06-SARIMA

November 5, 2021

1 SARIMA(p,d,q)(P,D,Q)m

2 Seasonal Autoregressive Integrated Moving Averages

We have finally reached one of the most fascinating aspects of time series analysis: seasonality.

Where ARIMA accepts the parameters (p, d, q), SARIMA accepts an additional set of parameters $(P, D, Q)m$ that specifically describe the seasonal components of the model. Here P, D and Q represent the seasonal regression, differencing and moving average coefficients, and m represents the number of data points (rows) in each seasonal cycle.

NOTE: The statsmodels implementation of SARIMA is called SARIMAX. The "X" added to the name means that the function also supports exogenous regressor variables. We'll cover these in the next section.

Related Functions:

sarimax.SARIMAX(endog[, exog, order, …]) sarimax.SARIMAXResults(model, params, …[, …]) Class to hold results from fitting a SARIMAX model.

For Further Reading:

Statsmodels Tutorial: Time Series Analysis by State Space Methods

[6]:
```
import pandas as pd
import numpy as np

#from statsmodels.tsa.statespace.sarimax import SARIMAX
#from statsmodels.graphics.tsaplots import plot_acf,plot_pacf # for determining␣
↪(p,q) orders
#from statsmodels.tsa.seasonal import seasonal_decompose      # for ETS Plots
#from pmdarima import auto_arima                              # for determining␣
↪ARIMA orders

# Ignore harmless warnings
import warnings
warnings.filterwarnings("ignore")
```

[7]:
```
'''

you'll often read in datasets and
```

```
they separate out the year and the month into columns like this.

So what we want to do is we want to figure out how we can use these two columns␣
↪to create a date time

index.

'''
# Load dataset
df = pd.read_csv('co2_mm_mlo.csv')
df
```

```
---------------------------------------------------------------------------
FileNotFoundError                         Traceback (most recent call last)
<ipython-input-7-f54d278abbc0> in <module>
     11 '''
     12 # Load dataset
---> 13 df = pd.read_csv('co2_mm_mlo.csv')
     14 df

/private/var/containers/Bundle/Application/AEFE78AE-0E04-4CBF-AE3D-3F11BBBC8C09/
↪Carnets-sci.app/Library/lib/python3.9/site-packages/pandas/io/parsers.py in␣
↪read_csv(filepath_or_buffer, sep, delimiter, header, names, index_col,␣
↪usecols, squeeze, prefix, mangle_dupe_cols, dtype, engine, converters,␣
↪true_values, false_values, skipinitialspace, skiprows, skipfooter, nrows,␣
↪na_values, keep_default_na, na_filter, verbose, skip_blank_lines, parse_dates,␣
↪infer_datetime_format, keep_date_col, date_parser, dayfirst, cache_dates,␣
↪iterator, chunksize, compression, thousands, decimal, lineterminator,␣
↪quotechar, quoting, doublequote, escapechar, comment, encoding, dialect,␣
↪error_bad_lines, warn_bad_lines, delim_whitespace, low_memory, memory_map,␣
↪float_precision, storage_options)
    608     kwds.update(kwds_defaults)
    609
--> 610     return _read(filepath_or_buffer, kwds)
    611
    612

/private/var/containers/Bundle/Application/AEFE78AE-0E04-4CBF-AE3D-3F11BBBC8C09/
↪Carnets-sci.app/Library/lib/python3.9/site-packages/pandas/io/parsers.py in␣
↪_read(filepath_or_buffer, kwds)
    460
    461     # Create the parser.
--> 462     parser = TextFileReader(filepath_or_buffer, **kwds)
    463
    464     if chunksize or iterator:

/private/var/containers/Bundle/Application/AEFE78AE-0E04-4CBF-AE3D-3F11BBBC8C09/
↪Carnets-sci.app/Library/lib/python3.9/site-packages/pandas/io/parsers.py in␣
↪__init__(self, f, engine, **kwds)
```

```
    817             self.options["has_index_names"] = kwds["has_index_names"]
    818
--> 819         self._engine = self._make_engine(self.engine)
    820
    821     def close(self):

/private/var/containers/Bundle/Application/AEFE78AE-0E04-4CBF-AE3D-3F11BBBC8C09/
 ↪Carnets-sci.app/Library/lib/python3.9/site-packages/pandas/io/parsers.py in␣
 ↪_make_engine(self, engine)
   1048             )
   1049         # error: Too many arguments for "ParserBase"
-> 1050         return mapping[engine](self.f, **self.options)  # type:␣
 ↪ignore[call-arg]
   1051
   1052     def _failover_to_python(self):

/private/var/containers/Bundle/Application/AEFE78AE-0E04-4CBF-AE3D-3F11BBBC8C09/
 ↪Carnets-sci.app/Library/lib/python3.9/site-packages/pandas/io/parsers.py in␣
 ↪__init__(self, src, **kwds)
   1865
   1866         # open handles
-> 1867         self._open_handles(src, kwds)
   1868         assert self.handles is not None
   1869         for key in ("storage_options", "encoding", "memory_map",␣
 ↪"compression"):

/private/var/containers/Bundle/Application/AEFE78AE-0E04-4CBF-AE3D-3F11BBBC8C09/
 ↪Carnets-sci.app/Library/lib/python3.9/site-packages/pandas/io/parsers.py in␣
 ↪_open_handles(self, src, kwds)
   1360         Let the readers open IOHanldes after they are done with their␣
 ↪potential raises.
   1361         """
-> 1362         self.handles = get_handle(

   1363             src,
   1364             "r",

/private/var/containers/Bundle/Application/AEFE78AE-0E04-4CBF-AE3D-3F11BBBC8C09/
 ↪Carnets-sci.app/Library/lib/python3.9/site-packages/pandas/io/common.py in␣
 ↪get_handle(path_or_buf, mode, encoding, compression, memory_map, is_text,␣
 ↪errors, storage_options)
    640                 errors = "replace"
    641             # Encoding
--> 642             handle = open(

    643                 handle,
    644                 ioargs.mode,

FileNotFoundError: [Errno 2] No such file or directory: 'co2_mm_mlo.csv'
```

