Guitar effects pedal (analogue)

by

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This Report is submitted in partial fulfilment of the requirements of the Honours Degree in Computer & Communications Engineering (DT021A) of the Technological University Dublin

May 23rd, 2022

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1 Introduction & Objectives

This project is about building an analogue guitar pedal, this is a device controlled by the guitarist's feet to change the sound of the guitar. In simple terms, a guitar pedal takes in the guitar signal as an input, applies a specified effect to the signal and this becomes the output.

This will require the research of various guitar pedal effects and how they change a guitar's output signal. This research will inform the choice of pedal to implement. Considerations include limited available components and project time window. The project will make use of simulation software such as Cadence PSpice software for circuit design and use signal processing software during design validation before building the pedal effect.

Figure 1 shows an example of a guitar effects pedal called the "VOX V847 Pedal", this is a Wah-Wah pedal and is an example of a typical guitar pedal.

Figure 1 VOX V847 Wah-Wah Pedal [1]

1.1 Objectives

The project aims to create one analogue guitar pedal. Once built, the aim was to plug a ukulele into the device and have a speaker connected at the output to hear to the effect in action.

A high-level view of various guitar pedal effects was investigated. This included an investigation into how they modify a signal in the time and frequency domains. One pedal effect was then chosen for implementation. This included more in-depth research into this effect along with practical considerations that would need to be considered during its implementation. The parameters of the ukulele signal were then investigated, and component values were calculated to work within a designed specification. After simulation on PSpice, the chosen pedal circuit was constructed on a breadboard for use with a ukulele and speaker. Difficulties were encountered in creating the analogue guitar pedal due to the finite selection of electronic components available on campus, ruling out some pedal designs. Other difficulties encountered were electrical noise, limitations of components themselves such as operating voltage & tolerances and component failures.

Objectives of the project:

- Investigate common guitar effects pedals.
- Breakdown pedal effects into categories and identify a suitable effect to implement.
- Thoroughly investigate the underlying mechanism which produces the chosen effect.
- Determine system input and output parameters
- Derive design equations for the filter.
- Use simulation software to validate the design parameters.
- Analyse filter e.g. frequency and phase responses, poles & zeros via MATLAB & PSpice.
- Implement the software-validated design on a breadboard for hardware verification.
- Build onto a prototyping board, attach audio jacks for input and output connections.

2 Ethical Considerations

When undertaking this project, it was essential to maintain a high degree of professional conduct. As an engineer this was done by being mindful of the practices and principles laid out by Engineer's Ireland Code of Ethics (EICE) [24]. This ensures an ethically and fully transparent project is undertaken. As part of this, the following topics were taken into consideration:

- Accessibility
- Appropriately credited work
- Efficient consumption of natural resources
- Intellectual property

2.1 Accessibility

EICE Code 1.1 and Code 1.4 highlight the importance of accessibility. The documentation of this project will contain schematic designs and MATLAB code. It is therefore important to ensure the designs are labelled appropriately and the code be as clear as possible for those not previously familiar with the MATLAB programming language. For the written documentation the European standard EN 17161:2019 was referred to in making the document accessible to all. This was accomplished with the use of headings to structure the document, changing the font were deemed necessary and using pre-set formats such as bullets, numbering and tables for formatting.

2.2 Appropriately credited work

EICE Code 1.3 states that all work and professional achievements must be properly credited. This means that as part of the project, it must be ensured that all credit is given for work not completed by the author and should be referenced properly using Institute of Electrical and Electronic Engineers (IEEE) referencing.

2.3 Efficient consumption of natural resources

EICE Code 2.3 states that members shall strive to accomplish the objectives of their work with the most efficient consumption of natural resources. Effort was made to use simulation software OrCAD PSpice as much as possible in the design process to verify schematic design before needing to conduct testing using equipment such as oscilloscope & function generator, by simulating the circuitry the waste electronics components were reduced. This action, however small, contributes towards reducing the fossil fuels which accounted for 87% of Irelands total primary energy supply in 2019 [26]. The domino effect of this impacts the United Nations sustainable development goal 13, in accordance with the Paris Agreement of reaching net-zero carbon dioxide CO2 emissions globally by 2050 [27].

2.4 Intellectual Property

During research into the pedals there was a prevalence of clone guitar pedals which leads to open the discussion about the ethics of creating clones. With readily available circuit designs available online for certain pedals, the barriers to preventing one from using the same designs are few. However, there are legal considerations that one must consider as the designs are likely to be patented and be the intellectual property (IP) of the brand or manufacturer. Infringing upon the IP of an organisation is not only morally unethical as an engineer but could include legal ramifications under areas of patent & design infringement.

It is important to be aware of patent, design and copyright law as an engineer, particularly during a project such as this where designs of guitar effect pedals are readily available online. The ACM Code of Ethics and Professional Conduct in section 2.3 states to "Know and respect existing rules pertaining to professional work " [25], the term 'rules' include national, and international laws and regulations. The EU Directive 98/71/EC is one of the directives which provide legal protection of designs within the European Union, which was kept in mind during the project if proprietary designs were to be used in any way.

3 Research

3.1 Breakdown of Pedal Effects

The aim is to create an analogue guitar effect, for this to happen research started by breaking down the guitar pedals into categories. Guitar pedals commonly use filters to produce a desired effect, first a selection of pedals were compiled into a mind map broken down by filter effect employed [2].

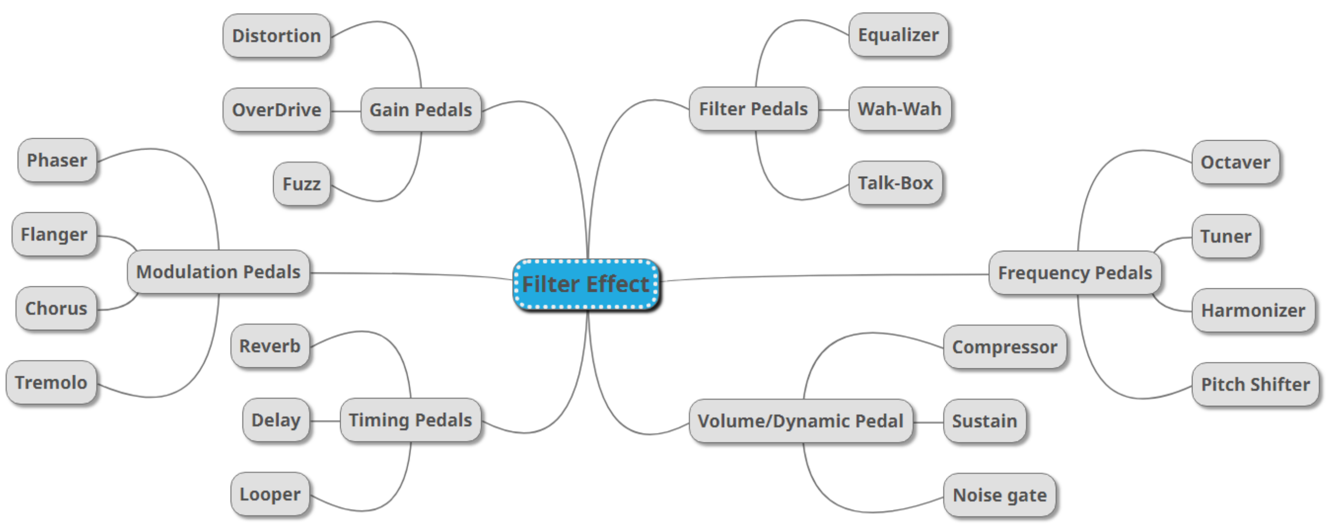


Figure 2 Mind map of filter effects with pedal types

Figure 2 contains a selection of pedal effects, broken down by filter effect employed. As can be seen there are categories of pedals that use a particular filter effect such as the timing pedals which employ a shift in the signal time domain to achieve the effect.

Table 1 contains a short description of the pedals from Figure 2, for longer descriptions refer to Appendix G.

Table 1 List of pedals with short descriptions

Pedal Name	**Overview**
Equalizer	An equalizer (EQ) is a filter that can isolate a range of frequencies for amplification, attenuation or allow through unchanged.
Wah-Wah	As the foot pedal moves it performs a frequency sweep on a bandpass filter. Rocking the pedal back and forth will create the "Wah-Wah effect".
Octaver	This is where the input guitar signal is mixed with a synthesised signal that is one or more octave lower or higher than the original.
Pitch Shifter	It works in real time to shift the frequencies of the input signal.
Compressor	These are used to reduce the dynamic range (the ratio of the loudest to the quietest discernible sound).
Noise Gate	Used to prevent unwanted noise from forming and getting amplified.
Reverb	This pedal simulates the effect caused by sound reflection.
Looper	This pedal records a time interval of the signal and plays it back in a loop.
Phasor	This pedal creates a copy of the signal, this copy is phase shifted and mixed back in with the original signal for the output.
Flanger	Like the Phasor, it also creates a copy of the signal. The copied signal is played at slower speed by modulating the delay time.
Overdrive	When the signal reaches max voltage level, it gets truncated causing frequencies to cut off, termed "clipping". Here signals are caused to clipped on purpose.
Distortion	Like the overdrive pedal except they use multiple stages for more clipping of the signal.

The Wah-Wah effect is the one that is chosen to move forward with for implementation. This is because it uses audio filters which will complement the knowledge gained during the course of study. Basic audio filters such as the low-pass, high-pass, band-pass and band-stop filters were covered during the course and now there is an opportunity to further enhance and use this knowledge in producing a guitar pedal for practical use.

Since the Wah-Wah effect modifies the frequencies in the guitar whose strings typically can produce up to a few hundred hertz in frequency, these changes in frequencies will be easily perceptible to the human ear which has a threshold of approximately 20 – 20,000 Hz [28].

For effects such as the delay and reverb which produces changes in time to the signal would require particular reliance on oscilloscope to ensure the changes in time to the signal occur at the designed time interval.

3.2 Analogue Technique to Produce Wah-Wah Effect

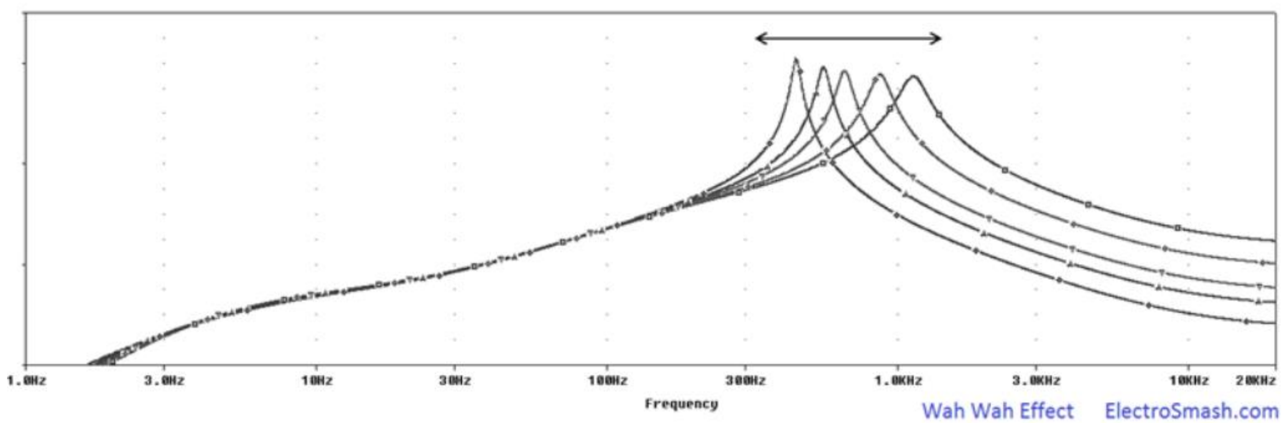


Figure 3 Vox V847 Wah-Wah Frequency Sweep Response [29]

In a guitar pedal the "Wah-Wah" effect is produced using a bandpass filter. This filter amplifies frequencies at the resonant frequency range while all other frequencies above and below this are attenuated. The Wah-Wah effect is controlled using a component whose electrical properties vary.

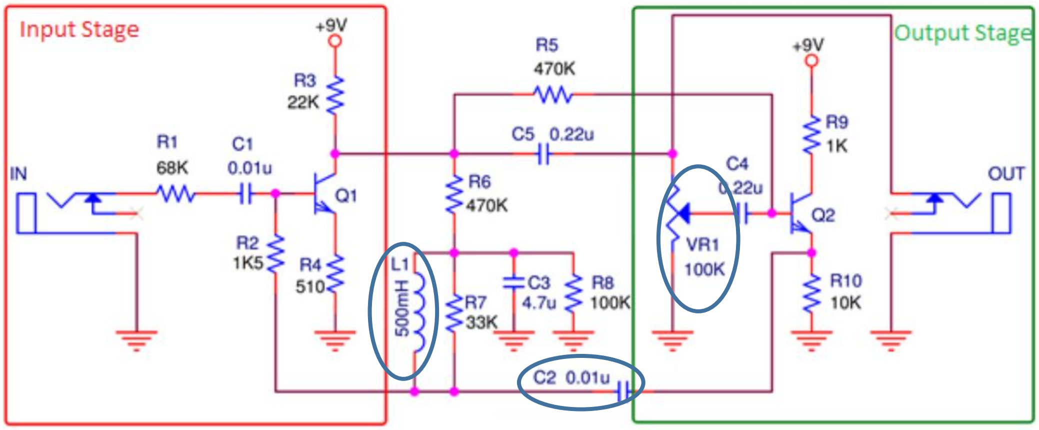


Figure 4 Vox V847 Wah-Wah Pedal Schematic [29]

The Vox V947 pedal in Figure 4 uses two cascaded transistor stages, in this design resonant frequency is controlled by the inductor capacitor (LC) filter made up of a fixed L1 and C2 are modulated using the variable resistor VR1 at the output stage.

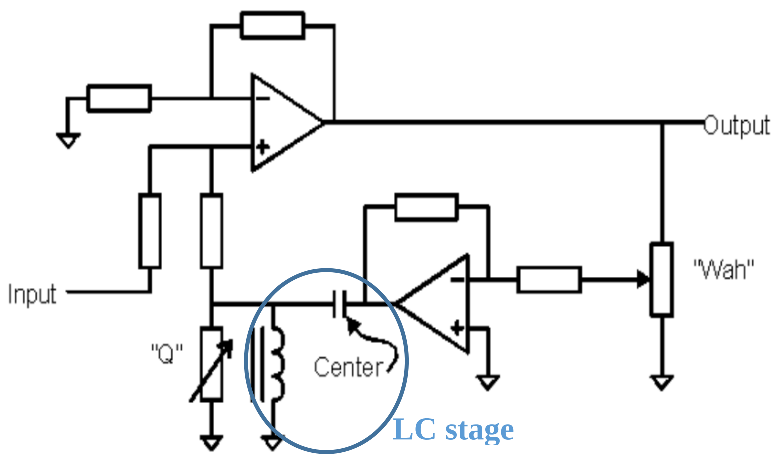


Figure 5 Op-amp Implementation of Inductor Wah-Wah [30]

In Figure 5 shows an implementation of the same effect by R.G Keen using two op-amps. Again, an LC filter stage is used with a variable resistor to control the Wah-Wah effect. There is also an option here to control the Q factor of the filter using a separate variable resistor.

3.3 Filter Background

Due to filters being key to creating many of the pedal effects researched in section 3.1. This section will provide an overview of filters, in relation to its output and input voltages along with the common types of filters and filter terms relevant to the project.

