Paper 1

Solution	Marks	Remarks
1. $9(h+6k)=7h+8$		
$9h+54k=7h+8$	1M	
$9h-7h=8-54k$	1M	for putting h on one side
$2h=8-54k$		
$h=4-27k$	1A	or equivalent
$9(h+6k)=7h+8$		
$h+6k=\dfrac{7h+8}{9}$	1M	
$h-\dfrac{7h}{9}=\dfrac{8}{9}-6k$	1M	for putting h on one side
$\dfrac{2h}{9}=\dfrac{8-54k}{9}$		
$2h=8-54k$		
$h=4-27k$	1A	or equivalent
	----------(3)	
2. $\dfrac{3}{7x-6}-\dfrac{2}{5x-4}$		
$=\dfrac{3(5x-4)-2(7x-6)}{(7x-6)(5x-4)}$	1M	
$=\dfrac{15x-12-14x+12}{(7x-6)(5x-4)}$	1M	
$=\dfrac{x}{(7x-6)(5x-4)}$	1A	or equivalent
	----------(3)	
3. $24^2+(13+r)^2=(17-3r)^2$	1M	
$576+169+26r+r^2=289-102r+9r^2$		
$8r^2-128r-456=0$	1M	for $ar^2+br+c=0$
$r^2-16r-57=0$		
$(r+3)(r-19)=0$		
$r=-3$ or $r=19$ (rejected)	1A	
Thus, we have $r=-3$.		
	----------(3)	
4. (a) $4m^2-9$		
$=(2m+3)(2m-3)$	1A	or equivalent
(b) $2m^2n+7mn-15n$		
$=n(2m^2+7m-15)$		
$=n(2m-3)(m+5)$	1A	or equivalent
(c) $4m^2-9-2m^2n-7mn+15n$		
$=4m^2-9-(2m^2n+7mn-15n)$		
$=(2m+3)(2m-3)-n(2m-3)(m+5)$	1M	for using the results of (a) and (b)
$=(2m-3)(2m-mn-5n+3)$	1A	or equivalent
	----------(4)	

Solution	Marks	Remarks
5. (a) Let $\$m$ be the marked price of the wallet.		
$(1-25\%)m = 690$	1M	
$m = \frac{690}{0.75}$		
$m = 920$	1A	
Thus, the marked price of the wallet is \$920 .		
(b) Let $\$c$ be the cost of the wallet.		
$(1+15\%)c = 690$	1M	
$c = \frac{690}{1.15}$		
$c = 600$	1A	
Thus, the cost of the wallet is \$600 .		
	----------(4)	
6. (a) $\frac{7x+26}{4} \le 2(3x-1)$		
$7x+26 \le 24x-8$		
$7x-24x \le -8-26$	1M	for putting x on one side
$-17x \le -34$		
$x \ge 2$	1A	
(b) $45-5x \ge 0$		
$x \le 9$	1A	
By (a), we have $2 \le x \le 9$.		
Thus, the required number is 8 .	1A	
	----------(4)	
7. Let $13k$ and $6k$ be the original number of adults and the original number of children in the playground respectively, where k is a positive constant.	1A	can be absorbed
$\frac{13k+9}{6k+24} = \frac{8}{7}$	1M+1A	
$91k-48k = 192-63$		
$k = 3$		
Thus, the original number of adults in the playground is 39 .	1A	
Let x and y be the original number of adults and the original number of children in the playground respectively.		
$\begin{cases} \frac{x}{y} = \frac{13}{6} \\ \frac{x+9}{y+24} = \frac{8}{7} \end{cases}$	1A+1A	
$\begin{cases} 6x = 13y \\ 7x-8y = 129 \end{cases}$		
So, we have $7x - 8\left(\frac{6x}{13}\right) = 129$.	1M	for getting a linear equation in x or y only
Solving, we have $x = 39$.	1A	
Thus, the original number of adults in the playground is 39 .		
	----------(4)	

Solution	Marks	Remarks
8. (a) 2	1A	
(b) Note that $360° - 54° - 90° - 144° = 72°$.		
The mean of the distribution		
$= \dfrac{2(144)+3(54)+5(72)+7(90)}{360}$	1M	
$= 4$	1A	
(c) The required probability		
$= \dfrac{72+90}{360}$	1M	
$= \dfrac{9}{20}$	1A	0.45
The required probability		
$= \dfrac{360-54-144}{360}$	1M	
$= \dfrac{9}{20}$	1A	0.45
	----------(5)	
9. (a) Note that the ratio of the radius of the larger sphere to the radius of the smaller sphere is $2:1$. So, the ratio of the volume of the larger sphere to the volume of the smaller sphere is $8:1$.	1M	
The volume of the larger sphere		
$= 324\pi\left(\dfrac{8}{1+8}\right)$		
$= 288\pi \text{ cm}^3$	1A	
(b) Let R cm be the radius of the larger sphere.		
$\dfrac{4}{3}\pi R^3 = 288\pi$	1M	
$R = 6$		
So, the radius of the smaller sphere is 3 cm .		
The sum of the surface areas of the two spheres		
$= 4\pi(6^2) + 4\pi(3^2)$	1M	
$= 180\pi \text{ cm}^2$	1A	
	----------(5)	

Solution	Marks	Remarks
10. (a) Let $h(x) = r + sx$, where r and s are non-zero constants.	1A	
So, we have $r - 2s = -96$ and $r + 5s = 72$.	1M	for either substitution
Solving, we have $r = -48$ and $s = 24$.	1A	for both correct
Thus, we have $h(x) = 24x - 48$.		
	----------(3)	
(b) $h(x) = 3x^2$		
$3x^2 - 24x + 48 = 0$	1M	
$x = 4$	1A	
	----------(2)	
11. (a) Let $ax + b$ be the required quotient, where a and b are constants.	1M	
Then, we have $p(x) = (ax + b)(2x^2 + 9x + 14)$.		
Note that $p(1) = 50$ and $p(-2) = -52$.	1M	for either one
Hence, we have $(a(1) + b)(2(1)^2 + 9(1) + 14) = 50$ and		
$(a(-2) + b)(2(-2)^2 + 9(-2) + 14) = -52$.		
So, we have $a + b = 2$ and $-2a + b = -13$.		
Solving, we have $a = 5$ and $b = -3$.	1A	for both correct
Thus, the required quotient is $5x - 3$.		
	----------(3)	
(b) $p(x) = 0$		
$(5x - 3)(2x^2 + 9x + 14) = 0$ (by (a))		
$5x - 3 = 0$ or $2x^2 + 9x + 14 = 0$		
$9^2 - 4(2)(14)$	1M	
$= -31$		
< 0		
So, the quadratic equation $2x^2 + 9x + 14 = 0$ does not have real roots.	1M	
Note that $\frac{3}{5}$ is a rational root of the equation $p(x) = 0$.		
Thus, the equation $p(x) = 0$ has 1 rational root.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
12. (a) $72-(60+c)=8$	1M	
$c=4$	1A	
	----------(2)	
(b) (i) $(80+b)-(50+a)>34$	1M	
$b-a>4$		
$\frac{50+a+60(2)+63+64(2)+68+69(3)+70+71(3)+72(2)+75+76+79+80+b}{20}=69$	1M	
Therefore, we have $a+b=7$.		
Thus, we have $\begin{cases}a=0\\b=7\end{cases}$ or $\begin{cases}a=1\\b=6\end{cases}$.	1A+1A	1A for one pair + 1A for all
(ii) By (b)(i), there are two cases.		
Case 1: $a=0$ and $b=7$ The standard deviation of the distribution ≈ 7.582875444	1M	either one
Case 2: $a=1$ and $b=6$ The standard deviation of the distribution ≈ 7.341661937		
Thus, the least possible standard deviation of the distribution is 7.34 seconds.	1A	f.t.
Note that $(50-69)^2+(87-69)^2>(51-69)^2+(86-69)^2$.	1M	
When $a=1$ and $b=6$, the standard deviation of the distribution is the least. The standard deviation ≈ 7.341661937		
Thus, the least possible standard deviation of the distribution is 7.34 seconds.	1A	f.t.
	----------(6)	

Solution	Marks	Remarks
13. (a) Note that $\angle ABF + \angle AED = 180°$.	1M	
So, we have $\angle ABF + 115° = 180°$.		
Hence, we have $\angle ABF = 65°$.		
Also note that $\angle ABC = 90°$.	1M	
Therefore, we have $\angle CBF + 65° = 90°$.		
Thus, we have $\angle CBF = 25°$.	1A	
$\angle AOD$ $= 360° - 2\angle AED$ $= 360° - 2(115°)$ $= 130°$	1M	
$\angle COD$ $= 180° - \angle AOD$ $= 180° - 130°$ $= 50°$	1M	
Since $2\angle CBF = \angle COD$, we have $\angle CBF = 25°$.	1A	
	----------(3)	
(b) $\angle ODF = \angle CBF = 25°$	1M	
$\angle OBF = \angle ODF = 25°$	1M	
$\angle DOF$ $= 2\angle CBF$ $= 2(25°)$ $= 50°$		
$\angle BOC$ $= 180° - \angle DOF - \angle OBF - \angle ODF$ $= 180° - 50° - 25° - 25°$ $= 80°$	1M	
The perimeter of the sector OBC $= \frac{80}{360}(2\pi(18)) + 2(18)$ $= 8\pi + 36$ $> 8(3) + 36$ $= 60$	1M	
Thus, the perimeter of the sector OBC is not less than 60 cm .	1A	f.t.
$\angle ODF = \angle CBF = 25°$	1M	
$\angle OBF = \angle ODF = 25°$	1M	
$\angle BOC$ $= 180° - \angle COD - \angle OBF - \angle ODF$ $= 180° - 50° - 25° - 25°$ $= 80°$	1M	
The perimeter of the sector OBC $= \frac{80}{360}(2\pi(18)) + 2(18)$ $= 8\pi + 36$ $> 8(3) + 36$ $= 60$	1M	
Thus, the perimeter of the sector OBC is not less than 60 cm .	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
14. **Marking Schemes for (a)(i) and (a)(ii) :**		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
(a) (i) $BC = BC$ (common side)		
$\angle BCG = \angle CBF$ (alt. $\angle$s, $CG // DB$)		
$\angle CBG = \angle BCF$ (alt. $\angle$s, $BG // EC$)		
$\Delta BCG \cong \Delta CBF$ (ASA)		
(ii) $\angle CBF = \angle EDF$ (alt. $\angle$s, $BC // ED$)		
$\angle BFC = \angle DFE$ (vert. opp. $\angle$s)		
$\angle BCF = \angle DEF$ ($\angle$ sum of Δ)		
$\Delta BCF \sim \Delta DEF$ (AAA)		(AA) (equiangular)
	----------(4)	
(b) (i) By (a)(i), we have $\angle BGC = \angle BFC$.		
Since $\angle BCF = \angle BGC$, we have $\angle BCF = \angle BFC$.		
Therefore, we have $BF = BC = \ell$.	1M	
Since $BD\cos 45° = \ell$, we have $BD = \sqrt{2}\ell$.		
DF $= BD - BF$ $= \sqrt{2}\ell - \ell$ $= (\sqrt{2}-1)\ell$	1A	
(ii) By (b)(i), ΔBCF is an isosceles triangle with $BC = BF$.		
By (a)(ii), ΔDEF is an isosceles triangle with $DE = DF$.		
AE $= AD - DE$ $= AD - DF$ $= \ell - (\sqrt{2}-1)\ell$ (by (b)(i))	1M	for using the result of (b)(i)
$= (2-\sqrt{2})\ell$ $> \left(2-\frac{3}{2}\right)\ell$ $= \frac{\ell}{2}$		
Note that $AE + DE = \ell$.		
So, we have $DE < \frac{\ell}{2}$.		
Since $DE = DF$, we have $DF < \frac{\ell}{2}$.		
Therefore, we have $AE > DF$.		
Thus, the claim is agreed.	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
15. The required number		
$= C_5^{32} - C_5^{11}$	1M+1M	$\begin{cases} \text{1M for } C_p^m - C_q^n \\ \text{+1M for either one} \end{cases}$
$= 200\,914$	1A	
The required number		
$= C_1^{21}C_4^{11} + C_2^{21}C_3^{11} + C_3^{21}C_2^{11} + C_4^{21}C_1^{11} + C_5^{21}$	1M+1M	$\begin{cases} \text{1M for considering 5 cases} \\ \text{+1M for either one} \end{cases}$
$= 200\,914$	1A	
	----------(3)	
16. (a) Putting $\beta = 5\alpha - 18$ in $\beta = \alpha^2 - 13\alpha + 63$, we have		
$5\alpha - 18 = \alpha^2 - 13\alpha + 63$	1M	
$\alpha^2 - 18\alpha + 81 = 0$		
Solving, we have $\alpha = 9$ and $\beta = 27$.	1A	for both correct
	----------(2)	
(b) Let $\mathrm{T}(n)$ be the nth term of the arithmetic sequence.		
Since $\mathrm{T}(1) = \log 9 = \log 3^2 = 2\log 3$ and $\mathrm{T}(2) = \log 27 = \log 3^3 = 3\log 3$,	1M	for either one
the common difference of the sequence is $\log 3$.		
$\mathrm{T}(1) + \mathrm{T}(2) + \mathrm{T}(3) + \cdots + \mathrm{T}(n) > 888$		
$2\log 3 + 3\log 3 + 4\log 3 + \cdots + (n+1)\log 3 > 888$		
$\frac{n}{2}\big(2(2\log 3) + (n-1)\log 3\big) > 888$	1M	
$(\log 3)n^2 + (3\log 3)n - 1776 > 0$		
$n < -62.52928981$ or $n > 59.52928981$	1M	
Thus, the least value of n is 60 .	1A	
Let $\mathrm{T}(n)$ be the nth term of the arithmetic sequence.		
Since $\mathrm{T}(1) = \log 9 = \log 3^2$ and $\mathrm{T}(2) = \log 27 = \log 3^3$,		
the common difference of the sequence is $\log 3$.		
$\mathrm{T}(1) + \mathrm{T}(2) + \mathrm{T}(3) + \cdots + \mathrm{T}(n) > 888$		
$\log 9 + \log 27 + \log 81 + \cdots + \log 3^{n+1} > 888$		
$\log 3^2 + \log 3^3 + \log 3^4 + \cdots + \log 3^{n+1} > 888$		
$\log\left(3^2 \cdot 3^3 \cdot 3^4 \cdots 3^{n+1}\right) > 888$	1M	
$\log\left(3^{2+3+4+\cdots+(n+1)}\right) > 888$		
$\log 3^{\frac{n(n+3)}{2}} > 888$	1M	
$3^{\frac{n(n+3)}{2}} > 10^{888}$		
$\frac{n(n+3)}{2} > \log_3 10^{888}$		
$n^2 + 3n - 2\log_3 10^{888} > 0$		
$n < -62.52928981$ or $n > 59.52928981$	1M	
Thus, the least value of n is 60 .	1A	
	----------(4)	

Solution	Marks	Remarks
17. (a) $\frac{r(CD)}{2}+\frac{r(DE)}{2}+\frac{r(CE)}{2}=a$	1M	
$r(CD+DE+CE)=2a$		
$pr=2a$	1	
	----------(2)	
(b) (i) Γ is the angle bisector of $\angle OHK$.	1M	
(ii) OH $=\sqrt{9^2+12^2}$ $=15$		
HK $=\sqrt{(9-14)^2+12^2}$ $=13$		
Note that the area of $\Delta OHK=\frac{14(12)}{2}=84$.		
Also note that the perimeter of $\Delta OHK=13+14+15=42$.		
Let r be the radius of the inscribed circle of ΔOHK .		
By (a), we have $42r=2(84)$.	1M	for using (a)
So, we have $r=4$.		
Let $(h, 4)$ be the coordinates of the in-centre of ΔOHK .		
Hence, we have $(15-h)+(14-h)=13$.	1M	
Therefore, we have $h=8$.		
The slope of Γ $=\frac{12-4}{9-8}$ $=8$		
The equation of Γ is		
$y-4=8(x-8)$	1M	
$8x-y-60=0$	1A	or equivalent
	----------(5)	

Solution	Marks	Remarks
18. (a) (i) By sine formula, we have		
$\frac{\sin\angle BAD}{BD}=\frac{\sin\angle ABD}{AD}$	1M	
$\frac{\sin\angle BAD}{12}=\frac{\sin 72^\circ}{13}$		
$\angle BAD\approx 61.38986936^\circ$ or $\angle BAD\approx 118.61013064^\circ$ (rejected)		
Thus, we have $\angle BAD\approx 61.4^\circ$.	1A	r.t. 61.4°
(ii) $\angle ADB\approx 180^\circ-72^\circ-61.38986936^\circ$		
$\angle ADB\approx 46.61013064^\circ$		
$\cos\angle ADB=\frac{AD-AP}{BD}$	1M	
$AP\approx 13-12\cos 46.61013064^\circ$		
$AP\approx 4.756491614$		
Note that $\angle CAP=60^\circ$.		
By cosine formula, we have		
$CP^2=AC^2+AP^2-2(AC)(AP)\cos\angle CAP$	1M	
$CP^2\approx 13^2+4.756491614^2-2(13)(4.756491614)\cos 60^\circ$		
$CP\approx 11.39253359$		
$CP\approx 11.4$ cm	1A	r.t. 11.4 cm
By sine formula, we have		
$\frac{AB}{\sin\angle ADB}=\frac{AD}{\sin\angle ABD}$		
$\frac{AB}{\sin(180^\circ-72^\circ-61.38986936^\circ)}\approx\frac{13}{\sin 72^\circ}$		
$AB\approx 9.933216094$		
$\cos\angle BAD=\frac{AP}{AB}$	1M	
$AP=AB\cos\angle BAD$		
$AP\approx 4.756491614$		
Note that $\angle CAP=60^\circ$.		
By cosine formula, we have		
$CP^2=AC^2+AP^2-2(AC)(AP)\cos\angle CAP$	1M	
$CP^2\approx 13^2+4.756491614^2-2(13)(4.756491614)\cos 60^\circ$		
$CP\approx 11.39253359$		
$CP\approx 11.4$ cm	1A	r.t. 11.4 cm
	----------(5)	
(b) AP^2+CP^2		
$\approx 4.756491614^2+11.39253359^2$		
≈ 152.4140341		
AC^2		
$=169$		
Hence, we have $AP^2+CP^2\neq AC^2$.		
Therefore, $\angle APC$ is not a right angle.	1M	
So, $\angle BPC$ is not the angle between the face ABD and the face ACD .		
Thus, the claim is not correct.	1A	f.t.
	----------(2)	

Solution	Marks	Remarks
19. (a) $f(4)$		
$=\frac{1}{1+k}\left(4^2+4(6k-2)+(9k+25)\right)$		
$=\frac{1}{1+k}(33+33k)$		
$=33$	1	
Thus, the graph of $y=f(x)$ passes through F.		
	----------(1)	
(b) (i) $g(x)$		
$=f(-x)+4$	1M	
$=\frac{1}{1+k}\left((-x)^2+(6k-2)(-x)+(9k+25)\right)+4$		
$=\frac{1}{1+k}\left(x^2-(6k-2)x+(3k-1)^2-(3k-1)^2+(9k+25)\right)+4$	1M	for completing the square
$=\frac{1}{k+1}\left((x-3k+1)^2-(k+1)(9k-24)\right)+4$		
$=\frac{1}{k+1}(x-(3k-1))^2+(28-9k)$	1M	
Thus, the coordinates of U are $(3k-1, 28-9k)$.	1A	
(ii) Note that the area of the circle passing through F and O is the least when FO is a diameter of the circle. If U lies on this circle, then we have $\angle FUO=90°$.	1M	
Under this case, we have $k\neq\frac{1}{3}$ and $k\neq\frac{5}{3}$.		
$\left(\frac{(28-9k)-0}{(3k-1)-0}\right)\left(\frac{33-(28-9k)}{4-(3k-1)}\right)=-1$	1M+1A	
$\frac{(28-9k)(5+9k)}{(3k-1)(5-3k)}=-1$		
$2k^2-5k-3=0$		
$k=3$ or $k=\frac{-1}{2}$ (rejected)	1A	
Thus, the area of the circle passing through F, O and U is the least when $k=3$.		
Note that the area of the circle passing through F and O is the least when FO is a diameter of the circle. Let M be the mid-point of FO. The coordinates of M $=\left(2,\frac{33}{2}\right)$	1M	
If U lies on this circle, then we have $FO=2MU$.		
$\sqrt{(0-4)^2+(0-33)^2}=2\sqrt{(2-(3k-1))^2+\left(\frac{33}{2}-(28-9k)\right)^2}$	1M+1A	
$2k^2-5k-3=0$		
$k=3$ or $k=\frac{-1}{2}$ (rejected)	1A	
Thus, the area of the circle passing through F, O and U is the least when $k=3$.		

Solution	Marks	Remarks
(iii) The coordinates of G are $(-4, 37)$.	1A	
The product of the slope of FG and the slope of GO $=\left(\frac{37-33}{-4-4}\right)\left(\frac{37-0}{-4-0}\right)$	1M	
$=\frac{37}{8}$ $\neq -1$ So, we have $\angle FGO \neq 90°$. Since $\angle FVO = 90°$, G does not lie on the circle passing through F, O and V. Thus, F, G, O and V are not concyclic.	1A	f.t.
When the area of the circle passing through F, O and V is the least, FO is a diameter of the circle. The coordinates of G are $(-4, 37)$.	1A	
FO^2 $= 1105$ GO^2 $= 1385$ FG^2 $= 80$	1M	any one
$FG^2 + GO^2$ $= 1465$ As $FG^2 + GO^2 \neq FO^2$, $\angle FGO$ is not a right angle. Since $\angle FVO = 90°$, G does not lie on the circle passing through F, O and V. Thus, F, G, O and V are not concyclic.	1A	f.t.
When the area of the circle passing through F, O and V is the least, FO is a diameter of the circle. The coordinates of the centre of the circle passing through F, O and V $=\left(2, \frac{33}{2}\right)$ Note that the circle passes through $(0, 0)$. Let $x^2 + y^2 + Dx + Ey = 0$ be the equation of the circle passing through F, O and V. So, we have $\frac{-D}{2} = 2$ and $\frac{-E}{2} = \frac{33}{2}$.	1M	
Solving, we have $D = -4$ and $E = -33$. Therefore, the equation of the circle passing through F, O and V is $x^2 + y^2 - 4x - 33y = 0$. Also note that the coordinates of G are $(-4, 37)$.	1A	
Since $(-4)^2 + (37)^2 - 4(-4) - 33(37) \neq 0$, G does not lie on the circle passing through F, O and V. Thus, F, G, O and V are not concyclic.	1A	f.t.
	----------(11)	