

# Functions and Graphs

## 函數及其圖像

### Exercise(練習)

1. If  $f(x) = -4x^2 + 3x$ , find the values of the function when

若  $f(x) = -4x^2 + 3x$ ，求在下列各情況中該函數的值。

(a)  $x = -1$ ,

(b)  $x = \frac{1}{2}$ .

2. If  $f(x) = 2x^2 + x$ , find the values of

若  $f(x) = 2x^2 + x$ ，求

(a)  $f\left(\frac{a}{3}\right)$ ,

(b)  $f(b-3)$ .

3. If  $f(x) = kx - x^2$  and  $f(5) = -5$ , find the values of

若  $f(x) = kx - x^2$  及  $f(5) = -5$ ，求

(a)  $k$ ,

(b)  $f(-5)$ .

4. If  $h(x) = x + \frac{1}{x^2 - 1}$ , find the values of

若  $h(x) = x + \frac{1}{x^2 - 1}$ ，求下列各題的值。

(a)  $h(-2)$ ,

(b)  $h(0)$ ,

(c)  $h(2)$ .

5. The figure shows the graph of  $y = ax + b$ .

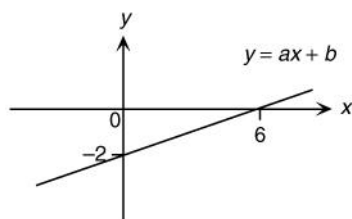
圖中所示為  $y = ax + b$  的圖像。

(a) Find the  $x$ -intercept and the  $y$ -intercept of the graph.

求圖像的  $x$  軸截距和  $y$  軸截距。

(b) Find the values of  $a$  and  $b$ .

求  $a$  和  $b$  的值。



6. (a) Plot the graph of the function  $y = \frac{x}{2} - 1$ .

繪畫函數  $y = \frac{x}{2} - 1$  的圖像。

(b) Hence, find the  $x$ -intercept and the  $y$ -intercept of  $y = \frac{x}{2} - 1$ .

由此，求  $y = \frac{x}{2} - 1$  的  $x$  軸截距和  $y$  軸截距。

7. (a) Plot the graph of  $y = -x^2 + 4x - 3$  from  $x = 0$  to  $x = 4$ .

繪畫  $y = -x^2 + 4x - 3$  在  $x = 0$  與  $x = 4$  之間的圖像。

(b) State the following features of the graph:

寫出該圖像的各項特性：

(i) Axis of symmetry

對稱軸

(ii) Coordinates of the vertex

頂點的坐標

(iii)  $y$ -intercept

$y$  軸截距

(iv) Direction of opening

開口方向

8. Determine the directions of opening and find the  $y$ -intercepts of the graphs of the following functions.

判斷下列各拋物線的開口方向，並求圖像的  $y$  軸截距。

(a)  $y = x^2 - 3x + 1$

(b)  $y = x(4 - x) - 2$

9. Find (a) the direction of opening,

(b) the vertex,

(c) the axis of symmetry

of the graph of  $y = -3(x + 1)^2 + 2$ .

求  $y = -3(x + 1)^2 + 2$  的圖像的

(a) 開口方向；

(b) 頂點的坐標；

(c) 對稱軸。

10. A marble is projected vertically upwards to the ceiling of a house from the floor. After  $t$  seconds, its height ( $h$  m) above the ground is given by:

一顆彈珠從地面向上拋起。 $t$  秒後，該顆彈珠距離地面的高度 ( $h$  m) 可由以下的公式計算：

$$h = -5t^2 + 5t + 1$$

(a) When will the marble attain its maximum height?

問該顆彈珠於何時會達到最高的高度？

(b) If the ceiling is 3 m above the ground, will the marble hit the ceiling?

若天花板距離地面 3 m，該顆彈珠會擊中天花板嗎？

11. Find the optimum value for each of the following quadratic functions and the axis of symmetry of their graphs.

求下列各二次函數的極值，並求其圖像的對稱軸。

(a)  $y = x^2 + 6x - 7$

(b)  $y = -2x^2 - 8x + 3$

12. Given  $y = 2(x - 3)^2 - 4$ , find

已知二次函數  $y = 2(x - 3)^2 - 4$ ，求

(a) its optimum value,

函數的極值；

(b) the direction of opening and the axis of symmetry of its graph.

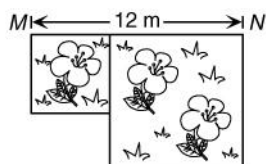
其圖像的開口方向和對稱軸。

13. Given that the minimum value of the function  $y = x^2 - 6x + k$  is  $-17$ , find the value of  $k$ .

已知函數  $y = x^2 - 6x + k$  的極小值是  $-17$ ，求  $k$  的值。

14. The figure shows a garden in the shape of two squares alongside each other. If  $MN$  is 12 m long, find the minimum area of the garden.

圖中的花園是由兩個正方形併合而成的。若  $MN$  的長度固定為 12 m，求花園的面積的極小值。



15. Solve  $2x^3 - 5x^2 + 3x \leq 2$  graphically.

利用圖解法解  $2x^3 - 5x^2 + 3x \leq 2$ 。

16. Solve the following inequalities graphically.

利用圖解法解下列各不等式。

(a)  $x^2 - 4x + 1 > -2$

(b)  $x^2 - 4x + 1 < -2$

17. It is given that  $f(x) = 2x^2 + 3$  and  $g(x) = 2x^2 - 1$ . If  $f(x)$  is transformed to  $g(x)$ , describe the effect of the transformation on the graph of  $y = f(x)$ .

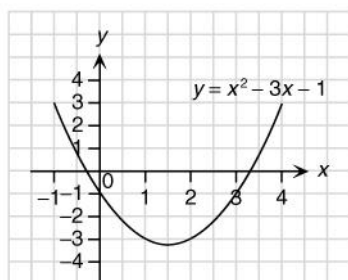
已知  $f(x) = 2x^2 + 3$  及  $g(x) = 2x^2 - 1$ 。描述當  $f(x)$  變換成  $g(x)$  時， $y = f(x)$  的圖像所受的影響。

18. Solve  $4^x \geq \frac{1}{4}$  graphically.

利用圖解法解  $4^x \geq \frac{1}{4}$ 。

19. Given the graph of  $y = x^2 - 3x - 1$ , solve the inequality  $x^2 - 3x + 2 > 0$  by adding a suitable straight line on the graph.

圖中所示為  $y = x^2 - 3x - 1$  的圖像。試在圖像中加上一條適當的直線，解不等式  $x^2 - 3x + 2 > 0$ 。



- 20. (a)** It is given that  $f(x) = x^2 + 2$ . Plot the graph of  $y = f(x)$  for  $-4 \leq x \leq 4$  and find the coordinates of the vertex of the graph.

已知  $f(x) = x^2 + 2$ 。繪畫  $y = f(x)$  在  $-4 \leq x \leq 4$  的圖像，並求它的頂點的坐標。

- (b)** Hence, if  $g(x) = f(x - 1)$ , find the coordinates of the vertex of the graph of  $y = g(x)$  without plotting the graph.

由此，若  $g(x) = f(x - 1)$ ，試不繪畫  $y = g(x)$  的圖像而求出它的頂點的坐標。

- 21.** The following table shows the tabular representation of a function  $f(x)$ :

以下為函數  $f(x)$  的表列形式：

$x$	-3	-2	-1	0	1	2	3	4
$f(x)$	7	2	-1	-2	-1	2	7	14

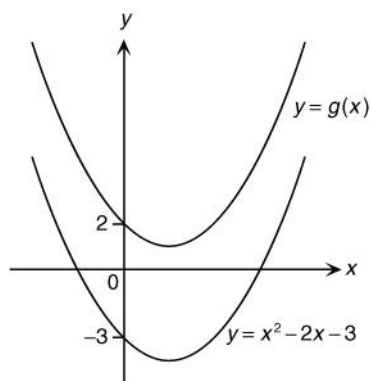
If the graph of  $y = g(x)$  is obtained by translating the graph of  $y = f(x)$  in the negative direction of the  $y$ -axis by 3 units, complete the following tabular representation of  $g(x)$ :

若  $y = f(x)$  的圖像沿  $y$  軸的負方向平移 3 單位，可得出  $y = g(x)$  的圖像。試完成以下  $g(x)$  的表列形式。

$x$	-3	-2	-1	0	1	2	3	4
$g(x)$								

- 22.** In the figure, the graph of  $y = g(x)$  is obtained by translating the graph of  $y = x^2 - 2x - 3$  in the positive direction of the  $y$ -axis. Find the symbolic representation of  $g(x)$ .

在圖中， $y = x^2 - 2x - 3$  的圖像沿  $y$  軸的正方向作平移變換，得出  $y = g(x)$  的圖像。試以符號形式表示  $g(x)$ 。

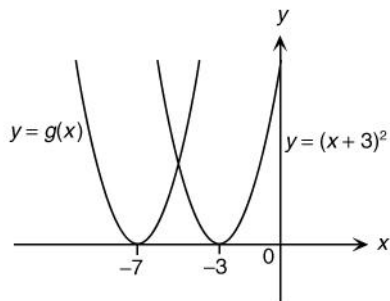


- 23.** If  $f(x) = x^2 - 2$  is transformed to  $g(x) = x^2 - 6x + 7$ , describe the effect of the transformation on the graph of  $y = f(x)$ .

若把  $f(x) = x^2 - 2$  變換成  $g(x) = x^2 - 6x + 7$ ，試描述  $y = f(x)$  的圖像所受的影響。

24. In the figure, the graph of  $y = g(x)$  is obtained by translating the graph of  $y = (x + 3)^2$  in the negative direction of the  $x$ -axis. Find the symbolic representation of  $g(x)$ .

在圖中， $y = (x + 3)^2$  的圖像沿  $x$  軸的負方向作平移變換後，可得出  $y = g(x)$  的圖像。試以符號形式表示  $g(x)$ 。



25. It is given that  $f(x) = x^2 + kx + 2$ .

已知  $f(x) = x^2 + kx + 2$ ，

(a) Find  $f(-x)$ .

求  $f(-x)$ ；

(b) If for any real number  $x$ ,  $f(x) = f(-x)$ , find the value of  $k$ .

對於任意的實數  $x$ ，若  $f(x) = f(-x)$ ，求  $k$  的值。

26. If  $f(x) = -x + 1$ , find the values of

若  $f(x) = -x + 1$ ，求

(a)  $f(0)$  and  $f(1)$ ,

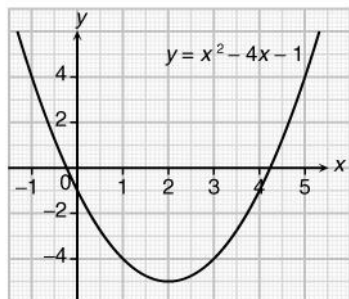
$f(0)$  和  $f(1)$  的值；

(b)  $\underbrace{f[f[f \cdots f[f(0)]] \cdots]}_{100 \text{ times}}$ .

$\underbrace{f[f[f \cdots f[f(0)]] \cdots]}_{100 \text{ 次}}$  的值。

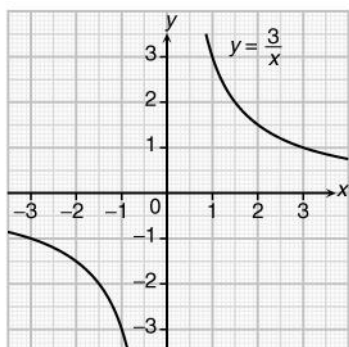
27. Given the graph of  $y = x^2 - 4x - 1$ , solve the inequality  $\sqrt{x^2 - 4x - 1} \leq 2$  by adding a suitable straight line on the graph.

圖中所示為  $y = x^2 - 4x - 1$  的圖像。試在圖像中加上一條適當的直線，解不等式  $\sqrt{x^2 - 4x - 1} \leq 2$ 。



28. Given the graph of  $y = \frac{3}{x}$ , solve the inequality  $\frac{3}{x} > 2$ .

圖中所示為  $y = \frac{3}{x}$  的圖像。試利用圖解法解不等式  $\frac{3}{x} > 2$ 。



## Pre-requisite Questions

### 預備測驗

1. Write down the linear equation  $4(1 - x) + 5y = 24$  in the form  $y = ax + b$ .

把二元一次方程  $4(1 - x) + 5y = 24$  寫成  $y = ax + b$  的形式。

2. If  $P(a, 2)$  and  $Q(0, b)$  both lie on the graph of  $x - y + 4 = 0$ , find the values of  $a$  and  $b$ .

若  $P(a, 2)$  和  $Q(0, b)$  均位於直線  $x - y + 4 = 0$  上，求  $a$  和  $b$  的值。

3. Write down the linear equation  $12x - 2y + 8 = 0$  in the form  $y = ax + b$ .

把二元一次方程  $12x - 2y + 8 = 0$  寫成  $y = ax + b$  的形式。

4. (a) Given  $y = 2x + 3$ , complete the following table.

已知  $y = 2x + 3$ ，完成下表。

$x$	-2	0	2
$y$			

- (b) Plot the graph of  $y = 2x + 3$  from  $x = -2$  to  $x = 2$ .

繪畫  $y = 2x + 3$  在  $x = -2$  與  $x = 2$  之間的圖像。

5. (a) Given  $y = -4x - 1$ , complete the following table.

已知  $y = -4x - 1$ ，完成下表。

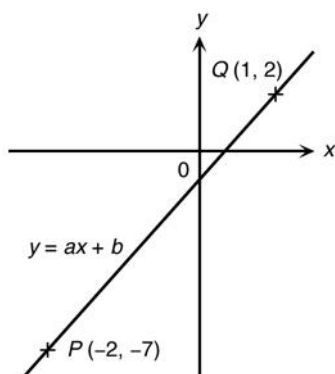
$x$	-1	0	1
$y$			

- (b) Plot the graph of  $y = -4x - 1$  from  $x = -1$  to  $x = 1$ .

