Solution	Marks	Remarks
1. $\dfrac{a+4}{3}=\dfrac{b+1}{2}$		
$2(a+4)=3(b+1)$	1M	
$2a+8=3b+3$		
$3b=2a+5$	1M	for putting b on one side
$b=\dfrac{2a+5}{3}$	1A	or equivalent
$\dfrac{a+4}{3}=\dfrac{b+1}{2}$		
$2\left(\dfrac{a+4}{3}\right)=b+1$	1M	
$\dfrac{2a+8}{3}=b+1$		
$b=\dfrac{2a+8}{3}-1$	1M	for putting b on one side
$b=\dfrac{2a+5}{3}$	1A	or equivalent
	----------(3)	
2. $\dfrac{xy^7}{(x^{-2}y^3)^4}$		
$=\dfrac{xy^7}{x^{-8}y^{12}}$	1M	for $(a^h)^k=a^{hk}$ or $(ab)^l=a^lb^l$
$=\dfrac{x^{1+8}}{y^{12-7}}$	1M	for $\dfrac{c^p}{c^q}=c^{p-q}$ or $d^{-r}=\dfrac{1}{d^r}$
$=\dfrac{x^9}{y^5}$	1A	
	----------(3)	
3. (a) 266	1A	
(b) 265.4	1A	
(c) 270	1A	
	----------(3)	

Solution	Marks	Remarks
4. Note that the probability of drawing a red ball is $\frac{8}{n+5+8}$.	1M	for denominator
$\frac{8}{n+5+8}=\frac{2}{5}$	1A	
$2n+26=40$		
$n=7$	1A	
	----------(3)	
5. (a) $9r^3-18r^2s$		
$=9r^2(r-2s)$	1A	or equivalent
(b) $9r^3-18r^2s-rs^2+2s^3$		
$=9r^2(r-2s)-rs^2+2s^3$	1M	for using the result of (a)
$=9r^2(r-2s)-s^2(r-2s)$		
$=(r-2s)(9r^2-s^2)$	1M	
$=(r-2s)(3r+s)(3r-s)$	1A	or equivalent
	----------(4)	
6. (a) $\frac{3-x}{2}>2x+7$		
$3-x>4x+14$		
$-5x>11$	1M	for putting x on one side
$x<\frac{-11}{5}$	1A	$x<-2.2$
$x+8\geq 0$		
$x\geq -8$		
Thus, the required range is $-8\leq x<\frac{-11}{5}$.	1A	$-8\leq x<-2.2$
(b) -3	1A	
	----------(4)	

Solution	Marks	Remarks
7. Let $\$x$ be the marked price of the vase.		
The cost of the vase		
$=\dfrac{x}{1+30\%}$	1M	
$=\$\left(\dfrac{10x}{13}\right)$		
The selling price of the vase		
$=(1-40\%)x$	1M	
$=\$\left(\dfrac{3x}{5}\right)$		
$\dfrac{10x}{13}-\dfrac{3x}{5}=88$	1M+1A	
$\dfrac{11x}{65}=88$		
$x=520$	1A	
Thus, the marked price of the vase is $\$520$.		
Let $\$c$ be the cost of the vase.		
The marked price of the vase		
$=(1+30\%)c$	1M	
$=\$\ 1.3c$		
The selling price of the vase		
$=(1-40\%)(1.3c)$	1M	
$=\$\ 0.78c$		
$c-0.78c=88$	1M+1A	
$0.22c=88$		
$c=400$		
The marked price of the vase		
$=1.3(400)$		
$=\$520$	1A	
	----------(5)	
8. x		
$=180^\circ-\theta$	1A	
$\angle ADE$		
$=x$	1M	
$=180^\circ-\theta$		
$\angle BED$		
$=x$	1M	
$=180^\circ-\theta$		
y		
$=180^\circ-\angle ADE-\angle BED$	1M	
$=180^\circ-(180^\circ-\theta)-(180^\circ-\theta)$		
$=2\theta-180^\circ$	1A	
	----------(5)	

