 x m,而它的面積是花園 PQRS 的 50%。



- (a) Set up a quadratic equation in x.建立一個以 x 為變數的二次方程。
- (**b**) Find the value of x. (Give your answer in surd form.) 求 x 的值。(答案以根式表示。)

44. The figure shows a rhombus EFGH of side (3x + 3) cm, where

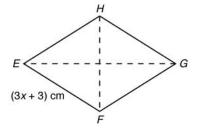
$$EG = (6x + 3)$$
 cm and $FH = (4x - 7)$ cm.

圖中所示為一邊長 (3x+3) cm 的菱形 EFGH, 其中

(a) Find the value of x.

求
$$x$$
 的值。

(b) Hence, find the area of the rhombus.



Level 2+ Questions 程度 2+ 題目

1. Given that p and q are the roots of the equation $ax^2 + bx + c = 0$, where p > q.

已知 p 和 q 是方程 $ax^2 + bx + c = 0$ 的根,其中 p > q。

- (a) By using the quadratic formula, express p and q in terms of a, b and c. 利用二次公式,以 $a \cdot b$ 和 c 表示 p 和 $q \circ$
- (b) Find p + q and pq in terms of a, b and c respectively. 試以 $a \cdot b$ 和 c 表示 p + q 和 pq \circ
- (c) By using the results of (b) or otherwise, find the sum and the product of the roots of $7x^2 + \frac{4}{3}x 5 = 0$ respectively.

利用 **(b)** 的結果,或其他方法,求 $7x^2 + \frac{4}{3}x - 5 = 0$ 的根之和及積。

- 2. (a)(i) Solve the equation $x^2 4px 4pq q^2 = 0$ by grouping the terms, where p and q are non-zero real numbers, and express your answers in terms of p and q. 利用合併項的方法解方程 $x^2 4px 4pq q^2 = 0$,其中 p 和 q 是非零實數,並以 p
 - (ii) Hence, if the discriminant of the equation $x^2 4px 4pq q^2 = 0$ is 0, suggest a pair of possible values of p and q.

由此,若方程 $x^2-4px-4pq-q^2=0$ 的判別式是 0,試舉出 p 和 q 的一組可能值。

- (b) By using the result of (a)(i), if the sum and the product of the roots of $x^2+2(1-a)x-(1+b)(2a+b-1)=0$ are 8 and 15 respectively, find the values of a and b. 利用 (a)(i) 的結果,若 $x^2+2(1-a)x-(1+b)(2a+b-1)=0$ 的根之和及積分別為 8 及 15,求 a 和 b 的值。
- 3. (a) Consider a quadratic equation in the form $px^2 + qx + r = 0$, where p and r are non-zero real numbers. 考慮形式為 $px^2 + qx + r = 0$ 的二次方程,其中 p 和 r 是非零實數。
 - (i) Show that the roots of the equation are $\frac{2r}{-q\pm\sqrt{q^2-4pr}}$. 證明該方程的根是 $\frac{2r}{-a\pm\sqrt{a^2-4pr}}$ 。
 - (ii) Hence, find the relationship between p and r if one of the roots of the equation is the reciprocal of the other.

由此,若方程的其中一個根是另一個根的倒數,求 p 和 r 之間的關係。

(b) It is given that the equation $2mx^2 - (m + 2n - 2)x + (n - 1) = 0$, where $m \ne 0$ and $n \ne 1$. If the product of two distinct real roots of the equation is 1, solve the equation.

已知方程 $2mx^2 - (m+2n-2)x + (n-1) = 0$,其中 $m \neq 0$ 及 $n \neq 1$ 。若方程的兩個相異實根的積是 1,解該方程。

4. (a) Solve $x^2 - x - 2 = 0$.

解
$$x^2 - x - 2 = 0$$
。

- (b) Hence, solve $(x^2-2x)^2-x^2+2x-2=0$. (Leave your answers in surd form.) 由此,解 $(x^2-2x)^2-x^2+2x-2=0$ 。(答案以根式表示。)
- 5. (a) Express $7+2\sqrt{12}$ in the form of $(a+\sqrt{b})^2$, where b>0. 把 $7+2\sqrt{12}$ 寫成 $(a+\sqrt{b})^2$ 的形式,其中 b>0。
 - (b) Given that the quadratic expression $3k(x^2 + x 2) (3x 19)$ is a perfect square, where k is an non-zero positive integer.

