

# Arithmetic and Geometric Sequences

## and their Summation

## 等差數列和等比數列及其求和法

### Exercises(練習)

**1 (a)** Find the general term of the arithmetic sequence 12, 7, 2, -3, ...

**(b)** If the  $k$ th term of the sequence is -38, find  $k$ .

**(a)** 求等差數列 12, 7, 2, -3, ... 的通項。

**(b)** 若該數列的第  $k$  項是-38，求  $k$  的值。

**2.** Consider the following sequence

1, 8, 27, 64, ...

**(a)** Determine the general term  $T(n)$  of the sequence.

**(b)** Write down the 5th term and the 7th term of the sequence.

考慮以下的數列：

1, 8, 27, 64, ...

**(a)** 寫出該數列的通項  $T(n)$ 。

**(b)** 求該數列的第 5 項和第 7 項。

**3.** Consider the following sequence

$\log 1, \log 4, \log 9, \log 16, \dots$

**(a)** Determine the general term  $T(n)$  of the sequence.

**(b)** Write down the 6th term and the 9th term of the sequence.

考慮以下的數列：

$\log 1, \log 4, \log 9, \log 16, \dots$

**(a)** 寫出該數列的通項  $T(n)$ 。

**(b)** 求該數列的第 6 項和第 9 項。

4. It is given that the first term  $a$  and the common difference  $d$  of an arithmetic sequence are 5 and 3 respectively.

(a) Find the general term  $T(n)$  of the sequence.

(b) Hence, find the 12th term and the 18th term of the sequence.

已知一個等差數列的首項  $a$  是 5 而公差  $d$  是 3。

(a) 求該數列的通項  $T(n)$ 。

(b) 由此，求該數列的第 12 項和第 18 項。

5. If 16 is the arithmetic mean between  $x$  and  $y$ , while  $y$  is the arithmetic mean between  $x$  and 1, find the values of  $x$  and  $y$ .

若 16 是  $x$  與  $y$  的等差中項，而  $y$  是  $x$  與 1 的等差中項，求  $x$  和  $y$  的值。

6. For an arithmetic sequence, it is given that  $T(21) = -16$  and  $T(25) = 20$ . Find

(a) the first term and the common difference of the sequence,

(b) the general term of the sequence.

在一個等差數列中， $T(21) = -16$  及  $T(25) = 20$ 。求

(a) 首項和公差；

(b) 通項。

7. Given an arithmetic sequence  $-57, -51, -45, \dots, 207$ .

(a) How many terms are there in the sequence?

(b) Find the value of  $m$  such that the  $m$ th term is the first positive term.

已知一個等差數列  $-57, -51, -45, \dots, 207$ 。

(a) 問該數列有多少項？

(b) 若第  $m$  項為該數列中的首個正數項，求  $m$  的值。

8. Given two numbers 15 and 63,

(a) insert two arithmetic means between these two numbers,

(b) insert three arithmetic means between these two numbers.

在 15 與 63 兩個數之間插入

(a) 兩個等差中項；

(b) 三個等差中項。

9. For a geometric sequence, it is given that  $T(3) = 5$  and  $T(6) = 135$ . Find

- (a) the first term and the common ratio of the sequence.
- (b) the general term of the sequence,
- (c) the value of  $\frac{T(2) + T(3)}{T(5)}$ .

在一個等比數列中， $T(3) = 5$  及  $T(6) = 135$ 。求

- (a) 該數列的首項和公比；
- (b) 該數列的通項；
- (c)  $\frac{T(2) + T(3)}{T(5)}$  的值。

10. It is given that the first term  $a$  and the common ratio  $R$  of a geometric sequence are 3 and 2 respectively.

- (a) Find the general term  $T(n)$  of the sequence.
- (b) If  $T(k) = 768$ , find the value of  $k$ .

已知一個等比數列的首項  $a$  是 3 而公比  $R$  是 2。

- (a) 求該數列的通項  $T(n)$ 。
- (b) 若  $T(k) = 768$ ，求  $k$  的值。

11. It is given that the first term  $a$  and the common ratio  $R$  of a geometric sequence are 2 and  $-4$  respectively.

- (a) Find the general term  $T(n)$  of the sequence.
- (b) Hence, find the 4th term and the 7th term of the sequence.

已知一個等比數列的首項  $a$  是 2 而公比  $R$  是  $-4$ 。

- (a) 求該數列的通項  $T(n)$ 。
- (b) 由此，求該數列的第 4 項和第 7 項。

12. Find the general term and the 12th term of the geometric sequence  $27, -9, 3, -1, \dots$

求等比數列  $27, -9, 3, -1, \dots$  的通項和第 12 項。

13. (a) Find the sum of the first 10 terms of an arithmetic series if its first term is 5 and the 10th term is 95.

(b) Find the sum of the first 12 terms of an arithmetic series if its first term is  $-31$  and the common difference is 3.

- (a) 若一個等差級數的首項是 5 而第 10 項是 95，求該等差級數首 10 項之和。
- (b) 若一個等差級數的首項是  $-31$  而公差是 3，求該等差級數首 12 項之和。

14. (a) Insert two geometric means between 54 and  $-128$ .

(b) Insert three geometric means between  $\frac{1}{2}$  and 8.

(a) 在 54 與  $-128$  之間插入兩個等比中項。

(b) 在  $\frac{1}{2}$  與 8 之間插入三個等比中項。

15. If the geometric mean between  $k$  and  $3k + 2$  is  $2k + 3$ , find the value(s) of  $k$ .

若  $2k + 3$  是  $k$  與  $3k + 2$  的等比中項，求  $k$  的值。

16. Given an arithmetic series  $(-17) + (-13) + (-9) + \dots$ ,

(a) find the 15th terms,

(b) find the sum of the first 15 terms,

(c) find the sum of the first 30 terms,

(d) hence, find the sum from the 16th term to the 30th term.

已知一個等差級數：

$(-17) + (-13) + (-9) + \dots$

(a) 求第 15 項。

(b) 求首 15 項之和。

(c) 求首 30 項之和。

(d) 由此，求第 16 項至第 30 項之和。

17. (a) Find the sum of the arithmetic series  $3 + 12 + 21 + \dots + 147$ .

(b) How many terms of the arithmetic series  $1 + 3 + 5 + \dots$  must be taken to give a sum of 2025?

(a) 求等差級數  $3 + 12 + 21 + \dots + 147$  各項之和。

(b) 對於等差級數  $1 + 3 + 5 + \dots$ ，問需要取多少項才可使級數之和等於 2025？

$$a = 1, d = 3 - 1 = 2, S(N) = 2025$$

18. Find the least number of terms of the geometric series  $3 + 15 + 75 + \dots$  that can give a sum greater than 5000.

對於等比級數  $3 + 15 + 75 + \dots$ ，問需要取多少項才可使級數之和大於 5000？

19. (a) Find the sum of the first 6 terms of a geometric series if its first term is 27 and the common ratio is  $\frac{1}{3}$ .

(b) Find the sum of the first 8 terms of a geometric series if its first term is 3 and the common ratio is 2.

(a) 若一個等比級數的首項是 27 而公比是  $\frac{1}{3}$ ，求該級數首 6 項之和。

(b) 若一個等比級數的首項是 3 而公比是 2，求該級數首 8 項之和。

20. (a) Find the sum of the terms of the geometric series  $2 + (-4) + 8 + (-16) + \dots + 128$ .

(b) How many terms of the series in (a) must be taken to give a sum of 342?

(a) 求等比級數  $2 + (-4) + 8 + (-16) + \dots + 128$  各項之和。

(b) 對於(a) 的等比級數，問需要取多少項才可使級數之和等於 342？

21. Find the sum to infinity of the following geometric series.

求下列等比級數的無限項之和。

(a)  $4 + 2 + 1 + \frac{1}{2} + \dots$

(b)  $9 - 3 + 1 - \frac{1}{3} + \dots$

22. In 15 years' time, Mr. So would like to save \$1 500 000 for his retirement. If he deposits a fixed amount of money at the beginning of each year at a fixed interest rate of 6% p.a., compounded yearly, how much should he deposit at the beginning of the year?

(Give your answer correct to the nearest \$10.)

蘇先生打算在 15 年內儲蓄\$1 500 000 作退休之用。若他在每年年初把一筆固定的款項存入銀行，年利率為 6%，且每年計算複利息一次，問他每年年初應把多少款項存入銀行？

(答案須準確至最接近的\$10。)

23. Express the following recurring decimals as fractions.

把下列各循環小數化為分數。

(a)  $0.\dot{4}$

(b)  $0.3\dot{7}\dot{5}$

(c)  $4.\dot{1}2\dot{5}$

24. In the figure,  $A_1B_1C_1D_1$  is a square of side 8 cm. A second square  $A_2B_2C_2D_2$  is formed by joining the mid-points of the sides of  $A_1B_1C_1D_1$  and this process is continued to form an infinite number of squares.

(a) Find

(i) the perimeters,

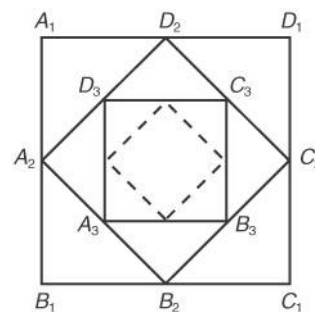
(ii) the areas

of  $A_1B_1C_1D_1$  and  $A_2B_2C_2D_2$ .

(b) Find the sum of

(i) the perimeters,

(ii) the areas



of all the squares formed (i.e.  $A_1B_1C_1D_1, A_2B_2C_2D_2, A_3B_3C_3D_3, \dots$ ).

