07-SARIMAX

November 5, 2021

1 SARIMAX

1.1 Seasonal AutoRegressive Integrated Moving Average with eXogenous regressors

So far the models we've looked at consider past values of a dataset and past errors to determine future trends, seasonality and forecasted values. We look now to models that encompass these non-seasonal (p,d,q) and seasonal (P,D,Q,m) factors, but introduce the idea that external factors (environmental, economic, etc.) can also influence a time series, and be used in forecasting.

Related Functions:

sarimax.SARIMAX(endog[, exog, order, …]) sarimax.SARIMAXResults(model, params, …[, …]) Class to hold results from fitting a SARIMAX model.

For Further Reading:

Statsmodels Tutorial: Time Series Analysis by State Space Methods Statsmodels Example: SARIMAX

[4]:
```
import pandas as pd
import numpy as np
%matplotlib inline

# Load specific forecasting tools
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.graphics.tsaplots import plot_acf,plot_pacf # for determining␣
↪(p,q) orders
from statsmodels.tsa.seasonal import seasonal_decompose      # for ETS Plots
from pmdarima import auto_arima                              # for determining␣
↪ARIMA orders

# Ignore harmless warnings
import warnings
warnings.filterwarnings("ignore")
```

[5]:
```
# Load dataset
df = pd.read_csv('../Data/RestaurantVisitors.
↪csv',index_col='date',parse_dates=True)
df.index.freq = 'D'
```

1.1.1 Inspect the data

For this section we've built a Restaurant Visitors dataset that was inspired by a recent Kaggle competition. The data considers daily visitors to four restaurants located in the United States, subject to American holidays. For the exogenous variable we'll see how holidays affect patronage. The dataset contains 478 days of restaurant data, plus an additional 39 days of holiday data for forecasting purposes.

[3]:
```
'''
OK, so for the exactions variables, we're going to see whether or not this␣
↪holiday
column actually affects this total column.

First off, we'll do a normal seasonal Arima model and then we'll add in this
seasonal aroma exogenous model adding in this holiday information.

'''
df.head()
```

[3]:
```
              weekday  holiday    holiday_name  …  rest3  rest4  total
date                                            …
2016-01-01     Friday        1  New Year's Day  …   67.0  139.0  296.0
2016-01-02   Saturday        0              na  …   43.0   85.0  191.0
2016-01-03     Sunday        0              na  …   66.0   81.0  202.0
2016-01-04     Monday        0              na  …   32.0   32.0  105.0
2016-01-05    Tuesday        0              na  …   38.0   43.0   98.0

[5 rows x 8 columns]
```

Notice that even though the restaurant visitor columns contain integer data, they appear as floats. This is because the bottom of the dataframe has 39 rows of NaN data to accommodate the extra holiday data we'll use for forecasting, and pandas won't allow NaN's as integers. We could leave it like this, but since we have to drop NaN values anyway, let's also convert the columns to dtype int64.

[6]:
```
df.tail()
```

[6]:
```
              weekday  holiday  holiday_name  rest1  rest2  rest3  rest4  total
date
2017-05-27   Saturday        0            na    NaN    NaN    NaN    NaN    NaN
2017-05-28     Sunday        0            na    NaN    NaN    NaN    NaN    NaN
2017-05-29     Monday        1  Memorial Day    NaN    NaN    NaN    NaN    NaN
2017-05-30    Tuesday        0            na    NaN    NaN    NaN    NaN    NaN
2017-05-31  Wednesday        0            na    NaN    NaN    NaN    NaN    NaN
```

[7]:
```
'''
So we're going to go ahead and drop that missing data since we can't really use
it for training purposes.
```

```
'''
df1 = df.dropna()
df1.tail()
```

[7]:

```
              weekday  holiday holiday_name  rest1  rest2  rest3  rest4  total
date
2017-04-18    Tuesday        0           na   30.0   30.0   13.0   18.0   91.0
2017-04-19  Wednesday        0           na   20.0   11.0   30.0   18.0   79.0
2017-04-20   Thursday        0           na   22.0    3.0   19.0   46.0   90.0
2017-04-21     Friday        0           na   38.0   53.0   36.0   38.0  165.0
2017-04-22   Saturday        0           na   97.0   20.0   50.0   59.0  226.0
```

[15]:

```
df1.columns
```

[15]:

```
Index(['weekday', 'holiday', 'holiday_name', 'rest1', 'rest2', 'rest3',
       'rest4', 'total'],
      dtype='object')
```

[8]:

```
'''
So what we're going do here is we're going to change the data type of
these last four columns, the restaurant numbers, as well as the total
and make them into integers.




'''

# Change the dtype of selected columns
cols = ['rest1','rest2','rest3','rest4','total']
for col in cols:
    df1[col] = df1[col].astype(int)
df1.head()
```

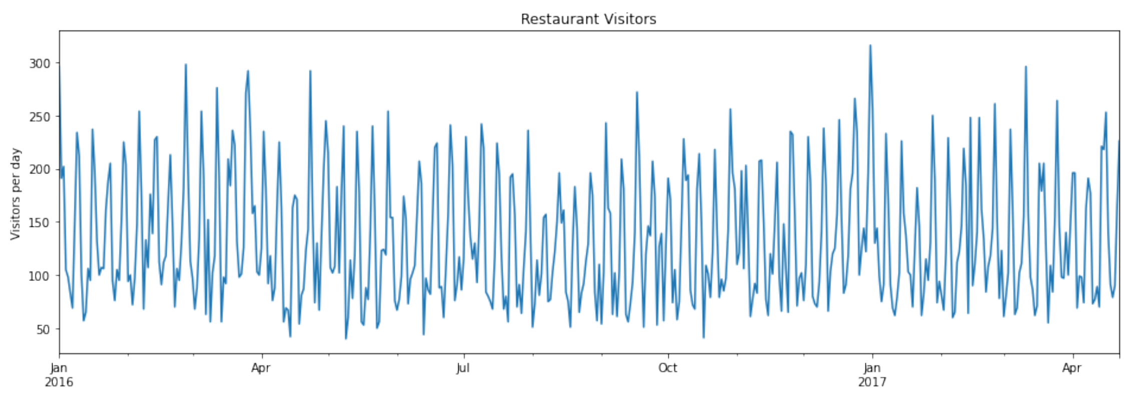
[8]:

```
             weekday  holiday    holiday_name  …  rest3  rest4  total
date                                           …
2016-01-01    Friday        1  New Year's Day  …     67    139    296
2016-01-02  Saturday        0              na  …     43     85    191
2016-01-03    Sunday        0              na  …     66     81    202
2016-01-04    Monday        0              na  …     32     32    105
2016-01-05   Tuesday        0              na  …     38     43     98

[5 rows x 8 columns]
```

[9]:

```
'''
Recall that what we're trying to do is we're trying to predict and forecast the

total number of visitors.So let's see what that looks like over time,
```

```
It does look like there's some sort of repeating peak.

It doesn't look like it's on a monthly level.

In fact, it's probably on a weekly level.

So maybe on the weekends there's a peak or maybe on Fridays or maybe on a happy␣
↪hour day.
'''
title='Restaurant Visitors'
ylabel='Visitors per day'
xlabel='' # we don't really need a label here

ax = df1['total'].plot(figsize=(16,5),title=title)
ax.autoscale(axis='x',tight=True)
ax.set(xlabel=xlabel, ylabel=ylabel);
```