Solution	Marks	Remarks
9. Let x minutes be the time required for the car to travel from city P to city Q. Then, the time required for the car to travel from city Q to city R is $(161-x)$ minutes.	1A	
$72\left(\frac{x}{60}\right)+90\left(\frac{161-x}{60}\right)=210$	1M+1A+1M	1M for changing unit 1M for getting a linear equation in one unknown
$18x=1890$		
$x=105$	1A	
Thus, the car takes 105 minutes to travel from city P to city Q.		
72 km/h $=\frac{72}{60}$ km/min $=1.2$ km/min	1M	either one
90 km/h $=\frac{90}{60}$ km/min $=1.5$ km/min		
Let x minutes and y minutes be the time required for the car to travel from city P to city Q and from city Q to city R respectively. So, we have $x+y=161$ and $1.2x+1.5y=210$.	1A+1A	
Therefore, we have $1.2x+1.5(161-x)=210$.	1M	for getting a linear equation in x or y only
Solving, we have $x=105$ and $y=56$.	1A	
Thus, the car takes 105 minutes to travel from city P to city Q.		
Let x hours be the time required for the car to travel from city P to city Q. Then, the car takes $\left(\frac{161}{60}-x\right)$ hours to travel from city Q to city R.	1M+1A	1M for changing unit
$72x+90\left(\frac{161}{60}-x\right)=210$	1A+1M	1M for getting a linear equation in one unknown
$x=1.75$	1A	
Thus, the car takes 1.75 hours to travel from city P to city Q.		
The time required for the car to travel from city P to city Q $=\dfrac{90\left(\frac{161}{60}\right)-210}{90-72}$	1M+1A +1M+1A	1M for fraction + 1A for numerator + 1M for changing unit + 1A for denominator
$=1.75$ hours	1A	
Let y km be the distance between city P and city Q. Then, the distance between city Q and city R is $(210-y)$ km.	1A	
$\frac{y}{72}+\frac{210-y}{90}=\frac{161}{60}$	1M+1A+1M	1M for changing unit 1M for getting a linear equation in one unknown
$y=126$		
The time required for the car to travel from city P to city Q $=\frac{126}{72}$		
$=1.75$ hours	1A	
	----------(5)	

Solution	Marks	Remarks
10. (a) $a-27=21$	1M	either one
$a=48$	1A	
$b-19=43$		
$b=62$	1A	
	----------(3)	
(b) Note that $38-20=18$.	1M	
Therefore, the least possible age of the clerks in team Y is 18 .		
The greatest possible range of the distribution of the ages of the clerks in the section		
$=62-18$		
$=44$		
$\neq 43$		
Thus, the claim is disagreed.	1A	f.t.
Suppose that the ages of the clerks in team Y are 18 , 19 , 38 , 38 and 38 .	1M	
Note that the range of the ages of the clerks in team Y is 20 .		
The range of the ages of the clerks in the section		
$=62-18$		
$=44$		
$\neq 43$		
Thus, the claim is disagreed.	1A	f.t.
	----------(2)	
11. (a) (i) 1	1A	
(ii) 8	1A	
	----------(2)	
(b) (i) 3	1A	
(ii) 19	1A	
	----------(2)	
(c) $\dfrac{0(k)+1(2)+2(9)+3(6)+4(7)}{k+2+9+6+7}=2$	1M	
$\dfrac{66}{k+24}=2$		
$2k+48=66$		
$k=9$	1A	
	----------(2)	

Solution	Marks	Remarks
12. (a) $f(3)=0$	1M	
$4(3)(3+1)^2+a(3)+b=0$		
$3a+b=-192$		
$f(-2)=2b+165$	1M	
$4(-2)(-2+1)^2+a(-2)+b=2b+165$		
$2a+b=-173$		
Solving, we have $a=-19$ and $b=-135$.	1A ----------(3)	for both correct
(b) $f(x)=0$		
$4x(x+1)^2-19x-135=0$		
$4x^3+8x^2-15x-135=0$		
$(x-3)(4x^2+20x+45)=0$	1M	for $(x-3)(px^2+qx+r)$
$x=3$ or $4x^2+20x+45=0$		
$20^2-4(4)(45)$	1M	
$=-320$		
<0		
So, the equation $4x^2+20x+45=0$ has no real roots.	1M	
Note that 3 is not an irrational number.		
Thus, the claim is disagreed.	1A ----------(4)	f.t.

Solution	Marks	Remarks
13. (a) $\angle ABE = 90°$ (given)		
$\angle DCE = 180° - \angle ABE$ (int. $\angle$s, $AB \parallel DC$)		
$\angle DCE = 90°$		
$\angle ABE = \angle DCE$		
$\angle BAE = 180° - \angle ABE - \angle AEB$ ($\angle$ sum of Δ)		
$\angle BAE = 90° - \angle AEB$		
$\angle AED = 90°$ (given)		
$\angle CED = 180° - \angle AED - \angle AEB$ (adj. $\angle$ s on st. line)		
$\angle CED = 90° - \angle AEB$		
$\angle BAE = \angle CED$		
$\angle AEB = \angle CDE$ ($\angle$ sum of Δ)		
$\Delta ABE \sim \Delta ECD$ (AAA)		(AA) (equiangular)
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
	----------(2)	
(b) (i) BE		
$= \sqrt{AE^2 - AB^2}$		
$= \sqrt{25^2 - 15^2}$		
$= 20$ cm		
$\frac{CD}{BE} = \frac{CE}{AB}$ (by (a))	1M	for using (a)
$\frac{CD}{20} = \frac{36}{15}$		
$CD = 48$ cm	1A	
(ii) The area of ΔADE		
$= \frac{1}{2}(AB + CD)(BC) - \frac{1}{2}(AB)(BE) - \frac{1}{2}(CD)(CE)$	1M	
$= \frac{1}{2}(15 + 48)(20 + 36) - \frac{1}{2}(15)(20) - \frac{1}{2}(48)(36)$		
$= 750$ cm^2	1A	
(iii) AD		
$= \sqrt{BC^2 + (CD - AB)^2}$		
$= \sqrt{(20 + 36)^2 + (48 - 15)^2}$		
$= 65$ cm		
The shortest distance from E to AD		
$= \frac{2(750)}{65}$	1M	
$= \frac{300}{13}$		
≈ 23.07692308 cm		
> 23 cm		
Thus, there is no point F lying on AD such that the distance between E and F is less than 23 cm .	1A	f.t.
	----------(6)	

Solution	Marks	Remarks
14. (a) The volume of water in the vessel		
$= \pi(8^2)(64)$	1M	
$= 4\,096\pi \text{ cm}^3$	1A	
	----------(2)	
(b) Let h cm be the depth of water in the vessel.		
Then, the radius of the water surface is $\frac{h}{3}$ cm .	1M	
$\frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h = 4\,096\pi$	1M+1A	
$h^3 = 110\,592$		
$h = 48$	1A	
Thus, the depth of water in the vessel is 48 cm .		
Let h cm be the depth of water in the vessel.		
The capacity of the vessel is $\frac{1}{3}\pi(20)^2(60) \text{ cm}^3$.	1M	
$\frac{1}{3}\pi(20)^2(60)\left(\frac{h}{60}\right)^3 = 4\,096\pi$	1M+1A	1M for $\left(\frac{h}{60}\right)^3$
$h^3 = 110\,592$		
$h = 48$		
Thus, the depth of water in the vessel is 48 cm .	1A	
	----------(4)	
(c) The volume not occupied by water in the vessel		
$= \frac{1}{3}\pi(20^2)(60) - 4\,096\pi$	1M	
$= 3\,904\pi \text{ cm}^3$		
The volume of the metal sphere		
$= \frac{4}{3}\pi(14^3)$	1M	
$= \frac{10\,976}{3}\pi \text{ cm}^3$		
$< 3\,904\pi \text{ cm}^3$		
Thus, the water will not overflow.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
15. (a) The required number		
$= P_8^8$		
$= 40\,320$	1A	
	----------(1)	
(b) The required number		
$= (P_2^4)(P_6^6)$	1M	
$= 8\,640$	1A	
	----------(2)	
16. (a) Let a and r be the 1st term and the common ratio of the sequence respectively.		
So, we have $ar^2 = 720$ and $ar^3 = 864$.	1M	for either one
Solving, we have $a = 500$.	1A	
Thus, the 1st term is 500 .		
	----------(2)	
(b) Note that $r = 1.2$.		
$500(1.2^n) + 500(1.2^{2n}) < 5 \times 10^{14}$		
$(1.2^n)^2 + (1.2^n) - 10^{12} < 0$	1M	
$\frac{-1-\sqrt{1^2-4(1)(-10^{12})}}{2(1)} < 1.2^n < \frac{-1+\sqrt{1^2-4(1)(-10^{12})}}{2(1)}$		
$\log 1.2^n < \log\left(\frac{-1+\sqrt{4\times 10^{12}+1}}{2}\right)$	1M	
$n\log 1.2 < \log\left(\frac{-1+\sqrt{4\times 10^{12}+1}}{2}\right)$		
$n < 75.77551608$		
Note that n is an integer.		
Thus, the greatest value of n is 75 .	1A	
	----------(3)	

Solution	Marks	Remarks
17. (a) By sine formula, we have		
$\dfrac{AD}{\sin\angle ABD}=\dfrac{AB}{\sin\angle ADB}$	1M	
$\dfrac{AD}{\sin 20^\circ}=\dfrac{60}{\sin(180^\circ-120^\circ-20^\circ)}$		
$AD\approx 31.92533317\text{ cm}$		
$AD\approx 31.9\text{ cm}$	1A	r.t. 31.9 cm
	----------(2)	
(b) (i) By cosine formula, we have		
$\cos\angle ABC=\dfrac{AB^2+BC^2-AC^2}{2(AB)(BC)}$	1M	
$\cos\angle ABC\approx\dfrac{60^2+(31.92533317)^2-40^2}{2(60)(31.92533317)}$		
$\angle ABC\approx 37.99207534^\circ$		
$\angle ABC\approx 38.0^\circ$	1A	r.t. 38.0°
(ii) In Figure 3(a), AP produced meets CD at Q, where P is the foot of the perpendicular from A to BD.		
Note that the required angle is $\angle APQ$ in Figure 3(b).	1M	for identifying the required angle
AP $=AD\sin\angle ADP$ $\approx 31.92533317\sin(180^\circ-120^\circ-20^\circ)$ $\approx 20.5212086\text{ cm}$	1M	either one
$DP^2=AD^2-AP^2$ $DP^2\approx(31.92533317)^2-(20.5212086)^2$ $DP\approx 24.45622407\text{ cm}$		
PQ $=DP\tan\angle PDQ$ $\approx(24.45622407)\tan 20^\circ$ $\approx 8.901337605\text{ cm}$		
$DQ^2=DP^2+PQ^2$ $DQ^2\approx(24.45622407)^2+(8.901337605)^2$ $DQ\approx 26.02577006\text{ cm}$		
Note that $\angle ADC=\angle ABC\approx 37.99207534^\circ$.		
By cosine formula, we have $AQ^2=AD^2+DQ^2-2(AD)(DQ)\cos\angle ADC$		
$AQ^2\approx(31.92533317)^2+(26.02577006)^2-2(31.92533317)(26.02577006)\cos 37.99207534^\circ$		
$AQ\approx 19.67076991\text{ cm}$		
By cosine formula, we have		
$\cos\angle APQ=\dfrac{AP^2+PQ^2-AQ^2}{2(AP)(PQ)}$		
$\cos\angle APQ\approx\dfrac{(20.5212086)^2+(8.901337605)^2-(19.67076991)^2}{2(20.5212086)(8.901337605)}$		
$\angle APQ\approx 71.91411397^\circ$		
$\angle APQ\approx 71.9^\circ$	1A	r.t. 71.9°
Thus, the required angle is 71.9°.		
	----------(5)	

Solution	Marks	Remarks
18. (a) Let $f(x) = ax^2 + bx$, where a and b are non-zero constants.	1A	
So, we have $4a + 2b = 60$ and $9a + 3b = 99$.	1M	for either substitution
Solving, we have $a = 3$ and $b = 24$.	1A	for both correct
Thus, we have $f(x) = 3x^2 + 24x$.		
	----------(3)	
(b) (i) $f(x)$ $= 3x^2 + 24x$ $= 3(x^2 + 8x)$		
$= 3(x^2 + 8x + 16 - 16)$	1M	
$= 3(x+4)^2 - 48$		
Thus, the coordinates of Q are $(-4, -48)$.	1A	
(ii) $(-4, 75)$	1M	
(iii) The slope of QS $= \frac{-48-0}{-4-56}$ $= \frac{4}{5}$		
The slope of RS $= \frac{75-0}{-4-56}$ $= \frac{-5}{4}$		
Hence, the product of the slope of QS and the slope of RS is -1 .	1M	
So, $\angle QSR$ is a right angle. Therefore, QR is a diameter of the circumcircle of ΔQRS . Note that P is the circumcentre of ΔQRS .		
Thus, P is the mid-point of the line segment joining Q and R .	1A	f.t.
$QS^2 + RS^2$ $= ((-4-56)^2 + (-48-0)^2) + ((-4-56)^2 + (75-0)^2)$ $= 15\,129$		
QR^2 $= (-48-75)^2$ $= 15\,129$		
Hence, we have $QS^2 + RS^2 = QR^2$.	1M	
So, $\angle QSR$ is a right angle. Therefore, QR is a diameter of the circumcircle of ΔQRS . Note that P is the circumcentre of ΔQRS .		
Thus, P is the mid-point of the line segment joining Q and R .	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
19. (a) The equation of C is $(x-8)^2+(y-2)^2=r^2$.	1A	$x^2+y^2-16x-4y+68-r^2=0$
Putting $y=\dfrac{kx-21}{5}$ in $(x-8)^2+(y-2)^2=r^2$, we have		
$(x-8)^2+\left(\dfrac{kx-21}{5}-2\right)^2=r^2$	1M	
$(k^2+25)x^2+(-62k-400)x+2\,561-25r^2=0$		
Note that L is a tangent to C .		
So, we have $(-62k-400)^2-4(k^2+25)(2\,561-25r^2)=0$.	1M	
Thus, we have $r^2=\dfrac{64k^2-496k+961}{k^2+25}$.	1A	$r^2=\dfrac{(8k-31)^2}{k^2+25}$
	----------(4)	
(b) (i) Since L passes through D, we have $18k-5(39)-21=0$.	1M	
Solving, we have $k=12$.		
By (a), we have $r^2=\dfrac{64(12)^2-496(12)+961}{12^2+25}$.	1M	for using the result of (a)
Thus, we have $r=5$.	1A	
(ii) Let G be the centre of C .		
Note that the coordinates of E are $\left(0,\dfrac{-21}{5}\right)$.	1M	
Also note that G is the in-centre of ΔDEF .		
$DG^2=(18-8)^2+(39-2)^2$	1M	either one
$DG=\sqrt{1\,469}$		
$\sin\angle EDG=\dfrac{r}{DG}$	1M	either one
$\sin\angle EDG=\dfrac{5}{\sqrt{1\,469}}$		
$\angle EDG\approx 7.49585764°$		
$EG^2=(8-0)^2+\left(2+\dfrac{21}{5}\right)^2$		
$EG=\dfrac{\sqrt{2\,561}}{5}$		
$\sin\angle DEG=\dfrac{r}{EG}$		
$\sin\angle DEG=\dfrac{25}{\sqrt{2\,561}}$		
$\angle DEG\approx 29.60445074°$		
Note that $\angle EDG=\angle FDG$ and $\angle DEG=\angle FEG$.	1M	for either one
$\angle DFE$		
$=180°-(\angle EDG+\angle FDG)-(\angle DEG+\angle FEG)$		
$\approx 180°-2(7.49585764°)-2(29.60445074°)$		
$\approx 105.7993832°$		
$>90°$		
Thus, ΔDEF is an obtuse-angled triangle.	1A	f.t.
	----------(8)	

Paper 2

Question No.	Key	Question No.	Key
1.	B (71)	26.	C (40)
2.	D (80)	27.	C (43)
3.	C (80)	28.	A (50)
4.	A (74)	29.	C (78)
5.	A (61)	30.	A (43)
6.	D (22)	31.	C (66)
7.	D (73)	32.	C (34)
8.	C (51)	33.	D (30)
9.	D (72)	34.	C (35)
10.	B (72)	35.	B (40)
11.	D (67)	36.	A (49)
12.	A (62)	37.	D (44)
13.	C (69)	38.	B (41)
14.	B (42)	39.	B (28)
15.	D (83)	40.	A (20)
16.	A (39)	41.	D (35)
17.	B (28)	42.	A (51)
18.	B (78)	43.	C (26)
19.	D (24)	44.	B (78)
20.	B (48)	45.	A (51)
21.	C (45)		
22.	B (45)		
23.	B (75)		
24.	A (56)		
25.	D (41)		

Note: Figures in brackets indicate the percentages of candidates choosing the correct answers.