Variations

變分

Exercises(練習)

1. Given that y varies directly as x, the following table shows some corresponding values of x and y.
已知 y 隨 x 而正變。下表列出了 x 和 y 的一些對應值：

x	4	8	12	16	20
y	2	4	6	8	10

(a) Find the variation constant.

(b) Plot the graph of y against x.

(c) When $y = 7$, find the value of x graphically.

(a) 求變分常數。

(b) 繪畫 y 對 x 的圖像。

(c) 利用圖解法，求當 $y = 7$ 時 x 的值。

2. The value ($P) of a piece of precious stone varies directly as the cube of its weight (w g) and $P = 2048$ when $w = 16$.

(a) Find an equation connecting P and w.

(b) **(i)** Find the value of a precious stone weighing 8 g.

(ii) Find the weight of a piece of precious stone if its value is $2916.

一枚寶石的價值 ($P) 隨其重量 ($w$ g) 的立方而正變。當 $w = 16$ 時，$P = 2048$。

(a) 求一個聯繫 P 和 w 的方程。

(b) **(i)** 求一枚重量為 8 g 的寶石的價值。

(ii) 若一枚寶石的價值為 $2916，求它的重量。

3. If y varies directly as x and $y = 21$ when $x = 12$, find

(a) an equation connecting x and y,

(b) the value of y when $x = 28$.

若 y 隨 x 而正變及當 $x = 12$ 時，$y = 21$，求

(a) 一個聯繫 x 和 y 的方程；

(b) 當 $x = 28$ 時 y 的值。

4. If $(z-2) \propto x^3$ and $z = 29$ when $x = 3$, find

(a) an equation connecting x and z,

(b) the value of x when $z = 66$.

若 $(z-2) \propto x^3$ 及當 $x = 3$ 時，$z = 29$，求

(a) 一個聯繫 x 和 z 的方程；

(b) 當 $z = 66$ 時 x 的值。

5. Given $y \propto \sqrt{x}$, find the percentage change in y when x is increased by 44%.

已知 $y \propto \sqrt{x}$。求當 x 增加 44% 時 y 的百分變化。

6. If y varies inversely as $\sqrt[3]{x}$ and $y = 2$ when $x = 8$, find

(a) an equation connecting x and y,

(b) the value of x when $y = \frac{1}{3}$.

若 y 隨 $\sqrt[3]{x}$ 而反變及當 $x = 8$ 時，$y = 2$，求

(a) 一個聯繫 x 和 y 的方程；

(b) 當 $y = \frac{1}{3}$ 時 x 的值。

7. Given that y varies inversely as x, the following table shows some corresponding values of x and y.

已知 y 隨 x 而反變。下表列出了 x 和 y 的一些對應值：

x	2	3	5	10	15	25
y	75	50	30	15	10	6

(a) Find the variation constant.

(b) Plot the graph of y against x.

(c) When $x = 6$, find the value of y graphically.

(a) 求變分常數。

(b) 繪畫 y 對 x 的圖像。

(c) 利用圖解法，求當 $x = 6$ 時 y 的值。

8. If y varies inversely as z^2 and z varies directly as $\sqrt{x}$, prove that y varies inversely as x.

若 y 隨 z^2 而反變及 z 隨 $\sqrt{x}$ 而正變，證明 y 隨 x 而反變。

9. Given y varies inversely as x^3, find the percentage change in y when x is increased by 25%.

已知 y 隨 x^3 而反變。求當 x 增加 25% 時 y 的百分變化。

10. In a journey, a car travels at uniform speed. The time (T hours) it takes to finish the journey varies inversely as the speed (V km/h) of the car. It takes 4 hours to finish the journey if the car travels at 25 km/h.

(a) Find an equation connecting T and V.

(b) **(i)** If the car takes 2 hours to finish the journey, what is the speed of the car?

(ii) Find the time it takes to finish the journey if the speed of the car is 75 km/h.

對於一段固定的車程，一輛以勻速行駛的汽車完成旅程所需的時間 (T 小時) 隨其速率 (V km/h) 而反變。若該輛汽車的速率為 25 km/h，則完成旅程所需的時間為 4 小時。

(a) 求一個聯繫 T 和 V 的方程。

(b) **(i)** 若該輛汽車完成旅程所需的時間為 2 小時，求它的速率。

(ii) 若該輛汽車的速率為 75 km/h，求它完成旅程所需的時間。

11. It is given that z varies directly as x^2 and inversely as $\sqrt{y}$, and $z = 72$ when $x = 6$ and $y = 4$.

(a) Express z in terms of x and y.

(b) Find the value of z when $x = 3$ and $y = 16$.

已知 z 隨 x^2 而正變且隨 $\sqrt{y}$ 而反變。當 $x = 6$ 及 $y = 4$ 時，$z = 72$。

(a) 試以 x 和 y 表示 z。

(b) 求當 $x = 3$ 及 $y = 16$ 時 z 的值。

12. If z varies jointly as x^3 and y, and $z = 64$ when $x = 2$ and $y = 4$, find

(a) an equation connecting x, y and z,

(b) the value of y when $x = 3$ and $z = 108$.

若 z 隨 x^3 和 y 而聯變，且當 $x = 2$ 及 $y = 4$ 時，$z = 64$，求

(a) 一個聯繫 x、y 和 z 的方程；

(b) 當 $x = 3$ 及 $z = 108$ 時 y 的值。

13. The rent per month ($\$R$) of an apartment varies directly as its area (A m^2) and inversely as the square root of its age (G years). If the rent of an apartment is \$4000 when its area is 100 m^2 and it is 9 years old, find the age of an apartment if its area is 80 m^2 and the rent is \$2400.

一個住宅單位的月租 ($\$R$) 隨其面積 ($A$ m^2) 而正變，且隨樓齡 (G 年) 的平方根而反變。若一個面積為 100 m^2，且樓齡為 9 年的住宅單位的月租是 \$4000，求一個面積為 80 m^2，而月租為 \$2400 的住宅單位的樓齡。

14. If z partly varies directly as x and partly varies directly as y^2, and $z = 18$ when $x = 3$ and $y = 2$, $z = 35$ when $x = 4$ and $y = 3$, find

(a) an equation connecting x, y and z,

(b) the value of z when $x = 2$ and $y = 4$.

已知 z 的一部分隨 x 正變，而另一部分則隨 y^2 正變。當 $x = 3$ 及 $y = 2$ 時，$z = 18$；當 $x = 4$ 及 $y = 3$ 時，$z = 35$。求

(a) 一個聯繫 x、y 和 z 的方程；

(b) 當 $x = 2$ 及 $y = 4$ 時 z 的值。

15. The value ($\$V$) of a crown varies jointly as the weight (w g) of gold in it and the number (N) of jewels on it. Given that two crowns A and B have weights of gold in the ratio 4 : 5, and their values are in the ratio 24 : 45. If crown A has 12 jewels on it, find the number of jewels on crown B.

一個皇冠的價值 ($\$V$) 隨所含的黃金重量 ($w$ g) 和寶石的數目 (N) 而聯變。

已知兩個皇冠 A 和 B 所含的黃金重量之比為 4：5，而其價值之比則為 24：45。若皇冠 A 上的寶石數目為 12 枚，求皇冠 B 上的寶石數目。

16. It is given that z varies directly as x^2 and inversely as $\sqrt[3]{y}$, and $z = 4$ when $x = 2$ and $y = 27$.

(a) Express z in terms of x and y.

(b) If x is decreased by 10% and y is increased by 33.1%, find the percentage change in z. (Give your answer correct to 2 decimal places.)

已知 z 隨 x^2 而正變且隨 $\sqrt[3]{y}$ 而反變。當 $x = 2$ 及 $y = 27$ 時，$z = 4$。

(a) 試以 x 和 y 表示 z。

(b) 若 x 減少 10% 而 y 增加 33.1%，求 z 的百分變化。
(答案須準確至二位小數。)

17, Given that $y \propto (ax^2 - 1)$, where a is a constant. If $y = 21$ when $x = 2$ and $y = 51$ when $x = 3$,

(a) find the value of a,

(b) find an equation connecting x and y,

(c) how does y change when the value of x is doubled?

(d) find the value of x when $y = 7x$.

已知 $y \propto (ax^2 - 1)$，其中 a 是一個常數。當 $x = 2$ 時，$y = 21$；當 $x = 3$ 時，$y = 51$。

(a) 求 a 的值。

(b) 求一個聯繫 x 和 y 的方程。

(c) 求當 x 增至兩倍時 y 的變化。

(d) 求當 $y = 7x$ 時 x 的值。

18. It is given that z partly varies directly as x and partly varies inversely as $\sqrt{y}$, and $z = 3$ when $x = 6$ and $y = 9$, $z = \frac{7}{6}$ when $x = 2$ and $y = 36$.

(a) Express z in terms of x and y.

(b) Find the value of y when $x = 3$ and $z = 19$.

已知 z 的一部分隨 x 正變，而另一部分則隨 $\sqrt{y}$ 反變。當 $x = 6$ 及 $y = 9$ 時，$z = 3$；當 $x = 2$ 及 $y = 36$ 時，$z = \frac{7}{6}$。

(a) 試以 x 和 y 表示 z。

(b) 求當 $x = 3$ 及 $z = 19$ 時 y 的值。

19. The cost ($\$C$) of holding a picnic is partly constant and partly varies directly as the number of participants (P). For 25 participants, the cost is \$1050, and for 20 participants, the cost is \$900.

(a) Find an equation connecting C and P.

(b) If the cost is \$1140, find the number of participants.

(c) If the number of participants obtained in (b) is halved, what is the cost?