在圖中， $A_1B_1C_1D_1$  是一個邊長為 8 cm 的正方形。連接  $A_1B_1C_1D_1$  各邊的中點後形成了第二個正方形  $A_2B_2C_2D_2$ 。在不斷重複這個過程下，會形成無限多個正方形。

(a) 求  $A_1B_1C_1D_1$  和  $A_2B_2C_2D_2$  的

(i) 周界；

(ii) 面積。

(b) 求所有正方形 (即  $A_1B_1C_1D_1, A_2B_2C_2D_2, A_3B_3C_3D_3, \dots$ ) 的

(i) 周界之和；

(ii) 面積之和。

25. How many positive terms are there in the arithmetic sequence 178, 175, 172, ...?

問等差數列 178, 175, 172, ... 有多少個正數項？

26. (a) Find the sum of the terms of the arithmetic series  $59 + 54 + 49 + \dots + 4$ .

(b) What is the maximum number of terms of the arithmetic series  $59 + 54 + 49 + \dots$  that can give a positive sum?

(a) 求等差級數  $59 + 54 + 49 + \dots + 4$  各項之和。

(b) 對於等差級數  $59 + 54 + 49 + \dots$ ，問最多可取多少項，使級數之和仍是正數？

27. It is given that the first term of a geometric sequence is 270 and its common ratio is positive. If  $T(3):T(7)=3:5$ , find  $T(13)$ .

已知一個等比數列的首項是 270，而公比是一個正數。若  $T(3):T(7)=3:5$ ，求  $T(13)$ 。

28. If  $p$  and  $q$  are the two geometric means between 2 and 50, find the value of  $\log p + \log q$ .

若  $p$  和  $q$  是 2 與 50 之間的兩個等比中項，求  $\log p + \log q$  的值。

29. If  $x$  is the geometric mean between  $\frac{a}{b}$  and  $\frac{b}{a}$ , find the common ratio of the geometric sequence formed.

若  $x$  是  $\frac{a}{b}$  與  $\frac{b}{a}$  的等比中項，求所形成的等比數列的公比。

30. A student is asked to find the sum to infinity of the geometric series

$6 + 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \dots$ , but he just adds up the first 6 terms to find the answer. Find the percentage error in his answer.

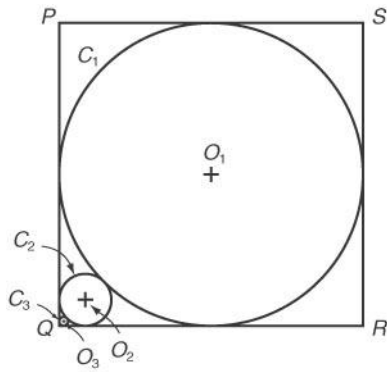
某學生需要求出  $6 + 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \dots$  的無限項之和，但他只把該級數的首 6 項加起來作為答案。求他的答案的百分誤差。

31. It is given that the sum of the first 3 terms and the sum of the first 6 terms of a geometric series are 6 and  $-42$  respectively. Find the sum of the first 10 terms of the series.

已知一個等比級數首 3 項之和是 6，而首 6 項之和是  $-42$ 。求該級數首 10 項之和。

31. In the figure,  $C_1$  is a circle with centre  $O_1$  inscribed in the square  $PQRS$  of side 4 cm;  $C_2$  is a circle with centre  $O_2$  which touches  $C_1$  externally and touches two sides of the square.  $C_3$  is a circle with centre  $O_3$  which touches  $C_2$  externally and touches two sides of the square. Following the pattern, an infinite number of circles  $C_4, C_5, C_6, \dots$  with centres  $O_4, O_5, O_6, \dots$  are formed. Let  $r_i$  be the radius of the circle  $C_i$  where  $i = 1, 2, 3, 4, \dots$

在圖中， $C_1$  是一個圓心為  $O_1$  的圓，它內切於一個邊長 4 cm 的正方形  $PQRS$ ； $C_2$  是一個以  $O_2$  為圓心的圓，它與  $C_1$  外切，且與正方形的兩邊相切； $C_3$  是一個以  $O_3$  為圓心的圓，它與  $C_2$  外切，且與正方形的兩邊相切。在不斷重複這個過程下，形成了無限個圓  $C_4, C_5, C_6, \dots$ ，它們的圓心分別為  $O_4, O_5, O_6, \dots$ 。設圓  $C_i$  的半徑為  $r_i$ ，其中  $i = 1, 2, 3, 4, \dots$ 。



(a) (i) Find  $r_1$  and  $r_2$

(ii) Hence, find the ratio  $\frac{r_2}{r_1}$ .

(b) Find the sum of the areas of all the circles formed (i.e.  $C_1, C_2, C_3, C_4, \dots$ ).

(Give your answer correct to 4 significant figures.)

(a) (i) 求  $r_1$  和  $r_2$ 。

- (ii) 由此，求  $\frac{r_2}{r_1}$  這個比。
- (b) 求所有圓（即圓  $C_1$ 、 $C_2$ 、 $C_3$ 、 $C_4$ 、...）的面積之和。  
（答案須準確至四位有效數字。）



## Pre-requisite Questions

## 預備測驗

1. If  $f(x) = x^2 - 2x - 1$ , find the values of the function when

若  $f(x) = x^2 - 2x - 1$ ，求在下列各情況中  $f(x)$  的值。

- (a)  $x = -3$ ,  
(b)  $x = 2$ .

2. If  $h(x) = (x + k)(2x - 3)$  and  $h(2) = -3$ , find the values of

若  $h(x) = (x + k)(2x - 3)$  及  $h(2) = -3$ ，求下列各題的值。

- (a)  $k$ ,  
(b)  $h(-5)$ .

3. If  $g(t) = \frac{t}{1+t^2}$ , find the values of the function when

若  $g(t) = \frac{t}{1+t^2}$ ，求在下列各情況中  $g(t)$  的值。

- (a)  $t = 2$ ,  
(b)  $t = \frac{1}{2}$ .

4. If  $f(x) = 2x - 3$  and  $g(x) = 3x^2 + 1$ , find the values of

若  $f(x) = 2x - 3$  及  $g(x) = 3x^2 + 1$ ，求下列各題的值。

- (a)  $f(0) \bullet g(-1)$   
(b)  $3f(4) - g(1)$

5. If  $f(x) = \sin x$  and  $g(x) = \cos x$ , find the values of

若  $f(x) = \sin x$  及  $g(x) = \cos x$ ，求下列各題的值。

- (a)  $g(30^\circ) + f(45^\circ)$   
(b)  $\frac{[f(30^\circ)]^2}{g(60^\circ)}$

6. It is given that  $h(x) = x^2 - 3x + 1$ . If  $g(x) = h(x + 2)$ , write down the symbolic representation of  $g(x)$ .

已知  $h(x) = x^2 - 3x + 1$ 。若  $g(x) = h(x + 2)$ ，試以符號形式表示  $g(x)$ 。

7. If  $f(t) = 2t^2 - mt + 9$  and  $f(-3) = 0$ , find the values of  
若  $f(t) = 2t^2 - mt + 9$  及  $f(-3) = 0$ , 求下列各題的值。  
(a)  $m$ ,  
(b)  $f(1)$ .
8. It is given that  $f(x) = a(x-3) + b$ . If  $f(0) = 1$  and  $f(1) = 0$ , find the values of  $a$  and  $b$ .  
已知  $f(x) = a(x-3) + b$ 。若  $f(0) = 1$  及  $f(1) = 0$ , 求  $a$  和  $b$  的值。
9. It is given that  $h(x) = (x-1)(x+2) - kx$  and  $h(k) = 3k$ .  
已知  $h(x) = (x-1)(x+2) - kx$  及  $h(k) = 3k$ 。  
(a) Find the value of  $k$ .  
(b) Solve for  $x$  if  $h(x) = 2x$ .
10. It is given that  $H(s) = s^2 + 2s$ . If  $V(s) = H(2s-1)$ , write down the symbolic representation of  $V(s)$ .  
已知  $H(s) = s^2 + 2s$ 。若  $V(s) = H(2s-1)$ , 試以符號形式表示  $V(s)$ 。
11. If  $s(t) = (t-4)^2 + 2(t+5)$ , find  $s(2t-3)$ .  
若  $s(t) = (t-4)^2 + 2(t+5)$ , 求  $s(2t-3)$  的值。
12. If  $f(x) = 3x-1$  and  $g(x) = -2x+5$ , find  
若  $f(x) = 3x-1$  及  $g(x) = -2x+5$ , 求  
(a)  $f(x) + g(x)$ ,  
(b)  $[f(x) + g(x)]^2 - g(x)$ .

## Level 1 Questions

## 程度 1 題目

1. Consider the sequence:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$$

- (a) Write down the next two terms of the sequence.
- (b) Determine the general term  $T(n)$  of the sequence.
- (c) Find the 9th term and 10th term of the sequence.

考慮以下的數列:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$$

- (a) 寫出接著的兩項。
- (b) 寫出通項  $T(n)$ 。
- (c) 求該數列的第 9 項和第 10 項。

2. Consider the sequence:

$$2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \dots$$

- (a) Determine the general term  $T(n)$  of the sequence,
- (b) Find the 6th term and 7th term of the sequence.

考慮以下的數列:

$$2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \dots$$

- (a) 寫出通項  $T(n)$ ;
- (b) 求第 6 項和第 7 項。

3. Consider the sequence:

$$1, 4, 9, 16, 25, \dots$$

- (a) Write down the next two terms of the sequence.
- (b) Determine the general term  $T(n)$  of the sequence.
- (c) Find the 7th term and 9th term of the sequence.

