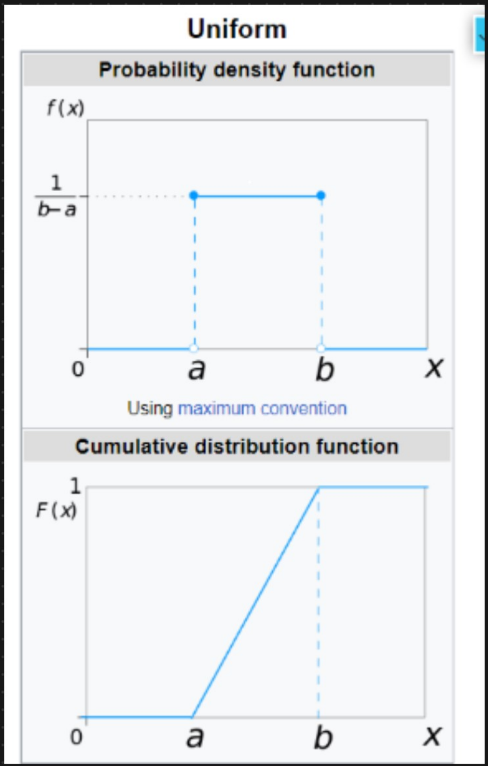
Uniform Distribution

① Continuous Uniform Distribution (pdf)

② Discrete Uniform Distribution (pmf)

① Continuous Uniform Distribution [Continuous Random Variable]

In probability theory and statistics, the continuous uniform distribution or rectangular distribution is a family of symmetric probability distributions. The distribution describes an experiment where there is an arbitrary outcome that lies between certain bounds. The bounds are defined by the parameters, a and b, which are the minimum and maximum values



Notation : $U(a,b)$

Parameters : $-\infty < a < b < \infty$

$$\text{Pdf} = \begin{cases} \dfrac{1}{b-a} & \text{for } x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Cdf} = \begin{cases} 0 & \text{for } x < a \\ \dfrac{x-a}{b-a} & \text{for } x \in [a,b] \\ 1 & \text{for } x > b \end{cases}$$

Mean $= \frac{1}{2}(a+b)$

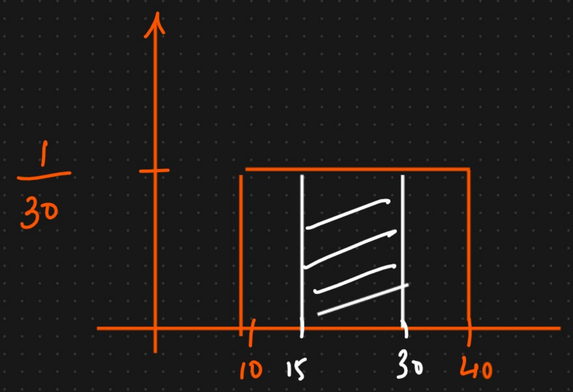
Variance $= \frac{1}{12}(b-a)^2$

Median $= \frac{1}{2}(a+b)$

Eg: The number of candies sold daily at a shop is uniformly distributed with a maximum of 40 and a minimum of 10

i) Probability of daily sales to fall between 15 and 30?

Ans)



$x_1 = 15$

$x_2 = 30$

$$Pr(15 \le x \le 30) = (x_2 - x_1) * \frac{1}{b-a}$$

$$= \cancel{15}^{1} * \frac{1}{\cancel{30}\,2}$$

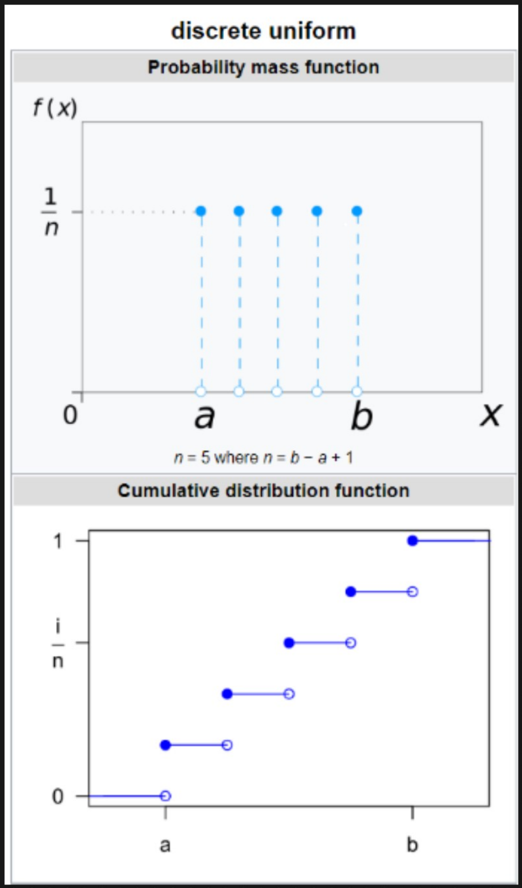
$$= 0.5$$

$$Pr(x \geq 20) = (40-20) * \frac{1}{30}$$

$$= 0.66 = 66\%$$

② Discrete Uniform Distribution (pmf)

In probability theory and statistics, the discrete uniform distribution is a symmetric probability distribution wherein a finite number of values are equally likely to be observed; every one of n values has equal probability 1/n. Another way of saying "discrete uniform distribution" would be "a known, finite number of outcomes equally likely to happen".



Eg: Rolling a dice

$\{1, 2, 3, 4, 5, 6\}$

$a = 1$ $\quad b = 6$

$Pr(1) = \frac{1}{6}$

$Pr(2) = \frac{1}{6}$

$Pr(3) = \frac{1}{6}$

$\frac{1}{n} \Rightarrow \boxed{n = b - a + 1}$

Notation $\quad U(a, b)$

Parameters $\quad a, b$ with $b \geq a$

PMF $\quad 1/n$

Mean $\quad \frac{a+b}{2}$

Median