舉辦一次旅行的成本 ($\$C$) 的一部分固定不變，而另一部分則隨參加人數 ($P$) 正變。若有 25 人參加，則所需成本是 \$1050；若有 20 人參加，則所需成本是 \$900。

(a) 求一個聯繫 C 和 P 的方程。

(b) 若旅行所需成本是 \$1140，求參加人數。

(c) 根據 **(b)** 中的情況，若參加人數減至一半，求旅行所需的成本。

20. The cost ($\$C$) of printing an invitation card is partly constant and partly varies inversely as the number of cards (n) printed. When 400 cards are printed, the cost of each card is \$6. When 600 cards are printed, the cost of each card is \$5.5.

(a) Express C in terms of n.

(b) Find the cost of each card if 500 cards are printed.

(c) Mr. Chan planned to print 500 cards, but finally he printed only 100 cards. What is the percentage change in the cost of each card?
(Give your answer correct to 2 decimal places.)

印製一張邀請卡的成本 ($\$C$) 的一部分固定不變，而另一部分則隨印製的數目 (n) 而反變。當印製 400 張邀請卡時，每張的成本是 \$6；當印製 600 張邀請卡時，每張的成本是 \$5.5。

(a) 試以 n 表示 C。

(b) 若所印製的邀請卡數目為 500 張，求每張邀請卡的成本。

(c) 陳先生打算印製 500 張邀請卡，但最後只印製了 100 張。求每張邀請卡的成本的百分變化。
(答案須準確至二位小數。)

21. The cost (\$$C$) of a gold-plating article varies directly as the square of the surface area (S cm^2) of the article and varies inversely as its weight (W g). The cost of a gold-plating article with surface area 8 cm^2 and weight 20 g is \$96.

(a) Express C in terms of S and W.

(b) If the cost of a gold-plating article with surface area 10 cm^2 is \$240, find the weight of the article.

(c) Find the percentage change in the cost of a gold-plating article when the surface area is halved and the weight is doubled.

一件鍍金物件的價值 (\$$C$) 隨它的表面面積 ($S$ cm^2) 的平方而正變，且隨它的重量 (W g) 而反變。已知一件表面面積為 8 cm^2 及重量為 20 g 的鍍金物件的價值是 \$96。

(a) 試以 S 和 W 表示 C。

(b) 若一件表面面積為 10 cm^2 的鍍金物件的價值是 \$240，求它的重量。

(c) 已知一件鍍金物件的表面面積減至一半，而其重量增至二倍。求該鍍金物件的價值的百分變化。

22. It is given that x varies directly as y^2 and y varies inversely as z^2.

(a) Show that x varies inversely as z^4.

(b) Find the percentage change in x when z is decreased by 5%.

(c) Find the percentage change in z when x is increased by 46.41%.

(Give your answers correct to 2 decimal places if necessary.)

已知 x 隨 y^2 而正變，且 y 隨 z^2 而反變。

(a) 證明 x 隨 z^4 而反變。

(b) 若 z 減少 5%，求 x 的百分變化。

(c) 若 x 增加 46.41%，求 z 的百分變化。

(如有需要，取答案準確至二位小數。)

23. The cost ($*C*) of holding a mini-concert is the sum of three parts. The first part is a constant of $2000, the second part varies directly as the number of participants (*x*), the third part varies directly as the square of the number of participants. The cost is $3000 when there are 100 participants and the cost is $2750 when there are 150 participants.

(a) Express *C* in terms of *x*.

(b) Find the cost when there are 20 participants.

(c) Find the number of participants when the cost is $2960.

(d) Find the maximum cost as the number of participants varies.

籌辦一次迷你演唱會的開支 ($*C*) 由三部分組成。第一部分為固定的金額 $2000，第二部分隨參加人數 (*x*) 正變，而第三部分則隨參加人數的平方正變。當參加人數是 100 人時，所需的開支是 $3000；當參加人數是 150 人時，所需的開支是 $2750。

(a) 試以 *x* 表示 *C*。

(b) 若參加人數為 20 人，求籌辦迷你演唱會的開支。

(c) **若所需的開支是** $2960，**求參加人數。**

(d) 求籌辦迷你演唱會的開支的最大可能值。

Pre-requisite Questions

預備測驗

1. Find the following ratios.

(a) 30 cm to 1 m

(b) 3 kg to 1500 g

(c) 1.5L to 300mL

求下列的比。

(a) 30 cm 與 1 m 的比；

(b) 3 kg 與 1500 g 的比；

(c) 1.5 L 與 300 mL 的比。

2. Express the following rates in the given units.

(a) A car travels 60 km in 2 hours (km/min).

(b) The cost of 200 VCDs is $700 ($/VCD).

(c) HKD100 is exchanged for 10.3 Euro (HKD/Euro).

(d) A man types 315 words in 7 minutes (words/minutes).

利用所給的單位表示下列的率。

(a) 一輛汽車在 2 小時內行駛了 60 km。 (km/min)

(b) 200 張雷射影碟的成本是 $700。 ($/張)

(c) 100 港元可兌換成 10.3 歐元。 (港元/歐元)

(d) 某人花了 7 分鐘來打 315 字。 (字/分鐘)

3. Express the following ratios in the simplest form:

簡化下列各比。

(a) $a : b = 18 : 24$

(b) $a : b = \frac{1}{6} : \frac{3}{8}$

(c) $a : b = 0.24 : \frac{3}{5}$

4. Every day, the time Alan, Eric and Tom spend on the Internet are 15 minutes, 45 minutes and 1 hour respectively. Find the ratio of the time that Alan, Eric and Tom spend on the Internet every day.

5. In a map of scale 1 : 200, the length and the width of a rectangular field are 6 cm and 5 cm respectively. Find the actual area of the field in m^2.
在比例尺為 1 : 200 地圖中，而長方形田地的長和闊分別是 6 cm 和 5 cm。 求該田地的真實面積 (單位為 m^2)。

6. If 1 Euro is equivalent to 9.73 HK dollars and 1 GBP is equivalent to 14.6 HK dollars, how many Euros are equivalent to 100 GBP? Correct your answer to the nearest Euro.
若 1 歐元相等於 9.73 港元及 1 英鎊相等於 14.6 港元，哪麼 100 英鎊相等於多少歐元? 答案須準確至最接近的歐元。

7. If $a : b = 3 : 4$ and $b : c = 2 : 5$, find the value of $a^2 : c^2$.
若 $\frac{1}{a} : \frac{1}{b} = 4 : 3$ 及 $a : c = 5 : 1$，求 $a : b : c$ 的值。

8. If $\frac{1}{a} : \frac{1}{b} = 4 : 3$ and $a : c = 5 : 1$, find the value of $a : b : c$.
雅倫、艾力和湯姆每天花在互聯網上的時間分別為 15 分鐘、45 分鐘和 1 小時。求雅倫、艾力和湯姆每天花在互聯網上的時間的比。

9. Find the value of x in each of the following proportions.
求下列各比例中 x 的值。
(a) $x : 3 = 7 : 6$
(b) $x : 6 = 24 : x$
(c) $(x + 1) : (2x – 7) = 7 : 9$

10. Find the value of x in each of the following proportions.
求下列各比例中 x 的值。
(a) $x : 6 = 7 : 2$
(b) $3 : 4 = 18 : x$
(c) $(3x + 1) : (2x – 7) = 2 : 1$

11. Find $x : y$ in each of the following.
在下列各題中，求 $x : y$。
(a) $4x^2 – 17xy + 4y^2 = 0$
(b) $2x^2 – 7xy + 6y^2 = 0$

12. Find $x : y$ in the following.

在下列各題中，求 $x : y$。

(a) $5x = 6y$

(b) $\frac{1}{x} : \frac{1}{y} = 4 : 5$

(c) $3(2x - y) = 4x + 3y$

(d) $\frac{2x+y}{3x-y} = \frac{3}{4}$

(e) $729^x = 81^y$

13. Find $x : y : z$ in each of the following.

在下列各題中，求 $x : y : z$。

(a) $x : y = 2 : 3$ and $y : z = 6 : 7$

(b) $x : y : z = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

(c) $3x = 2y = 7z$

14. \$777 is divided among 3 students A, B and C in the ratio $\frac{1}{4} : \frac{1}{5} : \frac{1}{6}$, how much does A get?

A、B 和 C 三位同學以 $\frac{1}{4} : \frac{1}{5} : \frac{1}{6}$ 的比分享一筆金錢 \$777，問 A 分得多少?

15. Find the two numbers if they are in the ratio

(a) 5 : 2 and their sum is 35.

(b) 9 : 7 and their difference is 8.

(a) 若兩個數的比為 5 : 2，而他們的和是 35，求該兩個數。

(b) 若兩個數的比為 9 : 7，而他們的差是 8，求該兩個數。

16. The interior angles of a quadrilateral are in the ratio 1 : 3 : 4 : 7. Find the difference between the largest and the smallest interior angles.

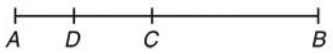
四邊形的內角的比是 1 : 3 : 4 : 7。求最大的內角與最小的內角之差。

***17*.** A mother's age and her daughter's age are now in the ratio 17 : 6. Ten years ago, the ratio of their ages was 12 : 1. Find their present ages.

. 某母親與她女兒現時歲數的比是 17 : 6。十年前，她們的歲數的比是 12 : 1。 求她們目前的歲數。

18. In the figure, AC : CB = 14 : 17 and AD : DC = 3 : 4.

在圖中，AC : CB = 14 : 17 及 AD : DC = 3 : 4。



(a) Find *AD* : *DC* : *CB*.

