Paper 1

Solution	Marks	Remarks
1. $(\alpha\beta^3)(\alpha^{-2}\beta^4)^5$		
$=(\alpha\beta^3)(\alpha^{-10}\beta^{20})$	1M	for $(a^h)^k = a^{hk}$ or $(ab)^l = a^l b^l$
$=\alpha^{-9}\beta^{23}$	1M	for $c^p c^q = c^{p+q}$ or $d^{-r} = \dfrac{1}{d^r}$
$=\dfrac{\beta^{23}}{\alpha^9}$	1A	
	----------(3)	
2. $\dfrac{4-3a}{b}=5$		
$4-3a=5b$	1M	
$-3a=5b-4$	1M	
$a=\dfrac{4-5b}{3}$	1A	or equivalent
$\dfrac{4-3a}{b}=5$		
$\dfrac{4}{b}-\dfrac{3a}{b}=5$		
$-3a=b\left(5-\dfrac{4}{b}\right)$	1M+1M	
$a=\dfrac{-b}{3}\left(5-\dfrac{4}{b}\right)$		
$a=\dfrac{4}{3}-\dfrac{5b}{3}$	1A	or equivalent
	----------(3)	
3. (a) $6x^2+xy-2y^2$		
$=(2x-y)(3x+2y)$	1A	or equivalent
(b) $8x-4y-6x^2-xy+2y^2$		
$=8x-4y-(2x-y)(3x+2y)$	1M	for using the result of (a)
$=4(2x-y)-(2x-y)(3x+2y)$		
$=(2x-y)(4-3x-2y)$	1A	or equivalent
	----------(3)	
4. (a) $\dfrac{7(x-2)}{5}+11>3(x-1)$		
$7x-14>15x-70$		
$-8x>-56$	1M	for putting x on one side
$x<7$	1A	
$x+4\geq 0$		
$x\geq -4$		
Thus, the required range is $-4\leq x<7$.	1A	
(b) 6	1A	
	----------(4)	

Solution	Marks	Remarks
5. Let x be the number of stickers owned by the girl.		
Then, the number of stickers owned by the boy is $3x$.	1A	
$2(3x-20)=x+20$	1M+1A	
$6x-40=x+20$		
$x=12$		
Thus, the total number of stickers owned by the boy and the girl is 48 .	1A	
Let x and y be the number of stickers owned by the girl and the number of stickers owned by the boy respectively.		
Then, we have $3x=y$ and $2(y-20)=x+20$.	1A+1A	
$2(3x-20)=x+20$	1M	
$6x-40=x+20$		
$x=12$		
$y=36$		
Thus, the total number of stickers owned by the boy and the girl is 48 .	1A	
Let n be the total number of stickers owned by the boy and the girl.		
Then, we have $2\left(\frac{3}{4}n-20\right)=\frac{1}{4}n+20$.	1M+1A+1A	
$\frac{5}{4}n=60$		
$n=48$	1A	
Thus, the total number of stickers owned by the boy and the girl is 48 .		
	----------(4)	
6. Let $\$x$ be the marked price of the shirt.		
The cost of the shirt		
$=\$(x-80)$	1M	
The selling price of the shirt		
$=(90\%)x$	1M	
$=\$0.9x$		
$0.9x=(x-80)(1+30\%)$	1M	
$0.9x=1.3x-104$		
$x=260$	1A	
Thus, the marked price of the shirt is $\$260$.		
Let $\$c$ be the cost of the shirt.		
The marked price of the shirt		
$=\$(c+80)$	1M	
The selling price of the shirt		
$=(c+80)(90\%)$	1M	
$=\$(0.9c+72)$		
$0.9c+72=(1+30\%)c$	1M	
$0.9c+72=1.3c$		
$c=180$		
Thus, the marked price of the shirt is $\$260$.	1A	
	----------(4)	

Solution	Marks	Remarks
7. (a) $\angle POQ$		
$= 140° - 80°$		
$= 60°$	1A	
(b) Since ΔOPQ is an equilateral triangle, we have $r = 21$.	1A	
(c) The perimeter of ΔOPQ		
$= 3(21)$	1M	
$= 63$	1A	
	----------(4)	
8. (a) $\angle CAE = \angle BDE$ (given)		
$\angle AEC = \angle DEB$ (common $\angle$)		
$\angle ACE = \angle DBE$ ($\angle$ sum of Δ)		
$\Delta ACE \sim \Delta DBE$ (AAA)		(AA) (equiangular)
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
(b) (i) $AC^2 + AE^2$		
$= 25^2 + 60^2$		
$= 4\,225$		
$= 65^2$		
$= CE^2$		
Thus, ΔACE is a right-angled triangle.	1A	f.t.
(ii) $\dfrac{DE}{AE} = \dfrac{BD}{AC}$	1M	
$\dfrac{DE}{60} = \dfrac{15}{25}$		
$DE = 36$ cm		
Note that $\angle BDE = 90°$.		
The area of ΔBDE		
$= \dfrac{15(36)}{2}$		
$= 270 \text{ cm}^2$	1A	
	----------(5)	
9. (a) $\dfrac{12+k+16}{12+k+16+9+11+4} = \dfrac{7}{10}$	1M	
$k = 28$	1A	
(b) The range		
$= 5$	1A	
The inter-quartile range		
$= 2$	1A	
The standard deviation		
≈ 1.43	1A	r.t. 1.43
	----------(5)	

Solution	Marks	Remarks
10. (a) Let $f(x)=m(x+4)^2+n$, where m and n are non-zero constants.	1M	
Since $f(-3)=0$ and $f(2)=105$, we have $m+n=0$ and $36m+n=105$.	1M	for either substitution
Solving, we have $m=3$ and $n=-3$.		
Thus, we have $f(0)=45$.	1A	
	----------(3)	
(b) (i) 48	1M	(a) + 3
(ii) For $f(x)+3=0$, we have		
$3(x+4)^2=0$	1M	
$x=-4$	1A	
Thus, the x-intercept of G is -4 .		
	----------(3)	
11. (a) The mean		
$=\dfrac{1(15)+2(9)+3(2)+4(5)+5(4)+6(2)+7(5)}{15+9+2+5+4+2+5}$	1M	
$=\dfrac{126}{42}$		
$=3$	1A	
	----------(2)	
(b) The median and the mode are 2 and 1 respectively.	1M	for either one
Thus, the median and the mode of the distribution are not equal.	1A	f.t.
	----------(2)	
(c) (i) 42	1A	
(ii) 11	1A	
(iii) 10	1A	
	----------(3)	

Solution	Marks	Remarks
12. (a) Let $p(x)=(x^2+x+1)(2x^2-37)+cx+c-1$.	1M	
$p(5)=0$	1M	
$(5^2+5+1)(2(5^2)-37)+5c+c-1=0$		
$6c+402=0$		
$c=-67$	1A	
	----------(3)	
(b) $p(x)$		
$=(x^2+x+1)(2x^2-37)-67x-68$ (by (a))		
$=2x^4+2x^3-35x^2-104x-105$		
$p(-3)$		
$=2(-3)^4+2(-3)^3-35(-3)^2-104(-3)-105$		
$=0$		
Thus, $x+3$ is a factor of $p(x)$.	1	
	----------(1)	
(c) By (b), we have $p(x)=2x^4+2x^3-35x^2-104x-105$.		
Therefore, we have $p(x)=(x+3)(x-5)(2x^2+6x+7)$.	1M	$p(x)=(x+3)(x-5)(lx^2+mx+n)$
$p(x)=0$		
$(x+3)(x-5)(2x^2+6x+7)=0$		
$x=-3$, $x=5$ or $2x^2+6x+7=0$		
$6^2-4(2)(7)$	1M	
$=-20$		
<0		
So, the roots of the equation $2x^2+6x+7=0$ are not real numbers.		
Thus, the claim is not correct.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
13. (a) Note that the coordinates of G are $(6,8)$.		
OG		
$=\sqrt{(6-0)^2+(8-0)^2}$	1M	
$=10$	1A ----------(2)	
(b) The radius of C		
$=\frac{1}{2}\sqrt{(-12)^2+(-16)^2+4(69)}$		
$=13$		
$>OG$ (by (a))		
Thus, O lies inside C.	1A ----------(1)	f.t.
(c) Since both M and N lie on Γ , we have $OM=GM$ and $ON=GN$. Note that $GM=GN$. So, we have $OM=GM=GN=ON$. Hence, the quadrilateral $OMGN$ is a rhombus. Let Q be the point of intersection of OG and MN.		
GQ		
$=\frac{1}{2}OG$		
$=\frac{1}{2}(10)$ (by (a))	1M	for using the result of (a)
$=5$		
Also note that $\angle GQM=90°$ and $GM=13$.		
MQ		
$=\sqrt{GM^2-GQ^2}$	1M	
$=\sqrt{13^2-5^2}$		
$=12$		
The area of the quadrilateral $OMGN$		
$=4\left(\frac{1}{2}(GQ)(MQ)\right)$	1M	
$=4\left(\frac{1}{2}(5)(12)\right)$		
$=120$	1A ----------(4)	