考慮以下的數列:

$$1, 4, 9, 16, 25, \dots$$

- (a) 寫出接著的兩項。
- (b) 寫出通項  $T(n)$ 。
- (c) 求該數列的第 7 項和第 9 項。

4. Consider the following sequence:

2, 6, 12, 20, ...

- (a) Determine the general term  $T(n)$  of the sequence.  
(b) Write down the 7th term and the 10th term of the sequence.

考慮以下的數列:

2, 6, 12, 20, ...

- (a) 寫出通項  $T(n)$ ;  
(b) 求第 7 項和第 10 項。

5. Consider the sequence:

0,  $\log 3$ ,  $\log 9$ ,  $\log 27$ , ...

- (a) Determine the general term  $T(n)$  of the sequence,  
(b) Find the 6th term and 7th term of the sequence.

考慮以下的數列:

0,  $\log 3$ ,  $\log 9$ ,  $\log 27$ , ...

- (a) 寫出通項  $T(n)$ ;  
(b) 求第 6 項和第 7 項。

6. It is given that the first term and the common difference of an arithmetic sequence are 3 and  $-4$  respectively. Find

- (a) the general term  $T(n)$  of the sequence,  
(b) the 6th term and the 36th term of the sequence.

已知一個等差數列的首項是 3 而公差是  $-4$ 。求

- (a) 通項  $T(n)$ ;  
(b) 第 6 項和第 36 項。

7. Consider the arithmetic sequence:

5, 13, 21, 29, ...

- (a) Find the general term of the sequence.  
(b) If the  $k$ th term of the sequence is 101, find  $k$ .

考慮下列的等差數列:

5, 13, 21, 29, ...

- (a) 求該數列的通項。  
(b) 若該數列的第  $k$  項是 101，求  $k$  的值。

8. It is given that the first term and the common difference of an arithmetic sequence are  $-5$  and  $4$  respectively. Find

- (a) the general term  $T(n)$  of the sequence,
- (b) the 5th term and the 20th term of the sequence.

已知一個等差數列的首項是 $-5$ 而公差是 $4$ 。求

- (a) 通項  $T(n)$ ;
- (b) 第 5 項和第 20 項。

9. It is given that the first term and the common difference of an arithmetic sequence are  $15$  and  $-3$  respectively. Find

- (a) the general term  $T(n)$  of the sequence,
- (b) the 6th term and the 33rd term of the sequence.

已知一個等差數列的首項是 $15$ 而公差是 $-3$ 。求

- (a) 通項  $T(n)$ ;
- (b) 第 6 項和第 33 項。

10. Consider the arithmetic sequence:

$-39, -31, -23, -15, \dots$

- (a) Find the general term of the sequence.
- (b) If the  $k$ th term of the sequence is  $25$ , find  $k$ .

考慮下列的等差數列:

$-39, -31, -23, -15, \dots$

- (a) 求該數列的通項。
- (b) 若該數列的第  $k$  項是  $25$ ，求  $k$  的值。

11. For an arithmetic sequence, it is given that  $T(11) = 35$  and  $T(18) = 56$ , find

- (a) the general term of the sequence,
- (b) the third term of the sequence.

在一個等差數列中，已知  $T(11) = 35$  及  $T(18) = 56$ ，求該數列的

- (a) 通項;
- (b) 第三項。

12. Find the number of terms in each of the following arithmetic sequences.

求以下各等差數列的項數。

(a)  $108, 101, 94, \dots, -25$

(b)  $33, 37, 41, \dots, 373$

13. For an arithmetic sequence, it is given that  $T(2) = 8$  and  $T(5) = 23$ , find the general term of the sequence.

在一個等差數列中，已知  $T(2) = 8$  及  $T(5) = 23$ ，求該數列的通項。

14. If the arithmetic mean between 9 and  $p$  is 16, find the value of  $p$ .

若 9 與  $p$  的等差中項是 16，求  $p$  的值。

15. If  $x + 1, 3x - 2, 2x + 4$  are in arithmetic sequence, find the value of  $x$ .

若  $x + 1, 3x - 2, 2x + 4$  是一個等差數列，求  $x$  的值。

16. Given two numbers 8 and 50.

(a) insert two arithmetic means between these two numbers,

(b) insert six arithmetic means between these two numbers.

在 8 與 50 兩個數之間插入

(a) 兩個等差中項;

(b) 六個等差中項。

17. Given two numbers  $-84$  and  $-14$ ,

(a) insert four arithmetic means between these two numbers,

(b) insert six arithmetic means between these two numbers.

在  $-84$  與  $-14$  兩個數之間插入

(a) 四個等差中項;

(b) 六個等差中項。

18. It is given that the first term and the common ratio of a geometric sequence are 2 and  $\frac{2}{3}$  respectively.

Find

- (a) the general term  $T(n)$  of the sequence,
- (b) the 3rd term and the 5th term of the sequence.

已知一個等比數列的首項是 2 而公比是  $\frac{2}{3}$ 。求該數列的

- (a) 通項  $T(n)$ ;
- (b) 第 3 項和第 5 項。

19. If 4,  $p$ ,  $q$ ,  $r$ , 16 are in arithmetic sequence, find the values of  $p$ ,  $q$  and  $r$ .

若 4,  $p$ ,  $q$ ,  $r$ , 16 是一個等差數列，求  $p$ 、 $q$  和  $r$  的值。

20. It is given that the first term and the common ratio of a geometric sequence are  $\frac{3}{4}$  and  $-\frac{1}{3}$

respectively. Find

- (a) the general term  $T(n)$  of the sequence,
- (b) the 4th term and the 5th term of the sequence.

已知一個等比數列的首項是  $\frac{3}{4}$  而公比是  $-\frac{1}{3}$ 。求該數列的

- (a) 通項  $T(n)$ ;
- (b) 第 4 項和第 5 項。

21. It is given that the first term and the common ratio of a geometric sequence are 8 and 2 respectively.

Find

- (a) the general term  $T(n)$  of the sequence,
- (b) the 4th term and the 7th term of the sequence.

已知一個等比數列的首項是 8 而公比是 2。求該數列的

- (a) 通項  $T(n)$ ;
- (b) 第 4 項和第 7 項。

22. It is given that the first term and the common ratio of a geometric sequence are 5 and  $-3$  respectively.

- (a) Find the general term  $T(n)$  of the sequence.
- (b) If  $T(k) = -135$ , find the value of  $k$ .

已知一個等比數列的首項是 5 而公比是  $-3$ 。求該數列的

- (a) 通項  $T(n)$ ;
- (b) 若  $T(k) = -135$ ，求  $k$  的值。

23. For a geometric sequence, it is given that  $T(3) = \frac{1}{6}$  and  $T(6) = \frac{9}{16}$ . Find

- (a) the first term and the common ratio,  
(b) the general term of the sequence.

在一個等比數列中，已知  $T(3) = \frac{1}{6}$  及  $T(6) = \frac{9}{16}$ 。求

- (a) 首項和公比；  
(b) 該數列的通項。

24. Find the number of terms in each of the following geometric sequences.

. 求以下各等比數列的項數。

- (a) 3, 6, 12, ..., 768

- (b)  $\frac{1}{5}, -\frac{3}{20}, \frac{9}{80}, \dots, \frac{729}{20480}$

- 25 Consider the geometric sequence,

$$6, -\frac{9}{2}, \frac{27}{8}, -\frac{81}{32}, \dots$$

- (a) Find the general term of the sequence.  
(b) Find the 8th term of the sequence.

. 考慮以下的等比數列：

$$6, -\frac{9}{2}, \frac{27}{8}, -\frac{81}{32}, \dots$$

- (a) 求該數列的通項。  
(b) 求該數列的第 8 項。

- 26 For a geometric sequence with positive common ratio, it is given that  $T(4) = 5$  and  $T(6) = 245$ .

Find

- (a) the first term and the common ratio,  
(b) the general term of the sequence,  
(c) the value of  $\frac{T(5) - T(4)}{T(3)}$ .

在一個公比為正數的等比數列中，已知  $T(4) = 5$  及  $T(6) = 245$ 。求

- (a) 首項和公比；  
(b) 該數列的通項；  
(c)  $\frac{T(5) - T(4)}{T(3)}$  的值。



27. It is given that the first term of a geometric sequence is 102 and its common ratio is positive. If  $T(6):T(11)=3:2$ , find  $T(11)$ .

已知一個公比為正數的等比數列的首項是 102。若  $T(6):T(11)=3:2$ ，求  $T(11)$ 。

28. If  $27, a, b, \frac{1}{27}$  are in geometric sequence, find the values of  $a$  and  $b$ .

若  $27, a, b, \frac{1}{27}$  是一個等比數列，求  $a$  和  $b$  的值。

29. If the geometric mean between  $\frac{x-1}{2}$  and  $3x+2$  is  $x$ , find the values of  $x$ .

若  $x$  是  $\frac{x-1}{2}$  與  $3x+2$  的等比中項，求  $x$  的值。

30. If  $2x-1, x, x-2$  are in geometric sequence, find the values of  $x$  (Leave your answer in surd form).

若  $2x-1, x, x-2$  是一個等比數列，求  $x$  的值。（答案以根式表示。）

31. Insert three geometric means between 1875 and 48.

在 1875 與 48 之間插入三個等比中項。

32. Insert four geometric means between 6 and  $-\frac{2}{81}$ .

在 6 與  $-\frac{2}{81}$  之間插入四個等比中項。

33. Find the sum of the first 16 terms of an arithmetic series if its second term is  $-8$  and the 8th term is 40.

若一個等差級數的第二項是  $-8$  而第 8 項是 40，求該等差級數首 16 項之和。

34. Find the sum of the first 15 terms of an arithmetic series if its first term is 4 and the 15th term is 26.

若一個等差級數的首項是 4 而第 15 項是 26，求該等差級數首 15 項之和。

35. Find the sum of the first 16 terms of an arithmetic series if its third term is 28 and the common difference is  $-9$ .