We need to combine two integer columns (year and month) into a DatetimeIndex. We can do this by passing a dictionary into pandas.to_datetime() with year, month and day values. For more information visit https://pandas.pydata.org/pandas-docs/stable/reference/api/pandas.to_datetime.html

[3]:
```
'''
We'll create a new column called Date and say, PD to date time, and then we can␣
↪actually pass in a

dictionary call of what the year should be and what the month should be.And␣
↪then if we want what the day should be

'''
df["date"]=pd.to_datetime({"year":df["year"],"month":df["month"],"day":1})
```

[4]:
```
'''
And now if we check out the head of our data frame, we notice we have this date␣
↪and it looks like it's

now a time stamp object,
'''
df
```

[4]:
```
     year  month  decimal_date  average  interpolated       date
0    1958      3      1958.208   315.71        315.71 1958-03-01
1    1958      4      1958.292   317.45        317.45 1958-04-01
2    1958      5      1958.375   317.50        317.50 1958-05-01
3    1958      6      1958.458      NaN        317.10 1958-06-01
4    1958      7      1958.542   315.86        315.86 1958-07-01
..    …     …            …        …            …          …
724  2018      7      2018.542   408.71        408.71 2018-07-01
725  2018      8      2018.625   406.99        406.99 2018-08-01
726  2018      9      2018.708   405.51        405.51 2018-09-01
727  2018     10      2018.792   406.00        406.00 2018-10-01
728  2018     11      2018.875   408.02        408.02 2018-11-01

[729 rows x 6 columns]
```

[5]:
```
'''
And you'll notice that the date column is, in fact, a date time object.
'''
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 729 entries, 0 to 728
Data columns (total 6 columns):
 #   Column        Non-Null Count  Dtype
---  ------        --------------  -----
 0   year          729 non-null    int64
```

```
 1   month         729 non-null    int64
 2   decimal_date  729 non-null    float64
 3   average       722 non-null    float64
 4   interpolated  729 non-null    float64
 5   date          729 non-null    datetime64[ns]
dtypes: datetime64[ns](1), float64(3), int64(2)
memory usage: 34.3 KB
```

