

Algorithm 1 Second-order Inside Algorithm.

- 1: **define:** $I, S, C \in \mathbb{R}^{n \times n \times B}$ ▷ B is batch size
- 2: **initialize:** $C_{i,i} = \log e^0 = 0, 0 \leq i \leq n$
- 3: **for** $w = 1$ **to** n **do** ▷ span width
- 4: **Batchify:** $0 \leq i; j = i + w \leq n$ (also for B)
- 5:
$$I_{i,j} = \log \left(\frac{e^{C_{i,i} + C_{j,i+1}} + \sum_{i < r < j} e^{I_{i,r} + S_{r,j} + s(i,r,j)}}{\sum_{i < r < j} e^{I_{i,r} + S_{r,j} + s(i,r,j)}} \right) + s(i,j)$$
- 6:
$$S_{i,j} = \log \sum_{i \leq r < j} e^{C_{i,r} + C_{j,r+1}}$$
- 7:
$$C_{i,j} = \log \sum_{i < r \leq j} e^{I_{i,r} + C_{r,j}}$$
- 8: **end for**
- 9: **return** $C_{0,n} \equiv \log Z$