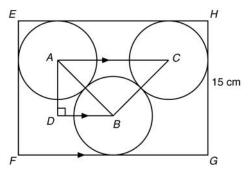
已知二次數式 $3k(x^2+x-2)-(3x-19)$ 為一個完全平方,其中 k 是一個非零正整數。

- (i) Find the value of k. 求 k 的值。
- (ii) By using the above results, solve the equation $3k(x^2+x-2)-(3x+1)=4\sqrt{3}-13$. (Leave your answers in surd form.) 利用以上的結果,解方程 $3k(x^2+x-2)-(3x+1)=4\sqrt{3}-13$ 。 (答案以根式表示。)
- 6. If the equation $2x^2 + (4k-1)x k + \frac{5}{4} = 0$ has no real roots, find the range of possible values of k.

若方程
$$2x^2 + (4k-1)x - k + \frac{5}{4} = 0$$
 沒有實根,求 k 值的可能範圍。

7. The figure shows a rectangle EFGH of width 15 cm with three identical circles of radii x cm inscribed in it. A, B and C are centres of the three circles respectively. Given that AC // DB // FG and △ADB is a right-angled isosceles triangle.

如圖所示,三個相等且半徑為 x cm 的圓內接於闊度為 15 cm 的長方形 $EFGH \circ A \circ B$ 和 C 分別是該三個圓的圓心。已知 AC // DB // FG,而 $\triangle ADB$ 是一個直角等腰三角形。



(a) (i) By considering \triangle ADB, set up a quadratic equation in x. 考慮 \triangle ADB, 試建立一個以 x 為變數的二次方程。

- (ii) Solve equation in (a)(i). (Leave your answers in surd form.) 解 (a)(i) 所得的方程。(答案以根式表示。)
- (b) Hence, find the area of \triangle ABC and the rectangle EFGH.

(Give your answers correct to 2 decimal places.)

由此,求 $\triangle ABC$ 和長方形 EFGH的面積。(答案須準確至兩位小數。)

8. (a) If both of the roots of the equations $rx^2 + sx + t = 0$ and $rx^2 + s'x + t' = 0$ are α and β , where α and β are non-zero real numbers and $\alpha \neq \beta$, prove that s = s' and t = t'.

若方程 $rx^2+sx+t=0$ 及 $rx^2+s'x+t'=0$ 的兩個根分別都是 α 和 β ,其中 α 和 β 是非 零實數,且 $\alpha\neq\beta$,證明 s=s' 及 t=t'。

- (b) (i) It is given that $y = p(x+1)^2 qx(x-1) 2p(x+1)$, where p and q are non-zero real numbers and $q \neq 2p$. Show that its graph intersects the x-axis at two points. 已知 $y = p(x+1)^2 qx(x-1) 2p(x+1)$,其中 p 和 q 是非零實數,且 $q \neq 2p$ 。證明 其圖像與 x 軸相交於兩點。
 - (ii) If the x-intercepts of the graph of $y = p(x+1)^2 qx(x-1) 2p(x+1)$ are -3p and $\frac{q}{p-q}$, find the values of p and q.

若 $y = p(x+1)^2 - qx(x-1) - 2p(x+1)$ 的圖像的 x 軸截距是 -3p 和 $\frac{q}{p-q}$,求 p 和 q 的值。

Multiple Choice Questions

多項選擇題

1. Which of the following expression is a perfect square?

下列哪個數式是完全平方?

A.
$$9x^2 - 12x + 4$$

B.
$$9x^2 - 12x - 4$$

C.
$$9x^2 - 6x + 4$$

D.
$$9x^2 - 6x - 4$$

2. Which of the following equations has no real roots?

下列哪個二次方程沒有實根?

A.
$$x^2 = 0$$

B.
$$2x^2 - x - 1 = 0$$

C.
$$x^2 + 4x + 5 = 0$$

D.
$$x^2 + 7x + 1 = 0$$

3. Solve the equation $x + \frac{1}{x} = 6 + \frac{1}{6}$.

解二次方程
$$x + \frac{1}{x} = 6 + \frac{1}{6}$$
。

$$\mathbf{A.} \quad \mathbf{x} = \mathbf{6}$$

B.
$$x = \frac{1}{6}$$

C.
$$x = 6 \text{ or } \frac{1}{6}$$

D.
$$x = -6$$

4. If β is a root of the equation $3x^2 + 2x - 3 = 0$, then $6\beta^2 + 4\beta - 7 =$

若 β 是方程 $3x^2 + 2x - 3 = 0$ 的一個根,則 $6\beta^2 + 4\beta - 7 =$

5. Solve the equation 2ax(2 - ax) + ax = 2, where a $\neq 0$.

解二次方程 2ax(2-ax) + ax = 2, 其中 $a \neq 0$ 。

$$\mathbf{A.} \quad \mathbf{x} = \mathbf{0}$$

B.
$$x = \frac{1}{2a}$$

C.
$$x = \frac{1}{2a} \text{ or } \frac{2}{a}$$

D.
$$x = 2a \text{ or } a$$

6. Which of the following equations has/have real root(s)?

下列哪個二次方程有實根?

I.
$$x^2 + 3x + 1 = 0$$

II.
$$x^2 + 2x + 1 = 0$$

III.
$$x^2 + 3x + 7 = 0$$

7. Solve the equation (x + 1)x = 3(x + 1).

解二次方程
$$(x+1)x = 3(x+1)$$
。

A.
$$x = -1$$

B.
$$x = -1 \text{ or } 3$$

C.
$$x = 3$$

D.
$$x = 1 \text{ or } 3$$

- 8. The roots of the equation $2x^2 + 3x 2 = 0$ are 二次方程 $2x^2 + 3x 2 = 0$ 的根是
 - **A.** $\frac{1}{2}$ and -2.
 - **B.** $-\frac{1}{2}$ and 2.
 - **C.** 1 and $\frac{1}{2}$.
 - **D.** $-\frac{1}{2}$ and -2.
- 9. Which of the following equations may represent the graph below?

圖中所示的可能是以下哪個二次方程的圖 像?

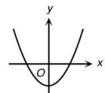
- **A.** $y = -x^2 + x + 6$
- **B.** $y = -x^2 x + 6$
- **C.** $y = -x^2 + x 6$
- **D.** $y = -x^2 x 6$
- 10. Set up a quadratic equation in x whose roots are 1 + 2p and -3p.

建立一個以 x 為變數,並以 1 + 2p 和 -3p 為根的二次方程。

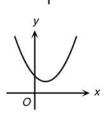
- **A.** $x^2 + 2px 3p = 0$
- **B.** $x^2 + (2p + 1)x + 3p = 0$
- $\mathbf{C.} \quad \mathbf{x}^2 + (\mathbf{p} 1)\mathbf{x} 6\mathbf{p}^2 3\mathbf{p} = 0$
- **D.** $x^2 + (1-p)x 6p^2 = 0$
- 11. Which of the following may represent the graph $y = x^2 + 3x 1$?

下列何者可表示 $y = x^2 + 3x - 1$ 的圖像?

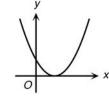
A.



B.



C.



D.



12. Set up a quadratic equation in x whose roots are

0 and
$$\frac{1}{4}$$
.

建立一個以 x 為變數,並以 0 和 $\frac{1}{4}$ 為根

的二次方程。

A.
$$(4x-1)^2=0$$

B.
$$4x^2 + x = 0$$

C.
$$4x^2 - 1 = 0$$

D.
$$4x^2 - x = 0$$

13. If the quadratic equation $x^2 - 7x + m + 1 = 0$ has real root(s), then

若二次方程 $x^2 - 7x + m + 1 = 0$ 有實根,則

A.
$$m < 7$$
.

B.
$$m \le \frac{45}{4}$$
.

C.
$$m \ge -\frac{45}{4}$$
.

D.
$$m \le \frac{7}{2}$$
.

14. The graph of $y = 2x^2 + kx + 18$ touches the x-axis. Find the value(s) of k.

若 $y = 2x^2 + kx + 18$ 的圖像與 x 軸只相交於一點,求 k 的值。

- **A.** 12
- **B.** -12
- **C.** 12 or –12
- **D.** 144
- 15. The figure shows the graph of $y = x^2 + 4x + c$. Which of the following is a possible value of c?

圖中所示為 $y = x^2 + 4x + c$ 的圖像。下列哪一個是 c 的可能值?

- **A.** 4
- **B.** 5
- **C.** -4
- **D.** 0
- 16. If 0 , which of the following quadratic equations has / have two distinct real roots?

若 0 ,下列哪個二次方程有兩個相異實根?

I.
$$(p^2 + 1)x^2 + (2p - 1)x + 1 = 0$$

II.
$$px^2 - (p+7)x + 7 = 0$$

III.
$$\frac{1}{2}x^2 - 3px + 4p^2 + 18 = 0$$

- A. I only 只有 I
- B. II only 只有 II
- C. I and III only 只有 I 及 III
- D. II and III only 只有 II 及 III
- 17. In the figure, the total surface area of cuboid is 102 cm^2 . Find the volume of the cuboid.

圖中長方體的總表面面積是 $102 \text{ cm}^2 \cdot$ 求它的體積。

- **A.** 67.5 cm^3
- **B.** 80 cm^3

- **C.** 92.5 cm^3
- **D.** 102 cm^3
- 18. In a Marathon race of 40 km, the speed of John is x km/h. If Peter runs faster than John by 0.8 km/h, then Peter can finish the race
 - 2 hours earlier than John. Which of the following is true?

在一場全長 40 km 的馬拉松比賽中,志剛 的速率是 x km/h。若家偉跑得較志剛快 0.8 km/h,則家偉會較志剛早 2 小時完成 賽事。問下列哪個數式是正確的?

A.
$$\frac{40}{x} - \frac{40}{x+0.8} = 2$$

B.
$$\frac{40}{x-8} - \frac{40}{x+8} = 2$$

C.
$$\frac{40}{x-0.8} - \frac{40}{x} = 2$$

D.
$$\frac{40}{x} = (2 - 0.8)x$$

19. If k < 0, then the quadratic equation

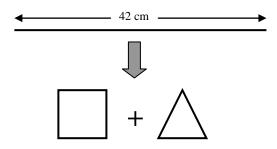
$$x^2 - x + 6k = 0 \text{ has}$$

若 k < 0, 則二次方程 $x^2 - x + 6k = 0$

- **A.** a double real root. 有一個二重實根。
- **B.** no real roots. 沒有實根。
- **C.** two negative roots. 有兩個負根。
- **D.** a positive root and a negative root. 有一個正根和一個負根。

20. As shown in the figure, a piece of wire of length 42 cm is cut into two parts and bent into a square and an equilateral triangle respectively. If the area of the square and that of the triangle are in the ratio $4:\sqrt{3}$, find the length of the side of the square.

如下圖所示,把一條長 42 cm 的鐵線剪成兩段,並分別圍成一個正方形和等邊三角形。若正方形面積:三個形面積 $= 4:\sqrt{3}$,求正方形的邊長。



- **A.** $2\sqrt{3}$ cm
- **B.** $\sqrt{6}$ cm
- **C.** 6 cm
- **D.** 12 cm
- 21. The area of a rectangular playground is

72a² m². If the perimeter of the playground is 41a m, find its dimensions.

- 一個長方形遊樂場的面積是 $72a^2$ m^2 。若該遊樂場的周界是 41a m,求它的大小。
- **A.** $(8a \times 9a) \text{ m}^2$
- **B.** $(6a \times 12a) \text{ m}^2$
- C. $(\frac{27}{4} \text{ a} \times \frac{32}{3} \text{ a}) \text{ m}^2$
- **D.** $(\frac{9}{2} a \times 16a) \text{ m}^2$