[6]:
```
'''
Well, we still need to do, though, is we want this to actually be the index.

And now if I check the head of the data frame, I have my date index
'''

df.set_index("date")
```

[6]:
```
            year  month  decimal_date  average  interpolated
date
1958-03-01  1958      3      1958.208   315.71        315.71
1958-04-01  1958      4      1958.292   317.45        317.45
1958-05-01  1958      5      1958.375   317.50        317.50
1958-06-01  1958      6      1958.458      NaN        317.10
1958-07-01  1958      7      1958.542   315.86        315.86
…            …     …            …        …            …
2018-07-01  2018      7      2018.542   408.71        408.71
2018-08-01  2018      8      2018.625   406.99        406.99
2018-09-01  2018      9      2018.708   405.51        405.51
2018-10-01  2018     10      2018.792   406.00        406.00
2018-11-01  2018     11      2018.875   408.02        408.02

[729 rows x 5 columns]
```

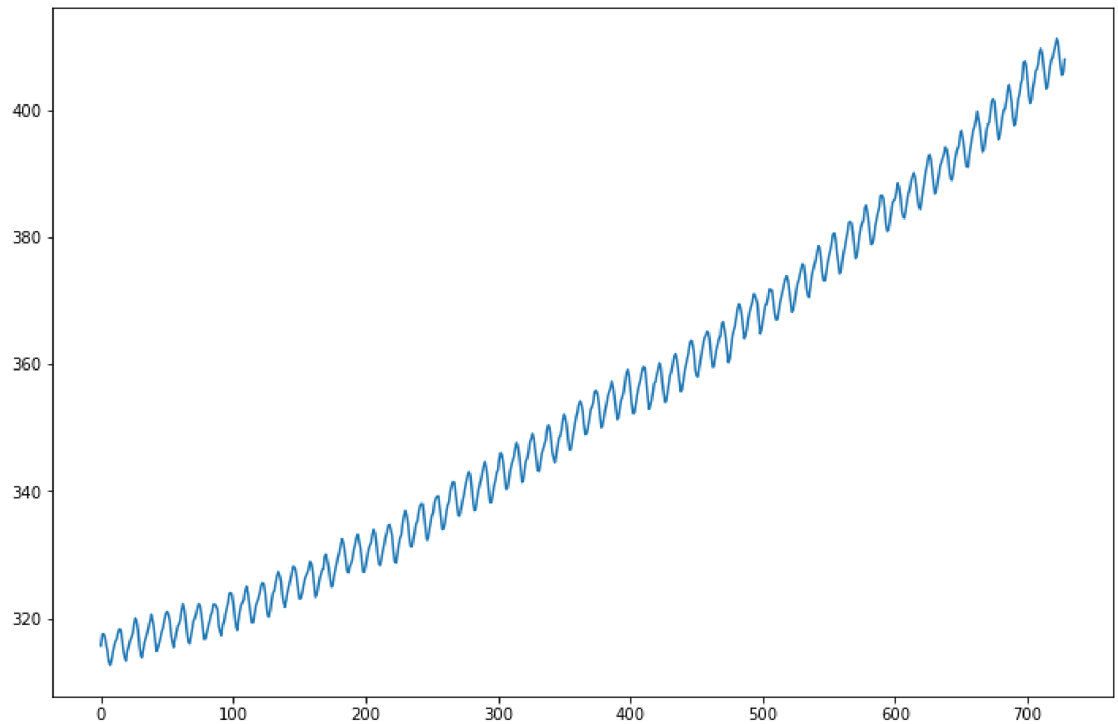
[7]:
```
'''
the last thing to do in order to use stats models is that my frequency.
'''

df.index.freq="MS"
```

2.1 Plot Source Data

[8]:
```
'''
Let's go ahead and plot out this data.

You'll notice that the average column is sometimes missing a few values.

So what they did instead is, they just interpolated it between the previous␣
↪points and some of the future
```

```
points to fill in that value.

So we'll go ahead and use the interpolated column that we were not missing any␣
↪points.

you should definitely see here that there are some clear seasonality as well as␣
↪some general upward trend.
'''
df["interpolated"].plot(figsize=(12,8))
```

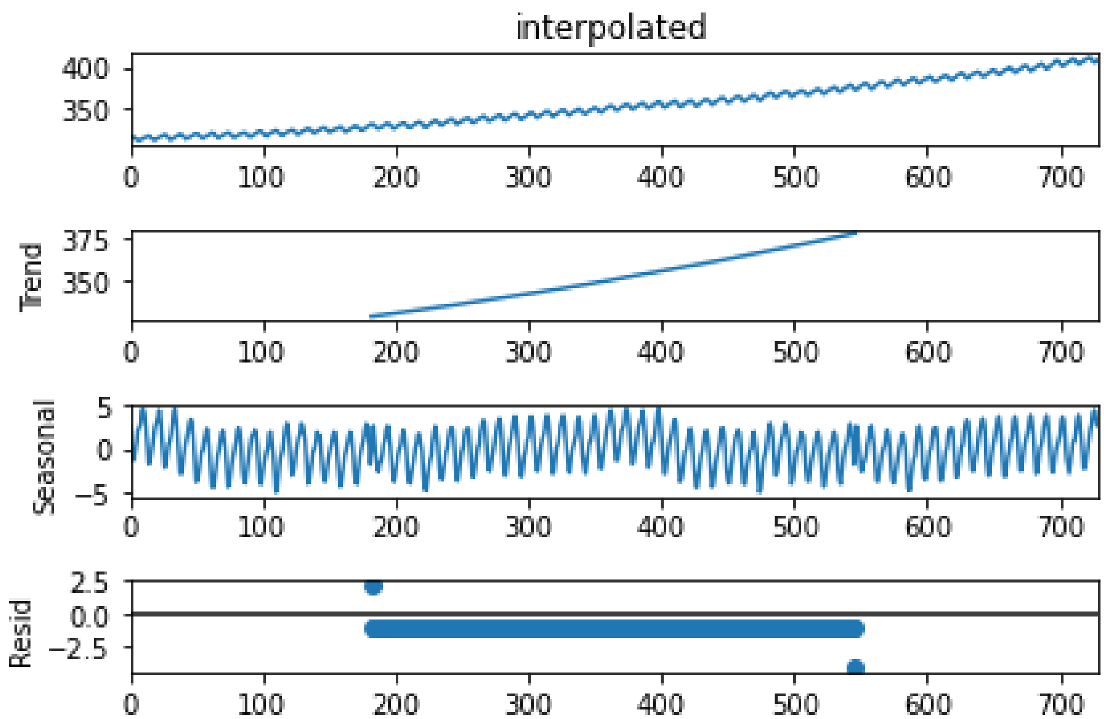
[8]: <AxesSubplot:>



2.1.1 Run an ETS Decomposition

[9]:
```
'''
And to confirm that there are some seasonality, we can run a decomposition so␣
↪we can say results,

go ahead and say seasonal decompose on that interpolated column.

And you can use either an additive model or a multiplicative model, but the key␣
↪thing to note here
```

```
is we'll definitely see a clear seasonal component.

So when we plot out this result, we can see here the observed values, the␣
↪general trend.

And definitely by the scale, it's going to be large enough that we want to take␣
↪that into account,

which is why we're using a seasonal Arima model.
'''

period=int(len(df)/2)
result=seasonal_decompose(df["interpolated"],model='add',period=period);
result.plot();
```



Although small in scale compared to the overall values, there is a definite annual seasonality.

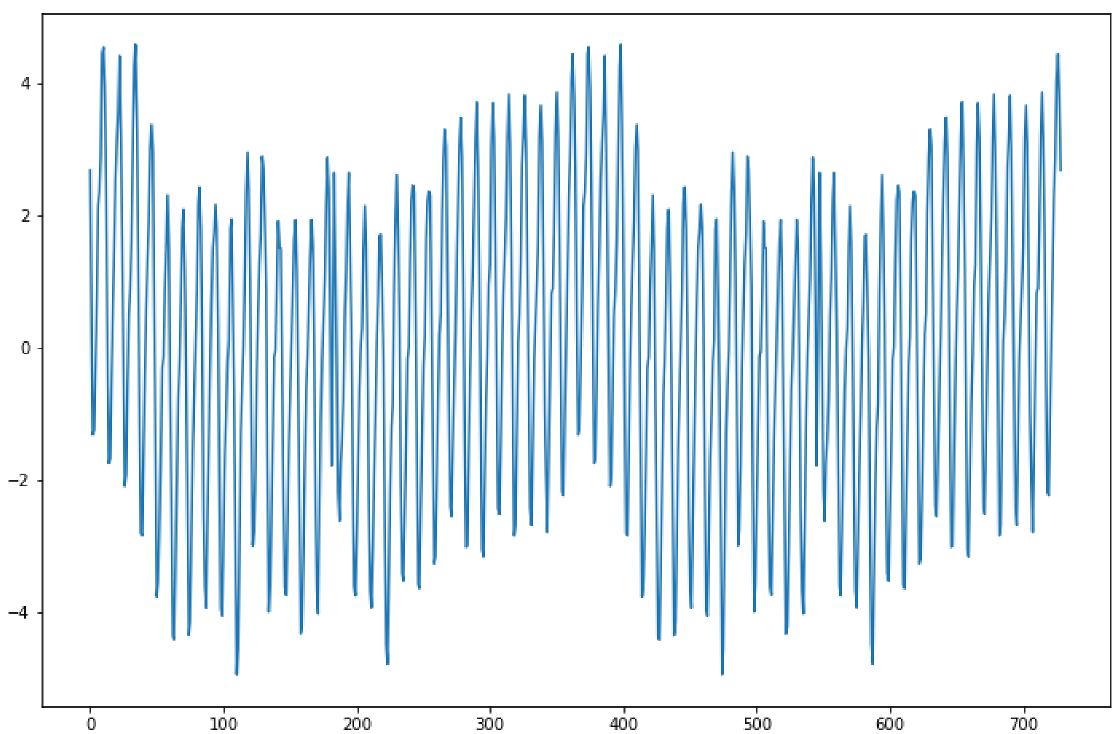
2.1.2 Run pmdarima.auto_arima to obtain recommended orders

This may take awhile as there are a lot more combinations to evaluate.

[11]:

```
'''
If you were unsure about your particular data set and the seasonality cycle (M)␣
 ↪of when you should set what

you should basically set equal to.

You could take the seasonal component of this result and then expand that, plot␣
 ↪it out into different

sizes and then judge from there.

Plot that out and then you could start looking and maybe zoom in on this to see␣
 ↪at what point does the

seasonal cycle repeat itself.

So we can see here the certain repetition.

You would just zoom in and see how many rows that take.
'''

result.seasonal.plot(figsize=(12,8))
```

[11]: <AxesSubplot:>



[10]:
```
'''
So because of that, let's go ahead and run the auto arima in order to obtain␣
↪the recommended orders.


we pass df['interpolated'] on to auto Arima and then we want to make sure that␣
↪we specify seasonals equal to true, even

though that technically is the default and because we specified seasonals equal␣
↪to true, we need to

make sure we state how many rows are there per period.

And in this case, the seasonal is happening every year.

So say M is equal to 12 since we have monthly data and there's 12 months per␣
↪year.
'''
auto_arima(df['interpolated'],seasonal=True,m=12).summary()
```

[10]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                                      SARIMAX Results
================================================================================
============
Dep. Variable:                                    y   No. Observations:
729
Model:             SARIMAX(0, 1, 3)x(1, 0, [1], 12)   Log Likelihood
-205.686
Date:                              Fri, 05 Nov 2021   AIC
423.371
Time:                                      15:07:23   BIC
450.913
Sample:                                           0   HQIC
433.998
                                              - 729
Covariance Type:                                opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ma.L1         -0.3564      0.036     -9.820      0.000      -0.428      -0.285
ma.L2         -0.0221      0.034     -0.658      0.511      -0.088       0.044
ma.L3         -0.0856      0.031     -2.756      0.006      -0.146      -0.025
ar.S.L12       0.9996      0.000   3082.714      0.000       0.999       1.000
ma.S.L12      -0.8671      0.021    -41.245      0.000      -0.908      -0.826
```

```
sigma2          0.0955      0.005     20.304      0.000       0.086       0.105
================================================================================
===
Ljung-Box (L1) (Q):                   0.07   Jarque-Bera (JB):
4.07
Prob(Q):                              0.79   Prob(JB):
0.13
Heteroskedasticity (H):               1.13   Skew:
0.00
Prob(H) (two-sided):                  0.33   Kurtosis:
3.37
================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
"""
```

[13]:
```
'''
Let's go ahead and do a train test, split on the data and test that our model,␣
 ↪see how it performs

in the test set and then forecast into the future.

So we have 729 rows.

Let's go ahead and set one year for testing.
'''
len(df)
```

[13]: 729

[14]:
```
'''
So that means our training is going to be the df.Loc from the beginning, all␣
 ↪the way to 717.
'''

train=df.iloc[:717]
test=df.iloc[717:]
```

Excellent! This provides an ARIMA Order of (0,1,3) combined with a seasonal order of (1,0,1,12) Now let's train & test the SARIMA(0,1,3)(1,0,1,12) model, evaluate it, then produce a forecast of future values. ### Split the data into train/test sets

2.1.3 Fit a SARIMA(0,1,3)(1,0,1,12) Model

[15]:
```python
'''
So we're going to do now create the model.

We're going to pass in the interpolated column from the training data.

And here we're going to specify two parameters.

One is the first order for the Arima.

So AR, I, and MA of the normal Arima model.

That is going to be this first component here of (0,1,3).

And then the other one we're going to do is the seasonal order.

And that one's going to be the second one here, which is (1 0 1 12)

then we'll fit the model and get those results, so we'll a model

that fit.

Check the results summary.

And this is basically the same results or very similar results to what was just
 ↪reported by Auto Arima,
'''

model=SARIMAX(train['interpolated'],order=(0,1,3),seasonal_order=(1,0,1,12))
results = model.fit()
results.summary()
```

[15]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                                      SARIMAX Results
================================================================================
============
Dep. Variable:                          interpolated   No. Observations:
717
Model:             SARIMAX(0, 1, 3)x(1, 0, [1], 12)   Log Likelihood
-201.196
Date:                               Fri, 05 Nov 2021   AIC
414.391
Time:                                       15:29:16   BIC
441.833
Sample:                                            0   HQIC
```

```
424.988
                                                              - 717
Covariance Type:                                                opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ma.L1         -0.3541      0.036     -9.908      0.000      -0.424      -0.284
ma.L2         -0.0246      0.035     -0.707      0.479      -0.093       0.044
ma.L3         -0.0874      0.035     -2.486      0.013      -0.156      -0.018
ar.S.L12       0.9996      0.000   2909.812      0.000       0.999       1.000
ma.S.L12      -0.8649      0.023    -37.958      0.000      -0.910      -0.820
sigma2         0.0951      0.005     20.274      0.000       0.086       0.104
================================================================================
===
Ljung-Box (L1) (Q):                   0.08   Jarque-Bera (JB):
4.28
Prob(Q):                              0.78   Prob(JB):
0.12
Heteroskedasticity (H):               1.15   Skew:
0.02
Prob(H) (two-sided):                  0.29   Kurtosis:
3.38
================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
"""
```

[21]:

```
'''
Now, let's go ahead and get predicted values for our test set range.

And now let's create some predictions by simply calling results.predict( the␣
 ↪start of the end and

we'll go ahead and say type is levels) to make sure we don't have any sort of␣
 ↪issues of any difference

in components.

And then we'll rename it.
'''
start=len(train)
end = len(train)+len(test)-1
predictions=results.predict(start,end,type='levels').rename("SARIMA␣
 ↪Predictions")
```

```
predictions
```

[21]:
```
717    406.610447
718    407.826126
719    408.579543
720    409.484660
721    411.043312
722    411.646671
723    410.865698
724    409.174774
725    407.074523
726    405.624632
727    405.896054
728    407.430499
Name: SARIMA Predictions, dtype: float64
```

Passing dynamic=False means that forecasts at each point are generated using the full history up to that point (all lagged values).

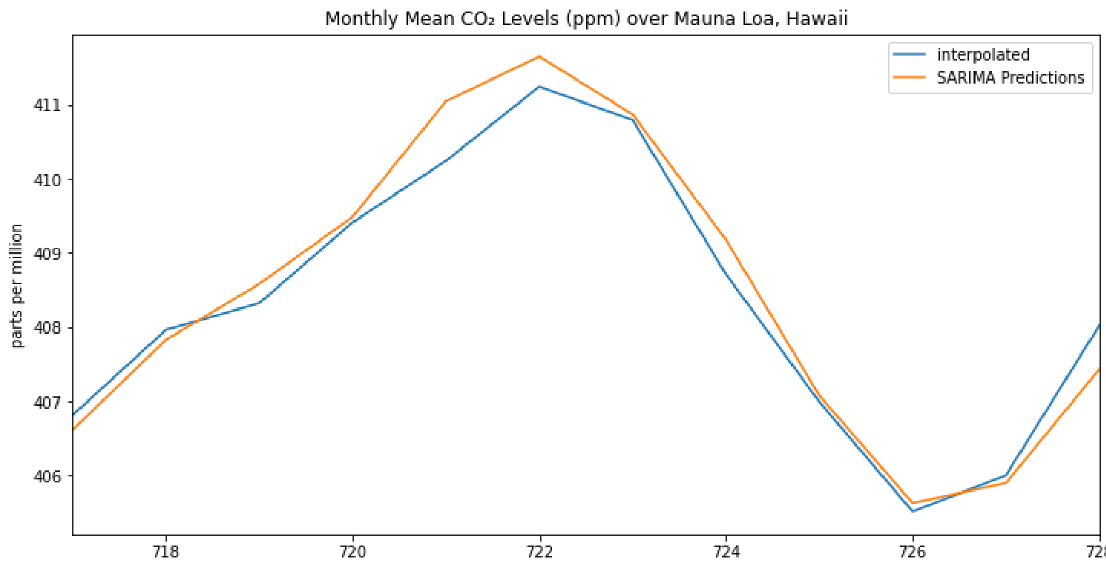
Passing typ='levels' predicts the levels of the original endogenous variables. If we'd used the default typ='linear' we would have seen linear predictions in terms of the differenced endogenous variables.

For more information on these arguments visit https://www.statsmodels.org/stable/generated/statsmodels.tsa.arin

[ ]:
```

```

[23]:
```
'''
So we run that, we have our predictions, so let's go ahead and plot them out␣
↪against the test results,

here we can see the real blue interpolating results and the Sarino predictions.

So as you can tell for predicting about a year out, we're actually a pretty␣
↪good.

Again, remember that our SARIMA model actually does not know what this data␣
↪should be.

And we can see compared to the real data, it's pretty on target.
'''
# Plot predictions against known values
title = 'Monthly Mean CO  Levels (ppm) over Mauna Loa, Hawaii'
ylabel='parts per million'
xlabel=''

ax = test['interpolated'].plot(legend=True,figsize=(12,6),title=title)
predictions.plot(legend=True)
ax.autoscale(axis='x',tight=True)
```

```
ax.set(xlabel=xlabel, ylabel=ylabel);
```



[42]:
```
'''
So if we actually want to evaluate the model, we can always do things such as␣
↪import root, mean squared

error.

'''

from sklearn.metrics import  mean_squared_error
error=mean_squared_error(test['interpolated'],predictions)
print(f'SARIMA(0,1,3)(1,0,1,12) MSE Error: {error:11.10}')
```

```
SARIMA(0,1,3)(1,0,1,12) MSE Error: 0.1284765781
```

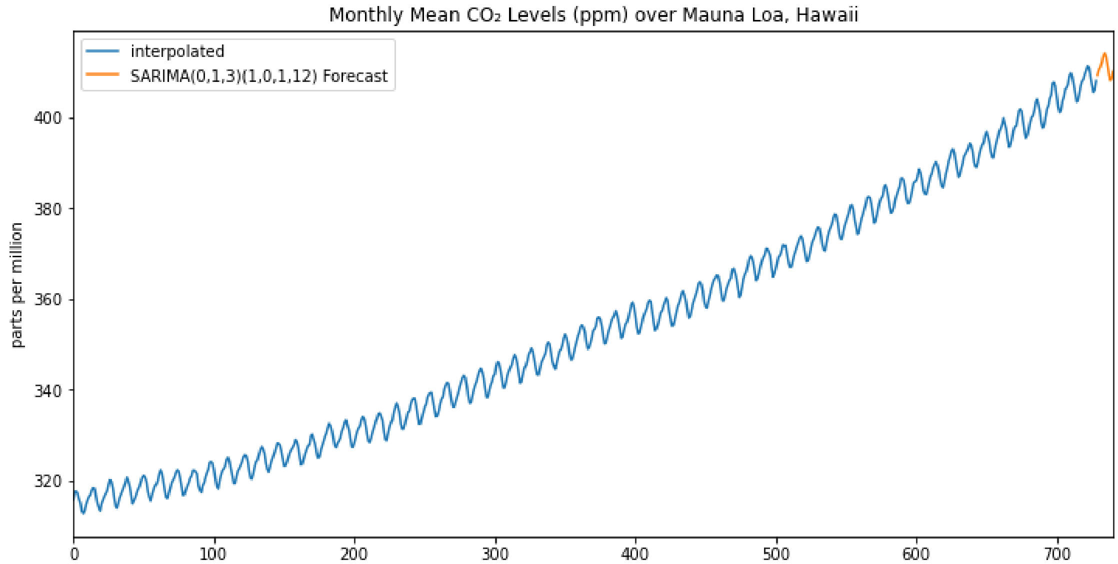
[43]:
```
'''
And then my error would be something like our RMSE.

Test interpolated compared to our predictions,
'''
from statsmodels.tools.eval_measures import rmse
error=rmse(test['interpolated'],predictions)
print(f'SARIMA(0,1,3)(1,0,1,12) RMSE Error: {error:11.10}')
```

```
SARIMA(0,1,3)(1,0,1,12) RMSE Error: 0.3584362958
```

2.1.4 Retrain the model on the full data, and forecast the future

[45]:
```python
'''
Fit Entire data into model

And let's go on to predict one year into the future.

And we don't want to output the different results.

We want to put the true results in the same units as the original data.

so type='levels'

'''
model=SARIMAX(df['interpolated'],order=(0,1,3),seasonal_order=(1,0,1,12))
results=model.fit()
fcast = results.predict(len(df),len(df)+11,typ='levels').
 ↪rename('SARIMA(0,1,3)(1,0,1,12) Forecast')
```

[46]:
```python
'''
So when we run that, we can see here at the very end what our forecast is.

'''

# Plot predictions against known values
title = 'Monthly Mean CO  Levels (ppm) over Mauna Loa, Hawaii'
ylabel='parts per million'
xlabel=''

ax = df['interpolated'].plot(legend=True,figsize=(12,6),title=title)
fcast.plot(legend=True)
ax.autoscale(axis='x',tight=True)
ax.set(xlabel=xlabel, ylabel=ylabel);
```



Monthly Mean CO₂ Levels (ppm) over Mauna Loa, Hawaii
interpolated
SARIMA(0,1,3)(1,0,1,12) Forecast
parts per million
400
380
360
340
320
0
100
200
300
400
500
600
700

[ ]: