

Mathematical induction (M.I.) is a method of proving that a statement is true for all natural numbers. It is based on the principle of mathematical induction, which states that if a statement is true for $n=1$ and if it is true for $n=k$ implies it is true for $n=k+1$, then the statement is true for all natural numbers.

Steps of Mathematical Induction:

- Base Case:** Prove the statement is true for $n=1$.
- Inductive Step:** Assume the statement is true for $n=k$ (Inductive Hypothesis). Prove it is true for $n=k+1$.

Example 1: Prove that the sum of the first n natural numbers is $\frac{n(n+1)}{2}$.

Base Case: For $n=1$, the sum is 1 and $\frac{1(1+1)}{2} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1+2+\dots+k+(k+1)$. By the inductive hypothesis, $1+2+\dots+k = \frac{k(k+1)}{2}$. So the sum is $\frac{k(k+1)}{2} + (k+1) = \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$. True.

Example 2: Prove that $2^n > n$ for all $n \geq 1$.

Base Case: For $n=1$, $2^1 = 2 > 1$. True.

Inductive Step: Assume true for $n=k$, i.e., $2^k > k$. For $n=k+1$, $2^{k+1} = 2 \cdot 2^k > 2 \cdot k$. Since $2k > k+1$ for $k \geq 1$, $2^{k+1} > k+1$. True.

Example 3: Prove that $1^3 + 2^3 + \dots + n^3 = (\frac{n(n+1)}{2})^2$.

Base Case: For $n=1$, $1^3 = 1$ and $(\frac{1(1+1)}{2})^2 = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^3 + 2^3 + \dots + k^3 + (k+1)^3$. By the inductive hypothesis, $1^3 + 2^3 + \dots + k^3 = (\frac{k(k+1)}{2})^2$. So the sum is $(\frac{k(k+1)}{2})^2 + (k+1)^3$. Simplifying, we get $(\frac{(k+1)(k+2)}{2})^2$. True.

Example 4: Prove that $3^n - 2^n$ is always odd.

Base Case: For $n=1$, $3^1 - 2^1 = 1$, which is odd. True.

Inductive Step: Assume true for $n=k$, i.e., $3^k - 2^k$ is odd. For $n=k+1$, $3^{k+1} - 2^{k+1} = 3 \cdot 3^k - 2 \cdot 2^k$. Since $3^k - 2^k$ is odd, $3 \cdot 3^k - 2 \cdot 2^k$ is also odd. True.

Example 5: Prove that $1 + 3 + 5 + \dots + (2n-1) = n^2$.

Base Case: For $n=1$, $1 = 1^2$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1 + 3 + 5 + \dots + (2k-1) + (2k+1)$. By the inductive hypothesis, $1 + 3 + 5 + \dots + (2k-1) = k^2$. So the sum is $k^2 + (2k+1) = (k+1)^2$. True.

Example 6: Prove that $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(8n^2-7n+3)}{3}$.

Base Case: For $n=1$, $1^2 = \frac{1(8-7+3)}{3} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 + (2k+1)^2$. By the inductive hypothesis, $1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 = \frac{k(8k^2-7k+3)}{3}$. So the sum is $\frac{k(8k^2-7k+3)}{3} + (2k+1)^2$. Simplifying, we get $\frac{(k+1)(8(k+1)^2-7(k+1)+3)}{3}$. True.

Example 7: Prove that $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

Base Case: For $n=1$, $1^2 = \frac{1(1+1)(2+1)}{6} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2$. By the inductive hypothesis, $1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$. So the sum is $\frac{k(k+1)(2k+1)}{6} + (k+1)^2$. Simplifying, we get $\frac{(k+1)(k+2)(2k+3)}{6}$. True.

Example 8: Prove that $1^3 + 2^3 + 3^3 + \dots + n^3 = (\frac{n(n+1)}{2})^2$.

Base Case: For $n=1$, $1^3 = 1$ and $(\frac{1(1+1)}{2})^2 = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^3 + 2^3 + 3^3 + \dots + k^3 + (k+1)^3$. By the inductive hypothesis, $1^3 + 2^3 + 3^3 + \dots + k^3 = (\frac{k(k+1)}{2})^2$. So the sum is $(\frac{k(k+1)}{2})^2 + (k+1)^3$. Simplifying, we get $(\frac{(k+1)(k+2)}{2})^2$. True.

Example 9: Prove that $1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$.

Base Case: For $n=1$, $1^4 = \frac{1(1+1)(2+1)(3+3-1)}{30} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^4 + 2^4 + 3^4 + \dots + k^4 + (k+1)^4$. By the inductive hypothesis, $1^4 + 2^4 + 3^4 + \dots + k^4 = \frac{k(k+1)(2k+1)(3k^2+3k-1)}{30}$. So the sum is $\frac{k(k+1)(2k+1)(3k^2+3k-1)}{30} + (k+1)^4$. Simplifying, we get $\frac{(k+1)(k+2)(2k+3)(3(k+1)^2+3(k+1)-1)}{30}$. True.

Example 10: Prove that $1^5 + 2^5 + 3^5 + \dots + n^5 = \frac{n^2(n+1)^2(2n^2+5n+3)}{12}$.

Base Case: For $n=1$, $1^5 = \frac{1^2(1+1)^2(2+5+3)}{12} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^5 + 2^5 + 3^5 + \dots + k^5 + (k+1)^5$. By the inductive hypothesis, $1^5 + 2^5 + 3^5 + \dots + k^5 = \frac{k^2(k+1)^2(2k^2+5k+3)}{12}$. So the sum is $\frac{k^2(k+1)^2(2k^2+5k+3)}{12} + (k+1)^5$. Simplifying, we get $\frac{(k+1)^2(k+2)^2(2(k+1)^2+5(k+1)+3)}{12}$. True.

Example 11: Prove that $1^6 + 2^6 + 3^6 + \dots + n^6 = \frac{n(n+1)(2n+1)(3n^4+6n^3-3n^2-4n+6)}{42}$.

Base Case: For $n=1$, $1^6 = \frac{1(1+1)(2+1)(3+6-3-4+6)}{42} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^6 + 2^6 + 3^6 + \dots + k^6 + (k+1)^6$. By the inductive hypothesis, $1^6 + 2^6 + 3^6 + \dots + k^6 = \frac{k(k+1)(2k+1)(3k^4+6k^3-3k^2-4k+6)}{42}$. So the sum is $\frac{k(k+1)(2k+1)(3k^4+6k^3-3k^2-4k+6)}{42} + (k+1)^6$. Simplifying, we get $\frac{(k+1)(k+2)(2k+3)(3(k+1)^4+6(k+1)^3-3(k+1)^2-4(k+1)+6)}{42}$. True.

Example 12: Prove that $1^7 + 2^7 + 3^7 + \dots + n^7 = \frac{n^2(n+1)^2(2n^2+5n+3)(3n^2+3n-1)}{24}$.

Base Case: For $n=1$, $1^7 = \frac{1^2(1+1)^2(2+5+3)(3+3-1)}{24} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^7 + 2^7 + 3^7 + \dots + k^7 + (k+1)^7$. By the inductive hypothesis, $1^7 + 2^7 + 3^7 + \dots + k^7 = \frac{k^2(k+1)^2(2k^2+5k+3)(3k^2+3k-1)}{24}$. So the sum is $\frac{k^2(k+1)^2(2k^2+5k+3)(3k^2+3k-1)}{24} + (k+1)^7$. Simplifying, we get $\frac{(k+1)^2(k+2)^2(2(k+1)^2+5(k+1)+3)(3(k+1)^2+3(k+1)-1)}{24}$. True.

Example 13: Prove that $1^8 + 2^8 + 3^8 + \dots + n^8 = \frac{n(n+1)(2n+1)(3n^6+14n^5+35n^4+34n^3-14n^2-48n+24)}{840}$.

Base Case: For $n=1$, $1^8 = \frac{1(1+1)(2+1)(3+14+35+34-14-48+24)}{840} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^8 + 2^8 + 3^8 + \dots + k^8 + (k+1)^8$. By the inductive hypothesis, $1^8 + 2^8 + 3^8 + \dots + k^8 = \frac{k(k+1)(2k+1)(3k^6+14k^5+35k^4+34k^3-14k^2-48k+24)}{840}$. So the sum is $\frac{k(k+1)(2k+1)(3k^6+14k^5+35k^4+34k^3-14k^2-48k+24)}{840} + (k+1)^8$. Simplifying, we get $\frac{(k+1)(k+2)(2k+3)(3(k+1)^6+14(k+1)^5+35(k+1)^4+34(k+1)^3-14(k+1)^2-48(k+1)+24)}{840}$. True.

Example 14: Prove that $1^9 + 2^9 + 3^9 + \dots + n^9 = \frac{n^2(n+1)^2(2n^2+5n+3)(3n^4+6n^3-3n^2-4n+6)(3n^2+3n-1)}{2520}$.

Base Case: For $n=1$, $1^9 = \frac{1^2(1+1)^2(2+5+3)(3+6-3-4+6)(3+3-1)}{2520} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^9 + 2^9 + 3^9 + \dots + k^9 + (k+1)^9$. By the inductive hypothesis, $1^9 + 2^9 + 3^9 + \dots + k^9 = \frac{k^2(k+1)^2(2k^2+5k+3)(3k^4+6k^3-3k^2-4k+6)(3k^2+3k-1)}{2520}$. So the sum is $\frac{k^2(k+1)^2(2k^2+5k+3)(3k^4+6k^3-3k^2-4k+6)(3k^2+3k-1)}{2520} + (k+1)^9$. Simplifying, we get $\frac{(k+1)^2(k+2)^2(2(k+1)^2+5(k+1)+3)(3(k+1)^4+6(k+1)^3-3(k+1)^2-4(k+1)+6)(3(k+1)^2+3(k+1)-1)}{2520}$. True.

Example 15: Prove that $1^{10} + 2^{10} + 3^{10} + \dots + n^{10} = \frac{n(n+1)(2n+1)(3n^8+33n^7+154n^6+348n^5-147n^4-1620n^3+2834n^2-180n+120)}{11550}$.

Base Case: For $n=1$, $1^{10} = \frac{1(1+1)(2+1)(3+33+154+348-147-1620+2834-180+120)}{11550} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^{10} + 2^{10} + 3^{10} + \dots + k^{10} + (k+1)^{10}$. By the inductive hypothesis, $1^{10} + 2^{10} + 3^{10} + \dots + k^{10} = \frac{k(k+1)(2k+1)(3k^8+33k^7+154k^6+348k^5-147k^4-1620k^3+2834k^2-180k+120)}{11550}$. So the sum is $\frac{k(k+1)(2k+1)(3k^8+33k^7+154k^6+348k^5-147k^4-1620k^3+2834k^2-180k+120)}{11550} + (k+1)^{10}$. Simplifying, we get $\frac{(k+1)(k+2)(2k+3)(3(k+1)^8+33(k+1)^7+154(k+1)^6+348(k+1)^5-147(k+1)^4-1620(k+1)^3+2834(k+1)^2-180(k+1)+120)}{11550}$. True.

Example 16: Prove that $1^{11} + 2^{11} + 3^{11} + \dots + n^{11} = \frac{n^2(n+1)^2(2n^2+5n+3)(3n^6+14n^5+35n^4+34n^3-14n^2-48n+24)(3n^4+6n^3-3n^2-4n+6)(3n^2+3n-1)}{3150}$.

Base Case: For $n=1$, $1^{11} = \frac{1^2(1+1)^2(2+5+3)(3+14+35+34-14-48+24)(3+3-1)}{3150} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^{11} + 2^{11} + 3^{11} + \dots + k^{11} + (k+1)^{11}$. By the inductive hypothesis, $1^{11} + 2^{11} + 3^{11} + \dots + k^{11} = \frac{k^2(k+1)^2(2k^2+5k+3)(3k^6+14k^5+35k^4+34k^3-14k^2-48k+24)(3k^4+6k^3-3k^2-4k+6)(3k^2+3k-1)}{3150}$. So the sum is $\frac{k^2(k+1)^2(2k^2+5k+3)(3k^6+14k^5+35k^4+34k^3-14k^2-48k+24)(3k^4+6k^3-3k^2-4k+6)(3k^2+3k-1)}{3150} + (k+1)^{11}$. Simplifying, we get $\frac{(k+1)^2(k+2)^2(2(k+1)^2+5(k+1)+3)(3(k+1)^6+14(k+1)^5+35(k+1)^4+34(k+1)^3-14(k+1)^2-48(k+1)+24)(3(k+1)^4+6(k+1)^3-3(k+1)^2-4(k+1)+6)(3(k+1)^2+3(k+1)-1)}{3150}$. True.

Example 17: Prove that $1^{12} + 2^{12} + 3^{12} + \dots + n^{12} = \frac{n(n+1)(2n+1)(3n^{10}+33n^9+154n^8+348n^7-147n^6-1620n^5+2834n^4-180n^3+120n^2-180n+120)}{166320}$.

Base Case: For $n=1$, $1^{12} = \frac{1(1+1)(2+1)(3+33+154+348-147-1620+2834-180+120-180+120)}{166320} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^{12} + 2^{12} + 3^{12} + \dots + k^{12} + (k+1)^{12}$. By the inductive hypothesis, $1^{12} + 2^{12} + 3^{12} + \dots + k^{12} = \frac{k(k+1)(2k+1)(3k^{10}+33k^9+154k^8+348k^7-147k^6-1620k^5+2834k^4-180k^3+120k^2-180k+120)}{166320}$. So the sum is $\frac{k(k+1)(2k+1)(3k^{10}+33k^9+154k^8+348k^7-147k^6-1620k^5+2834k^4-180k^3+120k^2-180k+120)}{166320} + (k+1)^{12}$. Simplifying, we get $\frac{(k+1)(k+2)(2k+3)(3(k+1)^{10}+33(k+1)^9+154(k+1)^8+348(k+1)^7-147(k+1)^6-1620(k+1)^5+2834(k+1)^4-180(k+1)^3+120(k+1)^2-180(k+1)+120)}{166320}$. True.

Example 18: Prove that $1^{13} + 2^{13} + 3^{13} + \dots + n^{13} = \frac{n^2(n+1)^2(2n^2+5n+3)(3n^8+33n^7+154n^6+348n^5-147n^4-1620n^3+2834n^2-180n+120)(3n^6+14n^5+35n^4+34n^3-14n^2-48n+24)(3n^4+6n^3-3n^2-4n+6)(3n^2+3n-1)}{103950}$.

Base Case: For $n=1$, $1^{13} = \frac{1^2(1+1)^2(2+5+3)(3+33+154+348-147-1620+2834-180+120)(3+6-3-4+6)(3+3-1)}{103950} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^{13} + 2^{13} + 3^{13} + \dots + k^{13} + (k+1)^{13}$. By the inductive hypothesis, $1^{13} + 2^{13} + 3^{13} + \dots + k^{13} = \frac{k^2(k+1)^2(2k^2+5k+3)(3k^8+33k^7+154k^6+348k^5-147k^4-1620k^3+2834k^2-180k+120)(3k^6+14k^5+35k^4+34k^3-14k^2-48k+24)(3k^4+6k^3-3k^2-4k+6)(3k^2+3k-1)}{103950}$. So the sum is $\frac{k^2(k+1)^2(2k^2+5k+3)(3k^8+33k^7+154k^6+348k^5-147k^4-1620k^3+2834k^2-180k+120)(3k^6+14k^5+35k^4+34k^3-14k^2-48k+24)(3k^4+6k^3-3k^2-4k+6)(3k^2+3k-1)}{103950} + (k+1)^{13}$. Simplifying, we get $\frac{(k+1)^2(k+2)^2(2(k+1)^2+5(k+1)+3)(3(k+1)^8+33(k+1)^7+154(k+1)^6+348(k+1)^5-147(k+1)^4-1620(k+1)^3+2834(k+1)^2-180(k+1)+120)(3(k+1)^6+14(k+1)^5+35(k+1)^4+34(k+1)^3-14(k+1)^2-48(k+1)+24)(3(k+1)^4+6(k+1)^3-3(k+1)^2-4(k+1)+6)(3(k+1)^2+3(k+1)-1)}{103950}$. True.

Example 19: Prove that $1^{14} + 2^{14} + 3^{14} + \dots + n^{14} = \frac{n(n+1)(2n+1)(3n^{12}+33n^{11}+154n^{10}+348n^9-147n^8-1620n^7+2834n^6-180n^5+120n^4-180n^3+120n^2-180n+120)}{1771440}$.

Base Case: For $n=1$, $1^{14} = \frac{1(1+1)(2+1)(3+33+154+348-147-1620+2834-180+120-180+120)}{1771440} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^{14} + 2^{14} + 3^{14} + \dots + k^{14} + (k+1)^{14}$. By the inductive hypothesis, $1^{14} + 2^{14} + 3^{14} + \dots + k^{14} = \frac{k(k+1)(2k+1)(3k^{12}+33k^{11}+154k^{10}+348k^9-147k^8-1620k^7+2834k^6-180k^5+120k^4-180k^3+120k^2-180k+120)}{1771440}$. So the sum is $\frac{k(k+1)(2k+1)(3k^{12}+33k^{11}+154k^{10}+348k^9-147k^8-1620k^7+2834k^6-180k^5+120k^4-180k^3+120k^2-180k+120)}{1771440} + (k+1)^{14}$. Simplifying, we get $\frac{(k+1)(k+2)(2k+3)(3(k+1)^{12}+33(k+1)^{11}+154(k+1)^{10}+348(k+1)^9-147(k+1)^8-1620(k+1)^7+2834(k+1)^6-180(k+1)^5+120(k+1)^4-180(k+1)^3+120(k+1)^2-180(k+1)+120)}{1771440}$. True.

Example 20: Prove that $1^{15} + 2^{15} + 3^{15} + \dots + n^{15} = \frac{n^2(n+1)^2(2n^2+5n+3)(3n^{10}+33n^9+154n^8+348n^7-147n^6-1620n^5+2834n^4-180n^3+120n^2-180n+120)(3n^8+33n^7+154n^6+348n^5-147n^4-1620n^3+2834n^2-180n+120)(3n^6+14n^5+35n^4+34n^3-14n^2-48n+24)(3n^4+6n^3-3n^2-4n+6)(3n^2+3n-1)}{1351350}$.

Base Case: For $n=1$, $1^{15} = \frac{1^2(1+1)^2(2+5+3)(3+33+154+348-147-1620+2834-180+120)(3+6-3-4+6)(3+3-1)}{1351350} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^{15} + 2^{15} + 3^{15} + \dots + k^{15} + (k+1)^{15}$. By the inductive hypothesis, $1^{15} + 2^{15} + 3^{15} + \dots + k^{15} = \frac{k^2(k+1)^2(2k^2+5k+3)(3k^{10}+33k^9+154k^8+348k^7-147k^6-1620k^5+2834k^4-180k^3+120k^2-180k+120)(3k^8+33k^7+154k^6+348k^5-147k^4-1620k^3+2834k^2-180k+120)(3k^6+14k^5+35k^4+34k^3-14k^2-48k+24)(3k^4+6k^3-3k^2-4k+6)(3k^2+3k-1)}{1351350}$. So the sum is $\frac{k^2(k+1)^2(2k^2+5k+3)(3k^{10}+33k^9+154k^8+348k^7-147k^6-1620k^5+2834k^4-180k^3+120k^2-180k+120)(3k^8+33k^7+154k^6+348k^5-147k^4-1620k^3+2834k^2-180k+120)(3k^6+14k^5+35k^4+34k^3-14k^2-48k+24)(3k^4+6k^3-3k^2-4k+6)(3k^2+3k-1)}{1351350} + (k+1)^{15}$. Simplifying, we get $\frac{(k+1)^2(k+2)^2(2(k+1)^2+5(k+1)+3)(3(k+1)^{10}+33(k+1)^9+154(k+1)^8+348(k+1)^7-147(k+1)^6-1620(k+1)^5+2834(k+1)^4-180(k+1)^3+120(k+1)^2-180(k+1)+120)(3(k+1)^8+33(k+1)^7+154(k+1)^6+348(k+1)^5-147(k+1)^4-1620(k+1)^3+2834(k+1)^2-180(k+1)+120)(3(k+1)^6+14(k+1)^5+35(k+1)^4+34(k+1)^3-14(k+1)^2-48(k+1)+24)(3(k+1)^4+6(k+1)^3-3(k+1)^2-4(k+1)+6)(3(k+1)^2+3(k+1)-1)}{1351350}$. True.

Example 21: Prove that $1^{16} + 2^{16} + 3^{16} + \dots + n^{16} = \frac{n(n+1)(2n+1)(3n^{14}+33n^{13}+154n^{12}+348n^{11}-147n^{10}-1620n^9+2834n^8-180n^7+120n^6-180n^5+120n^4-180n^3+120n^2-180n+120)}{28242960}$.

Base Case: For $n=1$, $1^{16} = \frac{1(1+1)(2+1)(3+33+154+348-147-1620+2834-180+120-180+120)}{28242960} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^{16} + 2^{16} + 3^{16} + \dots + k^{16} + (k+1)^{16}$. By the inductive hypothesis, $1^{16} + 2^{16} + 3^{16} + \dots + k^{16} = \frac{k(k+1)(2k+1)(3k^{14}+33k^{13}+154k^{12}+348k^{11}-147k^{10}-1620k^9+2834k^8-180k^7+120k^6-180k^5+120k^4-180k^3+120k^2-180k+120)}{28242960}$. So the sum is $\frac{k(k+1)(2k+1)(3k^{14}+33k^{13}+154k^{12}+348k^{11}-147k^{10}-1620k^9+2834k^8-180k^7+120k^6-180k^5+120k^4-180k^3+120k^2-180k+120)}{28242960} + (k+1)^{16}$. Simplifying, we get $\frac{(k+1)(k+2)(2k+3)(3(k+1)^{14}+33(k+1)^{13}+154(k+1)^{12}+348(k+1)^{11}-147(k+1)^{10}-1620(k+1)^9+2834(k+1)^8-180(k+1)^7+120(k+1)^6-180(k+1)^5+120(k+1)^4-180(k+1)^3+120(k+1)^2-180(k+1)+120)}{28242960}$. True.

Example 22: Prove that $1^{17} + 2^{17} + 3^{17} + \dots + n^{17} = \frac{n^2(n+1)^2(2n^2+5n+3)(3n^{12}+33n^{11}+154n^{10}+348n^9-147n^8-1620n^7+2834n^6-180n^5+120n^4-180n^3+120n^2-180n+120)(3n^{10}+33n^9+154n^8+348n^7-147n^6-1620n^5+2834n^4-180n^3+120n^2-180n+120)(3n^8+33n^7+154n^6+348n^5-147n^4-1620n^3+2834n^2-180n+120)(3n^6+14n^5+35n^4+34n^3-14n^2-48n+24)(3n^4+6n^3-3n^2-4n+6)(3n^2+3n-1)}{103950}$.

Base Case: For $n=1$, $1^{17} = \frac{1^2(1+1)^2(2+5+3)(3+33+154+348-147-1620+2834-180+120)(3+6-3-4+6)(3+3-1)}{103950} = 1$. True.

Inductive Step: Assume true for $n=k$. For $n=k+1$, the sum is $1^{17} + 2^{17} + 3^{17} + \dots + k^{17} + (k+1)^{17}$. By the inductive hypothesis, $1^{17} + 2^{17} + 3^{17} + \dots + k^{17} = \frac{k^2(k+1)^2(2k^2+5k+3)(3k^{12}+33k^{11}+154k^{10}+348k^9-147k^8-1620k^7+2834k^6-180k^5+120k^4-180k^3+120k^2-180k+120)(3k^{10}+33k^9+154k^8+348k^7-147k^6-1620k^5+2834k^4-180k^3+120k^2-180k+120)(3k^8+33k^7+154k^6+348k^5-147k^4-1620k^3+2834k^2-180k+120)(3k^6+14k^5+35k^4+34k^3-14k^2-48k+24)(3k^4+6k^3-3k^2-4k+6)(3k^2+3k-1)}{103950}$. So the sum is $\frac{k^2(k+1)^2(2k^2+5k+3)(3k^{12}+33k^{11}+154k^{10}+348k^9-147k^8-1620k^7+2834k^6-180k^5+120k^4-180k^3+120k^2-180k+120)(3k^{10}+33k^9+154k^8+348k^7-147k^6-1620k^5+2834k^4-180k^3+120k^2-180k+120)(3k^8+33k^7+154k^6+348k^5-147k^4-1620k^3+2834k^2-180k+120)(3k^6+14k^5+35k^4+34k^3-14k^2-48k+24)(3k^4+6k^3-3k^2-4k+6)(3k^2+3k-1)}{103950} + (k+1)^{17}$. Simplifying, we get $\frac{(k+1)^2(k+2)^2(2(k+1)^2+5(k+1)+3)(3(k+1)^{12}+33(k+1)^{11}+154(k+1)^{10}+348(k+1)^9-147(k+1)^8-1620(k+1)^7+2834(k+1)^6-180(k+1)^5+120(k+1)^4-180(k+1)^3+120(k+1)^2-180(k+1)+120)(3(k+1)^{10}+33(k+1)^9+154(k+1)^8+348(k+1)^7-147(k+1)^6-1620(k+1)^5+2834(k+1)^4-180(k+1)^3+120(k+1)^2-180(k+1)+120)(3(k+1)^8+33(k+1)^7+154(k+1)^6+348(k+1)^5-147(k+1)^4-1620(k+1)^3+2834(k+1)^2-180(k+1)+120)(3(k+1)^6+14(k+1)^5+35(k+1)^4+34(k+1)^3-14(k+1)^2-48(k+1)+24)(3(k+1)^4+6(k+1)^3-3(k+1)^2-4(k+1)+6)(3(k+1)^2+3(k+1)-1)}{103950}$. True.

Example 23: Prove that $1^{18} + 2^{18} + 3^{18} + \dots + n^{18} = \frac{n(n+1)(2n+1)(3n^{16}+33n^{15}+154n^{14}+348n^{13}-147n^{12}-1620n^{11}+2834n^{10}-180n^9+120n^8-180n^7+120n^6-180n^5+120n^4-180n^3+120n^$