Solution	Marks	Remarks
14. (a) Let r cm be the base radius of Y.		
$\frac{24\pi r^2}{3}=800\pi$	1M	
$r=10$	1A	
Thus, the base radius of Y is 10 cm .		
	----------(2)	
(b) The volume of Z		
$=\pi(10^2)(20)+800\pi$	1M	
$=2\,800\pi\text{ cm}^3$		
$\left(\frac{\text{The base radius of } Y}{\text{The base radius of } Z}\right)^3=\left(\frac{1}{2}\right)^3=\frac{1}{8}$		
$\frac{\text{The volume of } Y}{\text{The volume of } Z}=\frac{800}{2\,800}=\frac{2}{7}$		
$\frac{\text{The volume of } Y}{\text{The volume of } Z}\neq\left(\frac{\text{The base radius of } Y}{\text{The base radius of } Z}\right)^3$	1M	for comparing the two ratios
Thus, Y and Z are not similar.	1A	f.t.
	----------(3)	
(c) The curved surface area of X		
$=2\pi(10)(20)$	1M	
$=400\pi\text{ cm}^2$		
The curved surface area of Y		
$=\pi(10)\left(\sqrt{10^2+24^2}\right)$	1M	either one
$=260\pi\text{ cm}^2$		
Let h cm be the height of Z.		
$\frac{\pi(20^2)(h)}{3}=2\,800\pi$		
$h=21$		
Therefore, the height of Z is 21 cm .		
The curved surface area of Z		
$=\pi(20)\left(\sqrt{20^2+21^2}\right)$		
$=580\pi\text{ cm}^2$		
The sum of the curved surface area of X and the curved surface area of Y		
$=400\pi+260\pi$		
$=660\pi\text{ cm}^2$		
$>580\pi\text{ cm}^2$		
Thus, the claim is agreed.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
15. (a) The required number		
$= P_{10}^{10}$		
$= 3\,628\,800$	1A	
	----------(1)	
(b) The required probability		
$= \dfrac{7!\,C_3^8\,3!}{3\,628\,800}$	1M+1M	1M for denominator + 1M for $7!3!$
$= \dfrac{1\,693\,440}{3\,628\,800}$		
$= \dfrac{7}{15}$	1A	r.t. 0.467
	----------(3)	
16. (a) The slope of L_1		
$= \dfrac{6-3}{2-0}$		
$= \dfrac{3}{2}$		
The equation of L_1 is		
$y-3 = \dfrac{3}{2}(x-0)$	1M	either one
$3x-2y+6=0$	1A	either one
The equation of L_2 is		
$y-6 = \dfrac{-2}{3}(x-2)$		
$2x+3y-22=0$		
Thus, the system of inequalities is $\begin{cases} 3x-2y+6 \ge 0 \\ 2x+3y-22 \le 0 \\ y \ge 0 \end{cases}$.	1A	or equivalent
	----------(3)	
(b) Note that the vertices of R are the points $(-2,0)$, $(2,6)$ and $(11,0)$.		
When $x=-2$ and $y=0$, we have $8x-5y=-16$.	1M	any one
When $x=2$ and $y=6$, we have $8x-5y=-14$.		
When $x=11$ and $y=0$, we have $8x-5y=88$.		
Thus, the least value of $8x-5y$ is -16 .	1A	
	----------(2)	

Solution	Marks	Remarks
17. (a) Let d be the common difference of the arithmetic sequence.		
So, we have $A(1)+4d=26$ and $A(1)+11d=61$.	1M	for either one
Solving, we have $d=5$.		
Thus, we have $A(1)=6$.	1A	
	----------(2)	
(b) $\log_8(G(1)G(2)G(3)\cdots G(k))<999$		
$\frac{\log_2(G(1)G(2)G(3)\cdots G(k))}{\log_2 8}<999$	1M	
$\log_2(G(1)G(2)G(3)\cdots G(k))<2\,997$		
$\log_2 G(1)+\log_2 G(2)+\log_2 G(3)+\cdots+\log_2 G(k)<2\,997$	1M	
$A(1)+A(2)+A(3)+\cdots+A(k)<2\,997$		
$\frac{k}{2}(2(6)+(k-1)(5))<2\,997$	1M	
$5k^2+7k-5\,994<0$		
$\frac{-7-\sqrt{7^2-4(5)(-5\,994)}}{2(5)}<k<\frac{-7+\sqrt{7^2-4(5)(-5\,994)}}{2(5)}$	1M	
$-35.33076667<k<33.93076667$		
Thus, the greatest value of k is 33 .	1A	
	----------(5)	

Solution	Marks	Remarks
18. (a) Let P be a point lying on AD such that $AB // PC$.		
By sine formula, we have		
$\dfrac{CD}{\sin\angle CPD}=\dfrac{CP}{\sin\angle CDP}$	1M	
$\dfrac{CD}{\sin 50^\circ}=\dfrac{45}{\sin 70^\circ}$		
$CD \approx 36.68433611$ cm		
$CD \approx 36.7$ cm	1A	r.t. 36.7 cm
	----------(2)	
(b) (i) $AE = AB\cos\angle BAE = 45\cos 50^\circ \approx 28.92544244$ cm		
$DE = BC + CD\cos\angle CDE \approx 40 + 36.68433611\cos 70^\circ \approx 52.54678189$ cm		
AD		
$=\sqrt{AE^2+DE^2}$	1M	either one
$\approx \sqrt{(28.92544244)^2+(52.54678189)^2}$		
≈ 59.98204321 cm		
Note that $\angle ABC = 90^\circ$.		
AC		
$=\sqrt{AB^2+BC^2}$		
$=\sqrt{45^2+40^2}$		
≈ 60.20797289 cm		
By cosine formula, we have		
$\cos\angle CAD = \dfrac{AC^2+AD^2-CD^2}{2(AC)(AD)}$	1M	
$\cos\angle CAD \approx \dfrac{(60.20797289)^2+(59.98204321)^2-(36.68433611)^2}{2(60.20797289)(59.98204321)}$		
$\angle CAD \approx 35.54210789^\circ$		
$\angle CAD \approx 35.5^\circ$	1A	r.t. 35.5°
(ii) Let Q be the foot of the perpendicular from A to CD.		
The angle between the plane ACD and the plane $BCDE$ is $\angle AQE$.	1M	
$\dfrac{(AQ)(CD)}{2}=\dfrac{(AC)(AD)\sin\angle CAD}{2}$		
$\dfrac{(AQ)(36.68433611)}{2}\approx\dfrac{(60.20797289)(59.98204321)\sin 35.54210789^\circ}{2}$		
$AQ \approx 57.22631076$ cm		
$\sin\angle AQE = \dfrac{AE}{AQ}$		
$\sin\angle AQE \approx \dfrac{28.92544244}{57.22631076}$		
$\angle AQE \approx 30.36169732^\circ$		
Since $\angle AQE > 30^\circ$, the angle between the plane ACD and the plane $BCDE$ exceeds 30°.	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
19. (a) $f(x)$		
$= x^2 - 12kx - 14x + 36k^2 + 89k + 53$		
$= x^2 - 2(6k+7)x + (6k+7)^2 - (6k+7)^2 + 36k^2 + 89k + 53$	1M	
$= (x - 6k - 7)^2 + 5k + 4$		
Thus, the coordinates of Q are $(6k+7, 5k+4)$.	1A	
	----------(2)	
(b) $(7-6k, 5k+4)$	1M	
	----------(1)	
(c) (i) The slope of the straight line passing through Q and S $= \dfrac{5k+4-(4-3k)}{6k+7-7} = \dfrac{4}{3}$		
The required equation is		
$y - (4-3k) = \dfrac{4}{3}(x-7)$	1M	
$4x - 3y - 9k - 16 = 0$	1A	or equivalent
(ii) Let r be the radius of C .		
Note that $QS = RS$.		
So, the coordinates of the centre of C are $(7, 5k+4-r)$.	1M	
Hence, the equation of C is $(x-7)^2 + (y-5k-4+r)^2 = r^2$.		
Putting $y = \dfrac{4x-16}{3} - 3k$ in $(x-7)^2 + (y-5k-4+r)^2 = r^2$,		
we have $(x-7)^2 + \left(\dfrac{4x-16}{3} - 3k - 5k - 4 + r\right)^2 = r^2$	1M	
$25x^2 + (24r - 192k - 350)x + 576k^2 - 144kr + 1344k - 168r + 1225 = 0$		
Since QS is a tangent to C , we have		
$(24r - 192k - 350)^2 - 4(25)(576k^2 - 144kr + 1344k - 168r + 1225) = 0$	1M	
Simplifying, we have $r^2 + 9kr - 36k^2 = 0$.		
Therefore, we have $r = 3k$ or $r = -12k$ (rejected).		
Thus, the equation of C is		
$(x-7)^2 + (y-5k-4+3k)^2 = (3k)^2$		
$(x-7)^2 + (y-2k-4)^2 = 9k^2$	1A	$x^2 + y^2 - 14x - (4k+8)y - 5k^2 + 16k + 65 = 0$
(iii) For $ST // VU$, the slope of UV is equal to the slope of QS .	1M	
Therefore, we have $\dfrac{-14-(2k+4)}{-29-7} = \dfrac{4}{3}$.		
Solving, we have $k = 15$.	1A	
The coordinates of S and U are $(7, -41)$ and $(7, 34)$ respectively.		
The slope of SV $= \dfrac{-14+41}{-29-7} = \dfrac{-3}{4}$		
So, the product of the slope of QS and the slope of SV is -1 .		
Hence, we have $ST \perp SV$.		
Since $ST \perp TU$, we have $SV // TU$.		
When $k = 15$, we have $ST // VU$, $SV // TU$ and $ST \perp TU$.		
Thus, it is possible that $STUV$ is a rectangle.	1A	f.t.

Solution	Marks	Remarks
Note that T lies on QS and $QR = 12k = 2QT$. So, we have $QT = 6k$. Let $\left(t, \dfrac{4t-16}{3} - 3k\right)$ be the coordinates of T. $(t-6k-7)^2 + \left(\dfrac{4t-16}{3} - 3k - 5k - 4\right)^2 = (6k)^2$ $25(t-7)^2 - 300k(t-7) + 576k^2 = 0$ $t = \dfrac{12k}{5} + 7$ or $t = \dfrac{48k}{5} + 7$ (rejected) Hence, the coordinates of T are $\left(\dfrac{12k}{5} + 7, \dfrac{k}{5} + 4\right)$.		
For $ST = UV$, we have $\left(\dfrac{12k}{5} + 7 - 7\right)^2 + \left(\dfrac{k}{5} + 4 + 3k - 4\right)^2 = (7+29)^2 + (2k+4+14)^2$ Simplifying, we have $12k^2 - 72k - 1620 = 0$.	1M	
Solving, we have $k = 15$ or $k = -9$ (rejected). The coordinates of S, T and U are $(7, -41)$, $(43, 7)$ and $(7, 34)$ respectively. $SV^2 = (7+29)^2 + (-14+41)^2 = 2025$ $TU^2 = (7+29)^2 + (34-7)^2 = 2025$ Therefore, we have $SV = TU$. Also note that $ST \perp TU$.	1A	
When $k = 15$, we have $ST = UV$, $SV = TU$ and $ST \perp TU$. Thus, it is possible that $STUV$ is a rectangle.	1A	f.t.
	----------(9)	

Paper 2

Question No.	Key	Question No.	Key
1.	B (73)	26.	A (43)
2.	C (87)	27.	C (35)
3.	D (82)	28.	D (60)
4.	B (76)	29.	B (88)
5.	D (74)	30.	A (77)
6.	D (49)	31.	A (65)
7.	A (76)	32.	D (41)
8.	B (29)	33.	C (30)
9.	A (62)	34.	C (46)
10.	C (68)	35.	A (33)
11.	C (73)	36.	A (54)
12.	B (74)	37.	B (27)
13.	C (83)	38.	B (34)
14.	A (52)	39.	D (37)
15.	C (43)	40.	A (31)
16.	A (34)	41.	C (33)
17.	B (50)	42.	D (62)
18.	B (72)	43.	D (44)
19.	D (36)	44.	C (76)
20.	C (34)	45.	B (41)
21.	D (40)		
22.	A (56)		
23.	B (65)		
24.	D (64)		
25.	C (36)		

Note: *Figures in brackets indicate the percentages of candidates choosing the correct answers.*