若一個等差級數的第三項是 28 而公差是  $-9$ ，求該等差級數首 16 項之和。

36. Find the sum of the 29 arithmetic means between  $-30$  and 90.

求在  $-30$  與 90 之間的 29 個等差中項之和。

37. (a) Find the sum of the integers between 1 and 150 inclusive.  
 (b) Find the sum of the integers between 1 and 150 that are multiples of 7.  
 (c) Find the sum of the integers between 1 and 150 that are not multiples of 7.  
 (a) 求在 1 與 150 之間(包括 1 和 150)所有整數之和。  
 (b) 在 1 與 150 之間所有 7 的倍數之和。  
 (c) 求在 1 與 150 之間(包括 1 和 150)所有不能被 7 整除的整數之和。

38. Consider the arithmetic series  $-123-134-145-\dots-343$ .

- (a) Find the number of terms of the series.  
 (b) Find the sum of the series.

考慮等差級數  $-123-134-145-\dots-343$ 。

- (a) 求該級數的項數。  
 (b) 求該級數之和。

39. The sum of the first 8 terms of an arithmetic series is 212. If the 21st term is  $-23$ , find

- (a) the first term and the common difference of the series  
 (b) the sum of the first 15 terms of the series.

已知一個等差級數首 8 項之和是 212。若第 21 項是  $-23$ ，求

- (a) 該級數的首項和公差;  
 (b) 該級數首 15 項之和。

40. Find the sum of the geometric series  $-\frac{1}{81} + \frac{1}{27} - \frac{1}{9} + \dots + 243$ .

求等比級數  $-\frac{1}{81} + \frac{1}{27} - \frac{1}{9} + \dots + 243$  各項之和。

41. Find the sum of the first 3 terms of a geometric series if its first term is 49 and the common ratio is

$$-\frac{1}{7}.$$

若一個等比級數的首項是 49 而公比是  $-\frac{1}{7}$ ，求該級數首 3 項之和。

42. Find the sum of the first 10 terms of a geometric series if its first term is  $-5$  and the common ratio is 3.

若一個等比級數的首項是  $-5$  而公比是 3，求該級數首 10 項之和。

- 43.** Find the sum of first 5 terms of a geometric series if its second term is 6 and the common ratio is  $\frac{2}{3}$ .

若一個等比級數的第二項是 6 而公比是  $\frac{2}{3}$ ，求該級數首 5 項之和。

- 44.** The sum of the first two terms of a geometric series is 8, and the sum from the 3rd term to 4th term is  $\frac{8}{9}$ . Find the first term and the common ratio of the series.

已知一個等比級數首兩項之和是 8，而第 3 項至第 4 項之和是  $\frac{8}{9}$ 。求該級數的首項和公比。

- 45.** Find the sum of the geometric series  $8 + 4 + 2 + \dots + \frac{1}{16}$ .

求等比級數  $8 + 4 + 2 + \dots + \frac{1}{16}$  各項之和。

- 46.** Express the following recurring decimals as fractions.

把下列各循環小數化為分數。

- (a)  $0.\dot{5}$
- (b)  $0.0\dot{1}\dot{6}$
- (c)  $2.\dot{2}2\dot{5}$

- 47.** Find the least number of terms of the geometric series  $6 + 18 + 54 + \dots$  that can give a sum greater than 5000.

對於等比級數  $6 + 18 + 54 + \dots$ ，問最少需要取多少項才可使級數之和大於 5000？

- 48.** Find the sum to infinity of the following geometric series.

求下列等比級數的無限項之和。

- (a)  $27 + 18 + 12 + 8 + \dots$
- (b)  $64 - 16 + 4 - 1 + \dots$

- 49.** Given an geometric sequence  $24, -4, \frac{2}{3}, -\frac{1}{9}, \dots$ , find the sum of all the positive terms of the sequence.

對於等比數列  $24, -4, \frac{2}{3}, -\frac{1}{9}, \dots$ ，求該數列的所有正數項之和。

50. Find the first term of an infinite geometric series if the second term is 8 and the sum to infinity is 32.

若一個無限等比級數的第 2 項是 8，而無限項之和是 32，求該無限等比級數的首項。

51 If the sum to infinity of a geometric series is 5 and the common ratio of the series is  $\frac{1}{3}$ . Find

(a) the 5th term of the series,

(b) the sum of the first three terms of the series.

若一個等比級數的無限項之和是 5 而公比是  $\frac{1}{3}$ 。求該級數的

(a) 第 5 項;

(b) 首三項之和。

## Level 2 Questions

## 程度 2 題目

1. How many negative terms are there in the arithmetic sequence  $-203, -198, -193, \dots$ ?

問等差數列  $-203, -198, -193, \dots$  共有多少個負數項?

2. It is given that  $\sqrt{3}, \sqrt{12}, \sqrt{p}, \dots$  are in arithmetic sequence. Find

- (a) the value of  $p$ ,  
(b) the sixth term of the sequence.

已知  $\sqrt{3}, \sqrt{12}, \sqrt{p}, \dots$  是一個等差數列。求

- (a)  $p$  的值;  
(b) 該數列的第 6 項。

3. Given an arithmetic sequence  $88, 79, 70, \dots$ , if  $T(k)$  is the first negative term, find the value of  $k$ .

已知一個等差數列  $88, 79, 70, \dots$ 。若  $T(k)$  是該數列的首個負數項，求  $k$  的值。

4. For an arithmetic sequence, the 8th term is 17 times that of the 2nd term, and the 6th term is greater than 9 times that of the 2nd term by 8. Find

- (a) the general term of the arithmetic sequence,  
(b) the value of  $k$  if  $T(2k) - 2T(3) = 29$ .

已知一個等差數列的第 8 項是第 2 項的 17 倍，而數列的第 6 項又比第 2 項的 9 倍大 8。

- (a) 求該等差數列的通項。  
(b) 若  $T(2k) - 2T(3) = 29$ ，求  $k$  的值。

5. How many positive terms are there in the arithmetic sequence  $230, 212, 194, \dots$ ?

問等差數列  $230, 212, 194, \dots$  共有多少個正數項?

6. All interior angles of a convex quadrilateral form an arithmetic sequence and the smallest interior angle of the quadrilateral is  $60^\circ$ . Find the other three interior angles of the quadrilateral.

已知一個凸四邊形的所有內角形成一個等差數列，而最小的內角是  $60^\circ$ 。求該四邊形其餘的內角。

7. For an arithmetic sequence, it is given that  $T(1) = 8$  and  $T(5) = 3T(2)$ . Find

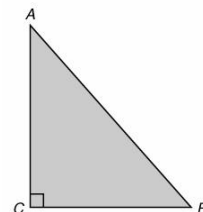
- (a) the general term of the sequence,  
(b) the 15th term of the sequence.

對於一個等差數列，已知  $T(1) = 8$  及  $T(5) = 3T(2)$ 。求該數列的

- (a) 通項；  
(b) 第 15 項。

8. In the figure,  $\triangle ABC$  is a right-angled triangle where  $AC > BC$  and  $\angle C = 90^\circ$ .  
If the lengths of  $BC, AC, AB$  are in arithmetic sequence, find  $AB : AC : BC$ .

在圖中， $\triangle ABC$  是一個直角三角形，其中  $AC > BC$  及  $\angle C = 90^\circ$ 。若  $BC$ 、 $AC$ 、 $AB$  的長度是一個等差數列，求  $AB : AC : BC$ 。



9. For an arithmetic sequence, it is given that  $T(8) = 15$  and  $T(5) + T(17) = 42$ . Find

- (a) the first term and the common difference of the sequence.  
(b) the general term of the sequence.  
(c) the smallest term of the sequence that is greater than 1000.

對於一個等差數列，已知  $T(8) = 15$  及  $T(5) + T(17) = 42$ 。求

- (a) 該數列的首項和公差；  
(b) 該數列的通項；  
(c) 該數列中僅大於 1000 的項。

10. Insert three arithmetic means, in terms of  $a$ , between  $a + 3$  and  $a^2 + 1$ ,

在  $a + 3$  與  $a^2 + 1$  之間插入三個等差中項，答案以  $a$  表示。

11. If  $x$  is the arithmetic mean between  $-7$  and  $y$ , while  $y$  is the arithmetic mean between  $x + 9$  and  $14$ , find the value of  $x$  and  $y$ .