[18]:
```
df1[df1['holiday']==1].index
```

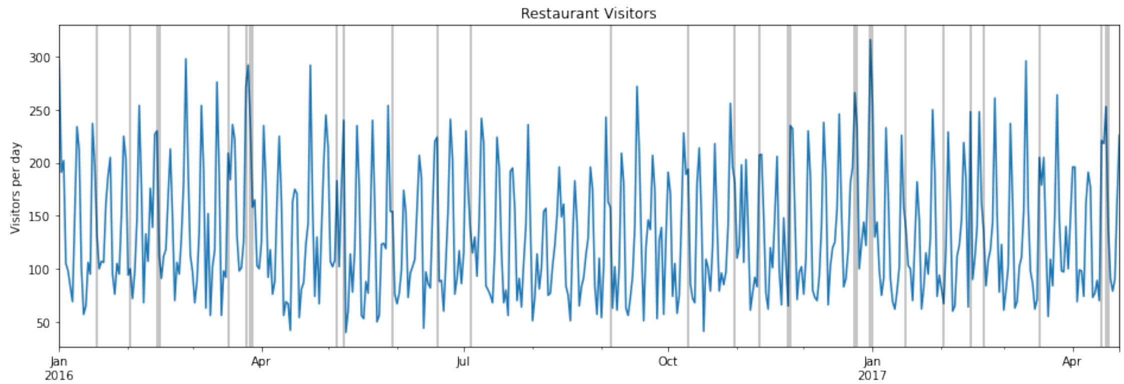
[18]:
```
DatetimeIndex(['2016-01-01', '2016-01-18', '2016-02-02', '2016-02-14',
               '2016-02-15', '2016-03-17', '2016-03-25', '2016-03-27',
               '2016-03-28', '2016-05-05', '2016-05-08', '2016-05-30',
               '2016-06-19', '2016-07-04', '2016-09-05', '2016-10-10',
               '2016-10-31', '2016-11-11', '2016-11-24', '2016-11-25',
               '2016-12-24', '2016-12-25', '2016-12-31', '2017-01-01',
               '2017-01-16', '2017-02-02', '2017-02-14', '2017-02-20',
               '2017-03-17', '2017-04-14', '2017-04-16', '2017-04-17'],
              dtype='datetime64[ns]', name='date', freq=None)
```

[20]:
```
'''
We don't really know for sure until we zoom in.
```

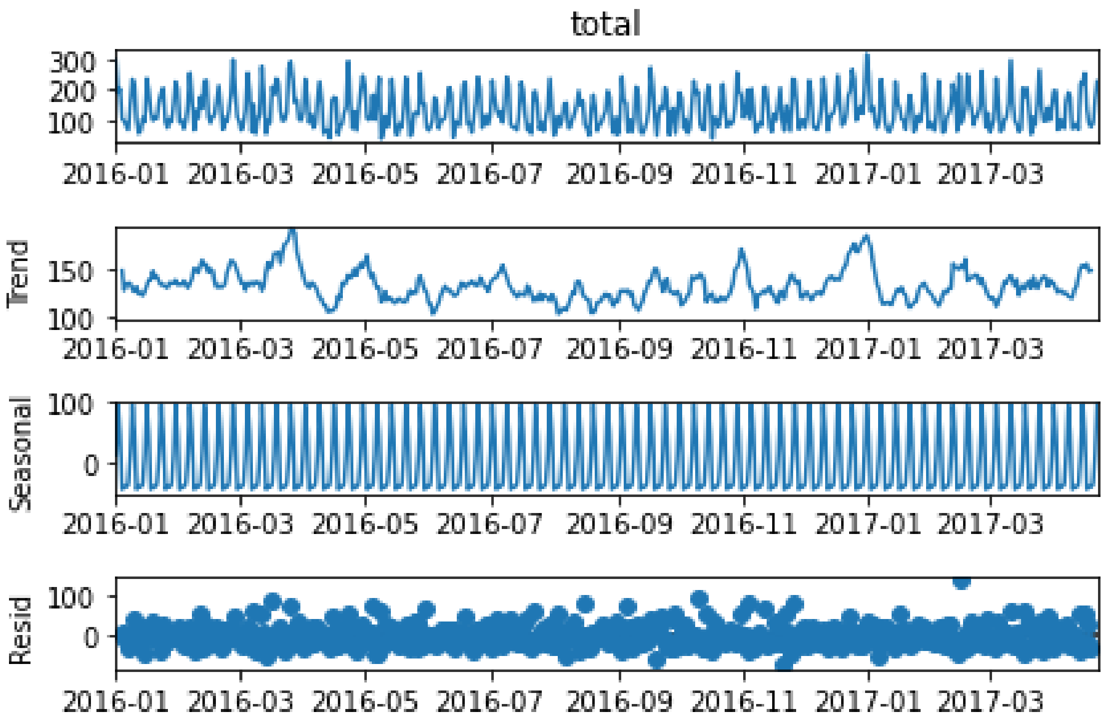
```
But what I'm also going to do is I'm going to write a little bit of code here␣
 ↪so I can overlap when

there is a holiday and see, maybe there's an effect there, maybe not.

It may be a little hard to tell visually, but hopefully when we run, our Suruma␣
 ↪X

model will actually be able to tell that.


df1.query('holiday==1').index

So I'm essentially going to query the data frame for when Holliday's equal to␣
 ↪one and
grab those index numbers.

So if you return that, it basically returns back only the dates where the one␣
 ↪happens
to be equal to holiday.

ax.axvline(x=x, color='k', alpha = 0.3)

And what I want to do is for all these index locations at these date time␣
 ↪stamps,
I want to add in a little vertical line on my plot.


We'll say ax.axvline, which basically is a matplotlib command to add a vertical␣
 ↪line onto
this axis object And then we'll say where X is equal today(x=x).

So essentially for every day in this date time index at a vertical line and␣
 ↪then
we can say what color we want it to be.

It's also the color equal to black, which is just color code, or you can also␣
 ↪type out black.
And then let's give it a little bit of transparency.

'''

title='Restaurant Visitors'
ylabel='Visitors per day'
xlabel='' # we don't really need a label here
```

```
ax = df1['total'].plot(figsize=(16,5),title=title)
ax.autoscale(axis='x',tight=True)
ax.set(xlabel=xlabel, ylabel=ylabel)
for x in df1.query('holiday==1').index:# for days where holiday == 1
                                        #or df1[df1['holiday']==1].index
       ax.axvline(x=x, color='k', alpha = 0.3)
       print(x)  ;# add a semi-transparent grey line
```

```
2016-01-01 00:00:00
2016-01-18 00:00:00
2016-02-02 00:00:00
2016-02-14 00:00:00
2016-02-15 00:00:00
2016-03-17 00:00:00
2016-03-25 00:00:00
2016-03-27 00:00:00
2016-03-28 00:00:00
2016-05-05 00:00:00
2016-05-08 00:00:00
2016-05-30 00:00:00
2016-06-19 00:00:00
2016-07-04 00:00:00
2016-09-05 00:00:00
2016-10-10 00:00:00
2016-10-31 00:00:00
2016-11-11 00:00:00
2016-11-24 00:00:00
2016-11-25 00:00:00
2016-12-24 00:00:00
2016-12-25 00:00:00
2016-12-31 00:00:00
2017-01-01 00:00:00
2017-01-16 00:00:00
2017-02-02 00:00:00
2017-02-14 00:00:00
2017-02-20 00:00:00
2017-03-17 00:00:00
2017-04-14 00:00:00
2017-04-16 00:00:00
2017-04-17 00:00:00
```