Figure 6 An Unknown filter with Input and Output

Figure 6 shows a voltage source connected to the terminals 1 & 1' of a system. If it is assumed the circuit is operating in the sinusoidal steady state, the input voltage V_1 and output voltage V_2 may be represented by the phasors θ:

$$|V_1| \angle\theta_1 \, and \, |V_2| \angle\theta_2$$

These may be used to define a transfer function such that:

$$Transfer\ function: T = \frac{output\ quantity}{input\ quantity} = \frac{V2}{V1}$$

Then the magnitude of the transfer function T and phase θ is:

$$Magnitude: \ |T| = \frac{|V_2|}{|V_1|} \ \& \ Phase: \theta = \theta_2 - \theta_1$$

In filter design, when describing frequency, we can use angular frequency (ω) or frequency in Hz. Angular frequency is measured in radians per second (rad/s) and can be found using:

$$\text{Radians per second}: \omega = 2\pi f, where\ f\ is\ in\ hertz$$

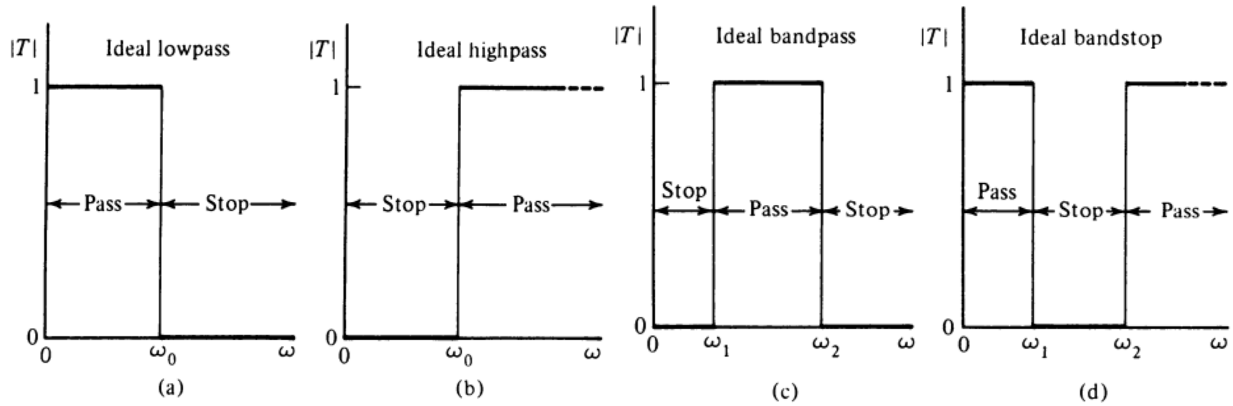


Figure 7 Ideal filters [14]

Filters are categorized in terms of ranges of frequencies, as *passbands* and *stop bands*.

It is the design of the passband & stopband which distinguish the four most common filters [14]:

1. An idealized *Lowpass* filter is one where the passband region extends from $\omega = 0$ to $\omega = \omega_0$ during this time the magnitude is 1. ω_0 is known as the *cut-off frequency* (Figure 7a).
2. An idealized *Highpass* filter is where the stopband region extends from $\omega = 0$ to $\omega = \omega_0$ and from ω_0 to infinity is the passband region (Figure 7b).
3. In an idealized *Bandpass* filter, the only the frequencies within the passband between ω_1 and ω_2 are allowed through (Figure 7c).
4. While an idealized *Stopband* will allow the frequencies outside the stopband region ω_1 and ω_2 through (Figure 7d).

3.3.1 Attenuation and Gain

For linear, time-invariant systems, complex signals can be split up into a set sinusoidal component, each with a unique frequency. A filter works to reject components of a signal by providing *attenuation* over a band of frequencies while retaining other components of a signal which it attenuates or adds *gain* to this region [14].

The unit of gain is the *decibel* (dB). When $|T|$ >1 this implies gain:

$$A = 20\, log\, |T|\, dB, \qquad where\ |T| > 1$$

When the gain is negative i.e., $|T| < 1$ this implies attenuation.

3.3.2 Quality Factor Q

For a bandpass filter the quality factor Q is defined as the ratio of the mid frequency f_m (also known as resonant frequency) to the bandwidth BW at the half power points of -3dB:

$$\text{Quality Factor: } Q = \frac{f_m}{BW}$$

It measures the selectivity of a bandpass filter, the higher the value of Q the smaller a value for bandwidth, making the filter more selective to frequency f_m. To show the influence Q has on a bandpass system it can observed in Figure 8 which shows the normalized gain response of a bandpass filter for different Q values.

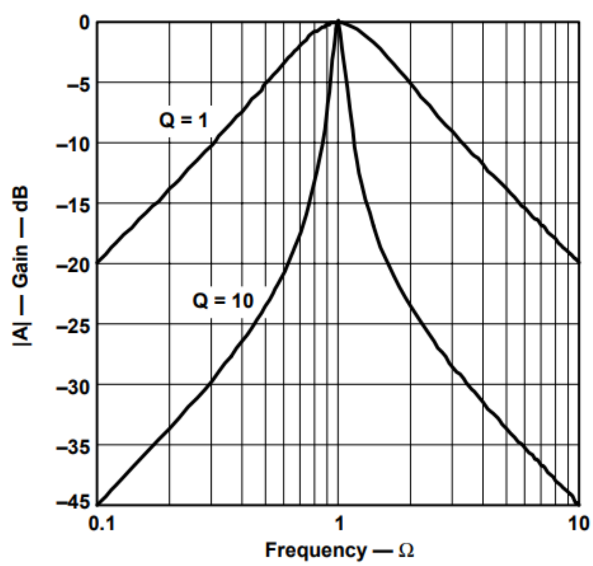


Figure 8 Normalized gain response for bandpass filter at different Q values [15]

Figure 8 shows that with a higher value Q = 10 the filter response gets steeper at the centre frequency, making it more selective to this frequency than when compared with the response at a lower Q value of 1. In the Wah-Wah pedal, the effect becomes better audible to the ear with higher values of Q due to better selectivity at frequency f_m and less of the side band frequencies getting through.

3.4 The Operational Amplifier

The operational amplifier (Op-Amp) has widespread use in analogue electronics, such as preforming arithmetic functions, linear and nonlinear. They are also commonly used in the implementation of active filters and amplifiers.

3.4.1 Ideal Op-Amps

The symbol for an op-amp is shown in Figure 9. It has two inputs, inverting and non-inverting, and one output. The op-amp requires an external power supply (+VDD & -VDD) to operate.

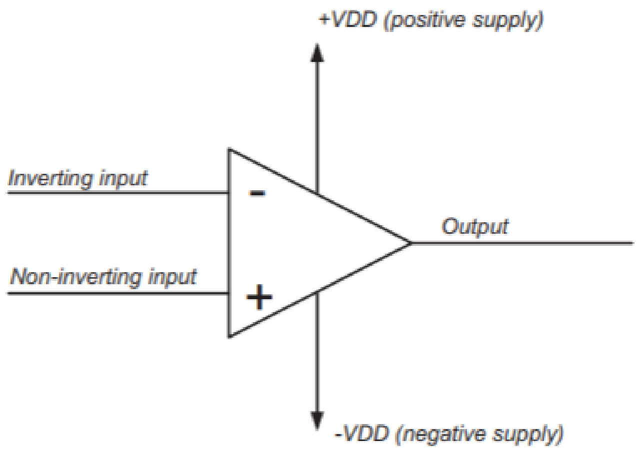


Figure 9 Op-Amp Symbol [16]

It takes around 5Hz change in frequency for humans to distinguish a frequency has changed. This allows for +/-0.5% deviation from a designed resonant frequency to occur without the user perceiving this change [31]. Therefore, for this project it was decided to be sufficient to use an ideal model of an op-amp for the design process so long as the output is within the 0.5% range of the designed resonant frequency during testing. Therefore, the following characteristics were assumed [16]:

a) The open loop gain is infinite. $A_{OL} \rightarrow \infty$.
b) The inverting and non-inverting inputs draw no current therefore, the *input impedance* is infinitely high. $Z_{input} = \infty$.
c) The output can provide an infinitely large current, therefore *output impedance* is zero. $Z_{input} = 0$.

d) *Bandwidth*: An ideal op-amp would have infinite bandwidth because it can react to signals of any frequency. However, bandwidth in a real op-amp refers to small signal bandwidth, that is, signals whose peek-to-peek amplitude is a fraction of the op-amp power supply. For example, signals of a 1 V for a ±15V powered op amp can be considered small signal amplitudes.

e) *Slew rate*: In a real op-amp, for signals which are comparable to the power supply rail magnitude, the op-amp takes a finite amount of time to react to large voltage swings. Sew rate is typically expressed in volts per microsecond.

f) *Offset voltage*: For real op-amps the negative and positive inputs cannot be perfectly matched, therefore this would be a nonzero value. Ideally the difference between them would be zero.

g) Bias current: For real op-amps the negative and positive inputs cannot be perfectly matched. Zero for an ideal op-amp.

h) *Offset current* (difference of bias currents at two inputs): For real op-amps the negative and positive inputs cannot be perfectly matched.

3.4.2 Slew rate

Since the output of the op-amp can only change its output voltage at a finite rate. It is an important op-amp characteristic when deciding on an op-amp to use, as it ensures the amplifier can provide an output that is an accurate representation of the input. Slew rate defines the maximum rate of change of an op-amps output voltage and is given units of volts per microsecond (V/μs). The slew rate for the LM324 is 0.3 V/μs this means that it can output 0.3 volts in a microsecond [17]. The slew rate can cause the output waveform to be distorted if the maximum rate of change is exceeded. The slew rate required for a system can be calculated from the formula below [23]:

$$slew\ rate\ =\ 2\pi fV, where\ f\ is\ frequency\ Hz\ \&\ V\ is\ amplitude\ volts$$

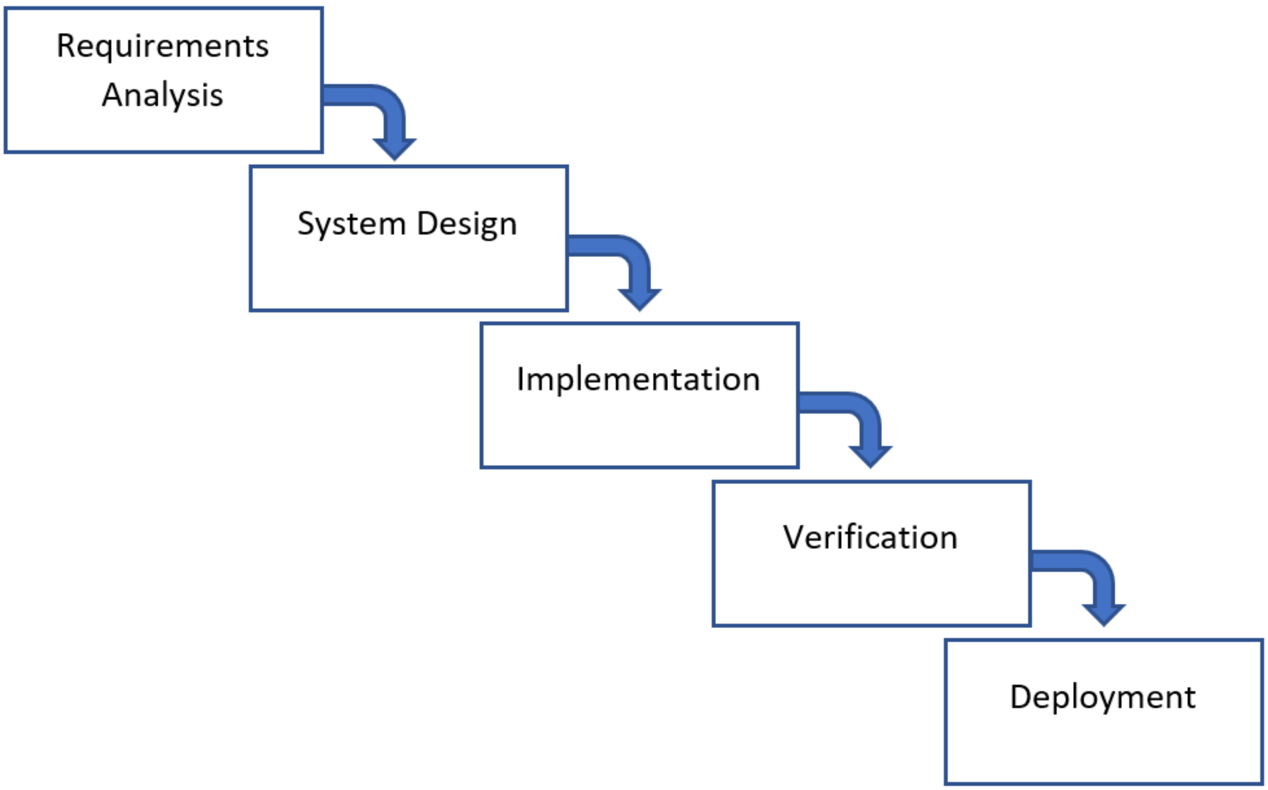
If it is assumed 0.5v as the peak amplitude the system would receive, given the slew rate of the LM324 to be 0.3 V/μs, we can work out the max frequency at which it can still faithfully represent signal.

$$slew\ rate\ \mathrm{V}/\mu\mathrm{s}\ \ =\ 2\pi fV\ \ => 0.3\ \mathrm{V}/\mu\mathrm{s}\ \ =\ 2\pi \cdot f \cdot 0.5v\ \ =>\ f\ =\ \frac{0.3}{\pi} \cdot 10^6 =\ 95.49\ kHz$$

Therefore, if the highest frequency component of our input signal is below 95 kHz, the op-amp will be able to keep up with the rate of change in the signal.

4 Methodology

With the research complete, there was a clear understanding of the factors that would be needed for the implementation of a Wah-Wah pedal. The filter would need to be a bandpass filter, the design must allow for a frequency sweep behaviour controlled via one component. Also, the Q value is to be sufficiently high in order to produce a short bandwidth that will make the filter more selective at the given resonant frequency.



The above waterfall model are stages for the guitar pedal project which were adopted during the development phase of the project, this model is suited for small scale projects where the objectives are well defined and development is done in phases, each phase must be complete before the next phase can occur [32]. The deployment stage represents the point where the pedal is ready for use with ukulele and speaker.

4.1 Requirements Analysis

First step was to tune the ukulele, this was done using a mobile application which listened to the strings played and presented a graphical user interface how of much out of tune the device was. The tuning pegs on the ukulele could then be used to tune to the correct musical note. The Wah-Wah filter that was to be created would be designed with the ukulele that is available on hand to test with. The next step was to determine the input frequency and voltage range the filter must be designed to work within. To do this audio recordings were taken of the ukulele with low and high frequency note played.

Methodology to obtaining ukulele harmonics:

1. Take an audio recording with the highest frequency note of the ukulele played.
2. Using the MATLAB code in Figure 10, produce the frequency plot.
3. Record the first, second and third harmonic frequency values.
4. Repeat steps 1-3 multiple times, resulting in multiple recordings (v1, v2, v3 etc)
5. The results recorded change with the strength applied when plucking the string, therefore the average value for each harmonic was taken.
6. Repeat steps 1-5 with the low frequency string.

```
[sample_data, sample_rate] = audioread('highv2.mp4');

m = length(sample_data);
n = pow2(nextpow2(m));
y = fft(sample_data, n);
f = (0:n-1)*(sample_rate/n);
amplitude = abs(y)/n;
figure(2)
plot(f(1:floor(n/2)),amplitude(1:floor(n/2)))
title('Frequency Domain Representation ')
xlabel('Frequency')
ylabel('Amplitude')
```

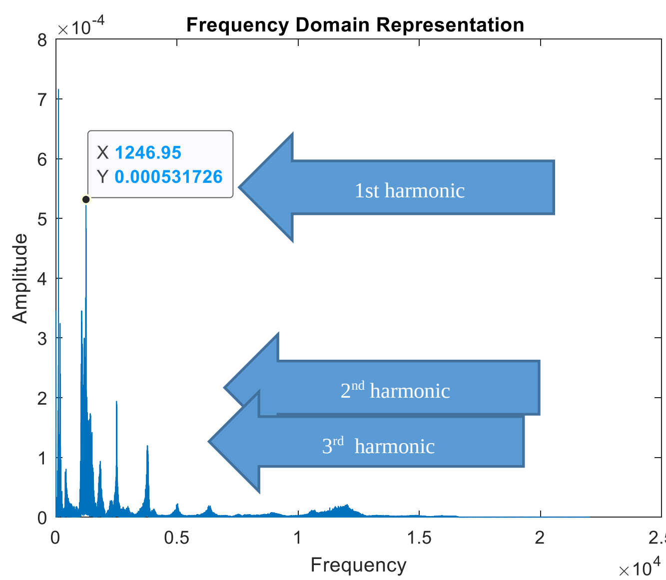


Figure 10 MATLAB Code used to obtain frequency domain plot of audio file: highv2.mp4

In Figure 10 from the plot of frequency domain, data is only collected from first 3 harmonics of the signal because they contain the most energy in terms of voltage and are easily distinguishable visually from the rest of the signal harmonics as the first harmonic is most prominent.

The tabulated results were put into an excel file, the data is shown in Table 3. It was decided to design the filter to work within the range for the first harmonic of low & high frequency notes because the first harmonic contains the most energy in terms of voltage, this can be seen in the Figure 10 where the first harmonic is almost three times more than the second harmonic.

Table 2 Tabulated results of ukulele harmonics

	HARMONIC IN HZ		
HIGH FREQUENCY NOTE	1st	2nd	3rd
HIGHV1	1041	2413	3730
HIGHV2	1246	2507	3775
HIGHV4	1065	2506	3767
HIGHV5	1061	2505	3773
HIGH_CLEANV1	1241	2487	3743
AVERAGE	**1130.8**	**2483.6**	**3757.6**
LOW FREQUENCY NOTE	1st	2nd	3rd
LOWV1	257	515	777
LOWV2	257	516	776
LOWV3	257	516	776
LOWV4	257	515	776
LOW_CLEANV1	256	512	773
AVERAGE	**256.8**	**514.8**	**775.6**

The high frequency note average value at the 1st harmonic is 1130 Hz

The low frequency note average value at the 1st harmonic is 256 Hz

Therefore, a range of 200Hz – 2 kHz will cover all the first harmonics and is now the range to design the system for. While the max amplitude was obtained by connecting an oscilloscope to the ukulele which showed it to be approximately 0.5 volts. The 2 kHz is chosen arbitrarily because it is a factor of 10 times the 200Hz.

4.2 Investigating Feasibility of Passive filter

A filter is termed a *passive filter* when it uses components such as resistors, capacitors, and inductors while an *active filter* uses active components like op-amps, in addition to resistors and capacitors, but not inductors.

Using a multimeter the output impedance of the ukulele is 510k Ω while the input resistance of the speaker is 221k Ω. One of the modules the student completed is Field and Circuit theory where the loading effect was covered; When one circuit is connected to another, there is a loading effect that must be considered due to differences in circuit impedances which would cause a percentage power loss in the transfer to the load system. Typically, impedance matching (both impedances are the same) is used to ensure that all the power from the output goes into the receiving system.

This section will investigate the loading effect in a simple resistor divider circuit shown in Figure 11 and how effective the op-amp is in eliminating this undesirable effect. This will also be informative in determining the feasibility of using a passive filter design for this project.

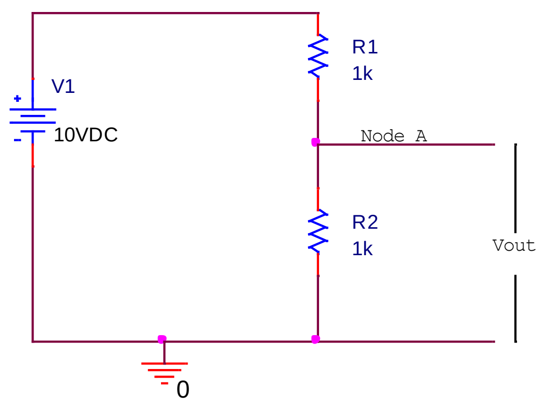


Figure 11 voltage divider configuration

$$Voltage\ Divider: V_{out} = V_{in} \frac{R2}{R2 + R1}$$

The voltage at V_{out} is: $V_{out} = 10v \frac{1K}{1K + 1K} = 5v$

Ideally this circuit will provide 5v to any load circuit connected across V_{out}. However, if a load circuit is connected such as in Figure 12, whose equivalent resistance is modelled by the load resistor R_{load} its value is successively incremented to observe if it can maintain the supply of 5v at node A.

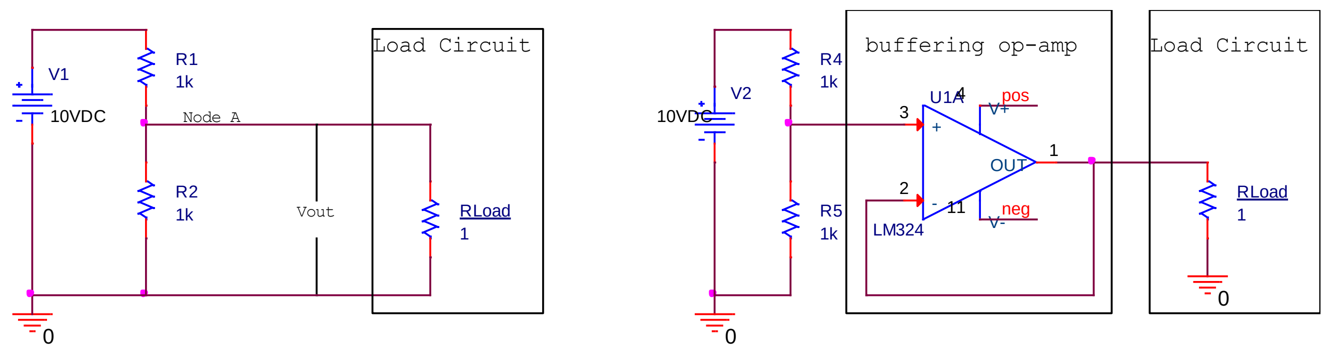


Figure 12 (Left) Resistor divider with resistor at node A, no buffer. (Right) resistor divider load with a resistor after buffering node A

Table 3 Simulation Results: Load effect on circuit with and without buffering op-amp

TEST NO.	R_{load} (ohms)	Voltage **without** buffering Op-amp	Voltage **with** buffering Op-amp
1	1	0.009998	5
2	100	0.8333	5
3	10,000	4.762	5
4	100,000	4.975	5
5	1,000,000	4.998	5

From Table 2 in observing the voltage without the buffering op-amp, it only approaches the ideal 5 V at node A only when the load resistance is a very high value such as 1 MΩ in test 5. It cannot supply 5V when the load resistance is a comparatively low value. Unless the load circuit has a very large resistance, If the load impedance is unknown then it is clear from the results obtained to best use a buffering op-amp that will ensure the intended output voltage. Based on the results from this investigation, a preference will be made for op-amp based designs as it will remove the need to conduct impedance matching with the speaker to be used, which is the intended load to the filter.

A return to the voltage divider equation for the circuit in Figure 11, it is observed that resistor R_{load} is now part of the circuit, and it is connected in parallel with R2 therefore this will affect the voltage out, the equation for voltage out becomes:

$$V_{out} = \frac{(R2||RLoad)}{R1+\ (R2||RLoad)}$$

An ideal op-amp will have infinite input resistance while real world op-amps employ a very high input impedance. The op-amp LM324 to be used has input impedance of 1G Ω [17], this is sufficiently high enough to come close to an ideal op-amp for this project.

4.3 System Design

In the initial design process, bandpass designs of active filter types such as the Sallen-Key and passive bandpass configurations were considered before the investigation in section 4.2 about the feasibility of passive designs. The design to be used would ideally not require inductors, as they are not readily available on campus which would require purchasing online and need to wait for delivery. It was ideal to commence circuit validation as soon as simulation verification is done. By using resistor-capacitor based designs flexibility is gained if changes are made to component values down the line, there will be no need to continually order online. Instead, use is made of the available component values on campus. Most important the design should allow for sweeping of the frequency range with one component to incorporate the sweeping of the frequency domain as this creates the "Wah-Wah" effect.

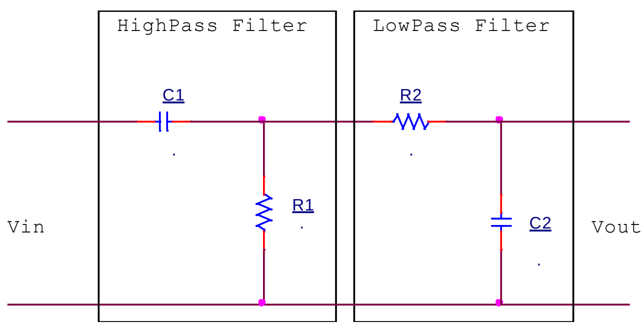


Figure 13 Cascading High Pass with a Low Pass to form Bandpass filter

$$HighPass\ cutoff: F_{Low} = \frac{1}{2\pi R1C1}\ , \qquad LowPass\ cutoff: F_{High} = \frac{1}{2\pi R2C2}$$

A passive cascading high pass with a low pass system as shown in Figure 13 was first considered, however issues of circuit loading was a concern after conducting the loading tests in section 4.2, also by observing the F_{low} & F_{High} equations, there would be two resistors that need to change in unison in order to sweep the frequency domain. This was seen as impractical for the project as coordinating a change in resistance for two components with one mechanical movement would present its own challenge in keeping the resistance values coordinated.

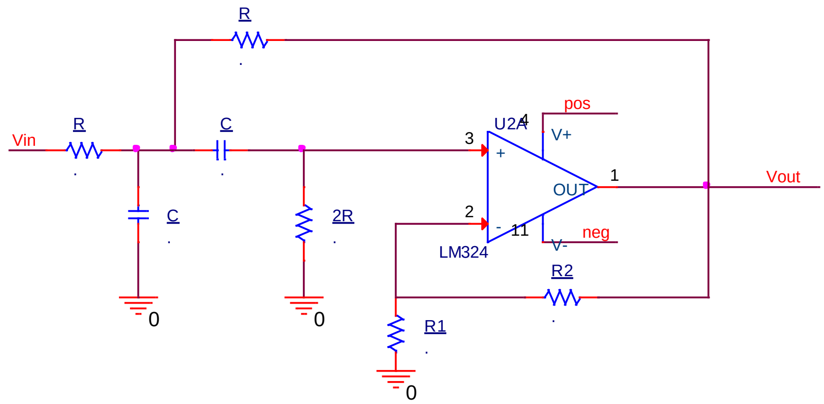


Figure 14 Sallen-Key bandpass filter

$$Sallen - Key\ Bandpass\ mid\ frequency\colon f_m = \frac{1}{2\pi RC}$$

Filter types such as the Sallen-Key bandpass filter shown in Figure 14 would require all the resistor components in the circuit to change in unison to provide the sweep. Instead, the schematic of a multiple feedback bandpass system was found, shown in Figure 15. It was ideal because of the ability to use one component to sweep the frequency range which provides the Wah-Wah effect also it does not use inductors which are not available on campus.

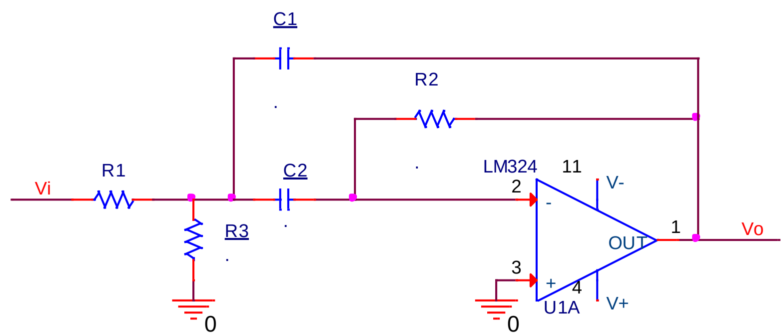


Figure 15 Multiple Feedback Band Pass filter

To derive the multi-feedback bandpass filter (BPF) transfer function with sweep resistor R3. The derivation could be simplified by recognising R1 and R3 form a voltage divider with equivalent resistance Req.

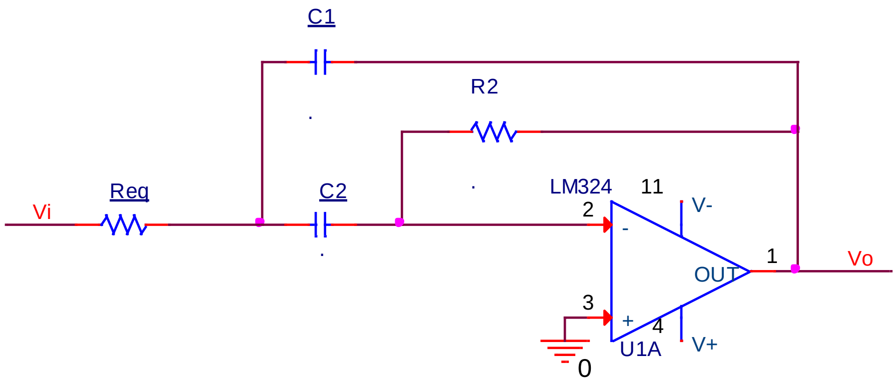


Figure 16 Multiple feedback BPF using equivalent resistance of R1 & R3

This produces the transfer function: (refer to Appendix A for step-by-step derivation)

$$\frac{Vo}{Vi} = \frac{sC2R2}{1 - s^2C1C2ReqR2 + sReq[C1 + C2]}$$

This transfer function is correct as it can be compared with the transfer function given by Sergio Franco [18] for the same configuration in Figure 16 where it is:

$$Multifeedback\ transfer\ function:\ T = \frac{-j\omega R2C2}{1 - \omega^2 ReqR2C1C2 + j\omega Req[C1 + C2]}$$

The transfer function that has been derived does not take into account the behaviour influenced by R1 & R3, the latter of which (R3) is used to sweep the frequency.

R1 & R3 is simply a voltage divider at the input of the circuit therefore:

$$Vo = -Vi * \frac{R3}{R1 + R3} * \frac{sC2R2}{1 - s^2C1C2ReqR2 + sReq[C1 + C2]}$$

$$\frac{Vo}{Vi} = \frac{-s\frac{R2R3}{R1 + R3}C2}{1 - s^2C1C2ReqR2 + sReq[C1 + C2]}$$

Next let C1 = C2 = C

$$\frac{Vo}{Vi} = \frac{-s\frac{R2R3}{R1 + R3}C}{1 - s^2ReqR2C^2 + s\ 2ReqC}$$

Lastly the voltage divider section made up of R1 & R3 shown in Figure 15 has an equivalent resistance equal to that of resistor Req from Figure 16 therefore a substitution will be made into the derived transfer function.

Substitute Req = (R1*R3)/(R1+R3) to give:

$$\frac{Vo}{Vi} = \frac{-s\frac{R2\ R3}{R1 + R3}C}{1 - s\frac{2\ R1\ R3}{R1 + R3}C + s^2\frac{R2\ R1\ R3}{R1 + R3}C^2}$$

The component design equations for R1 and R2 in terms of the resonant frequency ω_o and quality factor Q for the circuit in Figure 16 are: (Refer to Appendix C & D where this is derived)

$$Req = \frac{1}{2\omega_0 CQ} \quad \& \, R2 = \frac{2Q}{\omega_0 C}$$

By mathematical observation of the resonance gain magnitude: $|H_{0BP}| = -2Q^2$ (refer to Appendix B for derivation), this tells that the resonance gain will increase quadratically with Q. It was intended to design the filter with relatively high Q value therefore it is undesirable if the gain increases in a quadratic behaviour as the signal could reach and go beyond the max voltage of the op-amp power supply and damage the load circuit.

By adding the resistor R3 it does two things, firstly it stops the resonance gain from increasing quadratically with Q. Now it will be $|H_{OBP}| < 2Q^2$. Secondly resistor R3 is used to sweep the frequency range with a variable resistor.

After the substitution, the design equation for R2 remaining unchanged, while R1 & R3 design equations become:

$$R1 = \frac{Q}{H_0\omega_0\ C}\ , \quad R3 = \frac{R1}{(\ (2Q^2/H_0) - 1)}$$

4.4 Designing the pedal

The range the filter is to work in is: 200 Hz – 2k Hz. When designing the filter pick the resonant frequency of a value within this range. The chosen resonant frequency to design at was ω_o = 1100 Hz = 6911 rad/s which is within the range of operation. The quality factor Q is designed to be 10, The quality factor is set to a relative high value to produce a short bandwidth that will make the filter more selective at the given resonant frequency. Gain at resonant frequency is set to H_{0BP} = 0 dB = 1 V/V , there was no intention of adding any gain into the signal as it will be connected to a speaker which will already have gain built in.

Next step is to pick a capacitor value for the filter to then use the design equations to calculate the required resistor values of R1, R2 & R3.

Let C = 100nF

$$R2 = \frac{2Q}{\omega_0 C} = \frac{2 * 10}{6911 * 100\text{nF}} = 28.94\ k\Omega$$

$$R1 = \frac{Q}{H_0 \omega_0\ C} = \frac{10}{1 * 6911 * 100nF} = 14.47\ k\Omega$$

$$R3 = \frac{R1}{(\ (2Q^2/H_0) - 1)} = \frac{14.47k}{((2 \cdot 10^2/1) - 1)} = 72.71\ \Omega$$

The resistance value calculated of R3 is only for a resonant frequency of 1100 Hz, to find out the resistance range R3 must become for a frequency range of 200Hz to 2 kHz, get the resonant frequency equation in terms of R3 becomes:

$$Resonant\ \ frequency: \omega_0 = \frac{1}{\sqrt{Req\ R2\ C}}, where\ Req = \frac{R1R3}{R1 + R3}$$

$$Get\ in\ terms\ of\ R3 \text{ becomes}: R3 = \frac{R1}{(2\pi f C)^2 R2(R1 - 1)}$$

To find the required resistor range of R3, sub in the values of R1, R2 and capacitor C then the frequency to find the corresponding resistance of R3:

For 200 Hz, R3 must be 2.1k Ω.

For 2k Hz, R3 must be 21 Ω.

5 Outcomes & Analysis

The filter was first simulated on Cadence PSpice simulation software, this was the quickest route to verifying the design behaved as intended within the design parameters set. After this work moved onto constructing the filter onto a breadboard for validation and testing before the final step in testing with the ukulele and speaker connected.

5.1 Simulation Implementation

The nearest available matching resistors on campus to the calculated values are:

R1: 15k Ω with +/- 5% tolerance,

R2: 33k Ω with +/- 5% tolerance,

R3: variable resistor up to 2k (Recall R3 must go up to 2k Ω)

To reflect the physical circuit more accurately, the resistance values were measured using a multimeter and these are the values used during simulation of the filter shown in Figure 17.

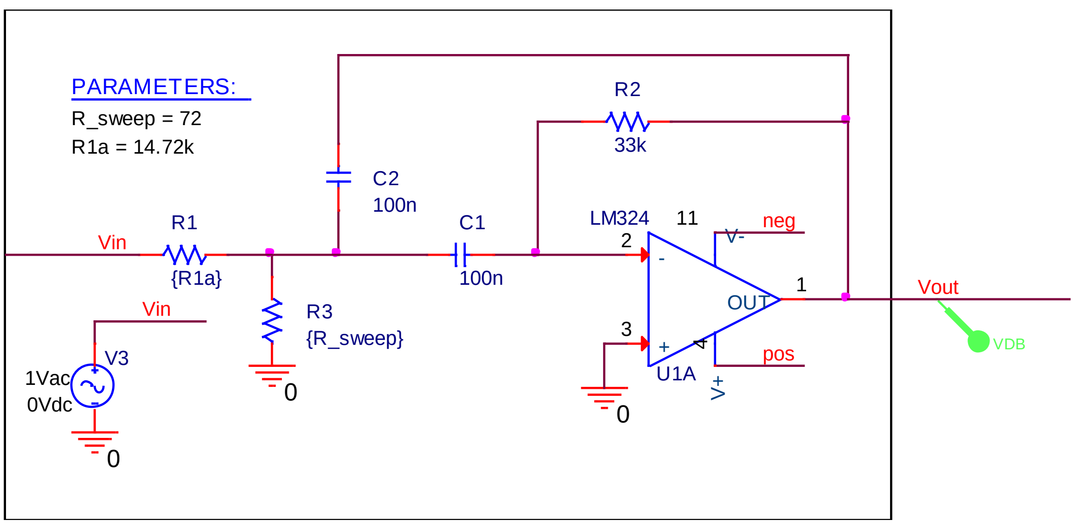


Figure 17 Multi-Feedback Band Pass Circuit Built in PSpice

It was not expected for the frequency response to perfectly match the ideal behaviour of how it was designed due factors such as using the closest matching resistor components and using ideal op-amp component model in PSpice.

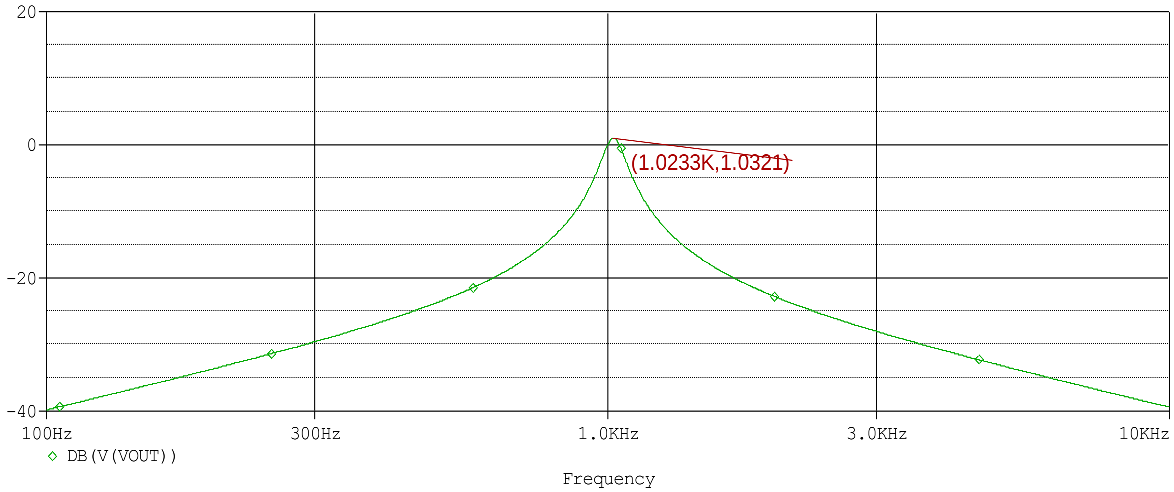


Figure 18 Frequency response of multi-feedback bandpass circuit

In Figure 18 it is observed the resonant frequency has deviated from the designed resonant frequency of 1.1k Hz to 1.02 kHz and there is a 1 dB gain at this point which are acceptable deviations from the expected values. The speaker to be connected is a brand called *Marshal* model: MS-2 which uses an input transistor type C3198 that can handle up to 5 V at the base-emitter junction therefore a 1dB gain represents 0.56 volts out assuming 0.5 V in. This is well within the tolerated range of the transistor inside [34].

Up next simulation was changed to allow the sweep resistor R3 labelled R_{sweep} to be automatically incremented, each execution was plotted on the same graph as shown in Figure 19 to observe the frequency response across the intended frequency range.

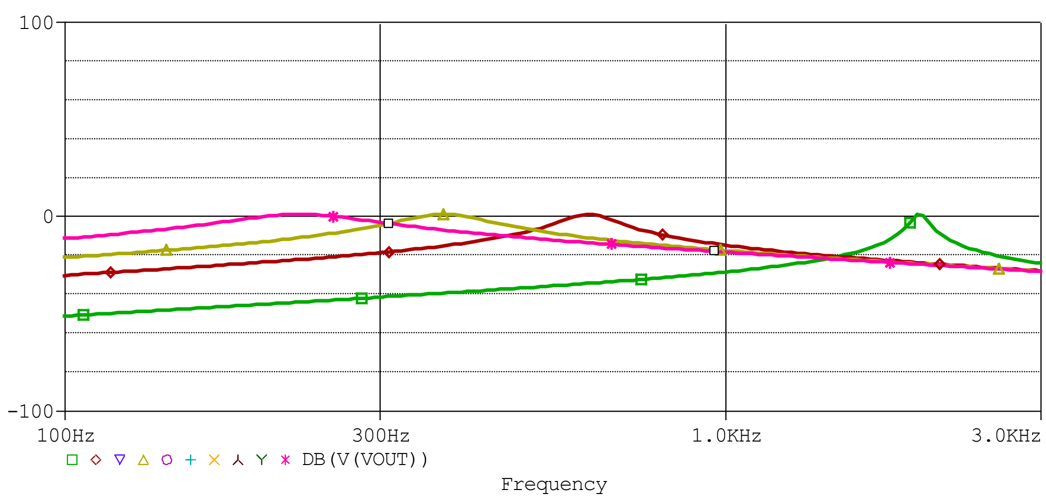


Figure 19 Frequency Response with R3 incremented between 20 Ω - 2K Ω

In Appendix F the behaviour observed in the frequency response of Figure 19 is verified in the time & frequency domain simulations by using a guitar audio file as input, where the sweep resistor is increased iteratively. The plots show the resonant frequency of the filter changing along the frequency domain to the expected resonant frequency values. Using MATLAB to represent the derived transfer function as H, it can then be used to obtain the bode magnitude and phase graphs where it is observed to be near identical results for the value of resonant frequency and the gain at resonant frequency previously obtained in simulation on Figure 18.

```
C = 100e-9; %100nF
R2 = 33e+3;  % kohms
R1 = 14.72e+3; % kohms
R3 = 72;

S_Numerator = -(R2*R3*C)/(R1+R3);
S = (2*R1*R3*C)/(R1+R3);
S2 = ((R2*R1*R3*C^2)/(R1+R3));

b = [ S_Numerator 0 ];
a = [S2 S 1 ];

H = tf(b,a);
figure(1)
bode(H)
```

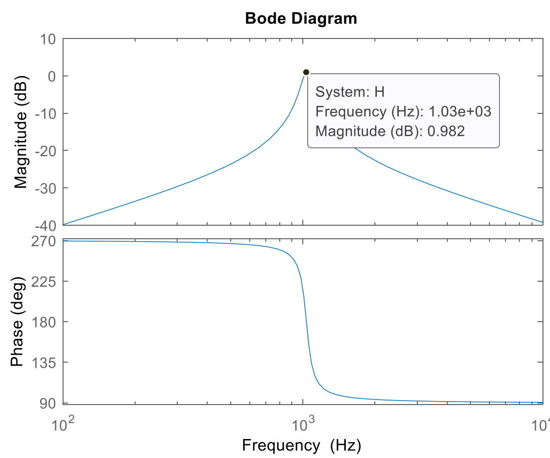


Figure 20 Bode Code and plots of multi-feedback transfer function

With analysis concluded in the time domain, work moved onto the frequency domain to check the stability of the system using the s-plane to plot the Laplace transform roots as shown in Figure 21.

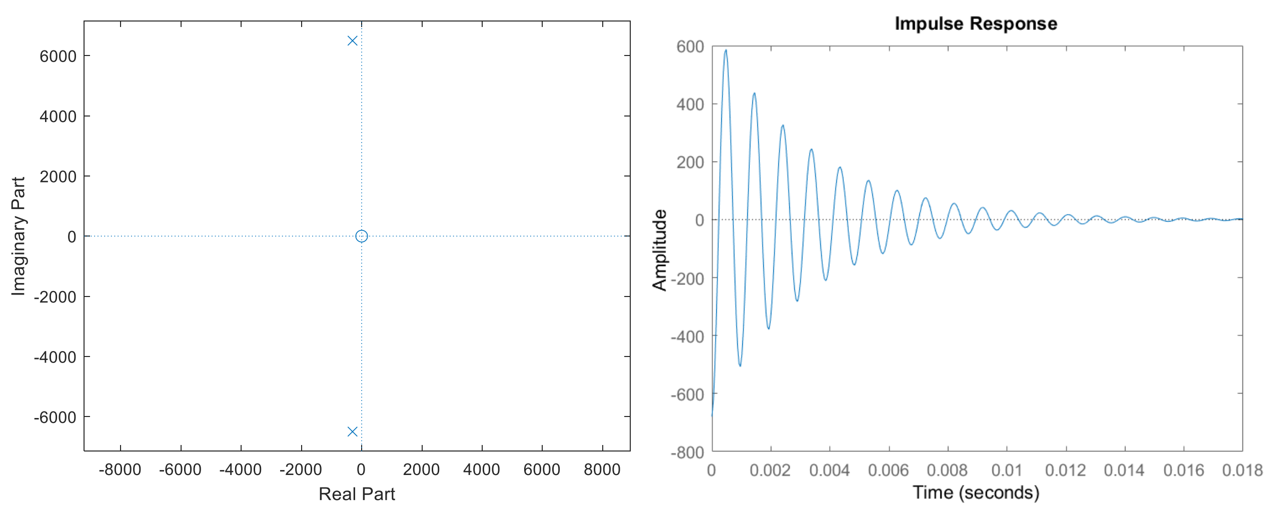


Figure 21 (Left) Roots of the BPF system viewed on S-plane, (Right) Impulse Response of Transfer Function

In reference to Figure 21 left, if the poles (marked by X) on the s-plane were on the right-hand side of the real axis, this implies the system to be unstable where the magnitude would increase indefinitely with respect to time [33]. This would cause the pedal to reach the max voltage as supplied by the op-amp power supply and could damage the speaker that is intended to be connected. However, this is not the case in this instance and shows the system intended to be built to be stable. Poles situated on the left imply a stable system where the magnitude will eventually decay to zero [33]. This behaviour is verified by the system impulse response on the right in Figure 22.

The impulse response of a system is the response to signal that is zero everywhere but at the origin (time: $t = 0$). This is useful in MATLAB as the response to more complicated signals can be found by using a convolution integral of the system impulse response. This convolution method was used to apply the guitar pedal effect to an audio recorded ukulele file on MATLAB to compare the output with the built system in section 5.4 .

5.2 Filter Construction

After design is complete and simulations verify the intended behaviour of the circuit, it was time to construct on a breadboard to conduct tests which will validate the simulations with hardware test results. The LM324 quad operational amplifier IC chip was used for the project.

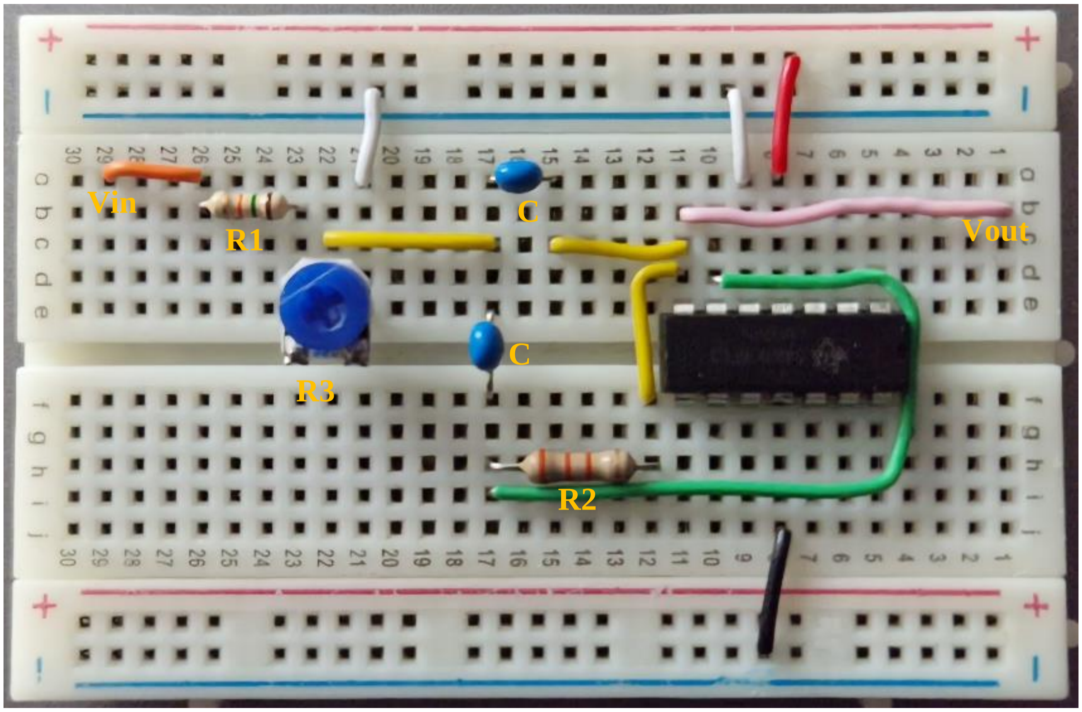


Figure 22 The Filter built onto Breadboard

The layout of the components when constructing onto the breadboard was kept closely to the PSpice circuit in Figure 17, the orange wire represents the input connection V_{in} into R1, and the pink wire represents the output of the filter V_{out}. The red and black are connections for +/- VCC power supply for the op-amp while the white colour wire is used for ground terminal in the circuit.

5.3 Validation Testing

The first round of testing was to verify the behaviour of the filter before connecting to the ukulele and speaker.

To do this the filters' input and output was connected to oscilloscope channels one and two. A function generator was used as the input source. The voltage input and output were recorded for the magnitude response at different frequency intervals. For the phase response the cursor was used to take the time difference (Td) between the input and output, due to this particular oscilloscope not having automatic phase measurement feature, to calculate phase; the time difference was multiplied by a value of 360 and the frequency value used during the test in order to get the phase.

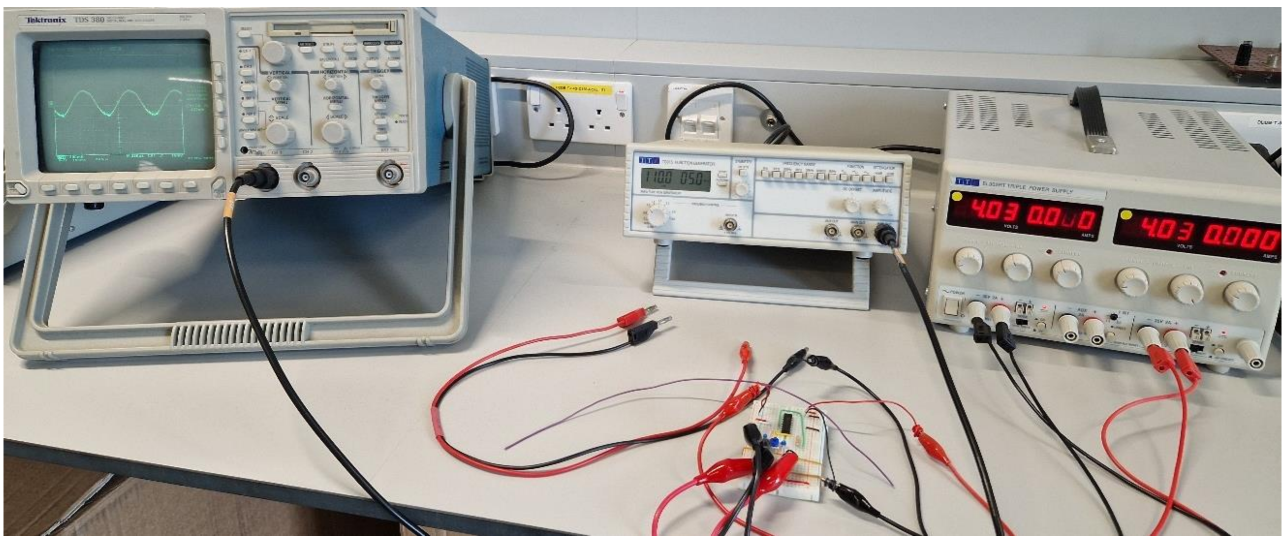


Figure 23 testbench for verifying the response of the filter

Calculation of the expected resonant frequency was done first; this was to be a reference of where the resonant frequency during testing is expected to appear. Using the resonant frequency equation to get in terms of frequency f and using the measured component values of the resistors where R3 was set to 150 Ω.

$$\omega_o = \frac{1}{\sqrt{ReqR2 \ C}}, where\ \omega_o \ = \ 2\pi f \ \&\ Req \ = \frac{R1R3}{R1+R3} \ \ becomes\ f = \frac{1}{\sqrt{ReqR2} \ \ C2\pi} \ = 719\ Hz$$

Therefore, the test results from the data obtained should show the resonant frequency to be at 719 Hz.

The analysis was conducted by incrementing the sinewave frequency from the signal generator and recording the output & input voltages and measuring the time delay between the signals using cursors on the oscilloscope. The tabulated results are included in Appendix H. From this data the bode magnitude plot is obtained in Figure 24.

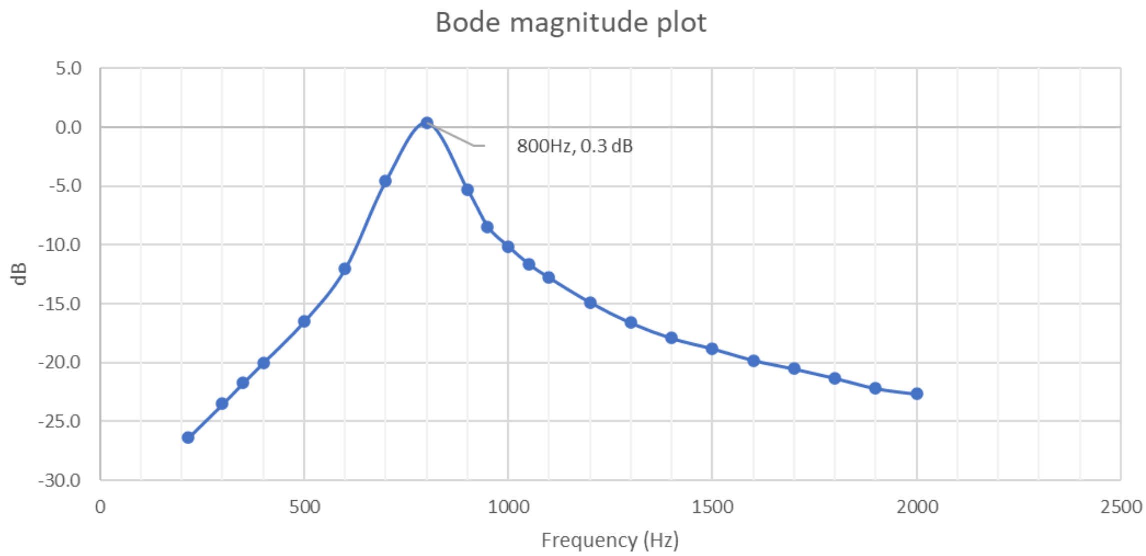


Figure 24 Bode Magnitude plot from Testing

The bode plot from testing shows a resonant frequency of 800 Hz instead of the expected 719 Hz which represnts an 11% error. Once way to double check this was to improve the calcuation of the expected resonant frequency, where the previously it was assumed the capictor values were at an ideal 100 nF. However just like resistors have tolerences the capictor capactiance can be measured and taken into account in the calculation of the expected resonant frequency.

Measured capictor values: C1 = 95nF & C2 = 96.4nF

$$\omega_o = \frac{1}{\sqrt{\frac{R1R3}{R1+R3}R2C1C2}}, \qquad becomes\ f = \frac{1}{\sqrt{\frac{R1R3}{R1+R3}R2C1C2}\ 2\pi} = 751\ Hz$$

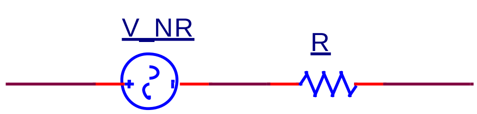
This means that the system actually had 6.5% error. This remaining error could be attributed to factors such as using ideal op-amp characteristics for the design process also parasitic capacitance is a factor due to conducting breadboard rows being close together creating a parasitic charge across pairs of rows that is not accounted for in the design equations.

5.4 Deployment

For deployment, 2 battery packs were connected to provide +/- 3v portable power supply to the op-amp also audio jack wires were connected for the ukulele and connection to the Marshall MS-2 speaker, this is shown in Figure 25.

Figure 25 Wah-Wah pedal connected to ukulele and Marshall MS-2 speaker

During live testing of the setup shown in Figure 26 there was issues where the connection of the audio jack to the speaker needed to be held by hand into the speaker, likely a fault in the cable. During ukulele play there was low electrical noise at the speaker output. Contributing factors of the electrical noise could be the equivalent input noise voltage for the LM324, the data sheet tells that under a test at 1 kHz produces 35 nV/ √ Hz [37], also the resistor noise can be a contributing factor [35] [36]. T is temperature in Kelvin, B is bandwidth in Hz:



$$Resistor\ Johnson\ Noise: V_{NR} = \sqrt{4kTBR}, where\ k\ is\ Boltzmann's\ Constant\ (1.38x10^{-23}\ J/K)$$

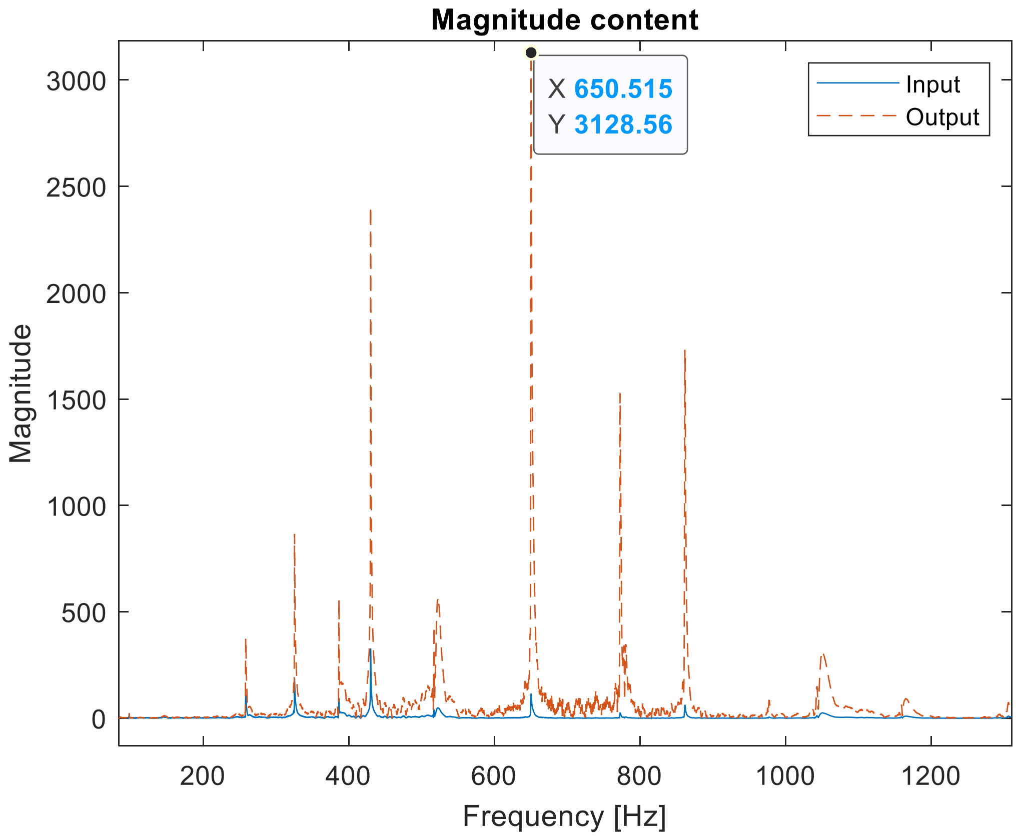


Figure 26 Wah-Wah effect applied to audio recorded file

Afterwards recording was made of the ukulele, this could then be passed into a MATLAB model (code is in Appendix I) of the Wah-Wah pedal where convolution can be used on the input audio file to apply the Wah-Wah effect to the input file for inspection of the frequency spectrum as shown in Figure 26 where there is a clear bandpass shape to the frequency spectrum of the output signal as can be observed by the single frequency at 650 Hz which is the most prominent magnitude while the side band frequencies are getting attenuated the further they are positioned away from this resonant frequency. Note the magnitude axis will not be a faithful representation as the recording device was not directly connected by cable which would have authentically recorded the voltage levels being played. The magnitude is of sound vibrations hitting the microphone which can vary in intensity depending on the distance causing larger magnitudes to appear in the recorded file.

6 Project Conclusion

This project was successful in creating an analogue guitar pedal effect. With the original goal of creating an analogue guitar pedal in mind, the objectives in section 1.1 were created to identify the tasks that would be required to be accomplished. By proceeding the development of the project in a waterfall type of model as laid out in Section 4 the methodology, progress was steady and continuously documented in the logbook. The pedal when connected to the speaker and ukulele produces the Wah-Wah effect, although may be difficult at times to discern with the electrical noise present in the system. Skills of research, analysis, documentation and requirements design were all used and further enhanced during the project.

Future work on this project would revisit the design process to account for electric noise as mentioned in section 5.4 and source closer matching resistor values to the design values. Also, the pedal can be made functional by soldering the circuit onto a prototyping board and creating a mechanical mechanism for a pedal to control the sweep resistor and enclosing the system into a 3D printed housing.

7 Bibliography

[1] "V847 Wah Pedal - Vox Amps", *Vox Amps*. [Online]. Available: https://voxamps.com/product/v847-wah-pedal/. [Accessed: 06- May- 2022].

[2] C. D. Schiebel, "The 23 Types of Guitar Pedals Explained (2022) - Guitar Lobby", *Guitar Lobby*, 2021. [Online]. Available: https://www.guitarlobby.com/types-of-guitar-pedals/. [Accessed: 04- Feb- 2022].

[3] "Music 101: What Is an Equalizer? Plus: Best Equalizer Settings for Drums and Guitar", *www.masterclass.com*,2021.[Online].Available: https://www.masterclass.com/articles/music-101-what-is-an-equalizer-plus-best-equalizer-settings-for-drums-and-guitar#:~:text=An%20equalizer%20(also%20called%20an,Car%20stereo%20systems. [Accessed: 03- Feb- 2022].

[4] "Guitar 101: What Is a Wah-Wah Pedal? Learn About the Different Types of Wah-Wah Pedals", *www.masterclass.com*, 2021. [Online]. Available: https://www.masterclass.com/articles/guitar-101-what-is-a-wah-wah-pedal-learn-about-the-different-types-of-wah-wah-pedals#how-does-the-wahwah-pedal-work. [Accessed: 04- Feb- 2022].

[5] "Music 101: What Is an Octave?", *www.masterclass.com*, 2021. [Online]. Available: https://www.masterclass.com/articles/music-101-what-is-an-octave#what-is-an-octave. [Accessed: 01- Feb- 2022].

[6] A. Fox, "What Are Pitch-Shifting Guitar Pedals & How Do They Work? | My New Microphone", *My New Microphone*. [Online]. Available: https://mynewmicrophone.com/what-are-pitch-shifting-guitar-pedals-how-do-they-work/. [Accessed: 02- Feb- 2022].

[7] A. Fox, "What Are Compressor Pedals (Guitar/Bass) & How Do They Work? | My New Microphone", *My New Microphone*. [Online]. Available: https://mynewmicrophone.com/what-are-compressor-pedals-guitar-bass-how-do-they-work/. [Accessed: 02- Feb- 2022].

[8] S. Beattie, "Guitar Noise Gate Pedals - Your Ultimate Guide from Andertons Music Co.", *Andertons Music Co.*. [Online]. Available: https://www.andertons.co.uk/guitar-noise-gate-

pedal-guide#:~:text=A%20noise%20gate%20pedal%20will,to%20prevent%20that%20annoying%20hum. [Accessed: 02- Feb- 2022].

[9] "How Does A Reverb Pedal Work? - Best Guitar Equipment Q&A", *Best Guitar Equipment*. [Online]. Available: https://www.bestguitarequipment.com/blog/how-does-a-reverb-pedal-work/. [Accessed: 03- Feb- 2022].

[10] A. Fox, "What Are Looper Pedals (Guitar/Bass FX) & How Do They Work? | My New Microphone", *My New Microphone*. [Online]. Available: https://mynewmicrophone.com/what-are-looper-pedals-guitar-bass-fx-how-do-they-work/. [Accessed: 01- Feb- 2022].

[11] J. Chiffey, "Flanger vs Phaser – Understanding the Key Differences - Guitar Space", *Guitar Space*, 2021. [Online]. Available: https://guitarspace.org/sound-pedals/flanger-vs-phaser/#:~:text=Phasers%20work%20is%20frequency%2Dbased,and%20plays%20both%20back%20together. [Accessed: 01- Feb- 2022].

[12] "Guitar 101: What Are Flangers and Phasers? Learn About the Best Flanger and Phaser Pedals for Electric Guitar Players", *www.masterclass.com*. [Online]. Available: https://www.masterclass.com/articles/guitar-101-what-are-flangers-and-phasers-learn-about-the-best-flanger-and-phaser-pedals-for-electric-guitar-players#:~:text=A%20flanger%20works%20by%20mixing,slightly%20slower%20than%20the%20other. [Accessed: 02- Feb- 2022].

[13] R. Pulgar, "The Difference Between Overdrive and Distortion Explained | Gearank", *Gearank.com*, 2020. [Online]. Available: https://www.gearank.com/articles/overdrive-v-distortion. [Accessed: 04- Feb- 2022].

[14] M. Van Valkenburg, *Analog filter design*. New York, N.Y.: CBS College Publishing, 1987, p. 17-25.

[15] R. Mancini and B. Carter, *Op amps for everyone*. Amsterdam: Newnes/Elsevier, 2009.

[16] E. Lipiansky, *Electrical, Electronics, and Digital Hardware Essentials for Scientists and*. John Wiley & Sons, 2013, p289-291.

[17] R. Michallick, Application Design Guidelines for LM324/LM358 Devices. Texas Instruments, 2019.

[18] S. Franco, *Design with operational amplifiers and analog integrated circuits*. New York, N.Y.: McGraw-Hill, 2002.

[19] *LMx24-N, LM2902-N Low-Power, Quad-Operational Amplifiers*. Texas Instruments, 2000.

[20] Texas Instruments, “LMx24-N, LM2902-N Low-Power, Quad-Operational Amplifiers,” LM324-N datasheet, March. 2000 [Revised Jan. 2015].

[21] P. Howorth, *The Wah Wah Book*. Milwaukee, Wis.: Hal Leonard Corp., 1994, p. 6.

[22] P. Pinti, F. Scholkmann, A. Hamilton, P. Burgess and I. Tachtsidis, "Current Status and Issues Regarding Pre-processing of fNIRS Neuroimaging Data: An Investigation of Diverse Signal Filtering Methods Within a General Linear Model Framework", *Frontiers in Human Neuroscience*, vol. 12, 2019. Available: 10.3389/fnhum.2018.00505.

[23] W. Jung, Op Amp applications. Norwood, Mass: Analog Devices, Inc, 2002, p. 148.

[24] “Code of Ethics | Engineers Ireland”, Engineersireland.ie, 2018. [Online]. Available:https://www.engineersireland.ie/Professionals/News-Insights/Resource-centre/Regulations-and-governance/Code-of-ethics.

[25] “Code of Ethics and Professional Conduct”, Acm.org, 2018. [Online]. Available: https://www.acm.org/code-of-ethics.

[26] "Overview - Energy in Ireland - CSO - Central Statistics Office", Cso.ie. [Online]. Available: https://www.cso.ie/en/releasesandpublications/ep/p-sdg7/irelandsunsdgs2019-reportonindicatorsforgoal7affordableandcleanenergy/overview-energyinireland/. [Accessed: 20-May- 2022].

[27] "— SDG Indicators", Unstats.un.org, 2022. [Online]. Available: https://unstats.un.org/sdgs/report/2021/goal-13/. [Accessed: 20- May- 2022].

[28] M. Wereski, "The Threshold of Hearing", STEAM, vol. 2, no. 1, pp. 1-4, 2015. Available: 10.5642/steam.20150201.20 [Accessed 20 May 2022].

[29] "ElectroSmash - Vox V847 Wah Wah", Electrosmash.com. [Online]. Available: https://www.electrosmash.com/vox-v847-wah-wah/pedals/filter/vox-v847-analysis.html. [Accessed: 20- May- 2022].

[30] "The Technology of Wah Pedals", Geofex.com. [Online]. Available: http://www.geofex.com/Article_Folders/wahpedl/wahped.htm#secretcap. [Accessed: 20- May- 2022].

[31] R. Dobie and S. Van Hemel, Hearing loss. Washington, DC: National Academies Press, 2005.

[32] M. Špundak, "Mixed Agile/Traditional Project Management Methodology – Reality or Illusion?", Procedia - Social and Behavioral Sciences, vol. 119, pp. 939-948, 2014. Available: 10.1016/j.sbspro.2014.03.105 [Accessed 20 May 2022].

[33] G. Hunt, Modern power station practice, 3rd ed. Oxford: Pergamon Pr., 1991, pp. 531 & 532.

[34] Elite Enterprises, "C3198 NPN Epitaxial Silicon Transistor", Part No: C3198, [online]. Available: https://datasheetspdf.com/pdf-file/1056951/Elite/C3198/1. [Accessed: 20- May- 2022].

[35] H. Zumbahlen, Linear Circuit Design Handbook. Burlington, MA: Newnes, 2008.

[36] W. Jung, Op amp applications handbook. Amsterdam: Elsevier, 2005.

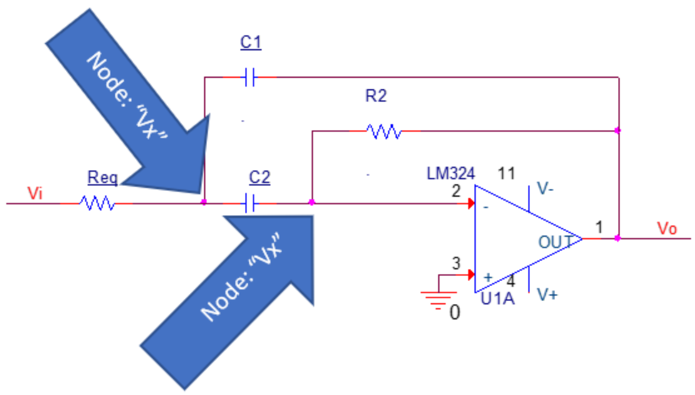
[37] 3PEAK, "LM321/LM358/LM324", Revision: 3, [online]. Available: https://datasheet.lcsc.com/szlcsc/3PEAK-LM358-SR_C93149.pdf. [Accessed: 20- May- 2022].

[38] C. Moler, "Cramer's rule on 2-by-2 systems", *ACM SIGNUM Newsletter*, vol. 9, no. 4, pp. 13-14, 1974. Available: 10.1145/1206085.1206090 [Accessed 15 March 2022].

8 Appendices

Appendix A. Deriving transfer function of MFB filter

multi-feedback bandpass filter circuit without resistor R3:



$$Node\ Vx: \frac{Vx - Vi}{Req} + \frac{Vx - 0}{1/sC2} + \frac{Vx - Vo}{1/sC1} = 0$$

$$\frac{Vx}{Req} - \frac{Vi}{Req} + sC2\,Vx + sC1\,Vx = sC1\,Vo$$

$$\frac{Vx}{ReqsC1} - \frac{Vi}{ReqsC1} + \frac{sC2Vx}{sC1} + \frac{sC1Vx}{sC1} = Vo$$

$$-\frac{Vi}{ReqsC1} + Vx[\frac{1}{ReqsC1} + \frac{sC2}{sC1} + 1] = Vo \qquad , Label\ this\ "A"$$

$$Node\ Vy: \frac{0 - Vx}{1/sC2} + \frac{0 - Vo}{R2} = 0$$

$$-sC2Vx = \frac{Vo}{R2}$$

$$-sC2VxR2 = Vo \qquad , Label\ this\ "B"$$

$$A: \ Vo = -\frac{Vi}{ReqsC1} + Vx[\frac{1}{ReqsC1} + \frac{sC2}{sC1} + 1]$$

$$B: \ Vo = -sC2VxR2$$

Algebraically manipulate A & B into a form for use with Cramer's rule. A & B become:

$$A:\ -Vx\left[\frac{1}{ReqsC1}+\frac{sC2}{sC1}+1\right]+Vo=-\frac{Vi}{ReqsC1}$$

$$B:\qquad Vx[sC2R2]+Vo=0$$

Now apply Cramer's rule (in Appendix E):

$$Vo=\frac{determinate\ of\ \begin{vmatrix}\left[\frac{1}{ReqsC1}+\frac{sC2}{sC1}+1\right] & -\frac{Vi}{ReqsC1}\\ [sC2R2] & 0\end{vmatrix}}{determinate\ of\ \begin{vmatrix}\left[\frac{1}{ReqsC1}+\frac{sC2}{sC1}+1\right] & 1\\ [sC2R2] & 1\end{vmatrix}}$$

Do Numerator first:

$$\left\{\left[\frac{1}{ReqsC1}+\frac{sC2}{sC1}+1\right]*[0]\right\}-\left\{[sC2R2]*\left[-\frac{Vi}{ReqsC1}\right]\right\}=\frac{Vi\,sC2R2}{sC1Req}$$

Denominator next is:

$$\left\{\left[\frac{1}{ReqsC1}+\frac{sC2}{sC1}+1\right]*[1]\right\}-\{[sC2R2]*[1]\}=\frac{1}{ReqsC1}+\frac{sC2}{sC1}+1-sC2R2$$

Now Vo/Vi becomes:

$$Vo=\frac{\frac{Vi\,sC2R2}{sC1Req}}{\frac{1}{ReqsC1}+\frac{sC2}{sC1}+1-sC2R2}$$

$$Vo=\frac{Vi\,sC2R2}{\frac{sC1Req}{ReqsC1}+\frac{sC2sC1Req}{sC1}+sC1Req-sC2R2sC1Req}$$

$$=\frac{Vi\,sC2R2}{1-s^2C1C2ReqR2+sReq[C1+C2]}$$

$$\frac{Vo}{Vi}=\frac{sC2R2}{1-s^2C1C2ReqR2+sReq[C1+C2]}$$

Appendix B. Deriving ω_0, Q, H_{0BP}

The standard second order band-pass is in the form shown below where H_{0BP} is the resonance frequency gain.

$$\mathrm{H}(j\omega) = H_{0BP}H_{BP}(j\omega)$$

$$H_{BP}(j\omega) = \frac{(j\omega/\omega_0)/Q}{1 - \ (\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

We can equate this with the derived transfer function: $\frac{Vo}{Vi} \ = \ \frac{sC2R2}{1 - s^2C1C2ReqR2 + sReq[C1+C2]}$

To find ω_0 :

$$\omega^2 ReqR2\ C1C2 \ = \ \frac{\omega}{\omega_0}^2$$

$$\sqrt{ReqR2\ C1C2} \ = \ \frac{1}{\omega_0}$$

$$\frac{1}{\sqrt{ReqR2\ C1C2}} \ = \ \omega_0$$

To find Q:

$$j\omega Req[C1 + C2] = \frac{j\omega}{\omega_0} \div Q$$

$$Req[C1 + C2] = \frac{1}{\omega_0 Q}$$

$$\frac{1}{Req[C1 + C2]} = \omega_0 Q$$

$$\omega_0 \ \frac{1}{Req[C1 + C2]} \ = \ Q$$

$$sub\ in\ \omega_0 = \ \frac{1}{\sqrt{ReqR2\ C1C2}}$$

$$\frac{\sqrt{ReqR2\ C1C2}}{Req[C1 + C2]} \ = \ Q$$

$$\frac{\sqrt{R2/Req}}{\sqrt{C1/C2} + \sqrt{C2/C1}} \ = \ Q$$

To find the resonance frequency gain, H_{0BP}:

$$-j\omega R2C2 = H_{0BP}\frac{j\omega}{\omega_0} \div Q$$

sub in $\frac{j\omega}{\omega_0} \div Q \ = j\omega Req[C1 + C2]$

$$-j\omega R2C2 = H_{0BP} \ * \ j\omega Req[C1 + C2]$$

$$\frac{-R2C2}{Req[C1 + C2]} = H_{0BP}$$

$$\frac{\frac{-R2C2}{Req}}{C1 + C2} = H_{0BP}$$

$$\frac{\frac{-R2}{Req}}{\frac{C1}{C2} + 1} = H_{0BP}$$

When C1 = C2 = C, these simplify into:

$$\frac{1}{\sqrt{ReqR2\ C}} \ = \ \omega_0, \quad 0.5\sqrt{\frac{R2}{Req}} \ = \ Q \ , \quad -2Q^2 = H_{0BP}$$

Appendix C. Design equation for R1 in terms of Q &ω_0

To derive the design equations for R1 in terms of Q & ω_0

we know:

$$\frac{1}{\sqrt{ReqR2}\ C} = \omega_0, \quad 0.5\sqrt{\frac{R2}{Req}} = Q$$

For Req design equation, get R2 on its own:

$$0.5\sqrt{\frac{R2}{Req}} = Q \Rightarrow \frac{0.5}{Q} = \frac{R2}{Req} \Rightarrow Req\,{\frac{Q}{0.5}}^{2} = R2$$

$$\frac{1}{\sqrt{ReqR2}\ C} = \omega_0 \Rightarrow \frac{1}{\omega_0 C} = \sqrt{ReqR2} \Rightarrow {\frac{1}{\omega_0 C}}^{2} = ReqR2 \Rightarrow \frac{1}{{\omega_0}^2 C^2\ Req} = R2$$

Now let them equal each other to find Req:

$$Req\,{\frac{Q}{0.5}}^{2} = R2 = \frac{1}{{\omega_0}^2 C^2\ Req}$$

$$Req\,{\frac{Q}{0.5}}^{2} = \frac{1}{{\omega_0}^2 C^2\ Req}$$

$${\omega_0}^2 C^2\ Req\ Req\,{\frac{Q}{0.5}}^{2} = 1$$

$$2\ Req[{\omega_0}^2 C^2 {\frac{Q}{0.5}}^{2}] = 1$$

$$2\ Req = \frac{1}{{\omega_0}^2 C^2 {\frac{Q}{0.5}}^{2}}$$

$$Req = \frac{1}{{\omega_0}^2 C^2 \frac{Q}{0.5}} = \frac{1}{2\omega_0 CQ}$$

Appendix D. Design equation for R2 in terms of Q & ω_0

To derive the design equations for R2 in terms of Q & ω_0

we know:

$$\frac{1}{\sqrt{ReqR2}\ C} = \omega_0, \quad 0.5\sqrt{\frac{R2}{Req}} = Q$$

For R2 design equation, get Req on its own:

$$0.5\sqrt{\frac{R2}{Req}} = Q \Rightarrow \frac{0.5}{Q} = \frac{R2}{Req} \Rightarrow \frac{R2}{(\frac{Q}{0.5})^2} = Req$$

$$\frac{1}{\sqrt{R1R2}\ C} = \omega_0 \Rightarrow \frac{1}{\omega_0 C} = \sqrt{R1R2} \Rightarrow \frac{1}{\omega_0 C}^2 = R1R2 \Rightarrow \frac{1}{\omega_0{}^2 C^2\ R2} = R1$$

Now let them equal each other to find R2:

$$\frac{R2}{(\frac{Q}{0.5})^2} = Req = \frac{1}{\omega_0{}^2 C^2\ R2}$$

$$\frac{R2}{(\frac{Q}{0.5})^2} = \frac{1}{\omega_0{}^2 C^2\ R2}$$

$$R2 = \frac{1}{\omega_0{}^2 C^2\ R2}(\frac{Q}{0.5})^2$$

$$R2\ \omega_0{}^2 C^2\ R2 = (\frac{Q}{0.5})^2$$

$$2\ R2[\omega_0{}^2 C^2] = (\frac{Q}{0.5})^2$$

$$R2[\omega_0{}^2 C^2] = (\frac{Q}{0.5})^2 \div 2 = \frac{Q^2}{0.5}$$

$$R2 = \frac{Q^2}{0.5\omega_0{}^2 C^2} = \frac{2Q}{0.5\omega_0{}^2 C^2} = \frac{2Q}{\omega_0 C}$$

Appendix E. Cramer's Rule [38]

Cramer's Rule:

EQ1: a_1x + b_1y = c_1

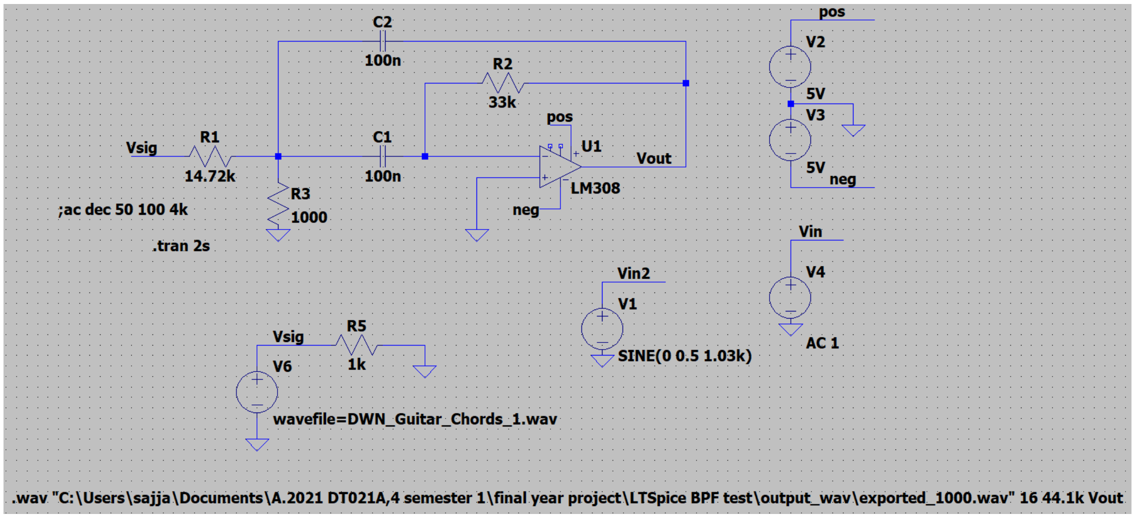
EQ2: a_2x + b_2y = c_2

$$x = \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_1 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1 b_2 - b_1 c_2}{a_1 b_2 - b_1 a_2}$$

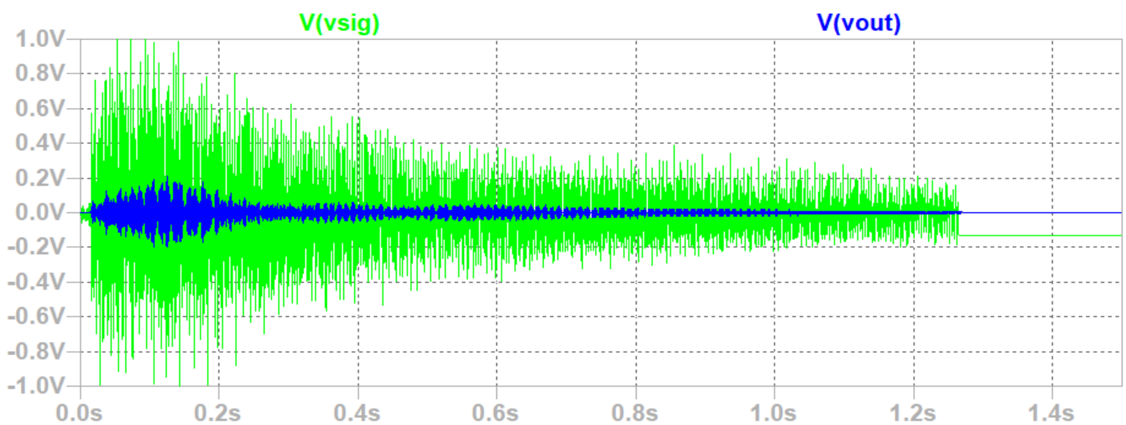
$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_1 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1 c_2 - c_1 a_2}{a_1 b_2 - b_1 a_2}$$

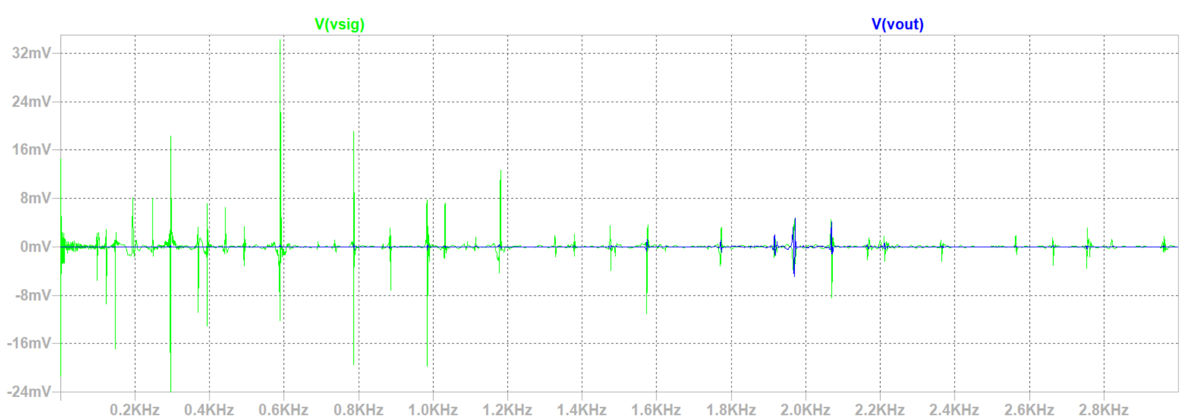
Appendix F. Iterative Frequency sweep on LTspice

Tests to show the sweeping of the filter in the time and frequency domains.

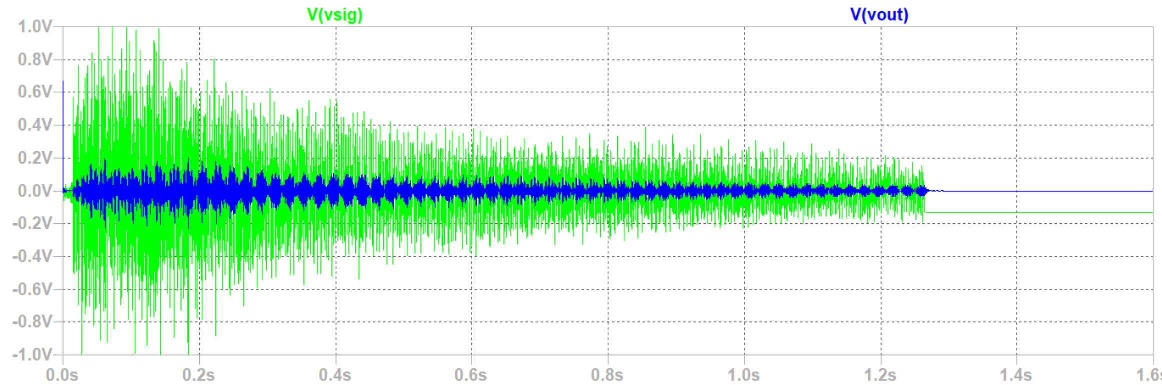


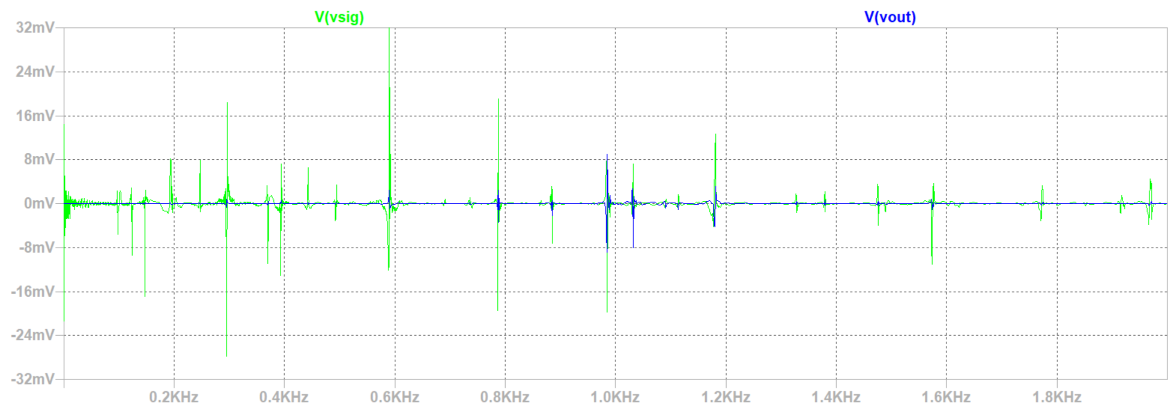
R3 at 20 ohms, resonant frequency = 1960 Hz



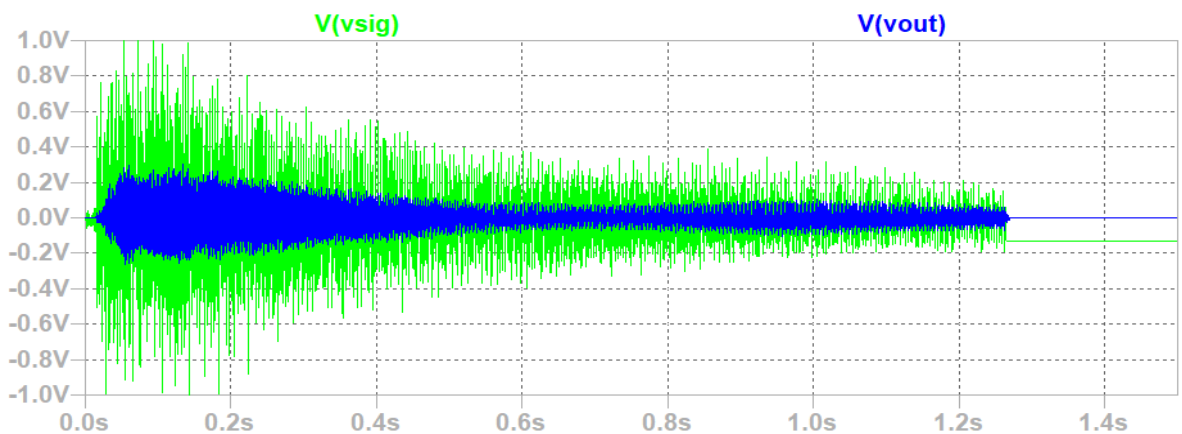


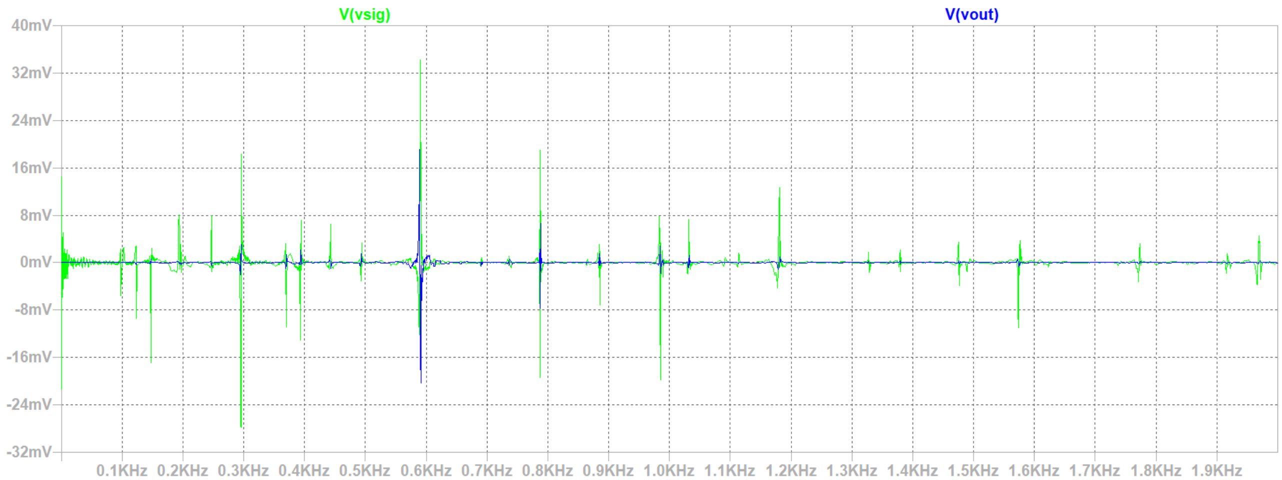
R3 at 70 ohms, resonant frequency = 1047 Hz



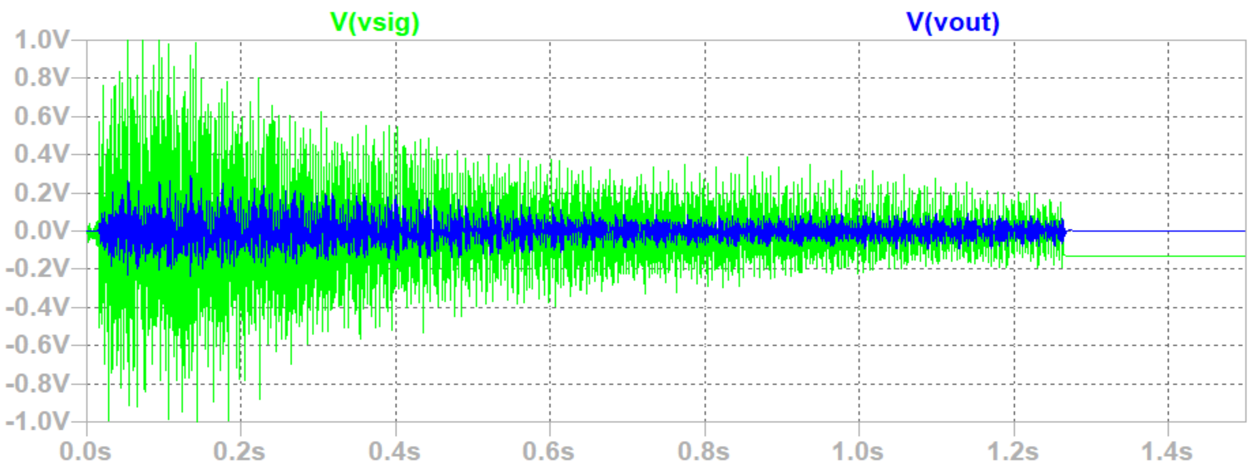


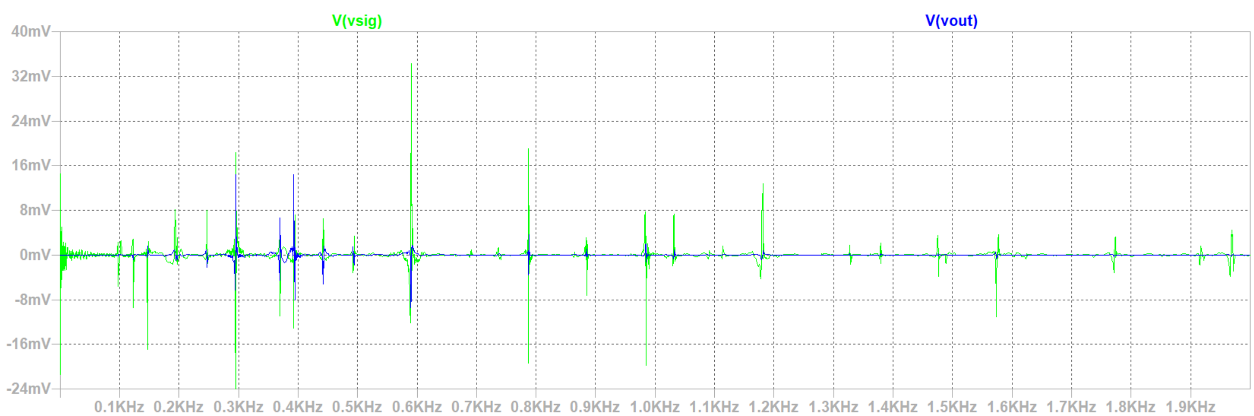
R3 at 200 ohms, resonant frequency = 620 Hz



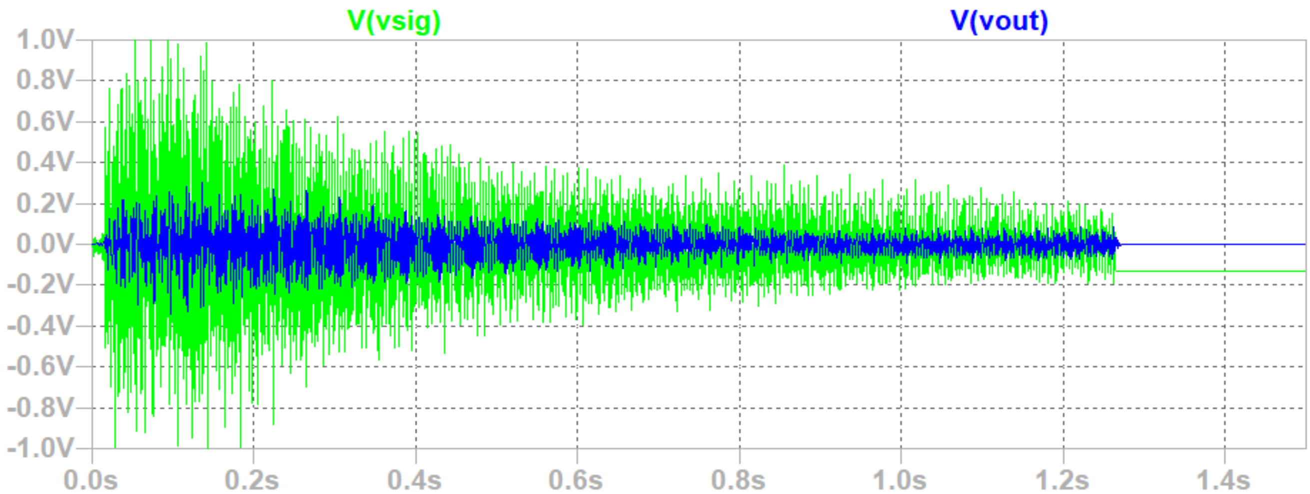


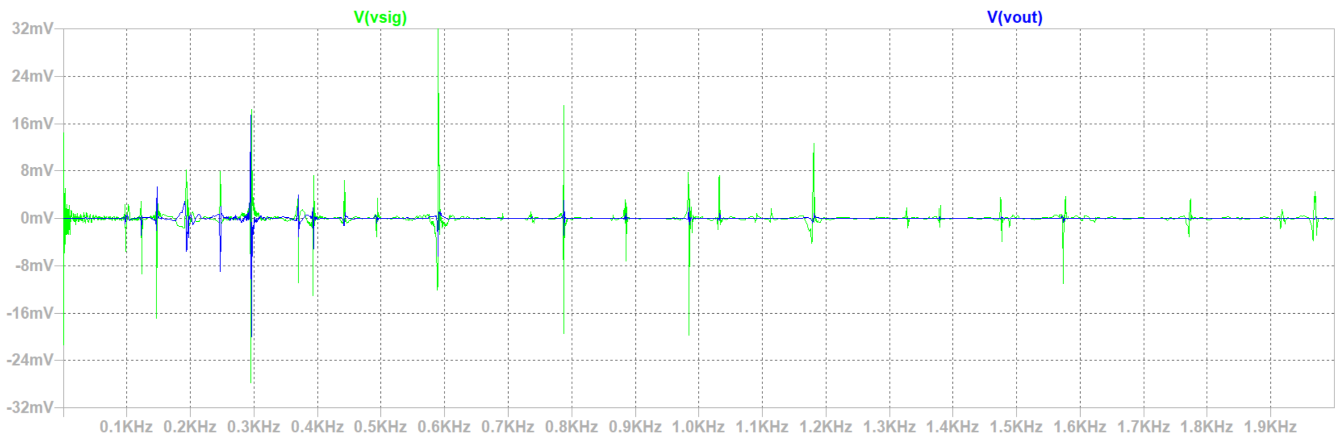
R3 at 500 ohms, resonant frequency = 390 Hz



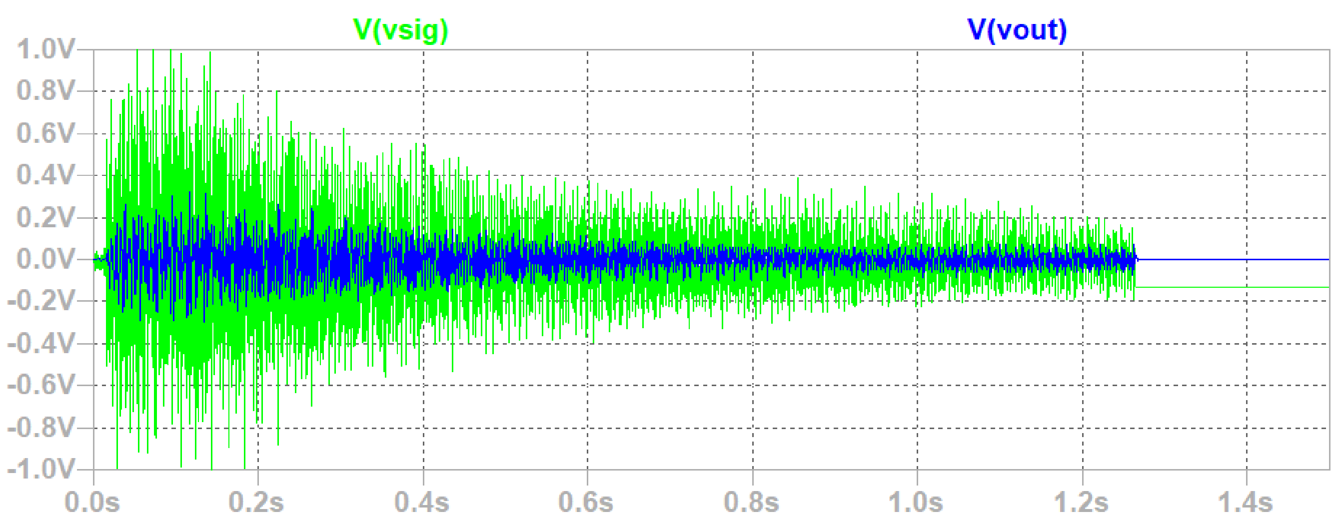


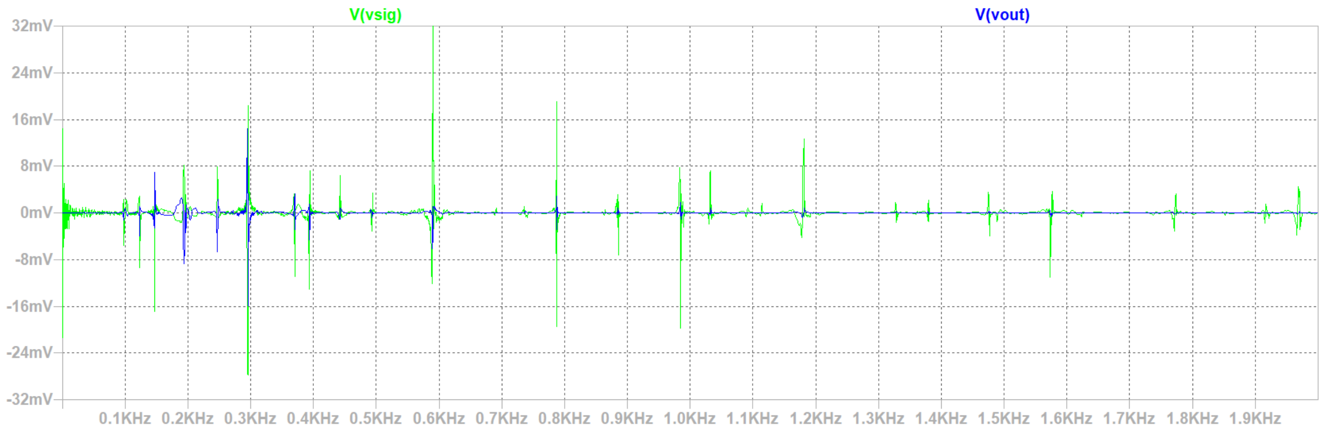
R3 at 1500 ohms, resonant frequency = 226Hz





R3 at 2000 ohms, resonant frequency = 196 Hz





Appendix G. Types of guitar pedals

<u>Filter Pedals</u>

Equalizer: An equalizer (EQ) is a filter that can isolate a range of frequencies for amplification, attenuation or allow though unchanged[3].

Wah-Wah: Inside this is a device called a potentiometer which is a variable resistor. As the rocker plate in a wah-wah pedal moves the potentiometer will enable to a degree either a low-pass or a high-pass filter. Rocking the pedal back and forth will create the “Wah effect”[4].

<u>Frequency Pedals</u>

Octaver: The octave effect is where the input guitar signal is mixed with a synthesised signal that is one or more octave lower or higher than the original. An Octave occurs in music when the higher note has a frequency that is twice that of its lower note[5].

Pitch Shifter: As the name suggests it, works in real time to shift the frequencies of the input signal. They can be set to only output the shifted signal or a combination of the shifted and original signal[6].

Volume/Dynamic Pedals

Compressor: These are used to reduce the dynamic range (the ratio of the loudest to the quietest discernible sound) of a signal by reducing the amplitude of the input signal by a certain ratio if it reaches beyond a set threshold[7].

Noise Gate: This is a common pedal used to attenuate a signal if it drops below a set threshold. It is typically used to prevent unwanted noise from forming and getting amplified by the amplifier[8].

Timing Pedals

Reverb: This pedal simulates the effect caused by sound reflection. Reverb decays the signal as opposed to an echo pedal which delays the signal[9].

Looper: This type of pedal records a time interval of the audio and plays it back in a loop. The pedal itself contains controls to start and stop the recording[10].

Modulation Pedals

Phasor: This pedal creates a copy of the signal; this copy is phase shifted and mixed back in with the original signal for the output[11].

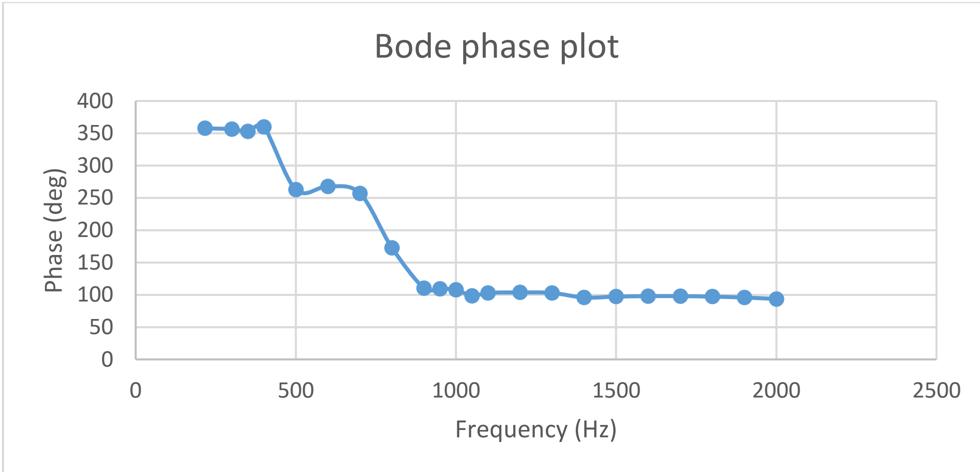
Flanger: This pedal is similar to the Phasor; it also creates a copy of the signal. The copied signal is played at slower speed by modulating the delay time of the copied signal using a low frequency oscillator (LFO). This creates a "whooshing" sound effect due to two recordings playing at the same time[12].

Gain Pedals

Overdrive: when the signal voltage level reaches the max limit of what the hardware can output. This leads to the signal getting truncated and some frequencies getting cut off. When this happens, it's termed "clipping", and this is an overdrive pedal. It causes the signals to clip on purpose[13].

Distortion: Like the overdrive pedal except they use multiple stages for more clipping of the signal[13].

Appendix H. Magnitude and Phase Data from Testing



	Measurements			Calculated			
Frequency (Hz)	Vin (mv)	Vout (mv)	Time delay, Td (ms)	phase (θ)	Vout/Vin	log(Vout/Vin)	20log(Vout/Vin) (dB)
216	624	30	4.6	358	0.05	-1.32	-26.4
300	688	46	3.3	356	0.07	-1.17	-23.5
350	488	40	2.8	353	0.08	-1.09	-21.7
400	488	48.8	2.5	360	0.10	-1.00	-20.0
500	488	73	1.46	263	0.15	-0.83	-16.5
600	488	122	1.24	268	0.25	-0.60	-12.0
700	488	288	1.02	257	0.59	-0.23	-4.6
800	488	508	0.6	173	1.04	0.02	0.3
900	488	264	0.34	110	0.54	-0.27	-5.3
950	488	184	0.32	109	0.38	-0.42	-8.5
1000	488	152	0.3	108	0.31	-0.51	-10.1
1050	488	128	0.26	98	0.26	-0.58	-11.6
1100	488	112	0.26	103	0.23	-0.64	-12.8
1200	488	88	0.24	104	0.18	-0.74	-14.9
1300	488	72	0.22	103	0.15	-0.83	-16.6
1400	488	62	0.19	96	0.13	-0.90	-17.9
1500	488	56	0.18	97	0.11	-0.94	-18.8
1600	488	50	0.17	98	0.10	-0.99	-19.8
1700	488	46	0.16	98	0.09	-1.03	-20.5
1800	488	42	0.15	97	0.09	-1.07	-21.3
1900	488	38	0.14	96	0.08	-1.11	-22.2
2000	488	36	0.13	94	0.07	-1.13	-22.6

Appendix I. Pedal Modelled in MATLAB to apply to audio file

```
[sigIn,fs] = audioread('finalonlystring1.M4A');
t = [0:length(sigIn)-1]*1/fs;

C1 = 95e-9;
C2 = 96.4e-9;
R2 = 33e+3;
R1 = 14.72e+3;
R3 = 150;  %sweep resistor
Re = (R1*R3)/(R1+R3);

num =[-R2*C2*R3/(R1+R3),  0];
den =[(R2*Re*C1*C2), Re*(C1+C2), 1];
H = tf(num,den);     %Transfer function defined
[impH] = impulse(H,t);  %getting impulse response

figure(1)
bode(H)%bode plots

sigOut = conv(sigIn, impH); %convolution integral using impulse
response
sigOut = sigOut(1:length(sigOut)/2 + 1)/abs(max(sigOut));

fax = [0:length(sigIn)-1]*fs/length(sigIn);
[Mag, Phase] = bode(H, 2*pi*fax);
Mag = 20*log10(squeeze(Mag));
Phase = squeeze(Phase);

SigIn = fft(sigIn);
SigOut = fft(sigOut);

sound(sigIn, fs) %listen to input file
pause(2)
sound(sigOut, fs) %listen to output file

figure(2) % Frequency-domain signals
clf
  plot(fax, abs(SigIn), "DisplayName", "Input")
  hold on
  plot(fax, abs(SigOut),'--', "DisplayName", "Output")
  xlabel("Frequency [Hz]")
  ylabel("Magnitude")
  title("Magnitude content")
  legend("show")
```

Appendix J. The Q factor & BW values across the frequency range

FREQ (HZ)	R3	REQ	Q	BW (HZ)
200	2188.53	1900.99	1.95	563.88
300	972.68	911.41	2.82	390.44
400	547.13	527.20	3.70	296.95
500	350.17	341.89	4.60	239.13
600	243.17	239.15	5.50	200.00
700	178.66	176.48	6.40	171.81
800	136.78	135.50	7.31	150.55
900	108.08	107.27	8.21	133.95
1000	87.54	87.01	9.12	120.64
1100	72.35	71.99	10.02	109.73
1200	60.79	60.54	10.93	100.63
1300	51.80	51.61	11.84	92.91
1400	44.66	44.53	12.75	86.30
1500	38.91	38.80	13.65	80.56
1600	34.20	34.12	14.56	75.54
1700	30.29	30.23	15.47	71.10
1800	27.02	26.97	16.38	67.16
1900	24.25	24.21	17.29	63.63
2000	21.89	21.85	18.19	60.46