若  $-7$  與  $y$  的等差中項是  $x$ ，而  $x + 9$  與  $14$  的等差中項是  $y$ ，求  $x$  和  $y$  的值。

$$\begin{aligned} y &= 2(3) + 7 \\ &= \underline{13} \end{aligned}$$

12. Insert four arithmetic means, in terms of  $m$ , between  $-3$  and  $2m$ .

在  $-3$  與  $2m$  之間插入四個等差中項，答案以  $m$  表示。

- 13.** Find the number of terms in the geometric sequence  
 $2, 10, 50, 250, \dots$   
 which is smaller than 5000.  
 求在等比數列  $2, 10, 50, 250, \dots$  中小於 5000 的項數。
- 14.** If  $\log(x+2)$  is the arithmetic mean between 1 and  $\log(x+3.1)$ , find the value of  $x$ .  
 若 1 與  $\log(x+3.1)$  的等差中項是  $\log(x+2)$ ，求  $x$  的值。
- 15.** If  $\sin \theta$  is the arithmetic mean between  $3\cos \theta$  and  $-\sin \theta$ , find  $\theta$  for  $0^\circ \leq \theta \leq 360^\circ$ .  
 若  $3\cos \theta$  與  $-\sin \theta$  的等差中項是  $\sin \theta$ ，其中  $0^\circ \leq \theta \leq 360^\circ$ ，求  $\theta$  的值。
- 16.** It is given that  $T(1), T(2), T(3), \dots, T(n), \dots$  are in geometric sequence. Show that the sequence  
 $2T(1), 2^2T(2), 2^3T(3), \dots, 2^nT(n), \dots$  is a geometric sequence.  
 已知  $T(1), T(2), T(3), \dots, T(n), \dots$  是一個等比數列。證明  $2T(1), 2^2T(2), 2^3T(3), \dots, 2^nT(n), \dots$  也是一個等比數列。
- 17.** The lengths of sides of  $\triangle ABC$  are in geometric sequence and the perimeter of the triangle is 38 cm.  
 If the shortest side of the triangle is  $\frac{4}{9}$  of its longest side, find  
 (a) the lengths of the three sides of the triangle,  
 (b) the area of the triangle.  
 $\triangle ABC$  的邊長是一個等比數列且其周界為 38 cm。若該三角形最短的邊的邊長是最長的邊的  $\frac{4}{9}$ ，求  
 (a) 該三角形三條邊的長度;  
 (b) 該三角形的面積。
- 18.** Find the number of terms in the geometric sequence  
 $800, 600, 450, \dots$   
 which is greater than 60.  
 求在等比數列  $800, 600, 450, \dots$  中大於 60 的項數。

19. It is given that the three interior angles of a triangle are in geometric sequence. If the smallest interior angle is one-fourth of the largest interior angle, find all the interior angles of the triangle.

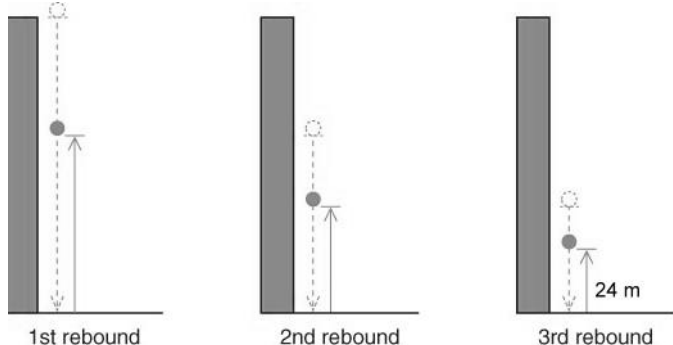
(Give your answers correct to 2 decimal places.)

已知一個三角形的三個內角形成一個等比數列。若最小的內角是最大的內角的四分之一，求該三角形的所有內角。

(答案須準確至二位小數。)

20. A ball is dropped vertically from the top of a tower. Each time it reaches the ground, it rebounds to a height which is  $\frac{2}{3}$  of its previous height. If the ball reaches a height of 24 m after the third rebound, find the height of the tower.

- 把一個皮球從一幢大廈的頂層垂直掉下來。當皮球到達地面時，它會垂直向上反彈至原來高度的  $\frac{2}{3}$ 。若皮球第 3 次反彈後所達到的高度是 24m，求該大廈的高度。



21. The value of a car is \$270 000 in the year 2005 and it is known that the value of the car decreases by 9% each year.

(a) Find the value of the car in the year 2017. (Give your answer correct to the nearest dollar.)

(b) When will the value of car half of that in the year 2005?

某私家車在 2005 年的售價是 \$270 000。已知該輛私家車的售價每年都減少 9 %。

(a) 求該輛私家車在 2017 年的售價。(答案須準確至最接近的元。)

(b) 問該輛私家車在哪一年的售價將會減至 2005 年時的售價的一半？

22. If  $p$ ,  $q$ ,  $r$  and  $s$  are the four geometric means between 2 and 50, find the value of  $\log p + \log q + \log r + \log s$ .

若  $p$ ,  $q$ ,  $r$  和  $s$  是 2 與 50 之間的四個等比中項，求  $\log p + \log q + \log r + \log s$  的值。



23. In a certain experiment, a kind of algae is placed in a pool. The area covered by the algae is doubled at every three hours. If it will take 18 hours to cover the pool, how long will it take for the algae to cover  $\frac{1}{3}$  of the pool? (Give your answer correct to 2 decimal places.)

在一次實驗中，某類海藻被放置於水池。每隔三小時，水池被海藻覆蓋的面積便增加一倍，而 18 小時後，整個水池的表面被海藻完全覆蓋。問需要多少時間，該海藻才可覆蓋  $\frac{1}{3}$  個水池？  
(答案須準確至二位小數)

24. (a) For  $b > 0$ , insert three geometric means, in terms of  $b$ , between  $\frac{1}{25b^2}$  and  $25b^2$ .

(b) If the general term of geometric sequence  $\dots, \frac{1}{25b^2}, x, y, z, 25b^2, \dots$  is  $3^{n-3}$ , find the value of  $b$ .

(a) 對於  $b > 0$ ，在  $\frac{1}{25b^2}$  與  $25b^2$  之間插入三個等比中項，答案以  $b$  表示。

(b) 若等比數列  $\dots, \frac{1}{25b^2}, x, y, z, 25b^2, \dots$  的通項是  $3^{n-3}$ ，求  $b$  的值。

25. How many terms must be taken from the arithmetic series  $47 + 43 + 39 + \dots$  to make a sum equals 24?

對於等差級數  $47 + 43 + 39 + \dots$ ，問需要取多少項才可使級數之和等於 24？

26. If  $\sin \theta$  is the geometric mean between  $\cos \theta$  and 1, find the value of  $\cos \theta$  for  $\theta$  is an acute angle.

若  $\sin \theta$  是  $\cos \theta$  和 1 的等比中項，其中  $\theta$  是一個銳角，求  $\cos \theta$  的值。

27. It is given that 15,  $x$ ,  $y$  are in arithmetic sequence and  $x$ ,  $y$ , 49 are in geometric sequence. Find the values of  $x$  and  $y$ .

已知 15,  $x$ ,  $y$  是一個等差數列，而  $x$ ,  $y$ , 49 是一個等比數列。求  $x$  和  $y$  的值。

28. (a) Find the sum of the terms of the arithmetic series  $-47 - 40 - 33 - \dots + 9$ .

(b) Find the maximum number of terms in the arithmetic series  $-47 - 40 - 33 - \dots$  that can give a negative sum.

(a) 求等差級數  $-47 - 40 - 33 - \dots + 9$  各項之和。

(b) 對於等差級數  $-47 - 40 - 33 - \dots$ ，問最多取多少項使到級數之和仍是負數？

29. It is given that  $1, k, \frac{1}{5}, \dots$  is a geometric sequence of positive real numbers.

- (a) Find the value of  $k$ . (Leave your answer in surd form.)
- (b) Find the general term of the sequence.
- (c) Find  $T(1) \times T(3) \times T(5) \times \dots \times T(2003)$ .

. 已知  $1, k, \frac{1}{5}, \dots$  是一個正實數的等比數列。

- (a) 求  $k$  的值。(答案以根式表示。)
- (b) 求該數列的通項。
- (c) 求  $T(1) \times T(3) \times T(5) \times \dots \times T(2003)$  的值。

30. The perimeter of an  $n$ -sided polygon is 160 cm. If the lengths of sides are arranged in an ascending order, they form an arithmetic sequence and the common difference is 4 cm. If the longest side of the polygon is 34 cm, find the number of sides of the polygon.

已知一個有  $n$  條邊的多邊形的周界是 160 cm。若將邊長由小至大排序，它們形成一個公差是 4 cm 的等差數列。若該多邊形最長的邊的邊長是 34 cm，求該多邊形的邊數。

31. The sum of the first  $n$  terms of an arithmetic series is  $7n^2 - 2n$ . Find the  $n$ th term of the series.

已知一個等差級數的首  $n$  項之和是  $7n^2 - 2n$ ，求該級數的第  $n$  項。

32. A boy saves \$1 on the first day, \$4 on the second day, \$7 on the third day and so on. If the boy saves \$590 in total on  $n$ th day, find  $n$ .

一名男孩在第一天儲蓄 \$1，第二天儲蓄 \$4，第三天儲蓄 \$7，餘此類推。若該男孩在第  $n$  天共儲存了 \$590，求  $n$  的值。

33. (a) Find the sum of the geometric series  $1 + 2 + 2^2 + \dots + 2^n$ .

(b) (i) By using the result of (a), simplify  $2 \times 2^2 \times 2^{2^2} \times 2^{2^3} \times \dots \times 2^{2^n}$ .

(ii) Find the least number of terms in  $2 \times 2^2 \times 2^{2^2} \times 2^{2^3} \times \dots \times 2^{2^n}$  such that its product is greater than 100 000 000.

(a) 求等比級數  $1 + 2 + 2^2 + \dots + 2^n$  各項之和。

(b) (i) 利用從(a)所得的結果，化簡  $2 \times 2^2 \times 2^{2^2} \times 2^{2^3} \times \dots \times 2^{2^n}$ 。

(ii) 問最少需要多少項才可使到  $2 \times 2^2 \times 2^{2^2} \times 2^{2^3} \times \dots \times 2^{2^n}$  的積大於 100 000 000？

34. It is given that  $a_1, a_2, \dots$  form a geometric sequence with  $a_2 - a_1 = 256$  and the sum to the infinity of the corresponding series is 144. Find the sum of all positive terms of the series.

已知  $a_1, a_2, \dots$  是一個等比數列，而  $a_2 - a_1 = 256$  及相應級數的無限項之和是 144。求該級數所有正數項之和。

35. It is given that the sum of the first 4 terms and the sum of the first 8 terms of a geometric series are 3 and 51 respectively. If the common ratio of the series is positive, find

- (a) the common ratio,  
(b) the sum of the first 10 terms of the series.

已知一個公比為正數的等比級數的首 4 項之和是 3，而首 8 項之和是 51。求

- (a) 公比;  
(b) 該級數首 10 項之和。

- 36 A car is now decelerating from 100 m/s to rest. It reduces half of its speed for every second. Find

- (a) the speed of the car in the 10th second,  
(b) the distance travelled in the 10th second,  
(c) the distance travelled before it comes to rest.

一輛汽車從 100 m/s 開始減速直至完全停下來。已知該輛汽車的速度每秒減少一半。求該輛汽車

- (a) 在第 10 秒的速度;  
(b) 在首 10 秒所行走的距離;  
(c) 在停下前所行走的距離。

37. The figure shows a right-angled triangle  $ABC$  with  $\angle ABC = 90^\circ$ .  $h_1, h_3, h_5, \dots$  are perpendiculars to  $AC$ ,  $h_2, h_4, h_6, \dots$  are perpendiculars to  $BC$ .

- (a) Show that  $h_1, h_2, h_3, \dots$  are in geometric sequence and express the common ratio in terms of  $\theta$ .

- (b) If  $AB = 20\text{ cm}$  and  $h_1 + h_2 + h_3 + \dots = 20(2\sqrt{3} + 3)\text{ cm}$ , find the value of  $\theta$ .

圖中所示為一個直角三角形  $ABC$ ，其中  $\angle ABC = 90^\circ$ ， $h_1, h_3, h_5, \dots$  是垂直於  $AC$  的線， $h_2, h_4, h_6, \dots$  是垂直於  $BC$  的線。

- (a) 證明  $h_1, h_2, h_3, \dots$  形成一個等比數列，並求該數列的公比，答案以  $\theta$  表示。  
(b) 若  $AB = 20\text{ cm}$  及  $h_1 + h_2 + h_3 + \dots = 20(2\sqrt{3} + 3)\text{ cm}$ ，求  $\theta$  的值。

38. Given an geometric series  $-64 + 16 + (-4) + \dots$

- (a) Find the sum to infinity of the series.
- (b) Find the sum of the first  $n$  terms of the series.
- (c) A student estimates the sum to infinity of the series by the answer in (b). If the relative error of the estimation is less than 0.0001, find the smallest value of  $n$ .

已知  $-64 + 16 + (-4) + \dots$  為一個等比級數。

- (a) 求該級數的無限項之和。
- (b) 求該級數的首  $n$  項之和。
- (c) 某學生利用 (b) 的結果來估算上列的等比級數的無限項之和。求  $n$  的最小值使估計值的相對誤差小於 0.0001。

39. A student is asked to find the sum to infinity of the geometric series  $12 + (-4) + \frac{4}{3} + (-\frac{4}{9}) + \dots$ . If he

just adds up the first 6 terms to find the answer, find the percentage error in his answer.

(Give your answer correct to 3 significant figures.)

- . 某學生需要求出級數  $12 + (-4) + \frac{4}{3} + \left(-\frac{4}{9}\right) + \dots$  的無限項之和，但他只把該級數的首 6 項加起來作為答案。求他的答案的百分誤差。(答案須準確至三位有效數字。)

## Level 2+ Questions

## 程度 2+ 題目

1. In the figure, an ant starts from  $O(0,0)$  and moves as follows:

- (i) Moving northwards and travels to  $A(0,3)$ .
- (ii) Moving eastwards and travels at a distance  $\frac{2}{3}$  of his northbound trip.
- (iii) Moving southwards and travels at a distance  $\frac{2}{3}$  of his eastbound trip.
- (iv) Moving westwards and travels at a distance  $\frac{2}{3}$  of his southbound trip.
- (v) Moving northwards and travels at a distance  $\frac{2}{3}$  of his westbound trip.

The ant repeats (ii) to (v) infinitely.

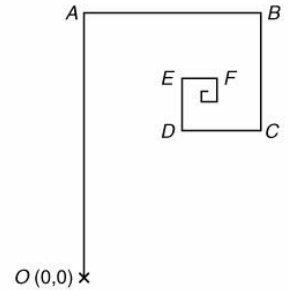
- (a) Find the total distance that the ant travels.
- (b) The ant finally reaches his nest. Find the coordinates of the nest.

在圖中，一隻螞蟻從  $O(0, 0)$  出發，並根據下列方法移動。

- (i) 向北移動，抵達  $A(0, 3)$ 。
- (ii) 向東移動，行走距離為牠在向北移動時的  $\frac{2}{3}$ 。
- (iii) 向南移動，行走距離為牠在向東移動時的  $\frac{2}{3}$ 。
- (iv) 向西移動，行走距離為牠在向南移動時的  $\frac{2}{3}$ 。
- (v) 向北移動，行走距離為牠在向西移動時的  $\frac{2}{3}$ 。

該隻螞蟻不斷重複(ii)至(v)。

- (a) 求牠所行走的總距離。
- (b) 該隻螞蟻最終回到牠的巢穴。求牠的巢穴的坐標。



2. Given an sequence  $1, 11, 111, 1111, \dots$

(a) Find the general term of the sequence.

(Note:  $\underbrace{111\dots1}_n$  is not accepted as an answer.)

(b) Using the result in (a), show that the integer  $\underbrace{111\dots1}_{2005}\underbrace{222\dots2}_{2005}$  is product of two consecutive integers.

(c) Find the general term of the sequence  $1, 12, 123, 1234, \dots$

已知一個數列  $1, 11, 111, 1111, \dots$

(a) 求該數列的通項。

(注意:  $\underbrace{111\dots1}_n$  不被接受為答案。)

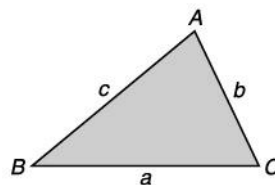
(b) 利用從 (a) 所得的結果，證明整數  $\underbrace{111\dots1}_{2005}\underbrace{222\dots2}_{2005}$  是兩個連續整數的積。

(c) 求數列  $1, 12, 123, 1234, \dots$  的通項。

3. In the figure,  $\triangle ABC$  is an acute-angled triangle. Let  $BC = a, CA = b, AB = c$  and area of  $\triangle ABC = S$ .

(a) Express  $\tan A$  in terms of  $a, b, c$  and  $S$ .

(b) If  $a^2, b^2, c^2$  are in arithmetic sequence, show that  $\frac{1}{\tan A}, \frac{1}{\tan B}, \frac{1}{\tan C}$  are also in arithmetic sequence.



在圖中， $\triangle ABC$  是一個銳角三角形。設  $BC = a, CA = b, AB = c$  及  $\triangle ABC$  的面積  $= S$ 。

(a) 以  $a, b, c$  和  $S$  表示  $\tan A$ 。

(b) 若  $a^2, b^2, c^2$  是一個等差數列，證明  $\frac{1}{\tan A}, \frac{1}{\tan B}, \frac{1}{\tan C}$  也是一個等差數列。

4. The terms of the geometric sequence  $1, 3, 3^2, 3^3, \dots$  are divided into the groups  $G_1, G_2, G_3, G_4, \dots$

$G_1 : 1$

$G_2 : 3, 3^2$

$G_3 : 3^3, 3^4, 3^5$

$G_4 : 3^6, 3^7, 3^8, 3^9$

$\vdots$

(a) Find the total number of elements from groups  $G_1$  to  $G_n$ .

(b) Find the sum of elements in group  $G_n$ .

把等比數列  $1, 3, 3^2, 3^3, \dots$  的項放置於組別  $G_1, G_2, G_3, G_4, \dots$  中。

$$G_1 : 1$$

$$G_2 : 3, 3^2$$

$$G_3 : 3^3, 3^4, 3^5$$

$$G_4 : 3^6, 3^7, 3^8, 3^9$$

⋮

- (a) 問組別  $G_1$  至  $G_n$  共有多少項？  
 (b) 求組別  $G_n$  內各項的總和。

5. Let  $n$  be an positive integer such that  $2^n - 1$  is a prime number.

- (a) Find all the factors of

(i)  $2^{n-1}$ ,

(ii)  $2^{n-1}(2^n - 1)$ .

- (b) If  $m = 2^{n-1}(2^n - 1)$ , find the sum of all the factors of  $m$ .

設  $n$  為一個正整數，使到  $2^n - 1$  是一個質數。

- (a) 求下列各數的所有因數。

(i)  $2^{n-1}$

(ii)  $2^{n-1}(2^n - 1)$

- (b) 若  $m = 2^{n-1}(2^n - 1)$ ，求  $m$  的所有因數之和。

6. In the figure,  $OAB$  is a right-angled triangle with  $\angle OAB = 90^\circ$ .  $B_1, B_2, B_3, \dots$  are points on  $OB$  such that  $ACB_1A_1, A_1C_1B_2A_2, A_2C_2B_3A_3, \dots$  are squares. Let  $S_1, S_2, S_3, \dots$  denote the area of  $ACB_1A_1, A_1C_1B_2A_2, A_2C_2B_3A_3, \dots$  respectively.

- (a) Show that  $S_1, S_2, S_3$  are in geometric sequence.

- (b) Express the sum of areas of all the squares in terms of  $S_1$  and  $S_2$ .

- (c) If  $S_1 = 1$  and the sum of areas of all the squares is 2, find  $\angle AOB$ .

在圖中， $OAB$  是一個直角三角形，且  $\angle OAB = 90^\circ$ 。 $B_1, B_2, B_3, \dots$  是  $OB$  上的點，使到

$ACB_1A_1, A_1C_1B_2A_2, A_2C_2B_3A_3, \dots$  是正方形。設  $ACB_1A_1, A_1C_1B_2A_2, A_2C_2B_3A_3, \dots$  的面積分別為  $S_1, S_2, S_3, \dots$ 。

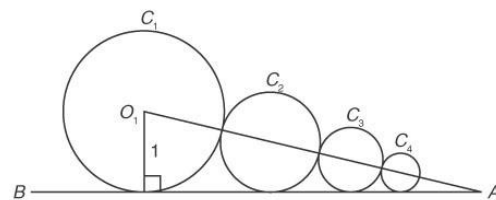
- (a) 證明  $S_1, S_2, S_3$  是一個等比數列。

- (b) 以  $S_1$  和  $S_2$  表示所有正方形的面積之和。

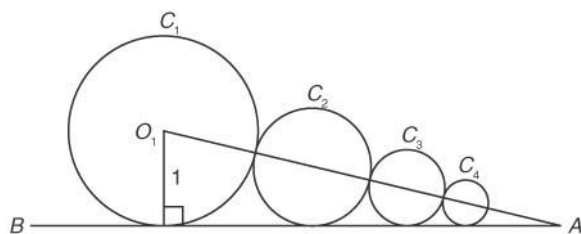
- (c) 若  $S_1 = 1$  及所有正方形的面積之和是 2，求  $\angle AOB$ 。

7. In the figure,  $C_1$  is a unit circle with centre  $O_1$  and touches the horizontal line  $AB$ . Circle  $C_2$  of radius  $r_2$  is constructed so that it touches both  $C_1$  and  $AB$  and its centre lies on  $O_1A$ . Circle  $C_3, C_4, C_5, \dots$  are constructed in a similar way. Denote  $\angle O_1AB$  by  $\theta$ .

- (a) Express  $r_2$  in terms of  $\theta$ .
- (b) Find the sum of areas of circles  $C_1, C_2, C_3, C_4, C_5, \dots$
- (c) If the length of  $O_1A$  doubles the radius of  $C_1$ , find the smallest values of  $n$  such that
- the radius of  $C_n$  is less than 0.01,
  - the sum of areas of circles  $C_n, C_{n+1}, C_{n+2}, \dots$  is less than  $0.01\pi$ .



在圖中， $C_1$ 是一個圓心為  $O_1$  的單位圓，並與水平線  $AB$  相切。作半徑為  $r_2$  的圓  $C_2$ ，使到它的圓心在  $O_1A$  上，並與圓  $C_1$  和  $AB$  相切。以同樣方法作  $C_3, C_4, C_5, \dots$ 。以  $\theta$  表示  $\angle O_1AB$ 。



- (a) 以  $\theta$  表示  $r_2$ 。
- (b) 求圓  $C_1, C_2, C_3, C_4, C_5, \dots$  的面積之和。
- (c) 若  $O_1A$  的長度是  $C_1$  的半徑的一倍，求  $n$  的最小值，使得
- $C_n$  的半徑小於 0.01;
  - $C_n, C_{n+1}, C_{n+2}, \dots$  的面積之和小於  $0.01\pi$ 。

$$\begin{aligned} \therefore \frac{O_1B_1}{O_2B_2} &= \frac{O_1A}{O_2A} \\ \frac{1}{r_2} &= \frac{\frac{1}{\sin \theta}}{\frac{1}{\sin \theta} - (r_2 + 1)} \\ \frac{1}{r_2} &= \frac{1}{1 - (r_2 + 1) \sin \theta} \\ 1 - (r_2 + 1) \sin \theta &= r_2 \\ r_2(1 + \sin \theta) &= 1 - \sin \theta \\ r_2 &= \frac{1 - \sin \theta}{1 + \sin \theta} \end{aligned}$$

$$\begin{aligned} \therefore r_n &= r_2^{n-1} \\ &= \left(\frac{1}{3}\right)^{n-1} \end{aligned}$$



## Multiple Choice Questions

## 多項選擇題

- 1 If the general term of a sequence is  $\frac{n(n-2)}{(n+3)^2}$ ,

find the 4th term of the sequence.

若一個數列的通項是  $\frac{n(n-2)}{(n+3)^2}$ ，求該數列  
的第 4 項。

- A.  $\frac{8}{49}$   
B.  $\frac{4}{49}$   
C.  $\frac{1}{2}$   
D.  $\frac{6}{49}$

2. Find the next term of the sequence  
6,13,27,55,...

求數列 6,13,27,55,... 接著的一項。

- A. 100  
B. 109  
C. 111  
D. 119

3. Which of the following is the general term of  
the sequence 5,8,11,14,...?

下列何者是數列 5,8,11,14,... 的通項?

- A.  $5n+3$   
B.  $3n+2$   
C.  $5n$   
D.  $4n+1$

4. Find the next term of the sequence

$\frac{2}{3}, \frac{8}{9}, \frac{32}{27}, \dots$

求數列  $\frac{2}{3}, \frac{8}{9}, \frac{32}{27}$  接著的一項。

- A.  $\frac{128}{81}$   
B.  $\frac{128}{192}$   
C.  $\frac{54}{81}$   
D.  $\frac{54}{192}$

(P01C14L06Q005)

5. Which of the following is/are in arithmetic  
sequence?

下列何者是等差數列?

- I.  $-1, 0, 1, 2, \dots$   
II.  $\log 2, \log 4, \log 8, \log 16, \dots$   
III.  $\frac{3}{4}, \frac{9}{8}, \frac{27}{16}, \frac{81}{32}, \dots$

- A. I only  
B. I and II only  
C. II and III only  
D. I, II and III

6. Which of the following about the arithmetic  
sequence  $T(1), T(2), \dots, T(n)$  is/are correct?

下列何者關於等差數列  $T(1), T(2), \dots, T(n)$   
的敘述是正確的?

- I.  $\frac{T(2)}{T(1)} = \frac{T(3)}{T(2)}$   
II.  $T(1) + T(2) + \dots + T(n) = \frac{T(1) + T(n)}{2}$   
III.  $T(n) = (n-1)T(2) + (2-n)T(1)$

- A. I only  
B. II only  
C. III only  
D. None of them are correct
7. If the sum of the first  $n$  terms of an arithmetic sequence is  $2n^2 + 3n$ , find the 5th term.  
若一個等差數列的首  $n$  項之和是  $2n^2 + 3n$ ，求第 5 項。
- A. 21  
B. 65  
C. 25  
D. 44
8. The general term of the arithmetic sequence 48, 40, 32, ... is  
等差數列 48, 40, 32, ... 的通項是
- A.  $8n$ .  
B.  $8(n+5)$ .  
C.  $48-n$ .  
D.  $8(7-n)$ .
9. Find the general term of the arithmetic sequence 7, 13, 19, ...  
求等差數列 7, 13, 19, ... 的通項。
- A.  $6n+13$   
B.  $7n-1$   
C.  $6n+1$   
D.  $7n+1$
10. Find the 8th term of the arithmetic sequence  $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \dots$   
求等差數列  $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \dots$  的第 8 項。
- A.  $\frac{25}{6}$   
B.  $\frac{13}{3}$   
C.  $\frac{23}{6}$   
D.  $\frac{2}{3}$
11. Let  $T(n)$  be the  $n$ th term of an arithmetic sequence. If  $T(3) = 26$  and  $T(2):T(12) = 1:4$ , find  $T(20)$ .  
設一個等差數列的第  $n$  項為  $T(n)$ 。若  $T(3) = 26$  及  $T(2):T(12) = 1:4$ ，求  $T(20)$  的值。
- A. 128  
B. 112  
C. 156  
D. 80
12. Find the number of terms in the arithmetic sequence 9, 3, -9, ..., -87.  
求等差數列 9, 3, -9, ..., -87 的項數。
- A. 15  
B. 16  
C. 17  
D. 18
13. Find the number of terms in the arithmetic sequence 15, 22, 29, ..., 169.  
求等差數列 15, 22, 29, ..., 169 的項數。
- A. 22  
B. 23  
C. 24  
D. 25

14. How many positive integers less than 300 are not divisible by 17?

問有多少個小於 300 的正整數不能被 17 整除？

- A. 283  
B. 282  
C. 281  
D. 280

15. The 6th term and the 13th term of an arithmetic sequence are 53 and 109 respectively.

Find the general term of the sequence.

已知一個等差數列的第 6 項是 53，而第 13 項是 109。求該數列的通項。

- A.  $7n + 11$   
B.  $9n - 8$   
C.  $16n + 31$   
D.  $8n + 5$

16. Which of the following are geometric sequences?

下列何者是等比數列？

I.  $\frac{4}{5}, \frac{8}{15}, \frac{16}{45}, \frac{32}{135}, \dots$

II.  $3, 6, 9, 12, \dots$

III.  $2^x, 2^{x+2}, 2^{x+4}, 2^{x+6}, \dots$

- A. I and II only  
B. I and III only  
C. II and III only  
D. I, II and III

17. If  $3x - 2, 4x, 2x - 7$  are three consecutive terms of an arithmetic sequence, find  $x$ .

若  $3x - 2, 4x, 2x - 7$  是一個等差數列的三個連續項，求  $x$  的值。

- A. 9

B. -9

C. 3

D. -3

18. If  $6p + 3, 2p + 1, 4p - 5$  are three consecutive terms of an arithmetic sequence, find  $p$ .

若  $6p + 3, 2p + 1, 4p - 5$  是一個等差數列的三個連續項，求  $p$  的值。

A.  $\frac{3}{8}$

B. 0

C.  $\frac{2}{3}$

D. 1

19. If four arithmetic means are inserted between  $m$  and  $3m$ , find the second arithmetic mean inserted.

若在  $m$  與  $3m$  之間插入四個等差中項，求插入的第二個等差中項。

A.  $\frac{7m}{5}$

B.  $2m$

C.  $\frac{9m}{5}$

D.  $\frac{2m}{5}$

20. If five arithmetic means are inserted between  $p$  and  $3q$ , find the sum of the arithmetic means inserted.

若在  $p$  與  $3q$  之間插入五個等差中項，求插入的等差中項之和。

A.  $5(p + 3q)$

B.  $\frac{3q - p}{6}$

C.  $2p + 9q$

D.  $\frac{5(p+3q)}{2}$

21. If the first term and the common ratio of a geometric sequence are  $a$  and  $R$  respectively, which of the following about the sequence are true?

若一個等比數列的首項是  $a$  而公比是  $R$ ，下列何者關於該數列的敘述是正確的？

I. General term =  $aR^n$

II. Sum of first  $n$  terms =  $\frac{a(1-R^n)}{1-R}$

III. Sum to infinity =  $\frac{a}{1-R}$  ( $-1 < R < 1$ )

I. 通項 =  $aR^n$

II. 首  $n$  項之和 =  $\frac{a(1-R^n)}{1-R}$

III. 無限項之和 =  $\frac{a}{1-R}$  ( $-1 < R < 1$ )

- A. I and II only  
B. I and III only  
C. II and III only  
D. I, II and III

22. Find the general term of the geometric

sequence  $\frac{2}{3}, -\frac{4}{15}, \frac{8}{75}, \dots$

求等比數列  $\frac{2}{3}, -\frac{4}{15}, \frac{8}{75}, \dots$  的通項。

A.  $-\frac{2}{3}\left(\frac{2}{5}\right)^{n-1}$

B.  $\frac{2}{3}\left(-\frac{2}{5}\right)^{n-1}$

C.  $\left(-\frac{4}{15}\right)^{n-1}$

D.  $\left(-\frac{4}{15}\right)^{n-1}$

23. Find the general term of the geometric

sequence  $\frac{1}{3}, 1, 3, 9, \dots$

求等比數列  $\frac{1}{3}, 1, 3, 9, \dots$  的通項。

A.  $3^n$

B.  $3^{n-1}$

C.  $3^{n-2}$

D.  $3^{n-3}$

24. Find the 8th term of the geometric sequence

$8, -\frac{16}{3}, \frac{32}{9}, \dots$

求等比數列  $8, -\frac{16}{3}, \frac{32}{9}, \dots$  的第 8 項。

A.  $\frac{1024}{2187}$

B.  $-\frac{1024}{2187}$

C.  $\frac{2048}{6561}$

D.  $-\frac{2048}{6561}$

25. It is given that  $3, 12, 48, \dots$  are in geometric

sequence. If  $T(k)$  is the first term in the sequence greater than 1200,  $k =$

已知  $3, 12, 48, \dots$  是一個等比數列。若  $T(k)$  是該數列中首個使得  $T(k) > 1200$  的項，則  $k =$

A. 5.

B. 6.

C. 7.

D. 8.

26. The sum of first  $n$  terms of a geometric sequence is  $-2(1-4^n)$ , find the third term of the sequence.

已知一個等比數列首  $n$  項之和是  $-2(1-4^n)$ ，求該數列的第 3 項。

- A. 96  
B. 384  
C. -96  
D. -384

- 27 Find the number of terms in the geometric sequence  $1, a^2, a^4, \dots, a^{4m+2}$ .

求等比數列  $1, a^2, a^4, \dots, a^{4m+2}$  的項數。

- A.  $2m+1$   
B.  $2m+2$   
C.  $4m+2$   
D.  $4m+3$

28. Find the number of terms in the geometric sequence  $\frac{2}{3}, -\frac{1}{2}, \frac{3}{8}, -\frac{9}{32}, \dots, \frac{2187}{32768}$ .

求等比數列  $\frac{2}{3}, -\frac{1}{2}, \frac{3}{8}, -\frac{9}{32}, \dots, \frac{2187}{32768}$  的項數。

- A. 6  
B. 7  
C. 8  
D. 9

29. If  $x+1, 3x+1, 5x+3$  are three consecutive terms of a geometric sequence, find the value(s) of  $x$ .

若  $x+1, 3x+1, 5x+3$  是一個等比數列的三個連續項，求  $x$  的值。

- A. -1  
B. 1

C.  $-1$  or  $\frac{1}{2}$

D.  $1$  or  $-\frac{1}{2}$

30. The third term and the sixth term of a geometric sequence are 18 and 486 respectively. Find the first term of the sequence.

已知一個等比數列的第 3 項是 18，而第 6 項是 486。求該數列的首項。

- A. 3  
B. 2  
C. 6  
D. 8

31. The second term and the fourth term of a geometric sequence of positive numbers are  $\frac{4}{3}$  and  $\frac{4}{27}$ , find the fifth term of the sequence.

已知一個由正數組成的等比數列的第 2 項是  $\frac{4}{3}$ ，而第 4 項是  $\frac{4}{27}$ ，求該數列的第 5 項。

- A.  $\frac{1}{81}$   
B.  $\frac{4}{243}$   
C.  $\frac{4}{81}$   
D.  $\frac{3}{245}$

32. If  $x$  and  $y$  are the geometric means between  $a$  and  $b$ ,  $s$  is the geometric mean between  $x$  and  $y$ , which of the following is/are **NOT** true?

若  $x$  和  $y$  是  $a$  與  $b$  之間的等比中項，而  $s$  是  $x$  與  $y$  的等比中項，下列何者並不正確？

- I.  $xy = ab$   
 II.  $s^2 = ab$   
 III.  $x^2 = ab$   
 A. I only  
 B. II only  
 C. III only  
 D. I, II and III

33. It is given that  $x$  and  $y$  are positive numbers. If  $x$  and  $y$  are the geometric means between 1 and 100, find  $\log x + \log y$ .

已知  $x$  和  $y$  是正數。若  $x$  和  $y$  是 1 與 100 之間的等比中項，求  $\log x + \log y$ 。

- A. 1  
 B. 2  
 C. 4  
 D. 8

34. Find the sum of first 20 terms of the arithmetic series  $18+14+10+6+\dots$

求等差級數  $18+14+10+6+\dots$  首 20 項之和。

- A. 1120  
 B. -800  
 C. -400  
 D. -580

35. Find the sum of the arithmetic series

求等差級數  $3+7+11+\dots+63$  各項之和。  
 $3+7+11+\dots+63$ .

- A. 528

- B. 495  
 C. 561  
 D. 1056

36. If the second term and the fifth term of a geometric series are 8 and 1 respectively, find the sum of the first 6 terms of the series.

若一個等比級數的第 2 項是 8，而第 5 項是 1，求該級數首 6 項之和。

- A. 15  
 B. 63  
 C.  $\frac{63}{2}$   
 D. 31

37. If the general term  $T(n)$  of an arithmetic sequence is  $4+3n$ , find  $T(4)+\dots+T(20)$ .

若一個等差數列的通項  $T(n)$  是  $4+3n$ ，求  $T(4)+\dots+T(20)$ 。

- A. 640  
 B. 680  
 C. 800  
 D. 1000

38.  $1+2+4+8+\dots+2^{3n} =$

- A.  $2^{3n} - 1$   
 B.  $2^{3n-1} - 1$   
 C.  $2^{3n+1} - 1$   
 D.  $2^{3n} + 1$

39. The 6th term of a geometric series is 486 and the common ratio is 3. Find the sum of the first 5 terms of the series.

已知一個等比級數的第 6 項是 486 而公比是 3。求該級數首 5 項之和。

- A. 242  
 B. 486  
 C. 162

D. 121

40. In a geometric series, if the sum of the first three terms is  $\frac{9}{4}$  and the sum to infinity is 2, find the first term of the series.

若一個等比級數的首 3 項之和是  $\frac{9}{4}$ ，而無限項之和是 2，求該級數的首項。

A.  $\frac{4}{3}$ 

B. 3

C. 4

D.  $-\frac{1}{2}$ 

41. Find the sum to infinity of the geometric

series  $1 + \frac{3}{10} + \frac{9}{100} + \frac{27}{1000} + \dots$

求等比級數  $1 + \frac{3}{10} + \frac{9}{100} + \frac{27}{1000} + \dots$  的無限項之和。

A.  $\frac{7}{10}$ B.  $\frac{8}{7}$ C.  $\frac{10}{7}$ D.  $\frac{10}{3}$ 

42. Find the sum of all positive terms in the

geometric sequence  $\frac{2}{3}, -\frac{4}{15}, \frac{8}{75}, \dots$

求等比數列  $\frac{2}{3}, -\frac{4}{15}, \frac{8}{75}, \dots$  中所有正數項之和。

A.  $\frac{5}{21}$ B.  $\frac{10}{21}$ C.  $\frac{50}{63}$ D.  $\frac{50}{87}$