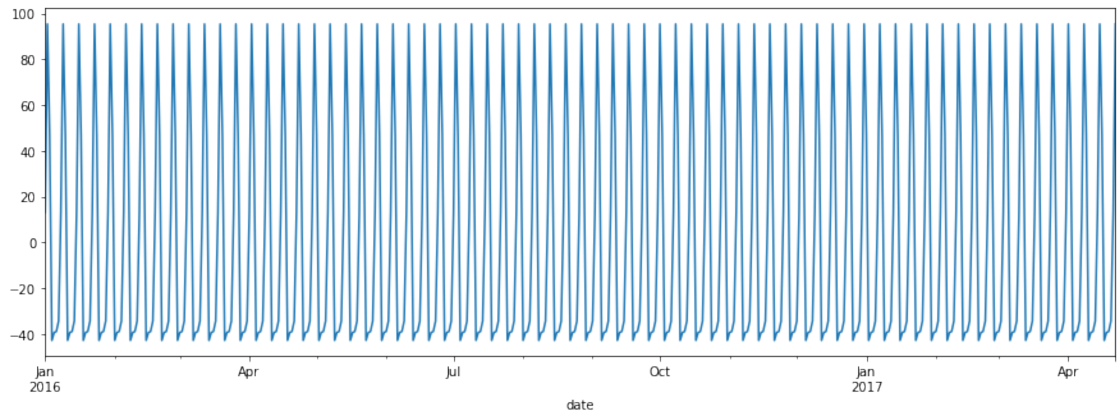


[22]:
```
'''

So keep that in mind that just visually right now, it's a little unclear␣
 ↪whether or not

this exogenous variable is really going to be predictive of how many visitors

show up to these restaurants.

However, intuitively, you should have some sort of idea that holidays probably␣
 ↪do matter

if there is a visit there or not.

Well, you also may want to do is since there is some indication of seasonality,

is run a ETS decomposition.


So there is the observed values,

the general trend, it looks like there's some sort of increase going on maybe␣
 ↪during

the holidays.A little hard to tell.

And there's definitely a seasonal component.
'''

result=seasonal_decompose(df1['total'])
result.plot();
```



[24]:
```
'''
So very strong seasonal component.

In fact, let's just take a look at that seasonal component


we can see the peaks and valleys here, and what you

can do is eventually if you kind of zoom in on the seasonal component, you'll

notice that it's it's weekly.

And in fact, you can kind of just tell that there's four seasonal periods per␣
↪month

indicating that there's four weeks per month.

So the seasonality of this data happens to be on a weekly basis, which makes␣
↪sense.
'''
result.seasonal.plot(figsize=(15,5))
```

[24]: <AxesSubplot:xlabel='date'>



[25]:

```

```

[25]: 478

1.1.2 Test for stationarity

[26]:

```
from statsmodels.tsa.stattools import adfuller

def adf_test(series,title=''):
    """
    Pass in a time series and an optional title, returns an ADF report
    """
    print(f'Augmented Dickey-Fuller Test: {title}')
    result = adfuller(series.dropna(),autolag='AIC') # .dropna() handles␣
 ↪differenced data

    labels = ['ADF test statistic','p-value','# lags used','# observations']
    out = pd.Series(result[0:4],index=labels)

    for key,val in result[4].items():
        out[f'critical value ({key})']=val

    print(out.to_string())          # .to_string() removes the line "dtype:␣
 ↪float64"

    if result[1] <= 0.05:
        print("Strong evidence against the null hypothesis")
        print("Reject the null hypothesis")
        print("Data has no unit root and is stationary")
    else:
        print("Weak evidence against the null hypothesis")
        print("Fail to reject the null hypothesis")
```

```
        print("Data has a unit root and is non-stationary")
```

[27]:
```
adf_test(df1['total'])
```

```
Augmented Dickey-Fuller Test:
ADF test statistic        -5.592497
p-value                    0.000001
# lags used               18.000000
# observations           459.000000
critical value (1%)       -3.444677
critical value (5%)       -2.867857
critical value (10%)      -2.570135
Strong evidence against the null hypothesis
Reject the null hypothesis
Data has no unit root and is stationary
```

1.1.3 Run pmdarima.auto_arima to obtain recommended orders

This may take awhile as there are a lot of combinations to evaluate.

