21. Find the value of $x^4 + y^4 + z^4$.

(a) 1925 (b) 1922

(c) 1920 (d) None of these

22. Every ten years, the Indian government counts all the people living in the country. Suppose that the director of the census has reported the following data on two neighbouring villages Chota Hazri and Mota Hazri:

Chota Hazri has 4,522 fewer males than Mota-Hazri.

Mota Hazri has 4,020 more females than males.

Chota Hazri has twice as many females as males.

Chota Hazri has 2,910 fewer females than Mota Hazri.

What is the total number of males in Chota Hazri?

(a) 11264 (b) 14174

(c) 5632 (d) 10154

23. At a certain fast food restaurant, Brian can buy 3 burgers, 7 shakes, and one order of fries for ` 120 exactly. At the same place, it would cost ` 164.50 for 4 burgers, 10 shakes, and one order of fries. How much would it cost for an ordinary meal of one burger, one shake, and one order of fries?

(a) ` 31 (b) ` 41

(c) ` 21 (d) Cannot be determined

24. If $x > 7$ and $y < -2$, then which of the following statements is true?

(a) $(x + 4y) > 1$ (b) $x > -4y$

(c) $-4x < 5y$ (d) None of these

25. For a Fibonacci sequence, from the third term onwards, each term in the sequence is the sum of the previous two terms in that sequence. If the difference in squares of seventh and sixth terms of this sequence is 517, what is tenth term of this sequence?

(a) 147 (b) 76

(c) 123 (d) Cannot be determined

Review Test 2

1. Let x, y be two positive numbers such that $x + y = 1$. Then, the minimum value of $\left(x+\frac{1}{x}\right)^2 + \left(y+\frac{1}{y}\right)^2$ is

   (a) 12 (b) 20
   (c) 12.5 (d) 13.3

2. Ujakar and Keshab attempted to solve a quadratic equation. Ujakar made a mistake in writing down the constant term. He ended up with the roots (4, 3). Keshab made a mistake in writing down the coefficient of x. He roots as (3, 2). What will be the exact roots of the original quadratic equation?

   (a) (6,1) (b) (–3,–4)
   (c) (4,3) (d) (–4,–3)

3. A change making machine contains 1 rupee, 2 rupee and 5 rupee coins. The total number of coins is 300. The amount is ` 960. If the number of 1 rupee coins and the number of 2 rupee coins are interchanged, the value comes down by ` 40. The total number of 5 rupee coins is:

   (a) 100 (b) 140
   (c) 60 (d) 150

4. If $x > 2$ and $y > –1$, then which of the following statements is necessarily true?

   (a) $xy > –2$ (b) $–x < 2y$
   (c) $xy < –2$ (d) $–x > 2y$

Directions for Questions 5 to 7: *A*, *B*, *C* and *D* collected one rupee coins following the given pattern.

Together they collected 100 coins.

Each one of them collected an even number of coins.

Each one of them colIected at least 10 coins.

No two of them collected the same number of coins.

5. The maximum number of coins collected by any one of them cannot exceed :

   (a) 64 (b) 36

   (c) 54 (d) None of these

6. If A collected 54 coins, then the difference in the number of coins between the one who collected maximum number of coins and the one who collected the second highest number of coins must be at least:

   (a) 12 (b) 24

   (c) 30 (d) None of these

7. If A collected 54 coins and B collected two more coins than twice the number of coins collected by C, then the number of coins collected by B could be:

   (a) 28 (b) 20

   (c) 26 (d) 22

Directions for Questions 8 and 9: A, B and C are three numbers. Let

@ (A, B) = Average of A and B,

/ (A, B) = Product of A and B, and

x (A, B) = The result of dividing A by B.

8. The sum of A and B is given by:

   (a) /(@ (A, B), 2) (b) x(@(A, B), 2)

   (c) @(/(A, B) , 2) (d) @ (x(A, B), 2)

9. Average of A, B and C is given by:

   (a) @(/@(/(B, A), 2), C), 3

   (b) x(@(/@(B, A), 3), C), 2)

   (c) /((x(@ (B, A), 2), C), 3)

   (d) /(x(@(/(@(B, A), 2), C), 3), 2)

Directions for Questions 10 and 11: For real numbers x and y, let $f(x, y)$ = Positive square root of $(x + y)$, if $(x + y)^{0.5}$ is real = $(x + y)^2$, otherwise $g(x, y) = (x + y)^2$, if $(x + y)^{0.5}$ is real = $-(x + y)$ otherwise

10. Which of the following expressions yields a positive value for every pair of non-zero real numbers (x, y)?

    (a) $f(x, y) - g(x, y)$
    (b) $f(x, y) - (g(x, y))^2$
    (c) $g(x, y) - (f(x, y))^2$
    (d) $f(x, y) + g(x, y)$

11. Under which of the following conditions is $f(x, y)$ necessarily greater than $g(x, y)$?

    (a) Both x and y are less than-1
    (b) Both x and y are positive
    (c) Both x and y are negative
    (d) $y > x$

Directions for Questions 12 to 14: For three distinct real numbers x, y and z, let

$$f(x, y, z) = \min(\max(x, y), \max(y, z), \max(z, x))$$
$$g(x, y, z) = \max(\min(x, y), \min(y, z), \min(z, x))$$
$$h(x, y, z) = \max(\max(x, y), \max(y, z), \max(z, x))$$
$$j(x, y, z) = \min(\min(x, y), \min(y, z), \min(z, x))$$
$$m(x, y, z) = \max(x, y, z)$$
$$n(x, y, z) = \min(x, y, z)$$

12. Which of the following is necessarily greater than 1?

    (a) $[h(x, y, z) - f(x, y, z)]/j(x, y, z)$
    (b) $j(x, y, z)/h(x, y, z)$
    (c) $f(x, y, z)/g(x, y, z)$
    (d) $[f(x, y, z) + h(x, y, z) - g(x, y, z)]/j(x, y, z)$

13. Which of the following expressions is necessarily equal to 1?

    (a) $[f(x, y, z) - m(x, y, z)]/[g(x, y, z) - h(x, y, z)]$

(b) $m(x, y, z) – f(x, y, z) – f(x, y, z)]/[g(x, y, z) – n(x, y, z)$

(c) $[j(x, y, z) – g(x, y, z)]/h(x, y, z)$

(d) $[f(x, y, z) – h(x, y, z)]/f(x, y, z)$

14. Which of the following expressions is in determinate?

(a) $[f(x, y, z) – h(x, y, z)]/[g(x, y, z) – j(x, y, z)]$

(b) $f(x, y, z) – h(x, y, z) + g(x, y, z) + j(x, y, z)]/[j(x, y, z) + h(x, y, z) – m(x, y, z) – n(x, y, z)]$

(c) $[g(x, y, z) – j(x, y, z)]/[f(x, y, z) – h(x, y, z)]$

(d) $[h(x, y, z) – f(x, y, z)]/[n(x, y, z) – g(x, y, z)]$

15.

x	1	2	3	4	5	6
y	4	8	16	28	44	64

In the above table, for suitable chosen constants a, b and c, which one of the following best describes the relation between y and x?

(a) $y = a + bx$

(b) $y = a + bx + cx^2$

(c) $y = e^{a+bx}$

(d) None of these

16. If $a_1 = 1$ and $a_{n+1} = 2a_n + 5$; $n = 1, 2$……., then a_{100} is equal to:

(a) $(5 \times 2^{99} – 6)$

(b) $(5 \times 2^{99} + 6)$

(c) $(6 \times 2^{99} + 5)$

(d) $(6 \times 2^{99} – 5)$

17. Let x, y and z be distinct integers that are odd and positive. Which one of the following statements cannot be true?

(a) xyz^2 is odd

(b) $(x – y)^2 z$ is even

(c) $(x + y – z)^2 (x + y)$ is even

(d) $(x – y) (y + z) (x + y – z)$ is odd

18. Let S be the set of prime numbers greater than or equal to 2 and less than 100. Multiply all elements of S. With how many consecutive zeros will the product end?

(a) 1

(b) 4

(c) 5

(d) 10

19. Each of the numbers x_1, x_2,………,x_n; $n \geq 4$, is equal to 1 or –1. Suppose

$x_1x_2x_3x_4 + x_2x_3x_4x_5 + x_3x_4x_5x_6 +$…………+ $x_{n-2}x_{n-1}x_n - x_1 + x_{n-1}x_nx_1x_2 + x_nx_1x_2x_3 = 0$, then,

(a) n is even

(b) n is odd

(c) n is an odd multiple of 3

(d) n is prime

For a real number x, let $f(x) = 1/(1 + x)$, if x is non-negative $= 1 + x$, if x is negative $f^n(x) = f(f^{n-1}(x))$;$n = 2, 3$…….

20. What is the value of the product $f(2)\, f^2(2)\, f^3(2)\, f^4(2)\, f^5(2)$?

(a) 1/13 (b) 3

(c) 1/18 (d) None of these

21. r is an integer ≥ 2. Then, what is the value of $f^{r-1}(-r) + f^r(-r) + f^{r+1}(-r)$?

(a) –1 (b) 0

(c) 1 (d) None of these

22. The set of all positive integers is the union of two disjoint subsets:

$\{f(1), f(2), \ldots, f(n), \ldots\}$ and $\{g(1), g(2), \ldots\ g(n), \ldots\}$, where $f(1) < f(2) < \ldots < f(n)$ ….., and $g(1) < g(2) < \ldots < g(n)$ ……., and $g(n) = (f(n)) + 1$ for all $n \geq 1$.

What is the value of $g(1)$?

(a) 0 (b) 2

(c) 1 (d) Cannot be determined

Directions for Questions 23 to 25: There are three bottles of water, *A*, *B*, *C*, whose capacities are 10*L*, 6*L*, and 4*L* respectively. For transferring water from one bottle to another and to drain out the bottles, there exists a piping system. The flow through these pipes is computer –controlled. The computer that controls the flow through these pipes can be fed with three types of instructions, as explained below.

Instruction type	**Explanation of the instruction**
Fill (X, Y)	Fill bottle labeled X from the water in bottle labeled Y where the remaining capacity of X is less than or equals to the amount of water in Y.
Empty (X, Y)	Empty out the water in bottle labeled X into bottle labeled Y, where the amount of water in X is less than or equals to remaining capacity of Y.
Drain (X)	Drain out all the water contained in bottle labeled X.

Initially, A is full with water, and B and C are empty.

23. After executing a sequence of three instructions, bottle A contains two liters of water. The first and the third of these instructions are shown below.

    First instruction: FILL (C, A)

    Third instruction: FILL (C, A)

    Then, which of the following statements about the instructions is true?

    (a) The second instruction is FILL(B, A)

    (b) The second instruction is EMPTY(C, B)

    (c) The second instruction transfers water from B to C

    (d) Cannot be determined

24. Consider the same sequence of three instructions and the same initial state mentioned above. Three more instructions are added at the end of the above sequence to have A contain 8L of water. In this total sequence of six instructions, the fourth one is DRAIN (A). This is the only DRAIN instruction in the entire sequence. At the end of the execution of the above sequence, how much water is contained in C?

(a) 2*L* (b) 4*L*

(c) 0 (d) None of these

25. For the question above, what is the amount of water contained in *B*?

(a) 2*L* (b) 4*L*

(c) 0 (d) None of these

Review Test 3

Directions for Questions 1 to 3: These are based on the functions defined below

$Q(a, b)$ = Quotient when a is divided by b

$R^2 (a, b)$ = Remainder when a is divided by b

$R (a, b) = a^2/b^2$

$SQ (a, b) = \sqrt{(a-1)/(b-1)}$

1. SQ (5, 10) – ? > 0

   (a) (8/3) R (5, 10)
   (b) R^2 (5, 10) + Q (5, 10)
   (c) R^2 (5, 10)/2
   (d) $\frac{1}{2}$ {R (2, 3) + SQ (17, 26)}

2. SQ (a, b) is same as

   (a) $bQ(a, b) + R^2(a)$
   (b) $\sqrt{R\,(a,b)-1}$
   (c) [R{(a – 1), (b – 1)}
   (d) $\sqrt{R\,(a,1)-1}/\sqrt{R\,(b,1)}$

3. Which of the following relations cannot be false?

   (a) $R(a, b) = R^2(a, b)\lozenge Q(a, b)$
   (b) $a^2 \lozenge Q(a, b) = b^2 \lozenge R^2(a, b)$
   (c) $a = R^2 (a, b) + y \lozenge Q (a, b)$
   (d) $SQ(a, b) = R(a, b) \lozenge R^2(a, b)$

Directions for Questions 4 to 7: *Answer the questions based on the following information:*

$W(a, b)$ = least of a and b

$M(a, b)$ = greatest of a and b

$N(a)$ = absolute value of a

4. Find the value of 1 + M [y + N {–W (x, y)}, N {y + W (M (x, y), N (y))}] given that x = 2 and y = – 3.

(a) 0 (b) 1

(c) 2 (d) 3

5. Given that $a > b$, then the relation $M\{N(x), W(x, y)\} = W[x, N\{M(x, y)\}]$ does not hold if

(a) $x > 0, y < 0, |x| > |y|$

(b) $x > 0, y < 0, |y| > |x|$

(c) $x > 0, y > 0$

(d) $x < 0, y < 0$

6. Which of the following must be correct for $x, y < 0$

(a) $N(W(x, y))$ £ $W(N(x), N(y))$

(b) $N(M(x, y)) > W(N(x), N(y))$

(c) $N(M(x, y)) = W(N(x), N(y))$

(d) $N(M(x, y)) < M(N(x), N(y))$

7. For what value of x is $W(x^2 + 2x, x + 2) < 0$?

(a) $-2 < x < 2$ (b) $-2 < x < 0$

(c) $x < -2$ (d) Both (2) and (3)

8. It is given that, $(a^{n-3} + a^{n-5}b^2 + \ldots + b^{n-3})pq = 0$, where p and q π 0 and n is odd then $\dfrac{(a^n - b^n)(a+b)}{(a^n + b^n)(a-b)}$ =?

(a) 1 (b) –1

(c) 3/2 (d) 0

9. If $f = \dfrac{1}{\log_2 \pi} + \dfrac{1}{\log_{4.5} \pi}$, which of the following is true?

(a) $f > 4$ (b) $2 < f < 4$

(c) $1 < f < 2$ (d) $0 < f < 1$

10. If $px + qy > rx + sy$, and $y, x, p, q, r, s > 0$ and if $x < y$, then which of the following must be true?

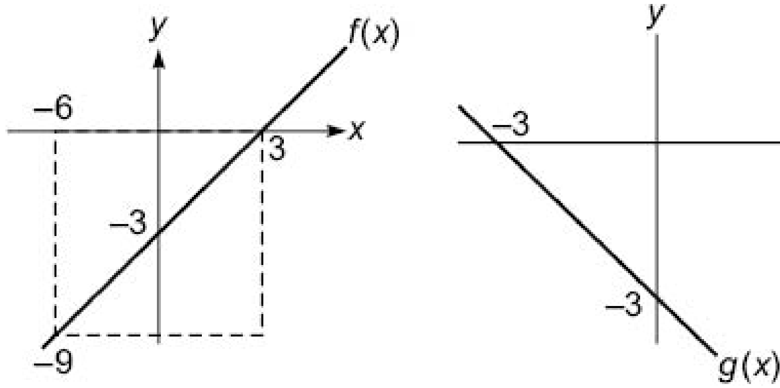
(a) $\frac{p}{q} > \frac{q}{r}$

(b) $p - q > r - s$

(c) $p + q > r + s$

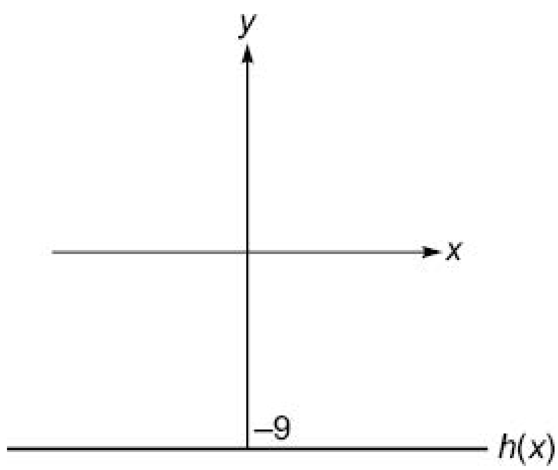
(d) $p + q < r + s$

Directions for Questions 11 to 13: $f(x)$ *and* $g(x)$ are defined by the graphs shown below:



Each of the following questions has a graph of function $h(x)$ with the answer choices expressing $h(x)$ in terms of a relationship of $f(x)$ or/and $g(x)$. Choose the alternative that could represents the relationship appropriately.

11.



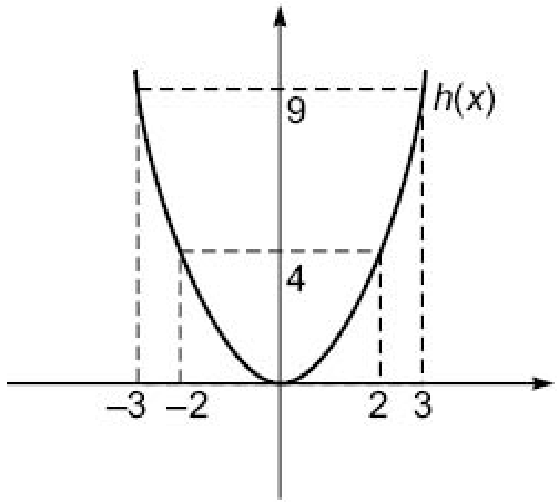
(a) $6f(x) + 6g(x)$

(b) $-1.5f(x) + 1.5g(x)$

(c) $1.5f(x) + 1.5g(x)$

(d) None of these

12.



(a) $9 - f(x)\, g(x)$

(b) $[f(x) + g(x) + 4]^2 + [f(x) - 2]^2 + [g(x)]$

(c) $[f(x) - g(x) + 4]^2 - 2$

(d) $f(x)g(x) - 9$

13. (a) $2 f(x) + g(x)$

(b) $f(x) + 2g(x) - 9$

(c) $\frac{3}{2} f(x) + \frac{g(x)}{2}$

(d) None of these

14. If $p^a = q^b = r^c$ and $\frac{p}{q} = \frac{q}{r}$, $\left(\frac{1}{a} + \frac{1}{c}\right) b = ?$

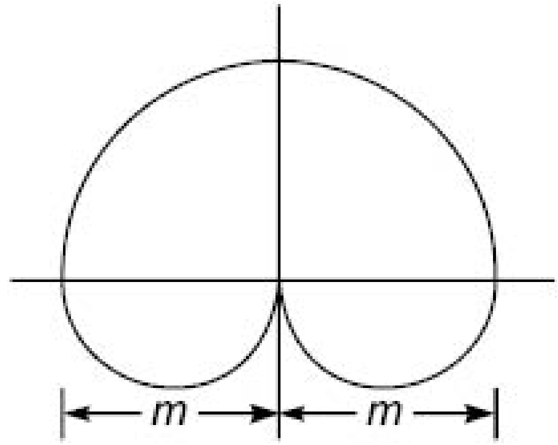
(a) 1

(b) 1/2

(c) 3/4

(d) 2

15. What could be the equation of the following curve?



(a) $(a^2 + b^2)^2 = m^2(a^2 + b^2)$

(b) $(a^2 + b^2 – mb)^2 = m^2(a^2 + b^2)$

(c) $a^2 + b^2 – mb = m^2(a^2 + b^2)$

(d) None of these

Directions for Questions 16 to 18: It is given that $f(x) = p^{x,}$ $g(x) = (–p)^x$; $h(x) = (1/p)^x$, $k(x) = (–1/p)^x$

16. Think of a situation where a function is odd or even, if the function is odd it is given a weightage of 1; otherwise, it is given a weightage of 0. What is the result if the weightages of four functions are added?

(a) 2 | (b) 0
(c) 1 | (d) –1

17. If ‘p’ and ‘x’ are both whole numbers other than 0 and 1, which of the functions must have the highest value?

(a) $g(x)$ only | (b) $f(x)$ only
(c) $g(x)$ and $h(x)$ both | (d) $h(x)$ and $k(x)$*both*

18. Which of the following is true if ‘p’ is a positive number and x is a real number?

(a) $\{ f(x) – h(x) \} / \{g(x) – k(x)\}$ is always positive

(b) $f(x).\ g(x)$ is always negative

(c) $f(x).\ h(x)$ is always greater than one

(d) $g(x).\ h(x)$ could exist outside the real domain

19. If x, and $y \geq 1$ and belong to set of integers then which of the following is true about the function $(xy)^n$?

(a) The function is odd if 'x' is even and 'y'is odd.

(b) The function is odd if 'x' is odd and 'y' is even.

(c) The function is odd if 'x' and 'y' both are odd.

(d) The function is even if 'x' and 'y' both are even.

20. If $f(a,b)$ = remainder left upon division of b by a, then the maximum value for $f(f(a, b), f(a + 1, b + 1)) \times f(f(a, \mathrm{b}), 0)$ is ….. (b and a are co-primes)

(a) $a - 1$ (b) a

(c) 0 (d) 1

Review Test 4

1. The number of solutions of $\frac{\log 5 + \log(y^2 + 1)}{\log(y - 2)} = 2$ is:

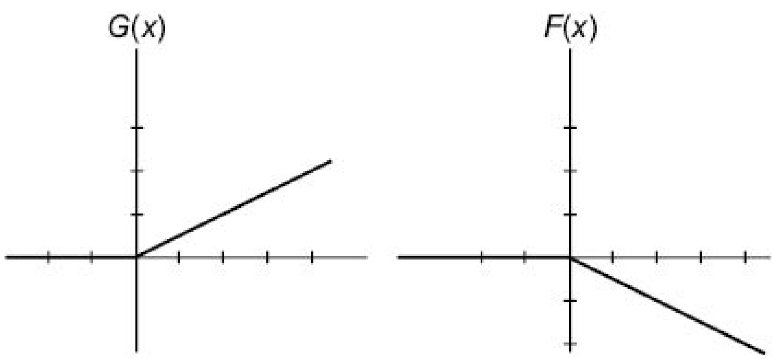
   (a) 3 (b) 2

   (c) 1 (d) None of these

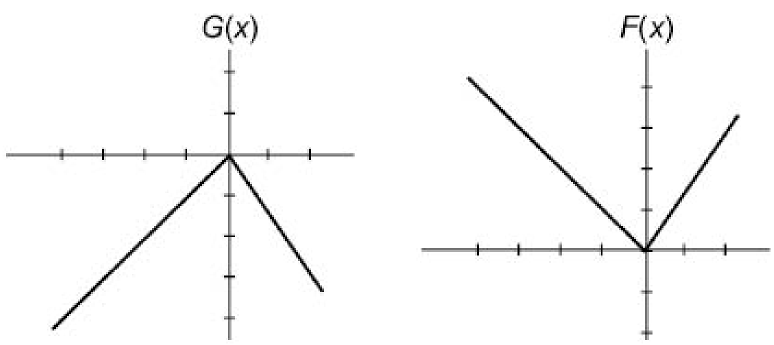
Information for Questions 2 and 3: Given below are two graphs labeled *F*(*x*) and *G*(*x*). Compare the graphs and give the answer in accordance to the options given below:

(a) $F(-x) = G(x) + x/3$
(b) $F(-x) = -G(x) - x/2$
(c) $F(-x) = -G(x) + x/2$
(d) $2F(-x) = +G(x) - x/3$

2.



3.



4. If *a* is a natural number which of the following statements is always true?

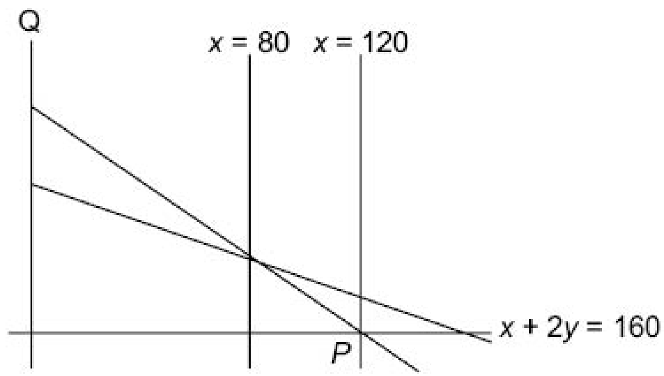
(a) $(a + 1)(a^2 + 1)$ is odd

(b) $9a^2 + 6a + 6$ is even

(c) $a^2 - 2a$ is even

(d) $a^2(a^2 + a) + 1$ is odd

5. In the figure below, equation of the line *PQ* is



(a) $x + y = 120$

(b) $2x + y = 120$

(c) $x + 2y = 120$

(d) $2x + y = 180$

6. For which of the following functions is $\frac{f(a) - f(b)}{a - b}$ constant for all the numbers '*a*' and '*b*', where *a* π b?

(a) $f(y) = 4y + 7$

(b) $f(y) = y + y^2$

(c) $f(y) = \cos y$

(d) $f(y) = \log_e y$

7. Given that $f(a, b, c) = \frac{a + b + c}{3}$ then

(a) $f(a, b, c) \geq \frac{|a| + |b| + |c|}{3}$

(b) $f(a, b, c) \geq \max(a, b, c)$

(c) $|f(a, b, c)| \geq \frac{|a + b + c|}{3}$

(d) $|f(a, b, c)|$ £ $\frac{|a| + |b| + |c|}{3}$

8. We are given two variables x and y. The values of the variables are $x = \frac{1}{a+b}$ and $y = \frac{3}{c+x}$. Find the value of the expression $\frac{7y}{x}$

(a) $\frac{21(a+b)^2}{ca+cb+1}$

(b) $\frac{3(a+b)}{7ab+ac}$

(c) $\frac{7}{3(ca+cb+1)}$

(d) None of these

9. If $p = \frac{12-|x-3|}{12+|x-3|}$ the maximum value that 'p' can attain is:

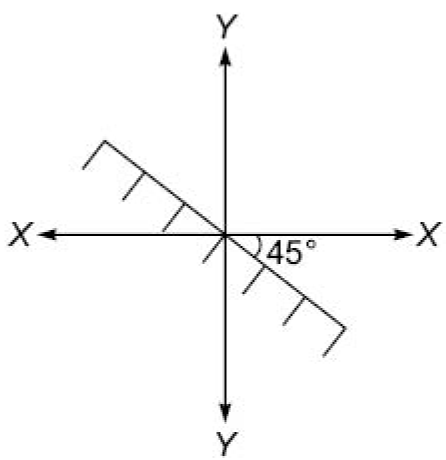
(a) 1

(b) 2

(c) 21

(d) 12

10.



Refer to the graph. What does the shaded portion represent?

(a) $x + y$ £ 0

(b) $x \geq y$

(c) $x + 1 \geq y + 1$

(d) $x + y \geq 0$

11. If a, b, c are positive numbers and it is known that $a^2 + b^2 + c^2 = 8$ then:

(a) $a^3 + b^3 + c3$ £ $16\sqrt{\frac{2}{3}}$

(b) $a^3 + b^3 + c^3 \geq 64$

(c) $a^3 + b^3 + c^3 \geq 16\sqrt{\frac{2}{3}}$

(d) $a^3 + b^3 + c^3$ £ 64

12. Find the value of the expression given in terms of variables 'x' and 'y'.

$$\frac{(x^2+(a-c)x-ac)\,(x^2-ax-bx+ab)\,(x+c)}{(x^2-a^2)\,(x^2-bx^2-c^2x+bc^2)}$$

(a) $\frac{(x-b)\,(x-c)}{(x-a)}$ (b) $\frac{(x+a)\,(x+c)}{(x-b)}$

(c) $\frac{(x+a)\,(x-b)}{(x-c)}$ (d) None of these

Directions for Questions 13 and 14: These questions are based on the relation given below:

$f^{a}(y) = f^{a-1}(y-1)$ where $a > 1$ (integer values only) and $f^{1}(y) = 2/y$ if 'y' is positive or $f^{1}(y) = 1/(y^{2} + 1)$ otherwise.

13. What is $f^{a}(a-1)$?

(a) 0 (b) 1

(c) 2 (d) Indeterminate.

14. What is the value of $f^{a}(a+1)$?

(a) 1 (b) a

(c) $2a$ (d) 2

15. Raman derived an equation to denote distance of a Haley's comet (x) in the form of a quadratic equation. Distance is given by solution of quadratic $x^2 + Bx + c = 0$. To determine constants of the above

equation for Haley's Comet, two separate series of experiments were conducted by Raman. Based on the data of first series, value of x obtained is (1, 8) and based on the second series of data, value of x obtained is (2, 10). Later on it is discovered that first series of data gave incorrect value of constant C while second series of data gave incorrect value of constant B. What is the set of actual distance of Haley's Comet found by Raman?

(a) (11, 3) (b) (6, 3)

(c) (4, 5) (d) (3,11)

16. 'a', 'b' and 'c' are three real numbers. Which of the following statements is/are always true?

(A) $(a - 1)(b - 1)(c - 1) < abc$.

(B) $(a^2 + b^2 + c^2)/2 \geq ca + cb - ab$

(C) $a^2b \div c$ is a real number

(a) Only A is true (b) Only B and C are true

(c) Only B is true (d) None is true

17. If we have f[g(y)] = g[f(y)], then which of the following is true?

(a) [f[f[g[g[g[g(y)]]]]]] = [f[g[g[f[f[g(y)]]]]]]

(b) [f[f[f[g[f[g(y)]]]]]] = f[f[g[g[g[f(y)]]]]]

(c) [g[f[g[g[g[f(y)]]]]]] = [f[g[g[f[f[f(y)]]]]]]

(d) [g[f[g[g[f[g[f(y)]]]]]]] = [f[g[g[g[g[g[f(y)]]]]]]]

18. $f(a) = \dfrac{a^8 - 1}{a^2 + 1}$ and $g(a) = \dfrac{a^4 - 3}{(a+1)^2}$, what is $f\left(\dfrac{1}{g(2)}\right)$?

(a) 0.652 (b) $\dfrac{1468}{2250}$

(c) $-\dfrac{734}{1625}$ (d) 0

19. Dev was solving a question from his mathematics book when he encountered the expression $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$ then $a^a\ b^b\ c^c$ is

(a) –1 (b) 1

(c) 0.5 (d) 2

20. The number of solutions of $\frac{\log 5 + \log(a^2+1)}{\log(a-2)} = 2$ is:

(a) 1/2 (b) 2

(c) 1 (d) None of these

Review Test 5

Directions for the Questions 1 to 3: Refer to the data given below and answer the questions.

Given $\frac{a}{b} = \frac{1}{2}$, $\frac{c}{d} = \frac{1}{3}$ and $z = \frac{a+c}{b+d}$, answer the questions below on limits of *z*.

1. If *y* ≥ 0 and *p* ≥ 0 then the limits of '*z*' are:

   (a) *z* £ 0 or *z* ≥ 1
   (b) $\frac{1}{3}$ £ *z* £ $\frac{1}{2}$
   (c) *z* ≥ $\frac{1}{2}$ or z £ $\frac{1}{3}$
   (d) 0 £ *z* £ 1

2. *c* £ 0 and 1/3 £ *z* £ 1/2 only if:

   (a) *a* > 1.5*c*
   (b) *c* > –1
   (c) *a* £ 0
   (d) *a* > –1.5c

3. If *a* = –31, which of the following value of '*d*' gives the highest value of '*z*'?

   (a) *d* = 72
   (b) *d* = 721
   (c) d = –31
   (d) *d* = 0

4. Find the integral solution of: $5y - 1 < (y + 1)^2 < (7y - 3)$

   (a) 2
   (b) $2 < y < 4$
   (c) $1 < y < 4$
   (d) 3

5. If $f(a) = \frac{a-1}{a+1}$, $x \geq 0$ and if $y = f\left(\frac{1}{a}\right)$ then

   (a) As '*a*' decreases, '*y*' decreases
   (b) As '*a*' increases, '*y*' decreases
   (c) As '*x*' increases, '*y*' increases
   (d) As '*x*' increases, '*y*' remains unchanged

6. If f and g are real functions defined by $f(a) = a + 2$ and $g(a) = 2a^2 + 5$, then fog is equal to

(a) $2a^2 + 7$ (b) $2a^2 + 5$

(c) $2(a + 2)^2 + 5$ (d) $2a + 5$

7. If 'p' and 'q' are the roots of the equation $x^2 - 10x + 16 = 0$, the value of $(1 - p)(1 - q)$ is

(a) –7 (b) 7

(c) 16 (d) –16

8. Given that 'a' and 'b' are positive real numbers such that $a + b = 1$, then what is the minimum value of $\sqrt{12+\frac{1}{a^2}}+\sqrt{12+\frac{1}{b^2}}$?

(a) 8 (b) 16

(c) 24 (d) 4

9. Let p, q and r be distinct positive integers satisfying $p < q < r$ and $p + q + r = k$. What is the smallest value of k that does *not* determine p, q, r uniquely?

(a) 9 (b) 6

(c) 7 (d) 8

10. Given odd positive integers p, q and r which of the following is not necessarily true?

(a) $p^2 q^2 r^2$ is odd (b) $3(p^2 + q^3)r^2$ is even

(c) $5p + q + r^4$ is odd (d) $r^2 (p^4 + q^4)/2$ is even

11. $f(a) = (a^2 + 1)(a^2 - 1)$ where $a = 1, 2, 3\ldots$ which of the following statement is not correct about $f(a)$?

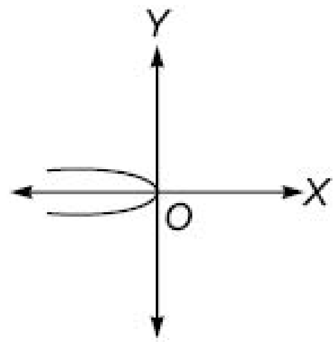
(a) $f(a)$ is always divisible by 5

(b) $f(a)$ is always divisible by 3

(c) $f(a)$ is always divisible by 30

(d) None of these

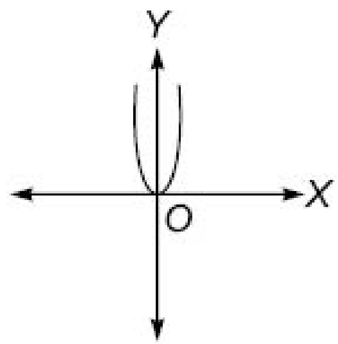
Directions for Questions 12 and 13: The following questions are based on the graph of parabola plotted on $x - y$ axes. Answer the questions according to given conditions if applicable and deductions from the graph.

12. 

The above graph represent the equations

(a) $y^2 = kx, k > 0$ (b) $y^2 = kx, k < 0$

(c) $x^2 = ky - 1, k < 0$ (d) $x^2 = ky - 1, k > 0$

13. 

The above graph represents the equation

(a) $x^2 = ky, k < 0$ (b) $y^2 = kx + 1, k > 0$

(c) $y^2 = kx + 1, k < 0$ (d) None of these

14. Mala while teaching her class on functions gives her students a question.

According to the question the functions are $f(x) = -x$, $g(x) = x$. She also provides her students with following functions also.

$f(x, y) = x - y$ and $g(x, y) = x + y$

Since she wants to test the grasp of her students on functions she asks them a simple question “which of the following is not true?” and provides her students with the following options. None of her students were able to answer the question in single attempt. Can you answer her question?

(a) $f[f(g(x, y))] = g(x, y)$

(b) $g[f[g[f(x, y)]]] = f(x, y)$

(c) $f(x) + g(x) + f(x, y) + g(x, y) = g(x) – f(x)$

(d) None of these

Directions for Questions 15 and 16: We are given that $f(x) = f(y)$ and
$f(x, y) = x + y$, if $x, y > 0$
$f(x, y) = xy$, if $x, y = 0$
$f(x, y) = x – y$, if $x, y < 0$
$f(x, y) = 0$, otherwise

15. Find the value of the following function: $f[f(2, 0), f(– 3, 2)] + f[f(– 6, – 3), f(2, 3)]$.

(a) 0 (b) 2

(c) –8 (d) None of these

16. Find the value of the following function:

$\{ f[f(1, 2), f(2, 3)]\} \times \{ f[f(1.6, 2.9), f(– 1, – 3)]\}$

(a) 12 (b) 36

(c) 48 (d) 52

17. Given that $f(a) = a (a + 1) (a + 2)$ where $a = 1, 2, 3, \ldots$. Then find $S = f(1) + f(2) + f(3) + \ldots\ldots + f(10)$?

(a) 4200 (b) 4290

(c) 4400 (d) None of these

18. We have three inequalities as:

(i) $2^a > a$ (ii) $2^a > 2a + 1$

(iii) $2^a > a^2$

For what natural numbers n are all the three inequalities satisfied?

(a) $a \geq 3$ (b) $a \geq 4$

(c) $a \geq 5$ (d) $a \geq 6$

19. For the curve $p^3 - 3pq + 2 = 0$, the set A of points on the curve at which the tangent to the curve is horizontal and the set B of points on the curve at which the tangent to the curve is vertical are respectively:

(a) (1, 1) and (0, 0) (b) (0, 0) and (1, 1)

(c) (1, 1) and null set (d) None of these

20. If $f(a) = a^2 - \frac{1}{a^2}$ and $g(a) = \frac{1}{\sqrt{f(a) - 4},}$ then the real domain for all values of 'a' such that $f(a)$ and $g(a)$ are both real and defined is represented by the inequality:

(a) $a^2 - a - 1 > 0$ (b) $a^4 - 4a^2 - 1 > 0$

(c) $a^2 - 4a - 1 > 0$ (d) None of these

ANSWER KEY

Review Test 1

1. (d)	2. (b)	3. (d)	4. (a)
5. (a)	6. (b)	7. (b)	8. (c)
9. (c)	10. (c)	11. (b)	12. (a)
13. (b)	14. (d)	15. (d)	16. (a)
17. (b)	18. (a)	19. (c)	20. (d)
21. (b)	22. (d)	23. (a)	24. (d)
25. (c)			

Review Test 2

1. (c)	2. (a)	3. (b)	4. (b)
5. (a)	6. (c)	7. (d)	8. (a)

9. (d)	10. (d)	11. (a)	12. (d)
13. (a)	14. (b)	15. (c)	16. (d)
17. (d)	18. (a)	19. (a)	20. (a)
21. (b)	22. (b)	23. (b)	24. (c)
25. (c)			

Review Test 3

1. (d)	2. (c)	3. (c)	4. (c)
5. (d)	6. (c)	7. (d)	8. (a)
9. (c)	10. (c)	11. (c)	12. (a)
13. (d)	14. (d)	15. (b)	16. (b)
17. (b)	18. (d)	19. (b)	20. (c)

Review Test 4

1. (d)	2. (c)	3. (b)	4. (d)
5. (a)	6. (a)	7. (d)	8. (a)
9. (a)	10. (a)	11. (c)	12. (d)
13. (b)	14. (a)	15. (c)	16. (c)
17. (c)	18. (d)	19. (b)	20. (c)

Review Test 5

1. (b)	2. (c)	3. (d)	4. (b)
5. (b)	6. (a)	7. (d)	8. (a)
9. (d)	10. (d)	11. (d)	12. (b)
13. (d)	14. (b)	15. (a)	16. (d)
17. (b)	18. (c)	19. (c)	20. (b)

BLOCK VI

COUNTING

Chapter 17
Permutations and Combinations

Chapter 18
Probability

Chapter 19
Set Theory

...BACK TO SCHOOL

In my experience, students can be divided into two broad categories on the basis of their ability in solving chapters of this block.

Category 1: Students who are comfortable in solving questions of this block, since they understand the underlying concepts well.

Category 2: Students who are not able to tackle questions of this block of chapters since they are not conversant with the counting tools and methods in this block.

If you belong to the second category of students, the main thing you would need to do is to familiarise yourself with the counting methods and techniques of Permutations and Combinations. Once you are through with the same, you would find yourself relatively comfortable at both Permutations & Combinations (P&C) and Probability—the chapters in this block which create the maximum problems for students. Set Theory being a relatively easier chapter, the Back to School section of this block concentrates mostly on P & C and Probability. However, before we start looking at these counting methods, right at the outset, I would want you to remove any negative experiences you might have had while trying to study P& C and Probability. So if you belong to the second category of students, you are advised to read on:

Look at the following table:

Suppose, I were to ask you to count the number of cells in the table above, how would you do it??

$5 \times 5 = 25$!!

I guess, all of you realise the fact that the number of cells in this table is given by the product of the number of rows and columns. However, if you ask a 5 year old child to count the same, he would be counting the number of cells physically. In fact, when you were 5 years old, you would also have required to do a physical count of the number of cells in the table. However, there must have come a point where you must have understood that in all such situations (no. of cells in a table, no. of students in a class, etc.) the total count is got by simply multiplying the number of rows and the number of columns. What I am interested in pointing out to you, is that the discovery of this process for this specific counting situation surely made your counting easier!! What used to take you much longer started taking you shorter times. Not only that, in situations where the count was too large (e.g. 100 rows × 48 columns) an infeasible count became extremely easy. So why am I telling you this??

Simply because just as the rows into columns tool for counting had the effect of making your count easier, so also the tools for counting which P& C describe will also have the effect of making counting easier for you in the specific situations of counting that you will encounter. My experience of training students shows that the only reason students have problems in this block is because it has not been explained properly to them in their +2 classes.

If you try to approach these chapters with the approach that it is not meant to complicate your life but to simplify it, you might end up finding out that there is nothing much to fear in this chapter.

THE TOOL BOX APPROACH TO P&C

While studying P&C, your primary objective should be to familiarise yourself with each and every counting situation. You also need to realise that there are a limited number of counting situations which you need to tackle. You can look at this as the process of the creation of what can be described as a counting tool box.

Once this tool box is created, you would be able to understand the basic situations for counting. Then solving tough questions becomes a matter of simplifying the language of the question into the language of the answer—a matter I would come back to later.

Let us now proceed to list out the specific counting situations which you need to get a hold on. You need to know and understand the following twelve situations of counting and the tools that are used in these situations. Please take note that these tools are explained in detail in various parts of the text on the Permutations and Combinations chapter. You are required to keep these in mind along with the specific situations in which these apply. Look at these as a kind of comprehensive list of situations in which you should know how to count using mathematical tools for your convenience.

The Twelve Counting Situations and Their Tools

Tool No. 1: **The ^nC_r tool**—This tool is used for the specific situation of counting the number of ways of selecting r things out of n **distinct** things.

Example: Selecting a team of 11 cricketers out of a team of 16 distinct players will be $^{16}C_{11}$.

Tool No. 2: The tool for counting the number of selections of r things out of n **identical** things. (Always 1)

Example: The number of ways of selecting 3 letters out of five A's. Note that in such cases there will only be one way of selecting them.

Tool No. 3: **The 2^n tool—**The tool for selecting any number (including 0) of things from n **distinct** things ($^nC_0 + {^nC_1} + {^nC_2} \ldots + {^nC_n} = 2^n$). Example the number of ways you can or cannot eat sweets at a party if there are ten sweets and you have the option of eating one piece of as many sweets as you like or even of not eating any sweet. Will be given by 2^{10}.

Corollary: **The $2^n - 1$ tool**—Used when zero selections are not allowed. For instance in the above situation if you are asked to eat at least one sweet and rejecting all sweets is not an option.

Tool No. 4: **The $n + 1$ tool—**The tool for selecting any number (including 0) of things from *n* **identical** things. For instance, if you have to select any number of letters from *A*, *A*, *A*, *A* and *A* then you can do it in six ways.

Tool No. 5: **The $^{n+r-1}C_{r-1}$ tool—**The tool for distributing n identical things amongst r people/groups such that any person/group might get any number (including 0). For instance, if you have to distribute 7 identical gifts amongst 5 children in a party then it can be done in $^{(7+5-1)}C_{(5-1)}$ ways. i.e. $^{11}C_4$ ways.

Tool No. 6: **The $^{n-1}C_{r-1}$ tool—**The tool for distributing n identical things amongst r people/groups such that any person/group might get any number (except 0).

Suppose in the above case you have to give at least one gift to each child it can be done in: $^6C_4$ ways.

Tool No. 7: **The mnp rule tool—**The tool for counting the number of ways of doing three things when each of them has to be done and there are m ways of doing the first thing, *n* ways of doing the second thing and p ways of doing the last thing. Suppose you have to go from Lucknow to Varanasi to Patna to Ranchi to Jamshedpur and there are 5 trains from Lucknow to Varanasi & there are 4 trains from Varanasi to Patna, six from Patna to Ranchi and 3 from Ranchi to Jamshedpur, then the number of ways of going from Lucknow to Jamshedpur through Varanasi, Patna & Ranchi is $5 \times 4 \times 6 \times 3 = 360$.

Note that this tool is extremely crucial and is used to form numbers and words.

For example how many 4 digit numbers can you form using the digits 0, 1, 2, 3, 4, 5 & 6 only without repeating any digits. In this situation the work outline is to choose a digit for the first place (=6, any one out of 1, 2, 3, 4, 5 & 6 as zero cannot be used there), then choosing a digit for the second place (=6, select any one out of 5 remaining digits plus 0), then a digit for the third place (5 ways) and a digit for the fourth place (4 ways).

Tool No 8: **The r! tool for arrangement—**This tool is used for counting the number of ways in which you can arrange r things in r places. Notice that this can be derived out of the mnp rule tool.

Tool No 9: **The AND rule tool—**Whenever you describe the counting situation and connect two different parts of the count by using the conjunction **'AND'**, you will always replace the **'AND'** with a multiplication sign between the two parts of the count. This is also used for solving Probability questions. For instance, suppose you have to choose a vowel **'AND'** a consonant from the letters of the word PERMIT, you can do it in $^4C_1 \times {}^2C_1$ ways.

Tool No 10: **The OR rule tool—**Whenever you describe the counting situation and connect two different parts of the count by using the conjunction **'OR**, you will always replace the '**OR'** with an Addition sign between the two parts of the count. This is also used for solving Probability questions. For instance, suppose you have to select either 2 vowels or a consonant from the letters of the word PERMIT, you can do it in $^4C_2 + {}^2C_1$ ways.

Tool No 11: **The Circular Permutation Tool [(n – 1)! Tool]—**This gives the number of ways in which n distinct things can be placed around a circle. The need to reduce the value of the factorial by 1 is due to the fact that in a circle there is no defined starting point.

Tool No 12: **The Circular Permutation Tool when there is no distinction between clockwise and anti clockwise arrangements [(n – 1)!/2 Tool]—**This gives the number of ways in which n distinct things can be placed around a circle when there is no distinction between clockwise and anti clockwise arrangements. In such a case the number of circular permutations are just divided by 2.

Two More Issues

(1) What about the ^nP_r tool? The ^nP_r tool is used to arrange r things amongst n. However, my experience shows that the introduction of this tool just adds to the confusion for most students. A little bit of smart thinking gets rid of all the problems that the ^nP_r tool creates in your mind. For this let us consider the difference between the ^nP_r and the ^nC_r tools.

$^nC_r = n!/[r! \times (n - r)!]$ while $^nP_r = n!/[(n - r)!]$.

A closer look would reveal that the relationship between the ^nP_r and the ^nC_r tools can be summarised as:

$^nP_r = {}^nC_r \times r!$

Read this as: If you have to arrange r things amongst n things, then simply select r things amongst n things and use the r! tool for the arrangement of the selected r items amongst themselves. This will result in the same answer as the ^nP_r tool.

In fact, for all questions involving arrangements always solve the question in two parts—First finish the selection within the question and then arrange the selected items amongst themselves. This will remove a lot of confusions in questions of this chapter.

(2) Treat Both Permutations and Combinations and Probability as English language chapters rather than Maths chapters: One of the key discoveries in getting stronger at this block is that after knowing the basic twelve counting situations and their tools (as explained above) you need to stop treating the questions in this chapter as questions in Mathematics and rather treat them as questions in English. Thus, while solving a question in these chapters your main concentration and effort should be on converting the question language into the answer language.

What do I mean by this?

Consider this:

Question Language: What is the number of ways of forming 4 letter words from 5 vowels and 6 consonants such that the word contains one vowel and three consonants?

To solve this question, first create the answer language in your mind:

Answer Language: Select one vowel out of 5 **AND** select three consonants out of 6 **AND** arrange the four selected letters amongst themselves.

Once you have this language the remaining part of the answer is just a matter of using the correct tools.

Thus, the answer is ${}^5C_1 \times {}^6C_3 \times 4!$

As you can see, the main issue in getting the solution to this question was working out the language by connecting the various parts of the solution using the conjunction AND (In this case, the conjunction OR was not required.). Once, you had the language all you had to do was use the correct tools to count.

The most difficult of questions are solved this way. This is why I advise you to treat this chapter as a language chapter and not as a chapter in Mathematics!!

Pre-assessment Test

1. How many words of 11 letters could be formed with all the vowels present in even places, using all the letters of the alphabet? (without repetitions)

   (a) ${}^{21}P_6.5!$ (b) $21!$

   (c) ${}^{21}P_5(5!)$ (d) ${}^{26}P_s$

2. A candidate is required to answer 6 out of 10 questions, which are divided into two groups each containing 5 questions, and he is not permitted to attempt more than 4 from each group. In how many ways can he make up his choice?

   (a) 300 (b) 200

   (c) 400 (d) 100

3. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then find the number of ways in which they can be seated.

   (a) $m! \times {}^{m+1}P_n$ (b) $m! \times m+1Cn$

   (c) $m! \times {}^{m}P_n$ (d) $m! \times {}^{m}C_n$

4. How many different 4-digit numbers can be made from the first 4 whole numbers, using each digit only once?

   (a) 20 (b) 18

   (c) 24 (d) 20

5. How many different 4-digit numbers can be made from the first 4 natural numbers, if repetition is allowed?

   (a) 128 (b) 512

   (c) 256 (d) None of these

6. How many six-lettered words starting with the letter T can be made from all the letters of the word TRAVEL?

(a) 5 (b) 5!

(c) 124 (d) None of these

7. In the question above, how many words can be made with T and L at the end positions?

(a) 24 (b) 5!

(c) 6! (d) None of these

8. If the letters of the word ATTEMPT are written down at random, find the probability that all the T's are together.

(a) 2/7 (b) 1/7

(c) 4/7 (d) 3/7

9. A bag contains 3 white and 2 red balls. One by one, two balls are drawn without replacing them. Find the probability that the second ball is red.

(a) 2/5 (b) 1/10

(c) 2/10 (d) 3/5

10. Three different numbers are selected from the set $X = \{1,2,3,4,\ldots,10\}$. What is the probability that the product of two of the numbers is equal to the third?

(a) 3/10 (b) 1/40

(c) 1/20 (d) 4/5

Directions for Questions 11 and 12: Read the passage below and solve the questions based on it.

India plays two matches each with New Zealand and South Africa. In any match, the probability of different outcomes for India is given below:

Outcome	*Win*	*Loss*	*Draw*
Probability	0.5	0.45	0.05
Points	2	0	1

Outcome of all the matches are independent of each other.

11. What is the probability of India getting at least 7 points in the contest? Assume South Africa & New Zealand Play 2 matches.

(a) 0.025 (b) 0.0875

(c) 0.0625 (d) 0.975

12. What is the probability of South Africa getting at least 4 points? Assume South Africa and New Zealand play 2 matches.

(a) 0.2025 (b) 0.0625

(c) 0.0425 (d) Can't be determined

13. If ${}^{n}C_4 = 126$, then $np_4 = ?$

(a) 12. (b) 126 * 2!

(c) 126 * 4! (d) None of these

14. The number of ways, in which a student can choose 5 courses out of 9 courses, if 2 courses are compulsory, is

(a) 53 (b) 35

(c) 34 (d) 32

15. What is the value of ${}^{18}C_{16}$?

(a) 5! (b) 6!

(c) 153 (d) Can't be determined

16. How many triangles can be drawn from *N* given points on a circle?

(a) N! (b) 3!

(c) N! / 3! (d) (N – 1)N(N – 2)/6

17. Following are the alphabets for which the alphabet and its mirror image are same. *A, H, I, M, 0, T, U, V, W, Y, X*. These alphabets are called as symmetrical. Other are called as unsymmetrical. A password containing 4 alphabets (without repetitions) is to be formed using symmetrical alphabets. How many such passwords can be formed?

(a) 720 (b) 330

(c) 7920 (d) Can’t be determined

18. How many 5 letter words can be formed out of 10 consonants and 4 vowels, such that each contains 3 consonants and 2 vowels?

(a) ${}^{10}P_3 \times {}^{4}P_2 \times 5!$ (b) ${}^{10}C_3 \times {}^{4}C_2 \times 5!$

(c) ${}^{10}P_3 \times {}^{4}C_2 \times 5!$ (d) ${}^{10}C_2 \times {}^{4}P_2 \times 5!$

19. A question paper consists of two sections *A* and *B* having respectively 3 and 4 questions. Four questions are to be solved to qualify in that paper. It is compulsory to solve at least one question from Section A and two questions from Section B. In how many ways can a candidate select the questions to qualify in that paper?

(a) 30 (b) 18

(c) 48 (d) 60

20. A student is allowed to select at most n books from a collection of $(2n + 1)$ books. If the total number of ways in which he can select at least one book is 63, find the value of n.

(a) 4 (b) 3

(c) 5 (d) 6

21. A man and a woman appear in an interview for two vacancies for the same post. The probability of a man’s selection is 1/4 and that of a woman’s selection is 1/3. What is the probability that both of them will be selected?

(a) 1/12 (b) 1/3

(c) 2/5 (d) 3/7

22. In Question 21, what will be the probability that only one of them will be selected?

(a) 11/12 (b) 1/2

(c) 7/12 (d) 5/12

23. In Question 21, what will be the probability that none of them will be selected?

(a) 1/2 (b) 1/3

(c) 2/5 (d) 3/7

24. How many different signals can be made by hoisting 5 different coloured flags one above the other, when any number of them may be hoisted a time?

(a) 2^5 (b) $^5P_5$

(c) 325 (d) none of these

25. Find the number of ways in which the letters of the word MACHINE can be arranged so that the vowels may occupy only odd positions.

(a) $4! \times 4!$ (b) $^7P_3 \times 4!$

(c) $^7P_4 \times 3!$ (d) none of these

ANSWER KEY

1. (a)	2. (b)	3. (a)	4. (b)
5. (c)	6. (b)	7. (d)	8. (b)
9. (a)	10. (b)	11. (b)	12. (d)
13. (c)	14. (b)	15. (c)	16. (d)
17. (c)	18. (b)	19. (a)	20. (b)
21. (a)	22. (d)	23. (a)	24. (c)
25. (a)			

Chapter 17

Permutations and Combinations

STANDARD THEORY

Factorial Notation! Or $\underline{|}$

$\underline{|n} = n(n-1)(n-2)\ldots 3.2.1$

$n! = \underline{n}(n-1)(n-2)\ldots 3.2.1$

= Product of n consecutive integers starting from 1.

1. $0! = 1$
2. Factorials of only Natural numbers are defined.

   $n!$ is defined only for $n \geq 0$

   $n!$ is not defined for $n < 0$

4. $^nC_r = 1$ when $n = r$.
5. Combinations (represented by ^nC_r) can be defined as the number of ways in which r things at a time can be **SELECTED** from amongst n things available for selection.

   The key word here is **SELECTION**. Please understand here that the order in which the r things are selected has no importance in the counting of combinations.

   ^nC_r = Number of combinations (selections) of n things taken r at a time.

   $^nC_r = n!/[r!(n-r)!]$; where $n \geq r$ (n is greater than or equal to r).

Some typical situations where selection/combination is used:

(a) Selection of people for a team, a party, a job, an office etc. (e.g. Selection of a cricket team of 11 from 16 members)

(b) Selection of a set of objects (like letters, hats, points pants, shirts, etc) from amongst another set available for selection.

In other words any selection in which the order of selection holds no importance is counted by using combinations.

6. Permutations (represented by ${}^{n}P_{r}$) can be defined as the number of ways in which *r* things at a time can be **SELECTED** & **ARRANGED** at a time from amongst *n* things.

   The key word here is **ARRANGEMENT**. Hence please understand here that the order in which the *r* things are arranged has critical importance in the counting of permutations.

   In other words permutations can also be referred to as an **ORDERED SELECTION**.

   ${}^{n}P_{r}$ = number of permutations (arrangements) of *n* things taken *r* at a time.

   ${}^{n}P_{r} = n!/\,(n - r)!;\ n \geq r$

Some typical situations where **ordered selection/ permutations** are used:

(a) Making words and numbers from a set of available letters and digits respectively

(b) Filling posts with people

(c) Selection of batting order of a cricket team of 11 from 16 members

(d) Putting distinct objects/people in distinct places, e.g. making people sit, putting letters in envelopes, finishing order in horse race, etc.)

The exact difference between selection and arrangement can be seen through the illustration below:

Selection

Suppose we have three men A, B and C out of which 2 men have to be selected to two posts.

This can be done in the following ways: AB, AC or BC (These three represent the basic selections of 2 people out of three which are possible. Physically they can be counted as 3 distinct selections. This value can also be got by using $3C_2$.

Note here that we are counting AB and BA as one single selection. So also AC and CA and BC and CB are considered to be the same instances of selection since the order of selection is not important.

Arrangement

Suppose we have three men A, B and C out of which 2 men have to be selected to the post of captain and vice captain of a team.

In this case we have to take AB and BA as two different instances since the order of the arrangement makes a difference in who is the captain and who is the vice captain.

Similarly, we have BC and CB and AC and CA as 4 more instances. Thus in all there could be 6 arrangements of 2 things out of three.

This is given by ${}^3P_2 = 6$.

7. **The Relationship Between Permutation & Combination:**

   When we look at the formulae for Permutations and Combinations and compare the two we see that,

   ${}^nP_r = r! \times {}^nC_r$

   $= {}^nC_r \times {}^rP_r$

   This in words can be said as:

   The permutation or arrangement of *r* things out of *n* is nothing but the selection of *r* things out of *n* followed by the arrangement of the *r* selected things amongst themselves.

8. **MNP Rule:** If there are three things to do and there are M ways of doing the first thing, N ways of doing the second thing and P ways of doing the third thing then there will be **M** × **N** × **P** ways of doing all the three things together.The works are mutually inclusive.

   This is used to for situations like:

The numbers 1, 2, 3, 4 and 5 are to be used for forming 3 digit numbers without repetition. In how many ways can this be done?

Using the MNP rule you can visualise this as: There are three things to doÆ The first digit can be selected in 5 distinct ways, the second can be selected in 4 ways and the third can be selected in 3 different ways. Hence, the total number of 3 digit numbers that can be formed are $5 \times 4 \times 3 = 60$

9. When the pieces of work are mutually exclusive, there are M+N+P ways of doing the complete work.

Important Results

The following results are important as they help in problem solving.

1. Number of permutations (or arrangements) of n different things taken all at a time = $n!$
2. Number of permutations of n things out of which P_1 are alike and are of one type, P_2 are alike and are of a second type and P_3 are alike and are of a third type and the rest are all different = $n! / P_1! P_2! P_3!$

   Illustration: The number of words formed with the letters of the word Allahabad.

   Solution: Total number of Letters = 9 of which A occurs four times, L occurs twice and the rest are all different.

   Total number of words formed = 9! / (4! 2! 1!)
3. Number of permutations of n different things taken r at a time when repetition is allowed = $n \times n \times n \times \ldots$ (r times) = n^r.

   Illustration: In how many ways can 4 rings be worn in the index, ring finger and middle finger if there is no restriction of the number of rings to be worn on any finger?

   Solution: Each of the 4 rings could be worn in 3 ways either on the index, ring or middle finger.

   So, four rings could be worn in $3 \times 3 \times 3 \times 3 = 3^4$ ways.
4. Number of selections of r things out of n identical things = 1

Illustration: In how many ways 5 marbles can be chosen out of 100 identical marbles?

Solution: Since, all the 100 marbles are identical

Hence, Number of ways to select 5 marbles = 1

5. Total number of selections of zero or more things out of k identical things = $k + 1$.

   This includes the case when zero articles are selected.

6. Total number of selections of zero or more things out of n different things =

   $^nC_o + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$

   $^nC_o + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

   Corollary: The number of selections of 1 or more things out of n different things = $^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - 1$

7. Number of ways of distributing n identical things among r persons when each person may get any number of things = $^{n+r-1}C_{r-1}$

   Imagine a situation where 27 marbles have to be distributed amongst 4 people such that each one of them can get any number of marbles (including zero marbles). Then for this situation we have, $n = 27$(no. of identical objects), $r = 4$ (no. of people) and the answer of the number of ways this can be achieved is given by: $^{n+r-1}C_{r-1} = {}^{30}C_3$.

8. Corollary: No. of ways of dividing n non distinct things to r distinct groups are:

   $^{n-1}C_{r-1}$ Æ For non-empty groups only

   Also, the number of ways in which n distinct things can be distributed to r different persons:

   $= r^n$

9. Number of ways of dividing $m + n$ different things in two groups containing m and n things respectively = $^{m+n}C_n \times {}^mC_m$ =

   $= (m + n)! / m!\, n!$

Or, ${}^{m+n}C_m \times {}^nC_n = (m + n)! / n!\, m!$

10. Number of ways of dividing $2n$ different things in two groups containing n things = $2n! / n!\, n!\, 2!$

11. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

12. ${}^nC_x = {}^nC_y$ fi $x = y$ or $x + y = n$

13. ${}^nC_r = {}^nC_{n-r}$

14. $r \,.\, {}^nC_r = n \,.\, {}^{n-1}C_{r-1}$

15. ${}^nC_r / (r + 1) = {}^{n+1}C_{r+1} /(n + 1)$

16. For ${}^nC_r$ to be greatest,

    (a) if n is even, $r = n/2$

    (b) if n is odd, $r = (n + 1)/2$ or $(n - 1)/2$

17. Number of selections of r things out of n different things

    (a) When k particular things are always included = ${}^{n-k}C_{r-k}$

    (b) When k particular things are excluded = ${}^{n-k}C_r$

    (c) When all the k particular things are not together in any selection

    $= {}^nC_r - {}^{n-k}C_{r-k}$

    No. of ways of doing a work with given restriction = total no. of ways of doing it — no. of ways of doing the same work with opposite restriction.

18. The total number of ways in which 0 to n things can be selected out of n things such that p are of one type, q are of another type and the balance r of different types is given by: $(p + 1)(q + 1)(2^r - 1)$.

19. Total number of ways of taking some or all out of $p + q + r$ things such that p are of one type and q are of another type and r of a third type

    $= (p + 1)(q + 1)(r + 1) - 1$

    [Only non-empty sets]

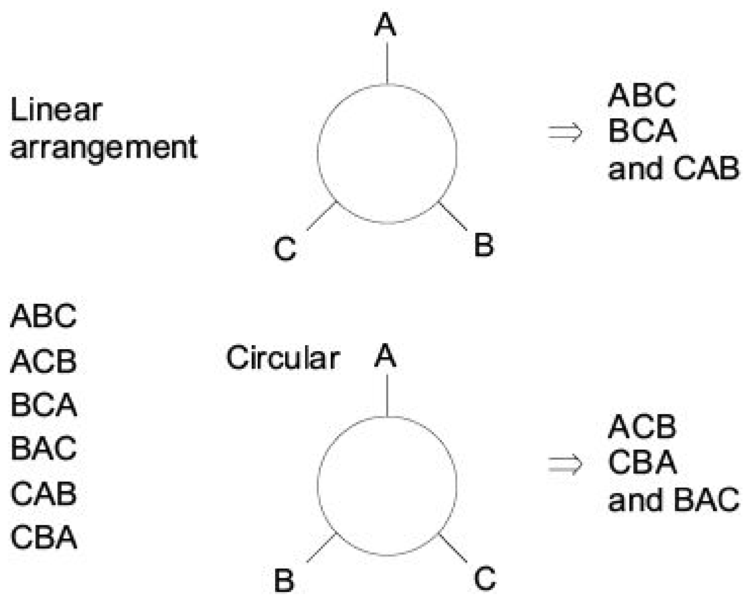
20. $\frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}$

21. Number of selections of k consecutive things out of n things in a row = $n - k + 1$

Circular Permutations

Consider two situations:

There are three A, B and C. In the first case, they are arranged linearly and in the other, around a circular table –



For the linear arrangement, each arrangement is a totally new way. For circular arrangements, three linear arrangements are represented by one and the same circular arrangement.

So, for six linear arrangements, there correspond only 2 circular arrangements. This happens because there is no concept of a starting point on a circular arrangement. (i.e., the starting point is not defined.)

Generalising the whole process, for $n!$, there corresponds to be $(1/n)\ n!$ ways.

Important Results

1. Number of ways of arranging n people on a circular track (circular arrangement) = $(n – 1)!$
2. When clockwise and anti-clockwise observation are not different then number of circular arrangements of n different things = $(n – 1)! /2$

   e.g. the case of a necklace with different beads, the same arrangement when looked at from the opposite side becomes anti-clockwise.
3. Number of selections of k consecutive things out of n things in a circle

   = n when $k < n$

   = 1 when $k = n$

Some More Results

1. Number of terms in $(a_1 + a_2 + \ldots + a_n)^m$ is ${}^{m+n-1}C_{n-1}$

   Illustration: Find the number of terms in $(a + b + c)^2$.

   Solution: $n = 3$, $m = 2$

   ${}^{m+n-1}C_{n-1} = {}^4C_2 = 6$

   Corollary: Number of terms in

   $(1 + x + x^2 + \ldots . x^n)^m$ is $mn + 1$
2. Number of zeroes ending the number represented by $n! = [n/5] + [n/5^2] + [n/5^3] + \ldots [n/5^x]$

   [] Shows greatest integer function where 5^x £ n

   Illustration: Find the number of zeroes at the end of 1000!

   Solution: $[1000/5] + [1000/5^2] + [1000/5^3] + [1000/5^4]$

   $200 + 40 + 8 + 1 = 249$

   Corollary: Exponent of 3 in $n! = [n!/3] + [n!/32] + [n!/3^3] + \ldots [n!/3^x]$ where 3^x £ n

   [] Shows greatest integer *fn*.

   Illustration: Find how many exponents of 3 will be there in 24!.

Solution: [24/3] + [24/3^2] = 8 + 2 = 10

3. Number of squares in a square of $n \times n$ side = $1^2 + 2^2 + 3^2 + 4^2 + \ldots. n^2$

Number of rectangles in a square of $n \times n$ side = $1^3 + 2^3 + 3^3 + 4^3 + \ldots. n^3$. (This includes the number of squares.)

Thus the number of squares and rectangles in the following figure are given by:

Number of squares = $1^2 + 2^2 + 3^2 = 14$ = Œn^2

Number of rectangles = $1^3 + 2^3 + 3^3 = 36$ (Œn)^2 = Œn^3 for the rectangle.

A rectangle having m rows and n columns:

The number of squares is given by: $m.n + (m - 1)(n - 1) + (m - 2)(n - 2) + \ldots..$ until any of $(m - x)$ or $(n - x)$ comes to 1.

The number of rectangles is given by: $(1 + 2 + \ldots\ldots + m)(1 + 2 + \ldots\ldots + n)$

WORKED-OUT PROBLEMS

In the following examples the solution is given upto the point of writing down the formula that will apply for the particular question. The student is expected to calculate the values after understanding the solution.

Problem 17.1 Find the number of permutations of 6 things taken 4 at a time.

Solution The answer will be given by ${}^6P_4$.

Problem 17.2 How many 3-digit numbers can be formed out of the digits 1, 2, 3, 4 and 5?

Solution Forming numbers requires an ordered selection. Hence, the answer will be ${}^5P_3$.

Problem 17.3 In how many ways can the 7 letters M, N, O, P, Q, R, S be arranged so that P and Q occupy continuous positions?

Solution For arranging the 7 letters keeping P and Q always together we have to view P and Q as one letter. Let this be denoted by $\underline{PQ}$.

Then, we have to arrange the letters M, N, O, $\underline{PQ}$, R and S in a linear arrangement. Here, it is like arranging 6 letters in 6 places (since 2 letters are counted as one). This can be done in 6! ways.

However, the solution is not complete at this point of time since in the count of 6! the internal arrangement between P and Q is neglected. This can be done in 2! ways. Hence, the required answer is 6! × 2!.

Task for the student: What would happen if the letters P, Q and R are to be together? (Ans: 5! × 3!)

What if P and Q are never together? (Answer will be given by the formula: Total number of ways – Number of ways they are always together)

Problem 17.4 Of the different words that can be formed from the letters of the words BEGINS how many begin with *B* and end with *S*?

Solution *B* & *S* are fixed at the start and the end positions. Hence, we have to arrange *E*, *G*, *I* and *N* amongst themselves. This can be done in 4! ways.

Task for the student: What will be the number of words that can be formed with the letters of the word BEGINS which have *B* and *S* at the extreme positions? (Ans: $4! \times 2!$)

Problem 17.5 In how many ways can the letters of the word VALEDICTORY be arranged, so that all the vowels are adjacent to each other?

Solution There are 4 vowels and 7 consonants in Valedictory. If these vowels have to be kept together, we have to consider AEIO as one letter. Then the problem transforms itself into arranging 8 letters amongst themselves (8! ways). Besides, we have to look at the internal arrangement of the 4 vowels amongst themselves. (4! ways)

Hence Answer = $8! \times 4!$.

Problem 17.6 If there are two kinds of hats, red and blue and at least 5 of each kind, in how many ways can the hats be put in each of 5 different boxes?

Solution The significance of at least 5 hats of each kind is that while putting a hat in each box, we have the option of putting either a red or a blue hat. (If this was not given, there would have been an uncertainty in the number of possibilities of putting a hat in a box.)

Thus in this question for every task of putting a hat in a box we have the possibility of either putting a red hat or a blue hat. The solution can then be looked at as: there are 5 tasks each of which can be done in 2 ways. Through the MNP rule we have the total number of ways = 2^5 (Answer).

Problem 17.7 In how many ways can 4 Indians and 4 Nepalese people be seated around a round table so that no two Indians are in adjacent positions?

Solution If we first put 4 Indians around the round table, we can do this in 3! ways.

Once the 4 Indians are placed around the round table, we have to place the four Nepalese around the same round table. Now, since the Indians are already placed we can do this in 4! ways (as the starting point is defined when we put the Indians. Try to visualize this around a circle for placing 2 Indians and 2 Nepalese.)

Hence, Answer = $3! \times 4!$

Problem 17.8 How many numbers greater than a million can be formed from the digits 1, 2, 3, 0, 4, 2, 3?

Solution In order to form a number greater than a million we should have a 7 digit number. Since we have only seven digits with us we cannot take 0 in the starting position. View this as 7 positions to fill:

_ _ _ _ _ _ _

To solve this question we first assume that the digits are all different. Then the first position can be filled in 6 ways (0 cannot be taken), the second in 6 ways (one of the 6 digits available for the first position was selected. Hence, we have 5 of those 6 digits available. Besides, we also have the zero as an additional digit), the third in 5 ways (6 available for the 2nd position – 1 taken for the second position.) and so on. Mathematically this can be written as:

$6 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6 \times 6!$

This would have been the answer had all the digits been distinct. But in this particular example we have two 2’s and two 3’s which are identical to each other. This complication is resolved as follows to get the answer:

$$\frac{6 \times 6}{2! \times 2!}$$

Problem 17.9 If there are 11 players to be selected from a team of 16, in how many ways can this be done?

Solution ${}^{16}C_{11}$.

Problem 17.10 In how many ways can 18 identical white and 16 identical black balls be arranged in a row so that no two black balls are together?

Solution When 18 identical white balls are put in a straight line, there will be 19 spaces created. Thus 16 black balls will have 19 places to fill in. This will give an answer of: ${}^{19}C_{16}$. (Since, the balls are identical the arrangement is not important.)

Problem 17.11 A mother with 7 children takes three at a time to a cinema. She goes with every group of three that she can form. How many times can she go to the cinema with distinct groups of three children?

Solution She will be able to do this as many times as she can form a set of three distinct children from amongst the seven children. This essentially means that the answer is the number of selections of 3 people out of 7 that can be done.

Hence, Answer = ${}^{7}C_{3}$.

Problem 17.12 For the above question, how many times will an individual child go to the cinema with her before a group is repeated?

Solution This can be viewed as: The child for whom we are trying to calculate the number of ways is already selected. Then, we have to select 2 more children from amongst the remaining 6 to complete the group. This can be done in ${}^{6}C_{2}$ ways.

Problem 17.13 How many different sums can be formed with the following coins:

5 rupee, 1 rupee, 50 paisa, 25 paisa, 10 paisa and 1 paisa?

Solution A distinct sum will be formed by selecting either 1 or 2 or 3 or 4 or 5 or all 6 coins.

But from the formula we have the answer to this as : $2^6 – 1$.

[Task for the student: How many different sums can be formed with the following coins:

5 rupee, 1 rupee, 50 paisa, 25 paisa, 10 paisa, 3 paisa, 2 paisa and 1 paisa?

Hint: You will have to subtract some values for double counted sums.]

Problem 17.14 A train is going from Mumbai to Pune and makes 5 stops on the way. 3 persons enter the train during the journey with 3 different tickets. How many different sets of tickets may they have had?

Solution Since the 3 persons are entering during the journey they could have entered at the:

1st station (from where they could have bought tickets for the 2nd, 3rd, 4th or 5th stations or for Pune Æ total of 5 tickets.)

2nd station (from where they could have bought tickets for the 3rd, 4th or 5th stations or for Pune Æ total of 4 tickets.)

3rd station (from where they could have bought tickets for the 4th or 5th stations or for Pune Æ total of 3 tickets.)

4th station (from where they could have bought tickets for the 5th station or for Pune Æ total of 2 tickets.)

5th station (from where they could have bought a ticket for Pune Æ total of 1 ticket.)

Thus, we can see that there are a total of 5 + 4 + 3 + 2 + 1 = 15 tickets available out of which 3 tickets were selected. This can be done in ${}^{15}C_3$ ways (Answer).

Problem 17.15 Find the number of diagonals and triangles formed in a decagon.

Solution A decagon has 10 vertices. A line is formed by selecting any two of the ten vertices. This can be done in ${}^{10}C_2$ ways. However, these ${}^{10}C_2$ lines also count the sides of the decagon.

Thus, the number of diagonals in a decagon is given by: ${}^{10}C_2 - 10$ (Answer)

Triangles are formed by selecting any three of the ten vertices of the decagon. This can be done in ${}^{10}C_3$ ways (Answer).

Problem 17.16 Out of 18 points in a plane, no three are in a straight line except 5 which are collinear. How many straight lines can be formed?

Solution If all 18 points were non-collinear then the answer would have been ${}^{18}C_2$. However, in this case ${}^{18}C_2$ has double counting since the 5 collinear points are also amongst the 18. These would have been counted as

$^5C_2$ whereas they should have been counted as 1. Thus, to remove the double counting and get the correct answer we need to adjust by reducing the count by $(^5C_2 - 1)$.

Hence, Answer = $^{18}C_2 - (^5C_2 - 1) = {}^{18}C_2 - {}^5C_2 + 1$

Problem 17.17 For the above situation, how many triangles can be formed?

Solution The triangles will be given by $^{18}C_3 - {}^5C_3$.

Problem 17.18 A question paper had ten questions. Each question could only be answered as True (T) or False (F). Each candidate answered all the questions. Yet, no two candidates wrote the answers in an identical sequence. How many different sequences of answers are possible?

(a) 20 (b) 40
(c) 512 (d) 1024

Solution 2^{10} = 1024 unique sequences are possible. Option (d) is correct.

Problem 17.19 When ten persons shake hands with one another, in how many ways is it possible?

(a) 20 (b) 25
(c) 40 (d) 45

Solution For n people there are always $^nC_2$ shake hands. Thus, for 10 people shaking hands with each other the number of ways would be $^{10}C_2$ = 45.

Problem 17.20 In how many ways can four children be made to stand in a line such that two of them, *A* and *B* are always together?

(a) 6 (b) 12
(c) 18 (d) 24

Solution If the children are ***A***, *B*, *C*, *D* we have to consider *A* & *B* as one child. This, would give us 3! ways of arranging *AB*, *C* and *D*. However, for every arrangement with *AB*, there would be a parallel arrangement with *BA*. Thus, the correct answer would be 3! × 2! = 12 ways. Option (b) is correct.

Problem 17.21 Each person's performance compared with all other persons is to be done to rank them subjectively. How many comparisons are needed to total, if there are 11 persons?

(a) 66 (b) 55

(c) 54 (d) 45

Solution There would be ${}^{11}C_2$ combinations of 2 people taken 2 at a time for comparison. ${}^{11}C_2 = 55$.

Problem 17.22 A person X has four notes of Rupee 1, 2, 5 and 10 denomination. The number of different sums of money she can form from them is

(a) 16 (b) 15

(c) 12 (d) 8

Solution $2^4 - 1 = 15$ sums of money can be formed. Option (b) is correct.

Problem 17.23 A person has 4 coins each of different denomination. What is the number of different sums of money the person can form (using one or more coins at a time)?

(a) 16 (b) 15

(c) 12 (d) 11

Solution $2^4 - 1 = 15$. Hence, option (b) is correct.

Problem 17.24 How many three-digit numbers can be generated from 1, 2, 3, 4, 5, 6, 7, 8, 9, such that the digits are in ascending order?

(a) 80 (b) 81

(c) 83 (d) 84

Solution Numbers starting with 12 – 7 numbers

Numbers starting with 13 – 6 numbers; 14 – 5, 15 – 4, 16 – 3, 17 – 2, 18 – 1. Thus total number of numbers starting from 1 is given by the sum of 1 to 7 = 28.

Number of numbers starting from 2- would be given by the sum of 1 to 6 = 21

Number of numbers starting from 3- sum of 1 to 5 = 15

Number of numbers starting from 4 – sum of 1 to 4 = 10

Number of numbers starting from 5 – sum of 1 to 3 = 6

Number of numbers starting from 6 = 1 + 2 = 3

Number of numbers starting from 7 = 1

Thus a total of: 28 + 21 + 15 + 10 + 6 + 3 + 1 = 84 such numbers. Option (d) is correct.

Problem 17.25 In a carrom board game competition, m boys n girls ($m > n > 1$) of a school participate in which every student has to play exactly one game with every other student. Out of the total games played, it was found that in 221 games one player was a boy and the other player was a girl.

Consider the following statements:

I. The total number of students that participated in the competition is 30.

II. The number of games in which both players were girls is 78.

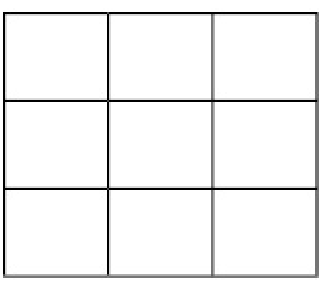
Which of the statements given above is/are correct?

(a) I only (b) II only

(c) Both I and II (d) Neither I nor II.

Solution The given condition can get achieved if we were to use 17 boys and 13 girls. In such a case both statement I and II are correct. Hence, option (c) is correct.

Problem 17.26



In how many different ways can all of 5 identical balls be placed in the cells shown above such that each row contains at least 1 ball?

(a) 64 (b) 81

(c) 84 (d) 108

Solution The placement of balls can be 3, 1, 1 and 2, 2, 1. For 3, 1, 1- If we place 3 balls in the top row, there would be ${}^3C_1$ ways of choosing a place for the ball in the second row and ${}^3C_1$ ways of choosing a place for the ball in the third row. Thus, ${}^3C_1 \times {}^3C_1 = 9$ ways. Similarly there would be 9 ways each if we were to place 3 balls in the second row and 3 balls in the third row. Thus, with the 3, 1, 1 distribution of 5 balls we would get 9 + 9 + 9 = 27 ways of placing the balls.

We now need to look at the 2, 2, 1 arrangement of balls. If we place 1 ball in the first row, we would need to place 2 balls each in the second and the third rows. In such a case, the number of ways of arranging the balls would be ${}^3C_1 \times {}^3C_2 \times {}^3C_2 = 27$ ways. (choosing 1 place out of 3 in the first row, 2 places out of 3 in the second row and 2 places out of 3 in the third row).

Similarly if we were to place 1 ball in the second row and 2 balls each in the first and third rows we would get 27 ways of placing the balls and another 27 ways of placing the balls if we place 1 ball in the third row and 2 balls each in the other two rows.

Thus with a 2, 2, 1 distribution of the 5 balls we would get 27 + 27 + 27 = 81 ways of placing the balls.

Hence, total number of ways = Number of ways of placing the balls with a 3,1,1 distribution of balls + number of ways of placing the balls with a 2, 2, 1 distribution of balls = 27 + 81 = 108.

Hence, option (d) is correct.

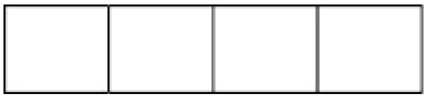
Problem 17.27 There are 6 different letter and 6 correspondingly addressed envelopes. If the letters are randomly put in the envelopes, what is the probability that exactly 5 letters go into the correctly addressed envelopes?

(a) Zero (b) 1/6

(c) 1/2 (d) 5/6

Solution If 5 letters go into the correct envelopes the sixth would automatically go into it's correct envelope. Thus, there is no possibility when exactly 5 letters are correct and 1 is wrong. Hence, option (a) is correct.

Problem 17.28



There are two identical red, two identical black and two identical white balls. In how many different ways can the balls be placed in the cells (each cell to contain one ball) shown above such that balls of the same colour do not occupy any two consecutive cells?

(a) 15 (b) 18

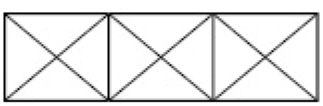
(c) 24 (d) 30

Solution In the first cell, we have 3 options of placing a ball. Suppose we were to place a red ball in the first cell- then the second cell can only be filled with either black or white – so 2 ways. Subsequently there would be 2 ways each of filling each of the cells (because we cannot put the colour we have already used in the previous cell).

Thus, the required number of ways would be $3 \times 2 \times 2 \times 2 = 24$ ways.

Hence, option (c) is correct.

Problem 17.29



How many different triangles are there in the figure shown above?

(a) 28 (b) 24

(c) 20 (d) 16

Solution Look for the smallest triangles first—there are 12 of them.

Then, look for the triangles which are equal to half the rectangle—there are 12 of them.

Besides, there are 4 bigger triangles (spanning across 2 rectangles).

Thus a total of 28 triangles can be seen in the figure.

Hence, option (a) is correct.

Problem 17.30 A teacher has to choose the maximum different groups of three students from a total of six students. Of these groups, in how many groups there will be included a particular student?

(a) 6 (b) 8

(c)1 0 (d) 12

Solution If the students are *A*, *B*, *C*, *D*, *E* and *F*- we can have ${}^6C_3$ groups in all. However, if we have to count groups in which a particular student (say *A*) is always selected- we would get ${}^5C_2$ = 10 ways of doing it. Hence, option (c) is correct.

Problem 17.31 Three dice (each having six faces with each face having one number from 1 to 6) are rolled. What is the number of possible outcomes such that at least one dice shows the number 2?

(a) 36 (b) 81

(c) 91 (d) 116

Solution **All 3 dice have twos** – 1 case.

Two dice have twos:

This can principally occur in 3 ways which can be broken into:

If the first two dice have 2- the third dice can have 1, 3, 4, 5 or 6 = 5 ways.

Similarly, if the first and third dice have 2, the second dice can have 5 outcomes Æ 5 ways and if the second and third dice have a 2, there would be another 5 ways. Thus a total of 15 outcomes if 2 dice have a 2.

With only 1 dice having a two- If the first dice has 2, the other two can have 5 × 5 = 25 outcomes.

Similarly 25 outcomes if the second dice has 2 and 25 outcomes if the third dice has 2. A total of 75 outcomes. Thus, a total of 1 + 15 + 75 = 91 possible outcomes with at least.

Hence, option (c) is correct.

Problem 17.32 All the six letters of the name SACHIN are arranged to form different words without repeating any letter in any one word. The words so formed are then arranged as in a dictionary. What will be the position of the word SACHIN in that sequence?

(a) 436 (b) 590

(c) 601 (d) 751

Solution All words staring with *A*, *C*, *H*, *I* and *N* would be before words starting with S. So we would have 5! Words (= 120 words) each starting with *A*, *C*, *H*, *I* and *N*. Thus, a total of 600 words would get completed before we start off with *S*. SACHIN would be the first word starting with *S*, because *A*, *C*, *H*, *I*, *N* in that order is the correct alphabetical sequence. Hence, Sachin would be the 601st word. Hence, option (c) is correct.

Problem 17.33 Five balls of different colours are to be placed in three different boxes such that any box contains at least one 1 ball. What is the maximum number of different ways in which this can be done?

(a) 90 (b) 120

(c) 150 (d) 180

Solution The arrangements can be [3 & 1 & 1 or 1 & 3 & 1 or 1 & 1 & 3] or 2 & 2 & 1 or 2 & 1 & 2 or 1 & 2 and 2.

Total number of ways = $3 \times {}^5C_3 \times {}^2C_1 \times {}^1C_1 + 3 \times {}^5C_2 \times {}^3C_2 \times {}^1C_1 = 60 + 90 = 150$ ways

Hence, option (c) is correct.

Problem 17.34 Amit has five friends: 3 girls and 2 boys. Amit's wife also has 5 friends : 3 boys and 2 girls. In how many maximum number of different ways can they invite 2 boys and 2 girls such that two of them are Amit's friends and two are his wife's?

(a) 24 (b) 38

(c) 46 (d) 58

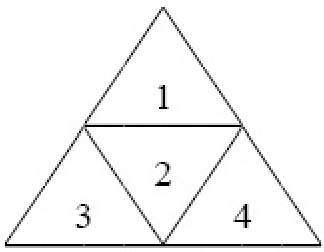
Solution The selection can be done in the following ways:

2 boys from Amit's friends and 2 girls from his wife's friends OR 1 boy & 1 girl from Amit's friends and 1 boy and 1 girl from his wife's friends OR 2 girls from Amit's friends and 2 boys from his wife's friends.

The number of ways would be:

$^2C_2 \times {}^2C_2 + {}^3C_1 \times {}^2C_1 \times {}^3C_1 \times {}^2C_1 + {}^3C_2 \times {}^3C_2 = 1 + 36 + 9 = 46$ ways.

Problem 17.35



In the given figure, what is the maximum number of different ways in which 8 identical balls can be placed in the small triangles 1, 2, 3 and 4 such that each triangle contains at least one ball?

(a) 32 (b) 35

(c) 44 (d) 56

Solution The ways of placing the balls would be 5, 1, 1, 1 (4!/3! = 4 ways); 4, 2, 1 & 1 (4!/2! = 12 ways); 3, 3, 1, 1 (4!/2! × 2! = 6 ways); 3, 2, 2, 1 (4!/2! = 12 ways) and 2, 2, 2, 2 (1 way). Total number of ways = 4 + 12 + 6 + 12 + 1 = 35 ways. Hence, option (b) is correct.

Problem 17.36 6 equidistant vertical lines are drawn on a board. 6 equidistant horizontal lines are also drawn on the board cutting the 6 vertical lines, and the distance between any two consecutive horizontal lines is equal to that between any two consecutive vertical lines. What is the maximum number of squares thus formed?

(a) 37 (b) 55

(c) 91 (d) 225

Solution The number of squares would be $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 91$. Hence, option (c) is correct.

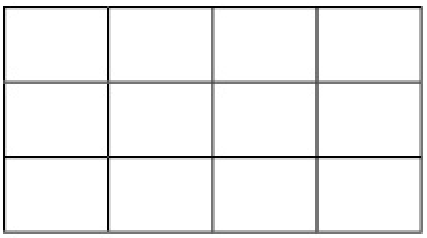
Problem 1.37 Groups each containing 3 boys are to be formed out of 5 boys—*A*, *B*, *C*, *D* and *E* such that no group contains both *C* and *D* together. What is the maximum number of such different groups?

(a) 5 (b) 6

(c) 7 (d) 8

Solution All groups – groups with C and D together = $^5C_3 - {}^3C_1 = 10 - 3 = 7$

Problem 17.38



In how many maximum different ways can 3 identical balls be placed in the 12 squares (each ball to be placed in the exact centre of the squares and only one ball is to be placed in one square) shown in the figure given above such that they do not lie along the same straight line ?

(a) 144 (b) 200

(c) 204 (d) 216

Solution The thought process for this question would be:

All arrangements ($^{12}C_3$) – Arrangements where all 3 balls are in the same row ($3 \times {}^4C_3$)– arrangements where all 3 balls are in the same straight line diagonally (4 arrangements) = $^{12}C_3 - 3 \times {}^4C_3 - 4 = 220 - 12 - 4 = 204$ ways.

Hence, option (c) is correct.

Problem 17.39 How many numbers are there in all from 6000 to 6999 (Both 6000 and 6999 included) having at least one of their digits repeated?

(a) 216 (b) 356

(c) 496 (d) 504

Solution All numbers – numbers having no numbers repeated = $1000 - 9 \times 8 \times 7 = 1000 - 504 = 496$ numbers. Hence, option (c) is correct.

Problem 17.40 Each of two women and three men is to occupy one chair out of eight chairs, each of which is numbered from one to eight. First, women are to occupy any two chairs from\those numbered one to four; and then the three men would occupy any three chairs out of the remaining six chairs. What is the maximum number of different ways in which this can be done?

(a) 40 (b) 132

(c) 1440 (d) 3660

Solution ${}^4C_2 \times 2! \times {}^6C_3 \times 3! = 6 \times 2 \times 20 \times 6 = 1440$. Hence, option (c) is correct.

Problem 17.41 A box contains five set of balls while there are three balls in each set. Each set of balls has one colour which is different from every other set. What is the least number of balls that must be removed from the box in order to claim with certainty that a pair of balls of the same colour has been removed?

(a) 6 (b) 7

(c) 9 (d) 11

Solution Let $C1$, $C2$, $C3$, $C4$ and $C5$ be the 5 distinct colours which have no repetition. For being definitely sure that we have picked up 2 balls of the same colour we need to consider the worst case situation.

Consider the following scenario:

Set 1	*Set 2*	*Set 3*	*Set 4*	*Set 5*
C1	C2	C3	C4	C5
C6	C8	C7	C9	C9
C7	C6	C10	C10	C8

In the above distribution of balls each set has exactly 1 ball which is unique in it’s colour while the colours of the other two balls are shared at least once in one of the other sets. In such a case, the worst scenario would be if we pick up the first 10 balls and they all turn out to be of different colours. The 11^{th} ball has to be of a colour which has already been taken. Thus,if we were to pick out 11 balls we would be sure of having at least 2 balls of the same colour. Hence, option (d) is correct.

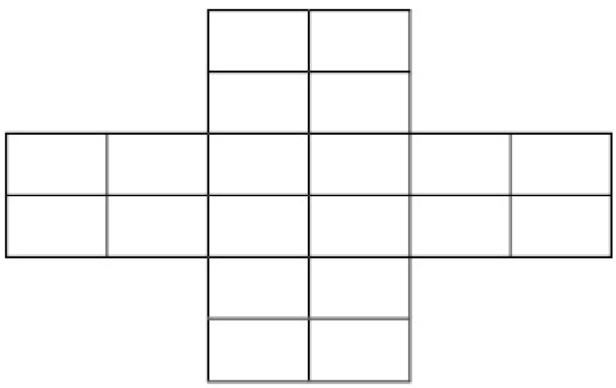
Problem 17.42 In a question paper, there are four multiple-choice questions. Each question has five choices with only one choice as the correct answer. What is the total number of ways in which a candidate will not get all the four answers correct?

(a) 19 (b) 120

(c) 624 (d) 1024

Solution 5^4 would be the total number of ways in which the questions can be answered. Out of these there would be only 1 way of getting all 4 correct. Thus, there would be 624 ways of not getting all answers correct.

Problem 17.43



Each of 8 identical balls is to be placed in the squares shown in the figure given in a horizontal direction such that one horizontal row contains 6 balls and the other horizontal row contains 2 balls. In how many maximum different ways can this be done?

(a) 38 (b) 28

(c) 16 (d) 14

Solution The 6 balls must be on either of the middle rows. This can be done in 2 ways. Once, we put the 6 balls in their single horizontal row- it becomes evident that for placing the 2 remaining balls on a straight line there are 2 principal options:

1. Placing the two balls in one of the four rows with two squares. In this case the number of ways of placing the balls in any particular row would be 1 way (since once you were to choose one of the 4 rows, the balls would automatically get placed as there are only two squares in each row.) Thus the total number of ways would be $2 \times 4 \times 1 = 8$ ways.

2. Placing the two balls in the other row with six squares. In this case the number of ways of placing the 2 balls in that row would be ${}^6C_2$. This would give us ${}^2C_1 \times 1 \times {}^6C_2 = 30$ ways. Total is 30 + 8 = 38 ways.

Hence, option (a) is correct.

Problem 17.44 In a tournament each of the participants was to play one match against each of the other participants. 3 players fell ill after each of them had played three matches and had to leave the tournament. What was the total number of participants at the beginning, if the total number of matches played was 75?

(a) 8 (b) 10
(c) 12 (d) 15

Solution The number of players at the start of the tournament cannot be 8, 10 or 12 because in each of these cases the total number of matches would be less than 75 (as ${}^8C_2$, ${}^{10}C_2$ and ${}^{12}C_2$ are all less than 75.) This only leaves 15 participants in the tournament as the only possibility.

Hence, option (d) is correct.

Problem 17.45 There are three parallel straight lines. Two points *A* and *B* are marked on the first line, points *C* and *D* are marked on the second line and points *E* and *F* are marked on the third line. Each of these six points can move to any position on its respective straight line.

Consider the following statements:

I. The maximum number of triangles that can be drawn by joining these points is 18.

II. The minimum number of triangles that can be drawn by joining these points is zero.

Which of the statements given above is/are correct?

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

Solution The maximum triangles would be in case all these 6 points are non-collinear. In such a case the number of triangles is ${}^6C_3 = 20$. Statement

I is incorrect.

Statement II is correct because if we take the position that *A* and *B* coincide on the first line, *C* & *D* coincide on the second line, *E* & *F* coincide on the third line and all these coincidences happen at 3 points which are on the same straight line- in such a case there would be 0 triangles formed. Hence, option (b) is correct.

Problem 17.46 A mixed doubles tennis game is to be played between two teams (each team consists of one male and one female). There are four married couples. No team is to consist of a husband and his wife. What is the maximum number of games that can be played?

(a) 12 (b) 21

(c) 36 (d) 42

Solution First select the two men. This can be done in ${}^4C_2$ ways. Let us say the men are *A*, *B*, *C* and *D* and their respective wives are *a*, *b*, *c* and *d*.

If we select *A* and *B* as the two men then while selecting the women there would be two cases as seen below:

Case 1:

A If *b* is selected to partner *A*

B There will be 3 choices for choosing *B*'s partner – viz *a*, *c* and *d*

Thus, total number of ways in this case = ${}^4C_2 \times 1 \times {}^3C_1 = 18$ ways.

Case 2:

A If either *c* or *d* is selected to partner *A*

B There will be 2 choices for choosing *B*'s partner – viz *a* and any one of *c* and *d*

Total number of ways of doing this = ${}^4C_2 \times 2 \times {}^2C_1 = 24$ ways.

Hence, the required answer is 18 + 24 = 42 ways.

Hence, option (d) is correct.

LEVEL OF DIFFICULTY (I)

1. How many numbers of 3-digits can be formed with the digits 1, 2, 3, 4, 5 (repetition of digits not allowed)?

   (a) 125 (b) 120

   (c) 60 (d) 150

2. How many numbers between 2000 and 3000 can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7 (repetition of digits not allowed?

   (a) 42 (b) 210

   (c) 336 (d) 440

3. In how many ways can a person send invitation cards to 6 of his friends if he has four servants to distribute the cards?

   (a) 6^4 (b) 4^6

   (c) 24 (d) 120

4. In how many ways can 5 prizes be distributed to 8 students if each student can get any number of prizes?

   (a) 40 (b) 5^8

   (c) 8^5 (d) 120

5. In how many ways can 7 Indians, 5 Pakistanis and 6 Dutch be seated in a row so that all persons of the same nationality sit together?

   (a) 3! (b) 7!5!6!

   (c) 3!.7!.5!.6! (d) 182

6. There are 5 routes to go from Allahabad to Patna & 4 ways to go from Patna to Kolkata, then how many ways are possible for going from Allahabad to Kolkata via Patna?

   (a) 20 (b) 5^4

(c) 4^5 (d) $5^4 + 4^5$

7. There are 4 qualifying examinations to enter into Oxford University: RAT, BAT, SAT, and PAT. An Engineer cannot go to Oxford University through BAT or SAT. A CA on the other hand can go to the Oxford University through the RAT, BAT & PAT but not through SAT. Further there are 3 ways to become a CA(viz., Foundation, Inter & Final). Find the ratio of number of ways in which an Engineer can make it to Oxford University to the number of ways a CA can make it to Oxford University.

   (a) 3:2 (b) 2:3

   (c) 2:9 (d) 9:2

8. How many straight lines can be formed from 8 non-collinear points on the X-Y plane?

   (a) 28 (b) 56

   (c) 18 (d) 19860

9. If ${}^nC_3 = {}^nC_8$, find n.

   (a) 11 (b) 12

   (c) 14 (d) 10

10. In how many ways can the letters of the word DELHI be arranged?

    (a) 119 (b) 120

    (c) 60 (d) 24

11. In how many ways can the letters of the word PATNA be rearranged?

    (a) 60 (b) 120

    (c) 119 (d) 59

12. For the arrangements of the letters of the word PATNA, how many words would start with the letter P?

    (a) 24 (b) 12

    (c) 60 (d) 120

13. In Question no.11, how many words will start with P and end with T?

(a) 3 (b) 6

(c) 11 (d) 12

14. If ${}^{n}C_4 = 70$, find n.

(a) 5 (b) 8

(c) 4 (d) 7

15. If ${}^{10}P_r = 720$, find r.

(a) 4 (b) 5

(c) 3 (d) 6

16. How many numbers of four digits can be formed with the digits 0, 1, 2, 3 (repetition of digits is not allowed?

(a) 18 (b) 24

(c) 64 (d) 192

17. How many numbers of four digits can be formed with the digits 0, 1, 2, 3 (repetition of digits being allowed?

(a) 12 (b) 108

(c) 256 (d) 192

18. How many numbers between 200 and 1200 can be formed with the digits 0, 1, 2, 3 (repetition of digits not allowed?

(a) 6 (b) 6

(c) 2 (d) 14

19. For the above question, how many numbers can be formed with the same digits if repetition of digits is allowed?

(a) 48 (b) 63

(c)32 (d) 14

20. If $(2n + 1)P_{(n-1)} : (2n - 1)P_n = 7:10$ find n

(a) 4 (b) 6

(c) 3 (d) 7

21. If ($^{28}C_{2r}$: $^{24}C_{2r-4}$) = 225:11 Find the value of r.

(a) 10 (b) 11

(c) 7 (d) 9

22. Arjit being a party animal wants to hold as many parties as possible amongst his 20 friends. However, his father has warned him that he will be financing his parties under the following conditions only:

(a) The invitees have to be amongst his 20 best friends.

(b) He cannot call the same set of friends to a party more than once.

(c) The number of invitees to every party have to be the same.

Given these constraints Arjit wants to hold the maximum number of parties. How many friends should he invite to each party?

(a) 11 (b) 8

(c) 10 (d) 12

23. In how many ways can 10 identical presents be distributed among 6 children so that each child gets at least one present?

(a) $^{15}C_5$ (b) $^{16}C_6$

(c) $^{9}C_5$ (d) 6^{10}

24. How many four digit numbers are possible, criteria being that all the four digits are odd?

(a) 125 (b) 625

(c) 45 (d) none of these

25. A captain and a vice-captain are to be chosen out of a team having eleven players. How many ways are there to achieve this?

(a) 10.9 (b) $^{11}C_2$

(c) 110 (d) 10.9!

26. There are five types of envelopes and four types of stamps in a post office. How many ways are there to buy an envelope and a stamp?

(a) 20 (b) 45

(c) 54 (d) 9

27. In how many ways can Ram choose a vowel and a consonant from the letters of the word ALLAHABAD?

(a) 4 (b) 6

(c) 9 (d) 5

28. There are three rooms in a motel: one single, one double and one for four persons. How many ways are there to house seven persons in these rooms?

(a) 7!/1!2!4! (b) 7!

(c) 7!/3 (d) 7!/3!

29. How many ways are there to choose four cards of different suits and different values from a deck of 52 cards?

(a) 13.12.11.10 (b) ${}^{52}C_4$

(c) 134 (d) 52.36.22.10

30. How many new words are possible from the letters of the word PERMUTATION?

(a) 11!/2! (b) (11!/2!) – 1

(c) 11! – 1 (d) None of these

31. A set of 15 different words are given. In how many ways is it possible to choose a subset of not more than 5 words?

(a) 4944 (b) 4^{15}

(c) 15^4 (d) 4943

32. In how many ways can 12 papers be arranged if the best and the worst paper never come together?

(a) 12!/2! (b) 12! – 11!

(c) (12! – 11!)/2 (d) 12! – 2.11!

33. In how many ways can the letters of the word 'EQUATION' be arranged so that all the vowels come together?

(a) ${}^9C_4 \, . \, {}^9C_5$ (b) 4!.5!

(c) 9!/5! (d) 9! – 4!5!

34. A man has 3 shirts, 4 trousers and 6 ties. What are the number of ways in which he can dress himself with a combination of all the three?

(a) 13 (b) 72

(c) 13!/3!.4!.6! (d) 3!.4!.6!

35. How many motor vehicle registration number of 4 digits can be formed with the digits 0, 1, 2, 3, 4, 5? (No digit being repeated.)

(a) 1080 (b) 120

(c) 300 (d) 360

36. How many motor vehicle registration number plates can be formed with the digits 1, 2, 3, 4, 5 (No digits being repeated) if it is given that registration number can have 1 to 5 digits?

(a) 100 (b) 120

(c) 325 (d) 205

Directions for Question 37 to 39: There are 25 points on a plane of which 7 are collinear. Now solve the following:

37. How many straight lines can be formed?

(a) 7 (b) 300

(c) 280 (d) none of these

38. How many triangles can be formed from these points?

(a) 453 (b) 2265

(c) 755 (d) none of these

39. How many quadrilaterals can be formed from these points?

(a) 5206 (b) 2603

(c) 13015 (d) None of these

40. There are ten subjects in the school day at St.Vincent's High School but the sixth standard students have only 5 periods in a day. In how many ways can we form a time table for the day for the sixth standard students if no subject is repeated?

(a) 510 (b) 105

(c) 252 (d) 30240

41. There are 8 consonants and 5 vowels in a word jumble. In how many ways can we form 5-letter words having three consonants and 2 vowels?

(a) 67200 (b) 8540

(c) 720 (d) None of these

42. How many batting orders are possible for the Indian cricket team if there is a squad of 15 to choose from such that Sachin Tendulkar is always chosen?

(a) 1001.11! (b) 364.11!

(c) 11! (d) 15.11!

43. There are 5 blue socks, 4 red socks and 3 green socks in Debu's wardrobe. He has to select 4 socks from this set. In how many ways can he do so?

(a) 245 (b) 120

(c) 495 (d) 60

44. A class prefect goes to meet the principal every week. His class has 30 people apart from him. If he has to take groups of three every time he goes to the principal, in how many weeks will he be able to go to the principal without repeating the group of same three which accompanies him?

(a) ${}^{30}P_3$ (b) ${}^{30}C_3$

(c) 30!/3 (d) None of these

45. For the above question if on the very first visit the principal appoints one of the boys accompanying him as the head boy of the school and lays down the condition that the class prefect has to be accompanied by the head boy every time he comes then for a maximum of how many weeks (including the first week) can the class prefect ensure that the principal sees a fresh group of three accompanying him?

(a) ${}^{30}C_2$ (b) ${}^{29}C_2$

(c) ${}^{29}C_3$ (d) None of these

46. How many distinct words can be formed out of the word PROWLING which start with R & end with W?

(a) 8!/2! (b) 6!2!

(c) 6! (d) None of these

47. How many 7-digit numbers are there having the digit 3 three times & the digit 5 four times?

(a) 7!/(3!)(5!) (b) $3^3 \times 5^5$

(c) 77 (d) 35

48. How many 7-digit numbers are there having the digit 3 three times & the digit 0 four times?

(a) 15 (b) $3^3 \times 4^4$

(c) 18 (d) None of these

49. From a set of three capital consonants, five small consonants and 4 small vowels how many words can be made each starting with a capital consonant and containing 3 small consonants and two small vowels.

(a) 3600 (b) 7200

(c) 21600 (d) 28800

50. Several teams take part in a competition, each of which must play one game with all the other teams. How many teams took part in the competition if they played 45 games in all?

(a) 5 (b) 10

(c) 15 (d) 20

51. In how many ways a selection can be made of at least one fruit out of 5 bananas, 4 mangoes and 4 almonds?

(a) 120 (b) 149

(c) 139 (d) 109

52. There are 5 different Jeffrey Archer books, 3 different Sidney Sheldon books and 6 different John Grisham books. The number of ways in which at least one book can be given away is

(a) $2^{10} – 1$ (b) $2^{11} – 1$

(c) $2^{12} – 1$ (d) $2^{14} – 1$

53. In the above problem, find the number of ways in which at least one book of each author can be given.

(a) $(2^5 – 1)(2^3 – 1)(2^8 – 1)$

(b) $(2^5 – 1)(2^3 – 1)(2^3 – 1)$

(c) $(2^5 – 1)(2^3 – 1)(2^3 – 1)$

(d) $(2^5 – 1)(2^3 – 1)(2^6 – 1)$

54. There is a question paper consisting of 15 questions. Each question has an internal choice of 2 options. In how many ways can a student attempt one or more questions of the given fifteen questions in the paper?

(a) 3^7 (b) 3^8

(c) 3^{15} (d) $3^{15} – 1$

55. How many numbers can be formed with the digits 1, 6, 7, 8, 6, 1 so that the odd digits always occupy the odd places?

(a) 15 (b) 12

(c) 18 (d) 20

56. There are five boys of McGraw-Hill Mindworkzz and three girls of I.I.M Lucknow who are sitting together to discuss a management

problem at a round table. In how many ways can they sit around the table so that no two girls are together?

(a) 1220 (b) 1400

(c) 1420 (d) 1440

57. Amita has three library cards and seven books of her interest in the library of Mindworkzz. Of these books she would not like to borrow the D.I. book, unless the Quants book is also borrowed. In how many ways can she take the three books to be borrowed?

(a) 15 (b) 20

(c) 25 (d) 30

58. From a group of 12 dancers, five have to be taken for a stage show. Among them Radha and Mohan decide either both of them would join or none of them would join. In how many ways can the 5 dancers be chosen?

(a) 190 (b) 210

(c) 278 (d) 372

59. Find the number of 6-digit numbers that can be found using the digits 1, 2, 3, 4, 5, 6 once such that the 6-digit number is divisible by its unit digit. (The unit digit is not 1.)

(a) 620 (b) 456

(c) 520 (d) 528

60. An urn contains 5 boxes. Each box contains 5 balls of different colours red, yellow, white, blue and black. Rangeela wants to pick up 5 balls of different colours, a different coloured ball from each box. If from the first box in the first draw, he has drawn a red ball and from the second box he has drawn a black ball, find the maximum number of trials that are needed to be made by Rangeela to accomplish his task if a ball picked is not replaced.

(a) 12 (b) 11

(c) 20 (d) 60

61. How many rounds of matches does a knock-out tennis tournament have if it starts with 64 players and every player needs to win 1 match to move at the next round?

(a) 5 (b) 6

(c) 7 (d) 64

62. There are *N* men sitting around a circular table at *N* distinct points. Every possible pair of men except the ones sitting adjacent to each other sings a 2 minute song one pair after other. If the total time taken is 88 minutes, then what is the value of *N*?

(a) 8 (b) 9

(c) 10 (d) 11

63. In a class with boys and girls a chess competition was played wherein every student had to play 1 game with every other student. It was observed that in 36 matches both the players were boys and in 66 matches both were girls. What is the number of matches in which 1 boy and 1 girl play against each other?

(a) 108 (b) 189

(c) 210 (d) 54

64. Zada has to distribute 15 chocolates among 5 of her children Sana, Ada, Jiya, Amir and Farhan. She has to make sure that Sana gets at least 3 and at most 6 chocolates. In how many ways can this be done?

(a) 495 (b) 77

(c) 417 (d) 435

65. Mr Shah has to divide his assets worth ` 30 crores in 3 parts to be given to three of his songs Ajay, Vijay and Arun ensuring that every son gets assets atleast worth ` 5 crores. In how many ways can this be done if it is given that the three sons should get their shares in multiples of ` 1 crore?

(a) 136 (b) 152

(c) 176 (d) 98

66. Three variables x, y, z have a sum of 30. All three of them are non-negative integers. If any two variables don't have the same value and exactly one variable has a value less than or equal to three, then find the number of possible solutions for the variables.

(a) 98 (b) 285

(c) 68 (d) 294

67. The letters of the word ALLAHABAD are rearranged to form new words and put in a dictionary. If the dictionary has only these words and one word on every page in alphabetical order then what is the page number on which the word LABADALAH comes?

(a) 6089 (b) 6088

(c) 6087 (d) 6086

68. If x, y and z can only take the values 1, 2, 3, 4, 5, 6, 7 then find the number of solutions of the equation $x + y + z = 12$.

(a) 36 (b) 37

(c) 38 (d) 31

69. There are nine points in a plane such that no three are collinear. Find the number of triangles that can be formed using these points as vertices.

(a) 81 (b) 90

(c) 9 (d) 84

70. There are nine points in a plane such that exactly three points out of them are collinear. Find the number of triangles that can be formed using these points as vertices.

(a) 81 (b) 90

(c) 9 (d) 83

71. If xy is a 2-digit number and u, v, x, y are digits, then find the number of solutions of the equation: $(xy)^2 = u! + v$

(a) 2 (b) 3

(c) 0 (d) 5

72. Ten points are marked on a straight line and eleven points are marked on another straight line. How many triangles can be constructed with vertices from among the above points?

(a) 495 (b) 550

(c) 1045 (d) 2475

73. For a scholarship, at the most n candidates out of $2n + 1$ can be selected. If the number of different ways of selection of at least one candidate is 63, the maximum number of candidates that can be selected for the scholarship is

(a) 3 (b) 4

(c) 2 (d) 5

74. One red flag, three white flags and two blue flags are arranged in a line such that,

(a) no two adjacent flags are of the same colour

(b) the flags at the two ends of the line are of different colours.

In how many different ways can the flags be arranged?

(a) 6 (b) 4

(c) 10 (d) 2

75. Sam has forgotten his friend's seven-digit telephone number. He remembers the following: the first three digits are either 635 or 674, the number is odd, and the number nine appears once. If Sam were to use a trial and error process to reach his friend, what is the minimum number of trials he has to make before he can be certain to succeed?

(a) 1000 (b) 2430

(c) 3402 (d) 3006

76. There are three cities A, B and C. Each of these cities is connected with the other two cities by at least one direct road. If a traveler wants to go from one city (origin) to another city (destination), she can do so either by traversing a road connecting the two cities

directly, or by traversing two roads, the first connecting the origin to the third city and the second connecting the third city to the destination. In all there are 33 routes from A to B (including those via C). Similarly there are 23 routes from B to C (including those via A). How many roads are there from A to C directly?

(a) 6 (b) 3

(c) 5 (d) 10

77. Let n be the number of different 5-digit numbers, divisible by 4 that can be formed with the digits 1, 2, 3, 4, 5 and 6, with no digit being repeated. What is the value of n?

(a) 144 (b) 168

(c) 192 (d) none of these

78. How many numbers greater than 0 and less than a million can be formed with the digits 0, 7 and 8?

(a) 486 (b) 1086

(c) 728 (d) none of these

79. In how many ways is it possible to choose a white square and a black square on a chess board so that the squares must not lie in the same row or column?

(a) 56 (b) 896

(c) 60 (d) 768

Directions for Questions 80 and 81: Answer these questions based on the information given below.

Each of the 11 letters *A*, *H*, *I*, *M*, *O*, *T*, *U*, *V*, *W*, *X* and *Z* appear same when looked at in a mirror. They are called symmetric letters. Other letters in the alphabet are asymmetric letters.

80. How many four-letter computer passwords can be formed using only the symmetric letters (no repetition allowed)?

(a) 7920 (b) 330

(c) 14640 (d) 419430

81. How many three-letter computer passwords can be formed (no repetition allowed) with at least one symmetric letter?

(a) 990
(b) 2730
(c) 12870
(d) 15600

82. Twenty seven persons attend a party. Which one of the following statements can never be true?

(a) There is a person in the party who is acquainted with all the twenty six others.
(b) Each person in the party has a different number of acquaintances.
(c) There is a person in the party who has an odd number of acquaintances.
(d) In the party, there is no set of three mutual acquaintances.

83. There are 6 boxes numbered 1, 2, ...6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is

(a) 5
(b) 21
(c) 33
(d) 60

84. How many numbers can be formed with odd digits 1, 3, 5, 7, 9 without repetition?

(a) 275
(b) 325
(c) 375
(d) 235

85. In how many ways five chocolates can be chosen from an unlimited number of Cadbury, Five-star, and Perk chocolates?

(a) 81
(b) 243
(c) 21
(d) 31

86. How many even numbers of four digits can be formed with the digits 1, 2, 3, 4, 5, 6 (repetitions of digits are allowed)?

(a) 648 (b) 180

(c) 1296 (d) 540

Directions for Questions 87 and 88: In a chess tournament there were two women participating and every participant played two games with the other participants. The number of games that the men played among themselves exceeded the number of games that the men played with the women by 66.

87. The number of participants in the tournament were?

    (a) 12 (b) 13

    (c) 15 (d) 11

88. The total number of games played in the tournament were?

    (a) 132 (b) 110

    (c) 156 (d) 210

LEVEL OF DIFFICULTY (II)

1. How many even numbers of four digits can be formed with the digits 1, 2, 3, 4, 5, 6 (repetitions of digits are allowed)?

   (a) 648 (b) 180

   (c) 1296 (d) 600

2. How many 4 digit numbers divisible by 5 can be formed with the digits 0, 1, 2, 3, 4, 5, 6 and 6?

   (a) 220 (b) 249

   (c) 432 (d) 288

3. There are 6 pups and 4 cats. In how many ways can they be seated in a row so that no cats sit together?

   (a) 6^4 (b) 10!/(4!).(6!)

   (c) $6! \times {}^7P_4$ (d) None of these

4. How many new words can be formed with the word MANAGEMENT all ending in *G*?

   (a) $10!/(2!)^4–1$ (b) $9!/(2!)^4$

   (c) $10!/(2!)^4$ (d) None of these

5. Find the total numbers of 9-digit numbers that can be formed all having different digits.

   (a) ${}^{10}P_9$ (b) 9!

   (c) 10!–9! (d) 9.9!

6. There are *V* lines parallel to the *x*-axis and '*W*' lines parallel to *y*-axis. How many rectangles can be formed with the intersection of these lines?

   (a) ${}^vP_2.{}^wP_2$ (b) ${}^vC_2.{}^wC_2$

(c) ${}^{v-2}C_2 . {}^{w-2}C_2$ (d) None of these

7. From 4 gentlemen and 4 ladies a committee of 5 is to be formed. Find the number of ways of doing so if the committee consists of a president, a vice-president and three secretaries?

(a) ${}^8P_5$ (b) 1120

(c) ${}^4C_2 \times {}^4C_3$ (d) None of these

8. In the above question, what will be the number of ways of selecting the committee with at least 3 women such that at least one woman holds the post of either a president or a vice-president?

(a) 420 (b) 610

(c) 256 (d) None of these

9. Find the number of ways of selecting the committee with a maximum of 2 women and having at the maximum one woman holding one of the two posts on the committee.

(a) 16 (b) 512

(c) 608 (d) 324

10. The crew of an 8 member rowing team is to be chosen from 12 men, of which 3 must row on one side only and 2 must row on the other side only. Find the number of ways of arranging the crew with 4 members on each side.

(a) 40,320 (b) 30,240

(c) 60,480 (d) None of these

11. In how many ways 5 MBA students and 6 Law students can be arranged together so that no two MBA students are side by side?

(a) $\frac{7!6!}{2!}$ (b) 6!.6!

(c) 5!.6! (d) ${}^{11}C_5$

12. The latest registration number issued by the Delhi Motor Vehicle Registration Authority is DL-5S 2234. If all the numbers and

alphabets before this have been used up, then find how many vehicles have a registration number starting with DL-5?

(a) 1,92,234 (b) 1,92,225

(c) 1,72,227 (d) None of these

13. There are 100 balls numbered n_1, n_2, n_3, n_4,… n_{100}. They are arranged in all possible ways. How many arrangements would be there in which n_{28} ball will always be before n_{29} ball and the two of them will be adjacent to each other?

(a) 99!/2! (b) 99!.2!

(c) 99! (d) None of these

14. Find the sum of the number of sides and number of diagonals of a hexagon.

(a) 210 (b) 15

(c) 6 (d) 9

15. A tea party is arranged for $2M$ people along two sides of a long table with M chairs on each side. R men wish to sit on one particular side and S on the other. In how many ways can they be seated (provided R, S £ M)

(a) ${}^{M}p_{R}.{}^{M}p_{S}$

(b) ${}^{M}p_{R}.{}^{M}p_{S}$ $({}^{2M-R-S}p_{2M-R-S})$

(c) ${}^{2M}P_{R}.\ {}^{2M-R}P_{S}$

(d) None of these

16. In how many ways can 'mn' things be distributed equally among n groups?

(a) ${}^{mn}P_{m}.\ {}^{mn}P_{n}$ (b) ${}^{mn}C_{m}.\ {}^{mn}C_{n}$

(c) $(mn)!/(m!)(.n!)$ (d) None of these

17. In how many ways can a selection be made of 5 letters out of 5As, 4Bs, 3Cs, 2Ds and 1E?

(a) 70 (b) 71

(c) ${}^{15}C_5$ (d) None of these

18. Find the number of ways of selecting 'n' articles out of $3n + 1$, out of which n are identical.

(a) 2^{2n-1} (b) ${}^{3n+1}C_n/n!$

(c) ${}^{3n+1}P_n/n!$ (d) None of these

19. The number of positive numbers of not more than 10 digits formed by using 0, 1, 2, 3 is

(a) $4^{10} - 1$ (b) 4^{10}

(c) $4^9 - 1$ (d) None of these

20. There is a number lock with four rings. How many attempts at the maximum would have to be made before getting the right number?

(a) 10^4 (b) 255

(c) $10^4 - 1$ (d) None of these

21. Find the number of numbers that can be formed using all the digits 1, 2, 3, 4, 3, 2, 1 only once so that the odd digits occupy odd places only.

(a) $4!/(2!)^2$ (b) $7!/(2!)^3$

(c) 1!.3!.5!.7! (d) None of these

22. There is a 7-digit telephone number with all different digits. If the digit at extreme right and extreme left are 5 and 6 respectively, find how many such telephone numbers are possible.

(a) 120 (b) 1,00,000

(c) 6720 (d) None of these

23. If a team of four persons is to be selected from 8 males and 8 females, then in how many ways can the selections be made to include at least one male.

(a) 3500 (b) 875

(c) 1200 (d) None of these

24. In the above question, in how many ways can the selections be made if it has to contain at the maximum three women?

(a) 1750
(b) 1200
(c) 875
(d) None of thes

25. How many figures are required to number a book containing 150 pages?

(a) 450
(b) 360
(c) 262
(d) None of these

26. There are 8 orators *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H*. In how many ways can the arrangements be made so that *A* always comes before *B* and *B* always comes before *C*.

(a) 8!/3!
(b) 8!/6!
(c) 5!.3!
(d) 8!/(5!.3!)

27. There are 4 letters and 4 envelopes. In how many ways can wrong choices be made?

(a) 4^3
(b) 4! – 1
(c) 16
(d) $4^4 - 1$

28. In the question above, find the number of ways in which only one letter goes in the wrong envelope?

(a) 4^3
(b) ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$
(c) ${}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3$
(d) 0

29. In question 27, find the number of ways in which only two letters go in the wrong envelopes?

(a) 4
(b) 5
(c) 6
(d) 3

30. A train is running between Patna to Howrah. Seven people enter the train somewhere between Patna and Howrah. It is given that nine stops are there in between Patna and Howrah. In how many ways can the tickets be purchased if no restriction is there with respect to

the number of tickets at any station? 2 people do not buy the same ticket.

(a) ${}^{45}C_7$ (b) ${}^{63}C_7$

(c) ${}^{56}C_7$ (d) ${}^{52}C_7$

31. There are seven pairs of black shoes and five pairs of white shoes. They all are put into a box and shoes are drawn one at a time. To ensure that at least one pair of black shoes are taken out, what is the number of shoes required to be drawn out?

(a) 12 (b) 13

(c) 7 (d) 18

32. In the above question, what is the minimum number of shoes required to be drawn out to get at least 1 pair of correct shoes (either white or black)?

(a) 12 (b) 7

(c) 13 (d) 18

33. In how many ways one white and one black rook can be placed on a chessboard so that they are never in an attacking position?

(a) 64×50 (b) 64×49

(c) 63×49 (d) None of these

34. How many 6-digit numbers have all their digits either all odd or all even?

(a) 31,250 (b) 28,125

(c) 15,625 (d) None of these

35. How many 6-digit numbers have at least 1 even digit?

(a) 884375 (b) 3600

(c) 880775 (d) 15624

36. How many 10-digit numbers have at least 2 equal digits?

(a) $9 \times {}^{10}C_2 \times 8!$ (b) $9.10^9 – 9.\ 9!$

(c) $9 \times 9!$ (d) None of these

37. On a triangle ABC, on the side AB, 5 points are marked, 6 points are marked on the side BC and 3 points are marked on the side AC (none of the points being the vertex of the triangle). How many triangles can be made by using these points?

(a) 90 (b) 333

(c) 328 (d) None of these

38. If we have to make 7 boys sit with 7 girls around a round table, then the number of different relative arrangements of boys and girls that we can make so that so that there are no two boys nor any two girls sitting next to each other is

(a) $2 \times (7!)^2$ (b) $7! \times 6!$

(c) $7! \times 7!$ (d) None of these

39. If we have to make 7 boys sit alternately with 7 girls around a round table which is numbered, then the number of ways in which this can be done is

(a) $2 \times (7!)^2$ (b) $7! \times 6!$

(c) $7! \times 7!$ (d) None of these

40. In the Suniti Building in Mumbai there are 12 floors plus the ground floor. 9 people get into the lift of the building on the ground floor. The lift does not stop on the first floor. If 2, 3 and 4 people alight from the lift on its upward journey, then in how many ways can they do so? (Assume they alight on different floors.)

(a) ${}^{11}C_3 \times {}^{3}P_3$ (b) ${}^{11}P_3 \times {}^{9}C_4 \times {}^{5}C_3$

(c) ${}^{10}P_3 \times {}^{9}C_4 \times {}^{5}C_3$ (d) ${}^{12}C_3$

Directions for Questions 41 and 42. There are 40 doctors in the surgical department of the AIIMS. In how many ways can they be arranged to form a team with:

41. 1 surgeon and an assistant

(a) 1260 (b) 1320

(c) 1440 (d) 1560

42. 1 surgeon and 4 assistants

(a) $40 \times {}^{39}C_4$ (b) $41 \times {}^{39}C_4$

(c) $41 \times {}^{40}C_4$ (d) None of these

43. In how many ways can 10 identical marbles be distributed among 6 children so that each child gets at least 1 marble?

(a) ${}^{15}C_5$ (b) ${}^{15}C_9$

(c) ${}^{10}C_5$ (d) ${}^{9}C_5$

44. Seven different objects must be divided among three people. In how many ways can this be done if one or two of them can get no objects?

(a) 15120 (b) 2187

(c) 3003 (d) 792

45. How many 6-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 and 7 so that the digits should not repeat?

(a) 720 (b) 1440

(c) 2160 (d) 6480

46. How many 6-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 and 7 so that the digits should not repeat and the second last digit is even?

(a) 720 (b) 320

(c) 2160 (d) 1440

47. How many 5-digit numbers that do not contain identical digits can be written by means of the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9?

(a) 6048 (b) 7560

(c) 5040 (d) 15,120

48. How many different 4-digit numbers are there which have the digits 1, 2, 3, 4, 5, 6, 7 and 8 such that the digit 1 appears exactly once.

(a) $7.\ {}^8P_4$ (b) ${}^8P_4$

(c) 4.7^3 (d) 7^3

49. How many different 7-digit numbers can be written using only three digits 1, 2 and 3 such that the digit 3 occurs twice in each number?

(a) ${}^7C_2.2^5$ (b) $7!/(2!)$

(c) $7!/(2!)^3$ (d) None of these

50. How many different 4-digit numbers can be written using the digits 1, 2, 3, 4, 5, 6, 7 and 8 only once such that the number 2 is contained once.

(a) 360 (b) 840

(c) 760 (d) 1260

LEVEL OF DIFFICULTY (III)

1. The number of ways in which four particular persons *A*, *B*, *C*, *D* and six more persons can stand in a queue so that *A* always stands before *B*, *B* always before *C* and *C* always before *D* is

   (a) 10!/4!
   (b) $^{10}P_4$
   (c) $^{10}C_4$
   (d) None of these

2. The number of circles that can be drawn out of 10 points of which 7 are collinear is

   (a) 130
   (b) 85
   (c) 45
   (d) Cannot be determined

3. How many different 9-digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?

   (a) 120
   (b) $9!/(2!)^3.3!$
   (c) $(4!)(2!)^3.3!$
   (d) None of these

4. How many diagonals are there in an *n*-sided polygon ($n > 3$)?

   (a) $(^nC_2 - n)$
   (b) $^nC_2$
   (c) $n(n - 1)/2$
   (d) None of these

5. A polygon has 54 diagonals. Find the number of sides.

   (a) 10
   (b) 14
   (c) 12
   (d) 9

6. The number of natural numbers of two or more than two digits in which digits from left to right are in increasing order is

   (a) 127
   (b) 128
   (c) 502
   (d) 512

7. In how many ways a cricketer can score 200 runs with fours and sixes only?

(a) 13 (b) 17

(c) 19 (d) 16

8. A dices is rolled six times. One, two, three, four, five and six appears on consecutive throws of dices. How many ways are possible of having 1 before 6?

(a) 120 (b) 360

(c) 240 (d) 280

9. The number of permutations of the letters *a, b, c, d, e, f, g* such that neither the pattern '*beg*' nor '*acd*' occurs is

(a) 4806 (b) 420

(c) 2408 (d) None of these

10. In how many ways can the letters of the English alphabet be arranged so that there are seven letters between the letters *A* and *B*?

(a) 31!.2! (b) ${}^{24}P_7.18!.2$

(c) 36.24! (d) None of these

11. There are 20 people among whom two are sisters. Find the number of ways in which we can arrange them around a circle so that there is exactly one person between the two sisters?

(a) 18! (b) 2!.19!

(c) 19! (d) None of these

12. There are 10 points on a straight line *AB* and 8 on another straight line, *AC* none of them being *A*. How many triangles can be formed with these points as vertices?

(a) 720 (b) 640

(c) 816 (d) None of these

13. In an examination, the maximum marks for each of the three papers is 50 each. The maximum marks for the fourth paper is 100. Find

the number of ways with which a student can score 60% marks in aggregate.

(a) 3,30,850 (b) 2,33,551

(c) 1,10,551 (d) None of these

14. How many rectangles can be formed out of a chessboard?

(a) 204 (b) 1230

(c) 1740 (d) None of these

15. On a board having 18 rows and 16 columns, find the number of squares.

(a) ${}^{18}C_2 . {}^{16}C_2$ (b) ${}^{18}P_2 . {}^{16}P_2$

(c) 18.16 + 17.15 + 16.14 + 15.13 + 14.12 +…….+ 4.2 + 3.1 (d) None of these

16. In the above question, find the number of rectangles.

(a) ${}^{18}C_2 . {}^{16}C_2$ (b) ${}^{18}P_2 . {}^{16}P_2$

(c) 171.136 (d) None of these

Directions for Questions 17 and 18: Read the passage below and answer the questions.

In the famous program *Kaun Banega Crorepati*, the host shakes hand with each participant once, while he shakes hands with each qualifier (amongst participant) twice more. Besides, the participants are required to shake hands once with each other, while the winner and the host each shake hands with all the guests once.

17. How many handshakes are there if there are 10 participants in all, 3 finalists and 60 spectators?

(a) 118 (b) 178

(c) 181 (d) 122

18. In the above question, what is the ratio of the number of handshakes involving the host to the number of handshakes not involving the host?

(a) 43 : 75 (b) 76 : 105

(c) 46 : 75 (d) None of these

19. What is the percentage increase in the total number of handshakes if all the guests are required to shake hands with each other once?

(a) 82.2% (b) 822%

(c) 97.7% (d) None of these

20. Two variants of the CAT paper are to be given to twelve students. In how many ways can the students be placed in two rows of six each so that there should be no identical variants side by side and that the students sitting one behind the other should have the same variant?

(a) $2 \times {}^{12}C_6 \times (6!)^2$ (b) $6! \times 6!$

(c) $7! \times 7!$ (d) None of these

21. For the above question, if there are now three variants of the test to be given to the twelve students (so that each variant is used for four students) and there should be no identical variants side by side and that the students sitting one behind the other should have the same variant. Find the number of ways this can be done.

(a) $6!^2$ (b) $6 \times 6! \times 6!$

(c) $6!^3$ (d) None of these

22. Five boys and three girls are sitting in a row of eight seats. In how many ways can they be seated so that not all girls sit side by side?

(a) 36,000 (b) 45,000

(c) 24,000 (d) None of these

23. How many natural numbers are there that are smaller than 10^4 and whose decimal notation consists only of the digits 0, 1, 2, 3 and 5, which are not repeated in any of these numbers?

(a) 32 (b) 164

(c) 31 (d) 212

24. Seven different objects must be divided among three people. In how many ways can this be done if one or two of them must get no objects?

(a) 381 (b) 36

(c) 84 (d) 180

25. Seven different objects must be divided among three people. In how many ways can this be done if at least one of them gets exactly 1 object?

(a) 2484 (b) 1218

(c) 729 (d) None of these

26. How many 4-digit numbers that are divisible by 4 can be formed from the digits 1, 2, 3, 4 and 5?

(a) 36 (b) 72

(c) 24 (d) None of these

27. How many natural numbers smaller than 10,000 are there in the decimal notation of which all the digits are different?

(a) 2682 (b) 4474

(c) 5274 (d) 1448

28. How many 4-digit numbers are there whose decimal notation contains not more than two distinct digits?

(a) 672 (b) 576

(c) 360 (d) 448

29. How many different 7-digit numbers are there the sum of whose digits are odd?

(a) 45.10^5 (b) 24.10^5

(c) 224320 (d) None of these

30. How many 6-digit numbers contain exactly 4 different digits?

(a) 4536 (b) 2,94,840

(c) 1,91,520 (d) None of these

31. How many numbers smaller than 2.10^8 and are divisible by 3 can be written by means of the digits 0, 1 and 2 (exclude single digit and double digit numbers)?

(a) 4369 (b) 4353

(c) 4373 (d) 4351

32. Six white and six black balls of the same size are distributed among ten urns so that there is at least one ball in each urn. What is the number of different distributions of the balls?

(a) 25,000 (b) 26,250

(c) 28,250 (d) 13,125

33. A bouquet has to be formed from 18 different flowers so that it should contain not less than three flowers. How many ways are there of doing this in?

(a) 5,24,288 (b) 2,62,144

(c) 2,61,972 (d) None of these

34. How many different numbers which are smaller than 2.10^8 can be formed using the digits 1 and 2 only?

(a) 766 (b) 94

(c) 92 (d) 126

35. How many distinct 6-digit numbers are there having 3 odd and 3 even digits?

(a) 55 (b) $(5.6)^3.(4.6)^3.3$

(c) 281250 (d) None of these

36. How many 8-digit numbers are there the sum of whose digits is even?

(a) 14400 (b) 4.5^5

(c) 45.10^6 (d) None of these

Directions for Questions 37 and 38: In a chess tournament there were two women participating and every participant played two games with the other participants. The number of games that the men played among themselves exceeded the number of games that the men played with the women by 66.

37. The number of participants in the tournament were:

(a) 12 (b) 13

(c) 15 (d) 11

38. The total number of games played in the tournament were:

(a) 132 (b) 110

(c) 156 (d) 210

39. There are 5 bottles of sherry and each have their respective caps. If you are asked to put the correct cap to the correct bottle then how many ways are there so that not a single cap is on the correct bottle?

(a) 44 (b) 55 – 1

(c) 5^5 (d) None of these

40. Amartya Banerjee has forgotten the telephone number of his best friend Abhijit Roy. All he remembers is that the number had 8-digits and ended with an odd number and had exactly one 9. How many possible numbers does Amartya have to try to be sure that he gets the correct number?

(a) 104.9^5 (b) 113.9^5

(c) 300.9^5 (d) $764.9^5.6!$

41. In Question 40, if Amartya is reminded by his friend Sharma that apart from what he remembered there was the additional fact that the last digit of the number was not repeated under any circumstance then how many possible numbers does Amartya have to try to be sure that he gets the correct number?

(a) $200.8^5 + 72.9^5$ (b) 8.96

(c) 36.85 + 7.96 (d) $36.8^5 + 8.9^6$

42. How many natural numbers not more than 4300 can be formed with the digits 0,1, 2, 3, 4 (if repetitions are allowed)?

(a) 574 (b) 570

(c) 575 (d) 569

43. How many natural numbers less than 4300 can be formed with the digits 0, 1, 2, 3, 4 (if repetitions are not allowed)?

(a) 113 (b) 158

(c) 154 (d) 119

44. How many even natural numbers divisible by 5 can be formed with the digits 0, 1, 2, 3, 4, 5, 6 (if repetitions of digits not allowed)?

(a) 1957 (b) 1956

(c) 1236 (d) 1235

45. There are 100 articles numbered n_1, n_2, n_3, n_4,… n_{100}. They are arranged in all possible ways. How many arrangements would be there in which n_{28} will always be before n_{29}.

(a) $5050 \times 99!$ (b) $5050 \times 98!$

(c) $4950 \times 98!$ (d) $4950 \times 99!$

46. The letters of the word PASTE are written in all possible orders and these words are written out as in a dictionary. Then the rank of the word SPATE is

(a) 432 (b) 86

(c) 59 (d) 446

47. The straight lines S_1, S_2, S_3 are in a parallel and lie in the same plane. A total number of A points on S_1; B points on S_2 and C points on S_3 are used to produce triangles. What is the maximum number of triangles formed?

(a) ${}^{A+B+C}C_3 - {}^{A}C_3 - {}^{B}C_3 - {}^{C}C_3 + 1$

(b) ${}^{A+B+C}C_3$

(c) ${}^{A+B+C}C_3 + 1$

(d) $({}^{A+B+C}C_3 - {}^{A}C_3 - {}^{B}C_3 - {}^{C}C_3)$

48. The sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The total number of triangles that can be constructed by using these points as vertices are

(a) 212 (b) 210

(c) 205 (d) 190

49. A library has 20 copies of CAGE; 12 copies each of RAGE Part1 and Part 2 ; 5 copies of PAGE Part 1, Part 2 and Part 3 and single copy of SAGE, DAGE and MAGE. In how many ways can these books be distributed?

(a) 62!/(20!)(12!)(5!) (b) 62!

(c) 62!/ (37)3 (d) 62!/(20!)(12!)2(5!)3

50. The AMS MOCK CAT test CATALYST 19 consists of four sections. Each section has a maximum of 45 marks. Find the number of ways in which a student can qualify in the AMS MOCK CAT if the qualifying marks is 90.

(a) 36,546 (b) 6296

(c) 64906 (d) None of these

ANSWER KEY

Level of Difficulty (I)

1. (c)	2. (b)	3. (b)	4. (c)
5. (c)	6. (a)	7. (b)	8. (a)
9. (a)	10. (b)	11. (d)	12. (b)
13. (a)	14. (b)	15. (c)	16. (a)
17. (d)	18. (d)	19. (b)	20. (c)
21. (c)	22. (c)	23. (c)	24. (b)
25. (c)	26. (a)	27. (a)	28. (a)
29. (a)	30. (b)	31. (a)	32. (d)

33. (b)	34. (b)	35. (d)	36. (c)
37. (c)	38. (b)	39. (d)	40. (d)
41. (a)	42. (a)	43. (c)	44. (b)
45. (b)	46. (c)	47. (d)	48. (a)
49. (c)	50. (b)	51. (b)	52. (d)
53. (d)	54. (d)	55. (c)	56. (d)
57. (c)	58. (d)	59. (d)	60. (a)
61. (b)	62. (d)	63. (a)	64. (d)
65. (a)	66. (d)	67. (a)	68. (b)
69. (d)	70. (d)	71. (b)	72. (c)
73. (a)	74. (a)	75. (c)	76. (a)
77. (c)	78. (c)	79. (d)	80. (a)
81. (c)	82. (b)	83. (b)	84. (b)
85. (b)	86. (a)	87. (b)	88. (d)

Level of Difficulty (II)

1. (a)	2. (b)	3. (c)	4. (b)
5. (d)	6. (b)	7. (b)	8. (d)
9. (b)	10. (c)	11. (a)	12. (d)
13. (c)	14. (b)	15. (b)	16. (d)
17. (b)	18. (d)	19. (a)	20. (c)
21. (d)	22. (c)	23. (d)	24. (a)
25. (d)	26. (a)	27. (b)	28. (d)
29. (c)	30. (a)	31. (d)	32. (c)
33. (b)	34. (b)	35. (a)	36. (b)
37. (b)	38. (b)	39. (a)	40. (b)
41. (d)	42. (a)	43. (d)	44. (b)
45. (c)	46. (a)	47. (d)	48. (c)
49. (a)	50. (b)		

Level of Difficulty (III)

1. (a)	2. (b)	3. (d)	4. (a)
5. (c)	6. (c)	7. (b)	8. (b)
9. (a)	10. (c)	11. (d)	12. (b)
13. (c)	14. (d)	15. (c)	16. (c)

17. (c)	18. (b)	19. (d)	20. (a)
21. (d)	22. (a)	23. (b)	24. (a)
25. (b)	26. (c)	27. (c)	28. (b)
29. (a)	30. (b)	31. (c)	32. (b)
33. (d)	34. (a)	35. (c)	36. (c)
37. (b)	38. (c)	39. (a)	40. (c)
41. (a)	42. (c)	43. (b)	44. (b)
45. (c)	46. (b)	47. (d)	48. (c)
49. (d)	50. (c)		

Hints

Level of Difficulty (III)

1. $^{10}C_4 \times 6! = 10!/4!$
2. For drawing a circle we need 3 non collinear points. This can be done in:

   $^3C_3 + {}^3C_2 \times {}^7C_1 + {}^3C_1 \times {}^7C_2 = 1 + 21 + 63 = 85$
3. The odd digits have to occupy even positions. This can be done in $\frac{4!}{2!2!}$ = 6 ways.

   The other digits have to occupy the other positions. This can be done in $\frac{5!}{3!2!}$ = 10 ways.

   Hence total number of rearrangements possible = 6 × 10 = 60.
4. The number of straight lines is $^nC_2$ out of which there are n sides. Hence, the number of diagonals is $^nC_2 - n$.
5. $^nC_2 - n = 54$.
6. We cannot take '0' since the smallest digit must be placed at the left most place. We have only 9 digits from which to select the numbers. First select any number of digits. Then for any selection there is only one possible arrangement where the required condition is met.

This can be done in ${}^9C_1 + {}^9C_2 + {}^9C_3 + ... + {}^9C_9$ ways = $2^9 - 1 = 511$ ways.

But we can't take numbers which have only one digit, hence the required answer is 511 – 9.

7. 200 runs can be scored by scoring only fours or through a combination of fours and sixes.

   Possibilities are 50×4, $47 \times 4 + 2 \times 6$, $44 \times 4 + 4 \times 6$ … A total of 17 ways.

8. Of the total arrangements possible (6!) exactly half would have 1 before 6. Thus, 6!/2 = 360.

9. Total number of permutations without any restrictions – Number of permutations having the '*acd*' pattern – Number of permutations having the '*beg*' pattern + Number of permutations having both the '*beg*' and '*acd*' patterns.

10. *A* and *B* can occupy the first and the ninth places, the second and the tenth places, the third and the eleventh place and so on… This can be done in 18 ways.

    A and *B* can be arranged in 2 ways.

    All the other 24 alphabets can be arranged in 24! ways.

    Hence the required answer = $2 \times 18 \times 24!$

11. First arrange the two sisters around a circle in such a way that there will be one seat vacant between them. [This can be done in 2! ways since the arrangement of the sisters is not circular.]

    Then, the other 18 people can be arranged on 18 seats in 18! ways.

12. ${}^{10}C_2 \times {}^8C_1 + {}^{10}C_1 \times {}^8C_2 = 360 + 280 = 640$

14. A chess board consists 9 parallel lines × 9 parallel lines. For a rectangle we need to select 2 parallel lines and two other parallel lines that are perpendicular to the first set. Hence, ${}^9C_2 \times {}^9C_2$

15-16. Based on direct formulae.

15. This is a direct result based question. Option (c) is correct. Refer to result no. 6 in Important Results 2.

16. $(1 + 2 + 3 + \ldots + 18)(1 + 2 + 3 + \ldots + 16)$

17-19. Based on simple counting according to the conditions given in the passage

17. ${}^{10}C_1 + 3 \times 2 + {}^{10}C_2 + 60 + 60 = 181$

18. Handshakes involving host = 76

    Hence, the required ratio is 76: 105.

19. The guests (Spectators) would shake hands ${}^{60}C_2$ times = 1770.

    Required percentage increase = 977.9%.

20. First select six people out of 12 for the first row. The other six automatically get selected for the second row. Arrange the two rows of people amongst themselves. Besides, the papers can be given in the pattern of 121212 or 212121. Hence the answer is $2 \times ({}^{12}C_6 \times 6! \times 6!)$.

21. The difference in this question from the previous question is the number of ways in which the papers can be distributed. This can be done by either distributing three different variants in the first three places of each row or by repeating the same variant in the first and the third places.

22. Required permutations = Total permutations with no condition – permutations with the conditions which we do not have to count.

23. We have to count natural numbers which have a maximum of 4 digits. The required answer will be given by:

    Number of single digit numbers + Number of two digit numbers + Number of three digit numbers + Number of four digit numbers.

24. Let the three people be A, B and C.

    If 1 person gets no objects, the 7 objects must be distributed such that each of the other two get 1 object at least.

    This can be done as 6 & 1, 5 & 2, 4 & 3 and their rearrangements.

    The answer would be

    $({}^{7}C_6 + {}^{7}C_5 + {}^{7}C_4) \times 3! = 378$

    Also, two people getting no objects can be done in 3 ways.

    Thus, the answer is 378 + 3 = 381

25. If only one gets 1 object

The remaining can be distributed as: (6,0) , (4, 2), (3, 3).

$({}^7C_1 \times {}^6C_6 \times 3! + {}^7C_1 \times {}^6C_4 \times 3! + {}^7C_1 \times {}^6C_3 \times 3!/2!)$

$= 42 + 630 + 420 + 1092.$

If 2 people get 1 object each:

${}^7C_1 \times {}^6C_1 \times {}^5C_5 \times 3!/2! = 126.$

Thus, a total of 1218.

26. Natural numbers which consist of the digits 1, 2, 3, 4. and 5 and are divisible by 4 must have either 12, 24, 32 or 52 in the last two places. For the other two places we have to arrange three digits in two places.

27. No. of 1 digit nos = 9

No. of 2 digit nos = 81

No. of 3 digit nos = $9 \times 9 \times 8 = 648$

No. of 4 digit nos = $9 \times 9 \times 8 \times 7 = 4536$

Total nos = 9 + 81 + 648 + 4536 = 5274

28. If the two digits are a and b then 4 digit numbers can be formed in the following patterns.

aabb; *aaab* or *aaaa*.

You will have to take two situations in each of the cases- first when the two digits are non zero digits and second when the two digits are zero.

29. For the total of the digits to be odd one of the following has to be true:

The number should contain 1 odd + 6 even or 3 odd + 4 even or 5 odd + 2 even or 7 odd digits. Count each case separately.

33. ${}^{18}C_4 + {}^{18}C_5 + {}^{18}C_6 + \ldots + {}^{18}C_{17} + {}^{18}C_{18}$

$= [{}^{18}C_0 + {}^{18}C_4 + \ldots + {}^{18}C_{18}] - [{}^{18}C_0 + {}^{18}C_1 + {}^{18}C_2 + {}^{18}C_3]$

$= 2^{18} - [1 + 18 + 153 + 816]$

$= 261158$

35. Total number of 6 digit numbers having 3 odd and 3 even digits (including zero in the left most place) = $5^3 \times 5^3$.

From this subtract the number of 5 digit numbers with 2 even digits and 3 odd digits (to take care of the extra counting due to zero)

36. There will be 5 types of numbers, viz. numbers which have

All eight digits even or six even and two odd digits or four even and four odd digits or two even and six odd digits or all eight odd digits. This will be further solved as below:

Eight even digits Æ $5^8 - 5^7 = 4 \times 5^7$

Six even and two odd digits Æ

when the left most digit is even Æ $4 \times {}^7C_5 \times 5^5 \times 5^5$

when the left most digit is odd Æ $5 \times {}^7C_6 \times 5^6 \times 5^1$

Four even and four odd digits Æ

when the left most digit is even Æ $4 \times {}^7C_5 \times 5^5 \times 5^4$

when the left most digit is odd Æ $5 \times {}^7C_4 \times 5^4 \times 5^3$

Two even and six odd digits Æ

when the left most digit is even Æ $4 \times {}^7C_1 \times 5 \times 5^6$

when the left most digit is odd Æ $5 \times {}^7C_2 \times 5^2 \times 5^5$

Eight odd digits Æ 58

37–38. Solve through options.

39. This question is based on a formula: The condition is that '*n*' things (each thing belonging to a particular place) have to be distributed in '*n*' places such that no particular thing is arranged in its correct place.

$n! - \frac{n!}{1!} + \frac{n!}{2!} - \frac{n!}{3!}$ sign of the terms will be alternate and the last term will be $\frac{n!}{n!}$.

However, this can also be solved through logic.

40. The possible cases for counting are:

Number of numbers when the units digit is nine + the number of numbers when neither the units digit nor the left most is nine + number of numbers when the left most digit is nine.

42. The condition is that we have to count the number of natural numbers not more than 4300.

The total possible numbers with the given digits = $5 \times 5 \times 5 \times 5 = 625 - 1 = 624$.

Subtract form this the number of natural number greater than 4300 which can be formed from the given digits = $1 \times 2 \times 5 \times 5 - 1 = 49$.

Hence, the required number of numbers = $624 - 49 = 575$.

43. The required answer will be given by

The number of one digit natural number + number of two digit natural numbers + the number of three digit natural numbers + the number of four digit natural number starting with 1, 2, or 3 + the number of four digit natural numbers starting with 4.

46. The following words will appear before SPATE. All words starting with A + All words starting with E + All words starting with P + All words starting with SA + All words starting with SE + SPAET

47. For the maximum possibility assume that no three points other than given in the question are in a straight line.

Hence, the total number of D's = ${}^{A+B+C}C_3 - {}^{A}C_3 - {}^{B}C_3 - {}^{C}C_3$

48. Use the formula $\frac{n!}{p!q!r!}$.

$n(E) = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times {}^{7}C_2$

$n(S) = 7^7$

$n(E) = 8 \times 7 + 8 \times 7 = 112$

$n(S) = 64C_2 = \frac{2 \times 8 \times 7 \times 2}{64 \times 63} = \frac{1}{18}$

49. Use the formula $\frac{n!}{p!q!r!}$

Solutions and Shortcuts

Level of Difficulty (I)

1. The number of numbers formed would be given by 5 × 4 × 3 (given that the first digit can be filled in 5 ways, the second in 4 ways and the third in 3 ways – MNP rule).
2. The first digit can only be 2 (1 way), the second digit can be filled in 7 ways, the third in 6 ways and the fourth in 5 ways. A total of 1 × 7 × 6 × 5 = 210 ways.
3. Each invitation card can be sent in 4 ways. Thus, 4 × 4 × 4 × 4 × 4 × 4 = 4^6.
4. In this case since nothing is mentioned about whether the prizes are identical or distinct we can take the prizes to be distinct (the most logical thought given the situation). Thus, each prize can be given in 8 ways — thus a total of 8^5 ways.
5. We need to assume that the 7 Indians are 1 person, so also for the 6 Dutch and the 5 Pakistanis. These 3 groups of people can be arranged amongst themselves in 3! ways. Also, within themselves the 7 Indians the 6 Dutch and the 5 Pakistanis can be arranged in 7!, 6! And 5! ways respectively. Thus, the answer is 3! × 7! × 6! × 5!.
6. Use the MNP rule to get the answer as 5 × 4 = 20.
7. An IITian can make it to IIMs in 2 ways, while a CA can make it through in 3 ways. Required ratio is 2:3. Option (b) is correct.
8. For a straight line we just need to select 2 points out of the 8 points available. ${}^8C_2$ would be the number of ways of doing this.
9. Use the property ${}^nC_r = {}^nC_{n-r}$ to see that the two values would be equal at $n = 11$ since ${}^{11}C_3 = {}^{11}C_8$.
10. There would be 5! ways of arranging the 5 letters. Thus, 5! = 120 ways.
11. Rearrangements do not count the original arrangements. Thus, 5!/2! – 1 = 59 ways of rearranging the letters of PATNA.
12. We need to count words starting with P. These words would be represented by P _ _ _ _.

The letters ATNA can be arranged in 4!/2! ways in the 4 places. A total of 12 ways.

13. P _ _ _ T. Missing letters have to be filled with A,N,A. 3!/2!.= 3 ways.
14. Trial and error would give us ${}^8C_4$ as the answer. ${}^8C_4$= 8 × 7 × 6 × 5/4 × 3 × 2 ×1 = 70.
15. ${}^{10}P_3$ would satisfy the value given as ${}^{10}P_3$ = 10 × 9 × 8 = 720.
16. 3 × 3 × 2 × 1 = 18
17. 3 × 4 × 4 × 4 = 192
18. Divide the numbers into three-digit numbers and 4- digit numbers—Number of 3 digit numbers = 2 × 3 × 2 = 12. Number of 4-digit numbers starting with 10 = 2 × 1 = 2. Total = 14 numbers.
19. 3-digit numbers = 2 × 4 × 4 –1 = 31 (–1 is because the number 200 cannot be counted) ; 4-digit numbers starting with 10 = 4 × 4 = 16, Number of 4 digit numbers starting with 11 = 4 × 4 = 16. Total numbers = 31 + 16 + 16 = 63.
20. At n = 3, the values convert to ${}^7P_2$ and ${}^5P_3$ whose values respectively are 42 & 60 giving us the required ratio.
21. At r = 7, the value becomes (28!/14! × 14!)/(24!/10! × 14!) Æ 225:11.
22. The maximum value of ${}^nC_r$ for a given value of n, happens when r is equal to the half of n. So if he wants to maximise the number of parties given that he has 20 friends, he should invite 10 to each party.
23. This is a typical case for the use of the formula ${}^{n-1}C_{r-1}$ with n = 10 and r = 6. So the answer would be given ${}^9C_5$.
24. For each digit there would be 5 options (viz 1, 3, 5, 7, 9). Hence, the total number of numbers would be 5 × 5 × 5 × 5 = 625.
25. ${}^{11}C_1 \times {}^{10}C_1$ = 110. Alternately, ${}^{11}C_2 \times 2!$
26. 5 × 4 = 20.

27. In the letters of the word ALLAHABAD there is only 1 vowel available for selection (A). Note that the fact that A is available 4 times has no impact on this fact. Also, there are 4 consonants available — viz: L, H, B and D. Thus, the number of ways of selecting a vowel and a consonant would be $1 \times {}^4C_1 = 4$.

28. Choose 1 person for the single room & from the remaining choose 2 people for the double room & from the remaining choose 4 people for the 4 persons room Æ ${}^7C_1 \times {}^6C_2 \times {}^4C_4$.

29. From the first suit there would be 13 options of selecting a card. From the second suite there would be 12 options, from the third suite there would be 11 options and from the fourth suite there would be 10 options for selecting a card. Thus, $13 \times 12 \times 11 \times 10$.

30. Number of 11 letter words formed from the letters P, E, R, M, U, T, A, T, I, O, N = 11!/2!.

    Number of new words formed = total words –1 = 11!/2!–1.

31. ${}^{15}C_0 + {}^{15}C_1 + {}^{15}C_2 + {}^{15}C_3 + {}^{15}C_4 + {}^{15}C_5 = 1 + 15 + 105 + 455 + 1365 + 3003 = 4944$

32. All arrangements – Arrangements with best and worst paper together = $12! - 2! \times 11!$.

33. The vowels EUAIO need to be considered as 1 letter to solve this. Thus, there would be 4! ways of arranging Q, T and N and the 5 vowels taken together. Also, there would be 5! ways of arranging the vowels amongst themselves. Thus, we have $4! \times 5!$.

34. ${}^3C_1 \times {}^4C_1 \times {}^6C_1 = 72$.

35. 4-digit Motor vehicle registration numbers can have 0 in the first digit. Thus, we have $6 \times 5 \times 4 \times 3 = 360$ ways.

36. Single digit numbers = 5

    Two digit numbers = $5 \times 4 = 20$

    Three digit numbers = $5 \times 4 \times 3 = 60$

    Four digit numbers = $5 \times 4 \times 3 \times 2 = 120$

    Five digit numbers = $5 \times 4 \times 3 \times 2 \times 1 = 120$

    Total = 5 + 20 + 60 + 120 + 120 = 325

37. $^{25}C_2 - {}^{7}C_2 + 1 = 280$

38. $^{25}C_3 - {}^{7}C_3 = 2265$

39. $^{25}C_4 - {}^{7}C_4 - {}^{7}C_3 \times {}^{18}C_1 = 11985$

40. $^{10}C^5 \times 5! = 30240$

41. $^{8}C_3 \times {}^{5}C_2 \times 5! = 67200$

42. The selection of the 11 player team can be done in $^{14}C_{10}$ ways. This results in the team of 11 players being completely chosen. The arrangements of these 11 players can be done in 11!.

    Total batting orders = $^{14}C_{10} \times 11! = 1001 \times 11!$

    (**Note:** Arrangement is required here because we are talking about forming batting orders).

43. $^{12}C_4 = 495$

44. $^{30}C_3$

45. $^{29}C_2$

46. R _ _ _ _ _ _ W. The letters to go into the spaces are P, O, L, I, N, G. Since all these letters are distinct the number of ways of arranging them would be 6!.

47. 7!/3! × 4! = 35

48. The number has to start with a 3 and then in the remaining 6 digits it should have two 3's and four 0's. This can be done in 6!/2! × 4! = 15 ways.

49. $^{3}C_1 \times {}^{5}C_3 \times {}^{4}C_2 \times 5! = 21600$

50. If the number of teams is n, then $^{n}C_2$ should be equal to 45. Trial and error gives us the value of n as 10.

51. From 5 bananas we have 6 choices available (0, 1, 2, 3, 4 or 5). Similarly 4 mangoes and 4 almonds can be chosen in 5 ways each.

    So total ways = 6 × 5 × 5 = 150 possible selections. But in this 150, there is one selection where no fruit is chosen.

    So required no. of ways = 150 – 1 = 149

Hence Option (b) is correct.

52. For each book we have two options, give or not give. Thus, we have a total of 2^{14} ways in which the 14 books can be decided upon. Out of this, there would be 1 way in which no book would be given. Thus, the number of ways is $2^{14} – 1$.

    Hence, Option (d) is correct.

53. The number of ways in which at least 1 Archer book is given is $(2^5–1)$. Similarly, for Sheldon and Grisham we have $(2^3–1)$ and $(2^6–1)$. Thus required answer would be the multiplication of the three. Hence, Option (d) is the correct answer.

54. For each question we have 3 choices of answering the question (2 internal choices + 1 non-attempt). Thus, there are a total of 3^{15} ways of answering the question paper. Out of this there is exactly one way in which the student does not answer any question. Thus there are a total of $3^{15} – 1$ ways in which at least one question is answered.

    Hence, Option (d) is correct.

55. The digits are 1, 6, 7, 8, 7, 6, 1. In this seven-digit no. there are four add places and three even places— OEOEOEO. The four odd digits 1, 7, 7, 1 can be arranged in four odd places in [4!/2! × 2] = 6 ways [as 1 and 7 are both occurring twice].

    The even digits 6, 8, 6 can be arranged in three even places in 3!/2! = 3 ways.

    Total no. of ways = 6 × 3 = 18.

    Hence, Option (c) is correct.

56. We have no girls together, let us first arrange the 5 boys and after that we can arrange the girls in the spaces between the boys.

    Number of ways of arranging the boys around a circle = [5 – 1]! = 24.

    Number of ways of arranging the girls would be by placing them in the 5 spaces that are formed between the boys. This can be done in $^5P_3$ ways = 60 ways.

    Total arrangements = 24 × 60 = 1440.

Hence, Option (d) is correct.

57. Books of interest = 7, books to be borrowed = 3

Case 1— Quants book is taken. Then D.I book can also be taken.

So Amita is to take 2 more books out of 6 which she can do in $^6C_2$ = 15 ways.

Case 2— If Quants book has not been taken, the D.I book would also not be taken.

So Amita will take three books out of 5 books. This can be done in $^5C_3$ = 10 ways.

So total ways = 15 + 10 = 25 ways.

Hence Option (c) is correct.

58. We have to select 5 out of 12.

If Radha and Mohan join- then we have to select only 5 – 2 = 3 dancers out of 12 – 2 = 10 which can be done in $^{10}C_3$ = 120 ways.

If Radha and Mohan do not join, then we have to select 5 out of 12 – 2 = 10-> $^{10}C_5$ = 252 ways.

Total number of ways = 120 + 252 = 372.

Hence, Option (d) is correct.

59. The unit digit can either be 2, 3, 4, 5 or 6.

When the unit digit is 2, the number would be even and hence will be divisible by 2. Hence all numbers with unit digit 2 will be included which is equal to 5! Or 120.

When the unit digit is 3, then in every case the sum of the digits of the number would be 21 which is a multiple of 3. Hence all numbers with unit digit 3 will be divisible by 3 and hence will be included. Total number of such numbers is 5! or 120.

Similarly for unit digit 5 and 6, the number of required numbers is 120 each.

When the unit digit is 4, then the number would be divisible by 4 only if the ten's digit is 2 or 6. Total number of such numbers is 2 × 4! or 48.

Hence total number of required numbers is $(4 \times 120) + 48 = 528$.

Hence, Option (d) is the answer.

60. As we need to find the maximum number of trials, so we have to assume that the required ball in every box is picked as late as possible. So in the third box, first two balls will be red and black. Hence third trial will give him the required ball. Similarly, in fourth box, he will get the required ball in fourth trial and in the fifth box, he will get the required ball in fifth trial. Hence maximum total number of trials required is $3 + 4 + 5 = 12$.

    Hence, Option (a) is the answer.

61. Since every player needs to win only 1 match to move to the next round, therefore the 1st round would have 32 matches between 64 players out of which 32 will be knocked out of the tournament and 32 will be moved to the next round. Similarly in 2nd round 16 matches will be played, in the 3rd round 8 matches will be played, in 4th round 4 matches, in 5th round 2 matches and the 6th round will be the final match. Hence total number of rounds will be 6 ($2^6 = 64$).

    Hence, option (b) is the answer.

62. Total number of pairs of men that can be selected if the adjacent ones are also selected is ${}^{N}C_2$. Total number of pairs of men selected if only the adjacent ones are selected is N. Hence total number of pairs of men selected if the adjacent ones are not selected is ${}^{N}C_2 - N$.

    Since the total time taken is 88 min, hence the number of pairs is 44.

    Hence, ${}^{N}C_2 - N = 44$ Æ $N = 11$.

    Hence, Option (d) is the answer.

63. Let the number of boys be B. Then ${}^{B}C_3 = 36$ Æ $B = 9$

    Let the number of girls be G. Then ${}^{G}C_2 = 66$ Æ $G = 12$.

    Therefore total number of students in the class $= 12 + 9 = 21$. Hence total number of matches $= {}^{21}C_2 = 210$. Hence, number of matches between 1 boy and 1 girl $= 210 - (36 + 66) = 108$.

    Hence, Option (a) is the answer.

64. We will first give 3 chocolates to Sana and 1 each to the other 4. So we have already distributed 7 chocolates. Now we are left with 8 chocolates that have to be distributed among 5 people. So number of ways possible is 12!/8!4! = 495. Out of these we have to eliminate cases in which Sana gets more than 6 chocolates. As we have already given her 3 chocolates that means we have to eliminate cases in which she gets 7 or 8 or 9 or 10 or 11 chocolates out of 495 cases. She would get 11 chocolates in one case, 10 chocolates in ${}^4C_1$ cases, 9 chocolates in ${}^5C_1$ cases, 8 chocolates in ${}^6C_2$ cases and 7 chocolates in ${}^7C_3$ cases. So, total number of cases that need to be eliminated is 60. So the required number of ways is 495 – 60 = 435.

    Hence, Option (d) is the answer.

65. Firstly we will give 5 crores each to the three sons. That will cover 15 crores out of 30 crores leaving behind 15 crores. Now 15 crores can be distributed in three people in 17!15!2! ways or 136 ways.

    Hence, Option (a) is the answer.

66. Let x = 3. Then y + z = 27. For the conditions given in the question, no. of solutions is 20.

    Similarly for x = 2 there will be 23 solutions, for x = 1 there will be 26 solutions and for x = 0, there will be 29 solutions. Therefore total 98 solutions are possible.

    Similarly for y = 3 to 0, there will be 98 solutions and for z = 3 to 0, there will be 98 solutions.

    Hence there will be total of 294 solutions. Hence, Option (d) is the answer.

67. No. of words starting with A = 8!/ 2!3! = 3360.

    No. of words starting with B = 8!/ 2!4! = 840

    No. of words starting with D = 8!/2!4! = 840

    No. of words starting with H = 8!/2!4! = 840

    Now words with L start.

    No. of words starting with LAA = 6!/2! = 180

    Now LAB starts and first word starts with LABA.

No. of words starting with LABAA = 4! = 24

After this the next words will be LABADAAHL, LABADAALH, LABADAHAL, LABADAHLA and hence, Option (a) is the answer.

68. We will consider $x = 7$ to $x = 1$.

For $x = 7$, $y + z = 5$. No. of solutions = 4

For $x = 6$, $y + z = 6$. No. of solution = 5

For $x = 5$, $y + z = 7$. No. of solutions = 6

For $x = 4$, $y + z = 8$. No. of solutions = 7

For $x = 3$, $y = z = 9$. No. of solutions = 6

For $x = 2$, $y + z = 10$. No. of solutions = 5

For $x = 1$, $y + z = 11$. No. of solutions = 4

Hence number of solutions = 37

Hence, Option (b) is the answer.

69. As no three points are collinear, therefore every combination of 3 points out of the nine points will give us a triangle. Hence, the answer is ${}^9C_3$ or 84.

Hence, Option (d) is correct.

70. The number of combinations of three points picked from the nine given points is ${}^9C_3$ or 84. All these combinations will result in a triangle except the combination of the three collinear points. Hence number of triangles formed will be 84 – 1 = 83.

Hence, Option (d) is the answer.

71. $(xy)^2 = u! + v$

Here xy is a two-digit number and maximum value of its square is 9801. 8! is a five-digit number => u is less than 8 and 4! is 24 which when added to a single digit will never give the square of a two-digit number. Hence u is greater than 4. So, possible values of u can be 5, 6 and 7.

If $u = 5$, $u! = 120 \Rightarrow (xy)^2 = u! + v \Rightarrow (xy)^2 = 120 + v = 120 + 1 = 121 = 11^2$

If $u = 6$, $u! = 720 \Rightarrow (xy)^2 = u! + v \Rightarrow (xy)^2 = 720 + v = 720 + 9 = 729 = 27^2$

If $u = 7$, $u! = 5040 \Rightarrow (xy)^2 = u! + v \Rightarrow (xy)^2 = 5040 + v = 5040 + 1 = 5041 = 71^2$

So there are three cases possible. Hence, 3 solutions exist for the given equation.

Hence, Option (b) is the correct answer.

72. In order to form triangles from the given points, we can either select 2 points from the first line and 1 point from the second OR select one point from the first line and 2 from the second.

This can be done in:

${}^{10}C_2 \times {}^{11}C_1 + {}^{10}C_1 \times {}^{11}C_2 = 495 + 550 = 1045$

73. If we have 'n' candidates who can be selected at the maximum, naturally, the answer to the question would also represent 'n'.

Hence we check for the first option. If $n = 3$, then $2n + 1 = 7$ and it means that there are 7 candidates to be chosen from. Since it is given that the number of ways of selection of at least 1 candidate is 63, we should try to see, whether selecting 1, 2 or 3 candidates from 7 indeed adds up to 63 ways. If it does this would be the correct answer.

${}^{7}C_1 + {}^{7}C_2 + {}^{7}C_3 = 7 + 21 + 35 = 63$. Thus, the first Option fits the situation and is hence correct.

74. This problem can be approached by putting the white flags in their possible positions. There are essentially 4 possibilities for placing the 3 white flags based on the condition that two flags of the same color cannot be together:

1, 3, 5; 1, 3, 6; 1, 4, 6 and 2, 4, 6.

Out of these 4 possible arrangements for the 3 white flags we cannot use 1, 3, 6 and 1, 4, 6 as these have the same color of flag at both ends- something which is not allowed according to the question. Thus there are only 2 possible ways of placing the white flags— 1, 3, 5 OR 2, 4, 6. In each of these 2 ways, there are a further 3 ways of placing the 1 red flag and the 2 blue flags. Thus we get a total of 6 ways. Option (a) is correct.

75. The possible numbers are:

635_ _ _ 9	9 in the units place	$9 \times 9 \times 9 = 729$ numbers
635 _ _ _ _	9 used before the units place	$3 \times 9 \times 9 \times 4 = 972$ numbers
674_ _ _ 9	9 in the units place	$9 \times 9 \times 9 = 729$ numbers
674 _ _ _ _	9 used before the units place	$3 \times 9 \times 9 \times 4 = 972$ numbers
Total		3402 numbers

76. We need to go through the options and use the MNP rule tool relating to Permutations and Combinations.

We can draw up the following possibilities table for the number of routes between each of the three towns.

If the first option is true, i.e., there are 6 routes between *A* to *C*:

A-C	*Possibilities for C-B*	*Possibilities for total routes A-C-B (Say X)*	*Possibilities for Total routes A–B (Y)*
6	5, 4, 3, 2, 1	30, 24, 18, 12, 6	3, 9, 15, 21, 27 Note: these values are derived based on the logic that X + Y = 33

We further know that there are 23 routes between *B* to *C*.

From the above combinations the possibilities for the routes between *B* to *C* are:

B–A (Y in the table above)	*A–C*	*B–A–C*	*B–C*	*Total*
3	6	18	5	23
9	6	54 not possible	4	
15	6	90 not possible	3	
21	6	126 not possible	2	
27	6	162 not possible	1	

It is obvious that the first possibility in the table above satisfies all conditions of the given situation. Option (a) is correct.

77. With the digits 1, 2, 3, 4, 5 and 6 the numbers divisible by 4 that can be formed are numbers ending in: 12, 16, 24, 32, 36, 52, 56 and 64.

Number of numbers ending in 12 are: $4 \times 3 \times 2 = 24$

Thus the number of numbers is $24 \times 8 = 192$

Option (c) is correct.

78. A million is 1000000 (i.e. the first seven digit number). So we need to find how many numbers of less than 7 digits can be formed using the digits 0,7 and 8.

Number of 1 digit numbers = 2

Number of 2 digit numbers = $2 \times 3 = 6$

Number of 3 digit numbers = $2 \times 3 \times 3 = 18$

Number of 4 digit numbers = $2 \times 3 \times 3 \times 3 = 54$

Number of 5 digit numbers = $2 \times 3 \times 3 \times 3 \times 3 = 162$

Number of 6 digit numbers = $2 \times 3 \times 3 \times 3 \times 3 \times 3 = 486$

Total number of numbers = 728. Option (c) is correct.

79. The white square can be selected in 32 ways and once the white square is selected 8 black squares become ineligible for selection. Hence, the black square can be selected in 24 ways. $32 \times 24 = 768$. Option (d) is correct.

80. Since there are 11 symmetric letters, the number of passwords that can be formed would be $11 \times 10 \times 9 \times 8 = 7920$. Option (a) is correct.

81. This would be given by the number of passwords having:

1 symmetric and 2 asymmetric letters + 2 symmetric and 1 asymmetric letter + 3 symmetric and 0 asymmetric letters

${}^{11}C_1 \times {}^{15}C_2 \times 3! + {}^{11}C_2 \times {}^{15}C_1 \times 3! + {}^{11}C_3 \times 3! = 11 \times 105 \times 6 + 55 \times 15 \times 6 + 11 \times 10 \times 9 = 6930 + 4950 + 990 = 12870$. Option (c) is correct.

82. Each of the first, third and fourth options can be obviously seen to be true— no mathematics needed there. Only the second option can never be true.

In order to think about this mathematically and numerically— think of a party of 3 persons say *A*, *B* and *C*. In order for the second condition to be possible, each person must know a different number of persons. In a party with 3 persons this is possible only if the numbers are 0, 1 and 2. If *A* knows both *B* and *C* (2), *B* and *C* both would know at least 1 person— hence it would not be possible to create the person knowing 0 people. The same can be verified with a group of 4 persons i.e., the minute you were to make 1 person know 3 persons it would not be possible for anyone in the group to know 0 persons and hence you would not be able to meet the condition that every person knows a different number of persons. Option (b) is correct.

83. With one green ball there would be six ways of doing this. With 2 green balls 5 ways, with 3 green balls 4 ways, with 4 green balls 3 ways, with 5 green balls 2 ways and with 6 green balls 1 way. So a total of 1 + 2 + 3 + 4 + 5 + 6 = 21 ways. Option (b) is correct.

84. One digit no. = 5; Two digit nos = 5 × 4 = 20; Three digit no = 5 × 4 × 3 = 60; four digit no = 5 × 4 × 3 × 2 = 120; Five digit no.= 5 × 4 × 3 × 2 × 1 = 120 Total number of nos = 325. Hence Option (b) is correct.

85. For each selection there are 3 ways of doing it. Thus, there are a total of 3 × 3 × 3 × 3 × 3 = 243. Hence, Option (b) is correct.

86. Number of even numbers = 6 × 6 × 6 × 3

87. Solve this one through options. If you pick up option (a) it gives you 12 participants in the tournament. This means that there are 10 men and 2 women. In this case there would be $2 \times {}^{10}C_2 = 90$ matches amongst the men and $2 \times {}^{10}C_1 \times {}^{2}C_1 = 40$ matches between 1 man and 1 woman. The difference between number of matches where both participants are men and the number of matches where 1 participant is a man and one is a woman is 90 – 40 = 50 – which is not what is given in the problem.

    With 15 participants Æ 11 men and 2 women.

In this case there would be $2 \times {}^{11}C_2 = 110$ matches amongst the men and $2 \times {}^{11}C_1 \times {}^{2}C_1 = 44$ matches between 1 man and 1 woman. The difference between number of matches where both participants are men and the number of matches where 1 participant is a man and one is a woman is 110 – 44 = 66 – which is the required value as given in the problem. Thus, option (b) is correct.

88. Based on the above thinking we get that since there are 15 players and each player plays each of the others twice, the number of games would be $2 \times {}^{15}C_2 = 2 \times 105 = 210$

Level of Difficulty (II)

1. Number of even numbers = 6 × 6 × 6 × 3
2. We need to think of this as: Number with two sixes or numbers with one six or number with no six.

   0, 1, 2, 3, 4, 5, 6

   Numbers with 2 sixes:

   Numbers ending *n* zero $\quad {}^{5}C_1 \times 3!/2! = 15$

   Numbers Ending in 5 and

   (a) Starting with $\quad 6\,{}^{5}C_1 \times 2! = 10$

   (b) Not starting with $\quad 6\,{}^{4}C_1$ (as zero is not allowed) = 4

   Number with 1 six or no sixes.

   Numbers ending in $\quad 0\,{}^{6}C_3 \times 3! = 120$

   Numbers ending in $\quad 5\,{}^{5}C_1 \times {}^{5}C_2 \times 2! = 100$

   Thus a total of 249 numbers.
3. First arrange 6 pups in 6 places in 6! ways.

   This will leave us with 7 places for 4 cats. Answer = $6! \times 7p_4$.
4. Arrangement of M, A, N, A, E, M, E, N, T is $\dfrac{9!}{2! \times 2! \times 2! \times 2!}$.
5. For nine places we have following number of arrangements.

   9 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2

6. For a rectangle, we need two pair of parallel lines which are perpendicular to each other. We need to select two parallel lines from '*v*' lines and 2 parallel lines from '*w*' lines. Hence required number of parallel lines is ${}^{V}C_2 \times {}^{W}C_2$.

7. From 8 people we have to *arrange* a group of 5 in which three are similar $\frac{8P_5}{3!}$ or $\frac{8C_5 \times 5!}{3!}$.

8. $\frac{4C_4 \times 4C_1 \times 5!}{3!} + \frac{4C_2 \times 4C_3 \times 5!}{3!} - 4C_2 \times 4C_3 \times 2C_2 \times 2!$

9. Since the number of men and women in the question is the same, there is no difference in solving this question and solving the previous one (question number 8) as committees having a maximum of 2 women would mean committees having a minimum of 3 men and committees having at maximum one woman holding the post of either president or vice president would mean at least 1 man holding one of the two posts.

   Thus, the answer would be:

   Number of committees with 4 men and 1 woman (including all arrangements of the committees) + Number of committees with 3 men and 2 women (including all arrangements of the committees)– Number of committees with 3 men and 2 women where both the women are occupying the two posts.

   $= ({}^4C_4 \times {}^4C_1 \times 5!)/3! + ({}^4C_3 \times {}^4C_2 \times 5!)/3! - ({}^4C_3 \times {}^4C_2 \times {}^2C_2 \times 2!)$

   $= 80 + 480 - 48 = 512$

10. ${}^7C_1 \times {}^6C_2 \times 4! \times 4! = 60480$

11. First make the six law students sit in a row. This can be done in 6! Ways. Then, there would be 7 places for the MBA students. We need to select 5 of these 7 places for 5 MBA students and then arrange these 5 students in those 5 places. This can be done in ${}^7C_5 \times 5!$ Ways.

    Thus, the answer is:

    $6! \times {}^7C_5 \times 5! = 7! \times 6!/2!$

12. The required answer will be given by counting the total number of registration numbers starting with DL-5A to DL-5R and the number of registration numbers starting with DL-5S that have to be counted.

13. Out of 100 balls arrange 99 balls (except n_{28}) amongst themselves. Now put n_{28} just before n_{29} in the above arrangement.

14. ${}^6C_2 = 15$.

15. We need to arrange R people on M chairs, S people on another set of M chairs and the remaining people on the remaining chairs. ${}^MP_R \times {}^MP_S \times {}^{2M-R-S}P_{2M-R-S}$.

16. Each group will consists of m things. This can be done in: ${}^{mn}C_m$ ◊ ${}^{mn-m}C_m$ ◊ ${}^{mn-2m}C_{m.......}{}^mC_m$

$$= \frac{mn!}{(mn-m)!m!} \cdot \frac{(mn-m)!}{(mn-2m)!m!} \cdots \frac{m!}{0!m!} = \frac{mn!}{(m!)^n}$$

Divide this by $n!$ since arrangements of the n groups amonst themselves is not required.

Required number of ways = $\dfrac{mn!}{(m!)^n \cdot n!}$

17. Number of ways of selecting 5 different letters = ${}^5C_5 = 1$

Number of ways of selecting 2 similar and 3 different letters = ${}^4C_1 \times {}^4C_3 = 16$

Number of ways of selecting 2 similar letters + 2 more similar letters and 1 different letter = ${}^4C_2 \times {}^3C_1 = 18$

Number of ways of selecting 3 similar letters and 2 different letters = ${}^3C_1 \times {}^4C_2 = 18$

Number of ways of selecting 3 similar letters and another 2 other similar letters = ${}^3C_1 \times {}^3C_1 = 9$

Number of ways of selecting 4 similar letters and 1 different letter = ${}^2C_1 \times {}^4C_1 = 8$

Number of ways of selecting 5 similar letters = ${}^{1}C_1 = 1$

Total number of ways = 1 + 16 + 18 + 18 + 9 + 8 + 1 = 71.

18. Divide $3n + 1$ articles in two groups.

    (i) n identical articles and the remaining

    (ii) $2n + 1$ non-identical articles

    We will select articles in two steps. Some from the first group and the rest from the second group.

Number of articles from first group	*Number of articles from second group*	*Number of ways.*
0	n	$1 \times {}^{2n+1}C_n$
1	$n-1$	$1 \times {}^{2n+1}C_{n-1}$
2	$n-2$	$1 \times {}^{2n+1}C_{n-2}$
3	$n-3$	$1 \times {}^{2n+1}C_{n-3}$
..	..	…….........
$n-1$	1	$1 \times {}^{2n+1}C_1$
n	0	$1 \times {}^{2n+1}C_0$

Total number of ways = ${}^{2n+1}C_n + {}^{2n+1}C_{n-1} + {}^{2n+1}C_{n-2} + \ldots\ {}^{2n+1}C_1 + {}^{2n+1}C_0 = \dfrac{2^{2n+1}}{2} = 2^{2n}$.

19. We have four options for every place including the left most.

    So the total number of numbers = $4 \times 4 \times 4 \times \ldots = 4^{10}$.

    We have to consider only positive numbers, so we don't consider one number in which all ten digits are zeroes.

20. Total number of attempts = 10^4 out of which one is correct.

21. For odd places, the number of arrangements = $\dfrac{4!}{2!2!}$

For even places, the number of arrangements = $\frac{3!}{2!}$

Hence the total number of arrangements = $\frac{4! \times 3!}{2! \times 2! \times 2!}$

22. The number would be of the form 6 ______ 5

The 5 missing digits have to be formed using the digits 0, 1, 2, 3, 4, 7, 8, 9 without repetition.

Thus, ${}^8C_5 \times 5! = 6720$

23. $1m + 3f = {}^8C_1 \times {}^8C_3 = 8 \times 56 = 448$

$2m + 2f = {}^8C_2 \times {}^8C_2 = 28 \times 28 = 784$

$3m + 1f\ {}^8C_3 \times {}^8C_1 = 56 \times 8 = 448$

$4m + 0f = {}^8C_4 \times {}^8C_0 = 70 \times 1 = 70$

Total = 1750

24. Solve this by dividing the solution into,

3 women and 1 man or

2 women and 2 men or

1 woman and 3 men or

0 woman and 4 men.

This will give us:

${}^8C_3 \times {}^8C_1 + {}^8C_2 \times {}^8C_2 + {}^8C_1 \times {}^8C_3 + {}^8C_0 \times {}^8C_4$

$= 448 + 784 + 448 + 70 = 1750$

25. For 1 to 9 we require 9 digits

For 10 to 99 we require 90×2 digits

For 100 to 150 we require 51×3 digits

26. Select any three places for *A*, *B* and *C*. They need no arrangement amongst themselves as A would always come before *B* and *B* would come before *C*.

The remaining 5 people have to be arranged in 5 places.

Thus, ${}^8C_3 \times 5! = 56 \times 120 = 67200$

Or

$\frac{8\,:}{3\,:\times 5\,:}$ × 5! = 8!/3!

27. Total number of choices = 4! out of which only one will be right.
28. At least two letters have to interchange their places for a wrong choice.
29. Select any two letters and interchange them ($^4C_2$).
30. $^{45}C_7$ (refer to solved example 16.14).
31. For one pair of black shoes we require one left black and one right black. Consider the worst case situation:

    $7LB + 5LW + 5RW + 1RB$ or

    $7RB + 5LW + 5RW + 1LB$ = 18 shoes
32. For one pair of correct shoes one of the possible combinations is $7LB + 5LW + 1R$ (B or W) = 13

    Some other cases are also possible with at least 13 shoes.
33. The first rook can be placed in any of the 64 squares and the second rook will then have only 49 places so that they are not attacking each other.
34. When all digits are odd.

    $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

    When all digits are even

    $4 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 4 \times 5^5$

    $5^6 + 4 \times 5^5 = 28125$
35. All six digit numbers – Six digit numbers with only odd digits.

    $= 900000 - 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 884375$.
36. “Total number of all 10-digits numbers – Total number of all 10-digits numbers with no digit repeated” will give the required answer.

    $= 9 \times 10^9 - 9 \times {}^9P_8$
37. There will be two types of triangles

    The first type will have its vertices on the three sides of the D*ABC*.

The second type will have two of it's vertices on the same side and the third vertex on any of the other two sides.

Hence, the required number of triangles

$= 6 \times 5 \times 3 + {}^6C_2 \times 8 + {}^5C_2 \times 9 + {}^3C_2 \times 11$

$= 90 + 120 + 90 + 33$

$= 333$

38. First step – arrange 7 boys around the table according to the circular permutations rule. i.e. in 6! ways.

    Second step – now we have 7 places and have to arrange 7 girls on these places. This can be done in ${}^7P_7$ ways. Hence, the total number of ways = 6! × 7!

39. 2 × 7! × 7! (Note: we do not need to use circular arrangements here because the seats are numbered.)

40. We just need to select the floors and the people who get down at each floor.

    The floors selection can be done in ${}^{11}C_3$ ways.

    The people selection is ${}^9C_4 \times {}^5C_3$.

    Also, the floors need to be arranged using 3!

    Thus, ${}^{11}C_3 \times {}^9C_4 \times {}^5C_3 \times 3!$ or ${}^{11}P_3 \times {}^9C_4 \times {}^5C_3$

41. To arrange a surgeon and an assistant we have ${}^{40}P_2$ ways.

42. To arrange a surgeon and 4 assistants we have or $40 \times {}^{39}C_4$ ways.

43. Give one marble to each of the six children. Then, the remaining 4 identical marbles can be distributed amongst the six children in ${}^{(4+6-1)}C_{(6-1)}$ ways.

44. Since it is possible to give no objects to one or two of them we would have 3 choices for giving each item.

    Thus, 3 × 3 × 3 × 3 × 3 × 3 = 2187.

45. For an even number the units digit should be either 2, 4 or 6. For the other five places we have six digits. Hence, the number of six digit numbers = ${}^6P_5 \times 3 = 2160$.

46. Visualize the number as:

_ _ _ _ _ _ _ _ _

This number has to have the last two digits even. Thus, ${}^3C_2 \times 2!$ will fill the last 2 digits.

For the remaining places : ${}^5C_4 \times 4!$

Thus, we have ${}^5C_4 \times 4! \times {}^3C_2 \times 2! = 720$

47. ${}^9C_5 \times 5! = 15120$

48. ${}^4C_1 \times 7 \times 7 \times 7 = {}^4C_1 \times 7^3$

49. Select the two positions for the two 3's. After that the remaining 5 places have to be filled using either 1 or 2.

Thus, ${}^7C_2 \times 2^5$

50. ${}^4C_1 \times {}^7C_3 \times 3! = 840$

Chapter 18 Probability

CONCEPT AND IMPORTANCE OF PROBABILITY

Probability is one of the most important mathematical concepts that we use/come across in our day-to-day life. Particularly important in business and economic situations, probability is also used by us in our personal lives. For a lot of students who are not in touch with Mathematics after their Xth/XIIth classes, this chapter, along with permutations and combinations, is seen as an indication that XIIth standard Mathematics appear in the MBA entrance exams. This leads to students taking negative approach while tacking/preparing for the Mathematics section. Students are advised to remember that the Math asked in MBA entrance is mainly logical while studying the chapter.

As I set out to explain the basics of this chapter, I intend to improve your concepts of probability to such an extent that you feel in total control of this topic.

For those who are reasonably strong, my advice would be to use this chapter both for revisiting the basic concepts as well as for extensive practice.

Probability means the chance of the occurrence of an event. In layman terms, we can say that it is the likelihood that something—that is defined as the event—will or will not occur. Thus probabilities can be estimated for each of the following events in our personal lives:

(a) the probability that an individual student of B.Com will clear the CAT,

(b) The chance that a candidate chosen at random will clear an interview,

(c) The chance that you will win a game of flush in cards if you have a trio of twos in a game where four people are playing,

(d) The likelihood of India's winning the football World Cup in 2014.

(e) The probability that a bulb will fuse in it's first day of operation, and so on.

The knowledge of these estimations helps individuals decide on the course of action they will take in their day-to-day life. For instance, your estimation/ judgement of the probability of your chances of winning the card game in Event *c* above will influence your decision about the amount of money you will be ready to invest in the stakes for the particular game. The application of probability to personal life helps in improving our decision making.

However, the use of probability is much more varied and has far reaching influence on the world of economics and business. Some instances of these are:

(a) the estimation of the probability of the success of a business project,

(b) the estimation of the probability of the success of an advertising campaign in boosting the profits of a company,

(c) the estimation of the probability of the death of a 25-year old man in the next 10 years and that of the death of a 55-year old man in the next 10 years leading to the calculation of the premiums for life insurance,

(d) the estimation of the probability of the increase in the market price of the share of a company, and so on.

UNDERLYING FACTORS FOR REAL-LIFE ESTIMATION OF PROBABILITY

The factors underlying an event often affect the probability of that event's occurrence. For instance, if we estimate the probability of India winning the

2015 World Cup as 0.14 based on certain expectations of outcomes, then this probability will definitely improve if we know that Sachin Tendulkar will score 800 runs in that particular World Cup.

As we now move towards the mathematical aspects of the chapter, one underlying factor that recurs in every question of probability is that whenever one is asked the question, what is the probability? the immediate question that arises/should arise in one's mind is the probability of what?

The answer to this question is the probability of the EVENT.

The EVENT is the cornerstone or the bottomline of probability. Hence, the first objective while trying to solve any question in probability is to define the event.

The event whose probability is to be found out is described in the question and the task of the student in trying to solve the problem is to define it.

In general, the student can either define the event narrowly or broadly. Narrow definitions of events are the building blocks of any probability problem and whenever there is a doubt about a problem, the student is advised to get into the narrowest form of the event definition.

The *difference* between the narrow and broad definition of event can be explained through an example:

Example: What is the probability of getting a number greater than 2, in a throw of a normal unbiased dice having 6 faces?

The broad definition of the event here is getting a number greater than 2 and this probability is given by 4/6. However, this event can also be broken down into its more basic definitions as:

The event is defined as getting 3 or 4 or 5 or 6. The individual probabilities of each of these are 1/6,1/6,1/6 and 1/6 respectively.

Hence, the required probability is 1/6 + 1/6 + 1/6 + 1/6 = 4/6 = 2/3.

Although in this example it seems highly trivial, the narrow event-definition approach is very effective in solving difficult problems on probability.

In general, event definition means breaking up the event to the most basic building blocks, which have to be connected together through the two English conjunctions—AND and OR.

The Use of the Conjunction AND (Tool No. 9)

Refer *Back to school* section. Whenever we use AND as the natural conjunction joining two separate parts of the event definition, we replace the AND by the multiplication sign.

Thus, if A AND B have to occur, and if the probability of their occurrence are $P(A)$ and $P(B)$ respectively, then the probability that A AND B occur is got by connecting $P(A)$ AND $P(B)$. Replacing the AND by multiplication sign we get the required probability as:

$P(A) \times P(B)$

Example: If we have the probability of A hitting a target as 1/3 and that of B hitting the target as 1/2, then the probability that both hit the target if one shot is taken by both of them is got by

Event Definition: A hits the target AND B hits the target.

Æ $P(A) \times P(B) = 1/3 \times 1/2 = 1/6$

(Note that since we use the conjunction AND in the definition of the event here, we multiply the individual probabilities that are connected through the conjunction AND.)

The Use of the Conjunction OR (Tool No. 10)

Refer Back to school section. Whenever we use OR as the natural conjunction joining two separate parts of the event definition, we replace the OR by the addition sign.

Thus, if A OR B have to occur, and if the probability of their occurrence are $P(A)$ and $P(B)$ respectively, then the probability that A OR B occur is got by connecting

$P(A)$ OR $P(B)$. Replacing the OR by addition sign, we get the required probability as

$P(A) + P(B)$

Example: If we have the probability of A winning a race as 1/3 and that of B winning the race as 1/2, then the probability that either A or B win a race is got by

Event Definition: *A* wins OR *B* wins.

Æ $P(A) + P(B) = 1/3 + 1/2 = 5/6$

(Note that since we use the conjunction OR in the definition of the event here, we add the individual probabilities that are connected through the conjunction OR.)

Combination of AND and OR

If two dice are thrown, what is the chance that the sum of the numbers is not less than 10.

Event Definition: The sum of the numbers is not less than 10 if it is either 10 OR 11 OR 12.

Which can be done by

(6 AND 4) OR (4 AND 6) OR (5 AND 5) OR (6 AND 5) OR (5 AND 6) OR (6 AND 6)

that is, $1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 = 6/36 = 1/6$

The bottomline is that no matter how complicated the problem on probability is, it can be broken up into its narrower parts, which can be connected by ANDs and ORs to get the event definition.

Once the event is defined, the probability of each narrow event within the broad event is calculated and all the narrow events are connected by Multiplication (for AND) or by Addition (for OR) to get the final solution.

Example: In a four game match between Kasporov and Anand, the probability that Anand wins a particular game is 2/5 and that of Kasporov winning a game is 3/5. Assuming that there is no probability of a draw in an individual game, what is the chance that the match is drawn (Score is 2–2).

For the match to be drawn, 2 games have to be won by each of the players. If '*A*' represents the event that Anand won a game and *K* represents the event that Kasporov won a game, the event definition for the match to end in a draw can be described as: [The student is advised to look at the use of narrow event definition.]

A&K&K) OR (A&K&A&K) OR (A&K&K&A)

OR (K&K&A&A) OR (K&A&K&A) OR (K&A&A&K)

This further translates into

$(2/5)^2(3/5)^2 + (2/5)^2(3/5)^2 + (2/5)^2(3/5)^2 + (2/5)^2(3/5)^2$

$+ (2/5)^2(3/5)^2 + (2/5)^2(3/5)^2$

$= (36/625) \times 6 = 216/625$

After a little bit of practice, you can also think about this directly as:

${}^4C_2 \times (2/5)^2 \times {}^2C_2 \times (3/5)^2 = 6 \times 1 \times 36/625 = 216/625$

Where, $4C_2$ gives us the number of ways in which Anand can win two games and $2C_2$ gives us the number of ways in which Kasporov can win the remaining 2 games (obviously, only one).

BASIC FACTS ABOUT PROBABILITY

For every event that can be defined, there is a corresponding non-event, which is the opposite of the event. The relationship between the event and the non-event is that they are mutually exclusive, that is, if the event occurs then the non-event does not occur and vice versa.

The event is denoted by E; the number of ways in which the event can occur is defined as $n(E)$ and the probability of the occurrence of the event is $P(E)$.

The non-event is denoted by E¢; the number of ways in which the non-event can occur is defined as $n(E$¢$)$ while the probability of the occurrence of the event is $P(E$¢$)$.

The following relationships hold true with respect to the event and the non-event.

$n(E) + n(E$¢$)$ = sample space representing all the possible events that can occur related to the activity.

$P(E) + P(E$¢$) = 1$

This means that if the event does not occur, then the non-event occurs.

Æ $P(E) = 1 – P(E$¢$)$

This is often very useful for the calculation of probabilities of events where it is easier to describe and count the non-event rather than the event.

Illustration

The probability that you get a total more than 3 in a throw of 2 dice.

Here, the event definition will be a long and tedious task, which will involve long counting. Hence, it would be more convenient to define the non-event and count the same.

Therefore, here the non-event will be defined as

A total not more than 3 Æ 2 or 3 Æ (1&1) OR (1&2) OR (2&1) = 1/36 + 1/36 + 1/36 = 3/36 = 1/12.

However, a word of caution especially for students not comfortable at mathematics: Take care while defining the non-event. Beware of a trap like Æ event definition: Total > 10 in two throws of a dice does not translate into a non-event of < 10 but instead into the non-event of $\leq$ 10.

SOME IMPORTANT CONSIDERATIONS WHILE DEFINING EVENT

Random Experiment An experiment whose outcome has to be among a set of events that are completely known but whose exact outcome is unknown is a random experiment (e.g. Throwing of a dice, tossing of a coin). Most questions on probability are based on random experiments.

Sample Space This is defined in the context of a random experiment and denotes the set representing all the possible outcomes of the random experiment. [e.g. Sample space when a coin is tossed is (Head, Tail). Sample space when a dice is thrown is (1, 2, 3, 4, 5, 6).]

Event The set representing the desired outcome of a random experiment is called the event. Note that the event is a subset of the sample space.

Non-event The outcome that is opposite the desired outcome is the non-event. Note that if the event occurs, the non-event does not occur and vice versa.

Impossible Event An event that can never occur is an impossible event. The probability of an impossible event is 0. e.g. (Probability of the occurrence of 7 when a dice with 6 faces numbered 1–6 is thrown).

Mutually Exclusive Events A set of events is mutually exclusive when the occurrence of any one of them means that the other events cannot occur. (If head appears on a coin, tail will not appear and vice versa.)

Equally Likely Events If two events have the same probability or chance of occurrence they are called equally likely events. (In a throw of a dice, the chance of 1 showing on the dice is equal to 2 is equal to 3 is equal to 4 is equal to 5 is equal to 6 appearing on the dice.)

Exhaustive Set of Events A set of events that includes all the possibilities of the sample space is said to be an exhaustive set of events. (e.g. In a throw of a dice the number is less than three or more than or equal to three.)

Independent Events An event is described as such if the occurrence of an event has no effect on the probability of the occurrence of another event. (If the first child of a couple is a boy, there is no effect on the chances of the second child being a boy.)

Conditional Probability It is the probability of the occurrence of an event A given that the event B has already occurred. This is denoted by $P(A|B)$. (E.g. The probability that in two throws of a dices we get a total of 7 or more, given that in the first throw of the dices the number 5 had occurred.)

The Concept of Odds For and Odds Against

Sometimes, probability is also viewed in terms of *odds* for and *odds against* an event.

Odds in favour of an event E is defined as: $\dfrac{P(E)}{P(E)'}$

Odds against an event is defined as: $\dfrac{P(E)'}{P(E)}$

Expectation: The expectation of an individual is defined as

Probability of winning × Reward of winning

Illustration: A man holds 20 out of the 500 tickets to a lottery. If the reward for the winning ticket is ` 1000, find the expectation of the man.

Solution: Expectation = Probability of winning × Reward of winning = $\frac{20}{500}$ × 1000 = ` 40.

ANOTHER APPROACH TO LOOK AT THE PROBABILITY PROBLEMS

The probability of an event is defined as

$$\frac{\text{Number of ways in which the event occurs}}{\text{Total number of outcomes possible}}$$

This means that the probability of any event can be got by counting the numerator and the denominator independently.

Hence, from this approach, the concentration shifts to counting the numerator and the denominator.

Thus for the example used above, the probability of a number > 2 appearing on a dice is:

$$\frac{\text{Number of ways in which the event occurs}}{\text{Total number of outcomes possible}} = \frac{4}{6}$$

The counting is done through any of

(a) The physical counting as illustrated above,

(b) The use of the concept of permutations,

(c) The use of the concept of combinations,

(d) The use of the MNP rule.

[*Refer to the chapter on Permutations and Combinations to understand b, c and d above*.]

WORKED-OUT PROBLEMS

Problem 18.1 In a throw of two dice, find the probability of getting one prime and one composite number.

Solution The probability of getting a prime number when a dices is thrown is 3/6 = 1/2. (This occurs when we get 2, 3 or 5 out of a possibility of getting 1, 2, 3, 4, 5 or 6.)

Similarly, in a throw of a dice, there are only 2 possibilities of getting composite numbers viz : 4 or 6 and this gives a probability of 1/3 for getting a composite number.

Now, let us look at defining the event. The event is—getting one prime and one composite number.

This can be got as:

The first number is prime and the second is composite OR the first number is composite and the second is prime.

$= (1/2) \times (1/3) + (1/3) \times (1/2) = 1/3$

Problem 18.2 Find the probability that a leap year chosen at random will have 53 Sundays.

Solution A leap year has 366 days. 52 complete weeks will have 364 days. The 365th day can be a Sunday (Probability = 1/7) OR the 366th day can be a Sunday (Probability = 1/7). Answer = 1/7 + 1/7 = 2/7.

Alternatively, you can think of this as: The favourable events will occur when we have Saturday and Sunday or Sunday and Monday as the 365th and 366th days respectively. (i.e. 2 possibilities of the event occurring). Besides, the total number of ways that can happen are Sunday and Monday

OR Monday and Tuesday … OR Friday and Saturday OR Saturday and Sunday.

Problem 18.3 There are two bags containing white and black balls. In the first bag, there are 8 white and 6 black balls and in the second bag, there are 4 white and 7 black balls. One ball is drawn at random from any of these two bags. Find the probability of this ball being black.

Solution The event definition here is: 1st bag and black ball OR 2nd Bag and Black Ball. The chances of picking up either the 1st OR the 2nd Bag are 1/2 each.

Besides, the chance of picking up a black ball from the first bag is 6/14 and the chance of picking up a black ball from the second bag is 7/11.

Thus, using these values and the ANDs and ORs we get:

$(1/2) \times (6/14) + (1/2) \times (7/11) = (3/14) + (7/22) = (66 + 98)/(308) = 164/308 = 41/77$

Problem 18.4 The letters of the word LUCKNOW are arranged among themselves. Find the probability of always having NOW in the word.

Solution The required probability will be given by the equation

= No. of words having NOW/Total no. of words

= 5!/7! = 1/42 [See the chapter of Permutations and

Combinations to understand the logic behind these values.]

Problem 18.5 A person has 3 children with at least one boy. Find the probability of having at least 2 boys among the children.

Solution The event is occurring under the following situations:

(a) Second is a boy and third is a girl OR

(b) Second is a girl and third is a boy OR

(c) Second is a boy and third is a boy

This will be represented by: $(1/2) \times (1/2) + (1/2) \times (1/2) + (1/2) \times (1/2) = 3/4$

Problem 18.6 Out of 13 applicants for a job, there are 5 women and 8 men. Two persons are to be selected for the job. The probability that at least one of the selected persons will be a woman is:

Solution The required probability will be given by

First is a woman and Second is a man OR

First is a man and Second is a woman OR

First is a woman and Second is a woman

i.e. $(5/13) \times (8/12) + (8/13) \times (5/12) + (5/13) \times (4/12) = 100/156 = 25/39$

Alternatively, we can define the non-event as: There are two men and no women. Then, probability of the non-event is

$(8/13) \times (7/12) = 56/156$

Hence, $P(E) = (1 - 56/156) = 100/156 = 25/39$

[***Note:*** This is a case of probability calculation where repetition is not allowed.]

Problem 18.7 The probability that A can solve the problem is 2/3 and B can solve it is 3/4. If both of them attempt the problem, then what is the probability that the problem gets solved.

Solution The event is defined as:

A solves the problem AND B does not solve the problem

OR

A doesn't solve the problem AND B solves the problem

OR

A solves the problem AND B solves the problem.

Numerically, this is equivalent to:

$(2/3) \times (1/4) + (1/3) \times (3/4) + (2/3) \times (3/4)$

$= (2/12) + (3/12) + (6/12) = 11/12$

Problem 18.8 Six positive numbers are taken at random and are multiplied together. Then what is the probability that the product ends in an odd digit other than 5.

Solution The event will occurs when all the numbers selected are ending in 1, 3, 7 or 9.

If we take numbers between 1 to 10 (both inclusive), we will have a positive occurrence if each of the six numbers selected are either 1, 3, 7 or 9.

The probability of any number selected being either of these 4 is 4/10 (4 positive events out of 10 possibilities)

[***Note:*** If we try to take numbers between 1 to 20, we will have a probability of 8/20 = 4/10. Hence, we can extrapolate up to infinity and say that the probability of any number selected ending in 1, 3, 7 or 9 so as to fulfill the requirement is 4/10.]

Hence, answer = $(0.4)^6$

Problem 18.9 The probability that Arjit will solve a problem is 1/5. What is the probability that he solves at least one problem out of ten problems?

Solution The non-event is defined as:

He solves no problems i.e. he doesn't solve the first problem and he doesn't solve the second problem … and he doesn't solve the tenth problem.

Probability of non-event = $(4/5)^{10}$

Hence, probability of the event is $1-(4/5)^{10}$

Problem 18.10 A carton contains 25 bulbs, 8 of which are defective. What is the probability that if a sample of 4 bulbs is chosen, exactly 2 of them will be defective?

Solution The probability that exactly two balls are defective and exactly two are not defective will be given by $(4C_2) \times (8/25) \times (7/24) \times (17/23) \times (16/22)$

Problem 18.11 Out of 40 consecutive integers, two are chosen at random. Find the probability that their sum is odd.

Solution Forty consecutive integers will have 20 odd and 20 even integers. The sum of 2 chosen integers will be odd, only if

(a) First is even and Second is odd OR

(b) First is odd and Second is even

Mathematically, the probability will be given by:

P(First is even) × P(Second is odd) + P(First is odd) × P(second is even)

= (20/40) × (20/39) + (20/40) × (20/39)

= $(2 \times 20^2/40 \times 39)$ = 20/39

Problem 18.12 An integer is chosen at random from the first 100 integers. What is the probability that this number will not be divisible by 5 or 8?

Solution For a number from 1 to 100 not be divisible by 5 or 8, we need to remove all the numbers that are divisible by 5 or 8.

Thus, we remove 5, 8, 10, 15, 16, 20, 24, 25, 30, 32, 35, 40, 45, 48, 50, 55, 56, 60, 64, 65, 70, 72, 75, 80, 85, 88, 90, 95, 96, and 100.

i.e. 30 numbers from the 100 are removed.

Hence, answer is 70/100 = 7/10 (required probability)

Alternatively, we could have counted the numbers as number of numbers divisible by 5 + number of numbers divisible by 8 – number of numbers divisible by both 5 or 8.

= 20 + 12 – 2 = 30

Problem 18.13 From a bag containing 8 green and 5 red balls, three are drawn one after the other. Find the probability of all three balls being green if

(a) the balls drawn are replaced before the next ball is picked

(b) the balls drawn are not replaced.

Solution

(a) When the balls drawn are replaced, we can see that the number of balls available for drawing out will be the same for every draw. This means that the probability of a green ball appearing in the first draw and a green ball appearing in the second draw as well as one appearing in the third draw are equal to each other.

Hence answer to the question above will be:

Required probability = $\frac{8}{13} \times \frac{8}{13} \times \frac{8}{13}$ = $(8^3/13^3)$

(b) When the balls are not replaced, the probability of drawing any color of ball for every fresh draw changes. Hence, the answer here will be:

Required probability = $\frac{8}{13} \times \frac{7}{12} \times \frac{6}{11}$

LEVEL OF DIFFICULTY (I)

1. In throwing a fair dice, what is the probability of getting the number ‘3’?

   (a) $\frac{1}{3}$ (b) $\frac{1}{6}$

   (c) $\frac{1}{9}$ (d) $\frac{1}{12}$

2. What is the chance of throwing a number greater than 4 with an ordinary dice whose faces are numbered from 1 to 6?

   (a) $\frac{1}{3}$ (b) $\frac{1}{6}$

   (c) $\frac{1}{9}$ (d) $\frac{1}{8}$

3. Find the chance of throwing at least one ace in a simple throw with two dice.

   (a) $\frac{1}{12}$ (b) $\frac{1}{3}$

   (c) $\frac{1}{4}$ (d) $\frac{11}{36}$

4. Find the chance of drawing 2 blue balls in succession from a bag containing 5 red and 7 blue balls, if the balls are not being replaced.

   (a) $\frac{3}{13}$ (b) $\frac{21}{64}$

   (c) $\frac{7}{22}$ (d) $\frac{21}{61}$

5. From a pack of 52 cards, two are drawn at random. Find the chance that one is a knave and the other a queen.

(a) $\frac{8}{663}$ (b) $\frac{1}{6}$

(c) $\frac{1}{9}$ (d) $\frac{1}{12}$

6. If a card is picked up at random from a pack of 52 cards. Find the probability that it is

(i) a spade.

(a) $\frac{1}{9}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{4}$

(ii) a king or queen.

(a) $\frac{3}{13}$ (b) $\frac{2}{13}$

(c) $\frac{7}{52}$ (d) $\frac{1}{169}$

(iii) 'a spade' or 'a king' or 'a queen'

(a) $\frac{21}{52}$ (b) $\frac{5}{13}$

(c) $\frac{19}{52}$ (d) $\frac{15}{52}$

7. Three coins are tossed. What is the probability of getting

(i) 2 Tails and 1 Head

(a) $\frac{1}{4}$ (b) $\frac{3}{8}$

(c) $\frac{2}{3}$ (d) $\frac{1}{8}$

(ii) 1 Tail and 2 Heads

(a) $\frac{3}{8}$ (b) 1

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

8. Three coins are tossed. What is the probability of getting

(i) neither 3 Heads nor 3 Tails?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

(ii) three heads

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

9. For the above question, the probability that there is at least one tail is:

(a) $\frac{2}{3}$ (b) $\frac{7}{8}$

(c) $\frac{3}{8}$ (d) $\frac{1}{2}$

10. Two fair dice are thrown. Find the probability of getting

(i) a number divisible by 2 or 4.

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(ii) a number divisible by 2 and 4.

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{5}{7}$

(iii) a prime number less than 8.

(a) $\frac{11}{13}$ (b) $\frac{1}{13}$

(c) $\frac{1}{4}$ (d) $\frac{13}{36}$

11. A bag contains 3 green and 7 white balls. Two balls are drawn from the bag in succession without replacement. What is the probability that

(i) both are white?

(a) $\frac{1}{7}$ (b) $\frac{5}{11}$

(c) $\frac{7}{11}$ (d) $\frac{7}{15}$

(ii) they are of different colour?

(a) $\frac{7}{15}$ (b) $\frac{7}{9}$

(c) $\frac{5}{11}$ (d) $\frac{7}{11}$

12. 100 students appeared for two examinations. 60 passed the first, 50 passed the second and 30 passed both. Find the probability that a student selected at random has failed in both the examinations?

(a) $\frac{1}{5}$ (b) $\frac{1}{7}$

(c) $\frac{5}{7}$ (d) $\frac{5}{6}$

13. What is the probability of throwing a number greater than 2 with a fair dice?

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$

(c) 1 (d) $\frac{3}{5}$

14. Three cards numbered 2, 4 and 8 are put into a box. If a card is drawn at random, what is the probability that the card drawn is

(i) a prime number?

(a) 1 (b) $\frac{1}{3}$

(c) $\frac{4}{5}$ (d) $\frac{5}{7}$

(ii) an even number?

(a) 1 (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{5}$

(iii) an odd number?

(a) 1 (b) 0

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

15. Two fair coins are tossed. Find the probability of obtaining

(i) 2 Heads

(a) 1 (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

(ii) 1 Head and 1 Tail

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(iii) 2 Tails

(a) 1 (b) $\frac{1}{4}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

16. In rolling two dices, find the probability that

(i) there is at least one '6'

(a) $\frac{11}{36}$ (b) $\frac{22}{36}$

(c) $\frac{15}{36}$ (d) $\frac{29}{36}$

(ii) the sum is 5

(a) $\frac{1}{4}$ (b) $\frac{1}{9}$

(c) $\frac{1}{2}$ (d) $\frac{1}{6}$

17. From a bag containing 4 white and 5 black balls a man draws 3 at random. What are the odds against these being all black?

(a) $\frac{5}{37}$ (b) $\frac{37}{5}$

(c) $\frac{11}{13}$ (d) $\frac{13}{37}$

18. Amit throws three dice in a special game of Ludo. If it is known that he needs 15 or higher in this throw to win then find the chance of his winning the game.

(a) $\frac{5}{54}$ (b) $\frac{17}{216}$

(c) $\frac{13}{216}$ (d) $\frac{15}{216}$

19. Find out the probability of forming 187 or 215 with the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 when only numbers of three digits are formed and when

(i) repetitions are not allowed

(a) $\frac{12}{504}$ (b) $\frac{18}{504}$

(c) $\frac{2}{504}$ (d) $\frac{24}{504}$

(ii) repetitions are allowed

(a) $\frac{2}{729}$ (b) $\frac{6}{729}$

(c) $\frac{11}{729}$ (d) $\frac{4}{729}$

20. In a horse race there were 18 horses numbered 1–18. The probability that horse 1 would win is 1/6, that 2 would win is 1/10 and that 3 would win is 1/8. Assuming that a tie is impossible, find the chance that one of the three will win.

(a) $\frac{47}{120}$ (b) $\frac{119}{120}$

(c) $\frac{11}{129}$ (d) $\frac{1}{5}$

21. Two balls are to be drawn from a bag containing 8 grey and 3 blue balls. Find the chance that they will both be blue.

(a) $\frac{1}{5}$ (b) $\frac{3}{55}$

(c) $\frac{11}{15}$ (d) $\frac{14}{45}$

22. Two fair dice are thrown. What is the probability of

(i) throwing a double?

(a) $\frac{1}{6}$ (b) 1

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

(ii) the sum is greater than 10

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$

(c) $\frac{1}{6}$ (d) $\frac{1}{12}$

(iii) the sum is less than 10?

(a) $\frac{5}{6}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

23. In a certain lottery the prize is ` 1 crore and 5000 tickets have been sold. What is the expectation of a man who holds 10 tickets?

(a) ` 20,000 (b) ` 25,000

(c) ` 30,000 (d) ` 15,000

24. Two letters are randomly chosen from the word LIME. Find the probability that the letters are *L* and *M*.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{6}$

Directions for Questions 25 to 27: Read the following passage and answer the questions based on it.

The Bangalore office of Infosys has 1200 executives. Of these, 880 subscribe to the *Time* magazine and 650 subscribe to the *Economist*. Each executive may subscribe to either the *Time* or the *Economist* or both. If an executive is picked at random, answer questions 25–27.

25. What is the probability that

(i) he has subscribed to the *Time* magazine.

(a) $\frac{11}{15}$ (b) $\frac{11}{12}$

(c) $\frac{7}{15}$ (d) $\frac{7}{11}$

(ii) he has subscribed to the *Economist*.

(a) $\frac{13}{21}$ (b) $\frac{13}{20}$

(c) $\frac{13}{24}$ (d) $\frac{12}{30}$

26. He has subscribed to both magazines.

(a) $\frac{22}{40}$ (b) $\frac{11}{40}$

(c) $\frac{12}{20}$ (d) $\frac{4}{20}$

27. If among the executives who have subscribed to the *Time* magazine, an executive is picked at random. What is the probability that he has also subscribed to the *Economist*?

(a) $\frac{3}{8}$ (b) $\frac{5}{8}$

(c) $\frac{2}{3}$ (d) $\frac{1}{8}$

28. A bag contains four black and five red balls. If three balls from the bag are chosen at random, what is the chance that they are all black?

(a) $\frac{1}{21}$ (b) $\frac{1}{20}$

(c) $\frac{2}{23}$ (d) $\frac{1}{9}$

29. If a number of two digits is formed with the digits 2, 3, 5, 7, 9 without repetition of digits, what is the probability that the number formed is 35?

(a) $\frac{1}{10}$ (b) $\frac{1}{20}$

(c) $\frac{2}{11}$ (d) $\frac{1}{11}$

30. From a pack of 52 playing cards, three cards are drawn at random. Find the probability of drawing a king, a queen and jack.

(a) $\frac{6}{5525}$ (b) $\frac{1}{13^3}$

(c) $\frac{1}{14^3}$ (d) $\frac{1}{15^3}$

31. A bag contains 20 balls marked 1 to 20. One ball is drawn at random. Find the probability that it is marked with a number multiple of 5 or 7.

(a) $\frac{3}{10}$ (b) $\frac{7}{10}$

(c) $\frac{1}{11}$ (d) $\frac{2}{3}$

32. A group of investigators took a fair sample of 1972 children from the general population and found that there are 1000 boys and 972 girls. If the investigators claim that their research is so accurate that the sex of a new born child can be predicted based on the ratio of the sample of the population, then what is the expectation in terms of the probability that a new child born will be a girl?

(a) $\frac{243}{250}$ (b) $\frac{250}{257}$

(c) $\frac{9}{10}$ (d) $\frac{243}{493}$

33. A bag contains 3 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that both are black?

(a) $\frac{1}{8}$ (b) $\frac{7}{40}$

(c) $\frac{12}{40}$ (d) $\frac{13}{40}$

34. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that

(i) all the three balls are of the same colour.

(a) $\frac{17}{240}$ (b) $\frac{5}{51}$

(c) $\frac{31}{204}$ (d) None of these

(ii) all the three balls are blue.

(a) $\frac{8}{51}$ (b) $\frac{50}{51}$

(c) $\frac{7}{102}$ (d) $\frac{13}{51}$

35. If *P*(*A*) = 1/3, *P*(*B*) = 1/2, *P*(*A* « *B*) = 1/4 then find *P*(*A*¢ » *B*¢)

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

36. *A* and *B* are two candidates seeking admission to the IIMs. The probability that *A* is selected is 0.5 and the probability that both *A* and *B* are selected is at most 0.3. Is it possible that the probability of *B* getting selected is 0.9.

(a) No (b) Yes

(c) Either (a) or (b) (d) Can’t say

37. The probability that a student will pass in Mathematics is 3/5 and the probability that he will pass in English is 1/3. If the probability that he will pass in both Mathematics and English is 1/8, what is the probability that he will pass in at least one subject?

(a) $\frac{97}{120}$ (b) $\frac{87}{120}$

(c) $\frac{53}{120}$ (d) $\frac{120}{297}$

38. The odds in favour of standing first of three students Amit, Vikas and Vivek appearing at an examination are 1 : 2. 2 : 5 and 1 : 7 respectively. What is the probability that either of them will stand first (assume that a tie for the first place is not possible).

(a) $\frac{168}{178}$ (b) $\frac{122}{168}$

(c) $\frac{5}{168}$ (d) $\frac{125}{168}$

39. *A*, *B*, *C* are three mutually exclusive and exhaustive events associated with a random experiment. Find $P(A)$ if it is given that $P(B) = 3/2\ P(A)$ and $P(C) = 1/2\ P(B)$.

(a) $\frac{4}{13}$ (b) $\frac{2}{3}$

(c) $\frac{12}{13}$ (d) $\frac{1}{13}$

40. *A* and *B* are two mutually exclusive events of an experiment. If *P*(*A*¢) = 0.65, *P*(*A* » *B*) = 0.65 and $P(B) = p$, find the value of p.

(a) 0.25 (b) 0.3

(c) 0.1 (d) 0.2

41. A bag contains 4 white and 2 black balls. Another contains 3 white and 5 black balls. If one ball is drawn from each bag, find the

probability that

(i) both are white.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

(ii) both are black.

(a) $\frac{3}{24}$ (b) $\frac{1}{24}$

(c) $\frac{3}{12}$ (d) $\frac{5}{24}$

(iii) one is white and one is black.

(a) $\frac{13}{24}$ (b) $\frac{15}{24}$

(c) $\frac{11}{21}$ (d) $\frac{1}{2}$

42. The odds against an event is 5 : 3 and the odds in favour of another independent event is 7 : 5. Find the probability that at least one of the two events will occur.

(a) $\frac{52}{96}$ (b) $\frac{69}{96}$

(c) $\frac{71}{96}$ (d) $\frac{13}{96}$

43. Kamal and Monica appeared for an interview for two vacancies. The probability of Kamal's selection is 1/3 and that of Monica's selection is 1/5. Find the probability that only one of them will be selected.

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{5}{9}$ (d) $\frac{2}{3}$

44. A husband and a wife appear in an interview for two vacancies for the same post. The probability of husband's selection is (1/7) and that of the wife's selection is 1/5. What is the probability that

(i) both of them will be selected?

(a) $\frac{1}{35}$ (b) $\frac{2}{35}$

(c) $\frac{3}{35}$ (d) $\frac{1}{7}$

(ii) one of them will be selected?

(a) $\frac{1}{7}$ (b) $\frac{3}{7}$

(c) $\frac{2}{7}$ (d) $\frac{5}{7}$

(iii) none of them will be selected?

(a) $\frac{24}{35}$ (b) $\frac{20}{35}$

(c) $\frac{21}{35}$ (d) $\frac{2}{7}$

(iv) at least one of them will be selected?

(a) $\frac{12}{35}$ (b) $\frac{11}{35}$

(c) $\frac{16}{35}$ (d) $\frac{1}{5}$

LEVEL OF DIFFICULTY (II)

1. Two fair dices are thrown. Given that the sum of the dice is less than or equal to 4, find the probability that only one dice shows two.

   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$

   (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

2. *A* can hit the target 3 times in 6 shots, *B* 2 times in 6 shots and *C* 4 times in 6 shots. They fire a volley. What is the probability that at least 2 shots hit?

   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$

   (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

3. There are two bags, one of them contains 5 red and 7 white balls and the other 3 red and 12 white balls, and a ball is to be drawn from one or the other of the two bags. Find the chance of drawing a red ball.

   (a) $\frac{37}{120}$ (b) $\frac{30}{120}$

   (c) $\frac{11}{120}$ (d) None of these

4. In two bags there are to be put altogether 5 red and 12 white balls, neither bag being empty. How must the balls be divided so as to give a person who draws one ball from either bag

   (i) the least chance of drawing a red ball?

(a) $\frac{3}{35}$ (b) $\frac{5}{32}$

(c) $\frac{7}{32}$ (d) $\frac{1}{16}$

(ii) the greatest chance of drawing a red ball?

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$

(c) $\frac{5}{8}$ (d) $\frac{5}{7}$

5. If 8 coins are tossed, what is the chance that one and only one will turn up Head?

(a) $\frac{1}{16}$ (b) $\frac{3}{35}$

(c) $\frac{3}{32}$ (d) $\frac{1}{32}$

6. What is the chance that a leap year, selected at random, will contain 53 Sundays?

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$

(c) $\frac{1}{7}$ (d) $\frac{5}{7}$

7. Out of all the 2-digit integers between 1 to 200, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7?

(a) $\frac{11}{90}$ (b) $\frac{33}{90}$

(c) $\frac{55}{90}$ (d) $\frac{77}{90}$

8. A child is asked to pick up 2 balloons from a box containing 10 blue and 15 red balloons. What is the probability of the child picking, at random, 2 balloons of different colours?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{3}{5}$

9. Tom and Dick are running in the same race; the probability of their winning are 1/5 and 1/2 respectively. Find the probability that

(i) either of them will win the race.

(a) $\frac{7}{10}$ (b) $\frac{3}{10}$

(c) $\frac{1}{5}$ (d) $\frac{7}{9}$

(ii) neither of them will win the race.

(a) $\frac{7}{10}$ (b) $\frac{3}{10}$

(c) $\frac{2}{5}$ (d) $\frac{4}{5}$

10. Two dice are thrown. If the total on the faces of the two dices are 6, find the probability that there are two odd numbers on the faces?

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{5}{9}$ (d) $\frac{3}{5}$

11. Amarnath appears in an exam that has 4 subjects. The chance he passes an individual subject's test is 0.8. What is the probability that he will

(i) pass in all the subjects?

(a) 0.8^4 (b) 0.3^4

(c) 0.7^3 (d) None of these

(ii) fail in all the subjects?

(a) 0.4^2 (b) 0.2^4

(c) 0.3^4 (d) None of these

(iii) pass in at least one of the subjects?

(a) 0.99984 (b) 0.9984

(c) 0.0016 (d) None of these

12. A box contains 2 tennis, 3 cricket and 4 squash balls. Three balls are drawn in succession with replacement. Find the probability that

(i) all are cricket balls.

(a) $\frac{1}{27}$ (b) $\frac{2}{27}$

(c) $\frac{25}{27}$ (d) $\frac{1}{8}$

(ii) the first is a tennis ball, the second is a cricket ball, the third is a squash ball.

(a) $\frac{8}{243}$ (b) $\frac{5}{243}$

(c) $\frac{4}{243}$ (d) $\frac{11}{243}$

(iii) all three are of the same type.

(a) $\frac{11}{81}$ (b) $\frac{1}{9}$

(c) $\frac{13}{81}$ (d) $\frac{17}{81}$

13. With the data in the above question, answer the questions when the balls are drawn in succession without replacement.

(i)

(a) $\frac{3}{84}$ (b) $\frac{1}{84}$

(c) $\frac{5}{84}$ (d) None of these

(ii)

(a) $\frac{2}{21}$ (b) $\frac{4}{21}$

(c) $\frac{1}{21}$ (d) $\frac{1}{9}$

(iii)

(a) $\frac{3}{84}$ (b) $\frac{1}{84}$

(c) $\frac{5}{84}$ (d) $\frac{11}{84}$

14. In the Mindworkzz library, there are 8 books by Stephen Covey and 1 book by Vinay Singh in shelf *A*. At the same time, there are 5 books by Stephen Covey in shelf *B*. One book is moved from shelf *A* to shelf *B*. *A* student picks up a book from shelf *B*. Find the probability that the book by Vinay Singh.

(i) is still in shelf *A*.

(a) $\frac{1}{3}$

(b) $\frac{8}{9}$

(c) $\frac{3}{4}$

(d) None of these

(ii) is in shelf *B*.

(a) $\frac{3}{54}$

(b) $\frac{4}{54}$

(c) $\frac{5}{54}$

(d) None of these

(iii) is taken by the student.

(a) $\frac{3}{54}$

(b) $\frac{1}{54}$

(c) $\frac{2}{27}$

(d) None of these

15. The ratio of number of officers and ladies in the Scorpion Squadron and in the Gunners Squadron are 3 : 1 and 2 : 5 respectively. An individual is selected to be the chairperson of their association. The chance that this individual is selected from the Scorpions is 2/3. Find the probability that the chairperson will be an officer.

(a) $\frac{25}{42}$

(b) $\frac{13}{43}$

(c) $\frac{11}{43}$

(d) $\frac{7}{42}$

16. A batch of 50 transistors contains 3 defective ones. Two transistors are selected at random from the batch and put into a radio set. What is the probability that

(i) both the transistors selected are defective?

(a) $\frac{4}{1225}$

(b) $\frac{3}{1225}$

(c) $\frac{124}{1224}$

(d) None of these

(ii) only one is defective?

(a) $\frac{141}{1225}$

(b) $\frac{121}{1225}$

(c) $\frac{123}{1224}$

(d) None of these

(iii) neither is defective?

(a) $\frac{1082}{1224}$

(b) $\frac{1081}{1225}$

(c) $\frac{1081}{1224}$

(d) None of these

17. The probability that a man will be alive in 35 years is $\frac{3}{5}$ and the probability that his wife will be alive is $\frac{3}{7}$. Find the probability that after 35 years.

(i) both will be alive.

(a) $\frac{2}{35}$

(b) $\frac{9}{35}$

(c) $\frac{6}{35}$

(d) $\frac{3}{35}$

(ii) only the man will be alive.

(a) $\frac{12}{35}$

(b) $\frac{11}{35}$

(c) $\frac{13}{35}$ (d) $\frac{8}{35}$

(iii) only the wife will be alive.

(a) $\frac{2}{35}$ (b) $\frac{3}{35}$

(c) $\frac{6}{35}$ (d) $\frac{11}{35}$

(iv) at least one will be alive.

(a) $\frac{27}{35}$ (b) $\frac{12}{35}$

(c) $\frac{11}{35}$ (d) $\frac{7}{35}$

18. *A* speaks the truth 3 out of 4 times, and *B* 5 out of 6 times. What is the probability that they will contradict each other in stating the same fact?

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{5}{6}$ (d) None of these

19. A party of *n* persons sit at a round table. Find the odds against two specified persons sitting next to each other.

(a) $\frac{n+1}{2}$ (b) $\frac{n-3}{2}$

(c) $\frac{n+3}{2}$ (d) None of these

20. If 4 whole numbers are taken at random and multiplied together, what is the chance that the last digit in the product is 1, 3, 7 or 9?

(a) $\frac{15}{653}$ (b) $\frac{12}{542}$

(c) $\frac{16}{625}$ (d) $\frac{17}{625}$

21. In four throws with a pair of dices what is the chance of throwing a double twice?

(a) $\frac{11}{216}$ (b) $\frac{25}{216}$

(c) $\frac{35}{126}$ (d) $\frac{41}{216}$

22. A life insurance company insured 25,000 young boys, 14,000 young girls and 16,000 young adults. The probability of death within 10 years of a young boy, young girl and a young adult are 0.02, 0.03 and 0.15 respectively. One of the insured persons dice. What is the probability that the dead person is a young boy?

(a) $\frac{36}{165}$ (b) $\frac{25}{166}$

(c) $\frac{26}{165}$ (d) $\frac{32}{165}$

23. Three groups of children contain 3 girls and 1 boy, 2 girls and 2 boys, and 1 girl and 2 boys respectively. One child is selected at random from each group. The probability that the three selected consist of 1 girl and 2 boys is

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$

(c) $\frac{5}{8}$ (d) $\frac{3}{5}$

24. A locker at the world famous WTC building can be opened by dialing a fixed three-digit code (between 000 and 999). Don, a

terrorist, only knows that the number is a three-digit number and has only one six. Using this information he tries to open the locker by dialing three digits at random. The probability that he succeeds in his endeavour is

(a) $\frac{1}{243}$ (b) $\frac{1}{900}$

(c) $\frac{1}{1000}$ (d) $\frac{3}{216}$

25. In a bag there are 12 black and 6 white balls. Two balls are chosen at random and the first one is found to be black. The probability that the second one is also black is:

(a) $\frac{11}{17}$ (b) $\frac{12}{17}$

(c) $\frac{13}{18}$ (d) None of these

26. In the above question, what is the probability that the second one is white?

(a) $\frac{3}{17}$ (b) $\frac{6}{17}$

(c) $\frac{5}{17}$ (d) $\frac{1}{17}$

27. A fair dice is tossed six times. Find the probability of getting a third six on the sixth throw.

(a) $\frac{{}^5C_2\, 5^2}{6^2}$ (b) $\frac{{}^5C_2\, 5^3}{6^6}$

(c) $\frac{{}^5C_3\, 5^2}{6^3}$ (d) $\frac{{}^5C_3\, 5^2}{6^6}$

28. In shuffling a pack of cards, four are accidentally dropped. Find the chance that the dropped cards should be one from each suit.

(a) $\frac{13^4}{^{52}C_4}$ (b) $\frac{12^4}{^{52}C_2}$

(c) $\frac{13^2}{^{34}C_2}$ (d) $\frac{12^2}{^{22}C_3}$

29. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with these vertices is equilateral is

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$

(c) $\frac{1}{5}$ (d) $\frac{4}{10}$

30. There are 5 red shoes and 4 black shoes in a sale. They have got all mixed up with each other. What is the probability of getting a matched shoe if two shoes are drawn at random?

(a) $\frac{6}{9}$ (b) $\frac{4}{9}$

(c) $\frac{2}{9}$ (d) $\frac{5}{9}$

31. A person draws a card from a pack of 52, replaces it and shuffles it. He continues doing it until he draws a heart. What is the probability that he has to make 3 trials?

(a) $\frac{9}{64}$ (b) $\frac{3}{64}$

(c) $\frac{5}{64}$ (d) $\frac{1}{64}$

32. For the above problem, what is the probability if he does not replace the cards?

(a) $\frac{274}{1700}$ (b) $\frac{123}{1720}$

(c) $\frac{247}{1700}$ (d) $\frac{234}{1500}$

33. An event *X* can happen with probability *P*, and event *Y* can happen with probability *P*¢. What is the probability that exactly one of them happens?

(a) *P* + *P*¢ – 2*PP*¢ (b) 2*PP*¢ – *P*¢+ *P*

(c) *P* – *P*¢ + 2*PP*¢ (d) 2*P*¢*P* – *P*¢ + *P*

34. In the above question, what is the probability that at least one of them happens?

(a) *P* + *P*¢ + *PP*¢ (b) *P* + *P*¢ – *PP*¢

(c) 2*PP*¢ – *P*¢ – *P* (d) *P* + *P*¢ – 2*PP*¢

35. Find the probability that a year chosen at random has 53 Mondays.

(a) $\frac{5}{28}$ (b) $\frac{3}{28}$

(c) $\frac{1}{28}$ (d) $\frac{3}{28}$

36. There are four machines and it is known that exactly two of them are faulty. They are tested one by one in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{5}{6}$

37. For the above question, the probability that exactly 3 tests will be required to identify the 2 faulty machines is

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

38. Seven white balls and three black balls are randomly placed in a row. Find the probability that no two black balls are placed adjacent to each other.

(a) $\frac{7}{15}$ (b) $\frac{2}{15}$

(c) $\frac{3}{7}$ (d) $\frac{2}{7}$

39. A fair coin is tossed repeatedly. If Head appears on the first four tosses then the probability of appearance of tail on the fifth toss is

(a) $\frac{1}{7}$ (b) $\frac{1}{2}$

(c) $\frac{3}{7}$ (d) $\frac{2}{3}$

40. The letters of the word ‘article’ are arranged at random. Find the probability that the vowels may occupy the even places.

(a) $\frac{2}{35}$ (b) $\frac{1}{35}$

(c) $\frac{3}{36}$ (d) $\frac{2}{34}$

41. What is the probability that four Ss come consecutively in the word MISSISSIPPI?

(a) $\frac{4}{165}$ (b) $\frac{2}{165}$

(c) $\frac{3}{165}$ (d) $\frac{1}{165}$

42. Eleven books, consisting of five Engineering books, four Mathematics books and two Physics books, are arranged in a shelf at random. What is the probability that the books of each kind are all together?

(a) $\frac{5}{1155}$ (b) $\frac{2}{1155}$

(c) $\frac{3}{1155}$ (d) $\frac{1}{1155}$

43. Three students appear at an examination of Mathematics. The probability of their success are 1/3, 1/4, 1/5 respectively. Find the probability of success of at least two.

(a) $\frac{1}{6}$ (b) $\frac{2}{5}$

(c) $\frac{3}{4}$ (d) $\frac{3}{5}$

44. A bag contains 8 white and 4 red balls. Five balls are drawn at random. What is the probability that two of them are red and 3 are white?

(a) $\frac{12}{44}$ (b) $\frac{14}{33}$

(c) $\frac{14}{34}$ (d) $\frac{15}{34}$

45. A team of 4 is to be constituted out of 5 girls and 6 boys. Find the probability that the team may have 3 girls.

(a) $\frac{4}{11}$ (b) $\frac{3}{11}$

(c) $\frac{5}{11}$ (d) $\frac{2}{11}$

46. 12 persons are seated around a round table. What is the probability that two particular persons sit together?

(a) $\frac{2}{11}$ (b) $\frac{1}{6}$

(c) $\frac{3}{11}$ (d) $\frac{3}{15}$

47. Six boys and six girls sit in a row randomly. Find the probability that all the six girls sit together.

(a) $\frac{3}{22}$ (b) $\frac{1}{132}$

(c) $\frac{1}{1584}$ (d) $\frac{1}{66}$

48. From a group of 7 men and 4 women a committee of 6 persons is formed. What is the probability that the committee will consist of exactly 2 women?

(a) $\frac{5}{11}$ (b) $\frac{3}{11}$

(c) $\frac{4}{11}$ (d) $\frac{2}{11}$

49. A bag contains 5 red, 4 green and 3 black balls. If three balls are drawn out of it at random, find the probability of drawing exactly 2 red balls.

(a) $\frac{7}{22}$ (b) $\frac{10}{33}$

(c) $\frac{7}{12}$ (d) $\frac{7}{11}$

50. A bag contains 100 tickets numbered 1, 2, 3, …, 100. If a ticket is drawn out of it at random, what is the probability that the ticket

drawn has the digit 2 appearing on it?

(a) $\frac{19}{100}$

(b) $\frac{21}{100}$

(c) $\frac{32}{100}$

(d) $\frac{23}{100}$

LEVEL OF DIFFICULTY (III)

1. Out of a pack of 52 cards one is lost; from the remainder of the pack, two cards are drawn and are found to be spades. Find the chance that the missing card is a spade.

   (a) $\frac{11}{50}$ (b) $\frac{11}{49}$

   (c) $\frac{10}{49}$ (d) $\frac{10}{50}$

2. *A* and *B* throw one dice for a stake of ` 11, which is to be won by the player who first throws a six. The game ends when the stake is won by *A* or *B*. If *A* has the first throw, what are their respective expectations?

   (a) 5 and 6 (b) 6 and 5

   (c) 11 and 0 (d) 9 and 2

3. Counters marked 1, 2, 3 are placed in a bag and one of them is withdrawn and replaced. The operation being repeated three times, what is the chance of obtaining a total of 6 in these three operations?

   (a) $\frac{11}{27}$ (b) $\frac{7}{27}$

   (c) $\frac{1}{27}$ (d) $\frac{5}{14}$

4. A speaks the truth 3 times out of 4, *B* 7 times out of 10. They both assert that a white ball is drawn from a bag containing 6 balls, all of different colours. Find the probability of the truth of the assertion.

   (a) $\frac{12}{49}$ (b) $\frac{3}{10}$

(c) $\frac{21}{40}$ (d) None of these

5. In a shirt factory, processes *A*, *B* and *C* respectively manufacture 25%, 35% and 40% of the total shirts. Of their respective productions, 5%, 4% and 2% of the shirts are defective. A shirt is selected at random from the production of a particular day. If it is found to be defective, what is the probability that it is manufactured by the process *C* ?

(a) $\frac{16}{69}$ (b) $\frac{25}{69}$

(c) $\frac{28}{69}$ (d) $\frac{27}{44}$

6. A pair of fair dice are rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is

(a) 0.45 (b) 0.4

(c) 0.5 (d) 0.7

7. For the above problem, the probability of 7 coming before 5 is:

(a) $\frac{3}{5}$ (b) 0.55

(c) 0.4 (d) 0.7

8. For the above problem, the probability of 4 coming before either 5 or 7 is:

(a) $\frac{3}{13}$ (b) $\frac{7}{13}$

(c) $\frac{11}{13}$ (d) $\frac{10}{13}$

9. The probability of a bomb hitting a bridge is 1/2 and two direct hits are needed to destroy it. The least number of bombs required so that the probability of the bridge being destroyed is greater than 0.9 is:

(a) 7 bombs

(b) 3 bombs

(c) 8 bombs

(d) 9 bombs

10. What is the probability of the destruction of the bridge if only 5 bombs are dropped?

(a) 62.32%

(b) 81.25%

(c) 45.23%

(d) 31.32%

11. Sanjay writes a letter to his friend from IIT, Kanpur. It is known that one out of '*n*' letters that are posted does not reach its destination. If Sanjay does not receive the reply to his letter, then what is the probability that Kesari did not receive Sanjay's letter? It is certain that Kesari will definitely reply to Sanjay's letter if he receives it.

(a) $\frac{n}{(2n-1)}$

(b) $\frac{n-1}{n}$

(c) $\frac{1}{n}$

(d) None of these

12. A word of 6 letters is formed from a set of 16 different letters of the English alphabet (with replacement). Find out the probability that exactly 2 letters are repeated.

(a) $\frac{225 \times 224 \times 156}{16^6}$

(b) $\frac{18080}{16^6}$

(c) $\frac{15 \times 224 \times 156}{16^6}$

(d) None of these

13. A number is chosen at random from the numbers 10 to 99. By seeing the number, a man will sing if the product of the digits is 12. If he chooses three numbers with replacement, then the probability that he will sing at least once is:

(a) $1-\left(\frac{43}{45}\right)^3$

(b) $\left(\frac{43}{45}\right)^3$

(c) $1-\frac{48\times 86}{90^3}$ (d) None of these

14. In a bag, there are ten black, eight white and five red balls. Three balls are chosen at random and one is found to be black. The probability that the rest two are white is. Find the probability that the remaining two balls are white.

(a) $\frac{8}{23}$ (b) $\frac{4}{33}$

(c) $\frac{10\times 8\times 7}{23\times 22\times 21}$ (d) $\frac{5}{23}$

15. In the above question, find the probability that the remaining two balls are red.

(a) $\frac{10}{231}$ (b) $\frac{12}{231}$

(c) $\frac{12}{363}$ (d) None of these

16. Ten tickets are numbered 1, 2, 3…, 10. Six tickets are selected at random one at a time with replacement. The probability that the largest number appearing on the selected ticket is 7 is:

(a) $\frac{(7^6-1)}{10^6}$ (b) $\frac{7^6-6^6}{10^6}$

(c) $\frac{6^6}{10^6}$ (d) None of these

17. A bag contains 15 tickets numbered 1 to 15. A ticket is drawn and replaced. Then one more ticket is drawn and replaced. The probability that first number drawn is even and second is odd is

(a) $\frac{56}{225}$ (b) $\frac{26}{578}$

(c) $\frac{57}{289}$

(d) None of these

18. Six blue balls are put in three boxes. The probability of putting balls in the boxes in equal numbers is

(a) $\frac{1}{21}$

(b) $\frac{1}{8}$

(c) $\frac{1}{28}$

(d) $\frac{1}{7}$

19. AMS employs 8 professors on their staff. Their respective probability of remaining in employment for 10 years are 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9. The probability that after 10 years at least 6 of them still work in AMS is

(a) 0.19

(b) 1.22

(c) 0.1

(d) None of these

20. A person draws a card from a pack of 52, replaces it and shuffles it. He continues doing so until he draws a heart. What is the probability that he has to make at least 3 trials?

(a) $\frac{3}{17}$

(b) $\frac{8}{19}$

(c) $\frac{2}{17}$

(d) $\frac{11}{16}$

21. Hilips, the largest white goods producer in India, uses a quality check scheme on produced items before they are sent into the market. The plan is as follows: A set of 20 articles is readied and 4 of them are chosen at random. If any one of them is found to be defective then the whole set is put under 100% screening again. If no defectives are found, the whole set is sent into the market. Find the probability that a box containing 4 defective articles will be sent into the market.

(a) $\frac{364}{969}$ (b) $\frac{364}{963}$

(c) $\frac{96}{969}$ (d) $\frac{343}{969}$

22. In the above question, what is the probability that a box containing only one defective will be sent back for screening?

(a) $\frac{2}{3}$ (b) $\frac{1}{5}$

(c) $\frac{2}{5}$ (d) $\frac{4}{5}$

23. If the integers m and n are chosen at random from 1 to 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{16}$ (d) $\frac{1}{6}$

24. Three numbers are chosen at random without replacement from (1, 2, 3 …, 10). The probability that the minimum number is 3 or the maximum number is 7 is

(a) $\frac{12}{37}$ (b) $\frac{11}{40}$

(c) $\frac{13}{35}$ (d) $\frac{14}{35}$

25. An unbiased dice with face values 1, 2, 3, 4, 5 and 6 is rolled four times. Out of the 4 face values obtained, find the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5.

(a) $\frac{16}{81}$

(b) $\frac{14}{6^4}$

(c) $\frac{16}{80}$

(d) None of these

26. Three faces of a dice are yellow, two faces are red and one face is blue. The dice is tossed three times. Find the probability that the colours yellow, red and blue appear in the first, second and the third toss respectively.

(a) $\frac{1}{18}$

(b) $\frac{1}{12}$

(c) $\frac{1}{9}$

(d) $\frac{1}{36}$

27. If from each of three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is

(a) $\frac{13}{32}$

(b) $\frac{12}{14}$

(c) $\frac{12}{25}$

(d) $\frac{3}{13}$

28. Probabilities that Rajesh passes in Math, Physics and Chemistry are m, p and c respectively. Of these subjects, Rajesh has a 75% chance of passing in at least one, 50% chance of passing in at least two and 40% chance of passing in exactly two. Find which of the following is true.

(a) $p + m + c = \frac{19}{20}$

(b) $p + m + c = \frac{27}{20}$

c) $pmc = \frac{1}{20}$

(d) $pmc = \frac{1}{8}$

29. There are 5 envelopes corresponding to 5 letters. If the letters are placed in the envelopes at random, what is the probability that all the letters are not placed in the right envelopes?

(a) $\frac{119}{120}$ (b) $\frac{59}{60}$

(c) $\frac{23}{24}$ (d) $\frac{4^5}{5^5}$

30. For the above question what is the probability that no single letter is placed in the right envelope.

(a) $\frac{12}{35}$ (b) $\frac{11}{30}$

(c) $\frac{12}{25}$ (d) $\frac{3}{12}$

31. An urn contains four tickets having numbers 112, 121, 211, 222 written on them. If one ticket is drawn at random and Ai (i = 1, 2, 3) be the event that the ith digit from left of the number on ticket drawn is 1, which of these can be said about the events A_1, A_2 and A_3?

(a) They are mutually exclusive

(b) A_1 and A_3 are not mutually exclusive to A_2

(c) A_1 and A_3 are mutually exclusive

(d) Both *b* and *c*

32. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will get an electric contract is 5/9. If the probability of getting at least one contract is 4/5, what is the probability that he will get both the contracts?

(a) $\frac{19}{45}$ (b) $\frac{13}{45}$

(c) $\frac{12}{35}$ (d) $\frac{11}{23}$

33. If *P*(*A*) = 3/7, *P*(*B*) = 1/2 and *P*(*A*¢ « B¢) = 1/14, then are *A* and *B* are mutually exclusive events?

(a) No (b) Yes

(c) Either yes or no (d) Cannot be determined

34. Six boys and six girls sit in a row at random. Find the probability that the boys and girls sit alternately.

(a) $\frac{1}{132}$ (b) $\frac{1}{462}$

(c) $\frac{1}{623}$ (d) $\frac{1}{231}$

35. A problem on mathematics is given to three students whose chances of solving it are 1/2, 1/3 and 1/4 respectively. What is the chance that the problem will be solved?

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

36. *A* and *B* throw a pair of dice alternately. A wins if he throws 6 before *B* throws 5 and *B* wins if he throws 5 before *A* throws 6. Find *B*'s chance of winning if *A* makes the first throw.

(a) $\frac{1}{2}$ (b) $\frac{5}{12}$

(c) $\frac{1}{3}$ (d) $\frac{5}{11}$

37. Two persons *A* and *B* toss a coin alternately till one of them gets Head and wins the game. Find *B*'s chance of winning if *A* tosses the

coin first.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) None of these

38. A bag contains 3 red and 4 white balls and another bag contains 4 red and 3 white balls. A dice is cast and if the face 1 or 3 turns up, a ball is taken from the first bag and if any other face turns up, a ball is taken from the second bag. Find the probability of drawing a red ball.

(a) $\frac{11}{20}$ (b) $\frac{12}{21}$

(c) $\frac{2}{11}$ (d) $\frac{11}{21}$

39. Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, and 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected consists of 1 girl and 2 boys is:

(a) $\frac{13}{32}$ (b) $\frac{12}{32}$

(c) $\frac{15}{32}$ (d) $\frac{11}{32}$

40. The probabilities of *A*, *B* and *C* solving a problem are $\frac{1}{3}$, $\frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

(a) $\frac{26}{65}$ (b) $\frac{25}{56}$

(c) $\frac{52}{65}$ (d) $\frac{25}{52}$

41. A bag contains 5 black and 3 red balls. A ball is taken out of the bag and is not returned to it. If this process is repeated three times, then what is the probability of drawing a black ball in the next draw of a ball?

(a) 0.7 (b) 0.625

(c) 0.1 (d) None of these

42. For question 41, what is the probability of drawing a red ball?

(a) 0.375 (b) 0.9

(c) 0.3 (d) 0.79

43. One bag contains 5 white and 4 black balls. Another bag contains 7 white and 9 black balls. A ball is transferred from the first bag to the second and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.

(a) $\frac{7}{18}$ (b) $\frac{5}{9}$

(c) $\frac{4}{9}$ (d) $\frac{11}{18}$

44. V Anand and Gary Kasparov play a series of 5 chess games. The probability that V Anand wins a game is 2/5 and the probability of Kasparov winning a game is 3/5. There is no probability of a draw. The series will be won by the person who wins 3 matches. Find the probability that Anand wins the series. (The series ends the moment when any of the two wins 3 matches.)

(a) $\frac{992}{3125}$ (b) $\frac{273}{625}$

(c) $\frac{1021}{3125}$ (d) $\frac{1081}{3125}$

45. There are 10 pairs of socks in a cupboard from which 4 individual socks are picked at random. The probability that there is at least one pair is.

(a) $\frac{195}{323}$ (b) $\frac{99}{323}$

(c) $\frac{198}{323}$ (d) $\frac{185}{323}$

46. A fair coin is tossed 10 times. Find the probability that two Heads do not occur consecutively.

(a) $\frac{1}{2^4}$ (b) $\frac{1}{2^3}$

(c) $\frac{1}{2^5}$ (d) None of these

47. In a room there are 7 persons. The chance that two of them were born on the same day of the week is

(a) $\frac{1080}{7^5}$ (b) $\frac{2160}{7^5}$

(c) $\frac{540}{7^4}$ (d) None of these

48. In a hand at a game of bridge what is the chance that the 4 kings are held by a specified player?

(a) $\frac{10}{4165}$ (b) $\frac{11}{4165}$

(c) $\frac{110}{4165}$ (d) None of these

49. One hundred identical coins each with probability P of showing up Heads are tossed once. If $0 < P < 1$ and the probability of Heads

showing on 50 coins is equal to that of Heads showing on 51 coins, then value of P is

(a) $\frac{1}{21}$ (b) $\frac{49}{101}$

(c) $\frac{50}{101}$ (d) $\frac{51}{101}$

50. Two small squares on a chess board are chosen at random. Find the probability that they have a common side:

(a) $\frac{1}{12}$ (b) $\frac{1}{18}$

(c) $\frac{2}{15}$ (d) $\frac{3}{14}$

ANSWER KEY

Level of Difficulty (I)

1. (b)	2. (a)	3. (d)	4. (c)
5. (a)	6(i). (c)	6(ii). (b)	6(iii). (c)
7(i). (b)	7(ii). (a)	8(i). (d)	8(ii). (a)
9. (b)	10(i). (a)	10(ii). (b)	10(iii). (d)
11(i). (d)	11(ii). (a)	12. (a)	13. (a)
14(i). (b)	14(ii). (a)	14(iii). (b)	15(i). (d)
15(ii). (a)	15(iii). (b)	16(i). (a)	16(ii). (b)
17. (b)	18. (a)	19(i). (c)	19(ii). (a)
20. (a)	21. (b)	22(i). (a)	22(ii). (d)
22(iii). (a)	23. (a)	24. (d)	25(i). (a)
25(ii). (c)	26. (b)	27. (a)	28. (a)
29. (b)	30. (a)	31. (a)	32. (d)
33. (b)	34(i). (b)	34(ii). (c)	35. (d)
36. (a)	37. (a)	38. (d)	39. (a)
40. (b)	41(i). (c)	41(ii). (d)	41(iii). (a)
42. (c)	43. (a)	44(i). (a)	44(ii). (c)

44(iii). (a)	44(iv). (b)

Level of Difficulty (II)

1. (d)	2. (a)	3. (a)	4(i) (b)
4(ii) (c)	5. (d)	6. (a)	7. (d)
8. (a)	9(i) (a)	9(ii) (b)	10. (d)
11(i) (a)	11(ii) (b)	11(iii) (b)	12(i) (a)
12(ii) (a)	12(iii) (a)	13(i) (b)	13(ii) (c)
13(iii) (c)	14(i) (b)	14(ii) (c)	14(iii) (b)
15. (a)	16(i) (b)	16(ii) (a)	16(iii) (b)
17(i) (b)	17(ii) (a)	17(iii) (c)	17(iv) (a)
18. (b)	19. (b)	20. (c)	21. (b)
22. (b)	23. (a)	24. (a)	25. (a)
26. (b)	27. (b)	28. (a)	29. (a)
30. (b)	31. (a)	32. (c)	33. (a)
34. (b)	35. (a)	36. (b)	37. (c)
38. (a)	39. (b)	40. (b)	41. (a)
42. (d)	43. (a)	44. (b)	45. (d)
46. (a)	47. (b)	48. (a)	49. (a)
50. (a)			

Level of Difficulty (III)

1. (a)	2. (b)	3. (b)	4. (c)
5. (a)	6. (b)	7. (a)	8. (a)
9. (a)	10. (b)	11. (a)	12. (a)
13. (a)	14. (b)	15. (a)	16. (b)
17. (a)	18. (c)	19. (a)	20. (d)
21. (a)	22. (b)	23. (a)	24. (b)
25. (a)	26. (d)	27. (a)	28. (b)
29. (a)	30. (b)	31. (b)	32. (a)
33. (b)	34. (b)	35. (b)	36. (d)
37. (a)	38. (d)	39. (b)	40. (b)
41. (a)	42. (a)	43. (c)	44. (a)
45. (b)	46. (b)	47. (b)	48. (b)
49. (d)	50. (b)		

Solutions and Shortcuts

Level of Difficulty (I)

1. Out of a total of 6 occurrences, 3 is one possibility = 1/6.
2. 5 or 6 out of a sample space of

   1, 2, 3, 4, 5 or 6 = 2/6 = 1/3
3. Event definition is:

   (1 and 1) or (1 and 2) or (1 and 3) or (1 and 4) or (1 and 5) or (1 and 6) or (2 and 1) or (3 and 1) or (4 and 1) or (5 and 1) or (6 and 1)

   Total 11 out of 36 possibilities = 11/36
4. Event definition: First is blue and second is blue = 7/12 × 6/11 = 7/22.
5. Knave and queen or Queen and Knave Æ

   4/52 × 4/51 + 4/52 × 4/51 = 8/663
6. (i) 13/52 = 1/4

   (ii) 4 kings and 4 queens out of 52 cards.

   Thus, 8/52 = 2/13.

   (iii) 13 spades + 3 kings Æ 19/52.
7. (i) Event definition is: *T* and *T* and *H* or *T* and *H* and *T* or *H* and *T* and *T* = 3 × 1/8 = 3/8.

   (ii) Same as above = 3/8.
8. (i) Probability of 3 heads = 1/8

   Also, Probability of 3 tails = 1/8.

   Required probability = 1– (1/8 + 1/8) = 6/8 = 3/4.

   (ii) *H* and *H* and *H* = 1/8.
9. At least one tail is the non-event for all heads.

   Thus, *P* (at least 1 tail) = 1 – *P*(all heads)

   = 1 – 1/8 = 7/8.
10. (i) Positive outcomes are 2(1 way), 4(3 ways), 6(5 ways) 8(5 ways), 10 (3 ways), 12 (1way).

    Thus, 18/36 = 1/2

(ii) Positive outcomes are: 4, 8 and 12

4 (3 ways), 8(5 ways) and 12(1 way)

Gives us 9/36 = 1/4.

(iii) Positive outcomes are 2(1 way), 3(2 ways), 5(4 ways), 7(6 ways). Total of 13 positive outcomes out of 36.

Thus, 13/36.

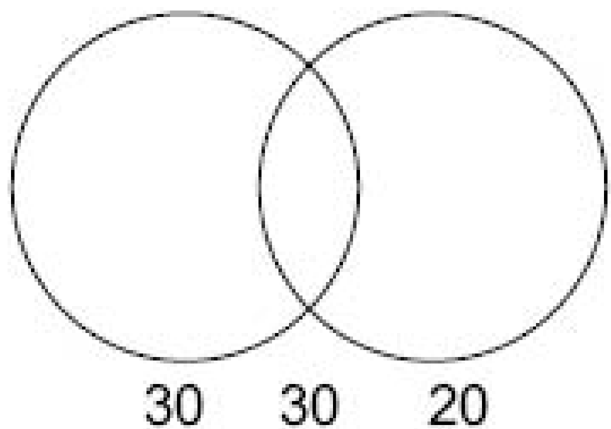
11. (i) First is white and second is white 7/10 × 6/9 = 7/15.

(ii) White and Green or Green and White

7/10 × 39 + 3/10 × 7/9

42/90 = 7/15.

12.



From the figure it is evident that 80 students passed at least 1 exam. Thus, 20 failed both and the required probability is 20/100 = 1/5.

13. 3 or 4 or 5 or 6 = 4/6 = 2/3

14. (i) Since 2 is the only prime number out of the three numbers, the answer would be 1/3

(ii) Since all the numbers are even, it is sure that the number drawn out is an even number. Hence, the required probability is 1.

(iii) Since there are no odd numbers amongst 2, 4 and 8, the required probability is 0.

15. (i) The event would be Head and Head Æ ½ × ½ = ¼

(ii) The event would be Head and Tail OR Tail and Head Æ = ½ × ½ + ½ × ½ = 1/8

(iii) The event would be Tail and Tail Æ ½ × ½ = ¼

16. (i) With a six on the first dice, there are 6 possibilities of outcomes that can appear on the other dice (viz. 6 & 1, 6 & 2, 6 & 3, 6 & 4, 6 & 5 and 6 & 6). At the same time with 6 on the second dice there are 5 more possibilities for outcomes on the first dice: (1 & 6, 2 & 6, 3 & 6, 4 & 6, 5 & 6)

Also, the total outcomes are 36. Hence, the required probability is 11/36.

(ii) Out of 36 outcomes, 5 can come in the following ways – 1 + 4; 2 + 3; 3 + 2 or 4 + 1 Æ 4/36 = 1/9.

17. Odds against an event = $\frac{p(E^2)}{p(E)}$

In this case, the event is: All black, i.e., First is black and second is black and third is black.

$P(E)$ = 5/9 × 4/8 × 3/7 = 60/504 = 5/42.

Odds against the event = 37/5.

18. Event definition is: 15 or 16 or 17 or 18.

15 can be got as: 5 and 5 and 5 (one way)

Or

6 and 5 and 4 (Six ways)

Or

6 and 6 and 3 (3 ways)

Total 10 ways.

16 can be got as: 6 and 6 and 4 (3 ways)

Or

6 and 5 and 5 (3 ways)

Total 6 ways.

17 has 3 ways and 18 has 1 way of appearing.

Thus, the required probability is: (10 + 6 + 3 + 1)/216 = 20/216 = 5/54.

19. (i) Positive outcomes = 2 (187 or 215)

Total outcomes = 9 × 8 × 7

Required probability = 2/504 = 1/252

(ii) = 2/729.

20. 1/6 + 1/10 + 1/8 = 47/120

21. The event definition would be given by:

First is blue and second is blue = 3/11 × 2/10 = 3/55

22. (i) There are six doubles (1, 1; 2 & 2; 3 & 3; 4 & 4; 5 & 5; 6 & 6) out of a total of 36 outcomes Æ 6/36 = 1/6

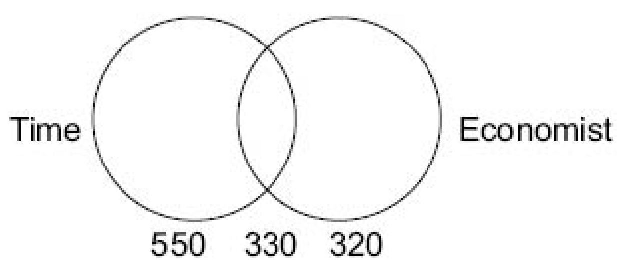
(ii) Sum greater than 10 means 11 or 12 Æ 3/36 = 1/12

(iii) Sum less than 10 is the non event for the case sum is 10 or 11 or 12. There are 3 ways of getting 10, 2 ways of getting 11 and 1 way of getting a sum of 12 in the throw of two dice. Thus, the required probability would be 1 – 6/36 = 5/6.

23. Expectation = Probability of winning × Reward of winning = (10/5000) × 1 crore = (1 crore/500) = 20000.

24. $1/{}^4C_2$ = 1/6.

25.



(i) 880/1200 = 11/15

(ii) 650/1200 = 13/24

26. 330/1200 = 11/40

27. 330/880 = 3/8

28. Black and Black and Black = 4/9 × 3/8 × 2/7

= 23/504 = 1/21.

29. $1/{}^5P_2$ = 1/20.

30. 6 × (4/52) × (3/51) × (2/50) = 6/5525.

31. Positive Outcomes are: 5, 7, 10, 14, 15 or 20

Thus, 6/20 = 3/10.

32. 972/1972 = 243/ 493.

33. Black and black = (7/16) × 6/15 = 7/40

34. (i) The required probability would be given by:

All are Red OR All are white OR All are Blue

= (6/18) × (5/17) × (4/16) + (4/18) × (3/17) × (2/16) + (8/18) × (7/17) × (6/16)

= 480/(18 × 17 × 16) 5/51

(ii) All blue = (8 × 7 × 6)/(18 × 17 × 16) = 7/102

35. The required value of the union of the two non events (of A and B) would be 1 – 1/4 = ¾

36. P (Both are selected) = $P(A) \times P(B)$

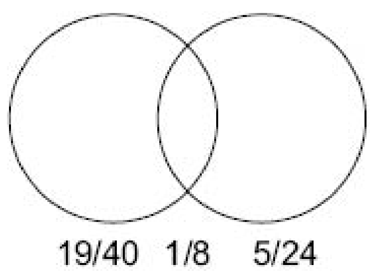
Since $P(A) = 0.5$, we get

0.3 = 0.5 × 0.6.

The maximum value of $P(B) = 0.6$.

Thus $P(B) = 0.9$ is not possible.

37.



We have: 19/40 + 1/8 + 5/24 = 97/120

38. P (Amit) = 1/3

P (Vikas) = 2/7

P (Vivek) = 1/8.

Required Probability = 1/3 + 2/7 + 1/8 = 125/168.

39. $P(A) + P(B) + P(C) = 1$ Æ $2P(B)/3 + P(B) + P(B)/2 = 1$ Æ $13P(B)/6 = 1$ Æ $P(B) = 6/13$. Hence, $P(A) = 4/13$

40. $P(A) = 1 – 0.65 = 0.35$.

Hence, $P(B) = 0.65 – 0.35 = 0.3$

41. (i) The required probability would be given by the event:

White from first bag and white from second bag = (4/6) × (3/8) = ¼

(ii) The required probability would be given by the event:

Black from first bag and black from second bag = (2/6) × (5/8) = 10/48 = 5/24

(iii) This would be the non event for the event [both are white OR both are black).

Thus, the required probability would be:

¼ – 5/24 = 13/24.

42. $P(E_1)$ 3/8

$P(E_2)$ = 7/12.

Event definition is: E_1 occurs and E_2 does not occur or E_1 occurs and E_2 occurs or E_2 occurs and E_1 does not occur.

(3/8) × (5/12) + (3/8) × (7/12) + (5/8) × (7/12) = 71/96.

43. Kamal is selected and Monica is not selected or Kamal is not selected and Monica is selected Æ (1/3) × (4/5) ¨ (2/3) × (1/5) = (6/15) = (2/5).

44. (i) 1/5 × 1/7 = 1/35

(ii) (1/5) × (6/7) + (4/5) × (1/7) = 10/35 = 2/7.

(iii) (4/5) × (6/7) = 24/35

(iv) Both selected or 1 selected = 1/35 + 2/7 = 11/35.

Level of Difficulty (II)

1. The possible outcomes are:

(1, 1); (1, 2); (2, 1), (2, 2); (3, 1); (1, 3).

Out of six cases, in two cases there is exactly one ‘2’

Thus, the correct answer is 2/6 = 1/3.

2. Event definition is *A* hits, *B* hits and *C* hits OR any two of the three hits.

3. The event can be defined as:

First bag is selected and red ball is drawn.

$1/2 \times 5/12 + ½ \times 3/15 = (5/24) + (3/30) = 37/120$

4. (a) For the least chance of drawing a red ball the distribution has to be 5 Red + 11 white in one bag and 1 white in the second bag. This gives us

$$\frac{1}{2} \times \frac{5}{16} + \frac{1}{2} \times 0 = \frac{5}{32}$$

(b) For the greatest chance of drawing a red ball the distribution has to be 1 Red in the first bag and 4 red + 12 white balls in the second bag. This gives us

$$\frac{1}{2} \times 1 + \frac{1}{2} \times \frac{4}{16} = \frac{5}{8}$$

5. One head and seven tails would have eight positions where the head can come.

Thus, $8 \times (1/2)^8 = (1/32)$

6. A leap year has 366 days – which means 52 completed weeks and 2 more days. The last two days can be (Sunday, Monday) or (Monday, Tuesday) or ……. (Saturday, Sunday).

2 scenarios out of 7 have a 53rd Sunday.

Thus, 2/7 is the required answer.

7. The count of the event will be given by:

The number of all 2 digit integers – the number of all 2 digit integers divisible by 7

8. Blue and Red or Red and Blue

$= (10/25) \times (15/24) + (15/25) \times (10/24) = (1/2)$.

9. (i) 1/5 + ½ = 7/10.

(ii) 1 – (7/10) = (3/10)

10. A total of six can be made in any of the following ways (1 + 5, 2 + 4, 3 + 3, 4 + 2, 5 + 1)

11. The event definitions are:

(a) Passes the first AND Passes the second AND Passes the third AND Passes the fourth

(b) Fails the first AND Fails the second AND Fails the third AND Fails the fourth

(c) Fails all is the non-event.

12. (i) First is cricket and second is cricket and third is cricket Æ (3/9) × (3/9) × (3/9) = (1/27)

(ii) (2/9) × (3/9) × (4/9) = (8/243)

(iii) All are cricket or all are tennis or all are squash.

$(3/9)^3 + (2/9)^3 + (4/9)^3 = (99/729) = (11/81)$

13. (i) (3/9) × (2/8) × (1/7) = (1/84)

(ii) (2/9) × (3/8) × (4/7) = (1/21)

(iii) (3/9) × (2/8) × (1/7) + 0 + (4/9) × (3/8) × (2/7)

30/504 = 5/84

14. The event definitions are

(i) The book transferred is by Stephen Covey

(ii) The book transferred is by Stephen Covey AND The book picked up is by Stephen Covey

(iii) The book transferred is by Stephen Covey AND The book picked up is by Vinay Singh.

15. (2/3) × (3/4) + (1/3) × (2/7) = (1/2) + (2/21) = (25/42)

16. (i) (3/50) × (2/49) = (3/1225)

(ii) (3/50) × (47/49) + 47/50 × (3/49) = (141/1225)

(iii) (47/50) × (46/49) = (1081/1225)

17. (i) (3/5) × (3/7) = (9/35)

(ii) (3/5) × 4/7) = (12/35)

(iii) (2/5) × (3/7) = (6/35)

(iv) (3/5) × (4/7) + (2/5) × (3/7) + (3/5) × (3/7) = 27/35

18. They will contradict each other if: A is true and B is false or A is false and B is true.

(3/4) × (1/6) + (1/4) × (5/6) = 1/3.

19. For the counting of the number of events, think of it as a circular arrangement with $n - 1$ people (by considering the two specified persons as one). This will give you $n(E) = (n - 2)! \times (2)!$

20. The whole numbers selected can only be 1, 3, 7 or 9 and cannot contain 2, 4, 6, 8, 0 or 5.

21. ${}^4C_2 \times (6/36)^2 \times (30/36)^2 = 6 \times (1/36) \times (25/36) = 25/216$.

22. The required probability will be given by the expression:

$$\frac{\text{The number of young boys who will die}}{\text{The total number of people who will die}}$$

23. Girl and Boy and Boy or Boy and Girl and Boy

    Or

    Boy and Boy and Girl

    $= (3/4) \times (2/4) \times (2/3) + (1/4) \times (2/4) \times 2/3)$

    $+ (1/4) \times (2/4) \times (1/3) = 18/48 = 3/8$.

24. $n(E) = 1$

    $n(S) = {}^3C_1 \times 9 \times 9 = 243$

25. 11/17 (if the first one is black, there will be 11 black balls left out of 17)

26. 6/17

27. There must have been two sixes in the first five throws. Thus, the answer is given by:

$${}^5C_2 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$$

28. $\dfrac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4}$

29. There will be ${}^6C_3$ triangles formed overall. Out of these visualise the number of equilateral triangles.

30. $(5/9) \times (4/8) + (4/9) \times (3/8) = 32/72 = 4/9$

    As matched shoes means both red or both black.

31. The event definition here is that first is not a heart and second is not a heart and third is a heart = (3/4) × (3/4) × (1/4) = 9/64.
32. (39/52) × (38/51) × (13/50) = 247/1700.
33. The event definition will be: Event *X* happens and *Y* doesn’t happen Or *Y* happens and *X* does not happen.
34. *X* happens and *Y* does not happen or *X* doesn’t happen and *Y* happens or *X* happens and *Y* happens

    P × (1 – *P*) + (1 – *P*) × *P*¢ + *P* × *P*¢ = *P* + *P*¢ – *PP*¢
35. Same logic as question 6.
36. (2/4) × (1/3) = 1/6. (Faulty and faulty)
37. Faulty and not Faulty and Faulty or Not Faulty and Faulty and Faulty = (2/4) × (2/3) × (1/2) + (2/4) × (2/3) × (1/2) = 1/3.
38. When you put the 7 balls with a gap between them in a row, you will have 8 spaces.
39. The appearance of head or tail on a toss is independent of previous occurrences.

    Hence, 1/2.
40. $\frac{3! \times 4!}{7!}$ = 1/35.
41. $P = \frac{\text{No. of arrangements with four 5s together}}{\text{Total No. of arrangements}}$

    $= \frac{[8!/(4! \times 2!)]}{[11!/(4! \times 4! \times 2!)]}$

    = 8! × 4!/11! = 24/990 = 4/165.
42. $\frac{(5! \times 4! \times 2! \times 3!)}{11!} = \frac{24 \times 2 \times 6}{11 \times 10 \times 9 \times 8 \times 7 \times 6}$ = 1/1155.
43. (1/3) × (1/4) × (4/5) + (1/3) × (3/4) × (1/5) + (2/3)

    × (1/4) × (1/5) + (1/3) × (1/4) × (1/5)

    = 10/60 = 1/6.
44. ${}^5C_3$ × [(8/12) × (7/11) × (6/10) × (4/9) × (3/8)] = 14/33.

45. There can be three girls and one boy.

46. $P = \dfrac{\text{Total no of ways in which two people sit together}}{\text{Total No. of ways}}$

$= (10! \times 2!)/11!$

47. Consider the 6 girls to be one person. Then the number of arrangements satisfying the condition is given by $n(E) = 7! \times 6!$

48. ${}^6C_2 \times [\,(7/11) \times (6/10) \times (5/9) \times (4/8) \times (4/7) \times (3/6)] = 5/11.$

49. The event definition is Red AND Red AND Not Red OR Red AND Not Red AND Red OR Not Red AND Red AND Red.

50. The numbers having 2 in them are: 2, 12, 22, 32….92 and 21, 23, 24, 25….29. Hence, $n(E) = 19$.

Level of Difficulty (III)

1. This problem has to be treated as if we are selecting the third card out of the 50 remaining cards.

   11 of these are spades.

   Hence, 11/50.

2.

	Chance that A will win	*Chance that B will win*
First throw	$\frac{1}{6}$	$\frac{5}{6} \times \frac{1}{6}$
Second throw	$\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$
Third throw	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

The chance that A will win is an infinite GP with a = $\frac{1}{6}$ and $r = \frac{25}{36}$

Similarly, the chance that B will win is an infinite GP with $a = \frac{5}{36}$ and $r = \frac{25}{36}$.

3. A total of 6 can be obtained by either (1 + 2 + 3) or by (2 + 2 + 2).
4. A is true and B is true = (3/4) × (7/10) = (21/40).
5. The required probability will be given by:

$$\frac{\text{Percentage of defective from } C}{\text{Percentage of defectives from } A, B \text{ and } C}$$

6. and 7.

We do not have to consider any sum other than 5 or 7 occurring.

A sum of 5 can be obtained by any of [4 + 1, 3 + 2, 2 + 3, 1 + 4]

Similarly a sum of 7 can be obtained by any of [6 + 1, 5 + 2, 4 + 3, 3 + 4, 2 + 5, 1 + 6]

For 6: $n(E) = 4$, $n(S) = 6 + 4$

$P = 0.4$

For 7: $n(E) = 6$

$n(S) = 6 + 4$ $P = 0.6$

8. The number of ways for a sum of 4 = 3 i.e. [3 + 1, 2 + 2, 1 + 3]
9. Try to find the number of ways in which 0 or 1 bomb hits the bridge if n bombs are thrown.

The required value of the number of bombs will be such that the probability of 0 or 1 bomb hitting the bridge should be less than 0.1.

10. The required probability will be given by: $1 - [{}^nC_0 + {}^nC_1]\left(\frac{1}{2}\right)^n$ for $n = 5$.
11. The required answer will be given by:

$$\frac{\text{P(Kesari does not receive the letter)}}{\text{P(Kesari does not receive the letter)} + \text{P(Kesari replied and Sanjay did not receive the reply)}}$$

12. $n(E) = {}^6C_2 \times 16 \times 15 \times 14 \times 13 \times 12$

$n(S) = 16^6$

13. The number of events for the condition that he will sing = 4. [34, 43, 26, 62]

The number of events in the sample = 90.

Probability that he will sing at least once

= 1 – Probability that he will not sing.

14. The required probability would be given by the event definition:

First is white and second is white = 8/22 × 7/21 = 4/33

15. The required probability would be given by the event definition:

First is red and second is red = 5/22 × 4/21 = 10/231

16. The number of ways in which 6 tickets can be selected from 10 tickets is 10^6.

The number of ways in which the selection can be done so that the condition is satisfied = $7^6 - 6^6$.

17. In the first draw, we have 7 even tickets out of 15 and in the second we have 8 odd tickets out of 15.

Thus, (7/15) × (8/15) = 56/225.

18. The total number of ways in which the 6 identical balls can be distributed amongst the three boxes such that each box can get 0, 1, 2, 3, 4, 5 or 6 balls is given by the formula: ${}^{n+r-1}C_r$ where n is the number of identical balls and r is the number of boxes. This will give the sample space.

19. Event definition:

Any six of them work AND four leave OR any seven work AND three leave OR any eight work AND two leave OR any nine work AND one leaves OR All ten work.

20. "To make at least 3 trials to draw a heart" implies that he didn't get a heart in the first two trials.

21. A box containing 4 defectives would get sent to the market if all the 4 articles selected are not defective.

Thus, ${}^{16}C_4/{}^{20}C_4 = 364/969$

22. If the box contains only 1 defective, it would be sent back if the defective is one amongst the 4 selected for testing.

    To ensure one of the 4 is selected , the no. of ways is ${}^{19}C_3$, while the total no. of selections of 4 out of 20 is ${}^{20}C_4$.

    Thus, ${}^{19}C_3/{}^{20}C_4 = 1/5$.

23. For divisibility by 5 we need the units digit to be either 0 or 5.

    The units digit in the powers of 7 follow the pattern – 7, 9, 3, 1, 7, 9, 3, 1, 7, 9…….

    Hence, divide 1 to 100 into four groups of 25 elements each as follows.

    A = 1, 5, 9, ------- Æ 25 elements

    B = 2, 6, 10, ------ Æ 25 elements

    C = 3, 7, 11, ------ Æ 25 elements

    D = 4, 8, 12, ------ Æ 25 elements

    Check the combination values of m and n so that $7^m + 7^n$ is divisible by 5.

24. P(minimum 3) or P (maximum 7)

    P(minimum 3) = ${}^{7}C_2/{}^{10}C_3 = 21/120$

    P(max 7) = ${}^{6}C_2/{}^{10}C_2 = 15/120$

Note: The logic for this can be explained for minimum 3 conditions as: Since the minimum value has to be 3, the remaining 2 numbers have to be selected from 4 to 10. This can be done in ${}^{7}C_2$ ways.

25. For the event to occur, the dice should show values from 2, 3, 4 or 5. This is similar to selection with repetition.
26. Yellow and Red and Blue = (3/6) × (2/6) × (1/6) = (6/216) = 1/36.
27. Two white and one black can be obtained only through the following three sequences:

Ball drawn from A and B are white and the ball drawn from C is Black. or

Ball drawn from A and C are white and the ball drawn from B is Black. or

Ball drawn from B and C are white and the ball drawn from A is Black.

28. At least one means (exactly one + exactly two + exactly three)

    At least two means (exactly two + exactly three)

    The problem gives the probabilities for passing in at least one, at least two and exactly two.

29. All four are not in the correct envelopes means that at least one of them is in a wrong envelope. A little consideration will show that one letter being placed in a wrong envelope is not possible, since it will have to be interchanged with some other letter.

    Since, there is only one way to put all the letters in the correct envelopes, we can say that the event of not all four letters going into the correct envelopes will be given by

    $5! - 1 = 119$

30. $n(E) = 44$

    $n(S) = 120$

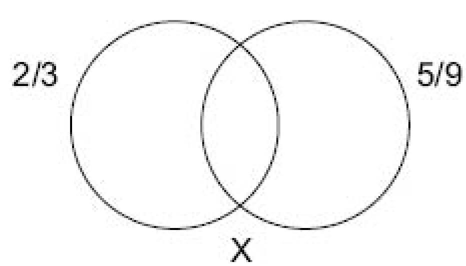
31. $A_1 = (112 \text{ or } 121)$

    $A_2 = (112 \text{ or } 211)$

    $A_3 = (121 \text{ or } 211)$

    Option (b) is correct.

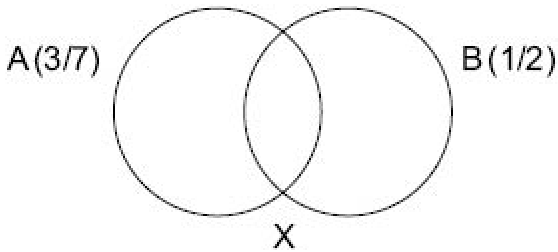
32.



From the venn diagram we get:

$(2/3) + (5/9) - x = 4/5$ Æ $x = (2/3) + (5/9) - (4/5) = 19/45$

33.



Also *P*(*A* » *B*) = 1 – *P*(*A*¢ « *B*¢)

(3/7) + (1/2) – *x* = 13/14 Æ *x* = 0

Thus, there is no interference between *A* and *B* as *P*(*A* « *B*) = *x* = 0. Hence, *A* and *B* are mutually exclusive.

34. The required probability will be given by $\frac{2 \times 6! \times 6!}{12!}$.

35. The non-event in this case is that the problem is not solved.

36-37. Are similar to Question No. 2 of LOD III.

38. The event definition will be:

The first bag is selected AND a red ball is selected. OR

The second bag is selected AND a red ball is selected.

39. The event definition is:

A girl is selected from the first group and one boy each are selected from the second and third groups. OR A girl is selected from the second group and one boy each are selected from the first and third groups. OR A girl is selected from the third group and one boy each are selected from the first and second groups.

40. The event definition will be:

A solves the problem and *B* and *C* do not solve the problem. OR *B* solves the problem and *A* and *C* do not solve the problem. OR *C* solves the problem and *A* and *B* do not solve the problem.

41. The three balls that are taken out can be either 3 black balls or 2 black and 1 red ball or 1 black and 2 red ball or 3 red balls.

Each of these will give their own probabilities of drawing a black ball.

42. 3 Blacks and 4^{th} is Red or 2 Blacks and 1 Red and 4^{th} is Red or 1 Black and 2 Reds and 4^{th} is Red

$= (5/8) \times (4/7) \times (3/6) \times (3/5) + {}^3C_1 \times (5/8) \times (4/7)$

$\times (3/6) \times (2/5) + {}^3C_1 \times (5/8) \times (3/7) \times (2/6) \times (1/5)$

$= \dfrac{180 + 360 + 90}{1680} = 630/1680 = 3/8 = 0.375$

43. The event definition would be: Ball transferred is white and white ball drawn

Or

Ball transferred is black and white ball is drawn.

The answer will be given by:

$(5/9) \times (8/17) + (4/9) \times (7/17) = 68/153 = 4/9$

44. Solve this on a similar pattern to the example given in the theory of this chapter.

45. Required probability = 1 – probability that no pair is selected

46. We can have a maximum of 5 heads.

For 0 heads Æ $P(E) = (1/2^{10}) \times 1$

For 1 heads Æ $P(E) = (1/2^{10}) \times 1$

For 2 heads and for them not to occur consecutively we will need to see the possible distribution of 8 tails and 2 heads.

Since the 2 heads do not need to occur consecutively, this would be given by (All – 2heads together)

Æ $({}^{10}C_8 - 9)$

$P(E) = \dfrac{({}^{10}C_8 - 9)}{2^{10}}$

Solving in this fashion, we would get $1/2^3$.

47. This can be got by taking the number of ways in which exactly two people are born on the same day divided by the total number of ways in which 7 people can be born in 7 days of a week. For the first part select two people from 7 in ${}^7C_2$ ways & select a day from

the week on which they have to be born in $^7C_1$ ways & for the remaining 5 people select 5 days out of the remaining six days of the week & then the number of arrangements of these 5 people in 5 days-thus a total of $^7C_2 \times {}^7C_1 \times {}^6C_5 \times 5!$ ways. Also, the number of ways in which seven people can be born on 7 days would be given by 7^7. Hence, the answer is given by: $({}^7C_2 \times {}^7C_1 \times {}^6C_5\ 5!/7^7) = 21 \times 7 \times 6 \times 120/7^7 = 3 \times 6 \times 120/7^5 = 2160/7^5$.

48. This can be got by defining the number of ways in which the player can get a deal of 13 cards if he gets all four kings divided by the number of ways in which the player can get a deal of 13 cards without any constraints from 52 cards:

The requisite value would be given by: $^{48}C_9/{}^{52}C_{13} = [48!/39! \times 9!] \times [13! \times 39!/52!] = (48! \times 13! \times 39!)/(39! \times 9! \times 52!) = 13 \times 12 \times 11 \times 10/52 \times 51 \times 50 \times 49 = 1 \times 11/17 \times 17 \times 5 \times 49 = 11/4165$

49. P of heads showing on 50 coins = $^{100}C_{50} \times P^{50}(1 - p)^{50}$

P of heads showing on 51 coins = $^{100}C_{51} \times P^{51}(1 - p)^{49}$

Both are equal

$^{100}C_{50} \times P^{50}(1 - P)^{50} = {}^{100}C_{51} \times P^{51}(1 - P)^{49}$

or $\dfrac{100 \times 49 \times \ldots \times 52 \times 51}{1 \times 2 \times \ldots \times 49 \times 50} \times (1 - P)$

$= \dfrac{100 \times 99 \times \ldots \times 53 \times 52}{1 \times 2 \times \ldots \times 48 \times 49} \times P$

or $\dfrac{51}{50} \times (1 - P) = P$

or $51 \times (1 - P) = 50\,P$

or $51 - 51\,P = 50\,P$

or $51 = 101\,P$

\ $P = \dfrac{51}{101}$

50. The common side could be horizontal or vertical. Accordingly, the number of ways the event can occur is.

$n(E) = 8 \times 7 + 8 \times 7 = 112$

$n(S) = 64C_2$

$= \dfrac{2 \times 8 \times 7 \times 2}{64 \times 63} = \dfrac{1}{18}$

19 Chapter Set Theory

Set Theory is an important concept of mathematics which is often asked in aptitude exams. There are two types of questions in this chapter:

(i) Numerical questions on set theory based on venn diagrams

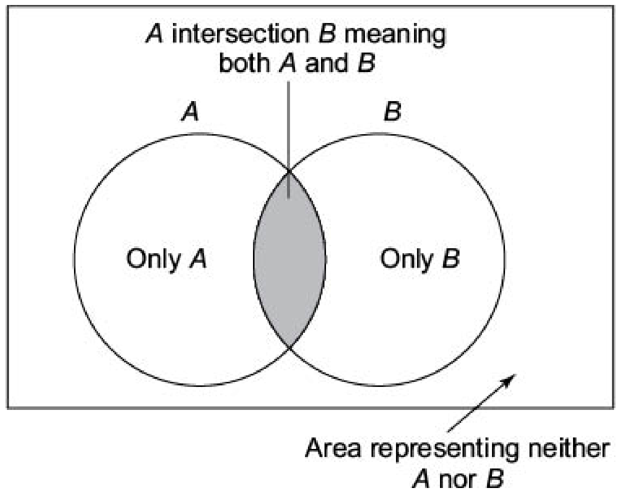
(ii) Logical questions based on set theory

Let us first take a look at some standard theoretical inputs related to set theory.

SET THEORY

Look at the following diagrams:

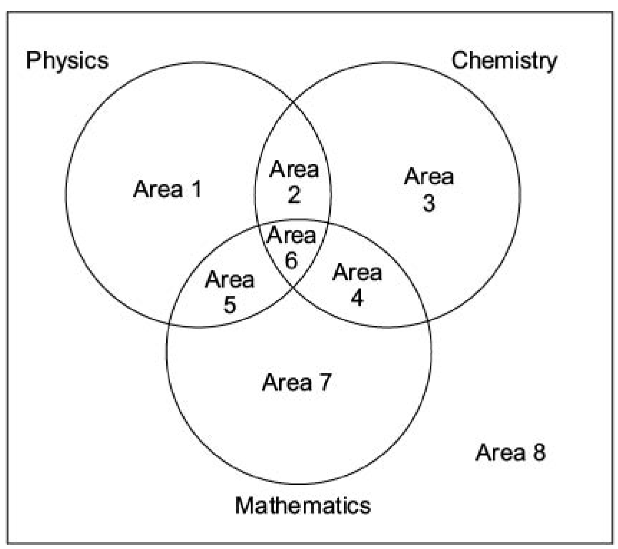
Figure 1: Refers to the situation where there are two attributes *A* and *B*. (Let's say *A* refers to people who passed in Physics and *B* refers to people who passed in Chemistry.) Then the shaded area shows the people who passed both in Physics and Chemistry.



In mathematical terms, the situation is represented as:

Total number of people who passed at least 1 subject = *A* + *B* – *A* « *B*

Figure 2: Refers to the situation where there are three attributes being measured. In the figure below, we are talking about people who passed Physics, Chemistry and/or Mathematics.



In the above figure, the following explain the respective areas:

Area 1: People who passed in Physics only

Area 2: People who passed in Physics and Chemistry only (in other words —people who passed Physics and Chemistry but not Mathematics)

Area 3: People who passed Chemistry only

Area 4: People who passed Chemistry and Mathematics only (also, can be described as people who passed Chemistry and Mathematics but not

Physics)

Area 5: People who passed Physics and Mathematics only (also, can be described as people who passed Physics and Mathematics but not Chemistry)

Area 6: People who passed Physics, Chemistry and Mathematics

Area 7: People who passed Mathematics only

Area 8: People who passed in no subjects

Also take note of the following language which there is normally confusion about:

People passing Physics and Chemistry—Represented by the sum of areas 2 and 6

People passing Physics and Maths—Represented by the sum of areas 5 and 6

People passing Chemistry and Maths—Represented by the sum of areas 4 and 6

People passing Physics—Represented by the sum of the areas 1, 2, 5 and 6

In mathematical terms, this means:

Total number of people who passed at least 1 subject =

P + C + M – P « C – P « M – C « M + P « C « M

Let us consider the following questions and see how these figures work in terms of real time problem solving:

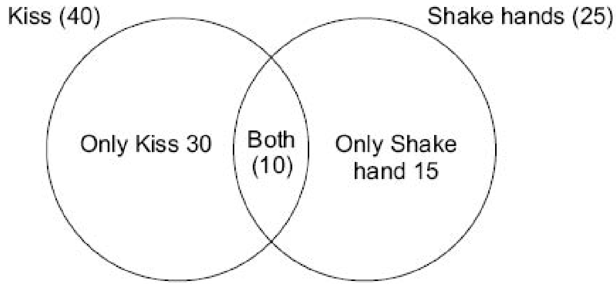
Illustration 1

At the birthday party of Sherry, a baby boy, 40 persons chose to kiss him and 25 chose to shake hands with him. 10 persons chose to both kiss him and shake hands with him. How many persons turned out at the party?

(a) 35 (b) 75

(c) 55 (d) 25

Solution:



From the figure, it is clear that the number of people at the party were 30 + 10 + 15 = 55.

We can of course solve this mathematically as below:

Let n(A) = No. of persons who kissed Sherry = 40

n(B) = No. of persons who shake hands with Sherry = 25

and n(A « B) = No. of persons who shook hands with Sherry and kissed him both = 10

Then using the formula, n(A » B) = n(A) + n(B) – n(A « B)

n(A » B) = 40 + 25 – 10 = 55

Illustration 2

Directions for Questions 1 to 4: Refer to the data below and answer the questions that follow:

In an examination 43% passed in Math, 52% passed in Physics and 52% passed in Chemistry. Only 8% students passed in all the three. 14% passed in Math and Physics and 21% passed in Math and Chemistry and 20% passed in Physics and Chemistry. Number of students who took the exam is 200.

Let Set P, Set C and Set M denotes the students who passed in Physics, Chemistry and Math respectively. Then

1. How many students passed in Math only?

   (a) 16 (b) 32

   (c) 48 (d) 80

2. Find the ratio of students passing in Math only to the students passing in Chemistry only?

   (a) 16:37 (b) 29:32

(c) 16:19 (d) 31:49

3. What is the ratio of the number of students passing in Physics only to the students passing in either Physics or Chemistry or both?

(a) 34/46 (b) 26/84

(c) 49/32 (d) None of these

4. A student is declared pass in the exam only if he/she clears at least two subjects. The number of students who were declared passed in this exam is?

(a) 33 (b) 66

(c) 39 (d) 78

Sol. Let P denote Physics, C denote Chemistry and M denote Maths.

% of students who passed in P and C only is given by

% of students who passed in P and C - % of students who passed all three = 20% – 8% = 12%

% of students who passed in P and M only is given by

% of students who passed in P and M - % of students who passed all three = 14% – 8% = 6%

% of students who passed in M and C only is:

% of students who passed in C and M - % of students who passed all three = 21% – 8% = 13%

So, % of students who passed in P only is given by:

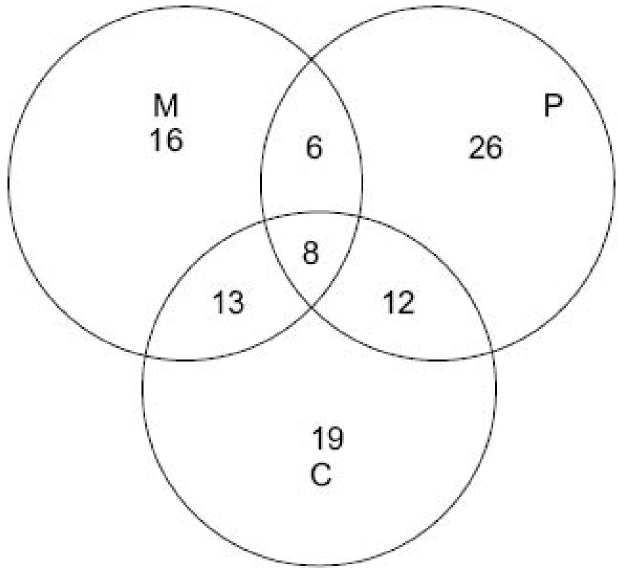
Total no. passing in P – No. Passing in P & C only – No. Passing P & M only – No. Passing in all threeÆ

52% – 12% – 6% – 8%- = 26%

% of students who passed in M only is:

Total no. passing in M – No. Passing in M & C only – No. Passing P & M only – No. Passing in all threeÆ

43% – 13% – 6% – 8%- = 16%



% of students who passed in Chemistry only is

Total no. passing in C – No. Passing in P & C only – No. Passing C & M only – No. Passing in all three Æ

52% – 12% – 13% – 8%– = 19%

The answers are:

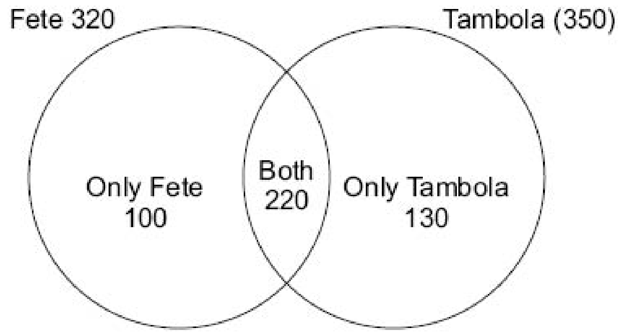
1. Only Math =16% = 32 people. Option (b) is correct.
2. Ratio of Only Math to Only Chemistry = 16:19. Option (c) is correct.
3. 26:84 is the required ratio. Option (b) is correct.
4. 39 % or 78 people. Option (d) is correct.

Illustration 3

In the Mindworkzz club all the members participate either in the Tambola or the Fete. 320 participate in the Fete, 350 participate in the Tambola and 220 participate in both. How many members does the club have?

(a) 410 (b) 550

(c) 440 (d) None of these



The total number of people = 100 + 220 + 130 = 450

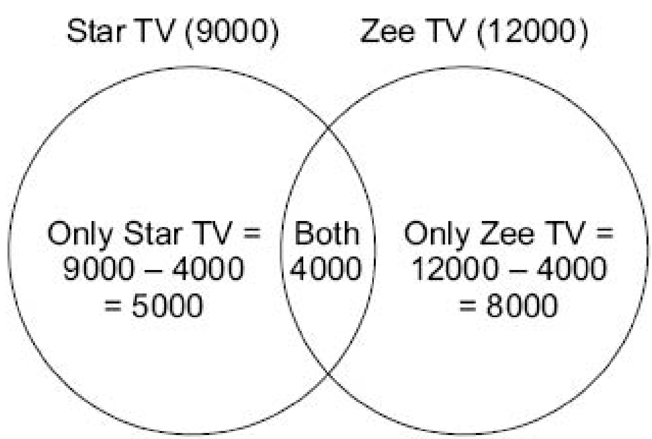
Option (d) is correct.

Illustration 4

There are 20000 people living in Defence Colony, Gurgaon. Out of them 9000 subscribe to Star TV Network and 12000 to Zee TV Network. If 4000 subscribe to both, how many do not subscribe to any of the two?

(a) 3000 (b) 2000

(c) 1000 (d) 4000



The required answer would be 20000 – 5000 – 4000 – 8000 = 3000.

Illustration 5

Directions for Questions 1 to 3: Refer to the data below and answer the questions that follow.

Last year, there were 3 sections in the Catalyst, a mock CAT paper. Out of them 33 students cleared the cut-off in Section 1, 34 students cleared the cut-off in Section 2 and 32 cleared the cut-off in Section 3. 10 students cleared the cut-off in Section 1 and Section 2, 9 cleared the cut-off in Section 2 and Section 3, 8 cleared the cut-off in Section 1 and Section 3. The number of people who cleared each section alone was equal and was 21 for each section.

1. How many cleared all the three sections?

   (a) 3 (b) 6

   (c) 5 (d) 7

2. How many cleared only one of the three sections?

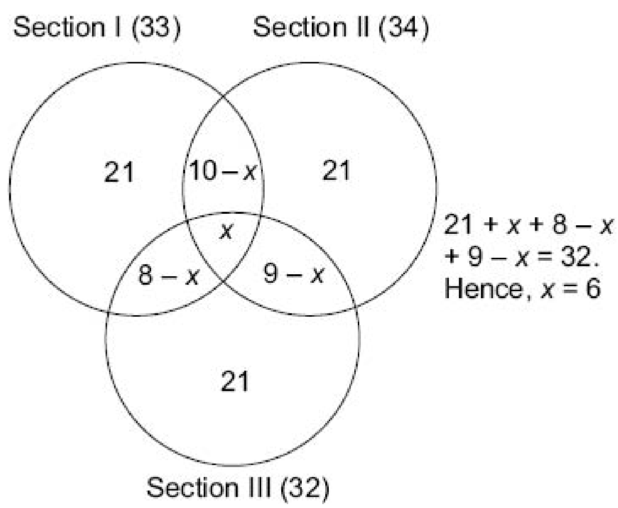
   (a) 21 (b) 63

   (c) 42 (d) 52

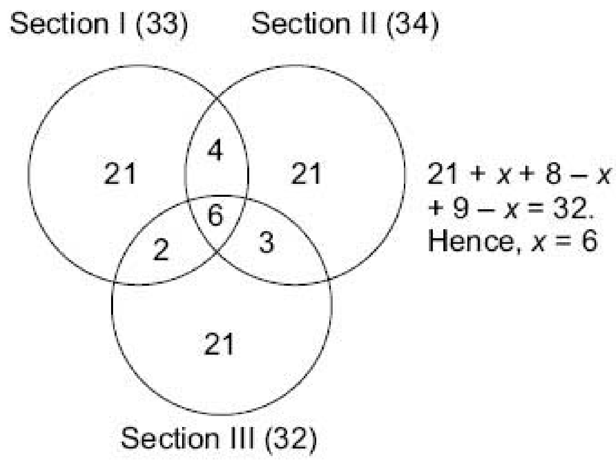
3. The ratio of the number of students clearing the cut-off in one or more of the sections to the number of students clearing the cutoff in Section 1 alone is?

   (a) 78/21 (b) 3

   (c) 73/21 (d) None of these



Since, x = 6, the figure becomes:



The answers would be:

1. 6. Option (b) is correct.
2. 21+21+21=63. Option (b) is correct.
3. (21+21+21+6+4+3+2)/21 = 78/21. Option (a) is correct.

Illustration 6

In a locality having 1500 households, 1000 watch Zee TV, 300 watch NDTV and 750 watch Star Plus. Based on this information answer the questions that follow:

1. The minimum number of households watching Zee TV and Star Plus is:

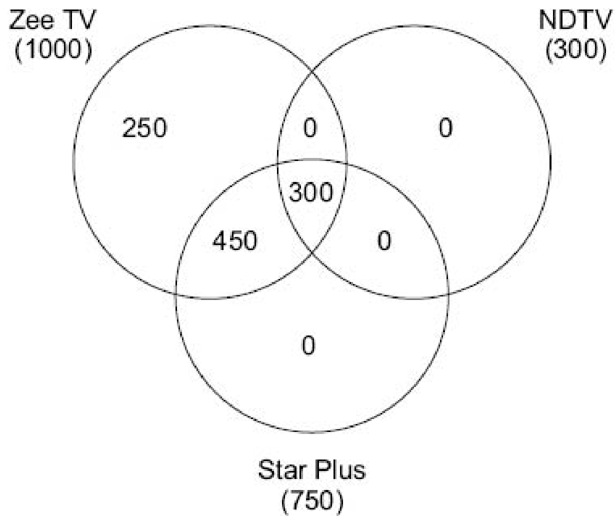
   Logic: If we try to consider each of the households watching Zee TV and Star Plus as independent of each other, we would get a total of 1000 + 750 = 1750 households. However, we have a total of only 1500 households in the locality and hence, there has to be a minimum interference of at least 250 households who would be watching both Zee TV and Star Plus. Hence, the answer to this question is 250.

2. The minimum number of households watching both Zee TV and NDTV is:

   In this case, the number of households watching Zee TV and NDTV can be separate from each other since there is no interference required between the households watching Zee TV and the households watching NDTV as their individual sum (1000 + 300) is smaller than the 1500 available households in the locality. Hence, the answer in this question is 0.

3. The maximum number of households who watch neither of the the three channels is:

   For this to occur the following situation would give us the required solution:



   As you can clearly see from the figure, all the requirements of each category of viewers is fulfilled by the given allocation of 1000 households. In this situation, the maximum number of households who do not watch any of the three channels is visible as 1500 –1000 = 500.

Illustration 7

1. In a school, 90% of the students faced problems in Mathematics, 80% of the students faced problems in Computers, 75% of the students faced problems in Sciences, and 70% of the students faced problems in Social Sciences. Find the minimum percent of the students who faced problems in all four subjects.

   Solution: In order to think about the minimum number of students who faced problems in all four subjects you would need to think of keeping the students who did not face a problem in any of the subjects separate from each other. We know that 30% of the students did not face problems in Social Sciences, 25% of the students did not face problems in Sciences, 20% students did not face problems in computers and 10% students did not face problems in Mathematics. If each of these were separate from each other, we would have 30+25+20+10=85% people who did not face a problem in one of the four subjects. Naturally, the remaining 15% would be students who faced problems in all four subjects. This represents the minimum percentage of students who faced problems in all the four subjects.

2. For the above question, find the maximum possible percentage of students who could have problems in all 4 subjects.

   In order to solve this, you need to consider the fact that 100 (%) people are counted 315 (%) times, which means that there is an extra count of 215 (%). When you put a student into the 'has problems in each of the four subjects' he is one student counted four times — an extra count of 3. Since, 215/3= 71 (quotient) we realise that if we have 71 students who have problems in all four subjects — we will have an extra count of 213 students. The remaining extra count of 2 more can be matched by putting 1 student in 'has problems in 3 subjects' or by putting 2 students in 'has problems in 2 subjects'. Thus, from the extra count angle, we have a limit of 71% students in the 'have problems in all four categories.'

   However, in this problem there is a constraint from another angle — i.e. there are only 70% students who have a problem in Social Sciences — and hence it is not possible for 71% students to have problems in all the four subjects. Hence, the maximum possible percentage of people who have a problem in all four subjects would be 70%.

3. In the above question if it is known that 10% of the students faced none of the above mentioned four problems, what would have been the minimum number of students who would have a problem in all four subjects?

   If there are 10% students who face none of the four problems, we realise that these 10% would be common to students who face no problems in Mathematics, students who face no problems in

Sciences, students who face no problems in Computers and students who face no problems in Social Sciences.

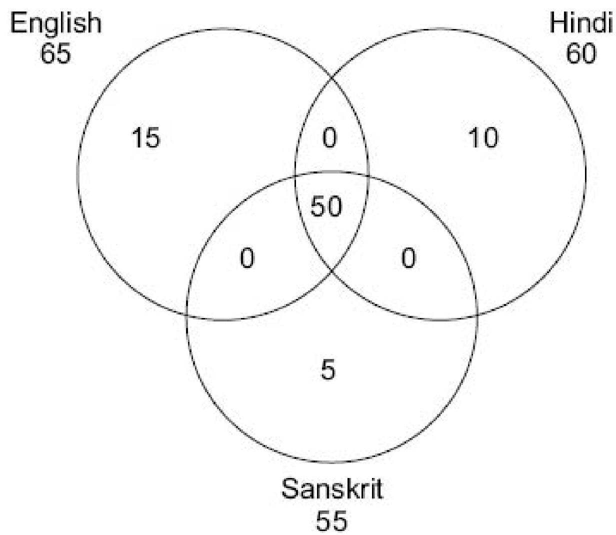
Now, we also know that overall there are 10% students who did not face a problem in Mathematics; 20% students who did not face a problem in computers; 25% students who did not face a problem in Sciences and 30% students who did not face a problem in Social Sciences. The 10% students who did not face a problem in any of the subjects would be common to each of these 4 counts. Out of the remaining 90% students, if we want to identify the minimum number of students who had a problem in all four subjects we will take the same approach as we took in the first question of this set — i.e. we try to keep the students having problems in the individual subjects separate from each other. This would result in: 0% additional students having no problem in Mathematics; 10% additional students having no problem in Computers; 15% additional students having no problem in Sciences and 20% additional students having no problem in Social Sciences. Thus, we would get a total of 45% (0+10+15+20=45) students who would have no problem in one of the four subjects. Thus, the minimum percentage of students who had a problem in all four subjects would be 90 – 45 = 45%.

Illustration 8

In a class of 80 students, each of them studies at least one language — English, Hindi and Sanskrit. It was found that 65 studied English, 60 studied Hindi and 55 studied Sanskrit.

1. Find the maximum number of people who study all three languages.

   This question again has to be dealt with from the perspective of extra counting. In this question, 80 students in the class are counted 65 + 60 + 55 = 180 times — an extra count of 100. If we put 50 people in the all three categories as shown below, we would get the maximum number of students who study all three languages.

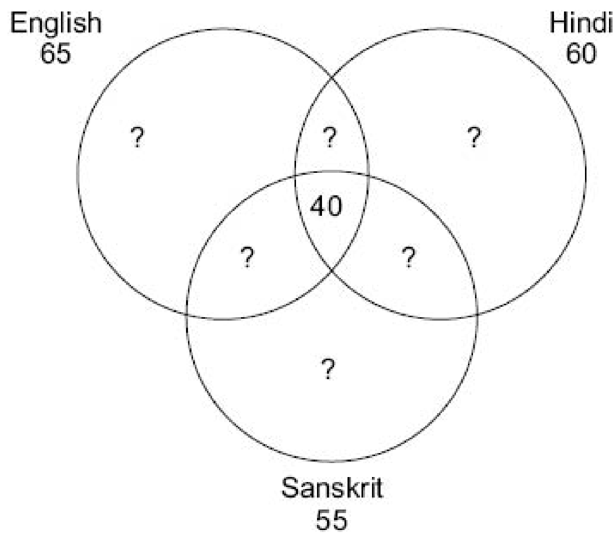


2. Find the minimum number of people who study all three languages.

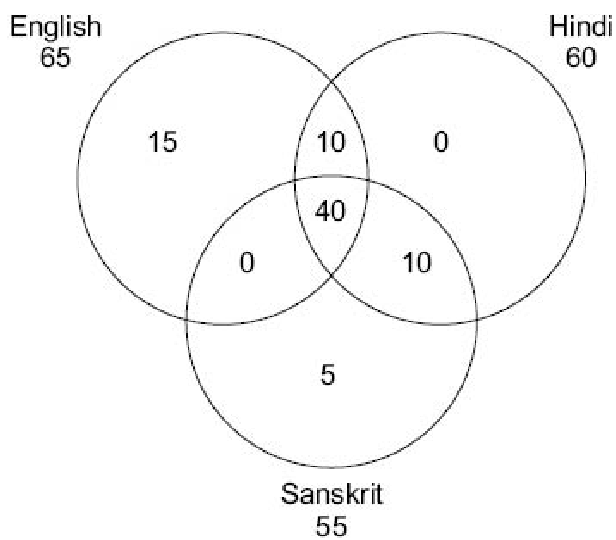
   In order to think about how many students are necessarily in the 'study all three languages' area of the figure (this thinking would lead us to the answer to the minimum number of people who study all three languages) we need to think about how many people we can shift out of the 'study all three category' for the previous question. When we try to do that, the following thought process emerges:

   Step 1: Let's take a random value for the all three categories (less than 50 of course) and see whether the numbers can be achieved. For this purpose we try to start with the value as 40 and see what happens. Before we move on, realise the basic situation in the question remains the same — 80 students have been counted 180 times — which means that there is an extra count of 100 students & also realise that when you put an individual student in the all three categories, you get an extra count of 2, while at the same time when you put an individual student into the 'exactly two languages category', he/she is counted twice — hence an extra count of 1.

   The starting figure we get looks something like this:



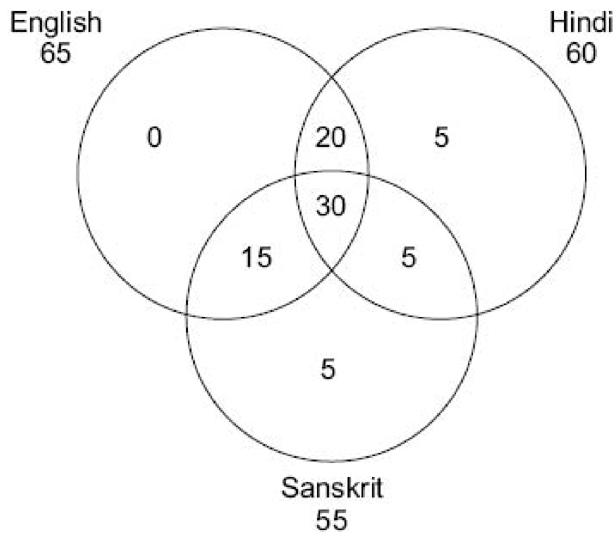
At this point, since we have placed 40 people in the all three categories, we have taken care of an extra count of 40 × 2 = 80. This leaves us with an extra count of 20 more to manage and as we can see in the above figure we have a lot of what can be described as 'slack' to achieve the required numbers. For instance, one solution we can think of from this point is as below:



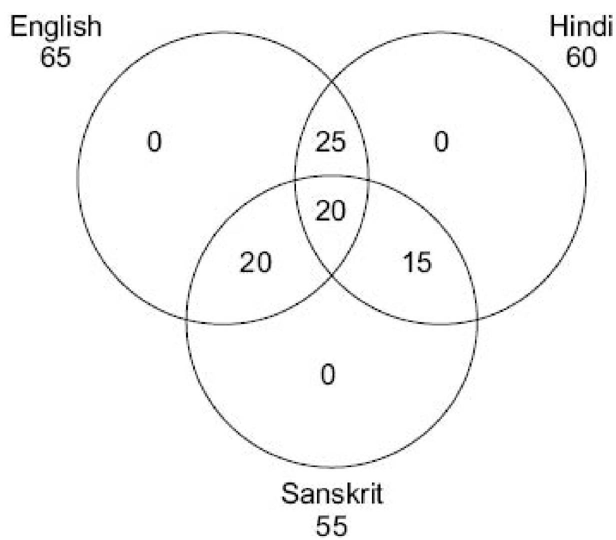
One look at this figure should tell you that the solution can be further optimised by reducing the middle value in the figure since there is still a lot of 'slack' in the figure — in the form of the number of students in the 'exactly one language category'. Also, you can easily see that there are many ways in which this solution could have been achieved with 40 in the middle. Hence, we go in search of a lower value in the middle.

So, we try to take an arbitrary value of 30 to see whether this is still achievable.

In this case we see the following as one of the possible ways to achieve this (again there is a lot of slack in this solution as the ‘only Hindi’ or the ‘only Sanskrit’ areas can be reallocated):



Trying the same solution for 20 in the middle we get the optimum solution:



We realise that this is the optimum solution since there is no ‘slack’ in this solution and hence, there is no scope for re-allocating numbers from one area to another.

Author's note: You might be justifiably thinking how do you do this kind of a random trial and error inside the exam? That's not the point of this question at this place. What I am trying to convey to you is that this is a critical thought structure which you need to have in your mind. Learn it here and do not worry about how you would think inside the exam — remember you would need to check only the four options to choose the best one. We are talking about a multiple choice test here.

Illustration 9

In a group of 120 athletes, the number of athletes who can play Tennis, Badminton, Squash and Table Tennis is 70, 50, 60 and 30 respectively. What is the maximum number of athletes who can play none of the games?

In order to think of the maximum number of athletes who can play none of the games, we can think of the fact that since there are 70 athletes who play tennis, fundamentally there are a maximum of 50 athletes who would be possibly in the 'can play none of the games'. No other constraint in the problem necessitates a reduction of this number and hence the answer to this question is 50.

LEVEL OF DIFFICULTY (I)

Directions for Questions 1 and 2: Refer to the data below and answer the questions that follow:

In the Indian athletic squad sent to the Olympics, 21 athletes were in the triathlon team; 26 were in the pentathlon team; and 29 were in the marathon team. 14 athletes can take part in triathlon and pentathlon; 12 can take part in marathon and triathlon; 15 can take part in pentathlon and marathon; and 8 can take part in all the three games.

1. How many players are there in all?

   (a) 35 (b) 43

   (c) 49 (d) none of these

2. How many were in the marathon team only?

   (a) 10 (b) 14

   (c) 18 (d) 15

Directions for Questions 3 and 4: Refer to the data below and answer the questions that follow.

In a test in which 120 students appeared, 90 passed in History, 65 passed in Sociology and 75 passed in Political Science. 30 students passed in only one subject and 55 students in only two. 5 students passed no subjects.

3. How many students passed in all the three subjects?

   (a) 25 (b) 30

   (c) 35 (d) Data insufficient

4. Find the number of students who passed in at least two subjects.

   (a) 85 (b) 95

   (c) 90 (d) Data insufficient

Directions for Questions 5 to 8: Refer to the data below and answer the questions that follow.

5% of the passengers who boarded Guwahati- New Delhi Rajdhani Express on 20th February, 2002 do not like coffee, tea and ice cream and 10% like all the three. 20% like coffee and tea, 25% like ice cream and coffee and 25% like ice cream and tea. 55% like coffee, 50% like tea and 50 % like ice cream.

5. The number of passengers who like only coffee is greater than the passengers who like only ice cream by

   (a) 50% (b) 100%

   (c) 25% (d) 0

6. The percentage of passengers who like both tea and ice cream but not coffee is

   (a) 15 (b) 5

   (c) 10 (d) 25

7. The percentage of passengers who like at least 2 of the 3 products is

   (a) 40 (b) 45

   (c) 50 (d) 60

8. If the number of passengers is 180, then the number of passengers who like ice cream only is

   (a) 10 (b) 18

   (c) 27 (d) 36

Directions for Questions 9 to 15: Refer to the data below and answer the questions that follow.

In a survey among students at all the IIMs, it was found that 48% preferred coffee, 54% liked tea and 64% smoked. Of the total, 28% liked coffee and tea, 32% smoked and drank tea and 30% smoked and drank coffee. Only 6% did none of these. If the total number of students is 2000 then find

9. The ratio of the number of students who like only coffee to the number who like only tea is

(a) 5:3 (b) 8:9

(c) 2:3 (d) 3:2

10. Number of students who like coffee and smoking but not tea is

(a) 600 (b) 240

(c) 280 (d) 360

11. The percentage of those who like coffee or tea but not smoking among those who like at least one of these is

(a) more than 30 (b) less than 30

(c) less than 25 (d) None of these

12. The percentage of those who like at least one of these is

(a) 100 (b) 90

(c) Nil (d) 94

13. The two items having the ratio 1:2 are

(a) Tea only and tea and smoking only.

(b) Coffee and smoking only and tea only.

(c) Coffee and tea but not smoking and smoking but not coffee and tea.

(d) None of these

14. The number of persons who like coffee and smoking only and the number who like tea only bear a ratio

(a) 1:2 (b) 1:1

(c) 5:1 (d) 2:1

15. Percentage of those who like tea and smoking but not coffee is

(a) 14 (b) 14.9

(c) less than 14 (d) more than 15

16. 30 monkeys went to a picnic. 25 monkeys chose to irritate cows while 20 chose to irritate buffaloes. How many chose to irritate both buffaloes and cows?

(a) 10 (b) 15

(c) 5 (d) 20

Directions for Questions 17 to 20: Refer to the data below and answer the questions that follow.

In the CBSE Board Exams last year, 53% passed in Biology, 61% passed in English, 60% in Social Studies, 24% in Biology & English, 35% in English & Social Studies, 27% in Biology and Social Studies and 5% in none.

17. Percentage of passes in all subjects is

    (a) Nil (b) 12

    (c) 7 (d) 10

18. If the number of students in the class is 200, how many passed in only one subject?

    (a) 48 (b) 46

    (c) more than 50 (d) less than 40

19. If the number of students in the class is 300, what will be the % change in the number of passes in only two subjects , if the original number of students is 200?

    (a) more than 50% (b) less than 50%

    (c) 50. (d) None of these

20. What is the ratio of percentage of passes in Biology and Social Studies but not English in relation to the percentage of passes in Social Studies and English but not Biology?

    (a) 5:7 (b) 7:5

    (c) 4:5 (d) None of these

Directions for Questions 21 to 25: Refer to the data below and answer the questions that follow.

In the McGraw-Hill Mindworkzz Quiz held last year, participants were free to choose their respective areas from which they were asked questions. Out of 880 participants, 224 chose Mythology, 240 chose Science and 336 chose

Sports, 64 chose both Sports and Science, 80 chose Mythology and Sports, 40 chose Mythology and Science and 24 chose all the three areas.

21. The percentage of participants who did not choose any area is

    (a) 23.59% (b) 30.25%

    (c) 37.46% (d) 27.27%

22. Of those participating, the percentage who choose only one area is

    (a) 60% (b) more than 60%

    (c) less than 60% (d) more than 75%

23. Number of participants who chose at least two areas is

    (a) 112 (b) 24

    (c) 136 (d) None of these

24. Which of the following areas shows a ratio of 1:8?

    (a) Mythology & Science but not Sports: Mythology only

    (b) Mythology & Sports but not Science: Science only

    (c) Science: Sports

    (d) None of these

25. The ratio of students choosing Sports & Science but not Mythology to Science but not Mythology & Sports is

    (a) 2:5 (b) 1:4

    (c) 1:5 (d) 1:2

Directions for Questions 26 to 30: Refer to the data below and answer the questions that follow.

The table here gives the distribution of students according to professional courses.

Courses	*STUDENTS*			
	English		*Maths*	
	MALES	*FEMALES*	*MALES*	*FEMALES*

Part-time MBA	30	10	50	10
Full-time MBA only	150	8	16	6
CA only	90	10	37	3
Full time MBA & CA	70	2	7	1

26. What is the percentage of Math students over English students?

    (a) 50.4 (b) 61.4

    (c) 49.4 (d) None of these

27. The average number of females in all the courses is (count people doing full-time MBA and CA as a separate course)

    (a) less than 12 (b) greater than 12

    (c) 12 (d) None of these

28. The ratio of the number of girls to the number of boys is

    (a) 5:36 (b) 1:9

    (c) 1:7.2 (d) None of these

29. The percentage increase in students of full-time MBA only over CA only is

    (a) less than 20 (b) less than 25

    (c) less than 30 (d) more than 30

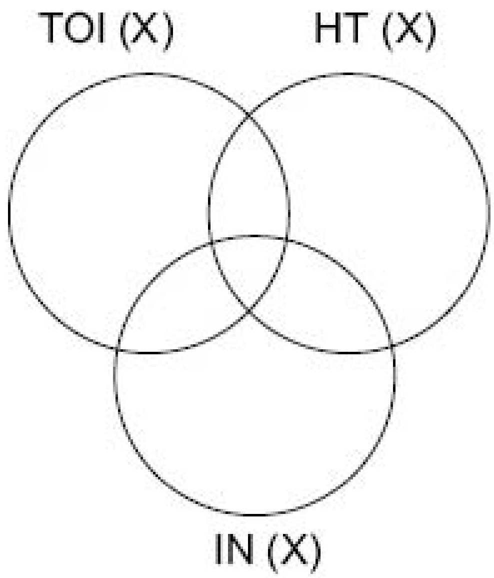
30. The number of students doing full-time MBA or CA is

    (a) 320 (b) 80

    (c) 160 (d) None of these.

Directions for Questions 31 to 34: Refer to the data below and answer the questions that follow:

A newspaper agent sells The TOI, HT and IN in equal numbers to 302 persons. Seven get HT & IN, twelve get The TOI & IN, nine get The TOI & HT and three get all the three newspapers. The details are given in the Venn diagram:



31. How many get only one paper?

(a) 280 (b) 327

(c) 109 (d) None of these

32. What percent get The TOI and The HT but not The IN

(a) more than 65% (b) less than 60%

(c) @ 64% (d) None of these.

33. The number of persons buying The TOI and The HT only, The TOI and The IN only and The HT and The IN only are in the ratio of

(a) 6:4:9 (b) 6:9:4

(c) 4:9:6 (d) None of these

34. The difference between the number reading The HT and The IN only and HT only is

(a) 77 (b) 78

(c) 83 (d) None of these.

35. A group of 78 people watch Zee TV, Star Plus or Sony. Of these, 36 watch Zee TV, 48 watch Star Plus and 32 watch Sony. If 14 people watch both Zee TV and Star Plus, 20 people watch both Star Plus and Sony, and 12 people watch both Sony and Zee TV find the ratio

of the number of people who watch only Zee TV to the number of people who watch only Sony.

(a) 9:4 (b) 3:2

(c) 5:3 (d) 7:4

Directions for Questions 36 and 37: Answer the questions based on the following information.

The following data was observed from a study of car complaints received from 180 respondents at Colonel Verma's car care workshop, viz., engine problem, transmission problem or mileage problem. Of those surveyed, there was no one who faced exactly two of these problems. There were 90 respondents who faced engine problems, 120 who faced transmission problems and 150 who faced mileage problems.

36. How many of them faced all the three problems?

(a) 45 (b) 60

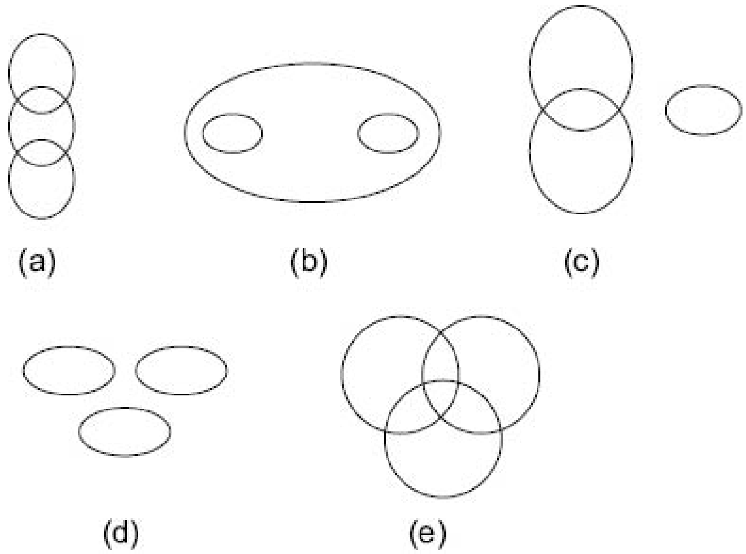
(c) 90 (d) 20

37. How many of them faced either transmission problems or engine problems?

(a) 30 (b) 60

(c) 90 (d) 40

Directions for Questions 38 to 42: given below are five diagrams one of which describes the relationship among the three classes given in each of the five questions that follow. You have to decide which of the diagrams is the most suitable for a particular set of classes.



38. Elephants, tigers, animals
39. Administrators, Doctors, Authors
40. Platinum, Copper, Gold
41. Gold, Platinum, Ornaments
42. Television, Radio, Mediums of Entertainment
43. Seventy percent of the employees in a multinational corporation have VCD players, 75 percent have microwave ovens, 80 percent have ACs and 85 percent have washing machines. At least what percentage of employees has all four gadgets?
    (a) 15
    (b) 5
    (c) 10
    (d) Cannot be determined

LEVEL OF DIFFICULTY (II)

1. At the Rosary Public School, there are 870 students in the senior secondary classes. The school is widely known for its science education and there is the facility for students to do practical training in any of the 4 sciences – viz: Physics, Chemistry, Biology or Social Sciences. Some students in the school however have no interest in the sciences and hence do not undertake practical training in any of the four sciences. While considering the popularity of various choices for opting for practical training for one or more of the choices offered, Mr. Arvindaksham, the school principal noticed something quite extraordinary. He noticed that for every student in the school who opts for practical training in at least M sciences, there are exactly three students who opt for practical training in at least (M – 1) sciences, for M = 2, 3 and 4. He also found that the number of students who opt for all four sciences was half the number of students who opt for none. Can you help him with the answer to "How many students in the school opt for exactly three sciences?"

   (a) 30 (b) 60

   (c) 90 (d) None of these

2. A bakery sells three kinds of pastries—pineappple, chocolate and black forest. On a particular day, the bakery owner sold the following number of pastries: 90 pineapple, 120 chocolate and 150 black forest. If none of the customers bought more than two pastries of each type, what is the minimum number of customers that must have visited the bakery that day?

   (a) 80 (b) 75

   (c) 60 (d) 90

3. Gauri Apartment housing society organised annual games, consisting of three games – viz: snooker, badminton and tennis. In all, 510 people were members in the apartments' society and they were invited to participate in the games — each person participating in as many games as he/she feels like. While viewing the statistics of the performance. Mr Capoor realised the following acts. The number of people who participated in at least two games was 52% more than those who participated in exactly one game. The number of people participating in 1, 2 or 3 games respectively was at least equal to 1. Being a numerically inclined person, he further noticed an interesting thing — the number of people who did not participate in any of the three games was the minimum possible integral value with these conditions. What was the maximum number of people who participated in exactly three games?

   (a) 298 (b) 300

   (c) 303 (d) 304

4. A school has 180 students in its senior section where foreign languages are offered to students as part of their syllabus. The foreign languages offered are: French, German and Chinese and the numbers of people studying each of these subjects are 80, 90 and 100 respectively. The number of students who study more than one of the three subjects is 50% more than the number of students who study all the three subjects. There are no students in the school who study none of the three subjects. Then how many students study all three foreign languages?

   (a) 18 (b) 24

   (c) 36 (d) 40

Directions for Questions 5 and 6: Answer the questions on the basis of the information given below.

In the second year, the Hampard Business School students are offered a choice of the specialisations they wish to study from amongst only three specialisations —Marketing, Finance and HR. The number of students who have specialised in only Marketing, only Finance and only HR are all numbers in an Arithmetic Progression—

in no particular order. Similarly, the number of students specialising in exactly two of the three types of subjects are also numbers that form an Arithmetic Progression.

The number of students specialising in all three subjects is one-twentieth of the number of students specialising in only Finance which in turn is half of the number of students studying only HR. The number of students studying both Marketing and Finance is 15, whereas the number of students studying both Finance and HR is 19. The number of students studying HR is 120, which is more than the number of students studying Marketing (which is a 2 digit number above 50). It is known that there are exactly 4 students who opt for none of these specialisations and opt only for general subjects.

5. What is the total number of students in the batch?

   (a) 223 (b) 233

   (c) 237 (d) Cannot be determined

6. What is the number of students specialising in both Marketing and HR?

   (a) 11 (b) 21

   (c) 23 (d) Cannot be determined

Directions for Questions 7 to 9: In the Stafford Public School, students had an option to study none or one or more of three foreign languages viz: French, Spanish and German. The total student strength in the school was 2116 students out of which 1320 students studied French and 408 students studied both French and Spanish. The number of people who studied German was found to be 180 higher than the number of students who studied Spanish. It was also observed that 108 students studied all three subjects.

7. What is the maximum possible number of students who did not study any of the three languages?

   (a) 890 (b) 796

   (c) 720 (d) None of these

8. What is the minimum possible number of students who did not study any of the three languages?

   (a) 316 (b) 0

   (c) 158 (d) None of these

9. If the number of students who used to speak only French was 1 more than the number of people who used to speak only German, then what could be the maximum number of people who used to speak only Spanish?

   (a) 413 (b) 398

   (c) 403 (d) 431

Directions for Questions 10 to 13: In the Vijayantkhand sports stadium, athletes choose from four different racquet games (apart from athletics which is compulsory for all). These are Tennis, Table Tennis, Squash and Badminton. It is known that 20% of the athletes practising there are not choosing any of the racquet sports. The four games given here are played by 460, 360, 360 and 440 students respectively. The number of athletes playing exactly 2 racquet games for any combination of two racquet games is 40. There are 60 athletes who play all the four games but in a strange coincidence, it was noticed that the number of people playing exactly 3 games was also equal to 20 for each combination of 3 games.

10. What is the number of athletes in the stadium?

    (a) 1140 (b) 1040

    (c) 1200 (d) 1300

11. What is the number of athletes in the stadium who play either only squash or only Tennis?

    (a) 120 (b) 220

(c) 340 (d) 440

12. How many athletes in the stadium participate in only athletics?

(a) 160 (b) 1040

(c) 260 (d) 220

13. If all the athletes were compulsorily asked to add one game to their existing list (except those who were already playing all the four games) — then what will be the number of athletes who would be playing all 4 games after this change?

(a) 80 (b) 100

(c) 120 (d) 140

Directions for Questions 14 and 15: Answer the questions on the basis of the following information.

In the Pattbhiraman family, a clan of 192 individuals, each person has at least one of the three Pattabhiraman characteristics—Blue eyes, Blonde hair, and sharp mind. It is also known that:

(i) The number of family members who have only blue eyes is equal to the number of family members who have only sharp minds and this number is also equal to twice the number of family members who have blue eyes and sharp minds but not blonde hair.

(ii) The number of family members who have exactly two of the three features is 50.

(iii) The number of family members who have blonde hair is 62.

(iv) Among those who have blonde hair, 26 family members have at least two of the three characteristics.

14. If the number of family members who have blue eyes is the maximum amongst the three characteristics, then what is the maximum possible number of family members who have both sharp minds and blonde hair but do not have blue eyes?

(a) 11 (b) 10

(c) 12 (d) Cannot be determined

15. Which additional piece of information is required to find the exact number of family members who have blonde hair and blue eyes but not sharp minds?

(a) The number of family members who have exactly one of the three characterisitcs is 140.

(b) Only two family members have all three characteristics.

(c) The number of family members who have sharp minds is 89.

(d) The number of family members who have only sharp minds is 52.

16. In a class of 97 students, each student plays at least one of the three games – Hockey, Cricket and Football. If 47 play Hockey, 53 play Cricket, 72 play Football and 15 play all the three games, what is the number of students who play exactly two games?

(a) 38 (b) 40

(c) 42 (d) 45

Directions for Questions 17 to 19: Answer the questions on the basis of the information given below.

In the ancient towns of Mohenjo Daro, a survey found that students were fond of three kinds of cold drinks (Pep, Cok and Thum). It was also found that there were three kinds of beverages that they liked (Tea, Cof and ColdCof).

The popluation of these towns was found to be 400000 people in all—and the survey was conducted on 10% of the population. Mr. Yadav, a data analyst observed the following things about the survey:

(i) The number of people in the survey who like exactly two cold drinks is five times the number of people who like all the three cold drinks.

(ii) The sum of the number of people in the survey who like Pep and 42% of those who like Cok but not Pep is equal to the number of people who like Tea.

(iii) The number of people in the survey who like Cof is equal to the sum of $3/8^{th}$ of those who like Cok and ½ of those who like Thum. This number is also equal to the number who like ColdCof.

(iv) 18500 people surveyed like Pep;

(v) 15000 like all the beverages and 3500 like all the cold drinks;

(vi) 14000 do not like Pep but like Thum;

(vii) 11000 like Pep and exactly one more cold drink;

(viii) 6000 like only Cok and the same number of people like Pep and Thum but not Cok.

17. The number of people in the survey who do like at least one of the three cold drinks?

(a) 38500 (b) 31500

(c) 32500 (d) 39500

18. What is the maximum number of people in the survey who like none of the three beverages?

(a) 24000 (b) 16000

(c) 12000 (d) Cannot be determined

19. What is the maximum number of people in the survey who like at least one of the three beverages?

(a) 7000 (b) 32,000

(c) 33000 (d) Cannot be determined

20. In a certain class of students, the number of students who drink only tea, only coffee, both tea and coffee and neither tea nor coffee are x, $2x$, $\frac{57}{x}$ and $\frac{57}{3x}$ respectively. The number of people who drink coffee can be

(a) 41 (b) 40

(c) 59 (d) Both a and c.

ANSWER KEY

Level of Difficulty (I)

1. (b)	2. (a)	3. (b)	4. (a)
5. (b)	6. (a)	7. (c)	8. (b)
9. (c)	10. (b)	11. (a)	12. (d)
13. (c)	14. (b)	15. (a)	16. (b)
17. (c)	18. (c)	19. (c)	20. (a)
21. (d)	22. (c)	23. (c)	24. (a)
25. (b)	26. (d)	27. (b)	28. (b)
29. (c)	30. (a)	31. (a)	32. (c)
33. (b)	34. (d)	35. (a)	36. (c)
37. (b)	38. (b)	39. (e)	40. (d)
41. (a)	42. (b)	43. (c)	

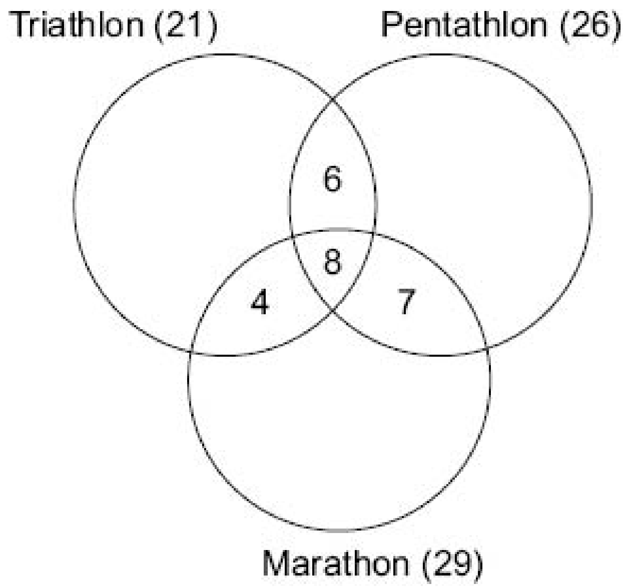
Level of Difficulty (II)

1. (b)	2. (b)	3. (c)	4. (c)
5. (d)	6. (c)	7. (b)	8. (b)
9. (d)	10. (d)	11. (c)	12. (c)
13. (d)	14. (a)	15. (c)	16. (d)
17. (a)	18. (b)	19. (c)	20. (d)

Solutions and Shortcuts

Level of Difficulty (I)

Solutions for Questions 1 and 2: Since there are 14 players who are in triathlon and pentathlon, and there are 8 who take part in all three games, there will be 6 who take part in only triathlon and pentathlon. Similarly, Only triathlon and marathon = 12 – 8=4 & Only Pentathlon and Marathon = 15 – 8 = 7.



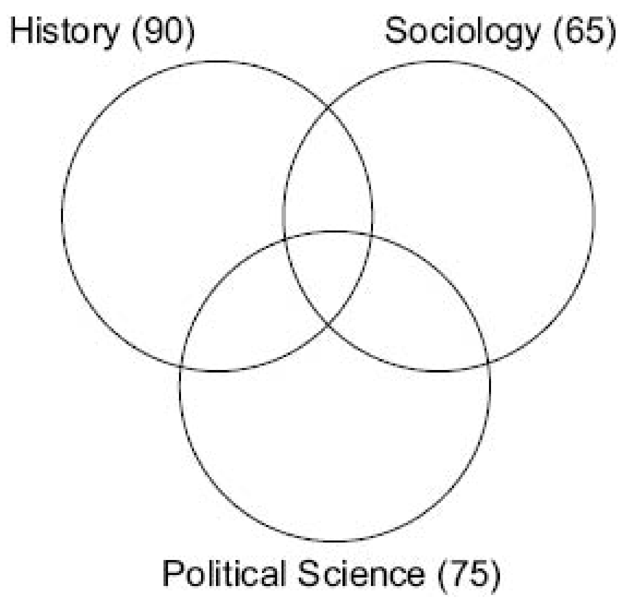
The figure above can be completed with values for each sport (only) plugged in:

The answers would be:

3 + 6 + 8 + 4 + 5 + 7 + 10 = 43. Option (b) is correct.

Option (a) is correct.

Solutions for Questions 3 and 4:



The given situation can be read as follows:

115 students are being counted 75+65+90= 230 times.

This means that there is an extra count of 115. This extra count of 115 can be created in 2 ways.

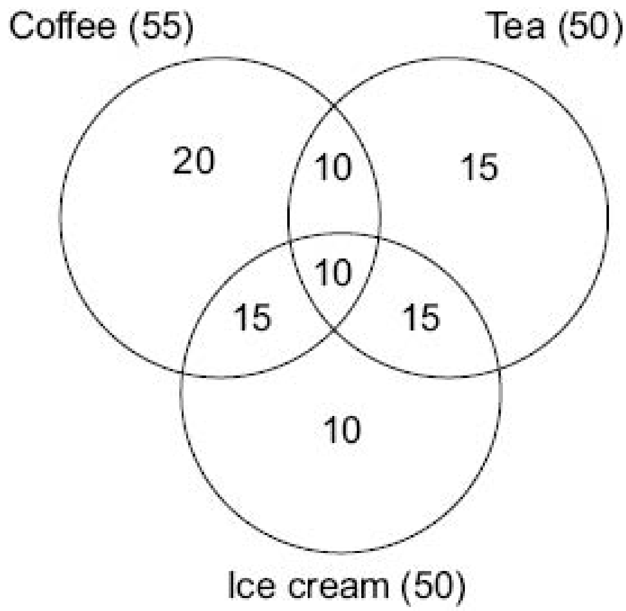
A. By putting people in the 'passed exactly two subjects' category. In such a case each person would get counted 2 times (double counted), i.e., an extra count of 1.
B. By putting people in the 'all three' category, each person put there would be triple counted. 1 person counted 3 times – meaning an extra count of 2 per person.

The problem tells us that there are 55 students who passed exactly two subjects. This means an extra count of 55 would be accounted for. This would leave an extra count of 115 – 55 = 60 more to be accounted for by 'passed all three' category. This can be done by using 30 people in the 'all 3' category.

Hence, the answers are:

3. Option (b)
4. Option (a)

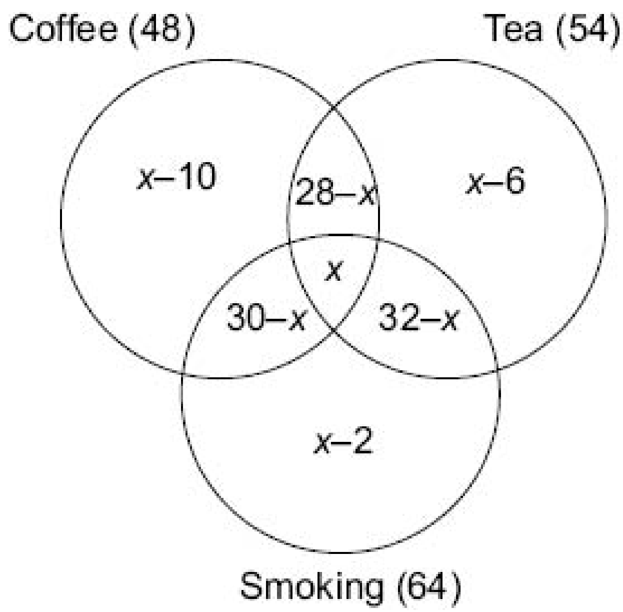
Solutions for Questions 5 to 8: Based on the information provided we would get the following figure:



The answers could be read off the figure as:

5. [(20 – 10)/10] * 100 = 100%. Option (b) is correct.
6. 15% (from the figure). Option (a) is correct.
7. 10+10+15+15=50%. Option (c) is correct.
8. Only ice cream is 10% of the total. Hence, 10% of 180 =18. Option (b) is correct.

Solutions for Questions 9 to 15: If you try to draw a figure for this question, the figure would be something like:



We can then solve this as:

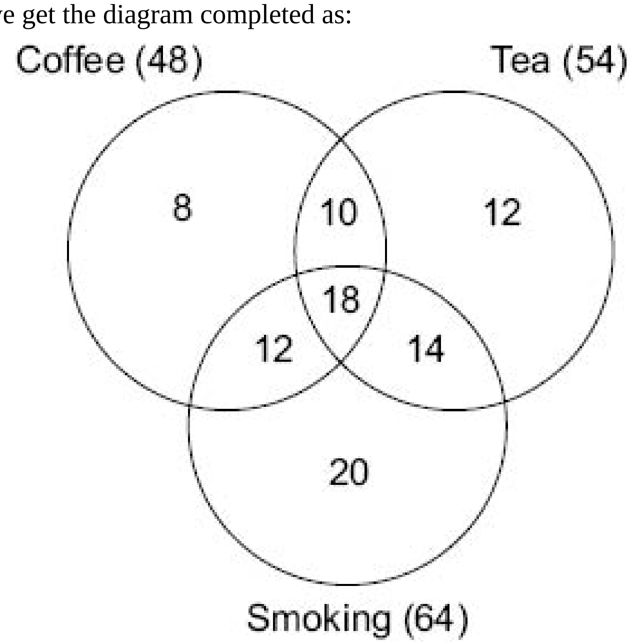
$x - 10 + 28 - x + x + 30 - x + x + 2 + 32 - x + x - 6 = 94$ Æ $x + 76 = 94$ Æ $x = 18$.

Note: In this question, since all the values for the use of the set theory formula are given, we can find the missing value of students who liked all three as follows:

94 = 48 + 54 + 64 – 28 – 32 – 30 + All three Æ All three = 18

As you can see this is a much more convenient way of solving this question, and the learning you take away for the 3 circle situation is that whenever you have all the values known and the only unknown value is the center value – it is wiser and more efficient to solve for the unknown using the formula rather than trying to solve through a venn diagram.

Based on this value of *x* we get the diagram completed as:



The answers then are:

9. 8:12 = 2:3 Æ Option (c) is correct.

10. 12 % of 2000 = 240. Option (b) is correct.
11. 30/94 Æ more than 30%. Option (a) is correct.
12. 94%. Option (d) is correct.
13. Option (c) is correct as the ratio turns out to be 10:20 in that case.
14. 12:12 = 1:1 Æ Option (b) is correct.
15. 14%. Option (a) is correct.
16. 30 = 25 + 20 – x Æ x = 15. Option (b) is correct.

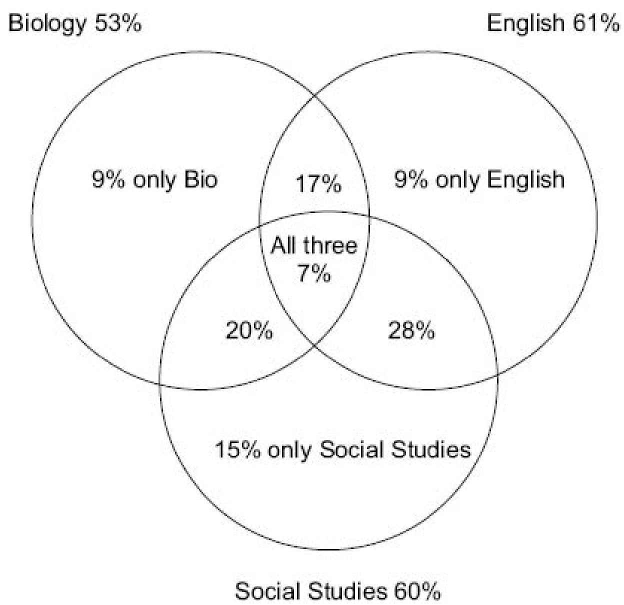
Solutions for Questions 17 to 20:

Let people who passed all three be x. Then:

53 + 61 + 60 – 24 – 35 – 27 + x = 95

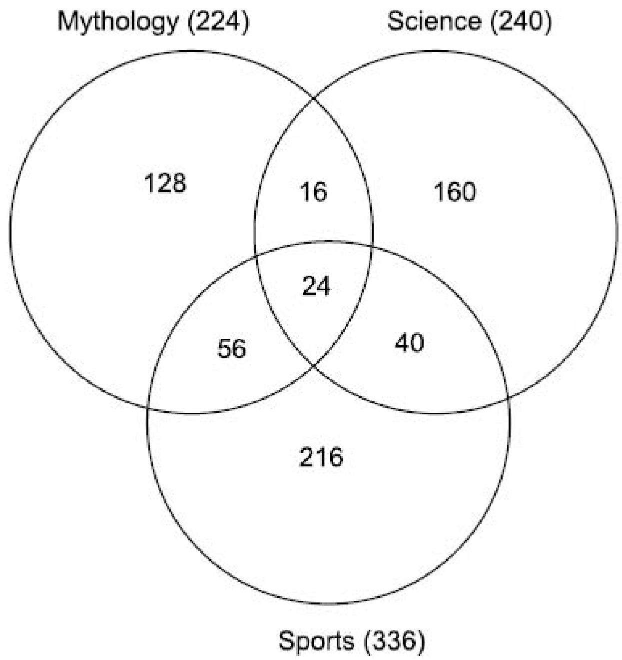
Æ x = 7.

The venn diagram in this case would become:



17. Option (c) is correct.
18. 33% of 200 = more than 50. Option (c) is correct.
19. If the number of students is increased by 50%, the number of students in each category would also be increased by 50%. Option (c) is correct.
20. 20:28 = 5:7. Option (a) is correct.

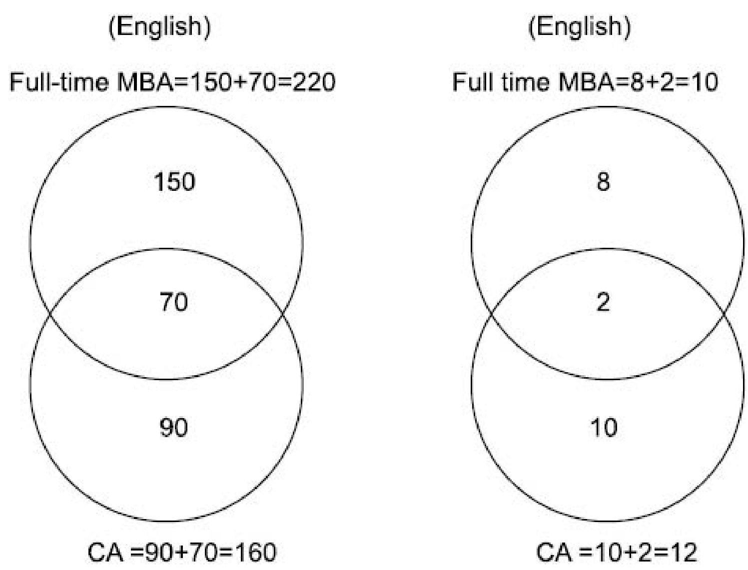
Solutions for Questions 21 to 25: The following figure would emerge on using all the information in the question:

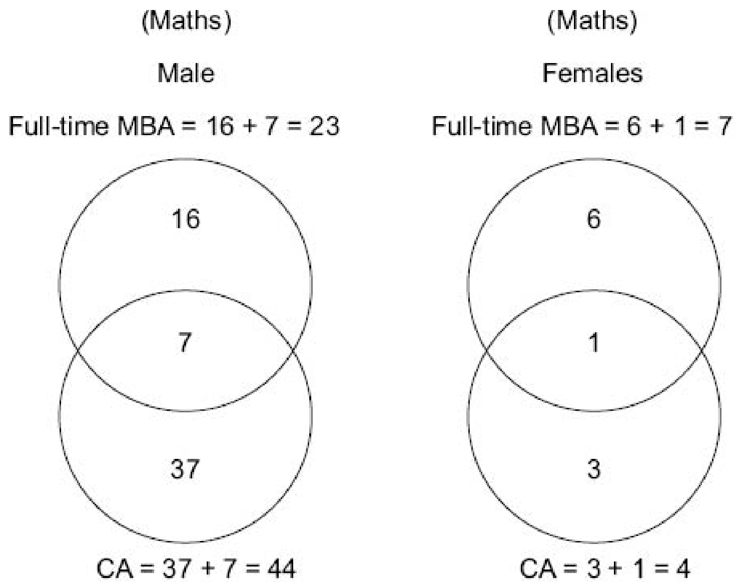


The answers would then be:

21. 240/880 = 27.27%. Option (d) is correct.
22. 504/880 = 57.27%. Hence, less than 60. Option (c) is correct.
23. 40 + 16 + 56 + 24 = 136. Option (c) is correct.
24. Option a gives us 16:128 = 1:8. Option (a) is hence correct.
25. 40:160 Æ 1:4. Option (b) is correct.

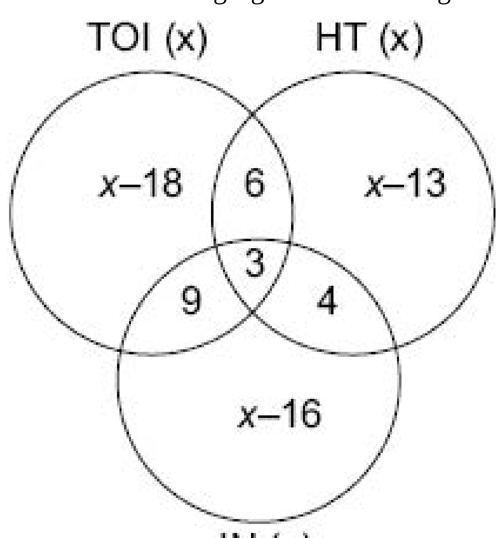
Solutions for Questions 26 to 30: The following Venn diagrams would emerge:





26. Math Students = 130. English Students =370
    130/370 = 35.13%. Option (d) is correct.
27. Number of Female Students = 10 + 8 + 10 + 2 + 10 + 6 + 3 + 1 = 50. Average number of females per course = 50/3 = 16.66. Option (b) is correct.
28. 50:450 = 1:9. Option (b) is correct.
29. 40/140 Æ 28.57%. Option (c) is correct.
30. From the figures, this value would be 150+8+ 90 + 10 + 16+6+37+3= 320. Option (a) is correct.

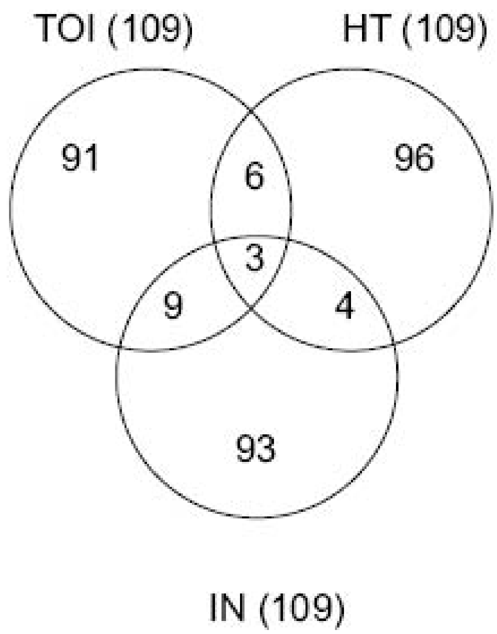
Solutions for Questions 31 to 34: The following figure would emerge-



Based on this figure we have:

$x + x - 13 + 4 + x - 16 = 302$ Æ $3x - 25 = 302$ Æ $x = 327$. Hence, $x = 109$.

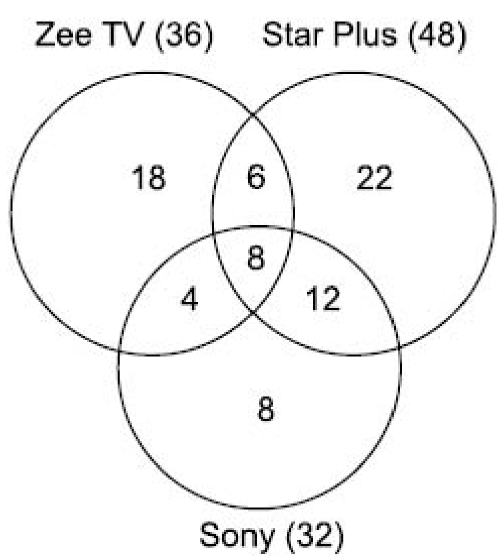
Consequently the figure becomes:



The answers are:

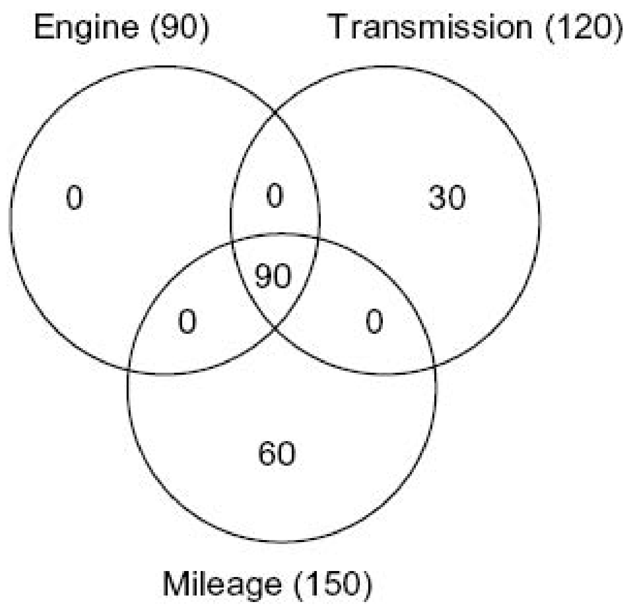
31. 91 + 93 + 96 = 280. Option (a) is correct.
32. 193/302 @ 64%
33. 6:9:4 is the required ratio. Option (b) is correct.
34. 96 – 4 = 92. Options (d) is correct.
35. 78 = 36 + 48 + 32 – 14 – 20 – 12 + x Æ x = 8.

The figure for this question would become:

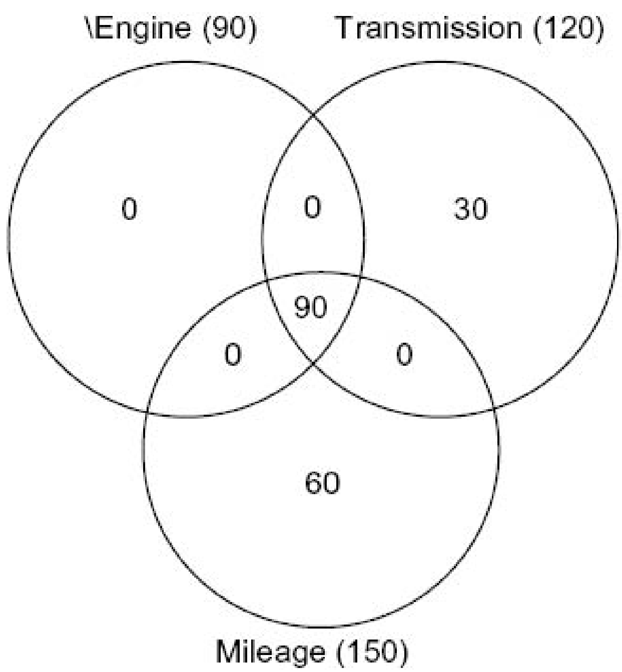


Required ratio is 18:8 Æ 9:4. Option (a) is correct.

36. Option (c)

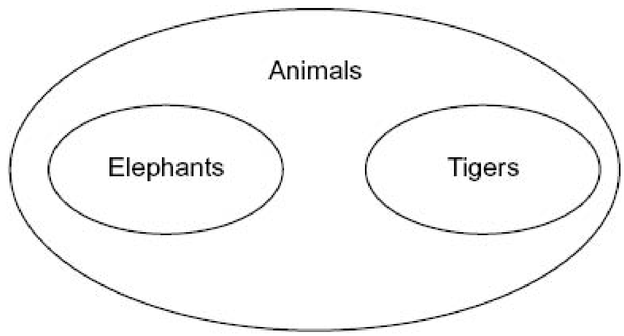


37. There are 30 such people. Option (b) is correct.

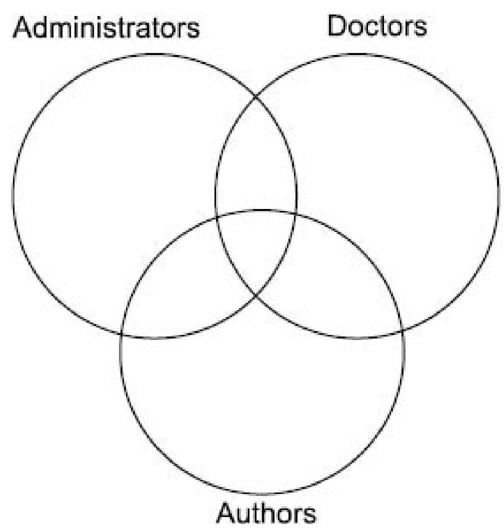


Solutions for Questions 38 to 42:

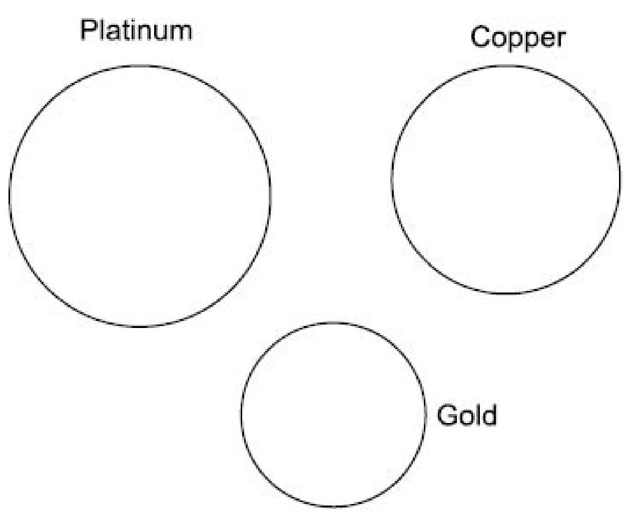
38. Option (b) is correct



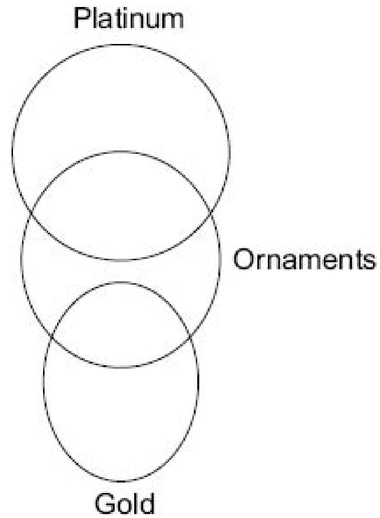
39. Option (e) is correct.



40. Option (d) is correct.



41. Option (a) is correct.



42. Option (b) is correct

Solution for Question 43:

43. The least percentage of people with all 4 gadgets would happen if all the employees who are not having any one of the four objects is mutually exclusive.

    Thus, 100 – 30 – 25 – 20 – 15 = 10

    Option (c) is correct

Level of Difficulty (II)

1. The key to think about this question is to understand what is meant by the statement —"for every student in the school who opts for practical training in at least M sciences, there are exactly three students who opt for practical training in at least (M – 1) sciences, for M = 2, 3 and 4"

   What this statement means is that if there are x students who opt for practical training in all 4 sciences, there would be $3x$ students who would opt for practical training in at least 3 sciences. Since opting for at least 3 sciences includes those who opted for exactly 3 sciences and those who opted for exactly 4 sciences—we can conclude from this that:

   The number of students who opted for exactly 3 sciences = Number of students who opted for at least 3 sciences – Number of students who opted for all 4 sciences = $3x - x = 2x$

   Thus, the number of students who opted for various number of science practicals can be summarised as below:

	Number of students who opted for at least n subjects	*Number of students who opted for exactly n subjects*
$n = 4$	x	x
$n = 3$	$3x$	$2x$
$n = 2$	$9x$	$6x$
$n = 1$	$27x$	$18x$

Also, number of students who opt for none of the sciences = twice the number of students who opt for exactly 4 sciences = $2x$.

Based on these deductions we can clearly identify that the number of students in the school would be: $x + 2x + 6x + 18x + 2x = 29x = 870$ Æ $x = 30$.

Hence, number of students who opted for exactly three sciences = $2x = 60$

2. (b) In order to estimate the minimum number of customers we need to assume that each customer must have bought the maximum number of pastries possible for him to purchase.

Since, the maximum number of pastries an individual could purchase is constrained by the information that no one bought more than two pastries of any one kind—this would occur under the following situation—First 45 people would buy 2 pastries of all three kinds, which would completely exhaust the 90 pineapple pastries and leave the bakery with 30 chocolate and 60 black forest pastries. The next 15 people would buy 2 pastries each of the available kinds and after this we would be left with 30 black forest pastries. 15 people would buy these pastries, each person buying 2 pastries each.

Thus, the total number of people (minimum) would be: 45 + 15 + 15 = 75.

3. (c) Let the number of people who participated in 0, 1, 2 and 3 games be A, B, C, D respectively. Then from the information we have:

C+D = 1.52 × B (Number of people who participate in at least 2 games is 52% higher than the number of people who participate in exactly one game)

A + B + C + D = 510 (Number of people invited to participate in the games is 510)

This gives us: A + 2.52B = 510 Æ B = $\frac{25}{63}$ (510 – A)

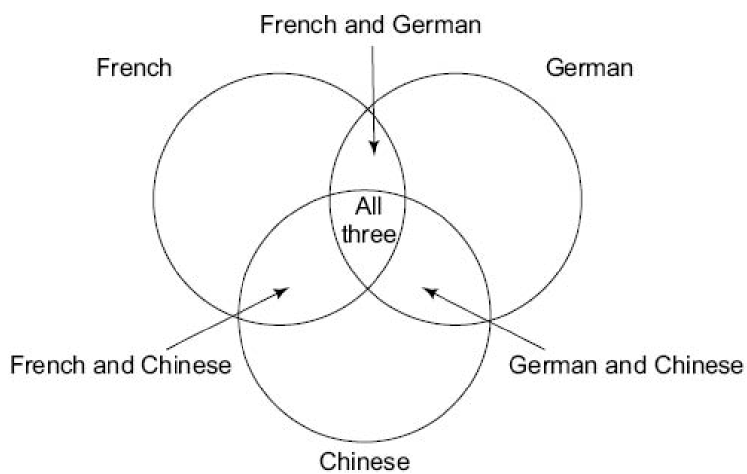
For A to be minimum, 510 – A should give us the largest multiple of 63. Since, 63×8 = 504 we have A = 6.

Also, 2.52B = 504, so B = 200 and C+D = 1.52B = 304.

For number of people participating in exactly 3 games to be maximum, the number of people participating in exactly 2 games has to be minimised and made equal to 1. Thus, the required answer = 304 – 1 = 303.

4. (c) In order to think about this question, the best way is to use the process of slack thinking. In this question, we have 180 students counted 270 times. This means that there is an extra count of 90 students. In a three circle venn diagram, extra counting can occur only due to exactly two regions (where 1 individual student would be counted in two subjects leading to an extra count of 1) and the exactly three region (where 1 individual student would be counted in 3 subjects leading to an extra count of 2).

This can be visualised in the figure below:



A student placed in the all three area will be counted three times when you count the number of students studying French, the number of students studying German and the number of students studying Chinese independently. Hence, HE/SHE would be counted three times—leading to an extra count of 2 for each individual places here.

A person placed in any of the three 'Exactly two' areas would be counted two times when we count the number of students studying French, the number of students studying German and the number of students studying Chinese independently. Hence, HE/SHE would be counted two times—leading to an extra count of 1 for each individual placed in any of these three areas.

The thought chain leading to the solution would go as follows:

(i) 180 students are counted 80 + 90 + 100 = 270 times.

(ii) This means that there is an extra count of 90 students.

(iii) Extra counts can fundamentally occur only from the 'exactly two' areas or the all three area in the figure.

(iv) We also know that 'The number of students who study more than one of the three subjects is 50% more than the number of students who study all the three subjects' hence we know that if there are a total of 'n' students studying all three subjects, there would be $1.5n$ students studying more than one subject. This in turn means that there must be $0.5n$ students who study two subjects.

(Since, number of students studying more than 1 subject = number of students studying two subjects + number of students studying three subjects.

i.e. $1.5n = n$ + number of students studying 2 subjects Æ number of students studying 2 subjects = $1.5n - n = 0.5n$)

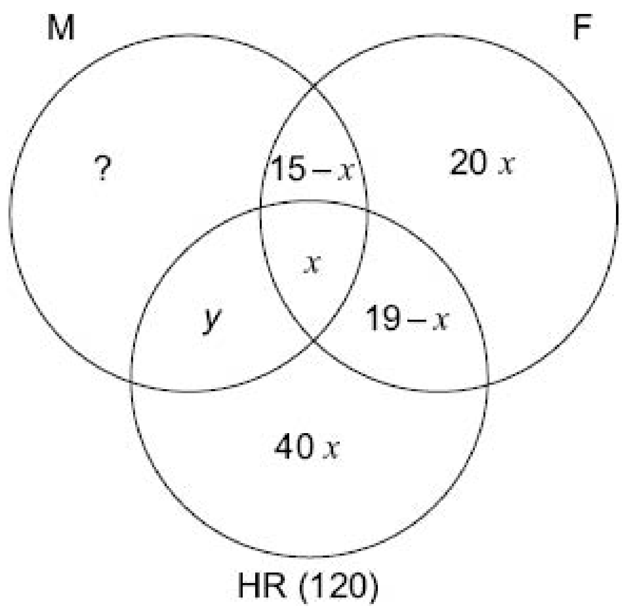
(v) The extra counts from the n students studying 3 subjects would amount to $n \times 2 = 2n$ – since each student is counted twice extra when he/she studies all three subjects.

(vi) The extra counts from the 0.5n students who study exactly two subjects would be equal to $0.5n \times 1 = 0.5\ n$.

(vii) Thus extra count = 90 = $2n + 0.5n$ Æ $n = 90/2.5 = 36$.

(viii)Hence, there must be 36 people studying all three subjects.

Solutions 5 and 6: The following would be the starting Venn diagram encapsulating the basic information:

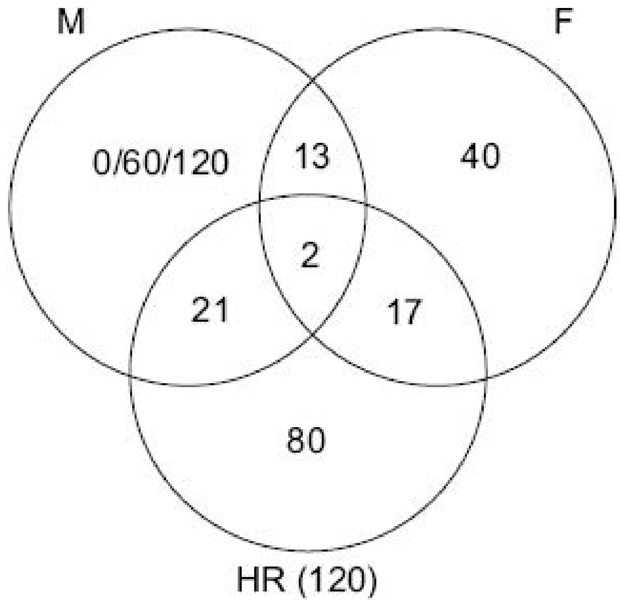


From this figure we get the following equation:

$40x + (19 - x) + x + y = 120$. This gives us $40x + y = 101$

Thinking about this equation, we can see that the value of x can be either 1 or 2. In case we put x as 1, we get $y = 61$ and then we have to also meet the additional condition that $15–x$, $19–x$ and y should form an AP which is obviously not possible (since it is not possible practically to build an AP having two positvie terms below 19 and the third term being 61. Hence, this option is rejected.

Moving forward, the other possible value of x from the equation is $x = 2$ in which case we get, $y = 21$ and $15 - x = 13$ and $19 - x = 17$. Thus, we get the AP 13, 17, 21 which satisfies the given conditions. Putting $x = 2$ and $y = 21$ in the figure, the venn diagram evolves to:



In this figure the value that only Marketing takes can either be 0, 60 or 120 (to satisfy the AP condition). However, since the total number of students in Marketing is a two digit number above 50, the number of people studying only marketing would be narrowed down to the only possibility which remains – viz 60.

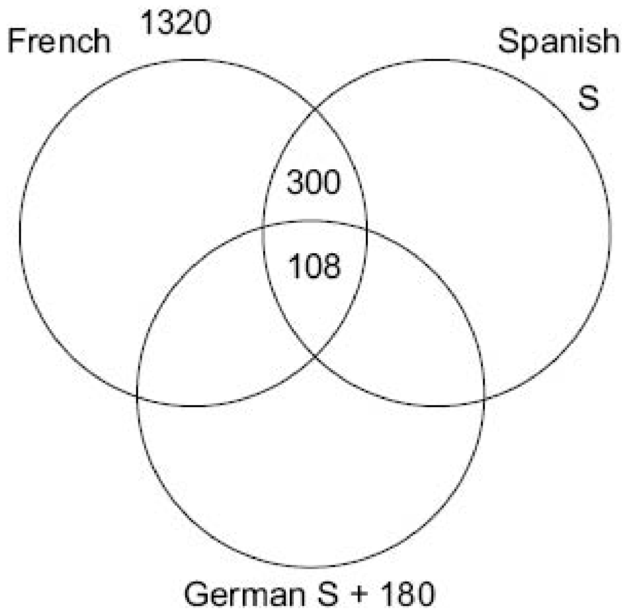
Thus, the number of students studying in the batch = 120 + 40 + 60 + 13 + 4 = 237.

The number of students specialising in both Marketing and HR is 21 + 2 = 23

5. (d) The total number of students is either 113 or 128.
6. (c) The number of students studying both Marketing and HR is 23.

Solutions 7 to 9:

7 & 8: In order to think about the possibility of the maximum and/or the minimum number of people who could be studying none of the three languages, you need to first think of the basic information in the question. The basic information in the question can be encapsulated by the following Venn diagram:



At this point we have the flexibility to try to put the remaining numbers into this Venn diagram while maintaining the constraints the question has placed on the relative numbers in the figure. In order to do this, we need to think of the objective with which we have to fill in the remaining numbers in the figure. At this stage you have to keep two constraints in mind while filling the remaining numbers:

(a) The remaining part of the French circle has to total 1320 – 408 = 912;

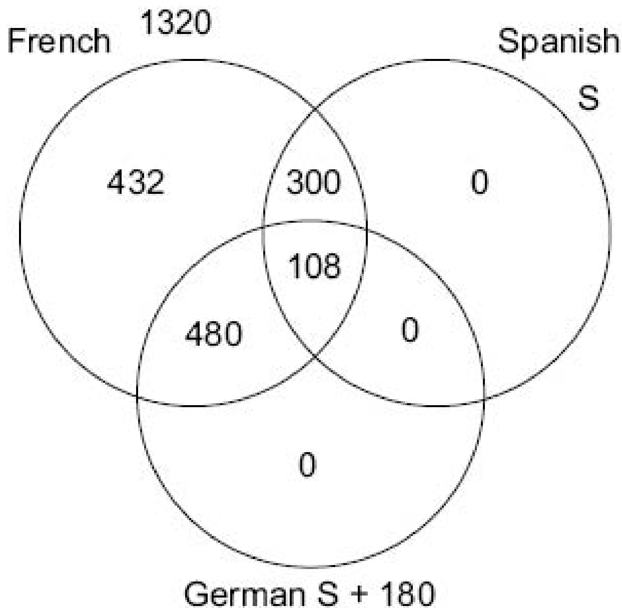
(b) The German circle has to be 180 more than the Spanish circle.

When we try to fill in the figure for making the number of students who did not study any of the three subjects maximum:

You can think of first filling the French circle by trying to think of how you would want to distribute the remaining 912 in that circle. When we want to maximise the number of students who study none of the three, we would need to use the minimum number of people inside the three circles—while making sure that all the constraints are met.

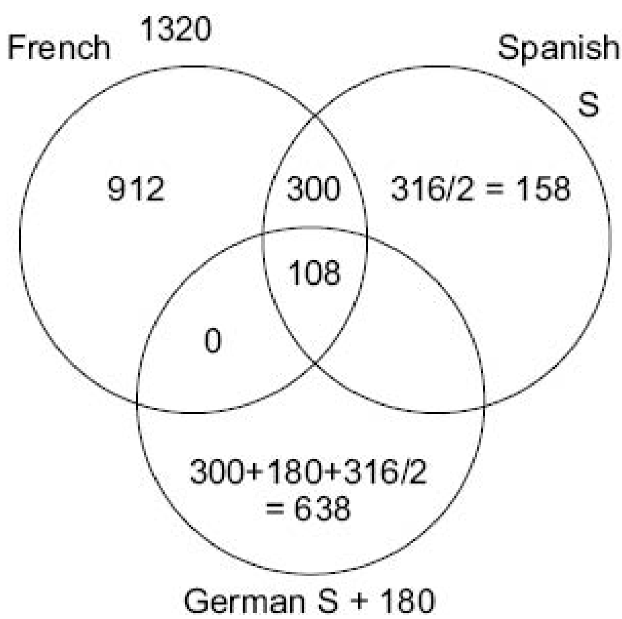
Since we have to forcefully fit in 912 into the remaining areas of the French circle, we need to see whether while doing the same we can also meet the second constraint.

This thinking would lead you to see the following solution possibility:



In this case we have ensured that the German total is 180 more than the Spanish total (as required) and at the same time the French circle has also reached the desired 1320. Hence, the number of students who study none of the three can be 2116–1320 = 796 (at maximum).

When minimising the number of students who have studied none of the three subjects, the objective would be to use the maximum number of students who can be used in order to meet the basic constraints. The answer in this case can be taken to as low as zero in the following case:



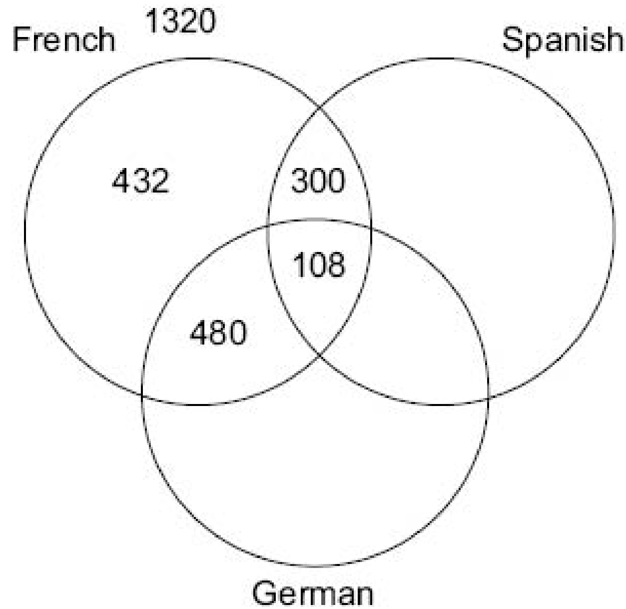
Note: While thinking about the numbers in this case, we first use the 912 in the 'only French' area. At this point we have 796 students left to be allocated. We first make the German circle 180 more than the Spanish circle (by taking the only German as 300 +180 to start with, this is accomplished). At this point, we are left with 316 more students, who can be allocated equally as 316 ÷ 2 for both the 'only German' and the 'only Spanish' areas.

Thus, the minimum number of students who study none of the three is 0.

Solution 9:

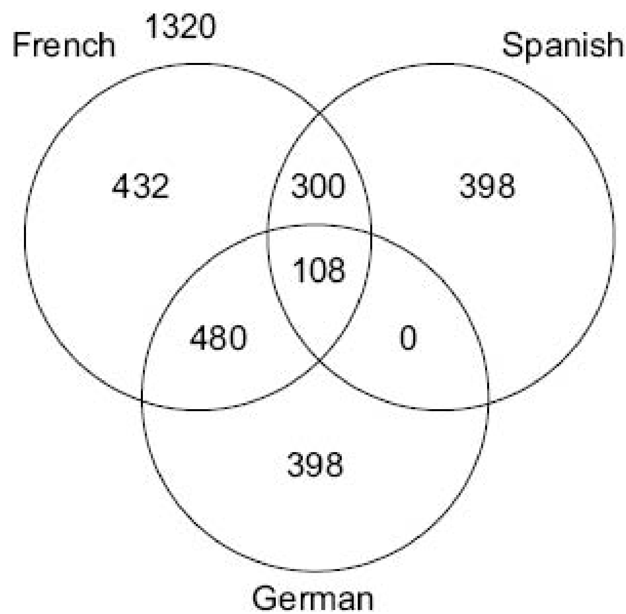
In order to think about this question, let us first see the situation we had in order to maintain all constraints.

If we try to fit in the remaining constraints in this situation we would get:

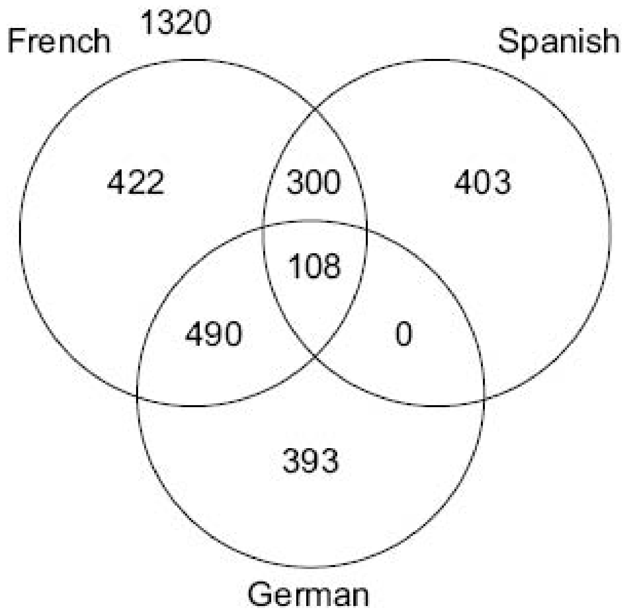


This leaves us with a slack of 796 people which would need to be divided equally since we cannot disturb the equilibrium of German being exactly 180 more than Spanish.

This gives us the following figure:



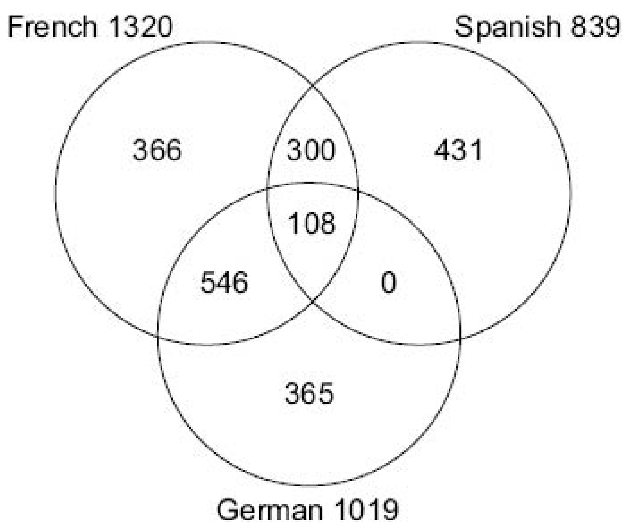
When you think about this situation, you realise that it is quite possible to increase Spanish if we reduce the only French area and reallocate the reduction into the ‘only French’ and German area. A reduction of 10 from the ‘only French’ area can be visualised as follows:



In this case, as you can see from the figure above, the number of students who study only Spanish has gone up by 5 (which is half of 10).

Since, there is still some gap between the 'only German' and the 'only French' areas in the figure, we should close that gap by reducing the 'only French' area as much as possible.

The following solution figure would emerge when we think that way:



Hence, the maximum possible for the only Spanish area is 431.

Solutions 10 to 13: The information given in the question can be encapsulated in the following way:

Game	*Only that game*	*2 games combination 1*	*2 games combination 2*	*2 games combination 3*	*3 games combination 1*	*3 games combination 2*	*3 games combination 3*	*All 4 games*
Tennis (460)	220	40	40	40	20	20	20	60
TT (360)	120	40	40	40	20	20	20	60

Squash (360)	120	40	40	40	20	20	20	60
Badminton (440)	200	40	40	40	20	20	20	60

From the above table, we can draw the following conclusions,, which can then be used to answer the questions asked.

The total number of athletes who play at least one of the four games = 220 + 120 + 120 + 200 + 40×6 + 20×4 + 60 = 1040

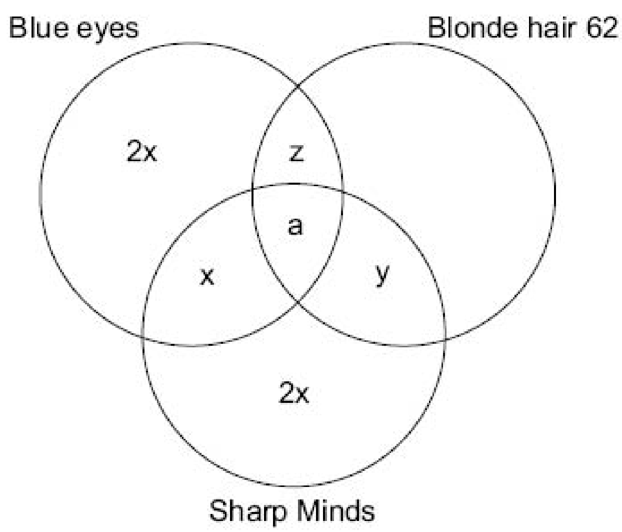
(Note : that in doing this calculation, we have used 40 × 6 for calculating how many unique people would be playing exactly two games—where 40 for each combination is given and there are ${}^4C_2$= 6 combinations of exactly two sports that exist. Similar logic applies to the 20 × 4 calculation for number of athletes playing exactly 3 sports.)

Also, since we know that the number of athletes who participate in none of these four games is 20% of the total number of athletes, we can calculate the total number of athletes who practise in the stadium as 5 × 1040 ÷ 4 = 1300.

Thus, the questions can be answered as follows:

10. The number of athletes in the stadium = 1300.
11. Only squash + only tennis = 120 + 220 = 340 (from the table)
12. Only athletics means none of the 4 games = total number of athletes – number of athletes who play at least one game = 1300 – 1040 = 260.
13. In case all the three game athletes would add one more game they would become 4 game athletes. Hence, the number of athletes who play all four games would be: Athletes playing 3 games earlier + athletes playing all 4 games earlier = 80 + 60 = 140

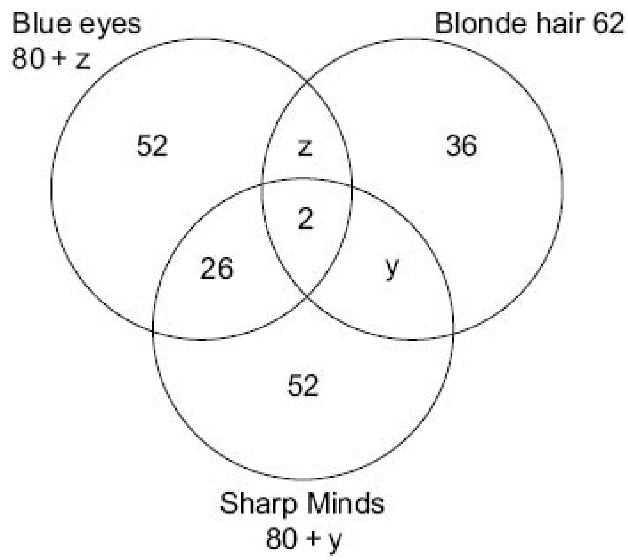
Solutions 14 and 15: The starting figure based on the information given in the question would look something as below:



From this figure we see a few equations:

$x + y + z = 50$; $a + y + z = 26$ Æ $x – 24 = a$.

Also, since, $5x + 62 = 192$, we get the value of x as 26. The figure would evolve as follows.



Based on this we can deduce the answer to the two questions as:

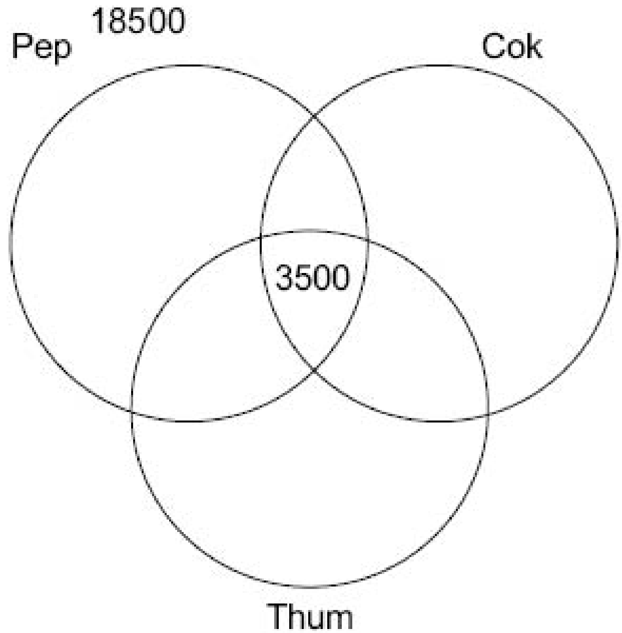
14. For the number of family members with blue eyes to be maximum, the family members with both sharp minds and blonde hair, but not blue eyes (represented by '*y*' in the figure), would be at maximum 11 because we would need to keep $z > y$. Hence, Option (a) is the correct answer.
15. If we are given the information in Option (c) we know the value of y would be 9 and hence, the value of z would be determined as 15. Hence, Option (c) provides us the information to determine the exact number of family members who have blonde hair and blue eyes but not sharp minds. Notice here that the information in each of the other options is already known to us.
16. Solve this again using slack thinking by using the following thought process:

    97 students are counted 47 + 53 + 72 = 172 times— which means that there is an extra count of 75 students (172 – 97 = 75). Now, since there are 15 students who are playing all the three games, they would be counted 45 times—hence they take care of an extra count of 15 × 2 = 30. (**Note:** in a 3 circle venn diagram situation, any person placed in the all three areas is counted thrice—hence he/she is counted two extra times).

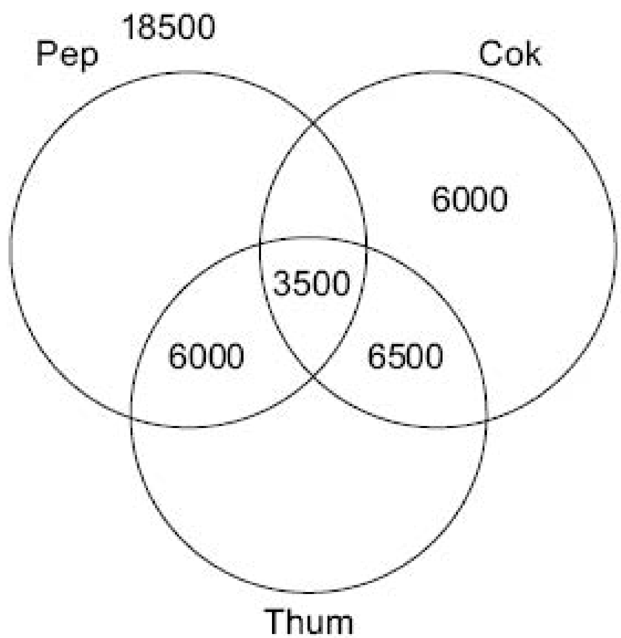
    This leaves us with an extra count of 45 to be managed—and the only way to do so is to place people in the exactly two areas. A person placed in the 'exactly two games area' would be counted once extra. Hence, with each student who goes into the 'exactly two games' areas it would be counted once extra. Thus, to manage an extra count of 45, we need to put 45 people in the 'exactly two' area. Option (d) is correct.

Solutions17 to 19: When you draw a Venn diagram for the three cold drinks, you realise as given here.

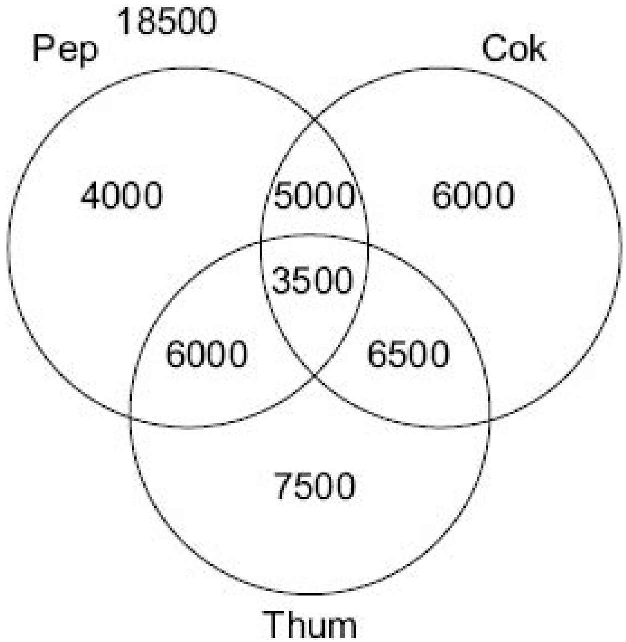
Once you fill in the basic information into the Venn diagram, you reach the following position:



At this point we know that since the 'all three area' is 3500, the value of the 'exactly two areas' would be 5 × 3500 = 17500. Also, we know that "11000 like Pep and exactly one more cold drink" which means that the area for Cok and Thum but not Pep is equal to 17500 – 11000 = 6500. Further, when you start adding the information : "6000 like only Cok and the same number of people like Pep and Thum but not Cok," the Venn Diagram transforms to the following:



Filling in the remaining gaps in the picture we get:



Note, we have used the following info here:

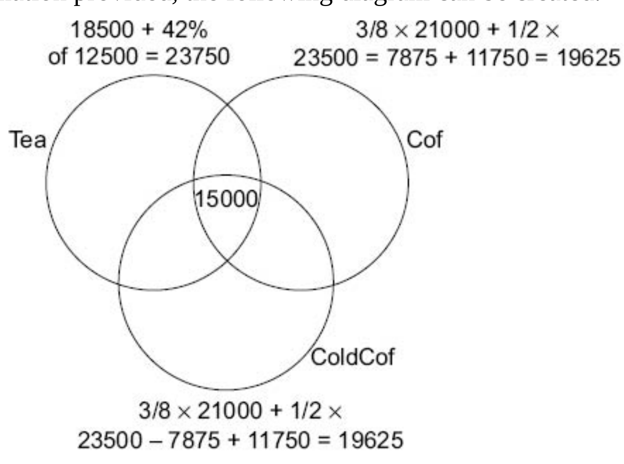
Thum but not Pep is 14000 and since we already know that Thum and Cok but not Pep is 6500, the value of ‘only Thum’ would be 14000 – 6500 = 7500.

We also know that the ‘exactly two’ areas add up to 17500 and we know that two of these three areas are 6500 and 6000 respectively. Thus, Pep and Cok but not Thum is 17500 – 6000 – 6500 = 5000.

Finally, the ‘only Pep’ area would be 18500 – 5000 – 6000 – 3500 = 4000.

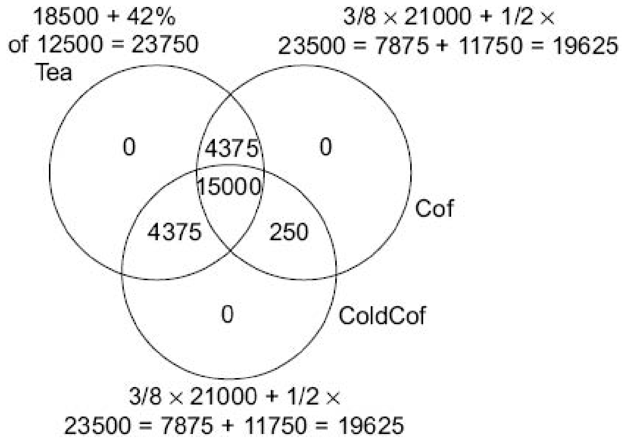
Once we have created the Venn diagram for the cold drinks, we can focus our attention to the Venn diagram for the beverages.

Based on the information provided, the following diagram can be created.



Based on these figures, the questions asked can be solved as follows:

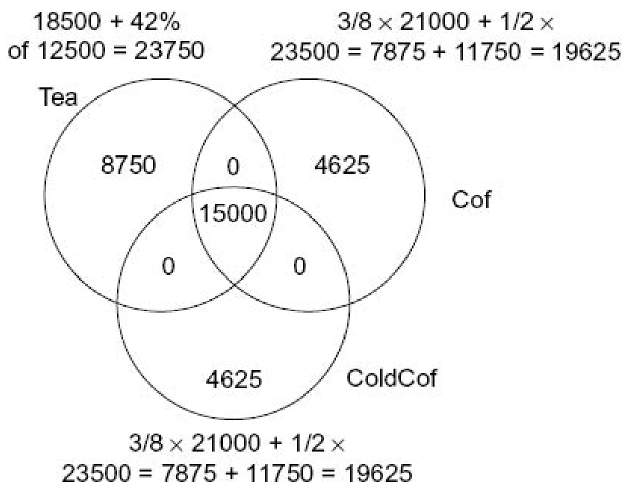
17. Option (a) is correct as the number of people who like at least one of the cold drinks is the sum of 18500 + 6000 + 6500 + 7500 = 38500.
18. For the number of people who do not like any of the beverages to be maximum, we have to ensure that the number of people used in order to meet the situation described by the beverage’s venn diagram should be minimum. This can be done by the following values in the inner areas of this venn diagram:



In this situation, the number of people used inside the Venn diagram to match upto all the values for this figure = 15000 + 4375 + 4375 + 250 = 24000.

Naturally, in this case the number of people who do not like any of the beverages is maximised at 40000 – 24000 = 16000. Option (b) is correct.

19. The solution for this situation would be given by the following figure:



The number of people who like at least one of the three beverages is:

15000 + 8750 +4625 + 4625 = 33000. Option (c) is correct.

20. The number of people cannot be a fraction in any situation. We can deduce that the values of x and $3x$ have to be factors of 57. This gives us that the values of x can only be either 1 or 19 (for both x and $3x$ to be a factor of 57).

So, the number of people who drink tea is equal to $2x + 57/x$ which can be 59 (if x = 1) or 41 (if x = 19).

Hence, Option (d) is correct.

TRAINING GROUND FOR BLOCK VI

HOW TO THINK IN PROBLEMS ON BLOCK VI

1. The probability that a randomly chosen positive divisor of 10^{29} is an integer multiple of 10^{23} is : *a*2/*b*2, then '*b* – *a*' would be:

(XAT 2014)

(a) 8 (b) 15

(c) 21 (d) 23

(e) 45

Solution: This question appeared in the XAT 2014 exam. The number $10^{29} = 2^{29} \times 5^{29}$

Factors or divisors of such a number would be of the form: $2^a \times 5^b$ where the values of *a* and *b* can be represented as 0 £ *a*, *b* £ 29, i.e., there are 30 × 30 = 900 possibilities when we talk about randomly selecting a positive divisor of 10^{29}.

Next, we need to think of numbers which are integral multiples of 10^{23}. Such numbers would be of the form 2*x* × 5*y* such that $x, y \geq 23$.

Hence, the number of values possible when the chosen divisor would also be an integer multiple of 10^{23} would be when $23 \leq x, y \leq 29$. There would be 7 × 7 = 49 such combinations.

Thus, the required probability is 49÷900. In the context of $a^2 \div b^2$, the values of *a* and *b* would come out as 7 and 30 respectively. The required difference between *a* and *b* is 23. Hence, Option (d) is correct.

2. Aditya has a total of 18 red and blue marbles in two bags (each bag has marbles of both colors). A marble is randomly drawn from the first bag followed by another randomly drawn from the second bag, the probability of both being red is 5/16. What is the probability of both marbles being blue?

(XAT 2014)

(a) 1/16 (b) 2/16

(c) 3/16 (d) 4/16

(e) None of the above

Solution: This problem has again appeared in XAT 2014. The problem most students face in such situations is to understand how to place how many balls of each colour in each bag. Since there is no directive given in the question that tells us how many balls are there and/or how many balls are placed in any bag the next thing that a mathematically oriented mind would do would be to try to assume some variables to represent the number of balls in each bag. However, if you try to do so on your own you would realise that that would be the wrong way to solve this quesiton as it would lead to extreme complexity while solving the problem. So how can we think alternately? Is there a smarter way to think about this question?

Yes indeed there is. Let me explain it to you here. In order to think about this problem, you would need to first think about how a fraction like 5/16 would emerge. The value of 5/16 = 10/32 = 15/48 = 20/64 = 25/80 = 30/96 and so on. Next, you need to understand that there are a total of 18 balls and this 18 has to be broken into two parts such that their product is one of the above denominators. Scanning the denominators we see the opportunity that the number 80 = 10×8 and hence we realise that the probability of both balls being red would happen in a situation where the structure of the calculation would look something like: $(r_1/10) \times (r_2/8)$. Next, to get 25 as the corresponding numerator with 80 as the denominator the values of r_1 and r_2 should both be 5. This means that there are 5 red balls out of ten in the first bag and 5 red balls out of 8 in the second bag. This further means that the number of blue balls would be 5 out of 8 and 3 out of 8. Thus, the correct answer would be: (5/10) × (3/8) = 25/80 = 5/16. Hence, Option (c) is the correct answer.

3. The scheduling officer for a local police department is trying to schedule additional patrol units in each of two neighbourhoods – southern and northern. She knows that on any given day, the probabilities of major crimes and minor crimes being committed in the northern neighbourhood were 0.418 and 0.612, respectively, and that the corresponding probabilities in the southern neighbourhood were 0.355 and 0.520. Assuming that all crimes occur independent of each other and likewise that crime in the two neighbourhoods are independent of each other, what is the probability that no crime of either type is committed in either neighbourhood on any given day?

(XAT 2011)

(a) 0.069 (b) 0.225

(c) 0.690 (d) 0.775

(e) None of the above

Solution: This question appeared in XAT 2011, and the key to solving this correctly is to look at the event definition. A major crime not occurring in the northern neighbourhood is the non-event for a major crime occurring in the nothern neighbourhood on any given day. Its probability would be (1–0.418) = 0.582.

The values of minor crime not occurring in the northern neighbourhood and a major crime not occurring in the southern neighbourhood and a minor crime not occurring in the northern neighbourhood would be (1–0.612); (1–0.355) and (1–0.520) respectively. The value of the required probability would be the probability of the event:

Major crime does not occur in the northern neighbourhood and minor crime does not occur in the northern neighbourhood and major crime does not occur in the southern neighbourhood and minor crime does not occur in the northern neighbourhood =

(1– 0.418) × (1– 0.612) × (1– 0.355) × (1– 0.520). Option (a) is the closest answer.

4. There are four machines in a factory. At exactly 8 pm, when the mechanic is about to leave the factory, he is informed that two of the four machines are not working properly. The mechanic is in a hurry, and decides that he will identify the two faulty machines before going home, and repair them next morning. It takes him twenty minutes to walk to the bus stop. The last bus leaves at 8 :32 pm. If it takes six minutes to identify whether a machine is defective or not, and if he decides to check the machines at random, what is the probability that the mechanic will be able to catch the last bus?

(a) 0 (b) 1/6

(c) 1/4 (d) 1/3

(e) 1

Solution: The first thing you look for in this question, is that obviously the mechanic has only 12 minutes to check the machines before he leaves to catch the bus. In 12 minutes, he can at best check two machines. He will be able to identify the two faulty machines under the following cases:

(The first machine checked is faulty AND the second machine checked is also faulty) OR (The first machine checked is working fine AND the second machine checked is also working fine)

Required probability = $\frac{2}{4}\times\frac{1}{3}+\frac{2}{4}\times\frac{1}{3}=\frac{1}{3}$

5. Little Pika who is five and half years old has just learnt addition. However, he does not know how to carry. For example, he can add 14 and 5, but he does not know how to add 14 and 7. How many pairs of consecutive integers between 1000 and 2000 (both 1000 and 2000 included) can Little Pika add?

(a) 150 (b) 155

(c) 156 (d) 258

(e) None of the above

Solution: This question again appeared in the XAT 2011 exam. If you try to observe the situations under which the addition of two consecutive four-digit numbers between 1000 and 2000 would come through without having a carry over value in the answer you would be able to identify the following situations – each of which differs from the other due to the way it is structured with respect to the values of the individual digits:

Category 1: 1000 + 1001; 1004 +1005, 1104 + 1105 and so on. A little bit of introspection should show you that in this case, the two numbers are 1abc and 1abd where $d = c + 1$. Also, for the sum to come out without any carry-overs, the values of *a*, *b* and *c* should be between 0 and 4 (including both). Thus, each of *a*, *b* and *c* gives us 5 values each – giving us a total of 5 × 5 × 5 = **125 such situations.**

Category 2: 1009 + 1010; 1019 + 1020; 1029 + 1030; 1409 + 1410. The general form of the first number here would be 1*ab* 9 with the values of *a* and *b* being between 0 and 4 (including both). Thus, each of *a*, *b* and *c* gives us 5 values each – giving us a total of 5 × 5 = **25 such situations**.

Category 3: 1099 + 1100; 1199 + 1200; 1299 + 1300; 1399 + 1400 and 1499 + 1500. There are only **5 such pairs.** Note that 1599 + 1600 would not work in this case as the addition of the hundreds' digit would become more than 10 and lead to a carry-over.

Category 4: 1999 + 2000 is the only other situation where the addition would not lead to a carry-over calculation. Hence, **1 more situation.**

The required answer = 125 + 25 + 5 + 1 = 156. Option (c) is the correct answer.

6. In the country of Twenty, there are exactly twenty cities, and there is exactly one direct road between any two cities. No two direct roads have an overlapping road segment. After the election dates are announced, candidates from their respective cities start visiting the other cities. The following are the rules that the election commission has laid down for the candidates:

- Each candidate must visit each of the other cities exactly once.
- Each candidate must use only the direct roads between two cities for going from one city to another.
- The candidate must return to his own city at the end of the campaign.
- No direct road between two cities would be used by more than one candidate.

The maximum possible number of candidates is

(a) 5 (b) 6

(c) 7 (d) 8

(e) 9

Solution: Again an XAT 2011 question. Although this question carried a very high weightage (it had 5 marks where 'normal questions' had 1 to 3 marks) it is not so difficult once you understand the logic of the question. The key to understanding this question is from two points.

(a) Since there is exactly one direct road between any pair of two cities – there would be a toal of $20C_2$ roads = 190 roads.

(b) The other key condition in this question is the one which talks about each candidate must visit each of the other cities exactly once and 'No direct road between two cities would be used by more than one candidate.' This means two things. i) Since each candidate visits each city exactly once, if there are '*c*' candidates, there would be a total of 20c roads used and since no road is repeated it means that the 20 roads Candidate A uses will be different from the 20 roads Candidate B uses and so on. Thus, the value of 20*c* £ 190 should be an inequality that must be satisfied.This gives us a maximum possible value of *c* as 9. Hence, Option (e) is correct.

7. In a bank the account numbers are all 8 digit numbers, and they all start with the digit 2. So, an account number can be represented as $2x_1x_2x_3x_4x_5x_6x_7$. An account number is considered to be a 'magic' number if $x_1x_2x_3$ is exactly the same as $x_4x_5x_6$ or $x_5x_6x_7$ or both, x_i can take values from 0 to 9, but 2 followed by seven 0s is not a valid account number. What is the maximum possible number of customers having a 'magic' account number?

(a) 9989
(b) 19980
(c) 19989
(d) 19999
(e) 19990

Solution: This question appeared in XAT 2011. In order to solve this question, we need to think of the kinds of numbers which would qualify as magic numbers. Given the definition of a magic number in the question, a number of form 2mnpmnpq would be a magic number while at the same time a number of the form 2mnpqmnp would also qualify as a magic number. In this situation, each of m,n and p can take any of the ten digit values from 0 to 9. Also, q would also have ten different possibilities from 0 to 9. Thus, the total number of numbers of the form 2mnpmnpq would be

$10^4 = 10000$. Similarly, the total number of numbers of the form 2mnpqmnp would also be $10^4 = 10000$. This gives us a total of 20000 numbers. However, in this count the numbers like 21111111, 22222222, 23333333, 24444444 etc have been counted under both the categories. Hence we need to remove these numbers once each (a total of 9 reductions). Also, the number 20000000 is not a valid number according to the question. This number needs to be removed from both the counts.

Hence, the final answer = 20000 – 9 – 2 = 19989.

8. If all letters of the word "CHCJL" be arranged in an English dictionary, what will be the 50th word?

(a) HCCLJ (b) LCCHJ

(c) LCCJH (d) JHCLC

(e) None of the above

Solution: A Xat 2010 question. In the English dictionary the ordering of the words would be in alphabetical order. Thus, words starting with C would be followed by words starting with H, followed by words starting with J and finally words starting with L. Words starting with C = 4! = 24; Words starting with H = 4! ÷ 2! = 12 words. Words starting with J = 4! ÷ 2! = 12 words. This gives us a total of 48 words. The 49^{th} and the 50^{th} words would start with L. The 49^{th} word would be the first word starting with L (=LCCHJ) and the 50^{th} word would be the 2^{nd} word starting with L – which would be LCCJH. Option (c) is correct.

9. The supervisor of a packaging unit of a milk plant is being pressurised to finish the job closer to the distribution time, thus giving the production staff more leeway to cater to last minute demand. He has the option of running the unit at normal speed or at 110% of normal – "fast speed". He estimates that he will be able to run at the higher speed 60% of time. The packet is twice as likely to be damaged at the higher speed which would mean temporarily stopping the process. If a packet on a randomly selected packaging runs has probability of 0.112 of damage, what is the probability that the packet will not be damaged at normal speed?

(a) 0.81 (b) 0.93

(c) 0.75 (d) 0.60

(e) None of the above

Solution: Again a XAT 2013 question. Let the probability of the package being damaged at normal speed be '*p*'. This means that the probability of the damage of a package when the unit is running at a fast speed is '2*p*'. Since, he is under pressure to complete the production quickly, we would need to assume that he runs the unit at fast speed for the maximum possible time (60% of the time).

Then, we have

Probability of damaged packet in all packaging runs

$= 0.6 \times 2p + 0.4 \times p = 0.112.$

fi $p = 0.07$

Probability of non damaged packets at normal speed

$= 1 - p = 1 - 0.07 = 0.93$. Option (b) is correct.

10. Let X be a four-digit positive integer such that the unit digit of X is prime and the product of all digits of X is also prime. How many such integers are possible?

(a) 4 (b) 8

(b) 12 (d) 24

(e) None of these

Solution: This one is an easy question as all you need to do is understand that given the unit digit is a prime number, it would mean that the number can only be of the form *abc*2; *abc*3 or *abc*5 or *abc*7. Further, for each of these, the product of the four digits $a \times b \times c \times$ units digit has to be prime. This can occur only if $a = b = c = 1$. Thus, there are only 4 such numbers viz: 1112, 1113, 1115 and 1117. Hence, Option (a) is correct.

11. The chance of India winning a cricket match against Australia is 1/6. What is the minimum number of matches India should play against Australia so that there is a fair chance of winning at least one match?

(a) 3 (b) 4

(c) 5 (d) 6

(e) None of the above

Solution: This is another question from the XAT 2009 test paper. A fair chance is defined when the probability of an event goes to above 0.5. If India plays 3 matches, the probability of at least one win will be given by the non-event of losing all matches. This would be:

$1 – (5/6)^3$ = 1–125/216 = 91/216 which is less than 0.5. Hence, Option (a) is rejected.

For four matches, the probability of winning at least 1 match would be:

$1– (5/6)^4$ = 1– 625/1296 = 671/216 which is more than 0.5. Hence, Option (b) is correct.

12. Two teams *Arrogant* and *Overconfident* are participating in a cricket tournament. The odds that team *Arrogant* will be champion is 5 to 3, and the odds that team *Overconfident* will be the champion is 1 to 4. What are the odds that either *Arrogant* or team *Overconfident* will become the champion?

(a) 3 to 2 (b) 5 to 2

(c) 6 to 1 (d) 7 to 1

(e) 33 to 7

Solution: You need to be clear about what odds for an event mean in order to solve this. Odds for team *Arrogant* to be champion being 5 to 3 means that the probability of team *Arrogant* being champion is 5/8. Similarly, the probability of team *Overconfident* being champion is 1/5 (based on odds of team *Overconfident* being champion being 1 to 4). Thus, the probability that either of the teams would be the champion would be

$$= \frac{5}{8} + \frac{1}{5} = \frac{33}{40}$$

This means that in 40 times, 33 times the event of one of the teams being champion would occur. Hence, the odds for one of the two given teams to be the chamion would be 33 to 7.

So required odds will be 33 to 7. Option (e) is correct.

13. Let X be a four-digit number with exactly three consecutive digits being same and is a multiple of 9. How many such X's are possible?

(a) 12 (b) 16

(c) 19 (d) 21

(e) None of the above

Solution: Since the number has to be a multiple of 9, the sum of the digits would be either 9 or 18 or 27. Also, the number would either be in the form aaab or baaa. For the sum of the digits to be 9, we would have the following cases:

$a = 1$ and $b = 6$ for the numbers 1116 and 6111;

$a = 2$ and $b = 3$ for the numbers 2223 and 3222;

$a = 3$ and $b = 0$ for the number 3330 and

$b = 9$ and $a = 0$ for the number 9000. We get a total of 6 such numbers.

Similarly for the sum of the digits to be 18 we will get:

3339, 9333; 4446, 6444; 5553, 3555; 6660. We get a total of 7 such numbers.

For the sum of the digits to be 27 we will get the numbers:

6669, 9666; 7776, 6777; 8883, 3888 and 9990. Thus, we get a total of 7 such numbers. Hence, the total number of numbers is 20. Option (e) is correct.

14. A shop sells two kinds of rolls—egg roll and mutton roll. Onion, tomato, carrot, chilli sauce and tomato sauce are the additional ingredients. You can have any combination of additional ingredients, or have standard rolls without any additional ingredients subject to the following constraints:

(a) You can have tomato sauce if you have an egg roll, but not if you have a mutton roll.

(b) If you have onion or tomato or both you can have chilli sauce, but not otherwise.

How many different rolls can be ordered according to these rules?

(a) 21 (b) 33

(c) 40 (d) 42

(e) None of the above.

Solution: Let the 5 additional ingredients onion, tomato, carrot, chilli sauce and tomato sauce are denoted by O, T, C, CS, TS respectively.

Number of ways of ordering the egg roll:

For the egg roll there are a total of 32 possibilities (with each ingredient being either present or not present – there being 5 ingredients the total number of possibilities of the combinations of the egg rolls would be equal to 2 × 2 × 2 × 2 × 2 = 32 ways).

However, out of these 32 instances, the following combinations are not possible due to the constraint given in Statement (b) which tells us that to have CS in the roll either of onion or tomato must be present (or both should be present). The combinations which are not possible are:

(CS) (CS, TS) (CS, C) (CS, C, TS)

Total number of ways egg roll can be ordered

= 32 – 4 = 28.

Number of ways of ordering the mutton roll:

Total number of cases for mutton roll without any constraints = 2 × 2 × 2 × 2 = 16 ways. Cases rejected due to constraint given in statement (b): (CS); (CS,C) Æ 16 – 2 = 14 cases.

Total number of ways or ordering a roll = 28 + 14 = 42. Option (d) is correct.

15. Steel Express stops at six stations between Howrah and Jamshedpur. Five passengers board at Howrah. Each passenger can get down at any station till Jamshedpur. The probability that all five persons will get down at different stations is:

(a) $\frac{^6P_5}{6^5}$ (b) $\frac{^6C_5}{6^5}$

(c) $\frac{^7P_5}{7^5}$ (d) $\frac{^6C_5}{7^5}$

(e) None of the above.

Solution: The required probability would be given by:

$$\frac{\left(\begin{array}{c}\text{Total number of ways in which 5 people can get down}\\ \text{at 5 different stations from amongst 7 stations}\end{array}\right)}{\left(\begin{array}{c}\text{Total number of ways in which 5 people can get down}\\ \text{at 7 stations}\end{array}\right)}$$

The value of the numerator would be ${}^7P_5$, while the value of the denominator would be 7^5. The correct answer would be Option (c).

16. In how many ways can 53 identical chocolates be distributed amongst 3 children– C_1, C_2 and C_3 – such that C_1 gets more chocolates than C_2 and C_2 gets more chocolates than C_3?

(a) 468 (b) 344

(c) 1404 (d) 234

Solution: 53 identical chocolates can be distributed amongst 3 children in ${}^{55}C_2$ ways =1485 ways ($n^+r^{-1}C\mathrm{r}_{-1}$ formula). Out of these ways of distributing 53 chocolates, the following distributions methods are not possible as they would have two values equal to each other– (0, 0, 53); (1, 1, 51); (2, 2, 49)….(26, 26, 1).

There are 27 such distributions, but when allocated to C_1, C_2 and C_3 respectively, each of these distributions can be allocated in 3 ways amongst them. Thus, C_1= 0, C_2 = 0 and C_3= 53 is counted differently from C_1 = 0, C_2 = 53 and C_3 = 0 and also from C_1 = 53, C_2 = 0 and C_3 = 0. This will remove 27 × 3 = 81 distributions from 1485, leaving us with 1404 distributions. These 1404 distributions are those where all three numbers are different from each other. However, whenever we have three different values allocated to three children, there can be 3! = 6 ways of allocating the three different values amongst the three people. For instance, the distribution of 10, 15 and 48 can be seen as follows:

C1	*C2*	*C3*	
48	15	10	Only case which meets the problems' requirement.

48	10	15
15	48	10
15	10	48
10	15	48
10	48	15

Hence, out of every six distributions counted in the 1404 distributions we currently have, we need to count only one. The answer can be arrived at by dividing 1404 ÷ 6 = 234. Option (d) is correct.

17. In a chess tournament at the ancient Olympic Games of Reposia, it was found that the number of European participants was twice the number of non-European participants. In a round robin format, each player played every other player exactly once. The tournament rules were such that no match ended in a draw – any conventional draws in chess were resolved in favour of the player who had used up the lower time. While analysing the results of the tournament, K.Gopal the tournament referee observed that the number of matches won by the non-European players was equal to the number of matches won by the European players. Which of the following can be the total number of matches in which a European player defeated a non-European player?

(a) 57 (b) 58

(c) 59 (d) 60

Solution: If we assume the number of non-European players to be n, the number of European players would be 2n. Then there would be three kinds of matches played –

Matches between two Europan players – a total of $^{2}nC_{2}$ matches – which would yield a European winner.

Matches between two non-European players – a total of nC_2 matches, – which would yield a non-European winner.

Matches, between a European and a non-European player = $2n^2$. These matches would have some European wins and some non-European wins.

Let the number of European wins amongst these matches be x, then the number of non-European wins = $2n^2 - x$.

Now, the problem clearly states that the number of European wins = Number of non-European wins

fi $\quad \frac{2n(2n-1)}{2} + x = \frac{n(n-1)}{2} + 2n^2 - x$

fi $\quad n(n + 1) = 4x$

This means that the value of 4 times the number of wins for a European player over a non-European player should be a product of two consecutive natural numbers (since n has to be a natural number).

Among the options, $n = 60$ is the only possible value as the value of $4 \times 60 = 15 \times 16$.

Hence, Option (d) is correct.

18. A man, starting from a point M in a park, takes exactly eight equal steps. Each step is in one of the four directions – East, West, North and South. What is the total number of ways in which the man ends up at point M after the eight steps?

(a) 4200 (b) 2520

(c) 4900 (d) 5120

Solution: For the man to reach back to his original point, the number of steps North should be equal to the number of steps South. Similarly, the number of steps East should be equal to the number of steps West.

The following cases would exist:

4 steps north and 4 steps south = $8!/(4! \times 4!) = 70$ ways;

3 steps north, 3 steps south, 1 step east and 1 step west = $8!/(3! \times 3!) = 1120$ ways;

2 steps north, 2 steps south, 2 steps east and 2 steps west = $8!/(2! \times 2! \times 2! \times 2!) = 2520$ ways;

1 step north, 1 step south, 3 steps east and 3 steps west = $8!/(3! \times 3!) = 1120$ ways;

4 Steps east and 4 steps west $=8!/(4! \times 4!) = 70$ ways;

Thus, the total number of ways = 70 × 2 + 1120 × 2 + 2520 = 140 + 2240 + 2520 = 4900 ways.

Option (c) is correct.

BLOCK REVIEW TESTS

Review Test 1

1. 18 guests have to be seated, half on each side of a long table. 4 particular guests desire to sit on one particular side and 3 others on the other side. Determine the number of ways in which the sitting arrangements can be made

   (a) ${}^{11}P_n \times (9!)^2$ (b) ${}^{11}C_5 \times (9!)^2$

   (c) ${}^{11}P_6 \times (9!)^2$ (d) None of these

2. If m parallel lines in a plane are intersected by a family of n parallel lines, find the number of parallelograms that can be formed.

   (a) $m^2 \times n^2$ (b) $m^{(m+1)}n^{(n+1)}/4$

   (c) ${}^mC_2 \times {}^nC_2$ (d) None of these

3. A father with eight children takes three at a time to the zoological garden, as often as he can without taking the same three children together more than once. How often will he go and how often will each child go?

   (a) ${}^8C_{3,}\ {}^7C_3$ (b) ${}^8C_{3,}\ {}^7C_2$

   (c) ${}^8P_{3,}\ {}^7C_3$ (d) ${}^8P_{3,}\ {}^7C_2$

4. A candidate is required to answer 7 questions out of 2 questions which are divided into two groups, each containing 6 questions. He

is not permitted to attempt more than 5 questions from either group. In how many different ways can he choose the 7 questions?

(a) 390 (b) 520

(c) 780 (d) None of these

5. Find the sum of all 5 digit numbers formed by the digits 1, 3, 5, 7, 9 when no digit is being repeated.

(a) 4444400 (b) 8888800

(c) 13333200 (d) 6666600

6. Consider a polygon of n sides. Find the number of triangles, none of whose sides is the side of the polygon.

(a) $nC_3 - 2n - n \times (n-4)C_1$

(b) $n(n-4)(n-5)/3$

(c) $n(n-4)(n-5)/6$

(d) $n(n - 1l(n - 2)/3$

7. The number of 4 digit numbers that can be formed using the digits 0, 2, 3, 5 without repetition is

(a) 18 (b) 20

(c) 24 (d) 20

8. Find the total number of words that can be made by using all the letters from the word MACHINE, using them only once.

(a) 7! (b) 5020

(c) 6040 (d) 7!/2

9. What is the total number of words that can be made by using all the letters of the word REKHA, using each letter only once?

(a) 240 (b) 4!

(c) 124 (d) 5!

10. How many different 5-digit numbers can be made from the first 5 natural numbers, using each digit only once?

(a) 240 (b) 4!

(c) 124 (d) 5!

11. There are 7 seats in a row. Three persons take seats at random. What is the probability that the middle seat is always occupied and no two persons are sitting on consecutive seats?

(a) 7/70 (b) 14/35

(c) 8/70 (d) 4/35

12. Let N = 33^x. where x is any natural no. What is the probability that the unit digit of N is 3?

(a) 1/4 (b) 1/3

(c) 1/5 (d) 1/2

13. Find the probability of drawing one ace in a single draw of one card out of 52 cards.

(a) 1/(52 × 4) (b) 1/4

(c) 1/52 (d) 1/13

14. In how many ways can a committee of 4 persons be made from a group of 10 people?

(a) 10! / 4! (b) 210

(c) 10! / 6! (d) None of these

15. In Question 14, what is the number of ways of forming the committee, if a particular member must be there in the committee?

(a) 12 (b) 84

(c) 9!/ 3! (d) None of these

16. A polygon has 54 diagonals. The numbers of sides of this polygon are

(a) 12 (b) 84

(c) 3. 3! (d) 4. 4!

17. An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it, the probabilities of hitting the plane at

first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane?

(a) 0.7654 (b) 0.6976

(c) 0.3024 (d) 0.2346

18. 7 white balls and 3 black balls are placed in a row at random. Find the probability that no two black balls are adjacent.

(a) 2/15 (b) 7/15

(c) 8/15 (d) 4/15

19. The probability that A can solve a problem is 3/10 and that B can solve is 5/7. If both of them attempt to solve the problem, what is the probability that the problem can be solved?

(a) 3/5 (b) 1/4

(c) 2/3 (d) 4/5

20. The sides *AB*, *BC*, *CA* of a triangle *ABC* have 3, 4 and 5 interior points respectively on them. Find the number of triangles that can be constructed using these points as vertices.

(a) 180 (b) 105

(c) 205 (d) 280

21. From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done if there is no restriction in its formation?

(a) 256 (b) 246

(c) 252 (d) 260

22. From 4 officers and 8 jawans in how many ways can 6 be chosen to include exactly one officer?

(a) ${}^{12}C_6$ (b) 1296

(c) 1344 (d) 224

23. From 4 officers and 8 jawans in how many ways can 6 be chosen to include atleast one officer?

(a) 868 (b) 924

(c) 896 (d) none of these

24. Two cards are drawn one after another from a pack of 52 ordinary cards. Find the probability that the first card is an ace and the second drawn is an honour card if the first card is not replaced while drawing the second.

(a) 12/13 (b) 12/51

(c) 1/663 (d) None of these

25. The probability that Andrews will be alive 15 years from now is 7/15 and that Bill will be alive 15 years from now is 7/10. What is the probability that both Andrews and Bill will be dead 15 years from now?

(a) 12/150 (b) 24/150

(c) 49/150 (d) 74/150

Review Test 2

1. A group consists of 100 people; 25 of them are women and 75 men; 20 of them are rich and the remaining poor; 40 of them are employed. The probability of selecting an employed rich woman is:

   (a) 0.05 (b) 0.04

   (c) 0.02 (d) 0.08

2. Out of 13 job applicants, there are 5 boys and 8 men. It is desired to choose 2 applicants for the job. The probability that at least one of the selected applicant will be a boy is:

   (a) 5/13 (b) 14/39

   (c) 25/39 (d) 10/13

3. Fours dogs and three pups stand in a queue. The probability that they will stand in alternate positions is:

   (a) 1/34 (b) 1/35

   (c) 1/17 (d) 1/68

4. Asha and Vinay play a number game where each is asked to select a number from 1 to 5. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is:

   (a) 1/25 (b) 24/25

   (c) 2/25 (d) None of these

5. The number of ways in which 6 British and 5 French can dine at a round table if no two French are to sit together is given by:

   (a) $6! \times 5!$ (b) $5! \times 4!$

   (c) 30 (d) $7! \times 5!$

6. A cricket team of 11 players is to be formed from 20 players including 6 bowlers and 3 wicketkeepers. Find the number of ways in which a team can be formed having exactly 4 bowlers and 2 wicketkeepers:

(a) 20790 (b) 6930

(c) 10790 (d) 360

7. Three boys and three girls are to be seated around a circular table. Among them one particular boy Rohit does not want any girl neighbour and one particular girl Shaivya does not want any boy neighbour. How many such arrangements are possible?

(a) 5 (b) 6

(c) 4 (d) 2

8. Words with five letters are formed from ten different letters of an alphabet. Then the number of words which have at least one letter repeated is

(a) 19670 (b) 39758

(c) 69760 (d) 99748

9. Sunil and Kapil toss a coin alternatively till one of them gets a head and wins the game. If Sunil starts the game, the probability that he (Sunil) will win is:

(a) 0.66 (b) 1

(c) 0.33 (d) None of these

10. The number of pallelograms that can be formed if 7 parallel horizontal lines intersect 6 parallel vertical lines, is:

(a) 42 (b) 294

(c) 315 (d) None of these

11. $1.3.5\ldots(2n-1)/2.4.6\ldots(2n)$ is equal to:

(a) $(2n)! \div 2^n(n!))^2$ (b) $(2n)! \div n!$

(c) $(2n-1) \div)\ n-1)!$ (d) 2^n

12. How many four-digit numbers, each divisible by 4 can be formed using the digits 1, 2, 3, 4 and 5 (repetitions allowed)?

(a) 100 (b) 150

(c) 125 (d) 75

13. A student is to answer 10 out of 13 questions in a test such that he/she must choose at least 4 from the first five questions. The number of choices available to him is:

(a) 140 (b) 280

(c) 196 (d) 346

14. The number of ways in which a committee of 3 ladies and 4 gentlemen can be appointed from a meeting consisting of 8 ladies and 7 gentlemen, if Mrs. Pushkar refuses to serve in a committee if Mr.Modi is its member, is

(a) 1960 (b) 3240

(c) 1540 (d) None of these

15. A room has 3 lamps. From a collection of 10 light bulbs of which 6 are not good, a person selects 3 at random and puts them in a socket. The probability that he will have light, is:

(a) 5/6 (b) 1/2

(c) 1/6 (d) None of these

16. Two different series of a question booklet for an aptitude test are to be given to twelve students. In how many ways can the students be placed in two rows of six each so that there should be no identical series side by side and that the students sitting one behind the other should have the same series?

(a) $2 \times {}^{12}C_6 \times (6!)^2$ (b) $6! \times 6!$

(c) $7! \times 7 \times$ (d) None of these

17. The letters of the word PROMISE are arranged so that no two of the vowels should come together. The total number of arrangements is:

(a) 49 (b) 1440

(c) 7 (d) 1898

18. Find the remainder left after dividing $1! + 2! + 3! + \ldots + 1000!$ by 7.

(a) 0 (b) 5

(c) 21 (d) 14

19. In the McGraw-Hill Mindworkzz mock test paper, there are two sections, each containing 4 questions. A candidate is required to attempt 5 questions but not more than 3 questions from any section. In how many ways can 5 questions be selected?

(a) 24 (b) 48

(c) 72 (d) 96

20. A bag contains 10 balls out of which 3 are pink and rest are orange. In how many ways can a random sample of 6 balls be drawn from the bag so that at the most 2 pink balls are included in the sample and no sample has all the 6 balls of the same colour?

(a) 105 (b) 168

(c) 189 (d) 120

Review Test 3

1. There is one grandfather, 5 sons and daughters and 8 grandchildren in a family. They are to be seated in a row for dinner. The grandchildren wish to occupy the 4 seats at each end and the grandfather refuses to have a grandchild on either side of him. Find the number of ways in which the family can be made to sit :

   (a) 11360 (b) 11520

   (c) 21530 (d) None of these

2. A bag of 200 electric switches contains 16 defective switches. One switch is taken out at random from the bag. Find the probability that the switch drawn is (i) defective (ii) non-defective?

   (a) (i) 2/25 (ii) 23/25

   (b) (i) 4/25 (ii) 21/25

   (c) (i) 3/25 (ii) 22/25

   (d) (i) 1/25 (ii) 20/25

3. In a group photograph at the Patna Women's College, all the seven teachers should sit in the first row and all the twenty students should sit in the second row. The two corners of the second row are reserved for the two tallest students, interchangeable only between them and if the middle seat of the front row is reserved for the principal. Find the number of possible arrangements:

   (a) $720 \times 18!$ (b) $1440 \times 18!$

   (c) $1370 \times 18!$ (d) None of these

4. In a certain town, all telephone numbers have six digits, the first two digits always being 53 or 54 or 56 or 82 or 84. The number of telephone numbers having all the six digits distinct is:

   (a) 8400 (b) 9200

   (c) 7200 (d) None of these

5. Three groups X, Y and Z are contesting for a position on the Board of Directors of a company. The probabilities of their winning are 0.5, 0.3 and 0.2 respectively. If the group X wins, then the probability of introducing a new product is 0.7 and the corresponding probabilities for group Y and Z are 0.6 and 0.5 respectively. The probability that the new product will be introduced is:

   (a) 0.52 (b) 0.74

   (c) 0.63 (d) None of these

6. A computer hardware firm manufactures two parts of the computer —CPU and Monitor. In the process of manufacture of the CPU, 9 out of 100 are likely to be defective. Similarly 5 out of 100 are likely to be defective in the process of manufacture of Monitor. The probability that the assembled part will not be defective is:

   (a) 0.8645 (b) 0.9645

   (c) 0.6243 (d) None of these

7. There are 4 applicants for the post of the Indian cricket captain and one of them is to be selected by the votes of the 5 wise men (also called as the selectors). The number of ways in which the votes can be given is:

   (a) 1048 (b) 1072

   (c) 1024 (d) None of these

8. In a Football Tournament, there were 171 matches played. Every two teams played one match with each other. The number of teams participating in the Tournament is:

   (a) 19 (b) 18

   (c) 17 (d) 16

9. Seven points lie on a circle. How many chords can be drawn by joining these points?

   (a) 22 (b) 21

   (c) 23 (d) 24

10. Ten different letters of an alphabet are given. Words with 5 letters are formed from these given letters. Find the number of words which have at least one letter repeated:

(a) 69760 (b) 30240

(c) 99748 (d) None of these

11. There are eight chairs marked A to H. Two girls and three boys wish to occupy one chair each. First, the girls chose the chairs from amongst the chairs marked A to D, then the boys selected the chairs from amongst the remaining, marked E to H. The number of possible arrangements is:

(a) $6C_3 \times 4C_3$ (b) ${}^4P_2 \times {}^4P_3$

(c) ${}^4C_3 \times {}^4P_3$ (d) ${}^4C_2 \times {}^4C_3$

12. Prabhjeet a fan of poker, wants to play poker every night for the maximum number of nights possible. The key constraint for him is that he has only 8 friends who are willing to play with him and he wants to hold poker games with distinct groups of friends. His mother has put the condition that in case the same group is repeated on any day, she would stop his poker games and make him study—something that he should be doing anyway. Being a mathematically-oriented person, Prabhjeet calculates the maximum number of nights on which he can play poker without violating his mother's condition. What is the value that he has calculated?

(a) 70 (b) 105

(c) 140 (d) 120

13. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways this can be done is:

(a) 216 (b) 240

(c) 3125 (d) 600

14. There are 6 equi-distant points A, B, C, D, E and F marked on a circle with radius M. How many convex pentagons of distinctly

different areas can be drawn using these points as vertices?

(a) ${}^6P_5$ (b) 1

(c) 55^5 (d) None of these

15. Five men A, B, C, D and E occupy seats in a row such that C and D sit next to each other. In how many possible ways can these five men sit?

(a) 24 (b) 48

(c) 72 (d) None of these

16. The number of zero at the end of 60! is

(a) 12 (b) 14

(c) 16 (d) 18

17. A department had 8 male and female employees each. A project team involving 3 male and 3 female members needs to be selected from the department employees. How many different project teams can be formed?

(a) 112896 (b) 3136

(c) 720 (d) 112

18. From a group of 7 men and 6 women, 5 persons are to be chosen to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

(a) 756 (b) 735

(c) 564 (d) None of these

19. The number of different ways that the letters of the word STALLION can be arranged so that the vowels always come together, is:

(a) 2160 (b) 720

(c) 4320 (d) 880

20. If a polygon has 54 diagonals, then find the number of its sides.

(a) 13 (b) 12

(c) 11 (d) 10

ANSWER KEY

Review Test 1

1. (b)	2. (c)	3. (b)	4. (c)
5. (d)	6. (a)	7. (a)	8. (a)
9. (d)	10. (d)	11. (d)	12. (a)
13. (d)	14. (b)	15. (b)	16. (a)
17. (b)	18. (b)	19. (d)	20. (c)
21. (c)	22. (d)	23. (c)	24. (d)
25. (b)			

Review Test 2

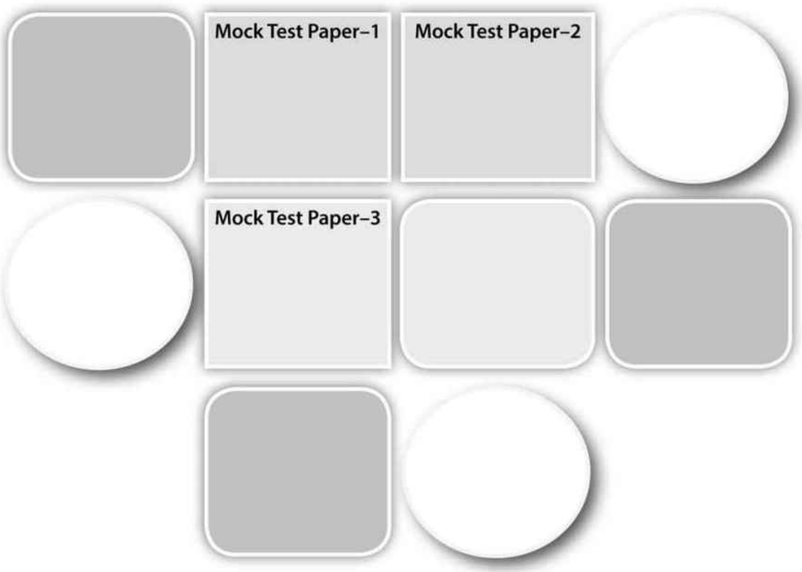
1. (c)	2. (c)	3. (b)	4. (b)
5. (b)	6. (a)	7. (c)	8. (c)
9. (c)	10. (c)	11. (a)	12. (c)
13. (a)	14. (d)	15. (d)	16. (b)
17. (b)	18. (b)	19. (b)	20. (b)

Review Test 3

1. (d)	2. (a)	3. (b)	4. (a)
5. (c)	6. (a)	7. (d)	8. (a)
9. (b)	10. (b)	11. (b)	12. (a)
13. (a)	14. (b)	15. (b)	16. (b)
17. (b)	18. (a)	19. (a)	20. (b)

Mock Test Papers

MOCK TEST PAPERS



The following section contains 3 mock tests based on the CAT 2011 pattern of QA. As mentioned elsewhere in the book, the papers varied greatly in difficulty levels—from the ridiculously easy to the tough. However, one thing common about the trend of difficulty in the papers was that after the initial few days, the quality of questions in the tests became more and more difficult—and became quite like the CAT papers of the past years. In the coming years, the CAT is expected to retain the degree of difficulty normally associated with the exam over the past decade and more and hence the tests I am providing are on the tougher side.

- You should take these tests only after you are through with your preparations of the entire book.
- Each question in these tests carries a weightage of +4 and –1.
- Take each of these tests in a limited time frame of 45 minutes (adhere strictly to the time limits).
- A net score of anywhere above 40 (with a maximum of 2 errors) in these tests would give you a high 99+ percentile in the exam. Note that in case you score more than 40 but have made 5–6 errors in doing so, your percentile would drop to around 90. Hence, the higher your percentage accuracy, the more the exam is going to reward you.
- Similarly:
  30+ with 100% accuracy can expect a percentile of 95+.
  30+ with 4-5 errors would give around 80 percentile.

For more practice and mock tests log on to www.mindworkzz.in.

1 Mock Test Paper—1

1. Works W1 and W2 are done by Priyanka and Sanjana. Priyanka takes 80% more time to do the work W1 alone than she takes to do it together with Sanjana. How much percent more time Sanjana will take to do the work W2 alone than she takes to do it together with Priyanka?

   (1) 125% (2) 180%

   (3) 20. (4) 80%

2. The value of the expression $(x^2 - x + 1)/(x - 1)$ cannot lie between?

   (1) (1, 3) (2) (–1, –3)

   (3) (–1, 3) (4) (–1, 2)

3. What is the maximum value of the function $y = \min(12 - x, 8 + x)$?

   (1) 12 (2) 10

   (3) 11 (4) 8

4. How many integral values for the set (x, y) would exist for the expression $|x - 4| + |y - 2| + = 5$?

   (1) 16 (2) 14

   (3) 12 (4) 20

5. A book contains 20 chapters. Each chapter has a different number of pages (each under 21). The first chapter starts on page 1 and each

chapter starts on a new page. What is the largest possible number of chapters that can begin on odd page numbers?

(1) 19 (2) 15

(3) 10 (4) 11

6. How many even three-digit integers have the property that their digits, read left to right, are not in a strictly increasing order?

(1) 420 (2) 416

(3) 412 (4) 422

7. An unlimited number of coupons bearing the digits 1, 2 and 3 are available. What is the possible number of ways of choosing 4 of these coupons so that they can not be used to make the number 123?

(1) 15 (2) 18

(3) 21 (4) 24

8. How many real solutions exist for the equation $3^x - 2x - 1 = 0$?

(1) 2 (2) 3

(3) 5 (4) 1

9. The number of rational points $x = p/5$ satisfying $\log(2x - 3/4)/\log x > 2$, where p is an integer and $gcd(p, 5) = 1$ is/are

(1) 2 (2) 3

(3) 5 (4) 1

10. Two schools play against each other in a grass court tennis tournament. Each school is represented by 8 students. Every game is a doubles game, and every possible pair from the first school must play one game against every possible pair from the second school. How many games will each student play?

(1) 196 (2) 180

(3) 192 (4) 164

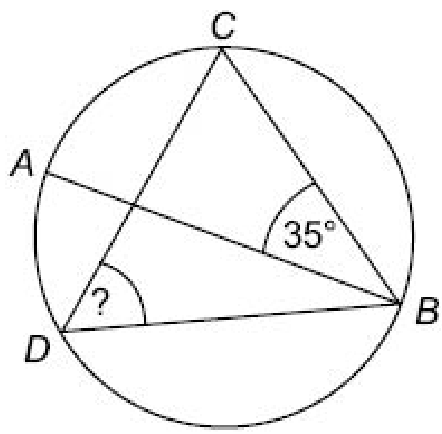
11. Consider the set $T_x = \{x, x + 1, x + 2, x + 3, x + 4, x + 5\}$. For $x = 1, 2, 3, 4 \ldots 999$. How many of these sets do not contain any 7 or any

integral multiple of 7?

(1) 121 (2) 143

(3) 144 (4) 145

12. In the figure below you can see points *A*, *B*, *C*, *D* on a circle. Chord *AB* is a diameter of this circle. The measure of angle *ABC* is 35°. The measure of angle *BDC* is:



(1) 35° (2) 45°

(3) 55° (4) 60°

13. There are two arithmetic progressions, A_1 and A_2, whose first terms are 3 and 5 respectively and whose common differences are 6 and 8 respectively. How many terms of the series are common in the first *n* terms of A_1 and A_2, if the sum of the n^{th} terms of A_1 and A_2 is equal to 6000?

(1) 101 (2) 105

(3) 107 (4) 111

14. Shaurya Sharma travels from Delhi to Lucknow at a speed of 100 Kmph and returns to Delhi at a speed of 50 Kmph. He again leaves for Lucknow immediately at a speed of 30 Kmph and goes back to Delhi at a speed of 60 Kmph. What is his average speed for the entire journey?

(1) 54 kmph (2) 48 kmph

(3) 56 kmph (4) 50 kmph

15. At 9 PM, Divya is driving her car at 100 km/h. At this velocity she has enough petrol to cover a distance of 80 km. Unfortunately the nearest petrol pump is 100 km away. The amount of petrol her car uses per km is proportional to the velocity of the car. What is the earliest time that Divya can arrive at the petrol pump?

(1) 10:12 pm (2) 10:15 pm

(3) 10:20 pm (4) 10:25 pm

16. The number y is defined as the sum of the digits of the number x, and z as the sum of the digits of the number y. Let 'A' be defined as the number of natural numbers x which satisfy the equation $x + y + z = 60$ and let 'B' be defined as the number of natural numbers x which satisfy the equation $x + y + z = 84$. Which of the following statements about A and B is/are correct?

(i) $A > B$ (ii) $A < B$

(iii) $A = B$ (iv) $A + B = 6$

(1) i & iv only (2) ii & iv only

(3) iii only (4) iii & iv only

17. The letters of the word HASTE are written in all possible orders and these words are written out as in dictionary. Then the dictionary rank of the word HEATS is:

(1) 52 (2) 54

(3) 56 (4) 58

18. Problems A, B and C were posed in a mathematical contest. 25 competitors solved at least one of the three. Amongst those who did not solve A, twice as many solved B as C. The number solving only A was one more than the number solving A and at least one other. The number solving just A equalled the number solving just B plus the number solving just C. How many solved just C?

(1) 2 (2) 4

(3) 6 (4) can not be determined

19. x, y are integers belonging to {1, 2, 3, 4, 5, 6, 7, 8, 9 … 15}. How many possible ratios of x/y can you get such that x/y is an integer?

(1) 22 (2) 40

(3) 42 (4) 44

20. Five friends Amit, Arun, Abhishek, Aishwarya and Azad buy lottery tickets having numbers 2, 4, 6, 8 and 10 respectively. Arun exchanges his ticket with Abhishek, Abhishek with Aishwarya, Aishwarya with Azad and Azad with Arun. Amit does not exchange his ticket. For three consecutive exchanges, the difference between the ticket numbers of two particular persons is constant at 2. After the fourth exchange, the difference in their ticket numbers will be

(1) 1 (2) 2

(3) 4 (4) Cannot be determined

ANSWER KEY

1. (1)	2. (3)	3. (2)	4. (4)
5. (2)	6. (2)	7. (3)	8. (1)
9. (2)	10. (1)	11. (2)	12. (3)
13. (3)	14. (4)	15. (2)	16. (4)
17. (3)	18. (1)	19. (4)	20. (2)

Solutions and Shortcuts

Solution 1: Level of Difficulty (2)

Let p be the amount of work Priyanka can do per day and s be amount of work Sanjana can do per day. According to the given statement ... ((w1/p) – (w1/(p + s)))/(w1/(p + s)) = 0.8 this gives s = 0.8p, what we were asked is, ((w2/s) – (w2/(p + s)))/(w2/(p + s)) substituting s = 0.8p, we get the required value as 1.25 i.e. 125%. Hence, choice (1) is the right answer.

Solution 2: Level of Difficulty (3)

The expression needs to be evaluated at different values of x and we can easily see that at x = 0, the value of the function becomes –1. Further at x =

0.5 we can find that the value is –3/2. So we can understand that the value of the function is reducing when we move to the right of 0. It can also be seen that to the left of 0 also there will be a drop in the value of the function. For instance at x = –0.1 also the value of the function will be less than –1. So obviously the function is reaching a kind of a maximum at –1 and is not going beyond that when the range of values are in this range.

It can be observed that after x = 1, the function will become positive. At x = 1.1 it can be seen that the value of the function would become around 10-11. As you would increase the value of x beyond 1, 1 the function would reduce in value. Also, it can be seen that after x = 1, the funtion would achieve its minimum value at x = 2 Æ where its value would be 3. After 2 the value would start increasing. Hence, the value of the function cannot be between –1 to +3. Hence, option 3 is the correct answer.

Solution 3: Level of Difficulty (1)

Equate 12 – x = 8 + x to give you the intersection point between the two lines 12 – x and 8 + x. The intersection occurs at a value of x as 2. It can be visualized by plotting both these lines that the maximum value of the given function would occur at x = 2. Hence, the correct answer would be 10.

Solution 4: Level of Difficulty (2)

Solutions would exist for the following structures of making the value of 5: 0 + 5 Æ This would happen if we take the value of x as 4 and y can take the values of 7 or –3. Hence, there would be 2 sets of integral (x, y) values giving us 0 + 5 = 5

1 + 4 Æ (5, 6), (5, –2), (3, 6), (3, –2) Æ four solutions

2 + 3 Æ four possibilities again

3 + 2 Æ four possibilities again

4 + 1 Æ four possibilities again

5 + 0 Æ 2 possibilities

Solution to Question 5: Level of Difficulty (2)

There would be 10 chapters with even number of pages. Place them to start with—each of them would start on an odd numbered page. After that start to place the chapters with an odd number of pages—the first one would start on an odd numbered page, the second on an even numbered page, the third on an odd numbered page and so on. Thus there would be 10 + 5 = 15 chapters out of 20 which can at the maximum start on an odd numbered page. Hence, option 2 is correct.

Solution to Question 6: Level of Difficulty (2)

For this question, you would have to count the actual number of numbers. In the hundreds, the first numbers you would find would be in the 120s. The first numbers are 124, 126, 128, 134, 136, 138, 146, 148, 156, 158, 168, 178.

In the 200s, the values would be 234, 236, 238, 246, 248, 256, 258, 268, 278

In the 300s the values would be 346, 348, 356, 358, 368, 378

In the 400s the values would be 456, 458, 468, 478

In the 500s there would be only 2 values.

1 value in the 600s and no value after that. Hence 34 values. But in all there are 450 even three digit numbers starting from 100, 102, 104 …998. Hence, the required answer is 450 – 34 = 416.

Solution 7: Level of Difficulty (2)

Each of the 3 places can take 3 letters fi 27. But we don't want the combination (1, 2, 3) fi 3! = 6 are out fi 27 – 6 = 21.

Solution 8: Level of Difficulty (3)

It can be seen by plotting the graph of this expression that the function $y = 3^x - 2x - 1$ would cut the x axis twice. Hence, the equation would have 2 real solutions.

Solution 9: Level of Difficulty (3)

$\log(2x - 3/4) > 2\log x$ solving we get 2 cases:

Case 1: When $x > 1$ fi $(2x - 3/4) > x^2$.

Case 2: When $x < 1$ fi $(2x - 3/4) < x^2$

Solving these inequalities we get: x lies in (3/8, 1/2) » (1, 3/2) viz. (0.375, 0.5) » (1, 1.5)

In these ranges we have two independent values which could be expressed as $p/5$ viz. 0.4 = 2/5 and 1.2 = 6/5. Since in both the cases, p is co prime with 5, we can say that both these cases satisfy the conditional requirements. Hence, choice (2) is the right answer.

Solution 10: Level of Difficulty (2)

Total matches being played = 8C2*8C2 = 28^2 = 784. Thus, a total of 784 × 4 = 3136 people are part of these 784 matches. Each of the 16 players would play in the same number of matches = 3136/16 = 196.

Hence, choice (1) is the right answer.

Solution 11: Level of Difficulty (2)

In this case you would get the sets like: {1, 2, 3, 4, 5, 6}, {8, 9, 10, 11, 12, 13}, {15, 16, 17, 18, 19, 20}…. As we can see the starting digits for each of these sets consists of an Arithmetic Progression as 1, 8, 15…. The next step is to find the number of terms in this series before 999. This can be done as: Series is 1, 8, 15, 22 … 995 …. And this series would have [(995 – 1)/7] + 1 = 143 terms.

Solution 12: Level of Difficulty (1)

Join AC fi < ACB = 90° fi < CAB = 55°. But < BDC = < CAB as they are subscribed by the same arc. Hence, choice (3) is the right answer.

Solution 13: Level of Difficulty (2)

Use $t_n = a + (n – 1)d$

$6000 = a_1 + a_2 + (n – 1)(d_1 + d_2)$

or, $n = 429$

so, we have two A.P. series

3, 9, 15, 21, ———, 2571 (Calculate t_n)

5, 13, 21, ———, 3429 (Calculate t_n)

so, common series is 21, 45 ———, x where x £ 2571

On solving, x comes for n = 107. Hence, choice (3) is the right answer.

Solution 14:

Let the distance between Delhi and Lucknow be 300 kms. The total time taken would be 3 + 6 + 10 + 5 = 24 hours. The total distance would be 1200 kms. Hence, the average speed is given by

Average speed = (total distance/total time)

= 1200/24 = 50 kmph

Hence, choice (4) is the right answer.

Solution 15: Level of Difficulty (2)

Divya needs to travel 100 km @ 80 km/h fi Time taken = 5/4 hours. Hence, choice (2) is the right answer.

Solution 16: Level of Difficulty (3)

Go through trial and error for both situations.

We can see that $x + y + z = 60$ is satisfied for $z = 44$, 47 and 50. Hence, $A = 3$

We can see that $x + y + z = 84$ is satisfied for $z = 73$, 70 and 67. Hence, $B = 3$.

Hence, choice (4) is the right answer.

Solution 17: Level of Difficulty (2)

The letters available to us are: *A*, *E H*, *S* and *T* in their alphabetical order. When we form 5 lettter words out of these, the following order of appearing in the dictionary would hold…

Words starting with *A*___4! = 24

Words starting with *E*___4! = 24

Words starting with *HA*___3! = 6

Words starting with *HE* … complete description:

First word: HEAST, Second word = HEATS…

Hence, the 56th word in the list would be HEATS. Option 3 is correct.

Solution 18: Level of Difficulty (2)

Let a solve just A, b solve just B, c solve just C, and d solve B and C but not A. Then $25 – a – b – c – d$ solve A and at least one of B or C. The conditions give: $b + d = 2(c + d)$; $a = 1 + 25 – a – b – c – d$; $a = b + c$. Eliminating a

and d, we get: $4b + c = 26$. But $d = b - 2c >= 0$, so $b = 6$, $c = 2$. Hence, choice (1) is the right answer.

Solution 19: Level of Difficulty (2)

If we take x as 2, we will get 2 integers,

With x as 3 we will get 2 integers,

With x as 4 we have 3 integers,

With x as 5 we have 2 integers,

With x as 6 we have 4 integers,

With x as 7, we have 2 integers,

With x as 8 we have 4 integers,

With x as 9 we have 3 integers,

With x as 10 we have 4 integers,

With x as 11 we have 2 integers,

With x as 12 we have 6 integers,

With x as 13 we have 2 integers

With x as 14 we have 4 integers,

With x as 15 we have 4 integers.

Thus there would be a total of 44 such integral ratios. Hence, the option 4 is correct.

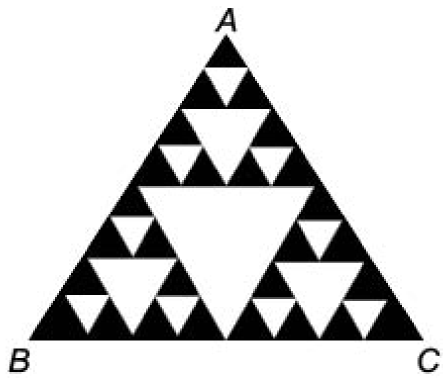
Solution 20: Level of Difficulty (1)

Name	*Amit*	*Arun*	*Abhishek*	*Aishwarya*	*Azad*
Initial	2	4	6	8	10
1st exchange	2	10	4	6	8
2nd exchange	2	8	10	4	6
3rd exchange	2	6	8	10	4
4th exchange	2	4	6	8	10

From the table, it is clear that the difference between Arun and Aishwarya's lottery ticket value is constant upto the 3rd exchange and equal to 2. After the 4^{th} exchange also it equals 2.

2 Mock Test Paper—2

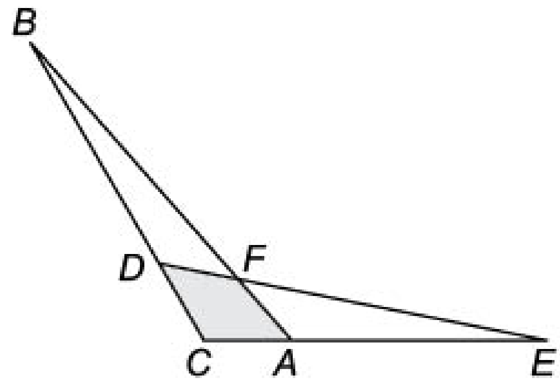
1. In the diagram, all triangles are equilateral. If $AB = 16$, then the total area of all the black triangles is



(1) $25\sqrt{3}$ (2) $27\sqrt{3}$

(3) $35\sqrt{3}$ (4) $37\sqrt{3}$

2. In the figure below $DC = AC = 1$ and $CB = CE = 4$. If the area of triangle ABC is equal to S then the area of the quadrilateral $AFDC$ is equal to:



(1) $\frac{S}{2}$ (2) $\frac{S}{4}$

(3) $\frac{S}{5}$ (4) $\frac{2S}{5}$

3. Find the number of zeroes in

$100^1 * 99^2 * 98^3 * 97^4 * 96^5 *$______$* 1^{100}$

(1) 970 (2) 1070

(3) 1120 (4) 1124

4. The crew of an 8-member rowing team is to be chosen from 12 men (M_1, M_2 …, M_{12}) and 8 women (W_1, W_2 …, W_8). There have to be 4 people on each side with at least one woman on each side. Further it is also known that on the right side of the boat (while going forward) W_1 and M_7 must be selected while on the left side of the boat M_2, M_3 and M_{10} must be selected. What is the number of ways in which the rowing team can be arranged?

(1) 1368 × 4! × 4! (2) 1200 × 4! × 4!

(3) 1120 × 4! × 4! (4) 728 × 4! × 4!

5. In a community of 20 families, every family is expected to have the number of offsprings in an Arithmetic Progression with a common difference of 2, starting with 2 offsprings in the first family. Further for every child in the family every family is expected to deposit the same number of gold coins as there are children in the family

towards a community contribution) for each child the family has. What will be the total number of gold coins collected in the community?

(1) 11480 (2) 11280

(3) 10880 (4) 10280

6. A boy plays a mathematical game wherein he tries to write the number 1998 into the sum of 2 or more consecutive positive even numbers (e.g. 1998 = 998 + 1000). In how many different ways can he do so?

(1) 5 (2) 6

(3) 7 (4) 8

7. Find the remainder when 2^{650} is divided by 224.

(1) 124 (2) 66

(3) 32 (4) 28

8. How many numbers are there between 100 and 1000, which have exactly one of their digits as 8?

(1) 300 (2) 1444

(3) 225 (4) 729

9. $f(n)$ is a function which is defined in three different domains as

$$f(n) = \begin{cases} 2n+4 \text{ if } n < -1 \\ 3n+5 \text{ if } -1 \le x < 1 \\ 2n-5 \text{ if } x \ge 1 \end{cases}$$

Find the value of (–5)?

(1) –20 (2) –12

(3) –32 (4) –36

10. Aman while getting bored in his school starts to draw lines on a fixed pattern. First, he draws a line 10 mm long. Then he constructs a square with this line as its diagonal. He then draws another square with a side of the first square as its diagonal. This process is

repeated 'x' times, until the perimeter of the x^{th} square is less than $\frac{1}{12.5}$ mm. What is the least value of 'n'?

(1) 16 (2) 18

(3) 20 (4) 22

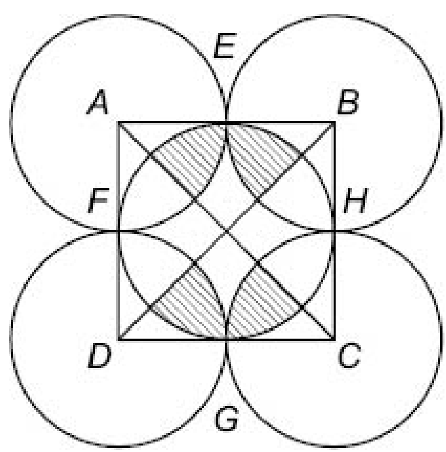
11. In the world championship of table tennis, players play best of 5 game matches. In every game the first player who scores 21 points wins. Service changes alternately between the 2 players after every five points. A player can score points both during his service and his opponent's service. Raman beat Ching Mai by 21-16 in a game. 24 of the 37 points played were won by the player serving. Who served first?

(1) Raman (2) Ching Mai

(3) Either could have served first

(4) Indeterminate

12. A square of side 12 m is drawn and a circle is inscribed in it. Now with



Now with each vertex as center, a circle is drawn passing through the midpoints of the two sides of the square, which meet at the vertex. Find the area of the shaded portion. (in m^2)

(1) $18p - 36$ (2) $36p - 72$

(3) $9p - 72$ (4) $18p - 72$

13. In a bombing of the Nathula pass, the Indian troops have to destroy a bridge on the pass. The bridge is such that it is destroyed when exactly 2 bombs hit it. A MIG-27 is dispatched in order to do the bombing. Flt Lt. Rakesh Sharma needs to ensure that there is at least 97% probability for the bridge to be destroyed. He knows that when he drops a bomb on the bridge the probability of the bomb hitting the bridge is 90%. Weather conditions and visibility being poor he is unable to see the bridge from his plane. How many bombs does he need to drop to be 95% sure that the bridge will be destroyed?

    (1) 3 (2) 4

    (3) 5 (4) 6

14. In a bombing of the Nathula pass, the Indian troops have to destroy a bridge on the pass. The bridge is such that it is destroyed when exactly 3 bombs hit it.

    Flt Lt. Rakesh Sharma for his extraordinary skills is given the responsibility to drop the bombs again. If he drops a maximum of 5 bombs and he knows that when he drops a bomb on the bridge the probability of the bomb hitting the bridge is 90%. What will be the probability that the bridge would have been destroyed? (assume the bridge is visible to Flt Lt Sharma this time).

    (1) 0.98348 (2) 0.98724

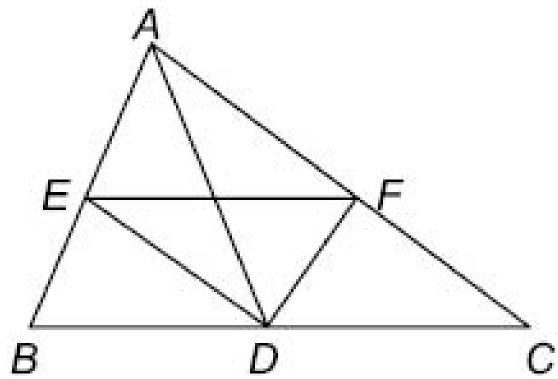
    (3) 0.99144 (4) 0.99348

15. In a peculiar chessboard containing 12×12 squares, what is the greatest number of 5×5 squares that can be traced?

    (1) 8 (2) 16

    (3) 64 (4) 32

16. In the following figure (not drawn to scale)–*DEF* = 42°. Find the other two angles of D*DEF* if *DE* and *DF* are the angle bisectors of –*ADB* and –*ADC* respectively.



(1) 28° and 11°0 (2) 65° and 73°

(3) 67° and 71° (4) 48° and 90°

17. The sides of a triangle have 4, 5 and 6 interior points marked on them respectively. The total number of triangles that can be formed using any of these points

(1) 371 (2) 415

(3) 286 (4) 421

18. The sides of a triangle have 4, 5 and 6 interior points marked on them respectively. The total number of triangles that can be formed using any of these points and/or the vertices would be?

(1) 704 (2) 415

(3) 684 (4) 421

19. The number of digits lying between 2000 and 7000 (including 2000 and 7000) which have at least two digits equal but at most 3 digits equal is:

(1) 2476 (2) 2474

(3) 2475 (4) 2473

20. The number of ways in which three distinct numbers in *AP* can be selected from 1, 2, 3, …, 24 is

(1) 144 (2) 276

(3) 572 (4) 132

ANSWER KEY

1. (4)	2. (4)	3. (4)	4. (1)
5. (1)	6. (3)	7. (3)	8. (3)
9. (4)	10. (2)	11. (1)	12. (2)
13. (1)	14. (3)	15. (3)	16. (4)
17. (4)	18. (1)	19. (1)	20. (4)

Solutions and Shortcuts

Solution 1: Level of Difficulty (1)

In the diagram, there are 27 black triangles. If the entire diagram was divided into the smallest size equilateral triangles, there would be 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64 equilateral triangles). Thus, 27/64 of D*ABC* is coloured black, so 37/64 is unshaded.

Area of triangle *ABC* = $\left\{\sqrt{(3)/4}\right\}$ × 16 × 16 = 64 $\sqrt{3}$. Hence, the area of the unshaded portion is 37 $\sqrt{3}$. Drop a perpendicular from *A*, meeting *BC* at *D*. Since *D ABC* is equilateral and *AB* = 16, then *BD* = *DC* = 8. Hence, Option 4 is correct.

Solution 2: Level of Difficulty (3)

The best way to solve problems like these is to assume specific case: Assume *ACB* and *ECB* to be right angled. Also assume that the point *C* is at the origin (0, 0). Then by the information we can conclude that *E* would be (4, 0), *B*(0, 4), *A*(1, 0) and *D*(0, 1).

Then the equations of the lines *AB* and *ED* would be 4*x* + *y* = 4 and *x* + 4*y* = 4 respectively.

The intersection point would be got by equating these lines: *F* = (4/5, 4/5).

Area of *CFA* is 2/5. Hence, area of *CAFD* = 2*2/5 = 4/5. But area of *ABC* = *S* = 2. Hence, choice (4) is the right answer.

Solution 3: Level of Difficulty (2)

= (1 + 6 + 11 + 16 + 26 + 31___+ 96) + (1 + 26 + 51 + 76)

= 20*48.5 + 4*38.5

= 970 + 154 = 1124

Solution 4: Level of Difficulty (3)

In this question you will first have to complete the selection of 4 people for either side and then arrange the rowers on each side (which would be done by using 4!)

The solution would depend on the following structure—the structure would vary based on whether you select 2 more men for the right side or you select 1 man and 1 woman for the right side or you select 2 women for the right side.

The solution would be given by:

$^{12}C_2 \times 4! \times {}^8C_1 \times 4! + {}^{12}C_1 \times {}^8C_1 \times 4! \times {}^7C_1 \times 4!$

$+ {}^8C_2 \times 4! \times {}^6C_1 \times 4! = 1368 \times 4! \times 4!$

Hence, Option 1 is the correct answer.

Solution 5: Level of Difficulty (3)

The number of children in the family would be: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32 ,34, 36, 38, 40.

Hence, the number of gold coins selected would be:

2 × 2 + 4 × 4 + 6 × 6 + … 40 × 40 = 4 + 16 + 36 + 64 + 100 + 144 + 196 + 256 + 324 + 400 + 484 + 576 + 676 + 784 + 900 + 1024 + 1156 + 1296 + 1444 + 1600 = 11480

Solution 6: Level of Difficulty (3)

For answering this question we need to plan the use of the factors of 1998.

$1998 = 2 \times 3^3 \times 37$ Æ 16 factors viz. 1 × 1998, 2 × 999, 3 × 666, 6 × 333, 9 × 222, 18 × 111, 27 × 74, 54 × 37.

Thus we could form 7 APs as follows:

(1) An AP with 2 terms and average 999

(2) An AP with 3 terms and average 666 and so on Æ 7 ways.

Solution 7: Level of Difficulty (1)

$2^{650} = 25$ ◊ $(2^3)^{215} = 32 \times 8^{215}/224 = 8^{215}/7$

gives us a remainder of 1. Since we have cut the numerator and denominator by 32 in the process, the actual remainder must be 32.

Solution 8: Level of Difficulty (2)

There are exactly 72 numbers in which 8 comes at the units place. Given by $8 \times 9 \times 1$

Similarly at the tens place there are $8 \times 1 \times 9$ ways

Also at the hundred's place there are $1 \times 9 \times 9 = 81$ ways

So, there are $72 + 72 + 81 = 225$ numbers between 100 and 1000 which have exactly one of their digits as 8.

Solution 9: Level of Difficulty (1)

$f(-3) = 2(-5) + 4 = -6$

$f(-6) = 2(-6) + 4 = -8$

$f(-8) = 2(-8) + 4 = -12$

$f(-12) = -20$

$f(-20) = -36$

Solution 10: Level of Difficulty (2)

(3) Diagonal of first square = 10 mm.

Side of first square = $10/\sqrt{2}$

Side of second square = $10 \times 1/(\sqrt{2})^2$ …

Side of x^{th} square = $10 \times 1/(\sqrt{2})^x$

fi $4 \times 10 \times 1/(\sqrt{2})^x < 0.08$ fi $1/(\sqrt{2})^x < 0.002$

fi $(1/2)^{x/2} < 2/1000$ fi x should be at least 18.

Solution 11: Level of Difficulty (3)

In all 37 services are made. Two cases are possible.

(a) Raman has served 20 times and Ching Mai has served 17 times.

(b) Ching Mai has served 20 times and Raman has served 17 times.

We would need to see what occurs in each case to understand which of these situations holds true.

Case (a): In this case Raman starts serving. Let X be the score obtained by Raman in his services and Y be the score obtained by Raman when Ching Mai serves. The following table would emerge:

Player who serves	*No. of balls serves*	*Scores of Raman*	*Scores of Ching Mai*
R	*C*		
R	20	X	$20 - X$
C	17	Y	$17 - Y$

Now, $X + Y = 21$, $X + 17 - Y = 24$.

fi $X = 14$ fi we get an acceptable solution.

Case (b): In this case Ching Mai starts serving.

Player who serves	*No. of balls serves*	*Scores of Raman*	*Scores of Ching Mai*
R	*C*		
C	20	X	$20 - X$
R	17	Y	$17 - Y$

fi $X + Y = 21$

fi $20 - X + Y = 24$

fi $Y = 12.5$ (inadmissible)

Hence Raman started first.

Solution 12: Level of Difficulty (2)

In order to solve the question you need to find the difference between the area of the quarter circle (inside the square) and the right angled triangle at the center of the inner circle having base and height equal to 6 m each. The required area would be 4 times this value.

Hence, the required answer would be given by:

$4x[36p/4 - 18] = 36p - 72$

Solution 13: Level of Difficulty (3)

The probability if he drops 3 bombs will be given by: Hit and Hit OR Miss and Hit and Hit OR Hit and Miss and Miss = $0.9 \times 0.9 + 0.1 \times 0.9 \times 0.9 + 0.9 \times 0.1 \times 0.9 = 0.81 + 0.081 + 0.081 = 0.972 > 0.97$.

Hence, 3 bombs would give him a probability of higher than 97% for the bridge to be destroyed.

Solution 14: Level of Difficulty (3)

The bridge would be destroyed under the following conditions: In 3 bombs OR In 4 Bombs Or In 5 Bombs Hit, Hit and Hit OR One miss and 3 hits (3 ways) OR 2 misses and 3 hits ($4C_2 = 6$ ways)

$= (0.9)^3 + 3 \times (0.9)^3 \times (0.1)^1 + 6 \times (0.9)^3 \times (0.1)^2$

$= 0.729 + 3 \times 0.0729 + 6 \times 0.00729 = 0.99144$.

Solution 15: Level of Difficulty (2)

The required answer would be given by $8 \times 8 = 64$ (Because if you take the top side of the square on the top side of the chess board, you would be able to trace out 8 squares. Similarly you would get exactly 8 squares in each row when you move to the next row of the chess board. This would continue till the 8^{th} row of the chess board is taken as the top row of the 5×5 square. Hence, $8 \times 8 = 64$).

Solution 16: Level of Difficulty (2)

The measure of the angle *EDF* has to be 90° since it should be half of the 180° angle. Hence, the required answer has to be option 4.

Solution 17: Level of Difficulty (3)

You can form triangles by taking 1 point from each side, or by taking 2 points from any 1 side and the third point from either of the other two sides.

This can be done in: $4 \times 5 \times 6 + {}^4C_2 \times {}^{11}C_1 + {}^5C_2 \times {}^{10}C_1 + {}^6C_2 \times {}^9C_1 = 120 + 66 + 100 + 135 = 421$

Solution 18: Level of Difficulty (3)

In order to solve this question, you will first need to solve assuming that neither of the three vertices (say *A*, *B* or *C*) are used. For this we can solve the following way: You can form triangles by taking 1 point from each side, or by taking 2 points from any 1 side and the third point from either of the other two sides.

This can be done in: $4 \times 5 \times 6 + {}^4C_2 \times {}^{11}C_1 + {}^5C_2 \times {}^{10}C_1 + {}^6C_2 \times {}^9C_1 = 120 + 66 + 100 + 135 = 421$

Then, you need to consider situations where you are either taking one of the vertices of the triangle *ABC* or taking two vertices from the triangle. Thus assuming 6 points on *AB*, 4 on *AC* and 5 on *BC* the following possibilities emerge: Taking $A - 5C_2 + 6 \times 5 + 6 \times 4 + 5 \times 4 = 10 + 30 + 24 + 20 = 84$

Taking $B - 4C_2 + 4 \times 5 + 4 \times 6 + 5 \times 6 = 80$

Taking $C - 6C_2 + 6 \times 4 + 6 \times 5 + 5 \times 4 = 89$

Taking *AB* – 9

Taking *AC* – 11

Taking *BC* – 10

Thus the total number of ways is: 421 + 84 + 80 + 89 + 9 + 11 + 10 = 704

Solution 19: Level of Difficulty (3)

Numbers having 3 digits equal—Starting with 2: Numbers with 3 twos-222_ Æ 9 numbers (and 3 such cases so a total of 27 such numbers) Numbers with 3 digits other than twos Æ like 2000, 2111, 2333 … 2999 = 9 numbers

Total of 36 numbers

Similarly, numbers starting with 3 and having 3 digits common would be 36

numbers starting with 4 and having 3 digits common would be 36

numbers starting with 5 and having 3 digits common would be 36

numbers starting with 6 and having 3 digits common would be 36

Total numbers having 3 digits equal = 180.

Numbers having 2 digits equal—Again we first count number of numbers that are starting with 2 and having 2 digits equal and then multiply the same by 5 to get the answer: In order to find the number of numbers starting with 2 and having 2 digits equal we need to divide the same into:

(a) Numbers starting with 2 and having 2 twos and two more digits equal $3C_1 \times 9 = 27$

OR

(b) Numbers starting with 2 and having 2 twos and two more digits which are not equal to one another-

$3C_1 \times 9C_1 \times 8C_1 = 216$

OR

(c) Number of numbers starting with 2 and having two digits which are same and a fourth digit which is other than 2 as well as different from the other digit which has been used for 2 digits same.

$9C_1 \times 3C_1 \times 8C_1 = 216$

Thus, the total number of such numbers is 459.

Totally, 459 × 5 + 180 + 1 (for 7000) = 2476

Solution 20: Level of Difficulty (2)

If we take common difference = 1, then (1, 2, 3), (2, 3, 4) … (22, 23, 24). There will be 22 combinations.

If we take common difference = 2, then total 20 combinations are possible i.e. (1, 3, 5), (2, 4, 6), (3, 5, 7), …, (20 , 22, 24).

If we take c.d. = 3, 18 combinations are possible

If we take c.d. = 4, 16 combinations are possible

(There will be two combinations in last i.e. (1, 12, 23) & (2, 13, 24) with c.d. = 11

So total no. of combinations

= 22 + 20 + 18 + 16 + … + 4 + 2

= 2 × (1 + 2 + 3 + … + 11)

$= 2 \times \frac{11 \times 12}{2} = 132$

3 Mock Test Paper—3

1. Find the number of solutions for x if 50 £ x £ 120 such that $\left[\frac{x}{6}\right] + \left[\frac{x}{5}\right] + \left[\frac{x}{10}\right] = \frac{7x}{15}$, where [$y$] indicates the greatest integer less than or equal to y.

   (1) 0 (2) 1

   (3) 2 (4) 3

2. The number of ordered pairs of positive integers (m, n) such that $\frac{1}{m} + \frac{1}{n} = \frac{1}{15}$ is:

   (1) 10 (2) 2

   (3) 4 (4) 9

3. Let 'f' be a function such that for real x & y, $f(x) \times f(y) = f(x + y)$ & $\frac{f(x)}{f(y)} = f(x - y)$ for all $f(y)$ π 0.

   Given that $f(0) = 1$ and $f(1) = 3$, then $f(x - 1) + f(-x - 1)$

   (1) cannot be negative.

   (2) has minimum value 2/3.

   (3) lies between 0 and 2/3.

(4) none of these.

4. Let $a = \frac{1^2}{1} + \frac{2^2}{3} + \frac{3^2}{5} + \ldots + \frac{1001^2}{2001}$ and $b = \frac{1^2}{3} + \frac{2^2}{5} + \frac{3^2}{7} + \ldots + \frac{1001^2}{2003}$. The integer closest to $a - b$ is

(1) 500 (2) 501

(3) 1000 (4) 1001

5. Which of the following statements is false?

(1) The product of three consecutive even numbers must be divisible by 48.

(2) x% of y% of z is same as z% of x% of y.

(3) The factorial of any natural number greater than 1 cannot be a perfect square.

(4) The numbers $(100)_2$, $(100)_3$, $(100)_4$, $(100)_5$ … and so on, when converted to decimal system are all in an arithmetic progression.

6. Find the length of the string wound on a cylinder of height 48 cm and a base diameter of $5\frac{1}{11}$ cm. The string makes exactly four complete turns round the cylinder while its two ends touch the cylinder's top and bottom.

(1) 192 cm (2) 80 cm

(3) 64 cm (4) Cannot be determined

7. In Coorg, the production of tea is three times the production of coffee. If a percent more tea and b percent more coffee were produced, the aggregate amount would be $5c$ percent more. But if b percent more tea and a percent more coffee were produced, the aggregate amount produced would be $3c$ percent more. What is the ratio $a : b$?

(1) 1 : 3 (2) 1 : 2

(3) 2 : 1 (4) 3 : 1

8. A girl wished to purchase m roses for n rupees. (m and n are integers). The shopkeeper offered to give the remaining 10 flowers also if she paid him a total of ` 2. This would have resulted in a saving of 80 paise per dozen for her. How many flowers did she wish to buy initially?

(1) 4 (2) 5

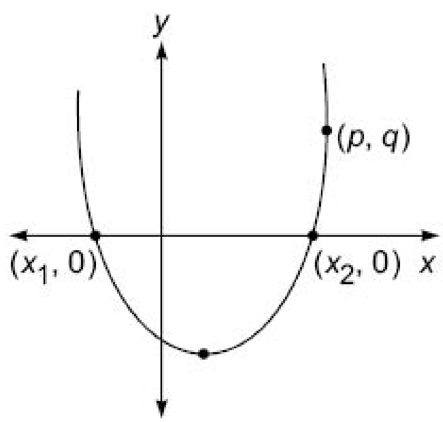
(3) 6 (4) 7

9. Samit went to the market with ` 100. If he buys three pens and six pencils he uses up all his money. On the other hand if he buys three pencils and six pens he would fall short by 20%. If he wants to buy equal number of pens & pencils, how many pencils can he buy?

(1) 4 (2) 5

(3) 6 (4) 7

Directions for Questions 10 and 11: The graph given below represents a quadratic expression, $f(x) = ax^2 + bx + c$. The minimum value of $f(x)$ is –2 and it occurs at $x = 2$



x_1 and x_2 are such that $x_1 + x_2 + 2x_1x_2 = 0$.

10. What is the nature of the roots of the equation $f(x) = 0$?

(1) Complex conjugates

(2) Rational

(3) Irrational

(4) Cannot be determined

11. Find the value of the function at $x = 3$

(1) 2 (2) – 1

(3) 5 (4) none of these

12. How many numbers are there between 100 and 1000, which have exactly one of their digits as 7?

(1) 300 (2) 1444

(3) 225 (4) 729

13. A group of men was employed to shift 545 crates. Every day after the first, 6 more men than the previous day were put on the job. Also, every day after the first, each man working, shifted 5 fewer crates than the number of crates moved by each man the previous day. The result was that during the latter part of the period, the number of crates shifted per day began to go down. 5 days were required to finish the work.

What was the number of crates shifted on the third day?

(1) 137 (2) 169

(3) 26 (4) 152

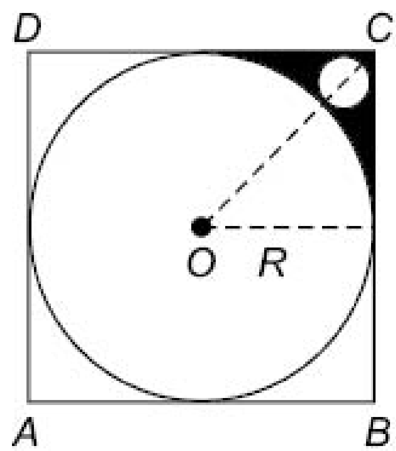
14. A group of men was employed to shift 545 crates. Every day after the first, 6 more men than the previous day were put on the job. Also, every day after the first, each man working, shifted 5 fewer crates than the number of crates moved by each man the previous day. The result was that during the latter part of the period, the number of crates shifted per day began to go down. 5 days were required to finish the work.

What was the number of men working on the fifth day?

(1) 13 (2) 25

(3) 19 (4) 75

15. In the figure given, the radius of the smaller circle is $\sqrt{2} - 1$ cm. Then the area of shaded region will be



(1) $3 - \sqrt{2}$ cm^2

(2) $\frac{3}{4}(4 - p) + \sqrt{2}(4 - \sqrt{2}p)$ cm^2

(3) $\frac{3}{4}(4 - 5p) + \frac{1}{\sqrt{2}}(4 - 3p)$ cm^2

(4) Data insufficient

16. Two sisters go up 40-step escalators. The older sister rides the up escalator, but can only take 10 steps up during the ride since it is quite crowded. The younger sister runs up the empty down escalator, arriving at the top at the same time as her sister. How many steps does the younger sister take?

(1) 70 (2) 60

(3) 80 (4) 70

17. In how many ways may the numbers {1, 2, 3, 4, 5, 6} be ordered such that no two consecutive terms have a sum which is divisible by 2 or 3?

(1) 6 (2) 12

(3) 15 (4) 14

18. Let $f(n)$ denote the square of the sum of the digits of n. Let $f^2(n)$ denote $f(f(n))$, $f^3(n)$ denote $f(f(f(n)))$ and so on. Then $f^{1998}(11) =$

(1) 49 (2) 56

(3) 169 (4) 16

19. A biologist catches a random sample of 60 fish from a lake, tags them and releases them. Six months later she catches a random sample of 70 fish and finds 3 are tagged. She assumes 25% of the fish in the lake on the earlier date have died or moved away and that 40% of the fish on the later date have arrived (or been born) since. What does she estimate as the number of fish in the lake on the earlier date?

(1) 840 (2) 280

(3) 560 (4) 750

20. Michael Jordan's probability of hitting any basketball shot is three times than mine, which never exceeds a third. To beat him in a game, I need to hit a shot myself and have Jordan miss the same shot. If I pick my shot optimally, what is the maximum probability of winning which I can attain?

(1) $\frac{1}{16}$ (2) $\frac{1}{12}$

(3) $\frac{5}{6}$ (4) 1/14

ANSWER KEY

1. (4)	2. (4)	3. (2)	4. (2)
5. (4)	6. (2)	7. (4)	8. (2)
9. (1)	10. (3)	11. (2)	12. (3)
13. (2)	14. (2)	15. (3)	16. (1)
17. (2)	18. (3)	19. (1)	20. (2)

Solutions and Shortcuts

Solution 1: Level of Difficulty (2)

Since a greatest integer function always delivers an integer value, $\frac{7x}{15}$ must be an integer.

So possible values of x lying in the given domain would be 60, 75, 90, 105 and 120. Out of these 60, 90 and 120 satisfy the given equation.

Solution 2: Level of Difficulty (2)

$\frac{1}{m}+\frac{1}{n}=\frac{1}{15}$ fi $\frac{m+n}{mn} = \frac{1}{15}$ fi $mn - 15(m + n) = 0$

Adding 225 to both sides we get, $mn - 15(m + n) + 225 = 225$ fi $(m - 15)(n - 15) = 225$.

Now we have to consider all the factors of 225 which are 1, 3, 5, 9, 15, 25, 45, 75 and 225. Hence 9 possible ordered pairs of (m, n) satisfy the given condition.

Solution 3: Level of Difficulty (2)

The given properties lead us to an exponential function, i.e. $f(x) = a^x$. Given that $f(1) = 3$ fi $a = 3$. So the given function is $f(x) = a^x$.

$f(x - 1) + f(-x - 1) = 3^{x-1} + 3^{-x-1}$

$= \frac{1}{3}(3^x + 3^{-x}) \geq \frac{1}{3}$ ◊ $2\sqrt{3^x \cdot 3^{-x}}$

[\ A.M ≥ G.M.]

fi $3^{x-1} + 3^{-x-1} \geq 2/3$.

Hence the minimum value of $3^{x-1} + 3^{-x-1} = 2/3$.

Solution 4: Level of Difficulty (2)

$a - b = \frac{1^2}{1} + \frac{2^2-1^2}{3} + \frac{3^2-2^2}{5} + \frac{4^2-3^2}{7} + \dots$

$+ \frac{1001^2-1000^2}{2001} - \frac{1001^2}{2003}$

Since   =   = 1 for all $k \geq 0$, it follows that

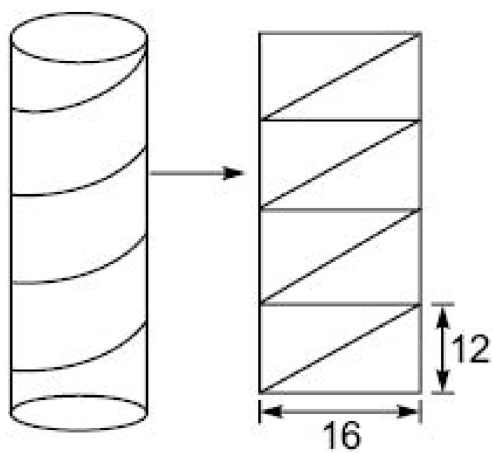
$a - b =$ –   ª 1001 – 500.25 ª 500.75

Hence, $a - b$ ª 501.

Solution 5: Level of Difficulty (2)

The numbers in option (4) can be written as 2^2, 3^2, 4^2, 5^2 … so on, which are not in an arithmetic progression. All the other statements always hold true.

Solution 6: Level of Difficulty (2)



The base circumference = $\frac{22}{7} \times \frac{56}{11}$ = 16 cm. The length of one complete turn = $\sqrt{16^2 + 12^2}$ = 20 cm. Hence, total length = 80 cm.

Solution 7: Level of Difficulty (2)

Let x be the production of tea and y be the production of coffee. It is given that $x = 3y$.

Also, $x(100 + a) + y(100 + b) = (x + y)(100 + 5c)$

Æ $x(a - 5c) = y(5c - b)$

i.e. $3a - 15c = 5c - b$

i.e. $3a + b = 20c$ (i)

Also, $x(100 + b) + y(100 + a) = (x + y)(100 + 3c)$

Æ $x(b - 3c) = y(3c - a)$

i.e. $3b - 9c = 3c - a$

i.e. $a + 3b = 12c$ (ii)

Eqn. (i) – 3 × Eqn. (ii) given,

$3a + b = 20c$

$a + 3b = 12c$

$-8b = -16c$

i.e. $b = 2c$

\ $3a = 18c$ Æ $a = 6c$

\ $\frac{a}{b} = \frac{6c}{2c} = \frac{3}{1}$

Hence (4).

Solution 8: Level of Difficulty (2)

$n = 1$; since n is an integer < 2.

\ $\frac{100}{m} - \frac{200}{m+10} = \frac{80}{12}$

\ $m = 5$.

Hence [2].

Solution 9: Level of Difficulty (1)

9 pens + 9 pencils would cost him ` 225. Hence, he can buy 4 pens and 4 pencils for `100.

Solution 10: Level of Difficulty (2)

The differentiation of the function would give:

$2ax + b = 0$ Æ $x = -b/2a = 2$ Æ $-b = 4a$ Æ $b = -4a$.

Hence, the expression would become: $x^2 - 4x + c$. At $x = 2$, it becomes $4 - 8 + c = -2$ Æ $c = 2$. Hence, the equation is $x^2 - 4x + 2$.

10. 3

It can be seen for this equation that the roots are irrational.

Solution 11: Level of Difficulty (1)

Æ $b = -4a$.

Hence, the expression would become: $x^2 - 4x + c$. At $x = 2$, it becomes $4 - 8 + c = -2$ Æ $c = 2$

Hence, the equation is $x^2 - 4x + 2$.

At $x = 3$, the function becomes $11 - 12 = -1$.

Solution 12: Level of Difficulty (2)

Solution 13: Level of Difficulty (2)

Consider a three digit number in which 7 comes exactly once i.e. from 107 to 997.

If you fix seven at the units place and use the other digits for filling in the other 2 places you will get 72 numbers in which 7 comes at unit place i.e. $\underline{8} \times \underline{9} \times \underline{_7}$

Similarly, at the ten's place there would be i.e. $\underline{8} \times \underline{_7} \times 9 = 72$ ways

And also at the hundred's place there would be $\underline{_7} \times \underline{9} \times 9 = 81$ ways

So, the total numbers from 100 to 1000, there are $72 + 72 + 81 = 225$ numbers in which they have exactly one of their digit is 7.

Solution 13: Level of Difficulty (2)

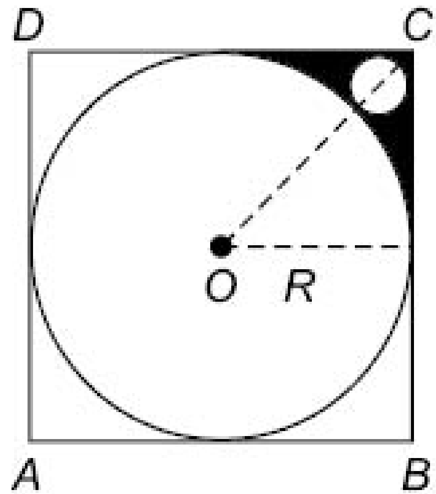
Solve through trial and error (2)

Solution 14: Level of Difficulty (2)

Solve through trial and error (2)

Solution 15: Level of Difficulty (3)

Let radius of bigger circle be R.



\ $OC = R\sqrt{2}$ cm.

If ‘r’ be the radius of smaller circle. Then $R\sqrt{2} = R + r + r\sqrt{2}$ (By Fig.)

$R(\sqrt{2} - 1) = r(\sqrt{2} + 1)$

$R(\sqrt{2} - 1) = (\sqrt{2} - 1)(\sqrt{2} + 1)$ (\ $r = \sqrt{2} - 1$)

fi $R = (\sqrt{2} + 1)$ cm.

Area of shaded part $= R^2 - \dfrac{\pi R^2}{4} - pr^2$

$= (\sqrt{2} + 1)^2 - \dfrac{\pi}{6}(\sqrt{2} + 1)^2 - p(\sqrt{2} - 1)^2$

$= 2 + 1 + 2\sqrt{2} - p\left(\dfrac{2+1+2\sqrt{2}}{4} + 2 + 1 - 2\sqrt{2}\right)$

$= 3 + 2\sqrt{2} - \dfrac{\pi}{4}(3 + 2\sqrt{2} + 12 - 8\sqrt{2})$

$= 3 + 2\sqrt{2} - \dfrac{\pi}{4}(15 - 6\sqrt{2})$

$= 3\left(1 - 5\dfrac{\pi}{4}\right) + 2\sqrt{2}\left(1 - 3\dfrac{\pi}{4}\right)$

$= \dfrac{3}{4}(4 - 5p) + \dfrac{1}{\sqrt{2}}(4 - 3p)$

Solution 16: Level of Difficulty (2)

In the time it takes the older sister to reach the top, the up escalator has carried her forward 30 steps, because the total length of the escalator is 40 steps and she took 10 steps herself. Therefore, in the same time the down escalator pushes the younger sister back 30 steps. To make up this set-back and cover the original 40 steps separating her from the top the younger sister must make a total of 70 steps.

Solution 17: Level of Difficulty (2)

Each of the numbers from 1 to 6 can only have two possible neighbours to avoid sums divisible by 2 or 3. For example, 5 may neighbour only 2 or 6. If we write the numbers 1, 4, 3, 2, 5, and 6 in that order in a ring then each number will be next to its two possible neighbors. Therefore, to order the six numbers successfully we need only start at any point along the ring (6 choices) and list the numbers as they appear around the ring in either direction (2 choices) for a total of 12 possible orderings.

Solution 18: Level of Difficulty (1)

We find $f^4(11) = 169$, $f^6(11) = 169 \setminus f^{1998}(11) = 169$.

Solution 19: Level of Difficulty (2)

Suppose there are N fishes initially. Then after 6 months, $N \times 75\%/60\% = 5N/4$ fishes, and $60 \times 75\% = 45$ tagged. Estimate $45/(5N/4) = 3/70$, or $N = 840$

Solution 20: Level of Difficulty (2)

If *I* choose a shot that *I* will make with probability p (where p is between 0 and 1/3), then Michael Jordan will make the same shot with probability $3p$. Hence, the probability that *I* make a shot that Jordan subsequently misses is $p(1 - 3p)$. The graph of this function is a parabola which equals zero when $p = 0$ and $p = \frac{1}{3}$. By symmetry the vertex (maximum) is midway between the two, at $p = \frac{1}{6}$.

Hence, the best chance *I* have of winning the game is $\frac{1}{12}$.

Model Test Paper

MODEL TEST PAPER

(base on the Latest Online Pattern)

Instructions: Time: 70 mins in each section

This section contains 1 model test paper. The paper has 2 sections having 20 questions each. In order to reach the qualifying score, you would need to solve at least 36 questions at 100% accuracy. However at an accuracy of 90%, you would need around 45 attempts with at least 40 correct questions.

While taking the test you have to ensure that you have to spend the entire 70 mins allocated for a section within the section itself. You are not allowed to move between the sections while solving a section.

Model Test Paper

Section I: Quantitative Aptitude and Data Interpretation

1. I bought 7 apples, 9 oranges and 5 guavas. Rajan bought 11 apples, 10 guavas and 18 oranges for an amount which was three-fourth more than what I had paid. What per cent of the total amount paid by me was paid for the apples?

   (a) 37.5% (b) 62.5%

   (c) 58.3% (d) 45%

2. Find the sum of all the roots of the given function:

   $F(x) = |x - 2|^2 + |x - 2| - 2$

   (a) 4 (b) 3

   (c) 5 (d) not possible to find

3. The equation $(x + 1)^{0.5} - (x - 1)^{0.5} = (4x - 1)^{0.5}$ has:

   (a) no solutions (b) one solution

   (c) two solutions (d) more than two solution

4. A man has 7 friends comprising 4 ladies and 3 gentlemen. His wife also has 7 friends comprising 4 gentlemen and 3 ladies. Find the no. of ways in which 6 people can be invited by man and his wife such

that among them 3 are ladies , 3 are gentlemen, 3 are man’s friends and 3 are wife’s friends.

(a) 324 (b) 400

(c) 475 (d) 485

5. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all 5 balls. In how many different ways we can place balls so that no box remains empty?

(a) 60 (b) 150

(c) 300 (d) 600

6. Find the sum of the series

$1/(1 + 1^2 + 1^4) + 2/(1 + 2^2 + 2^4) + 3/(1 + 3^2 + 3^4) + \ldots\ldots\ldots\ldots$ till infinity

(a) 1/3 (b) 1/2

(c) 2/3 (d) 3/4

7. Ishant and Ganguly were warming up for a match practice. So both of them were taking a round of the cricket stadium which had a circumference of 1000 m. If they both maintain a uniform speed throughout the run and Ishant crosses Ganguly for the first time in 3.5 mins., then what is the time taken by Ishant to cover 4 laps of the ground? Both have the same starting speed and as per coach’s orders both have to complete 4 laps of the ground. Also Ishant being a fast bowler runs at twice the speed of Ganguly.

(a) 20 min (b) 15 min

(c) 10 min (d) 7 min

8. The college auditorium is to be painted. But due to lack of funds it couldn’t be done. So a sincere student took the initiative and decided to paint the audi. Seeing his enthusiasm other students joined him. He is the only person doing the job on the first day. On the second day 2 more boys join him. On the third day 3 more boys join the group of the previous day and so on.....In this manner, the job is finished in exactly 25 days. How many days would 15

workers take to do the same job, given that one man is twice as efficient as a boy.

(a) 142 (b) 143

(c) 145 (d) 147

9. Three cards, each with a positive integer written on it, are lying face-down on a table. Twinkle, Raveena, and Akshay are told that

(a) the numbers are all different,

(b) they sum to 13, and

(c) they are in increasing order, left to right

First, Twinkle looks at the number on the leftmost card and says, “I don’t have enough information to determine the other two numbers.” Then Akshay looks at the number on the rightmost card and says, “I don’t have enough information to determine the other two numbers.” Finally, Raveena looks at the number on the middle card and says, “I don’t have enough information to determine the other two numbers.” Assume that each person knows that the other two reason perfectly and hears their comments. What number is on the middle card?

(a) 2 (b) 3

(c) 4 (d) 5

10. Let a, b, c, d, e, f, g and h be distinct elements in the set {–7, –5, –3, –2, 2, 4, 6, 13}. What is the minimum possible value of $(a + b + c + d)^2 + (e + f + g + h)^2$?

(a) 34 (b) 50

(c) 30 (d) 40

11. A sum of money gets doubled in little over 4 years at X % SI. Had the rate been Y % the sum would have been thrice in little less than 7 years. Then the least possible value of (Y-X), given that it is an integer will be

(a) 4 (b) 5

(c) 2 (d) 6

Directions for Questions 12 and 13: A, B, and C are three persons on a circular track in random order. They start moving simultaneously with constant speeds such that after some time t, A, B, C are at the initial positions of B, C and A respectively (none of them having completed a full revolution during the time).

12. If A, B and C can complete a full rotation of a circular track in 4, 6 and 12 minutes respectively, then t (in seconds) can be:

    (a) 60 (b) 240

    (c) 120 (d) both 120 and 240 seconds are possible

13. If A, B and C come again to their initial positions simultaneously next time after 1 hour; then t can not be equal to

    (a) 10 minutes (b) 12 minutes

    (c) 15 minutes (d) 18 minutes

14. For each real number x, let $f(x)$ be the minimum of the numbers $4x + 1$, $x + 2$, and $-2x + 4$. Then the maximum value of $f(x)$ is

    (a) 1/3 (b) 1/2

    (c) 2/3 (d) 8/3

Questions (15 and 16): A group of students are asked to take at least one subject from Physics, Chemistry, and Mathematics for their courses.

15. The number of students taking exclusively one subject is 50 while the number of students taking exclusively two subjects is 30. The number of students taking physics, chemistry, and mathematics is 45, 44, and 48 respectively. The number of students taking all subjects is

    (a) 9 (b) 10

    (c) 11 (d) 13

16. The number of students taking physics, chemistry and mathematics is 42, 45 and 49 respectively while the number of students taking

physics or chemistry is 65, that taking chemistry or mathematics is 70 and that taking physics or mathematics is 72. What is the number of students not taking all 3 subjects?

(a) 71 (b) 77

(c) 58 (d) cannot be determined

17. Lets consider all irreducible fractions below 1, of type p/q where $q = 99$ and p/q is positive. If S denotes the sum of all such fractions denoted by p/q , then the value of S is best represented by

(a) 45 (b) 40

(c) 50 (d) 44

18. 900 distinct 'n' digit numbers are to be formed by using only 3 digits 2,5,7. Find the smallest value of 'n' for which this is possible.

(a) 5 (b) 6

(c) 7 (d) 8

19. Let $g(x) = 1 + x - [x]$ and $f(x) = -1$ for $x < 0$, 0 for $x = 0$, 1 for $x > 0$. Find $f(g(x))$.

(a) x (b) 0

(c) 1 (d) $f(x)$

20. 301! is divisible by $(30!)^n$. What is the max. possible integral value of n?

(a) 13 (b) 16

(c) 9 (d) 10

Directions for Questions 21 to 23: Answer the questions on the basis of data given in the table:

The table below shows the sales and expenditure of three companies—KESARI foods, KASTURI foods and KABULI foods for the financial year 2004-2005. These companies are located in the different parts of the world. Expenditure break-ups are in percentage. Closely analyse the data and answer the questions.

Particulars	*Kesari foods (Sales ` 7.5)*	*Kasturi foods (Sales ` 1.9)*	*Kabuli foods (Sales ` 6.9)*
Operating Profit	7	9	9
Interest paid	10	11	12
Rental	**21**	**19**	**18**
Taxes	8	8	11
Salaries	**12**	**9**	**12**
Raw material cost	**25**	**18**	**19**
Power and Electricity	5	10	7
Labour cost	5	6	4
Transportation	**3**	**2**	**3**
Maintenance	2	5	2
Miscellaneous	2	3	3
Total	**100**	**100**	100

* Sales to be read as ` million/month

* Sales break ups are in percentages (%)

21. What can be concluded about the expenditure in salaries, raw material, transportation and rental of all the three companies listed above in the table?

    (a) Percentage wise Kabuli food is spending more than each of the other two.

    (b) Revenue wise Kesari foods is spending more than Kasturi and Kabuli together.

    (c) Revenue wise Kasturi and Kabuli are spending less then Kesari foods.

    (d) Percentage wise Kasturi spends more than Kesari, which in turn spends more than Kabuli.

22. If the interest paid by KESARI FOODS is reduced by 1%, and everything else remains the same, its operating profit will:

(a) increase by 1% per cent.

(b) not change.

(c) increase by 0.0075 million

(d) increase by 2% per cent

23. If the average monthly salary in Kabuli FOODS is ` 9,517.24, then how many employees does Kabuli have?

(a) 28 (b) 48

(c) 81 (d) 87

Directions for Questions 24 to 27: Performance study of various brands of soap is given below with their cost/ 100 gm.

Brand	*% increase in skin softness* S_1	*% increase in natural fairness* S_2	*% reduction in skin diseases* S_3	*% reduction in foul smell* S_4	*% reduction in dry skin* S_5	*Cost* C
B_1	55	49	35	30	30	100
B_2	20	25	40	35	32	50
B_3	69	63	45	20	15	50
B_4	51	48	55	40	38	25
B_5	38	30	25	22	24	100

Benefit/Rupee spent for a soap is measured by a variable known as 'R' where

$R_1 = S_1$/C, $R_2 = S_2/C$, $R_3 = S_3/C$ and so on.

24. "Soap Effectiveness Index" = $(R_1 + R_2 + R_3 + R_4 + R_5)/100$. With respect to the "Soap Effectiveness Index" if the soap brands are arranged in the increasing order, then which of the options, satisfy?

(a) B_1, B_2, B_3, B_4 (b) B_2, B_1, B_3, B_4

(c) B_2, B_1, B_3, B_5 (d) B_1, B_3, B_4, B_5

25. A family wants to purchase 3 different kinds of soaps for different requirements. They want to make a choice of 3 soaps according to the following condition that any of the three soaps selected should have the highest value among all the brands in any quality criteria excluding cost. Which of the following option satisfies their requirement?

(a) B_3, B_1, B_5 (b) B_4, B_2, B_1

(c) B_3, B_4, B_1 (d) None of these

26. In the case of Brand "B3", the company starts a scheme under which the customers get 1 soap free for every 4 purchased. After the change has come into effect, in how many cases for the Brand 'B3', the value of the quantities S_1, S_2, S_3, S_4, S_5, R_1, R_2, R_3, R_4 and R_5 is higher in comparison to Brand "B4"?

(a) 4 (b) 3

(c) 2 (d) 1

27. A family would like to change the soap brand that they are using. The condition the family members make is that the new brand should have the highest value in at least 3 out the first four quality parameters (S_1, S_2, S_3, S_4) and for the fifth column (S_5), the value of R should be at least greater than 80% of the 'R' value of the old brand. If they are using brand B_2 now, then which brand can they replace it with ?

(a) B_3 (b) B_1

(c) B_4 (d) No brand satisfies the above criteria

Directions for Questions 28 to 30: Given below is table indicating the individual scores of ten players of two different basketball teams in single match between them. Each had 5 players and all the scoring shots carried either two or three points. The team scoring the maximum points won. Players scored for their own teams only.

Player	*Points*
Arun	15
Birender	17
Chiranjeev	23
Dhruv	18
Eleswarappu	13
Fardeen	16
Geetam	19
Hasan	24
Ishwar	15
Jeetu	20

The difference between the lowest and the highest individual scores in both the teams is the same.

The highest individual scores from the teams had the same number of scoring shots.

Arun, Dhruv, Fardeen and Geetam had an even number of scoring shots comprising both 2 and 3 –pointers.

Birender, Eleswarappu, and Ishwar had an odd number of scoring shots comprising both 2 and 3 –pointers.

Chiranjeev was another player in his team to score 8 points in 4 shots.

28. What is the difference between the numbers of 3 –pointer shots scored by both the teams?

(a) 4 (b) 5

(c) 6 (d) 7

29. Which of the following could be a difference between the total 2 –pointer and 3 –pointer shots scored by both the teams together?

(a) 7 (b) 8

(c) 11 (d) 10

30. Who among the given players had the highest ratio of points scored to number of scoring shots?

(a) Arun (b) Chiranjeev

(c) Eleswarappu (d) Fardeen

Section Ii: Verbal Ability and Logical Reasoning

Passage I

In this passage from a novel, the narrator has been reading letters of his grandmother. Susan Ward was a young woman—a writer and a mother—whose husband Oliver was working as a mining engineer in Leadville, in the West. Here, the narrator imagines Susan Ward as she spends the winter with her family in Milton, New York, before rejoining her husband in the spring.

From the parental burrow, Leadville seemed so far away that it was only half-real. Unwrapping her apple-cheeked son after a sleigh ride down the lane, she had difficulty in believing that she had ever lived anywhere but here in Milton.

She felt how the placid industry of her days matched the placid industry of all the days that had passed over that farm through six generations. Present and past were less continuous than synonymous. She did not have to come at her grandparents through a time machine. Her own life and that of the grandfather she was writing about showed her similar figures in an identical landscape. At the milldam where she had learned to skate she pulled her little boy on his sled, and they watched a weasel snow-white for winter flirt his black-tripped tail in and out of the mill's timbers. She might have been watching with her grandfather's eyes.

Watching a wintry sky die out beyond black elms, she could not make her mind restore the sight of the western mountains at sunset from her cabin door, or the cabin itself, or Oliver, or their friends. Who were those glittering people intent on raiding the continent for money or for scientific knowledge? What illusion was it that she bridged between this world and that? She paused sometimes; cleaning the room she had always called Grandma's Room, and thought with astonishment of the memory of

Oliver's great revolver lying on the dresser when he, already a thoroughing Westerner, had come to the house to court her.

The town of Milton was dim and gentle, molded by gentle lives, the current of change as slow through it, as the seep of water through a bog. More than once she thought how wrong those women in San Francisco had been, convinced that their old homes did not welcome them on their return. Last year when Oliver's professional future was uncertain, she would have agreed. Now, with the future assured in the form of Oliver's appointment as manager of the Adelaide mine in Leadville, the comfortable past asserted itself unchanged. Need for her husband, like worry over him, was turned low. Absorbed in her child and in the writing of her book, she was sunk in her affection for home. Even the signs of mutability that sometimes jolted her—the whiteness of her mother's hair, the worn patience of her sister's face, the morose silences of her brothers-in-law, now so long and black that the women worried about him in low voices—could not more than briefly interrupt the deep security and peace.

I wonder if ever again Americans can have that experience of returning to a home place so intimately known, profoundly felt, deeply loved, and absolutely submitted to. It is not quite true that you can't go home again. But it gets less likely. We have had too many divorces, we have consumed too much transpiration, we have lived too shallowly in too many places. I doubt that any one of my son's generation could comprehend the home feelings of someone like Susan Ward. Despite her unwillingness to live separately from her husband, she could probably have stayed on indefinitely in Milton, visited only occasionally by an asteroid husband. Or she would have picked up the old home and remade it into a new place. What she resisted was being a woman with no real home.

When frontier historians theorize about the uprooted, the lawless, the purseless, and the socially cut-off who emigrated to the West, they are not talking about people like my grandmother, so much that was cherished and loved, women like her had to give up; and the more they gave it up, the more they carried it helplessly with them. It was a process like ionization: what was subtracted from one pole was added to the other. For that sort of pioneer, the west was not a new country being created, but an old one being reproduced; in that sense our pioneer women were always more realistic than our pioneer men. The moderns, carrying little baggage of the cultural kind, not even living in traditional air, but breathing in to their space helmets, a scientific mixture of synthetic gases (and polluted at that) are the true pioneers. Their circuitry seems to include no domestic sentiment; they have had their empathy removed. Their computers have no ghostly feedback of home, Sweet Home. How marvelously free they are! How unutterably deprived!

31. In the beginning of the first paragraph, the phrase “parental burrow” suggests

    (a) a lack of luxurious accommodations

    (b) an atmosphere of peaceful security

    (c) the work required to sustain a home

    (d) the loss of privacy

32. It can be said that Ward “did not have to come at her grandparents through machine” because

    (a) she was deeply immersed in the history and literature of the period of their lives.

    (b) her life in Milton closely resembled theirs

    (c) as a writer she could intuitively sense their lives

    (d) she possessed written accounts of their lives

33. The reference to the grandfather’s eyes at the end of the second paragraph indicates that Ward

    (a) longed to see nature as her ancestors did

    (b) felt that her grandfather would approve of her life choices

    (c) was seeing something her grandfather himself might well have seen

    (d) longed to let her grandfather know what she was experiencing

34. The reference to a bog in the first part of the fifth paragraph serves to convey a sense of the

    (a) natural setting of the town of Milton

(b) deliberate pace of life in Milton

(c) confinement that Ward first felt in Milton

(d) vague foreboding that permeated Milton

35. Ward came to feel differently from “those women in San Francisco” because

(a) the rigors of life in West made life in East seem more pleasant

(b) the problems in her sister’s life made her more content with the situation in her own life

(c) her own career as a writer had become more important to her

(d) she was free to enjoy her surroundings as she was confident about her ancestral home always welcoming her.

36. The word “sunk” (Paragraph 5) in the passage conveys the degree to which Ward

(a) is depressed about being separated from her husband

(b) is concerned about her son’s social development

(c) allows herself to be totally engrossed in what she is doing

(d) lets down her defenses to free her creativity

Passage II

When one company acquires another, the larger firm usually takes over the smaller one. There are exceptions to the rule, however, such as the Good—Value Mart merger.

Once the leading chain of supermarkets in New England, Good Food Stores has been steadily declining for twenty years. Their sales fell from 25 per cent to 5 per cent of the market. During the same period (1958-1978), earning plunged from over 8 million to a loss of 23 million. With such a track record, Good Food stores hardly seemed a likely candidate for acquisition.

But the Midwest-based Value Mart chain acquired Good Food earlier this year with high hopes of turning around the New England chain. Value Mart is a privately owned chain of supermarkets, with approximately sixty stores throughout the Midwest. The average sales for last year ran a little over 6 million per store. On the other hand, Good Food is a publicly held company that averaged sales of 4 million in each of its 230 stores. In the last five years, Value Mart's earnings reached nearly 15 million, while Good Food lost 40 million during that same period.

The chairman of Value Mart, Harold Brown, is an old hand at putting ailing supermarkets back on their feet. In 1963, Mr. Brown helped turn around an old Cincinnati chain; he changed it from an unprofitable, out of date store to a market leader with a profit of 7 million in just six years. After leaving the Cincinnati chain, Brown, along with thirty other investors, bought Value Mart and turned it around, so that in the past year it has surpassed even the Cincinnati chain in sales. Brown's two success stories now account for 60 percent of the supermarket business in the Cincinnati area.

Although Brown's track record has been good (and was, In fact, the reason Value Mart was able to obtain the necessary funds to acquire Good Food), there has been some speculation on whether Value Mart has undertaken more than it can handle. A powerful New England trade member wonders whether Value Mart has the know-how and strength to take on the leading established New England chains.

Good Food's problems started in the late 1950's and early 1960's, when they failed to follow their competitors move to the suburbs and large shopping centers. This put Good Food behind in market share.

Good Food then made another bad move in 1962 in an attempt to regain some of its lost market share. They acquired a wavering division of supermarkets in the New York area and tried to establish themselves there, while maintaining a policy of low investment and high prices back home in New England. The strategy did not work, and consequently Good Food had to pull to out of New York. In addition, Good Food has lost customers in New England.

Good Food's woes were increased by mismanagement. Complacent task forces were formed to "study" problems instead of dealing with them immediately. Coupled with this was a group of directors who were, for the most part, not industry experts but bankers and lawyers.

The choices facing Good Food in 1976 were as follows: sell, if possible; liquidate; or push onwards. The decision was made to seek a merger partner, and Value Mart was contacted through Good Food's investment counselor. Given Good Food's unpromising situation, it seems surprising that Value Mart was interested. Mr. Brown, however, fresh out of a Cincinnati price war, realized that his home market was saturated and that acquisition or territorial expansion was the necessary means for growth.

Value Mart took charge of the situation and began to reorganise even before the merger was completed. Their top executives took over key positions in the Good Food organisation, a move that included ousting Good Food's president. In an attempt to drastically cut down on administrative expenses, over two hundred jobs at the management level were abolished. (it is expected that this alone will result in savings of over 3 million).

Another change was placing control of grocery merchandising and buying in the hands of those at the corporate level, rather than dividing this function amongst Good Food's store managers and executives. This facilitated the introduction of Value Mart's tried-and-true policy of "deal buying," or taking advantage of cut-rate prices to buy huge quantities of canned or packaged goods. Because "buying" in such quantity necessitates ample inventory space, construction was begun the day after the merger, to increase Good Food's warehousing facilities to the tune of 7 million. The new warehouse will be the largest supermarket warehouse in the United States, and it is expected to save in cost and avoid out-of-stock problems.

The Value Mart strategy for turning around the New England chain also involves deemphasizing nonfood items and hence attracting customers by placing emphasis on the quality of its produce (fruits and vegetables) and meats. Problems at the store level are being corrected by extending work hours at individual stores, cleaning up dirty premises(Good Food had reduced personnel in its stores in an attempt to cut labor costs, resulting in dirty stores and low morale among the employees), and teaching store managers how to repackage and maintain their fresh produce.

Visits to each Good Food store by a Value Mart senior vice-president of operations resulted in control and, equally as important, a demonstrated and direct interest by top management in individual store operations.

This top-level involvement in daily store routine is a far cry from the old Good Food "hands off" approach. The Midwestern senior vice-president not only visits each store to make suggestions, but also comes back unannounced to check on the implementations of changes.

Mr. Brown's immediate objective is to increase business from present customers. He estimates an increase in volume of over 100 million if sales per customer can be improved by only 10 percent. Once sales are up and stores are operating smoothly, Brown plans to renovate about sixty-five of Good Food's largest stores, a move that should bring in an additional 6- 7 million in weekly sales.

Finally, the older and smaller Good Food stores will be given a facelift with the increased revenue from the redone larger stores. Value Mart intends to feed its profits back into the entire operation to keep it going and constantly moving ahead.

37. What was the principal reason for Value Mart's interest in the declining Good Food chain?

    (a) Value Mart wanted to establish a foothold in the east.

    (b) Brown was tempted by the challenge offered by Good Food.

    (c) Brown saw Good Food as a means for growth.

    (d) The acquisition price was very low.

38. What was Good Food's first bad move that led to eventual decline?

    (a) Over expansion

    (b) Failure to maintain good relations with suppliers

    (c) failure to follow the trend to the suburbs

    (d) failure to maintain quality merchandise

39. In the early 1960s Good Food tried to enter the New York area. Why was their strategy unsuccessful?

    A. because of low investment in their home territory

    B. because of high prices in their home territory

    C. because of lack of prior knowledge about the New York market

    (a) A only

(b) B only

(c) A and B only

(d) B and C only

40. What was Value Mart’s initial move in reorganising Good Food?

(a) placing Value Mart’s top executives in key Good Food positions.

(b) cutting expenses

(c) making buying and merchandising corporate-level functions

(d) improving produce quality

41. According to the passage, what is “deal buying”?

(a) buying packaged goods in the huge quantities from wholesalers

(b) buying packaged goods in huge quantities at cut-rate prices

(c) buying packaged goods from small dealers at reduced prices

(d) buying produce from big, out –of-town farmers at reduced prices

42. One of the most important and constructive reforms in National Politics is the abolition of the post of the State Ministers in the various departments.

Each of the following, if true, would strengthen the above argument, except

(a) There are few, if any, specific duties or responsibilities assigned to the State Minister in any department.

(b) A historian claimed that the post was “superfluous.”

(c) People of Cabinet minister caliber normally refuse the post if offered a ministership in the guise of a state minister.

(d) The office is used as a means of appeasing regional parties, by giving their MPs ministerial status and perks without giving the many significant responsibilities.

43. Jaya and Devika are both successful women who are also members of a socially disadvantaged section of the society. Jaya has a firm belief in positive discrimination. By positive discrimination she believes that the negative discrimination that society has subjected her section of the society to can only be offset through reverse discrimination. She believes that if positions of economic, social and political eminence, power and honor are offered principally to historically disadvantaged sections of society, then these groups will begin to play a more significant role in society today.

Devika, on the other hand, feels that she has succeeded in her chosen field of work on her hard work and on her own merits. She thinks that the principle of positive discrimination is flawed since it will result in the lowering of standards and decreases competition between similarly qualified personnel who will expect to achieve positions because of their factors other than rather than their suitability for the particular position.

Which of the following best sums up Jaya's argument?

(a) Positive discrimination will encourage more people to apply for jobs, previously unavailable to them.

(b) Positive discrimination will give extra opportunities to socially disadvantaged sections of the society.

(c) Quality and professionalism will improve because of the greater number of positions held by members of minority groups.

(d) Positive discrimination will remove deep rooted prejudices against the weaker sections of society from the work arena.

Directions for Questions 44 to 46: The sentences given in each question, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a letter. Choose the most logical order of sentences from among the given choices to construct a coherent paragraph.

44.

1. In the twentieth century, John Maynard Keynes has been the most important scholar working in the tradition of the classical political economists.

A. But his interest, like theirs, was in the analysis of the great issues of his day, the greatest of which in the inter-war period was not growth but unemployment, a problem so acute at the time that in desperation the Germans turned to Hitler and to fascism.

B. The very future of Western democracies was placed at risk.

C. Keynes was concerned not just to understand unemployment intellectually, but to put forward practical suggestions as to how the problem could be solved.

D. This does not mean that he agreed with everything they wrote.

6. He believed fervently that, for all its faults, Western liberal democracy offered the best hope for the world, and he saw himself working to save it.

(a) DCBA (b) BACD

(c) BCDA (d) DABC

45.

1. For centuries philosophers have dealt with aspects of humanness, of humanity. But, surprisingly, there is no agreed-upon definition of the quality of humanness.

A. It is my conviction that we are beginning to identify these components, that we can see the gradual emergence of humanness in our evolutionary history.

B. But if this sense of humanity came into being in the course of evolutionary history, then it must have component parts, and they in turn must be identifiable.

C. Those who tried to define humanness found themselves molding Jell-O.: it kept slipping through the fingers.

D. It hardly seemed necessary, partly because it appeared so obvious: humanness is what we feel about ourselves.

6. I am therefore perplexed by, and impatient with, a popular alternative view that is championed by several scholars.

(a) ABCD (b) DCBA

(c) BCDA (d) CBDA

46.

A. Economists see the world as a machine.

B. A very complicated one perhaps, but nevertheless a machine, whose workings can be understood by putting together carefully and meticulously its component parts.

C. A lever pulled in a certain part of the machine with certain strength will have regular and predictable outcomes elsewhere in the machine.

D. The behaviour of the system as a whole can be deducted from a simple aggregation of these components.

(a) ABDC (b) ABCD

(c) ACBD (d) ADBC

Directions for Questions 47 to 50: Fill in the blanks with the appropriate words from the options given below.

47. The King has ---------- to a proposal to enhance the powers of the council of ministers.

(a) coincided (b) allied

(c) assented (d) opined

48. The disciplinary committee has --------- the use of detention after classes as a punishment for bad behaviour.

(a) extended (b) authorized

(c) clamped (d) embargo

49. The judge ---------- the use of capital punishment for serious crimes.

(a) franchised (b) endorse

(c) agreed (d) condoned

50. The committee were in favour of the proposal but the president -------- it.

(a) vetoed (b) countenance

(c) sanctioned (d) condoned

For Questions 51 to 55 read the following instructions: Sunday morning, local pediatrician Dr. Raju Sarma had appointments with five infants scheduled at 9:00, 9:30, 10:00, 10:30, and 11:00. Each of the five, including the Oke baby, is a different number--1, 2, 3, 4, or 5--of months old. The following notes are available.

1. Immediately after seeing infant Bimal, Dr. Sarma examined the Michaels infant, who is 2 months younger than Bimal.
2. Eshwarya isn't the one of the five who is 1 month old.
3. The doctor saw Dhruv later in the morning than the 1-month-old.
4. The 9:30 appointment was with the 3-month-old baby.
5. The Lavande infant isn't the one who is 5 months of age.
6. Dr. Sarma saw the Nalini infant, then examined Amar, who is 2 months older than the Nalini baby.
7. The pediatrician's 10:00 appointment was with Charu, who isn't the Michaels or the Nalini baby.
8. The Pattabhiraman baby isn't the 1-month-old and wasn't the doctor's 9:00 examinee.

51. How old (in months) is infant Bimal?

(a) 2 (b) 3

(c) 4 (d) 5

52. At what time did infant Eshwarya meet the doctor?

(a) 9 (b) 9:30

(c) 10 (d) 10:30

53. Whose baby is infant Charu?

(a) Oke (b) Michaels

(c) Lavande (d) Nalini

54. Which of the following is not correct with respect to infant Dhruv?

(a) He is Nalini's baby.

(b) He is 2 months old.

(c) His appointment was at 11.00.

(d) He is not the Oke baby.

55. Who was the last infant to meet the doctor?

(a) Amar (b) Dhruv

(c) Charu (d) Eshwarya

Directions for Questions 56 to 58: Read the information given below and answer the questions that follow.

Four men - Amar, Bijay, Chetan and Derek and four women - Preeti, Queen, Raveena and Surabhi are sitting for a formal get together, in such a way that they face the centre and form a circle. No two women and no two men are next to each other, Amar is to the immediate left of Raveena, who is opposite to Queen. Preeti and Queen have only Chetan dancing between them. Preeti is sitting opposite to Surabhi, who is sitting to the immediate right of Bijay.

56. If Bijay is the only person sitting between Queen and Surabhi, then who is opposite to him?

(a) Amar (b) Queen

(c) Chetan (d) Derek

57. Which of the following is an acceptable arrangement of the invitees in clock\vise direction (names of individuals are only represented by their first letters in the options given below)?

(a) C, Q, P, D, S, B, A and R

(b) Q, C, P, D, R, A, S and B.

(c) A, R, D , P, C, Q, B and S

(d) D, R, A, S, B, Q, P and C.

58. Which of the following pairs are opposite to each other?

(a) Queen and Raveena

(b) Surabhi and Preeti

(c) Chetan and Derek

(d) both options (a) and (b) are correct

Directions for Questions 59 and 60: Six persons-Akshay, Bobby, Celina, Dimple, Esha, and Faisal took up a job with XYZ Consultants in a week from Monday to Saturday. Each of them joined for different posts on different days. The posts were of-Clerk, Officer, Technician, Manager, Supervisor and Sales Executive though not in the same order.

Faisal joined as a Manager on the first day. Bobby joined as a Supervisor but neither on Wednesday nor Friday. Dimple joined as a Technician on Thursday. The officer joined the firm on Wednesday. Esha joined as a clerk on Tuesday. Akshay joined as a Sales Executive.

59. Who joined the firm on Wednesday?

(a) Bobby (b) Celina

(c) Esha (d) Data inadequate

60. Who was the last to join the firm?

(a) Esha (b) Faisal

(c) Bobby (d) Akshay

ANSWER KEY

1. (c)	2. (a)	3. (a)	4. (d)
5. (c)	6. (b)	7. (d)	8. (d)
9. (c)	10. (a)	11. (b)	12. (d)
13. (d)	14. (d)	15. (a)	16. (d)
17. (b)	18. (c)	19. (c)	20. (d)

21. (b)	22. (c)	23. (d)	24. (a)
25. (d)	26. (c)	27. (d)	28. (c)
29. (d)	30. (d)	31. (b)	32. (b)
33. (c)	34. (b)	35. (d)	36. (c)
37. (c)	38. (c)	39. (c)	40. (a)
41. (b)	42. (b)	43. (b)	44. (d)
45. (b)	46. (a)	47. (c)	48. (b)
49. (d)	50. (a)	51. (d)	52. (b)
53. (c)	54. (c)	55. (a)	56. (d)
57. (b)	58. (d)	59. (b)	60. (c)

Solutions

Section I

1. Let us look at the two equations. Let (7 apples + 9 oranges + 5 guavas) cost ` $x$ … (1). Hence, (11 apples + 18 oranges + 10 guavas) will cost ` $1.75x$ … (2). Had, in the second case, Rajan decided to buy 14 apples instead of 11, the quantity of each one of them would have doubled over the first case and hence it would have cost me ` $2x$. So (14 apples + 18 oranges + 10 guavas ) = ` $2x$ … (3)

   Now subtracting the second equation from the third, we get 3 apples cost ` 0.25x. Since 3 apples cost Re 0.25x, 7 of them will cost Re 0.583x. This is the amount that I spent on apples. Hence, fraction of the total amount paid = 0.583 = 58.3%.

   Hence, Option (c) is correct.

2. Let $|x – 2|= y$

   $y^2 + y – 2 = 0$

   $y = -2,1$

   $|x – 2|= 1$

   $x – 2 = 1, x – 2 = -1$

   $x = 3, 1$

   sum of roots=4

   Hence, Option(a) is correct.

3. Squaring the given equation

$x + 1 + x - 1 - 2(x^2 - 1)^{0.5} = 4x - 1$

$4(x^2 - 1) = 4x^2 + 1 - 4x$

$x = 5/4$

but this value of x does not satisfy the given equation

Hence, Option (a) is correct.

4. case 1: ${}^4C_3\,{}^4C_3 = 16$

   case 2: $({}^4C_2\,{}^3C_1)({}^3C_1{}^4C_2) = 324$

   case 3: $({}^4C_1{}^3C_2)({}^3C_2{}^4C_1) = 144$

   case4: ${}^3C_3\,{}^3C_3 = 1$

   total = 485

   Hence, Option (d) is correct.

5. Since no box remains empty possible balls can be distributed in manner(2, 2, 1) or (3, 1, 1)

   i.e. $({}^5C_2\,{}^3C_2\,{}^1\mathrm{C}_1 + {}^5C_3\,{}^2C_1\,{}^1C_1)3! = 300$

   Hence, Option (c) is correct.

6. solution nth term = $n/(1 + n^2 + n^4) = n/((n^2 + 1)^2\text{-}n^2) = n/(n^2 - n + 1)(n^2 + n + 1)$

   $½(1/(n^2 - n + 1)–1/(n^2 + n + 1))$

   sum =1/2

   Hence, Option (b) is correct.

   You can also solve this question using values.

7. The ratio of the speeds of Ishant and Ganguly is 2 : 1. Hence they should meet at only

   one point on the circumference i.e. the starting point (As the difference in the ratio in reduced form is 1). For the two of them to meet for the first time, Ishant should have completed one complete round over Ganguly. Since the two of them meet for the first time after 3.5 mins, Ishant should have completed 2 rounds (i.e. 2000 m) and Ganguly should have completed 1 round. (i.e. 1000 m) in this time. Thus, Ishant would complete the race (i.e. 4000 m) in 7 min. Hence, Option (d) is correct.

8. Lets say one boy can do the x part of the work in 1 day.. On the first day only 1 boy works, he can do xth part of the work. On the second day 2 more join him...together the boys can do $x(1 + 2)$ of the work on second day. On third day 3 more boys join the group...they can do $x(1 + 2 + 3)$ part o the work on 3rd day....and so on...Hence we have the sum $x(1) + x(1 + 2) + x(1 + 2 + 3)$.....so on till 25 terms = 1 (as the work is completed in 25 days) now this is nothing but the summation x * sigma[$n(n + 1)/2$] from $n = 1$ to $n = 25$ this can be spit into $x/2$*(sigma (n^2) + sigma(n)) $n = 1$ to $n = 25$ applying formula for sigma(n ^ 2) = $n(n + 1)(2n + 1)/6$ and sigma(n) = $n(n + 1)/2$ and substituting $n = 25$, we get x*2925 = 1 or $x = 1/2925$. Hence one boy can do the work in 2925 days. One man can do it in half the no. of days, ie 1462.5 = 1463 days. and so 10 men can do it in 146.3 = 147 days.

Hence, Option (d) is correct.

9. The possibilities are:

1,2,10

1,3,9

1,4,8

1,5,7

2,3,8

2,4,7

2,5,6

3,4,6

When Twinkle is not able to respond after seeing the left most card, we can assume that 3, 4, 6 was not the correct order. Akshay knows this when he sees his card, and hence he must not have seen any of 10,9 or 6. This leaves us with the possible combinations 1, 4, 8;

1, 5, 7; 2, 3, 8 and 2, 4, 7. If Raveena had seen a 3 or 5 she would have known all the cards. Since she does not answer, she must have seen a 4 on the middle card.

Hence, Option (c) is correct.

10. The sum of the positives is 25 and the negatives is 17. The best way to distribute the +8 is 5 and 3 by taking 13, 2, –5 and –7 in the first bracket and the remaining values 4, 6, –2 and –3 in the second bracket. The required answer would be 34.

    Hence, Option (a) is correct.

11. The values at which this would occur would be X = 24% and Y = 29%.

    Hence, Option (b) is correct

12. This question is based on the concept that the net distance travelled by all three of them (added up) together is going to be either 1 full round (For the circular order ABC) or 2 full rounds (For the circular order ACB). It can be seen that with the given times for completing a round, if t = 120, then the net distance covered by them is equal to 1 complete round. For t = 240 seconds it becomes two complete rounds.

    Hence, Option (d) is correct.

13. Using the logic that the net distance traveled by all three of them (added up) together is going to be either 1 full round (For the circular order ABC) or 2 full rounds (For the circular order ACB). Since, all three of them are at their respective starting points after an hour, the times required by each of them must be a factor of 60 minutes. (& Can only be 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 or 60 minutes). With these values, we can see that for t equal to 10,12 or 15 minutes it is possible to have either 1 complete round or 2 complete rounds as the total distance traveled by all three of them. Only 18 minutes gives no possibility of creating a total of 1 complete round or 2 complete rounds. Option (d) is correct.

14. Plot the graph of the three lines and realize the maximum point for $f(x)$ would be at the intersection of $x + 2$ and $-2x + 4$. This occurs at $x = 2/3$, and $f(x)$ is 8/3 at that point. Hence, Option (d) is correct.

15. Solve using options. If you put 9 in the middle (all three) you get 36 for only Physics & Physics+ 1 more subject. Similarly, 35 for only

Chemistry & Chemistry+ one more subject, 39 for Mathematics & Mathematics + 1 more subject.

The sum of these numbers should be equal to [number of students taking only 1 subject + 2 times number of students taking exactly two subjects] = 50 + 30 + 30 = 110.

Since, 36 + 35 + 39 is also 110, this is the correct answer. The other options won't work here.

Hence, Option (a) is correct.

16. If you were to plot a venn diagram for this question you would realize that we do not know the number of students having only Physics and Chemistry, Only Chemistry and Mathematics and also only Physics and Mathematics. Also, we do not know the value of the total number of students. Hence, we cannot determine the answer to this question. Hence, Option (d) is correct.

17. The required sum would be given by: (1/99 + 2/99 +….+ 98/99) – (11/99 + 22/99 +…+ 88/99)– (9/99 + 18/99 + … 90/99) = 40. Option (b) is correct.

18. We have to find the value of n for which $3^n > 900$ Æ $n > 7$

    Hence, Option (c) is correct.

19. $g(x)>1$ for all x

    $f(g(x)) = 1$

    Option (c) is correct.

20. When we expand 30!...we get $1 \times 2 \times 3 ... 30$. $2^{26} \times 3^{14} \times 5^7 \times 7^4 \times 11^2 \times 13^2 \times 17^1 \times 19^1 \times 23^1 \times 29^1$

    From this list it is clear that the constraint for the maximum value of n would be either due to the 2's, the 3's, the 5's or the 29's.

    301! has 150 + 75 + 37 + 18 + 9 + 4 + 2 + 1 = 296 twos and hence has [296/26]=11 instances of 2^{26}.

    301! has 100 + 33 + 11 + 3 + 1 = 148 threes and hence has [148/14]=10 instances of 3^{14}.

    301! has 60 + 12 + 2 = 74 fives and hence has [74/7] = 10 instances of 5^7.

301! has 10 instances of 29^1.

Hence, the correct answer would be 10. Hence, Option (d) is correct.

21. It can be seen that Option (b) is correct, as the expenditure of Kesari Foods on these 4 categories are 4.575 while the combined expenditure of Kasturi and Kabuli is lower than that. Hence, Option (b) is correct.
22. If there is a 1% reduction in interest, there would be an increment of 0.0075 million in the operating profit. Hence, Option (c) is correct.
23. 0.12 × 6900000/9517.24 = 87. Hence, Option (d) is correct.
24. Soap Effectiveness Index for B_1 is (199/100)/100 = 0.0199

    Soap Effectiveness Index for B2 is (152/50)/100 = 0.0304

    Soap Effectiveness Index for B3 is (212/50)/100 = =0.0424

    Soap Effectiveness Index for B4 is (232/25)/100 = =0.0928

    Soap Effectiveness Index for B5 is (139/100)/100 = =0.0139

    Hence, B_1, B_2, B_3, B_4 is the correct order.

    Option (a) is correct.
25. Quality wise the value of R_1, R_2, R_3, R_4 and R_5 are all maximum only for B4. Hence, the correct option is None of these. Option (d) is correct.
26. B3 is better than B_4 in S_1 and S_2 only. For all other values, B_4 has a higher value (in spite of the scheme which has the effect of B_4's cost coming down to 40.) Option (c) is correct.
27. Only Brand B_3 satisfies the first condition (of having the highest value in at least 3 of the first four categories- however, the brand does not satisfy the other criteria. Hence, Option (d) is correct.
28. The only way to divide the teams would be given by the following table:

WINNING TEAM			*LOSING TEAM*		
Player Name	*Points*	*Scoring shots break up*	*Player Name*	*Points*	*Scoring shots break up*

Hasan	24		Jeetu	20	
Chiranjeev	23	5-4	Fardeen (Even)	16	4-2
Geetam (Even)	19	3-5	Arun (Even)	15	3-3
Dhruv (Even)	18	2-6	Ishwar (Odd)	15	1-6
Birender (Odd)	17	3-4	Eleswarappu (Odd)	13	3-2

Further, Hasan and Jeetu having the same number of scoring shots means three possibilities for them:

	Hasan (24 points)	*Jeetu (20 points)*
Possibility 1	8-0	4-4
Possibility 2	6-3	2-7
Possibility 3	4-6	0-10

Based on the above tables we can conclude that the difference between the number of 3 pointers would always be 6.

Hence, Option (c) is correct.

29. The only way to divide the teams would be given by the following table:

WINNING TEAM			*LOSING TEAM*		
Player Name	*Points*	*Scoring shots break up*	*Player Name*	*Points*	*Scoring shots break up*
Hasan	24		Jeetu	20	
Chiranjeev	23	5-4	Fardeen (Even)	16	4-2
Geetam (Even)	19	3-5	Arun (Even)	15	3-3

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Further, Hasan and Jeetu having the same number of scoring shots means three possibilities for them:

	Hasan (24 points)	*Jeetu (20 points)*
Possibility 1	8-0	4-4
Possibility 2	6-3	2-7
Possibility 3	4-6	0-10

Based on the above tables we can conclude that the difference between the number of 2 pointers and 3 pointers for the teams could be either 0, 10 or 20. Only option (d) gives a possible value of this difference.

30. The only way to divide the teams would be given by the following table:

WINNING TEAM			*LOSING TEAM*		
Player Name	*Points*	*Scoring shots break up*	*Player Name*	*Points*	*Scoring shots break up*
Hasan	24		Jeetu	20	
Chiranjeev	23	5-4	Fardeen (Even)	16	4-2
Geetam (Even)	19	3-5	Arun (Even)	15	3-3
Dhruv (Even)	18	2-6	Ishwar (Odd)	15	1-6
Birender (Odd)	17	3-4	Eleswarappu (Odd)	13	3-2

From the above table, it is clear that Fardeen has the highest ratio of 2.66.

Option (d) is correct.

Section II

31. Option (b);The phrase is used in order to suggest the peaceful atmosphere.
32. Option (b); Third paragraph, second line clearly suggests that the writer's life resembled his grandparents'
33. Option (c); She was imitating her grandfather.
34. Option (b); In the whole fifth paragraph, the reference is being given to the peace in milton.
35. Option (d); As per the paragraph 5^{th}, the Ward was free to enjoy her life.
36. Option (c);[clearly mentioned in the passage]
37. Option (c) This is an indirect question and the answer lies in the 8^{th} and 9^{th} paragraph.
38. Option (c) Given in sixth paragraph
39. Option (c) This is also gives in the seventh paragraph
40. Option (a) This is given in the tenth paragraph
41. Option (b)This is given in the eleventh paragraph
42. The correct answer is option (b). [This is clearly an irrelevant option]
43. The correct answer is (b).[this is the summary which covers the main idea.]

44. In the above paragraph, the starting sentence is already defined. The sentence sequence 1DA is using a double contrast- A kind of flip flop argument. Statement D opposes(contrasts) the idea of 1, while statement A again contrasts the opposition of D. In this question, recognizing this structure is sufficient to get to the correct answer. Option (d) is correct.

45. In the above question, the key sentence sequence is the BA sequence, which is in the form of an idea transformation since, B

introduces the concept of the component parts, while A refers to these components in a different context altogether. Hence, the correct answer is option (b).

46. The sentence sequence AB is in the form of an idea elaboration, while the sentence sequence DC is in the form of generic to specific. Hence, the correct answer is (a).
47. Option (c) [assented in the sense of agreed]
48. Option (b) [given permission, thus authorised is the answer]
49. Option (d) [in the sense of - to forgive]
50. Option (a) [in the sense of disagree/not allow]
51. Option (d) is correct.
52. Option (b) is correct.
53. Option (c) is correct.
54. Option (c) is correct.
55. Option (a) is correct.

Solutions for 56 to 58:

The two possible distributions of the eight in clockwise fashion are:

Possibility 1: APCQDSBR

Possibility 2: ASBQCPDR

Based on these arrangements the answers are:

56. This case is referring to possibility 2. Derek would be opposite Bijay (gap of 3 people between any 2 opposite persons on the table). Option (d) is correct.
57. Possibility 2 is reflected in Option (b). Hence, Option (b) is correct.
58. Preeti and Surabhi are always opposite each other as are Queen and Raveena (in both cases). Hence, Option (d) is correct.

Solutions for 59 to 60:

Six persons A, B, C, D, E & F on six days, Monday to Saturday, six posts –Clerk (C), Officer (O), Technician (T), Manager (M), Supervisor (S) and Sales Executive (SE).

The second paragraph gives the clues directly to fit in the following table:

A Æ Sales Executive	
B Æ	X Wed, X Friday
C Æ	
D Æ Technician	Thursday
E Æ Clerk	Tuesday
F Æ Manager	Monday

We also know that the officer joined on Wednesday. It can only be Celina (C). Thus, the table will look like:

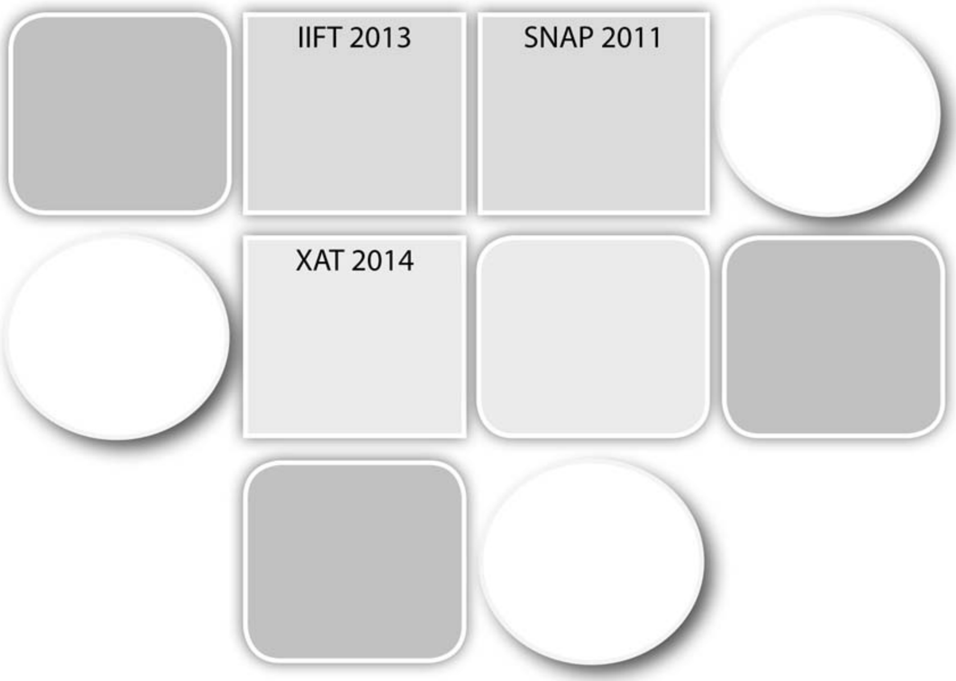
A Æ Sales Executive	Friday
B Æ Supervisor	Saturday
C Æ Officer	Wednesday
D Æ Technician	Thursday
E Æ Clerk	Tuesday
F Æ Manager	Monday

Thus, the answers are:

59. Celina. Option (b) is correct.
60. Bobby. Option (c) is correct.

Solved Papers

SOLVED PAPERS



Solved Quantitative Aptitude Questions from Other Competitive Examinations (IIFT, SNAP and XAT)

IIFT 2013

1. Suppose there are 4 bags. Bag 1 contains 1 black and $a^2 - 6a + 9$ red balls, bag 2 contains 3 black and $a^2 - 6a + 7$ red balls, bag 3 contains 5 black and $a^2 - 6a + 5$ red balls and bag 4 contains 7 black and $a^2 - 6a + 3$ red balls. A ball is drawn at random from a randomly chosen bag. The maximum value of probability that the selected ball is black, is

   (a) $16/a^2 - 6a + 10$ (b) $20/ a^2 - 6a + 10$

   (c) 1/16 (d) None of the above

2. If the product of the integers a, b, c and d is 3094 and if $1 < a < b < c < d$, what is the product of b and c?

   (a) 26 (b) 91

   (c) 133 (d) 221

3. Mrs. Sonia buys ` 249.00 worth of candies for the children of a school. For each girl she gets a strawberry flavoured candy priced at ` 3.30 per candy; each boy receives a chocolate flavoured candy priced at ` 2.90 per candy. How many candies of each type did she buy?

   (a) 21, 57 (b) 57, 21

   (c) 37, 51 (d) 27, 51

4. There is a triangular building (ABC) located in the heart of Jaipur, the Pink City. The length of the wall in east (BC) direction is 397 feet. If the length of south wall (AB) is perfect cube, the length of southwest wall (AC) is a power of three, and the length of wall in southwest (AC) is thrice the length of side AB, determine the perimeter of this triangular building.

(a) 3209 feet (b) 3213 feet

(c) 3773 feet (d) 3313 feet

5. Out of 8 consonants and 5 vowels how many words can be made, each containing 4 consonants and 3 vowels?

(a) 700 (b) 504000

(c) 3528000 (d) 7056000

6. If $x^2 + 3x - 10$ is a factor of $3x^4 + 2x^3 - ax^2 + bx - a + b - 4$, then the closest approximate values of a and b are

(a) 25, 43 (b) 52, 43

(c) 52, 67 (d) None of the above

7. If the product of n positive integers is n^n, then their sum is

(a) A negative integer (b) Equal to n

(c) Equal to $n + 1/n$ (d) Never less than n^2

8. A tennis ball is initially dropped from a height of 180 m. After striking the ground, it rebounds (3/5)th of the height from which it has fallen. The total distance that the ball travels before it comes to rest is

(a) 540 m (c) 600 m

(c) 720 m (d) 900 m

9. In a sports meet for senior citizens organised by the Rotary Club in Kolkata, 9 married couples participated in table tennis mixed double event. The number of ways in which the mixed double team can be made, so that no husband and wife play in the same set, is

(a) 1512 (b) 1240

(c) 960 (d) 640

10. Two trains P and Q are scheduled to reach New Delhi railway station at 10.00 AM. The probability that train P and train Q will be late is 7/9 and 11/27 respectively. The probability that train Q will be late, given that train P is late, is 8/9. Then the probability that neither train will be late on a particular day is

(a) 40/81 (b) 41/81

(c) 77/81 (d) 77/243

11. A survey was conducted to test relative aptitudes in quantitative and logical reasoning of MBA applicants. It is perceived (prior to the survey) that 80 percent of MBA applicants are extremely good in logical reasoning, while only 20 percent are extremely good in quantitative aptitude. Further, it is believed that those with strong quantitative knowledge are also sound in data interpretation, with conditional probability as high as 0.87. However, some MBA applicants who are extremely good in logical reasoning can be also good in data interpretation, with conditional probability 0.15. An applicant surveyed is found to be strong in data interpretation. The probability that the applicant is also strong in quantitative aptitude is

(a) 0.4 (b) 0.6

(c) 0.8 (d) 0.9

12. Your friend's cap is in the shape of a right circular cone of base radius 14 cm and height 26.5 cm. The approximate area of the sheet required to make 7 such caps is

(a) 6750 sq cm (b) 7280 sq cm

(c) 8860 sq cm (d) 9240 sq cm

13. In an engineering college there is a rectangular garden of dimensions 34 m by 21 m. Two mutually perpendicular walking corridors of 4 m width have been made in the central part and flowers have been grown in the rest of the garden. The area under the flowers is

(a) 320 sq m (b) 400 sq m

(c) 510 sq m (d) 630 sq m

14. If decreasing 70 by X percent yields the same result as increasing 60 by X percent, then X percent of 50 is

(a) 3.84

(b) 4.82

(c) 7.10

(d) The data is insufficient to answer the question

15. A rod is cut into 3 equal parts. The resulting portions are then cut into 12, 18 and 32 equal parts, respectively. If each of the resulting portions have integer length, the minimum length of the rod is

(a) 6912 units (b) 864 units

(c) 288 units (d) 240 units

16. If $\log_{10}x - \log_{10}\sqrt[3]{x} = 6\log_x 10$ then the value of x is

(a) 10 (b) 30

(c) 100 (d) 1000

17. A mother along with her two sons is entrusted with the task of cooking *biryani* for a family get-together. It takes 30 minutes for all three of them cooking together to complete 50 percent of the task. The cooking can also be completed if the two sons start cooking together and the elder son leaves after 1 hour and the younger son cooks for further 3 hours. If the mother needs 1 hour less than the elder son to complete the cooking, how much cooking does the mother complete in an hour?

(a) 33.33% (b) 50%

(c) 66.67% (d) None of the above

18. It was a rainy morning in Delhi when Rohit drove his mother to a dentist in his Maruti Alto. They started at 8.30 AM from home and Rohit maintained the speed of the vehicle at 30 km/hr. However, while returning from the doctor’s chamber, rain intensified and the

vehicle could not move due to severe water logging. With no other alternative, Rohit kept the vehicle outside the doctor's chamber and returned home along with his mother in a rickshaw at a speed of 12 km/hr. They reached home at 1.30 PM. If they stayed at the doctor's chamber for the dental check-up for 48 minutes, the distance of the doctor's chamber from Rohit's house is

(a) 15 km (b) 30 km

(c) 36 km (d) 45 km

19. Two alloys of aluminium have different percentages of aluminium in them. The first one weighs 8 kg and the second one weighs 16 kg. One piece each of equal weight was cut off from both the alloys and first piece was alloyed with the second alloy and the second piece alloyed with the first one. As a result, the percentage of aluminium became the same in the resulting two new alloys. What was the weight of each cut-off piece?

(a) 3.33 kg (b) 4.67 kg

(c) 5.33 kg (d) None of the above

20. Three years ago, your close friend had won a lottery of ` 1 crore. He purchased a flat for ` 40 lakhs, a car for ` 20 lakhs and shares worth ` 10 lakhs. He put the remaining money in a bank deposit that pays compound interest @ 12 percent per annum. If today, he sells off the flat, the car and the shares at certain percentage of their original value and withdraws his entire money from the bank, the total gain in his assets is 5%. The closest approximate percentage of the original value at which he sold off the three items is

(a) 60 percent (b) 75 percent

(c) 90 percent (d) 105 percent

21. If $\log_{13}\log_{21}\{\sqrt{x+21}+\sqrt{x}\} = 0$, then the value of x is

(a) 21 (b) 13

(c) 81 (d) None of the above

22. If x is real, the smallest value of the expression

$3x^2 - 4x + 7$ is :

(a) 2/3 (b) 3/4

(c) 7/9 (d) None of the above

23. The average of 7 consecutive numbers is P. If the next three numbers are also added, the average shall

(a) Remain unchanged (b) Increase by 1

(c) Increase by 1.5 (d) Increase by 2

24. The duration of the journey from your home to the College in the local train varies directly as the distance and inversely as the velocity. The velocity varies directly as the square root of the diesel used per km., and inversely as the number of carriages in the train. If, in a journey of 70 km. in 45 minutes with 15 carriages, 10 litres of diesel is required, then the diesel that will be consumed in a journey of 50 km in half an hour with 18 carriages is

(a) 2.9 litres (b) 11.8 litres

(c) 15.7 litres (d) None of the above

25. The capacity of tap *Y* is 60% more than that of *X*. If both the taps are opened simultaneously, they take 40 hours to fill the rank. The time taken by *Y* alone to fill the tank is

(a) 60 hours (b) 65 hours

(c) 70 hours (d) 75 hours

ANSWER KEY

1. (d)	2. (b)	3. (b)	4. (d)
5. (c)	6. (c)	7. (d)	8. (c)
9. (a)	10. (b)	11. (b)	12. (d)
13. (c)	14. (d)	15. (c)	16. (d)
17. (b)	18. (c)	19. (c)	20. (c)
21. (d)	22. (d)	23. (c)	24. (b)
25. (b)			

Solutions

1. The number of balls every bag is $a^2 - 6a + 10$. The probability of picking up a black ball would be defined as:

   Probability of picking up bag 1 × Probability of picking up a black ball + Probability of picking up bag 2 × Probability of picking up a black ball + Probability of picking up bag 3 × Probability of picking up a black ball + Probability of picking up bag 4 × Probability of picking up a black ball

   $= ¼ \times [1/(a^2 - 6a + 10)] + ¼ \times [3/(a^2 - 6a + 10)] + ¼ \times [5/(a^2 - 6a + 10)] + ¼ \times [7/(a^2 - 6a + 10)] = ¼ [16/(a^2 - 6a + 10)] = [4/(a^2 - 6a + 10)]$.

   Hence, Option (d) is correct.

2. The key in this question is to look for factor pairs of 3094. If we do a prime factor search of 3094, we get 3094 = 2 × 7 × 13 × 17. These are clearly the values of *a*, *b*, *c* and *d* in that order. Thus, the value of the product of *b* and *c* is 13 × 7 =91. Hence, Option (b) is correct.

3. Solve this one through options. If you use Option (a) you get:

   21 × 3.3 + 57 × 2.9 = 234.6 π 249.

   Option (b) gives us: 57 × 3.3 + 21 × 2.9 = 249.

   Hence, Option (b) is correct.

4. The perimeter P = AB + BC +AC.

   Of these BC = 397, AC = $3n$ and AB = a^3.

   Thus, P = $397 + 3n + a^3$

   We also know that AC = 3 × AB

   $3n = 3 \times a^3$ Æ

   $3n^{-1} = a^3$.

   So, P = $397 + 3n + 3n^{-1}$

   Æ $(P - 397) = 3n^{-1} \times (3 + 1) = 3n^{-1} \times 4$

   This means that $(P - 397)$ should be a multiple of 3 and 4 simultaneously.

Checking the options, only Option (d) meets the criteria because if we put $P = 3313$ we get $P - 397 = 2916$ which is a multiple of 3 and 4. The other options do not satisfy this requirement and hence Option (d) is the correct answer.

5. We would need to select 4 consonants and 3 vowels and arrange them. This can be done in ${}^8C_4 \times {}^5C_3 \times 7! = 70 \times 10 \times 5040 = 3528000$ words.

6. The equation $x^2 + 3x - 10$ has two factors 2 and –5. Putting $x = 2$ in the expression $3x^4 + 2x^3 - ax^2 + bx - a + b - 4$, we get: $48 + 16 - 4a + 2b - a + b - 4 = 0$ Æ $5a - 3b = 0$ ...(i)

   We also have: $x = -5$.

   Putting $x = 2$ in the expression $3x^4 + 2x^3 - ax^2 + bx - a + b - 4$, we get: $26a + 4b = 1621$ ...(ii)

   Solving (i) and (ii) simultaneously we get $a = 52$ and $b = 67$. Option (c) is correct.

7. Since we are talking about the calculation of the sum of n positive integers, it cannot be negative or a fraction. Thus, Options (a) and (c) are ruled out. Further, Option (b) can be rejected by taking a sample value of n as 4 – it gives us *nn* as 256. A possible value of the 4 integers could be: 1, 2, 16 and 8. Obviously their sum is not equal to the value of *n* and hence we can also reject Option (b). This leaves us only with Option (d) as a possible answer. Hence, Option (d) is correct.

8. The total distance traveled would be the infinite sum of the 'falling down' distance plus the infinite sum of the 'bouncing up' distance traveled by the ball. The answer would be got by:

   $180 \div (1 - 3/5) + 108 \div (1 - 3/5) = 450 + 270 = 720$. Hence, Option (c) is correct.

9. The number of ways of selecting 2 men and 2 women (such that there would be no husband wife pair in the same set) would be ${}^9C_2 \times {}^7C_2$. Further, there are two ways in which we can form the two teams for the same set of players. Thus, the correct answer would be: ${}^9C_2 \times {}^7C_2 \times 2 = 36 \times 21 \times 2 = 1512$. Hence, Option (a) is correct.

10. Let the probability of *P* coming late = *P*(*P*); the probability of *Q* coming late = *P*(*Q*). Then:

   P(*P* » *Q*) = *P*(*P*) + *P*(*Q*) – *P*(*P* « *Q*)

   Where *P*(*P* » *Q*) means the situation where at least one of the two trains *P* and *Q* is late and *P*(*P* « *Q*) means the situation where both the trains *P* and *Q* are late.

   The value of *P*(*P* « *Q*) = 7/9 × 8/9 = 56/81.

   This gives us:

   P (*P* » *Q*) = 7/9 + 11/27 – 56/81 = (63 + 33 – 56)/81 = 40/81.

   Thus, the probability that none of the two trains is late = 1 – Probability that at least 1 train is late.

   The required answer = 1– 40/81 = 41/81.

   Hence, Option (b) is correct.

11. Out of every 100 applicants, 80 are extremely good at Logical Reasoning while 20 are extremely good at Quantitative Aptitude. Further, since the probability of someone who is extremely good at Quantitative Aptitude also being strong at Data Interpretation is 0.87, we can expect 20 × 0.87 = 17.4 people out of 20 to be strong at DI.

   Also, since the related probability for someone being strong at DI if he/she is strong at LR is 0.15, we get 80 × 0.15 =12 people who would be expected to be strong at DI (from the 80 who are strong at LR).

   Thus, we get a total of 29.4 people in every 100 who would be strong at DI. Naturally, if we find someone who is strong at DI, the probability he/she would also be strong at QA = 17.4/29.4 ª 0.6. Hence, Option (b) is correct.

12. The surface area of the sheet required to make the cap would be equal to the lateral surface area of the cap which is given by the formula p × *r* × *l*. Since the base radius is 14 cm and height is 26.5 cm, the value of ‘*l*’ can be calculated using $r^2 + h^2 = l^2$ Æ *l* = 29.97 ª 30.

   Thus, 7 caps would require 7 × p × 14 × 30 = 9240 cm^2.

Hence, Option (d) is correct.

13. The area of the garden is 34 × 21 = 714 sq. m. Out of this the area covered by the paths = 4 × 34 + 4 × 21 – 4 × 4 = 204 sq. m. The remaining area being covered by flowers would be equal to: 714 – 204 = 510 sq.m.

14. Solve this one through options. One reading of the question sentence should tell you clearly that there would be a value of 'X' that would satisfy this – and hence Option (d) can be rejected. This leaves us to check the first three options. Rather than trying to solve the question by creating complex mathematical processes, it is better to solve this using the options. The following process would help you identify whether an option is correct or not.

    For Option (c): X % of 50 = 7.10 implies that the value of X must be 14.2%.

    If we now visualise the equation 70 – 14.2% of 70 = 60 + 14.2% of 60, we can clearly realise that the LHS and the RHS of the above equation do not match. Hence, this option can be rejected.

    For Option (b): X % of 50 = 4.82 implies that the value of X must be 9.64 %.

    If we now visualise the equation 70 – 9.64 % of 70 = 60 + 9.64 % of 60, we can clearly realise that the LHS and the RHS of the above equation do not match. Hence, this option can be rejected.

15. Solve this question using options. The correct option would be the smallest number amongst the given options, whose $1/3^{rd}$ value would be divisible by 12, 18 as well as by 32. If we try the lowest option first our thought process would go as follows:

    Option (d): If rod length = 240, then $1/3^{rd}$ of the rod length = 80. But 80 is not divisible by 12. Hence, option (d) is wrong.

    Option (c): If rod length = 288, then $1/3^{rd}$ of the rod length = 96. But 96 is not divisible by 18. Hence, option (c) is wrong.

    For Option (b), rod length = 864, we get $1/3^{rd}$ of the rod length = 288 which is a multiple of 12, 18 and 32 simultaneously. Hence, Option (c) is correct.

16. $\log_{10}x - \log_{10} \sqrt[3]{x} = 6 \log_x 10$ Æ $\log_{10} x - ⅓ \times \log_{10} x$

$= 6/(\log_{10} x)$;

Put $\log_{10}x = m$, gives us:

$m - ⅓ \times m = 6/m$ Æ $m^2 - ⅓ \times m^2 = 6$ Æ $m^2 = 9$ Æ $m = \pm 3$.

We can reject the value of $m = -3$ because that would involve x to be a decimal value between 0 and 1 (which is ruled out from the options).

Thus, $m = 3$, which means that $\log_{10} x = 3$ Æ $x = 10^3$

$= 1000$.

Hence, Option (d) is correct.

17. The total work for them is 100% per hour. If we try to go through the options and try Option (b) which is the easiest to check – we get: The mother's work per hour = 50%. This would mean that the mother requires 2 hours to complete the task. Since it is given that the elder son takes one hour more than his mother to complete the task, it would imply that the elder son's time taken for the task = 3 hours Æ elder son's work per hour = 33.33%. Then the younger son's work per hour would be 100–50–33.33= 16.66% per hour.

With these working rates, we need to confirm whether the last condition mentioned in the problem matches with these working rates for the two sons. The condition states that – "The cooking can also be completed if the two sons start cooking together and the elder son leaves after 1 hour and the younger son cooks for further 3 hours." If we go by this – we will get that the elder son would do 33.33% of the work in the first hour, while the younger son would cook for 4 hours and do 66.66% of the work – thus completing 100% of the work together (since 33.333% + 66.666% = 100%).

Thus, Option (b) is the correct option.

18. The equation that one needs to solve for this question is:

$d/30 + d/12 = 4.2$. (4.2 here, means that the total time taken for the travel is 4 hours 12 minutes – since he comes home in exactly 5 hours, which includes the 48 minutes at the doctor's chamber.)

The value of '*d*' can then be checked from the options – the best option which fits the value of $d = 36$. Hence, Option (c) is correct.

19. The main focus in this problem should be on understanding that the cut off piece's weight should be such that the alloyed pieces that are created should have the same ratio weight wise for each of the two alloys.

    Thus, if the weight of the cut off pieces is '*w*' each, then:

    $(8–w)/w = w/(16–w)$.

    Again to solve this, it is better to try to use the options given in the question to see which one fits the above equation.

    Using Option (c), we can see that if we take $w = 5.33$, we get:

    $2.66/5.33 = 5.33/10.66 = ½$.

    Hence, Option (c) is correct.

20. 30 lakh invested @ 12% per annum compounded, would approximately become 42 lakh in three years. Since, he gets back 1.05 crore – his return on his remaining 70 lakh would be equal to 1.05 crore – 42 lakh ª 63 lakh – which is around 90% of the value of his investment of 70 lakh. Thus, Option (c) is the correct answer.

21. $\log_{13}\log_{21}\left\{\sqrt{x+21}+\sqrt{x}\right\} = 0$ implies:

    $\log_{21}\left\{\sqrt{x+21}+\sqrt{x}\right\} = 13^0 = 1$ implies:

    $\left\{\sqrt{x+21}+\sqrt{x}\right\} = 21^1$

    The value of x can be visualised to be 100 because at $x = 100$, LHS = RHS.

    Hence, Option (d) is correct.

22. The given expression would have its minima at the value of x got by differentiating the quadratic expression with respect to x and equating the resultant linear expression to 0. We get $6x–4 = 0$, which means that the minima would occur at $x = 2/3$.

    The minimum value would be got by putting $x = 2/3$ in the given expression.

    We get, minimum value = $12/9 – 8/3 + 7 = 17/3$.

    Hence, Option (d) is correct.

23. It can be experimentally verified by taking the values of the 7 integers as 1, 2, 3, 4, 5, 6 and 7 to get an average of 4, while the average of the group if the next three numbers are included would become 5.5 (the group of numbers would be 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10).

    Thus, the average increases by 1.5.

    Hence, Option (c) is correct.

24. Let Distance of the journey = D, Diesel used per km = A and the number of carriages = N. Then we have Time, $T = \dfrac{K \times D \times N}{\sqrt{A}}$

    From the first set of values given in the question, we have T = 45, D = 70, N = 15 and A = 1/7. Putting these values in the equation we get: $K = \dfrac{3}{70\sqrt{7}}$

    The equation then becomes, $T = \dfrac{3 \times D \times N}{70\sqrt{7} \times \sqrt{A}}$.

    In the second case, D = 50 km, N = 18 and T = 30 minutes.

    Solving for A, we get the value of A = 11.8.

    Hence, Option (b) is correct.

25. Let the work done by Tap X in an hour be w. Then the work done by Tap Y in an hour would be 1.6 w. Together, they would do 2.6 w work in an hour. Thus, the total work in 40 hours = 40 × 2.6 w.

    This amount of work would be done by Y alone in $\dfrac{40 \times 2.6w}{1.6w}$ hours = 65 hours.

    Hence, Option (b) is correct.

SNAP 2011

Note: The SNAP 2011 paper had 40 questions on the **Quantitative Aptitude and Data Interpretation** section. Apart from the solutions to the 31 questions in Quantitative Aptitude given here, the solutions to the 9 questions of data interpretation are available in my book—***How to Prepare for Data Interpretation for the CAT*** also published by McGraw Hill.

1. A train travelling at 36 kmph crosses a platform in 20 seconds and a man standing on the platform in 10 seconds. What is the length of the platform in meters?

   (a) 240 meters (b) 100 meters
   (c) 200 meters (d) 300 meters

2. By walking at 4/5th of his usual speed, a man reaches office 10 minutes later than usual. What is his usual time?

   (a) 20 min (b) 40 min
   (c) 30 min (d) 50 min

3. A man and a woman 81 miles apart from each other, start travelling towards each other at the same time. If the man covers 5 miles per hour to the women's 4 mile per hour, how far will the woman have travelled when they meet?

   (a) 27 (b) 36

(c) 45 (d) None of these

4. Two people were walking in opposite directions. Both of them walked 6 miles forward then took right and walked 8 miles. How far is each from starting positions?

   (a) 14 miles and 14 miles

   (b) 10 miles and 10 miles

   (c) 6 miles and 6 miles

   (d) 12 miles and 12 miles

5. Four men and three women can do a job in 6 days. When 5 men and 6 women work on the same job, the work gets completed in 4 days. How long will 2 women and 3 men take to do the job?

   (a) 18 (b) 10

   (c) 8.3 (d) 12

6. Ram completes 60% of a task in 15 days and then takes the help of Rahim and Rachel. Rahim is 50% as efficient as Ram is and Rachel is 50% as efficient as Rahim is. In how many more days will they complete the work?

   (a) $\frac{121}{3}$ (b) $\frac{51}{7}$

   (c) $\frac{40}{7}$ (d) $\frac{65}{7}$

7. A and B can do a piece of work in 21 and 24 days respectively. They start the work together and after some days A leaves the work and B completes the remaining work in 9 days. After how many days did A leave?

   (a) 5 (b) 7

   (c) 8 (d) 6

8. A trader makes a profit equal to the selling price of 75 articles when he sells 100 of the articles. What % profit does he make in the transaction?

(a) 33.33% (b) 75%

(c) 300% (d) 150%

9. In a 100 m race, if A gives B a start of 20 meters, then A wins the race by 5 seconds. Alternatively, if A gives B a start of 40 meters the race ends in a dead heat. How long does A take to run 200 m?

(a) 10 seconds (b) 20 seconds

(c) 30 seconds (d) 40 seconds

10. A 4 cm cube is cut into 1cm cubes. What is the percentage increase in the surface area after such cutting?

(a) 4% (b) 300%

(c) 75% (d) 400%

11. A number G236G0 can be divided by 36 if G is:

(a) 8

(b) 6

(c) 1

(d) More than one values are possible

12. Amit can do a work in 12 days and Sagar in 15 days. If they work on it together for 4 days, then the fraction of the work that is left is:

(a) 3/20 (b) 3/5

(c) 2/5 (d) 2/20

13. A rectangular park 60 m long and 40 m wide has two concrete crossroads running in the middle of the park and rest of the park has been used as a lawn. If the area of the lawn is 2109 sq. m, then what is the width of the road?

(a) 2.91 m (b) 3 m

(c) 5.82 m (d) None of these

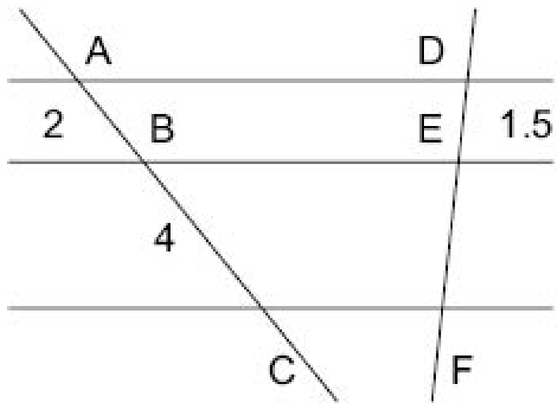
14. A bag contains 5 white and 3 black balls; another bag contains 4 white and 5 black balls. From any one of these bags a single draw of

two balls is made. Find the probability that one of them would be white and other black.

(a) 275/504 (b) 5/18

(c) 5/9 (d) None of these

15. Three parallel lines are cut by two transversals as shown in the given figure. If AB = 2 cm, BC = 4 cm and DE = 1.5 cm, then the length of EF is:



(a) 2 cm (b) 3 cm

(c) 3.5 cm (d) 4 cm

16. $\log_{10}10 + \log_{10}10^2 + \ldots. + \log_{10}10n$

(a) $n^2 + 1$ (b) $n^2 - 1$

(c) $\frac{n^2+n}{2} \cdot \frac{n(n+1)}{3}$ (d) $\frac{(n^2+n)}{2}$

17. The sum of a number and its reciprocal is thrice the difference of the number and its reciprocal. The number is:

(a) $\pm\sqrt{2}$ (b) $\pm\frac{1}{\sqrt{2}}$

(c) $\pm\frac{1}{\sqrt{3}}$ (d) $\pm\sqrt{3}$

18. The total number of natural numbers that lie between 10 and 300 and are divisible by 9 is

(a) 32 (b) 30

(c) 33 (d) 34

19. If ${}^{n}C_{x} = 56$ and ${}^{n}P_{x} = 336$, find n and x.

(a) 7, 3 (b) 8, 4

(c) 8, 3 (d) 9, 6

20. One side of an equilateral triangle is 24 cm. The midpoints of its sides are joined to form another triangle whose midpoints are in turn joined to form still another triangle. This process continues indefinitely. Find the sum of the perimeters of all the triangles.

(a) 144 cm (b) 72 cm

(c) 536 cm (d) 676 cm

21. The probability that a leap year selected at random contains either 53 Sundays or 53 Mondays, is:

(a) 17/53 (b) 1/53

(c) 3/7 (d) None of these

22. Find the intercepts made by the line $3x + 4y - 12 = 0$ on the axes:

(a) 2 and 3 (b) 4 and 3

(c) 3 and 5 (d) None of these

23. The average of 4 distinct prime numbers a, b, c, d is 35, where $a < b < c < d$. a and d are equidistant from 36 and b and c are equidistant from 34 and a, b are equidistant from 30 and c and d are equidistant from 40. The difference between a and d is:

(a) 30 (b) 14

(c) 21 (d) Cannot be determined

24. Ramsukh bhai sells *rasgulla* (a favourite Indian sweets) at ` 15 per kg. A *rasgulla* is made up of flour and sugar in the ratio 5 : 3. The ratio of price of sugar and flour is 7 : 3 (per kg). Thus he earns $66\frac{2}{3}$% profit. What is the cost price of sugar?

(a) ` 10/kg (b) ` 9/kg

(c) ` 18/kg (d) ` 14/kg

25. A reduction of 20% in the price of sugar enables a person to purchase 6 kg more for ` 240. What is the original price per kg of sugar?

(a) ` 10/kg (b) ` 8/kg

(c) ` 6/kg (d) ` 5/kg

26. A solid sphere is melted and recast into a right circular cone with a base radius equal to the radius of the sphere. What is the ratio of the height and radius of the cone so formed?

(a) 4 : 3 (b) 2 : 3

(c) 3 : 4 (d) None of these

27. The speed of scooter, car and train are in the ratio of 1 : 4 : 16. If all of them cover equal distance then the ratio of time taken/velocity for each of the vehicle is:

(a) 256 : 16 : 1 (b) 1 : 4 : 16

(c) 16 : 4 : 1 (d) 16 : 1 : 4

28. B is twice efficient as A and A can do a piece of work in 15 days. A started the work and after a few days B joined him. They completed the work in 11 days, from the starting. For how many days did they work together?

(a) 1 day (b) 2 days

(c) 6 days (d) 5 days

29. A, B, C and D purchased a restaurant for ` 56 lakhs. The contribution of B, C and D together is 460% that of A, alone. The contribution of A, C and D together is 366.66% that of B's contribution and the contribution of C is 40% that of A, B and D together. The amount contributed by D is:

(a) 10 lakhs (b) 12 lakhs

(c) 16 lakhs (d) 18 lakhs

30. The salary of Raju and Ram is 20% and 30% less than the salary of Saroj respectively. By what percent is the salary of Raju more than the salary of Ram?

(a) 33.33% (b) 50%

(c) 15.18% (d) 14.28%

31. The radius of a wire is decreased to one-third and its volume remains the same. The new length is how many times the original length?

(a) 2 times (b) 4 times

(c) 5 times (d) 9 times

ANSWER KEY

1. (b)	2. (b)	3. (b)	4. (b)
5. (c)	6. (c)	7. (b)	8. (b)
9. (c)	10. (b)	11. (a)	12. (c)
13. (b)	14. (a)	15. (b)	16. (d)
17. (a)	18. (a)	19. (c)	20. (a)
21. (c)	22. (b)	23. (b)	24. (d)
25. (a)	26. (d)	27. (a)	28. (b)
29. (d)	30. (d)	31. (d)	

Solutions

1. The speed of the train in m/s would be $36 \times 5 \div 18 = 10$ m/s. If the train takes 10 seconds to cross a man on the platform, it means that the length of the train = $10 \times 10 = 100$ m.

   Also, while crossing the platform, the distance traveled by the train is equal to: Length of Train + Length of Platform.

   Hence, Length of Platform + 100 = 20×10 Æ Length of Platform = 100 meters. Option (b) is the correct answer.

2. As the distance is constant, the speed and time are inversely proportional to each other. Walking at $4/5^{th}$ of his usual speed would

mean that the man would take $5/4^{th}$ of his usual time. Thus, the extra time taken = $5/4^{th}$ of $t - t = 1/4^{th}$ of t (where t = normal time taken).

But the extra time taken is given as 10 minutes. Hence, $t/4 = 100$ Æ $t = 40$ minutes.

Hence, Option (b) is correct.

3. Their relative speed would be equal to 9 miles per hour (as they are approaching each other). Thus, they would meet in $81 \div 9 = 9$ hours. The woman would travel $9 \times 4 = 36$ miles in that time. Hence, Option (b) is correct.

4. Since both have them have taken a 90^o turn, it clearly implies that 6 miles and 8 miles (the distances they have walked) would be the base and height of a right-angled triangle. Their distance from the starting point would be calculated by the hypotenuse of each right-angled triangle. The appropriate Pythagoras triplet would be 6, 8, 10 and hence their distance from the start would be 10 miles each.

   Hence, Option (b) is correct.

5. 24 man days + 18 woman days = 20 man days + 24 woman days Æ 1 man-day = 1.5 woman days.

   In terms of woman days the total work would be:

   24 man days + 18 woman days = 36 woman days + 18 woman days = 54 woman days.

   2 women and 3 men would be equivalent to 2 women + 4.5 women = 6.5 women.

   The number of days required would be:

   54/6.5 = 8.3 days. Hence, Option (c) is correct.

6. Since Ram has completed 60% of the task in 15 days – this gives us two pieces of information –

   (i) That 40% of the task is left;

   (ii) Ram's rate of work is 4% per hour.

   Then, from the information in the problem we can see that Rahim's work rate would be 2% per annum, while Rachel's work rate would be 1% per annum. Thus, their combined rate of work would be 7% per hour giving us 40/7 as the required answer.

Option (c) is correct.

7. In the 9 days that he works alone, B would do $\frac{9}{24}$ of the work. This means that A and B together have done $\frac{15}{24}$ of the work. Thus,

$$\frac{d}{24}+\frac{d}{21}=\frac{15}{24}$$

Solving the above equation we get $d = 7$.

Hence, Option (b) is correct.

8. The profit percent = $\frac{75}{100}$ × 100 = 75. Hence, Option (b) is correct.

9. From the second statement, we have that in the time A runs 100 m, B would run 60 m. Thus, their ratio of speeds is 5:3. From this point you can start checking with the options in order to get to the correct answer.

   If we check for Option (a) we see that, A's speed = 20m/s, B's speed = 12 m/s. This would satisfy the second condition – i.e., if A gives B a start of 40 meters the race ends in a dead heat.

   We need to see whether these values satisfy the first condition too – i.e., if A gives B a start of 20 meters, then A wins the race by 5 seconds.

   A would complete the race in 5 seconds and B at that time would have reached from 20 m to 80 m (as he would simultaneously have traveled 60 m in 5 seconds @ 12 m/s). To cover the remaining 20 m, A would not be taking 5 seconds. Hence, we can reject this option.

   The same thinking can be applied to the other options to get the correct answer.

   For Option (c): A's speed = 6.666 m/s, B's speed = 4 m/s

   We need to see whether these values satisfy the first condition too – i.e., if A gives B a start of 20 meters, then A wins the race by 5 seconds.

A would complete the race in 15 seconds and B at that time would have reached from 20 m to 80 m (as he would simultaneously have traveled 60 m in 15 seconds @ 4 m/s). To cover the remaining 20 m, A would be taking 5 seconds. Hence, this option is correct.

Option (c) is the correct answer.

10. The initial surface area = 6 × 4 × 4 = 96 sq. cm. The new surface area = 64 × 6 × 1 × 1 = 384 sq. cm. The surface area is becoming 4 times the original, which means that there is a 300% increase in the surface area. Hence, Option (b) is correct.

11. With $G = 8$, divisibility can be verified. Hence, Option (a) is correct.

12. Amit's work per day = 8.33%, Sagar's work per day = 6.66%. Combined work per day = 15%. In 4 days they would complete 15 × 4 = 60% work. Fraction of work left = 40 out of 100 or 2 out of 5. Hence, Option (c) is correct.

13. The total area of the field is 2400 sq. m. If we try to keep the width of the road as 3 m, we can see that the total area of the roads = 3 × 60 + 3 × 40 – 3 × 3 = 291sq. m. – leaving 2109 sq. m for the lawn. Hence, Option (b) is correct.

14. The event definition would be:

First bag & white ball & black ball OR First bag & black ball & white ball

OR

Second bag & white ball & black ball OR Second bag & black ball & white ball

$$= \frac{1}{2}\times\frac{5}{8}\times\frac{3}{7}+\frac{1}{2}\times\frac{3}{8}\times\frac{5}{7}+\frac{1}{2}\times\frac{4}{9}\times\frac{5}{8}+\frac{1}{2}\times\frac{5}{9}\times\frac{4}{8}$$

$$= \frac{30}{112}+\frac{40}{144}=\frac{275}{504}$$

15. The ratio of AB to BC would be equal to the ratio of DE to EF. Since, BC is twice the value of AB, EF would also be twice the value of DE and hence, EF = 3 cm. Hence, Option (b) is correct.

16. The value of the expression = 1 + 2 + 3 +…n = sum of the first n natural numbers $\frac{n\times(n+1)}{2}=\frac{n^2+n}{2}$. Hence, Option (d) is correct.

17. Solve through options by checking for the given conditions. The value of $\pm\sqrt{2}$ fits the conditions. Hence, Option (a) is the correct answer.

18. We need to find the number of terms in the series:

    18, 27, 36, … 297

    There are 32 terms in this series since it starts with 9 × 2 and ends with 9 × 33.

    Hence, Option (a) is correct.

19. We know that the value of $^nC_x \times r! = {}^nP_x$

    Since, the value of ^nP_x is 6 times the value of $^nC_{x,}$ we know that r = 3. Checking the options, we can see that n = 8 and r = 3 fits the given values. Hence, Option (c) is the correct answer.

20. The second perimeter would be half the first perimeter. In order to get the answer to this question, we need the infinite sum of the geometric progression: 72 +36 +18 + 9 +….. This is a GP with a = 72 and r = ½ . The infinite sum of the given series = 144. Hence, Option (a) is correct.

21. A leap year has 366 days, which means that it has 52 completed weeks + 2 extra days. Thus, if 1st January is a Saturday, 30th December would also be a Saturday. For a 53rd Sunday or Monday, the year should start either from a Saturday, Sunday or a Monday in which case, the last two days of the year would be (Saturday, Sunday) or (Sunday, Monday) or (Monday, Tuesday) respectively. The required probability is 3/7. Hence, Option (c) is correct.

22. For the x-intercept y = 0 and hence the x-intercept = 4. Similarly, for the y-intercept, the value of x = 0 and hence the y-intercept = 3. Hence, Option (b) is correct.

23. A quick look at the various prime numbers between the 20s to the 40s gives us the following list: 23, 29, 31, 37, 39, 41, 43, 47. The

given conditions are satisfied by the set of numbers: 29, 31, 37 and 43. Thus, a = 29 and d = 43 and hence the required difference between and d is 14. Hence, Option (b) is correct.

24. 8 kg of *rasgulla* would give a revenue of `120 and a cost of `72. (Note: Cost should be 3/5 of the revenue for a profit of 66.66%.) 8 kg of the sweet would also require 5 kg of flour and 3 kg of sugar. Thus, the cost of 5 kg flour + 3 kg sugar = `72. The price of sugar that satisfies this condition in association with the additional condition that the ratio of the price of sugar to flour is 7:3 is `14/kg.

    Hence, Option (d) is correct.

25. Reduction of 20% in the price of sugar would increase the quantity by 25% = increase of 6 kg (for the same cost). Thus, the original quantity = 24 kg and the new quantity = 30 kg. The price of sugar (original) = 240 ÷ 24 = `10/ kg.

26. The volume of the sphere = $\frac{4\pi r^3}{3}$. The volume of a right circular cone = $\frac{\pi r^3 h}{3}$.

    Since the radii of the sphere and the cone are equal, when we equate the two volumes we get $4r = h$. Hence, the ratio of height to radius for the right circular cone is 4:1. Hence, Option (d) is correct.

27. If the speeds are 1 kmph, 4 kmph and 16 kmph respectively, they would cover a distance of (say 16 km) in 16 hours, 4 hours and 1 hour respectively. The ratio of time taken to velocity would be (16/1): (4/4) : (1/16) = 16:1:1/16 = 256:16:1. Hence, Option (a) is correct.

28. A’s work = 6.66% per hour. Thus, B’s work = 13.33% per hour. Combined work = 20% per hour. Checking the options: Option (a) does not work because 10 × 6.66 + 20 × 1 π 100.

    For Option (b) we get: 9 × 6.66 + 2 × 20 = 60 +40 = 100%.

    Hence, Option (b) is correct.

29. From the given statements we can work out the values of the individual investments as follows:

Statement: The contribution of B, C and D together is 460% that of A, alone means that A’s investment = 56 ÷ 5.6 = 10 lakhs.

Statement: The contribution of A, C and D together is 366.66% that of B’s contribution means that B’s investment = 56 ÷ 4.6666 = 12 lakhs.

Statement: The contribution of C is 40% that of A, B and D together implies that C’s investment = 56 × 40 ÷ 140 (unitary method) = 16 lakhs

Hence, D’s investment = 56 –10 – 12 – 16 = 18 lakhs.

Hence, Option (d) is correct.

30. If Saroj is taken as 100, Raju would be 80 and Ram would be 70. The salary of Raju would be 14.28% more than the salary of Ram. Hence, Option (d) is correct.

31. If the wire is $1/3^{rd}$ in terms of the radius, per unit length the volume of material required would be $1/9^{th}$. Thus, the length of the wire—if the volume has to be kept the same—would be 9 times the original length. Hence, Option (d) is correct.

XAT 2014

Note: XAT 2014 had 31 questions on the **Quantitative Aptitude** section. Of these there were 21 questions directly on Quantitative Aptitude and 9 questions were on Data Interpretation. In this book, the paper consists of the 22 questions from QA part. The 9 question on DI for XAT 2014 can be found in my book—***How to Prepare for Data Interpretation for the CAT*** *also* published by McGraw Hill.

1. x, 17, $3x - y^2 - 2$, and $3x + y^2 - 30$, are four consecutive terms of an increasing arithmetic sequence. The sum of the four numbers is divisible by:

   (a) 2 (b) 3
   (c) 5 (d) 7
   (e) 11

2. In quadrilateral PQRS, PQ = 5 units, QR = 17 units, RS = 5 units, and PS = 9 units. The length of the diagonal QS can be:

   (a) > 10 and < 12 (b) > 12 and < 14
   (c) > 14 and < 16 (d) > 16 and < 18
   (e) Cannot be determined

3. Consider the formula,

$S = \dfrac{\alpha \times \omega}{\tau + \rho \times \omega}$, where all the parameters are positive integers. If *w* is increased and *a*, *t* and *r* are kept constant, then *S*:

(a) increases

(b) decreases

(c) increases and then decreases

(d) decreases and then increases

(e) Cannot be determined

4. Prof. Suman takes a number of quizzes for a course. All the quizzes are out of 100. A student can get an A grade in the course if the average of her scores is more than or equal to 90. Grade B is awarded to a student if the average of her scores is between 87 and 89 (both included). If the average is below 87, the student gets a C grade.

   Ramesh is preparing for the last quiz and he realises that he will score a minimum of 97 to get an A grade. After the quiz, he realises that he will score 70, and he will just manage a B. How many quizzes did Prof. Suman take?

   (a) 6 (b) 7

   (c) 8 (d) 9

   (e) None of these

5. A polynomial "$ax^3 + bx^2 + cx + d$" intersects *x*-axis at 1 and –1, and *y*-axis at 2. The value of *b* is:

   (a) – 2 (b) 0

   (c) 1 (d) 2

   (e) Cannot be determined

6. The sum of the possible values of *X* in the equation $|X + 7| + |X - 8| = 16$ is:

   (a) 0 (b) 1

(c) 2 (d) 3

(e) None of the above

7. There are two windows on the wall of a building that need repairs. A ladder 30 m long is placed against a wall such that it just reaches the first window which is 26 m high. The foot of the ladder is at Point A. After the first window is fixed, the foot of the ladder is pushed backwards to Point B so that the ladder can reach the second window. The angle made by the ladder with the ground is reduced by half, as a result of pushing the ladder. The distance between points A and B is

   (a) < 9 m

   (b) ≥ 9 m and < 9.5 m

   (c) ≥ 9.5 m and < 10 m

   (d) ≥ 10 m and < 10.5 m

   (e) ≥ 10.5 m

8. Amitabh picks a random integer between 1 and 999, doubles it and gives the result to Sashi. Each time Sashi gets a number from Amitabh, he adds 50 to the number, and gives the result back to Amitabh, who doubles the number again. The first person, whose result is more than 1000, loses the game. Let '*x*' be the smallest initial number that results in a win for Amitabh. The sum of the digits of '*x*' is:

   (a) 3 (b) 5

   (c) 7 (d) 9

   (e) None of these

9. Consider four natural numbers: x, y, $x + y$, and $x - y$. Two statements are provided below:

   I. All four numbers are prime numbers.

   II. The arithmetic mean of the numbers is greater than 4.

   Which of the following statements would be sufficient to determine the sum of the four numbers?

(a) Statement I

(b) Statement II

(c) Statement I and Statement II

(d) Neither Statement I nor Statement II

(e) Either Statement I or Statement II

10. Triangle *ABC* is a right-angled triangle. *D* and *E* are mid points of *AB* and *BC* respectively. Read the following statements.

I. $AE = 19$

II. $CD = 22$

III. Angle *B* is a right angle.

Which of the following statements would be sufficient to determine the length of *AC*?

(a) Statement I and Statement II

(b) Statement I and Statement III

(c) Statement II and III

(d) Statement III alone

(e) All three statements

11. There are two circles C_1 and C_2 of radii 3 and 8 units respectively. The common internal tangent, *T*, touches the circles at Points P_1 and P_2 respectively. The line joining the centers of the circles intersects *T* at *X*. The distance of *X* from the center of the smaller circle is 5 units. What is the length of the line segment P_1P_2?

(a) ≤ 13 (b) > 13 and ≤ 14

(c) > 14 and ≤ 15 (d) > 15 and ≤ 16

(e) > 16

12. The probability that a randomly chosen positive divisor of 10^{29} is an integer multiple of 10^{23} is: a^2/b^2, then '$b - a$' would be:

(a) 8 (b) 15

(c) 21　　　(d) 23

(e) 45

13. Circle C_1 has a radius of 3 units. The line segment PQ is the only diameter of the circle which is parallel to the X-axis. P and Q are points on curves given by the equations $y= a^x$ and $y = 2a^x$ respectively, where $a < 1$. The value of a is:

(a) $1/\sqrt[6]{2}$　　　(b) $1/\sqrt[6]{3}$

(c) $1/\sqrt[3]{6}$　　　(d) $1/\sqrt{6}$

(e) None of these

14. There are two squares S_1 and S_2 with areas 8 and 9 units, respectively. S_1 is inscribed within S_2, with one corner of S_1 on each side of S_2. The corners of the smaller square divides the sides of the bigger square into two segments, one of length 'a' and the other of length 'b', where, $b > a$.

A possible value of 'b/a', is:

(a) ≥ 5 and < 8　　　(b) ≥ 8 and < 11

(c) ≥ 11 and < 14　　　(d) ≥ 14 and < 17

(e) > 17

15. Diameter of the base of a water-filled inverted right circular cone is 26 cm. A cylindrical pipe, 5 mm in radius, is attached to the surface of the cone at a point. The perpendicular distance between the point and the base (the top) is 15 cm. The distance from the edge of the base to the point is 17 cm, along the surface. If water flows at the rate of 10 meters per minute through the pipe, how much time would elapse before water stops coming out of the pipe?

(a) < 4.5 minutes

(b) ≥ 4.5 minutes but < 4.8 minutes

(c) ≥ 4.8 minutes but < 5 minutes

(d) ≥ 5 minutes but < 5.2 minutes

(e) ≥ 5.2 minutes

16. Aditya has a total of 18 red and blue marbles in two bags (each bag has marbles of both colors). A marble is randomly drawn from the first bag followed by another randomly drawn from the second bag, the probability of both being red is 5/16. What is the probability of both marbles being blue?

(a) 1/16 (b) 2/16

(c) 3/16 (d) 4/16

(e) None of these

17. Consider a rectangle *ABCD* of area 90 units. The points *P* and *Q* trisect *AB*, and *R* bisects *CD*. The diagonal *AC* intersects the line segments *PR* and *QR* at *M* and *N* respectively. What is the area of the quadrilateral *PQMN* ?

(a) > 9.5 and ≤ 10 (b) > 10 and ≤ 10.5

(c) > 10.5 and ≤ 11 (d) > 11 and ≤ 11.5

(e) > 11.5

18. Two numbers, $297B$ and $792B$, belong to base B number system. If the first number is a factor of the second number then the value of B is:

(a) 11 (b) 12

(c) 15 (d) 17

(e) 19

19. A teacher noticed a strange distribution of marks in the exam. There were only three distinct scores: 6, 8 and 20. The mode of the distribution was 8. The sum of the scores of all the students was 504. The number of students in the most populated category was equal to the sum of the number of students with lowest score and twice the number of students with the highest score. The total number of students in the class was:

(a) 50 (b) 51

(c) 53 (d) 56

(e) 57

20. Read the following instruction carefully and answer the question that follows:

Expression $\sum_{n=1}^{13} \frac{1}{n}$ can also be written as $\frac{x}{13!}$

What would be the remainder if x is divided by 11?

(a) 2 (b) 4

(c) 7 (d) 9

(e) None of the above

21. A rectangular swimming pool is 48 m long and 20 m wide. The shallow edge of the pool is 1 m deep. For every 2.6 m that one walks up the inclined base of the swimming pool, one gains an elevation of 1 m. What is the volume of water (in cubic meters), in the swimming pool? Assume that the pool is filled up to the brim.

(a) 528 (b) 960

(c) 6790 (d) 10560

(e) 12960

22. The value of the expression:

$\sum_{i=2}^{100} \frac{1}{\log_i 100!}$ is:

(a) 0.01 (b) 0.1

(c) 1 (d) 10

(e) 100

ANSWER KEY

1. (a)	2. (b)	3. (a)	4. (d)
5. (a)	6. (b)	7. (e)	8. (c)

9. (a)	10. (e)	11. (c)	12. (d)
13. (a)	14. (d)	15. (d)	16. (c)
17. (d)	18. (e)	19. (e)	20. (d)
21. (d)	22. (c)		

Solutions

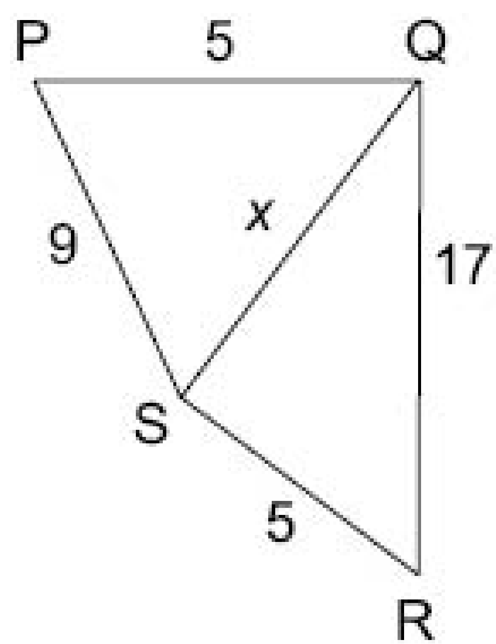
1. Given that one of the terms of the AP is 17, we can identify that the AP would either have all four odd numbers or have two odd and two even numbers.

   Either ways the sum of the AP for four numbers would be even and hence would be divisible by 2.

Note: we can safely assume here that since the terms of the AP are based on values of the variables x and y, all the four terms of the AP would be integers and there would be no possibility of some of the terms having decimal values.

Hence, Option (a) is correct.

2. Since, PQ, QR, RS and PS are consecutive sides in the quadrilateral, we would get a figure for the quadrilateral as:



In this figure we have two relationships for the range of the value of SQ (based on the fact that the sum of two sides of a triangle would always be greater than the third side).

From Triangle SRQ: 5 + QS >17 Æ 5 + x >17 Æ x > 12

From Triangle PQS: x < 5 + 9 Æ x < 14.

Hence, 12 < x < 14. Hence, Option (b) is correct.

3. The expression can be modified to:

$$\frac{1}{S} = \frac{\tau + \rho \times \omega}{\alpha\omega} = \frac{\tau}{\alpha\omega} + \frac{\rho}{\alpha} = \frac{K_1}{\omega} + K_2$$

It can be clearly seen that when w is increased, $\frac{1}{S}$ would decrease.

Hence, S would increase if we increase w.

Hence, Option (a) is correct.

4. The difference between just getting an A grade and just getting a B grade would be equal to (number of quizzes × 3). The difference between the required score to manage an A grade and the score achieved to just manage a B grade as given in the problem's information is equal to 97 – 70 = 27.

Thus, number of quizzes × 3 = 27. Hence, number of quizzes is 9.

Hence, Option (d) is correct.

5. When a polynomial intersects the x-axis, the value of $y = 0$. When a polynomial intersects the y-axis, the value of $x = 0$.

If we take the polynomial as $y = ax^3 + bx^2 + cx + d$, we get:

Intersects x-axis at $x = 1$ implies: $a + b + c + d = 0$;

Intersects x-axis at $x = –1$ implies: $–a + b – c + d = 0$;

Hence, $2(b + d) = 0$ or simply put $(b + d) = 0$.

Intersects y-axis at $y = 2$ implies: $0 + d = 2$ Æ $d = 2$. Hence, $b = –2$. Hence, Option (a) is correct.

6. You need to consider the value of the expression for three ranges of values of X.

If $X \geq 8$, $|X + 7|$ and $|X – 8|$ are both non-negative.

If $-7 \leq X < 8$, $|X + 7|$ is positive and $|X - 8|$ is negative.

& If $X < -7$, $|X + 7|$ and $|X - 8|$ are both negative.

For the first case:

If $X \geq 8$, $|X + 7|$ and $|X - 8|$ are both non-negative.

$|X + 7| = X + 7$ and $|X - 8| = X - 8$.

Hence, $|X + 7| + |X - 8| = 16$;

Implies $X + 7 + X - 8 = 16$

Æ $2X - 1 = 16$

\ $x = 8.5$.

For the second case:

If $-7 \leq X < 8$, $|X + 7|$ is positive and $|X - 8|$ is negative.

$|X + 7| = X + 7$ and $|X - 8| = 8 - X$.

Hence, $X + 7 + 8 - X = 16$. This leads to the ridiculous outcome $15 = 16$.

Of course this is not possible and hence we can rule out this range – i.e. there would be no value of X between –7 £ x < 8 that would satisfy this equation.

For the third case:

If $X < -7$, $|X + 7|$ and $|X - 8|$ are both negative.

$|X + 7| = -7-X$ and $|X - 8| = 8 - X$.

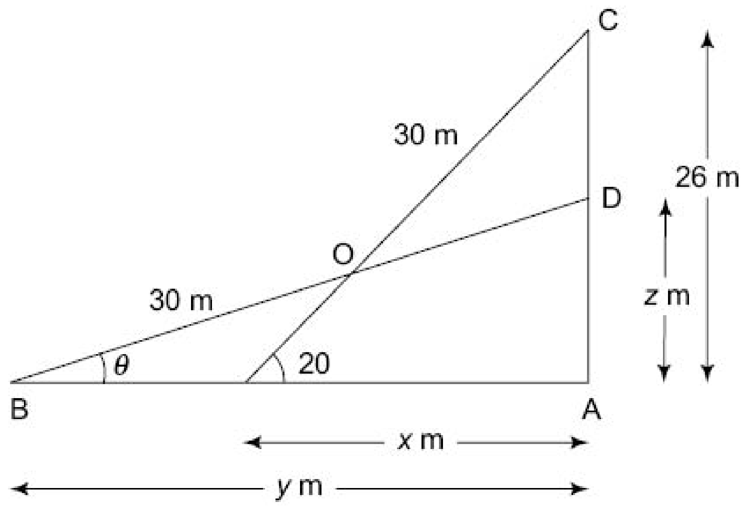
Thus, we get:

$-7 - X + 8 - X = 16$ Æ $2X = -15$ Æ $X = -7.5$.

Thus, we have two possible values of X : i.e. 8.5 and –7.5 and their sum = 8.5 – 7.5 = 1.

Hence Option (b) is correct.

7.



From the figure we have:

cos q = *y*/ 30 and cos 2q = *x*/30.

Solving using Pyhtagoras theorem we get:

x = $\sqrt{30^2 - 26^2} = \sqrt{224}$ ª 15. Thus, the value of the angle 2q is approximately equal to 60^o. Hence, q ª 30^o.

Solving cos 30 = *y*/30, we get *y* = $15\sqrt{3}$ m.

The distance between A and B would be $15\sqrt{3}$ – 15

= 15 × 0.73 ª 10.95.

Hence, Option (e) is correct.

8. Let the number chosen by Amitabh be X. Then the flow of the numbers would go as follows:

	Amitabh	*Sashi*
Round 1	2X	2X + 50
Round 2	4X + 100	4X + 150
Round 3	8X + 300	8X + 350
Round 4	16X + 700	16X + 750
Round 5	32X + 1500 (Obviously the game cannot continue beyond this point as the value would definitely have crossed 1000 by this time.)	

The latest that Amitabh can win is when Sashi gets a value of 16X +750. For 16X +750 > 1000, the smallest possible value of X = 16. Hence, sum of the digits of X = 1 + 6 = 7. Hence, Option (c) is correct.

9. If we take only Statement I, for all four numbers to be prime one of them must be even and hence equal to 2. Only in such an event do we get $x + y$ and $x–y$ as odd numbers and only if they are odd can all the four numbers be prime. A little bit of trial and error then gives us we get $x = 5$, $y = 2$, $x + y = 7$ and $x – y = 3$. There is not other case of $x–y$, x and $x+y$ being prime as if we take y as 2, these numbers become $x–2$, x and $x+2$ and hence represent three consecutive odd numbers. (After 3, 5, 7 there is no situation where three consecutive odd numbers are all prime.)

   Hence, Statement I is sufficient.

   Statement II can be easily rejected as all it is giving us is that the sum of the four numbers is greater than 16. As we can easily imagine there are infinite sets of four such numbers, which have a sum greater than 16.

   Hence, Option (a) is correct.

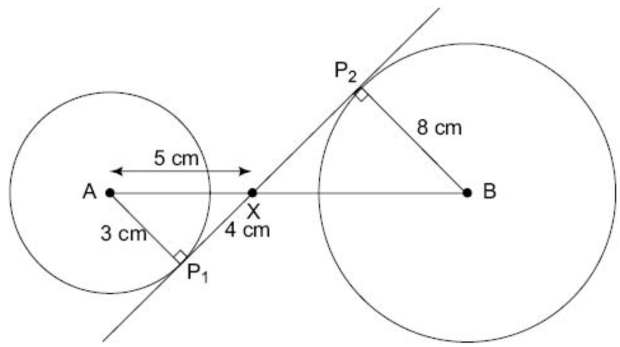
10. First things first while solving this. If we do not include Statement III, we do not know which angle is a right angle and hence cannot uniquely calculate the value of the Side AC.

    Also, Statement III alone does not give us anything. Hence, we can reject Options (a) and (d). Options (b) and (c) give us similar set of information – i.e., the value of 1 median and the fact that B is the right angle in the triangle. This is also clearly not sufficient to answer the question.

    For Option (e): We have 2 medians of a right-angled triangle, and we know B is the right angle. Hence, we can find AC.

    Hence, Option (e) is the correct answer.

11. Let A, B be the centre of the two circles with radius 3 cm and 8 cm respectively.



The use of the Pythagoras triplet 3, 4, 5 is self evident from the figure.

In order to find P_1P_2 we need to find the value of XP_2.

The similarity between triangles AP_1X and BP_2X implies that $\frac{AP_1}{BP_2} = \frac{P_1X}{P_2X}$.

Since, we know three of the lengths in the above equation we get:

$\frac{3}{8} = \frac{4}{P_2X}$ Æ $P_2X = 4 \times 2.66 = 10.66$.

Thus, the length of $T = 4 + 10.66 = 14.66$

Hence, Option (c) is correct.

12. $10^{29} = 5^{29} \times 2^{29}$. Thus, the number of divisors of $10^{29} = (29 + 1)(29 + 1) = 30 \times 30$.

Now, if we look at divisors of 10^{29}, which satisfy the given condition, we need to get divisors of the form: as $N \times 10^{23}$. The number of such divisors can be traced based on the number of divisors of N, such that $10^{29} = N \times 10^{23}$.

$N = 10^6 = 5^6 \times 2^6$ which would have $7 \times 7 = 49$ divisors. Hence, 10^{29} would have 49 divisors that are also divisible by 10^{23}.

The probability that a randomly chosen divisor of 10^{29} is divisible by 10^{23} is $49/900 = a^2/b^2$.

Thus, $a = 7$ and $b = 30$ and consequently $b - a = 23$.

Hence, Option (d) is correct.

13. The crux of the solution to this problem lies in understanding the fact that the Points P and Q would have the same Y-coordinates (since the diameter PQ is parallel to the X-axis) while their X co-ordinates would differ by 6 (since the diameter of the circle is 6).

We thus have two possible cases:

Case 1: If P lies on the curve $y = a^x$ and Q lies on the curve $y = 2a^x$

Then, the difference between the x-coordinates of being 6, we get:

$y = 2ax - a^{(x + 6)}$. We get $a = 2^{1/6}$

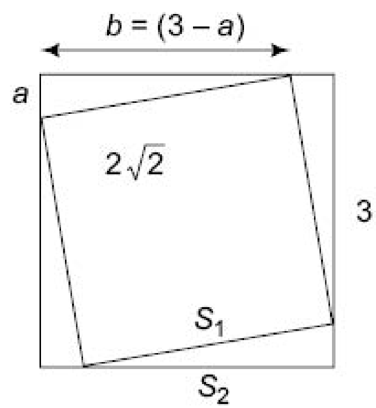
Case 2: If Q lies on the curve $y= a^x$ and P lies on the curve $y = 2a^x$

Then, the difference between the x-coordinates of being 6, we get:

$y = ax - 2a^{(x + 6)}$. We get $a = (1/2)^{1/6}$

Since, we are given that $a < 1$, only Case 2 is possible and hence Option (a) is correct.

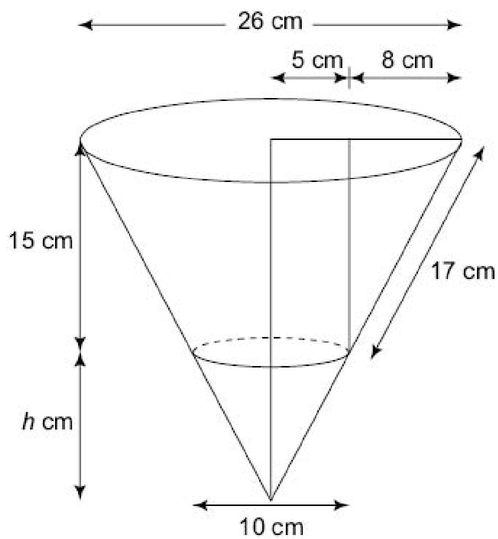
14. If you visualise a figure for this situation, you would be able to see something as follows:



Solving through Pythagoras theorem, we will get $a = 0.18$ and $b = 3 - 0.18 = 2.82$.

Hence, the value of $b/a = 2.82/0.18 = 15.666$. Hence, Option (d) is correct.

15. The following figure would exemplify the situation, with the pipe attached at a height of h from the apex (bottom) of the cone.



In the above figure the cones with height *h* and the cone with height *h* + 15 are similar to each other. Hence using similarity we will get:

$$\frac{10}{26} = \frac{h}{h+15}$$

Æ 10*h* + 150 = 26*h* Æ 16*h* = 150 Æ = 9.375 cm.

Based on this information we can then calculate the volume of water that would flow out from the pipe as: Total volume of the cone with height (*h* + 15) – Volume of cone with height *h*.

$$= \frac{1}{3}\pi\left[\left(13^2\times(15+9.375)\right)-(5^2\times 9.375)\right]$$

Calculating this we get the volume of water that overflows = 1295*p* cm^3

Further, the rate at which the water flows out of the hole per minute is given by: *p* × (0.5^2 × 1000) cm^3

Hence, the required time can be given as,

$$\frac{1295\pi}{\pi\times(0.5^2\times 1000)} = 5.18 \text{ min}$$

Hence, Option (d) is the correct answer.

16. Let there be ‘*r*’ red marbles in the first bag and ‘*R*’ red marbles in the second bag.

Also, let there be 'b' blue marbles in the first bag and 'B' blue marbles in the second bag.

Then $(r + b) + (R + B) = 18$

The number of ways of selecting 1 marble from the first bag and one marble from the second bag = $(r + b) \times (R + B)$.

Also, the number of ways of selecting a red marble from the first bag and a red marble from the second bag = $r \times R$.

Hence, the probability of selecting two red marbles would be given by:

$$\frac{r \times R}{(r+b)\times(R+B)} = \frac{5}{16}.$$

This equation shows us that the value of $(r + b) \times (R + B)$ must be a multiple of 16 while the value of $r \times R$ must be a multiple of 5.

We now know that $(r + \mathrm{b}) + (R + B) = 18$ and $(r + b) \times (R + B)$ = a multiple of 16.

If we try to break down 18 into two parts, such that their product is a multiple of 16, we get two cases:

Case I: $16 \times 2 = 32$ and Case II: $10 \times 8 = 80$.

Analysing Case I:

If $(r + b) \times (R + B) = 32$, then $r \times R = 10$ (in order to maintain the ratio of 5/16 between the two).

There would be 16 balls in one bag and 2 balls in the second bag.

The second bag must contain 1 red and 1 blue ball. i.e., $R = 1$ and $B = 1$

Since $r \times R = 10$ we get $r = 10$ and $b = 6$.

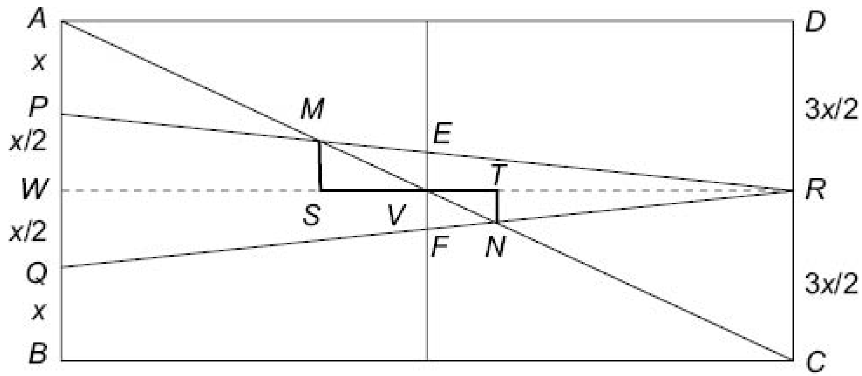
The probability of both balls being blue in this case would turn out as:

$$\frac{1}{2}\times\frac{6}{16} = \frac{3}{16}.$$

If we try to solve the other case in the same way, we get $r = 5$, $R = 5$ and $b = 3$ and $B = 5$ and the probability of both balls being blue becomes $\frac{1}{2}\times\frac{3}{8}=\frac{3}{16}$

We get the same answer in both the cases. Hence, Option (c) is correct.

17. The given situation can be visualised based on the following figure:



The area of the quadrilateral *PQNM* = Area of *PQFE* – Area Triangle *MVS* + Area Triangle *VFN*.

So our focus to find the area of the required quadrilateral should shift to the area of the three individual components on the right hand side of the above equation.

Finding the area of quadrilateral PQFE: Being a paralleogram, the required area would be given by:

$\frac{1}{2}$ × *sum of parallel sides* × *perpendicular distance between the parallel sides*.

In the figure, let side *AB* = 3*x* and side *BC* = *y*. Then, for the quadrilateral *PQFE*, the perpendicular distance between the parallel sides would be *y*/2.

Further, the sum of parallel sides would be equal to *PQ* + *EF* = $x+\frac{x}{2}=\frac{3x}{2}$

Area of *PQFE* = $\frac{1}{2}\times\frac{3x}{2}\times\frac{y}{2}$

We know that the area of the rectangle is $3x \times y = 90$.

Hence, area of *PQFE* = 90 ÷ 8 =11.25.

Finding the area of the triangle MSV= Finding the area of the triangle MEV.

$\frac{1}{2}$ × base × height = $\frac{1}{2}$ × *EV* × height

Using similarity between *APM* and *VEM*, we can see that since *AP* = *x* and *EV* = *x*/4, the ratios of the lengths of *APM* and *VEM* would be 4:1.

Thus, if the height of *APM* = 4*h*, the height of *VEM* = *h* and also 4*h* + *h* = *y*/2 Æ *h* = height of *VEM* with base *VE* = *y*/10.

Hence, area triangle *MEV* = $\frac{1}{2}$ × *EV* × height = $\frac{xy}{80}$

= 0.375.

Similarly, the area of triangle *VFN* ª 0.27.

Thus, the required area = 11.25 – 0.375 + 0.27

ª 11.145.

Hence, Option (d) is correct.

18. Solve this question through options. The correct option would give the number 792_B as a multiple of the number 297_B

Now $792_B = B^2 \times 7 + B \times 9 + 2$

And $297_B = B^2 \times 2 + B \times 9 + 7$.

If, we put B = 19 from Option (e) we get:

$792_B = B^2 \times 7 + B \times 9 + 2 = 27000$ and $297_B = B^2 \times 2 + B \times 9 + 7 = 900$. This option satisfies the condition given in the problem. Hence, Option (e) is the correct answer.

19. Let the number of students scoring 6, 8 and 20 be *a*, *b*, and *c* respectively.

So, 6*a* + 8*b* + 20*c* = 504

Also, since ‘The number of students in the most populated category was equal to the sum of the number of students with lowest score and twice the number of students with the highest score' we get: $a + 2c = b$.

Thus, $14a + 36c = 504$.

By trying out values we get that:

$c = 7$ and $a = 18$ which gives us $b = 32$.

Therefore, total number of students = 18 + 32 + 7 = 57

Hence, Option (e) is the correct answer.

20. By equating the summation to $\frac{x}{13!}$ we get:

$$x = \frac{13!}{1} + \frac{13!}{2} + \frac{13!}{3} + \cdots + \frac{13!}{11} + \frac{13!}{12} + \frac{13!}{13}$$

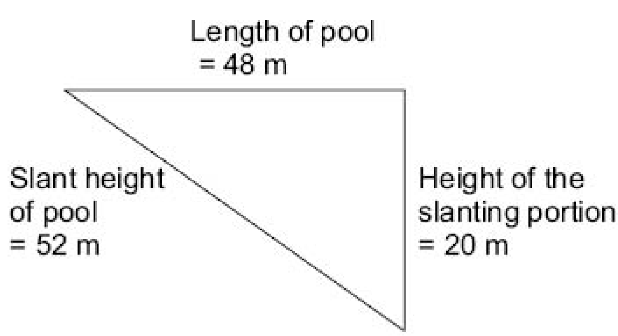
All the terms in x are divisible by 11 except 13!/11

Thus, the remainder of x divided by 11 would be given by the remainder of:

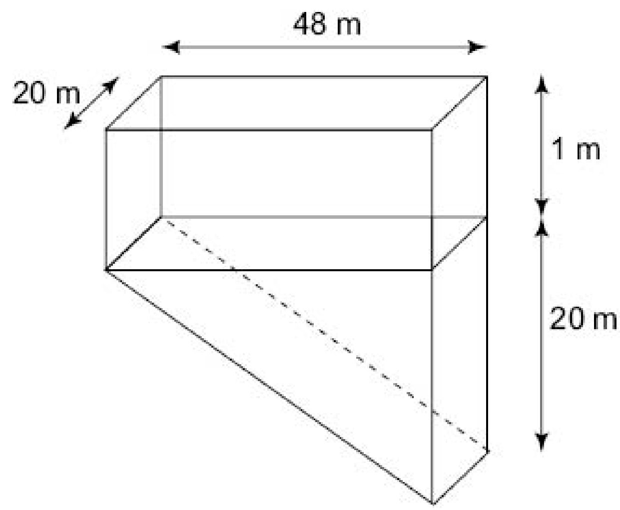
1.2.3.4.5.6.7.8.9.10.12.13 ÷ 11 = Rem (10! × 12 × 13) ÷ 11 = Rem (–1 × 1 × 2) ÷ 11. The required remainder is –2 = 9. Hence, Option (d) is correct.

Note: Rem $\left(\frac{(p-1)!}{p}\right) = -1$

21. For every 2.6 m that one walks along the slanting part of the pool, there is a height of 1 m that is gained. Also, since the length of the pool is 48 m we get the following dimensions of the pool.



The pool would look as given in the figure below:



The volume of water in the pool = volume of the upper part + volume of the slanted triangular vessel

$\left(\frac{1}{2} \times 48 \times 20\right) \times 20 + (48 \times 20 \times 1)$

fi 48 × 20 × 11

fi10560 m^3

Hence, Option (d) is the correct answer.

22. The given expression would be equal to: $\log_{100!}2 + \log_{100!}3 + \log_{100!}4 + \ldots + \log_{100!}100$

$= \log_{100!}100! = 1.$

Hence, Option (c) is the correct answer.