(b) If *AD* = 20 cm, find *DB*.

(a) 求 *AD* : *DC* : *CB*。

(b) 若 *AD* = 20 cm，求 *DB*。

19. The ratio of two numbers is 3 : 2. If the larger number is decreased by 9 and the smaller number is increased by 4, the two numbers will be equal. Find the two numbers.

兩個數的比為 3 : 2。 若較大的數減少 9 及較小的數增加 4，則它們便會相等。 求該兩個數。

Level 1 Questions
程度 1 題目

***1*.** Given that $y \propto x$, complete the tables below.

. 已知 $y \propto x$，試完成下列各表。

(a)

x	0	$\frac{1}{2}$	$\frac{3}{4}$	
y			6	14

(b)

x	1		25	
y		4	10	16

2. If y varies directly as x^3 and $x = 2$ when $y = 16$, find

(a) an equation connecting x and y,

(b) the value of x when $y = –54$.

若 y 隨 x^3 而正變及當 $y = 16$ 時，$x = 2$，求

(a) 一個聯繫 x 和 y 的方程；

(b) 當 $y = –54$ 時 x 的值。

3. Given that y varies directly as x, complete the tables below.

已知 y 隨 x 而正變，試完成下列各表。

(a)

x	1	2		4
y	3		15	

(b)

x	2	4		
y	6.2		24.8	310

4. In each of the following, given that y varies directly as $\sqrt{x}$, find

(i) the variation constant,

(ii) an equation connecting x and y.

在下列各題中，已知 y 隨 $\sqrt{x}$ 而正變。 求

(i) 變分常數；

(ii) 一個聯繫 x 和 y 的方程。

(a)

x	1	4	9	16	25	36
y	2	4	6	8	10	12

(b)

x	9	36	81	144	225	324
y	1	2	3	4	5	6

5. In each of the following, given that $y \propto (x + 1)$, find

(i) the variation constant,

(ii) an equation connecting x and y.

在下列各題中，已知 $y \propto (x + 1)$。 求

(i) 變分常數；

(ii) 一個聯繫 x 和 y 的方程。

(a)

x	3	5	7	9	11	13
y	6	9	12	15	18	21

(b)

x	1	2	3	4	5	6
y	$\frac{3}{4}$	$\frac{9}{8}$	$\frac{3}{2}$	$\frac{15}{8}$	$\frac{9}{4}$	$\frac{21}{8}$

6. If t varies directly as $\sqrt{s}$ and $t = 20$ when $s = 16$, find

(a) an equation connecting t and s,

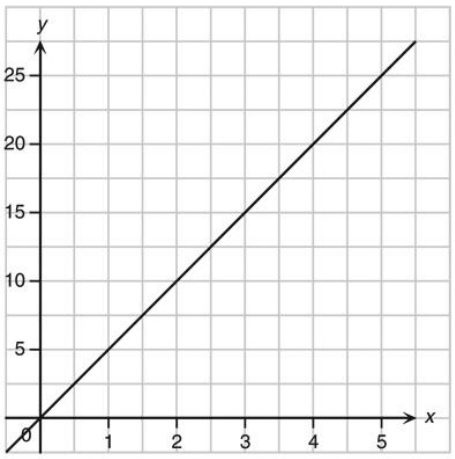
(b) the value of s when $t = 30$.

. 若 t 隨 $\sqrt{s}$ 而正變及當 $s = 16$ 時，$t = 20$，求

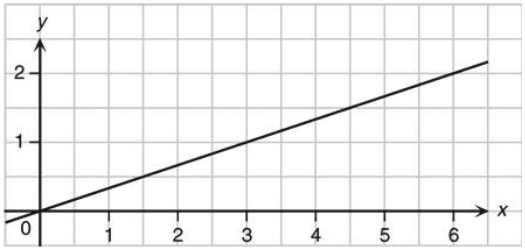
(a) 一個聯繫 t 和 s 的方程；

(b) 當 $t = 30$ 時 s 的值。

7. It is given y varies directly as x, and the figure below shows the graph of y against x.



8. It is given y varies directly as x, and the figure below shows the graph of y against x.
已知 y 隨 x 而正變，而下圖所示為 y 對 x 的圖像。



Find an equation connecting x and y.
求一個聯繫 x 和 y 的方程。

9. It is given $p \propto \sqrt{q}$. If q is decreased by 19%, find the percentage change in p.
. 已知 $p \propto \sqrt{q}$。若 q 減少 19%，求 p 的百分變化。

10. It is given $y \propto x^2$. If x is increased by 21%, find the percentage change in y.
已知 $y \propto x^2$。 若 x 增加 21%，求 y 的百分變化。

11. The postal cost (\$$C$) of a parcel varies directly as the square root of its weight (w kg). If the postal cost of a parcel of 16 kg is \$28, find
(a) an equation connecting C and w,
(b) the postal cost of a parcel of 36 kg.
一個包裹的運輸成本 (\$$C$) 隨它的重量 ($w$ kg) 的平方根而正變。若一個重 16 kg 的包裹的運輸成本為 \$28，求
(a) 一個聯繫 C 和 w 的方程；
(b) 一個重 36 kg 的包裹的運輸成本。

12. The cost (\$$C$) of painting the surface of a sphere varies directly as the square of its radius (r cm). If the cost of painting a sphere of radius 3 cm is \$36, find the cost of painting a sphere of radius 7 cm.
在一個球體的表面髹上油漆的成本 (\$$C$) 隨它的半徑 ($r$ cm) 的平方而正變。若把一個半徑為 3 cm 的球體髹上油漆的成本為 \$36，問把一個半徑為 7 cm 的球體髹上油漆的成本是多少?

13. Given that y varies inversely as $\sqrt{x}$, complete the tables below.

已知 y 隨 $\sqrt{x}$ 而反變，試完成下列各表。

(a)

x	4	25		
y	50		$\frac{50}{3}$	$\frac{25}{3}$

(b)

x		36	81	
y	9		4	$\frac{9}{4}$

14. Given that y varies inversely as x, complete the tables below.

. 已知 y 隨 x 而反變，試完成下列各表。

(a)

x		6	8	
y	8	4		24

(b)

x	2	5	10	
y		8		2

***15*.** In each of the following, given that y varies inversely as $(x + 3)$, find

(i) the variation constant,

(ii) an equation connecting x and y.

在下列各題中，已知 y 隨 $(x + 3)$ 而反變。求

(i) 變分常數；

(ii) 一個聯繫 x 和 y 的方程。

(a)

x	1	3	5	6	9
y	3	2	$\frac{3}{2}$	$\frac{4}{3}$	1

(b)

x	0	1	2	3	4
y	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{7}$

16. In each of the following, given that y varies inversely as x^2, find

(i) the variation constant,

(ii) an equation connecting x and y.

在下列各題中，已知 y 隨 x^2 而反變。求

(i) 變分常數；

(ii) 一個聯繫 x 和 y 的方程。

(a)

x	2	4	6	8	10	12
y	180	45	20	11.25	7.2	5

(b)

x	1	2	3	4	5	6
y	900	225	100	56.25	36	25

17. If y varies inversely as $(x-2)$ and $y = 6$ when $x = 4$, find

(a) an equation connecting x and y,

(b) the value of x when $y = 8$.

若 y 隨 $(x-2)$ 而反變及當 $x = 4$ 時，$y = 6$，求

(a) 一個聯繫 x 和 y 的方程；

(b) 當 $y = 8$ 時 x 的值。

18. If x varies inversely as $(3y-5)$, and $x = 1$ when $y = 4$, find

(a) an equation connecting x and y,

(b) the value of y when $x = \frac{7}{4}$.

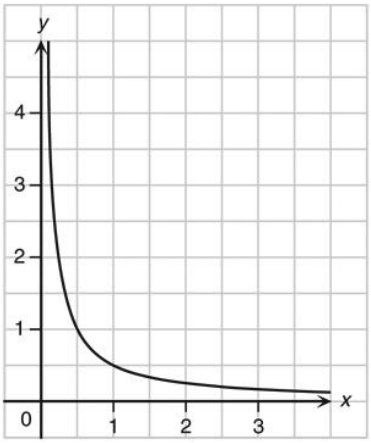
若 x 隨 $(3y-5)$ 而反變及當 $y = 4$ 時，$x = 1$，求

(a) 一個聯繫 x 和 y 的方程；

(b) 當 $x = \frac{7}{4}$ 時 y 的值。

19. It is given y varies inversely as x, and the figure below shows the graph of y against x.

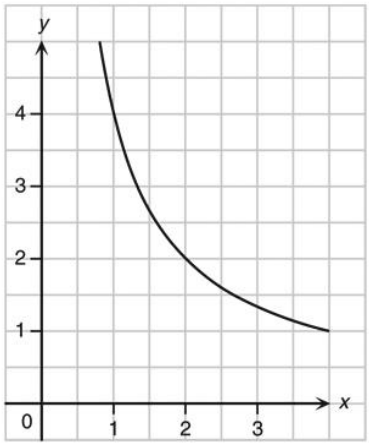
已知 y 隨 x 而反變，而下圖所示為 y 對 x 的圖像。



Find an equation connecting x and y.

求一個聯繫 x 和 y 的方程。

20 It is given y varies inversely as x, and the figure below shows the graph of y against x.
已知 y 隨 x 而反變，而下圖所示為 y 對 x 的圖像。



Find an equation connecting x and y.
求一個聯繫 x 和 y 的方程。

21 For a tunnel of a given length, the average speed of a car to travel through the tunnel varies inversely as the time required. When the speed is 50 ms^{-1}, the time required is 10 minutes. Find the time required to travel through the tunnel if the speed is 75 ms^{-1}.
對於一條已知長度的隧道，一輛汽車穿越它的平均速度隨所需時間而反變。當速度是 50 ms^{-1} 時，所需時間是 10 分鐘。 若速度是 75 ms^{-1}，求穿越該隧道的所需時間。

22 It is given that y varies inversely as x^3. If x is increased by 25%, find the percentage change in y.
已知 y 隨 x^3 而反變。若 x 增加 25%，求 y 的百分變化。

23. It is given that $y \propto \frac{1}{x^2}$. If x is decreased by 10%, find the percentage change in y. Give your answer correct to 3 significant figures.
已知 $y \propto \frac{1}{x^2}$。若 x 減少 10%，求 y 的百分變化。 取答案準確至三位有效數字。

***24*.** Express each of the following statements as an equation. Use the letter k to denote the variation constant.

(a) y varies directly as $(x + 1)$.

(b) z varies jointly as $\sqrt{x}$ and y.

(c) z varies jointly as x^2 and $\sqrt[3]{y}$.

(d) z varies inversely as x and directly as t^3.

把下列各變分表示成一個方程，並以字母 k 代表變分常數。

(a) y 隨 $(x + 1)$ 而正變。

(b) z 隨 $\sqrt{x}$ 和 y 而聯變。

(c) z 隨 x^2 和 $\sqrt[3]{y}$ 而聯變。

(d) z 隨 x 而反變且隨 t^3 而正變。

25. The number of days required to finish a certain task varies inversely as the number of workers. If there are 12 workers, they can finish the task in 15 days. Find the number of workers needed so that the task is finished in 9 days.

. 完成一項工程所需的日數隨工人的數目而反變。若有 12 名工人參與該項工程，則工程需時 15 天便可完成。若該項工程須在 9 天內完成，問最少需要多少名工人?

26. If s varies jointly as t^2 and a, and $s = 100$ when $a = 2$ and $t = 5$, find

(a) an equation connecting s, t and a,

(b) the value of s when $a = 3$ and $t = 2$.

. 若 s 隨 t^2 和 a 而聯變，且當 $a = 2$ 及 $t = 5$ 時，$s = 100$，求

(a) 一個聯繫 s、t 和 a 的方程；

(b) 當 $a = 3$ 及 $t = 2$ 時 s 的值。

27. If z varies directly as x^2 and inversely as y, and $z = 8$ when $x = 2$ and $y = 3$, find

(a) an equation connecting x, y and z,

(b) the value of x when $y = 2$ and $z = 27$.

若 z 隨 x^2 而正變且隨 y 而反變，當 $x = 2$ 及 $y = 3$ 時， $z = 8$，求

(a) 一個聯繫 x、y 和 z 的方程；

(b) 當 $y = 2$ 及 $z = 27$ 時 x 的值。

28. It is given that P varies inversely as q and directly as r^2. Find the percentage change in P if q is increased by 20% and r is decreased by 20%.

已知 P 隨 q 而反變，且隨 r^2 而正變。 若 q 增加 20% 而 r 減少 20%，求 P 的百分變化。

29. If y varies directly as x^3 and inversely as z^2, and $y = 32$ when $x = 2$ and $z = 1$, find y when $x = 1$ and $z = 3$.

. 若 y 隨 x^3 而正變且隨 z^2 而反變，當 $x = 2$ 及 $z = 1$ 時，$y = 32$，求當 $x = 1$ 及 $z = 3$ 時 y 的值。

30. The volume (V) of a gas varies directly as its temperature (T) and inversely as its pressure (P). Find the percentage change in V if T is increased by 20% and P is decreased by 40%.

汽體的體積 (V) 隨它的溫度 (T) 而正變，且隨它的壓力 (P) 而反變。 若 T 增加 20% 而 P 減少 40%，求 V 的百分變化。

31. It is given that a partly varies directly as the square of b and partly varies inversely as c, and $a = 4$ when $b = c = 1$, $a = 5$ when $b = 1$ and $c = 2$. Express a in terms of b and c.

已知 a 的一部分隨 b 的平方正變，而另一部分則隨 c 反變。當 $b = c = 1$ 時，$a = 4$；當 $b = 1$ 及 $c = 2$ 時，$a = 5$ 。試以 b 和 c 表示 a。

32. S varies jointly as p^2 and q, and $S = 48$ when $p = 2$ and $q = 4$.

(a) Find the equation connecting p, q and S.

(b) Find the percentage change in S if p and q are both increased by 20%.

S 隨 p^2 和 q 而聯變，且當 $p = 2$ 及 $q = 4$ 時，$S = 48$。

(a) 求一個聯繫 p、q 和 S 的方程。

(b) 若 p 和 q 同時增加 20%，求 S 的百分變化。

33. It is given that y partly varies directly as x and partly varies inversely as x, and $y = 7$ when $x = 1$, $y = -2$ when $x = \frac{1}{4}$.

(a) Express y in terms of x.

(b) Find the value of y when $x = 2$.

已知 y 的一部分隨 x 正變，而另一部分則隨 x 反變。 當 $x = 1$ 時， $y = 7$；當 $x = \frac{1}{4}$ 時，$y = -2$。

(a) 試以 x 表示 y。

(b) 當 $x = 2$ 時 y 的值。

34. It is given that w is partly constant and partly varies directly as v, and $w = 4$ when $v = 2$, $w = 6$ when $v = 8$. Find

(a) the equation connecting w and v,

(b) the value of v when $w = 12$.

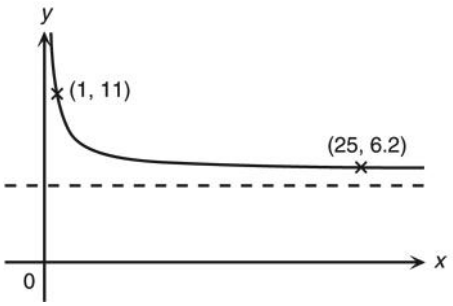
已知 w 的一部分固定不變，而另一部分則隨 v 正變。 當 $v = 2$ 時，$w = 4$；當 $v = 8$ 時，$w = 6$。求

(a) 一個聯繫 w 和 v 的方程；

(b) 當 $w = 12$ 時 v 的值。

35. It is given that y is partly constant and partly varies inversely as x. The figure below shows the graph of y against x. Express y in terms of x.

已知 y 的一部分固定不變，而另一部分則隨 x 反變。下圖所示為 y 對 x 的圖像。 試以 x 表示 y。



36. The friction (F) acting on a traveling car is partly constant and partly varies directly as the square of speed (V). When $V = 40$, $F = 98$. When $V = 50$, $F = 125$. Find

(a) the equation connecting V and F,

(b) the value of V when $F = 242$.

一輛汽車行駛時受到的阻力 (F) 的一部分固定不變，而另一部分則隨行駛速度 (V) 的平方正變。當 $V = 40$ 時，$F = 98$；當 $V = 50$ 時，$F = 125$。求

(a) 一個聯繫 V 和 F 的方程；

(b) 當 $F = 242$ 時 V 的值。

37. The cost (\$$C$) of holding a birthday party in a fastfood restaurant is partly constant and partly varies directly as the number (n) of people joining the party. For a group of 20 people, the cost of the party is \$1700. For a group of 30 people, the cost of the party is \$2300. Find

(a) the equation connecting C and n.

(b) the cost of the party for a group of 40 people.

在某快餐店舉行一次生日派對的費用 (\$$C$) 的一部分固定不變，而另一部分則隨參加人數 ($n$) 正變。 若有 20 人參加派對，則派對的費用是 \$1700；若有 30 人參加派對，則派對的費用是 \$2300。求

(a) 一個聯繫 C 和 n 的方程；

(b) 40 人參加派對的費用。

Level 2 Questions
程度 2 題目

1. If $S \propto u^2$ and $u \propto t$, show that $S \propto t^2$.
若 $S \propto u^2$ 及 $u \propto t$，證明 $S \propto t^2$。

2. It is given that $y + 3$ varies directly as $x^2 + 2x$. When $x = 2$, $y = 5$. Find
(a) an equation connecting *x* and *y*,
(b) the value of *y* when $x = -2$,
(c) the value of *x* when $y = -4$.
已知 $y + 3$ 隨 $x^2 + 2x$ 而正變。當 $x = 2$ 時，$y = 5$。求
(a) 一個聯繫 x 和 y 的方程；
(b) 當 $x = -2$ 時 y 的值；
(c) 當 $y = -4$ 時 x 的值。

3. It is given that $z \propto y$ and $y \propto x^2$. When $y = 2$, $z = 4$. When $x = 6$, $y = 12$. Find
(a) an equation connecting *x* and *z*,
(b) the value of *z* when $x = 9$.
已知 $z \propto y$ 及 $y \propto x^2$。當 $y = 2$ 時，$z = 4$；當 $x = 6$ 時，$y = 12$。求
(a) 一個聯繫 x 和 z 的方程；
(b) 當 $x = 9$ 時 z 的值。

4. It is given that $y + 5$ varies directly as $x^2 + x$. If $y = -1$ when $x = 1$, find
(a) an equation connecting *x* and *y*,
(b) the value of *y* when $x = -2$,
(c) the values of *x* when $y = 19$.
已知 $y + 5$ 隨 $x^2 + x$ 而正變。若當 $x = 1$ 時，$y = -1$，求
(a) 一個聯繫 x 和 y 的方程；
(b) 當 $x = -2$ 時 y 的值；
(c) 當 $y = 19$ 時 x 的值。

5 If $y \propto x^4$ and $y = 4$ when $x = t$, $y = 324$ when $x = 2t - 1$, find
(a) the values of *t*,
(b) an equation connecting *x* and *y* when *t* is positive.
若 $y \propto x^4$ 及當 $x = t$ 時，$y = 4$；當 $x = 2t - 1$ 時，$y = 324$，求
(a) t 的值；
(b) 當 t 是一個正數時，一個聯繫 x 和 y 的方程。

6. It is given that y varies directly as $(2x + a)$ and $y = 9$ when $x = 1$, $y = 15$ when $x = 2$, find

(a) the value of a,

(b) the relation between x and y,

(c) the value of x when $y = 24$.

已知 y 隨 $(2x + a)$ 而正變及當 $x = 1$ 時，$y = 9$；當 $x = 2$ 時，$y = 15$，求

(a) a 的值；

(b) x 和 y 之間的關係；

(c) 當 $y = 24$ 時 x 的值。

7. It is given that y varies directly as $(3x+a)^2$, where $a > 0$. When $x = 0$, $y = 4$. When $x = 2$, $y = 196$.

(a) Find the value of a.

(b) Express y in terms of x.

(c) Find the value of x when $y = 48x$.

已知 y 隨 $(3x + a)^2$ 而正變，其中 $a > 0$。當 $x=0$ 時，$y=4$；當 $x = 2$ 時，$y = 196$。

(a) 求 a 的值。

(b) 試以 x 表示 y。

(c) 求當 $y = 48x$ 時 x 的值。

8. The weight (W kg) of a certain metal rod varies directly as the cube of its length (L m). When the length of the rod is 5 m, the weight of the rod is 75 kg.

(a) Express W in terms of V.

(b) If the weight of the rod is increased by 60%, find the percentage increase in its length. Give your answer correct to 3 significant figures.

某金屬棒的重量 (W kg) 隨它的長度 (L m) 的立方而正變。 當該金屬棒的長度是 5 m 時，它的重量是 75 kg。

(a) 試以 V 表示 W。

(b) 若該金屬棒的重量增加 60%，求它的長度的百分增加。 取答案準確至三位有效數字。

9. The energy (E kJ) needed to raise the temperature of a liquid by 20°C varies directly as the mass of the liquid (m kg). When the mass of the liquid is 0.25 kg, the energy need is 8750 kJ.

(a) Express E in terms of m.

(b) If the mass of the liquid increases from 0.25 kg to 0.3 kg, find the percentage change in the energy to raise the temperature of the liquid by 20°C.

把某一種溶液的溫度提高 20°C 所需的能量隨它的質量 (m kg) 而正變。當該溶液的質量是 0.25 kg 時，所需的能量為 8750 kJ。

(a) 試以 m 表示 E。

(b) 若該溶液的質量由 0.25 kg 增至 0.3 kg，求把它的溫度提高 20°C 所需的能量的百分變化。

10. A factory produces a certain kind of cylindrical vessel. It is given that the volume (V mL) of the vessel varies directly as the square of its base radius (r cm). When the base radius of the vessel is 3 cm, the volume of the vessel is 288 mL.

(a) Express V in terms of r.

(b) The factory wants to increase the volume of the vessel by 28%, what is the corresponding percentage increase in the base radius of the vessel? Give your answer correct to 3 significant figures.

某工廠生產一種圓柱形的容器。已知該容器的容量 (V mL) 隨它的底半徑 (r cm) 的平方而正變。當該容器的底半徑是 3 cm 時，它的容量是 288 mL。

(a) 試以 r 表示 V。

(b) 工廠要增加該容器的容量 28%，問它的底半徑相應的百分增加是多少? 取答案準確至三位有效數字。

11. If $\sqrt[3]{y}$ varies inversely as $(x-a)$ and $y=8$ when $x=-2$, $y=1$ when $x=-10$, find

(a) the value of a,

(b) the equation connecting x and y,

(c) the value of x when $y=-64$.

若 $\sqrt[3]{y}$ 隨 $(x-a)$ 而反變及當 $x=-2$ 時， $y=8$；當 $x=-10$ 時，$y=1$，求

(a) a 的值；

(b) 一個聯繫 x 和 y 的方程；

(c) 當 $y=-64$ 時 x 的值。

12. If C varies inversely as $(p-2)^2$ and $C=12$ when $p=4$, find

(a) an equation connecting C and p,

(b) the values of p when $C=75$.

若 C 隨 $(p-2)^2$ 而反變及當 $p=4$ 時，$C=12$，求

(a) 一個聯繫 C 和 p 的方程；

(b) 當 $C=75$ 時 p 的值。

13. If m varies inversely as n and $m=1$ when $n=10$, find

(a) an equation connecting m and n.

(b) the values of m and n when $m-n=3$.

若 m 隨 n 而反變及當 $n=10$ 時，$m=1$，求

(a) 一個聯繫 m 和 n 的方程；

(b) 當 $m-n=3$ 時 m 和 n 的值。

14. It is given that y varies inversely as x and $y=24$ when $x=p$, $y=16$ when $x=q$, find

(a) $p:q$,

(b) the value of y when $x=q-p$.

已知 y 隨 x 而反變及當 $x=p$ 時，$y=24$；當 $x=q$ 時，$y=16$，求

(a) $p:q$；

(b) 當 $x=q-p$ 時 y 的值。

15. If y varies inversely as $(ax+2)$ and $y=1$ when $x=2$, $y=2$ when $x=\frac{2}{3}$, find

(a) the value of a,

(b) the equation connecting x and y.

若 y 隨 $(ax+2)$ 而反變及當 $x=2$ 時，$y=1$；當 $x=\frac{2}{3}$ 時，$y=2$，求

(a) a 的值；

(b) 一個聯繫 x 和 y 的方程。

16. It is given that $D\propto\frac{1}{w^2}$ and $D=x-3$ when $w=5$, $D=2x+3$ when $w=1$. Find

(a) the value of x,

(b) the equation connecting D and w.

. 已知 $D\propto\frac{1}{w^2}$ 及當 $w=5$ 時，$D=x-3$；當 $w=1$ 時，$D=2x+3$。 求

(a) x 的值；

(b) 一個聯繫 D 和 w 的方程。

17 The cost ($\$C$) of producing an electric device varies inversely as its weight (w g). When the weight of the device is 120 g, the cost of the device is $3600.

(a) Find an equation connecting C and w.

(b) If the weight of the device decreases from 120 g to 100 g, find the percentage change in the cost of producing the device.

生產一件電器用品的成本 ($\$C$) 隨它的重量 ($w$ g) 而反變。當該電器用品的重量是 120 g 時，它的生產成本是 $3600。

(a) 求一個聯繫 C 和 w 的方程。

(b) 若該電器用品的重量由 120 g 減至 100 g，求它的生產成本的百分變化。

***18*.** The selling price ($\$C$) of a car varies inversely as the square of its age (a years). It is known that the selling price of a 5-year-old car is $30 000. Find

(a) the equation connecting a and C,

(b) the selling price of a 10-year-old car,

(c) the age of the car so that the selling price is $20 000.

(Give your answer correct to 3 significant figures if necessary.)

一輛汽車的售價 ($\$C$) 隨它的出產年數 ($a$ 年) 的平方而反變。 已知一輛出產了五年的汽車的售價是 $30 000。求

(a) 一個聯繫 a 和 C 的方程；

(b) 一輛出產了十年的汽車的售價；

(c) 一輛售價為 $20 000 的汽車的出產年數。

(如有需要，取答案準確至三位有效數字。)

19. For a cylinder of fixed volume, the square of its base radius (r cm) varies inversely as its height (h cm). If the height of the cylinder is increased by 21%, find the percentage change in its base radius. (Give your answer correct to 3 significant figures if necessary.)

對於一個固定體積的圓柱體，其底半徑 (r cm) 的平方隨它的高 (h cm) 而反變。若該圓柱體的高增加了 21%，求其底半徑的百分變化。 (如有需要，取答案準確至三位有效數字。)

20. It is given that x varies jointly as y^2 and z, and $x = 9$ when $y = 2$ and $z = 3$.

(a) Express x in terms of y and z.

(b) If y is halved and z is doubled, find the new value of x to its original value.

已知 x 隨 y^2 和 z 而聯變，且當 $y = 2$ 及 $z = 3$ 時，$x = 9$。

(a) 試以 y 和 z 表示 x。

(b) 若 y 減半及 z 增至二倍，求 x 的新值與其原值的比。

21. It is given that R varies directly as x and y^2, and inversely as z^3. If $R = 6$ when $x = 6$, $y = 9$ and $z = 3$, find

(a) an equation connecting R, x, y and z,

(b) the values of y when $R = 18$, $x = 24$ and $z = 2$.

已知 R 隨 x 和 y^2 而正變，且隨 z^3 而反變。若當 $x = 6$、$y = 9$ 及 $z = 3$ 時，$R = 6$，求

(a) 一個聯繫 R、x、y 和 z 的方程；

(b) 當 $R = 18$，$x = 24$ 及 $z = 2$ 時 y 的值。

22. It is given that p varies directly as the square root of r and the cube of s, and inversely as the square of q. When $q = 2$, $r = 4$ and $s = \frac{1}{3}$, $p = \frac{1}{2}$.

(a) Express p in terms of q, r and s.

(b) If $p = \frac{q}{2} = r = 3s$, find the value of s.

已知 p 隨 r 的平方根和 s 的立方而正變，且隨 q 的平方而反變。當 $q = 2$，$r = 4$ 及 $s = \frac{1}{3}$，$p = \frac{1}{2}$。

(a) 試以 q、r 和 s 表示 p。

(b) 若 $p = \frac{q}{2} = r = 3s$，求 s 的值。

23. It is given that Q varies jointly as v and $\sqrt{u}$, and $Q = 20$ when $v = 6$ and $u = 25$. Find

(a) an equation connecting Q, u and v,

(b) the percentage change in v if the values of Q and u are both doubled.

已知 Q 隨 v 和 $\sqrt{u}$ 而聯變，且當 $v = 6$ 及 $u = 25$ 時，$Q = 20$。

(a) 求一個聯繫 Q、u 和 v 的方程。

(b) 若 Q 和 u 的值增至二倍，求 v 的百分變化。

24. The cost (\$$C$) of making a cylindrical can varies jointly as its height (h cm) and the square of the base radius (r cm). The cost of making cylindrical can A and that of can B is in the ratio 6 : 5, and the heights of can A and can B is in the ratio 8 : 15. Find the base radius of can B if the base radius of can A is 6 cm.

生產一個圓柱形罐子的成本 (\$$C$) 隨其高 ($h$ cm) 和其底半徑 (r cm) 的平方而聯變。生產圓柱形罐子 A 和罐子 B 的成本的比是 6 : 5，而罐子 A 和罐子 B 的高度的比是 8 : 15。若罐子 A 的底半徑是 6 cm，求罐子 B 的的底半徑。

25. The pressure (P) of water flowing through a cylindrical pipe varies directly as the length (l) of the pipe and inversely as the square of its radius (r). If $P = 90$ when $r = 2$ and $l = 3$, find

(a) the value of r when $P = 200$ and $l = 5$,

(b) the percentage change in P when l is increased by 10% and r is decreased by 10%,

(c) the percentage change in r when P is halved and l is tripled.

(Give your answer correct to 3 significant figures if necessary.)

. 圓柱形水管的水壓 (P) 隨它的長度 (l) 而正變，且隨它的半徑 (r) 的平方而反變。若當 $r = 2$ 及 $l = 3$ 時，$P = 90$，求

(a) 當 $P = 200$ 及 $l = 5$ 時 r 的值；

(b) 當 l 增加 10% 及 r 減少 10% 時 P 的百分變化；

(c) 當 P 減半及 l 增至三倍時 r 的百分變化。

(如有需要，取答案準確至三位有效數字。)

26. The selling price of a silver coin varies jointly as the thickness (t cm) and the square of the diameter (d cm). The ratio of selling prices of silver coins A to silver coin B is 4 : 27, and the ratio of thickness of coin A to coin B is 16 : 3. Find the diameter of the smaller one, if that of the larger one is 6 cm.

一個銀幣的售價隨其厚度 (t cm) 和其直徑 (d cm) 的平方而聯變。銀幣 A 和銀幣 B 的售價的比是 4 : 27，而銀幣 A 和銀幣 B 的厚度的比是 16 : 3。 若較大的銀幣的直徑是 6 cm，求較小的銀幣的直徑。

27. It is given that y partly varies directly as x and partly varies directly as x^2. When $x=2$, $y=18$. When $x=3$, $y=33$.

(a) Express y in terms of x.

(b) Find the value of y when $x=-1$.

(c) Find the value of x when $y=3$.

已知 y 的一部分隨 x 正變，而另一部分則隨 x^2 正變。 當 $x = 2$ 時，$y = 18$；當 $x = 3$ 時，$y = 33$。

(a) 試以 x 表示 y。

(b) 求當 $x = -1$ 時 y 的值。

(c) 求當 $y = 3$ 時 x 的值。

28. It is given that z varies directly as x and inversely as y^2. If x varies inversely as t and y varies directly as t, show that $z \propto x^3$.

已知 z 隨 x 而正變且隨 y^2 而反變。 若 x 隨 t 反變，而 y 隨 t 正變，證明 $z \propto x^3$。

29. The cost (\$*C*) of manufacturing an article is made up of a fixed amount (\$*A*) and an operation cost. The operation cost varies inversely as the number of articles (*n*) produced. If 50 articles are produced, the cost of manufacturing an article is \$30. If 100 articles are produced, the cost of manufacturing an article is \$25.

(a) Find the value of *A*.

(b) Express *C* in terms of *n*.

(c) The original cost of manufacturing an article is \$30. If the manufacturer wants to lower the cost of an article by 20%, how many articles should be produced?

一篇文章的製作成本 (\$*C*) 包括一筆固定的金額 (\$*A*) 及營運成本，而該營運成本隨文章的製作量 (*n*) 反變。若要製作 50 篇文章，則一篇文章的製作成本是 \$30；若要製作 100 篇文章，則一篇文章的製作成本是 \$25。

(a) 求 *A* 的值。

(b) 試以 *n* 表示 *C*。

(c) 一篇文章原本的製作成本是 \$30。若製造商要減少該文章的製作成本 20%，問要製作多少篇文章?

30. It is given that *y* partly varies directly as *x* and partly varies directly as x^2. When $x=1$, $y=1$. When $x=2$, $y=-2$.

(a) Express *y* in terms of *x*.

(b) Find the value of *y* when $x=-5$.

(c) Express *y* in the form of $a(x+b)^2+c$, where *a*, *b* and *c* are constants.
Hence determine the maximum value of *y*.

已知 *y* 的一部分隨 *x* 正變，而另一部分則隨 x^2 正變。當 $x = 1$ 時，$y = 1$；當 $x = 2$ 時，$y = -2$。

(a) 試以 *x* 表示 *y*。

(b) 求當 $x = -5$ 時 *y* 的值。

(c) 把 *y* 寫成 $a(x + b)^2 + c$ 的形式，其中 *a*、*b* 和 *c* 是常數。
由此，求 *y* 的最大值。

31. It is given that y is partly constant, partly varies directly as x and partly varies directly as x^2. When $x=0$, $y=-7$. When $x=1$, $y=-6$. When $x=2$, $y=-9$.

(a) Express y in terms of x.

(b) Express y in the form of $a(x+b)^2+c$, where a, b and c are constants.
Hence determine the maximum value of y.

已知 y 的一部分固定不變，一部分隨 x 正變，而另一部分則隨 x^2 正變。當 $x=0$ 時，$y=-7$；當 $x=1$ 時，$y=-6$；當 $x=2$ 時，$y=-9$。

(a) 試以 x 表示 y。

(b) 把 y 寫成 $a(x+b)^2+c$ 的形式，其中 a、b 和 c 是常數。
由此，求 y 的最大值。

32. The unit cost (\$$C$) of making a plastic bag is partly constant and partly varies inversely as the total number (n) of plastic bags made. When 200 bags are made, the cost of one bag is \$1; when 400 bags are made, the cost of one bag is \$0.7.

(a) Express C in terms of n.

(b) How many plastic bags must be made if the cost of making one plastic bag is \$0.6?

(c) Find the cost of one bag if 1200 bags are made.

一個塑膠袋的製造成本 (\$$C$) 的一部分固定不變，而另一部分則隨其總製造量 (n) 反變。當製造 200 個塑膠袋時，一個塑膠袋的成本是 \$1；當製造 400 個塑膠袋時，一個塑膠袋的成本是 \$0.7。

(a) 試以 n 表示 C。

(b) 若要把一個塑膠袋的製造成本降低至 \$0.6，問要製造塑膠袋多少個?

(c) 若要製造 1200 個塑膠袋，求一個塑膠袋的製造成本。

33. The running cost (\$$C$) of a language course in a school is partly constant and partly varies directly as the square of the number of students (n) enrolled. When there are 10 students enrolled in the course, the running cost is \$6100.

(a) Express C in terms of n.

(b) Find the running cost of the course if there are 50 students enrolled.

(c) The tuition fee for each student of the course is \$350. If there are 50 students enrolled the course, what is the profit percentage for running the course? (Give your answer correct to 3 significant figures)

一間學校的語言課程的經營成本 (\$$C$) 的一部分固定不變，而另一部分則隨參加該課程的學生數目 (n) 的平方正變。當有 10 個學生參加該課程時，課程的經營成本是 \$6100。

(a) 試以 n 表示 C。

(b) 若有 50 個學生參加該課程，求課程的經營成本。

(c) 該課程的每一個學生的學費是 \$350。若有 50 個學生參加該課程，求經營該課程的盈利百分率? (取答案準確至三位有效數字。)

34. The unit cost of making a school badge is partly constant and partly varies inversely as the total number of badges made. When 400 badges are made, the cost of one badge is \$0.8; when 600 badges are made, the cost of one badge is \$0.7.

(a) How many badges must be made if the cost of one badge is \$0.6?

(b) Find the cost of a badge if 1000 badges are made.

一個校徽的製造成本的一部分固定不變，而另一部分則隨其總製造量反變。當製造 400 個校徽時，一個校徽的製造成本是 \$0.8；當製造 600 個校徽時，一個校徽的製造成本是 \$0.7。

(a) 若要把一個校徽的製造成本降低至 \$0.6，問要製造校徽多少個?

(b) 若要製造 1000 個校徽，求一個校徽的製造成本。

35. A salesman's monthly income is partly constant and partly varies directly as his sales in this month. When the amount of his sales is \$100 000, his income in that month is \$9000. When the amount of his sales is \$120 000, his income in that month is \$10 000.

(a) If the salesman's income is \$$p$ and the amount of his sales in that month is \$$q$, find an equation connecting p and q.

(b) If the amount of his sales in a months is \$300 000, find his income for that month.

(c) If the salesman wants to have an income of \$28 000, what should be the amount of his sales?

. 一個推銷員每月的收入的一部分固定不變，而另一部分則隨他該月的銷售額正變。 當他的銷售額是 \$100 000 時，他該月的收入是 \$9000；當他的銷售額是 \$120 000 時，他該月的收入是 \$10 000。

(a) 若該推銷員的收入是 \$$p$ 及他該月的銷售額是 \$$q$，求一個聯繫 p 和 q 的方程。

(b) 若該推銷員的銷售額是 \$300 000，求他該月的收入。

(c) 若該推銷員該月的收入為 \$28 000，問他的銷售額是多少?

Level 2+ Questions
程度 2+ 題目

1. It is given that C partly varies directly as t^3 and partly varies directly as t^2. When $t = 1$, $C = 11$. When $t = 2$, $C = 52$. Find

(a) the equation connecting C and t,

(b) the values of t when $C = 27$.

已知 C 的一部分隨 t^3 正變，而另一部分則隨 t^2 正變。當 $t = 1$ 時，$C = 11$。當 $t = 2$ 時，$C = 52$。求

(a) 一個聯繫 C 和 t 的方程；

(b) 當 $C = 27$ 時 t 的值。

2. The cost (\$$C$) of a certain cubical block varies directly as the square of its length (ℓ cm). The selling price (\$$S$) of the block varies directly as its length. When the length is 50 cm, there is no profit (\$$P$) made when the block is sold. What should be the length of the block so as to obtain the maximum profit?

某一立方體的成本 (\$$C$) 隨它的長度 ($\ell$ cm) 的平方正變，而它的售價 (S) 則隨它的長度反變。當長度是 50 cm 時，售出該立方體並無盈利 (\$$P$)。若要獲得最高的盈利，問該立方體的長度應是多少?

3. **(a)** If $(y^2 - x^2)$ varies inversely as $\left(\frac{1}{x^2} - \frac{1}{y^2}\right)$, show that $(y^2 - x^2) \propto x^2y^2$.

(b) If $(y^2 + x^2)$ varies inversely as $\left(\frac{1}{x^2} - \frac{1}{y^2}\right)$, show that $(y^4 - x^4) \propto x^2y^2$.

(a) 若 $(y^2 - x^2)$ 隨 $\left(\frac{1}{x^2} - \frac{1}{y^2}\right)$ 而反變，證明 $(y^2 - x^2) \propto x^2y^2$。

(b) 若 $(y^2 + x^2)$ 隨 $\left(\frac{1}{x^2} - \frac{1}{y^2}\right)$ 而反變，證明 $(y^4 - x^4) \propto x^2y^2$。

4. In a company, the total expenditure ($\$T$) for a training course is shared in the ratio 1 : 3 by the staff enrolled in the course and the company respectively. It is known that the total expenditure of the training course is partly constant and partly varies directly as the number of the staff (n) enrolled in the course. If 60 staff are enrolled in the course, the company has to pay \$40 500. If 40 staff are enrolled in the course, the company has to pay \$28 500.

(a) Find an equation connecting T and n.

(b) If 50 staff are enrolled in the course, how much should each staff pay for the training course?

(c) If the company has to pay \$49 500 for the course, find the number of staff enrolled in the course.

在某公司中，一個培訓課程的總支出 ($\$T$) 是由參加課程的員工與該公司以 1 : 3 的比來分擔。已知該培訓課程的總支出的一部分固定不變，而另一部分則隨參加課程的員工數目 (n) 正變。若有 60 名員工參加課程，則該公司要支付 \$40 500；若有 40 名員工參加課程，則該公司要支付 \$28 500。

(a) 求一個聯繫 T 和 n 的方程。

(b) 若有 50 名員工參加課程，問每一個員工要支付多少?

(c) 若該公司為培訓課程支付 \$49 500，求參加課程的員工數目。

5. It is given that the sum (S) of the first n positive integers (i.e. $S = 1 + 2 + 3 + \ldots + n$) partly varies directly as n and partly varies directly as n^2.

(a) Find an equation connecting S and n.

(b) Hence, or otherwise, find the value of $101 + 102 + \ldots 160$.

已知首 n 個正整數的總和 (S) (即 $S = 1 + 2 + 3 + \ldots + n$) 的一部分隨 n 正變，而另一部分則隨 n^2 正變。

(a) 求一個聯繫 S 和 n 的方程。

(b) 由此，或以其他方法，求 $101 + 102 + \ldots 160$ 的值。

6. It is given that x, y and z are all positive real numbers. z varies directly as $(x - y)$ and inversely as xy.

When $x = 2$ and $y = 1$, $z = \frac{1}{12}$.

(a) Express z in terms of x and y.

(b) If $y = \frac{10}{x}$ and $z > \frac{1}{20}$, find the range of values of x.

已知 x、y 和 z 是正有理數。z 隨 $(x - y)$ 而正變且隨 xy 而反變。當 $x = 2$ 及 $y = 1$ 時，

$z = \frac{1}{12}$。

(a) 試以 x 和 y 表示 z。

(b) 若 $y = \frac{10}{x}$ 及 $z > \frac{1}{20}$，求 x 的值的範圍。

Multiple Choice Questions
多項選擇題

1. x and y are two changing quantities and the table below shows some of their corresponding values.

下表所示為兩個變量 x 和 y 的一些對應值。

x	2	3	5	6
y	16	36	100	144

Which of the following best describes the relation between x and y?

A. y varies directly as x.

B. y varies inversely as x.

C. y varies directly as x^2.

D. y varies inversely as x^2.

下列何者可能是 x 與 y 之間的關係?

A. y 隨 x 而正變。

B. y 隨 x 而反變。

C. y 隨 x^2 而正變。

D. y 隨 x^2 而反變。

2. If $y \propto \sqrt{x}$ and $y = 10$ when $x = 4$, find x when $y = 5$.

若 $y \propto \sqrt{x}$ 及當 $x = 4$ 時，$y = 10$，求當 $y = 5$ 時 x 的值。

A. –1

B. 1

C. –2

D. 2

3. If y varies directly as x, which of the following must be true?

若 y 隨 x 而正變，下列哪(些)項是正確的?

I. $(y+2) \propto (x+2)$

II. $y \propto \frac{2}{3}x$

III. $y^3 \propto x^3$

A. I only

B. II only

C. I and III only

D. II and III only

4. If $(y - 1)$ varies directly as $(x + 1)$ and $y = 3$ when $x = 4$, find y when $x = 14$.

若 $(y - 1)$ 隨 $(x + 1)$ 而正變及當 $x = 4$ 時，$y = 3$，求當 $x = 14$ 時 y 的值。

A. 9

B. 7

C. 6

D. 5

5. x and y are two changing quantities and the table below shows some of their corresponding values.

下表所示為兩個變量 x 和 y 的一些對應值。

x	1	4	5	8
y	3	9	11	17

Which of the following best describes the relation between x and y?

A. y varies directly as x.

B. y varies directly as $(2x + 1)$.

C. y varies directly as $(x + 1)$.

D. y varies directly as x^2.

下列何者可能是 x 與 y 之間的關係?

A. y 隨 x 而正變。

B. y 隨 $(2x + 1)$ 而正變。

C. y 隨 $(x + 1)$ 而正變。

D. y 隨 x^2 而正變。

6. If y varies directly as $(2x + 1)$ and $y = 21$ when $x = 3$, find x when $y = 39$.
若 y 隨 $(2x + 1)$ 而正變及當 $x = 3$ 時，$y = 21$，求當 $y = 39$ 時 x 的值。

A. 6
B. 10
C. 19.5
D. 21

7. If $x : y = 1 : 2$ and $y : z = 3 : 2$, which of the following must be true?
若 $x : y = 1 : 2$ 及 $y : z = 3 : 2$，下列哪(些)項是正確的?

I. x varies inversely as y.
II. y varies directly as z.
III. z varies directly as x.
I. x 隨 y 而反變。
II. y 隨 z 而正變。
III. z 隨 x 而正變。

A. I only
B. III only
C. I and II only

8. It is given that y varies directly as x^2. If x is decreased by 10%, then y is
已知 y 隨 x^2 而正變。若 x 減少 10%，則 y

A. decreased by 10%.
B. decreased by 100%.
C. decreased by 19%.
D. decreased by 81%.
A. 減少 10%。
B. 減少 100%。
C. 減少 19%。
D. 減少 81%。

9. If x varies inversely as $(4y - 5)^2$ and $x = 32$ when $y = 2$, find y when $x = 18$.
若 x 隨 $(4y - 5)^2$ 而反變及當 $y = 2$ 時，$x = 32$，求當 $x = 18$ 時 y 的值。

A. $\frac{9}{4}$
B. $\frac{9}{4}$ or $-\frac{9}{4}$
C. $\frac{9}{4}$ or $\frac{1}{4}$
D. $\frac{9}{4}$ or $-\frac{1}{4}$

10. It is given that $(y + 1)^2$ varies directly as x. If y is increased from 10 to 11, then x is
已知 $(y + 1)^2$ 隨 x 而正變。若 y 由 10 增至 11，則 x

A. increased by 11%.
B. increased by 21%.
C. increased by 79%.
D. increased by 89%.
A. 增加 11%。
B. 增加 21%。
C. 增加 79%。
D. 增加 89%。

11. If y varies inversely as $\sqrt{x}$ and $y = 4$ when $x = 9$, find y when $x = 36$.
若 y 隨 $\sqrt{x}$ 而反變 及當 $x = 9$ 時，$y = 4$，求當 $x = 36$ 時 y 的值。

A. 2
B. 4
C. 9
D. 36

12. x and y are two changing quantities and the table below shows some of their corresponding values.

下表所示為兩個變量 x 和 y 的一些對應值。

x	–1728	216	–64	27
y	1	–2	3	–4

Which of the following best describes the relation between x and y?

下列何者可能是 x 與 y 之間的關係?

A. $x \propto \frac{1}{\sqrt{y}}$

B. $x \propto \frac{1}{\sqrt[3]{y}}$

C. $x \propto \frac{1}{y^2}$

D. $x \propto \frac{1}{y^3}$

13. If z varies directly as $(x^2 - x)$ and $z = 1$ when $x = 2$, find z when $x = 5$.

若 z 隨 $(x^2 - x)$ 而正變及當 $x = 2$ 時，$z = 1$，求當 $x = 5$ 時 z 的值。

A. 10

B. 20

C. 25

D. 30

14. x and y are two changing quantities and the table below shows some of their corresponding values.

下表所示為兩個變量 x 和 y 的一些對應值。

x	1	2	4	25
y	1	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{1}{5}$

Which of the following best describes the relation between x and y?

下列何者可能是 x 與 y 之間的關係?

A. $y^2 \propto x$

B. $y^2 \propto \frac{1}{x}$

C. $x^2 \propto y$

D. $x^2 \propto \frac{1}{y}$

15. If $z \propto \frac{1}{y}$ and $y \propto x+1$, then which of the following must be true?

若 $z \propto \frac{1}{y}$ 及 $y \propto x+1$，則下列哪項是正確的?

A. $z \propto x$

B. $z \propto x+1$

C. $z \propto \frac{1}{x}+1$

D. $z \propto \frac{1}{x+1}$

16 If z varies directly as x^2 and varies inversely as y and $z = 2$ when $x = 1$ and $y = 2$, find the value of y when $x = 2$ and $z = 32$.

若 z 隨 x^2 而正變且隨 y 而反變。當 $x = 1$ 及 $y = 2$ 時，$z = 2$。求當 $x = 2$ 及 $z = 32$ 時 y 的值。

A. 4

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

17. It is given that x varies inversely as y^3. If y is increased by 10%, find the percentage change in x correct to 3 significant figures.
已知 x 隨 y^3 而反變。若 y 增加 10%，求 x 的百分變化，準確至三位有效數字。

A. –24.9%
B. –9.09%
C. +24.9%
D. +9.09%

18. If z varies jointly as $\sqrt{x}$ and y^2 and $z = 16$ when $x = 4$ and $y = 4$, find the value of z when $x = 81$ and $y = 6$.
若 z 隨 $\sqrt{x}$ 和 y^2 而聯變。當 $x = 4$ 及 $y = 4$ 時，$z = 16$，求當 $x = 81$ 及 $y = 6$ 時 z 的值。

A. 84
B. 162
C. 288
D. 486

19. It is given that p varies inversely as $(3q – 1)$. If q is increased from 2 to 3, find the percentage change in p.
已知 p 隨 $(3q – 1)$ 而反變。若 q 由 2 增至 3，求 p 的百分變化。

A. $+33\frac{1}{3}\%$
B. $-33\frac{1}{3}\%$
C. $+37\frac{1}{2}\%$
D. $-37\frac{1}{2}\%$

20. It is given that z varies jointly as x and y. If the value of y is kept constant, then which of the following must be true?
已知 z 隨 x 和 y 而聯變。若 y 的值固定不變，則下列哪項是正確的?

A. $x \propto y$
B. $x \propto \frac{1}{y}$
C. $z \propto y$
D. $z \propto x$

21. If a varies directly as $\sqrt{b}$ and inversely as c^2, then which of the following must be true?
若 a 隨 $\sqrt{b}$ 而正變且隨 c^2 而反變，則下列哪項是正確的?

A. $\frac{a\sqrt{b}}{c^2}$ = constant
B. $\frac{\sqrt{b}c^2}{a}$ = constant
C. $\frac{\sqrt{b}}{ac^2}$ = constant
D. $a\sqrt{b}c^2$ = constant
A. $\frac{a\sqrt{b}}{c^2}$ = 常數
B. $\frac{\sqrt{b}c^2}{a}$ = 常數
C. $\frac{\sqrt{b}}{ac^2}$ = 常數
D. $a\sqrt{b}c^2$ = 常數

C

22 It is given that p varies directly as q and inversely as r^3. If the value of q and r are doubled, find the ratio of the new value of p to the original one.
已知 p 隨 q 而正變且隨 r^3 而反變。若 q 和 r 的值增至二倍，求 p 的新值與其原值的比。

A. 1 : 8
B. 1 : 4
C. 8 : 1
D. 4 : 1

23. It is given that x varies directly as y^2 and inversely as $\sqrt{z}$. If y is increased by 10% and z is decreased by 19%, find the percentage increases in x correct to 3 significant figures.
已知 x 隨 y^2 而正變且隨 $\sqrt{z}$ 而反變。若 y 增加 10% 及 z 減少 19%，求 x 的百分變化，準確至三位有效數字。

A. 22.2%
B. 34.4%
C. 35.8%
D. 49.4%

24. If $x \propto z^2$ and $y \propto \frac{1}{z}$, which of the following must be true?
若 $x \propto z^2$ 及 $y \propto \frac{1}{z}$，下列哪些項是正確的?

I. $x \propto \frac{1}{y^2}$

II. $\frac{x}{y} \propto z^3$

III. $(x-y) \propto \left(z^2 - \frac{1}{z}\right)$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III

25. It is given that z varies directly as x^2 and inversely as y^3. If the value of y is doubled and the value of x is tripled, find the ratio of the new value of z to the original one.
已知 z 隨 x^2 而正變且隨 y^3 而反變。若 y 的值增至二倍及 x 的值增至三倍，求 z 的新值與其原值的比。

A. 8 : 1
B. 9 : 1
C. 8 : 9
D. 9 : 8

26. It is given that w varies jointly as x and y. If x is increased by 10% and y is decreased by 10%, find the percentage change in w.
已知 w 隨 x 和 y 而聯變。若 x 增加 10% 及 y 減少 10%，求 w 的百分變化。

A. –1%
B. +1%
C. 0%
D. 19%

27. It is given that y partly varies directly as x and partly varies directly as x^2. When $x = 1$, $y = 3$. When $x = 2$, $y = 8$. Find the relation between x and y.
已知 y 的一部分隨 x 正變，而另一部分則隨 x^2 正變。當 $x = 1$ 時，$y = 3$；當 $x = 2$ 時，$y = 8$。求 x 與 y 之間的關係。

A. $y = x + 2x^2$
B. $y = 2(x + x^2)$
C. $y = 2x + x^2$
D. $y = x + 4x^2$

28. It is given that y is partly constant and partly varies directly as x. When $x = 0$, $y = 2$ and when $x = 1$, $y = 8$. Find y when $x = 2$.
已知 y 的一部分固定不變，而另一部分則隨 x 正變。當 $x = 0$ 時，$y = 2$ 及當 $x = 1$ 時，$y = 8$。求當 $x = 2$ 時 y 的值。

A. 8
B. 14
C. 16
D. 32

29. It is given that z partly varies directly as x and partly varies directly as y. When $x = 1$ and $y = 1$, $z = 5$. When $x = 1$ and $y = 2$, $z = 4$. Find the relation between x, y and z.
已知 z 的一部分隨 x 正變，而另一部分則隨 y 反變。當 $x = 1$ 及 $y = 1$ 時，$z = 5$；當 $x = 1$ 及 $y = 2$ 時，$z = 4$。求 x、y 與 z 之間的關係。

A. $z = 3x + \frac{2}{y}$

B. $z = x + \frac{1}{y}$

C. $z = x + \frac{4}{y}$

D. $z = x + \frac{6}{y}$

30. It is given that y partly varies directly as x^2 and partly varies inversely as $\sqrt{x}$. If k_1 and k_2 are non-zero constants, which of the following equations describes the relation between x and y?
已知 y 的一部分隨 x^2 正變，而另一部分則隨 $\sqrt{x}$ 反變。若 k_1 和 k_2 是非零常數，問下列何者可能是 x 與 y 之間的關係?

A. $y = k_1x^2 + \frac{k_2}{\sqrt{x}}$

B. $y = \frac{k_1}{x^2} + k_2\sqrt{x}$

C. $y = k_1x^2 + k_2\sqrt{x}$

D. $y = \frac{k_1}{x^2} + \frac{k_2}{\sqrt{x}}$

31. It is given that y is partly constant and partly varies directly as x^2. If k_1 and k_2 are non-zero constants, which of the following equations describes the relation between x and y?
已知 y 的一部分固定不變，而另一部分則隨 x^2 正變。若 k_1 和 k_2 是非零常數，問下列何者可能是 x 與 y 之間的關係?

A. $y = k_1k_2x^2$

B. $y = k_1 + \frac{k_2}{x_2}$

C. $y = (k_1 + k_2x)^2$

D. $y = k_1 + k_2x^2$

32. If $x^2 = k + y_2$ and k is a non-zero constant, which of the following must be true?
若 $x^2 = k + y^2$ 及 k 是非零常數，下列哪項是正確的?

A. $(x^2 - y^2) \propto (x^2 + y^2)$

B. $(x^2 - y^2) \propto \frac{1}{(x^2 + y^2)}$

C. $(x - y) \propto (x + y)$

D. $(x - y) \propto \frac{1}{(x + y)}$

33. It is given that y is partly constant and partly varies inversely as x^2. When $x = 1$, $y = 20$ and when $x = 3$, $y = 4$. Find y when $x = –2$.
已知 y 的一部分固定不變，而另一部分則隨 x^2 反變。當 $x = 1$ 時，$y = 20$ 及當 $x = 3$ 時，$y = 4$。求當 $x = –2$ 時 y 的值。

A. –6.5

B. 6.5

C. –2.5

D. 2.5