[28]:
```
'''
And since we believe it to be weekly, given the plots that we just saw of, say,
M is equal to seven.
'''

# For SARIMA Orders we set seasonal=True and pass in an m value
auto_arima(df1['total'],seasonal=True,m=7).summary()
```

[28]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                                      SARIMAX Results
================================================================================
===========
Dep. Variable:                                  y   No. Observations:
478
Model:             SARIMAX(1, 0, 0)x(1, 0, [1], 7)   Log Likelihood
-2383.417
Date:                            Fri, 05 Nov 2021   AIC
4776.834
Time:                                    20:23:50   BIC
4797.682
Sample:                                         0   HQIC
4785.030
                                            - 478
Covariance Type:                              opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
```

```
intercept       6.8706      2.094      3.281      0.001       2.767      10.975
ar.L1           0.1549      0.047      3.272      0.001       0.062       0.248
ar.S.L7         0.9436      0.017     55.586      0.000       0.910       0.977
ma.S.L7        -0.6912      0.057    -12.084      0.000      -0.803      -0.579
sigma2       1308.0797     83.835     15.603      0.000    1143.766    1472.393
================================================================================
===
Ljung-Box (L1) (Q):                   0.29   Jarque-Bera (JB):
60.79
Prob(Q):                              0.59   Prob(JB):
0.00
Heteroskedasticity (H):               0.86   Skew:
0.72
Prob(H) (two-sided):                  0.34   Kurtosis:
3.99
================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
"""
```

Excellent! This provides an ARIMA Order of (1,0,0) and a seasonal order of (2,0,0,7) Now let's train & test the SARIMA model, evaluate it, then compare the result to a model that uses an exogenous variable. ### Split the data into train/test sets We'll assign 42 days (6 weeks) to the test set so that it includes several holidays.

[29]:
```
'''
we're first

going to fit to just a classic Suruma based model.

So we'll just take a seasonal or remodel and see how it performs.

So the first thing going to do is do a train test split.

So we're going to try to forecast a month into the future for restaurant␣
↪visits,

which means our test set should also be about a month.
'''
len(df1)
```

[29]: 478

[30]:
```
# Set four weeks for testing
train = df1.iloc[:436]
test = df1.iloc[436:]
```

1.1.4 Fit a SARIMA(1,0,0)(1,0,1,7) Model

NOTE: To avoid a ValueError: non-invertible starting MA parameters found we're going to set enforce_invertibility to False.

[32]:
```
'''

So we already split the data into a training set and a test set, and now we␣
↪have our orders.

So it's time to actually fit this model.

now I'm only passing in the training data that it can fairly evaluate this␣
↪model,


my order for the p,d,q terms, for the Arima portion of the model are just (1,␣
↪0, 0) which

was reported back by auto Arima.

for the seasonal order of the model will go ahead say (1,0,1,7)

enforce_invertibility=False:

And then the last parameter I need to provide here is this parameter of
inforce Inevitability.

Now, the reason we have to enforce convertibility equal to false here is mainly

due to the way built stat's models library.

Essentially, we already know about the auto regression representation, where␣
↪the most
recent error can be written as a linear function of current and past␣
↪observations.

So we already know that we can write out this linear function.

And the key part is for inconvertible process.Theta here is less than one.
And so the most recent observations have higher weight than observations from␣
↪the more
distant past.
```

```
Which makes sense, right?

That more recent data should hold the higher weight then further out data into␣
↪the past.

However, when you have a process where theta is greater than one, then the␣
↪weights
increase as lags increase, which actually means the opposite.

That the more distance observations have greater influence on the current␣
↪error,
which is sometimes a peculiar situation.

And when Theta is equal to exactly one, then the weights are constant and size␣
↪and the
distance observations have the same influence as the recent observations.

So these last two situations typically don't make much sense, and so we prefer␣
↪this
inevitable process.

However, the way that stat's models has built out the Sorina X model internally
It will try to force convertibility by forcing this theta to be less than one.
And in some particular situations, that actually doesn't make sense and it'll␣
↪force an error.