繪畫  $y = -4x - 1$  在  $x = -1$  與  $x = 1$  之間的圖像。

6. If  $P(-2, -7)$  and  $Q(1, 2)$  both lie on the graph of  $y = ax + b$ , find the values of  $a$  and  $b$ .

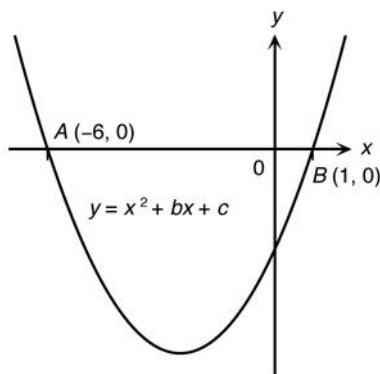
若  $P(-2, -7)$  和  $Q(1, 2)$  均位於直線  $y = ax + b$  上，求  $a$  和  $b$  的值。





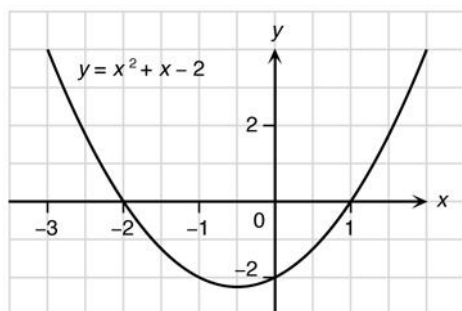
7. The figure shows the graph of  $y = x^2 + bx + c$  which cuts the  $x$ -axis at  $A(-6, 0)$  and  $B(1, 0)$ . Find the values of  $b$  and  $c$ .

圖中所示為  $y = x^2 + bx + c$  的圖像，該圖像與  $x$  軸相交於  $A(-6, 0)$  和  $B(1, 0)$ 。求  $b$  和  $c$  的值。



8. The figure shows the graph of  $y = x^2 + x - 2$ .

圖中所示為  $y = x^2 + x - 2$  的圖像。



- (a) Using the graph, state the number of real roots of  $x^2 + x = 2$ .  
根據以上的圖像，寫出方程  $x^2 + x = 2$  的實根的數目。
- (b) Solve the equation  $x^2 + x = 2$  graphically.  
利用圖解法解方程  $x^2 + x = 2$ 。
9. (a) Plot the graph of  $y = x^2 - 4x - 5$  from  $x = -2$  to  $x = 6$ .  
繪畫  $y = x^2 - 4x - 5$  在  $x = -2$  與  $x = 6$  之間的圖像。
- (b) Hence, solve the equation  $x^2 - 4x - 5 = 0$  graphically.
10. (a) Plot the graph of  $y = x^2 - 4x + 4$  from  $x = -1$  to  $x = 5$ .  
繪畫  $y = x^2 - 4x + 4$  在  $x = -1$  與  $x = 5$  之間的圖像。
- (b) Hence, solve the equation  $x^2 + 2 = 2(2x - 1)$  graphically.  
由此，利用圖解法解方程  $x^2 + 2 = 2(2x - 1)$ 。

- 11.** Given that the following expressions are perfect squares, find the value of  $p$ . Rewrite the expressions in the form  $(x \pm m)^2$ .

已知下列各數式均為完全平方，求  $p$  的值，並把數式寫成  $(x \pm m)^2$  的形式。

**(a)**  $x^2 + 3x + p$

**(b)**  $x^2 - \frac{x}{2} + p$

- 12.** Given that the following expressions are perfect squares, find the value of  $p$ . Rewrite the expressions in the form  $(x \pm m)^2$ .

已知下列各數式均為完全平方，求  $p$  的值，並把數式寫成  $(x \pm m)^2$  的形式。

**(a)**  $x^2 + 6x + p$

**(b)**  $x^2 - 10x + p$

## Level 1 Questions

### 程度 1 題目

1. If  $f(x) = 2x + 5$ , find the values of the function when

若  $f(x) = 2x + 5$ ，求在下列各情況中該函數的值。

(a)  $x = 0$ ,

(b)  $x = 5$ ,

(c)  $x = -3$ .

2. It is given that  $f(x) = 3x^2$ .

已知  $f(x) = 3x^2$ 。

(a) Find the values of  $f(1)$  and  $f(2)$ .

求  $f(1)$  和  $f(2)$  的值。

(b) Does the relation  $f(1) + f(1) = f(2)$  hold?

判斷  $f(1) + f(1) = f(2)$  是否成立。

3. If  $g(x) = 1 - (x + 1)(x - 2)$ , find the values of  $g(x)$  when

若  $g(x) = 1 - (x + 1)(x - 2)$ ，求在下列各情況中該函數的值。

(a)  $x = 2$ ,

(b)  $x = 0$ ,

(c)  $x = -4$ .

4. It is given that  $f(x) = (x + 3)(kx - 3)$  and  $g(x) = (x - 1)(x + 6)$ .

已知  $f(x) = (x + 3)(kx - 3)$  及  $g(x) = (x - 1)(x + 6)$ 。

(a) If  $f(3) = 18$ , find the value of  $k$ .

若  $f(3) = 18$ ，求  $k$  的值。

(b) If  $h(x) = g(2x)$ , write down the symbolic representation of  $h(x)$ .

若  $h(x) = g(2x)$ ，試以符號形式表示  $h(x)$ 。

(c) Solve the equation  $2f(x) - h(x) = 0$ .

解方程  $2f(x) - h(x) = 0$ 。

5. If  $h(x) = \frac{1}{ax} + 2x^2$  and  $h(5) = 100$ , find the value of  $a$ .

若  $h(x) = \frac{1}{ax} + 2x^2$  及  $h(5) = 100$ ，求  $a$  的值。

6. If  $h(x) = \frac{x-5}{x+5}$ , find the values of

若  $h(x) = \frac{x-5}{x+5}$ ，求下列各題的值。

(a)  $h(0)$ ,

(b)  $h(a+5)$ ,

(c)  $h\left(\frac{1}{b}\right)$ .

7. It is given that  $f(x) = ax^2 + 2x + 1$  and  $g(x) = x^2 + 3x$ .

已知  $f(x) = ax^2 + 2x + 1$  及  $g(x) = x^2 + 3x$ 。

(a) If  $f(-2) = 1$ , find the value of  $a$ .

若  $f(-2) = 1$ ，求  $a$  的值。

(b) If  $h(x) = g(x-1)$ , find the symbolic representation of  $h(x)$ .

若  $h(x) = g(x-1)$ ，試以符號形式表示  $h(x)$ 。

(c) Solve the equation  $f(x) = h(x) + 2$ .

解方程  $f(x) = h(x) + 2$ 。

8. Determine the directions of opening and find the  $y$ -intercepts of the graphs of the following functions.

判斷下列各函數的圖像的開口方向，並求圖像的  $y$  軸截距。

(a)  $y = x^2 - 4x + 6$

(b)  $y = (1 - 2x)^2 - 3$

9. (a) Plot the graph of the function  $y = 2x + 5$ .

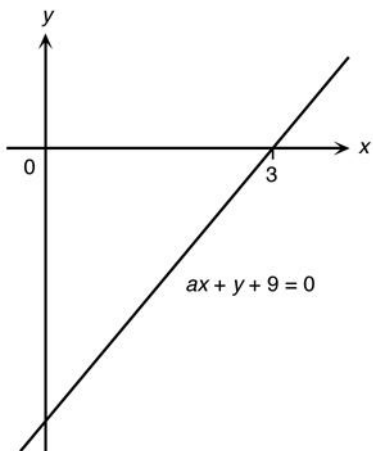
繪畫函數  $y = 2x + 5$  的圖像。

(b) Hence, find the  $x$ -intercept and the  $y$ -intercept of the graph.

由此，求該圖像的  $x$  軸截距和  $y$  軸截距。

10. The figure shows the graph of  $ax + y + 9 = 0$ .

圖中所示為  $ax + y + 9 = 0$  的圖像。

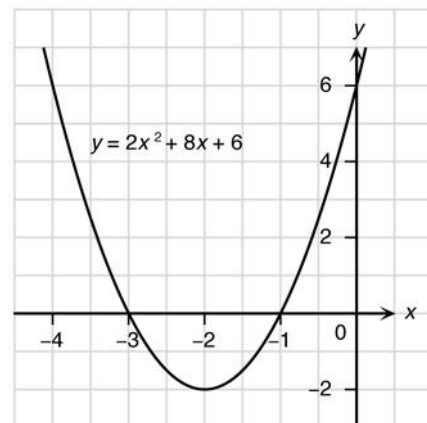


- (a) Find the x-intercept and the y-intercept of the graph.  
求圖像的  $x$  軸截距和  $y$  軸截距。
- (b) Find the value of  $a$ .  
求  $a$  的值。

11. The figure shows the graph of  $y = 2x^2 + 8x + 6$ . State the following features of the graph:

圖中所示為  $y = 2x^2 + 8x + 6$  的圖像，寫出該圖像的各項特性：

- (a) Axis of symmetry 對稱軸
- (b) Coordinates of the vertex 頂點的坐標
- (c) y-intercept  $y$  軸截距
- (d) Direction of opening 開口方向



12. Determine the directions of opening and find the y-intercepts of the graphs of the following functions.

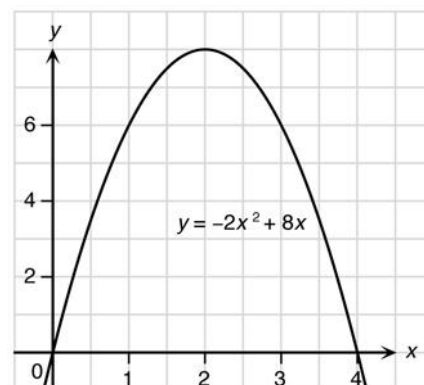
判斷下列各函數的圖像的開口方向，並求圖像的  $y$  軸截距。

- (a)  $y = -3x^2 + 10x - 7$
- (b)  $y = (x - 8)(1 + 2x) + 6$

13. The figure shows the graph of  $y = -2x^2 + 8x$ . State the following features of the graph:

圖中所示為  $y = -2x^2 + 8x$  的圖像，寫出該圖像的各項特性：

- (a) Axis of symmetry 對稱軸
- (b) Coordinates of the vertex 頂點的坐標
- (c) y-intercept  $y$  軸截距
- (d) Direction of opening 開口方向



14. For each of the following quadratic functions,

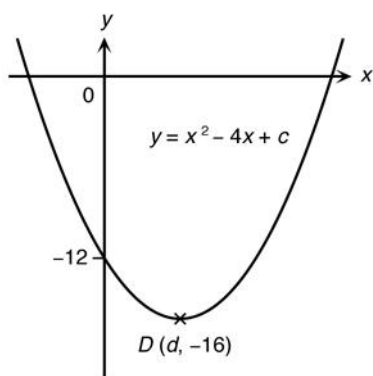
對於下列各二次函數，求其圖像的

- find (i) the direction of opening, 開口方向；  
 (ii) the coordinates of the vertex, 頂點的坐標；  
 (iii) the axis of symmetry 對稱軸。

of its graph.

15. The figure shows the graph of  $y = x^2 - 4x + c$ .  $D(d, -16)$  is the minimum point of the graph.

圖中所示為  $y = x^2 - 4x + c$  的圖像，該圖像的最低點是  $D(d, -16)$ 。



- (a) Find the values of  $c$  and  $d$ .

求  $c$  和  $d$  的值。

- (b) State the following features of the graph:

寫出該圖像的各項特性：

- (i) Axis of symmetry 對稱軸  
 (ii) Coordinates of the vertex 頂點的坐標  
 (iii)  $y$ -intercept  $y$  軸截距  
 (iv) Direction of opening 開口方向

16. Find the optimum value for each of the following quadratic functions and the axis of symmetry of their graphs.

求下列各二次函數的極值，並求其圖像的對稱軸。

(a)  $y = x^2 - 10x + 27$

(b)  $y = -2x^2 - 16x + 1$

17. Find the optimum values of the following quadratic functions.

求下列各二次函數的極值。

(a)  $y = -(x - 5)^2 - 17$

(b)  $y = (x + 2)^2 + 1$

18. Given that the maximum value of the function  $y = (4 - 2x)(x + k) + 2(k - 10)x$  is  $-4$ .

已知函數  $y = (4 - 2x)(x + k) + 2(k - 10)x$  的極大值是  $-4$ 。

(a) Find the value of  $k$ .

求  $k$  的值。

(b) State the axis of symmetry of its graph.

寫出其圖像的對稱軸。

19. For the quadratic function  $y = -3x^2 - 12x - 20$ ,

對於二次函數  $y = -3x^2 - 12x - 20$ ，

(a) find the optimum value of the function,

求函數的極值；

(b) state (i) the direction of opening,

(ii) the coordinates of the vertex,

(iii) the axis of symmetry

of its graph.

(b) 寫出其圖像的

(i) 開口方向；

(ii) 頂點的坐標；

(iii) 對稱軸。

20. Given that the maximum value of the function  $y = -3x^2 + 6x + p$  is 11.

已知函數  $y = -3x^2 + 6x + p$  的極大值是 11。

(a) Find the value of  $p$ .

求  $p$  的值。

(b) State the coordinates of the vertex of its graph.

寫出其圖像的頂點的坐標。

21. Given that the difference between two numbers is 10, find the minimum value of the product of these two numbers.

已知兩個數之差是 10，求它們的積的極小值。

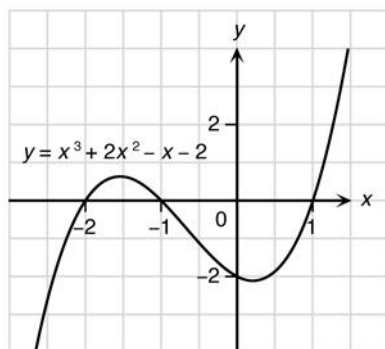
22. If the sum of two numbers is 12, find the maximum value of the product of these two numbers.

若兩個數之和是 12，求它們的積的極大值。

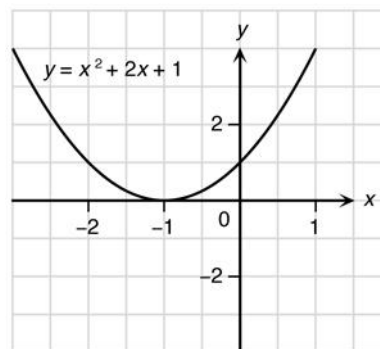
23. Describe the properties (number of  $x$ -intercepts, axis of symmetry and maximum or minimum point) of the following graphs.

描述下列各圖像的特性 ( $x$  軸截距的數目、對稱軸和最高點/最低點)。

(a)



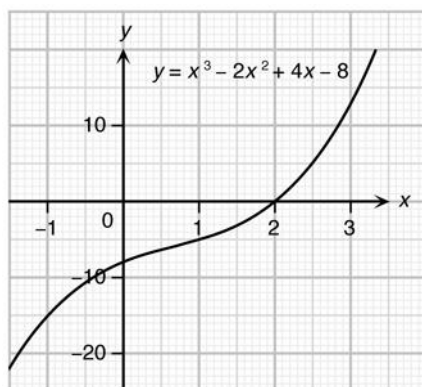
(b)



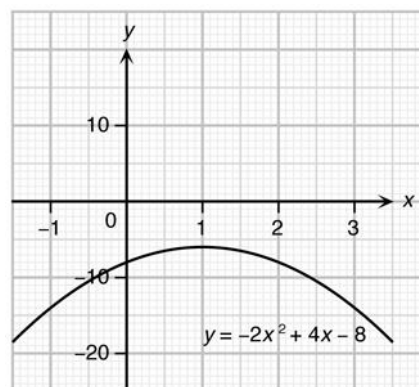
24. Describe the properties (number of  $x$ -intercepts, axis of symmetry and maximum or minimum point) of the following graphs.

描述下列各圖像的特性 ( $x$  軸截距的數目、對稱軸和最高點/最低點)。

(a)



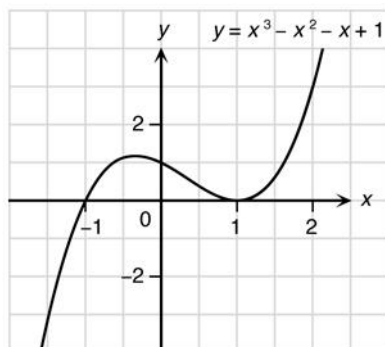
(b)



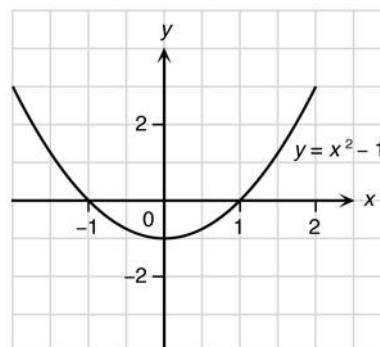
25. Describe the properties (number of  $x$ -intercepts, axis of symmetry and maximum or minimum point) of the following graphs.

描述下列各圖像的特性 ( $x$  軸截距的數目、對稱軸和最高點/最低點)。

(a)



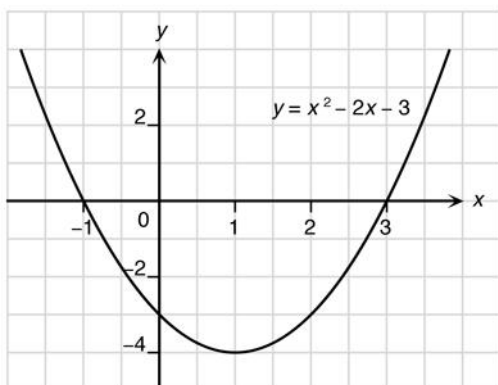
(b)





26. The figure shows the graph of  $y = x^2 - 2x - 3$ . Solve the inequality  $x^2 - 2x - 3 < -3$  by drawing a suitable straight line on the graph.

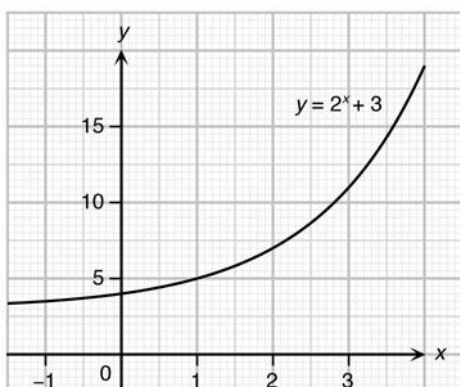
圖中所示為  $y = x^2 - 2x - 3$  的圖像。試在圖像中加上一條適當的直線，解不等式  $x^2 - 2x - 3 < -3$ 。



27. The figure shows the graph of  $y = 2^x + 3$ . Solve the inequality  $2^x + 3 \geq 7$  by drawing a suitable straight line on the graph. (Give your answer correct to 1 decimal place.)

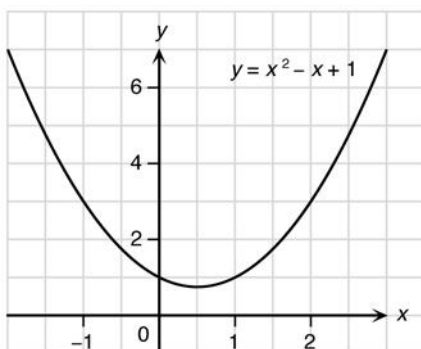
圖中所示為  $y = 2^x + 3$  的圖像。試在圖像中加上一條適當的直線，解不等式  $2^x + 3 \geq 7$ 。

(答案須準確至一位小數。)



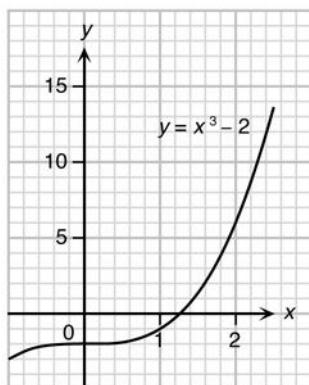
28. The figure shows the graph of  $y = x^2 - x + 1$ . Solve the inequality  $x^2 - x + 1 > 3$  by drawing a suitable straight line on the graph.

圖中所示為  $y = x^2 - x + 1$  的圖像。試在圖像中加上一條適當的直線，解不等式  $x^2 - x + 1 > 3$ 。



29. The figure shows the graph of  $y = x^3 - 2$ . Solve the inequality  $x^3 - 2 \leq 6$  by drawing a suitable straight line on the graph. (Give your answer correct to the nearest 0.2.)

圖中所示為  $y = x^3 - 2$  的圖像。試在圖像中加上一條適當的直線，解不等式  $x^3 - 2 \leq 6$ 。  
(答案須準確至最接近的 0.2。)



30. In each of the following, if  $f(x)$  is transformed to  $g(x)$ , describe the effect of the transformation on the graph of  $y = f(x)$ .

對於下列各題，若把  $f(x)$  變換成  $g(x)$ ，試描述  $y = f(x)$  的圖像所受的影響。

(a)  $g(x) = f(x) - 2$

(b)  $g(x) = f(x + 2)$

31. It is given that the graphs of  $y = x^2 + 5x + 9$  and  $y = 4$ . Suggest an inequality that can be solved.

已知  $y = x^2 + 5x + 9$  及  $y = 4$  的圖像。試舉出一個可求出解的不等式。

32. The following table shows the tabular representation of a function  $f(x)$ :

以下為函數  $f(x)$  的表列形式：

$x$	0	1	2	3	4	5
$f(x)$	-3	-2	-1	0	1	2

If the graph of  $y = g(x)$  is obtained by translating the graph of  $y = f(x)$  in the positive direction of the  $x$ -axis by 1 unit, complete the following table.

若把  $y = f(x)$  的圖像沿  $x$  軸的正方向平移 1 單位，可變換成  $y = g(x)$  的圖像，試完成下表。

$x$	1	2	3	4
$g(x)$				

33. The following table shows the tabular representation of a function  $f(x)$ :

以下為函數  $f(x)$  的表列形式：

$x$	0	1	2	3	4
$f(x)$	3	4	6	9	13

If the graph of  $y = g(x)$  is obtained by translating the graph of  $y = f(x)$  in the negative direction of the  $y$ -axis by 7 units, complete the following table.

若把  $y = f(x)$  的圖像沿  $y$  軸的負方向平移 7 單位，可變換成  $y = g(x)$  的圖像，試完成下表。

$x$	0	1	2	3	4
$g(x)$					

34. If  $f(x) = x^2 - x - 6$  is transformed to  $g(x) = x^2 - x + 36$ , describe the effect of the transformation on the graph of  $y = f(x)$ .

描述當  $f(x) = x^2 - x - 6$  變換成  $g(x) = x^2 - x + 36$  時， $y = f(x)$  的圖像所受的影響。

35. If  $f(x) = (3x + 5)^2$  is transformed to  $g(x) = 9x^2 + 12x + 4$ , describe the effect of the transformation on the graph of  $y = f(x)$ .

描述當  $f(x) = (3x + 5)^2$  變換成  $g(x) = 9x^2 + 12x + 4$  時， $y = f(x)$  的圖像所受的影響。

36. It is given that  $f(x) = (2 - 3x)^2$ . If the graph of  $y = g(x)$  is obtained by translating the graph of  $y = f(x)$  in each of the following directions, express  $g(x)$  in the form  $ax^2 + bx + c$ .

已知  $f(x) = (2 - 3x)^2$ 。若把  $y = f(x)$  的圖像沿下列的方向變換成  $y = g(x)$  的圖像，試把  $g(x)$  寫成  $ax^2 + bx + c$  的形式。

(a) Translated in the positive direction of the  $y$ -axis by 4 units.

沿  $y$  軸的正方向平移 4 單位。

(b) Translated in the negative direction of the  $x$ -axis by 3 units.

沿  $x$  軸的負方向平移 3 單位。

37. It is given that  $f(x) = x^2 - 2x + 3$ . If the graph of  $y = g(x)$  is obtained by translating the graph of  $y = f(x)$  in each of the following directions, find the symbolic representation of  $g(x)$ .

已知  $f(x) = x^2 - 2x + 3$ 。若把  $y = f(x)$  的圖像沿下列的方向變換成  $y = g(x)$  的圖像，試以符號形式表示  $g(x)$ 。

(a) Translated in the negative direction of the  $y$ -axis by 2 units.

沿  $y$  軸的負方向平移 2 單位。

(b) Translated in the positive direction of the  $x$ -axis by 4 units.

沿  $x$  軸的正方向平移 4 單位。

## Level 2 Questions

### 程度 2 題目

1. It is given that  $g(x) = 4x - 9$ . Find the values of  $x$  if  $g(x) - g\left(\frac{1}{x}\right) = 0$ .

已知  $g(x) = 4x - 9$ 。若  $g(x) - g\left(\frac{1}{x}\right) = 0$ ，求  $x$  的值。

2. It is given that  $f(x) = (x + 2k)^2$  and  $g(x) = 5k + x$ .

已知  $f(x) = (x + 2k)^2$  及  $g(x) = 5k + x$ 。

- (a) If  $f(0) - 3g(3) = k - 25$ , find the value of  $k$ .

若  $f(0) - 3g(3) = k - 25$ ，求  $k$  的值。

- (b) Solve the equation  $f(x) - 2g(x) = 3$ .

解方程  $f(x) - 2g(x) = 3$ 。

3. If  $f(x) = x^2 + 3x - 15$ , find the values of

若  $f(x) = x^2 + 3x - 15$ ，求下列各題的值。

(a)  $f(3)$ ,

(b)  $f(-a)$ ,

(c)  $f(2b + 1)$ .

4. If  $f(x) = x^2 - \frac{1}{x^2}$ , show that  $f\left(\frac{1}{x}\right) = -f(x)$ .

若  $f(x) = x^2 - \frac{1}{x^2}$ ，證明  $f\left(\frac{1}{x}\right) = -f(x)$ 。

5. It is given that  $f(x) = (x + k)(x - 1) - 2x$  and  $f(k) = k^2 - 3$ . Find the value(s) of  $k$ .

已知  $f(x) = (x + k)(x - 1) - 2x$  及  $f(k) = k^2 - 3$ ，求  $k$  的值。

6. It is given that  $f(x) = (x - a)(x - b) + 5$ . If  $f(a) = b$  and  $f(2b) = 3b$ , find the values of  $a$  and  $b$ .

已知  $f(x) = (x - a)(x - b) + 5$ 。若  $f(a) = b$  及  $f(2b) = 3b$ ，求  $a$  和  $b$  的值。

7. Let  $g(x) = \left(\frac{x-1}{x+1}\right)^2$ .

設  $g(x) = \left(\frac{x-1}{x+1}\right)^2$ 。

(a) Find  $g\left(\frac{x-1}{x+1}\right)$ .

求  $g\left(\frac{x-1}{x+1}\right)$ 。

(b) Hence, find the value of  $g\left(\frac{5}{7}\right)$ .

由此，求  $g\left(\frac{5}{7}\right)$  的值。

8. It is given that  $f(x) = x^2 - 3x + 2$ .

已知  $f(x) = x^2 - 3x + 2$ 。

(a) Find the values of  $f(2a)$  and  $f(a+2)$ .

求  $f(2a)$  和  $f(a+2)$  的值。

(b) If  $f(2a) = f(a+2) + 2f(a)$ , find the value(s) of  $a$ .

若  $f(2a) = f(a+2) + 2f(a)$ ，求  $a$  的值。

9. (a) Plot the graph of  $y = 3x^2 - 12x + 9$  from  $x = 0$  to  $x = 4$ .

繪畫  $y = 3x^2 - 12x + 9$  在  $x = 0$  與  $x = 4$  之間的圖像。

(b) State (i) the axis of symmetry,

(ii) the coordinates of the vertex,

(iii) the y-intercept,

(iv) the direction of opening

of the graph.

(b) 寫出該圖像的

(i) 對稱軸；

(ii) 頂點的坐標；

(iii) y 軸截距；

(iv) 開口方向。

10. It is given that  $f(x) = ax^2 + bx + 3$ .

已知  $f(x) = ax^2 + bx + 3$ 。

(a) If  $f(-1) = 1$  and  $f(3) = 33$ , find the values of  $a$  and  $b$ .

若  $f(-1) = 1$  及  $f(3) = 33$ ，求  $a$  和  $b$  的值。

(b) If  $g\left(\frac{x}{2}\right) = f(x)$ , find the symbolic representation of  $g(x)$ .

若  $g\left(\frac{x}{2}\right) = f(x)$ ，試以符號形式表示  $g(x)$ 。

(c) Hence, find the values of  $g(5)$  and  $g(p-1)$ .

由此，求  $g(5)$  和  $g(p-1)$  的值。

11. (a) Plot the graph of  $y = (2x-3)^2 + 2(1-2x)$  from  $x = 0$  to  $x = 4$ .

繪畫  $y = (2x-3)^2 + 2(1-2x)$  在  $x = 0$  與  $x = 4$  之間的圖像。

(b) State (i) the axis of symmetry,

(ii) the coordinates of the vertex,

(iii) the y-intercept,

(iv) the direction of opening

of the graph.

(b) 寫出該圖像的

(i) 對稱軸；

(ii) 頂點的坐標；

(iii) y 軸截距；

(iv) 開口方向。

12. (a) Plot the graph of  $y = 9 - (x+5)(x-3)$  from  $x = -6$  to  $x = 4$ .

繪畫  $y = 9 - (x+5)(x-3)$  在  $x = -6$  與  $x = 4$  之間的圖像。

(b) State (i) the axis of symmetry,

(ii) the coordinates of the vertex,

(iii) the y-intercept,

(iv) the direction of opening

of the graph.

(b) 寫出該圖像的

(i) 對稱軸；

(ii) 頂點的坐標；

(iii) y 軸截距；

(iv) 開口方向。

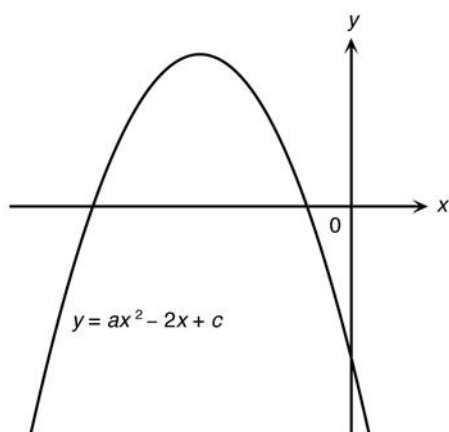
13. (a) Plot the graph of  $y = -x^2 - 2x + 15$  from  $x = -6$  to  $x = 4$ .  
繪畫  $y = -x^2 - 2x + 15$  在  $x = -6$  與  $x = 4$  之間的圖像。

- (b) State (i) the axis of symmetry,  
(ii) the coordinates of the vertex,  
(iii) the y-intercept,  
(iv) the direction of opening  
of the graph.

- (b) 寫出該圖像的  
(i) 對稱軸；  
(ii) 頂點的坐標；  
(iii) y 軸截距；  
(iv) 開口方向。

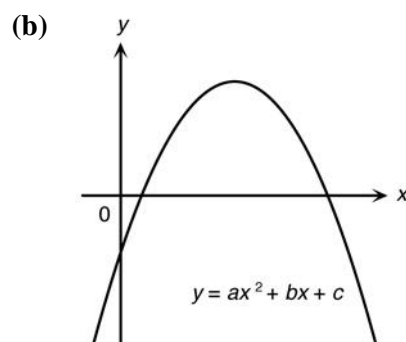
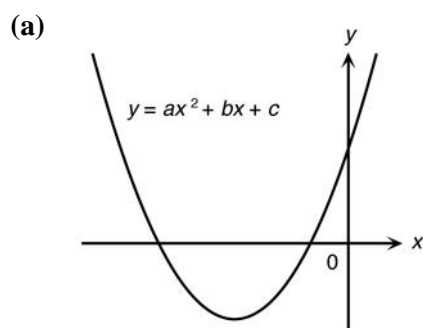
14. The figure shows the graph of  $y = ax^2 - 2x + c$ . Show that  $0 < ac < 1$ .

圖中所示為  $y = ax^2 - 2x + c$  的圖像。試證明  $0 < ac < 1$ 。



15. For each of the following graphs of  $y = ax^2 + bx + c$ , state the signs of  $a$  and  $c$ .

對於下列各  $y = ax^2 + bx + c$  的圖像，判斷  $a$  和  $c$  的正負號。



16. For the quadratic function  $y = (3x - 2)(4 - x) - 2(x - 2)$ ,

對於二次函數  $y = (3x - 2)(4 - x) - 2(x - 2)$ ，

- (a) find the optimum value of the function,  
求函數的極值；
- (b) state (i) the coordinates of the vertex,  
(ii) the axis of symmetry,  
(iii) the direction of opening  
of its graph.
- (b) 寫出其圖像的
  - (i) 頂點的坐標；
  - (ii) 對稱軸；
  - (iii) 開口方向。

17. For the quadratic function  $y = (x + 3)^2 - 2(5 + 2x)$ ,

對於二次函數  $y = (x + 3)^2 - 2(5 + 2x)$ ，

- (a) find the optimum value of the function,  
求函數的極值；
- (b) state (i) the coordinates of the vertex,  
(ii) the axis of symmetry,  
(iii) the direction of opening  
of its graph.
- (b) 寫出其圖像的
  - (i) 頂點的坐標；
  - (ii) 對稱軸；
  - (iii) 開口方向。

18. Given that the minimum value of the function  $y = (x - k)^2 + 6k(x - 1)$  is 3.

已知函數  $y = (x - k)^2 + 6k(x - 1)$  的極小值是 3。

- (a) Find the value of  $k$ .  
求  $k$  的值。
- (b) State the axis of symmetry of its graph.  
寫出其圖像的對稱軸。

19. Given that the axis of symmetry of the graph of  $y = x^2 - 4qx + 6q^2$  is  $x = 6$ .

已知  $y = x^2 - 4qx + 6q^2$  的圖像的對稱軸是  $x = 6$ 。

- (a) Find the value of  $q$ .  
求  $q$  的值。
- (b) Find the optimum value of  $y$ .  
求  $y$  的極值。



20. It is given that the total length of all the sides of the two cubes is 12 cm.

已知兩個正方體的所有邊長之和是 12 cm。

(a) Find the minimum value of the total volume of the two cubes.

求該兩個正方體的總體積的最小值。

(b) Find the lengths of a side of the two cubes when the total volume of the two cubes is minimum.

求當該兩個正方體的總體積達到最小值時，各正方體的邊長。

21. If the optimum value of the function  $y = x^2 - 3px + 21$  is  $6p$ , where  $p$  is positive, find the axis of symmetry of its graph.

若函數  $y = x^2 - 3px + 21$  的極值是  $6p$ ，其中  $p$  為正數，求其圖像的對稱軸。

22. The profit (\$  $P$ ) of holding a party with  $x$  tickets sold is given by:

已知某派對售出  $x$  張入場券，而舉辦該派對所得的盈利 (\$  $P$ ) 可由以下的公式計算：

$$P = 2400x - 80x^2$$

(a) How many tickets are sold when the profit is maximum?

求當盈利達到最高時，所售出的入場券數目。

(b) What is the maximum profit of holding a party?

舉辦該派對最多可獲得多少盈利？

23. Suggest a quadratic function such that the following conditions are satisfied:

試舉出一個符合下列條件的二次函數：

(1) The maximum value of the function is 5.

該函數的極大值是 5。

(2) The axis of symmetry of its graph is  $x = 1$ .

其圖像的對稱軸是  $x = 1$ 。

(3) The  $y$ -intercept of its graph is a positive integer.

其圖像的  $y$  軸截距是一個正整數。

24. In the figure, a rectangular picture  $ABCD$  of perimeter 160cm is hung by a piece of rope  $AED$  at the peg  $E$ , where  $AE = ED$ . It is given that the shortest distance between the peg and the picture is 15 cm.

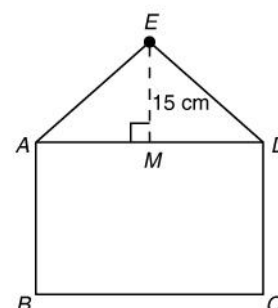
如圖所示，建強利用掛釘  $E$  和一條繩子  $AED$ ，其中  $AE = ED$ ，把一幅周界為 160 cm 的長方形圖畫  $ABCD$  固定於牆上。已知掛釘與圖畫的短距離為 15 cm。

(a) Find the maximum possible area of the picture.

求該圖畫的最大可能面積。

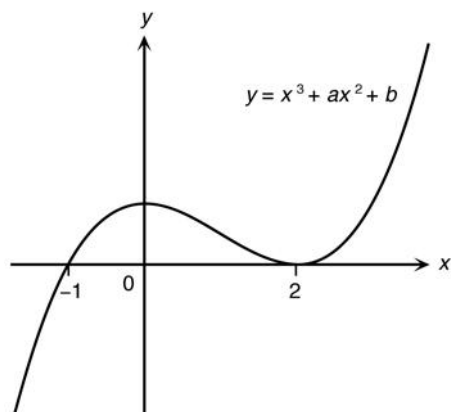
(b) Find the length of the rope when the area of the picture is maximum.

求當該圖畫的面積達到最大值時，繩子的長度。



25. The figure shows the graph of the function  $y = x^3 + ax^2 + b$ .

圖中所示為函數  $y = x^3 + ax^2 + b$  的圖像。



(a) Find the values of  $a$  and  $b$ .

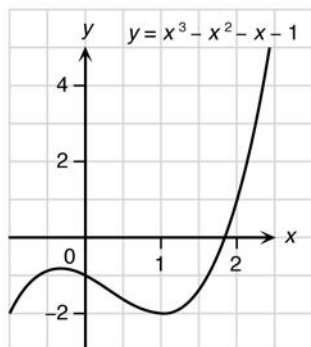
求  $a$  和  $b$  的值。

(b) If  $C(0, c)$  and  $D(-2, d)$  lie on the graph, find the values of  $c$  and  $d$ .

若  $C(0, c)$  和  $D(-2, d)$  均位於該圖像上，求  $c$  和  $d$  的值。

26. The figure shows the graph of the function  $y = x^3 - x^2 - x - 1$ .

圖中所示為函數  $y = x^3 - x^2 - x - 1$  的圖像。



(a) Find the minimum value of the function for  $0 \leq x \leq 2$ .

求該函數在  $0 \leq x \leq 2$  的極小值。

(b) Is the value obtained in (a) a minimum value of the function?

問從 (a) 所得的值是該函數的極小值嗎？

27. Given the graph of  $y = x^3 - 2x - 1$ . Find the equation of the straight line that should be added on the graph in order to solve  $2x^3 - 4x \geq 8$ .

已知  $y = x^3 - 2x - 1$  的圖像。問應在該圖像中加上哪一條直線，從而解  $2x^3 - 4x \geq 8$ ？

28. Given the graph of  $y = x^2 + 7x + 5$ . Find the equation of the straight line that should be added on the graph in order to solve  $x^2 + 7x < 1$ .

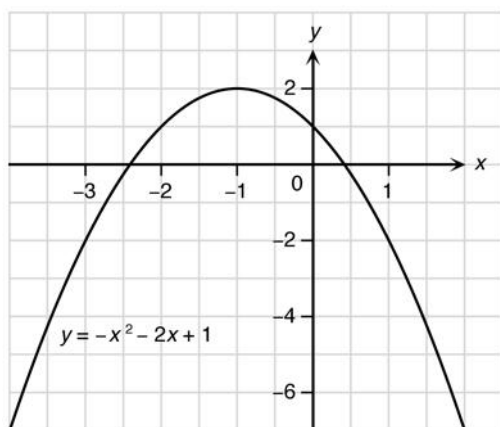
已知  $y = x^2 + 7x + 5$  的圖像。問應在該圖像中加上哪一條直線，從而解  $x^2 + 7x < 1$ ？

29. Solve the inequality  $-3x^2 + 6x + 2 \leq 2$  graphically.

利用圖解法解不等式  $-3x^2 + 6x + 2 \leq 2$ 。

30. The figure shows the graph of  $y = -x^2 - 2x + 1$ .

圖中所示為  $y = -x^2 - 2x + 1$  的圖像。



- (a) Solve  $(x - 1)^2 + 4x < 0$  graphically.

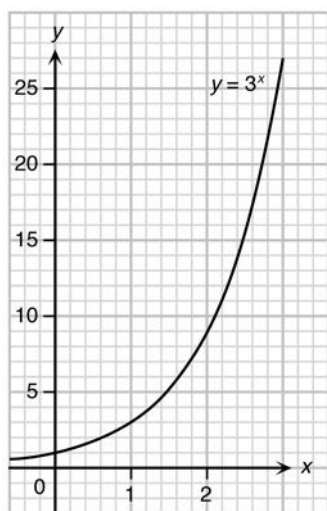
利用圖解法解  $(x - 1)^2 + 4x < 0$ 。

- (b) Find the largest integer  $x$  that satisfies  $x(x + 2) \leq 4$ .

求一個最大的整數  $x$ ，使  $x(x + 2) \leq 4$ 。

31. The figure shows the graph of  $y = 3^x$ .

圖中所示為  $y = 3^x$  的圖像。



- (a) Solve  $3^x - 9 < 0$  graphically.

(Give your answer correct to the nearest 0.2.)

利用圖解法解  $3^x - 9 < 0$ 。(答案須準確至最接近的 0.2。)

- (b) Find the smallest integer  $x$  that satisfies  $-3^x + 10 < -5$ .

求一個最小的整數  $x$ ，使  $-3^x + 10 < -5$ 。

32. Solve the inequality  $x^2 + 5x - 4 > -10$  graphically.

利用圖解法解不等式  $x^2 + 5x - 4 > -10$ 。

33. (a) Plot the graph of  $y = x^2 + 3x - 4$ .

繪畫  $y = x^2 + 3x - 4$  的圖像。

- (b) Hence, find the largest integer  $k$  such that the inequality  $(x + 2)(x - 2) > k - 3x$  is always true.

由此，求一個最大的整數  $k$ ，使  $(x + 2)(x - 2) > k - 3x$ 。

34. It is given that  $f(x) = x^2 + 4x + 1$  and  $g(x) = x^2 + 10x + 22$ . If  $f(x)$  is transformed to  $g(x)$ , describe the effect of the transformation on the graph of  $y = f(x)$ .

已知  $f(x) = x^2 + 4x + 1$  及  $g(x) = x^2 + 10x + 22$ 。描述當  $f(x)$  變換成  $g(x)$  時， $y = f(x)$  的圖像所受的影響。

35. It is given that  $f(x) = x^2 + 5$ ,  $g(x) = f(x) - 8$  and  $h(x) = g(x + 2)$ .

已知  $f(x) = x^2 + 5$ ， $g(x) = f(x) - 8$  及  $h(x) = g(x + 2)$ 。

- (a) (i) Describe the effect of the transformation on the graph of  $y = f(x)$  when  $f(x)$  is transformed to  $g(x)$ .

描述當  $f(x)$  變換成  $g(x)$  時， $y = f(x)$  的圖像所受的影響。

- (ii) Find the symbolic representation of  $g(x)$ .

試以符號形式表示  $g(x)$ 。

- (b) (i) Describe the effect of the transformation on the graph of  $y = g(x)$  when  $g(x)$  is transformed to  $h(x)$ .

描述當  $g(x)$  變換成  $h(x)$  時， $y = g(x)$  的圖像所受的影響。

- (ii) Find the symbolic representation of  $h(x)$ .

試以符號形式表示  $h(x)$ 。

36. (a) Given that the function  $f(x) = x^2 + 8x + 12$ , find the  $x$ -intercept(s) of its graph.

已知函數  $f(x) = x^2 + 8x + 12$ ，求其圖像的  $x$  軸截距。

- (b) Hence, if the graph of  $y = g(x)$  is obtained by translating the graph of  $y = f(x)$  in the negative direction of the  $x$ -axis by 2 units, find the  $x$ -intercepts of the graph of  $y = g(x)$ .

由此，若把  $y = f(x)$  的圖像沿  $x$  軸的負方向平移 2 單位，可得出  $y = g(x)$  的圖像，求  $y = g(x)$  的圖像的  $x$  軸截距。

37. It is given that  $f(x) = -x^2 + 3x - 29$ ,  $g(x) = f(x - 3)$  and  $h(x) = g(x) + 3$ .

已知  $f(x) = -x^2 + 3x - 29$ ， $g(x) = f(x - 3)$  及  $h(x) = g(x) + 3$ 。

- (a) (i) Describe the effect of the transformation on the graph of  $y = f(x)$  when  $f(x)$  is transformed to  $g(x)$ .

描述當  $f(x)$  變換成  $g(x)$  時， $y = f(x)$  的圖像所受的影響。

- (ii) Find the symbolic representation of  $g(x)$ .

試以符號形式表示  $g(x)$ 。

- (b) (i) Describe the effect of the transformation on the graph of  $y = g(x)$  when  $g(x)$  is transformed to  $h(x)$ .

描述當  $g(x)$  變換成  $h(x)$  時， $y = g(x)$  的圖像所受的影響。

- (ii) Find the symbolic representation of  $h(x)$ .

試以符號形式表示  $h(x)$ 。

38. Given that  $f(x) = 2x^2 - 3x - 4$  is transformed to  $g(x) = 2x^2 - 3x + 1$  first, then to  $h(x) = 2x^2 - 11x + 15$ .

Describe the effect of the transformation on the graph of  $y = f(x)$  when  $f(x)$  is transformed to  $h(x)$ .

已知  $f(x) = 2x^2 - 3x - 4$  先給變換成  $g(x) = 2x^2 - 3x + 1$ ，然後再變換成  $h(x) = 2x^2 - 11x + 15$ 。描述當  $f(x)$  變換成  $h(x)$  時， $y = f(x)$  的圖像所受的影響。

39. (a) Given that the function  $f(x) = x^2 - 3x + 10$ , find the y-intercept of its graph.

已知函數  $f(x) = x^2 - 3x + 10$ ，求其圖像的  $y$  軸截距。

- (b) Hence, if  $g(x) = f(x) - 2$ , find the y-intercept of the graph of  $y = g(x)$ .

由此，若  $g(x) = f(x) - 2$ ，求  $y = g(x)$  的圖像的  $y$  軸截距。

40. (a) It is given that  $f(x) = x^2 + 4x - 5$ . Plot the graph of  $y = f(x)$  for  $-6 \leq x \leq 2$  and find the coordinates of the vertex of the graph.

已知  $f(x) = x^2 + 4x - 5$ 。繪畫  $y = f(x)$  在  $-6 \leq x \leq 2$  的圖像，並求該圖像的頂點的坐標。

- (b) It is given that  $g(x) = f(x + 1)$ .

已知  $g(x) = f(x + 1)$ 。

- (i) Describe the effect of the transformation on the graph of  $y = f(x)$  when  $f(x)$  is transformed to  $g(x)$ .

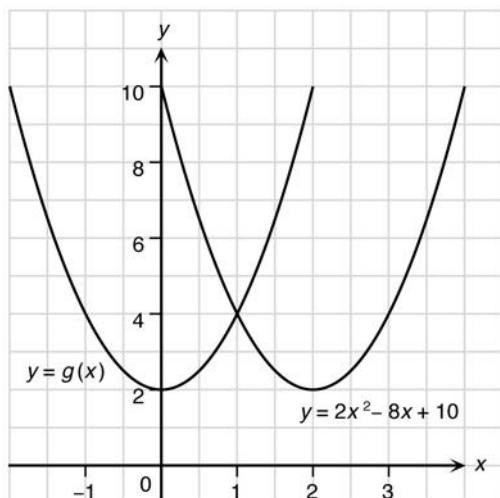
描述當  $f(x)$  變換成  $g(x)$  時， $y = f(x)$  的圖像所受的影響。

- (ii) By the result obtained in (a), find the coordinates of the vertex of the graph of  $y = g(x)$ .

利用 (a) 所得的結果，求  $y = g(x)$  的圖像的頂點的坐標。

41. In the figure, the graph of  $y = g(x)$  is obtained by translating the graph of  $y = 2x^2 - 8x + 10$ .

在圖中，把  $y = 2x^2 - 8x + 10$  的圖像作平移變換，可得出  $y = g(x)$  的圖像。



- (a) Describe the effect of the transformation on the graph of  $y = 2x^2 - 8x + 10$ .

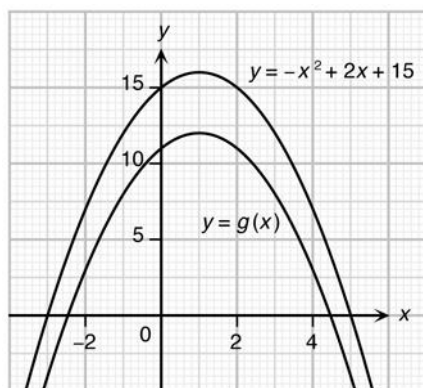
描述變換後  $y = 2x^2 - 8x + 10$  的圖像所受的影響。

- (b) Find the symbolic representation of  $g(x)$ .

試以符號形式表示  $g(x)$ 。

42. In the figure, the graph of  $y = g(x)$  is obtained by translating the graph of  $y = -x^2 + 2x + 15$ .

在圖中，把  $y = -x^2 + 2x + 15$  的圖像作平移變換，可得出  $y = g(x)$  的圖像。



- (a) Describe the effect of the transformation on the graph of  $y = -x^2 + 2x + 15$ .

描述變換後  $y = -x^2 + 2x + 15$  的圖像所受的影響。

- (b) Find the symbolic representation of  $g(x)$ .

試以符號形式表示  $g(x)$ 。

43. It is given that  $g(x) = 2x^2 + 5x - 3$  and  $g(x) = f(x - 2)$ .

已知  $g(x) = 2x^2 + 5x - 3$  及  $g(x) = f(x - 2)$ 。

- (a) Solve  $g(x) = 0$ .

解  $g(x) = 0$ 。

- (b) Hence, solve  $f(x) = 0$ .

由此，解  $f(x) = 0$ 。

44. It is given that  $f(x) = 2x^2 - 4x - 1$  is transformed to  $g(x)$ . If the coordinates of the vertex of the graph of  $y = g(x)$  are  $(-1, -3)$ , find the symbolic representation of  $g(x)$ .

已知  $f(x) = 2x^2 - 4x - 1$  給變換成  $g(x)$ 。若  $y = g(x)$  的圖像的頂點的坐標是  $(-1, -3)$ ，試以符號形式表示  $g(x)$ 。

45. It is given that the coordinates of the vertices of the graphs of  $y = f(x)$  and  $y = g(x)$  are  $(h, k)$  and  $(h + 3, k)$  respectively. If  $f(x)$  is transformed to  $g(x)$ ,

已知  $y = f(x)$  及  $y = g(x)$  的圖像的頂點的坐標分別是  $(h, k)$  和  $(h + 3, k)$ 。若把  $f(x)$  變換成  $g(x)$ ，

- (a) describe the effect of the transformation on the graph of  $y = f(x)$ ,

描述變換後  $y = f(x)$  的圖像所受的影響；

- (b) find the symbolic relation between  $f(x)$  and  $g(x)$ .

試以符號形式表示  $f(x)$  與  $g(x)$  之間的關係。

46. It is given that  $f(x) = 4x^2 - 12x + 9$  and  $g(x) = f(x + 3)$ .

已知  $f(x) = 4x^2 - 12x + 9$  及  $g(x) = f(x + 3)$ 。

- (a) Solve  $f(x) = 0$ .

解  $f(x) = 0$ 。

- (b) Hence, solve  $g(x) = 0$ .

由此，解  $g(x) = 0$ 。

47. The following table shows the tabular representations of the functions  $g(x)$  and  $h(x)$ . If  $g(x)$  is transformed to  $h(x)$ , find the symbolic relation between  $g(x)$  and  $h(x)$ .

以下所示為函數  $g(x)$  和  $h(x)$  的表列形式。若把  $g(x)$  變換成  $h(x)$ ，試以符號形式表示  $g(x)$  與  $h(x)$  之間的關係。

$x$	-3	-2	-1	0	1	2	3
$g(x)$	14	6	0	-4	-6	-6	-4
$h(x)$	24	14	6	0	-4	-6	-6

48. The following table shows the tabular representations of the functions  $f(x)$  and  $g(x)$ . If  $f(x)$  is transformed to  $g(x)$ , find the symbolic relation between  $f(x)$  and  $g(x)$ .

以下所示為函數  $f(x)$  和  $g(x)$  的表列形式。若把  $f(x)$  變換成  $g(x)$ ，試以符號形式表示  $f(x)$  與  $g(x)$  之間的關係。

$x$	-2	-1	0	1	2
$f(x)$	-5	-1	-1	-5	-13
$g(x)$	-2	2	2	-2	-10

49. The following tables show the tabular representations of the functions  $p(x)$  and  $q(x)$ .

以下所示為函數  $p(x)$  和  $q(x)$  的表列形式。

$x$	-3	-2	-1	0	1	2	3
$p(x)$	35	21	11	5	3	5	11

$x$	-3	-2	-1	0	1	2	3
$q(x)$	2	2	2	2	2	2	2

- (a) (i) Find the symbolic representation of  $q(x)$ .  
試以符號形式表示  $q(x)$ 。
- (ii) Given that  $p(x)$  is a quadratic function. Find the symbolic representation of  $p(x)$ .  
已知  $p(x)$  為二次函數，試以符號形式表示  $p(x)$ 。
- (b) (i) If  $r(x) = p(x) + q(x)$ , describe the effect of the transformation on the graph of  $y = p(x)$  when  $p(x)$  is transformed to  $r(x)$ .  
若  $r(x) = p(x) + q(x)$ ，試描述當  $p(x)$  變換成  $r(x)$  時， $y = p(x)$  的圖像所受的影響。
- (ii) If the graph of  $y = s(x)$  is obtained by translating the graph of  $y = r(x)$  in the positive direction of the  $x$ -axis by 6 units, find the symbolic relation between  $r(x)$  and  $s(x)$ .  
若把  $y = r(x)$  的圖像沿  $x$  軸的正方向平移 6 單位，可得出  $y = s(x)$  的圖像，試以符號形式表示  $r(x)$  與  $s(x)$  之間的關係。
- (c) Hence, find the symbolic representations of  $r(x)$  and  $s(x)$ .  
由此，試以符號形式表示  $r(x)$  和  $s(x)$ 。



## Level 2+ Questions

### 程度 2+ 題目

1. It is given that  $p(x) = x^2 + 1$ ,  $q(x) = 2x - 5$  and  $r(x) = 4x^2 - 12x + 10$ .

已知  $p(x) = x^2 + 1$ ,  $q(x) = 2x - 5$  及  $r(x) = 4x^2 - 12x + 10$ 。

- (a) Show that  $r(x) = p[q(x) + 2]$ .

證明  $r(x) = p[q(x) + 2]$ 。

- (b) Find the symbolic representation of  $q[p(x) + 2]$ .

試以符號形式表示  $q[p(x) + 2]$ 。

- (c) Find the range of possible values of  $x$  if  $r(x) \leq 2q[p(x) + 2]$ .

若  $r(x) \leq 2q[p(x) + 2]$ , 求  $x$  值的可能範圍。

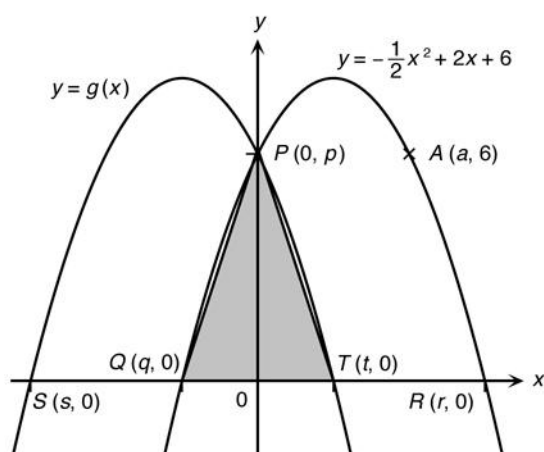
- (d) For the minimum integral value of  $x$  in (c), find the value of  $r[p(x) + q(x)]$ .

對於從 (c) 所得的  $x$  的最小整數值, 求  $r[p(x) + q(x)]$  的值。

2. Let  $f(x) = -\frac{1}{2}x^2 + 2x + 6$ . In the figure, the graph of  $y = g(x)$  is obtained by translating the graph of

$y = f(x)$  in the direction of the  $x$ -axis. The graphs of  $y = f(x)$  and  $y = g(x)$  intersect at  $P(0, p)$ . The graph of  $y = f(x)$  cuts the  $x$ -axis at  $Q(q, 0)$  and  $R(r, 0)$ . The graph of  $y = g(x)$  cuts the  $x$ -axis at  $S(s, 0)$  and  $T(t, 0)$ . It is given that  $A(a, 6)$  lies on the graph of  $y = f(x)$ .

設  $f(x) = -\frac{1}{2}x^2 + 2x + 6$ 。在圖中, 把  $y = f(x)$  的圖像沿  $x$  軸作平移變換, 可得出  $y = g(x)$  的圖像。 $y = f(x)$  與  $y = g(x)$  的圖像相交於  $P(0, p)$ 。 $y = f(x)$  的圖像與  $x$  軸相交於  $Q(q, 0)$  和  $R(r, 0)$ , 而  $y = g(x)$  的圖像與  $x$  軸相交於  $S(s, 0)$  和  $T(t, 0)$ 。已知  $A(a, 6)$  位於  $y = f(x)$  的圖像上。



- (a) Find the values of  $a$ ,  $p$ ,  $q$  and  $r$ .

求  $a$ 、 $p$ 、 $q$  和  $r$  的值。

- (b) (i) Find the symbolic relation between  $f(x)$  and  $g(x)$ .

試以符號形式表示  $f(x)$  與  $g(x)$  之間的關係。

- (ii) Hence, find the values of  $s$  and  $t$ .

由此，求  $s$  和  $t$  的值。

- (c) (i) Find the area of  $\triangle PQT$ .

求  $\triangle PQT$  的面積。

- (ii) If  $f(x)$  is transformed to  $g'(x)$  such that the area of  $\triangle PQT$  formed is smaller than that in

(c)(i), suggest a possible function  $g'(x)$ .

若把  $f(x)$  變換成  $g'(x)$ ，使形成的  $\triangle PQT$  的面積小於從 (c)(i) 所得的面積，試舉出一個可能的函數  $g'(x)$ 。

3. (a) Given that  $f(x) = (x-a)(x-b)$  and  $a-b=k$ . Prove that the minimum value of  $f(x)$  is  $-\frac{k^2}{4}$ .

已知  $f(x) = (x-a)(x-b)$  及  $a-b=k$ 。證明  $f(x)$  的極小值是  $-\frac{k^2}{4}$ 。

- (b) Let  $g(x) = x^2 - 6x + 5$  and  $h(x) = (x-3)(x-b)$ , where  $b < 3$ .

設  $g(x) = x^2 - 6x + 5$  及  $h(x) = (x-3)(x-b)$ ，其中  $b < 3$ 。

- (i) By the result obtained in (a), if  $g(x)$  and  $h(x)$  have the same minimum value, find the value of  $b$

利用 (a) 所得的結果，若  $g(x)$  和  $h(x)$  的極小值相等，求  $b$  的值。

- (ii) Hence, if  $h(x) = g(x+m) + n$ , find the values of  $m$  and  $n$ .

由此，若  $h(x) = g(x+m) + n$ ，求  $m$  和  $n$  的值。

4. (a) Let  $f(x) = a(x-h)^2 + k$  and  $g(x) = b(x-m)^2 + n$ . If the graph of  $y = g(x)$  is obtained by translating the graph of  $y = f(x)$ , prove that  $a = b$ .

設  $f(x) = a(x-h)^2 + k$  及  $g(x) = b(x-m)^2 + n$ 。若把  $y = f(x)$  的圖像作平移變換，可得出  $y = g(x)$  的圖像，證明  $a = b$ 。

- (b) Let  $F(x) = c(x-s)^2 + t$  and  $G(x) = 3x^2 - 12x + 16$ . It is given that the graph of  $y = G(x)$  is obtained by translating the graph of  $y = F(x)$  in the positive direction of the  $x$ -axis by 3 units, and then in the negative direction of the  $y$ -axis by 2 units. Find the values of  $c$ ,  $s$  and  $t$ .

設  $F(x) = c(x-s)^2 + t$  及  $G(x) = 3x^2 - 12x + 16$ 。已知把  $y = F(x)$  的圖像先沿  $x$  軸的正方向平移 3 單位，然後再沿  $y$  軸的負方向平移 2 單位，可得出  $y = G(x)$  的圖像。求  $c$ 、 $s$  和  $t$  的值。

- (c) Suggest quadratic functions  $f(x)$  and  $g(x)$  such that the following conditions are satisfied:

試舉出符合下列條件的二次函數  $f(x)$  和  $g(x)$ ：

- (1) The graph of  $y = f(x)$  passes through  $(1, 2)$ .

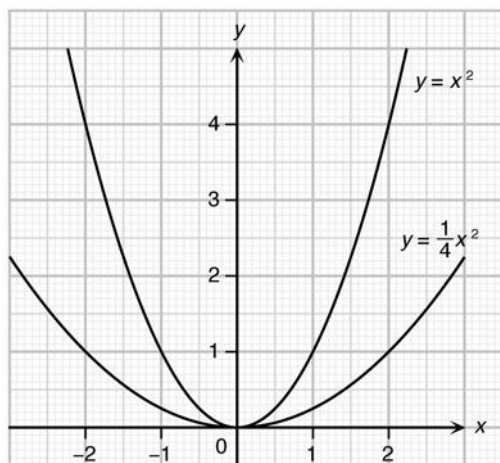
$y = f(x)$  的圖像通過  $(1, 2)$ 。

- (2) The graph of  $y = g(x)$  is obtained by translating the graph of  $y = f(x)$  in the negative direction of the  $x$ -axis, and then in the positive direction of the  $y$ -axis

把  $y = f(x)$  的圖像先沿  $x$  軸的負方向平移，然後再沿  $y$  軸的正方向平移，可得出  $y = g(x)$  的圖像。

5. The figure shows the graphs of  $y = x^2$  and  $y = \frac{1}{4}x^2$ .

圖中所示為  $y = x^2$  及  $y = \frac{1}{4}x^2$  的圖像。



- (a) (i) Solve  $x^2 < 1$  by adding a suitable straight line on the graph.  
試在圖像中加上一條直線，解  $x^2 < 1$ 。
- (ii) Hence, solve  $(x - 2)(x + 2) < 0$ .  
(Give your answers correct to 1 decimal place.)  
由此，解  $(x - 2)(x + 2) < 0$ 。(答案須準確至一位小數。)
- (b) (i) Suggest a quadratic function  $f(x)$  such that the following conditions are satisfied:  
試舉出一個符合下列條件的二次函數  $f(x)$ ：
- (1) The coordinates of the vertex of its graph are  $(0, 0)$ .  
其圖像的頂點的坐標是  $(0, 0)$ 。
- (2)  $f(x) < 1$  for  $-k < x < k$ , where  $0 < k < 1$   
對於  $-k < x < k$ ,  $f(x) < 1$ , 其中  $0 < k < 1$ 。
- (ii) Sketch the graph of  $y = f(x)$  obtained in (b)(i).  
試繪畫從 (b)(i) 所得的  $y = f(x)$  的圖像。
- (c) For each of the following quadratic function  $g(x)$ , determine whether the inequality  $g(x) > 1$  is always true.  
對於下列各二次函數  $g(x)$ ，判斷不等式  $g(x) > 1$  是否成立。
- (i)  $g(x) = x^2 - 2x + 6$
- (ii)  $g(x) = -x^2 + 2x + 4$
- (d) Suggest a quadratic function  $h(x)$  such that the inequality  $h(x) < 4$  is always true.  
試舉出一個二次函數  $h(x)$ ，使  $h(x) < 4$ 。

6. (a) It is given that  $27x^2 - 66x + 40 \equiv A(3x - 2)^2 + B(3x - 2) + C$ . Find the values of  $A$ ,  $B$  and  $C$ .  
 已知  $27x^2 - 66x + 40 \equiv A(3x - 2)^2 + B(3x - 2) + C$ 。求  $A$ 、 $B$  和  $C$  的值。
- (b) Hence, if  $f(3x - 2) = 27x^2 - 66x + 40$ , find the symbolic representation of  $f(x)$ .  
 由此，若  $f(3x - 2) = 27x^2 - 66x + 40$ ，試以符號形式表示  $f(x)$ 。
- (c) If the graph of  $y = f(x)$  is obtained by translating the graph of  $y = g(x)$  in the positive direction of the  $y$ -axis by 3 units, find  
 若把  $y = g(x)$  的圖像沿  $y$  軸的正方向平移 3 單位，可得出  $y = f(x)$  的圖像，
- (i) the symbolic relation between  $g(x)$  and  $f(x)$ ,  
 試以符號形式表示  $g(x)$  與  $f(x)$  之間的關係；
- (ii) the symbolic representation of  $g(x)$ .  
 試以符號形式表示  $g(x)$ 。
- (d) If the graph of  $y = g(x)$  is obtained by translating the graph of  $y = h(x)$  in the positive direction of the  $x$ -axis by 2 units, find  
 若把  $y = h(x)$  的圖像沿  $x$  軸的正方向平移 2 單位，可得出  $y = g(x)$  的圖像，
- (i) the symbolic relation between  $h(x)$  and  $g(x)$ ,  
 試以符號形式表示  $h(x)$  與  $g(x)$  之間的關係；
- (ii) the symbolic representation of  $h(x)$ .  
 試以符號形式表示  $h(x)$ 。
7. (a) Sandy throws a ball  $P$  upwards. After  $t_p$  seconds, the height ( $h_p$  m) of the ball  $P$  above the ground is given by  
 家琪垂直向上拋起球  $P$ 。  $t_p$  秒後，球  $P$  距離地面的高度 ( $h_p$  m) 可由以下的公式計算：
- $$h_p = -5t_p^2 + 20t_p + 13.$$
- (i) When will the ball  $P$  attain its maximum height?  
 問球  $P$  於何時會達到最高的高度？
- (ii) What is the maximum height reached by the ball  $P$ ?  
 求球  $P$  所達到的最高高度。
- (b) When the ball  $P$  attains its maximum height, Janis throws another ball  $Q$ . After  $t_Q$  seconds, the height ( $h_Q$  m) of the ball  $Q$  above the ground is given by  
 當球  $P$  達到最高的高度時，詠賢垂直向上拋起球  $Q$ 。  $t_Q$  秒後，球  $Q$  距離地面的高度 ( $h_Q$  m) 可由以下的公式計算：
- $$h_Q = -5t_Q^2 + 15t_Q + 13.$$
- (i) Find the maximum height that can be reached by the ball  $Q$ .  
 求球  $Q$  所達到的最高高度。
- (ii) Find the height of the ball  $P$  when the ball  $Q$  attains its maximum height.  
 求當球  $Q$  達到最高的高度時，球  $P$  的高度。
- (c) Who will receive her ball first? After how many seconds the other one will receive her ball?  
 若兩人均把拋出的球接回，誰的球先被接回？兩者在時間上的差距是多少？

8. (a) It is given that  $y = ax^2 + kax + c$ , where  $a \neq 0$ . Express the coordinates of the vertex of the graph of  $y = ax^2 + kax + c$  in terms of  $a$ ,  $c$  and  $k$ .

已知  $y = ax^2 + kax + c$ ，其中  $a \neq 0$ 。試以  $a$ 、 $c$  和  $k$  表示  $y = ax^2 + kax + c$  的圖像的頂點的坐標。

- (b) Let  $f(x) = 2x^2 - 6x + 3$ ,  $g(x) = x^2 - 3x - 2$  and  $h(x) = f(x) + g(x)$ .

設  $f(x) = 2x^2 - 6x + 3$ ， $g(x) = x^2 - 3x - 2$  及  $h(x) = f(x) + g(x)$ 。

- (i) By the result obtained in (a), find the minimum values of  $f(x)$  and  $g(x)$ .

利用 (a) 所得的結果，求  $f(x)$  和  $g(x)$  的極小值。

- (ii) Show that the minimum value of  $h(x)$  is equal to the sum of the minimum values of  $f(x)$  and  $g(x)$ .

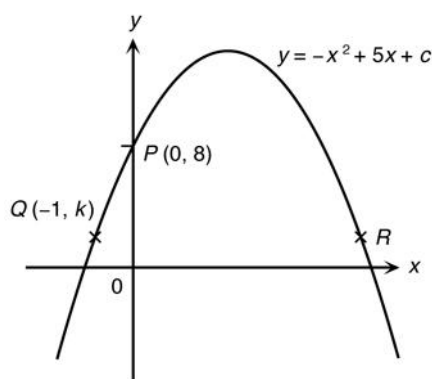
證明  $h(x)$  的極小值相等於  $f(x)$  及  $g(x)$  的極小值之和。

- (c) Suggest two quadratic functions such that the result obtained in (b)(ii) is not true.

試舉出兩個二次函數，使 (b)(ii) 所得的結果不成立。

9. The figure shows the graph of  $y = -x^2 + 5x + c$  which cuts the  $y$ -axis at  $P(0, 8)$ , and passes through  $Q(-1, k)$ . It is given that  $Q$  is reflected about the axis of symmetry of the graph to  $R$ .

圖中所示為  $y = -x^2 + 5x + c$  的圖像，該圖像與  $y$  軸相交於  $P(0, 8)$ ，且通過  $Q(-1, k)$ 。已知  $Q$  沿圖像的對稱軸反射至  $R$ 。



- (a) Find the values of  $c$  and  $k$ .

求  $c$  和  $k$  的值。

- (b) (i) Find the axis of symmetry of the graph.

求圖像的對稱軸。

- (ii) Hence, find the coordinates of  $R$ .

由此，求  $R$  的坐標。

- (c) Solve  $x(x - 5) \geq c - k$ .

解  $x(x - 5) \geq c - k$ 。

- (d) (i) Plot the graph of  $y = -x^2 + 5x + c$  from  $x = -2$  to  $x = 7$ .

繪畫  $y = -x^2 + 5x + c$  在  $x = -2$  與  $x = 7$  之間的圖像。

- (ii) Solve  $x^2 - 5x - c + 4k \leq 0$  by adding a suitable straight line on the graph.

試在圖像中加上一條直線，解  $x^2 - 5x - c + 4k \leq 0$ 。

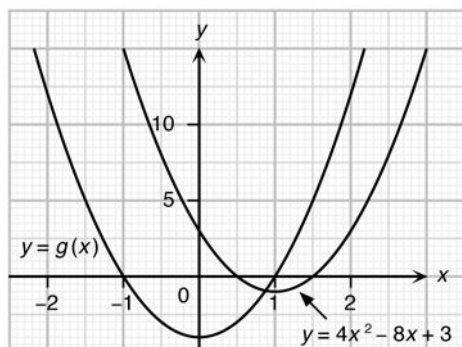
- (e) Using the result obtained in (d), find the possible values of integer  $x$  that satisfy

$$k + 6 < 3(x + 3) - (1 - x)^2 < 7k.$$

利用 (d) 所得的結果，求  $x$  的可能整數值，使  $k + 6 < 3(x + 3) - (1 - x)^2 < 7k$ 。

10. In the figure, the graph of  $y = g(x)$  is obtained by translating the graph of  $y = f(x)$ . It is given that  $f(x) = 4x^2 - 8x + 3$ .

在圖中，把  $y = f(x)$  的圖像作平移變換，可得出  $y = g(x)$  的圖像。已知  $f(x) = 4x^2 - 8x + 3$ 。



- (a) (i) Find the symbolic relation between  $f(x)$  and  $g(x)$ .  
試以符號形式表示  $f(x)$  與  $g(x)$  之間的關係。
- (ii) Find the symbolic representation of  $g(x)$ .  
試以符號形式表示  $g(x)$ 。
- (b) If the graph of  $y = h(x)$  is obtained by translating the graph of  $y = g(x)$  in the positive direction of the  $x$ -axis by 2 units,  
若把  $y = g(x)$  的圖像沿  $x$  軸的正方向平移 2 單位，可得出  $y = h(x)$  的圖像，
- (i) find the symbolic relation between  $g(x)$  and  $h(x)$ ,  
試以符號形式表示  $g(x)$  與  $h(x)$  之間的關係；
- (ii) find the symbolic representation of  $h(x)$ .  
試以符號形式表示  $h(x)$ 。
- (c) (i) Solve the inequality  $g(x) > 12$  by adding a suitable straight line on the graph.  
試在圖像中加上一條直線，解不等式  $g(x) > 12$ 。
- (ii) Hence, solve the inequality  $(x - 3)(x - 1) > 3$  without adding any straight lines on the graph.  
由此，試不在圖像中加上任何直線，解不等式  $(x - 3)(x - 1) > 3$ 。

## Multiple Choice Questions

### 多項選擇題

1. If  $f(x) = ax + b$ , then  $f(ax + b) =$   
若  $f(x) = ax + b$ ，則  $f(ax + b) =$

A.  $ax + b$ .  
B.  $a^2x + ab + b$ .  
C.  $a^2x^2 + 2ax + b^2$ .  
D.  $2ax + 2b$ .

2. Which of the following is NOT a function of  $x$ ?

下列何者不是  $x$  的函數？

A.  $y = 5 - x$   
B.  $y = x^2 + 9x - 12$   
C.  $y = x^3 + \frac{1}{x}$   
D.  $y^2 = 4x$

3. If  $f(x) = x^3 + kx^2 + kx + 1$  and  $f(1) + f(-1) = 8$ ,  
find the value of  $k$ .

若  $f(x) = x^3 + kx^2 + kx + 1$  及  
 $f(1) + f(-1) = 8$ ，求  $k$  的值。

A. 0  
B. 1  
C. 3  
D. 8

4. If  $f(x) = (x - 1)(2x + 3)$ , find the value of  $f(5)$ .

若  $f(x) = (x - 1)(2x + 3)$ ，求  $f(5)$  的值。

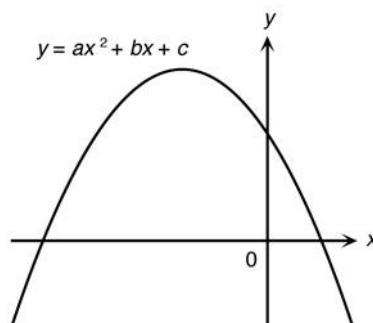
A. 7  
B. 5  
C. 52  
D. 20

5. If  $g(x + 1) = 2x^2 + 4x + 2$ , then  $g(x) =$   
若  $g(x + 1) = 2x^2 + 4x + 2$ ，則  $g(x) =$

A.  $x^2$ .  
B.  $2x^2$ .  
C.  $2x^2 + 4x + 2$ .  
D.  $2x^2 + 8x + 8$ .

6. The figure shows the graph of  
 $y = ax^2 + bx + c$ . Determine the signs of  $a$  and  
 $c$ .

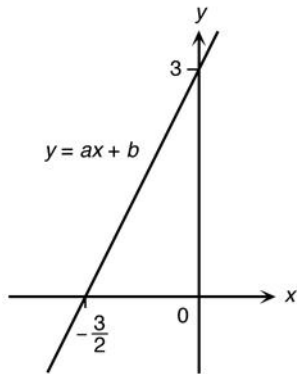
圖中所示為  $y = ax^2 + bx + c$  的圖像。判斷  
 $a$  和  $c$  的正負號。



A.  $a < 0, c < 0$   
B.  $a > 0, c < 0$   
C.  $a < 0, c > 0$   
D.  $a > 0, c > 0$

7. The figure shows the graph of  $y = ax + b$ . If  $f(x) = ax + b$ , find the value of  $f(3)$ .

圖中所示為  $y = ax + b$  的圖像。若  $f(x) = ax + b$ ，求  $f(3)$  的值。

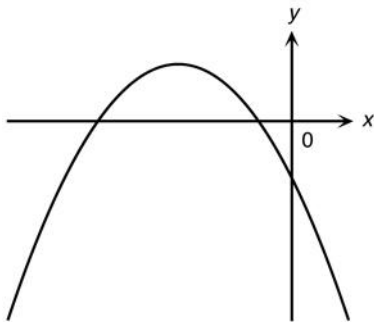


- A.  $-\frac{3}{2}$   
 B. 0  
 C. 9  
 D. -9

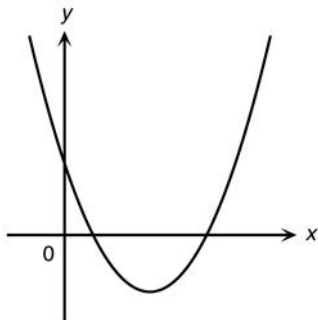
8. Which of the following may represent the graph of  $y = ax^2 + bx + c$ , where  $ac < 0$ ?

下列何者可表示  $y = ax^2 + bx + c$  的圖像，其中  $ac < 0$ ？

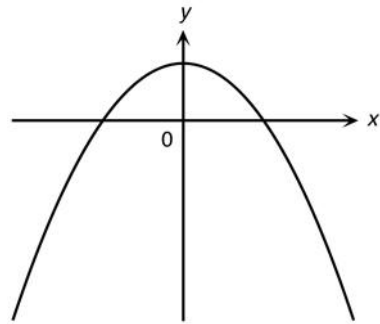
A.



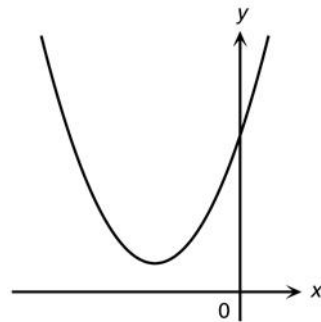
B.



C.



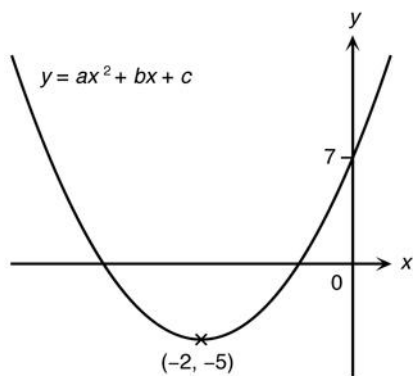
D.





9. The figure shows the graph of  $y = ax^2 + bx + c$ . The coordinates of the vertex of the graph is  $(-2, -5)$ . Which of the following must be true?

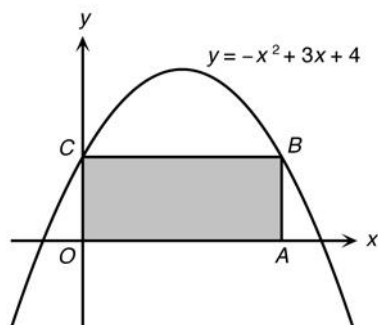
圖中所示為  $y = ax^2 + bx + c$  的圖像。圖像的頂點的坐標是  $(-2, -5)$ 。下列何者必為正確？



- I. The axis of symmetry is  $x = -5$ .  
 圖像的對稱軸是  $x = -5$ 。  
 II. The graph opens downwards.  
 圖像的開口向下。  
 III.  $ac > 0$   
 A. I only 只有 I  
 B. II only 只有 II  
 C. III only 只有 III  
 D. I and III only 只有 I 及 III

10. The figure shows the graph of  $y = -x^2 + 3x + 4$ . Find the area of the rectangle  $OABC$ .

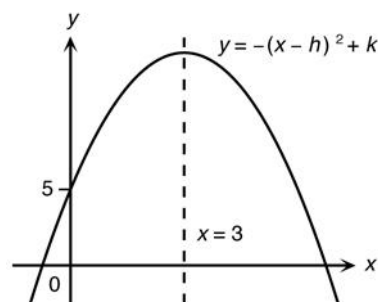
圖中所示為  $y = -x^2 + 3x + 4$  的圖像。求長方形  $OABC$  的面積。



- A. 9  
 B. 12  
 C. 16  
 D. 6

11. In the figure, the axis of symmetry of the graph of  $y = -(x - h)^2 + k$  is  $x = 3$ . Find the optimum value of  $y$ .

在圖中， $y = -(x - h)^2 + k$  的圖像的對稱軸是  $x = 3$ 。求  $y$  的極值。



- A. 4  
 B. 8  
 C. 12  
 D. 14

12. Which of the following functions has a maximum value of  $-2$ ?

下列哪一個函數的極大值是  $-2$ ？

- A.  $f(x) = (x + 1)^2 - 2$   
 B.  $f(x) = -(x - 2)^2 + 1$   
 C.  $f(x) = -(x + 1)^2 - 2$   
 D.  $f(x) = (x - 2)^2 + 1$

13. It is given that the function

$y = -(x+1)^2 + 4(x+1) + 5$ . Which of the following about its graph must be true?

已知函數  $y = -(x+1)^2 + 4(x+1) + 5$ 。對於該函數的圖像，下列何者必為正確？

- I. The axis of symmetry is  $x = -3$ .

圖像的對稱軸是  $x = -3$ 。

- II. The graph opens downwards.

圖像的開口向下。

- III. The coordinates of the vertex are  $(1, 9)$ .

圖像的頂點的坐標是  $(1, 9)$ 。

- A. II only 只有 II

- B. I and II only 只有 I 及 II

- C. I and III only 只有 I 及 III

- D. II and III only 只有 II 及 III

14. It is given that the perimeters of two rectangles are 36 m and 44 m respectively. What is the difference between their maximum areas?

已知兩個長方形的周界分別是 36 m 和 44 m，它們的最大面積相差多少？

- A.  $40 \text{ m}^2$

- B.  $64 \text{ m}^2$

- C.  $81 \text{ m}^2$

- D.  $121 \text{ m}^2$

15. It is given that  $f(x) = 4x^2 - 4x + 13$ . Which of the following must be true?

已知  $f(x) = 4x^2 - 4x + 13$ 。下列何者必為正確？

- I. The minimum value of  $f(x)$  is 12.

$f(x)$  的極小值是 12。

- II. The axis of symmetry of the graph of

$$y = f(x) \text{ is } x = -\frac{1}{2}.$$

$y = f(x)$  的圖像的對稱軸是  $x = -\frac{1}{2}$ 。

- III. The coordinates of the vertex of the

graph of  $y = f(-x)$  is  $(-\frac{1}{2}, 12)$ .

$y = f(-x)$  的圖像的頂點的坐標是

$(-\frac{1}{2}, 12)$ 。

- A. I and II only 只有 I 及 II

- B. I and III only 只有 I 及 III

- C. II and III only 只有 II 及 III

- D. I, II and III I、II 及 III

16. Find the optimum value of

$$y = \frac{2}{-(x-4)^2 + 6}.$$

求  $y = \frac{2}{-(x-4)^2 + 6}$  的極值。

- A. maximum value of  $y = 6$

$y$  的極大值 = 6

- B. minimum value of  $y = \frac{1}{3}$

$y$  的極小值 =  $\frac{1}{3}$

- C. maximum value of  $y = 2$

$y$  的極小值 = 2

- D. minimum value of  $y = 3$

$y$  的極小值 = 3

17. If the axis of symmetry of the graph of

$y = 2x^2 + kx + 9$  is  $x = \frac{3}{2}$ , find the value of  $k$ .

若  $y = 2x^2 + kx + 9$  的圖像的對稱軸是

$x = \frac{3}{2}$ ，求  $k$  的值。

- A. 3

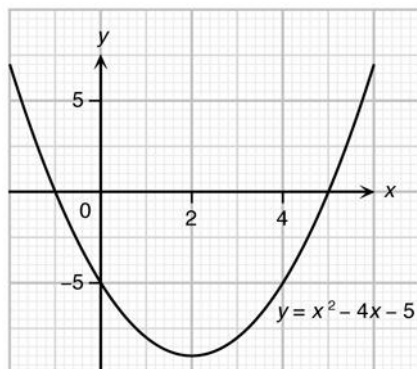
- B. 6

- C. -3

- D. -6

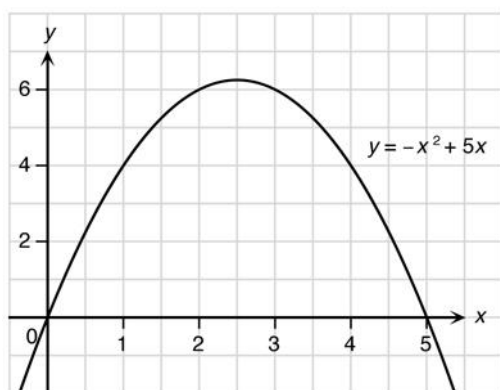
18. The figure shows the graph of  $y = x^2 - 4x - 5$ . Find the largest integer  $x$  that satisfies  $x^2 < 4(x + 2)$ .

圖中所示為  $y = x^2 - 4x - 5$  的圖像。求一個最大的整數  $x$ ，使  $x^2 < 4(x + 2)$ 。



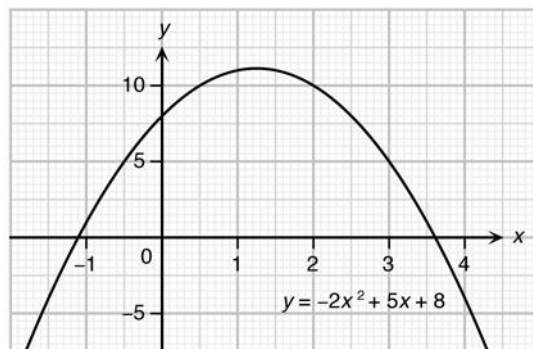
- A. -2  
B. 0  
C. 4  
D. 5

19. The figure shows the graph of  $y = -x^2 + 5x$ . Solve  $x^2 - 5x + 6 < 0$ .  
圖中所示為  $y = -x^2 + 5x$  的圖像。解  $x^2 - 5x + 6 < 0$ 。



- A.  $2 < x < 3$   
B.  $x < 2$  or  $x > 3$   
C.  $-1 < x < 6$   
D.  $x < -1$  or  $x > 6$

20. The figure shows the graph of  $y = -2x^2 + 5x + 8$ . Find the number of integral solutions of the inequality  $5x > 2(x^2 + 1)$ .  
圖中所示為  $y = -2x^2 + 5x + 8$  的圖像。問不等式  $5x > 2(x^2 + 1)$  有多少個整數解？



- A. 0  
B. 1  
C. 2  
D. 3

21. Which of the graphs of the following functions has the maximum number of  $x$ -intercepts?

下列哪一個函數的圖像有最多  $x$  軸截距？

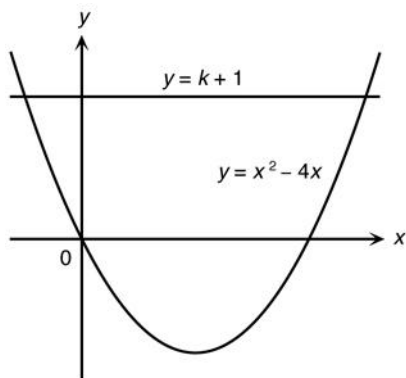
- A.  $y = 5x^2 - 6x + 8$   
B.  $y = 9 - 2x$   
C.  $y = x^3$   
D.  $y = 10x - x^2$

22. Which of the following inequality can be solved by adding the straight line  $y = -3$  on the graph of  $y = 3x^3 + 2x^2 - 5$ ?

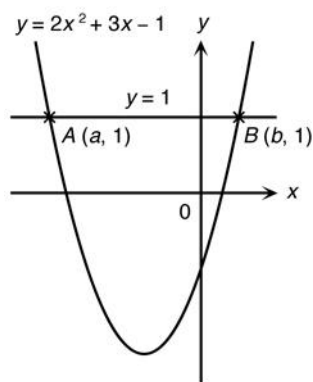
在  $y = 3x^3 + 2x^2 - 5$  的圖像中加上直線  $y = -3$ ，可解下列哪一個不等式？

- A.  $3x^3 + 2x^2 - 8 > 0$   
B.  $2x^2 \leq -3x^3 + 2$   
C.  $3 - 2x^2 < 3x^3$   
D.  $-3x^3 - 2x^2 \geq 2$

23. The figure shows the graph of  $y = x^2 - 4x$  which has the minimum value of  $-k$ . Which of the following inequality can be solved by adding the straight line  $y = k + 1$  on the graph?
- 圖中所示為  $y = x^2 - 4x$  的圖像，該圖像的極小值為  $-k$ 。在圖像中加上直線  $y = k + 1$  可解下列哪一個不等式？



- A.  $x(x - 2) \leq 2x + 3$   
 B.  $x(x - 2) \leq 2x - 3$   
 C.  $x(x - 3) \geq x + 5$   
 D.  $x(x - 3) \geq x - 5$
24. If  $f(x) = x^2 - 7x - 4$  is transformed to  $g(x) = x^2 - 7x + 4$ , find the symbolic relation between  $f(x)$  and  $g(x)$ .
- 若  $f(x) = x^2 - 7x - 4$  給變換成  $g(x) = x^2 - 7x + 4$ ，試以符號形式表示  $f(x)$  與  $g(x)$  之間的關係。
- A.  $g(x) = f(x) - 8$   
 B.  $g(x) = f(x) + 8$   
 C.  $g(x) = f(x - 8)$   
 D.  $g(x) = f(x + 8)$
25. In the figure, the graphs of  $y = 2x^2 + 3x - 1$  and  $y = 1$  intersect at  $A(a, 1)$  and  $B(b, 1)$ . Which of the following is true?
- 在圖中， $y = 2x^2 + 3x - 1$  與  $y = 1$  的圖像相交於  $A(a, 1)$  和  $B(b, 1)$ 。下列何者必為正確？



- A.  $\begin{cases} 2x^2 + 3x > -2, \text{ where } x < a \text{ or } x > b \\ 2x^2 + 3x \leq -2, \text{ where } a \leq x \leq b \end{cases}$   
 B.  $\begin{cases} 2x^2 > -3x + 2, \text{ where } x < a \text{ or } x > b \\ 2x^2 \leq -3x + 2, \text{ where } a \leq x \leq b \end{cases}$   
 C.  $\begin{cases} 2x^2 + 3x - 2 < 0, \text{ where } x < a \text{ or } x > b \\ 2x^2 + 3x - 2 \geq 0, \text{ where } a \leq x \leq b \end{cases}$   
 D.  $\begin{cases} 2(x^2 + 1) < -3x, \text{ where } x < a \text{ or } x > b \\ 2(x^2 + 1) \geq -3x, \text{ where } a \leq x \leq b \end{cases}$
26. If the graph of  $y = g(x)$  is obtained by translating the graph of  $y = f(x)$  in the negative direction of the  $x$ -axis by 2 units, find the symbolic relation between  $f(x)$  and  $g(x)$ .
- 若把  $y = f(x)$  的圖像沿  $x$  軸的負方向平移 2 單位，可得出  $y = g(x)$  的圖像，試以符號形式表示  $f(x)$  與  $g(x)$  之間的關係。
- A.  $g(x) = f(x) - 2$   
 B.  $g(x) = f(x) + 2$   
 C.  $g(x) = f(x - 2)$   
 D.  $g(x) = f(x + 2)$

27. It is given that  $f(x) = x^2 - 7x + 10$ . If the graph of  $y = g(x)$  is obtained by translating the graph of  $y = f(x)$  in the negative direction of the  $y$ -axis by 6 units, find the coordinates of the vertex of the graph of  $y = g(x)$ .

已知  $f(x) = x^2 - 7x + 10$ 。若把  $y = f(x)$  的圖像沿  $y$  軸的負方向平移 6 單位，可得出  $y = g(x)$  的圖像，求  $y = g(x)$  的圖像的頂點的坐標。

- A.  $(\frac{19}{2}, -\frac{9}{4})$   
 B.  $(-\frac{5}{2}, -\frac{9}{4})$   
 C.  $(\frac{7}{2}, \frac{15}{4})$   
 D.  $(\frac{7}{2}, -\frac{33}{4})$

28. The following tables show the tabular representations of functions

以下所示為函數  $f(x) = (1 - 2x)^2 + x(x + 5)$  及  $g(x)$  的表列形式。

$f(x) = (1 - 2x)^2 + x(x + 5)$  and  $g(x)$ .

$x$	0	1	2	3	4
$f(x)$	1	7	23	49	85

$x$	1	2	3	4	5
$g(x)$	1	7	23	49	85

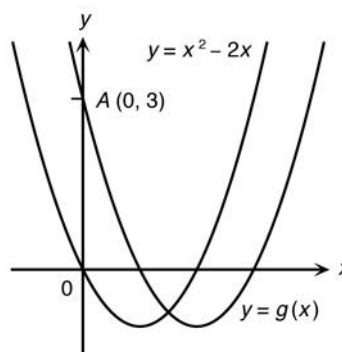
If  $f(x)$  is transformed to  $g(x)$ , find the symbolic representation of  $g(x)$ .

若  $f(x)$  給變換成  $g(x)$ ，試以符號形式表示  $g(x)$ 。

- A.  $g(x) = 5x^2 + x + 2$   
 B.  $g(x) = 5x^2 + x$   
 C.  $g(x) = 5x^2 + 11x + 7$   
 D.  $g(x) = 5x^2 - 9x + 5$

29. In the figure, the graph of  $y = g(x)$  is obtained by translating the graph of  $y = x^2 - 2x$  in the direction of the  $x$ -axis. If  $A(0, 3)$  lies on the graph of  $y = g(x)$ , find the symbolic representation of  $g(x)$ .

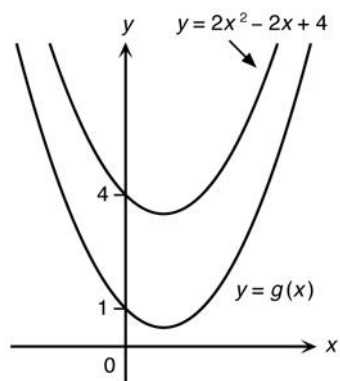
在圖中，把  $y = x^2 - 2x$  的圖像沿  $x$  軸的方向平移，可得出  $y = g(x)$  的圖像。若  $A(0, 3)$  位於  $y = g(x)$  的圖像上，試以符號形式表示  $g(x)$ 。



- A.  $g(x) = x^2 - 1$   
 B.  $g(x) = x^2 + 4x + 3$   
 C.  $g(x) = x^2 - 8x + 15$   
 D.  $g(x) = x^2 - 4x + 3$

30. In the figure, the graph of  $y = g(x)$  is obtained by translating the graph of  $y = 2x^2 - 2x + 4$  downwards. Which of the following is the symbolic representation of  $g(x)$ ?

在圖中，把  $y = 2x^2 - 2x + 4$  的圖像下移，可得出  $y = g(x)$  的圖像。下列何者為  $g(x)$  的符號表示形式？



- A.  $g(x) = 2x^2 - 2x + 7$   
B.  $g(x) = 2x^2 - 2x + 1$   
C.  $g(x) = 2x^2 + 10x + 16$   
D.  $g(x) = 2x^2 - 14x + 28$