So what we're going to do here is in order to avoid all those issues, we'll say
in force convertibility equal to false and a way to understand whether or not␣
↪you
need to do that is simply run the model and see if you get the error.

And the error you get is called value error, non-convertible starting M.A␣
↪parameters found.

So again, if you ever get the error.
When you're running one of these models that auto Auriemma suggested of value␣
↪error,
non-convertible starting M.A parameters found, that's totally OK.

Just inside your SARIMAX call, go ahead and say inforce inevitability equal to
false and then that should remove that error.



'''
```

```python
model=SARIMAX(train["total"],order=(1,0,0),seasonal_order=(1,0,1,7),enforce_invertibility=Fals
results=model.fit()
results.summary()
```

[32]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                                      SARIMAX Results
================================================================================
===========
Dep. Variable:                                total   No. Observations:
436
Model:             SARIMAX(1, 0, 0)x(1, 0, [1], 7)   Log Likelihood
-2155.511
Date:                              Fri, 05 Nov 2021   AIC
4319.023
Time:                                      20:46:14   BIC
4335.333
Sample:                                  01-01-2016   HQIC
4325.460
                                       - 03-11-2017
Covariance Type:                                opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          0.2194      0.043      5.066      0.000       0.134       0.304
ar.S.L7        0.9999      0.000   8044.788      0.000       1.000       1.000
ma.S.L7       -0.9405      0.023    -40.042      0.000      -0.987      -0.894
sigma2      1068.9003     54.351     19.667      0.000     962.374    1175.426
================================================================================
===
Ljung-Box (L1) (Q):                   1.02   Jarque-Bera (JB):
106.78
Prob(Q):                              0.31   Prob(JB):
0.00
Heteroskedasticity (H):               0.92   Skew:
0.77
Prob(H) (two-sided):                  0.61   Kurtosis:
4.87
================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
"""
```

[35]:
```
'''
And we're going to do now is get predicted values into the future for our test␣
↪set.

'''
start=len(train)
end=len(train)+len(test)-1

predictions=results.predict(start,end,dynamic=False).
↪rename('SARIMA(1,0,0)(1,0,1,7) Predictions')
predictions
```

[35]:
```
2017-03-12    186.304875
2017-03-13     90.985696
2017-03-14    100.278090
2017-03-15     96.887285
2017-03-16     98.669207
2017-03-17    140.777198
2017-03-18    235.249041
2017-03-19    172.961080
2017-03-20     88.051462
2017-03-21     99.623108
2017-03-22     96.733057
2017-03-23     98.624473
2017-03-24    140.750608
2017-03-25    235.214428
2017-03-26    172.936397
2017-03-27     88.038991
2017-03-28     99.609063
2017-03-29     96.719431
2017-03-30     98.610583
2017-03-31    140.730786
2017-04-01    235.181304
2017-04-02    172.912044
2017-04-03     88.026593
2017-04-04     99.595035
2017-04-05     96.705811
2017-04-06     98.596696
2017-04-07    140.710967
2017-04-08    235.148184
2017-04-09    172.887693
2017-04-10     88.014196
2017-04-11     99.581009
2017-04-12     96.692192
2017-04-13     98.582811
2017-04-14    140.691152
2017-04-15    235.115069
```

```
2017-04-16    172.863346
2017-04-17     88.001801
2017-04-18     99.566986
2017-04-19     96.678575
2017-04-20     98.568928
2017-04-21    140.671339
2017-04-22    235.081959
Freq: D, Name: SARIMA(1,0,0)(1,0,1,7) Predictions, dtype: float64
```

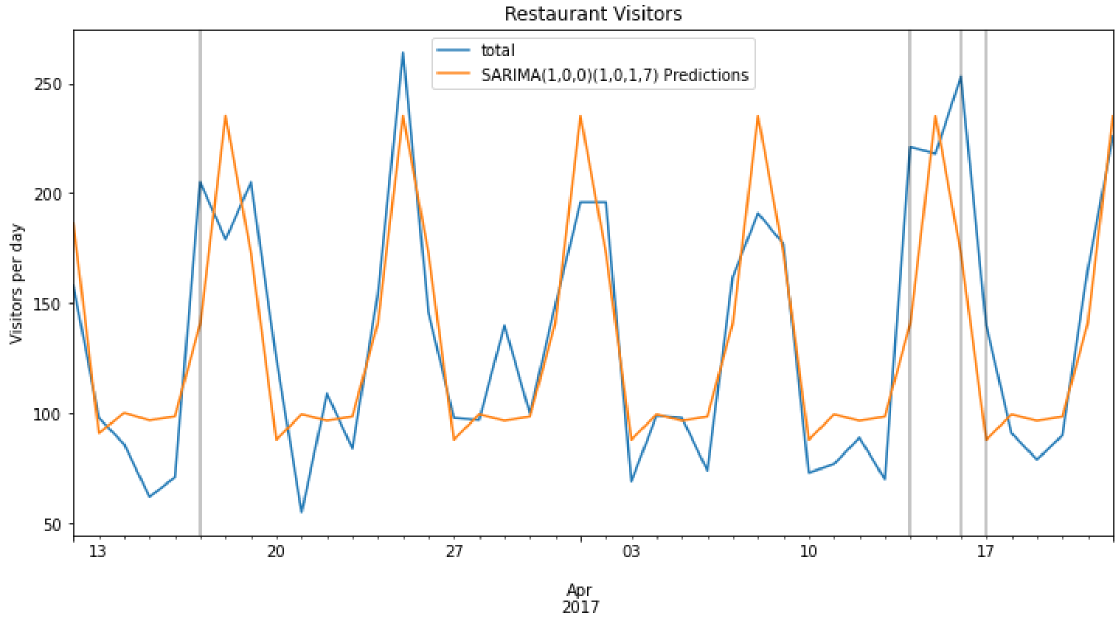
Passing dynamic=False means that forecasts at each point are generated using the full history up to that point (all lagged values).

For more information on these arguments visit https://www.statsmodels.org/stable/generated/statsmodels.tsa.arim

[36]:

```
'''

So we can see here in that second week, we're really doing quite well.

But you know this for this particular week, there's a dip or maybe even a peak␣
↪that
we didn't actually grab.

So would it be interesting to see if for the situations where we didn't do such␣
↪a
great job predicting?


And now we can see there was one holiday here that we didn't pick up on and␣
↪there's
three holidays here in the USA for this last one, and you'll notice it kind of␣
↪like
converges with these peaks.

So it'll be interesting to see if adding in the holidays as exogenous variables
would actually improve our model.
'''

# Plot predictions against known values
title='Restaurant Visitors'
ylabel='Visitors per day'
xlabel=''

ax = test['total'].plot(legend=True,figsize=(12,6),title=title)
predictions.plot(legend=True)
ax.autoscale(axis='x',tight=True)
ax.set(xlabel=xlabel, ylabel=ylabel)
for x in test.query('holiday==1').index:
```

```
    ax.axvline(x=x, color='k', alpha = 0.3);
```



1.1.5 Evaluate the Model

[37]:
```
'''
The last thing I want to do is evaluate the model quantitatively
using root mean squared error.
'''

from statsmodels.tools.eval_measures import mse,rmse

error1 = mse(test['total'], predictions)
error2 = rmse(test['total'], predictions)

print(f'SARIMA(1,0,0)(1,0,1,7) MSE Error: {error1:11.10}')
print(f'SARIMA(1,0,0)(1,0,1,7) RMSE Error: {error2:11.10}')
```

```
SARIMA(1,0,0)(1,0,1,7) MSE Error: 1021.947188
SARIMA(1,0,0)(1,0,1,7) RMSE Error: 31.96790872
```

[ ]: