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How to Prepare for QUANTITATIVE APTITUDE for the CAT

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QUANTITATIVE APTITUDE *for the*

CAT

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How to Prepare for QUANTITATIVE APTITUDE *for the* CAT

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Dedicated to my Father (Mr MK Sharma), Mother (Mrs Renu Sharma) and Brother (Mr Ravi Sharma)

who have instilled in me the courage to have my convictions

and to stand by them

Preface to the Sixth Edition

With the evolution of the CAT in its online avatar, I felt the need to create a comprehensive and updated book that caters to CAT aspirants. Some of its salient features are given below.

1. Questions from this book: Over the past decade, it has been noticed that a minimum of 10–20% questions in CAT and other major management entrance examinations have been directly taken from the questions provided in this book. Furthermore, it has been seen that between 2003 (when the book was first released) to 2013, 80–90% of the questions in CAT and other top management entrance tests were covered in this book.

The CAT having gone online saw no change in this trend. Many questions in each of the test papers that the CAT has administered in its online avatar since 2009 have been covered in this book.

In fact, 2009 onwards, the onset of the CAT online pattern has created a significant shift in terms of the CAT preparation process. This is because, 2009 was the first year where there were multiple CAT papers to study, analyse and base our writing and preparation process on. In subsequent years, with the increase of the CAT window, the number of papers every year has gone up to around 30–40 papers. Thus, I am now richer by the experience of around 150 plus test papers when it comes to understanding what I need to provide to my readers for their preparation. It is on the basis of this rather rich insight that I have based the changes in this edition. (Note: Similar changes have been incorporated in my other books *How to prepare for Verbal Ability and Reading Comprehension for the CAT, How to Prepare for Data Interpretation for the CAT, and How to Prepare for Logical Reasoning for the CAT).*

2. Not too many changes in the pattern: Looking rationally into the paper patterns of the 150 plus CAT papers in the past period, there have not been too many changes with regard to the pattern of the examination as compared to previous years. While the QA in the initial years of Online CAT examination (between 2009–2010) was slightly on the easier side, the standard of the questions from 2011 onwards has become pretty much ‘CAT standard’. Hence, the LOD I, LOD II and LOD III scheme of questions followed in this book is even more relevant now than ever before.

In fact, in the near future, the CAT is expected to shift to an adaptive format (like the GMAT) where every test-taker would get a different set of questions. In the adaptive format in the future, question banks of varying difficulty levels would be loaded into the computers and questions would appear for the student one-by-one. The difficulty level of the next question would increase if the previous one has been answered correctly and vice versa.

In order to do this, the examiners would need to build a database of questions which would be parallel to the LOD scheme followed in this book. These factors make this book and the content within all the more relevant for CAT aspirants.

How does Merging Quantitative Aptitude & Data Interpretation change the preparation process?

As you must be aware, the CAT 2011 introduced a new pattern shift by reducing the number of sections from 3 to 2. Before CAT 2011, there used to be three sections containing 20 questions each in the CAT exam namely:

Section 1: Quantitative Aptitude

Section 2: Verbal Ability & Reading Comprehension

Section 3: Data Interpretation & Logical reasoning.

In the new pattern the number of sections was reduced to ***2 sections*** *containing 30 questions each, namely*

Section 1: Quantitative Aptitude & Data Interpretation

Section 2: Verbal Ability & Logical reasoning.

In many ways, this does not really change your process of preparation. This is because, even though, on the surface the number of sections was changed, the actual number of questions under each category remained the same. In other words, while the three-section CAT used to have 20 questions on Quantitative Aptitude, 20 questions on Verbal Ability and

Reading Comprehension, 10-12 questions on Data Interpretation and 8-10 questions on Logical Reasoning, the new two-section examination has 20 questions on Quantitative Aptitude and 10 questions on Data Interpretation merged together under one section, while the 20 questions on Verbal Ability and Reading Comprehension and the 10 questions on Logical reasoning were merged together under the other section.

Thus, the emphasis of your preparation does not really need to change at all. The only change that might occur is in the way you may want to strategise to use the time available to you in each section in order to beat your competition.

3. The need for greater variety in your preparation: Prior to the CAT going online, preparing for QA used to be a battle for Blocks 1, 4, 5 and 6. Even out of these, if someone did Blocks I and V well, he stood a strong chance at QA section.

However, as explained in details in the *introductory note* to the *online CAT*, the new avatar of this exam requires the aspirant to be much more balanced in the context of portion coverage.

4. The tougher level of the CAT exam: As already stated above, the quality of questions asked in the CAT over the past couple of years has become extremely good—requiring an upgradation of your grasp of concepts and understanding of each particular topic to a level not required before. This shift has necessitated that we do more through this book.

5. I have also come to know that many readers use this book for their preparation of other important management entrance exams (like XAT, IIFT, CMAT, MAT, SNAP, etc). So now, I have also included/modified the contents so that aspirants of the above exams need not look for any resource beyond this book for strengthening their hold on the quantitative aptitude section.

Apart from management entrance examinations, the book also has relevance for aspirants of UPSC and state civil services, Bank PO exams, GATE, Engineering Placement exams, etc. In short the scope of this book has considerably widened to cover the entire subject of quantitative aptitude that finds a resonance for all career aspirants.

The book you now hold in your hand has always been written keeping in mind the avowed objective of developing your quantitative intelligence to a point where you can quickly scale the height of preparation in each chapter of the portion.

Key features:

1. **Comprehensive solutions** (wherever relevant) to questions in all LODs of all chapters.
2. Based on an assessment of any logic I have missed in any chapter, I have introduced extra questions for readers in the form of **additional block-wise practice tests**. The questions in these tests have been carefully selected to ensure that I do not miss out on any probable question type.

3 In some chapters, where I felt that there is some deficiency in the number and variety of questions (due to the increased difficulty level of the CAT) based on the concepts of the chapter, we have **introduced new questions into the LODs** of the chapter.

4. At some places, the need was felt to introduce an entire additional exercise on concepts of a chapter. This too has been accomplished in this revision.
5. **The training ground**: Perhaps the biggest differentiator in this book is the introduction of the 'Training Ground' – which is an area through which I teach the readers real time higher-end problem solving. The training ground is a section where I tell you exactly how to think inside the examination hall when faced with questions of varying difficulty levels. Hence, a must read for all management aspirants.

Logic of the Training Ground

The quality of the questions in the Quantitative Aptitude section (especially in CAT & XAT) is of such a high quality level that even if you know the basics of each chapter within a particular block, it might not be enough to reach a point where you might be able to solve the questions from the chapter/block. In order to have a grip on any chapter/block and be able to handle application-based questions in the actual examination, you would need to raise your level of thinking and ideation in that chapter/block to the point where you are able to tackle any twists and turns that can be thrown up by it.

For this edition, the training ground has been introduced into four of the major blocks of chapters of this book – and you can expect a very extensive training ground section especially for Block V and Block VI. (Block V covers the chapters on Functions, Inequalities, Logs and Quadratic & Other Equations, while Block VI covers the chapters on Permutations and Combinations, Probability and Set Theory).

6. An introductory write up on the QA section of the online CAT to give you a holistic view of how to approach the online CAT from the perspective of the examinee.

The book is now totally in sync with the new trend and pattern of the examination.

Ultimately the endeavour is to provide a one-stop solution for CAT and MBA exam aspirants to tackle the QA section of all major management entrance exams—an endeavour I feel I have managed to do pretty well.

Through this book, I am confident of giving you—the reader—an invaluable resource for enhancing your QA section score drastically. Contained in this book is the very best advisory for each and every question type. Your job is simple—to ensure that you follow the process contained in this advisory.

KEY POINTS FOR YOUR PREPARATION

Outline and Strategy

The first aspect I would like to deal with here is to focus on helping you with the formulation of your strategy with respect to the portion to be covered for the Quantitative Aptitude section of the various management entrance exams including the CAT, XAT, CMAT, IIFT, and other examinations.

Let us start by trying to understand some of the key areas in Quantitative Aptitude (QA).

Tackling each portion

My experience shows that very often students look at the vast number of chapters and concepts to be studied for QA and get disheartened. This is especially true for students who do not have a strong traditional background in Mathematics. Indeed if you were to look at it with a chapter-wise approach, you can easily define the course to be studied by dividing it into 20+ chapters—preparation for which is such a long-drawn effort that it ends up draining the student's energy enthusiasm and motivation.

It is in this context and for this precise reason that I have divided this book into six manageable blocks—the approach being rationalising the chapters and grouping them according to the amount of shared concepts these chapters have amongst each other.

The outline as defined in the index to this book would divide your work into 6 major areas to prepare for. For your convenience and strategising I have put down the relative importance of each of these six blocks into perspective:

Block I: Number Systems and Progressions

Importance: **Very High** for CAT, XAT, IIFT, FMS & **High** for MAT, CMAT, SNAP, IRMA, etc.

Block II: Averages and Alligations

Importance: **Low** for QA in CAT, XAT, IIFT, but **High** for Data Interpretation as a lot of questions in DI are based on the concepts of averages and alligations. Also **High** for MAT, CMAT, IRMA, NMIMS, etc.

Block III: Percentages, Ratio, Proportion and Variation, Time and Work, Time, Speed and Distance. (Subsidiary but almost redundant chapters in this block – Interest and Profit & Loss)

Importance: **Moderate to High** for QA in CAT, XAT, IIFT, and **Very High** for Data Interpretation (DI) as DI is almost entirely based on the concept of Percentages and Ratio and Proportions. **Very High** for MAT, CMAT, IRMA, NMIMS, etc.

Note: *The chapter on "Time, Speed and Distance" is extremely important for these exams (especially for the CAT as this chapter has been a constant presence in the CAT for almost a decade.)*

Block IV: Geometry, Mensuration and Coordinate Geometry

Importance: **Very High** for CAT, XAT. **Average** for MAT, CMAT, SNAP, IRMA, IIFT, etc.

Block V: Functions, Inequalities, Logs and Quadratic Equations

Importance: **Very High** for CAT, XAT. **Low** for MAT, CMAT, IRMA, NMIMS, etc.

Block VI: Permutations and Combinations, Probability and Set Theory

Importance: **Very High** for CAT, XAT, IIFT etc. **Average** for MAT, CMAT, IRMA etc.

Based on the experience of the online CAT, the strategic preparation imperative for you should be to do at least four blocks and if possible up to 6 blocks "really well".

What does it mean to prepare a block "really well"? This is something I feel needs emphasis here.

Well what I mean to say is that do not just focus on studying the theory in each of these areas but develop an intuitive knowledge of all problem scenarios which emerge out of each block.

Only then would you be able to reach a situation in the exam—that when the question presents itself to you in the exam—you would have had the logic for the same worked out before hand. This is something that can make a huge difference to your chances in the CAT.

Analysing Your Knowledge Level

The first thing you need to focus on is an analysis of your knowledge level in each of these seven parts. In each of the above areas, first analyse your level of knowledge/ability. In order to do so the typical question you should ask yourself is: For the next 100 questions I face in each of these areas, how many would I be able to handle comfortably?

Think of a number as an answer to this question for each of the six blocks.

Based on your answer, the following analysis would provide you a thumb rule which would tell you how much of a knowledge issue you have:

1. 90+: You know pretty much every question type and variant in the area. You should focus your energies on other aspects rather than knowledge improvement in the area.
2. 80+: Maybe you need to increase your exposure to questions a little bit; around 200–300 more questions in that area would be sufficient.
3. 60–80+: You have a significant knowledge issue in the area. You might need to go back to the basics, but it is less likely to be a theory issue but more of an exposure to questions issue.
4. <60: You need to work on both theory and exposure to questions.

Needless to say, the target and objective for preparations has to be to reach the 90+ range as explained above in any block you intend to do "really well".

Looking beyond Ability (Quick Reflexes)

A common frustrating experience for test-takers while taking the test is to not being able to solve a known question/logic and subsequently, not being able to score marks in questions which they knew.

In order to handle this problem, you would need to work on your reactions and reflexes when faced with QA questions. Once you have solved your knowledge/ability issue in a particular block, your next step is to improve your reactions and reflexes while solving a question. Needless to say you would need to do this block wise.

So obviously the main issue is how to improve reflexes and reactions.

(a) For every block, once you have solved the LODs and the Pre-Assessment/Review tests, the most crucial exercise in this context would be a comprehensive revision and review of each and every question you have solved in that block. Solve every question of every LOD and Pre-Assessment/Review test again and review the logic/process of problem solving used. This need to be done to the point where you almost "recollect" the logic of the question and are able to recognise the same if it is used again in a different context/problem.

(b) A thorough revision on the theory of the block.

1. Improve your ability to select what you know and leave what you do not

In the context of an examination where the required scores for 99 percentile would be 60–70% attempts with 100% accuracy, it is easy to see that perfect knowledge is perhaps not needed in order to crack the CAT. Hence, even if you have around 60–70% knowledge of the questions in an average test, you are perhaps good enough to crack the exam. A good way to test whether you have sufficient knowledge would be to pick up 10–20 test papers and divide your QA section into blocks of 5 questions each. Then test your knowledge by looking at the average number of questions you know. If on an average for every 5 QA test questions that you pick up, if you know more than 3, then the prognosis would be that you have adequate knowledge for cracking the CAT. Thus, while you may want to move towards knowing 5 out of 5 in this context, there are other things that you should focus on—developing your ability to decide on whether you are going to be able to solve a question while reading it for the first time. This would help you stop *fishing* during the test. (*Fishing* can be described as the activity of trying to solve a question without knowing whether you would actually complete the question.)

Your mind should give you a clear indication of whether you would be able to do the last step in a question, before you start doing it. In that sense you should be able to clearly define three types of outcomes when you finish reading a question for the first time:

(a) **I see a clear flowchart** and the steps are manageable—Obviously you need to go on and solve these questions.

(b) **I see a clear flowchart** but the steps are too lengthy—In this case you need to see where you stand in your test-time and attempt-wise.

(c) **I do not see a clear flowchart but I can try as I see a starting point**—This is potentially the most dangerous situation for you in the duration of the test, as once you get sucked into a question, there is a strong tendency to lose track of the time you are using up while trying the question. My advice is that while taking the test you should not even start doing such questions.

(d) **I see no flowchart and no starting point to the question**—Obviously you should leave such questions and in fact if these are limited to around 20–30% of the paper there is no problem and you need not worry about them.

2. Focus on thorough knowledge of 'problem scenarios' rather than theoretical learning

To illustrate this, I would like to start with a few examples.

Consider the following string of 3 questions. Before I come to my main point here, I would like you to start by solving these questions before looking at the explanations provided:

1. A boy starts adding consecutive natural numbers starting from 1. After some time he reaches a total of 1000 when he realises that he has double counted a number. Find the number double counted.
2. A boy starts adding consecutive natural numbers starting from 1. He reaches a total of 575 when he realises that he has missed a number. What can be said about the number missed?
3. Find the 288th term of the series: ABBCCCDDDDEEEEEFFFFFFG....

We can now start to look at each of these 3 questions:

1. Consider the fact that when you add numbers as stated above (1+2+3+4+......) the result is known as a triangular number. Hence, numbers like 1, 1+2=3, 1+2+3=6 and so on are triangular numbers. This question asks us to consider the possibility of making the mistake of double counting a number. So instead of 1+2+3+4 if you were to do by error 1+2+3+3+4 you would realise that the number you would get would be 13 which would be more than 10 (which should have been your correct addition) and less than 15 (the sum of 1 to 5) which is the next triangular number. And the double counted value could be achieved by spotting 10 as the immediately lower

value—and the difference between 10 and 13 would give you the required double counted number.

To carry forward this logic into the given question, we should realise that we are just bothered about finding the last triangular number below 1000—and in trying to work this out is where we really apply our intelligence.

Before one writes about that though, one fully realises that a lot of readers (especially aspirants with an engineering background at this point are thinking about $n \times (n + 1)/2$. Knowing that process, one chooses to write about the alternate way to think about in this question.

$$1 + 2 + 3 + 4 ... + 10 = 55;$$

Hence, we can easily see that 11+12+13+14+15+...+20 would equal 155 and the sum of 21 to 30 would equal 255 and so on.

Thus, in trying to find the last triangular number below 1000 we can just do: 55+155+255+355 = 820 (which is the sum of the first 40 natural numbers) and since we have still not reached close to 1000 we start by adding more numbers as: 820 + 41 + 42 + 43 + 44 = 990 and the difference between 990 and 1000 is 10 which is the required answer.

2. For this question we would just need to carry the learning from the previous question forward and realise that when we miss a number, we actually get a total which is lower than the correct total. Hence, if we want to find the number missed all we need to do is to find the first triangular number greater than 575. This can be got simply by 55+155+255+31+32+33+34 = 595, so the number missed has to be 20.

3. In this question all you would need to notice is that in the series ABBCCCDDDDEEEEEF...

A ends after the first term; B ends after the third (1+2) term; C ends after the sixth (1+2+3) term and so on. So we can infer that what we are looking at is how many numbers need to be added before we get to a number just below 288. So 55 + 155+21+22+23 gives us 276 which pretty much means that the 24th alphabet (i.e. x) would be running in this series when we reach the 288th term.

So looking at the three questions above and the solutions, one wants the reader to only answer one specific question:

How much does knowing the first question and developing your thought ability and your intelligence help you in solving the second and the third one? I hope you see the connection. For your information, the three questions presented above were asked in CAT 2001, CAT 2002 and CAT 2003!!!

CONCLUDING NOTE

You sit in front of your CAT question paper and the first question comes in front of you. If you have identified the logic of the question or seen the question itself earlier, your entire QA preparation is fructified. In fact, every question/logic (that you would face in your test) which you have seen earlier represents a triumph of your preparation process. It is for this very reaction that you prepare for an aptitude exam like the CAT. Any other preparation is quite worthless.

Your battle for CAT would be won if you get a “YES I KNOW THIS PATTERN/LOGIC” reaction to 50–60% of the questions in your test.

Contained in this book is the finest collection questions which you would hope to find anywhere. Remember, each question solved needs to be a learning experience—one that is to be kept in your mind for future problem solving. Adopt this approach with the problems contained in this book and I am quite confident that you would KNOW over 50% of your actual CAT test paper since you have already solved something like that before!!

All The BEST !!!!!!

ARUN SHARMA

Preface to the First Edition

Over the last few years, as a trainer of CAT and other aptitude tests, I have felt the need for a comprehensive book on the subject. Students appearing for the CAT and other aptitude tests usually struggle for appropriate study material to prepare for this vital section of the examination.

This book comes as a humble attempt to fulfil this gap.

Structure of the book

The book is divided into 19 chapters and five test papers. Each chapter is divided into three broad parts:

(a) Theory
(b) Solved examples
(c) Chapter end exercises (LODs I, II & III), with answer key

The questions in the chapter end exercises have been categorised into three levels of difficulty, viz, Level of Difficulty I, Level of Difficulty II and Level of Difficulty III.

Level of Difficulty I (LOD I): These are the basic types of questions pertaining to the chapter. A majority of the MBA entrance tests would test the student with LOD I questions. Tests which ask LOD I questions include MAT, IMT, IRMA, IIFT, NIFT, CET Maharashtra, Bank PO examinations, BBA, BCA, Law, and so on. Besides, there are about 10 questions of LOD I type in the CAT nowadays.

Level of Difficulty II (LOD II): These are questions, which are more advanced than the LOD I questions. These questions test all basic as well as applied concepts in the chapter. *LOD II questions are closest to the difficulty levels of the CAT.* Hence, the objective of LOD II questions should be to:

(a) Clearly understand the concept which underlies the question.
(b) Create a judgment of time required for different mental processes.

(c) Identify the time guzzlers.
(d) Reinforce application of a method in mental processes through the question.
(e) Learn to flowchart complex questions.

Level of Difficulty III (LOD III): LOD III questions build on the previous questions and are a step beyond the LOD II questions. Although they are also normally more difficult than the average CAT question, approximately 5–10 LOD III questions could be asked in the CAT every year. Hence, the learning objectives at LOD III are to:

(a) Learn applications of the basic concepts at the highest level.
(b) Sharpen the flowcharting skills learnt at LOD II.
(c) Use each question as a learning opportunity.

One should not be disheartened if he/she is unable to solve LOD III questions. These questions are extremely tough and uncommon in the CAT and other aptitude tests. Questions in actual tests will appear very simple and elementary if one can solve LOD III questions.

Approach Taken in Writing This Book

In my experience, the 'math skill' of students appearing for CAT can be classified into three levels:

Level 1: Students who are weak at Mathematics

Level 2: Students who are average at Mathematics

Level 3: Students who are strong at Mathematics

This book has been written keeping in mind all the three kinds of students.

From my experience I have given below my perspective of what one should aim for (based on the category that he/she belongs to). It is important to clearly understand the starting level and accordingly define strategy for the QA section.

Level 1: Students who are weak at mathematics: Typically, these are students who were weak at mathematics in school and/or have left mathematics after their 10th or 12th class. They face a mental block in mathematics and have problems in writing equations. They also have severe problems in understanding mathematical language and are unable to convert the mathematical language into mathematical equations. They make mistakes even in interpretation of the most basic statements in mathematics (leave alone the complex statements). Besides, these students also have problems in solving equations. They suffer from the insecurity of knowing that they are unable to solve most problems which they face.

Level 2: Students who are average at mathematics: These students lie between the Level 1 and Level 3 students.

Level 3: Students who are strong at mathematics: These are the students who have got strong, structured and logical thinking ability. They not only understand the basic repetitive statements in mathematics but also complex statements. They are able to create their own flowcharts to arrive at solutions of these complex mathematical situations.

There are two alternative approaches that a student can take in solving this book.

Approach 1: “Start with basic concepts, solved examples then move on to LOD I, then LOD II in the chapter. Do not go into LOD III in the chapter in the first go. Complete all 19 chapters and then re-start with Chapter1 – review the basic concepts, resolve LOD I and LOD II, then move on to LOD III. This approach is advocated for students who are weak to average in mathematics (i.e. students of Level 1 and Level 2).

After completing the theory and practice exercises of the book for a second time, go to the practice sets 1–5 provided at the end of book. Set a time limit of 40 minutes for each set and take the tests. The questions contained in the sets are questions which have appeared in the CAT over the last 5 years (based on memory).”

Approach 2: “Start with the basic concepts, solved examples and then go through the exercises of LOD I, LOD II and LOD III. This is recommended for students who have strong concepts in mathematics (Level 3 students).”

Then go to the 5 practice tests given at the end of the book and take them one by one (time limit of 40 minutes for each test).”

An Important Point

Each of the questions contained in the LOD I, LOD II and LOD III exercises in the chapters have immense learning value. Hence, the approach that one takes while solving the questions should be one of learning. The reader should try to clearly understand the interpretation of each sentence used in the construction of the questions.

In other words the learning in every chapter should not be restricted to the solved examples or the theory contained in the chapter, but should continue through each of the questions contained in the exercises.

In conclusion, this is a book which is unique in approach and coverage. Any CAT aspirant who goes through the questions contained in this book in the manner advised in this book would get a distinct advantage when he/she faces the CAT.

ARUN SHARMA

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The text and questions contained in this book is the manifestation of a learning process that began for me over 18 years ago. During the intervening period I have come across some wonderful people in the form of relatives, friends or acquaintances from whom I have learnt immensely. While it is not possible to name them individually, I would like to express a deep sense of gratitude towards them.

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Arun Sharma

Online CAT: From the QA Perspective

Welcome to the world of online CAT!

The advent of the online version of the Common Admission Test (CAT) in 2009 and beyond brought with it a whole lot of opinions and views about what has changed in the examination and what should be the ideal preparation pattern. Therefore, one objective in this revised edition of this widely read book is to look at the issues that an aspirant needs to consider while preparing for the online CAT. I would like to discuss this issue in the following parts:

1. *What has changed?* A comprehensive analysis of what are the critical dimensions of the changes that have taken place in the CAT in its online avatar and what it means for the aspirant, both in respect of positive and negative factors, taking into account the following:
   (a) Changes in the Test-Taking Experience
   (b) Changes in the Exam pattern
   (c) Changes in the Marking process
2. *What does all this mean for the Preparation Process?* How it should change in the context of an online examination and how has it remained constant- whether online or paper-and-pen?

While doing so I have taken the help of a varied experiential sample of test-takers from across India and also my own personal experience of taking (and may I add dominating) the CAT. Given below are some of the implications of the online version of the CAT in the context of the Quantitative Aptitude section (which this book is all about).

Note: In this book, whenever I refer to preparation for the Quantitative Aptitude Section, I am referring to the preparation for the approximately 20 questions on Quantitative Aptitude, that are being asked under the Quantitative Aptitude and Data Interpretation sections of the CAT.

I. WHAT HAS CHANGED?

The 'experience' of taking the test

1. Cleaner & More Efficient: Compared to the paper-and-pen based CAT, the online version is much cleaner as the clarity of questions, their visibility, as well as the overall feel of the question solving experience is much better. Consequently, the efficiency (of the thought processes) is much higher, leading to a much superior test-solving experience.

2. Space Management on the Table: In the paper-and-pen version, the aspirant had to typically manage the test paper, admit card, watch, pencils (at least 2), eraser, sharpener as well as the answer sheet on the table. To add to their woes, the paper-and-pen versions of the exam were mostly conducted in schools. Very often the aspirants had to contend with the additional challenge of managing all this paraphernalia on a school boy's small table. In addition, if luck did not run your way and you were made to sit in a classroom meant for juniors, (between classes 3 to 6) you really had a challenge.

Most of these problems have disappeared in the new version. The fact that computer terminals at most colleges and universities are of standard shape and size eliminates the imbalance created due to non-uniformity. Besides, while writing the online version of the CAT, all you need to manage on the table are the mouse, the key board, a pencil and a sheet of paper for rough work; no watches, erasers, sharpeners and most importantly, no test paper and no answer sheet.

3. Moving Questions in the Test: Unlike the paper-and-pen version, where test-takers could scan the whole question paper in one look, in the online CAT, aspirants had to move one question at a time. This had both its advantages and disadvantages in terms of the overall test experience. The obvious disadvantage that most aspirants faced was the fact that since you could not really see the whole paper in one look, you could not make a judgment about the balance, the difficulty level or the portion wise question distribution in the paper. (Although I am referring to the quantitative aptitude section here, this was also true for all the sections in the exam)

Ironically, the biggest advantage for the examinee in terms of the online CAT was exactly the same i.e. since you could not see the paper entirely at one go, the only option while taking the test was to look at the questions one by one.

This turned out to be a huge advantage because of two main reasons mentioned below:

Higher Focus while Solving an Individual Question: Not knowing the exact number of questions from various areas and not being able to estimate the difficulty level of the paper, left individuals with no choice but to focus on the one question that was visible to them on the screen. The result was that achieving the all important ‘tunnel vision’ while solving a question was much easier. The immediate result of this was that the focus on the `problem at hand’ was infinitely more in the online version than in the conventional paper-and-pen format. Thus, ironically, not knowing the pattern of the paper resulted in giving examinees their best chance to solve a question.

The main reason for this was that while solving the question in front of the computer screen the experience of the previous question was totally blanked out. In the paper-and-pen version, students who had a negative experience while solving a question or two carried that negativity to the next question.

Thus the specific advantage of the online version was that “forgetting” a bad experience was relatively easier. The moment you navigated away from the question in front of you, it went away from your mind as well. So much so, that remembering a question that was just two questions back was close to impossible. Naturally the ‘carry over’ emotions from a previous negative experience were significantly reduced.

The Imperative for Faster Navigation (less time wasted on unsolvable questions): Since the examinees had not seen the full question paper right at the beginning, the imperative to move to the next question was extremely strong. This resulted in students seeing a higher percentage of the questions in the online test than in the paper-and-pen version.

Author's Note: One of the problems I had noticed in the paper-and-pen version was that most examinees were not able to 'see' the entire paper. i.e. the fraction of the quantitative aptitude section that they were able to process was a fraction of the entire test paper. As a result they used to miss out on a large number of sitters! On an average, out of a 5-page question paper in quantitative aptitude, students were able to process at most upto 2–3 pages. So they would naturally miss out on all easy ones on the pages they did not process. A lot of time would get wasted in questions that they tried and were unable to solve or even if they solved, they were unable to get them correct.

Part of this time mismanagement also occurred due to the fact that they did not have the clock ticking on the screen in front of them. Therefore, they naturally lost track of how much time they had spent in attempting to solve a question. A good percentage of the time the aspirants used to spend in the QA section was spent in trying to solve a question which they were eventually unable to solve.

All this changed for the better in the online version. There was a greater imperative to move to the next question due to the twin facts that you had not seen the entire paper as you were moving from one question to the other, and that the ticking clock was omnipresent in front of your eyes on the screen. As a result, you were aware of the exact amount of time you had spent on a particular question. The net result was that after trying a question for maybe 60 to 90 seconds, in case you did not have a clue about what to do in the same, you moved to the next question. Thus time management improved drastically for the examinee.

I believe this is one of the main reasons why a lot of students who were trying to compare the two versions of the CAT said that the online version was easier. Since the amount of time spent in questions which they were eventually not able to solve, reduced drastically, they got a feeling that they were solving questions all the time as opposed to the paper-and-pen version where aspirants used to have an overall negative experience of the test (as they would end up spending a lot of time in attempting "unsolvable" questions.

4. Mark/Unmark Button & the Review Button: A very important feature in the online version was the introduction of the REVIEW button. In the paper-and-pen version, it was extremely difficult to track the number of your attempts and especially so in the context of questions that you were unsure about and/or questions which you wanted to come back to. There was simply no way in which you could keep a track of those and as a result there was effectively 'no second chance' at a question.

This too changed in the online CAT. For every question, apart from the facility to answer it, you also had a MARK button, which would give you easy access to the question at the end of the paper. When you have completed the paper (reached the last question in the paper), you also got access to a review screen that in one visual showed you all the questions you had solved as well as all the questions you had marked with the MARK button. So going back to a specific question in the paper was just the click of the mouse away.

To sum up, the net effect of the online CAT was a superior test-taking experience — something that gives you a chance to be more in control of your test— and thus aim for a higher score assuming that the same set of questions would have been asked in the paper-and-pen version.

What has changed in terms of the exam pattern?

Having seen the specific changes that have occurred in terms of the test-taking experience, let us now examine another crucial aspect.

Changes in Exam Pattern: Obviously for the purpose of this book, the analysis will pertain to the QA portion only. In order to read a similar analysis with respect to the other sections namely, *Verbal Ability* & *Logical Reasoning* you can refer to my book on these subjects, also published by McGraw Hill. The major changes in the pattern of the Quantitative Aptitude paper can be summarised through the following points:

1. More balanced portion coverage
2. Reduction in number of questions
3. Lack of uniformity
4. Higher percentage cutoffs

1. More Balanced Portion Coverage: As per the scheme followed in this book, the QA portion can be divided into 6 major parts (or blocks as I call them in this book).

The underlying constant that used to exist in the paper-and-pen version (through the entire decade prior to the first online CAT) was the prominence of Block I and Block V. (Block I comprising Number Systems and Progressions and Block V comprising the chapters on Functions, Inequalities, Quadratic and other Equations and Logarithms.)

In each of the years from 1999 to 2008, the QA section required you to get a net score of approximately 30 – 40% of the total marks in order to score a high 90 percentile in this section.

In the light of this fact, the importance of Block I and Block V can be gauged from the table below:

Block	***Weightage (as a % of total marks)***
Block I	30 – 50%
Block V	15 – 50%
Combined weightage of Blocks I & V	60 – 80%

Add to this, the chapter on Time, Speed and Distance with a minimum weightage of 5–10% and you pretty much had the QA section well covered. In a nutshell, QA for CAT preparation had become “do 10 chapters well”.

However, this scenario has changed in the context of the online version of the exam.

The balance of weightage of questions shifted and each of the six blocks have become important. The aspirant of CAT online version saw a weightage distribution of the kind illustrated below.

Block	***Total Out of 20 Questions***
Block I	3 – 4 questions
Blocks II & III	3 – 5 questions
Block IV	3 – 6 questions
Block V	4 – 5 questions
Block VI	1 – 3 questions

2. Reduction in number of Questions: The second major change in the QA section is the reduction of questions to 20. From 55 questions in the late nineties to 50 between 2000 to 2003 to 30 & then 25 in the last years of the paper-and-pen version, the number of questions has further gone down to 20 in the online version. Naturally, this reduced the amount of choice the aspirant had for leaving out a question.

For instance in CAT 2003 out of 50 questions, you needed to solve 15 to get to the cut off. This meant that at 100% accuracy you could afford to leave 35 questions. This scenario has now changed drastically as in evident from the following table.

Year	***No. of Questions in QA***	***Number of Marks***	***Cut off at (approx number of marks)***	***No. of Questions you could leave @ 100% accuracy***	***No. of Questions you could leave @ 90% accuracy***
CAT 1999	55	55	16-18	37+	32+
CAT 2000–04	50	50	12-14	36+	32+
CAT 2005	30	50	12-14	20+	16+
2006–08	25	100	28–32	17+	14+
Online CAT 2009 & 2010	20	80	40–48	8+	5+
Online CAT 2011–2013	20 (QA) + 10 (DI)	80 (QA) + 40 (DI)	68–72	12+	8+

QA &
DI
Section

- As you can see, there is very little elbowroom available in the online version now to leave out questions and expect a good percentile score.
- The expectation in the future is that students taking the CAT would have to really use their mathematical intelligence and attempt as many questions as possible in order to get a top percentile in the test.

3. Lack of uniformity: The third major factor in terms of paper pattern was the lack of uniformity of the test paper. Different students got tests with differing difficulty levels. The papers on the first few days of CAT 2009 were quite simple, but after the third day most papers had a pretty good difficulty level.

An issue that is being discussed widely on the Net is fairness. A lot of voices rose against the CAT committee and the online version of the exam questioning the fairness of the testing process.

The key criticism was: In the context of multiple papers with varying difficulty levels, how would the IIMs judge fairly between students who solved a high percentage of the questions in an easy test versus students who were able to solve a lower number of questions in a more difficult paper? The answer to this is really simple. Since the population size of each paper was significantly large, the IIMs could easily define individual percentiles in each test and ensure fairness to all.

The key point to be noted here is that there are infinite statistical ways through which processes like this can be made fair to everyone. As a future CAT aspirant, however, what you need to worry about is preparing diligently and facing the exam with a positive attitude.

4. Higher percentage cutoffs: In the online version, aspirants wasted less time in questions which they thought was unsolvable and moved on to those they could solve. The result— most students were able to raise their scores in this section significantly.

Consequently sectional cut offs which used to be in the range of 30% of the net marks rose to around 40 – 45% of the marks.

Changes in the Marking process

The key change that an analysis of CAT 2009 results showed was that there was an increased emphasis on accuracy. Mistakes were heavily penalised. This was evident from the fact that two students solving the same test paper (December 3 evening slot) scored:

(a) 14 attempts 1 incorrect – score 98.23 percentile

(b) 19 attempts 3 incorrect – score 92.6 percentile

There were innumerable such examples where students solving more questions with higher errors scored significantly lower than students who attempted much less but got most correct.

Hence, the key learning for you while preparing is to focus on improving your accuracy as well as the belief in your process of solving. This is especially true while preparing for the QA section. While solving a QA question, you should be able to know that if your process is correct then so would your answer would also be correct. The need to check the answer to a QA question is something that is only required for minds weak in Quantitative Aptitude. This is where an under-prepared aspirant loses out to the best—in the knowledge of whether what they are solving is correct or not.

Unfortunately, most students I see are more interested in seeing the answer to the question as soon as they solve the same. This is a habit I would strongly discourage you from. The ideal preparation process for you should be:

(a) solve the question,

(b) review your process and tell yourself, “if your process is correct, so is your answer”, and

(c) only check your answer after you have reviewed your process.

This is important, because when you are solving a QA question inside the CAT, you would not have the cushion to ‘look’ at the answer. The only thing you have is the question and the process you use in solving the same.

Your mind should be able to tell you whether the answer you have got is correct or not. This is a key difference in solving questions in practice and solving them under exam pressure.

Hence, developing more confidence in your QA problem-solving processes becomes a key ingredient and objective of your preparation process for this section.

II. WHAT DOES ALL THIS MEAN FOR THE PREPARATION PROCESS? HOW HAS IT CHANGED AND HOW HAS IT REMAINED CONSTANT?

Let us look at this aspect in two broad parts:

1. What are the changes that need to happen in the preparation processes for the online CAT vis-a-vis the preparation process for the traditional paper-and-pen version?
2. What are the things and issues that remain constant in the preparation process?

1. For the first question, the specific things that come to my mind are:
   - **(a) More Balanced portion coverage needed:** As explained above, in the paper-and-pen version the best approach for Quantitative Aptitude preparation was “do 10 chapters well — really well”. In fact, even 4 chapters done well were mostly sufficient to crack this section. However, in the new online version, since the weightage of distribution of questions is much more even, this approach is no longer going to work. Hence, the need to cover all aspects of the portion well and not ignore any particular portion is perhaps the first and the biggest change that needs to be done in the preparation process.
   - **(b) Need to cover the basics well, namely, speedily solving LOD I questions and the ability to think through LOD II and LOD III questions:** In the early years (1980s and upto the late 1990s), the CAT used to be essentially a speed test (including the QA section). There were times when the paper used to

consist of upto 225+ questions to be solved in 120 minutes. Questions used to be one-liners and could be solved in 1–2 steps. The key differentiation used to be the speed at which the aspirants could solve questions. However, from late 1990s onwards the QA section of the CAT had become a real test of quantitative intelligence. Questions ceased to be one-liners unless you had a very high degree of mathematical understanding and intelligence. The online CAT in its first year tended to be a mix of both these extremes. Papers consisted of between 4–6 one-liners topped up by LOD II and LOD III questions. So while most aspirants found 4–6 very easy questions in each paper, they also had to really use their mathematical strengths to cross 10–12 attempts. In the future, as the IIMs improve the quality of the database of quantitative questions, one can expect the quality of the questions to improve drastically and hence the LOD II and LOD III questions contained in this book would be an extremely important resource to solve for maximising your score in this section in the exam. [In fact after the first 3–4 days of the exam in 2009, examinees taking the test on the subsequent days found the paper to be of really good quality].

For the future CAT aspirants and the readers of my books, my advice is short and simple. Cover both the flanks—the short cuts and quicker methods to solve the easier LOD I questions *and* improving your mathematical and logical intelligence to cover the higher end questions of LOD II and LOD III level.

(c) **The need to take computer based tests in order to be able to think on the computer:** Thinking and solving questions from the computer screen is a slightly different experience than solving from a physical book. Thus students and aspirants are advised to experience this change by going for online solving experience. It is in this context that we have tied up with www.mindworkzz.in to give our readers a feel of the online problem solving experience. However, in spite of these seemingly big external changes, my personal opinion is that the changes are mostly external in nature.

2. The essence of preparation of the Quantitative Aptitude section remains the same in a lot many fundamental ways. Some of these that come readily to mind are:

   (a) **The need to develop mental structures for the CAT:** QA preparation has always been associated with the development of the mathematical thinking processes and thought structures for specific mathematical situations. The smart CAT aspirant is able to create mathematical thoughts in his mind to situations that he would encounter in the exam.

   The whole battle for QA preparation in the CAT can be essentially summarised in terms of the quality of pre formed logic that you have to the specific questions that you are going to face in the exam. In other words, your battle in QA section is won if you have during your preparation process, encountered the logic to the question which is in front of you. Hence, the focus of your QA preparation process has to be on creating the logics for as many questions and mathematical situations as possible. You would have well and truly won your battle at the CAT in case you encounter 15 'known' mathematical situations out of the 20 questions in your set. Hence, the **imperative to form "thought algorithms" for standard and non-standard mathematical situations related to various chapters and concepts in the portion remains as strong as ever.** In fact, if anything this imperative is expanded to the entire portion base due to the wider and more balanced portion coverage in the exam.

   (b) **The need for thoroughness in your preparation:** This is again something that does not change.

   The key point you need to remember is that the CAT still remains a test of your intelligence and an aspirant should focus on this aspect. This book provides plenty of mathematical thinking situations and alternatives related to each and every part of this section that help you hone your skills in the QA section of the examination.

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Block IV: Geometry

Block V: Algebra

Block VI: Counting

FIRST THINGS FIRST

DEVELOPING YOUR CALCULATIONS

This special section contains the best available approaches for all kind of calculations that you are likely to face in the CAT or any other aptitude examination...

Chapter 1 Additions and Subtractions (As an Extension of Additions)

Chapter 2 Multiplications

Chapter 3 Divisions, Percentage Calculations and Ratio Comparisons

Chapter 4 Squares and Cubes of Numbers

Developing your ability to calculate well should be one of your major thrust areas for your preparation strategy for Quantitative Aptitude. In fact, most of the times (in most coaching programs and books) this area is totally bypassed leaving the student broadly to develop his/her own methods to calculate faster. Needless to say, your work in trying to develop your ability to calculate would always be greatly superior if you are guided properly with approaches that have been tested and have stood the test of time. The following advisory contained in this special section of the book not just aims to give you the best advice for each and every type of calculation, but also gives you a comprehensive plan to develop your calculation speed—for every conceivable type of calculation.

My focus throughout this special note on calculations is to help you to develop the relevant calculations only, viz., calculations that you are likely to encounter inside the CAT based on the experiences of the past CAT examinations. For this purpose this section has been divided into the following chapters:

Chapter 1: ADDITIONS and SUBTRACTIONS *Ideas for developing your ability to add & subtract well;*

Chapter 2: MULTIPLICATIONS *Ideas for developing your ability to multiply well;*

Chapter 3: DIVISIONS, PERCENTAGE CALCULATIONS and RATIO COMPARISONS *Ideas for developing your ability to divide well as well as to compare ratios more efficiently;*

Chapter 4: SQUARES AND CUBES OF NUMBERS

Chapter 1
Additions and Subtractions (As an Extension of Additions)

IDEAS FOR ADDING AND SUBTRACTING WELL

Addition is perhaps the most critical skill when it comes to developing your calculations. As you would see through the discussions in the remaining chapters of this section of the book, if you have the ability to add well you would be able to handle all the other kinds of calculations with consummate ease.

Skill 1 for addition: The ability to react with the addition of two numbers when you see them.

The first and foremost skill in the development of your addition abilities is the ability to react to 2 two digit numbers when you come across them. You simply have to develop the ability to react with their totals whenever you come across 2 two digit numbers.

For instance, suppose I were to give you two numbers at random—5, 7 and ask you to **STOP!! STOP YOUR MIND BEFORE IT GIVES YOU THE SUM OF THESE TWO NUMBERS!!** What happened? Were you able to stop your mind from saying 12? No! of course not you would say.

TRY AGAIN: 12 + 7 STOP YOUR MIND!! You could not do it again!!

TRY AGAIN: 15+12 STOP!! Could not?

TRY AGAIN: 88+ 73 = ?? STOP!! If you belong to the normal category of what I call “addition disabled aspirants” you did not even start, did you?

TRY AGAIN: 57 + 95 =??

TRY AGAIN: 78+88 =??

What went wrong? You are not used to such big numbers, you would say. Well, if you are serious about your ability to crack aptitude exams, you better make this start to happen in your mind. You would know what I mean if you just try to look at a 5 year old child who has just learnt to add, struggle with a calculation like 12 + 7 on his fingers or his abacus.

His struggle with something like 12 + 7 or even 15 + 12 would be akin to the average aspirant's ability to react to 88+ 78. However, just as you know 15 + 12 is not a special skill so also 88+78 is not a special skill. It is just a function of how much you practice your calculations especially in the domain of 2 digit additions.

So what am I trying to tell you here?

All I am trying to communicate to you is to tell you to work on developing your ability to react to 2 two digit numbers with their addition as soon as these numbers hit your mind. What I am trying to tell you that the moment you make your mind adept at saying something like 74 + 87 = 161 just the way you would do 9 + 6 =15 you would have made a significant movement in your mind's ability to crack aptitude exams.

Why do I say that—you might be justified in asking me at this point of time? In order to answer your question I would like to present the following argument to you:

In numerical questions, a normal student/aspirant would be roughly calculating for approximately 50% of the time that he/she takes to solve a question. This means that half the total time that you would spend in solving questions of basic numeracy or data interpretation would essentially go into calculations. Thus, if the test paper consists of say 30-40% questions on basic numeracy and data interpretation you would be expected to spend somewhere between 36 to 48 minutes on these questions—which would in turn translate to approximately 18 to 24 minutes in calculations inside the test paper.

So the contention is this: If you can improve your calculation speed to 5x, the time you would require to do the same calculations would come down to 1/5th of your original time. In other words, 18 minutes would come down to 3.6 minutes—a saving of 14.4 minutes; 24 minutes would come down to 4.8 minutes—a saving of 19.2 minutes just by improving your calculation speed!!

In an exam like the CAT where you would always run out of time (rather than running out of solvable questions) 19 extra minutes could easily mean anywhere between 15–20 marks (even if you able to solve an additional 6–7 questions in this time).

15–20 extra marks in the prelims exam could very well make the difference between getting a chance to write the mains examination in the same year versus going back to the drawing board and preparing for the prelims for another year!!

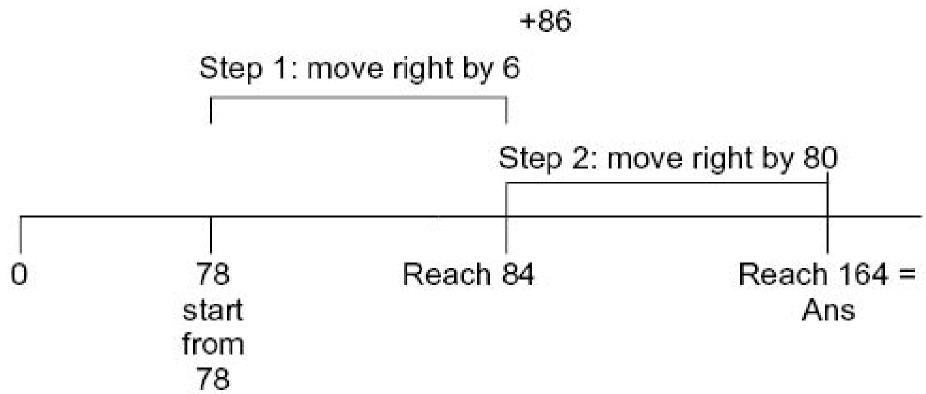
Addition being the mother of all calculations has the potential of giving you the extra edge you require to dominate this all important examination.

Over the next few chapters in this section of the book, all I am going to show you is how knowing additions well would have an impact on each and every calculation type that you might encounter in this exam and indeed for all aptitude tests. However, before we go that far you need to develop your ability to add well.

Let us look at the simple calculation of 78 + 88. For eternity you have been constrained to doing this as follows using the carry over method:

$$\begin{array}{r} {}^{1} \\ 7\ 8 \\ +\ 8\ 6 \\ \hline 1\ 6\ 4 \end{array}$$

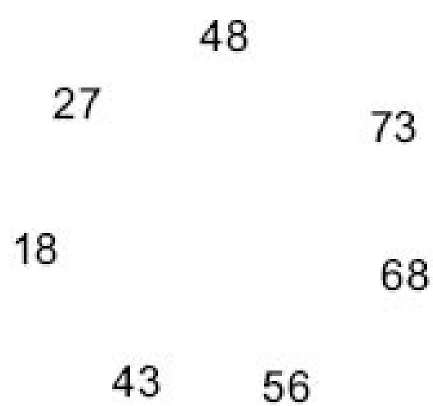
The problem with this thought is that no matter how many times you practice this process you would still be required to write it down. The other option of doing this same addition is to think on the number line as this:



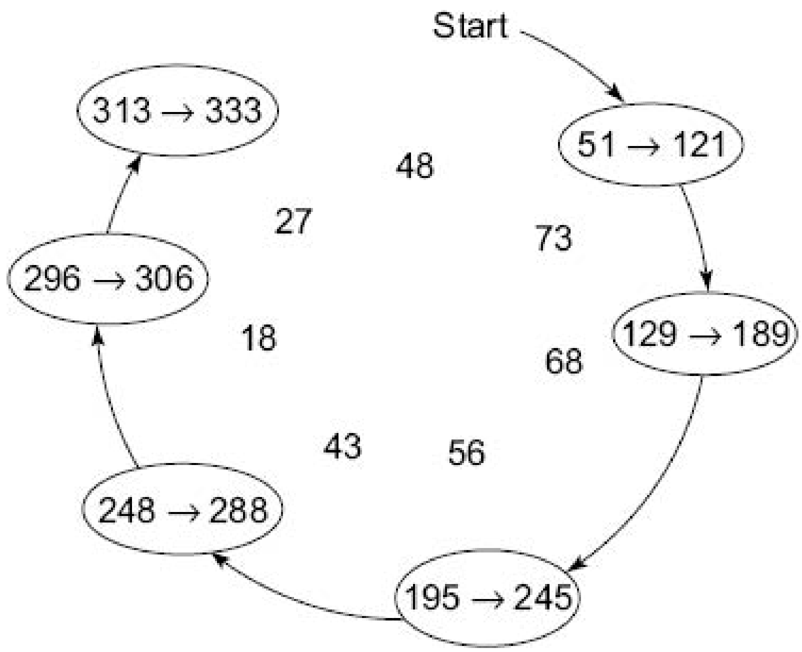
78 + 86 – how to think of this addition problem

As you can see, the above thinking in an addition situation requires no carry over and after some practice would require no writing at all. It is just an extension of how you are able to naturally react to 5+11 so also you can train your mind to react to 58+63 and react with a two step thought (as 61 Æ 121—with practice this can be done inside a fraction of a second. It is just a matter of how much you are willing to push your mind for this). Once you can do that your next target is to be able to add multiple 2 digit numbers written randomly on a single page:

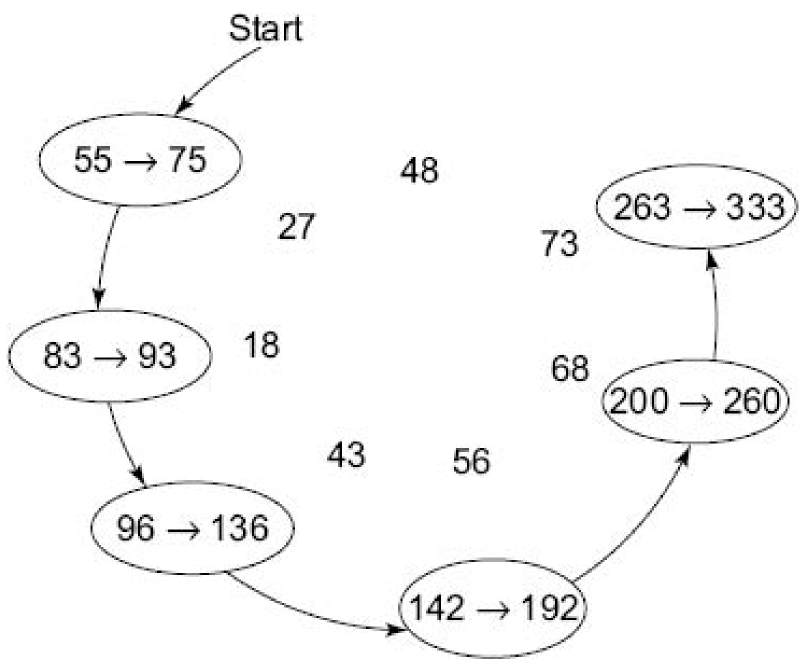
Try this: Add the following



In order to do this addition your thinking should go like this:



Alternately you may also do this the other way. The result would be quite the same:



While you are trying to work on this addition you would realize the following about your abilities to add (if you belong to the normal category of aspirants')

1. Something like 121 + 68 would be easier than 189 + 56 because the latter requires you to shift hundreds— something that the former

does not require you to do.

2. Something like 48 + 27 would be easier for you to do initially than 136 + 56; and 136 + 56 would be easier than 543 + 48 because your mind would be more comfortable with smaller numbers than you would be with larger numbers.

However as you start practicing your additions, these additions would become automatic for your mind—as they would then fall into the range where your mind can react with the answers. That is the point to which we would want you to target your skill levels for additions.

To put it in other terms, you would need to work on your additions in such a way that 10 numbers written around a circle (as shown above) should be done in around 10-12 seconds in your mind.

Till the time your addition skill levels reach that point, I would want you to work aggressively on your addition ability.

The following 10 × 10 table done at least once daily might be a good way to work on your additions:

	59	*68*	*77*	*96*	*84*	*32*	*17*	*69*	*81*	*38*	*TOTALS*
48											
54				= 96 + 54 = 150							
67											
89											
56											
73						105					
88											
24											
47											
96											
TOTALS											

Inside the table you would broadly do two things:

(a) For each cell you would add the values in the corresponding row and the corresponding column in order to get the value inside the cell. Thus, the second row and 4^{th} column intersection would give you 54+96=150, the sixth row and the sixth column would add to 73+32 = 105 as shown in the table.

(b) Add the total of the 10 numbers seen in each row after you finish doing the values inside the cells in the total. This would give you the final total of the row. Repeat the same process for the addition of the 10 numbers in the columns.

By this time, I guess you would have realized that we are targeting two broad addition skills—

(i) Your ability to react with the total when you see two 2 digit numbers (like 57+78=135)

(ii) Your ability to add multiple 2 digit numbers if they are given to you consecutively (like 57+78+43+65+91+38+44+18+64+72=570 in 8–10 seconds)

You might require around 1–2 months of regular practice to get proficient at this. However once you acquire this skill, every conceivable calculation that any aptitude exam can throw at you (or indeed has thrown at you over the past 20 years) would be very much within your zone.

How do you do larger additions?

One you have the skills to handle two digit additions as specified above handling bigger additions should be a cakewalk.

Suppose you were adding:

57436 + 64123 + 44586 + 78304 + 84653 + 5836. In order to do this, first add the thousands. 57 + 64 (=121) + 44 (=165) + 78 (=243) + 84 (=327) + 5 (=332). Thus, you have an interim answer of 332 thousands. At this stage you know that your answer would be 332000 + a maximum of 6000 (as there are 6 numbers whose last 3 digits you have neglected). If a range of 332000 to 338000 suffices for you in the addition based on the closeness of the options, you would be through with your calculation at this point. In the event that you need to get to a closer answer than this, the next step would involve taking the 100s digit into account.

Thus for the above calculation: 57**4**36+ 64**1**23 +44**5**86+78**3**04+84**6**53 +5**8**36 when you add the hundreds, you get 4+1+5+3+6+8 = 27 hundreds. Your answer gets refined to 334700 and at this point you also know that the upper limit of the addition has to be a maximum of 600 more than 334700 i.e. the answer lies between 334700 to 335300. In case this accuracy level is still not sufficient you may then look at the last 2 digits of the numbers. Our experience tells us that normally that would not be required.

However, in case you still need to add these digits-it would amount to 2 digit additions again. So you would need to add 574**36**+641**23**+445**86**+783**04**+846**53** +58**36** Æ **36+23 (=59) +86 (=145) + 4 (=149) + 53 (=202) +36 (=238).**

Thus, the correct total would be 334700 + 238 = 334938 and while doing this entire calculation we have not gone above 2 digit additions anywhere.

Apart from that, the biggest advantage of the process explained above is that in this process, you could stop the moment you had an answer that was sufficient in the context of the provided options.

SUBTRACTIONS—JUST AN EXTENSION OF ADDITIONS

The better your additions are, the better you would be able to implement the process explained for subtractions. So, a piece of advice from me—make sure that you have worked on your additions seriously for at least 15 days before you attempt to internalize the process for subtractions that is explained in this chapter.

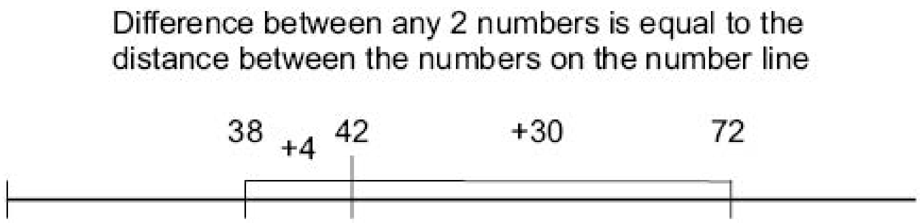
Throughout school you have always used the conventional carry over method of subtracting. But, I am here to show you that you have an option —something that would be much faster and much more superior to the current process you are using. What is it you would ask me? Well what would you do in case you are trying to subtract 38 from 72?

The conventional process tells us to do this as:

	Carry over 1
7	2

–	3	8
	3	4

Well, the alternative and much faster way of thinking about subtractions is shown on the number line below:



The principle used for doing subtractions this way is that the difference between any two numbers can be seen as the distance between them on the number line.

Thus, imagine you are standing on the number 38 on the number line and you are looking towards 72. To make your calculation easy, your first target has to be to reach a number ending with 2. When you start to move to the right from 38, the first number you see that ends in 2 is the number 42. To move from 38 to 42 you need to cover a distance of +4 (as shown in the figure).Once you are at 42, your next target is to move from 42 to 72. The distance between 42 to 72 is 30.

Thus, the subtraction's value for the numbers 72–38 would be 34.

Consider, the following examples:

Illustration 1 95 – 39

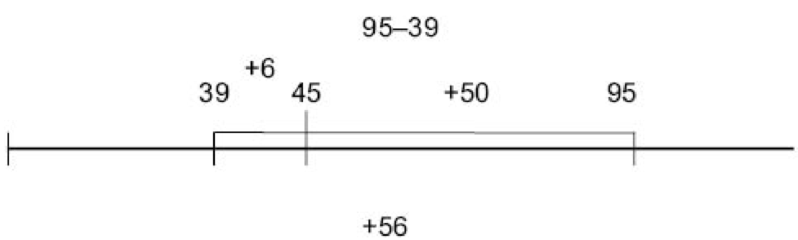
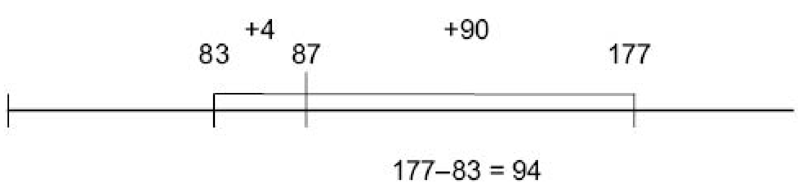


Illustration 2 177 – 83



Alternately:

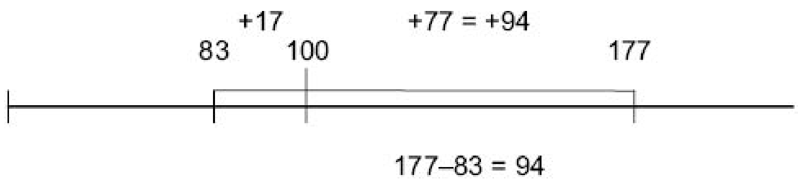
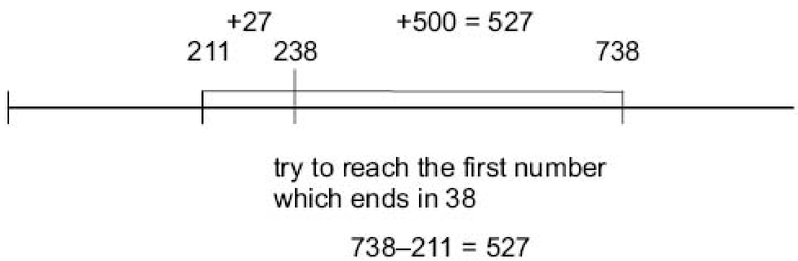


Illustration 3 738 – 211



In this case the first objective is to reach the first number ending in 38 as you start moving to the right of 211. The first such number to the right of 211 being 238, first reach 238 (by adding 27 to 211) and then move from 238 to 738 (adding 500 to 238 to reach 738)

In case you need an intermediate number before reaching 238 you can also think of doing the following:

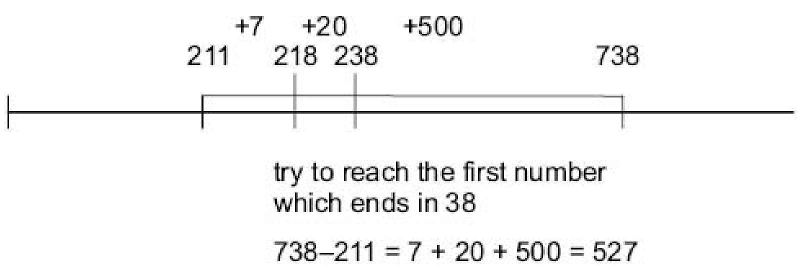
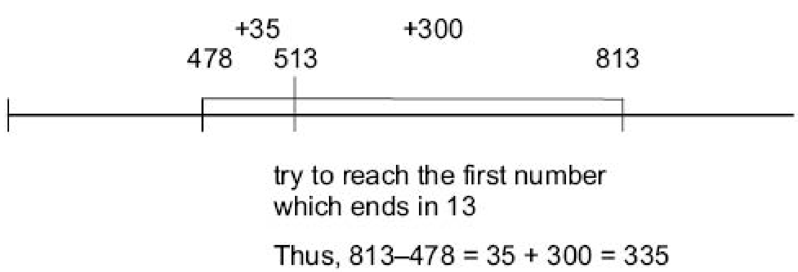
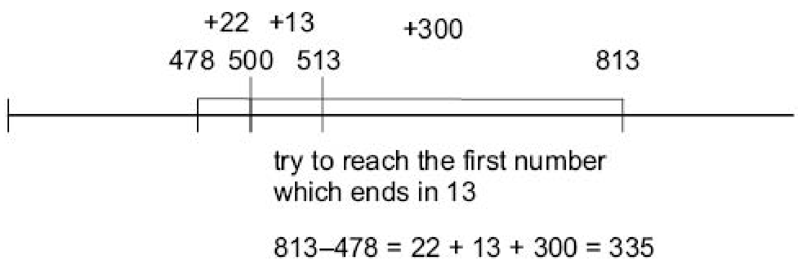


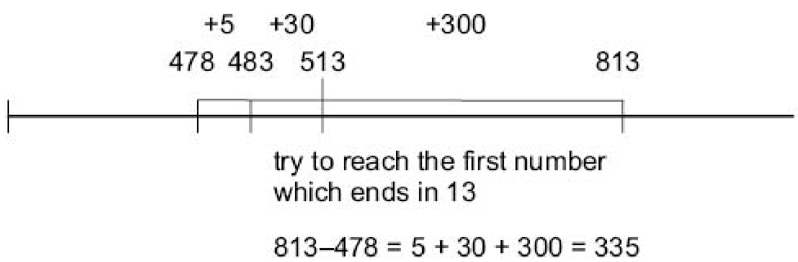
Illustration 4 813 – 478



Alternately, this thought can also be done as:



Also, you could have done it as follows:



Even if we were to get 4 digit numbers, you would still be able to use this process quite easily.

2 Chapter Multiplications

Multiplications are the next calculation which we need to look at—these are obviously crucial because most questions in Mathematics do involve multiplications.

The fundamental view of multiplication is essentially that when we need to add a certain number consecutively—say we want to add the number 17 seven times:

i.e. 17 + 17 + 17 + 17 + 17 + 17 + 17 it can also be more conveniently done by using 17 × 7.

Normally in aptitude exams like the CAT, multiplications would be restricted to 2 digits multiplied by 2 digits, 2 digits multiplied by 3 digits and 3 digits multiplied by 3 digits.

So what are the short cuts that are available in Multiplications? Let us take a look at the various options you have in order to multiply.

1. The straight line method of multiplying two numbers (From Vedic Mathematics and also from the Trachtenberg System of Speed Mathematics)

Let us take an example to explain this process:

Suppose you were multiplying two 2 digit numbers like 43 × 78.

The multiplication would be done in the following manner:

$$\begin{array}{r} 4\ 3 \\ \times\ 7\ 8 \\ \hline \end{array}$$

Step 1: Finding the Unit's digit

The first objective would be to get the unit's digit. In order to do this we just need to multiply the units' digit of both the numbers. Thus, 3 × 8 would give us 24. Hence, we would write 4 in the units' digit of the answer and carry over the digit 2 to the tens place as follows:

$$\begin{array}{r} 4\ 3 \\ \times\ 7\ 8 \\ \hline 4 \end{array}$$

2 carry over to the tens place

At this point we know that the units' digit is 4 and also that there is a carry over of 2 to the tens place of the answer.

Step 2: Finding the tens' place digit

$$\begin{array}{r} 4\ 3 \\ \times\ 7\ 8 \\ \hline 5\ 4 \end{array}$$

5 carry over to
hundreds place

Thought Process:

4 × 8 + 3 × 7 = 32 + 21 = 53

53 + 2 (from carry over) = 55

Thus we write 5 in the tens place and carry over 5 to the hundreds place

In the above case, we have multiplied the units digit of the second number with the tens digit of the first number and added the multiplication of the units digit of the first number with the tens digit of the second number. Thus we would get:

8 (units digit of the second number) × 4 (tens digit of the first number) + 7 (tens digit of the second number) × 3 (units digit of the first number + 2 (carry over from the units' digit calculation)

= 32 + 21 + 2 = 55.

We write down 5 in the tens' digit of the answer and carry over 5 to the hundreds digit of the answer.

Step 3: Finding the hundreds' place digit

$$
\begin{array}{r}
4\;3 \\
\times\;7\;8 \\
\hline
3\;3\;5\;4
\end{array}
$$

Thought Process:

4 × 7 = 28

28 + 5 = 33

Since, 4 and 7 are the last digits on the left in both the numbers this is the last calculation in this problem and hence we can write 33 for the remaining 2 digits in the answer

Thus, the answer to the question is 3354.

With a little bit of practice you can do these kinds of calculations mentally without having to write the intermediate steps.

Let us consider another example where the number of digits is larger:

Suppose you were trying to find the product of 43578 × 6921

Step 1: Finding the units digit

Units digit: 1 × 8 = 8

$$
\begin{array}{r}
4\;3\;5\;7\;8 \\
\times\;6\;9\;2\;1 \\
\hline
8
\end{array}
$$

Step 2: Finding the tens digit

$$
\begin{array}{r}
4\;3\;5\;7\;8 \\
\times\;6\;9\;2\;1 \\
\hline
3\;8
\end{array}
$$

Carry over 2

Thought Process:

Tens digit would come by multiplying tens with units and units with tens

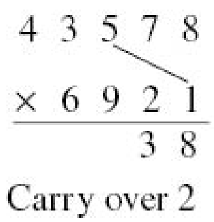
$7 \times 1 + 2 \times 8 = 7 + 16 = 23$

In order to think about this, we can think of the first pair - by thinking about which number would multiply 1 (units digit of the second number) to make it into tens.

Once, you have spotted the first pair the next pair would get spotted by moving right on the upper number (43578) and moving left on the lower number (6921)

Step 3: Finding the hundreds digit

Let us look at the broken down thought process for this step:



Thought Process:

Locate the first pair that would give you your hundreds digit.

For this first think of what you need to multiply the digit in the units place of the second number (digit 1) with to get the hundreds digit of the answer.

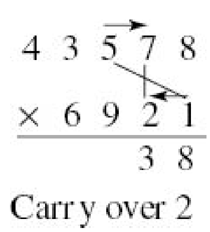
Since:

Units × Hundreds = Hundreds

We need to pair 1 with 5 in the upper number as shown in the figure.

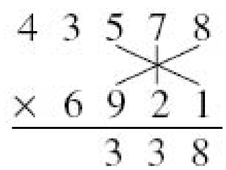
Once you have identified 5×1 as the first pair of digits, to identify the next pair, move 1 to the right of the upper number and move 1 to the left of the lower number.

Thus, you should be able to get 7×2 as your next pair.



For the last pair, you can again repeat the above thought- move to the right in the upper number and move to the left in the lower number.

Thus, the final thought for this situation would look like:

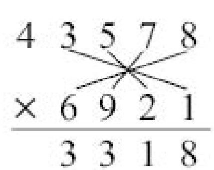


9 carry over to thousands place

> ***Thought Process:***
>
> First pair: 5 × 1
>
> Second pair: 7 × 2 (move right on upper number and move left on the lower number)
>
> Third pair: 8 × 9 (move right on upper number and move left on the lower number)
>
> Thus, 5 × 1 + 7 × 2 + 8 × 9 = 91
>
> 91 + 2 (carry over) = 93
>
> Hundreds place digit would be 3 and carry over to the thousands place would be 9

Step 4: Finding the thousands digit

We would follow the same process as above. For doing the same first identify the first pair as 3 × 1 (thousands from the first number multiplied by units from the second number) and then start moving right and moving left on both the numbers to find the other pairs.



13 carry over to
ten thousands place

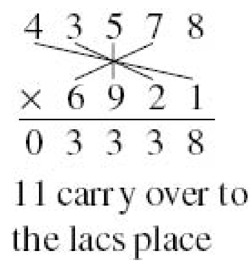
> ***Thought Process:***
>
> 3 × 1 + 5 × 2 + 7 × 9 + 8 × 6 = 124
>
> thousands × units + hundreds × tens + tens × hundreds + units × thousands

124 + 9 (from the carry over) = 133

put down 3 as the thousands place digit and carry over 13 to the ten thousands place

Step 5: Finding the ten thousands digit

4 × 1 would be the first pair here followed by 3 × 2; 5 × 9 and 7 × 6 as shown below:



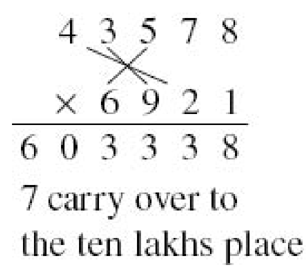
Thought Process:

4 × 1 + 3 × 2 + 5 × 9 + 7 × 6 = 97

97 + 13 (from the carry over of the previous step) = 110.

Hence, 0 becomes the digit which would come into the answer and 11 would be carried over

Step 6: Finding the digit in the lakhs' place



Thought Process:

Since we have already used up 4 × 1 (the left most digit in the first number multiplied with the right most digit in the second number, it is no longer possible to use the digit 1 (units digit of the second number) to form a pair.

Use this as a signal to fix the digit 4 from the first number and start to write down the pairs by pairing this digit 4 (ten thousands' digit of the first number) with the corresponding digit of the second number to form the lacs digit.

It is evident that ten thousands × tens = lacs.

Thus, the first pair is 4 × 2.

Note that you could also think of the first pair as 4 × 2 by realising that since we have used 4 × 1 as the first pair for the previous digit we can use the number to the left of 1 to form 4 × 2.

Subsequent pairs would be 3 × 9 and 5 × 6.

Thus, 4 × 2 + 3 × 9 + 5 × 6 = 8 + 27 + 30 = 65

65 + 11 (carry over) = 76.

Thus, 6 becomes the lacs digit and we get a carry over of 7.

Step 7: Finding the ten lakh's digit

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times\ 6\ 9\ 2\ 1 \\ \hline 1\ 6\ 0\ 3\ 3\ 3\ 8 \end{array}$$

6 carry over to the next place

Thought Process:

First pair: 4 × 9

Next pair: 3 × 6

Thus, 4 × 9 + 3 × 6 = 54

54 + 7 (from the carry over) = 61

1 becomes the next digit in the answer and we carry over 6.

Step 8: Finding the next digit

$$\begin{array}{r} 4\ 3\ 5\ 7\ 8 \\ \times\ 6\ 9\ 2\ 1 \\ \hline 3\ 0\ 1\ 6\ 0\ 3\ 3\ 3\ 8 \end{array}$$

Thought Process:

4 × 6 = 24

24 + 6 (from the carry over = 30)

This is the last step because we have multiplied the left most two digits in the number.

The above process of multiplication- although it looks extremely attractive and magical – especially for larger numbers, it's actual usage in the examination context might actually be quite low. This is because there are better ways of doing multiplication of 2 to 3 digits and larger multiplications might not be required to be executed in an exam like the CAT.

However, in order to solve questions where you might be asked to find the hundreds or even the thousands' digit of a big multiplication like the one showed above, this might be your only option.

Let us look at a few more alternative approaches in order to calculate multiplication problems.

2. Using squares to multiply two numbers

In this approach the usage of the mathematical result $a^2 - b^2 = (a - b)(a + b)$ helps us to find the result of a multiplication.

For instance 18 × 22 can be done using $20^2 - 2^2 = 400 - 4 = 396$, taking $a = 20$ and $b = 2$

Similarly, $22 \times 28 = 25^2 - 3^2 = 625 - 9 = 616$

$35 \times 47 = 41^2 - 6^2 = 1681 - 36 = 1645$

In case the difference between the two numbers is not even, we can still use this process by modifying it thus:

$24 \times 33 = 24 \times 32 + 24 = 28^2 - 4^2 + 24$

$= 784 - 16 + 24 = 792$

However, obviously this process might be ineffective in the following cases:

(i) If the values of the squares required to calculate an multiplication are difficult to ascertain (For two digit numbers, we can bypass this by knowing the short cut to calculate the squares of 2 digit numbers- **You may want to look at the methods given in Chapter 4 of this**

part to find out the squares of 2 digit numbers in order to be more effective with these kinds of calculations.)

(ii) When one is trying to multiply two numbers which are very far from each other, there might be other processes for multiplying them that might be better than this process. For instance, if you are trying to multiply 24 × 92 trying to do it as $58^2 - 34^2$ obviously would not be a very convenient process.

(iii) Also, in case one moves into trying to multiply larger numbers, obviously this process would fail. For instance 283 × 305 would definitely not be a convenient calculation if we use this process.

3. Multiplying numbers close to 100 and 1000

A specific method exists for multiplying two numbers which are both close to 100 or 1000 or 10000.

For us, the most important would be to multiply 2 numbers which are close to 100.

The following example will detail this process for you:

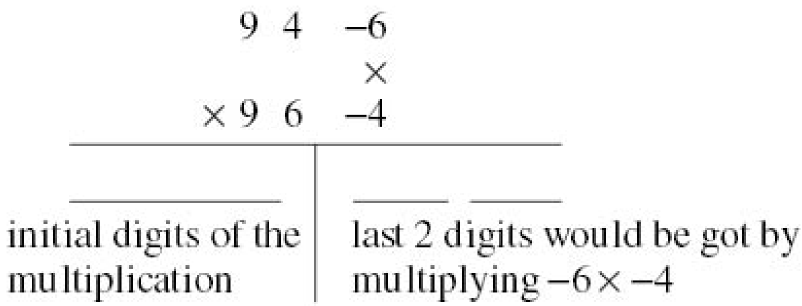
Let us say you are trying to multiply 94 × 96.

Step 1: Calculate the difference from 100 for both numbers and write them down (or visualize them) as follows:

$$\begin{array}{rr} & \text{Difference} \\ & \text{from } 100 \\ 94 & -6 \\ \times\ 96 & -4 \\ \hline \end{array}$$

Step 2: The answer would be calculated in two steps-

(a) The last two digits of the answer would be calculated by multiplying –6 × –4 to get 24.



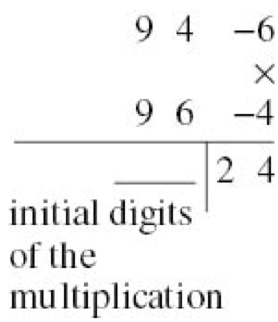
Note here that we divide the answer into two parts:

Last 2 digits and Initial digits

(In case the numbers were close to 1000 we would divide the calculation into the last three digits and the initial digits)

When we multiply –6 × –4 we get 24 and hence we would write that as our last 2 digits in the answer.

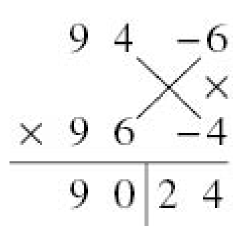
We would then reach the following stage of the multiplication:



The next task is to find the initial digits of the answer:

This can be done by cross adding 94 + (–4) or 96 + (–6) to get the digits as 90 as shown in the figure below:

Difference from 100



Thought Process:

Add along the diagonal connecting line shown in the figure to get the value of the initial digits of the answer:

Thus we would get 94 + (–4) = 90

or

96 + (–6) = 90

Thus, our initial digits of the answer are 90.

Let us take another example to illustrate a few more points which might arise in such a calculation:

Let us say, you were doing 102 × 103.

1 0 2	+2
1 0 3	+3
1 0 5	0 6
Initial digits	Last 2 digits

Thought Process:

In this case, the second part of the answer (the last two digits) turns out to be 2 × 3 = 6. In such a case, since we know that the second part of the answer has to be compulsorily in 2 digits, we would naturally need to take it as 06.

The initial digits of the answer would be got by cross addition:

102 + 3 = 103 + 2 = 105

Consider: 84 × 88

8 4	–16
8 8	–12
7 3	9 2
Initial digits	Last 2 digits

7392 is the answer

Thought Process:

In this case, the difference from 100 for the two numbers are –12 and –16 respectively.

We multiply them to get the last 2 digits of the number. However, –12 × –16 = 192 which is a 3 digit number.

Hence, retain 92 as the last 2 digits and carry over 1 to the initial digits.

Then while finding the initial digits you would need to add this carry over when you are writing the answer.

Thus, initial digits:

84 + (–12) + 1 (from the carry over) = 72 + 1 = 73

Alternately:

88 + (–16) + 1 (from the carry over) = 72 + 1 = 73

Now consider the situation where one number is above 100 and the other below 100.

For instance:

92 × 104

The following figure would show you what to do in this case:

1 0 4		+4
× 9 2		−8
9 6	0 0	
−	3 2	
9 5	6 8	
Initial digits	Last 2 digits	

Thought Process:

The problem in this calculation is that +4 × (–8) = –32 and hence cannot be directly written as the last two digits of the answer.

In this case, first leave the last 2 digits as 00 and find the initial digits of the answer.

Initial digits:

104 + (–8) = 96 = 92 + 4

When you write 96 having kept the last 2 digits of the number as 00, the meaning of the number's value would be 9600.

Now, from this subtract = 4 × (–8) = –32 to get the answer as 9600 – 32 = 9568.

For a multiplication like 994 × 996 the only adjustment you would need to do would be to look at the second part of the answer as a 3- digit number:

The following would make the process clear to you for such cases:

$$\begin{array}{r|l} 9\ 9\ 4 & \quad -6 \\ \times\ 9\ 9\ 6 & \quad -4 \\ \hline 9\ 9\ 0 & 0\ 2\ 4 \\ \hline \text{Initial} & \text{Last 3} \\ \text{digits} & \text{digits} \end{array}$$

Thought Process:

Last 3 digits = –6 × –4 = 24 Æ Hence, we write it as 024

Initial digits: Cross addition of 994 + (–4) = 990

Alternately, 996 + (–6) = 990

Thus, the answer would be 990024.

Note:

(i) The above process for multiplication is extremely good in cases when the two numbers are close to any power of 10 (like 100,1000,10000 etc)

However, when the numbers are far away from a power of 10, the process becomes infeasible.

Thus, this process would not be effective at all in the case of 62 × 34.

(ii) For finding squares of numbers between 80 to 120, this process is extremely good and hence you should use this whenever you are faced with the task of finding the square of a number in this range.

For instance, if you are multiplying 91 × 91 you can easily see the answer as 8281.

4. Using additions to multiply

Consider the following view of an option for multiplying

Let us say we are trying to multiply 83 × 32

This can be converted most conveniently into 80 × 30 + 3 × 30 + 2 × 83 = 2400 + 90 + 166 = 2656

This could also have been done as: 83 × 30 + 2 × 83 = 2490 + 166 = 2656

However in the case of 77 × 48 the second conversion shown above might not be so easy to execute- while the first one would be much easier:

70 × 40 + 7 × 40 + 8 × 77 = 70 × 40 + 7 × 40 + 8 × 70 + 8 × 7 = 2800 + 280 + 560 + 56 = 3696.

The advantage of this type of conversion is that at no point of time in the above calculation are you doing anything more than single digit × single digit multiplication.

5. Use of percentages to multiply

Another option that you have can be explained as below:

Let us say you are trying to find 43 × 78.

In order to calculate 43 × 78 first calculate 43% of 78 as follows:

43% of 78 = 10% of 78 + 10% of 78 +10% of 78 +10% of 78 + 1% of 78 + 1% of 78 + 1% of 78 = 7.8 + 7.8 + 7.8 + 7.8 + 0.78 + 0.78 + 0.78 = 33.34

This can be done using: 7 × 4 =28 as the integer part.

For adding the decimals, consider all the decimals as two digit numbers. In the addition if your total is a 2 digit number, write that down in the decimals place of the answer. If the number is a 3 digit number, carry over the hundreds' digit to the integer part of the answer.

Thus, in this case you would get:

80 + 80 + 80 + 80 + 78 + 78 + 78 = 554. This 554 actually means 5.54 in the context that we have written down 0.80 as 80.

Thus, the total is 33.54.

We have found that 43% of 78 is 33.54 and our entire addition has been done in single and 2 digits. We of course realize that 43% of 78 being the same as 0.43 × 78 the digits for 43 × 78 would be the same as the digits for what we have calculated.

Now, the only thing that remains is to put the decimals back where they belong.

There are many ways to think about this- perhaps the easiest being that 43 × 78 should have 4 as it's units digit and hence the correct answer is 3354.

Of course, this could also have been done by calculating 78% of 43 as 21.5 + 10.75 + 0.43 + 0.43 + 0.43 = 31 + 2.54 = 33.54 Æ Hence, the answer is

3354.

You can even handle 2 digit × 3 digit multiplication through the same process:

Suppose you were multiplying 324 × 82, instead of doing the multiplication as given, find 324 % of 82.

The question converts to: 3.24 × 82 = 82 × 3 + 8.2 + 8.2 + 0.82 + 0.82 + 0.82 + 0.82 = 246 + 16 + 3.68 = 265.68.

Hence, the answer is 26568.

We would encourage you to try to multiply 2 digits × 2 digits and 2 digits × 3 digits and 3 digits × 3 digits by the methods you find most suitable amongst those given above.

In my view, the use of percentages to multiply is the most powerful tool for carrying out the kinds of multiplications you would come across in Aptitude exams. Once you can master how to think about the decimals digits in these calculations, it has the potential to give you a significant time saving in your examination.

Obviously, when you convert a multiplication into an addition using any of the two processes given above, the speed and efficiency of your calculation would depend largely on your ability to add well. If your 2 digit additions are good (or if you have made your additions of 2 digit numbers good by using the process given in the chapter on additions) you would find the addition processes given here the best.

The simple reason is because this process has the advantage of being the most versatile- in the sense that it is not dependant on particular types of numbers.

Besides, after enough practice you would be able to do 2 digits × 2 digits and 2 digits × 3 digits and 3 digits × 3 digits orally.

3 Chapter
Divisions, Percentage Calculations and Ratio Comparisons

CALCULATING DECIMAL VALUES FOR DIVISION QUESTIONS USING PERCENTAGE CALCULATIONS

I have chosen to club these two together because they are actually parallel to each other—in the sense that for any ratio we can calculate two values—the percentage value and the decimal value. The digits in the decimal value and the percentage value of any ratio would always be the same. Hence, calculating the percentage value of a ratio and the decimal value of the ratio would be the same thing.

How do you calculate the percentage value of a ratio?

PERCENTAGE RULE FOR CALCULATING PERCENTAGE VALUES THROUGH ADDITIONS

Illustrated below is a powerful method of calculating percentages. In my opinion, the ability to calculate percentage through this method depends on your ability to handle 2 digit additions. Unless you develop the skill to add 2 digit additions in your mind, you are always likely to face problems in calculating percentage through the method illustrated below. In fact, trying this method without being strong at 2-digit additions/subtractions (including

2 digits after decimal point) would prove to be a disadvantage in your attempt at calculating percentages fast.

This process, essentially being a commonsense process, is best illustrated through a few examples:

Example What is the percentage value of the ratio: 53/81?

Solution The process involves removing all the 100%, 50%, 10%, 1%, 0.1% and so forth of the denominator from the numerator.

Thus, 53/81 can be rewritten as: (40.5 + 12.5)/81 = 40.5/81 + 12.5/81 = 50% + 12.5/81

= 50% + (8.1 + 4.4)/81 = 50% + 10% + 4.4/81

= 60% + 4.4/81

At this stage you know that the answer to the question lies between 60 – 70% (Since 4.4 is less than 10% of 81)

At this stage, you know that the answer to the calculation will be in the form: 6*a*.*bcde* ….

All you need to do is find out the value of the missing digits.

In order to do this, calculate the percentage value of 4.4/81 through the normal process of multiplying the numerator by 100.

Thus the % value of $\frac{4.4}{81} = \frac{4.4 \times 100}{81} = \frac{440}{81}$

[Note: Use the multiplication by 100, once you have the 10% range. This step reduces the decimal calculations.]

Thus $\frac{440}{81}$ = 5% with a remainder of 35

Our answer is now refined to 65.*bcde.* (1% Range)

Next, in order to find the next digit (first one after the decimal) add a zero to the remainder;

Hence, the value of '*b*' will be the quotient of

b Æ 350/81 = 4 Remainder 26

Answer: 65.4*cde* (0.1% Range)

c Æ 260/81 = 3 Remainder 17

Answer: 65.43 (0.01% Range)

and so forth.

The advantages of this process are two fold:

(1) You only calculate as long as you need to in order to eliminate the options. Thus, in case there was only a single option between 60–70% in the above question, you could have stopped your calculations right there.

(2) This process allows you to go through with the calculations as long as you need to.

However, remember what I had advised you right at the start: Strong Addition skills are a primary requirement for using this method properly.

To illustrate another example:

What is the percentage value of the ratio $\frac{223}{72}$?

223/72 Æ 300 – 310% Remainder 7

700/72 Æ 9. Hence 309 – 310%, Remainder 52

520/72 Æ 7. Hence, 309.7, Remainder 16

160/72 Æ 2. Hence, 309.72 Remainder 16

Hence, 309.7222 (2 recurs since we enter an infinite loop of 160/72 calculations).

In my view, percentage rule (as I call it) is one of the best ways to calculate percentages since it gives you the flexibility to calculate the percentage value up to as many digits after decimals as you are required to and at the same time allows you to stop the moment you attain the required accuracy range.

Of course I hope you realize that when you get 53/81 = 65.43 % the decimal value of the same would be 0.6543 and for 223/72, the decimal value would be 3.097222.

The kind of exam that the CAT is, I do not think you would not need to calculate ratios beyond 2 digits divided by 2 digits. In other words, if you are trying to calculate 5372/8164, you can take an approximation of this ratio as 53/81 and calculate the percentage value as shown in the process above. The accuracy in the calculation of 53/81 instead of 5372/8164 would be quite sufficient to answer questions on ratio values that the CAT may throw up in Quantitative Aptitude for even in Data Interpretation questions.

RATIO COMPARISONS

CALCULATION METHODS related to RATIOS

(A) Calculation methods for Ratio comparisons:

There could be four broad cases when you might be required to do ratio comparisons:

The table below clearly illustrates these:

	Numerator	*Denominator*	*Ratio*	*Calculations*
Case 1	Increases	Decreases	Increase	Not required
Case 2	Increases	Increases	May Increase or Decrease	Required
Case 3	Decreases	Increases	Decreases	Not required
Case 4	Decreases	Decreases	May Increase or Decrease	Required

In cases 2 and 4 in the table, calculations will be necessitated. In such a situation, the following processes can be used for ratio comparisons.

1. The Cross Multiplication Method

Two ratios can be compared using the cross multiplication method as follows. Suppose you have to compare

12/15 with 15/19

Then, to test which ratio is higher cross multiply and compare 12 × 19 and 15 × 17.

If 12 × 19 is bigger the Ratio 12/17 will be bigger. If 15 × 17 is higher, the ratio 15/19 will be higher.

In this case, 15 × 17 being higher, the Ratio 15/19 is higher.

Note: In real time usage (esp. in D.I.) this method is highly impractical and calculating the product might be more cumbersome than calculating the percentage values.

Thus, this method will not be able to tell you the answer if you have to compare $\frac{3743}{5624}$ with $\frac{3821}{5783}$

2. Percentage Value Comparison Method

Suppose you have to compare: $\frac{173}{212}$ with $\frac{181}{241}$

In such a case just by estimating the 10% ranges for each ratio you can clearly see that —

the first ratio is > 80% while the second ratio is < 80%

Hence, the first ratio is obviously greater.

This method is extremely convenient if the two ratios have their values in different 10% ranges.

However, this problem will become slightly more difficult, if the two ratios fall in the same 10% range. Thus, if you had to compare $^{173}/_{212}$ with $^{181}/_{225}$, both the values would give values between 80 – 90%. The next step would be to calculate the 1% range.

The first ratio here is 81 – 82% while the second ratio lies between 80 – 81%

Hence the first ratio is the larger of the two.

Note: For this method to be effective for you, you will first need to master the percentage rule method for calculating the percentage value of a ratio. Hence if you cannot see that 169.6 is 80% of 212 or for that matter that 81% of 212 is 171.72 and 82% is 173.84 you will not be able to use this method effectively. (This is also true for the next method.) However, once you can calculate percentage values of 3 digit ratios to 1% range, there is not much that can stop you in comparing ratios. The CAT and all other aptitude exams normally do not challenge you to calculate further than the 1% range when you are looking at ratio comparisons.

3. Numerator Denominator Percentage Change Method

There is another way in which you can compare close ratios like 173/212 and 181/225. For this method, you need to calculate the percentage changes in the numerator and the denominator.

Thus:

173 Æ 181 is a % increase of 4 – 5%

While 212 Æ 225 is a % increase of 6 – 7%.

In this case, since the denominator is increasing more than the numerator, the second ratio is smaller.

This method is the most powerful method for comparing close ratios—provided you are good with your percentage rule calculations.

(B) Method for calculating the value of a percentage change in the ratio:

PCG (Percentage Change Graphic) gives us a convenient method to calculate the value of the percentage change in a ratio.

Suppose, you have to calculate the percentage change between 2 ratios. This has to be done in two stages as:

$$\text{Original Ratio} \xrightarrow[\text{numerator}]{\text{Effect of}} \text{Intermediate Ratio}$$

$$\xrightarrow[\text{Denominator}]{\text{Effect of}} \text{Final Ratio}$$

Thus if 20/40 becomes 22/50

Effect of numerator = 20 Æ 22(10% increase)

Effect of denominator = 50 Æ 40(25% decrease) (reverse fashion)

Overall effect on the ratio:

$$100 \xrightarrow[\substack{\text{Numerator}\\\text{Effect}}]{10\%\uparrow} 110 \xrightarrow[\substack{\text{Denominator}\\\text{Effect}}]{25\%\downarrow} 82.5$$

Hence, overall effect = 17.5% decrease.

4 Chapter
Squares and Cubes of Numbers

SQUARES AND SQUARE ROOTS

When any number is multiplied by itself, it is called as the square of the number.

Thus, $3 \times 3 = 3^2 = 9$

Squares have a very important role to play in mathematics. In the context of preparing for CAT and other similar aptitude exams, it might be a good idea to be able to recollect the squares of 2 digit numbers.

Let us now go through the following table carefully:

Table 4.1

Number	*Square*	*Number*	*Square*	*Number*	*Square*
1	1	35	1225	69	4761
2	4	36	1296	70	4900
3	9	37	1369	71	5041
4	16	38	1444	72	5184
5	25	39	1521	73	5329
6	36	40	1600	74	5476
7	49	41	1681	75	5625
8	64	42	1764	76	5776

Number	*Square*	*Number*	*Square*	*Number*	*Square*
9	81	43	1849	77	5929
10	100	44	1936	78	6084
11	121	45	2025	79	6241
12	144	46	2116	80	6400
13	169	47	2209	81	6561
14	196	48	2304	82	6724
15	225	49	2401	83	6889
16	256	50	2500	84	7056
17	289	51	2601	85	7225
18	324	52	2704	86	7396
19	361	53	2809	87	7561
20	400	54	2916	88	7744
21	441	55	3025	89	7921
22	484	56	3136	90	8100
23	529	57	3249	91	8281
24	576	58	3364	92	8464
25	625	59	3481	93	8649
26	676	60	3600	94	8836
27	729	61	3721	95	9025
28	784	62	3844	96	9216
29	841	63	3969	97	9409
30	900	64	4096	98	9604
31	961	65	4225	99	9801
32	1024	66	4356	100	10000
33	1089	67	4489		
34	1156	68	4624		

Number	*Square*	*Number*	*Square*	*Number*	*Square*

So, how does one get these numbers onto one's finger tips? Does one memorize these values or is there a simpler way?

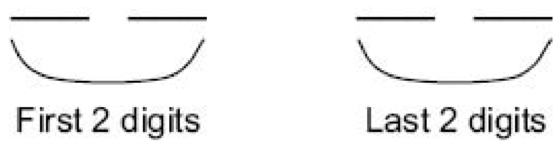
Yes indeed! There is a very convenient process when it comes to memorising the squares of the first 100 numbers.

First of all, you are expected to memorise the squares of the first 30 numbers. In my experience, I have normally seen that most students already know this. The problem arises with numbers after 30. You do not need to worry about that. Just follow the following processes and you will know all squares upto 100.

Trick 1: For squares from 51 to 80 – (**Note:** This method depends on your memory of the first thirty squares.)

The process is best explained through an example.

Suppose, you have to get an answer for the value of 67^2. Look at 67 as (50 + 17). The 4 digit answer will have two parts as follows:



The last two digits will be the same as the last two digits of the square of the number 17. (The value 17 is derived by looking at the difference of 67 with respect to 50.)

Since, $17^2 = 289$, you can say that the last two digits of 67^2 will be 89. (i.e. the last 2 digits of 289.) Also, you will need to carry over the '2' in the hundreds place of 289 to the first part of the number.

The first two digits of the answer will be got by adding 17 (which is got from 67 – 50) and adding the carry over (2 in this case) to the number 25. (Standard number to be used in all cases.) Hence, the first two digits of the answer will be given by 25 + 17 + 2 = 44.

Hence, the answer is $67^2 = 4489$.

Similarly, suppose you have to find 76^2.

Step 1: 76 = 50 + 26.

Step 2: 26^2 is 676. Hence, the last 2 digits of the answer will be 76 and we will carry over 6.

Step 3: The first two digits of the answer will be 25 + 26 + 6 = 57.

Hence, the answer is 5776.

This technique will take care of squares from 51 to 80 (if you remember the squares from 1 to 30). You are advised to use this process and see the answers for yourself.

SQUARES FOR NUMBERS FROM 31 TO 50

Such numbers can be treated in the form (50 – *x*) and the above process modified to get the values of squares from 31 to 50. Again, to explain we will use an example. Suppose you have to find the square of 41.

Step 1: Look at 41 as (50 – 9).

Again, similar to what we did above, realise that the answer has two parts—the first two and the last two digits.

Step 2: The last two digits are got by the last two digits in the value of $(–9)^2$ = 81. Hence, 81 will represent the last two digits of 41^2.

Step 3: The first two digits are derived by 25 – 9 = 16 (where 25 is a standard number to be used in all cases and – 9 comes from the fact that (50 – 9) = 41).

Hence, the answer is 1681.

Note*:* In case there had been a carry over from the last two digits it would have been added to 16 to get the answer.

For example, in finding the value of 36^2 we look at 36 = (50 – 14)

Now, $(–14)^2$ = 196. Hence, the last 2 digits of the answer will be 96. The number ‘1’ in the hundreds place will have to be carried over to the first 2 digits of the answer.

The, first two digits will be 25 – 14 + 1 = 12

Hence, $36^2 = 1296$.

With this process, you are equipped to find the squares of numbers from 31 to 50.

FINDING SQUARES OF NUMBERS BETWEEN 81 TO 100

Suppose you have to find the value of 82^2. The following process will give you the answers.

Step 1: Look at 82 as (100 – 18). The answer will have 4 digits whose values will be got by focusing on getting the value of the last two digits and that of the first two digits.

Step 2: The value of the last two digits will be equal to the last two digits of $(-18)^2$.

Since, $(-18)^2 = 324$, the last two digits of 82^2 will be 24. The '3' in the hundreds place of $(-18)^2$ will be carried over to the other part of the answer (i.e. the first two digits).

Step 3: The first two digits will be got by: 82 + (– 18) + 3 Where 82 is the original number; (– 18) is the number obtained by looking at 82 as (100 – *x*); and 3 is the carry over from $(-18)^2$.

Similarly, 87^2 will give you the following thought process:

87 = 100 – 13 Æ $(-13)^2 = 169$. Hence, 69 are the last two digits of the answer Æ Carry over 1. Consequently, 87 + (–13) + 1 = 75 will be the first 2 digits of the answer.

Hence, $87^2 = 7569$.

With these three processes you will be able to derive the square of any number up to 100.

Properties of squares:

1. When a perfect square is written as a product of its prime factors each prime factor will appear an even number of times.

2. The difference between the squares of two consecutive natural numbers is always equal to the sum of the natural numbers. Thus, $41^2 – 40^2 = (40 + 41) = 81$.

   This property is very useful when used in the opposite direction—i.e. Given that the difference between the squares of two consecutive integers is 81, you should immediately realise that the numbers should be 40 and 41.
3. The square of a number ending in 1, 5 or 6 also ends in 1, 5 or 6 respectively.
4. The square of any number ending in 5: The last two digits will always be 25. The digits before that in the answer will be got by multiplying the digits leading up to the digit 5 in the number by 1 more than itself.

   Illustration:

   85^2 = ___25.

   The missing digits in the above answer will be got by $8 \times (8 + 1) = 8 \times 9 = 72$. Hence, the square of 85 is given by 7225.

   Similarly, 135^2 = ___25. The missing digits are $13 \times 14 = 182$. Hence, $135^2 = 18225$.
5. The value of a perfect square has to end in 1, 4, 5, 6, 9 or an even number of zeros. In other words, a perfect square cannot end in 2, 3, 7, or 8 or an odd number of zeros.
6. If the units digit of the square of a number is 1, then the number should end in 1 or 9.
7. If the units digit of the square of a number is 4, then the units digit of the number is 2 or 8.
8. If the units digit of the square of a number is 9, then the units digit of the number is 3 or 7.
9. If the units of the square of a number is 6, then the unit’s digit of the number is 4 or 6.
10. The sum of the squares of the first ‘n’ natural numbers is given by

    $[(n)(n + 1)(2n + 1)]/6$.
11. The square of a number is always non-negative.

12. Normally, by squaring any number we increase the value of the number. The only integers for which this is not true are 0 and 1. (In these cases squaring the number has no effect on the value of the number).

    Further, for values between 0 to 1, squaring the number reduces the value of the number. For example $0.5^2 < 0.5$.

Say, you have to Find the Square Root of a Given Number

Say 7056

Step 1: Write down the number 7056 as a product of its

Prime factors.$7056 = 2 \times 2 \times 2 \times 2 \times 21 \times 21$

$= 2^4 \times 21^2$

Step 2: The required square root is obtained by halving the values of the powers.

Hence, $\sqrt{7056} = 2^2 \times 21^1$

CUBES AND CUBE ROOTS

When a number is multiplied with itself two times, we get the cube of the number.

Thus, $x \times x \times x = x^3$

Method to find out the cubes of 2 digit numbers: The answer has to consist of 4 parts, each of which has to be calculated separately.

The first part of the answer will be given by the cube of the ten's digit.

Suppose you have to find the cube of 28.

The first step is to find the cube of 2 and write it down.

$2^3 = 8$.

The next three parts of the number will be derived as follows.

Derive the values 32, 128 and 512.

(by creating a G. P. of 4 terms with the first term in this case as 8, and a common ratio got by calculating the ratio of the unit's digit of the number with its tens digit. In this case the ratio is 8/2 = 4.)

Now, write the 4 terms in a straight line as below. Also, to the middle two terms add double the value.

	8	32	128	512
+		64	256	
	21	9	5	2

(carry over 51)

(8 + 13) (32 + 64 + 43 =139) (128 + 256 + 51 = 435)

Carry over 13 (Carry over 43)

Hence, $28^3 = 21952$

Properties of Cubes

1. When a perfect cube is written in its standard form the values of the powers on each prime factor will be a multiple of 3.
2. In order to find the cube root of a number, first write it in its standard form and then divide all powers by 3.

   Thus, the cube root of $3^6 \times 5^9 \times 17^3 \times 2^6$ is

   given by $3^2 \times 5^3 \times 17 \times 2^2$
3. The cubes of all numbers (integers and decimals) greater than 1 are greater than the number itself.
4. $0^3 = 0$, $1^3 = 1$ and $-1^3 = -1$. These are the only three instances where the cube of the number is equal to the number itself.
5. The value of the cubes of a number between 0 and 1 is lower than the number itself. Thus, $0.5^3 < 0.5^2 < 0.5$.
6. The cube of a number between 0 and –1 is greater than the number itself. $(-0.2)^3 > -0.2$.
7. The cube of any number less than –1, is always lower than the number. Thus, $(-1.5)^3 < (-1.5)$.

BLOCK I

NUMBERS

Chapter 1
Number Systems

Chapter 2
Progressions

As already mentioned in the introductory note, Block I constituted the most crucial aspect of the Quantitative Aptitude Section in the paper & pen version of the CAT. Throughout the decade 1999 to 2008, almost 30–50% of the total questions in every CAT paper came from the two chapters given in this block. However, the online CAT has shifted this weightage around, and consequently, the importance of Block I has been reduced to around 20–25% of the total marks in the section. Thus, although Block I remains an important block for your preparations, it has lost its pre-eminence (as reflected in the strategy—"Do Block I well and you can qualify the QA section"). However, this does not change the need for you to go through this block in great depth.

Thus my advice to you is: *Go through this block in depth and try to gain clarity of concepts as well as exposure to questions for honing your ability to do well in this area.*

...BACK TO SCHOOL

- **Chapters in this Block: Number Systems and Progressions**
- **Block Importance – 20–25%**

The importance of this block can be gauged from the table below:

Year	*% of Marks from Block 1*	*Qualifying Score (approx. score for 96 percentile)*
2000	48%	35%
2001	36%	35%
2002	36%	35%
2003 (cancelled)	30%	32%
2003 (retest)	34%	35%
2004	32%	35%
2005	40%	35%
2006	32%	40%
2007	24%	32%
2008	40%	35%
Online CAT 2009–2013	15–30%	60% with no errors

As you can see from the table above, doing well in this block alone could give you a definite edge and take you a long way to qualifying the QA section. Although the online CAT has significantly varied the relative importance to this block, the importance of this block remains high. Besides, there is a good chance that once the IIMs get their act together in the context of the online CAT and its question databases—the pre eminence of this block of chapters might return.

Hence, understanding the concepts involved in these chapters properly and strengthening your problem solving experience could go a long way towards a good score.

Before we move into the individual chapters of this block, let us first organise our thinking by looking at the core concepts that we had learnt in school with respect to these chapters.

Pre-assessment Test

This test consists of 25 questions based on the chapters of BLOCK I (Number Systems and Progressions). Do your best in trying to solve each question.

The time limit to be followed for this test is 30 minutes. However, after the 30 minutes is over continue solving till you have spent enough time and paid sufficient attention to each question. After you finish thinking about each and every question of the test, check your scores. Then go through the SCORE INTERPRETATION ALGORITHM given at the end of the test to understand the way in which you need to approach the chapters inside this block.

1. The number of integers n satisfying $-n + 2 \geq 0$ and $2n \geq 4$ is

   (a) 0 (b) 1

   (c) 2 (d) 3

2. The sum of two integers is 10 and the sum of their reciprocals is 5/12. Then the larger of these integers is

   (a) 2 (b) 4

   (c) 6 (d) 8

3. If x is a positive integer such that $2x + 12$ is perfectly divisible by x, then the number of possible values of x is

   (a) 2 (b) 5

   (c) 6 (d) 12

4. Let K be a positive integer such that $k + 4$ is divisible by 7. Then the smallest positive integer n, greater than 2, such that $k + 2n$ is divisible by 7 equals.

   (a) 9 (b) 7

   (c) 5 (d) 3

5. $2^{73} - 2^{72} - 2^{71}$ is the same as

   (a) 2^{69} (b) 2^{70}

(c) 2^{71} (d) 2^{72}

6. Three times the first of three consecutive odd integers is 3 more than twice the third. What is the third integer?

   (a) 15 (b) 9

   (c) 11 (d) 5

7. x, y and z are three positive integers such that $x > y > z$. Which of the following is closest to the product xyz?

   (a) $xy(z – 1)$ (b) $(x – 1)yz$

   (c) $(x – y)xy$ (d) $x(y + 1)z$

8. A positive integer is said to be a prime number if it is not divisible by any positive integer other than itself and 1. Let p be a prime number greater than 5, then $(p^2 – 1)$ is

   (a) never divisible by 6.

   (b) always divisible by 6, and may or may not be divisible by 12.

   (c) always divisible by 12, and may or may not be divisible by 24.

   (d) always divisible by 24.

9. Iqbal dealt some cards to Mushtaq and himself from a full pack of playing cards and laid the rest aside. Iqbal then said to Mushtaq "If you give me a certain number of your cards, I will have four times as many cards as you will have. If I give you the same number of cards, I will have thrice as many cards as you will have". Of the given choices, which could represent the number of cards with Iqbal?

   (a) 9 (b) 31

   (c) 12 (d) 35

10. In Sivakasi, each boy's quota of match sticks to fill into boxes is not more than 200 per session. If he reduces the number of sticks per box by 25, he can fill 3 more boxes with the total number of

sticks assigned to him. Which of the following is the possible number of sticks assigned to each boy?

(a) 200 (b) 150

(c) 125 (d) 175

11. A lord got an order from a garment manufacturer for 480 Denim Shirts. He bought 12 sewing machines and appointed some expert tailors to do the job. However, many didn't report for duty. As a result, each of those who did, had to stitch 32 more shirts than originally planned by Alord, with equal distribution of work. How many tailors had been appointed earlier and how many had not reported for work?

(a) 12, 4 (b) 10, 3

(c) 10, 4 (d) None of these

12. How many 3-digit even numbers can you form such that if one of the digits is 5, the following digit must be 7?

(a) 5 (b) 405

(c) 365 (d) 495

13. To decide whether a number of n digits is divisible by 7, we can define a process by which its magnitude is reduced as follows: (i_1, i_2, i_3…..i_n are the digits of the number, starting from the most significant digit).

$i_1 i_2 \ldots i_n$ fi $i_1 . 3^{n-1} + \mathrm{i}_{2,} 3^{n-2} + \ldots. + I_n 3^0$.

e.g. 259 fi $2.3^2 + 5.3^1 + 9.3^0 = 18 + 15 + 9 = 42$

Ultimately the resulting number will be seven after repeating the above process a certain number of times.

After how many such stages, does the number 203 reduce to 7?

(a) 2 (b) 3

(c) 4 (d) 1

14. A teacher teaching students of third standard gave a simple multiplication exercise to the kids. But one kid reversed the digits of both the numbers and carried out the multiplication and found that the product was exactly the same as the one expected by the teacher. Only one of the following pairs of numbers will fit in the description of the exercise. Which one is that?

(a) 14, 22 (b) 13, 62

(c) 19, 33 (d) 42, 28

15. If 8 + 12 = 2, 7 + 14 = 3 then 10 + 18 =?

(a) 10 (b) 4

(c) 6 (d) 18

16. Find the minimum integral value of n such that the division $55n/124$ leaves no remainder.

(a) 124 (b) 123

(c) 31 (d) 62

17. What is the value of k for which the following system of equations has no solution:

$2x – 8y = 3$; and $kx + 4y = 10$.

(a) –2 (b) 1

(c) –1 (d) 2

18. A positive integer is said to be a prime if it is not divisible by any positive integer other than itself and one. Let p be a prime number strictly greater than 3. Then, when $p^2 + 17$ is divided by 12, the remainder is

(a) 6 (b) 1

(c) 0 (d) 8

19. A man sells chocolates that come in boxes. Either full boxes or half a box of chocolates can be bought from him. A customer comes and buys half the number of boxes the seller has plus half a box. A second customer comes and buys half the remaining

number of boxes plus half a box. After this, the seller is left with no chocolates box. How many chocolates boxes did the seller have before the first customer came?

(a) 2 (b) 3

(c) 4 (d) 3.5

20. X and Y are playing a game. There are eleven 50 paise coins on the table and each player must pick up at least one coin but not more than five. The person picking up the last coin loses. X starts. How many should he pick up at the start to ensure a win no matter what strategy Y employs?

(a) 4 (b) 3

(c) 2 (d) 5

21. If $a < b$, which of the following is always true?

(a) $a < (a + b) / 2 < b$

(b) $a < ab/2 < b$

(c) $a < b^2 - a^2 < b$

(d) $a < ab < b$

22. The money order commission is calculated as follows. From ` X to be sent by money order, subtract 0.01 and divide by 10. Get the quotient and add 1 to it, if the result is Y, the money order commission is ` 0.5Y. If a person sends two money orders to Aurangabad and Bhatinda for ` 71 and ` 48 respectively, the total commission will be

(a) ` 7.00 (b) ` 6.50

(c) ` 6.00 (d) ` 7.50

23. The auto fare in Ahmedabad has the following formula based upon the meter reading. The meter reading is rounded up to the next higher multiple of 4. For instance, if the meter reading is 37 paise, it is rounded up to 40 paise. The resultant is multiplied by 12. The final result is rounded off to nearest multiple of 25 paise. If 53 paise is the meter reading what will be the actual fare?

(a) ` 6.75 (b) ` 6.50

(c) ` 6.25 (d) ` 7.50

24. Juhi and Bhagyashree were playing simple mathematical puzzles. Juhi wrote a two digit number and asked Bhayashree to guess it. Juhi also indicated that the number is exactly thrice the product of its digits. What was the number that Juhi wrote?

(a) 36 (b) 24

(c) 12 (d) 48

25. It is desired to extract the maximum power of 3 from 24!, where $n!$ = $n.(n-1) . (n-2) \ldots 3.2.1$. What will be the exponent of 3?

(a) 8 (b) 9

(c) 11 (d) 10

ANSWER KEY

1. (b)	2. (c)	3. (c)	4. (a)
5. (c)	6. (a)	7. (b)	8. (d)
9. (b)	10. (b)	11. (c)	12. (c)
13. (a)	14. (b)	15. (a)	16. (a)
17. (c)	18. (a)	19. (b)	20. (a)
21. (a)	22. (b)	23. (a)	24. (b)
25. (d)			

Solutions

1. The only value that will satisfy will be 2.
2. ¼ + 1/6 will give you 5/12.
3. The possible values are 1, 2, 3, 4, 6 and 12. (i.e. the factors of 12)
4. k will be a number of the form $7n + 3$. Hence, if you take the value of n as 9, $k + 2n$ will become $7n + 3 + 18 = 7n + 21$. This number will be divisible by 7. The numbers 3, 5 and 7 do not provide us with this solution.

5. $2^{73} – 2^{72} – 2^{71} = 2^{71} (2^2 – 2 – 1) = 2^{71}(1)$. Hence option (c) is correct.
6. Solve through options.
7. The closest value will be option (b), since the percentage change will be lowest when the largest number is reduced by one.
8. This is a property of prime numbers greater than 5.
9. He could have dealt a total of 40 cards, in which case Mushtaq would get 9 cards. On getting one card from Mushtaq, the ratio would become 4:1, while on giving away one card to Mushtaq, the ratio would become 3:1.
10. Looking at the options you realise that the correct answer should be a multiple of 25 and 50 both. The option that satisfies the condition of increasing the number of boxes by 3 is 150. (This is found through trial and error.)
11. Trial and error gives you option 3 as the correct answer.
12. Given that the number must have a 57 in it and should be even at the same time, the only numbers possible are 570, 572, 574, 576 and 578. Also, if there is no 5 in the number, you will get 360 more numbers.
13. 203 becomes $\rightarrow 2.3^2 + 0 + 3.3^0 = 21 \rightarrow 2.3^1 + 1.3^0 = 7$. Hence, clearly two steps are required.
14. Trial and error will give option (b) as the correct answer, since $13 \times 62 = 26 \times 31$
15. The solutions are defined as the sum of digits of the answer. Hence, 10 is correct.
16. There are no common factors between 55 and 124. Hence the answer should be 124.
17. At $k = –1$, the two equations become inconsistent with respect to each other and there will then be no solution to the system of equations.
18. Try with 5, 7, 11. In each case the remainder is 6.
19. Trial and error gives you the answer as 3 Option (b) is correct.
20. Picking up 4 coins will ensure that he wins the game.

21. Option (a) is correct (since the average of any two numbers lies between the numbers.
22. 8/2 + 5/2 = 6.5.
23. The answer will be 56 × 12 = 672 → 675. Hence, ` 6.75.
24. The given condition is satisfied only for 24.
25. The answer will be given by 8 + 2 = 10.

(This logic is explained in the Number Systems chapter)

SCORE INTERPRETATION ALGORITHM FOR PRE-ASSESSMENT TEST OF BLOCK I

(Use a similar process for Blocks II to VI on the basis of your performance).

If You Scored: < 7: (In Unlimited Time)

Step One

Go through the block one **Back to School** Section carefully. Grasp each of the concepts explained in that part carefully. In fact I would recommend that you go back to your Mathematics school books (ICSE/ CBSE) Class 8, 9 and 10 if you feel you need it.

Step Two

Move into the first chapter of the block, viz, Number Systems. When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

Then move onto the LOD 1 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1 of Number systems. While doing so do not think about the time requirement. Once you finish solving LOD 1, revise the questions and their solution processes.

Step Three

After finishing LOD 1 of number systems, move into Chapter 2 of this block—Progressions and repeat the process, viz: chapter theory comprehensively followed by solving LOD 1 questions.

Step Four

Go to the first and second review tests given at the end of the block and solve them. While doing so, first look at the score you get within the mentioned time limit. Then continue to solve the test further without a time limit and try to evaluate the improvement in your unlimited time score.

In case the growth in your score is not significant, go back to the theory of each chapter and review each of the LOD 1 questions for both the chapters.

Step Five

Move to LOD 2 and repeat the process that you followed in LOD 1—first in the chapter of Number Systems, then in the chapter on Progressions. Concentrate on understanding each and every question and its underlying concept.

Step Six

Go to the third to fifth review tests given at the end of the block and solve them. Again, while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

Step Seven

Move to LOD 3 only after you have solved and understood each of the questions in LOD 1 and LOD 2. Repeat the process that you followed in LOD 1—first in the chapter of Number Systems, then with the Chapter on Progressions.

If You Scored: 7–15 (In Unlimited Time)

Although you are better than the person following the instructions above, obviously there is a lot of scope for the development of your score. You will need to work both on your concepts as well as speed. Initially emphasize more on the concept development aspect of your preparations, then move your emphasis onto speed development. The following process is recommended for you:

Step One

Go through the **block one Back to School Section** carefully. Revise each of the concepts explained in that part. Going through your VIIIth, IXth and

Xth standard books will be an optional exercise for you. It will be recommended in case you scored in single digits, while if your score is in two digits, I leave the choice to you.

Step Two

Move into the first chapter of the block. Viz Number Systems. When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

Then move onto the LOD 1 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1 of Number Systems. Once you finish solving LOD 1, revise the questions and their solution processes.

Step Three

After finishing LOD 1 of number systems, move into Chapter 2 of this block—Progressions and repeat the process, viz: Chapter theory comprehensively followed by solving LOD 1 questions.

Step Four

Go to the first and second review tests given at the end of the block and solve them. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your score.

Step Five

Move to LOD 2 and repeat the process that you followed in LOD 1—first with the chapter on Number Systems, then with the chapter on Progressions.

Step Six

Go to the third to fifth review tests given at the end of the block and solve it. Again while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

In case the growth in your score is not significant, go back to the theory of both the chapters and re-solve LOD 1 and LOD 2 of both the chapters.

While doing so concentrate more on the LOD 2 questions.

Step Seven

Move to LOD 3 and repeat the process that you followed in LOD 1—first with the chapter on Number Systems, then with the Chapter on Progressions.

If You Scored 15+ (In Unlimited Time)

Obviously you are much better than the first two categories of students. Hence unlike them, your focus should be on developing your speed by picking up the shorter processes explained in this book. Besides, you might also need to pick up concepts that might be hazy in your mind. The following process of development is recommended for you:

Step One

Quickly review the concepts given in the block one Back to School Section. Only go deeper into a concept in case you find it new. Going back to school level books is not required for you.

Step Two

Move into the first chapter of the block: Number Systems. Go through the theory explained there carefully. Concentrate specifically on clearly understanding the concepts which are new to you. Work out the short cuts and in fact try to expand your thinking by trying to think of alternative (and expanded) lines of questioning with respect to the concept you are studying.

Then move onto the LOD 1 exercises. Solve each and every question provided under LOD 1 of Number Systems. While doing so, try to think of variations that you can visualize in the same questions and how you would handle them.

Step Three

After finishing LOD 1 of number systems, move into Chapter 2 of this block, (Progressions) and repeat the process, viz: Chapter theory with emphasis on picking up things that you are unaware of, followed by solving LOD 1 questions and thinking about their possible variations.

Step Four

Move to LOD 2 and repeat the process that you followed in LOD 1—first in the chapter of Number Systems, then with the Chapter on Progressions.

Step Five

Go to the first to fifth review tests—given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your score.

Step Six

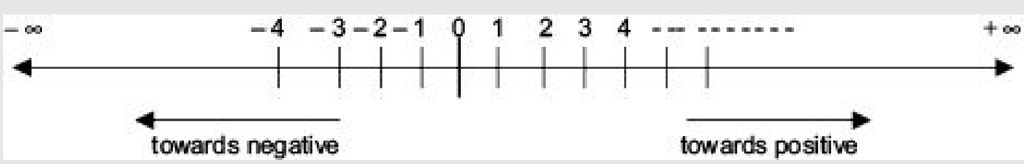
Move to LOD 3 and repeat the process that you followed in LOD 1—first in the chapter of Number Systems, then with the Chapter on Progressions.

THE NUMBER LINE

Core Concepts

I. The concept of the number line is one of the most crucial concepts in Basic Numeracy.

The number line is a line that starts from zero and goes towards positive infinity when it moves to the right and towards negative infinity when it moves to the left.



The difference between the values of any two points on the number line also gives the distance between the points.

Thus, for example if we look at the distance between the points + 3 and – 2, it will be given by their difference. 3 – (– 2) = 3 + 2 = 5.

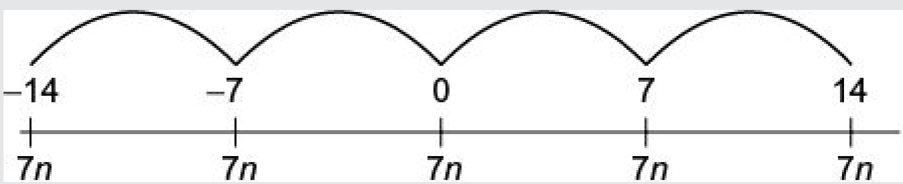
II. Types of numbers—

We will be looking at the types of numbers in detail again when we go into the chapter of number systems. Let us first work out in our minds the various types of numbers. While doing so do not fail to notice that most of these number types occur in pairs (i.e. the definition of one of them, defines the other automatically).

Note here that the number line is one of the most critical concepts in understanding and grasping numeracy and indeed mathematics.

Multiples on the Number Line

All tables and multiples of every number can be visualised on the number line. Thus, multiples of 7 on the number line would be seen as follows:

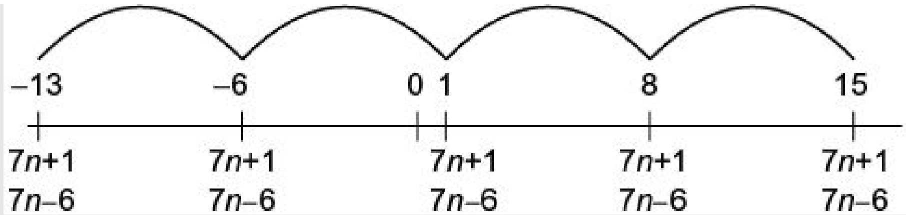


In order to visualize this you can imagine a frog jumping consistently 7 units every time. If it lands on –14, the next landing would be on –7, then on 0, then 7 and finally at 14. This is how you can visualise the table of 7 (in mathematical terms we can also refer to this as $7n$ – meaning the set of numbers which are multiples of 7 or in other words the set of numbers which are divisible by 7).

You can similarly visualise $5n$, $4n$, $8n$ and so on—practically all tables on the number line as above.

What do We Understand by $7n + 1$ and Other such Notations & the Equivalence of $7n + 1$ and $7n - 6$

Look at the following figure closely:



Our 7*n* frog is made to first land on –13 and then asked to keep doing it's stuff (jumping 7 to the right everytime). What is the result? It lands on –6, + 1, 8, 15 and it's next landing would be on 22, 29 and so forth. These numbers cannot be described as 7*n*, but rather they all have a single property which is constant for all numbers.

They can be described as: "One more than a multiple of 7" and in mathematical terms such numbers are also called as $7n + 1$. Alternately these numbers also have the property that they are "6 less than multiples of 7" and in mathematical terms such numbers can also be called as $7n – 6$.

That is why in mathematics, we say that the set of numbers represented by $7n + 1$ is the same as the set of numbers represented by $7n – 6$.

The Implication in Terms of Remainders

This concept can also be talked about in the context of remainders.

When a number which can be described as $7n + 1$ or $7n – 6$ (like the numbers 8, 15, –6, –13, –20, –27, –34, –41…) is divided by 7, the remainder in every case is seen to be 1. For some people reconciling the fact that the remainder when –27 is divided by 7 the remainder is 1, seems difficult on the surface. Note that this needs to be done because about remainders we should know that remainders are always non-negative.

However, the following thinking would give you the remainder in every case:

27/7, remainder is 6.

–27/7, remainder is –6. In the context of dividing by 7, a remainder of –6 means a remainder of 7 – 6 = 1.

Let us look at another example:

What is the remainder when –29 is divided by 8.

First reaction 29/8 Æ remainder 5, –29/8 Æ remainder –5, hence actual remainder is 8 – 5 = 3.

The student is advised to practice more such situations and get comfortable in converting positive remainders to negative remainders and vice versa.

Even and Odd Numbers

The meaning of 2n and 2n + 1: $2n$ means a number which is a multiple of the number 2. Since, this can be visualised as a frog starting from the origin and jumping 2 units to the right in every jump, you can also say that this frog represents $2n$.

(**Note**: Multiples of 2, are even numbers. Hence, $2n$ is also used to denote even numbers.)

So, what does $2n + 1$ mean?

Well, simply put, if you place the above frog on the point represented by the number 1 on the number line then the frog will reach points such as 3, 5, 7, 9, 11 …..and so on. This essentially means that the points the frog now reaches are displaced by 1 unit to the right of the $2n$ frog. In mathematical terms, this is represented as $2n + 1$.

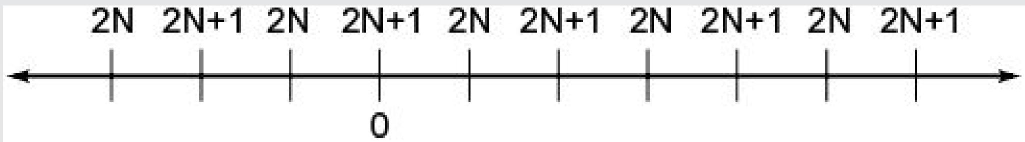
In other words, $2n + 1$ also represents numbers which leave a remainder of 1, when divided by 2. (**Note**: This is also the definition of an odd number. Hence, in Mathematics ($2n + 1$) is used to denote an odd number. Also note that taken together $2n$ and $2n + 1$ denote the entire set of integers. i.e. all integers from – • to + • on the number line can be denoted by either $2n$ or $2n + 1$. This happens because when we divide any integer by 2, there are only two results possible with respect to the remainder obtained, viz: A remainder of zero ($2n$) or a remainder of one ($2n + 1$).

This concept can be expanded to represent integers with respect to any number. Thus, in terms of 3, we can only have three types of integers $3n$, $3n + 1$ or $3n + 2$ (depending on whether the integer leaves a remainder 0, 1 or 2 respectively when divided by 3.) Similarly, with respect to 4, we have 4 possibilities—$4n$, $4n + 1$, $4n + 2$ or $4n + 3$.

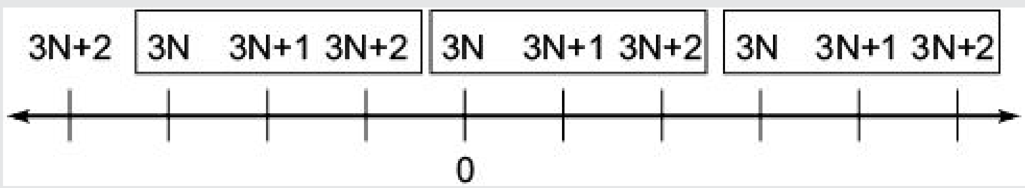
Needless to say, from these representations above, the representations 2*n* and 2*n* + 1 (which can also be represented as 2*n* – 1) have great significance in Mathematics as they represent even and odd numbers respectively. Similarly, we use the concept of 4*n* to check whether a year is a leap year or not.

These representations can be seen on the number line as follows:

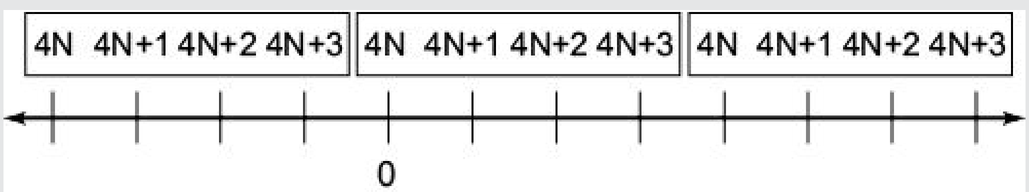
Representation of 2*n* and 2*n* + 1:



Representation of 3*n*, 3*n* + 1 and 3*n* + 2:



Representation of 4*n*, 4*n* + 1, 4*n* + 2 and 4*n* + 3:



One Particular Number can be a Multiple of more than 1 Number—The Concept of Common Multiples

Of course one of the things you should notice as you go through the above discussion is that individual numbers can indeed be multiples of more than 1 number—and actually often are.

Thus, for instance the number 14 is a multiple of both 7 and 2—hence 14 can be called as a common multiple of 2 and 7. Obviously, I think you can visualise more such numbers which can be classified as common multiples of 2 and 7?? 28,42,56 and in fact the list is infinite—i.e. the numbers never end. Thus, the common multiples of 2 and 7 can be represented by the infinite set:

{14,28,42,56,70…………1400……..14000……140000……and so on}

The LCM and It's Significance

From the above list, the number 14 (which is the lowest number in the set of Common multiples of 2 and 7) has a lot of significance in Mathematics. It represents what is commonly known as the **Least Common Multiple (LCM)** of 2 and 7. It is the first number which is a multiple of both 2 and 7 and there are a variety of questions in numeracy which you would come across—not only when you solve questions based on number systems (where the LCM has it's dedicated set of questions), but applications of the LCM are seen even in chapters like Time, Speed Distance, Time and Work etc.

So what did school teach us about the process of finding LCM?

Before you start to review/ relearn that process you first need to know about prime factors of a number.

I hope at this point you recognize the difference between finding factors of a number and prime factors of a number.

Simply put, finding factors of a number means finding the divisors of the number. Thus, for instance the factors of the number 80 would be the numbers 1, 2, 4, 5, 8, 10, 16, 20, 40 and 80. On the other hand finding the prime factors of the same number would mean writing the number 80 as $2^4 \times 5^1$. This form of writing the number is also called the **STANDARD FORM** or the **CANONICAL FORM** of the number.

The school process of writing down the standard form of a number:

Now this is something I would think most of you would remember and recognize:

Finding the prime factors of the number 80

2	80

2	40
2	20
2	10
5	5
	1

The prime factors of the number 80 are: $2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5^1$.

Exercise for Self-practice

Write down the standard form of the following numbers.

(a) 20 (b) 44 (c) 142 (d) 200

(e) 24 (f) 324 (g) 120 (h) 84

(i) 371 (j) 143 (k) 339 (l) 147

(m) 1715

Finding the LCM of two or more numbers: The school process

Step 1: Write down the prime factor form of all the numbers;

Let us say that you have 3 numbers whose standard forms are:

$2^4 \times 3^2 \times 5^3 \times 7^1$

$2^3 \times 3^1 \times 5^2 \times 11^2$

$2^4 \times 3^2 \times 5^1 \times 13^1$

To write down the LCM of these numbers write down all the prime numbers and multiply them with their highest available powers. The resultant number would be the LCM of these numbers.

Thus, in the above case the LCM would be:

$2^4 \times 3^2 \times 5^3 \times 7^1 \times 11^2 \times 13^1$

Note: Short cuts to a lot of these processes have been explained in the main chapters of the book.

Divisors (Factors) of a number

As we already mentioned, finding the factors or divisors of a number are one and the same thing. In order to find factors of a number, the key is to spot the factors below the square root of the number. Once you have found them, the factors above the square root would be automatically seen. Consider this for factors of the number 80:

Factors below the square root *of 80 (8.xxx) Hence, factors up to and including 8*
1
2
4
5
8

Once you can visualise the list on the left, the factors on the right would be seen automatically.

Factors below the square root *of 80 (8.xxx) Hence, factors up to and including 8*	***Factors above the square root***
1	80
2	40
4	20
5	16
8	10
	Note that these will be seen automatically the moment you have the list on the left.

Common Divisors Between 2 Numbers

List of Common Divisors

When we write down the factors of two numbers, we can look for the common elements within the two lists.

For instance, the factors of 80 are (1, 2, 4, 5, 8, 10, 16, 20, 40 and 80) while the factors of 144 are (1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72 and 144)

If you observe the two lists closely you will get the following list of common factors or common divisors between the two:

List of common factors of 80 and 144: (1, 2, 4, 8, 16)

The number 16 being the highest in this list of common factors (or divisors) is also called the Highest Common Factor or the Greatest Common Divisor. In short it is denoted as HCF or GCD. It has a lot of significance in terms of quantitative thinking—as the HCF is used in a multitude of problems and hence being able to spot the HCF of two or more numbers is one of the critical operations in Mathematics. You would be continuously seeing applications of the HCF in problems in the chapter of Number systems and right through the chapters of arithmetic (defined as word based problems in this book).

Rounding off and its use for approximation

One of the important things that we learnt in school was the use of approximation in order to calculate.

Thus, 72 × 53 can be approximated to 70 × 50 and hence seen as 3500.

Similarly, the addition of 48 + 53 + 61 + 89 can be taken as 50 + 50 + 60 + 90 = 250

Number Types You Should Know

Integers and Decimals: All numbers that do not have a decimal in them are called integers. Thus, –3, –17, + 4, + 13, + 1473, 0 etc are all integers.

Obviously, decimal numbers are numbers which have a decimal value attached to them. Thus, 1.3, 14.76, – 12.24 etc are all decimal numbers, since they have certain values after the decimal point.

Before we move ahead, let us pause a brief while, to further understand decimals. As you shall see, the concept of decimals is closely related to the concept of division and divisibility. Suppose, I have 4 pieces of bread which I want to divide equally between two people. It is easy for me to do this, since I can give two whole pieces to each of them.

However, if we alter the situation in such a way, that I now have 5 pieces of bread to distribute equally amongst 2 people. What do I do?

I give two whole pieces each, to each of them. The 5th piece has to be divided equally between the two. I can no longer do this, without in some way breaking the 5th piece into 2 parts. This is the elementary situation that gives rise to the need for decimals in mathematics.

Going back to the situation above, my only option is to divide the 5th piece into two equal parts (which in quants are called as halves).

This concept has huge implications for problem solving especially once you recognise that a half (i.e. a '.5' in the decimal) only comes when you divide a whole into two parts.

Thus, in fact, all standards decimals emerge out of certain fixed divisors.

Hence, for example, the divisor 2, gives rise to the decimal · 5.

Similarly the divisor 3 gives rise to the decimals ·33333 and ·66666, etc.

Prime numbers and Composite numbers Amongst natural numbers, there are three broad divisions–

Unity It is representative of the number 1.

Prime numbers These are numbers which have no divisors/ factors apart from 1 and itself.

Composite numbers On the other hand, are numbers, which have at least one more divisor apart from 1 and itself.

Note: A brief word about factors/ division—A number X is said to divide Y (or is said to be a divisor or factor of Y) when the division of Y/X leaves no remainder.

All composite numbers have the property that they can be written as a product of their prime factors.

Thus, for instance, the number 40 can be represented as: $40 = 2 \times 2 \times 2 \times 5$ or $40 = 2^3 \times 5^1$

This form of writing is called as the **standard form** of the composite number.

The difference between Rational and Irrational numbers: This difference is one of the critical but unfortunately one of the less well understood differences in elementary Mathematics.

The definition of Rational numbers: Numbers which can be expressed in the form p/q where q π 0 are called rational numbers.

Obviously, numbers which cannot be represented in the form p/q are called as irrational numbers.

However, one of the less well understood issues in this regard is what does this mean?

The difference becomes clear when the values of decimals are examined in details:

Consider the following numbers.

(1) 4.2,

(2) 4.333….,

(3) 4.1472576345…….

What is the difference between the decimal values of the three numbers above?

To put it simply, the first number has what can be described as a finite decimal value. Such numbers can be expressed in the form p/q easily. Since 4.2 can be first written as 42/10 and then converted to 21/5.

Similarly, numbers like 4.5732 can be represented as 45732/10000. Thus, numbers having a finite terminating decimal value are rational.

Now, let us consider the decimal value: 4.3333……..

Such decimal values will continue endlessly, i.e. they have no end. Hence, they are called **infinite decimals** (or non-terminating decimals).

But, we can easily see that the number 4.333.... can be represented as 13/3. Hence, this number is also rational. In fact, all numbers which have infinite decimal values, but have any recurring form within them can be represented in the p/q form.

For example the value of the number: 1.14814814814…. is 93/81.

(What I mean to say is that whenever you have any recurring decimal number, even if the value of 'q' might not be obvious, but it will always exist.)

Thus, we can conclude that all numbers whose decimal values are infinite (non-terminating) but which have a recurring pattern within them are rational numbers.

This leaves us with the third kind of decimal values, viz. **Infinite non-recurring decimal values**. These decimals neither have a recurring pattern, nor do they have an end—they go on endlessly. For such numbers it is not possible to find the value of a denominator 'q' which can be used in order to represent them as p/q. Hence, such numbers are called as irrational numbers.

In day-to-day mathematics, we come across numbers like $\sqrt{3}$, $\sqrt{5}$, $\sqrt[3]{7}$, p, e, etc. which are irrational numbers since they do not have a p/q representation.

Note: $\sqrt{3}$ can also be represented as $3^{1/2}$, just as $\sqrt[3]{7}$ can be represented as $7^{1/3}$.

An Important Tip:

Rational and Irrational numbers do not mix.: This means that in case you get a situation where an irrational number has appeared while solving a question, it will remain till the end of the solution. It can only be removed from the solution if it is multiplied or divided by the same irrational number.

Consider an example: The area of an equilateral triangle is given by the formula ($\sqrt{3}$/4) × a^2 (where a is the side of the equilateral triangle). Since, $\sqrt{3}$ is an irrational number, it remains in the answer till the end.

Hence, the area of an equilateral triangle will always have a $\sqrt{3}$ as part of the answer.

Before we move ahead we need to understand one final thing about recurring decimals.

As I have already mentioned, recurring decimals have the property of being able to be represented in the *p*/*q* form. The question that arises is—Is there any process to convert a recurring decimal into a proper fraction?

Yes, there is. In fact, in order to understand how this operates, you first need to understand that there are two kinds of recurring decimals. The process for converting an infinite recurring decimal into a fraction basically varies for both of these types. Let's look at these one by one.

Type 1—Pure recurring decimals: These are recurring decimals where the recurrence starts immediately after the decimal point.

For example:

0.5555… = $0.\overline{5}$

3.242424… = $3.\overline{24}$

5.362362… = $5.\overline{362}$

The process for converting these decimals to fractions can be illustrated as:

0.5555 = 5/9

3.242424 = 3 + (24/99)

5.362362 = 5 + (362/999)

A little bit of introspection will tell you that what we have done is nothing but to put down the recurring part of the decimal as it is and dividing it by a group of 9's. Also the number of 9's in this group equals the number of digits in the recurring part of the decimal.

Thus, in the second case, the fraction is derived by dividing 24 by 99. (24 being the recurring part of the decimal and 99 having 2 nines because the number of digits in 24 is 2.)

Similarly, 0.43576254357625…. = $\frac{4357625}{9999999}$

Type 2—Impure recurring decimals: Unlike pure recurring decimals, in these decimals, the recurrence occurs after a certain number of digits in the decimal. The process to convert these into a fraction is also best illustrated by an example:

Consider the decimal 0.435424242

= $0.435\overline{42}$

The fractional value of the same will be given by: (43542 – 435)/99000. This can be understood in two steps.

Step 1: Subtract the non-recurring initial part of the decimal (in this case, it is 435) from the number formed by writing down the starting digits of the decimal value upto the digit where the recurring decimals are written for the first time;

Expanding the meaning—

Note: For 0.435424242, subtract 435 from 43542

Step 2: The number thus obtained, has to be divided by a number formed as follows; Write down as many 9's as the number of digits in the recurring part of the decimal. (in this case, since the recurring part '42' has 2 digits, we write down 2 9's.) These nines have to be followed by as many zeroes as the number of digits in the non recurring part of the decimal value. (In this case, the non recurring part of the decimal value is '435'. Since, 435 has 3 digits, attach three zeroes to the two nines to get the number to divide the result of the first step.)

Hence divide 43542 – 435 by 99000 to get the fraction.

Similarly, for 3.436213213 we get $\frac{436213-436}{999000}$.

Mixed Fractions

A *mixed number* is a whole number plus a fraction. Here are a few mixed numbers:

$1\frac{1}{2} \quad 1\frac{1}{4} \quad 2\frac{1}{3} \quad 2\frac{2}{7} \quad 5\frac{4}{5}$

In order to convert a mixed fraction to a proper fraction you do the following conversion process.

$1\frac{1}{2}$ = (1 × 2 + 1)/2 = 3/2

$2\frac{1}{3}$ = (2 × 3 + 1)/3 = 7/3

Similiarly, $1\frac{1}{4}$ = 5/4

$2\frac{2}{7}$ = 16/7

$5\frac{4}{5}$ = 29/5 and so on.

i.e. multiply the whole number part of the mixed number by the denominator of the fractional part and add the resultant to the numerator of the fractional part to get the numerator of the proper fraction. The denominator of the proper fraction would be the same as the denominator of the mixed fraction.

OPERATIONS ON NUMBERS

Exponents and Powers

Exponents, or powers, are an important part of math as they are necessary to indicate that a number is multiplied by itself for a given number of times.

When a number is multiplied by itself it gives the ‘square of the number’.

Thus, $n \times n = n^2$ (for example $3 \times 3 = 3^2$)

If the same number is multiplied by itself twice we get the cube of the number.

Thus, $n \times n \times n = n^3$ (for example $3 \times 3 \times 3 = 3^3$)

$n \times n \times n \times n = n^4$ and so on.

With respect to powers of numbers, there are 5 basic rules which you should know:

For any number ‘*n*’ the following rules would apply:

Rule 1: $n^a \times n^b = n^{(a+b)}$. Thus, $4^3 \times 4^5 = 4^8$

Rule 2: $n^a / n^b = n^{a-b}$. Thus, $3^9/3^4 = 3^5$

Rule 3: $(n^a)^b = n^{ab}$. Thus, $(3^2)^4 = 3^8$.

Rule 4: $n^{(-a)} = 1/n^a$. Thus, $3^{-4} = 1/3^4$.

Rule 5: $n^0 = 1$. Thus, $5^0 = 1$.

General Form of Writing 2-3 Digit Numbers

In mathematics many a time we have to use algebraic equations in order to solve questions. In such cases an important concept is the way we represent two or three digit numbers in equation form.

For instance, suppose we have a 2 digit number with the digits ‘*A*B’.

In order to write this in the form of an equation, we have to use:

$10A + B$. This is because in the number ‘*AB*’ the digit *A* is occupying the tens place. Hence, in order to represent the value of the number ‘*AB*’ in the form of an equation- we can write $10A + B$.

Thus, the number $29 = 2 \times 10 + 9 \times 1$

Similarly, for a three digit number with the digits *A*, *B* and *C* respectively – the number ‘*ABC* ’ can be represented as below:

$ABC = 100\,A + 10\,B + C$.

Thus, $243 = 2 \times 100 + 4 \times 10 + 3 \times 1$

The BODMAS Rule: It is used for the ordering of mathematical operations in a mathematical situation:

In any mathematical situation, the first thing to be considered is Brackets followed by Division, Multiplication, Addition and Subtraction in that order.

Thus $3 \times 5 - 2 = 15 - 2 = 13$

Also, $3 \times 5 - 6 \prod 3 = 15 - 2 = 13$

Also, $3 \times (5 - 6) \prod 3 = 3 \times (-1) \prod 3 = -1$.

Operations on Odd and Even numbers

ODD *SEVENS*

Odd × odd	= Odd
Odd + odd	= Even
Odd – odd	= Even
Odd ÷ odd	= odd

Even × Even	= Even
Even + Even	= Even
Even – even	= Even
Even ÷ even	= Even or odd

ODDS & EVENS

Odd × Even	= Even
Odd + Even	= Odd
Odd – Even	= Odd
Even ÷ odd	= Even

Odd ∏ Even → Not divisible

SERIES OF NUMBERS

In many instances in Mathematics we are presented with a series of numbers formed simply when a group of numbers is written together. The following are examples of series:

1. 3, 5, 8, 12, 17…
2. 3, 7, 11, 15, 19…(Such series where the next term is derived by adding a certain fixed value to the previous number are called as Arithmetic Progressions).
3. 5, 10, 20, 40 …..(Such series where the next term is derived by multiplying the previous term by a fixed value are called as Geometric Progressions).

   (**Note**: You will study AP and GP in details in the chapter of progressions which is chapter 2 of this block.)
4. 2, 7, 22, 67 ….
5. 1/3, 1/5, 1/7, 1/9, 1/11…
6. $1/1^2$, $1/2^2$, $1/3^2$, $¼^2$, $1/5^2$…
7. $1/1^3$, $1/3^3$, $1/5^3$…

Remember the following points at this stage:

1. AP and GP are two specific instances of series. They are studied in details only because they have many applications and have defined

rules.

2. Based on the behaviour of their sums, series can be classified as:

Divergent: These are series whose sum to '*n*' terms keeps increasing and reaches infinity for infinite terms.

Convergent: Convergent series have the property that their sum tends to approach an upper limit/lower limit as you include more terms in the series. They have the additional property that even when infinite terms of the series are included they will only reach that value and not cross it.

For example consider the series;

$1/1^2 + 1/3^2 + 1/5^2 + 1/7^2 \ldots$

It is evident that subsequent terms of this series keep getting smaller. Hence, their value becomes negligible after a few terms of the series are taken into account.

If taken to infinite terms, the sum of this series will reach a value which it will never cross. Such series are called convergent, because their sum converges to a limit and only reaches that limit for infinite terms.

Note: Questions on finding infinite sums of convergent series are very commonly asked in most aptitude exams including CAT and XAT.

NOTE TO THE READER: NOW THAT YOU ARE THROUGH WITH THE *BACK TO SCHOOL* SECTION, YOU ARE READY TO PROCEED INTO THE CHAPTERS OF THIS BLOCK. HAPPY SOLVING!!

Chapter 1 Number Systems

INTRODUCTION

The chapter on Number Systems is amongst the most important chapters in the entire syllabus of Quantitative Aptitude for the CAT examination (and also for other parallel MBA entrance exams).Students are advised to go through this chapter with utmost care understanding each concept and question type on this topic. The CAT has consistently contained anything between 20–40% of the marks based on questions taken from this chapter. Naturally, this chapter becomes one of the most crucial as far as your quest to reach close to the qualification score in the section of Quantitative Aptitude and Data Interpretation is concerned.

Hence, going through this chapter and its concepts properly is imperative for you. It would be a good idea to first go through the basic definitions of all types of numbers. Also closely follow the solved examples based on various concepts discussed in the chapter. Also, the approach and attitude while solving questions on this chapter is to try to maximize your learning experience out of every question. Hence, do not just try to solve the questions but also try to think of alternative processes in order to solve the same question. Refer to hints or solutions only as a last resort.

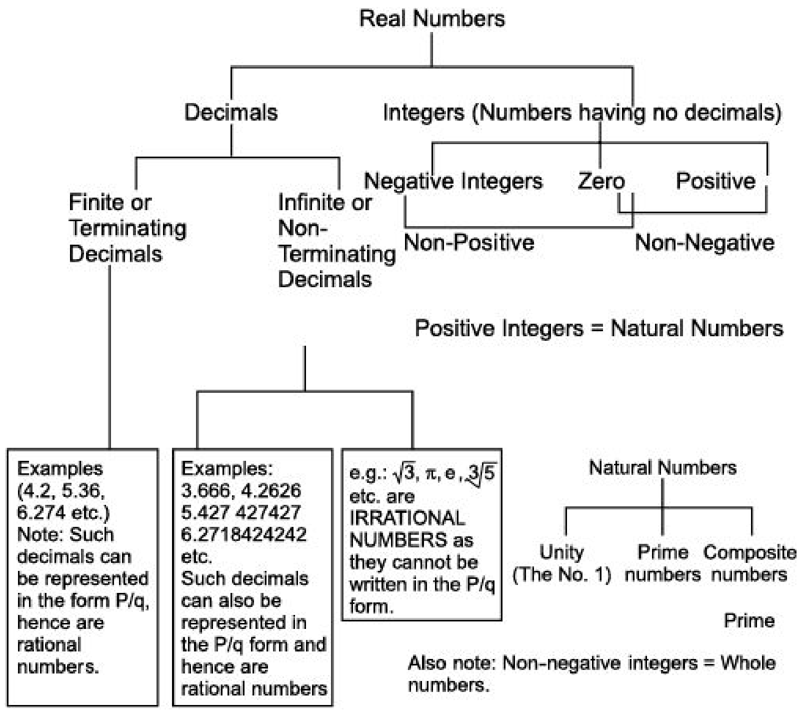
To start off, the following pictorial representation of the types of numbers will help you improve your quality of comprehension of different types of numbers.

DEFINITION

Natural Numbers These are the numbers (1, 2, 3, etc.) that are used for counting. In other words, all positive integers are natural numbers.

There are infinite natural numbers and the number 1 is the least natural number.

Examples of natural numbers: 1, 2, 4, 8, 32, 23, 4321 and so on.



The following numbers are examples of numbers that are not natural: –2, –31, 2.38, 0 and so on.

Based on divisibility, there could be two types of natural numbers: *Prime* and *Composite*.

Prime Numbers A natural number larger than unity is a prime number if it does not have other divisors except for itself and unity.

Note: Unity (i.e. 1) is not a prime number.

Some Properties of Prime Numbers

- The lowest prime number is 2.

- 2 is also the only even prime number.
- The lowest odd prime number is 3.
- The remainder when a prime number $p \geq 5$ is divided by 6 is 1 or 5. However, if a number on being divided by 6 gives a remainder of 1 or 5 the number need not be prime. Thus, this can be referred to as a necessary but not sufficient condition.
- The remainder of the division of the square of a prime number $p \geq 5$ divided by 24 is 1.
- For prime numbers $p > 3$, $p^2 - 1$ is divisible by 24.
- Prime Numbers between 1 to 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
- Prime Numbers between 100 to 200 are: 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199.
- If a and b are any two odd primes then $a^2 - b^2$ is composite. Also, $a^2 + b^2$ is composite.
- The remainder of the division of the square of a prime number $p \geq 5$ divided by 12 is 1.

SHORT CUT PROCESS

To Check Whether a Number is Prime or Not

To check whether a number N is prime, adopt the following process.

(a) Take the square root of the number.

(b) Round of the square root to the immediately lower integer. Call this number z. For example if you have to check for 181, its square root will be 13.____ Hence, the value of z, in this case will be 13.

(c) Check for divisibility of the number N by all prime numbers below z. If there is no prime number below the value of z which divides N then the number N will be prime.

To illustrate :-

The value of $\sqrt{239}$ lies between 15 to 16. Hence, take the value of z as 15.

Prime numbers less than 16 are 2, 3, 5, 7, 11 and 13, 239 is not divisible by any of these. Hence you can conclude that 239 is a prime number.

A Brief Look into why this Works?

Suppose you are asked to find the factors of the number 40.

An untrained mind will find the factors as : 1, 2, 4, 5, 8, 10, 20 and 40.

The same task will be performed by a trained mind as follows:

1×40

2×20

4×10

and 5×8

i.e., The discovery of one factor will automatically yield the other factor. In other words, factors will appear in terms of what can be called as factor pairs. The locating of one factor, will automatically pinpoint the other one for you. Thus, in the example above, when you find 5 as a factor of 40, you will automatically get 8 too as a factor.

Now take a look again at the pairs in the example above. If you compare the values in each pair with the square root of 40 (i.e. 6. ___) you will find that for each pair the number in the left column is lower than the square root of 40, while the number in the right column is higher than the square root of 40.

This is a property for all numbers and is always true.

Hence, we can now phrase this as: Whenever you have to find the factors of any number N, you will get the factors in pairs (i.e. factor pairs). Further, the factor pairs will be such that in each pair of factors, one of the factors will be lower than the square root of N while the other will be higher than the square root of N.

As a result of this fact one need not make any effort to find the factors of a number above the square root of the number. These come automatically. All you need to do is to find the factors below the square root of the number.

Extending this logic, we can say that if we are not able to find a factor of a number upto the value of its square root, we will not be able to find any factor above the square root and the number under consideration will be a prime number. This is the reason why when we need to check whether a number is prime, we have to check for factors only below the square root.

But, we have said that you need to check for divisibility only with the prime numbers below (and including) the square root of the number. What logic will explain this:

Let us look at an example to understand why you need to look only at prime numbers below the square root.

Uptil now, we have deduced that in order to check whether a number is prime, we just need to do a factor search below (and including) the square root.

Thus, for example, in order to find whether 181 is a prime number, we need to check with the numbers = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13.

The first thing you will realise, when you first look at the list above is that all even numbers will get eliminated automatically (since no even number can divide an odd number and of course you will check a number for being prime only if it is odd!)

This will leave you with the numbers 3, 5, 7, 9, 11 and 13 to check 181.

Why do we not need to check with composite numbers below the square root? This will again be understood best if explained in the context of the example above. The only composite number in the list above is 9. You do not need to check with 9, because when you checked N for divisibility with 3 you would get either of two cases:

Case I: If N is divisible by 3: In such a case, N will automatically become non-prime and you can stop your checking. Hence, you will not need to check for the divisibility of the number by 9.

Case II: N is not divisible by 3: If N is not divisible by 3, it is obvious that it will not be divisible by 9. Hence, you will not need to check for the divisibility of the number by 9.

Thus, in either case, checking for divisibility by a composite number (9 in this case) will become useless. This will be true for all composite numbers.

Hence, when we have to check whether a number *N* is prime or not, we need to only check for its divisibility by prime factors below the square root of *N*.

Finding Prime Numbers: The Short Cut

Using the logic that we have to look at only the prime numbers below the square root in order to check whether a number is prime, we can actually cut short the time for finding whether a number is prime drastically.

Before I start to explain this, you should perhaps realise that in an examination like the CAT, or any other aptitude test for that matter whenever you would need to be checking for whether a number is prime or not, you would typically be checking 2 digit or maximum 3 digit numbers in the range of 100 to 200.

Also, one would never really need to check with the prime number 5, because divisibility by 5 would automatically be visible and thus, there is no danger of anyone ever declaring a number like 35 to be prime. Hence, in the list of prime numbers below the square root we would never include 5 as a number to check with.

Checking Whether a Number is Prime (For Numbers below 49)

The only number you would need to check for divisibility with is the number 3. Thus, 47 is prime because it is not divisible by 3.

Checking Whether a Number is Prime (For Numbers above 49 and below 121)

Naturally you would need to check this with 3 and 7. But if you remember that 77, 91 and 119 are not prime, you would be able to spot the prime numbers below 121 by just checking for divisibility with the number 3.

Why? Well, the odd numbers between 49 and 121 which are divisible by 7 are 63, 77, 91, 105 and 119. Out of these perhaps 91 and 119 are the only numbers that you can mistakenly declare as prime. 77 and 105 are so obviously not-prime that you would never be in danger of declaring them prime.

Thus, for numbers between 49 and 121 you can find whether a number is prime or not by just dividing by 3 and checking for its divisibility.

For example:

61, is prime because it is not divisible by 3 and it is neither 91 nor 119.

Checking Whether a Number is Prime (For Numbers above 121 and below 169)

Naturally you would need to check this with 3, 7 and 11. But if you remember that 133,143 and 161 are not prime, you would be able to spot the prime numbers between 121 to 169 by just checking for divisibility with the number 3.

Why? The same logic as explained above. The odd numbers between 121 and 169 which are divisible by either 7 or 11 are 133,143,147,161 and 165. Out of these 133,143 and 161 are the only numbers that you can mistakenly declare as prime if you do not check for 7 or 11. The number 147 would be found to be not prime when you check its divisibility by 3 while the number 165 you would never need to check for, for obvious reasons.

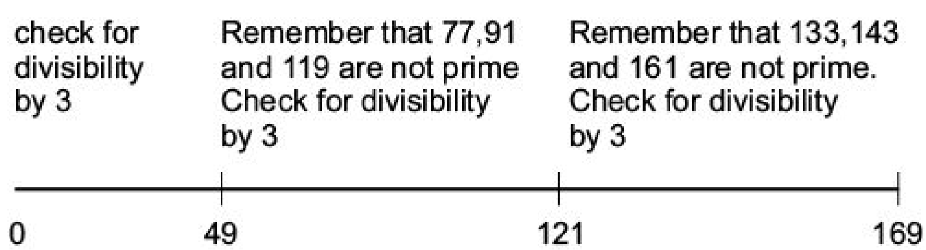
Thus, for numbers between 121 and 169 you can find whether a number is prime or not by just dividing by 3 and checking for its divisibility.

For example:

149, is prime because it is not divisible by 3 and it is neither 133,143 nor 161.

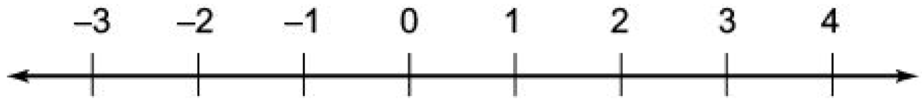
Thus, we have been able to go all the way till 169 with just checking for divisibility with the number 3.

This logic can be represented on the number line as follows:



Integers A set which consists of natural numbers, negative integers (–1, –2, –3…–*n*…) and zero is known as the set of integers. The numbers belonging to this set are known as integers.

Integers can be visualised on the number line:



Note: Positive integers are the same thing as natural numbers.

The moment you define integers, you automatically define **decimals.**

Decimals

A *decimal* number is a number with a *decimal point* in it, like these: 1.5, 3.21, 4.173, 5.1 etc

The number to the left of the decimal is an ordinary whole number. The first number to the right of the decimal is the number of tenths (1/10's). The second is the number of hundredths (1/100's) and so on. So, for the number 5.1, this is a shorthand way of writing the mixed number $5\frac{1}{10}$. 3.27 is the same as 3 + 2/10 + 7/100.

A word on where decimals originate from

Consider the situation where there are 5 children and you have to distribute 10 chocolates between them in such a way that all the chocolates should be distributed and each child should get an equal number of chocolates? How would you do it? Well, simple—divide 10 by 5 to get 2 chocolates per child.

Now consider what if you had to do the same thing with 9 chocolates amongst 5 children? In such a case you would not be able to give an integral number of chocolates to each person. You would give 1 chocolate each to all the 5 and the 'remainder' 4 would have to be divided into 5 parts. 4 out of 5 would give rise to the decimal 0.8 and hence you would give 1.8 chocolates to each child. That is how the concept of decimals enters mathematics in the first place.

Taking this concept further, you can realize that the decimal value of any fraction essentially emerges out of the remainder when the numerator of the fraction is divided by the denominator. Also, since we know that each divisor has a few defined remainders possible, there would be a limited set of decimals that each denominator gives rise to.

Thus, for example the divisor 4 gives rise to only 4 remainders (viz. 0,1,2 and 3) and hence it would give rise to exactly 4 decimal values when it divides any integer. These values are:

0 (when the remainder is 0)

.25 (when the remainder is 1)

.50 (when the remainder is 2)

.75 (when the remainder is 3)

There would be similar connotations for all integral divisors—although the key is to know the decimals that the following divisors give you:

Primary list:

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16

Secondary list:

18, 20, 24, 25, 30, 40, 50, 60, 80, 90, 120

Composite Numbers It is a natural number that has at least one divisor different from unity and itself.

Every composite number *n* can be factored into its prime factors. (This is sometimes called the canonical form of a number.)

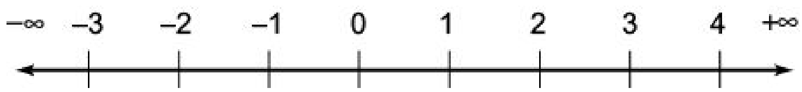
In mathematical terms: $n = p_1^m \, . \, p_2^n \ldots p_k^s$, where p_1, $p_2 \ldots p_k$ are prime numbers called factors and m, $n \ldots k$ are natural numbers.

Thus, $24 = 2^3$ ◊ 3, $84 = 7$ ◊ 3 ◊ 2^2 etc.

This representation of a composite number is known as the standard form of a composite number. It is an extermely useful form of seeing a composite number as we shall see.

Whole Numbers The set of numbers that includes all natural numbers and the number zero are called whole numbers. Whole number are also called as Non-negative integers.

The Concept of the Number Line The number line is a straight line between negative infinity on the left to positive infinity to the right.



The distance between any two points on the number line is got by subtracting the lower value from the higher value. Alternately, we can also start with the lower number and find the required addition to reach the higher number.

For example: The distance between the points 7 and –4 will be $7 – (–4) = 11$.

Real Numbers All numbers that can be represented on the number line are called real numbers. Every real number can be approximately replaced with a terminating decimal.

The following operations of addition, subtraction, multiplication and division are valid for both whole numbers and real numbers: [For any real or whole numbers a, b and c].

(a) Commutative property of addition: $a + b = b + a$.

(b) Associative property of addition: $(a + b) + c = a + (b + c)$.

(c) Commutative property of multiplication: $a \lozenge b = b \lozenge a$.

(d) Associative property of multiplication: $(a \lozenge b) \lozenge c = a \lozenge (b \lozenge c)$.

(e) Distributive property of multiplication with respect to addition: $(a + b) c = ac + bc$.

(f) Subtraction and division are defined as the inverse operations to addition and multiplication respectively.

Thus if $a + b = c$, then $c – b = a$ and if $q = a/b$ then $b \lozenge q = a$ (where b π 0).

Division by zero is not possible since there is no number q for which $b \lozenge q$ equals a non zero number a.

Rational Numbers A rational number is defined as a number of the form a/b where a and b are integers and b π 0.

The set of rational numbers encloses the set of integers and fractions. The rules given above for addition, subtraction, multiplication and division also apply on rational numbers.

Rational numbers that are not integral will have decimal values. These values can be of two types:

(a) **Terminating (or finite) decimal fractions:** For example, 17/4 = 4.25, 21/5 = 4.2 and so forth.

(b) **Non-terminating decimal fractions:** Amongst non- terminating decimal fractions there are two types of decimal values:

(i) *Non-terminating periodic fractions*: These are non- terminating decimal fractions of the type x ◊ $a_1a_2a_3a_4 \ldots a_na_1a_2a_3a_4 \ldots a_na_1a_2a_3a_4 \ldots a_n$. For example $\frac{16}{3}$ = 5.3333, 15.23232323, 14.287628762 876 … and so on.

(ii) *Non-terminating non-periodic fractions*: These are of the form x ◊ $b_1b_2b_3b_4 \ldots b_nc_1c_2c_3 \ldots c_n$. For example: 5.2731687143725186....

Of the above categories, terminating decimal and non-terminating periodic decimal fractions belong to the set of rational numbers.

Irrational Numbers Fractions, that are non-terminating, non-periodic fractions, are irrational numbers.

Some examples of irrational numbers are $\sqrt{2}$, $\sqrt{3}$ etc. In other words, all square and cube roots of natural numbers that are not squares and cubes of natural numbers are irrational. Other irrational numbers include p, e and so on.

Every positive irrational number has a negative irrational number corresponding to it.

All operations of addition, subtraction, multiplication and division applicable to rational numbers are also applicable to irrational numbers.

As briefly stated in the Back to School section, whenever an expression contains a rational and an irrational number together, the two have to be carried together till the end. In other words, an irrational number once it appears in the solution of a question will continue to appear till the end of the question. This concept is particularly useful in Geometry For example: If you are asked to find the ratio of the area of a circle to that of an equilateral triangle, you can expect to see a $p / \sqrt{3}$ in the answer. This is because the area of a circle will always have a *p* component in it, while that of an equilateral triangle will always have $\sqrt{3}$.

You should realise that once an irrational number appears in the solution of a question, it can only disappear if it is multiplied or divided by the same irrational number.

THE CONCEPT OF GCD (GREATEST COMMON DIVISOR OR HIGHEST COMMON FACTOR)

Consider two natural numbers n, and n_2.

If the numbers n_1 and n_2 are exactly divisible by the same number x, then x is a common divisor of n_1 and n_2.

The highest of all the common divisors of n_1 and n_2 is called as the GCD or the HCF. This is denoted as GCD (n_1, n_2).

Rules for Finding the GCD of Two Numbers n_1 and n_2

(a) Find the standard form of the numbers n_1 and n_2.

(b) Write out all prime factors that are common to the standard forms of the numbers n_1 and n_2.

(c) Raise each of the common prime factors listed above to the lesser of the powers in which it appears in the standard forms of the numbers n_1 and n_2.

(d) The product of the results of the previous step will be the GCD of n_1 and n_2.

Illustration: Find the GCD of 150, 210, 375.

Step 1: Writing down the standard form of numbers

$150 = 5 \times 5 \times 3 \times 2$

$210 = 5 \times 2 \times 7 \times 3$

$375 = 5 \times 5 \times 5 \times 3$

Step 2: Writing Prime factors common to all the three numbers is $5^1 \times 3^1$

Step 3: This will give the same result, i.e. $5^1 \times 3^1$

Step 4: Hence, the HCF will be $5 \times 3 = 15$

For practice, find the HCF of the following:

(a) 78, 39, 195

(b) 440, 140, 390

(c) 198, 121, 1331

SHORTCUT FOR FINDING THE HCF

The above 'school' process of finding the HCF (or the GCD) of a set of numbers is however extremely cumbersome and time taking. Let us take a look at a much faster way of finding the HCF of a set of numbers.

Suppose you were required to find the HCF of 39,78 and 195 (first of the practice problems above)

Logic The HCF of these numbers would necessarily have to be a factor (divisor) of the difference between any pair of numbers from the above 3. i.e. the HCF has to be a factor of (78 – 39 = 39) as well as of (195 – 39 = 156) and (195 – 78 = 117). Why?

Well the logic is simple if you were to consider the tables of numbers on the number line.

For any two numbers on the number line, a common divisor would be one which divides both. However, for any number to be able to divide both the numbers, it can only do so if it is a factor of the difference between the two numbers. Got it??

Take an example:

Let us say we take the numbers 68 and 119. The difference between them being 51, it is not possible for any number outside the factor list of 51 to divide both 68 and 119. Thus, for example a number like 4, which divides 68 can never divide any number which is 51 away from 68- because 4 is not a factor of 51.

Only factors of 51, i.e. 51,17,3 and 1 ***'could'*** divide both these numbers simultaneously.

Hence, getting back to the HCF problem we were trying to tackle—take the difference between any two numbers of the set—of course if you want to reduce your calculations in the situation, take the difference between the two closest numbers. In this case that would be the difference between 78 and 39 =39.

The HCF has then to be a factor of this number. In order to find the factors quickly remember to use the fact we learnt in the back to school section of this part of the book—that whenever we have to find the list of factors/divisors for any number we have to search the factors below the square root and the factors above the square root would be automatically visible)

A factor search of the number 39 yields the following factors:

1 × 39

3 × 13

Hence, one of these 4 numbers has to be the HCF of the numbers 39,78 and 195. Since we are trying to locate the **Highest** common factor—we would begin our search from the highest number (viz:39)

Check for divisibility by 39 Any one number out of 39 and 78 and also check the number 195 for divisibility by 39. You would find all the three numbers are divisible by 39 and hence 39 can be safely taken to be the correct answer for the HCF of 39,78 and 195.

Suppose the numbers were:

39, 78 and 182?

The HCF would still be a factor of 78–39=39. The probable candidates for the HCF's value would still remain 1,3,13 and 39.

When you check for divisibility of all these numbers by 39, you would realize that 182 is not divisible and hence 39 would not be the HCF in this case.

The next check would be with the number 13. It can be seen that 13 divides 39 (hence would automatically divide 78- no need to check that) and also divides 182. Hence, 13 would be the required HCF of the three numbers.

Typical questions where HCF is used directly

Question 1: The sides of a hexagonal field are 216, 423, 1215, 1422, 2169 and 2223 meters. Find the greatest length of tape that would be able to exactly measure each of these sides without having to use fractions/parts of the tape?

In this question we are required to identify the HCF of the numbers 216,423,1215, 1422, 2169 and 2223.

In order to do that, we first find the smallest difference between any two of these numbers. It can be seen that the difference between 2223-2169 = 54. Thus, the required HCF would be a factor of the number 54.

The factors of 54 are:

1×54

2×27

3×18

6×9

One of these 8 numbers has to be the HCF of the 6 numbers. 54 cannot be the HCF because the numbers 423 and 2223 being odd numbers would not be divisible by any even number. Thus, we do not need to check any even numbers in the list.

27 does not divide 423 and hence cannot be the HCF. 18 can be skipped as it is even.

Checking for 9:

9 divides 216,423,1215,1422 and 2169. Hence, it would become the HCF. (Note: we do not need to check 2223 once we know that 2169 is divisible by 9)

Question 2: A nursery has 363,429 and 693 plants respectively of 3 distinct varieties. It is desired to place these plants in straight rows of plants of 1 variety only so that the number of rows required is the minimum. What is the size of each row and how many rows would be required?

The size of each row would be the HCF of 363, 429 and 693. Difference between 363 and 429 =66. Factors of 66 are 66, 33, 22, 11, 6, 3, 2, 1.

66 need not be checked as it is even and 363 is odd. 33 divides 363, hence would automatically divide 429 and also divides 693. Hence, 33 is the correct answer for the size of each row.

For how many rows would be required we need to follow the following process:

Minimum number of rows required = 363/33 + 429/33 + 693/33 = 11 + 13 + 21 = 45 rows.

THE CONCEPT OF LCM (LEAST COMMON MULTIPLE)

Let n_1, and n_2 be two natural numbers distinct from each other. The smallest natural number n that is exactly divisible by n_1 and n_2 is called the Least Common Multiple (LCM) of n_1 and n_2 and is designated as LCM (n_1, n_2).

Rule for Finding the LCM of two Numbers n_1 and n_2

(a) Find the standard form of the numbers n_1 and n_2.

(b) Write out all the prime factors, which are contained in the standard forms of either of the numbers.

(c) Raise each of the prime factors listed above to the highest of the powers in which it appears in the standard forms of the numbers n_1 and n_2.

(d) The product of results of the previous step will be the LCM of n_1 and n_2.

Illustration: Find the LCM of 150, 210, 375.

Step 1: Writing down the standard form of numbers

150 = 5 × 5 × 3 × 2

210 = 5 × 2 × 7 × 3

375 = 5 × 5 × 5 × 3

Step 2: Write down all the prime factors: that appear at least once in any of the numbers: 5, 3, 2, 7.

Step 3: Raise each of the prime factors to their highest available power (considering each to the numbers).

The LCM = $2^1 \times 3^1 \times 5^3 \times 7^1 = 5250$.

Important Rule:

GCD (n_1, n_2). LCM $(n_1, n_2) = n_1 \Diamond n_2$

i.e. The product of the HCF and the LCM equals the product of the numbers.

SHORT CUT FOR FINDING THE LCM

The LCM (least common multiple) again has a much faster way of doing it than what we learnt in school.

The process has to do with the use of co-prime numbers.

Before we look at the process, let us take a fresh look at what co-prime numbers are:

Co-prime numbers are any two numbers which have an HCF of 1, i.e. when two numbers have no common prime factor apart from the number 1, they are called co-prime or relatively prime to each other.

Some Rules for Co-primes

2 Numbers being co-prime

(i) Two consecutive natural numbers are always co-prime (Example 5, 6; 82, 83; 749, 750 and so on)

(ii) Two consecutive odd numbers are always co-prime (Examples: 7, 9; 51, 53; 513, 515 and so on)

(iii) Two prime numbers are always co-prime (Examples: 13, 17; 53, 71 and so on)

(iv) One prime number and another composite number (such that the composite number is not a multiple of the prime number) are always co-prime (Examples: 17, 38; 23, 49 and so on, but note that 17 and 51 are not co-prime)

3 or more numbers being co-prime with each other means that all possible pairs of the numbers would be co-prime with each other.

Thus, 47, 49, 51 and 52 are co-prime since each of the 6 pairs (47,49); (47,51); (47,52); (49,51); (49,52) and (51,52) are co-prime.

Rules for Spotting three Co-prime Numbers

(i) Three consecutive odd numbers are always co-prime (Examples: 15, 17, 19; 51, 53, 55 and so on)

(ii) Three consecutive natural numbers with the first one being odd (Examples: 15, 16, 17; 21, 22, 23; 41, 42, 43 and so on). Note that 22, 23, 24 are not co-prime

(iii) Two consecutive natural numbers along-with the next odd number such that the first no. is even (examples: 22, 23, 25; 52, 53, 55; 68, 69, 71 and so on)

(iv) Three prime numbers (Examples: 17, 23, 29; 13, 31, 43 and so on)

So what do co-prime numbers have to do with LCMs?

By using the logic of co-prime numbers, you can actually bypass the need to take out the prime factors of the set of numbers for which you are trying to find the LCM. How?

The following process will make it clear:

Let us say that you were trying to find the LCM of 9,10,12 and 15.

The LCM can be directly written as: $9 \times 10 \times 2$. The thinking that gives you the value of the LCM is as follows:

Step 1: If you can see a set of 2 or more co-prime numbers in the set of numbers for which you are finding the LCM- write them down by multiplying them.

So in the above situation, since we can see that 9 and 10 are co-prime to each other we can start off writing the LCM by writing 9 × 10 as the first step.

Step 2: For each of the other numbers, consider what part of them have already been taken into the answer and what part remains outside the answer. In case you see any part of the other numbers such that it is not a part of the value of the LCM you are writing—such a part would need to be taken into the answer of the LCM.

The process will be clear once you see what we do (and how we think) with the remaining 2 numbers in the above problem.

At this point when we have written down 9 × 10 we already have taken into account the numbers 9 and 10 leaving us to account for 12 and 15.

Thought about 12: 12 is 2 × 2 × 3

9 × 10 already has a 3 and one 2 in it prime factors. However, the number 12 has two 2's. This means that one of the two 2's of the number 12 is still not accounted for in our answer. Hence, we need to modify the LCM by multiplying the existing 9 × 10 by a 2. With this change the LCM now becomes:

9 × 10 × 2

Thought about 15: 15 is 5 × 3

9 × 10 × 2 already has a 5 and a 3. Hence, there is no need to add anything to the existing answer.

Thus, 9 × 10 × 2 would become the correct answer for the LCM of the numbers 9, 10, 12 and 15.

What if the numbers were: 9, 10, 12 and 25

Step 1: 9 and 10 are co-prime

Hence, the starting value is 9 × 10

Thought about 12: 12 is 2 × 2 × 3

9 × 10 already has a 3 and one 2 in it prime factors. However, the number 12 has two 2's. This means that one of the two 2's of the number 12 is still not accounted for in our answer. Hence, we need to modify the LCM by multiplying the existing 9 × 10 by a 2. With this change the LCM now becomes:

$9 \times 10 \times 2$

Thought about 25: 25 is 5×5

$9 \times 10 \times 2$ has only one 5. Hence, we need to add another 5 to the answer.

Thus, $9 \times 10 \times 2 \times 5$ would become the correct answer for the LCM of the numbers 9, 10, 12 and 25.

Rule for Finding out HCF and LCM of Fractions

(A) HCF of two or more fractions is given by:

$$\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

(B) LCM of two or more fractions is given by:

$$\frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

Typical questions on LCMs

You would be able to see most of the standard questions on LCMs in the practice exercise on HCF and LCM given below.

Rules for HCF: If the HCF of x and y is G, then the HCF of

(i) x, $(x + y)$ is also G

(ii) x, $(x - y)$ is also G

HCF and LCM

Practice Exercise

(Typical questions asked in Exams)

1. Find the common factors for the numbers.

   (a) 24 and 64

   (b) 42, 294 and 882

   (c) 60, 120 and 220

2. Find the HCF of

(a) 420 and 1782

(b) 36 and 48

(c) 54, 72, 198

(d) 62, 186 and 279

3. Find the LCM of

(a) 13, 23 and 48

(b) 24, 36, 44 and 62

(c) 22, 33, 45, and 72

(d) 13, 17, 21 and 33

4. Find the series of common multiples of

(a) 54 and 36

(b) 33, 45 and 60

[*Hint*: Find the LCM and then create an Arithmetic progression with the first term as the LCM and the common difference also as the LCM.]

5. The LCM of two numbers is 936. If their HCF is 4 and one of the numbers is 72, the other is:

(a) 42

(b) 52

(c) 62

(d) None of these

[Answer: (b). Use HCF × LCM = product of numbers.]

6. Two alarm clocks ring their alarms at regular intervals of 50 seconds and 48 seconds. If they first beep together at 12 noon, at what time will they beep again for the first time?

(a) 12:10 P.M.

(b) 12:12 P.M.

(c) 12:11 P.M.

(d) None of these

[Answer: (d). The LCM of 50 and 48 being 1200, the two clocks will ring again after 1200 seconds.]

7. 4 Bells toll together at 9:00 A.M. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours?

(a) 3

(b) 4

(c) 5

(d) 6

[Answer: (c). The LCM of 7, 8, 11 and 12 is 1848. Hence, the answer will be got by the quotient of the ratio (10800)/(1848) Æ 5.]

8. On Ashok Marg three consecutive traffic lights change after 36, 42 and 72 seconds respectively. If the lights are first switched on at 9:00 A.M. sharp, at what time will they change simultaneously?

(a) 9 : 08 : 04 (b) 9 : 08 : 24

(c) 9 : 08 : 44 (d) None of these

[Answer (b). The LCM of 36,42 and 72 is 504. Hence, the lights will change simultaneously after 8 minutes and 24 seconds.]

9. The HCF of 2472, 1284 and a third number '*N*' is 12. If their LCM is $2^3 \times 3^2 \times 5^1 \times 103 \times 107$, then the number '*N*' is:

(a) $2^2 \times 3^2 \times 7^1$ (b) $2^2 \times 3^3 \times 103$

(c) $2^2 \times 3^2 \times 5^1$ (d) None of these

[Answer: (c)]

10. Two equilateral triangles have the sides of lengths 34 and 85 respectively.

(a) The greatest length of tape that can measure both of them exactly is:

[Answer: HCF of 34 and 85 is 17.]

(b) How many such equal parts can be measured?

[Answer: $\frac{34}{17} \times 3 + \frac{85}{17} \times 3 = 2 \times 3 + 5 \times 3 = 21$]

11. Two numbers are in the ratio 17:13. If their HCF is 15, what are the numbers?

(Answer: 17 × 15 and 13 × 15 i.e. 255 and 195 respectively.) [Note : This can be done when the numbers are co-prime.]

12. A forester wants to plant 44 apple trees, 66 banana trees and 110 mango trees in equal rows (in terms of number of trees). Also, he wants to make distinct rows of trees (i.e. only one type of tree in one row). The number of rows (minimum) that are required are:

(a) 2 (b) 3

(c) 10 (d) 11

[Answer: (c) 44/22 + 66/22 + 110/22 (Since 22 is the HCF)]

13. Three runners running around a circular track can complete one revolution in 2, 4 and 5.5 hours respectively. When will they meet at the starting point?

(a) 22 (b) 33

(c) 11 (d) 44

(The answer will be the LCM of 2, 4 and 11/2. This will give you 44 as the answer).

14. The HCF and LCM of two numbers are 33 and 264 respectively. When the first number is divided by 2, the quotient is 33. The other number is?

(a) 66 (b) 132

(c) 198 (d) 99

[Answer: $33 \times 264 = 66 \times n$. Hence, $n = 132$]

15. The greatest number which will divide: 4003, 4126 and 4249:

(a) 43 (b) 41

(c) 45 (d) None of these

The answer will be the HCF of the three numbers. (1 in this case)

16. Which of the following represents the largest 4 digit number which can be added to 7249 in order to make the derived number divisible by each of 12, 14, 21, 33, and 54.

(a) 9123 (b) 9383

(c) 8727 (d) None of these

[Answer: The LCM of the numbers 12, 14, 21, 33 and 54 is 8316. Hence, in order for the condition to be satisfied we need to get the number as:

$7249 + n = 8316 \times 2$

Hence, $n = 9383$.]

17. Find the greatest number of 5 digits, that will give us a remainder of 5, when divided by 8 and 9 respectively.

(a) 99931 (b) 99941

(c) 99725 (d) None of these

[Answer: The LCM of 8 and 9 is 72. The largest 5 digit multiple of 72 is 99936. Hence, the required answer is 99941.]

18. The least perfect square number which is divisible by 3, 4, 6, 8, 10 and 11 is:

Solution: The number should have at least one 3, three 2's, one 5 and one 11 for it to be divisible by 3, 4, 6, 8, 10 and 11.

Further, each of the prime factors should be having an even power in order to be a perfect square. Thus, the correct answer will be: $3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 11 \times 11$

19. Find the greatest number of four digits which when divided by 10, 11, 15 and 22 leaves 3, 4, 8 and 15 as remainders respectively.

(a) 9907 (b) 9903

(c) 9893 (d) None of these

[Answer: First find the greatest 4 digit multiple of the LCM of 10, 11, 15 and 22. (In this case it is 9900). Then, subtract 7 from it to give the answer.]

20. Find the HCF of $(3^{125} – 1)$ and $(3^{35} – 1)$.

[Answer: The solution of this question is based on the rule that:

The HCF of $(a^m – 1)$ and $(a^n – 1)$ is given by $(a^{\text{HCF of m, n}} – 1)$

Thus, in this question the answer is: $(3^5 – 1)$. Since 5 is the HCF of 35 and 125.]

21. What will be the least possible number of the planks, if three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length?

(a) 7 (b) 8

(c) 22 (d) None of these

22. Find the greatest number, which will divide 215, 167 and 135 so as to leave the same remainder in each case.

(a) 64 (b) 32

(c) 24 (d) 16

23. Find the L.C.M of 2.5, 0.5 and 0.175

(a) 2.5 (b) 5

(c) 7.5 (d) 17.5

24. The L.C.M of 4.5; 0.009; and 0.18 = ?

(a) 4.5 (b) 45

(c) 0.225 (d) 2.25

25. The L.C.M of two numbers is 1890 and their H.C.F is 30. If one of them is 270, the other will be

(a) 210 (b) 220

(c) 310 (d) 320

26. What is the smallest number which when increased by 6 is divisible by 36, 63 and 108?

(a) 750 (b) 752

(c) 754 (d) 756

27. The smallest square number, which is exactly divisible by 2, 3, 4, – 9, 6, 18, 36 and 60, is

(a) 900 (b) 1600

(c) 3600 (d) None of these

28. The H.C.F of two numbers is 11, and their L.C.M is 616. If one of the numbers is 88, find the other.

(a) 77 (b) 87

(c) 97 (d) None of these

29. What is the greatest possible rate at which a man can walk 51 km and 85 km in an exact number of minutes?

(a) 11 km/min (b) 13 km/min

(c) 17 km/min (d) None of these

30. The HCF and LCM of two numbers are 12 and 144 respectively. If one of the numbers is 36, the other number is

(a) 4 (b) 48

(c) 72 (d) 432

ANSWER KEY

21. (c)	22. (d)	23. (d)	24. (a)	25. (a)
26. (a)	27. (a)	28. (a)	29. (c)	30. (b)

Solutions

4. (a) The first common multiple is also the LCM. The LCM of 36 and 54 would be 108. The next common multiple would be 216, 324 and so on. Thus, the required series would be 108, 216, 324, 432, 540, 648….

   (b) The LCM of 33, 45 and 60 = 60 × 3 × 11 = 1980. Thus, the required series is: 1980, 3960, 5940…

5. LCM × HCF = $936 \times 4 = N_1 \times N_2$ Æ

   $936 \times 4 = 72 \times N_2$ Æ $N_2 = 13 \times 4 = 52$. Option (b) is correct.

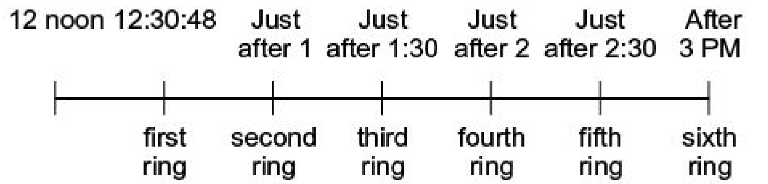
6. The first bell toll will be after the time elapsed is the LCM of 50 and 48. The LCM of 50 and 48 is 50 × 24 = 1200. Hence, the first time they would toll together after 12 noon would be exactly 1200 seconds or 20 minutes later. Option (d) is correct.

7. The LCM of 7, 8, 11 and 12 is given by 12 × 11 × 2 × 7 = 264 × 7 = 1848. 1848 seconds is 30 minutes 48 seconds. Hence, the 4 bells would toll together every 30 minutes 48 seconds.

The number of times they would toll together in the next 3 hours would be given by the quotient of the division:

$3 \times 60 \times 60 / 1848$ Æ 5 times

Alternately, by thinking of 1848 seconds as 30 minutes 48 seconds you can also solve the same question by thinking as follows:



Since the sixth ring is after 3 PM, we can say that the bells would toll 5 times in the next 3 hours

9. $2472 = 2^3 \times 103 \times 3$; $1284 = 2^2 \times 107 \times 3$. Since the HCF is 12, the number must have a component of $2^2 \times 3^1$ at the very least in it. Also, since the LCM is $2^3 \times 3^2 \times 5^1 \times 103 \times 107$ we can see that the minimum requirement in the required number has to be $3^2 \times 5^1$. Combining these two requirements we get that the number should have $2^2 \times 3^2 \times 5^1$ at the minimum and the power of 2 could also be 2^3 while we could also have either one of 103^1 and/or 107^1 as a part of the required number.

   Thus, for instance, the number could also be $2^3 \times 3^2 \times 5^1 \times 103^1 \times 107^1$. The question has asked us- what ***‘could’*** the number be?

   Option (c) gives us a possible value of the number and is hence the correct answer.

21. The least possible number of planks would occur when we divide each plank into a length equal to the HCF of 42, 49 and 63. The HCF of these numbers is clearly 7- and this should be the size of each plank. Number of planks in this case would be: $42/7 + 49/7 + 63/7 = 6 + 7 + 9 = 22$ planks. Hence, option (c) is correct.

22. Trial and error would give us that the number 16 would leave the same remainder of 7 in all the three cases. Hence, option (d) is correct.

23. The numbers are 5/2, ½ and 175/1000 = 7/40. The LCM of three fractions is given by the formula:

    LCM of numerators/HCF of denominators = (LCM of 5, 1 and 7)/(HCF of 2 and 40) = 35/2 = 17.5

24. Use the same process as for question 23 for the numbers: 9/2; 9/1000 and 9/50.

    (LCM of 9, 9, 9)/ (HCF of 2, 100, 50) = 9/2 = 4.5

25. 1890 × 30 = 270 × N_2 Æ N_2 = 210. Hence, option (a) is correct.

26. The LCM of 36, 63 and 108 is 756. Hence, the required number is 750. Option (a) is correct.

27. The LCM of the given numbers is 180. Hence, all multiples of 180 would be divisible by all of these numbers. Checking the series 180, 360, 540, 720, 900 we can see that 900 is the first perfect square in the list. Option (a) is correct.

28. Using the property HCF × LCM = product of the numbers, we get:

    616 × 11 = 88 × N_2 Æ N_2 = 77. Option (a) is correct.

29. The answer would be given by the HCF of 51 and 85 – which is 17. Hence, option (c) is correct.

30. Using the property HCF × LCM = product of the numbers, we get:

    12 × 144 = 36 × N_2 Æ N_2 = 48. Option (b) is correct.

DIVISIBILITY

A number *x* is said to be divisible by another number '*y*' if it is completely divisible by *Y* (i.e. it should leave no remainder).

In general it can be said that any integer *I*, when divided by a natural number *N*, there exist a unique pair of numbers *Q* and *R* which are called the quotient and Remainder respectively.

Thus, $I = QN + R$.

For any integer *I* and any natural number *n* there is a unique pair of numbers *a* and *b* such that:

$I = QN + R$

Where Q is an integer and N is a natural number or zero and $0 \geq R < N$ (i.e. remainder has to be a whole number less than N.)

If the remainder is zero we say that the number I is divisible by N.

When $R \pi 0$, we say that the number I is divisible by N with a remainder.

Thus, 25/8 can be written as: $25 = 3 \times 8 + 1$ (3 is the quotient and 1 is the remainder)

While, –25/7 will be written as $-25 = 7 \times (-4) + 3$ (–4 is the quotient and 3 is the remainder)

Note: An integer $b \pi 0$ is said to divide an integer a if there exists another integer c such that:

$a = bc$

It is important to explain at this point a couple of concepts with respect to the situation, when we divide a negative number by a natural number N.

Suppose, we divide – 32 by 7. Contrary to what you might expect, the remainder in this case is + 3 (and not – 4). This is because the remainder is always non negative.

Thus, –32/7 gives quotient as – 5 and remainder as + 3.

The relationship between the remainder and the decimal:

1. Suppose we divide 42 by 5. The result has a quotient of 8 and remainder of 2.

   But 42/5 = 8.4. As you can see, the answer has an integer part and a decimal part. The integer part being 8 (equals the quotient), the decimal part is 0.4 (and is given by 2/5).

   Since, we have also seen that for any divisor N, the set of remainders obeys the inequality $0 £ R < N$, we should realise that any divisor N, will yield exactly N possible remainders. (For example If the divisor is 3, we have 3 possible remainders 0, 1 and 2. Further, when 3 is the divisor we can have only 3 possible decimal values .00, .333 & 0.666 corresponding to remainders of 0, 1 or 2. I would want you to remember this concept when you study the fraction to percentage conversion table in the chapter on percentages.

2. In the case of –42 being divided by 5, the value is – 8.4. In this case the interpretation should be thus:

   The integer part is – 9 (which is also the quotient of this division) and the decimal part is 0.6 (corresponding to 3/5) Notice that since the remainder cannot be negative, the decimal too cannot be negative.

Theorems of Divisibility

(a) If a is divisible by b then ac is also divisible by b.

(b) If a is divisible by b and b is divisible by c then a is divisible by c.

(c) If a and b are natural numbers such that a is divisible by b and b is divisible by a then $a = b$.

(d) If n is divisible by d and m is divisible by d then $(m + n)$ and $(m - n)$ are both divisible by d. This has an important implication. Suppose 28 and 742 are both divisible by 7. Then (742 + 28) as well as (742 – 28) are divisible by 7. (and in fact so is + 28 – 742).

(e) If a is divisible by b and c is divisible by d then ac is divisible by bd.

(f) The highest power of a prime number p, which divides $n!$ exactly is given by

$$[n/p] + [n/p^2] + [n/p^3] + \dots$$

where $[x]$ denotes the greatest integer less than or equal to x.

As we have already seen earlier –

Any composite number can be written down as a product of its prime factors. (Also called standard form)

Thus, for example the number 1240 can be written as $2^3 \times 31^1 \times 5^1$.

The standard form of any number has a huge amount of information stored in it. The best way to understand the information stored in the standard form of a number is to look at concrete examples. As a reader I want you to understand each of the processes defined below and use them to solve similar questions given in the exercise that follows and beyond:

1. **Using the standard form of a number to find the sum and the number of factors of the number:**

(a) **Sum of factors of a number:**

Suppose, we have to find the sum of factors and the number of factors of 240.

$240 = 2^4 \times 3^1 \times 5^1$

The sum of factors will be given by:

$(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1)(5^0 + 5^1)$

$= 31 \times 4 \times 6 = 744$

Note: This is a standard process, wherein you create the same number of brackets as the number of distinct prime factors the number contains and then each bracket is filled with the sum of all the powers of the respective prime number starting from 0 to the highest power of that prime number contained in the standard form.

Thus, for 240, we create 3 brackets—one each for 2, 3 and 5. Further in the bracket corresponding to 2 we write $(2^0 + 2^1 + 2^2 + 2^3 + 2^4)$.

Hence, for example for the number $40 = 2^3 \times 5^1$, the sum of factors will be given by: $(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$ {2 brackets since 40 has 2 distinct prime factors 2 and 5}

(b) **Number of factors of the number:**

Let us explore the sum of factors of 40 in a different context.

$(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$

$= 2^0 \times 5^0 + 2^0 \times 5^1 + 2^1 \times 5^0 + 2^1 \times 5^1 + 2^2 \times 5^0 + 2^2 \times 5^1 + 2^3 \times 5^0 + 2^3 \times 5^1$

$= 1 + 5 + 2 + 10 + 4 + 20 + 8 + 40 = 90$

A clear look at the numbers above will make you realize that it is nothing but the addition of the factors of 40

Hence, we realise that the number of terms in the expansion of $(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$ will give us the number of factors of 40. Hence, 40 has $4 \times 2 = 8$ factors.

Note: The moment you realise that $40 = 2^3 \times 5^1$ the answer for the number of factors can be got by $(3 + 1)(1 + 1) = 8$

2. Sum and Number of even and odd factors of a number.

Suppose, you are trying to find out the number of factors of a number represented in the standard form by: $2^3 \times 3^4 \times 5^2 \times 7^3$

As you are already aware the answer to the question is $(3 + 1)(4 + 1)(2 + 1)(3 + 1)$ and is based on the logic that the number of terms will be the same as the number of terms in the expansion: $(2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$.

Now, suppose you have to find out the sum of the even factors of this number. The only change you need to do in this respect will be evident below. The answer will be given by:

$(2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$

Note: We have eliminated 2^0 from the original answer. By eliminating 2^0 from the expression for the sum of all factors you are ensuring that you have only even numbers in the expansion of the expression.

Consequently, the number of even factors will be given by: $(3)(4 + 1)(2 + 1)(3 + 1)$

i.e. Since 2^0 is eliminated, we do not add 1 in the bracket corresponding to 2.

Let us now try to expand our thinking to try to think about the number of odd factors for a number.

In this case, we just have to do the opposite of what we did for even numbers. The following step will make it clear:

Odd factors of the number whose standard form is : $2^3 \times 3^4 \times 5^2 \times 7^3$

Sum of odd factors = $(2^0)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$

i.e.: Ignore all powers of 2. The result of the expansion of the above expression will be the complete set of odd factors of the number.

Consequently, the number of odd factors for the number will be given by the number of terms in the expansion of the above expression.

Thus, the number of odd factors for the number $2^3 \times 3^4 \times 5^2 \times 7^3 = 1 \times (4 + 1)\ (2 + 1)\ (3 + 1)$.

3. **Sum and number of factors satisfying other conditions for any composite number**

These are best explained through examples:

(i) Find the sum and the number of factors of 1200 such that the factors are divisible by 15.

Solution : $1200 = 2^4 \times 5^2 \times 3^1$.

For a factor to be divisible by 15 it should compulsorily have 3^1 and 5^1 in it. Thus, sum of factors divisible by 15 = $(2^0 + 2^1 + 2^2 + 2^3 + 2^4) \times (5^1 + 5^2)\ (3^1)$ and consequently the number of factors will be given by $5 \times 2 \times 1 = 10$.

(What we have done is ensure that in every individual term of the expansion, there is a minimum of $3^1 \times 5^1$. This is done by removing powers of 3 and 5 which are below 1.)

Task for the student: Physically verify the answers to the question above and try to convert the logic into a mental algorithm.

NOTE FROM THE AUTHOR—The need for thought algorithms:

I have often observed that the key difference between understanding a concept and actually applying it under examination pressure, is the presence or absence of a mental thought algorithm which clarifies the concept to you in your mind. The thought algorithm is a personal representation of a concept—and any concept that you read/understand in this book (or elsewhere) will remain an external concept till it remains in someone else's words. The moment the thought becomes internalised the concept becomes yours to apply and use.

Practice Exercise on Factors

For the number 2450 find.

1. The sum and number of all factors.
2. The sum and number of even factors.
3. The sum and number of odd factors.
4. The sum and number of factors divisible by 5
5. The sum and number of factors divisible by 35.
6. The sum and number of factors divisible by 245.

For the number 7200 find.

7. The sum and number of all factors.
8. The sum and number of even factors.
9. The sum and number of odd factors.
10. The sum and number of factors divisible by 25.
11. The sum and number of factors divisible by 40.
12. The sum and number of factors divisible by 150.
13. The sum and number of factors not divisible by 75.
14. The sum and number of factors not divisible by 24.
15. Find the number of divisors of 1728.

    (a) 18 (b) 30

    (c) 28 (d) 20
16. Find the number of divisors of 1080 excluding the divisors, which are perfect squares.

    (a) 28 (b) 29

    (c) 30 (d) 31
17. Find the number of divisors of 544 excluding 1 and 544.

    (a) 12 (b) 18

    (c) 11 (d) 10
18. Find the number of divisors 544 which are greater than 3.

(a) 15 (b) 10

(c) 12 (d) None of these.

19. Find the sum of divisors of 544 excluding 1 and 544.

(a) 1089 (b) 545

(c) 589 (d) 1134

20. Find the sum of divisors of 544 which are perfect squares.

(a) 32 (b) 64

(c) 42 (d) 21

21. Find the sum of odd divisors of 544.

(a) 18 (b) 34

(c) 68 (d) 36

22. Find the sum of even divisors of 4096.

(a) 8192 (b) 6144

(c) 8190 (d) 6142

23. Find the sum the sums of divisors of 144 and 160.

(a) 589 (b) 781

(c) 735 (d) None of these

24. Find the sum of the sum of even divisors of 96 and the sum of odd divisors of 3600.

(a) 639 (b) 735

(c) 651 (d) 589

ANSWER KEY

15. (c)	16. (a)	17. (d)	18. (b)	19. (c)
20. (d)	21. (a)	22. (c)	23. (b)	24. (c)

Solutions

Solutions to Questions 1 to 6:

$2450 = 50 \times 49 = 2^1 \times 5^2 \times 7^2$

1. Sum and number of all factors:

   Sum of factors = $(2^0 + 2^1)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2)$

   Number of factors = $2 \times 3 \times 3 = 18$

2. Sum of all even factors:

   $(2^1)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2)$

   Number of even factors = $1 \times 3 \times 3 = 9$

3. Sum of all odd factors:

   $(2^0)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2)$

   Number of odd factors = $1 \times 3 \times 3 = 9$

4. Sum of factors divisible by 5:

   $(2^0 + 2^1)(5^1 + 5^2)(7^0 + 7^1 + 7^2)$

   Number of factors divisible by 5 = $2 \times 2 \times 3 = 12$

5. Sum of factors divisible by 35:

   $(2^0 + 2^1)(5^1 + 5^2)(7^1 + 7^2)$

   Number of factors divisible by 35 = $2 \times 2 \times 2 = 8$

6. Sum of all factors divisible by 245:

   $(2^0 + 2^1)(5^1 + 5^2)(7^2)$

   Number of factors divisible by 245 = $2 \times 2 \times 1 = 4$

Solutions to Questions 7 to 14:

$7200 = 72 \times 100 = 12 \times 6 \times 100 = 2^5 \times 3^2 \times 5^2$

7. **Sum and number of all factors:**

   Sum of factors = $(2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2)$

   Number of factors = $6 \times 3 \times 3 = 54$

8. **Sum and number of even factors:**

   Sum of even factors = $(2^1 + 2^2 + 2^3 + 2^4 + 2^5)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2)$

   Number of even factors = $5 \times 3 \times 3 = 45$

9. **Sum and number of odd factors:**

Sum of odd factors = $(2^0)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2)$

Number of odd factors = $1 \times 3 \times 3 = 9$

10. **Sum and number of factors divisible by 25:**

Sum of factors divisible by 25= $(2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)(3^0 + 3^1 + 3^2)(5^2)$

Number of factors divisible by 25 = $6 \times 3 \times 1 = 18$

11. **Sum and number of factors divisible by 40:**

Sum of factors divisible by 40 = $(2^3 + 2^4 + 2^5)(3^0 + 3^1 + 3^2)(5^1 + 5^2)$

Number of factors = $3 \times 3 \times 2 = 18$

12. **Sum and number of factors divisible by 150:**

Sum of factors divisible by 150 = $(2^1 + 2^2 + 2^3 + 2^4 + 2^5)(3^1 + 3^2)(5^2)$

Number of factors divisible by 150 = $5 \times 2 \times 1 = 10$

13. **Sum and number of factors not divisible by 75:**

Sum of factors not divisible by 75 = Sum of all factors – Sum of factors divisible by 75 =

$(2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2) - (2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)(3^1 + 3^2)(5^2)$

Number of factors not divisible by 75 = Number of factors of 7200 – Number of factors of 7200 which are divisible by 75 = $6 \times 3 \times 3 - 6 \times 2 \times 1 = 54 - 12 = 42$

14. **Sum and number of factors not divisible by 24:**

Sum of factors not divisible by 24 = Sum of all factors – Sum of factors divisible by 24 =

$(2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2) - (2^3 + 2^4 + 2^5)(3^1 + 3^2)(5^0 + 5^1 + 5^2)$

Number of factors not divisible by 24 = Number of factors of 7200 – Number of factors of 7200 which are divisible by 24 = $6 \times 3 \times 3 - 3 \times 2 \times 3 = 54 - 18 = 36$

15. **Number of divisors of 1728**

$1728 = 4 \times 432 = 16 \times 108 = 64 \times 27 = 2^6 \times 3^3$

Number of factors $= 7 \times 4 = 28$. Option (a) is correct.

16. $1080 = 108 \times 10 = 27 \times 4 \times 10 = 3^3 \times 2^3 \times 5^1$

Number of factors $= 4 \times 4 \times 2 = 32$.

In order to see the number of factors of 1080 which are perfect squares we need to visualize the structure for writing down the sum of perfect square factors of 1080.

This would be given by:

Sum of all perfect square factors of 1080 $= (2^0 + 2^2)(3^0 + 3^2)(5^0)$.

From the above structure it is clear that the number of perfect square factors is going to be $2 \times 2 \times 1 = 4$

Thus, the number of factors of 1080 which are not perfect squares are equal to $32 - 4 = 28$.

Option (a) is correct.

17. $544 = 17^1 \times 2^5$. Hence, the total number of factors of 544 is $2 \times 6 = 12$. But we have to count factors excluding 1 and 544. Thus, we need to remove 2 factors from this. The required answer is $12 - 2 = 10$. Option (d) is correct.

18. Using the fact that 544 has a total of 12 factors and the numbers 1 and 2 are the two factors which are lower than 3, we would get a total of 10 factors greater than 3. Option (b) is correct.

19. The required answer would be given by: Sum of all factors of 544 – 1 – 544 $= (2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)(17^0 + 17^1) - 545 = 63 \times 18 - 545 = 589$. Option (c) is correct.

20. Sum of divisors of 544 which are perfect square is:

$(2^0 + 2^2 + 2^4)(17^0) = 21$. Option (d) is correct.

21. Sum of odd divisors of 544 =

$(2^0)(17^0 + 17^1) = 18$. Option (a) is correct.

22. $4096 = 2^{12}$.

Sum of even divisors $= (2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9 + 2^{10} + 2^{11} + 2^{12}) = 2^{13} - 2 = 8190$

23. $144 = 2^4 \times 3^2$ Æ Sum of divisors of 144 = $(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1 + 3^2) = 31 \times 13 = 403$

$160 = 2^5 \times 5^1$ Æ Sum of divisors of 160 = $(2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)(5^0 + 5^1) = 63 \times 6 = 378$.

Sum of the two = 403 + 378 = 781.

24. $96 = 2^5 \times 3^1$. Sum of even divisors of 96 = $(2^1 + 2^2 + 2^3 + 2^4 + 2^5)(3^0 + 3^1) = 62 \times 4 = 248$

$3600 = 2^4 \times 5^2 \times 3^2$. Sum of odd divisors of 3600 = $(2^0)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2) = 13 \times 31 = 403$

Sum of the two = 248 + 403 = 651.

Option (c) is correct.

NUMBER OF ZEROES IN AN EXPRESSION

Suppose you have to find the number of zeroes in a product: $24 \times 32 \times 17 \times 23 \times 19 = (2^3 \times 3^1) \times (2^5) \times 17^1 \times 23 \times 19$.

As you can notice, this product will have no zeroes because it has no 5 in it.

However, if you have an expression like: $8 \times 15 \times 23 \times 17 \times 25 \times 22$

The above expression can be rewritten in the standard form as:

$2^3 \times 3^1 \times 5^1 \times 23 \times 17 \times 5^2 \times 2^1 \times 11^1$

Zeroes are formed by a combination of 2×5. Hence, the number of zeroes will depend on the number of pairs of 2's and 5's that can be formed.

In the above product, there are four twos and three fives. Hence, we shall be able to form only three pairs of (2×5). Hence, there will be 3 zeroes in the product.

Finding the Number of Zeroes in a Factorial Value

Suppose you had to find the number of zeroes in 6!.

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = (3 \times 2) \times (5) \times (2 \times 2) \times (3) \times (2) \times (1)$.

The above expression will have only one pair of 5×2, since there is only one 5 and an abundance of 2's.

It is clear that in any factorial value, the number of 5's will always be lesser than the number of 2's. Hence, all we need to do is to count the number of 5's. The process for this is explained in Solved Examples 1.1 to 1.3.

Exercise for Self-practice

Find the number of zeroes in the following cases:

1. 47!
2. 58!
3. 13 × 15 × 22 × 125 × 44 × 35 × 11
4. 12 × 15 × 5 × 24 × 13 × 17
5. 173!
6. 144! × 5 × 15 × 22 × 11 × 44 × 135
7. 148!
8. 1093!
9. 1132!
10. 1142! × 348! × 17!

Solutions

1. 47/5 Æ Quotient 9. 9/5 Æ Quotient Æ 1. 9 + 1 = 10 zeroes.
2. 58/5 Æ Quotient 11. 11/5 Æ Quotient Æ 2. 11 + 2 = 13 zeroes.
3. The given expression has five 5's and three 2's. Thus, there would be three zeroes in the expression.
4. The given expression has two 5's and five 2's. Thus, there would be two zeroes in the expression.
5. 173/5 Æ Quotient 34. 34/5 Æ Quotient 6. 6/5 Æ Quotient 1. 34 + 6 + 1 = 41 zeroes.
6. 144! Would have 28 + 5 + 1 = 34 zeroes and the remaining part of the expression would have three zeroes. A total of 34 + 3 = 37 zeroes.
7. 148/5 Æ Quotient 29. 29/5 Æ Quotient 5. 5/5 Æ Quotient 1. 29 + 5 + 1 = 35 zeroes.

8. 1093/5 Æ Quotient 218. 218/5 Æ Quotient 43. 43/5 Æ Quotient 8. 8/5 Æ Quotient 1. 218 + 43 + 8 + 1 = 270 zeroes.
9. 1132/5 Æ Quotient 226. 226/5 Æ Quotient 45. 45/5 Æ Quotient 9. 9/5 Æ Quotient 1. 226 + 45 + 9 + 1 = 281 zeroes.
10. 1142/5 Æ Quotient 228. 228/5 Æ Quotient 45. 45/5 Æ Quotient 9. 9/5 Æ Quotient 1. 228 + 45 + 9 + 1 = 284 zeroes.

    348/5 Æ Quotient 69. 69/5 Æ Quotient 13. 13/5 Æ Quotient 2. 69 + 13 + 2 = 84 zeroes.

    17/5 Æ Quotient 3 Æ 3 zeroes.

    Thus, the total number of zeroes in the expression is: 284 + 84 + 3 = 371 zeroes.

A special implication: Suppose you were to find the number of zeroes in the value of the following factorial values:

45!, 46!, 47!, 48!, 49!

What do you notice? The number of zeroes in each of the cases will be equal to 10. Why does this happen? It is not difficult to understand that the number of fives in any of these factorials is equal to 10. The number of zeroes will only change at 50! (It will become 12).

In fact, this will be true for all factorial values between two consecutive products of 5.

Thus, 50!, 51!, 52!, 53! And 54! will have 12 zeroes (since they all have 12 fives).

Similarly, 55!, 56!, 57!, 58! And 59! will each have 13 zeroes.

Apart from this fact, did you notice another thing? That while there are 10 zeroes in 49! there are directly 12 zeroes in 50!. This means that there is no value of a factorial which will give 11 zeroes. This occurs because to get 50! we multiply the value of 49! by 50. When you do so, the result is that we introduce two 5’s in the product. Hence, the number of zeroes jumps by two (since we never had any paucity of twos.)

Note: at 124! you will get 24 + 4 fi 28 zeroes.

At 125! you will get 25 + 5 + 1 = 31 zeroes. (A jump of 3 zeroes.)

Exercise for Self-practice

1. n! has 23 zeroes. What is the maximum possible value of n?
2. n! has 13 zeroes. The highest and least values of n are?
3. Find the number of zeroes in the product $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \times \ldots. \times 49^{49}$
4. Find the number of zeroes in:

   $100^1 \times 99^2 \times 98^3 \times 97^4 \times \ldots\ldots\ldots. \times 1^{100}$
5. Find the number of zeroes in:

   $1^{1!} \times 2^{2!} \times 3^{3!} \times 4^{4!} \times 5^{5!} \times \ldots\ldots..10^{10!}$
6. Find the number of zeroes in the value of:

   $2^2 \times 5^4 \times 4^6 \times 10^8 \times 6^{10} \times 15^{12} \times 8^{14} \times 20^{16} \times 10^{18} \times 25^{20}$
7. What is the number of zeroes in the following:

   (a) 3200 + 1000 + 40000 + 32000 + 15000000

   (b) 3200 × 1000 × 40000 × 32000 × 16000000

Solutions

1. This can never happen because at 99! number of zeroes is 22 and at 100! the number of zeroes is 24.
2. 59 and 55 respectively.
3. The fives will be less than the twos. Hence, we need to count only the fives.

   Thus : $5^5 \times 10^{10} \times 15^{15} \times 20^{20} \times 25^{25} \times 30^{30} \times 35^{35} \times 40^{40} \times 45^{45}$

   gives us: 5 + 10 + 15 + 20 + 25 + 25 + 30 + 35 + 40 + 45 fives. Thus, the product has 250 zeroes.
4. Again the key here is to count the number of fives. This can get done by:

   $100^1 \times 95^6 \times 90^{11} \times 85^{16} \times 80^{21} \times 75^{26} \times \ldots\ldots 5^{96}$

   (1 + 6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 + …….. + 96) + (1 + 26 + 51 + 76)

= 20 × 48.5 + 4 × 38.5 (Using sum of A.P. ex-plained in the next chapter.)

= 970 + 154 = 1124.

5. The answer will be the number of 5's. Hence, it will be 5! + 10!

6. The number of fives is again lesser than the number of twos.

   The number of 5's will be given by the power of 5 in the product:

   $5^4 \times 10^8 \times 15^{12} \times 20^{16} \times 10^{18} \times 25^{20}$

   = 4 + 8 + 12 + 16 + 18 + 40 = 98.

7. A. The number of zeroes in the sum will be two, since:

$$\begin{array}{r} 3200 \\ 1000 \\ 40000 \\ 32000 \\ 15076200 \\ 15152400 \end{array}$$

   Thus, in such cases the number of zeroes will be the least number of zeroes amongst the numbers.

   Exception: 3200 + 1800 = 5000 (three zeroes, not two).

   B. The number of zeroes will be:

   2 + 3 + 4 + 3 + 6 = 18.

An extension of the process for finding the number of zeroes.

Consider the following questions:

1. Find the highest power of 5 which is contained in the value of 127!
2. When 127! is divided by 5^n the result is an integer. Find the highest possible value for *n*.
3. Find the number of zeroes in 127!

In each of the above cases, the value of the answer will be given by:

[127/5] + [127/25] + [127/125]

= 25 + 5 + 1 = 31

This process can be extended to questions related to other prime numbers. For example:

Find the highest power of:

1. 3 which completely divides 38!

   Solution: [38/3] + [38/3^2] + 38/3^3] = 12 + 4 + 1 = 17

2. 7 which is contained in 57!

   [57/7] + [57/7^2] = 8 + 1 = 9.

This process changes when the divisor is not a prime number. You are first advised to go through worked out problems 1.4, 1.5, 1.6 and 1.19.

Now try to solve the following exercise:

1. Find the highest power of 7 which divides 81!
2. Find the highest power of 42 which divides 122!
3. Find the highest power of 84 which divides 342!
4. Find the highest power of 175 which divides 344!
5. Find the highest power of 360 which divides 520!

Solutions

1. 81/7 Æ Quotient 11. 11/7 Æ Quotient 1. Highest power of 7 in 81! = 11 + 1 =12.
2. In order to check for the highest power of 42, we need to realize that 42 is 2 × 3 × 7. In 122! The least power between 2,3 and 7 would obviously be for 7. Thus, we need to find the number of 7's in 122! (or in other words- the highest power of 7 in 122!).

   This can be done by:

   122/7 Æ Quotient 17. 17/7 Æ Quotient 2. Highest power of 7 in 122! = 17 + 2 = 19.
3. 84 = 2 × 2 × 3 × 7. This means we need to think of which amongst 2^2, 3 and 7 would appear the least number of times in 342! It is evident that there would be more 2^2s and 3's than 7's in any factorial value (Because if you write any factorial 1 × 2 × 3 × 4 × 5 × 6 × **7** × 8 × 9 × 10 × 11 × 12 × 13 × **14** × 15…you can clearly see that

before a 7 or it's multiple appears in the multiplication, there are at least two 2's and one 3 which appear beforehand.)

Hence, in order to solve this question we just need to find the power of 7 in 342!

This can be done as:

342/7 Æ Quotient 48. 48/7 Æ Quotient 6. 6/7 Æ Quotient 0. Highest power of 7 in 342! = 48 + 6 = 54.

4. 175 = 5 × 5 × 7. This means we need to think of which amongst 5^2 and 7 would appear the least number of times in 175! In this case it is not immediately evident that whether there would be more 5^2s or more 7's in any factorial value (Because if you write any factorial 1 × 2 × 3 × 4 × 5 × 6 × **7** × 8 × 9 × 10 × 11 × 12 × 13 × 14 × 15…you can clearly see that although the 5's appear more frequently than the 7's it is not evident that we would have at least two fives before the 7 appears.) Hence, in this question we would need to check for both the number of 5^2s and the number of 7's.

   Number of 7's in 344!

   344/7 Æ Quotient 49. 49/7 Æ Quotient 7. 7/7 Æ Quotient 1. Highest power of 7 in 344! = 49 + 7 + 1 = 57.

   In order to find the number of 5^2s in 344! We first need to find the number of 5's in 344!

   344/5 Æ Quotient 68. 68/5 Æ Quotient 13. 13/5 Æ Quotient 2. Number of 5's in 344! = 68 + 13 + 2 = 83.

   83 fives would obviously mean [83/2] = 41 5^2s

   Thus, there are 41 5^2s and 57 7's in 344!

   Since, the number of 5^2s are lower, they would determine the highest power of 175 that would divide 344!

   The answer is 41.

5. 360 = 5 × 2 × 2 × 2 × 3 × 3. This means we need to think of which amongst 2^3, 3^2 and 5 would appear the least number of times in 520! In this case it is not immediately evident which of these three would

appear least number of times. Hence, in this question we would need to check for all three – 2^3s 3^2s and 5s.

Number of 5's in 520!

520/5 Æ Quotient 104. 104/5 Æ Quotient 20. 20/5 Æ Quotient 4. Highest power of 5 in 520! = 104 + 20 + 4 = 128.

In order to find the number of 3^2s in 520! we first need to find the number of 3's in 520!

520/3 Æ Quotient 173. 173/3 Æ Quotient 57. 57/3 Æ Quotient 19. 19/3 Æ Quotient 6. 6/3 Æ Quotient 2. 2/3 Æ Quotient 0. Number of 3's in 520! = 173 + 57 + 19 + 6 + 2 = 257.

257 threes would obviously mean [257/2]= 128 3^2s.

In order to find the number of 2^3s in 520! we first need to find the number of 2's in 520!

520/2 Æ Quotient 260. 260/2 Æ Quotient 130. 130/2 Æ Quotient 65. 65/2 Æ Quotient 32. 32/2 Æ Quotient 16. 16/2 Æ Quotient 8. 8/2 Æ Quotient 4. 4/2 Æ Quotient 2. 2/2 Æ Quotient 1. 1/2 Æ Quotient 0.

Number of 2's in 520! = 260 + 130 +65 + 32 +16 + 8 + 4 + 2 + 1 = 518. 518 twos would obviously mean [518/3]= 172 2^3s.

Thus, there are 128 3^2s, 128 5's and 172 2^3's in 520!

The highest power of 360 that would divide 520! would be the least of 128, 128 and 172.

The answer is 128.

Exercise for Self-practice

1. Find the maximum value of n such that 157! is perfectly divisible by 10^n.

   (a) 37 (b) 38

   (c) 16 (d) – 1.15

2. Find the maximum value of n such that 157! is perfectly divisible by 12^n.

   (a) 77 (b) 76

   (c) 75 (d) 78

3. Find the maximum value of n such that 157! is perfectly divisible by 18^n.

   (a) 37 (b) 38

   (c) 39 (d) 40

4. Find the maximum value of n such that 50! is perfectly divisible by 2520^n.

   (a) 6 (b) 8

   (c) 7 (d) None of these

5. Find the maximum value of n such that 50! is perfectly divisible by 12600^n.

   (a) 7 (b) 6

   (c) 8 (d) None of these

6. Find the maximum value of n such that 77! is perfectly divisible by 720^n.

   (a) 35 (b) 18

(c) 17 (d) 36

7. Find the maximum value of n such that

$42 \times 57 \times 92 \times 91 \times 52 \times 62 \times 63 \times 64 \times 65 \times 66 \times 67$ is perfectly divisible by 42^n.

(a) 4 (b) 3

(c) 5 (d) 6

8. Find the maximum value of n such that

$570 \times 60 \times 30 \times 90 \times 100 \times 500 \times 700 \times 343 \times 720 \times 81$ is perfectly divisible by 30^n.

(a) 12 (b) 11

(c) 14 (d) 13

9. Find the maximum value of *n* such that

$77 \times 42 \times 37 \times 57 \times 30 \times 90 \times 70 \times 2400 \times 2402 \times 243 \times 343$ is perfectly divisible by 21^n.

(a) 9 (b) 11

(c) 10 (d) 6

Find the number of consecutive zeroes at the end of the following numbers.

10. 72!

(a) 17 (b) 9

(c) 8 (d) 16

11. $77! \times 42!$

(a) 24 (b) 9

(c) 27 (d) 18

12. $100! + 200!$

(a) 73 (b) 24

(c) 11 (d) 22

13. $57 \times 60 \times 30 \times 15625 \times 4096 \times 625 \times 875 \times 975$

(a) 6 (b) 16

(c) 17 (d) 15

14. $1! \times 2! \times 3! \times 4! \times 5! \times$ - - - - - - - - $\times 50!$

(a) 235 (b) 12

(c) 262 (d) 105

15. $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \times 7^7 \times 8^8 \times 9^9 \times 10^{10}$.

(a) 25 (b) 15

(c) 10 (d) 20

16. $100! \times 200!$

(a) 49 (b) 73

(c) 132 (d) 33

ANSWER KEY

1. (b)	2. (c)	3. (a)	4. (b)	5. (b)
6. (c)	7. (b)	8. (b)	9. (d)	10. (d)
11. (c)	12. (b)	13. (d)	14. (c)	15. (b)
16. (b)				

Solutions

1. [157/5] = 31. [31/5] = 6. [6/5] = 1. 31 + 6 + 1 = 38. Option (b) is correct.

2. No of 2's in 157! = [157/2] + [157/4] + [157/8]… + [157/128] = 78 + 39 + 19 + 9 + 4 + 2 + 1 = 152. Hence, the number of 2^2s would be [152/2] = 76.

   Number of 3's in 157! = 52 + 17 + 5 +1 = 75.

   The answer would be given by the lower of these values. Hence, 75 (Option c) is correct.

3. From the above solution:

   Number of 2's in 157! = 152

Number of 3^2s in 157! = [75/2]=37.

Hence, option (a) is correct.

4. $2520 = 7 \times 3^2 \times 2^3 \times 5$.

   The value of n would be given by the value of the number of 7s in 50!

   This value is equal to [50/7] + [50/49] = 7 + 1 = 8

   Option (b) is correct.

5. $12600 = 7 \times 3^2 \times 2^3 \times 5^2$

   The value of '*n*' would depend on which of number of 7s and number of 5^2s is lower in 50!.

   Number of 7's in 50! = 8. Note here that if we check for 7's we do not need to check for 3^2s as there would be at least two 3's before a 7 comes in every factorial's value. Similarly, there would always be at least three 2's before a 7 comes in any factorial's value. Thus, the number of 3^2s and the number of 2^3s can never be lower than the number of 7s in any factorial's value.

   Number of 5s in 50! = 10 + 2 =12. Hence, the number of 5^2s in 50! = [12/2] = 6.

   6 will be the answer as the number of 5^2s is lower than the number of 7's.

   Option (b) is correct.

6. $720 = 2^4 \times 5^1 \times 3^2$

   In 77! there would be 38 + 19 + 9 + 4 + 2 + 1 = 73 twos Æ hence [73/4] = 18 2^4s

   In 77! there would be 25 + 8 + 2 = 35 threes Æ hence [35/2] = 17 3^2s

   In 77! there would be 15 + 3 = 18 fives

   Since 17 is the least of these values, option (c) is correct.

7. In the expression given, there are three 7's and more than three 2's and 3's. Thus, Option (b) is correct.

8. Checking for the number of 2’s, 3’s and 5’s in the given expression you can see that the minimum is for the number of 3’s (there are 11 of them while there are 12 5’s and more than 11 2’s) Hence, option (b) is correct.
9. The number of 7’s in the number is 6, while there are six 3’s too. Option (d) is correct.
10. The number of zeroes would depend on the number of 5’s in the value of the factorial.

    72! Æ 14 + 2 = 16. Option (d) is correct.
11. The number of zeroes would depend on the number of 5’s in the value of the factorial.

    77! × 42! Æ 15 + 3 = 18 (for 77!) and 8 + 1 = 9 (for 42!).

    Thus, the total number of zeroes in the given expression would be 18 + 9 = 27. Option (c) is correct.
12. The number of zeroes would depend on the number of 5’s in the value of the factorial.

    100! would end in 20 + 4 = 24 zeroes

    200! Would end in 40 + 8 + 1 = 49 zeroes.

    When you add the two numbers (one with 24 zeroes and the other with 49 zeroes at it’s end), the resultant total would end in 24 zeroes. Option (b) is correct.
13. The given expression has fifteen 2’s and seventeen 5’s. The number of zeroes would be 15 as the number of 2’s is lower in this case. Option (d) is correct.
14. 1! to 4! would have no zeroes while 5! to 9! All the values would have 1 zero. Thus, a total of 5 zeroes till 9!. Going further 10! to 14! would have two zeroes each – so a total of 10 zeroes would come out of the product of 10! × 11! × 12! × 13! × 14!.

    Continuing this line of thought further we get:

    Number of zeroes between 15! × 16!... × 19! = 3 + 3 + 3 + 3 + 3 = 3 × 5 = 15

    Number of zeroes between 20! × 21!... × 24! = 4 × 5 = 20

    Number of zeroes between 25! × 26!... × 29! = 6 × 5 = 30

Number of zeroes between 30! × 31!... × 34! = 7 × 5 = 35

Number of zeroes between 35! × 36!... × 39! = 8 × 5 = 40

Number of zeroes between 40! × 41!... × 44! = 9 × 5 = 45

Number of zeroes between 45! × 46!... × 49! = 10 × 5 = 50

Number of zeroes for 50! = 12

Thus, the total number of zeroes for the expression 1! × 2! × 3! …. × 50! = 5 + 10 +15 + 20 + 30 + 35 + 40 + 45 + 50 + 12 = 262 zeroes. Option (c) is correct.

15. The number of 5's is `15 while the number of 2's is much more. Option (b) is correct.

16. The number of zeroes would depend on the number of 5's in the value of the factorial.

    100! would end in 20 + 4 = 24 zeroes

    200! Would end in 40 + 8 + 1 = 49 zeroes.

    When you multiply the two numbers (one with 24 zeroes and the other with 49 zeroes at it's end), the resultant total would end in 24 + 49 = 73 zeroes. Option (b) is correct.

Co-Prime or Relatively Prime Numbers Two or more numbers that do not have a common factor are known as co-prime or relatively prime. In other words, these numbers have a highest common factor of unity.

If two numbers *m* and *n* are relatively prime and the natural number *x* is divisible by both *m* and *n* independently then the number *x* is also divisible by *mn*.

Key Concept 1: The spotting of two numbers as co-prime has a very important implication in the context of the two numbers being in the denominators of fractions.

The concept is again best understood through an example:

Suppose, you are doing an operation of the following format – *M*/8 + *N*/9 where *M* & *N* are integers.

What are the chances of the result being an integer, if *M* is not divisible by 8 and *N* is not divisible by 9? A little bit of thought will make you realise that the chances are zero. The reason for this is that 8 and 9 are co-prime and the decimals of co-prime numbers never match each other.

Note: this will not be the case in the case of:

$M/3 + N/27$.

In this case even if 3 and 27 are not dividing M and N respectively, there is a possibility of the values of M and N being such that you have an integral answer.

For instance: 5/3 + 36/27 = 81/27 = 3

The result will never be integral if the two denominators are co-prime.

Note: This holds true even for expressions of the nature $A/7 – B/6$ etc.

This has huge implications for problem solving especially in the case of solving linear equations related to word based problems. Students are advised to try to use these throughout Blocks I, II and III of this book.

Example: Find all five-digit numbers of the form 34 x 5Y that are divisible by 36.

Solution: 36 is a product of two co-primes 4 and 9. Hence, if 34 x 5y is divisible by 4 and 9, it will also be divisible by 36. Hence, for divisibility by 4, we have that the value of y can be 2 or 6. Also, if y is 2 the number becomes 34 x 52. For this to be divisible by 9, the addition of $3 + 4 + x + 5 + 2$ should be divisible by 9. For this x can be 4.

Hence the number 34452 is divisible by 36.

Also for $y = 6$, the number 34 x 56 will be divisible by 36 when the addition of the digits is divisible by 9. This will happen when x is 0 or 9. Hence, the numbers 34056 and 34956 will be divisible by 36.

Exercise for Self-practice

Find all numbers of the form 56x3y that are divisible by 36.

Find all numbers of the form 72xy that are divisible by 45.

Find all numbers of the form 135xy that are divisible by 45.

Find all numbers of the form 517xy that are divisible by 89.

Divisibility Rules

Divisibility by 2 or 5: A number is divisible by 2 or 5 if the last digit is divisible by 2 or 5.

Divisibility by 3 (or 9): All such numbers the sum of whose digits are divisible by 3 (or 9) are divisible by 3 (or 9).

Divisibility by 4: A number is divisible by 4 if the last 2 digits are divisible by 4.

Divisibility by 6: A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

Divisibility by 8: A number is divisible by 8 if the last 3 digits of the number are divisible by 8.

Divisibility by 11: A number is divisible by 11 if the difference of the sum of the digits in the odd places and the sum of the digits in the even places is zero or is divisible by 11.

Divisibility by 12: All numbers divisible by 3 and 4 are divisible by 12.

Divisibility by 7, 11 or 13: The integer *n* is divisible by 7, 11 or 13 if and only if the difference of the number of its thousands and the remainder of its division by 1000 is divisible by 7, 11 or 13.

For Example: 473312 is divisible by 7 since the difference between 473 – 312 = 161 is divisible by 7.

Even Numbers: All integers that are divisible by 2 are even numbers. They are also denoted by $2n$.

Example: 2, 4, 6, 12, 122, –2, –4, –12.

Also note that zero is an even number.

2 is the lowest positive even number.

Odd Numbers: All integers that are not divisible by 2 are odd numbers. Odd number leave a remainder of 1 on being divided by 2. They are denoted by $2n + 1$ or $2n – 1$.

Lowest positive odd number is 1.

Example: –1, –3, –7, –35, 3, 11, etc.

Complex Numbers: The arithmetic combination of real numbers and imaginary numbers are called complex numbers.

Alternately: All numbers of the form $a + ib$, where $i = \div -1$ are called complex number.

Twin Primes: A pair of prime numbers are said to be twin prime when they differ by 2.

Example: 3 and 5 are Twin Primes, so also are 11 and 13.

Perfect Numbers: A number n is said to be a perfect number if the sum of all the divisors of n (including n) is equal to $2n$.

Example: $6 = 1 \times 2 \times 3$ sum of the divisors $= 1 + 2 + 3 + 6 = 12 = 2 \times 6$

$28 = 1, 2, 4, 7, 14, 28, = 56 = 2 \times 28$

Task for student: Find all perfect numbers below 1000.

Mixed Numbers: A number that has both an integral and a fractional part is known as a mixed number.

Triangular Numbers: A number which can be represented as the sum of consecutive natural numbers starting with 1 are called as triangular numbers.

e.g.: $1 + 2 + 3 + 4 = 10$. So, 10 is a triangular number.

Certain Rules

1. Of n consecutive whole numbers $a, a + 1 \ldots a + n - 1$, one and only one is divisible by n.
2. *Mixed numbers:* A number that has both the integral and fractional part is known as mixed number.
3. If a number n can be represented as the product of two numbers p and q, that is, $n = p \lozenge q$, then we say that the number n is divisible by p and by q and each of the numbers p and q is a divisor of the number n. Also, each factor of p and q would be a divisor of n.
4. Any number n can be represented in the decimal system of numbers as

   $N = a_k \times 10^k + a_{k-1} \times 10^{k-1} + \ldots + a_i \times 10 + a_0$

Example: 2738 can be written as: $2 \times 10^3 + 7 \times 10^2 + 3 \times 10^1 + 8 \times 10^0$.

5. 3^n will always have an even number of tens. (Example: 2 in 27,8 in 81,24 in 243,72 in 729 and so on.)
6. A sum of 5 consecutive whole numbers will always be divisible by 5.
7. The difference between 2 two digit numbers:

   $(xy) - (yx)$ will be divisible by 9
8. The square of an odd number when divided by 8 will always leave a remainder of 1.
9. The product of 3 consecutive natural numbers is divisible by 6.
10. The product of 3 consecutive natural numbers the first of which is even is divisible by 24.
11. Products:

    Odd × odd = odd

    Odd × even = even

    Even × even = even
12. All numbers not divisible by 3 have the property that their square will have a remainder of 1 when divided by 3.
13. $(a^2 + b^2)/(b^2 + c^2) = (a^2/b^2)$ if $a/b = b/c$.
14. The product of any r consecutive integers (numbers) is divisible by $r!$
15. If m and n are two integers then $(m + n)!$ is divisible by $m!n!$
16. Difference between any number and the number obtained by writing the digits in reverse order is divisible by 9. (for any number of digits)
17. Any number written in the form $10^n - 1$ is divisible by 3 and 9.
18. If a numerical expression contains no parentheses, first the operations of the third stage (involution or raising a number to a power) are performed, then the operations of the second stage (multiplication and division) and, finally, the operations of the first stage (addition and subtraction) are performed. In this case the

operations of one and the same stage are performed in the sequence indicated by the notation. If an expression contains parentheses, then the operation indicated in the parentheses are to be performed first and then all the remaining operations. In this case operations of the numbers in parentheses as well as standing without parentheses are performed in the order indicated above.

If a fractional expression is evaluated, then the operations indicated in the numerator and denominator of the function are performed and the first result is divided by the second.

19. $(a)^n/(a + 1)$ leaves a remainder of

    a if n is odd

    1 if n is even

20. $(a + 1)^n/a$ will always give a remainder of 1.
21. For any natural number n, n^5 has the same units digit as n has.
22. For any natural number: $n^3 – n$ is divisible by 6.
23. The remainder of $\frac{1\times 2\times 3\times 4\times \cdots (n-1)}{n}$ gives a remainder of (n – 1)

THE REMAINDER THEOREM

Consider the following question:

17 × 23.

Suppose you have to find the remainder of this expression when divided by 12.

We can write this as:

17 × 23 = (12 + 5) × (12 + 11)

Which when expanded gives us:

12 × 12 + 12 × 11 + 5 × 12 + 5 × 11

You will realise that, when this expression is divided by 12, the remainder will only depend on the last term above:

Thus, $\frac{12\times12+12\times11+5\times12+5\times11}{12}$ gives the same remainder as $\frac{5\times11}{12}$

Hence, 7.

This is the remainder when 17 × 23 is divided by 12.

Learning Point: In order to find the remainder of 17 × 23 when divided by 12, you need to look at the individual remainders of 17 and 23 when divided by 12. The respective remainders (5 and 11) will give you the remainder of the original expression when divided by 12.

Mathematically, this can be written as:

The remainder of the expression $[A \times B \times C + D \times E]/M$, will be the same as the remainder of the expression $[A_R \times B_R \times C_R + D_R \times E_R]/M$.

Where A_R is the remainder when A is divided by M,

B_R is the remainder when B is divided by M,

C_R is the remainder when C is divided by M

D_R is the remainder when D is divided by M and

E_R is the remainder when E is divided by M,

We call this transformation as the remainder theorem transformation and denote it by the sign $\xrightarrow{R}$

Thus, the remainder of

1421 × 1423 × 1425 when divided by 12 can be given as:

$$\frac{1421\times1423\times1425}{12} \xrightarrow{R} \frac{5\times7\times9}{12} = \frac{35\times9}{12} \xrightarrow{R} \frac{11\times9}{12}.$$

$\xrightarrow{R}$ gives us a remainder of 3.

In the above question, we have used a series of remainder theorem transformations (denoted by $\xrightarrow{R}$) and equality transformations to transform a difficult looking expression into a simple expression.

Try to solve the following questions on Remainder theorem:

Find the remainder in each of the following cases:

1. $17 \times 23 \times 126 \times 38$ divided by 8.
2. $243 \times 245 \times 247 \times 249 \times 251$ divided by 12.
3. $\frac{173 \times 261}{13} + \frac{248 \times 249 \times 250}{15}$.
4. $\frac{1021 \times 2021 \times 3021}{14}$.
5. $\frac{37 \times 43 \times 51}{7} + \frac{137 \times 143 \times 151}{9}$.

USING NEGATIVE REMAINDERS

Consider the following question:

Find the remainder when: 14×15 is divided by 8.

The obvious approach in this case would be

$$\frac{14 \times 15}{8} \xrightarrow{R} \frac{6 \times 7}{8} = \frac{42}{8} \xrightarrow{R} 2 \text{ (Answer).}$$

However there is another option by which you can solve the same question:

When 14 is divided by 8, the remainder is normally seen as + 6. However, there might be times when using the negative value of the remainder might give us more convenience. Which is why you should know the following process:

Concept Note: Remainders by definition are always non-negative. Hence, even when we divide a number like – 27 by 5 we say that the remainder is 3 (and not – 2). However, looking at the negative value of the remainder—it has its own advantages in Mathematics as it results in reducing calculations.

Thus, when a number like 13 is divided by 8, the remainder being 5, the negative remainder is – 3.

Note: It is in this context that we mention numbers like 13, 21, 29, etc. as $8n + 5$ or $8n - 3$ numbers.

Thus $\frac{14\times15}{8}$ will give us $\frac{-2\times-1}{8}$ *R* Æ 2.

Consider the advantage this process will give you in the following question:

$\frac{51\times52}{53} \xrightarrow{R} \frac{-2\times-1}{53} \xrightarrow{R} 2.$

(The alternative will involve long calculations. Hence, the principle is that you should use negative remainders wherever you can. They can make life much simpler!!!)

What if the Answer Comes Out Negative

For instance, $\frac{62\times63\times64}{66} \xrightarrow{R} \frac{-4\times-3\times-2}{66}$ *R* Æ $\frac{-24}{66}$.

But, we know that a remainder of –24, equals a remainder of 42 when divided by 66. Hence, the answer is 42.

Of course nothing stops you from using positive and negative remainders at the same time in order to solve the same question –

Thus $\frac{17\times19}{9} \xrightarrow{R} \frac{(-1)\times(1)}{9}$ *R* Æ –1 *R* Æ 8.

Dealing with large powers There are two tools which are effective in order to deal with large powers –

(A) If you can express the expression in the form $\frac{(ax+1)^n}{a}$, the remainder will become 1 directly. In such a case, no matter how large the value of the power n is, the remainder is 1.

For instance, $\frac{\left(37^{12635}\right)}{9} \xrightarrow{R} \frac{\left(1^{12635}\right)}{9} \xrightarrow{R}$ 1. In such a case the value of the power does not matter.

(B) $\frac{(ax-1)^n}{a}$. In such a case using –1 as the remainder it will be evident that the remainder will be +1 if n is even and it will be –1 (Hence a

– 1) when n is odd.

e.g.: $\frac{31^{127}}{8} \xrightarrow{R} \frac{(-1)^{127}}{8} \xrightarrow{R} \frac{(-1)}{8} \xrightarrow{R} 7$

ANOTHER IMPORTANT POINT

Suppose you were asked to find the remainder of 14 divided by 4. It is clearly visible that the answer should be 2.

But consider the following process:

$14/4 = 7/2 \xrightarrow{R} 1$ (The answer has changed!!)

What has happened?

We have transformed 14/4 into 7/2 by dividing the numerator and the denominator by 2. The result is that the original remainder 2 is also divided by 2 giving us 1 as the remainder. In order to take care of this problem, we need to reverse the effect of the division of the remainder by 2. This is done by multiplying the final remainder by 2 to get the correct answer.

Note: In any question on remainder theorem, you should try to cancel out parts of the numerator and denominator as much as you can, since it directly reduces the calculations required.

AN APPLICATION OF REMAINDER THEOREM

Finding the last two digits of an expression:

Suppose you had to find the last 2 digits of the expression:

$22 \times 31 \times 44 \times 27 \times 37 \times 43$

The remainder the above expression will give when it is divided by 100 is the answer to the above question.

Hence, to answer the question above find the remainder of the expression when it is divided by 100.

Solution: $\frac{22 \times 31 \times 44 \times 27 \times 37 \times 43}{100}$

$= \frac{22 \times 31 \times 11 \times 27 \times 37 \times 43}{25}$ (on dividing by 4)

$\xrightarrow{R} \frac{22 \times 6 \times 11 \times 2 \times 12 \times 18}{25} = \frac{132 \times 22 \times 216}{25}$

$\xrightarrow{R} \frac{7 \times 22 \times 16}{25}$

$= \frac{154 \times 16}{25} \xrightarrow{R} \frac{4 \times 16}{25} \xrightarrow{R} 14$

Thus the remainder being 14, (after division by 4). The actual remainder should be 56.

[Don't forget to multiply by 4 !!]

Hence, the last 2 digits of the answer will be 56.

Using negative remainders here would have helped further.

Note: Similarly finding the last three digits of an expression means finding the remainder when the expression is divided by 1000.

Exercise for Self-practice

1. Find the remainder when 73 + 75 + 78 + 57 + 197 is divided by 34.

   (a) 32 (b) 4

   (c) 15 (d) 28

2. Find the remainder when 73 × 75 × 78 × 57 × 197 is divided by 34.

   (a) 22 (b) 30

   (c) 15 (d) 28

3. Find the remainder when 73 × 75 × 78 × 57 × 197 × 37 is divided by 34.

   (a) 32 (b) 30

   (c) 15 (d) 28

4. Find the remainder when 43^{197} is divided by 7.

(a) 2 (b) 4

(c) 6 (d) 1

5. Find the remainder when 51^{203} is divided by 7.

(a) 4 (b) 2

(c) 1 (d) 6

6. Find the remainder when 59^{28} is divided by 7.

(a) 2 (b) 4

(c) 6 (d) 1

7. Find the remainder when 67^{99} is divided by 7.

(a) 2 (b) 4

(c) 6 (d) 1

8. Find the remainder when 75^{80} is divided by 7.

(a) 4 (b) 3

(c) 2 (d) 6

9. Find the remainder when 41^{77} is divided by 7.

(a) 2 (b) 1

(c) 6 (d) 4

10. Find the remainder when 21^{875} is divided by 17.

(a) 8 (b) 13

(c) 16 (d) 9

11. Find the remainder when 54^{124} is divided by 17.

(a) 4 (b) 5

(c) 13 (d) 15

12. Find the remainder when 83^{261} is divided by 17.

(a) 13 (b) 9

(c) 8 (d) 2

13. Find the remainder when 25^{102} is divided by 17.

(a) 13 (b) 15

(c) 4 (d) 2

ANSWER KEY

1. (b)	2. (a)	3. (a)	4. (d)	5. (a)
6. (b)	7. (d)	8. (a)	9. (c)	10. (b)
11. (a)	12. (d)	13. (c)		

Solutions

1. The remainder would be given by: (5 + 7 + 10 + 23 + 27)/34 = 72/34 Æ remainder = 4. Option (b) is correct.
2. The remainder would be given by: (5 × 7 × 10 × 23 × 27)/34 Æ 35 × 230 × 27/ 34 Æ 1 × 26 × 27/34 = 702/34 Æ remainder = 22. Option (a) is correct.
3. The remainder would be given by: (5 × 7 × 10 × 23 × 27 × 3)/34 Æ 35 × 230 × 27 × 3/34 Æ 1 × 26 × 81/34 Æ 26 × 13/34 = 338/34 Æ remainder = 32. Option (a) is correct.
4. $43^{197}/7$ Æ $1^{197}/7$ Æ remainder =1. Option (d) is correct.
5. $51^{203}/7$ Æ $2^{203}/7 = (2^3)^{67} \times 2^2/7 = 8^{67} \times 4/7$ Æ remainder = 4. Option (a) is correct.
6. $59^{28}/7$ Æ $3^{28}/7 = (3^6)^4 \times 3^4/7 = 729^6 \times 81/7$ Æ remainder = 4. Option (b) is correct.
7. $67^{99}/7$ Æ $4^{99}/7 = (4^3)^{33}/7 = 64^{33}/7$ Æ remainder = 1. Option (d) is correct.
8. $75^{80}/7$ Æ $5^{80}/7 = (5^6)^{13} \times 5^{2}/7$ Æ $1^{13} \times 25/7$ Æ remainder = 4. Option (a) is correct.
9. $41^{77}/7$ Æ $6^{77}/7$ Æ remainder = 6 (as the expression is in the form $a^n/(a + 1)$. Option (c) is correct.

10. $21^{875}/17$ Æ $4^{875}/17 = (4^4)^n \times 4^3/17 = 256^n \times 64/17$ Æ $1^n \times 13/17$ Æ remainder =13. Option (b) is correct.

11. $54^{124}/17$ Æ $3^{124}/17$. At this point, like in each of the other questions solved above, we need to plan the power of 3 which would give us a convenient remainder of either 1 or –1. As we start to look for remainders that powers of 3 would have when divided by 17, we get that at the power 3^6 the remainder is 15. If we convert this to –2 we will get that at the fourth power of 3^6, we should get a 16/17 situation (as $-2 \times -2 \times -2 \times -2 = 16$). This means that at a power of 3^{24} we are getting a remainder of 16 or –1. Naturally then if we double the power to 3^{48}, the remainder would be 1.

    With this thinking we can restart solving the problem:

    $3^{124}/17 = 3^{48} \times 3^{48} \times 3^{24} \times 3^4/17$ Æ $1 \times 1 \times 16 \times 81/17$ Æ $16 \times 13/17 = 208/17$ Æ remainder = 4. Option (a) is correct.

    (Note that if we are dividing a number by 17 and if we see the remainder as 15, we can logically say that the remainder is –2 – even though negative remainders are not allowed in mathematics)

12. Using the logic developed in Question 11 above, we have $83^{261}/17$ Æ $15^{261}/17$ Æ

    $(-2)^{261}/17$ Æ $(-2^4)^{65} \times (-2)/17$ Æ $16^{65} \times (-2)/17$ Æ $(-1) \times (-2)/17$ Æ remainder = 2. Option (d) is correct.

13. $25^{102}/17$ Æ $8^{102}/17 = 2^{306}/17 = (2^4)^{76} \times 2^2/17$ Æ $16^{76} \times 4/17$ Æ $1 \times 4/17$ Æ remainder = 4. Option (c) is correct.

BASE SYSTEM

All the work we carry out in our number system is called as the decimal system. In other words we work in the decimal system. Why is it called decimal?? It is because there are 10 digits in the system 0–9.

However, depending on the number of digits contained in the base system other number systems are also possible. Thus a number system with base 2 is called the binary number system and will have only two digits 0 and 1.

Some of the most commonly used systems are: Binary (base 2), Octal (base 8), Hexadecimal (base 16).

Binary system has 2 digits : 0, 1. Octal has 8 digits : – 0, 1, 2, 3, ... 7.

Hexadecimal has 16 digits – 0, 1, 2, ... 9, *A*, *B*, *C*, *D*, *E*, *F*.

Where *A* has a value 10, *B* = 11 and so on.

Before coming to the questions asked under this category, let us first look at a few issues with regard to converting numbers between different base systems.

1. Conversion from any base system into decimal:

Suppose you have to write the decimal equivalent of the base 8 number 146_8.

In such a case, follow the following structure for conversion:

$146_8 = 1 \times 8^2 + 4 \times 8^1 + 6 \times 8^0$

$= 64 + 32 + 6 = 102.$

Note: If you remember the process, for writing the value of any random number, say 146, in our decimal system (base 10) we use: $1 \times 10^2 + 4 \times 10^1 + 6 \times 10^0$. All you need to change, in case you are trying to write the value of the number in base 8, is that you replace 10 with 8 in every power.

Try to write the decimal equivalents of the following numbers:

143_5, 143_6, 143_7, 143_8, 143_9

1256_7, 1256_8, 1256_9.

2. Conversion of a number in decimals into any base:

Suppose you have to find out the value of the decimal number 347 in base 6. The following process is to be adopted:

Step 1: Find the highest power of the base (6 in this case) that is contained in 347. In this case you will realise that the value of $6^3 = 216$ is contained in 347, while the value of $6^4 = 1296$ is not contained in 347. Hence, we realise that the highest power of 6 contained in 347 is 3. This should make you realise that the number has to be constructed by using the powers 6^0, 6^1, 6^2, 6^3 respectively. Hence, a 4-digit number.

Structure of number: - - - -

Step 2: We now need to investigate how many times each of the powers of 6 is contained in 347. For this we first start with the highest power as found above. Thus we can see that 6^3 (216) is contained in 347 once. Hence our number now becomes:

1 - - -

That is, we now know that the first digit of the number is 1. Besides, when we have written the number 1 in this place, we have accounted for a value of 216 out of 347. This leaves us with 131 to account for.

We now need to look for the number of times 6^2 is contained in 131. We can easily see that $6^2 = 36$ is contained in 131 three times. Thus, we write 3 as the next digit of our number which will now look like:

1 3 - -

In other words we now know that the first two digits of the number are 13. Besides, when we have written the number 3 in this place, we have accounted for a value of 108 out of 131 which was left to be accounted for. This leaves us with 131–108 = 23 to account for.

We now need to look for the number of times 6^1 is contained in 23. We can easily see that $6^1 = 6$ is contained in 23 three times. Thus, we write 3 as the next digit of our number which will now look like:

1 3 3 -

In other words we now know that the first three digits of the number are 133. Besides, when we have written the number 3 in this place, we have accounted for a value of 18 out of the 23 which was left to be accounted for. This leaves us with 23 – 18 = 5 to account for.

The last digit of the number corresponds to $6^0 = 1$. In order to make a value of 5 in this place we will obviously need to use this power of 6, 5 times thus giving us the final digit as 5. Hence, our number is:

1 3 3 5.

A few points you should know about base systems:

(1) In single digits there is no difference between the value of the number—whichever base we take. For example, the equality $5_6 = 5_7 = 5_8 = 5_9 = 5_{10}$.

(2) Suppose you have a number in base *x*. When you convert this number into its decimal value, the value should be such that when it is divided by *x*, the remainder should be equal to the units digit of the number in base *x*.

In other words, 342_8 will be a number of the form $8n + 2$ in base 10. You can use this principle for checking your conversion calculations.

The following table gives a list of decimal values and their binary, octal and hexadecimal equivalents:

Decimal	*Binary*	*Octal*	*Hexadecimal*
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B

Illustrations

1. The number of *x* digit numbers in *n*th base system will be

(a) n^x (b) n^{x-1}

(c) $n^x - n$ (d) $nx - n^{(x-1)}$

Solution Base Æ *n*, digit Æ *x*

So, required number of numbers = $nx - n^{(x-1)}$

2. The number of 2 digit numbers in binary system is

(a) 2 (b) 90

(c) 10 (d) 4

Solution By using the formula, we get the required number of numbers = $2^2 - 2^1 = 2$

fi Option (a)

3. The number of 5 digit numbers in binary system is

(a) 48 (b) 16

(c) 32 (d) 20

Solution Required number of numbers = $2^5 - 2^4 = 32 - 16 = 16$

fi Option (b)

4. I celebrate my birthday on 12th January on earth. On which date would I have to celebrate my birthday if I were on a planet where binary system is being used for counting. (The number of days, months and years are same on both the planets.)

(a) 11th Jan (b) 111th Jan

(c) 110th Jan (d) 1100th Jan

Solution On earth (decimal system is used). 12th Jan fi 12th Jan

The 12th day on the planet where binary system is being used will be called

$(12)_{10} = (?)_2$

$$= \frac{1}{2^3}\,\frac{1}{2^2}\,\frac{0}{2^1}\,\frac{0}{2^0}$$

i.e., 1100th day on that planet

So, 12th January on earth = 1100th January on that planet

fi Option (d)

5. My year of birth is 1982. What would the year have been instead of 1982 if base 12 were used (for counting) instead of decimal system?

(a) 1182 (b) 1022

(c) 2082 (d) 1192

Solution The required answer will equal to $(1982)_{10} = (?)_{12}$.

$= \frac{1}{12^3}\frac{1}{12^2}\frac{9}{12^1}\frac{2}{12^0}$ Æ

$1 \times 12^3 + 1 \times 12^2 + 9 \times 12^1 + 2 \times 9^0 = 1728 + 144 + 108 + 2 = 1982$

Hence, the number $(1192)_{12}$ represents 1982 in our base system.

fi Option (d)

6. 203 in base 5 when converted to base 8, becomes

(a) 61 (b) 53

(c) 145 (d) 65

Solution $(203)_5 = (?)_{10}$

$= 2 \times 5^2 + 0 \times 5^1 + 3 \times 5^0$

$= 50 + 0 + 3 = 53$

Now,

$(53)_{10} = (?)8$

$= \frac{6}{8^1}\frac{5}{8^0}$

$= (203)_5 = (65)_8$

fi Option (d)

7. $(52)_7 + 46_8 = (?)_{10}$

(a) $(75)_{10}$ (b) $(50)_{10}$

(c) $(39)_{39}$ (d) $(28)_{10}$

Solution $(52)_7 = (5 \times 7^1 + 2 \times 7^0)_{10} = (37)_{10}$

also, $(46)_8 = (4 \times 8^1 + 6 \times 8^0)_{10} = (38)_{10}$

sum $= (75)_{10}$

fi Option (a)

8. $(23)_5 + (47)_9 = (?)_8$

(a) 70 (b) 35

(c) 64 (d) 18

Solution $(23)_5 = (2 \times 5^1 + 3 \times 5^0)_{10} = (13)_{10} = (1 \times 8^1 + 5 \times 8^0)_8 = (15)_8$

also,$(47)_9 = (4 \times 9^1 + 7 \times 9^0)_{10} = (43)_{10}$

$= (5 \times 8^1 + 3 \times 8^0)_8 = (53)_8$

sum = $(13)_{10} + (43)_{10} = (56)_{10}$ Æ $(70)_8$

fi Option (a)

9. $(11)_2 + (22)_3 + (33)_4 + (44)_5 + (55)_6 + (66)_7 + (77)_8 + (88)_9 = (?)_{10}$

(a) 396 (b) 276

(c) 250 (d) 342

Solution $(11)_2 = (1 \times 2^1 + 1 \times 2^0)_{10} = (3)_{10}$

$(22)_3 = (2 \times 3^1 + 2 \times 3^0)_{10} = (8)_{10}$

$(33)_4 = (3 \times 4^1 + 3 \times 4^0)_{10} = (15)_{10}$

$(44)_5 = (4 \times 5^1 + 4 \times 5^0)_{10} = (24)_{10}$

$(55)_6 = (5 \times 6^1 + 5 \times 6^0)_{10} = (35)_{10}$

$(66)_7 = (6 \times 7^1 + 6 \times 7^0)_{10} = (48)_{10}$

$(77)_8 = (7 \times 8^1 + 7 \times 8^0)_{10} = (63)_{10}$

$(88)_9 = (8 \times 9^1 + 8 \times 9^0)_{10} = (80)_{10}$

sum = $(276)_{10}$

fi Option (b)

10. $(24)_5 \times (32)_5 = (?)_5$

(a) 1423 (b) 1422

(c) 1420 (d) 1323

Solution $(24)_5 = 14_{10}$ and $32_5 = 17_{10}$. Hence, the required answer can be got by $14 \times 17 = 238_{10} = 1 \times 5^3 + 4 \times 5^2 + 2 \times 5^1 + 3 \times 5^0$ Æ 1423 as the correct answer.

Alternately, you could multiply directly in base 5 as follows:

(2 4)

$\times \quad (3\ 2)$

$(1\ 4\ 2\ 3)$

Unit's digit of the answer would correspond to: 4 × 2 = 8 Æ 13_5. Hence, we write 3 in the units place and carry over 1.

(Note that in this process when we are doing 4 × 2 we are effectively multiplying individual digits of one number with individual digits of the other number. In such a case we can write 4 × 2 = 8 by assuming that both the numbers are in decimal system as the value of a single digit in any base is equal.)

The tens digit will be got by: 2 × 2 + 4 × 3 = 16 + 1 = 17 Æ 32_5

Hence, we write 2 in the tens place and carry over 3 to the hundreds place.

Where we get 3 × 2 + 3 = 9 Æ 14

Hence, the answer is 14.

fi Option (a)

11. In base 8, the greatest four digit perfect square is

(a) 9801 (b) 1024

(c) 8701 (d) 7601

Solution In base 10, the greatest 4 digit perfect square = 9801

In base 9, the greatest 4 digits perfect square = 8701

In base 8, the greatest 4 digits perfect square = 7601

Alternately, multiply $(77)_8 \times (77)_8$ to get 7601 as the answer.

Unit's Digit

(A) The unit's digit of an expression will be got by getting the remainder when the expression is divided by 10.

Thus for example if we have to find the units digit of the expression:

17 × 22 × 36 × 54 × 27 × 31 × 63

We try to find the remainder –

$$\frac{17 \times 22 \times 36 \times 54 \times 27 \times 31 \times 63}{10}$$

$$\xrightarrow{R} \frac{7 \times 2 \times 6 \times 4 \times 7 \times 3}{10}$$

$$= \frac{14 \times 24 \times 21}{10} \xrightarrow{R} \frac{4 \times 4 \times 1}{10} = \frac{16}{10} \xrightarrow{R} 6.$$

Hence, the required answer is 6.

This could have been directly got by multiplying: 7 × 2 × 6 × 4 × 7 × 1 × 3 and only accounting for the units' digit.

(B) Unit's digits in the contexts of powers –

Study the following table carefully.

Unit's digit when 'N' is raised to a power

Number Ending With	*Value of power*								
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1	1	1	1	1	1	1	1	1	1
2	2	4	8	6	2	4	8	6	2
3	3	9	7	1	3	9	7	1	3
4	4	6	4	6	4	6	4	6	4
5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6
7	7	9	3	1	7	9	3	1	7
8	8	4	2	6	8	4	2	6	8
9	9	1	9	1	9	1	9	1	9
0	0	0	0	0	0	0	0	0	0

In the table above, if you look at the columns corresponding to the power 5 or 9 you will realize that the unit's digit for all numbers is repeated (i.e. it is 1 for 1, 2 for, 3 for 3….9 for 9.)

This means that whenever we have any number whose unit's digit is '*x*' and it is raised to a power of the form 4*n* + 1, the value of the unit's digit of the answer will be the same as the original units digit.

Illustrations: $(1273)^{101}$ will give a unit's digit of 3. $(1547)^{25}$ will give a units digit of 7 and so forth.

Thus, the above table can be modified into the form –

Value of power

Number ending in	*If the value of the Power is*			
	$4n + 1$	$4n + 2$	$4n + 3$	$4n$
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1

[Remember, at this point that we had said (in the Back to School section of Part 1) that all natural numbers can be expressed in the form $4n + x$. Hence, with the help of the logic that helps us build this table, we can easily derive the units digit of any number when it is raised to a power.)

A special Case

Question: What will be the unit's digit of $(1273)^{122!}$?

Solution: 122! is a number of the form $4n$. Hence, the answer should be 1. [**Note:** 1 here is derived by thinking of it as 3 (for $4n + 1$), 9 (for $4n + 2$), 7 (for $4n + 3$), 1(for $4n$)]

Exercise for Self-practice

Find the Units digit in each of the following cases:

1. $2^2 \times 4^4 \times 6^6 \times 8^8$

2. $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \ldots. \times 100^{100}$

3. $17 \times 23 \times 51 \times 32 + 15 \times 17 \times 16 \times 22$

4. $13 \times 17 \times 22 \times 34 + 12 \times 6 \times 4 \times 3 - 13 \times 33$

5. $37^{123} \times 43^{144} \times 57^{226} \times 32^{127} \times 52^{5!}$

6. $67 \times 37 \times 43 \times 91 \times 42 \times 33 \times 42$

   (a) 2 (b) 6

   (c) 8 (d) 4

7. $67 \times 35 \times 43 \times 91 \times 47 \times 33 \times 49$

   (a) 1 (b) 9

   (c) 5 (d) 6

8. $67 \times 35 \times 45 \times 91 \times 42 \times 33 \times 81$

   (a) 2 (b) 4

   (c) 0 (d) 8

9. $67 \times 35 \times 45 + 91 \times 42 \times 33 \times 82$

   (a) 8 (b) 7

   (c) 0 (d) 5

10. $(52)^{97} \times (43)^{72}$

   (a) 2 (b) 6

   (c) 8 (d) 4

11. $(55)^{75} \times (93)^{175} \times (107)^{275}$

   (a) 7 (b) 3

   (c) 5 (d) 0

12. $(173)^{45} \times (152)^{77} \times (777)^{999}$

   (a) 2 (b) 4

   (c) 8 (d) 6

13. $81 \times 82 \times 83 \times 84 \times 86 \times 87 \times 88 \times 89$

(a) 0 (b) 6

(c) 2 (d) 4

14. $82^{43} \times 83^{44} \times 84^{97} \times 86^{98} \times 87^{105} \times 88^{94}$

(a) 2 (b) 6

(c) 4 (d) 8

15. $432 \times 532 + 532 \times 974 + 537 \times 531 + 947 \times 997$

(a) 5 (b) 6

(c) 9 (d) 8

ANSWER KEY

6. (d)	7. (c)	8. (c)	9. (b)	10. (a)
11. (c)	12. (c)	13. (b)	14. (b)	15. (d)

Solutions

1. The units digit would be given by the units digit of the multiplication of 4 x 6 x 6 x6 = 4
2. 0
3. 7 × 3 × 1 × 2 + 0 Æ 2 + 0 = 2
4. 8 + 4 – 9 Æ 3
5. 3 × 1 × 9 × 8 × 6 = 6
6. 7 × 7 × 3 × 1 × 2 × 3 × 2 = 4
7. Since we have a 5 multiplied with odd numbers, the units digit would naturally be 5.
8. 5 × 2 Æ 0
9. 5 + 2 Æ 7
10. 2 × 1 Æ 2
11. 5 × 7 × 3 Æ 5
12. 3 × 2 × 3 Æ 8

13. 2 × 3 × 4 × 6 × 7 × 8 × 9 Æ 6
14. 8 × 1 × 4 × 6 × 7 × 4 Æ 6
15. 4 + 8 + 7 + 9 Æ 8

WORKED-OUT PROBLEMS

Problem 1.1 Find the number of zeroes in the factorial of the number 18.

Solution 18! contains 15 and 5, which combined with one even number give zeroes. Also, 10 is also contained in 18!, which will give an additional zero. Hence, 18! contains 3 zeroes and the last digit will always be zero.

Problem 1.2 Find the numbers of zeroes in 27!

Solution $27! = 27 \times 26 \times 25 \times \ldots \times 20 \times \ldots \times 15 \times \ldots \times 10 \times \ldots \times 5 \times \ldots \times 1$.

A zero can be formed by combining any number containing 5 multiplied by any even number. Similarly, everytime a number ending in zero is found in the product, it will add an additional zero. For this problem, note that $25 = 5 \times 5$ will give 2 zeroes and zeroes will also be got by 20, 15, 10 and 5. Hence 27! will have 6 zeroes.

Short-cut method: Number of zeroes is 27! Æ [27/5] + [27/25]

where [*x*] indicates the integer just lower than the fraction

Hence, $[27/5] = 5$ and $[27/5^2] = 1$, 6 zeroes

Problem 1.3 Find the number of zeroes in 137!

Solution $[137/5] + [137/5^2] + [137/5^3]$

$= 27 + 5 + 1 = 33$ zeroes

(since the restriction on the number of zeroes is due to the number of fives.)

Exercise for Self-practice

Find the number of zeroes in

(a) 81! (b) 100! (c) 51!

Answers

(a) 19 (b) 24 (c) 12

Problem 1.4 What exact power of 5 divides 87!?

Solution [87/5] + [87/25] = 17 + 3 = 20

Problem 1.5 What power of 8 exactly divides 25!?

Solution If 8 were a prime number, the answer should be [25/8] = 3. But since 8 is not prime, use the following process.

The prime factors of 8 is $2 \times 2 \times 2$. For divisibility by 8, we need three twos. So, everytime we can find 3 twos, we add one to the power of 8 that divides 25! To count how we get 3 twos, we do the following. All even numbers will give one 'two' at least [25/2] = 12

Also, all numbers in 25! divisible by 2^2 will give an additional two [25/2^2] = 6

Further, all numbers in 25! divisible by 2^3 will give a third two. Hence [25!/2^3] = 3

And all numbers in 25! divisible by 2^4 will give a fourth two. Hence [25!/2^4] = 1

Hence, total number of twos in 25! is 22. For a number to be divided by 8, we need three twos. Hence, since 25! has 22 twos, it will be divided by 8 seven times.

Problem 1.6 What power of 15 divides 87! exactly?

Solution $15 = 5 \times 3$. Hence, everytime we can form a pair of one 5 and one 3, we will count one.

87! contains – [87/5] + [87/5^2] = 17 + 3 = 20 fives

Also 87! contains – [87/3] + [87/3^2] + [87/3^3] + [87/3^4] = 29 +…(more than 20 threes).

Hence, 15 will divide 87! twenty times since the restriction on the power is because of the number of 5s and not the number of 3s.

In fact, it is not very difficult to see that in the case of all factors being prime, we just have to look for the highest prime number to provide the restriction for the power of the denominator.

Hence, in this case we did not need to check for anything but the number of 5s.

Exercise for Self-practice

(a) What power of 30 will exactly divide 128!

Hints: $[128/5] + [128/5^2] + [128/5^3]$

(b) What power of 210 will exactly divide 142!

Problem 1.7 Find the last digit in the expression $(36472)^{123!} \times (34767)^{76!}$.

Solution If we try to formulate a pattern for 2 and its powers and their units digit, we see that the units digit for the powers of 2 goes as: 2, 4, 8, 6, 2, 4, 8, 6, 2, 4, 8, 6 and so on. The number 2 when raised to a power of $4n + 1$ will always give a units digit of 2. This also means that the units digit for 2^{4n} will always end in 6. The power of 36472 is 123! . 123! can be written in the form $4n$. Hence, $(36472)^{123!}$ will end in 6.

The second part of the expression is $(34767)^{76!}$. The units digit depends on the power of 7. If we try to formulate a pattern for 7 and its powers and their units digit, we see that the units digit for the powers of 7 go as: 7 9 3 1 7 9 3 1 and so on. This means that the units digit of the expression 7^{4n} will always be 1.

Since 76! can be written as a multiple of 4 as $4n$, we can conclude that the unit's digit in $(34767)^{76!}$ is 1.

Hence the units digit of $(36472)^{123!} \times (34767)^{76!}$ will be 6.

Counting

Problem 1.8 Find the number of numbers between 100 to 200 if

(i) Both 100 and 200 are counted.

(ii) Only one of 100 and 200 is counted.

(iii) Neither 100 nor 200 is counted.

Solution

(i) Both ends included-Solution: 200 – 100 + 1 = 101

(ii) One end included-Solution: 200 – 100 = 100

(iii) Both ends excluded-Solution: 200 – 100 – 1 = 99.

Problem 1.9 Find the number of even numbers between 122 and 242 if:

(i) Both ends are included.

(ii) Only one end is included.

(iii) Neither end is included.

Solution

(i) Both ends included—Solution: (242 – 122)/2 = 60 + 1 = 61

(ii) One end included-Solution: (242 – 122)/2 = 60

(iii) Both ends excluded-Solution: (242 – 122)/2 – 1 = 59

Exercise for Self-practice

(a) Find the number of numbers between 140 to 259, both included, which are divisible by 7.

(b) Find the number of numbers between 100 to 200, that are divisible by 3.

Problem 1.10 Find the number of numbers between 300 to 400 (both included), that are not divisible by 2, 3, 4, and 5.

Solution Total numbers: 101

Step 1: Not divisible by 2 = All even numbers rejected: 51

Numbers left: 50.

Step 2: Of which: divisible by 3 = first number 300, last number 399. But even numbers have already been removed, hence count out only odd numbers between 300 and 400 divisible by 3. This gives us that:

First number 303, last number 399, common difference 6

So, remove: [(399 – 303)/6] + 1 = 17.

\ 50 – 17 = 33 numbers left.

We do not need to remove additional terms for divisibility by 4 since this would eliminate only even numbers (which have already been eliminated)

Step 3: Remove from 33 numbers left all odd numbers that are divisible by 5 and not divisible by 3.

Between 300 to 400, the first odd number divisible by 5 is 305 and the last is 395 (since both ends are counted, we have 10 such numbers as: [(395 – 305)/10 + 1 = 10].

However, some of these 10 numbers have already been removed to get to 33 numbers.

Operation left: Of these 10 numbers, 305, 315…395, reduce all numbers that are also divisible by 3. Quick perusal shows that the numbers start with 315 and have common difference 30.

Hence [(Last number – First number)/Difference + 1] = [(375 – 315)/30 + 1] = 3

These 3 numbers were already removed from the original 100. Hence, for numbers divisible by 5, we need to remove only those numbers that are odd, divisible by 5 but not by 3. There are 7 such numbers between 300 and 400.

So numbers left are: 33 – 7 = 26.

Exercise for Self-practice

Find the number of numbers between 100 to 400 which are divisible by either 2, 3, 5 and 7.

Problem 1.11 Find the number of zeroes in the following multiplication: $5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 45 \times 50$.

Solution The number of zeroes depends on the number of fives and the number of twos. Here, close scrutiny shows that the number of twos is the constraint. The expression can be written as

$5 \times (5 \times 2) \times (5 \times 3) \times (5 \times 2 \times 2) \times (5 \times 5) \times (5 \times 2 \times 3) \times (5 \times 7) \times (5 \times 2 \times 2 \times 2) \times (5 \times 3 \times 3) \times (5 \times 5 \times 2)$

Number of 5s – 12, Number of 2s – 8.

Hence: 8 zeroes.

Problem 1.12 Find the remainder for [(73 × 79 × 81)/11].

Solution The remainder for the expression: [(73 × 79 × 81)/11] will be the same as the remainder for [(7 × 2 × 4)/11]

That is, 56/11 fi remainder = 1

Problem 1.13 Find the remainder for (3^{560}/8).

Solution (3^{560}/8) = [$(3^2)^{280}$/8] = (9^{280}/8)

= [9.9.9…(280 times)]/8

remainder for above expression = remainder for [1.1.1…(280 times)]/8 fi remainder = 1.

Problem 1.14 Find the remainder when (2222^{5555} + 5555^{2222})/7.

Solution This is of the form: [(2222^{5555})/7 + (5555^{2222})/7]

We now proceed to find the individual remainder of : (2222^{5555})/7. Let the remainder be R_1.

When 2222 is divided by 7, it leaves a remainder of 3.

Hence, for remainder purpose (2222^{5555})/7 $\xrightarrow{R}$ (3^{5555}/7) = (3.3^{5554})/7 = [$3(3^2)^{2777}$]/7 = [$3.(7 + 2)^{2777}$]/7 $\xrightarrow{R}$ (3.2^{2777})/7 = (3.2^2 ◊ 2^{2775})/7 = [3.2^2 ◊ $(2^3)^{925}$]/7

= [3.2^2 ◊ $(8)^{925}$] /7 $\xrightarrow{R}$ (12/7) Remainder = 5.

Similarly, (5555^{2222})/7 $\xrightarrow{R}$ (4^{2222})/7 = [$(2^2)^{2222}$]/7 = $(2)^{4444}$/7 = (2.2^{4443})/7 = [$2.(2^3)^{1481}$]/7 = [$2.(8)^{1481}$]/7 $\xrightarrow{R}$ [$2.(1)^{1481}$]/7 Æ 2 (remainder).

Hence, (2222^{5555})/7 + (5555^{2222})/7 $\xrightarrow{R}$ (5 + 2)/7 fi Remainder = 0

Problem 1.15 Find the GCD and the LCM of the numbers 126, 540 and 630.

Solution The standard forms of the numbers are:

126 Æ 3 × 3 × 7 × 2 Æ 3^2 × 7 × 2

540 Æ 3 × 3 × 3 × 2 × 2 × 5 Æ 2^2 × 3^3 × 5

630 Æ 3 × 3 × 5 × 2 × 7 Æ 2 × 3^2 × 5 × 7

For GCD we use Intersection of prime factors and the lowest power of all factors that appear in all three numbers. 2 × 3^2 = 18.

For LCM Æ Union of prime factors and highest power of all factors that appear in any one of the three numbers fi 2^2 × 3^3 × 5 × 7 = 3780.

Exercise for Self-practice

Find the GCD and the LCM of the following numbers:

(i) 360, 8400 (ii) 120, 144

(iii) 275, 180, 372, 156 (iv) 70, 112

(v) 75, 114 (vi) 544, 720

Problem 1.16 The ratio of the factorial of a number *x* to the square of the factorial of another number, which when increased by 50% gives the required number, is 1.25. Find the number *x*.

(a) 6 (b) 5

(c) 9 (d) None of these

Solution Solve through options: Check for the conditions mentioned. When we check for option (a) we get 6 ! = 720 and $(4!)^2$ = 576 and we have 6 !/$(4!)^2$ = 1.25, which is the required ratio.

Hence the answer is **(a)**

Problem 1.17 Three numbers *A*, *B* and *C* are such that the difference between the highest and the second highest two-digit numbers formed by using two of *A*, *B* and *C* is 5. Also, the smallest two two-digit numbers differ by 2. If *B* > *A* > *C* then what is the value of *B*?

(a) 1 (b) 6 (c) 7 (d) 8

Solution Since B is the largest digit, option (a) is rejected. Check for option (b).

If B is 6, then the two largest two-digit numbers are 65 and 60 (Since, their difference is 5) and we have $B = 6$, $A = 5$ and $C = 0$.

But with this solution we are unable to meet the second condition. Hence (b) is not the answer. We also realise here that C cannot be 0.

Check for option (c).

B is 7, then the nos. are 76 and 71 or 75 and 70. In both these cases, the smallest two two-digit numbers do not differ by 2.

Hence, the answer is not (c).

Hence, option **(d)** is the answer

[To confirm, put $B = 8$, then the solution $A = 6$ and $C = 1$ satisfies the 2nd condition.]

Problem 1.18 Find the remainder when $2851 \times (2862)^2 \times (2873)^3$ is divided by 23.

Solution We use the remainder theorem to solve the problem. Using the theorem, we see that the following expressions have the same remainder.

fi $\dfrac{2851 \times (2862)^2 \times (2873)^3}{23}$

fi $\dfrac{22 \times 10 \times 10 \times 21 \times 21 \times 21}{23}$

fi $\dfrac{22 \times 8 \times 441 \times 21}{23}$ fi $\dfrac{22 \times 21 \times 8 \times 4}{23}$

fi $\dfrac{462 \times 32}{23}$ fi $\dfrac{2 \times 9}{23}$ fi Remainder is **18.**

Problem 1.19 For what maximum value of n will the expression $\dfrac{10200\,!}{504^n}$ be an integer?

Solution For $\frac{10200!}{504^n}$ to be a integer, we need to look at the prime factors of 504 Æ

$504 = 3^2 \times 7 \times 8 = 2^3 \times 3^2 \times 7$

We thus have to look for the number of 7s, the number of 2^3s and the number of 3^2s that are contained in 10200!. The lowest of these will be the constraint value for *n*.

To find the number of 2^3s we need to find the number of 2s as

$$\left[\frac{10200}{2}\right]+\left[\frac{10200}{4}\right]+\left[\frac{10200}{8}\right]+\left[\frac{10200}{16}\right]+\left[\frac{10200}{32}\right]$$

$$+\left[\frac{10200}{64}\right]+\left[\frac{10200}{128}\right]+\left[\frac{10200}{256}\right]+\left[\frac{10200}{512}\right]+\left[\frac{10200}{1024}\right]$$

$$+\left[\frac{10200}{2048}\right]+\left[\frac{10200}{4096}\right]+\left[\frac{10200}{8192}\right]$$

where [ ] is the greatest integer function.

= 5100 + 2550 + 1275 + 637 + 318 + 159 + 79 + 39 + 19 + 9 + 4 + 2 + 1

Number of twos = 10192

Hence, number of 2^3 = 3397

Similarly, we find the number of 3s as

Number of threes = $\left[\frac{10200}{3}\right]+\left[\frac{10200}{9}\right]+\left[\frac{10200}{27}\right]$

$$\left[\frac{10200}{81}\right]+\left[\frac{10200}{243}\right]+\left[\frac{10200}{729}\right]+\left[\frac{10200}{2187}\right]$$

$$+\left[\frac{10200}{6561}\right]$$

= 3400 + 1133 + 377 + 125 + 41 + 13 + 4 + 1

Number of threes = 5094

\ Number of 3^2 = 2547

Similarly we find the number of 7s as

$$\left[\frac{10200}{7}\right]+\left[\frac{10200}{49}\right]+\left[\frac{10200}{343}\right]+\left[\frac{10200}{2401}\right]$$

= 1457 + 208 + 29 + 4 = 1698.

Thus, we have, 1698 sevens, 2547 nines and 3397 eights contained in 10200!.

The required value of *n* will be given by the lowest of these three [The student is expected to explore why this happens]

Hence, answer = **1698**.

Short cut We will look only for the number of 7s in this case. *Reason*: 7 > 3 × 2. So, the number of 7s must always be less than the number of 2^3.

And 7 > 2 × 3, so the number of 7s must be less than the number of 3^2.

Recollect that earlier we had talked about the finding of powers when the divisor only had prime factors. There we had seen that we needed to check only for the highest prime as the restriction had to lie there.

In cases of the divisors having composite factors, we have to be slightly careful in estimating the factor that will reflect the restriction. In the above example, we saw a case where even though 7 was the lowest factor (in relation to 8 and 9), the restriction was still placed by 7 rather than by 9 (as would be expected based on the previous process of taking the highest number).

Problem 1.20 Find the units digit of the expression: $78^{5562} \times 56^{256} \times 97^{1250}$.

Solution We can get the units digits in the expression by looking at the patterns followed by 78, 56 and 97 when they are raised to high powers.

In fact, for the last digit we just need to consider the units digit of each part of the product.

A number (like 78) having 8 as the units digit will yield units digit as

78^1 Æ 8 78^5 Æ 8 8^{4n+1} Æ 8

78^2 Æ 4 78^6 Æ 4 8^{4n+2} Æ 4

78^3 Æ 2 78^7 Æ 2 Hence 78^{5562} will yield four as the units digit

78^4 Æ 6 78^8 Æ 6

Similarly,

56^1 Æ 6 Æ 56^{256} will yield 6 as the units digit.

56^2 Æ 6

56^3 Æ 6

Similarly,

97^1 Æ 7 7^{4n+1} Æ 7

97^2 Æ 9 7^{4n+2} Æ 9

97^3 Æ 3 Hence, 97^{1250} will yield a units digit of 9.

97^4 Æ 1

Hence, the required units digit is given by 4 × 6 × 9 Æ **6** (answer).

Problem 1.21 Find the GCD and the LCM of the numbers *P* and *Q* where $P = 2^3 \times 5^3 \times 7^2$ and $Q = 3^3 \times 5^4$.

Solution GCD or HCF is given by the lowest powers of the common factors.

Thus,GCD = 5^3.

LCM is given by the highest powers of all factors available.

Thus,LCM = $2^3 \times 3^3 \times 5^4 \times 7^2$

Problem 1.22 A school has 378 girl students and 675 boy students. The school is divided into strictly boys or strictly girls sections. All sections in the school have the same number of students. Given this information, what are the minimum number of sections in the school.

Solution The answer will be given by the HCF of 378 and 675.

$378 = 2 \times 3^3 \times 7$

$675 = 3^3 \times 5^2$

Hence, HCF of the two is $3^3 = 27$.

Hence, the number of sections is given by: $\frac{378}{27} + \frac{675}{27}$ = 14 + 25 = **39 sections**.

Problem 1.23 The difference between the number of numbers from 2 to 100 which are not divisible by any other number except 1 and itself and the numbers which are divisible by at least one more number along with 1 and itself.

(a) 25 (b) 50

(c) 49 (d) can't be determine

Solution From 2 to 100.

The number of numbers which are divisible by 1 and itself only = 25

Also, the number of numbers which are divisible by at least one more number except 1 and itself (i.e composite numbers) 99 – 25 = 74

So, required difference = 74 – 25 = 49

fi **Option (c)**

Problem 1.24 If the sum of (2*n* +1) prime numbers where *n* Œ*N* is an even number, then one of the prime numbers must be

(a) 2 (b) 3

(c) 5 (d) 7

Solution For any *n* Œ *N*, 2*n* + 1 is odd.

Also, it is given in the problem that the sum of an odd number of prime numbers = even. Since all prime numbers except 2 are odd, the above condition will only be fulfilled if we have an (odd + odd + even) structure of addition. Since, the sum of the three prime numbers is said to be even, we have to include one even prime number. Hence 2 being the only even prime number must be included.

If we add odd number of prime numbers, not including 2 (two), we will always get an odd number, because

$$\frac{\text{odd} + \text{odd} + \text{odd} + \text{-----} + \text{odd}}{\text{(an odd number of times)}} = \text{odd number}$$

fi **Option (a)**

Problem 1.25 What will be the difference between the largest and smallest four digit number made by using distinct single digit prime numbers?

(a) 1800 (b) 4499

(c) 4495 (d) 5175

Solution Required largest number Æ 7532

Required smallest number Æ 2357

Difference Æ 5175

fi **Option (d)**

Problem 1.26 The difference between the two three-digit numbers *XYZ* and *ZYX* will be equal to

(a) difference between X and Z i.e. $|x - z|$

(b) sum of X and z i.e $(X + Z)$

(c) 9 × difference between X and Z

(d) 99 × difference between X and Z

Solution From the property of numbers, it is known that on reversing a three digit number, the difference (of both the numbers) will be divisible by 99. Also, it is known that this difference will be equal to 99 × difference between the units and hundreds digits of the three digit number. fi Option (d)

Problem 1.27 When the difference between the number 842 and its reverse is divided by 99, the remainder will be

(a) 0 (b) 1

(c) 74 (d) 17

Solution From the property (used in the above question) we can say that the difference will be divisible by 99

fi Remainder = 0 (zero)

fi Option (a)

Problem 1.28 When the difference between the number 783 and its reverse is divided by 99, the quotient will be

(a) 1 (b) 10

(c) 3 (d) 4

Solution The quotient will be the difference between extreme digits of 783, i.e. 7 – 3 = 4 (This again is a property which you should know.)

fi Option (d)

Problem 1.29 A long Part of wood of same length when cut into equal pieces each of 242 cms, leaves a small piece of length 98 cms. If this Part were cut into equal pieces each of 22 cms, the length of the leftover wood would be

(a) 76 cm (b) 12 cm

(c) 11 cm (d) 10 cm

Solution As 242 is divisible by 22, so the required length of left wood will be equal to the remainder when 98 is divided by 22:

Hence, 10 [98/22; remainder 10]

fi Option (d)

Problem 1.30 Find the number of numbers from 1 to 100 which are not divisible by 2.

(a) 51 (b) 50

(c) 49 (d) 48

Solution The 1^{st} number from 1 to 100, not divisible by 2 is 1 and the last number from 1 to 100, not divisible by 2 is 99.

Every alternate number (i.e, at the gap of 2) will not be divisible by 2 from 1 to 99. (1, 2, 3, - - - - , 95, 97, 99)

So, the required number of nos = $\dfrac{\text{last no.} - \text{first no.}}{\text{gap (or step)}} + 1 = \dfrac{99-1}{2} + 1 = 50$

fi Option (b)

Alternate method

Total number of nos from 1 to 100 = 100 (i)

Now, if we count number of numbers from 1 to 100 which are divisible by 2 and subtract that from the total number of numbers from 1 to 100, as a result we will find the number of numbers from 1 to 100 which are not divisible by 2.

To count the number of nos from 1 to 100 which are divisible by 2:

The 1^{st} number which is divisible by 2 = 2

The last number which is divisible by 2 = 100

(2, 4, 6, - - - - , 96, 98, 100)

Gap/step between two consecutive numbers = 2

So, the number of numbers which are divisible by 2 = = $\dfrac{\text{last no.} - \text{first no.}}{\text{gap (or step)}} + 1 = \dfrac{100-2}{2} + 1 = 50$ (ii)

So, from (i) & (ii)

Required number of numbers = 100 – 50 = 50

fi Option (b)

Problem 1.31 Find the number of numbers from 1 to 100 which are not divisible by any one of 2 & 3.

(a) 16 (b) 17

(c) 18 (d) 33

Solution From 1 to 100

Number of numbers not divisible by 2 & 3 = Total number of numbers– number of numbers divisible by either 2 or 3.

Now, total number of numbers = 100 (ii)

For number of numbers divisible by either 2 or 3:

Number of numbers divisible by 2 = $\frac{\text{last no.} - \text{first no.}}{\text{gap (or step)}} + 1$ = $\frac{100-2}{2} + 1 = 50$

Now, the number of numbers divisible by 3 (but not by 2, as it has already been counted)

1st such no. = 3 and the gap will be 6. Hence 2nd such no. will be 9, 3rd no. would be 15 and the last number would be 99. Hence this series is 3, 9, 15, ..., 93, 99

So, the number of numbers divisible by 3 (but not by 2) = $\frac{\text{last such no.} - \text{first such no.}}{\text{gap/step}} + 1 = \frac{99-3}{6} + 1 = 17$

Hence, the number of numbers divisible by either 2 or 3 = 50 + 17 = 67

So, from (i), (ii) & (iii) required number of numbers = 100 – 67 = 33

fi Option (d)

Problem 1.32 Find the number of numbers from 1 to 100 which are not divisible by any one of 2, 3, and 5.

(a) 26	(b) 27
(c) 29	(d) 32

Solution From the above question, we have found out that

From 1 to 100, number of numbers divisible by 2 = 50 (i)

Number of numbers divisible by 3 (but not by 2) = 17 (ii)

Now, we have to find out the number of numbers which are divisible by 5 (but not by 2 and 3). Numbers which are divisible by 5

(5) 10 15 20 (25) 30 (35) 40 45 50 (55) 60 (65) 70 75 80 (85) 90 (95) 100

That is, there are 7 such numbers (iii)

Another way to find out the number of numbers that are divisible by 5 but not 2 and 3 is to first only consider odd multiples of 5.

You will get the series of 10 numbers: 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95

From amongst these we need to exclude multiples of 3. In other words, we need to find the number of common elements between the above series and the series of odd multiples of 3, viz, 3, 9, 15, 21 …. 99.

This situation is the same as finding the number of common elements between the two series for which we need to first observe that the first such number is 15. Then the common terms between these two series will themselves form an arithmetic series and this series will have a common difference which is the LCM of the common differences of the two series. (In this case the common difference of the two series are 10 and 6 respectively and their LCM being 30, the series of common terms between the two series will be 15, 45 and 75.) Thus, there will be 3 terms out of the 10 terms of the series 5, 15, 25…95 which will be divisible by 3 and hence need to be excluded from the count of numbers which are divisible by 5 but not 2 or 3.

Hence, the required answer would be: 100 – 50 – 17 – 7 = 26

fi Option (a)

Problem 1.33 Find the number of numbers from 1 to 100 which are not divisible by any one of 2, 3, 5 & 7.

(a) 22	(b) 24
(c) 23	(d) 27

Solution From the above question we have seen that from 1 to 100.

number of numbers divisible by 2 = 50 (i)

number of numbers divisible by 3 but not by 2 = 17 (ii)

number of numbers divisible by 5 but not by 2 and 3 = 7 (iii)

number of numbers divisible by 7 but not by 2, 3 & 5;

such nos are 7, 49, 77, 91 = 4 nos (iv)

Required number of numbers = Total number of numbers from 1 to 100 – {(i) + (ii) + (iii) + (iv)}

= 100 – (50 + 17 + 7 + 4)

= 22

fi Option (a)

Problem 1.34 What will be the remainder when – 34 is divided by 5?

(a) 1 (b) 4

(c) 2 (d) – 4

Solution – 34 = 5 × (– 6) + (– 4)

Remainder = – 4, but it is wrong because remainder cannot be negative.

So,–34 = 5 × (– 7) + 1

fi Option (a)

Alternately, when you see a remainder of – 4 when the number is divided by 5, the required remainder will be equal to 5 – 4 = 1.

Problem 1.35 What will be the remainder when – 24.8 is divided by 6?

(a) 0.8 (b) 5.2

(c) –0.8 (d) – 5.2

Solution – 24.8 = 6 × (– 4) + (– 0.8)

Negative remainder, so not correct – 24.8 = 6 × (– 5) + 5.2

Positive value of remainder, so correct

fi Option (b)

Problem 1.36 If *p* is divided by *q*, then the maximum possible difference between the minimum possible and maximum possible remainder can be?

(a) $p - q$ (b) $p - 1$

(c) $q - 1$ (d) None of these

Solution $\frac{p}{q}$ minimum possible remainder = 0 (when q exactly divides P)

Maximum possible remainder = $q - 1$

So, required maximum possible difference = $(q - 1) - 0 = (q - 1)$

fi Option (c)

Problem 1.37 Find the remainder when 2^{256} is divided by 17.

(a) 0 (b) 1

(c) 3 (d) 5

Solution $\frac{2^{256}}{17} = \frac{(2^4)^{64}}{17} = \frac{16^{64}}{17}$ fi $R = 1$

Q $\frac{a^n}{a+1}$; $R = 1$

when n Æ even

fi Option (b)

Problem 1.38 Find the difference between the remainders when 7^{84} is divided by 342 & 344.

(a) 0 (b) 1

(c) 3 (d) 5

Solution $\frac{7^{84}}{342} = \frac{(7^3)^{28}}{342} = \frac{343^{28}}{342}$ fi $R = 1$

also, $\frac{7^{84}}{344} = \frac{(7^3)^{28}}{344} = \frac{343^{28}}{344}$ fi $R = 1$

The required difference between the remainders = 1 – 1 = 0

fi Option (a)

Problem 1.39 What will be the value of x for $\frac{(100^{17}-1)+(10^{34}+x)}{9}$; the remainder = 0

(a) 3 (b) 6

(c) 9 (d) 8

Solution $\frac{(100^{17}-1)+(10^{34}+x)}{9}$

$100^{17} - 1 = \frac{1000\ldots00-1}{17 \text{ zeroes}} = \frac{9999\ldots99}{16 \text{ nines}}$ fi divisible by 9 fi $R = 0$

Since the first part of the expression is giving a remainder of 0, the second part should also give 0 as a remainder if the entire remainder of the expression has to be 0. Hence, we now evaluate the second part of the numerator.

$10^{34} + x = \underbrace{1000\ldots00 + x}_{34\text{ zeroes}} = \underbrace{1000\ldots00x}_{33\text{ zeroes}}$

with x at the right most place. In order for this number to be divisible by 9, the sum of digits should be divisible by 9.

fi $1 + 0 + 0 \ldots + 0 + x$ should be divisible by 9.

fi $1 + x$ should be divisible by 9 fi $x = 8$

fi Option (d)

LEVEL OF DIFFICULTY (I)

1. The last digit of the number obtained by multiplying the numbers 81 × 82 × 83 × 84 × 85 × 86 × 87 × 88 × 89 will be

   (a) 0 (b) 9

   (c) 7 (d) 2

2. The sum of the digits of a two-digit number is 10, while when the digits are reversed, the number decreases by 54. Find the changed number.

   (a) 28 (b) 19

   (c) 37 (d) 46

3. When we multiply a certain two-digit number by the sum of its digits, 405 is achieved. If you multiply the number written in reverse order of the same digits by the sum of the digits, we get 486. Find the number.

   (a) 81 (b) 45

   (c) 36 (d) 54

4. The sum of two numbers is 15 and their geometric mean is 20% lower than their arithmetic mean. Find the numbers.

   (a) 11, 4 (b) 12, 3

   (c) 13, 2 (d) 10, 5

5. The difference between two numbers is 48 and the difference between the arithmetic mean and the geometric mean is two more than half of 1/3 of 96. Find the numbers.

   (a) 49, 1 (b) 12, 60

   (c) 50, 2 (d) 36, 84

6. If A381 is divisible by 11, find the value of the smallest natural number A.

(a) 5 (b) 6

(c) 7 (d) 9

7. If 381A is divisible by 9, find the value of smallest natural number A.

(a) 5 (b) 5

(c) 7 (d) 6

8. What will be the remainder obtained when $(9^6 + 1)$ will be divided by 8?

(a) 0 (b) 3

(c) 7 (d) 2

9. Find the ratio between the LCM and HCF of 5, 15 and 20.

(a) 8 : 1 (b) 14 : 3

(c) 12 : 2 (d) 12 : 1

10. Find the LCM of 5/2, 8/9, 11/14.

(a) 280 (b) 360

(c) 420 (d) None of these

11. If the number A is even, which of the following will be true?

(a) 3A will always be divisible by 6

(b) 3A + 5 will always be divisible by 11

(c) $(A^2 + 3)/4$ will be divisible by 7

(d) All of these

12. A five-digit number is taken. Sum of the first four digits (excluding the number at the units digit) equals sum of all the five digits. Which of the following will not divide this number necessarily?

(a) 10 (b) 2

(c) 4 (d) 5

13. A number 15B is divisible by 6. Which of these will be true about the positive integer B?

(a) B will be even

(b) B will be odd

(c) B will be divisble by 6

(d) Both (a) and (c)

14. Two numbers $P = 2^3.3^{10}.5$ and $Q = 2^5.3^1.7^1$ are given. Find the GCD of P and Q.

(a) 2.3.5.7 (b) $3.\ 2^2$

(c) $2^2.3^2$ (d) $2^3.3$

15. Find the units digit of the expression $25^{6251} + 36^{528} + 73^{54}$.

(a) 4 (b) 0

(c) 6 (d) 5

16. Find the units digit of the expression $55^{725} + 73^{5810} + 22^{853}$.

(a) 4 (b) 0

(c) 6 (d) 5

17. Find the units digit of the expression $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$.

(a) 1 (b) 9

(c) 7 (d) 0

18. Find the units digit of the expression $11^1.12^2.13^3.\ 14^4.15^5.16^6$.

(a) 4 (b) 3

(c) 7 (d) 0

19. Find the number of zeroes at the end of 1090!

(a) 270 (b) 268

(c) 269 (d) 271

20. If 146! is divisible by 5^n, then find the maximum value of n.

(a) 34 (b) 35

(c) 36 (d) 37

21. Find the number of divisors of 1420.

(a) 14 (b) 15

(c) 13 (d) 12

22. Find the HCF and LCM of the polynomials $(x^2 - 5x + 6)$ and $(x^2 - 7x + 10)$.

(a) $(x - 2)$, $(x - 2)(x - 3)(x - 5)$

(b) $(x - 2)$, $(x - 2)(x - 3)$

(c) $(x - 3)$, $(x - 2)(x - 3)(x - 5)$

(d) $(x - 2)$, $(x - 2)(x - 3)(x - 5)^2$

Directions for Questions 23 to 25: Given two different prime numbers P and Q, find the number of divisors of the following:

23. $P.Q$

(a) 2 (b) 4

(c) 6 (d) 8

24. P^2Q

(a) 2 (b) 4

(c) 6 (d) 8

25. P^3Q^2

(a) 2 (b) 4

(c) 6 (d) 12

26. The sides of a pentagonal field (not regular) are 1737 metres, 2160 metres, 2358 metres, 1422 metres and 2214 metres respectively. Find the greatest length of the tape by which the five sides may be measured completely.

(a) 7 (b) 13

(c) 11 (d) 9

27. There are 576 boys and 448 girls in a school that are to be divided into equal sections of either boys or girls alone. Find the minimum total number of sections thus formed.

(a) 24 (b) 32

(c) 16 (d) 20

28. A milkman has three different qualities of milk. 403 gallons of 1st quality, 465 gallons of 2nd quality and 496 gallons of 3rd quality. Find the least possible number of bottles of equal size in which different milk of different qualities can be filled without mixing.

(a) 34 (b) 46

(c) 26 (d) 44

29. What is the greatest number of 4 digits that when divided by any of the numbers 6, 9, 12, 17 leaves a remainder of 1?

(a) 9997 (b) 9793

(c) 9895 (d) 9487

30. Find the least number that when divided by 16, 18 and 20 leaves a remainder 4 in each case, but is completely divisible by 7.

(a) 364 (b) 2254

(c) 2964 (d) 2884

31. Four bells ring at the intervals of 6, 8, 12 and 18 seconds. They start ringing together at 12’O’ clock. After how many seconds will they ring together again?

(a) 72 (b) 84

(c) 60 (d) 48

32. For Question 31, find how many times will they ring together during the next 12 minutes. (including the 12 minute mark)

(a) 9 (b) 10

(c) 11 (d) 12

33. The units digit of the expression $125^{813} \times 553^{3703} \times 4532^{828}$ is

(a) 4 (b) 2

(c) 0 (d) 5

34. Which of the following is not a perfect square?

(a) 1,00,856 (b) 3,25,137

(c) 9,45,729 (d) All of these

35. Which of the following can never be in the ending of a perfect square?

(a) 6 (b) 00

(c) x 000 where x is a natural number (d) 1

36. The LCM of 5, 8,12, 20 will not be a multiple of

(a) 3 (b) 9

(c) 8 (d) 5

37. Find the number of divisors of 720 (including 1 and 720).

(a) 25 (b) 28

(c) 29 (d) 30

38. The LCM of $(16 – x^2)$ and $(x^2 + x – 6)$ is

(a) $(x – 3)\ (x + 3)\ (4 – x^2)$

(b) $4\ (4 – x^2)\ (x + 3)$

(c) $(4 – x^2)\ (x – 3)$

(d) None of these

39. GCD of $x^2 – 4$ and $x^2 + x – 6$ is

(a) $x + 2$ (b) $x – 2$

(c) $x^2 – 2$ (d) $x^2 + 2$

40. The number A is not divisible by 3. Which of the following will not be divisible by 3?

(a) $9 \times A$ (b) $2 \times A$

(c) $18 \times A$ (d) $24 \times A$

41. Find the remainder when the number 9^{100} is divided by 8.

(a) 1 (b) 2

(c) 0 (d) 4

42. Find the remainder of 2^{1000} when divided by 3.

(a) 1 (b) 2

(c) 4 (d) 6

43. Decompose the number 20 into two terms such that their product is the greatest.

(a) $x_1 = x_2 = 10$ (b) $x_1 = 5, x_2 = 15$

(c) $x_1 = 16, x_2 = 4$ (d) $x_1 = 8, x_2 = 12$

44. Find the number of zeroes at the end of 50!

(a) 13 (b) 11

(c) 5 (d) 12

45. Which of the following can be a number divisible by 24?

(a) 4,32,15,604 (b) 25,61,284

(c) 13,62,480 (d) All of these

46. For a number to be divisible by 88, it should be

(a) Divisible by 22 and 8

(b) Divisible by 11 and 8

(c) Divisible by 11 and thrice by 2

(d) All of these

47. Find the number of divisors of 10800.

(a) 57 (b) 60

(c) 72 (d) 64

48. Find the GCD of the polynomials $(x + 3)^2 (x - 2)(x + 1)^2$ and $(x + 1)^3 (x + 3) (x + 4)$.

(a) $(x + 3)^3 (x + 1)^2 (x - 2) (x + 4)$

(b) $(x + 3) (x - 2) (x + 1)(x + 4)$

(c) $(x + 3) (x + 1)^2$

(d) $(x + 1) (x + 3)^2$

49. Find the LCM of $(x + 3) (6x^2 + 5x + 4)$ and $(2x^2 + 7x + 3) (x + 3)$

(a) $(2x + 1)(x + 3) (3x + 4)$

(b) $(4x^2 - 1) (x + 3)^2 (3x + 4)$

(c) $(4x^2 - 1)(x + 3) (3x + 4)$

(d) $(2x - 1) (x + 3) (3x + 4)$

50. The product of three consecutive natural numbers, the first of which is an even number, is always divisible by

(a) 12 (b) 24

(c) 6 (d) All of these

51. Some birds settled on the branches of a tree. First, they sat one to a branch and there was one bird too many. Next they sat two to a branch and there was one branch too many. How many branches were there?

(a) 3 (b) 4

(c) 5 (d) 6

52. The square of a number greater than 1000 that is not divisible by three, when divided by three, leaves a remainder of

(a) 1 always (b) 2 always

(c) 0 (d) either 1 or 2

53. The value of the expression $(15^3 \lozenge 21^2)/(35^2 \lozenge 3^4)$ is

(a) 3 (b) 15

(c) 21 (d) 12

54. If $A = \left(\frac{-3}{4}\right)^3$, $B = \left(\frac{-2}{5}\right)^2$, $C = (0.3)^2$, $D = (-1.2)^2$ then

(a) $A > B > C > D$ (b) $D > A > B > C$

(c) $D > B > C > A$ (d) $D > C > A > B$

55. If $2 < x < 4$ and $1 < y < 3$, then find the ratio of the upper limit for $x + y$ and the lower limit of $x - y$.

(a) 6 (b) 7

(c) 8 (d) None of these

56. The sum of the squares of the digits constituting a positive two-digit number is 13. If we subtract 9 from that number, we shall get a number written by the same digits in the reverse order. Find the number.

(a) 12 (b) 32

(c) 42 (d) 52

57. The product of a natural number by the number written by the same digits in the reverse order is 2430. Find the numbers.

(a) 54 and 45 (b) 56 and 65

(c) 53 and 35 (d) 85 and 58

58. Find two natural numbers whose difference is 66 and the least common multiple is 360.

(a) 120 and 54 (b) 90 and 24

(c) 180 and 114 (d) 130 and 64

59. Find the pairs of natural numbers whose least common multiple is 78 and the greatest common divisor is 13.

(a) 58 and 13 or 16 and 29

(b) 68 and 23 or 36 and 49

(c) 18 and 73 or 56 and 93

(d) 78 and 13 or 26 and 39

60. Find two natural numbers whose sum is 85 and the least common multiple is 102.

(a) 30 and 55
(b) 17 and 68
(c) 35 and 55
(d) 51 and 34

61. Find the pairs of natural numbers the difference of whose squares is 55.

(a) 28 and 27 or 8 and 3
(b) 18 and 17 or 18 and 13
(c) 8 and 27 or 8 and 33
(d) 9 and 18 or 8 and 27

62. Which of these is greater?

(a) 54^4 or 21^{12}
(b) $(0.4)^4$ or $(0.8)^3$

63. Is it possible for a common fraction whose numerator is less than the denominator to be equal to a fraction whose numerator is greater than the denominator?

(a) Yes
(b) No

64. What digits should be put in place of *c* in 38*c* to make it divisible by

(1) 2
(2) 3
(3) 4
(4) 5
(5) 6
(6) 9
(7) 10

65. Find the LCM and HCF of the following numbers: (54, 81, 135 and 189), (156, 195) and (1950, 5670 and 3900)

66. The last digit in the expansions of the three digit number $(34x)^{43}$ and $(34x)^{44}$ are 7 and 1 respectively. What can be said about the value of x?

(a) $x = 5$
(b) $x = 3$
(c) $x = 6$
(d) $x = 2$

Directions for Questions 67 and 68: Amitesh buys a pen, a pencil and an eraser for ` 41. If the least cost of any of the three items is ` 12 and it is known that a pen costs less than a pencil and an eraser costs more than a pencil, answer the following questions:

67. What is the cost of the pen?

    (a) 12 (b) 13

    (c) 14 (d) 15

68. If it is known that the eraser's cost is not divisible by 4, the cost of the pencil could be:

    (a) 12 (b) 13

    (c) 14 (d) 15

69. A naughty boy Amrit watches an innings of Sachin Tendulkar and acts according to the number of runs he sees Sachin scoring. The details of these are given below.

1 run	Place an orange in the basket
2 runs	Place a mango in the basket
3 runs	Place a pear in the basket
4 runs	Remove a pear and a mango from the basket

One fine day, at the start of the match, the basket is empty. The sequence of runs scored by Sachin in that innings are given as 11232411234232341121314. At the end of the above innings, how many more oranges were there compared to mangoes inside the basket? (The Basket was empty initially).

    (a) 4 (b) 5

    (c) 6 (d) 7

70. In the famous Bel Air Apartments in Ranchi, there are three watchmen meant to protect the precious fruits in the campus. However, one day a thief got in without being noticed and stole some precious mangoes. On the way out however, he was

confronted by the three watchmen, the first two of whom asked him to part with 1/3rd of the fruits and one more. The last asked him to part with 1/5th of the mangoes and 4 more. As a result he had no mangoes left. What was the number of mangoes he had stolen?

(a) 12 (b) 13

(c) 15 (d) None of these

71. A hundred and twenty digit number is formed by writing the first x natural numbers in front of each other as 12345678910111213… Find the remainder when this number is divided by 8.

(a) 6 (b) 7

(c) 2 (d) 0

72. A test has 80 questions. There is one mark for a correct answer, while there is a negative penalty of –1/2 for a wrong answer and –1/4 for an unattempted question. What is the number of questions answered correctly, if the student has scored a net total of 34.5 marks?

(a) 45 (b) 48

(c) 54 (d) Cannot be determined

73. For Question 72, if it is known that he has left 10 questions unanswered, the number of correct answers are:

(a) 45 (b) 48

(c) 54 (d) Cannot be determined

74. Three mangoes, four guavas and five watermelons cost ` 750. Ten watermelons, six mangoes and 9 guavas cost `1580. What is the cost of six mangoes, ten watermelons and 4 guavas?

(a) 1280 (b) 1180

(c) 1080 (d) Cannot be determined

75. From a number M subtract 1. Take the reciprocal of the result to get the value of 'N'. Then which of the following is necessarily true?

(a) $0 \leq M^N \leq 2$ (b) $M^N > 3$

(c) $1 < M^N < 3$ (d) $1 < M^N < 5$

76. The cost of four mangoes, six guavas and sixteen watermelons is ` 500, while the cost of seven magoes, nine guavas and nineteen watermelons is ` 620. What is the cost of one mango, one guava and one watermelon?

(a) 120 (b) 40

(c) 150 (d) Cannot be determined

77. For the question above, what is the cost of a mango?

(a) 20 (b) 14

(c) 15 (d) Cannot be determined

78. The following is known about three real numbers, x, y and z.

$-4 \le x \le 4$, $-8 \le y \le 2$ and $-8 \le z \le 2$. Then the range of values that $M = xz/y$ can take is best represented by:

(a) $-\bullet < x < \bullet$ (b) $-16 \le x \le 8$

(c) $-8 \le x \le 8$ (d) $-16 \le x \le 16$

79. A man sold 38 pieces of clothing (combined in the form of shirts, trousers and ties). If he sold at least 11 pieces of each item and he sold more shirts than trousers and more trousers than ties, then the number of ties that he must have sold is:

(a) Exactly 11 (b) At least 11

(c) At least 12 (d) Cannot be determined

80. For Question 79, find the number of shirts he must have sold.

(a) At least 13 (b) At least 14

(c) At least 15 (d) At most 16.

81. Find the least number which when divided by 12, 15, 18 or 20 leaves in each case a remainder 4.

(a) 124 (b) 364

(c) 184 (d) None of these

82. What is the least number by which 2800 should be multiplied so that the product may be a perfect square?

(a) 2 (b) 7

(c) 14 (d) None of these

83. The least number of 4 digits which is a perfect square is:

(a) 1064 (b) 1040

(c) 1024 (d) 1012

84. The least multiple of 7 which leaves a remainder of 4 when divided by 6, 9, 15 and 18 is

(a) 94 (b) 184

(c) 364 (d) 74

85. What is the least 3 digit number that when divided by 2, 3, 4, 5 or 6 leaves a remainder of 1?

(a) 131 (b) 161

(c) 121 (d) None of these

86. The highest common factor of 70 and 245 is equal to

(a) 35 (b) 45

(c) 55 (d) 65

87. Find the least number, which must be subtracted from 7147 to make it a perfect square.

(a) 86 (b) 89

(c) 91 (d) 93

88. Find the least square number which is divisible by 6, 8 and 15

(a) 2500 (b) 3600

(c) 4900 (d) 4500

89. Find the least number by which 30492 must be multiplied or divided so as to make it a

perfect square.

(a) 11 (b) 7

(c) 3 (d) 2

90. The greatest 4-digit number exactly divisible by 88 is

(a) 8888 (b) 9768

(c) 9944 (d) 9988

91. By how much is three fourth of 116 greater than four fifth of 45?

(a) 31 (b) 41

(c) 46 (d) None of these

92. If 5625 plants are to be arranged in such a way that there are as many rows as there are plants in a row, the number of rows will be:

(a) 95 (b) 85

(c) 65 (d) None of these

93. A boy took a seven digit number ending in 9 and raised it to an even power greater than 2000. He then took the number 17 and raised it to a power which leaves the remainder 1 when divided by 4. If he now multiples both the numbers, what will be the unit's digit of the number he so obtains?

(a) 7 (b) 9

(c) 3 (d) Cannot be determined

94. Two friends were discussing their marks in an examination. While doing so they realized that both the numbers had the same prime factors, although Raveesh got a score which had two more factors than Harish. If their marks are represented by one of the options as given below, which of the following options would correctly represent the number of marks they got?

(a) 30,60 (b) 20,80

(c) 40,80 (d) 20,60

95. A number is such that when divided by 3, 5, 6, or 7 it leaves the remainder 1, 3, 4, or 5 respectively. Which is the largest number below 4000 that satisfies this property?

(a) 3358 (b) 3988

(c) 3778 (d) 2938

96. A number when divided by 2,3 and 4 leaves a remainder of 1. Find the least number (after 1) that satisfies this requirement.

(a) 25 (b) 13

(c) 37 (d) 17

97. A number when divided by 2, 3 and 4 leaves a remainder of 1. Find the second lowest number (not counting 1) that satisfies this requirement.

(a) 25 (b) 13

(c) 37 (d) 17

98. A number when divided by 2, 3 and 4 leaves a remainder of 1. Find the highest 2 digit number that satisfies this requirement.

(a) 91 (b) 93

(c) 97 (d) 95

99. A number when divided by 2,3 and 4 leaves a remainder of 1. Find the highest 3 digit number that satisfies this requirement.

(a) 991 (b) 993

(c) 997 (d) 995

100. A frog is sitting on vertex A of a square $ABCD$. It starts jumping to the immediately adjacent vertex on either side in random fashion and stops when it reaches point C. In how many ways can it reach point C if it makes exactly 7 jumps?

(a) 1 (b) 3

(c) 5 (d) 0

101. Three bells ring at intervals of 5 seconds, 6 seconds and 7 seconds respectively. If they toll together for the first time at 9 AM in the morning, after what interval of time will they together ring again for the first time?

(a) After 30 seconds (b) After 42 seconds

(c) After 35 seconds (d) After 210 seconds

102. For the question above, how many times would they ring, together in the next 1 hour?

(a) 17 (b) 18

(c) 19 (d) None of these

103. A garrison has three kinds of soldiers. There are 66 soldiers of the first kind, 110 soldiers of the second kind and 242 soldiers of the third kind. It is desired to be arranging these soldiers in equal rows such that each row contains the same number of soldiers and there is only 1 kind of soldier in each row. What is the maximum number of soldiers who can be placed in each row?

(a) 11 (b) 1

(c) 22 (d) 33

104. For the question above, what are the minimum number of rows that would be required to be formed?

(a) 11 (b) 19

(c) 18 (d) None of these

105. A milkman produces three kinds of milk. On a particular day, he has 170 liters, 102 liters and 374 liters of the three kinds of milk. He wants to bottle them in bottles of equal sizes- so that each of the three varieties of milk would be completed bottled. How many bottle sizes are possible such that the bottle size in terms of liters is an integer?

(a) 1 (b) 2

(c) 4 (d) 34

106. For the above question, what is the size of the largest bottle which can be used?

(a) 1 (b) 2

(c) 17 (d) 34

107. For Question 105, what are the minimum number of bottles that would be required?

(a) 11 (b) 19

(c) 18 (d) None of these

108. Find the number of zeroes at the end of 100!

(a) 20 (b) 23

(c) 24 (d) 25

109. Find the number of zeroes at the end of 122!

(a) 20 (b) 23

(c) 24 (d) 28

110. Find the number of zeroes at the end of 1400!

(a) 347 (b) 336

(c) 349 (d) 348

111. Find the number of zeroes at the end of 380!

(a) 90 (b) 91

(c) 94 (d) 95

112. Find the number of zeroes at the end of 72!

(a) 14 (b) 15

(c) 16 (d) 17

113. The highest power of 3 that completely divides 40! is

(a) 18 (b) 15

(c) 16 (d) 17

114. $53!/3^n$ is an integer. Find the highest possible value of n for this to be true.

(a) 19 (b) 21

(c) 23 (d) 24

115. The highest power of 7 that completely divides 80! is:

(a) 12 (b) 13

(c) 14 (d) 15

116. $115!/7^n$ is an integer. Find the highest possible value of n for this to be true.

(a) 15 (b) 17

(c) 16 (d) 18

117. The highest power of 12 that completely divides 122! is:

(a) 54 (b) 56

(c) 57 (d) 58

118. $155!/20^n$ is an integer. Find the highest possible value of n for this to be true.

(a) 77 (b) 38

(c) 75 (d) 37

119. The minimum value of x so that $x^2/1024$ is an integer is:

(a) 4 (b) 32

(c) 16 (d) 64

120. Find the sum of all 2 digit natural numbers which leave a remainder of 3 when divided by 7.

(a) 650 (b) 663

(c) 676 (d) 702

121. How many numbers between 1 and 200 are exactly divisible by exactly two of 3, 9 and 27?

(a) 14 (b) 15

(c) 16 (d) 17

122. A number N is squared to give a value of S. The minimum value of $N + S$ would happen when N is

(a) –0.3 (b) –0.5

(c) –0.7 (d) None of these

123. $L = x + y$ where x and y are prime numbers. Which of the following statement/s is/are true?

(i) The unit's digit of L cannot be 5

(ii) The units digit of L cannot be 0.

(iii) L cannot be odd.

(a) All three (b) Only iii

(c) only ii (d) None

124. XYZ is a 3 digit number such that when we calculate the difference between the two three digit numbers $XYZ - YXZ$ the difference is exactly 90. How many possible values exist for the digits X and Y?

(a) 9 (b) 8

(c) 7 (d) 6

125. What is the sum of all even numbers between 1 and 100 (both included)?

(a) 2450 (b) 2500

(c) 2600 (d) 2550

126. The least number which can be added to 763 so that it is completely divisible by 57 is:

(a) 35 (b) 22

(c) 15 (d) 25

127. The least number which can be subtracted from 763 so that it is completely divisible by 57 is:

(a) 35 (b) 22

(c) 15 (d) 25

128. The least number which can be added to 8441 so that it is completely divisible by 57 is?

(a) 42 (b) 15

(c) 5 (d) 52

129. The least number which can be subtracted from 8441 so that it is completely divisible by 57 is:

(a) 3 (b) 4

(c) 5 (d) 6

130. Find the least number of 5 digits that is exactly divisible by 79

(a) 10003 (b) 10033

(c) 10043 (d) None of these

131. Find the maximum number of 5 digits that is exactly divisible by 79:

(a) 99925 (b) 99935

(c) 99945 (d) 99955

132. The nearest integer to 773 which is exactly divisible by 12 is:

(a) 768 (b) 772

(c) 776 (d) None of these

133. A number when divided by 84 leaves a remainder of 57. What is the remainder when the same number is divided by 12?

(a) 7 (b) 8

(c) 9 (d) Cannot be determined

134. A number when divided by 84 leaves a remainder of 57. What is the remainder when the same number is divided by 11?

(a) 2 (b) 7

(c) 8 (d) Cannot be determined

135. 511 and 667 when divided by the same number, leave the same remainder. How many numbers can be used as the divisor in order to make this occur?

(a) 14 (b) 12

(c) 10 (d) 8

136. How many numbers between 200 and 400 are divisible by 13?

(a) 14 (b) 15

(c) 16 (d) 17

137. A boy was trying to find $5/8^{th}$ of a number. Unfortunately, he found out $8/5^{th}$ of the number and realized that the difference between the answer he got and the correct answer is 39. What was the number?

(a) 38 (b) 39

(c) 40 (d) 52

138. The sum of two numbers is equal to thrice their difference. If the smaller of the numbers is 10 find the other number.

(a) 15 (b) 20

(c) 40 (d) None of these

139. $4^{11} + 4^{12} + 4^{13} + 4^{14} + 4^{15}$ is divisible by which of the following?

(a) 11 (b) 31

(c) 341 (d) All of the above

140. The product of two numbers is 7168 and their HCF is 16. How many pairs of numbers are possible such that the above conditions are satisfied?

(a) 2 (b) 3

(c) 4 (d) 6

LEVEL OF DIFFICULTY (II)

1. The arithmetic mean of two numbers is smaller by 24 than the larger of the two numbers and the GM of the same numbers exceeds by 12 the smaller of the numbers. Find the numbers.

   (a) 6 and 54 (b) 8 and 56

   (c) 12 and 60 (d) 7 and 55

2. Find the number of numbers between 200 and 300, both included, which are not divisible by 2, 3, 4 and 5.

   (a) 27 (b) 26

   (c) 25 (d) 28

3. Given x and n are integers, $(15n^3 + 6n^2 + 5n + x)/n$ is not an integer for what condition?

   (a) n is positive

   (b) x is divisible by n

   (c) x is not divisible by n

   (d) (a) and (c)

4. The unit digit in the expression $36^{234}*33^{512}*39^{180} - 54^{29}*25^{123}*31^{512}$ will be

   (a) 8 (b) 0

   (c) 6 (d) 5

5. The difference of $10^{25} - 7$ and $10^{24} + x$ is divisible by 3 for x = ?

   (a) 3 (b) 2

   (c) 4 (d) 6

6. Find the value of x in $\sqrt{x+2\sqrt{x+2\sqrt{x+2\sqrt{3x}}}} = x$.

   (a) 1 (b) 3

(c) 6 (d) 12

(e) 9

7. If a number is multiplied by 22 and the same number is added to it, then we get a number that is half the square of that number. Find the number

(a) 45 (b) 46

(c) 47 (d) data insufficient

8. $12^{55}/3^{11} + 8^{48}/16^{18}$ will give the digit at units place as

(a) 4 (b) 6

(c) 8 (d) 0

9. The mean of 1, 2, 2^2…. 2^{31} lies in between

(a) 2^{24} to 2^{25} (b) 2^{25} to 2^{26}

(c) 2^{26} to 2^{27} (d) 2^{29} to 2^{30}

10. *xy* is a number that is divided by *ab* where $xy < ab$ and gives a result 0.*xyxyxy*… then *ab* equals

(a) 11 (b) 33

(c) 99 (d) 66

11. A number *xy* is multiplied by another number *ab* and the result comes as *pqr*, where $r = 2y$, $q = 2(x + y)$ and $p = 2x$ where $x, y < 5$, q π 0. The value of *ab* may be:

(a) 11 (b) 13

(c) 31 (d) 22

12. [*x*] denotes the greatest integer value just below *x* and {*x*} its fractional value. The sum of $[x]^3$ and $\{x\}^2$ is –7.91. Find *x*.

(a) –2.03 (b) –1.97

(c) –2.97 (d) –1.7

13. $16^5 + 2^{15}$ is divisible by

(a) 31 (b) 13

(c) 27 (d) 33

14. If $AB + XY = 1XP$, where A π 0 and all the letters signify different digits from 0 to 9, then the value of A is:

(a) 6 (b) 7

(c) 9 (d) 8

Directions for questions 15 and 16: Find the possible integral values of x.

15. $|x - 3| + 2|x + 1| = 4$

(a) 1 (b) –1

(c) 3 (d) 2

16. $x^2 + |x - 1| = 1$

(a) 1 (b) –1

(c) 0 (d) 1 or 0

17. If $4^{n+1} + x$ and $4^{2n} - x$ are divisible by 5, n being an even integer, find the least value of x.

(a) 1 (b) 2

(c) 3 (d) 0

18. If the sum of the numbers $(a25)^2$ and a^3 is divisible by 9, then which of the following may be a value for a ?

(a) 1 (b) 7

(c) 9 (d) There is no value

19. If $| x - 4 | + | y - 4 | = 4$, then how many integer values can the set (x, y) have?

(a) Infinite (b) 5

(c) 16 (d) 9

20. $[3^{32}/50]$ gives remainder and {.} denotes the fractional part of that. The fractional part is of the form (0 ◊ bx). The value of x could be

(a) 2 (b) 4

(c) 6 (d) 8

21. The sum of two numbers is 20 and their geometric mean is 20% lower than their arithmetic mean. Find the ratio of the numbers.

(a) 4 : 1 (b) 9 : 1

(c) 1 : 1 (d) 17 : 3

22. The highest power on 990 that will exactly divide 1090! is

(a) 101 (b) 100

(c) 108 (d) 109

23. If 146! is divisible by 6^n, then find the maximum value of n.

(a) 74 (b) 70

(c) 76 (d) 75

24. The last two digits in the multiplication of 35 ◊ 34 ◊ 33 ◊ 32 ◊ 31 ◊ 30 ◊ 29 ◊ 28 ◊ 27 ◊ 26 is

(a) 00 (b) 40

(c) 30 (d) 10

25. The expression $333^{555} + 555^{333}$ is divisible by

(a) 2 (b) 3

(c) 37 (d) All of these

26. $[x]$ denotes the greatest integer value just below x and $\{x\}$ its fractional value. The sum of $[x]^2$ and $\{x\}^1$ is 25.16. Find x.

(a) 5.16 (b) –4.84

(c) Both (a) and (b) (d) 4.84

27. If we add the square of the digit in the tens place of a positive two-digit number to the product of the digits of that number, we shall get 52, and if we add the square of the digit in the units place to the

same product of the digits, we shall get 117. Find the two-digit number.

(a) 18 (b) 39

(c) 49 (d) 28

28. Find two numbers such that their sum, their product and the differences of their squares are equal.

(a) $\left(\frac{3+\sqrt{3}}{2}\right)$ and $\left(\frac{1+\sqrt{2}}{2}\right)$ or $\left(\frac{3+\sqrt{2}}{2}\right)$ and $\left(\frac{1+\sqrt{2}}{2}\right)$

(b) $\left(\frac{3+\sqrt{7}}{2}\right)$ and $\left(\frac{1+\sqrt{7}}{2}\right)$ or $\left(\frac{3+\sqrt{6}}{2}\right)$ and $\left(\frac{1-\sqrt{6}}{2}\right)$

(c) $\left(\frac{3-\sqrt{5}}{2}\right)$ and $\left(\frac{1-\sqrt{5}}{2}\right)$ or $\left(\frac{3+\sqrt{5}}{2}\right)$ and $\left(\frac{1+\sqrt{5}}{2}\right)$

(d) All of these

29. The sum of the digits of a three-digit number is 17, and the sum of the squares of its digits is 109. If we subtract 495 from that number, we shall get a number consisting of the same digits written in the reverse order. Find the number.

(a) 773 (b) 863

(c) 683 (d) 944

30. Find the number of zeros in the product: $1^1 \times 2^2 \times 3^3 \times 4^4 \times ...... 98^{98} \times 99^{99} \times 100^{100}$

(a) 1200 (b) 1300

(c) 1050 (d) 1225

31. Find the pairs of natural numbers whose greatest common divisor is 5 and the least common multiple is 105.

(a) 5 and 105 or 15 and 35

(b) 6 and 105 or 16 and 35

(c) 5 and 15 or 15 and 135

(d) 5 and 20 or 15 and 35

32. The denominator of an irreducible fraction is greater than the numerator by 2. If we reduce the numerator of the reciprocal fraction by 3 and subtract the given fraction from the resulting one, we get 1/15. Find the given fraction.

(a) $\frac{2}{4}$ (b) $\frac{3}{5}$

(c) $\frac{5}{7}$ (d) $\frac{7}{9}$

33. A two-digit number exceeds by 19 the sum of the squares of its digits and by 44 the double product of its digits. Find the number.

(a) 72 (b) 62

(c) 22 (d) 12

34. The sum of the squares of the digits constituting a two-digit positive number is 2.5 times as large as the sum of its digits and is larger by unity than the trebled product of its digits. Find the number.

(a) 13 and 31 (b) 12 and 21

(c) 22 and 33 (d) 14 and 41

35. The units digit of a two-digit number is greater than its tens digit by 2, and the product of that number by the sum of its digits is 144. Find the number.

(a) 14 (b) 24

(c) 46 (d) 35

36. Find the number of zeroes in the product: $5 \times 10 \times 25 \times 40 \times 50 \times 55 \times 65 \times 125 \times 80$

(a) 8 (b) 9

(c) 12 (d) 13

37. The highest power of 45 that will exactly divide 123! is

(a) 28 (b) 30

(c) 31 (d) 59

38. Three numbers are such that the second is as much lesser than the third as the first is lesser than the second. If the product of the two smaller numbers is 85 and the product of two larger numbers is 115 find the middle number.

(a) 9 (b) 8

(c) 12 (d) 10

39. Find the smallest natural number *n* such that *n*! is divisible by 990.

(a) 3 (b) 5

(c) 11 (d) 12

40. $\sqrt{x}\sqrt{y} = \sqrt{xy}$ is true only when

(a) $x > 0$, $y > 0$ (b) $x > 0$ and $y < 0$

(c) $x < 0$ and $y > 0$ (d) All of these

Directions for Questions 41 to 60: Read the instructions below and solve the questions based on this.

In an examination situation, always solve the following type of questions by substituting the given options, to arrive at the solution.

However, as you can see, there are no options given in the questions here since these are meant to be an exercise in equation writing (which I believe is a primary skill required to do well in aptitude exams testing mathematical aptitude). Indeed, if these questions had options for them, they would be rated as LOD 1 questions. But since the option-based solution technique is removed here, I have placed these in the LOD 2 category.

41. Find the two-digit number that meets the following criteria. If the number in the units place exceeds, the number in its tens by 2 and the product of the required number with the sum of its digits is equal to 144.

42. The product of the digits of a two-digit number is twice as large as the sum of its digits. If we subtract 27 from the required number, we

get a number consisting of the same digits written in the reverse order. Find the number?

43. The product of the digits of a two-digit number is one-third that number. If we add 18 to the required number, we get a number consisting of the same digits written in the reverse order. Find the number?

44. The sum of the squares of the digits of a two-digit number is 13. If we subtract 9 from that number, we get a number consisting of the same digits written in the reverse order. Find the number?

45. A two-digit number is thrice as large as the sum of its digits, and the square of that sum is equal to the trebled required number. Find the number?

46. Find a two-digit number that exceeds by 12 the sum of the squares of its digits and by 16 the doubled product of its digits.

47. The sum of the squares of the digits constituting a two-digit number is 10, and the product of the required number by the number consisting of the same digits written in the reverse order is 403. Find the 2 numbers that satisfy these conditions?

48. If we divide a two-digit number by the sum of its digits, we get 4 as a quotient and 3 as a remainder. Now, if we divide that two-digit number by the product of its digits, we get 3 as a quotient and 5 as a remainder. Find the two-digit number.

49. There is a natural number that becomes equal to the square of a natural number when 100 is added to it, and to the square of another natural number when 169 is added to it. Find the number?

50. Find two natural numbers whose sum is 85 and whose least common multiple is 102.

51. Find two-three digit numbers whose sum is a multiple of 504 and the quotient is a multiple of 6.

52. The difference between the digits in a two-digit number is equal to 2, and the sum of the squares of the same digits is 52. Find all the possible numbers?

53. If we divide a given two-digit number by the product of its digits, we obtain 3 as a quotient and 9 as a remainder. If we subtract the product of the digits constituting the number, from the square of the sum of its digits, we obtain the given number. Find the number.

54. Find the three-digit number if it is known that the sum of its digits is 17 and the sum of the squares of its digits is 109. If we subtract 495 from this number, we obtain a number consisting of the same digits written in reverse order.

55. The sum of the cubes of the digits constituting a two-digit number is 243 and the product of the sum of its digits by the product of its digits is 162. Find the two two-digit number?

56. The difference between two numbers is 16. What can be said about the total numbers divisible by 7 that can lie in between these two numbers.

57. Arrange the following in descending order:

111^4, 110.109.108.107, 109.110.112.113

58. If 3 £ *x* £ 5 and 4 £ *y* £ 7. Find the greatest value of *xy* and the least value of *x*/*y*.

59. Which of these is greater:

(a) 200^{300} or 300^{200} or 400^{150}

(b) 5^{100} and 2^{200}

(c) 10^{20} and 40^{10}

60. The sum of the two numbers is equal to 15 and their arithmetic mean is 25 per cent greater than its geometric mean. Find the numbers.

61. Define a number K such that it is the sum of the squares of the first M natural numbers.(i.e. $K = 1^2 + 2^2 +\ldots.+ M^2$) where $M < 55$. How many values of M exist such that K is divisible by 4?

(a) 10 (b) 11

(c) 12 (d) None of these

62. M is a two digit number which has the property that:

The product of factorials of it's digits > sum of factorials of its digits

How many values of M exist?

(a) 56 (b) 64

(c) 63 (d) None of these

63. A natural number when increased by 50% has its number of factors unchanged. However, when the value of the number is reduced by 75%, the number of factors is reduced by 66.66%. One such number could be:

(a) 32 (b) 84

(c) 126 (d) None of these

64. Find the 28383^{rd} term of the series: 123456789101112….

(a) 3 (b) 4

(c) 9 (d) 7

65. If you form a subset of integers chosen from between 1 to 3000, such that no two integers add up to a multiple of nine, what can be the maximum number of elements in the subset. (Include both 1 and 3000.)

(a) 1668 (b) 1332

(c) 1333 (d) 1336

66. The series of numbers (1,1/2,1/3,1/4 …………….1/1972) is taken. Now two numbers are taken from this series (the first two) say x, y. Then the operation $x + y + x.y$ is performed to get a consolidated number. The process is repeated. What will be the value of the set after all the numbers are consolidated into one number.

(a) 1970 (b) 1971

(c) 1972 (d) None of these

67. K is a three digit number such that the ratio of the number to the sum of its digits is least. What is the difference between the hundreds and the tens digits of K?

(a) 9 (b) 8

(c) 7 (d) None of these

68. In Question 67, what can be said about the difference between the tens and the units digit?

(a) 0 (b) 1

(c) 2 (d) None of these

69. For the above question, for how many values of K will the ratio be the highest?

(a) 9 (b) 8

(c) 7 (d) None of these

70. A triangular number is defined as a number which has the property of being expressed as a sum of consecutive natural numbers starting with 1. How many triangular numbers less than 1000, have the property that they are the difference of squares of two consecutive natural numbers?

(a) 20 (b) 21

(c) 22 (d) 23

71. x and y are two positive integers. Then what will be the sum of the coefficients of the expansion of the expression $(x + y)^{44}$? Answer: 2^{44}

(a) 2^{43} (b) $2^{43} + 1$

(c) 2^{44} (d) $2^{44} - 1$

72. What is the remainder when $9 + 9^2 + 9^3 + \ldots.9^{2n+1}$ is divided by 6?

(a) 1 (b) 2

(c) 3 (d) 4

73. The remainder when the number 123456789101112 ……484950 is divided by 16 is:

(a) 3 (b) 4

(c) 5 (d) 6

74. What is the highest power of 3 available in the expression 58! – 38!

(a) 17 (b) 18

(c) 19 (d) None of these

75. Find the remainder when the number represented by 22334 raised to the power $(1^2 + 2^2 + ..+ 66^2)$ is divided by 5?

(a) 2 (b) 4

(c) 1 (d) None of these

76. What is the total number of divisors of the number $12^{33} \times 34^{23} \times 2^{70}$?

(a) 4658. (b) 9316

(c) 2744 (d) None of these

77. For Question 76, which of the following will represent the sum of factors of the number (such that only odd factors are counted)?

(a) $\frac{(3^{34}-1)}{2} \times \frac{(17^{24}-1)}{16}$ (b) $(3^{34}–1) \times (17^{24} - 1)$

(c) $\frac{(3^{34}-1)}{33}$

(d) None of these

78. What is the remainder when $(1!)^3 + (2!)^3 + (3!)^3 + (4!)^3 + \ldots (1152!)^3$ is divided by 1152?

(a) 125

(b) 225

(c) 325

(d) 205

79. A set *S* is formed by including some of the first One thousand natural numbers. *S* contains the maximum number of numbers such that they satisfy the following conditions:

1. No number of the set *S* is prime.
2. When the numbers of the set *S* are selected two at a time, we always see co prime numbers

What is the number of elements in the set S?

(a) 11

(b) 12

(c) 13

(d) 7

Find the last two digits of the following numbers

80. $101 \times 102 \times 103 \times 197 \times 198 \times 199$

(a) 54

(b) 74

(c) 64

(d) 84

81. $65 \times 29 \times 37 \times 63 \times 71 \times 87$

(a) 05

(b) 95

(c) 15

(d) 25

82. $65 \times 29 \times 37 \times 63 \times 71 \times 87 \times 85$

(a) 25

(b) 35

(c) 75

(d) 85

83. $65 \times 29 \times 37 \times 63 \times 71 \times 87 \times 62$

(a) 70

(b) 30

(c) 10 (d) 90

84. $75 \times 35 \times 47 \times 63 \times 71 \times 87 \times 82$

(a) 50 (b) 70

(c) 30 (d) 90

85. $(201 \times 202 \times 203 \times 204 \times 246 \times 247 \times 248 \times 249)^2$

(a) 36 (b) 56

(c) 76 (d) 16

86. Find the remainder when 7^{99} is divided by 2400.

(a) 1 (b) 343

(c) 49 (d) 7

87. Find the remainder when $(10^3 + 9^3)^{752}$ is divided by 12^3.

(a) 729 (b) 1000

(c) 752 (d) 1

88. Arun, Bikas and Chetakar have a total of 80 coins among them. Arun triples the number of coins with the others by giving them some coins from his own collection. Next, Bikas repeats the same process. After this Bikas now has 20 coins. Find the number of coins he had at the beginning?

(a) 22 (b) 20

(c) 18 (d) 24

89. The super computer at Ram Mohan Roy Seminary takes an input of a number N and a X where X is a factor of the number N. In a particular case N is equal to $83p\ 796161q$ and X is equal to 11 where $0 < p < q$, find the sum of remainders when N is divided by $(p + q)$ and p successively.

(a) 6 (b) 3

(c) 2 (d) 9

90. On March 1st 2016, Sherry saved Re.1. Everyday starting from March 2nd 2016, he saved Re.1 more than the previous day. Find

the first date after March 1st 2016 at the end of which his total savings will be a perfect square.

(a) 17th March 2016
(b) 18th April 2016
(c) 26th March 2016
(d) None of these

91. What is the rightmost digit preceding the zeroes in the value of 20^{53}?

(a) 2
(b) 8
(c) 1
(d) 4

92. What is the remainder when 2(8!) – 21(6!) divides 14(7!) + 14(13!)?

(a) 1
(b) 7!
(c) 8!
(d) 9!

93. How many integer values of x and y are there such that $4x + 7y = 3$, while $|x| < 500$ and $|y| < 500$?

(a) 144
(b) 141
(c) 143
(d) 142

94. If $n = 1 + m$, where m is the product of four consecutive positive integers, then which of the following is/are true?

(A) n is odd
(B) n is not a multiple of 3
(C) n is a perfect square

(a) All three
(b) *A* and *B* only
(c) A and C only
(d) None of these

95. How many two-digit numbers less than or equal to 50, have the product of the factorials of their digits less than or equal to the sum of the factorials of their digits?

(a) 18
(b) 16

(c) 15

(d) None of these

96. A candidate takes a test and attempts all the 100 questions in it. While any correct answer fetches 1 mark, wrong answers are penalised as follows; one-tenth of the questions carry 1/10 negative mark each, one-fifth of the questions carry 1/5 negative marks each and the rest of the questions carry ½ negative mark each. Unattempted questions carry no marks. What is the difference between the maximum and the minimum marks that he can score?

(a) 100

(b) 120

(c) 140

(d) None of these

Directions for Questions 97 to 99: A mock test is taken at Mindworkzz. The test paper comprises of questions in three levels of difficulty—LOD1, LOD2 and LOD 3.

The following table gives the details of the positive and negative marks attached to each question type:

Difficulty level	*Positive marks for answering the question correctly*	*Negative marks for answering the question wrongly*
LOD 1	4	2
LOD 2	3	1.5
LOD 3	2	1

The test had 200 questions with 80 on LOD 1 and 60 each on LOD 2 and LOD 3.

97. If a student has solved 100 questions exactly and scored 120 marks, the maximum number of incorrect questions that he/she might have marked is:

(a) 44

(b) 56

(c) 60

(d) None of these

98. If Amit attempted the least number of questions and got a total of 130 marks, and if it is known that he attempted at least one of every type, then the number of questions he must have attempted is:

(a) 34 (b) 35

(c) 36 (d) None of these

99. In the above question, what is the least number of questions he might have got incorrect?

(a) 0 (b) 1

(c) 2 (d) None of these

100. Amitabh has a certain number of toffees, such that if he distributes them amongst ten children he has nine left, if he distributes amongst 9 children he would have 8 left, if he distributes amongst 8 children he would have 7 left … and so on until if he distributes amongst 5 children he should have 4 left. What is the second lowest number of toffees he could have with him?

(a) 2519 (b) 7559

(c) 8249 (d) 5039

LEVEL OF DIFFICULTY (III)

1. What two-digit number is less than the sum of the square of its digits by 11 and exceeds their doubled product by 5?

   (a) 15, 95 (b) 95

   (c) Both (a) and (b) (d) 15, 95 and 12345

2. Find the lower of the two successive natural numbers if the square of the sum of those numbers exceeds the sum of their squares by 112.

   (a) 6 (b) 7

   (c) 8 (d) 9

3. First we increased the denominator of a positive fraction by 3 and then we decreased it by 5. The sum of the resulting fractions proves to be equal to 19/42. Find the denominator of the fraction if its numerator is 2.

   (a) 7 (b) 8

   (c) 12 (d) 9

4. Find the last two digits of: $15 \times 37 \times 63 \times 51 \times 97 \times 17$.

   (a) 35 (b) 45

   (c) 55 (d) 85

5. Let us consider a fraction whose denominator is smaller than the square of the numerator by unity. If we add 2 to the numerator and the denominator, the fraction will exceed 1/3. If we subtract 3 from the numerator and the denominator, the fraction will be positive but smaller than 1/10. Find the value.

   (a) $\frac{3}{8}$ (b) $\frac{4}{15}$

(c) $\frac{5}{24}$ (d) $\frac{6}{35}$

6. Find the sum of all three-digit numbers that give a remainder of 4 when they are divided by 5.

(a) 98,270 (b) 99,270

(c) 1,02,090 (d) 90,270

7. Find the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7.

(a) 686 (b) 676

(c) 666 (d) 656

8. Find the sum of all odd three-digit numbers that are divisible by 5.

(a) 50,500 (b) 50,250

(c) 50,000 (d) 49,500

9. The product of a two-digit number by a number consisting of the same digits written in the reverse order is equal to 2430. Find the lower number.

(a) 54 (b) 52

(c) 63 (d) 45

10. Find the lowest of three numbers as described: If the cube of the first number exceeds their product by 2, the cube of the second number is smaller than their product by 3, and the cube of the third number exceeds their product by 3.

(a) $3^{1/3}$ (b) $9^{1/3}$

(c) 2 (d) Any of these

(e) None of these

11. How many pairs of natural numbers are there the difference of whose squares is 45?

(a) 1 (b) 2

(c) 3 (d) 4

12. Find all two-digit numbers such that the sum of the digits constituting the number is not less than 7; the sum of the squares of the digits is not greater than 30; the number consisting of the same digits written in the reverse order is not larger than half the given number.

(a) 52 (b) 51

(c) 49 (d) 53

13. In a four-digit number, the sum of the digits in the thousands, hundreds and tens is equal to 14, and the sum of the digits in the units, tens and hundreds is equal to 15. Among all the numbers satisfying these conditions, find the number the sum of the squares of whose digits is the greatest.

(a) 2572 (b) 1863

(c) 2573 (d) None of these

14. In a four-digit number, the sum of the digits in the thousands and tens is equal to 4, the sum of the digits in the hundreds and the units is 15, and the digit of the units exceeds by 7 the digit of the thousands. Among all the numbers satisfying these conditions, find the number the sum of the product of whose digit of the thousands by the digit of the units and the product of the digit of the hundreds by that of the tens assumes the least value.

(a) 4708 (b) 1738

(c) 2629 (d) 1812

15. If we divide a two-digit number by a number consisting of the same digits written in the reverse order, we get 4 as a quotient and 15 as a remainder. If we subtract 1 from the given number, we get the sum of the squares of the digits constituting that number. Find the number.

(a) 71 (b) 83

(c) 99 (d) None of these

16. Find the two-digit number the quotient of whose division by the product of its digits is equal to 8/3, and the difference between the required number and the number consisting of the same digits written in the reverse order is 18

(a) 86 (b) 42

(c) 75 (d) None of these

17. Find the two-digit number if it is known that the ratio of the required number and the sum of its digits is 8 as also the quotient of the product of its digits and that of the sum is 14/9.

(a) 54 (b) 72

(c) 27 (d) 45

18. If we divide the unknown two-digit number by the number consisting of the same digits written in the reverse order, we get 4 as a quotient and 3 as a remainder. If we divide the required number by the sum of its digits, we get 8 as a quotient and 7 as a remainder. Find the number.

(a) 81 (b) 91

(c) 71 (d) 72

19. The last two-digits in the multiplication $122 \times 123 \times 125 \times 127 \times 129$ will be

(a) 20 (b) 50

(c) 30 (d) 40

20. The remainder obtained when $43^{101} + 23^{101}$ is divided by 66 is:

(a) 2 (b) 10

(c) 5 (d) 0

21. The last three-digits of the multiplication 12345×54321 will be

(a) 865 (b) 745

(c) 845 (d) 945

22. The sum of the digits of a three-digit number is 12. If we subtract 495 from the number consisting of the same digits written in reverse order, we shall get the required number. Find that three-digit number if the sum of all pairwise products of the digits constituting that number is 41.

(a) 156 (b) 237

(c) 197 (d) Both (a) and (b)

23. A three-digit positive integer *abc* is such that $a^2 + b^2 + c^2 = 74$. a is equal to the doubled sum of the digits in the tens and units places. Find the number if it is known that the difference between that number and the number written by the same digits in the reverse order is 495.

(a) 813 (b) 349

(c) 613 (d) 713

24. Represent the number 1.25 as a product of three positive factors so that the product of the first factor by the square of the second is equal to 5 if we have to get the lowest possible sum of the three factors.

(a) $x_1 = 2.25$, $x_2 = 5$, $x_3 = 0.2$

(b) $x_1 = 1.25$, $x_2 = 4$, $x_3 = 4.5$

(c) $x_1 = 1.25$, $x_2 = 2$, $x_3 = 0.5$

(d) $x_1 = 1.25$, $x_2 = 4$, $x_3 = 2$

25. Find a number x such that the sum of that number and its square is the least.

(a) –0.5 (b) 0.5

(c) –1.5 (d) 1.5

26. When $2222^{5555} + 5555^{2222}$ is divided by 7, the remainder is

(a) 0 (b) 2

(c) 4 (d) 5

27. If x is a number of five-digits which when divided by 8, 12, 15 and 20 leaves respectively 5, 9, 12 and 17 as remainders, then find x such that it is the lowest such number.

(a) 10017 (b) 10057

(c) 10097 (d) 10137

28. $3^{2n} - 1$ is divisible by 2^{n+3} for $n =$

(a) 1 (b) 2

(c) 3 (d) None of these

29. $10^n - \left(5+\sqrt{17}\right)^n$ is divisible by 2^{n+2} for what whole number value of n?

(a) 2 (b) 3

(c) 7 (d) None of these

30. $\dfrac{32^{32^{32}}}{9}$ will leave a remainder:

(a) 4 (b) 7

(c) 1 (d) 2

31. Find the remainder that the number 1989 ◊ 1990 ◊ 1992^3 gives when divided by 7.

(a) 0 (b) 1

(c) 5 (d) 2

32. Find the remainder of 2^{100} when divided by 3.

(a) 3 (b) 0

(c) 1 (d) 2

33. Find the remainder when the number 3^{1989} is divided by 7.

(a) 1 (b) 5

(c) 6 (d) 4

34. Find the last digit of the number $1^2 + 2^2 + \ldots + 99^2$.

(a) 0 (b) 1

(c) 2 (d) 3

35. Find gcd ($2^{100} - 1$, $2^{120} - 1$).

(a) $2^{20} - 1$ (b) $2^{40} - 1$

(c) $2^{60} - 1$ (d) $2^{10} - 1$

36. Find the gcd (111…11 hundred ones ; 11…11 sixty ones).

(a) 111…forty ones (b) 111…twenty five ones

(c) 111…twenty ones (d) 111…sixty ones

37. Find the last digit of the number $1^3 + 2^3 + 3^3 + 4^3 \ldots + 99^3$.

(a) 0 (b) 1

(c) 2 (d) 5

38. Find the GCD of the numbers $2n + 13$ and $n + 7$.

(a) 1 (b) 2

(c) 3 (d) 4

39. $\dfrac{32^{32^{32}}}{7}$

(a) 4 (b) 2

(c) 1 (d) 3

40. The remainder when $10^{10} + 10^{100} + 10^{1000} + \ldots + 10^{10000000000}$ is divided by 7 is

(a) 0 (b) 1

(c) 2 (d) 5

41. n is a number, such that $2n$ has 28 factors and $3n$ has 30 factors. $6n$ has.

(a) 35 (b) 32

(c) 28 (d) None of these

42. Suppose the sum of n consecutive integers is $x + (x + 1) + (x + 2) + (x + 3) + \ldots + (x + (n - 1)) = 1000$, then which of the following cannot be true about the number of terms n

(a) The number of terms can be 16

(b) The number of terms can be 5

(c) The number of terms can be 25

(d) The number of terms can be 20

43. The remainder when $2^2 + 22^2 + 222^2 + 2222^2 + \ldots\ldots.(222\ldots..49 \text{ twos})^2$ is divided by 9 is:

(a) 2 (b) 5

(c) 6 (d) 7

44. $N = 202 \times 20002 \times 200000002 \times 20000000000000002 \times 200000000\ldots.2$ (31 zeroes) The sum of digits in this multiplication will be:

(a) 112 (b) 160

(c) 144 (d) Cannot be determined

45. Twenty five sets of problems on Data Interpretation– one each for the DI sections of 25 CATALYST tests were prepared by the AMS research team. The DI section of each CATALYST contained 50 questions of which exactly 35 questions were unique, i.e. they had not been used in the DI section of any of the other 24 CATALYSTs. What could be the maximum possible number of questions prepared for the DI sections of all the 25 CATALYSTs put together?

(a) 1100 (b) 975

(c) 1070 (d) 1055

46. In the above question, what could be the minimum possible number of questions prepared?

(a) 890 (b) 875

(c) 975 (d) None of these

Directions for Questions 47 to 49: At a particular time in the twenty first century there were seven bowlers in the Indian cricket team's list of 16 players short listed to play the next world cup. Statisticians discovered that that if you looked at the number of wickets taken by any of the 7 bowlers of the current Indian cricket team, the number of wickets taken by them had a strange property. The numbers were such that for any team selection of 11 players (having 1 to 7 bowlers) by using the number of wickets taken by each bowler and attaching coefficients of +1, 0, or –1 to each value available and adding the resultant values, any number from 1 to 1093, both included could be formed. If we denote W_1, W_2, W_3, W_4, W_5, W_6 and W_7 as the 7 values in the ascending order what could be the answer to the following questions:

47. Find the value of $W_1 + 2W_2 + 3W_3 + 4W_4 + 5W_5 + 6W_6$.

    (a) 2005 (b) 1995

    (c) 1985 (d) None of these

48. Find the index of the largest power of 3 contained in the product $W_1 W_2 W_3 W_4 W_5 W_6 W_7$.

    (a) 15 (b) 10

    (c) 21 (d) 6

49. If the sum of the seven coefficients is 0, find the smallest number that can be obtained.

    (a) – 1067 (b) – 729

    (c) – 1040 (d) – 1053

Directions for Questions 50 and 51: Answer these questions on the basis of the information given below.

In the ancient game of Honololo the task involves solving a puzzle asked by the chief of the tribe. Anybody answering the puzzle correctly is given the hand of the most beautiful maiden of the tribe. Unfortunately, for the youth of the tribe, solving the puzzle is not a cakewalk since the chief is the greatest mathematician of the tribe.

In one such competition the chief called everyone to attention and announced openly:

"A three-digit number '*mnp*' is a perfect square and the number of factors it has is also a perfect square. It is also known that the digits *m*, *n* and *p* are all distinct. Now answer my questions and win the maiden's hand."

50. If $(m + n + p)$ is also a perfect square, what is the number of factors of the six-digit number *mnpmnp*?

    (a) 32 (b) 72

    (c) 48 (d) Cannot be determined

51. If the fourth power of the product of the digits of the number *mnp* is not divisible by 5, what is the number of factors of the nine-digit number, *mnpmnpmnp*?

    (a) 32 (b) 72

    (c) 48 (d) Cannot be determined

52. In a cricket tournament organised by the ICC, a total of 15 teams participated. Australia, as usual won the tournament by scoring the maximum number of points. The tournament is organised as a single round robin tournament—where each team plays with every other team exactly once. 3 points are awarded for a win, 2 points are awarded for a tie/washed out match and 1 point is awarded for a loss. Zimbabwe had the lowest score (in terms of points) at the end of the tournament. Zimbabwe scored a total of 21 points. All the 15 national teams got a distinct score (in terms of points scored). It is also known that at least one match played by the Australian team was tied/washed out. Which of the following is always true for the Australian team?

    (a) It had at least two ties/washouts.

    (b) It had a maximum of 3 losses.

    (c) It had a maximum of 9 wins.

    (d) All of the above.

53. What is the remainder when 128^{1000} is divided by 153?

(a) 103 (b) 145

(c) 118 (d) 52

54. Find the remainder when $\mathbf{50^{51^{52}}}$ is divided by 11.

(a) 6 (b) 4

(c) 7 (d) 3

55. Find the remainder when $\mathbf{32^{33^{34}}}$ is divided by 11.

(a) 5 (b) 4

(c) 10 (d) 1

56. Find the remainder when $\mathbf{30^{72^{87}}}$ is divided by 11.

(a) 5 (b) 9

(c) 6 (d) 3

57. Find the remainder when $\mathbf{50^{56^{52}}}$ is divided by 11.

(a) 7 (b) 5

(c) 9 (d) 10

58. Find the remainder when $\mathbf{33^{34^{35}}}$ is divided by 7.

(a) 5 (b) 4

(c) 6 (d) 2

59. Let S_m denote the sum of the squares of the first m natural numbers. For how many values of $m < 100$, is S_m a multiple of 4?

(a) 50 (b) 25

(c) 36 (d) 24

60. For the above question, for how many values will the sum of cubes of the first m natural numbers be a multiple of 5 (if $m < 50$)?

(a) 20 (b) 21

(c) 22 (d) None of these

61. How many integer values of x and y satisfy the expression $4x + 7y = 3$ where $|x| < 1000$ and $|y| < 1000$?

(a) 284 (b) 285

(c) 286 (d) None of these

ANSWER KEY

Level of Difficulty (I)

1. (a)	2. (a)	3. (b)	4. (b)
5. (a)	6. (c)	7. (d)	8. (d)
9. (d)	10. (d)	11. (a)	12. (c)
13. (d)	14. (d)	15. (b)	16. (c)
17. (b)	18. (d)	19. (a)	20. (b)
21. (d)	22. (a)	23. (b)	24. (c)
25. (d)	26. (d)	27. (c)	28. (d)
29. (b)	30. (d)	31. (a)	32. (b)
33. (c)	34. (d)	35. (c)	36. (b)
37. (d)	38. (d)	39. (b)	40. (b)
41. (a)	42. (a)	43. (a)	44. (d)
45. (c)	46. (d)	47. (b)	48. (c)
49. (c)	50. (d)	51. (a)	52. (a)
53. (b)	54. (c)	55. (d)	56. (b)
57. (a)	58. (b)	59. (d)	60. (d)
61. (a)	62. (a) Æ $(21)^{12}$		(b) Æ $(0.8)^3$
63. (b)			

64.
1. Æ 0, 2, 4, 6, 8
2. Æ 1, 4, 7
3. Æ 0, 4, 8
4. Æ 0, 5
5. Æ 4
6. Æ 7
7. Æ 0

65.
LCM Æ 17010
HCF Æ 27

LCM Æ 780
HCF Æ 29
LCM Æ 245700
HCF Æ 30

66. (b)	67. (a)	68. (c)	69. (c)
70. (c)	71. (a)	72. (d)	73. (b)
74. (b)	75. (a)	76. (b)	77. (d)
78. (a)	79. (a)	80. (b)	81. (c)
82. (b)	83. (c)	84. (c)	85. (c)
86. (a)	87. (c)	88. (b)	89. (b)
90. (c)	91. (d)	92. (d)	93. (a)
94. (c)	95. (c)	96. (b)	97. (a)
98. (c)	99. (c)	100. (d)	101. (d)
102. (a)	103. (c)	104. (b)	105. (c)
106. (d)	107. (b)	108. (c)	109. (d)
110. (c)	111. (c)	112. (c)	113. (a)
114. (c)	115. (a)	116. (d)	117. (d)
118. (b)	119. (b)	120. (c)	121. (b)
122. (b)	123. (d)	124. (a)	125. (d)
126. (a)	127. (b)	128. (d)	129. (c)
130. (b)	131. (b)	132. (a)	133. (c)
134. (d)	135. (b)	136. (b)	137. (c)
138. (b)	139. (d)	140. (a)	

Level of Difficulty (II)

1. (a)	2. (b)	3. (c)	4. (b)
5. (b)	6. (b)	7. (b)	8. (d)
9. (c)	10. (c)	11. (d)	12. (d)
13. (d)	14. (c)	15. (b)	16. (d)
17. (a)	18. (d)	19. (c)	20. (a)
21. (a)	22. (c)	23. (b)	24. (a)
25. (d)	26. (c)	27. (c)	28. (d)
29. (b)	30. (b)	31. (a)	32. (b)
33. (a)	34. (a)	35. (b)	36. (b)
37. (a)	38. (d)	39. (c)	40. (a)

41. (24)	42. (63)	43. (24)	44. (32)
45. (27)	46. (64)	47. (13, 31)	48. (23)
49. (1056)	50. (51, 34)	51. (144, 864)	52. (46, 64)
53. (63)	54. (863)	55. (36, 63)	
56.	may be 2 or 3 depending upon the numbers		
57.	111^4 > 109.110.112.113 > 110.109.108.107		
58.	greatest Æ 35 least 3/7		
59. (a) 200^{300}	(b) 5^{100}	(c) 10^{20}	60. (12, 3)
61. (c)	62. (c)	63. (b)	64. (a)
65. (d)	66. (c)	67. (b)	68. (a)
69. (a)	70. (b)	71. (c)	72. (c)
73. (d)	74. (a)	75. (b)	76. (d)
77. (a)	78. (b)	79. (b)	80. (c)
81. (b)	82. (c)	83. (d)	84. (a)
85. (c)	86. (b)	87. (d)	88. (b)
89. (d)	90. (d)	91. (a)	92. (b)
93. (c)	94. (a)	95. (a)	96. (c)
97. (b)	98. (a)	99. (a)	100. (d)

Level of Difficulty (III)

1. (a)	2. (b)	3. (d)	4. (a)
5. (b)	6. (b)	7. (b)	8. (d)
9. (d)	10. (a)	11. (c)	12. (a)
13. (d)	14. (b)	15. (d)	16. (d)
17. (b)	18. (c)	19. (b)	20. (d)
21. (b)	22. (d)	23. (a)	24. (c)
25. (a)	26. (a)	27. (d)	28. (d)
29. (d)	30. (a)	31. (d)	32. (c)
33. (c)	34. (a)	35. (a)	36. (c)
37. (a)	38. (a)	39. (a)	40. (d)
41. (a)	42. (d)	43. (c)	44. (b)
45. (d)	46. (a)	47. (a)	48. (c)
49. (c)	50. (d)	51. (d)	52. (b)
53. (d)	54. (a)	55. (c)	56. (a)
57. (b)	58. (d)	59. (d)	60. (d)

61. (b)

Solutions and Shortcuts

Level of Difficulty (I)

1. The units digit in this case would obviously be ‘0’ because the given expression has a pair of 2 and 5 in it’s prime factors.
2. When you read the sentence “when the digits are reversed, the number decreases by 54, you should automatically get two reactions going in your mind.

   (i) The difference between the digits would be 54/9 = 6.

   (ii) Since the number ‘decreases’- the tens digit of the number would be larger than the units digit.

   Also, since we know that the sum of the digits is 10, we get that the digits must be 8 and 2 and the number must be 82. Thus, the changed number is 28.
3. The two numbers should be factors of 405. A factor search will yield the factors. (look only for 2 digit factors of 405 with sum of digits between 1 to 19).

   Also $405 = 5 \times 3^4$. Hence: 15×27

   45×9 are the only two options.

   From these factors pairs only the second pair gives us the desired result.

   i.e. Number × sum of digits = 405.

   Hence, the answer is 45.
4. You can solve this question by using options. It can be seen that Option (b) 12,3 fits the situation perfectly as their Arithmetic mean = 7.5 and their geometric mean = 6 and the geometric mean is 20% less than the arithmetic mean
5. Two more than half of 1/3rd of 96 = 18. Also since we are given that the difference between the AM and GM is 18, it means that the GM must be an integer. From amongst the options, only option (a) gives

us a GM which is an integer. Thus, checking for option 1, we get the GM=7 and AM=18.

6. For the number A381 to be divisible by 11, the sum of the even placed digits and the odds placed digits should be either 0 or a multiple of 11. This means that (A + 8) – (3 + 1) should be a multiple of 11 – as it is not possible to make it zero. Thus, the smallest value that A can take (and in fact the only value it can take) is 7. Option (c) is correct.
7. For 381A to be divisible by 9, the sum of the digits 3 + 8 + 1 + A should be divisible by 9. For that to happen A should be 6. Option (d) is correct.
8. 9^6 when divided by 8, would give a remainder of 1. Hence, the required answer would be 2.
9. LCM of 5, 15 and 20 = 60. HCF of 5, 15 and 20 = 5. The required ratio is 60:5 = 12:1
10. LCM of 5/2, 8/9 and 11/14 would be given by: (LCM of numerators)/(HCF of denominators)

    = 440/1 = 440
11. Only the first option can be verified to be true in this case. If A is even, 3A would always be divisible by 6 as it would be divisible by both 2 and 3. Options b and c can be seen to be incorrect by assuming the value of A as 4.
12. The essence of this question is in the fact that the last digit of the number is 0. Naturally, the number is necessarily divisible by 2,5 and 10. Only 4 does not necessarily divide it.
13. B would necessarily be even- as the possible values of B for the three digit number 15B to be divisible by 6 are 0 and 6. Also, the condition stated in option (c) is also seen to be true in this case – as both 0 and 6 are divisible by 6. Thus, option (d) is correct.
14. For the GCD take the least powers of all common prime factors.

    Thus, the required answer would be $2^3 \times 3$
15. The units digit would be given by 5 + 6 + 9 (numbers ending in 5 and 6 would always end in 5 and 6 irrespective of the power and 3^{54}

will give a units digit equivalent to 3^{4n+2} which would give us a unit digit of 3^2 i.e.9)

16. The respective units digits for the three parts of the expression would be:

    5 + 9 + 2 =16 Æ required answer is 6. Option (c) is correct.

17. The respective units digits for the six parts of the expression would be:

    1 + 4 + 7 + 6 + 5 + 6 =29 Æ required answer is 9. Option (b) is correct.

18. The respective units digits for the six parts of the expression would be:

    1 × 4 × 7 × 6 × 5 × 6 Æ required answer is 0. Option (d) is correct.

19. The number of zeroes would be given by adding the quotients when we successively divide 1090 by 5:

    1090/5 + 218/5 + 43/5 + 8/5 = 218 + 43 + 8 + 1 = 270. Option (a) is correct.

20. The number of 5's in 146! can be got by [146/5] + [29/5] + [5/5]= 29+5+1 = 35

21. $1420 = 142 \times 10 = 2^2 \times 71^1 \times 5^1$.

    Thus, the number of factors of the number would be (2 + 1) (1 + 1) (1 + 1) = 3 × 2 × 2 = 12.

    Option (d) is correct.

22. $(x^2 - 5x + 6) = (x - 2)(x - 3)$

    & $(x^2 - 7x + 10) = (x - 5)(x - 2)$

    Required HCF = $(x - 2)$; required LCM = $(x - 2)(x - 3)(x - 5)$.

    Option (a) is correct.

23. Since both *P* and *Q* are prime numbers, the number of factors would be (1 + 1)(1 + 1) = 4.

24. Since both *P* and *Q* are prime numbers, the number of factors would be (2 + 1)(1 + 1) = 6.

25. Since both *P* and *Q* are prime numbers, the number of factors would be (3 + 1)(2 + 1) = 12.

26. The sides of the pentagon being 1422, 1737, 2160, 2214 and 2358, the least difference between any two numbers is 54. Hence, the correct answer will be a factor of 54.

    Further, since there are some odd numbers in the list, the answer should be an odd factor of 54.

    Hence, check with 27, 9 and 3 in that order. You will get 9 as the HCF.

27. The HCF of 576 and 448 is 32. Hence, each section should have 32 children. The number of sections would be given by: 576/32 + 448/32 = 18 + 14 = 32. Option (b) is correct.

28. The HCF of the given numbers is 31 and hence the number of bottles required would be:

    403/31 + 465/31 + 496/31 = 13 + 15 + 16 = 44. Option (d) is correct.

29. The LCM of the 4 numbers is 612. The highest 4 digit number which would be a common multiple of all these 4 numbers is 9792. Hence, the correct answer is 9793.

30. The LCM of 16, 18 and 20 is 720. The numbers which would give a remainder of 4, when divided by 16, 18 and 20 would be given by the series:

    724, 1444, 2164, 2884 and so on. Checking each of these numbers for divisibility by 7, it can be seen that 2884 is the least number in the series that is divisible by 7 and hence is the correct answer. Option (d) is correct.

31. They will ring together again after a time which would be the LCM of 6, 8, 12 and 18. The required LCM = 72. Hence, they would ring together after 72 seconds. Option (a) is correct.

32. 720/72 = 10 times. Option (b) is correct.

33. $5 \times 7 \times 6 = 0$. Option (c) is correct.

34. All these numbers can be verified to not be perfect squares. Option (d) is correct.

35. A perfect square can never end in an odd number of zeroes. Option (c) is correct.

36. It is obvious that the LCM of 5,12,18 and 20 would never be a multiple of 9. At the same time it has to be a multiple of each of 3, 8 and 5. Option (b) is correct.

37. $720 = 2^4 \times 3^2 \times 5^1$. Number of factors = 5 × 3 × 2 = 30. Option (d) is correct.

38. $16 - x^2 = (4 - x)(4 + x)$ and $x^2 + x - 6 = (x + 3)(x - 2)$

The required LCM = $(4 - x)(4 + x)(x + 3)(x - 2)$.

Option (d) is correct.

39. $x^2 - 4 = (x - 2)(x + 2)$ and $x^2 + x - 6 = (x + 3)(x - 2)$

GCD or HCF of these expressions = $(x - 2)$.

Option (b) is correct.

40. If A is not divisible by 3, it is obvious that 2A would also not be divisible by 3, as 2A would have no '3' in it.

41. $9^{100}/8 = (8 + 1)^{100}/8$ Æ Since this is of the form $(a + 1)^n/a$, the Remainder = 1. Option (a) is correct.

42. 2^{1000} /3 is of the form $(a)^{\text{EVEN POWER}}/(a + 1)$. The remainder = 1 in this case as the power is even. Option (a) is correct.

43. The condition for the product to be the greatest is if the two terms are equal. Thus, the break up in option (a) would give us the highest product of the two parts. Option (a) is correct.

44. 50/5 =10, 10/5 =2.

Thus, the required answer would be 10 + 2 = 12. Option (d) is correct.

45. Checking each of the options it can be seen that the value in option (c)[viz: 1362480]is divisible by 24.

46. Any number divisible by 88, has to be necessarily divisible by 11, 2, 4, 8, 44 and 22. Thus, each of the first three options is correct.

47. $10800 = 108 \times 100 = 3^3 \times 2^4 \times 5^2$.

The number of divisors would be: (3 + 1) (4 + 1) (2 + 1) = 4 × 5 × 3 = 60 divisors. Option (b) is correct.

48. The GCD (also known as HCF) would be got by multiplying the least powers of all common factors of the two polynomials. The

common factors are $(x + 3)$ – least power 1, and $(x + 1)$ – least power 2. Thus, the answer would be $(x + 3)(x + 1)^2$. Option (c) is correct.

49. For the LCM of polynomials write down the highest powers of all available factors of all the polynomials.

    The correct answer would be $(x + 3)(3x + 4)(4x^2 - 1)$

50. Three consecutive natural numbers, starting with an even number would always have at least three 2's as their prime factors and also would have at least one multiple of 3 in them. Thus, 6, 12 and 24 would each divide the product.

51. When the birds sat one on a branch, there was one extra bird. When they sat 2 to a branch one branch was extra.

    To find the number of branches, go through options. Checking option (a), if there were 3 branches, there would be 4 birds. (this would leave one bird without branch as per the question.)

    When 4 birds would sit 2 to a branch there would be 1 branch free (as per the question). Hence, the answer (a) is correct.

52. The number would either be $(3n + 1)^2$ or $(3n + 2)^2$. In the expansion of each of these the only term which would not be divisible by 3 would be the square of 1 and 2 respectively. When divided by 3, both of these give 1 as remainder.

53. The given expression can be written as:

    $5^3 \times 3^3 \times 3^2 \times 7^2/5^2 \times 7^2 \times 3^4 = 5^3 \times 3^5 \times 7^2/5^2 \times 7^2 \times 3^4 = 15$. Option (b) is correct.

54. $D = 1.44$, $C = 0.09$, $B = 0.16$, while the value of A is negative.

    Thus, $D > B > C > A$ is the required order. Option (c) is correct.

55. The upper limit for $x + y = 4 + 3 = 7$. The lower limit of $x - y = 2 - 3 = -1$. Required ratio $= 7/-1 = -7$.

    Option (d) is correct.

56. For the sum of squares of digits to be 13, it is obvious that the digits should be 2 and 3. So the number can only be 23 or 32. Further, the number being referred to has to be 32 since the reduction of 9, reverses the digits.

57. trying the value in the options you get that the product of $54 \times 45 = 2430$. Option (a) is correct.

58. Option (b) can be verified to be true as the LCM of 90 and 24 is indeed 360.

59. The pairs given in option (d) 78 and 13 and 26 and 39 meet both the conditions of LCM of 78 and HCF of 13. Option (d) is correct.

60. Solve using options. Option (d) 51 and 34 satisfies the required conditions.

61. $28^2 - 27^2 = 55$ and so also $8^2 - 3^2 = 55$. Option (a) is correct.

62. (a) $21^{12} = (21^3)^4$

    Since $21^3 > 54$, $21^{12} > 54^4$.

    (b) $(0.4)^4 = (4/10)^4 = 1024/10000 = 0.1024$.

    $(0.8)^3 = (8/10)^3 = 512/1000 = 0.512$

    Hence, $(0.8)^3 > (0.4)^4$.

63. This is never possible.

64. 1. $c = 0, 2, 4, 6$ or 8 would make $38c$ as even and hence divisible by 2.
    2. $c = 1, 4$ or 7 are possible values to make $38c$ divisible by 3.
    3. $c = 0, 4$ or 8 would make the number end in 80, 84 or 88 and would hence be divisible by 4.
    4. $c = 0$ or 5 would make the number 380 or 385 – in which case it would be divisible by 5.
    5. For the number to become divisible by 6, it should be even and divisible by 3. From the values 1, 4 and 7 which make the number divisible by 3, we only have $c = 4$ making it even. Thus, $c = 4$.
    6. For the number to be divisible by 9, $3 + 8 + c$ should be a multiple of 9. $c = 7$ is the only value of c which can make the number divisible by 9.
    7. Obviously $c = 0$ is the correct answer.

65. Use the standard process to solve for LCM and HCF.

66. For $34x^{43}$ to be ending in 7, x has to be 3 (as $43 = 4n + 3$). Option (b) is correct.

Solutions for 67 & 68:

The given condition says that Pen<Pencil<Eraser.

Also, since the least cost of the three is `12, if we allocate a minimum of 12 to each we use up 36 out of the 41 available. The remaining 5 can be distributed as 0,1,4 or 0,2 and 3 giving possible values of Case 1: 12,13 and 16 or Case 2: 12,14 and 15.

67. In both cases, the cost of the pen is 12.

68. If the cost of the eraser is not divisible by 4, it means that Case 2 holds true. For this case, the cost of the pencil is 14.

69. Amrit would place eight oranges in the basket (as there are eight 1's).

For the mangoes, he would place six mangoes (number of 2's) and remove four mangoes (number of 4's) from the basket. Thus, there would be 2 mangoes and 8 oranges in the basket.

A total of 8 – 2 = 6 extra oranges in the basket. Option (c) is correct.

70. Solve using trial and error – Option (c) fits the situation as if we start with 15 mangoes, the following structure would take place:

Start with 15 mangoes Æ First watchman takes 1/3rd + 1 more = 5 + 1 = 6 mangoes Æ 9 mangoes left.

Second watchman takes Æ 1/3rd + 1 more = 3 + 1 = 4 mangoes Æ 5 mangoes left.

Third watchman takes Æ 1/5th + 4 more = 1 + 4 = 5 mangoes Æ 0 mangoes left.

71. The last 3 digits of the number would determine the remainder when it is divided by 8. The number upto the 120th digit would be 1234567891011… 646. 646 divided by 8 gives us a remainder of 6.

72. There would be multiple ways of scoring 34.5 marks. Think about this as follows:

If he solves 80 and gets all 80 correct, he would end up scoring 80 marks.

With every question that would go wrong his score would fall down by: 1.5 marks (he would lose the 1 mark he is gaining and further attract a penalty of 0.5 marks).

Also, for every question he does not attempt his score would fall down by: 1.25 marks (he would lose the 1 mark he is gaining and further attract a penalty of 0.25 marks).

Thus, his score would drop @ 1.5 and @1.25 marks for every wrong and every unattempted question respectively.

Also, to get a total of 34.5 marks overall he has to lose 45.5 marks.

There are many possible combinations of non attempts and wrongs through which he can possibly lose 45.5 marks—for example:

17 wrongs (loses 25.5 marks) and 16 non-attempts (loses 20 marks)

12 wrongs (loses 18 marks) and 22 non attempts (loses 27.5 marks)

Hence, we cannot answer this question uniquely and the answer is Option (d).

73. Continuing the thought process for the previous question our thinking would go as follows:

10 questions unanswered Æ loses 12.5 marks

To lose another 33 marks he needs to get 22 incorrects.

Thus, the number of corrects would be 80 – 10 – 22= 48. Option (b) is correct.

74. $3M + 4G + 5W = 750$ (i)

$6M + 9G + 10W = 1580$ (ii)

Adding the two equations we get:

$9M + 13G + 15W = 2330$ (iii)

Dividing this expression by 3 we get:

$3M + 4.33G + 5W = 776.666$ (iv)

(iv) - (i) Æ $0.33\ G = 26.666$ Æ $G = 80$

Now, if we look at the equation (i) and multiply it by 2, we get: $6M + 8G + 10W = 1500$. If we subtract the cost of 4 guavas from this we would get:

$6M + 4G + 10W = 1500 – 320 = 1180$

Option (b) is correct.

75. If you try a value of M as 5, N would become ¼. It can be seen that $5^{1/4}$ (which would be the value of M^N) would be around 1.4 and hence, less than 2.

If you try for possible values of M^N by increasing the value of M, you would get $6^{1/5}$, $7^{1/6}$, $8^{1/7}$, $9^{1/8}$ and so on. In each of these cases you can clearly see that the value of M^N would always be getting consecutively smaller than the previous value.

If you tried to go for values of M such that they are lower than 5, you would get the following values for M^N:

$4^{1/3}$, $3^{1/2}$ and the last value would be $2^{1/1}$. In this case, we can clearly see that the value of the expression M^N is increasing. However, it ends at the value of 2 (for $2^{1/1}$) and hence that is the maximum value that M^N can take. Option (a) is correct.

76. (7M + 9G + 19W) – (4M + 6G + 16W) = 120. Hence, 1M + 1G + 1W = 40

77. The cost of a mango cannot be uniquely determined here because we have only 2 equations between 3 variables, and there is no way to eliminate one variable.

78. Since the value of y varies between –8 to 2, it is evident that if we take a very small value for y, say 0.0000000000000000000000001, and we take normal integral values for x and z, the expression xz/y would become either positive or negative infinity (depending on how you manage the signs of the numbers x, y and z).

79. Ties < Trousers < Shirts. Since each of the three is minimum 11, the total would be a minimum of 33 (for all 3). The remaining 5 need to be distributed amongst ties, trousers and shirts so that they can maintain the inequality Ties < Trousers < Shirts

This can be achieved with 11 ties, and the remaining 27 pieces of clothing distributed between trousers and shirts such that the shirts are greater than the trousers.

This can be done in at least 2 ways: 12 trousers and 15 shirts; 13 trousers and 14 shirts.

If you try to go for 12 ties, the remaining 26 pieces of clothing need to be distributed amongst shirts and trousers such that the shirts are greater than the trousers and both are greater than 12.

With only 26 pieces of clothing to be distributed between shirts and trousers this is not possible. Hence, the number of ties has to be exactly 11. Option (a) is correct.

80. The number of shirts would be at least 14 as the two distributions possible are: 11, 12, 15 and 11, 13, 14. Option (b) is correct.

81. The LCM of 12, 15, 18 and 20 is 180. Thus, the least number would be 184.

Option (c) is correct.

82. 2800 = 20 × 20 × 7. Thus, we need to multiply or divide with 7 in order to make it a perfect square.

83. The answer is given by $\sqrt{1024}$ = 32.

This can be experimentally verified as $30^2 = 900$, $31^2 = 961$ and $32^2 = 1024$. Hence, 1024 is the required answer. Option (c) is correct.

84. First find the LCM of 6, 9, 15 and 18. Their LCM = 18 × 5 = 90.

The series of numbers which would leave a remainder of 4 when divided by 6, 9, 15 and 18 would be given by:

LCM + 4; 2 × LCM + 4; 3 × LCM + 4; 4 × LCM + 4; 5 × LCM + 4 and so on .

Thus, this series would be:

94, 184, 274, 364, 454….

The other constraint in the problem is to find a number which also has the property of being divisible by 7. Checking each of the numbers in the series above for their divisibility by 7, we see that 364 is the least value which is also divisible by 7. Option (c) is correct.

85. LCM of 2, 3, 4, 5 and 6 = 6 × 5 × 2 =60 (Refer to the shortcut process for LCM given in the chapter notes).

Thus, the series 61, 121, 181 etc would give us a remainder 1 when divided by 2, 3, 4, 5 and 6.

The least 3 digit number in this series is 121. Option (c) is correct.

86. $70 = 2 \times 5 \times 7$; $245 = 5 \times 7 \times 7$.

HCF = $5 \times 7 = 35$. Option (a) is correct.

87. 7056 is the closest perfect square below 7147. Hence, $7147 - 7056 = 91$ is the required answer. Option (c) is correct.

88. If you check each of the options, you can clearly see that the number 3600 is divisible by each of these numbers. Option (b) is correct.

Alternately, you can also think about this question as:

The LCM of 6, 8 and 15 = 120. Thus, we need to look for a perfect square in the series of multiples of 120. 120, 240, 360, 480, 600, 720…., the first number which is a perfect square is: 3600.

89. $30492 = 2^2 \times 3^2 \times 7^1 \times 11^2$.

For a number to be a perfect square each of the prime factors in the standard form of the number needs to be raised to an even power. Thus, we need to multiply or divide the number by 7 so that we either make it: $2^2 \times 3^2 \times 7^2 \times 11^2$ (if we multiply the number by 7) or

We make it: $2^2 \times 3^2 \times 11^2$. (if we divide the number by 7).

Option (b) is correct.

90. $88 \times 113 = 9944$ is the greatest 4 digit number exactly divisible by 88. Option (c) is correct.

91. $3/4^{th}$ of $116 = ¾ \times 116 = 87$

$4/5^{th}$ of $45 = 4/5 \times 45 = 36$.

Required difference = 51.

Option (d) is correct.

92. The correct arrangement would be 75 plants in a row and 75 rows since 5625 is the square of 75.

93. $9^{\text{EVEN POWER}} \times 7^{4n+1}$ Æ $1 \times 7 = 7$ as the units digit of the multiplication.

Option (a) is correct.

94. It can be seen that for 40 and 80 the number of factors are 8 and 10 respectively. Thus option (c) satisfies the condition.

95. In order to solve this question you need to realize that remainders of 1, 3, 4 and 5 in the case of 3, 5, 6 and 7 respectively, means remainders of –2 in each case. In order to find the number which leaves remainder –2 when divided by these numbers you need to first find the LCM of 3, 5, 6 and 7 and subtract 2 from them. Since the LCM is 210, the first such number which satisfies this condition is 208. However, the question has asked us to find the largest such number below 4000. So you need to look at multiples of the LCM and subtract 2. The required number is 3990 – 2 = 3988

96. The number would be given by the (LCM of 2, 3 and 4) +1 Æ which is 12 + 1 = 13. Option (b) is correct.

97. The number would be given by the 2 × (LCM of 2, 3 and 4) +1 Æ which is 24 + 1 = 25. Option (a) is correct.

98. In order to solve this you need to find the last 2 digit number in the series got by the logic:

    (LCM of 2, 3, 4) + 1; 2 × (LCM of 2, 3, 4) + 1; 3 × (LCM of 2, 3, 4) + 1 …

    i.e. you need to find the last 2 digit number in the series:

    13, 25, 37, 49….

    In order to do so, you can do one of the following:

    (a) Complete the series by writing the next numbers as:

        61, 73, 85, 97 to see that 97 is the required answer.

    (b) Complete the series by adding a larger multiple of 12 so that you reach closer to 100 faster.

        Thus, if you have seen 13, 25, 37,,,,, you can straightaway add any multiple of 12 to get a number close to 100 in the series in one jump.

        Thus, if you were to add 12 × 4 = 48 to 37 you would reach a value of 85 (and because you have added a multiple of 12 to 37 you can be sure that 85 would also be on the same series.

        Thus, the thinking in this case would go as follows:

        13, 25, 37, …, 85, 97. Hence, the number is 97.

If you look at the two processes above- it would seem that there is not much difference between the two, but the real difference would be seen and felt if you would try to solve a question which might have asked you to find the last 3 digit number in the series. (as you would see in the next question). In such a case, getting to the number would be much faster if you use a multiple of 12 to jump ahead on the series rather than writing each number one by one.

(c) For the third way of solving this, you can see that all the numbers in the series:

13, 25, 37… are of the form $12n + 1$. Thus, you are required to find a number which is of the form $12n + 1$ and is just below 100.

For this purpose, you can try to first see what is the remainder when 100 is divided by 12.

Since the remainder is 4, you can realize that the number 100 is a number of the form $12n + 4$.

Obviously then, if 100 is of the form $12n + 4$, the largest $12n + 1$ number just below 100 would occur at a value which would be 3 less than 100. (This occurs because the distance between $12n + 4$ and $12n + 1$ on the number line is 3.)

Thus the answer is $100 – 3 = 97$.

Hence, Option (c) is correct.

99. In order to solve this you need to find the last 3 digit number in the series got by the logic:

(LCM of 2, 3, 4) +1; 2 × (LCM of 2, 3, 4) +1; 3 × (LCM of 2, 3, 4) + 1 …

i.e. you need to find the last 3 digit number in the series:

13, 25, 37, 49….

In order to do so, you can do one of the following:

(a) Try to complete the series by writing the next numbers as:

61, 73, 85, 97, 109… However, you can easily see that this process would be unnecessarily too long and hence infeasible to solve this question.

(b) Complete the series by adding a larger multiple of 12 so that you reach closer to 1000 faster.

This is what we were hinting at in the previous question. If we use a multiple of 12 to write a number which will come later in the series, then we can reach close to 1000 in a few steps. Some of the ways of doing this are shown below:

(i) 13, 25, 37, …….. 997 (we add $12 \times 80 = 960$ to 37 to get to $37 + 960 = 997$ which can be seen as the last 3 digit number as the next number would cross 1000).

(ii) 13, 25, 37, …… (add 600)….637, …(add 120)… 757,,,,, (add 120),,,, 877, ….(add 120)….**997.** This is the required answer.

(iii) 13, 25, 37, ……(add 120)….157, …(add 120)…277…… (add 120)…..397….(add 120)……517…. (add 120)… 637……(add 120)…757,,,,, (add 120),,,, 877, ….(add 120)….**997.** This is the required answer.

What you need to notice is that all the processes shown above are correct. So while one of them might be more efficient than the other, as far as you ensure that you add a number which is a multiple of 12(the common difference) you would always be correct.

(c) Of course you can also do this by using remainders. For this, you can see that all the numbers in the series:

13, 25, 37….are of the form $12n + 1$. Thus, you are required to find a number which is of the form $12n + 1$ and is just below 1000.

For this purpose, you can try to first see what is the remainder when 1000 is divided by 12.

Since the remainder is 4, you can realize that the number 1000 is a number of the form $12n + 4$.

Obviously then, if 1000 is of the form $12n + 4$, the largest $12n + 1$ number just below 1000 would occur at a value which would be 3 less than 1000. (This occurs because the distance between $12n + 4$ and $12n + 1$ on the number line is 3.)

Thus the answer is 1000 – 3 = 997.

Hence, Option (c) is correct.

100. The logic of this question is that the frog can never reach point C if it makes an odd number of jumps. Since, the question has asked us to find out in how many ways can the frog reach point C in exactly 7 jumps, the answer would naturally be 0. Option (d) is correct.

101. They would ring together again after a time interval which would be the LCM of 5, 6 and 7. Since the LCM is 210, option (d) is the correct answer.

102. Since they would ring together every 210 seconds, their ringing together would happen at time intervals denoted by the following series- 210, 420, 630, 840, 1050, 1260, 1470, 1680, 1890, 2100, 2310, 2520, 2730, 2940, 3150, 3360, 3570 – a total of 17 times. This answer can also be calculated by taking the quotient of 3600/210 = 17. Option (a) is correct.

103. The maximum number of soldiers would be given by the HCF of 66, 110 and 242. The HCF of these numbers can be found to be 22 and hence, option (c) is correct.

104. The minimum number of rows would happen when the number of soldiers in each row is the maximum. Since, the HCF is 22 the number of soldiers in each row is 22. Then the total number of rows would be given by:

     66/22 + 110/22 + 242/22 = 3 + 5 + 11 = 19 rows. Option (b) is correct.

105. The Number of bottle sizes possible would be given by the number of factors of the HCF of 170, 102 and 374. Since, the HCF of these numbers is 34, the bottle sizes that are possible would be the divisors of 34 which are 1 liter, 2 liters, 17 liters and 34 liters respectively. Thus, a total of 4 bottle sizes are possible. Option (c) is correct.

106. The size of the largest bottle that can be used is obviously 34 liters (HCF of 170, 102 and 374). Option (d) is correct.

107. The minimum number of bottles required would be: 170/34 + 102/34 + 374/34 = 5 + 3 + 11 = 19. Option (b) is correct.

108. The answer would be given by the quotients of 100/5 + 100/25 = 20 + 4 = 24. Option (c) is correct.

The logic of how to think about Questions 108 to 118 has been given in the theory in the chapter. Please have a relook at that in case you have doubts about any of the solutions till Question 118.

109. 24 + 4 = 28. Option (d) is correct.

110. 280 + 56 + 11 + 2 = 349. Option (c) is correct.

111. 76 + 15 + 3 = 94. Option (c) is correct.

112. 14 + 2 = 16. Option (c) is correct.

113. 13 + 4 + 1 = 18. Option (a) is correct.

114. 17 + 5 + 1 = 23. Option (c) is correct.

115. 11 + 1 = 12. Option (a) is correct.

116. 16 + 2 = 18. Option (d) is correct.

117. The number of 3's in 122! = 40 +13 + 4 + 1 = 58. The number of 2's in 122! = 61 + 30 + 15 + 7 + 3 + 1 = 117. The number of 2^2s is hence equal to the quotient of 117/2 = 58. We have to choose the lower one between 58 and 58. Since both are equal, 58 would be the correct answer. Hence, Option (d) is correct.

118. The power of 20 which would divide 155! would be given by the power of 5's which would divide 155! since 20 = $2^2 \times 5$ and the number of 2^2s in any factorial would always be greater than the number of 5s in the factorial. 31 + 6 + 1 = 38. Option (b) is correct.

119. 1024 = 2^{10}. Hence, x has to be a number with power of 2 greater than or equal to 5. Since, we are asked for the minimum value, it must be 5. Thus, option (b) is correct.

120. The two digit numbers that would leave a remainder of 3 when divided by 7 would be the numbers 10, 17, 24, 31, 38, 45, …94. The sum of these numbers would be given by the formula

(number of numbers × average of the numbers) = There are 13 numbers in the series and their average is 52. Thus, the required answer is $13 \times 52 = 676$. Option (c) is correct.

(Note the logic used here is that of sum of an Arithmetic Progression and is explained in details in the next chapter).

121. All numbers divisible by 27 would also be divisible by 3 and 9. Numbers divisible by 9 but not by 27 would be divisible by 3 and 9 only and need to be counted to give us our answer.

 The numbers which satisfy the given condition are: 9, 18, 36, 45, 63, 72, 90, 99, 117, 126, 144, 153, 171, 180 and 198. There are 15 such numbers.

 Alternately, you could also think of this as:

 Between 1 to 200 there are 22 multiples of 9. But not all these 22 have to be counted as multiples of 27 need to be excluded from the count. There are 7 multiples of 27 between 1 and 200. Thus, the answer would be given by $22 - 7 = 15$. Option (b) is correct.

122. The required minimum happens when we use (–0.5) as the value of N. $(-0.5)^2 + (-0.5) = 0.25 - 0.5 = -0.25$ is the least possible value for the sum of any number and it's square. Option (b) is correct.

123. Each of the statements are false as we can have the sum of 2 prime numbers ending in 5, 0 and the sum can also be odd. Option (d) is correct.

124. This occurs for values such as: 103 – 013; 213 – 123; 324 – 234 etc where it can be seen that the value of X is 1 more than Y. The possible pairs of values for X and Y are: 1, 0; 2, 1; 3, 2…9, 8 – a total of nine pairs of values. Option (a) is correct.

125. The required sum would be given by the formula $n(n + 1)$ for the first n even numbers. In this case it would be $50 \times 51 = 2550$. Option (d) is correct.

126. 763/57 leaves a remainder of 22 when it is divided by 57. Thus, if we were to add 35 to this number the number we obtain would be completely divisible by 57. Option (a) is correct.

127. Since, 763/57 leaves a remainder of 22, we would need to subtract 22 from 763 in order to get a number divisible by 57. Option (b) is correct.

128. 8441/57 leaves a remainder of 5. Thus, if we were to add 52 to this number the number we obtain would be completely divisible by 57. Option (d) is correct.

129. Since, 8441/57 leaves a remainder of 5. We would need to subtract 5 from 8441 in order to get a number divisible by 57. Option (c) is correct.

130. 10000 divided by 79 leaves a remainder of 46. Hence, if we were to add 33 to 10000 we would get a number divisible by 79. The correct answer is 10033. Option (b) is correct.

131. 100000 divided by 79 leaves a remainder of 65. Hence, if we were to subtract 65 from 100000 we would get a number divisible by 79. The correct answer is 99935. Option (b) is correct.

132. It can be seen that in the multiples of 12, the number closest to 773 is 768. Option (a) is correct.

133. Since 12 is a divisor of 84, the required remainder would be got by dividing 57 by 12. The required answer is 9. Option (c) is correct

134. Since 11 does not divide 84, there are many possible answers for this question and hence we cannot determine one unique value for the answer. Option (d) is thus correct.

135. The numbers that can do so are going to be factors of the difference between 511 and 667 i.e. 156. The factors of 156 are 1,2,3,4,6, 12,13, 26, 39, 52,78,156. There are 12 such numbers. Option (b) is correct.

136. The multiples of 13 between 200 and 400 would be represented by the series:

    208, 221, 234, 247, 260, 273, 286, 299, 312, 325, 338, 351, 364, 377 and 390

    There are a total of 15 numbers in the above series. Option (b) is correct.

    Note: The above series is an Arithmetic Progression. The process of finding the number of terms in an Arithmetic Progression are defined in the chapter on Progressions.

137. $8n/5 - 5n/8 = 39n/40 = 39$. Solve for n to get the value of $n = 40$. Option (c) is correct.

138. $x + y = 3(x - y)$ Æ $2x = 4y$. If we take y as 10, we would get the value of x as 20. Option (b) is correct.

139. $4^{11} + 4^{12} + 4^{13} + 4^{14} + 4^{15} = .\ 4^{11}\ (1 + 4^1 + 4^2 + 4^3 + 4^4) = 4^{11} \times 341$. The factors of 341 are:

1, 11, 31 and 341. Thus, we can see that the values in each of the three options would divide the expression. $4^{11} + 4^{12} + 4^{13} + 4^{14} + 4^{15}$. Thus, option (d) is correct.

140. Since the numbers have their HCF as 16,both the numbers have to be multiples of 16 (i.e. 2^4).

$7168 = 2^{10} \times 7^1$

In order to visualise the required possible pairs of numbers we need to look at the prime factors of 7168 in the following fashion:

$7168 = 2^{10} \times 7^1 = (2^4 \times 2^4) \times 2^2 \times 7^1 = (16 \times 16) \times 2 \times 2 \times 7$ It is then a matter of distributing the 2 extra twos and the 1 extra seven in $2^2 \times 7^1$ between the two numbers given by 16 and 16 inside the bracket. The possible pairs are:

32×224; 64×112; 16×448. Thus there are 3 distinct pairs of numbers which are multiples of 16 and whose product is 7168. However, out of these the pair 32×224 has it's HCF as 32 and hence does not satisfy the given conditions. Thus there are two pairs of numbers that would satisfy the condition that their HCF is 16 and their product is 7168. Option (a) is correct.

Level of Difficulty (II)

1. If a and b are two numbers, then their Arithmetic mean is given by $(a + b)/2$ while their geometric mean is given by $(ab)^{0.5}$. Using the options to meet the conditions we can see that for the numbers in the first option (6 and 54) the AM being 30, is 24 less than the larger number while the GM being 18, is 12 more than the smaller number. Option (a) is correct.

2. Use the principal of counting given in the theory of the chapter. Start with 101 numbers (i.e. all numbers between 200 and 300 both included) and subtract the number of numbers which are divisible

by 2 (viz. [(300 – 200)/2] + 1 = 51 numbers), the number of numbers which are divisible by 3 but not by 2 (Note: This would be given by the number of terms in the series 201, 207, … 297. This series has 17 terms) and the number of numbers which are divisible by 5 but not by 2 and 3. (The numbers are 205, 215, 235, 245, 265, 275, 295. A total of 7 numbers)

Thus, the required answer is given by 101 – 51 – 17 – 7 = 26. Option (b) is correct.

3. Since $15n^3$, $6n^2$ and $5n$ would all be divisible by n, the condition for the expression to not be divisible by n would be if x is not divisible by n. Option (c) is correct.

4. It can be seen that the first expression is larger than the second one. Hence, the required answer would be given by the (units digit of the first expression – units digit of the second expression) = 6 – 0 = 6. Option (b) is correct.

5. Suppose you were to solve the same question for $10^3 - 7$ and $10^2 + x$.

   $10^3 - 7 = 993$ and $10^2 + x = 100 + x$.

   Difference = $993 - x$

   For $10^4 - 7$ and $10^3 + x$

   The difference would be $9993 - (1000 + x)$

   $= (8993 - x)$

   For $10^5 - 7$ and $10^3 + x$

   Difference: $99993 - (10000 + x) = 89993 - x$

   You should realize that the difference for the given question would be 8999 . . . . . 9 3 – x. For this difference to be divisible by 3, x must be 2 (since that is the only option which will give you a sum of digits divisible by 3.)

6. The value of x should be such that the left hand side after completely removing the square root signs should be an integer. For this to happen, first of all the square root of $3x$ should be an integer. Only 3 and 12 from the options satisfy this requirement. If we try to put x as 12, we get the square root of $3x$ as 6. Then the next point at

which we need to remove the square root sign would be 12+2(6) = 24 whose square root would be an irrational number. This leaves us with only 1 possible value (x = 3). Checking for this value of x we can see that the expression is satisfied as LHS = RHS.

7. If the number is n, we will get that $22n + n = 23n$ is half the square of the number n. Thus, we have

   $n^2 = 46\, n$ Æ $n = 46$

8. $12^{55}/3^{11} = 3^{44}.4^{55}$ Æ 4 as units place.

   Similarly, $8^{48}/16^{18} = 2^{72}$ Æ 6 as the units place.

   Hence, 0 is the answer.

9. $1 + 2 + 2^2 + \ldots + 2^{31} = 2^{32} - 1$

   Hence, the average will be: $\frac{2^{32}-1}{32} = 2^{27} - 1/2^5$ which lies between 2^{26} and 2^{27}.

   Hence the answer will be (c).

10. The denominator 99 has the property that the decimals it gives rise to are of the form 0.$xyxyxy$. This question is based on this property of 99. Option c is correct.

11. The value of b has to be 2 since, $r = 2y$. Hence, option d is the only choice.

12. For $[x]^3 + \{x\}^2$ to give – 7.91,

    $[x]^3$ should give – 8 (hence, $[x]$ should be –2]

    Further, $\{x\}^2$ should be + 0.09.

    Both these conditions are satisfied by –1.7.

    Hence option (d) is correct.

13. $16^5 + 2^{15} = 2^{20} + 2^{15} = 2^{15}(2^5 + 1)$ Æ Hence, is divisible by 33.

14. The interpretation of the situation $AB + XY = 1XP$ is that the tens digit in XY is repeated in the value of the solution (i.e. $1XP$). Thus for instance if X was 2, it would mean we are adding a 2 digit number AB to a number in the 20’s to get a number in the 120’s. This can only happen if AB is in the 90’s which means that A is 9.

15. $|x - 3| + 2|x + 1| = 4$ can happen under three broad conditions.

(a) When $2|x + 1| = 0$, then $|x - 3|$ should be equal to 4.

Putting $x = -1$, both these conditions are satisfied.

(b) When $2|x + 1| = 2$, x should be 0, then $|x - 3|$ should also be 2. This does not happen.

(c) When $2|x + 1| = 4$, x should be + 1 or – 3, in either case $|x - 3|$ which should be zero does not give the desired value.

16. At a value of $x = 0$ we can see that the expression $x^2 + |x - 1| = 1$ Æ $0 + 1 = 1$. Hence, $x = 0$ satisfies the given expression. Also at $x = 1$, we get $1 + 0 = 1$. Option (d) is correct.

17. 4^{n+1} represents an odd power of 4 (and hence would end in 4). Similarly, 4^{2n} represents an even power of 4 (and hence would end in 6). Thus, the least number 'x' that would make both $4^{n+1} + x$ and $4^{2n} - x$ divisible by 5 would be for $x = 1$.

18. Check for each value of the options to see that the expression does not become divisible by 9 for any of the initial options. Thus, there is no value that satisfies the divisibility by 9 case.

19. The expression would have solutions based on a structure of:

4 + 0; 3 + 1; 2 + 2; 1 + 3 or 0 + 4.

There will be 2*1 = 2 solutions for 4 + 0 as in this case x can take the values of 8 and 0, while y can take a value of 4;

Similarly there would be 2*2 = 4 solutions for 3 + 1 as in this case x can take the values of 7 or 1, while y can take a value of 5 or 3;

Thus, the total number of solutions can be visualised as:

2 (for 4 + 0) + 4 (for 3 + 1) + 4 (for 2 + 2) + 4 (for 1 + 3) + 2 (for (0 + 2) = 16 solutions for the set (x, y) where both x and y are integers.

20. The numerator of $3^{32}/50$ would be a number that would end in 1. Consequently, the decimal of the form $.bx$ would always give us a value of x as 2.

21. If we assume the numbers as 16 and 4 based on 4:1 (in option a), the AM would be 10 and the $GM = 8$ a difference of 20% as stipulated in the question. Option (a) is correct.

22. $990 = 11 \times 3^2 \times 2 \times 5$. The highest power of 990 which would divide 1090! would be the power of 11 available in 990. This is given by [1090/11] + [1090/121] = 99 + 9 = 108
23. For finding the highest power of 6 that divides 146!, we need to get the number of 3's that would divide 146!. The same can be got by: [146/3] + [48/3] + [16/3] + [5/3] = 70.
24. There would be two fives and more than two twos in the prime factors of the numbers in the multiplication. Thus, we would get a total of 2 zeroes.
25. Both 333^{555} and 555^{333} are divisible by 3,37 and 111. Further, the sum of the two would be an even number and hence divisible by 2. Thus, all the four options divide the given number.
26. Both the values of options a and b satisfy the given expression. As for 5.16, the value of $[x]^2 = 25$ and the value of $\{x\} = 0.16$. Thus, $[x]^2 + \{x\}^1 = 25.16$

    Similarly for a value of $x = –4.84$, the value of $[x] = –5$ and hence $[x]^2 = 25$ and the value of $\{x\} = 0.16$. Thus, $[x]^2 + \{x\}^1 = 25.16$
27. The given conditions can be seen to be true for the number 49. Option (c) is correct.
28. Solve this question through options. Also realize that $a \times b = a + b$ only occurs for the situation $2 \times 2 = 2 + 2$. Hence, clearly the answer has to be none of these.
29. 863 satisfies each of the conditions and can be spotted through checking of the options.
30. The number of zeroes would be given by counting the number of 5's. The relevant numbers for counting the number of 5's in the product would be given by:

    5^5; 10^{10}, 15^{15}, 20^{20}, 25^{25}…and so on till 100^{100}

    The number of 5's in these values would be given by:

    (5 + 10 + 15 + 20 + 50 + 30 + 35 + 40 + 45 + 100 + 55 + 60 + 65 + 70 + 150 + 80 + 85 + 90 + 95 + 200)

    This can also be written as:

(5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 + 55 + 60 + 65 + 70 + 75 + 80 + 85 + 90 + 95 + 100) + (25 + 50 + 75 + 100)

= 1050 + 250 = 1300

31. Option (a) is correct as the LCM of 5 and 105 is 105 and their HCF is 5. Also for the pair of values, 15 and 35 the HCF is 5 and the LCM is 105.

32. Solve using options. Using option b = 3/5 and performing the given operation we get:

    2/3 – 3/5 = (10 – 9)/15 = 1/15. Option (b) is hence correct.

33. Both the conditions are satisfied for option (a) = 72 as the number 72 exceeds the sum of squares of the digits by 19 and also 72 exceeds the doubled product of it's digits by 44.

34. Solve by checking the given options. 31 and 13 are possible values of the number as defined by the problem.

35. The given conditions are satisfied for the number 24.

36. The number of 2's in the given expression is lower than the number of 5's. The number of 2's in the product is 9 and hence that is the number of zeroes.

37. $45 = 3^2 \times 5$. Hence, we need to count the number of 3^2's and 5's that can be made out of 123!.

    Number of 3's = 41 + 13 + 4 + 1 = 59 Æ Number of 3^2's = 29

    Number of 5's = 24 + 4 = 28.

    The required answer is the lower of the two (viz. 28 and 29). Hence, option (a) 28 is correct.

38. The first sentence means that the numbers are in an arithmetic progression. From the statements and a little bit of visualization, you can see that 8.5, 10 and 11.5 can be the three values we are looking for – and hence the middle value is 10.

39. $990 = 11 \times 3^2 \times 5 \times 2$. For n! to be divisible by 990, the value of n! should have an 11 in it. Since, 11 itself is a prime number, hence the value of n should be at least 11.

40. For the expression to hold true, x and y should both be positive.

41. Since, we are not given options here we should go ahead by looking within the factors of 144 (especially the two digit ones)

The relevant factors are 72,48, 36, 24, 18 and 12. Thinking this way creates an option for you where there is none available and from this list of numbers you can easily identify 24 as the required answer.

42–46. Write simple equations for each of the questions and solve.

47. Since the sum of squares of the digits of the two digit number is 10, the only possibility of the numbers are 31 and 13.

48. If the number is '*ab*' we have the following equations:

$(10a + b) = 4 (a + b) + 3$ Æ $6a – 3b = 3$

$(10a + b) = 3 (a \times b) + 5.$

Obviously we would need to solve these two equations in order to get the values of the digits a and b respectively. However, it might not be a very prudent decision to try to follow this process- as it might turn out to be too cumbersome.

A better approach to think here is:

From the first statement we know that the number is of the form $4n + 3$. Thus, the number has to be a term in the series 11, 15, 19, 23, 27…

Also from the second statement we know that the number must be a $3n + 5$ number.

Thus, the numbers could be 11, 14, 17, 20, 23….

Common terms of the above two series would be probable values of the number.

It can be seen that the common terms in the two series are: 11, 23, 35, 47, 59, 71, 83 and 95. One of these numbers has to be the number we are looking for.

If we try these values one by one, we can easily seen that the value of the two digit number should be 23 since Æ 23/ (2 + 3) Æ Quotient as 4 and remainder = 3.

Similarly, if we look at the other condition given in the problem we would get the following-

23/6 Æ quotient as 3 and remainder = 5.

Thus, the value of the missing number would be 23.

49. We can see from the description that the number (say X) must be such that $X + 100$ and $X + 169$ both must be perfect squares. Thus we are looking for two perfect squares which are 69 apart from each other. This would happen for 34^2 and 35^2 since their difference would be $(35 – 34)(35 + 34) = 69$.

50. Since their least common multiple is 102, we need to look for two factors of 102 such that they add up to 85. 51 and 34 can be easily spotted as the numbers.

51. If one number is x, the other should be $6x$ or $12x$ or $18x$ or $24x$ and so on. Also, their sum should be either 504 or 1008 or 1512. (Note: the next multiple of 504 = 2016 cannot be the sum of two three digit numbers.

52. Obviously 46 and 64 are the possible numbers.

53. The key here is to look for numbers which are more than three times but less than four times the product of their digits. Also, the product of the digits should be greater than 9 so as to leave a remainder of 9 when the number is divided by the product of it's digits.

    In the 10s, 20s and 30s there is no number which gives a quotient of 3 when it is divided by the product of it's digits. In the 40s, 43 is the only number which has a quotient of 3 when divided by 12 (product of it's digits). But 43/12 does not give us a remainder of 9 as required.

    In the 50s the number 53 divided by 15 leaves a remainder of 8, while in the 60s, 63 divided by 18 gives us a remainder of 9 as required.

54. The first thing to use while solving this question is to look at the information that the sum of squares of the three digits is 109. A little bit of trial and error shows us that this can only occur if the digits are 8, 6 and 3. Using the other information we get that the number must be 863 since, 863 – 495 = 368.

55. It is obvious that the only condition where the cubes of 3 numbers add up to 243 is when we add the cubes of 3 and 6. Hence, the numbers possible are 36 and 63.

56. There would definitely be two numbers and in case we take the first number as $7n - 1$, there would be three numbers – (as can be seen when we take the first number as 27 and the other number is 43).

57. Between 111^4, $110 \times 109 \times 108 \times 107$, $109 \times 110 \times 112 \times 113$.

It can be easily seen that

$111 \times 111 \times 111 \times 111 > 110 \times 109 \times 108 \times 107$

also $109 \times 110 \times 112 \times 113 > 109 \times 110 \times 108 \times 107$

Further the product $111 \times 111 \times 111 \times 111 > 109 \times 110 \times 112 \times 113$ (since, the sum of the parts of the product are equal on the LHS and the RHS and the numbers on the LHS are closer to each other than the numbers on the RHS.

58. Both x and y should be highest for xy to be maximum. Similarly x should be minimum and y should be maximum for x/y to be minimum.

59. $200^{300} = (200^6)^{50}$

$300^{200} = (300^4)^{50}$

$400^{150} = (400^3)^{50}$

Hence 200^{300} is greater.

61. The sum of squares of the first n natural numbers is given by $n(n + 1)(2n + 1)/6$.

For this number to be divisible by 4, the product of $n(n + 1)(2n + 1)$ should be a multiple of 8. Out of n, $(n + 1)$ and $(2n + 1)$ only one of n or $(n + 1)$ can be even and $(2n + 1)$ would always be odd.

Thus, either n or $(n + 1)$ should be a multiple of 8.

This happens if we use n = 7, 8, 15, 16, 23, 24, 31, 32, 39, 40, 47, 48. Hence, 12 such numbers.

62. In the 20s the numbers are: 23 to 29

In the 30s the numbers are: 32 to 39

Subsequently the numbers are 42 to 49, 52 to 59, 62 to 69, 72 to 79, 82 to 89 and 92 to 99.

A total of 63 numbers.

63. You need to solve this question using trial and error.

For 32 (option 1):

$32 = 2^5$. Hence 6 factors. On increasing by 50%, $48 = 2^4 \times 3^1$ has 10 factors. Thus the number of factors is increasing when the number is increased by 50% which is not what the question is defining for the number. Hence, 32 is not the correct answer.

Checking for option (b) 84.

$84 = 2^2 \times 3^1 \times 7^1$ Æ (2 + 1) (1 + 1) (1 + 1) = 12 factors

On increasing by 50% Æ $126 = 2^1 \times 32 \times 7^1$ Æ (1 + 1) (2 + 1) (1 + 1) = 12 factors. (no change in number of factors).

Second Condition: When the value of the number is reduced by 75% Æ 84 would become 21 ($3^1 \times 7^1$) and the number of factors would be 2 × 2 = 4 – a reduction of 66.66% in the number of factors.

64. There will be 9 single digit numbers using 9 digits, 90 two digit numbers using 180 digits, 900 three digit numbers using 2700 digits. Thus, when the number 999 would be written, a total of 2889 digits would have been used up. Thus, we would need to look for the 25494th digit when we write all 4 digit numbers. Since, 25494/4 = 6373.5 we can conclude that the first 6373 four digit numbers would be used up for writing the first 25492 digits. The second digit of the 6374th four digit number would be the required answer. Since, the 6374th four digit number is 7373, the required digit is 3.

65. In order to solve this question, think of the numbers grouped in groups of 9 as:

{1, 2, 3, 4, 5, 6, 7, 8, 9} {10, 11, 12…..18} and so on till {2989, 2990…2997} – A total of 333 complete sets. From each set we can take 4 numbers giving us a total of 333 × 4 = 1332 numbers.

Apart from this, we can also take exactly 1 multiple of 9 (any one) and also the last 3 numbers viz 2998, 2999 and 3000. Thus, there would be a total of 1332 + 4 = 1336 numbers.

66. It can be seen that for only 2 numbers (1 and ½) the consolidated number would be 1 + ½ + ½ = 2

For 3 numbers, (1, ½ , 1/3) the number would be 3. Thus, for the given series the consolidated number would be 1972.

67. The value of K would be 199 and hence, the required difference is 9 – 1 = 8

68. 9 – 9 = 0 would be the difference between the units and the tens digits.

69. The highest ratio would be a ratio of 100 in the numbers, 100, 200, 300, 400, 500, 600, 700, 800 and 900. Thus a total of 9 numbers.

70. Basically every odd triangular number would have this property, that it is the difference of squares of two consecutive natural numbers. Thus, we need to find the number of triangular numbers that are odd.

    3, 15, 21, 45, 55, 91, 105, 153, 171, 231, 253, 325, 351, 435, 465, 561, 595, 703, 741, 861, 903 – A total of 21 numbers.

71. The coefficients would be ${}^{44}C_0$, ${}^{44}C_1$, ${}^{44}C_2$ and so on till ${}^{44}C_{44}$. The sum of these coefficients would be 2^{44} (since the value of ${}^{n}C_0 + {}^{n}C_1 + {}^{n}C_2 + \ldots {}^{n}C_n = 2^n$)

72. The remainder of each power of 9 when divided by 6 would be 3. Thus, for (2n+1) powers of 9, there would be an odd number of 3's. Hence, the remainder would be 3.

73. The remainder when a number is divided by 16 is given by the remainder of the last 4 digits divided by 16 (because 10000 is a multiple of 16. This principle is very similar in logic to why we look at last 2 digits for divisibility by 4 and the last 3 digits for divisibility by 8). Thus, the required answer would be the remainder of 4950/16 which is 6.

74. $58! - 38! = 38!\,(58 \times 57 \times 56 \times 55 \times \ldots. \times 39 - 1)$ Æ $38!\,(3n - 1)$ since the expression inside the bracket would be a $3n - 1$ kind of number. Thus, the number of 3's would depend only on the number of 3's in 38! Æ 12 + 4 + 1 = 17.

75. The given expression can be seen as $(22334^{\text{ODD POWER}})/5$, since the sum of $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 66^2$ can be seen to be an odd number. The remainder would always be 4 in such a case.

76. $12^{33} \times 34^{23} \times 2^{70} = 2^{159} \times 3^{33} \times 17^{23}$. The number of factors would be $160 \times 34 \times 24 = 130560$. Thus, option (d) is correct.

77. Option (a) is correct.

78. 1152 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 = $2^7 \times 3^2$. Essentially every number starting from $4!^3$ would be divisible perfectly by 1152 since each number after that would have at least 7 twos and 2 threes.

Thus, the required remainder is got by the first three terms:

(1 + 8 + 216)/1152 = 225/1152 gives us 225 as the required remainder.

79. We can take only perfect squares of odd numbers and one perfect square of an even number. Thus, for instance we can take numbers like 1, 4, 9, 25, 49, 121, 169, 289, 361, 529, 841 and 961. A total of 12 such numbers can be taken.

80. (101 × 102 × 103 × 197 × 198 × 199)/100 Æ [1 × 2 × 3 × (–3) × (–2) × (–1)]/100 Æ –36 as remainder Æ remainder is 64.

81. [65 × 29 × 37 × 63 × 71 × 87]/100 Æ [–35 × 29 × 37 × –37 × –29 × –13]/100 Æ [35 × 29 × 37 × 37 × 29 × 13]/100 = [1015 × 1369 × 377]/100 Æ 15 × 69 × 77/100 Æ remainder as 95.

82. [65 × 29 × 37 × 63 × 71 × 87 × 85]/100 Æ [–35 × 29 × 37 × –37 × –29 × –13 × –15]/100 Æ [35 × 29 × 37 × 37 × 29 × 13 × –15]/100 = [1015 × 1369 × 377 × –15]/100 Æ [15 × 69 × 77 × – 15]/100 Æ remainder as 75.

83. [65 × 29 × 37 × 63 × 71 × 87 × 62]/100 Æ [–35 × 29 × 37 × –37 × –29 × –13 × 62]/100 Æ [35 × 29 × 37 × 37 × 29 × 13 × 62]/100 = [1015 × 1369 × 377 × 62]/100 Æ [15 × 69 × 77 × 62]/100 = [1035 × 4774]/100 Æ 35 × 74/100 Æ remainder as 90.

84. [75 × 35 × 47 × 63 × 71 × 87 × 82]/100 = [3 × 35 × 47 × 63 × 71 × 87 × 41]/2 Æ remainder = 1.

Hence, required remainder = 1 x 50 = 50.

85. For this question we need to find the remainder of:

(201 × 202 × 203 × 204 × 246 × 247 × 248 × 249) × (201 × 202 × 203 × 204 × 246 × 247 × 248 × 249) divided by 100.

(201 × 202 × 203 × 204 × 246 × 247 × 248 × 249) × (201 × 202 × 203 × 204 × 246 × 247 × 248 × 249)/100 = (201 × 101 × 203 × 102

× 246 × 247 × 248 × 249) × (201 × 202 × 203 × 204 × 246 × 247 × 248 × 249)/25

Æ (1 × 1 × 3 × 2 × –4 × –3 × –2 × –1) × (1 × 2 × 3 × 4 × –4 × –3 × –2 × –1)/25 = 144 × 576/25 Æ (19 × 1)/25 = remainder 19.

19 × 4 = 76 is the actual remainder (since we divided by 4 during the process of finding the remainder.

86. 7^4/2400 gives us a remainder of 1. Thus, the remainder of 7^{99}/2400 would depend on the remainder of 7^3/2400 Æ remainder = 343.

87. The numerator can be written as $(1729)^{752}$/1728 Æ remainder as 1.

88. Bikas's movement in terms of the number of coins would be:

B Æ 3B (when Arun triples everyone's coins) Æ B.

Think of this as: When Bikas triples everyone's coins, and is left with 20 it means that the other 3 have 60 coins after their coins are tripled. This means that before the tripling by Bikas, the other three must have had 20 coins—meaning Bikas must have had 60 coins.

But 60 = 3B Æ B = 20.

89. For 83*p*796161*q* to be a multiple of 11 (here *X* is 11) we should have the difference between the sum of odd placed digits and even placed digits should be 0 or a multiple of 11.

$(8 + p + 9 + 1 + 1) - (3 + 7 + 6 + 6 + q) = (19 + p) - (22 + q)$. For this difference to be 0, *p* should be 3 more than *q* which cannot occur since $0 < p < q$.

The only way in which $(19 + p) - (22 + q)$ can be a multiple of 11 is if we target a value of –11 for the expression. One such possibility is if we take *p* as 1 and *q* as 9.

The number would be 8317961619. On successive division by $(p + q) = 10$ and 1 the sum of remainders would be 9.

90. $n(n + 1)/2$ should be a perfect square. The first value of *n* when this occurs would be for $n = 8$. Thus, on the 8th of March the required condition would come true.

91. We have to find the unit's digit of 2^{53} Æ 2^{4n+1} Æ 2 as the units digit.

92. [7! (14 + 14 × 13 × 12 × 11 × 10 × 9 × 8)]/[7! (16 – 3)] = [(14 + 14 × 13 × 12 × 11 × 10 × 9 × 8)]/[(13)] Æ remainder 1.

Hence, the original remainder must be 7! (because for the sake of simplification of the numbers in the question we have cut the 7! From the numerator and the denominator in the first step.

93. $x = 6$ and $y = 3$ is one pair of values where the given condition is met.

After that you should be able to spot that if you were to increase x by 7, y would also increase by 4. The number of such pairs would depend on how many terms are there in the series

–498, –491, ….. –1, 6, 13, 20, … 489, 496. The series has 994/7 + 1 = 143 terms and hence there would be 143 pairs of values for (x, y) which would satisfy the equation.

94. All three conditions can be seen to be true.

95. Product of factorials < Sum of factorials would occur for any number that has either 0 or 1 in it.

The required numbers uptil and including 50 are: 10 to 19, 20, 21, 30, 31, 40, 41, 50. Besides for the number 22, the product of factorials of the digits would be equal to the sum of factorials of the digits. Thus a total of 18 numbers.

96. The maximum marks he can score is: 100 (if he gets all correct).

The minimum marks he can score would be given by: 10 × (–0.1) + 20 × (–0.2) + 70 × (–0.5) = – 40.

The difference between the two values would be 100 – (–40) = 140 marks.

Logic for Questions 97 to 99:

If a student solved 200 questions and got everything correct he would score a total of 620 marks. By getting a LOD 1 question wrong he would lose 4+2=6 marks, while by not solving an LOD 1 question he would lose 4 marks.

Similarly for LOD 2 questions, Loss of marks = 4.5 (for wrong answers) and loss of marks= 3 (for not solved)

Similarly for LOD 3 questions, Loss of marks = 3 (for wrong answers) and loss of marks = 2 (for not solved)

Since, he has got 120 marks from 100 questions solved he has to lose 500 marks (from the maximum possible total of 620) by combining to lose marks through 100 questions not solved and some questions wrong.

97. It can be seen through a little bit of trial and error with the options, that if he got 44 questions of LOD 1 correct and 56 questions of LOD 3 wrong he would end up scoring $44 \times 4 - 56 \times 1 = 176 - 56 = 120$. In such a case he would have got the maximum possible incorrects with the given score.
98. $32 \times 4 + 1 \times 3 - 1 \times 1 = 130$ (in this case he has solved 32 corrects from LOD 1, 1 correct from LOD 2 and 1 incorrect from LOD 3). Thus, a total of 34 attempts.
99. In the above case he gets 1 question incorrect. However, he can also get 130 marks by $30 \times 4 + 2 \times 3 + 2 \times 2$ where he gets 30 LOD 1 questions correct, and 2 questions correct each from LOD 2 and LOD 3).

    The least number of incorrects would be 0.
100. The least number would be (LCM of 10, 9, 8, 7, 6 and 5 – 1) = 2519. The second least number = $2520 \times 2 - 1 = 5039$.

Hints

Level of Difficulty (III)

1–5. Solve through options.

6. Use AP with first term 104 and last term 999 and common difference 5.
7. Find the first 2 digit number which gives a remainder of 3 when divided by 7 and then find the largest such number (10 and 94 respectively). Use Arithmetic Progression formulae to add the numbers.
8. Use AP with first term 105 and last term 995 and common difference is 10.

10. The cubes of the numbers are $x - 3$, $x + 2$ and $x + 3$. Use options and you will see that (a) is the answer.

11. $(x^2 - y^2) = 45$. i.e. $(x - y)(x + y) = 45$. The factors of 45 possible are, 15, 3; 9, 5 and 45, 1.

    Hence, the numbers are 9 and 6, or 7 and 2 or 23 and 22.

12–18. Use options to check the given conditions.

19. The answer will be 50 since, 125*122 will give 50 as the last two digits.

20. The remainder theorem is to be used.

    $43^{101} + 23^{101} = (43 + 23)\,(.\ .\ .\ .\ .)$

    Hence, when divided by 66 the remainder will be zero.

21. The unit's digit will be $1 \times 5 = 5$ (no carry over.) The tens digit will be (4*1 + 5*2) = 4 (carry over 1). The hundreds digit will be (3*1 + 4*2 + 5*1) = 6 + 1 (carried over) = 7. Hence, answer is 745.

22–25. Use options to solve.

26. Use the rule of indices and remainder theorem.

27. Options are not provided as it is an LOD 3 question. If they were there you should have used options.

28. Use trial and error

29. Use options.

30. Use remainder theorem and look at patterns by applying the rules of indices.

    We get the value as:

    $5 \xrightarrow[9]{32.32.32\ldots\ldots\ldots.32\text{ times}}$

    $7 \xrightarrow[9]{32.32.32\ldots\ldots\ldots.31\text{ times}}$

    $4 \xrightarrow[9]{32.32.32\ldots\ldots\ldots.30\text{ times}}$

    $7 \xrightarrow[9]{32.32\ldots.29\text{ times}}$

    Æ Looking at the pattern we will get 4 as the final remainder.

31. Use the remainder theorem and get the remainder as: 1 × 2 × 4 × 4 × 4/7 = 128/7 Æ 2 is the remainder. 32.

32. $2^{100}/3 = (2^4)^{25}/3$ Æ 1.

33. Use the remainder theorem and try finding the patterns.

34. Find the last digit of the number got by adding $1^2 + 2^2 + \ldots 9^2$ (you will get 5 here). Then multiply by 10 to get zero as the answer.

35. $(2^{100} - 1)$ and $(2^{120} - 1)$ will yield the GCD as $2^{20} - 1$. (This has been explained in the theory of GCDs).

36. The GCDs of 100 ones and 60 ones will be twenty ones because 20 is the GCD of sixty and Hundred.

39. $\dfrac{32^{32^{32}}}{7} \to \dfrac{4^{32^{32}}}{7}$

But $4^3/7$ gives us a remainder of 1.

Hence we need to convert $4^{32^{32}}$ into $4^{3n + x}$ (Think why!!)

Here again, we will be more interested in finding the value of x rather than n, since the remainder only depends on the value of x.

[Concept Note: When we start to write as the remainder.

$4^{32^{32}}$ in the form $4^3 \times 4^3 \times 4^3 \ldots\ldots n$ times $\times 4^x$ we are not bothered about how many times we can write 4^3 since it will continuously give us 1 every time as the remainder.

Chapter 2 Progressions

The chapter on progressions essentially yields common-sense based questions in examinations.

Questions in the CAT and other aptitude exams mostly appear from either Arithmetic Progressions (more common) or from Geometric Progressions.

The chapter of progressions is a logical and natural extension of the chapter on Number Systems, since there is such a lot of commonality of logic between the problems associated with these two chapters.

ARITHMETIC PROGRESSIONS

Quantities are said to be in arithmetic progression when they increase or decrease by a common difference.

Thus each of the following series forms an arithmetic progression:

3, 7, 11, 15,…

8, 2, –4, –10,…

$a, a + d, a + 2d, a + 3d, \ldots$

The common difference is found by subtracting any term of the series from the next term.

That is, common difference of an $AP = (t_N - t_{N-1})$.

In the first of the above examples the common difference is 4; in the second it is –6; in the third it is d.

If we examine the series a, $a + d$, $a + 2d$, $a + 3d$,… we notice that *in any term the coefficient of d is always less by one than the position of that term in the series*.

Thus the rth term of an arithmetic progression is given by $T_r = a + (r - 1)d$.

If n be the number of terms, and if L denotes the last term or the nth term, we have

$$L = a + (n - 1)d$$

To Find the Sum of the given Number of Terms in an Arithmetic Progression

Let a denote the first term d, the common difference, and n the total number of terms. Also, let L denote the last term, and S the required sum; then

$$S = \frac{n(a + L)}{2} \quad (1)$$

$$L = a + (n - 1)d \quad (2)$$

$$S = \frac{n}{2} \times [2a + (n - 1)d] \quad (3)$$

If any two terms of an arithmetical progression be given, the series can be completely determined; for this data results in two simultaneous equations, the solution of which will give the first term and the common difference.

When three quantities are in arithmetic progression, the middle one is said to be the **arithmetic mean** of the other two.

Thus a is the arithmetic mean between $a - d$ and $a + d$. So, when it is required to arbitrarily consider three numbers in *AP* take $a - d$, a and $a + d$ as the three numbers as this reduces one unknown thereby making the solution easier.

To Find the Arithmetic Mean between any Two given Quantities

Let a and b be two quantities and A be their arithmetic mean. Then since a, A, b, are in *AP*. We must have

$b - A = A - a$

Each being equal to the common difference;

This gives us $A = \frac{(a+b)}{2}$

Between two given quantities it is always possible to insert any number of terms such that the whole series thus formed shall be in *AP*. The terms thus inserted are called the **arithmetic means**.

To Insert a given Number of Arithmetic Means between Two given Quantities

Let a and b be the given quantities and n be the number of means.

Including the extremes, the number of terms will then be $n + 2$ so that we have to find a series of $n + 2$ terms in *AP*, of which a is the first, and b is the last term.

Let d be the common difference;

then b = the $(n + 2)$th term

$= a + (n + 1)d$

Hence, $d = \frac{(b-a)}{(n+1)}$

and the required means are

$$a + \frac{(b-a)}{n-1}, a + \frac{2(b-a)}{n+1}, \cdots a + \frac{n(b-a)}{n+1}$$

Till now we have studied APs in their mathematical context. This was important for you to understand the basic mathematical construct of A.Ps. However, you need to understand that questions on A.P. are seldom solved on a mathematical basis, (Especially under the time pressure that you are likely to face in the CAT and other aptitude exams). In such situations the mathematical processes for solving progressions based questions are likely to fail or at the very least, be very tedious. Hence, understanding the following logical aspects about Arithmetic Progressions is likely to help you solve questions based on APs in the context of an aptitude exam.

Let us look at these issues one by one:

1. Process for finding the nth term of an A.P.

Suppose you have to find the 17th term of the

A.P. 3, 7, 11…….

The conventional mathematical process for this question would involve using the formula.

$T_n = a + (n – 1)\ d$

Thus, for the 17th term we would do

$T_{17} = 3 + (17 – 1) \times 4 = 3 + 16 \times 4 = 67$

Most students would mechanically insert the values for *a*, *n* and *d* and get this answer.

However, if you replace the above process with a thought algorithm, you will get the answer much faster.

The algorithm goes like this:

In order to find the 17th term of the above sequence add the common difference to the first term, sixteen times. (**Note:** Sixteen, since it is one less than 17).

Similarly, in order to find the 37th term of the A.P. 3, 11 …, All you need to do is add the common difference (8 in this case), 36 times.

Thus, the answer is 288 + 3 = 291.

(**Note:** You ultimately end up doing the same thing, but you are at an advantage since the entire solution process is reactionary.)

2. Average of an A.P. and Corresponding terms of the A.P.

Consider the A.P., 2, 6, 10, 14, 18, 22. If you try to find the average of these six numbers you will get: Average = (2 + 6 + 10 + 14 + 18 + 22)/6 = 12

Notice that 12 is also the average of the first and the last terms of the A.P. In fact, it is also the average of 6 and 18 (which correspond to the second and 5th terms of the A.P.). Further, 12 is also the average of the 3rd and 4th terms of the A.P.

(**Note:** In this A.P. of six terms, the average was the same as the average of the 1st and 6th terms. It was also given by the average of the 2nd and the 5th terms, as well as that of the 3rd and 4th terms.)

We can call each of these pairs as "CORRESPONDING TERMS" in an A.P.

What you need to understand is that every A.P. has an average.

And for any A.P., the average of any pair of corresponding terms will also be the average of the A.P.

If you try to notice the sum of the term numbers of the pair of corresponding terms given above:

1st and 6th (so that 1 + 6 = 7)

2nd and 5th(hence, 2 + 5 =7)

3rd and 4th (hence, 3 + 4 = 7)

Note: In each of these cases, the sum of the term numbers for the terms in a corresponding pair is one greater than the number of terms of the A.P.

This rule will hold true for all A.P.s.

For example, if an A.P. has 23 terms then for instance, you can predict that the 7th term will have the 17th term as its corresponding term, or for that matter the 9th term will have the 15th term as its corresponding term. (Since 24 is one more than 23 and 7 + 17 = 9 + 15 = 24.)

3. Process for finding the sum of an A.P.

Once you can find a pair of corresponding terms for any A.P., you can easily find the sum of the A.P. by using the property of averages:

i.e. Sum = Number of terms × Average.

In fact, this is the best process for finding the sum of an A.P. It is much more superior than the process of finding the sum of an A.P. using the expression $\frac{n}{2}(2a+(n-1)d)$.

4. Finding the common difference of an A.P., given 2 terms of an A.P.

Suppose you were given that an A.P. had its 3rd term as 8 and its 8th term as 28. You should visualize this A.P. as – , – , 8 , – , – , –, – , 28.

From the above figure, you can easily visualize that to move from the third term to the eighth term, (8 to 28) you need to add the common difference five times. The net addition being 20, the common difference should be 4.

Illustration: Find the sum of an A.P. of 17 terms, whose 3rd term is 8 and 8th term is 28.

Solution: Since we know the third term and the eighth term, we can find the common difference as 4 by the process illustrated above.

The total = 17 × Average of the A.P.

Our objective now shifts into the finding of the average of the A.P. In order to do so, we need to identify either the 10th term (which will be the corresponding term for the 8th term) or the 15th term (which will be the corresponding term for the 3rd term.)

Again: Since the 8th term is 28 and $d = 4$, the 10th term becomes $28 + 4 + 4 = 36$.

Thus, the average of the A.P.

= Average of 8th and 10th terms

$= (28 + 36)/2 = 32$.

Hence, the required answer is sum of the A.P. $= 17 \times 32 = 544$.

The logic that has applied here is that the difference in the term numbers will give you the number of times the common difference is used to get from one to the other term.

For instance, if you know that the difference between the 7th term and 12th term of an AP is –30, you should realize that 5 times the common difference will be equal to –30. (Since $12 - 7 = 5$).

Hence, $d = -6$.

Note: Replace this algorithmic thinking in lieu of the mathematical thinking of:

12^{th} term $= a + 11d$

7^{th} term $= a + 6d$

Hence, difference $= -30 = (a + 11d) - (a + 6d)$

– 30 = 5d

\ d = –6.

5. Types of APs:Increasing and Decreasing A.P.s.

Depending on whether ‘d’ is positive or negative, an A.P. can be increasing or decreasing.

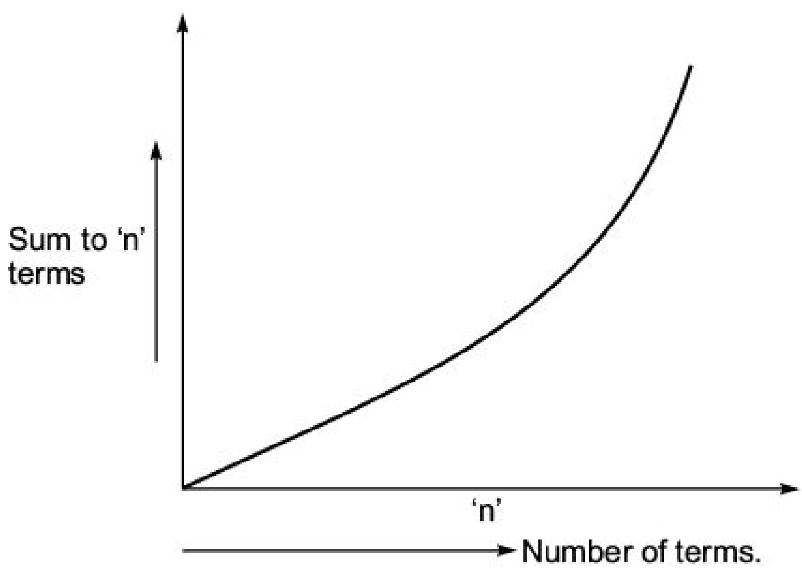
Let us explore these two types of A.P.s further:

(A) Increasing A.P.s:

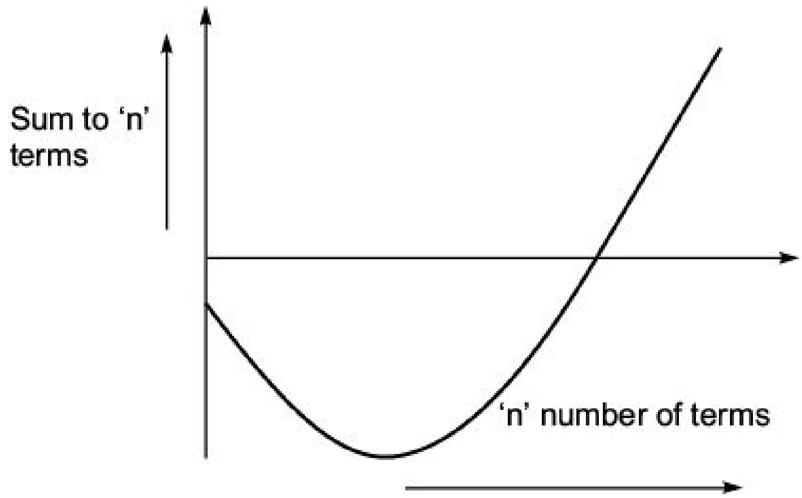
Every term of an increasing AP is greater than the previous term.

Depending on the value of the first term, we can construct two graphs for sum of an increasing A.P.

Case 1: When the first term of the increasing A.P. is positive. In such a case the sum of the A.P. will show a continuously increasing graph which will look like the one shown in the figure below:



Case 2: When the first term of the increasing A.P. is negative. In such a case, the Sum of the A.P. plotted against the number of terms will give the following figure:



The specific case of the sum to n_1 terms being equal to the sum to n_2 terms.

In the series case 2 above, there is a possibility of the sum to '*n*' terms being repeated for 2 values of '*n*'. However, this will not necessarily occur.

This issue will get clear through the following example:

Consider the following series:

Series 1: –12, –8, –4, 0, 4, 8, 12

As is evident the sum to 2 terms and the sum to 5 terms in this case is the same. Similarly, the sum to 3 terms is the same as the sum to 4 terms. This can be written as:

$S_2 = S_5$ and $S_3 = S_4$.

In other words the sum to n_1 terms is the same as the sum to n_2 terms.

Such situations arise for increasing A.P.'s where the first term is negative. But as we have already stated that this does not happen for all such cases.

Consider the following A.P.s.

Series 2: –8, –3, +2, + 7, + 12…

Series 3: –13, –7, –1, + 5, + 11…

Series 4: –12, –6, 0, 6, 12 …

Series 5: –15, –9, –3, + 3, 9, 15 …

Series 6: –20, –12, –4, 4 , 12, …

If you check the series listed above, you will realize that this occurrence happens in the case of Series 1, Series 4, Series 5 and Series 6 while in the case of Series 2 and Series 3 the same value is not repeated for the sum of the Series.

A clear look at the two series will reveal that this phenomenon occurs in series which have what can be called a balance about the number zero.

Another issue to notice is that in Series 4,

$S_2 = S_3$ and $S_1 = S_4$

While in series 5

$S_1 = S_5$ and $S_2 = S_4$.

In the first case (where '0' is part of the series) the sum is equal for two terms such that one of them is odd and the other is even.

In the second case on the other hand (when '0' is not part of the series) the sum is equal for two terms such that both are odd or both are even.

Also notice that the sum of the term numbers which exhibit equal sums is constant for a given A.P.

Consider the following question which appeared in CAT 2004 and is based on this logic:

The sum to 12 terms of an A.P. is equal to the sum to 18 terms. What will be the sum to 30 terms for this series?

Solution: If $S_{12} = S_{18}$, $S_{11} = S_{19\ldots}$ and $S_0 = S_{30}$

But Sum to zero terms for any series will always be 0. Hence $S_{30} = 0$.

Note: The solution to this problem does not take more than 10 seconds if you know this logic

(B) Decreasing A.P.s.

Similar to the cases of the increasing A.Ps, we can have two cases for decreasing APs –

Case 1– Decreasing A.P. with first term negative.

Case 2– Decreasing A.P. with first term positive.

I leave it to the reader to understand these cases and deduce that whatever was true for increasing A.Ps with first term negative will also be true for decreasing APs with first term positive.

GEOMETRIC PROGRESSION

Quantities are said to be in Geometric Progression when they increase or decrease by a constant factor.

The constant factor is also called the *common ratio* and it is found by dividing any term by the term immediately preceding it.

If we examine the series $a, ar, ar^2, ar^3, ar^4, \ldots$

we notice that in any term the index of r is always less by one than the number of the term in the series.

If n be the number of terms and if l denote the last, or nth term, we have

$l = ar^{n-1}$

When three quantities are in geometrical progression, the middle one is called the geometric mean between the other two. While arbitrarily choosing three numbers in GP, we take a/r, a and a/r. This makes it easier since we come down to two variables for the three terms.

To Find the Geometric Mean between two given Quantities

Let a and b be the two quantities; G the geometric mean. Then since a, G, b are in GP,

$b/G = G/a$

Each being equal to the common ratio

$G^2 = ab$

Hence $G = \sqrt{ab}$

To Insert a given Number of Geometric Means between two given Quantities

Let a and b be the given quantities and n the required number of means to be inserted. In all there will be $n + 2$ terms so that we have to find a series of $n + 2$ terms in GP of which a is the first and b the last.

Let r be the common ratio;

Then $\quad b$ = the $(n + 2)$th term = ar^{n+1};

\ $\quad r^{(n+1)} = \frac{b}{a}$

\ $\quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ (1)

Hence the required number of means are ar, ar^2, … ar^n, where r has the value found in (1).

To Find the Sum of a Number of Terms in a Geometric Progression

Let a be the first term, r the common ratio, n the number of terms, and S_n be the sum to n terms.

If $r > 1$, then

$$S_n = \frac{a(1-r^n)}{(1-r)} \qquad (1)$$

If $r < 1$, then

$$S_n = \frac{a(1-r^n)}{(1-r)} \qquad (2)$$

Note: It will be convenient to remember both forms given above for *S*. Number (2) will be used in all cases except when r is positive and greater than **one**.

Sum of an infinite geometric progression when $r < 1$

$$S_{\bullet} = \frac{a}{(1-r)}$$

Obviously, this formula is used only when the common ratio of the GP is less than one.

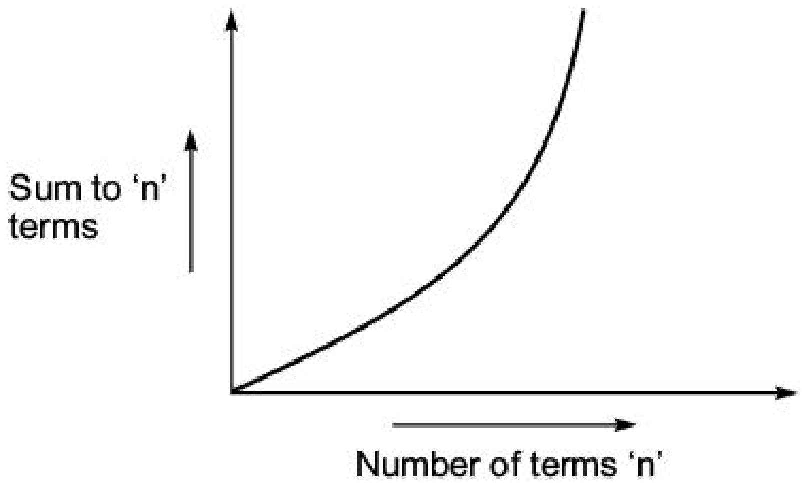
Similar to APs, GPs can also be logically viewed. Based on the value of the common ratio and its first term a G.P. might have one of the following structures:

(1) Increasing GPs type 1:

A G.P. with first term positive and common ratio greater than 1. This is the most common type of G.P.

e.g: 3, 6, 12, 24…(A G.P. with first term 3 and common ratio 2)

The plot of the sum of the series with respect to the number of terms in such a case will appear as follows:



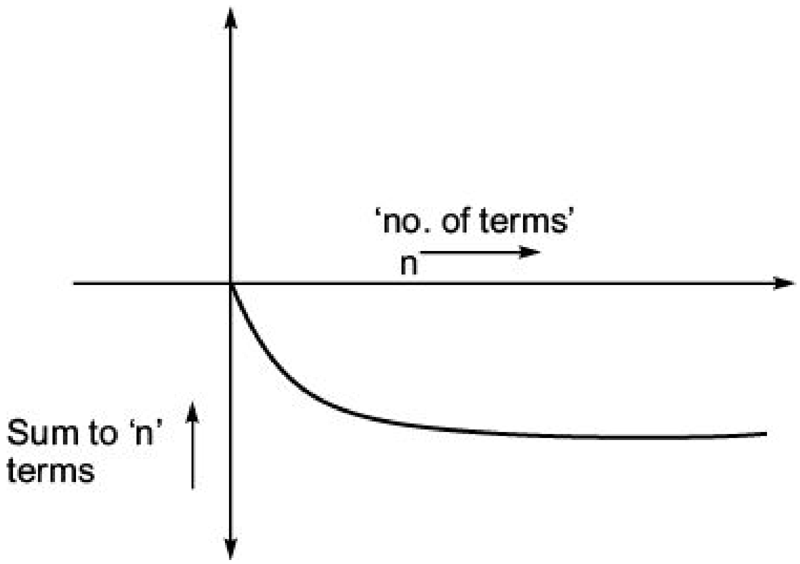
(2) Increasing GPs type 2:

A G.P. with first term negative and common ratio less than 1.

e.g: –8, –4,–2, –1, – ……

As you can see in this GP all terms are greater than their previous terms.

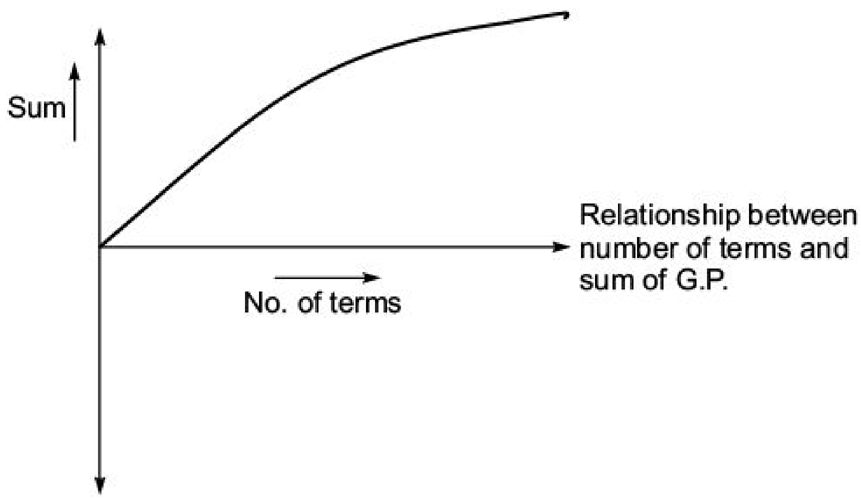
[The following figure will illustrate the relationship between the number of terms and the sum to '*n*' terms in this case]



(3) Decreasing G.Ps type 1:

These GPs have their first term positive and common ratio less than 1.

e.g: 12, 6, 3, 1.5, 0.75 …..

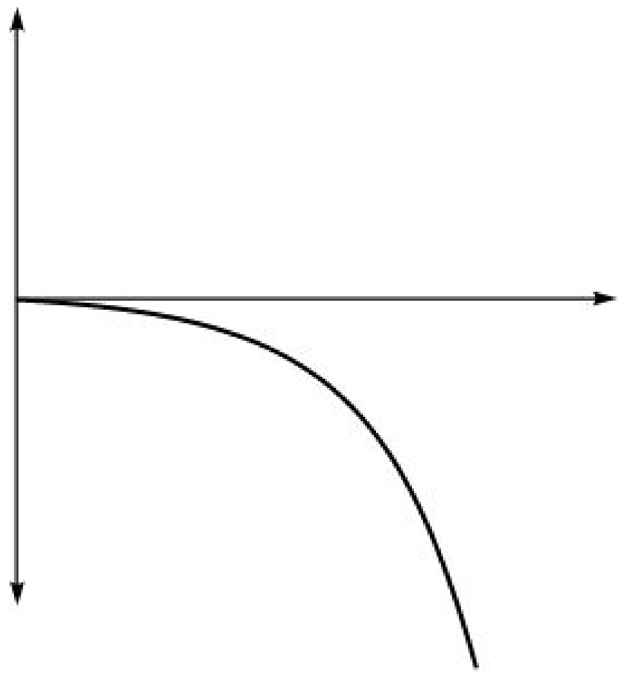


(4) Decreasing GPs type 2:

First term negative and common ratio greater than 1.

e.g: –2, –6, –18 ….

In this case the relationship looks like.



HARMONIC PROGRESSION

Three quantities *a*, *b*, *c* are said to be in Harmonic Progression when *a/c* = $\frac{(a-b)}{(b-c)}$.

In general, if *a*, *b*, *c*, *d* are in AP then 1/*a*, 1/*b*, 1/*c* and 1/*d* are all in HP.

Any number of quantities are said to be in harmonic progression when every three consecutive terms are in harmonic progression.

The reciprocals of quantities in harmonic progression are in arithmetic progression. This can be proved as:

By definition, if *a*, *b*, *c* are in harmonic progression,

$$\frac{a}{c} = \frac{(a-b)}{(b-c)}$$

\ $a(b - c) = c(a - b)$,

dividing every term by *abc*, we get

$$\left[\frac{1}{c} - \frac{1}{b} = \frac{1}{b} - \frac{1}{a}\right]$$

which proves the proposition.

There is no general formula for the sum of any number of quantities in harmonic progression. Questions in HP are generally solved by inverting the terms, and making use of the properties of the corresponding AP.

To Find the Harmonic Mean between two given Quantities

Let *a*, *b* be the two quantities, *H* their harmonic mean; then 1/*a*, 1/*H* and 1/*b* are in A.P.;

\ $\frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H}$

$\frac{2}{H} = \frac{1}{a} + \frac{1}{b}$

i.e. $\boxed{H = \frac{2ab}{(a+b)}}$

THEOREMS RELATED WITH PROGRESSIONS

If *A*, *G*, *H* are the arithmetic, geometric, and harmonic means between *a* and *b*, we have

$$A = \left(\frac{a+b}{2}\right) \quad (1)$$

$$G = \sqrt{ab} \quad (2)$$

$$H = \frac{2ab}{(a+b)} \quad (3)$$

Therefore, $A \times H = \frac{(a+b)}{2} \times \frac{2ab}{(a+b)} = ab = G^2$

that is, *G is the geometric mean between A and H.*

From these results we see that

$$A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{(a+b-2\sqrt{ab})}{2}$$

$$= \left[\frac{(\sqrt{a}-\sqrt{b})}{\sqrt{2}}\right]^2$$

which is positive if a and b are positive. Therefore, the arithmetic mean of any two positive quantities is greater than their geometric mean.

Also from the equation $G^2 = AH$, we see that G is intermediate in value between A and H; and it has been proved that $A > G$, therefore $G > H$ and $A > G > H$.

The arithmetic, geometric, and harmonic means between any two positive quantities are in descending order of magnitude.

As we have already seen in the Back to school section of this block there are some number series which have a continuously decreasing value from one term to the next – and such series have the property that they have what can be defined as the sum of infinite terms. Questions on such series are very common in most aptitude exams. Even though they cannot be strictly said to be under the domain of progressions, we choose to deal with them here.

Consider the following question which appeared in CAT 2003.

Find the infinite sum of the series:

$$1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + \ldots\ldots.$$

(a) 27/14 (b) 21/13

(c) 49/27 (d) 256/147

Solution: Such questions have two alternative widely divergent processes to solve them.

The first relies on mathematics using algebraic solving. Unfortunately this process being overly mathematical requires a lot of writing and hence is not advisable to be used in an aptitude exam.

The other process is one where we try to predict the approximate value of the sum by taking into account the first few significant terms. (This approach is possible to use because of the fact that in such series we invariably reach the point where the value of the next term becomes insignificant and does not add substantially to the sum). After adding the significant terms we are in a position to guess the approximate value of the sum of the series.

Let us look at the above question in order to understand the process.

In the given series the values of the terms are:

First term = 1

Second term = 4/7 = 0.57

Third term = 9/63 = 0.14

Fourth term = 16/343 = 0.04

Fifth term = 25/2401 = 0.01

Addition upto the fifth term is approximately 1.76

Options (b) and (d) are smaller than 1.76 in value and hence cannot be correct.

That leaves us with options 1 and 3

Option 1 has a value of 1.92 approximately while option 3 has a value of 1.81 approximately.

At this point you need to make a decision about how much value the remaining terms of the series would add to 1.76 (sum of the first 5 terms)

Looking at the pattern we can predict that the sixth term will be

$36/7^5$ = 36/16807 = 0.002 (approx.)

And the seventh term would be $49/7^6$ = 49/117649 = 0.0004 (approx.).

The eighth term will obviously become much smaller.

It can be clearly visualized that the residual terms in the series are highly insignificant. Based on this judgement you realize that the answer will not reach 1.92 and will be restricted to 1.81. Hence the answer will be option 3.

Try using this process to solve other questions of this nature whenever you come across them. (There are a few such questions inserted in the LOD exercises of this chapter)

Useful Results

1. If the same quantity be added to, or subtracted from, all the terms of an AP, the resulting terms will form an *AP*, but with the same common difference as before.
2. If all the terms of an AP be multiplied or divided by the same quantity, the resulting terms will form an AP, but with a new common difference, which will be the multiplication/division of the old common difference. (as the case may be)
3. If all the terms of a GP be multiplied or divided by the same quantity, the resulting terms will form a GP with the same common ratio as before.
4. If a, b, c, d, …. are in GP, they are also in continued proportion, since, by definition,

   $a/b = b/c = c/d = \ldots = 1/r$

   Conversely, a series of quantities in continued proportion may be represented by x, xr, xr^2,…
5. If you have to assume 3 terms in *AP*, assume them as

   $a - d$, a, $a + d$ or as a, $a + d$ and $a + 2d$

   For assuming 4 terms of an AP we use: $a - 3d$, $a - d$, $a + d$ and $a + 3d$

   For assuming 5 terms of an *AP*, take them as:

   $a - 2d$, $a - d$, a, $a + d$, $a + 2d$.

   These are the most convenient in terms of problem solving.
6. For assuming three terms of a GP assume them as

   a, ar and ar^2 or as a/r, a and ar
7. To find the sum of the first n natural numbers

   Let the sum be denoted by S; then

   $S = 1 + 2 + 3 + \ldots + n$, is given by

   $S = \dfrac{n(n+1)}{2}$
8. To find the sum of the squares of the first n natural numbers

   Let the sum be denoted by S; then

$S = 1^2 + 2^2 + 3^2 + \ldots.. + n^2$

This is given by : $S = \left\{\frac{n(n+1)(2n+1)}{6}\right\}$

9. To find the sum of the cubes of the first *n* natural numbers.

   Let the sum be denoted by *S*; then

   $S = 1^3 + 2^3 + 3^3 + \ldots + n^3$

   $S = \left[\frac{n(n+1)}{2}\right]^2$

   Thus, the sum of the cubes of the first *n* natural numbers is equal to the square of the sum of these numbers.

10. To find the sum of the first *n* odd natural numbers.

    $S = 1 + 3 + 5 + \ldots + (2n - 1)$ Æ n^2

11. To find the sum of the first *n* even natural numbers.

    $S = 2 + 4 + 6 + \ldots + 2n$ Æ $n(n + 1)$

    $= n^2 + n$

12. To find the sum of odd numbers £ *n* where *n* is a natural number:

    Case A: If *n* is odd Æ $[(n + 1)/2]^2$

    Case B: If *n* is even Æ $[n/2]^2$

13. To find the sum of even numbers £ *n* where *n* is a natural number:

    Case A: If *n* is odd Æ $\{(n/2)[(n/2) + 1]\}$

    Case B: If *n* is even Æ $[(n - 1)/2][(n + 1)/2]$

14. Number of terms in a count:

    - If we are counting in steps of 1 from n_1 to n_2 including both the end points, we get $(n_2 - n_1) + 1$ numbers.
    - If we are counting in steps of 1 from n_1 to n_2 including only one end, we get $(n_2 - n_1)$ numbers.
    - If we are counting in steps of 1 from n_1 to n_2 excluding both ends, we get $(n_2 - n_1) - 1$ numbers.

Example: Between 16 and 25 both included there are 9 + 1 = 10 numbers.

Between 100 and 200 both excluded there are 100 – 1 = 99 numbers.

- If we are counting in steps of 2 from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/2] + 1$ numbers.
- If we are counting in steps of 2 from n_1 to n_2 including only one end, we get $[(n_2 - n_1)/2]$ numbers.
- If we are counting in steps of 2 from n_1 to n_2 excluding both ends, we get $[(n_2 - n_1)/2] - 1$ numbers.
- If we are counting in steps of 3 from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/3] + 1$ numbers.
- If we are counting in steps of 3 from n_1 to n_2 including only one end, we get $[(n_2 - n_1)/3]$ numbers.
- If we are counting in steps of 3 from n_1 to n_2, excluding both ends, we get $[(n_2 - n_1)/3] - 1$ numbers.

Example: Number of numbers between 100 and 200 divisible by three.

Solution: The first number is 102 and the last number is 198. Hence, answer = (96/3) + 1 = 33 (since both 102 and 198 are included).

Alternately, highest number below 100 that is divisible by 3 is 99, and the lowest number above 200 which is divisible by 3 is 201.

Hence, 201 – 99 = 102 Æ 102/3 = 34 Æ Answer = 34 – 1 = 33 (Since both ends are not included.)

In General

- If we are counting in steps of x from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/x] + 1$ numbers.
- If we are counting in steps of "x" from n_1 to n_2 including only one end, we get $(n_2 - n_1)/x$ numbers.

- If we are counting in steps of "x" from n_1 to n_2 excluding both ends, we get $[(n_2 - n_1)/x] - 1$ numbers.

  For instance, if we have to find how many terms are there in the series 107, 114, 121, 128 ... 254, then we have

  (254 – 107)/7 + 1 = 147/7 + 1 = 21 + 1 = 22 terms in the series

  Of course, an appropriate adjustment will have to be made when n_2 does not fall into the series. This will be done as follows:

  For instance, if we have to find how many terms of the series 107, 114, 121, 128 ... are below 258, then we have by the formula:

  (258 – 107)/7 + 1 = 151/7 + 1 = 21.57 + 1. = 22.57. This will be adjusted by taking the lower integral value = 22. Æ The number of terms in the series below 258.

The student is advised to try and experiment on these principles to get a clear picture.

WORKED-OUT PROBLEMS

Problem 2.1 Two persons—Ramu Dhobi and Kalu Mochi have joined Donkey-work Associates. Ramu Dhobi and Kalu Mochi started with an initial salary of ` 500 and ` 640 respectively with annual increments of ` 25 and ` 20 each respectively. In which year will Ramu Dhobi start earning more salary than Kalu Mochi?

Solution The current difference between the salaries of the two is ` 140. The annual rate of reduction of this dfference is ` 5 per year. At this rate, it will take Ramu Dhobi 28 years to equalise his salary with Kalu Dhobi's salary.

Thus, in the 29th year he will earn more.

This problem should be solved while reading and the thought process should be 140/5 = 28. Hence, answer is 29th year.

Problem 2.2 Find the value of the expression

1 – 6 + 2 – 7 + 3 – 8 +……… to 100 terms

(a) –250 (b) –500

(c) –450 (d) –300

Solution The series (1 – 6 + 2 – 7 + 3 – 8 +……… to 100 terms) can be rewritten as:

fi (1 + 2 + 3 + … to 50 terms) – (6 + 7 + 8 + … to 50 terms)

Both these are *AP*'s with values of *a* and *d* as Æ

a = 1, n = 50 and d = 1 and a = 6, n = 50 and d = 1 respectively.

Using the formula for sum of an *AP* we get:

Æ 25(2 + 49) – 25(12 + 49)

Æ 25(51 – 61) = –250

Alternatively, we can do this faster by considering (1 – 6), (2 – 7), and so on as one unit or one term.

$1 - 6 = 2 - 7 = \ldots = -5$. Thus the above series is equivalent to a series of fifty –5's added to each other.

So, $(1 - 6) + (2 - 7) + (3 - 8) + \ldots$ 50 terms $= -5 \times 50 =$ **–250**

Problem 2.3 Find the sum of all numbers divisible by 6 in between 100 to 400.

Solution Here 1st term $= a = 102$ (which is the 1st term greater than 100 that is divisible by 6.)

The last term less than 400, which is divisible by 6 is 396.

The number of terms in the *AP;* 102, 108, 114…396 is given by $[(396 - 102)/6] + 1 = 50$ numbers.

Common difference $= d = 6$

So, $S = 25\,(204 + 294) = 12450$

Problem 2.4 If x, y, z are in *GP*, then $1/(1 + \log_{10}x)$, $1/\,(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ will be in:

(a) AP (b) GP

(c) HP (d) Cannot be said

Solution Go through the options.

Checking option (a), the three will be in AP if the 2nd expression is the average of the 1st and 3rd expressions. This can be mathematically written as

$2/(1 + \log_{10}y) = [1/(1 + \log_{10}x)] + [1/(1 + \log_{10}z)]$

$$= \frac{[1 + (1 + \log_{10} x) + 1 + (1 + \log_{10} z)]}{[(1 + \log_{10} x)(1 + \log_{10} z)}$$

$$= \frac{[2 + \log_{10} xz]}{(1 + \log_{10} x)\,(1 + \log_{10} z)}$$

Applying our judgement, there seems to be no indication that we are going to get a solution.

Checking option (b)

$[1/(1 + \log_{10}y)]^2 = [1/(1 + \log_{10}x)]\,[1/(1 + \log_{10}z)]$

$= [1/(1 + \log_{10}(x + z) + \log_{10}xz)]$

Again we are trapped and any solution is not in sight.

Checking option (c).

$1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ are in *HP* then $1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ will be in AP.

So, $\log_{10}x$, $\log_{10}y$ and $\log_{10}z$ will also be in AP.

Hence, $2 \log_{10}y = \log_{10}x + \log_{10}z$

fi$y^2 = xz$ which is given.

So, **(c)** is the correct option.

Alternatively, you could have solved through the following process.

x, y and z are given as logarithmic functions.

Assume$x = 1$, $y = 10$ and $z = 100$ as x, y, z are in GP

So,$1 + \log_{10}x = 1$, $1 + \log_{10}y = 2$ and $1 + \log_{10}z = 3$

fi Thus we find that since 1, 2 and 3 are in AP, we can assume that

$1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ are in AP

fi Hence, by definition of an HP we have that $1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ are in *HP*. Hence, option (c) is the required answer.

Author's Note: In my experience I have always found that the toughest equations and factorisations get solved very easily when there are options, by assuming values in place of the variables in the equation. The values of the variables should be taken in such a manner that the basic restrictions put on the variables should be respected. For example, if an expression in three variables a, b and c is given and it is mentioned that $a + b + c = 0$ then the values that you assume for a, b and c should satisfy this restriction. Hence, you should look at values like 1, 2 and –3 or 2, –1, –1 etc.

This process is especially useful in the case where the question as well as the options both contain expressions. Factorisation and advanced techniques of maths are then not required. This process will be very beneficial for students who are weak at Mathematics.

Problem 2.5 Find t_{10} and S_{10} for the following series:

1, 8, 15,…

Solution This is an AP with first term 1 and common difference 7.

$t_{10} = a + (n - 1)\, d = 1 + 9 \times 7 = 64$

$S_{10} = \dfrac{n\,[2a + (n-1)d]}{2}$

$= \dfrac{10\,[21 + (10-1)7]}{2} = 325$

Alternatively, if the number of terms is small, you can count it directly.

Problem 2.6 Find t_{18} and S_{18} for the following series:

2, 8, 32, …

Solution This is a GP with first term 2 and common ratio 4.

$t_{18} = ar^{n-1} = 2.4^{17}$

$S_{18} = \dfrac{a(r^n - 1)}{r - 1} = \dfrac{2(4^{18} - 1)}{(4-1)}$

Problem 2.7 Is the series 1, 4,… to n terms an *AP*, or a GP, or an HP, or a series which cannot be determined?

Solution *To determine any progression, we should have at least three terms*.

If the series is an *AP* then the next term of this series will be 7

Again, if the next term is 16, then this will be a GP series (1, 4, 16 …)

So, *we cannot determine* the nature of the progression of this series.

Problem 2.8 Find the sum to 200 terms of the series

1 + 4 + 6 + 5 + 11 + 6 +…

(a) 30,200 (b) 29,800

(c) 30,200 (d) None of these

Solution Spot that the above series is a combination of two *AP*s.

The 1st *AP* is (1 + 6 + 11 +…) and the 2nd *AP* is (4 + 5 + 6 +…)

Since the terms of the two series alternate, *S* = (1 + 6 + 11 +… to 100 terms) + (4 + 5 + 6 + … to 100 terms)

= $\frac{100[2 \times 1 + 99 \times 5]}{2} + \frac{100[2 \times 4 + 99 \times 1]}{2}$ Æ (Using the formula for the sum of an *AP*)

= 50[497 + 107] = 50[604] = 30200

Alternatively, we can treat every two consecutive terms as one.

So we will have a total of 100 terms of the nature:

(1 + 4) + (6 + 5) + (11 + 6) … Æ 5, 11, 17…

Now, a = 5, d = 6 and n = 100

Hence the sum of the given series is

$S = \frac{100}{2} \times [2 \times 5 + 99 \times 6]$

= 50[604] = 30200

Problem 2.9 How many terms of the series –12, –9, –6,… must be taken that the sum may be 54?

Solution Here S = 54, a = –12, d = 3, n is unknown and has to be calculated. To do so we use the formula for the sum of an *AP* and get.

$54 = \frac{[2(-12) + (n-1)3]n}{2}$

or $108 = -24n - 3n + 3n^2$ or $3n^2 - 27n - 108 = 0$

or $n^2 - 9n - 36 = 0$, or $n^2 - 12n + 3n - 36 = 0$

$n(n - 12) + 3(n - 12) = 0$ fi $(n + 3)(n - 12) = 0$

The value of n (the number of terms) cannot be negative. Hence –3 is rejected.

So we have $n = 12$

Alternatively, we can directly add up individual terms and keep adding manually till we get a sum of 54. We will observe that this will occur after adding 12 terms. (In this case, as also in all cases where the number of terms is mentally manageable, mentally adding the terms till we get the required sum will turn out to be much faster than the equation based process.

Problem 2.10 Find the sum of n terms of the series 1.2.4 + 2.3.5 + 3.4.6 + …

(a) $n(n + 1)(n + 2)$

(b) $(n(n + 1)/12)(3n^2 + 19n + 26)$

(c) $((n + 1)(n + 2)(n + 3))/4$

(d) $(n^2 (n + 1)(n + 2)(n + 3))/3$

Solution In order to solve such problems in the examination, the option-based approach is the best. Even if you can find out the required expression mathematically, it is advisable to solve through the options as this will end up saving a lot of time for you. Use the options as follows:

If we put $n = 1$, we should get the sum as 1.2.4 = 8. By substituting $n = 1$ in each of the four options we will get the following values for the sum to 1 term:

Option (a) gives a value of: 6

Option (b) gives a value of: 8

Option (c) gives a value of: 6

Option (d) gives a value of: 8

From this check we can reject the options (a) and (c).

Now put $n = 2$. You can see that up to 2 terms, the expression is 1.2.4 + 2.3.5 = 38

The correct option should also give 38 if we put $n = 2$ in the expression. Since, (a) and (c) have already been rejected, we only need to check for options (b) and (d).

Option (b) gives a value of 38

Option (d) gives a value of 80.

Hence, we can reject option (d) and get (b) as the answer.

Note: **The above process is very effective for solving questions having options. The student should try to keep an eye open for the possibility**

of solving questions through options. In my opinion, approximately 50–75% of the questions asked in CAT in the QA section can be solved with options (at least partially).

LEVEL OF DIFFICULTY (I)

1. How many terms are there in the AP 20, 25, 30,… 130.

   (a) 22 (b) 23

   (c) 21 (d) 24

2. Bobby was appointed to Mindworkzz in the pay scale of ` 7000–500–12,500. Find how many years he will take to reach the maximum of the scale.

   (a) 11 years (b) 10 years

   (c) 9 years (d) 8 years

3. Find the 1st term of an AP whose 8th and 12th terms are respectively 39 and 59.

   (a) 5 (b) 6

   (c) 4 (d) 3

4. A number of squares are described whose perimetres are in GP. Then their sides will be in

   (a) AP (b) GP

   (c) HP (d) Nothing can be said

5. There is an *AP* 1, 3, 5…. Which term of this AP is 55?

   (a) 27th (b) 26th

   (c) 25th (d) 28th

6. How many terms are identical in the two *AP*s 1, 3, 5,… up to 120 terms and 3, 6, 9,… up to 80 terms?

   (a) 38 (b) 39

   (c) 40 (d) 41

7. Find the lowest number in an AP such that the sum of all the terms is 105 and greatest term is 6 times the least.

(a) 5 (b) 10

(c) 15 (d) (a), (b) & (c)

8. Find the 15th term of the sequence 20, 15, 10, …

(a) –45 (b) –55

(c) –50 (d) 0

9. A sum of money kept in a bank amounts to ` 1240 in 4 years and ` 1600 in 10 years at simple Interest. Find the sum.

(a) ` 800 (b) ` 900

(c) ` 1150 (d) ` 1000

10. A number 15 is divided into three parts which are in *AP* and the sum of their squares is 83. Find the smallest number.

(a) 5 (b) 3

(c) 6 (d) 8

11. The sum of the first 16 terms of an *AP* whose first term and third term are 5 and 15 respectively is

(a) 600 (b) 765

(c) 640 (d) 680

12. The number of terms of the series 54 + 51 + 48 +… such that the sum is 513 is

(a) 18 (b) 19

(c) Both *a* and *b* (d) 15

13. The least value of n for which the sum of the series 5 + 8 + 11… n terms is not less than 670 is

(a) 20 (b) 19

(c) 22 (d) 21

14. A man receives ` 60 for the first week and ` 3 more each week than the preceding week. How much does he earn by the 20th week?

(a) ` 1770 (b) ` 1620

(c) ` 1890 (d) ` 1790

15. How many terms are there in the GP 5, 20, 80, 320,… 20480?

(a) 6 (b) 5

(c) 7 (d) 8

16. A boy agrees to work at the rate of one rupee on the first day, two rupees on the second day, four rupees on the third day and so on. How much will the boy get if he starts working on the 1st of February and finishes on the 20th of February?

(a) 2^{20} (b) $2^{20} – 1$

(c) $2^{19} – 1$ (d) 2^{19}

17. If the fifth term of a GP is 81 and first term is 16, what will be the 4th term of the GP?

(a) 36 (b) 18

(c) 54 (d) 24

18. The seventh term of a GP is 8 times the fourth term. What will be the first term when its fifth term is 48?

(a) 4 (b) 3

(c) 5 (d) 2

19. The sum of three numbers in a GP is 14 and the sum of their squares is 84. Find the largest number.

(a) 8 (b) 6

(c) 4 (d) 12

20. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this AP?

(a) 4551 (b) 10091

(c) 7881 (d) 13531

21. How many natural numbers between 300 to 500 are multiples of 7?

(a) 29 (b) 28

(c) 27 (d) 30

22. The sum of the first and the third term of a geometric progression is 20 and the sum of its first three terms is 26. Find the progression.

(a) 2, 6, 18,… (b) 18, 6, 2,…

(c) Both of these (d) None of these

23. If a man saves ` 4 more each year than he did the year before and if he saves ` 20 in the first year, after how many years will his savings be more than ` 1000 altogether?

(a) 19 years (b) 20 years

(c) 21years (d) 18 years

24. A man's salary is ` 800 per month in the first year. He has joined in the scale of 800–40–1600. After how many years will his savings be ` 64,800?

(a) 8 years (b) 7 years

(c) 6 years (d) Cannot be determined

25. The 4th and 10th term of an GP are 1/3 and 243 respectively. Find the 2nd term.

(a) 3 (b) 1

(c) 1/27 (d) 1/9

26. The 7th and 21st terms of an AP are 6 and –22 respectively. Find the 26th term.

(a) –34 (b) –32

(c) –12 (d) –10

27. The sum of 5 numbers in AP is 30 and the sum of their squares is 220. Which of the following is the third term?

(a) 5 (b) 6

(c) 8 (d) 9

28. Find the sum of all numbers in between 10–50 excluding all those numbers which are divisible by 8. (include 10 and 50 for counting.)

(a) 1070 (b) 1220

(c) 1320 (d) 1160

29. The sum of the first four terms of an AP is 28 and sum of the first eight terms of the same AP is 88. Find the sum of the first 16 terms of the AP?

(a) 346 (b) 340

(c) 304 (d) 268

30. Find the general term of the GP with the third term 1 and the seventh term 8.

(a) $(2^{3/4})^{n-3}$ (b) $(2^{3/2})^{n-3}$

(c) $(2^{3/4})^{3-n}$ (d) $(2^{3/4})^{2-n}$

31. Find the number of terms of the series 1/81, –1/27, 1/9,… –729.

(a) 11 (b) 12

(c) 10 (d) 13

32. Four geometric means are inserted between 1/8 and 128. Find the third geometric mean.

(a) 4 (b) 16

(c) 32 (d) 8

33. *A* and *B* are two numbers whose *AM* is 25 and *GM* is 7. Which of the following may be a value of *A*?

(a) 10 (b) 20

(c) 49 (d) 25

34. Two numbers *A* and *B* are such that their *GM* is 20% lower than their *AM*. Find the ratio between the numbers.

(a) 3 : 2 (b) 4 : 1

(c) 2 : 1 (d) 3 : 1

35. A man saves ` 100 in January 2014 and increases his saving by ` 50 every month over the previous month. What is the annual saving for

the man in the year 2014?

(a) ` 4200 (b) ` 4500

(c) ` 4000 (d) ` 4100

36. In a nuclear power plant a technician is allowed an interval of maximum 100 minutes. A timer with a bell rings at specific intervals of time such that the minutes when the timer rings are not divisible by 2, 3, 5 and 7. The last alarm rings with a buzzer to give time for decontamination of the technician. How many times will the bell ring within these 100 minutes and what is the value of the last minute when the bell rings for the last time in a 100 minute shift?

(a) 25 times, 89 (b) 21 times, 97

(c) 22 times, 97 (d) 19 times, 97

37. How many zeroes will be there at the end of the expression $(2!)^{2!} + (4!)^{4!} + (8!)^{8!} + (9!)^{9!} + (10!)^{10!} + (11!)^{11!}$?

(a) $(8!)^{8!} + (9!)^{9!} + (10!)^{10!} + (11!)^{11!}$

(b) 10^{101} (c) 4! + 6! + 8! + 2 (10!)

(d) $(0!)^{0!}$

38. The 1st, 8th and 22nd terms of an AP are three consecutive terms of a GP. Find the common ratio of the GP, given that the sum of the first twenty-two terms of the AP is 385.

(a) Either 1 or1/2 (b) 2

(c) 1 (d) Either 1 or 2

39. The internal angles of a plane polygon are in AP. The smallest angle is 100^o and the common difference is 10^o. Find the number of sides of the polygon.

(a) 8 (b) 9

(c) Either 8 or 9 (d) None of these

40. After striking a floor a rubber ball rebounds $(7/8)^{th}$ of the height from which it has fallen. Find the total distance that it travels before coming to rest, if it is gently dropped from a height of 420 meters.

(a) 2940 (b) 6300

(c) 1080 (d) 3360

41. Each of the series 13 + 15 + 17+…. and 14 + 17 + 20+… is continued to 100 terms. Find how many terms are identical between the two series.

(a) 35 (b) 34

(c) 32 (d) 33

42. Jack and Jill were playing mathematical puzzles with each other. Jill drew a square of sides 8 cm and then kept on drawing squares inside the squares by joining the mid points of the squares. She continued this process indefinitely. Jill asked Jack to determine the sum of the areas of all the squares that she drew. If Jack answered correctly then what would be his answer?

(a) 128 (b) 64

(c) 256 (d) 32

43. How many terms of the series 1 + 3 + 5 + 7 + …… amount to 123454321?

(a) 11101 (b) 11011

(c) 10111 (d) 11111

44. A student takes a test consisting of 100 questions with differential marking is told that each question after the first is worth 4 marks more than the preceding question. If the third question of the test is worth 9 marks. What is the maximum score that the student can obtain by attempting 98 questions?

(a) 19698 (b) 19306

(c) 9900 (d) None of these

45. In an infinite geometric progression, each term is equal to 2 times the sum of the terms that follow. If the first term of the series is 8, find the sum of the series?

(a) 12 (b) 32/3

(c) 34/3 (d) Data inadequate

46. What is the maximum sum of the terms in the arithmetic progression 25, 24 ½ , 24, ………..?

(a) 637 ½ (b) 625

(c) 662 ½ (d) 650

47. An equilateral triangle is drawn by joining the midpoints of the sides of another equilateral triangle. A third equilateral triangle is drawn inside the second one joining the midpoints of the sides of the second equilateral triangle, and the process continues infinitely. Find the sum of the perimeters of all the equilateral triangles, if the side of the largest equilateral triangle is 24 units.

(a) 288 units (b) 72 units

(c) 36 units (d) 144 units

48. The sum of the first two terms of an infinite geometric series is 18. Also, each term of the series is seven times the sum of all the terms that follow. Find the first term and the common ratio of the series respectively.

(a) 16, 1/8 (b) 15, 1/5

(c) 12, 1/2 (d) 8, 1/16

49. Find the 33^{rd} term of the sequence: 3, 8, 9, 13, 15, 18, 21, 23…

(a) 93 (b) 99

(c) 105 (d) 83

50. For the above question, find the sum of the series till 33 terms.

(a) 728 (b) 860

(c) 1595 (d) 1583

LEVEL OF DIFFICULTY (II)

1. If a times the ath term of an AP is equal to b times the bth term, find the $(a + b)$th term.

   (a) 0 (b) $a^2 – b^2$

   (c) $a – b$ (d) 1

2. A number 20 is divided into four parts that are in AP such that the product of the first and fourth is to the product of the second and third is 2 : 3. Find the largest part.

   (a) 12 (b) 4

   (c) 8 (d) 9

3. Find the value of the expression: 1 – 4 + 5 – 8 … to 50 terms.

   (a) –150 (b) –75

   (c) –50 (d) 75

4. If a clock strikes once at one o’clock, twice at two o’clock and twelve times at 12 o’clock and again once at one o’clock and so on, how many times will the bell be struck in the course of 2 days?

   (a) 156 (b) 312

   (c) 78 (d) 288

5. What will be the maximum sum of 44, 42, 40, … ?

   (a) 502 (b) 504

   (c) 506 (d) 500

6. Find the sum of the integers between 1 and 200 that are multiples of 7.

   (a) 2742 (b) 2842

   (c) 2646 (d) 2546

7. If the mth term of an AP is $1/n$ and nth term is $1/m$, then find the sum to mn terms.

(a) $(mn - 1)/4$ (b) $(mn + 1)/4$

(c) $(mn + 1)/2$ (d) $(mn - 1)/2$

8. Find the sum of all odd numbers lying between 100 and 200.

(a) 7500 (b) 2450

(c) 2550 (d) 2650

9. Find the sum of all integers of 3 digits that are divisible by 7.

(a) 69,336 (b) 71,336

(c) 70,336 (d) 72,336

10. The first and the last terms of an AP are 107 and 253. If there are five terms in this sequence, find the sum of sequence.

(a) 1080 (b) 720

(c) 900 (d) 620

11. Find the value of $1 - 2 - 3 + 2 - 3 - 4 + \ldots +$ upto 100 terms.

(a) –694 (b) –626

(c) –624 (d) –549

12. What will be the sum to n terms of the series $8 + 88 + 888 + \ldots$?

(a) $\dfrac{8(10^n - 9n)}{81}$ (b) $\dfrac{8(10^{n+1} - 10 - 9n)}{81}$

(c) $8(10^{n-1} - 10)$ (d) $8(10^{n+1} - 10)$

13. If a, b, c are in GP, then $\log a$, $\log b$, $\log c$ are in

(a) AP (b) GP

(c) HP (d) None of these

14. After striking the floor, a rubber ball rebounds to 4/5th of the height from which it has fallen. Find the total distance that it travels before coming to rest if it has been gently dropped from a height of 120 metres.

(a) 540 metres (b) 960 metres

(c) 1080 metres (d) 1020 metres

15. If x be the first term, y be the nth term and p be the product of n terms of a GP, then the value of p^2 will be

(a) $(xy)^{n-1}$ (b) $(xy)^n$

(c) $(xy)^{1-n}$ (d) $(xy)^{n/2}$

16. The sum of an infinite GP whose common ratio is numerically less than 1 is 32 and the sum of the first two terms is 24. What will be the third term?

(a) 2 (b) 16

(c) 8 (d) 4

17. What will be the value of $x^{1/2}.x^{1/4}.x^{1/8}\ldots$ to infinity.

(a) x^2 (b) x

(c) $x^{3/2}$ (d) x^3

18. Find the sum to n terms of the series

1.2.3 + 2.3.4 + 3.4.5 +…

(a) $(n + 1)(n + 2)(n + 3)/3$

(b) $n(n + 1)(2n + 2)(n + 2)/4$

(c) $n(n + 1)(n + 2)$

(d) $n(n + 1)(n + 2)(n + 3)/4$

19. Determine the first term of the geometric progression, the sum of whose first term and third term is 40 and the sum of the second term and fourth term is 80.

(a) 12 (b) 16

(c) 8 (d) 4

20. Find the second term of an AP if the sum of its first five even terms is equal to 15 and the sum of the first three terms is equal to –3.

(a) –3 (b) –2

(c) –1 (d) 0

21. The sum of the second and the fifth term of an AP is 8 and that of the third and the seventh term is 14. Find the eleventh term.

(a) 19 (b) 17

(c) 15 (d) 16

22. How many terms of an AP must be taken for their sum to be equal to 120 if its third term is 9 and the difference between the seventh and the second term is 20?

(a) 6 (b) 9

(c) 7 (d) 8

23. Four numbers are inserted between the numbers 4 and 39 such that an AP results. Find the biggest of these four numbers.

(a) 31.5 (b) 31

(c) 32 (d) 30

24. Find the sum of all three-digit natural numbers, which on being divided by 5, leave a remainder equal to 4.

(a) 57,270 (b) 96,780

(c) 49,680 (d) 99,270

25. The sum of the first three terms of the arithmetic progression is 30 and the sum of the squares of the first term and the second term of the same progression is 116. Find the seventh term of the progression if its fifth term is known to be exactly divisible by 14.

(a) 36 (b) 40

(c) 43 (d) 22

26. *A* and *B* set out to meet each other from two places 165 km apart. *A* travels 15 km the first day, 14 km the second day, 13 km the third day and so on. *B* travels 10 km the first day, 12 km the second day, 14 km the third day and so on. After how many days will they meet?

(a) 8 days (b) 5 days

(c) 6 days (d) 7 days

27. If a man saves ` 1000 each year and invests at the end of the year at 5% compound interest, how much will the amount be at the end of 15 years?

(a) ` 21,478 (b) ` 21,578

(c) ` 22,578 (d) ` 22,478

28. If sum to *n* terms of a series is given by (*n* + 8), then its second term will be given by

(a) 10 (b) 9

(c) 8 (d) 1

29. If *A* is the sum of the *n* terms of the series 1 + 1/4 + 1/16 + … and *B* is the sum of 2*n* terms of the series 1 + 1/2 + 1/4 + …, then find the value of *A*/*B*.

(a) 1/3 (b) 1/2

(c) 2/3 (d) 3/4

30. A man receives a pension starting with ` 100 for the first year. Each year he receives 90% of what he received the previous year. Find the maximum total amount he can receive even if he lives forever.

(a) ` 1100 (b) ` 1000

(c) ` 1200 (d) ` 900

31. The sum of the series represented as:

1/1 × 5 + 1/5 × 9 + 1/9 × 13 ….. + 1/221 × 225 is

(a) 28/221 (b) 56/221

(c) 56/225 (d) None of these

32. The sum of the series

1/($\sqrt{2}$ + $\sqrt{1}$) + 1/($\sqrt{2}$ + $\sqrt{3}$) +……………+ 1/($\sqrt{120}$ + $\sqrt{121}$) is:

(a) 10 (b) 11

(c) 12 (d) None of these

33. Find the infinite sum of the series 1/1 + 1/3 + 1/6 + 1/10 + 1/15 ….

(a) 2 (b) 2.25

(c) 3 (d) 4

34. The sum of the series 5 × 8 + 8 × 11 + 11 × 14 upto n terms will be:

(a) $(n + 1)[3(n + 1)^2 + 6(n + 1) + 1] – 10$

(b) $(n + 1)[3(n + 1)^2 + 6(n + 1) + 1] + 10$

(c) $(n + 1)[3(n + 1) + 6(n + 1)^2 + 1] – 10$

(d) $(n + 1)[3(n + 1) + 6(n + 1)^2 + 1] + 10$

35. The sum of the series: ½ + 1/6 + 1/12 + 1/20 + …1/156 + 1/182 is:

(a) 12/13 (b) 13/14

(c) 14/13 (d) None of these

36. For the above question 35, what is the sum of the series if taken to infinite terms:

(a) 1.1 (b) 1

(c) 14/13 (d) None of these

Directions for Questions 37 to 39: Answer these questions based on the following information.

There are 250 integers a_1, a_2…….a_{250}, not all of them necessarily different. Let the greatest integer of these 250 integers be referred to as Max, and the smallest integer be referred to as Min. The integers a_1 through a_{124} form sequence A, and the rest form sequence B. Each member of A is less than or equal to each member of B.

37. All values in A are changed in sign, while those in B remain unchanged. Which of the following statements is true?

(a) Every member of A is greater than or equal to every member of B.

(b) Max is in A.

(c) If all numbers originally in A and B had the same sign, then after the change of sign, the largest number of A and B is in A.

(d) None of these

38. Elements of A are in ascending order, and those of B are in descending order. a_{124} and a_{125} are interchanged. Then which of the following statements is true?

(a) A continues to be in ascending order.

(b) B continues to be in descending order

(c) A continues to be in ascending order and B in descending order

(d) None of the above

39. Every element of A is made greater than or equal to every element of B by adding to each element of A an integer x. Then, x cannot be less than:

(a) 2^{10}

(b) the smallest value of B

(c) the largest value of B

(d) (Max-Min)

40. Rohit drew a rectangular grid of 529 cells, arranged in 23 Rows and 23 columns, and filled each cell with a number. The numbers with which he filled each cell were such that the numbers of each row taken from left to right formed an arithmetic series and the numbers of each column taken from top to bottom also formed an arithmetic series. The seventh and the seventeenth numbers of the fifth row were 47 and 63 respectively, while the seventh and the seventeenth numbers of the fifteenth row were 53 and 77 respectively. What is the sum of all the numbers in the grid?

(a) 32798 (b) 65596

(c) 52900 (d) None of these

41. How many three digit numbers have the property that their digits taken from left to right form an Arithmetic or a Geometric

Progression?

(a) 15 (b) 36

(c) 20 (d) 42

Directions for Questions 42 and 43: These questions are based on the following data.

At Burger King—a famous fast food centre on Main Street in Pune, burgers are made only on an automatic burger making machine. The machine continuously makes different sorts of burgers by adding different sorts of fillings on a common bread. The machine makes the burgers at the rate of 1 burger per half a minute. The various fillings are added to the burgers in the following manner. The 1st, 5th, 9th, ... burgers are filled with a chicken patty; the 2nd, 9th, 16th, ... burgers with vegetable patty; the 1st, 5th, 9th, ... burgers with mushroom patty; and the rest with plain cheese and tomato fillings.

The machine makes exactly 660 burgers per day.

42. How many burgers per day are made with cheese and tomato as fillings?

(a) 424 (b) 236

(c) 237 (d) None of these

43. How many burgers are made with all three fillings Chicken, vegetable and mushroom?

(a) 23 (b) 24

(c) 25 (d) 26

44. An arithmetic progression P consists of n terms. From the progression three different progressions P_1 P_2 and P_3 are created such that P_1 is obtained by the 1st, 4th ,7th...... terms of P, P_2 has the 2nd, 5th, 8th,......terms of P and P_3 has the 3rd, 6th, 9th, ...terms of P. It is found that of P_1, P_2 and P_3 two progressions have the property that their average is itself a term of the original Progression P. Which of the following can be a possible value of n?

(a) 20 (b) 26

(c) 36 (d) Both (a)and (b)

45. For the above question, if the Common Difference between the terms of P_1 is 6, what is the common difference of P?

(a) 2 (b) 3

(c) 6 (d) Cannot be determined

LEVEL OF DIFFICULTY (III)

1. If in any decreasing arithmetic progression, sum of all its terms, except for the first term, is equal to –36, the sum of all its terms, except for the last term, is zero, and the difference of the tenth and the sixth term is equal to – 16, then what will be first term of this series?

   (a) 16 (b) 20
   (b) –16 (d) –20

2. The sum of all terms of the arithmetic progression having ten terms except for the first term, is 99, and except for the sixth term, 89. Find the third term of the progression if the sum of the first and the fifth term is equal to 10.

   (a) 15 (b) 5
   (c) 8 (d) 10

3. Product of the fourth term and the fifth term of an arithmetic progression is 456. Division of the ninth term by the fourth term of the progression gives quotient as 11 and the remainder as 10. Find the first term of the progression.

   (a) – 52 (b) – 42
   (c) – 56 (d) – 66

4. A number of saplings are lying at a place by the side of a straight road. These are to be planted in a straight line at a distance interval of 10 meters between two consecutive saplings. Mithilesh, the country's greatest forester, can carry only one sapling at a time and has to move back to the original point to get the next sapling. In this manner he covers a total distance of 1.32 kms. How many saplings does he plant in the process if he ends at the starting point?

   (a) 15 (b) 14
   (c) 13 (d) 12

5. A geometric progression consists of 500 terms. Sum of the terms occupying the odd places is P_1 and the sum of the terms occupying the even places is P_2. Find the common ratio.

(a) P_2/P_1 (b) P_1/P_2

(c) $P_2 + P_1/P_1$ (d) $P_2 + P_1/P_2$

6. The sum of the first ten terms of the geometric progression is S_1 and the sum of the next ten terms (11th through 20th) is S_2. Find the common ratio.

(a) $(S_1/S_2)^{1/10}$ (b) $-(S_1/S_2)^{1/10}$

(c) $\pm \sqrt[10]{S_2/S_1}$ (d) $(S_1/S_2)^{1/5}$

7. The first and the third terms of an arithmetic progression are equal, respectively, to the first and the third term of a geometric progression, and the second term of the arithmetic progression exceeds the second term of the geometric progression by 0.25. Calculate the sum of the first five terms of the arithmetic progression if its first term is equal to 2.

(a) 2.25 or 25 (b) 2.5 or 27.5

(c) 1.5 (d) 3.25

8. If (2 + 4 + 6 +… 50 terms)/(1 + 3 + 5 +… n terms) = 51/2, then find the value of n .

(a) 12 (b) 13

(c) 9 (d) 10

9. (666…. n digits)^2 + (888… n digits) is equal to

(a) $(10^n - 1) \times \frac{4}{9}$ (b) $(10^{2n} - 1) \times \frac{4}{9}$

(c) $\frac{4(10^n - 10^{n-1} - 1)}{9}$ (d) $\frac{4(10^n + 1)}{9}$

10. The interior angles of a polygon are in *AP*. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.

(a) 7 (b) 8

(c) 9 (d) 10

11. Find the sum to *n* terms of the series 11 + 103 + 1005 + …

(a) $\frac{10(10^n - 1)}{9} + 1$ (b) $\frac{10(10^n - 1)}{9} + n$

(c) $\frac{10(10^n - 1)}{9} + n^2$ (d) $\frac{10(10^n + 1)}{11} + n^2$

12. The sum of the first term and the fifth term of an *AP* is 26 and the product of the second term by the fourth term is 160. Find the sum of the first seven terms of this *AP*.

(a) 110 (b) 114

(c) 112 (d) 116

13. The sum of the third and the ninth term of an *AP* is 10. Find a possible sum of the first 11 terms of this *AP*.

(a) 55 (b) 44

(c) 66 (d) 48

14. The sum of the squares of the fifth and the eleventh term of an *AP* is 3 and the product of the second and the fourteenth term is equal it *P*. Find the product of the first and the fifteenth term of the *AP*.

(a) (58*P* – 39)/45 (b) (98*P* + 39)/72

(c) (116*P* – 39)/90 (d) (98*P* + 39)/90

15. If the ratio of harmonic mean of two numbers to their geometric mean is 12 : 13, find the ratio of the numbers.

(a) 4/9 or 9/4 (b) 2/3 or 3/2

(c) 2/5 or 5/2 (d) 3/4 or 4/5

16. Find the sum of the series $1.2 + 2.2^2 + 3.2^3 + \ldots + 100.\ 2^{100}$.

(a) $100.2^{101} + 2$ (b) $99.2^{100} + 2$

(c) $99.2^{101} + 2$ (d) None of these

17. The sequence $[x_n]$ is a *GP* with $x_2/x_4 = 1/4$ and $x_1 + x_4 = 108$. What will be the value of x_3?

(a) 42 (b) 48

(c) 44 (d) 56

18. If x, y, z are in GP and a^x, b^y and c^z are equal, then a, b, c are in

(a) *AP* (b) *GP*

(c) *HP* (d) None of these

19. Find the sum of all possible whole number divisors of 720.

(a) 2012 (b) 2624

(c) 2210 (d) 2418

20. Sum to n terms of the series $\log m + \log m^2/n + \log m^3/n^2 + \log m^4/n^3 \ldots$ is

(a) $\log\left(\frac{m^{n+1}}{n^{n-1}}\right)^{\frac{n}{2}}$ (b) $\log\left(\frac{n^{n-1}}{m^{n+1}}\right)^{\frac{n}{2}}$

(c) $\log\left(\frac{m^n}{n^n}\right)^{\frac{n}{2}}$ (d) $\log\left(\frac{m^{1-n}}{n^{1-m}}\right)^{\frac{n}{2}}$

21. The sum of first 20 and first 50 terms of an *AP* is 20 and 2550. Find the eleventh term of a *GP* whose first term is the same as the *AP* and the common ratio of the *GP* is equal to the common difference of the *AP*.

(a) 560 (b) 512

(c) 1024 (d) 3072

22. If three positive real numbers x, y, z are in *AP* such that $xyz = 4$, then what will be the minimum value of y?

(a) $2^{1/3}$ (b) $2^{2/3}$

(c) $2^{1/4}$ (d) $2^{3/4}$

23. If a_n be the nth term of an AP and if $a_7 = 15$, then the value of the common difference that would make $a_2a_7a_{12}$ greatest is

(a) 3 (b) 3/2

(c) 7 (d) 0

24. If $a_1, a_2, a_3 \ldots a_n$ are in AP, where $a_i > 0$, then what will be the value of the expression

$1/(\sqrt{a_1} + \sqrt{a_2}) + 1/(\sqrt{a_2} + \sqrt{a_3}) + 1/(\sqrt{a_3} + \sqrt{a_4}) + \ldots$ to n terms?

(a) $(1 - n)/(\sqrt{a_1} + \sqrt{a_n})$

(b) $(n - 1)/(\sqrt{a_1} + \sqrt{a_n})$

(c) $(n - 1)/(\sqrt{a_1} - \sqrt{a_n})$

(d) $(1 - n)/(\sqrt{a_1} + \sqrt{a_n})$

25. If the first two terms of a HP are 2/5 and 12/13 respectively, which of the following terms is the largest term?

(a) 4th term (b) 5th term

(c) 6th term (d) 7th term

26. One side of a staircase is to be closed in by rectangular planks from the floor to each step. The width of each plank is 9 inches and their height are successively 6 inches, 12 inches, 18 inches and so on. There are 24 planks required in total. Find the area in square feet.

(a) 112.5 (b) 107

(c) 118.5 (d) 105

27. The middle points of the sides of a triangle are joined forming a second triangle. Again a third triangle is formed by joining the middle points of this second triangle and this process is repeated

infinitely. If the perimetre and area of the outer triangle are P and A respectively, what will be the sum of perimetres of triangles thus formed?

(a) $2P$ (b) P^2

(c) $3P$ (d) $p^2/2$

28. In Problem 27, find the sum of areas of all the triangles.

(a) $\frac{4}{5}$ A (b) $\frac{4}{3}$ A

(c) $\frac{3}{4}$ A (d) $\frac{5}{4}$ A

29. A square has a side of 40 cm. Another square is formed by joining the mid-points of the sides of the given square and this process is repeated infinitely. Find the perimetre of all the squares thus formed.

(a) $160 (1 + \sqrt{2})$ (b) $160 (2 + \sqrt{2})$

(c) $160 (2 - \sqrt{2})$ (d) $160 (1 - \sqrt{2})$

30. In problem 29, find the area of all the squares thus formed.

(a) 1600 (b) 2400

(c) 2800 (d) 3200

31. The sum of the first n terms of the arithmetic progression is equal to half the sum of the next n terms of the same progression. Find the ratio of the sum of the first $3n$ terms of the progression to the sum of its first n terms.

(a) 5 (b) 6

(c) 7 (d) 8

32. In a certain colony of cancerous cells, each cell breaks into two new cells every hour. If there is a single productive cell at the start and this process continues for 9 hours, how many cells will the colony

have at the end of 9 hours? It is known that the life of an individual cell is 20 hours.

(a) $2^9 - 1$ (b) 2^{10}

(c) 2^9 (d) $2^{10} - 1$

33. Find the sum of all three-digit whole numbers less than 500 that leave a remainder of 2 when they are divided by 3.

(a) 49637 (b) 39767

(c) 49634 (d) 39770

34. If a be the arithmetic mean and b, c be the two geometric means between any two positive numbers, then $(b^3 + c^3)/abc$ equals

(a) $(ab)^{1/2}/c$ (b) 1

(c) a^2c/b (d) None of these

35. If p, q, r are three consecutive distinct natural numbers then the expression $(q + r - p)(p + r - q)(p + q - r)$ is

(a) Positive (b) Negative

(c) Non-positive (d) Non-negative

ANSWER KEY

Level of Difficulty (I)

1. (b)	2. (a)	3. (c)	4. (b)
5. (d)	6. (c)	7. (d)	8. (c)
9. (d)	10. (b)	11. (d)	12. (c)
13. (a)	14. (a)	15. (c)	16. (b)
17. (c)	18. (b)	19. (a)	20. (c)
21. (a)	22. (c)	23. (a)	24. (d)
25. (c)	26. (b)	27. (b)	28. (a)
29. (c)	30. (a)	31. (a)	32. (d)
33. (c)	34. (b)	35. (b)	36. (c)
37. (d)	38. (b)	39. (c)	40. (b)
41. (d)	42. (a)	43. (d)	44. (d)

45. (a)	46. (a)	47. (d)	48. (a)
49. (b)	50. (c)		

Level of Difficulty (II)

1. (a)	2. (c)	3. (b)	4. (b)
5. (c)	6. (b)	7. (c)	8. (a)
9. (c)	10. (c)	11. (b)	12. (b)
13. (a)	14. (c)	15. (b)	16. (d)
17. (b)	18. (d)	19. (c)	20. (c)
21. (a)	22. (d)	23. (c)	24. (d)
25. (b)	26. (c)	27. (b)	28. (d)
29. (c)	30. (b)	31. (c)	32. (a)
33. (a)	34. (a)	35. (b)	36. (b)
37. (d)	38. (a)	39. (d)	40. (a)
41. (d)	42. (a)	43. (b)	44. (d)
45. (a)			

Level of Difficulty (III)

1. (a)	2. (b)	3. (d)	4. (d)
5. (a)	6. (c)	7. (b)	8. (d)
9. (b)	10. (c)	11. (c)	12. (c)
13. (a)	14. (d)	15. (d)	16. (d)
17. (b)	18. (b)	19. (d)	20. (a)
21. (d)	22. (b)	23. (d)	24. (b)
25. (d)	26. (a)	27. (a)	28. (b)
29. (b)	30. (d)	31. (b)	32. (d)
33. (b)	34. (d)	35. (d)	

Hints

Level of Difficulty (II)

2. $a + (a + d) + (a + 2d) + (a + 3d) = 20$
   and $a(a + 3d) = (a + d)(a + 2d)$
4. Calculate the sum of an AP with first term 1, common difference 1 and last term 12. Multiply this sum by 4 for 2 days.

5. The maximum sum will occur when the last term is either 2 or 0.
6. Visualise the AP as 7, 14…196.
9. The AP is 105, 112 …994.
10. The common difference is $\frac{146}{5}$ = 29.2.
11. See the terms of the series in 33 blocks of 3 each. This will give the AP – 4, –5, –6…–33. Further, the hundredth term will be 34.
12. Solve through options.
14. The first drop is 120 metres. After this the ball will rise by 96 metres and fall by 96 meters. This process will continue in the form of an infinite GP with common ratio 0.8 and first term 96.

    The required answer will be got by

    120 + 96 * 1.25 * 2
15. Take any GP and solve by using values.
18. Solve by using values to check options.
22. The difference between the seventh and third term is given by

    $(a + 6d) – (a + d)$
23. $\frac{(39 - 4)}{5}$ = 7.
27. The required answer will be by adding 20 terms of the GP starting with the first term as 1000 and the common ratio as 1.05.
30. Visualise it as an infinite GP with common ratio 0.9.

Level of Difficulty (III)

1. Difference between the tenth and the sixth term = –16

    or $(a + 9d) – (a + 5d) = –16$

    Æ $d = –4$
2. Sum of the first term and the fifth term = 10

    or $a + a + 4d = 10$

    or $a + 2d = 5$ (1)

    and, the sum of all terms of the AP except for the 1st term = 99 or $9a + 45d = 99$

$a + 5d = 11$ (2)

Solve (1) and (2) to get the answer.

3. The second statement gives the equation as $a + 8d = 2(a + 3d) + 6$

   or $a - 2d = 6$

   Now, use the options to find the value of d, and put these values to check the equation obtained from the first statement.

   i.e. $(a + 2d)(a + 5d) = 406$

4. To plant the 1st sapling, Mithilesh will cover 20 m; to plant the 2nd sapling he will cover 40 m and so on. But for the last sapling, he will cover only the distance from the starting point to the place where the sapling has to be planted.

5. Assume a series having a few number of terms e.g.

   1, 2, 4, 8, 16, 32…

   Now sum of all the terms at the even places = 42 (P_2)

   and sum of all the terms at the odd places = 21 (P_1)

   common ratio of this series = $\frac{42}{21} = 2 = P_2/P_1$.

6. Use the same process as illustrated above.

7. Check the options by putting $n = 1, 2, \ldots$ and then equate it with the original equation given in question.

9. For 1 term, the value should be:

   $6^2 + 8 = 44$

   Only option (b) gives 44 for $n = 1$

10. Sum of the AP for n sides = Sum of interior angles of a polygon of n sides.

    $$\frac{n}{2} \times (2a + (n - 1)d) = (2n - 4) \times 90$$

    where $a = 120°$ and $d = 5°$.

11. Solve using options to check for the correct answer.

12. $a + (a + 4d) = 26$ and $(a + d)(a + 3d) = 160$

Alternatively, you can try to look at the factors of 160 and create an AP such that it meets the criteria.

Thus, 160 can be written as

2×80

4×40

8×20

10×16

and so on.

If we consider 8×20, then one possibility is that $d = 6$ and the first and fifth terms are 2 and 26. But $2 + 26$ π 28. Hence, this cannot be the correct factors.

Try 10×16. This will give you,

$a = 7$, $d = 3$ and 5th term = 19

and $7 + 19 = 26$ satisfies the condition

13. A possible AP satisfying this condition is

0, 1, 2, 3, 4, 5, 6, 7, 8, …

14. Assume the fifth term as $(a + 4d)$, the eleventh term as $(a + 10d)$, the second term as $(a + d)$, the fourteenth term as $(a + 13d)$, the first term as (a) and the fifteenth term as $(a + 14d)$.

Then, First term × Fifteenth term

$= a \times (a + 14d) = a^2 + 14ad$

Also $(a + d)(a + 13d) = P$

and $(a + 4d)^2 + (a + 10d)^2 = 3$

16. The solution (from the options) has got something to do with either 2^{100} or 2^{101} for 100 terms. Hence, for 3 terms recreate the options and crosscheck with the actual sum.

For 3 terms: Sum = 2 + 8 + 24 = 34.

(a) $100 \times 2^{101} + 2$ for 100 terms becomes $3 \times 2^4 + 2$ for 3 terms.

= 48 + 2 = 50 π 34. Hence is not correct.

(b) $99 \times 2^{100} + 2$ for 100 terms becomes $2 \times 2^3 + 2$ for 3 terms.

But this does not give 34. Hence is not correct.

(c) $99 \times 2^{101} + 2$ Æ $2 \times 2^4 + 2 = 34$

(d) $100 \times 2^{100} + 2$ Æ $3 \times 2^3 + 2$ π 34.

Hence, option (c) is correct.

17. $r = 2$ and $a + ar^3 = 108$.
20. Solve by checking options and using principles of logarithms.
21. The sum of the first 20 terms will be

    $a + (a + d) + (a + 2d) \dots + (a + 19d)$

    i.e.$20a + 190d = 400$

    Similarly, use the sum to fifty terms.
23. For a product $a \times b \times c$ to be maximum, given $a + b + c$ = constant, the condition is $a = b = c$.
27. The length of sides of successive triangles form a GP with common ratio 1/3.
28. The area of successive triangles form an infinite GP with a common ratio 1/4.
29. Common ration = $1/\sqrt{2}$.
31. Ration of sum of the first $2n$ terms to the first n terms is equal to 3.

    Thus,

    $$\frac{2n\left(\frac{a_1 + a_{2n}}{2}\right)}{n\left(\frac{a_1 + a_n}{2}\right)} = 3$$

    Solve to get, $2a = (n + 1)d$.

    Put values for a and n to get a value for d and check for the conditions given in the question.
33. Visualize the AP and solve using standard formulae.
34. Take any values for the numbers.

    Say, the two positive numbers are 1 and 27.

    Then, $a = 14$, $b = 3$ and $c = 9$.

Solutions and Shortcuts

Level of Difficulty (I)

1. In order to count the number of terms in the AP, use the short cut:

   [(last term – first term)/ common difference] + 1. In this case it would become:

   [(130 – 20)/5] +1 = 23. Option (b) is correct.

2. 7000 – 500 – 12500 means that the starting scale is 7000 and there is an increment of 500 every year. Since, the total increment required to reach the top of his scale is 5500, the number of years required would be 5500/500 = 11. Option (a) is correct.

3. Since the 8th and the 12th terms of the AP are given as 39 and 59 respectively, the difference between the two terms would equal 4 times the common difference. Thus we get $4d = 59 - 39 = 20$. This gives us $d = 5$. Also, the 8^{th} term in the AP is represented by $a + 7d$, we get:

   $a + 7d = 39$ Æ $a + 7 \times 5 = 39$ Æ $a = 4$. Option (c) is correct.

4. If we take the sum of the sides we get the perimeters of the squares. Thus, if the side of the respective squares are a_1, a_2, a_3, a_4… their perimeters would be $4a_1$, $4a_2$, $4a_3$, $4a_4$. Since the perimeters are in GP, the sides would also be in GP.

5. The number of terms in a series are found by:

   $$\frac{\text{Difference between first and last terms}}{\text{Common Difference}} + 1$$

6. The first common term is 3, the next will be 9 (Notice that the second common term is exactly 6 away from the first common term. 6 is also the LCM of 2 and 3 which are the respective common differences of the two series.)

   Thus, the common terms will be given by the A.P 3, 9, 15 ….., last term. To find the answer you need to find the last term that will be common to the two series.

   The first series is 3, 5, 7 … 239

   While the second series is 3, 6, 9 ….. 240.

   Hence, the last common term is 237.

Thus our answer becomes $\frac{237-3}{6} + 1 = 40$

7. Trying Option (a),

   We get least term 5 and largest term 30 (since the largest term is 6 times the least term).

   The average of the A.P becomes (5 + 30)/2 = 17.5

   Thus, 17.5 × n = 105 gives us:

   to get a total of 105 we need n = 6 i.e. 6 terms in this A.P. That means the A.P. should look like:

   5, _ , _ , _, _, 30.

   It can be easily seen that the common difference should be 5. The A.P, 5, 10, 15, 20, 25, 30 fits the situation.

   The same process used for option (b) gives us the A.P. 10, 35, 60. (10 + 35 + 60 = 105) and in the third option 15, 90 (15 + 90 = 105).

   Hence, all the three options are correct.

8. The first term is 20 and the common difference is –5, thus the 15th term is:

   20 + 14 × (–5) = –50. Option (c) is correct.

9. The difference between the amounts at the end of 4 years and 10 years will be the simple interest on the initial capital for 6 years.

   Hence, 360/6 = 60 =(simple interest.)

   Also, the Simple Interest for 4 years when added to the sum gives 1240 as the amount.

   Hence, the original sum must be 1000.

10. The three parts are 3, 5 and 7 since $3^2 + 5^2 + 7^2 = 83$. Since, we want the smallest number, the answer would be 3. Option (b) is correct.

11. $a = 5$, $a + 2d = 15$ means $d = 5$. The 16th term would be $a + 15d = 5 + 75 = 80$. The sum of the series would be given by:

    [16/2] × [5 + 80] = 16 × 42.5 = 680. Option (d) is correct.

12. Use trial and error by using various values from the options.

If you find the sum of the series till 18 terms the value is 513. So also for 19 terms the value of the sum would be 513. Option (c) is correct.

13. Solve this question through trial and error by using values of n from the options:

    For 19 terms, the series would be 5 + 8 + 11 + …. + 59 which would give us a sum for the series as 19 × 32 = 608. The next term (20th term of the series) would be 62. Thus, 608 + 62 = 670 would be the sum to 20 terms. It can thus be concluded that for 20 terms the value of the sum of the series is not less than 670. Option (a) is correct.

14. His total earnings would be 60 + 63 + 66 + … + 117 = ` 1770. Option (a) is correct.

15. The series would be 5, 20, 80, 320, 1280, 5120, 20480. Thus, there are a total of 7 terms in the series. Option (c) is correct.

16. Sum of a G.P. with first term 1 and common ratio 2 and no. of terms 20.

    $$\frac{1\times(2^{20}-1)}{(2-1)} = 2^{20} - 1$$

17. $16r^4 = 81$ Æ $r^4 = 81/16$ Æ $r = 3/2$. Thus, 4th term = $ar^3 = 16 \times (3/2)^3 = 54$. Option (c) is correct.

18. In the case of a G.P. the 7th term is derived by multiplying the fourth term thrice by the common ratio. (**Note:** this is very similar to what we had seen in the case of an A.P.)

    Since, the seventh term is derived by multiplying the fourth term by 8, the relationship.

    $r^3 = 8$ must be true.

    Hence, $r = 2$

    If the fifth term is 48, the series in reverse from the fifth to the first term will look like:

    48, 24, 12, 6, 3. Hence, option (b) is correct.

19. Visualising the squares below 84, we can see that the only way to get the sum of 3 squares as 84 is: $2^2 + 4^2 + 8^2 = 4 + 16 + 64$. The

largest number is 8. Option (a) is correct.

20. The series would be given by: 1, 5, 9… which essentially means that all the numbers in the series are of the form $4n + 1$. Only the value in option (c) is a $4n + 1$ number and is hence the correct answer.

21. The series will be 301, 308, …….. 497

    Hence, Answer = $\frac{196}{7} + 1 = 29$

22. The answer to this question can be seen from the options. Both 2, 6, 18 and 18, 6, 2 satisfy the required conditions- viz: GP with sum of first and third terms as 20. Thus, option (c) is correct.

23. We need the sum of the series 20 + 24 + 28 to cross 1000. Trying out the options, we can see that in 20 years the sum of his savings would be: 20 + 24 + 28 + … + 96. The sum of this series would be $20 \times 58 = 1160$. If we remove the 20th year we will get the series for savings for 19 years. The series would be 20 + 24 + 28 + …. 92. Sum of the series would be 1160 – 96 = 1064. If we remove the 19th year's savings the savings would be 1064 – 92 which would go below 1000. Thus, after 19 years his savings would cross 1000. Option (a) is correct.

24. The answer to this question cannot be determined because the question is talking about income and asking about savings. You cannot solve this unless you know the value of the expenditure the man incurs over the years. Thus, "Cannot be Determined" is the correct answer.

25. Similar to what we saw in question 18,

    4th term $\times r^6$ = 10th term.

    The 4th term here is $3^{-1}$and the tenth term is 3^5.

    Hence $3^{-1} \times r^6 = 3^5$

    Gives us: $r = 3$.

    Hence, the second term will be given by (fourth term/r^2)

    $^1/_3 \times {}^1/_3 \times {}^1/_3 = {}^1/_{27}$

[**Note:** To go forward in a G.P. you multiply by the common ratio, to go backward in a G.P. you divide by the common ratio.]

26. $a + 6d = 6$ and $a + 20d = -22$. Solving we get $14d = -28$ Æ $d = -2$. 26th term = 21st term + $5d = -22 + 5 \times (-2) = -32$. Option (b) is correct.

27. Since the sum of 5 numbers in AP is 30, their average would be 6. The average of 5 terms in AP is also equal to the value of the 3rd term (logic of the middle term of an AP). Hence, the third term's value would be 6. Option (b) is correct.

28. The answer will be given by:

$[10 + 11 + 12 + \ldots\ldots.. + 50] - [16 + 24 + \ldots + 48]$

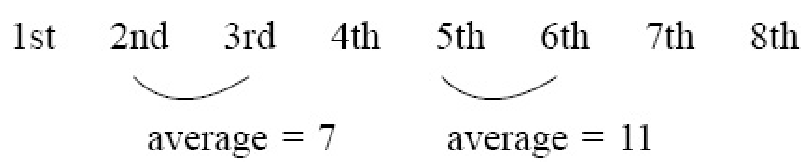
$= 41 \times 30 - 32 \times 5$

$= 1230 - 160 = 1070.$

29. Think like this:

The average of the first 4 terms is 7, while the average of the first 8 terms must be 11.

Now visualize this :



Hence, $d = 4/2 = 2$ [Note: understand this as a property of an A.P.]

Hence, the average of the 6th and 7th terms = 15 and the average of the 8th and 9th term = 19

But this (19) also represents the average of the 16 term A.P.

Hence, required answer = $16 \times 19 = 304$.

30. Go through the options. The correct option should give value as 1, when $n = 3$ and as 8 when $n = 8$.

Only option (a) satisfies both conditions.

31. The series is: 1/81, –1/27, 1/9, –1/3, 1, –3, 9, –27, 81, –243, 729. There are 11 terms in the series. Option (a) is correct.

32. $1/8 \times r^5 = 128$ Æ $r^5 = 128 \times 8 = 1024$ Æ $r = 4$. Thus, the series would be 1/8, 1/2, 2, 8, 32, 128. The third geometric mean would be 8. Option (d) is correct.

33. AM = 25 means that their sum is 50 and GM = 7 means their product is 49. The numbers can only be 49 and 1. Option (c) is correct.

34. Trial and error gives us that for option (b):

    With the ratio 4:1, the numbers can be taken as $4x$ and $1x$. Their AM would be $2.5x$ and their GM would be $2x$. The GM can be seen to be 20% lower than the AM. Option (b) is thus the correct answer.

35. The total savings would be given by the sum of the series:

    100 + 150 + 200 + 650 = 12 × 375 = ` 4500. Option (b) is correct.

36. In order to find how many times the alarm rings we need to find the number of numbers below 100 which are not divisible by 2,3, 5 or 7. This can be found by:

    100 – (numbers divisible by 2) – (numbers divisible by 3 but not by 2) – (numbers divisible by 5 but not by 2 or 3) – (numbers divisible by 7 but not by 2 or 3 or 5).

    Numbers divisible by 2 up to 100 would be represented by the series 2, 4, 6, 8, 10..100 Æ A total of 50 numbers.

    Numbers divisible by 3 but not by 2 up to 100 would be represented by the series 3, 9, 15, 21…99 Æ A total of 17 numbers. Note use short cut for finding the number of number in this series :

    [(last term – first term)/ common difference] + 1 = [(99 – 3)/6] + 1 = 16 + 1 = 17.

    Numbers divisible by 5 but not by 2 or 3: Numbers divisible by 5 but not by 2 up to 100 would be represented by the series 5, 15, 25, 35…95 Æ A total of 10 numbers. But from these numbers, the numbers 15, 45 and 75 are also divisible by 3. Thus, we are left with 10 – 3 = 7 new numbers which are divisible by 5 but not by 2 and 3.

    Numbers divisible by 7, but not by 2, 3 or 5:

    Numbers divisible by 7 but not by 2 upto 100 would be represented by the series 7, 21, 35, 49, 63, 77, 91 Æ A total of 7 numbers. But from these numbers we should not count 21, 35 and 63 as they are divisible by either 3 or 5. Thus a total of 7 – 3 = 4 numbers are divisible by 7 but not by 2, 3 or 5.

37. For looking at the zeroes in the expression we should be able to see that the number of zeroes in the third term onwards is going to be very high. Thus, the number of zeroes in the expression would be given by the number of zeroes in:

    $4 + 24^{24}$. 24^{24} has a unit digit 6. Hence, the number of zeroes in the expression would be 1. Option (d) is correct.

38. Since the sum of 22 terms of the AP is 385, the average of the numbers in the AP would be 385/22 = 17.5. This means that the sum of the first and last terms of the AP would be 2 × 17.5 = 35. Trial and error gives us the terms of the required GP as 7, 14, 28. Thus, the common ratio of the GP can be 2.

39. The sum of the interior angles of a polygon are multiples of 180 and are given by $(n - 1) \times 180$ where n is the number of sides of the polygon. Thus, the sum of interior angles of a polygon would be a member of the series: 180, 360, 540, 720, 900, 1080, 1260

    The sum of the series with first term 100 and common difference 10 would keep increasing when we take more and more terms of the series. In order to see the number of sides of the polygon, we should get a situation where the sum of the series represented by 100 + 110 + 120… should become a multiple of 180. The number of sides in the polygon would then be the number of terms in the series 100, 110, 120 at that point.

    If we explore the sums of the series represented by

    100 + 110 + 120…

    We realize that the sum of the series becomes a multiple of 180 for 8 terms as well as for 9 terms.

    It can be seen in: 100 + 110 + 120 + 130 + 140 + 150 + 160 + 170 = 1080

    Or 100 + 110 + 120 + 130 + 140 + 150 + 160 + 170 + 180 = 1260.

40. The sum of the total distance it travels would be given by the infinite sum of the series:

420 × 8/1 + 367.5 × 8/1 = 3360 + 2940 = 6300. Option (b) is correct.

41. The two series till their hundredth terms are 13, 15, 17….211 and 14, 17, 20…311. The common terms of the series would be given by the series 17, 23, 29….209. The number of terms in this series of common terms would be 192/6 + 1 = 33. Option (d) is correct.

42. The area of the first square would be 64 sq cm. The second square would give 32, the third one 16 and so on. The infinite sum of the geometric progression 64 + 32 + 16 + 8… = 128. Option (a) is correct.

43. It can be seen that for the series the average of two terms is 2, for 3 terms the average is 3 and so on. Thus, the sum to 2 terms is 2^2, for 3 terms it is 3^2 and so on. For 11111 terms it would be 11111^2 = 123454321. Option (d) is correct.

44. The maximum score would be the sum of the series 9 + 13 + …. + 389 + 393 + 397 = 98 × 406/2 = 19894. Option (d) is correct.

45. The series would be 8, 8/3, 8/9 and so on. The sum of the infinite series would be 8/(1 – 1/3) = 8 × 3/2 = 12. Option (a) is correct.

46. The maximum sum would occur when we take the sum of all the positive terms of the series.

    The series 25, 24.5, 24, 23.5, 23, …….. 1, 0.5, 0 has 51 terms. The sum of the series would be given by:

    n × average = 51 × 12.5 = 637.5

    Option (a) is correct.

47. The side of the first equilateral triangle being 24 units, the first perimeter is 72 units. The second perimeter would be half of that and so on.

    72, 36, 18 …

    The infinite sum of this series = 72(1 – 1/2) = 144. Option (d) is correct.

48. Solve using options. Option (a) fits the situation as 16 + 2 + 2/8 + 2/64 meets the conditions of the question. Option (a) is correct.

49. The 33rd term of the sequence would be the 17th term of the sequence 3, 9, 15, 21 ….

The 17th term of the sequence would be $3 + 6 \times 16 = 99$.

50. The sum to 33 terms of the sequence would be:

The sum to 17 terms of the sequence 3, 9, 15, 21, …99 + The sum to 16 terms of the sequence 8, 13, 18, 83.

The required sum would be $17 \times 51 + 16 \times 45.5 = 867 + 728 = 1595$.

Level of Difficulty (II)

1. Identify an A.P. which satisfies the given condition.

Suppose we are talking about the second and third terms of the A.P.

Then an A.P. with second term 3 and third term 2 satisfies the condition.

a times the *a*th term = *b* times the *b*th term.

In this case the value of $a = 2$ and $b = 3$.

Hence, for the $(a + b)^{th}$ term, we have to find the fifth term.

It is clear that the fifth term of this A.P. must be zero.

Check the other three options to see whether any option gives 0 when $a = 2$ and $b = 3$.

Since none of the options b, c or d gives zero for this particular value, option (a) is correct.

2. Since the four parts of the number are in *AP* and their sum is 20, the average of the four parts must be 5. Looking at the options for the largest part, only the value of 8 fits in, as it leads us to think of the *AP* 2, 4, 6, 8. In this case, the ratio of the product of the first and fourth (2×8) to the product of the first and second (4×6) are equal. The ratio becomes 2:3.

3. View: 1 – 4 + 5 – 8 + 9 – 12 . . . . . 50 terms as

(1 – 4) + (5 – 8) + (9 – 12) …… 25 terms.

Hence, –3 + –3 + –3 . . . . 25 terms

$= 25 \times -3 = -75$.

4. In the course of 2 days the clock would strike 1 four times, 2 four times, 3 four times and so on. Thus, the total number of times the clock would strike would be:

   4 + 8 + 12 + ..48 = 26 × 12 = 312. Option (b) is correct.

5. Since this is a decreasing A.P. with first term positive, the maximum sum will occur upto the point where the progression remains non – negative.

   44, 42, 40 ….. 0

   Hence, 23 terms × 22 = 506.

6. The sum of the required series of integers would be given by 7 + 14 + 21 + ….196 = 28 × 101.5 = 2842. Option (b) is correct.

7. A little number juggling would give you 2nd term is 1/3 and 3rd term is ½ is a possible situation that satisfies the condition.

   The A.P. will become:

   1/6, 1/3, 1/2, 2/3, 5/6, 1

   or in decimal terms, 0.166, 0.333, 0.5, 0.666, 0.833, 1

   Sum to 6 terms = 3.5

   Check the option with $m = 2$ and $n = 3$. Only option (c) gives 3.5. Hence, must be the answer.

8. 101 + 103 + 105 + … 199 = 150 × 50 = 7500

9. The required sum would be given by the sum of the series 105, 112, 119, …. 994. The number of terms in this series = (994 – 105)/7 + 1 = 127 + 1 = 128. The sum of the series = 128 × (average of 105 and 994) = 70336. Option (c) is correct.

10. 5 × average of 107 and 253 = 5 × 180 = 900. Option (c) is correct.

11. The first 100 terms of this series can be viewed as:

    (1 – 2 –3) + (2 – 3 –4) + ... + (33 – 34 – 35) + 34

    The first 33 terms of the above series (indicated inside the brackets) will give an A.P.: –4, –5, –6 .... –36

    Sum of this A.P. = 33 × –20 = –660

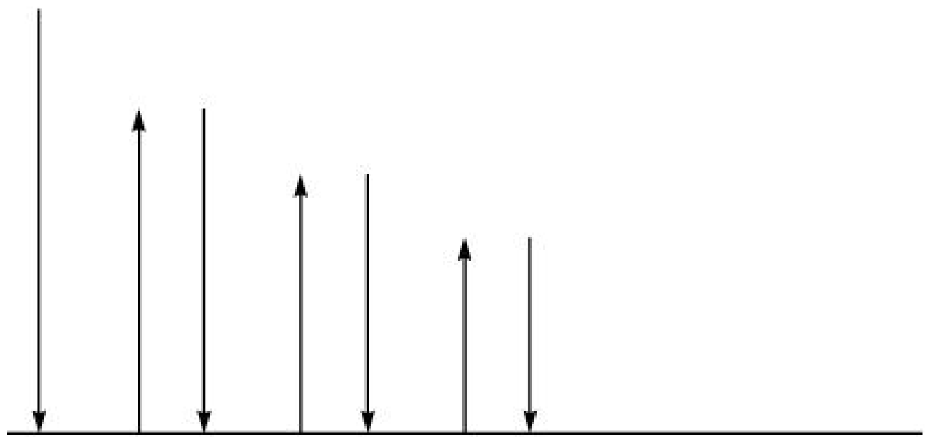
    Answer = –660 + 34 = –626

12. Solve this one through trial and error. For $n = 2$ terms the sum upto 2 terms is equal to 96. Putting $n = 2$, in the options it can be seen that for option (b) the sum to two terms would be given by

$8 \times (1000 - 10 - 18)/81 = 8 \times 972/81 = 8 \times 12 = 96$.

13. If we take the values of a, b, and c as 10,100 and 1000 respectively, we get log a, log b and log c as 1, 2 and 3 respectively. This clearly shows that the values of log a, log b and log c are in AP.

14. The path of the rubber ball is:



In the figure above, every bounce is 4/5th of the previous drop.

In the above movement, there are two infinite G.Ps (The GP representing the falling distances and the GP representing the Rising distances.)

The required answer: (Using $a/(1-r)$ formula)

$$\frac{120}{1/5} + \frac{96}{1/5} = 1080$$

15. Solve this for a sample GP. Let us say we take the GP as 2, 6, 18, 54. x, the first term is 2, let $n = 3$ then the 3rd term $y = 18$ and the product of 3 terms $p = 2 \times 6 \times 18 = 216 = 6^3$. The value of $p^2 = 216 \times 216 = 6^6$.

Putting these values in the options we have:

Option (a) gives us $(xy)^{n-1} = 36^2$ which is not equal to the value of p^2 we have from the options

Option (b) gives us $(xy)^n = 36^3 = 6^6$ which is equal to the value of p^2 we have from the options.

It can be experimentally verified that the other options yield values of p^2 which are different from 6^6 and hence we can conclude that option (b) is correct.

16. Trying to plug in values we can see that the infinite sum of the GP 16, 8, 4, 2… is 32 and hence the third term is 4.

17. The expression can be written as $x^{(1/2\ +\ ¼\ +\ 1/8\ +1/16\ldots..)} = x^{\text{INFINTE SUM OF THE GP}} = x^1$. Option (b) is correct.

18. For $n = 1$, the sum should be 6. Option (b), (c) and (d) all give 6 as the answer.

    For $n = 2$, the sum should be 30.

    Only option *d* gives this value. Hence must be the answer.

19. From the facts given in the question it is self evident that the common ratio of the GP must be 2 (as the sum of the 2nd and 4th term is twice the sum of the first and third term). After realizing this, the question is about trying to match the correct sums by taking values from the options.

    The GP formed from option (c) with a common ratio of 2 is: 8,16,32,64 and this GP satisfies the conditions of the problem- sum of 1st and 3rd term is 40 and sum of 2nd and 4th term is 80.

20. Since the sum of the first five even terms is 15, we have that the 2nd, 4th, 6th, 8th and 10th term of the AP should add up to 15. We also need to understand that these 5 terms of the AP would also be in an AP by themselves and hence, the value of the 6th term (being the middle term of the AP) would be the average of 15 over 5 terms. Thus, the value of the 6th term is 3. Also, since the sum of the first three terms of the AP is –3, we get that the 2nd term would have a value of –1. Thus, the AP can be visualized as:

    _, -1, _,_,_,3,….

    Thus, it is obvious that the AP would be –2, –1, 0, 1, 2, 3. The second term is –1. Thus, option (c) is correct.

21. Second term = $a + d$, Fifth term = $a + 4d$; third term = $a + 2d$, seventh term = $a + 6d$.

Thus, $2a + 5d = 8$ and $2a + 8d = 14$ Æ $d = 2$ and $a = -1$.

The eleventh term = $a + 10d = -1 + 20 = 19$. Thus, option (a) is correct.

22. If the difference between the seventh and the second term is 20, it means that the common difference is equal to 4. Since, the third term is 9, the *AP* would be 1, 5, 9, 13, 17, 21, 25, 29 and the sum to 8 terms for this *AP* would be 120. Thus, option (d) is correct.

23. $5d = 35$ Æ $d = 7$. Thus, the numbers are 4, 11, 18, 25, 32, 39. The largest number is 32. Option (c) is correct.

24. Find sum of the series:

104, 109, 114 999

Average × n = 551.5 × 180 = 99270

25. Since the sum of the first three terms of the *AP* is 30, the average of the *AP* till 3 terms would be 30/3 = 10. The value of the second term would be equal to this average and hence the second term is 10. Using the information about the sum of squares of the first and second terms being 116, we have that the first term must be 4. Thus, the AP has a first term of 4 and a common difference of 6. The seventh term would be 40. Thus, option (b) is correct.

26. The combined travel would be 25 on the first day, 26 on the second day, 27 on the third day, 28 on the fourth day, 29 on the fifth day and 30 on the sixth day. They meet after 6 days. Option (c) is correct.

27. This is a calculation intensive problem and you are not supposed to know how to do the calculations in this question mentally. The problem has been put here to test your concepts about whether you recognize how this is a question of GPs. If you feel like, you can use a calculator/ computer spreadsheet to get the answer to this question.

The logic of the question would hinge on the fact that the value of the investment of the fifteenth year would be 1000. At the end of the 15^{th} year, the investment of the 14^{th} year would be equal to 1000×1.05, the 13^{th} year's investment would amount to 1000×1.05^2 and so on till the first year's investment which would amount to 1000 × 1.05^{14} after 15 years.

Thus, you need to calculate the sum of the GP:

1000, 1000 × 1.05, 1000 × 1.05^2, 1000 × 1.05^3 for 15 terms.

28. Since, sum to n terms is given by (n + 8),

    Sum to 1 terms = 9

    Sum to 2 terms = 10

    Thus, the 2nd term must be 1.

29. Solve this question by looking at hypothetical values for n and $2n$ terms. Suppose, we take the sum to 1 (n = 1) term of the first series and the sum to 2 terms ($2n$ = 2) of the second series we would get A/B as 1/1.5 = 2/3.

    For n = 2 and $2n$ = 4 we get A = 1.25 and B = 1.875 and A/B = 1.25/1.875 = 2/3.

    Thus, we can conclude that the required ratio is always constant at 2/3 and hence the correct option is (c).

30. We need to find the infinite sum of the GP: 100, 90, 81…(first term = 100 and common ratio= 0.9)

    We get: Infinite sum of the series as 1000. Thus, option (b) is correct.

31. Questions such as these have to be solved on the basis of a reading of the pattern of the question. The sum upto the first term is: 1/5. Upto the second term it is 2/9 and upto the third term it is 3/13. It can be easily seen that for the first term, second term and third term the numerators are 1, 2 and 3 respectively. Also, for 1/5 – the 5 is the second value in the denominator of 1/1 × 5 (the first term); for 2/9 also the same pattern is followed—as 9 comes out of the denominator of the second term of series and for 3/13 the 13 comes out of the denominator of the third term of the series and so on. The given series has 56 terms and hence the correct answer would be 56/225.

32. Solve this on the same pattern as Question 31 and you can easily see that for the first term sum of the series is $\sqrt{2}-1$, for 2 terms we

have the sum as $\sqrt{3}-1$ and so on. For the given series of 120 terms the sum would be $\sqrt{121}-1$ = 10.

Option (a) is correct.

33. If you look for a few more terms in the series, the series is:

1, 1/3, 1/6, 1/10, 1/15, 1/21, 1/28, 1/36, 1/45, 1/55, 1/66, 1/78, 1/91, 1/105, 1/120, 1/136, 1/153 and so on. If you estimate the values of the individual terms it can be seen that the sum would tend to 2 and would not be good enough to reach even 2.25.

Thus, option (a) is correct.

34. Solve this using trial and error. For 1 term the sum should be 40 and we get 40 only from the first option when we put n = 1. Thus, option (a) is correct.

35. For this question too you would need to read the pattern of the values being followed. The given sum has 13 terms.

It can be seen that the sum to 1 term = ½

Sum to 2 terms = 2/3

Sum to 3 terms = ¾

Hence, the sum to 13 terms would be 13/14.

36. The sum to infinite terms would tend to 1 because we would get (infinity)/(infinity + 1).

37. All members of *A* are smaller than all members of *B*. In order to visualize the effect of the change in sign in *A*, assume that *A* is {1, 2, 3…124} and *B* is {126, 127…250}. It can be seen that for this assumption of values neither options (a), (b) or (c) is correct.

38. If elements of *A* are in ascending order a_{124} would be the largest value in *A*. Also a_{125} would be the largest value in *B*. On interchanging a_{124} and a_{125}, *A* continues to be in ascending order, but *B* would lose its descending order arrangement since a_{124} would be the least value in *B*. Hence, option *a* is correct.

39. Since the minimum is in *A* and the maximum is in *B*, the value of *x* cannot be *less than Max-Min.*

40. It is evident that the whole question is built around Arithmetic progressions. The 5th row has an average of 55, while the 15th row has an average of 65. Since even column wise each column is arranged in an *AP* we can conclude the following:

1st row – average 51 – total = 23 × 51

2nd row – average 52 – total 23 × 52…..

23rd row – average 73 – total 23 × 73

The overall total can be got by using averages as:

23 × 23 × 62 = 32798

41. The numbers forming an *AP* would be:

123, 135, 147, 159, 210, 234, 246, 258, 321, 345, 357, 369, 432, 420, 456, 468, 543, 531, 567, 579, 654, 642, 630, 678, 765, 753, 741, 789, 876, 864, 852, 840, 987, 975, 963 and 951. A total of 36 numbers.

If we count the GPs we get:

124, 139, 248, 421, 931, 842—a total of 6 numbers.

Hence, we have a total of 42 3 digit numbers where the digits are either APs or GPs.

Thus, option (d) is correct.

42. Total burgers made = 660

Burgers with chicken and mushroom patty = 165 (Number of terms in the series 1, 5, 9…657)

Burgers with vegetable patty = 95 (Number of terms in the series 2, 9, 16, …660)

Burgers with chicken, mushroom and vegetable patty = 24 (Number of terms in the series 9, 37, 65….653)

Required answer = 660 – 165 – 95 + 24 = 424

43. From the above question, we have 24 such burgers.

44. The key to this question is what you understand from the statement- ‘for two progressions out of P_1, P_2 and P_3 the average is itself a term of the original progression *P*.’ For option (a) which tells us that the Progression *P* has 20 terms, we can see that P_1 would have 7

terms, P_2 would have 7 terms and P_3 would have 6 terms. Since, both P_1 and P_2 have an odd number of terms we can see that for P_1 and P_2 their 4^{th} terms (being the middle terms for an *AP* with 7 terms) would be equal to their average. Since, all the terms of P_1, P_2 and P_3 have been taken out of the original *AP P*, we can see that for P_1 and P_2 their average itself would be a term of the original progression *P*. This would not occur for P_3 as P_3 has an even number of terms. Thus, 20 is a correct value for *n*.

Similarly, if we go for $n = 26$ from the second option we get:

P_1, P_2 and P_3 would have 9, 9 and 8 terms respectively and the same condition would be met here too.

For $n = 36$ from the third option, the three progressions would have 12 terms each and none of them would have an odd number of terms.

Thus, option (d) is correct as both options (a) and (b) satisfy the conditions given in the problem.

45. Since, P_1 is formed out of every third term of *P*, the common difference of P_1 would be three times the common difference of *P*. Thus, the common difference of *P* would be 2.

TRAINING GROUND FOR BLOCK I

HOW TO THINK IN PROBLEMS ON BLOCK I

1. A number x is such that it can be expressed as $a + b + c = x$ where a, b, and c are factors of x. How many numbers below 200 have this property?

(a) 31 (b) 32

(c) 33 (d) 5

Solution: In order to think about this question you need to think about whether the number can be divided by the initial numbers below the square root like 2, 3, 4, 5… and so on.

Let us say, if we think of a number that is not divisible by 2, in such a case if we take the number to be divisible by 3, 5 and 7, then the largest factors of x that we will get would be $\frac{x}{3}, \frac{x}{5}$ and $\frac{x}{7}$.

Even in this situation, the percentage value of these factors as a percentage of x would only be: for $\frac{x}{3}$ = 33.33%, $\frac{x}{5}$ = 20% and $\frac{x}{7}$ = 14.28%

Hence, if we try to think of a situation where $a = \frac{x}{3}$, $b = \frac{x}{5}$, and $c = \frac{x}{7}$ the value of $a + b + c$ would give us only (33.33% + 20% + 14.28%) = 67.61% of x, which is not equal to 100%

Since the problem states that $a + b + c = x$, the value of $a + b + c$ should have added up to 100% of x.

The only situation, for this to occur would be if

$a = \frac{x}{2}$ = 50% of x $b = \frac{x}{3}$ = 33.33% of x and

$c = \frac{x}{6}$ = 16.66% of x.

This means that 2, 3 and 6 should divide x necessarily. In other words x should be a multiple of 6.The multiples of 6 below 200 are 6, 12, 18 … 198, a total of 33 numbers.

Hence the correct answer is c.

2. Find the sum of all 3 digit numbers that leave a remainder of 3 when divided by 7.

(a) 70821 (b) 60821

(c) 50521 (d) 80821

Solution: In order to solve this question you need to visualise the series of numbers, which satisfy this condition of the remainder 3 when divided by 7.

The first number in 3 digits for this condition is 101 and the next will be 108, followed by 115, 122 …

This series would have its highest value in 3 digits as 997.

The number of terms in this series would be 129 (using the logic that for any AP, the number of terms is given by $\frac{D}{d} + 1$)

Also the average value of this series is the average of the first and the last term i.e., the average of 101 and 997. Hence the required sum = 549 × 129 = 70821.

Hence the correct Option is (a).

3. How many times would the digit 6 be used in numbering a book of 639 pages?

(a) 100 (b) 124

(c) 150 (d) 164

Solution: In order to solve this question you should count the digit 6 appearing in units digit, separately from the instances of the digit 6 appearing in the tens place and appearing in the hundreds place.

When you want to find out the number of times 6 appears in the unit digit, you will have to make a series as follows: 6, 16, 26, 36 … 636.

It should be evident to you that the above series has 64 terms because it starts from 06, 16, 26 … and continues till 636. The digit 6 will appear once in the unit digit for each of these 64 numbers.

Next you need to look at how many times the digit 6 appears in the ten's place.

In order to do this we will need to look at instances when 6 appears in the tens place. These will be in 6 different ranges 60s, 160s, 260s, 360s, 460s, 560s and in each of these ranges there are 10 numbers each with exactly one instance of the digit 6 in the tens place, a total of 60 times.

Lastly, we need to look at the number of instances where 6 appears in the hundreds place. For this we need to form the series 600, 601, 602, 603 … 639. This series will have 40 numbers each with exactly one instance of the digit 6 appearing in the hundreds place.

Therefore the required answer would be 64 + 60 + 40 = 164.

Hence, Option (d) is correct.

4. A number written in base 3 is 100100100100100100. What will be the value of this number in base 27?

(a) 999999 (b) 900000

(c) 989999 (d) 888888

The number can be visualised as:

1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
3^{17}	3^{16}	3^{15}	3^{14}	3^{13}	3^{12}	3^{11}	3^{10}	3^9	3^8	3^7	3^6	3^5	3^4	3^3	3^2	3^1	3^0

Now in the base 27, we can visualise this number as:

—	—	—	—	—	—
27^5	27^4	27^3	27^2	27^1	27^0

When you want to write this number in Base 27 the unit digit of the number will have to account for the value of 3^2 in the above number. Since the unit digit of the number in Base 27 will correspond to 27^0 we would have to use 27^0, 9 times to make a value 3^2. The number would now become:

—	—	—	—	—	9
27^5	27^4	27^3	27^2	27^1	27^0

Similarly, the number of times that we will have to use 27^1 in order to make the value of 3^5 (or 243) will be $\frac{243}{27}$ = 9. Hence the second last digit of the number will also be 9.

Similarly, to make a value of 3^8 using 27^2 the number of times we will have to use 27^2 will be given by $\frac{3^8}{27^2} = \frac{3^8}{3^6}$ = 9. The number would now look as:

—	—	—	9	9	9

27^5 27^4 27^3 27^2 27^1 27^0

It can be further predicted that each of subsequent 1's in the original number will equal 9 in the number, which is being written in Base 27. Since the original number in Base 3 has 18 digits, the left most '1' in that number will be covered by a number corresponding to 27^5 in the new number (i.e. 3^{15}). Hence the required number will be a 6-digit number 999999. Hence, Option (a) is correct.

5. Let $x = 1640$, $y = 1728$ and $z = 448$. How many natural numbers are there that divide at least one amongst x, y, z.

(a) 47 (b) 48

(c) 49 (d) 50

Solution: 1640 can be prime factorised as $2^3 \times 5^1 \times 41^1$. This number would have a total of 16 factors.

Similarly, 1728 can be prime factorised as $2^6 \times 3^3$. Hence, it would have 28 factors. While $448 = 2^6 \times 7^1$ would have 14 factors. Thus there are a total of (16 + 28 + 14) = 58 factors amongst x, y and z.

However, some of these factors must be common between x, y, z. Hence, in order to find the number of natural numbers that would divide at least one amongst x, y, z, we will need to account for double and triple counted numbers amongst these 58 numbers (by reducing the count by 2 for each triple counted number and by reducing the count by 1 for each double counted number).

The number of cases of triple counting would be for all the common factors of (x, y, z). This number can be estimated by finding the HCF of x, y and z and counting the number of factors of the HCF. The HCF of 1640, 1728 and 448 is 8 and hence the factors of 8, i.e., 1, 2, 4 and 8 itself must have got counted in each of the 3 counts done above. Thus each of these 4 numbers should get subtracted twice to remove the triple count. This leaves us with $58 - 4 \times 2 = 50$ numbers.

We still need to eliminate numbers that have been counted twice, i.e., numbers, which belong to the factors of any two of these numbers (while counting this we need to ensure that we do not count the triple counted numbers again). This can be visualised in the following way:

Number of common factors that are common to only 1640 and 1728 and not to 448 (x and y but not z):

$1640 = 2^3 \times 5^1 \times 41^1$

$1728 = 2^6 \times 3^3$

It can be seen from these 2 standard forms of the numbers that the highest common factors of these 2 numbers is 8. Hence there is no new number to be subtracted for double counting in this case.

The case of 1640 and 448 is similar because $1640 = 2^3 \times 5^1 \times 41^1$ while $448 = 2^6 \times 7^1$ and HCF = 2^3 and hence they will not give any more numbers as common factors apart from 1, 2, 4 and 8.

Thus there is no need of adjustment for the pair 1640 and 448.

Finally when we look for 1728 and 448, we realise that the HCF = $2^6 = 64$ and hence the common factors between 448 and 1728 are 1, 2, 4, 8, 16, 32, 64. But we are looking for factors which are common for 1728 and 448 but not common to 1640 to estimate the double counting error for this case.

Hence we can eliminate the number 1, 2, 4, 8 from this list and conclude that there are only 3 numbers 16, 32 and 64 that divide both 1728 and 448 but do not divide 1640.

If we subtract these numbers once each, from the 50 numbers we will end up with $50 – 3 \times 1 = 47$.

The complete answer can be visualised as $16 + 28 + 14 – 4 \times 2 – 3 \times 1 = 47$.

Hence, Option (a) is correct.

6. How many times will the digit 6 be used when we write all the six digit numbers?

(a) 5,50,000 (b) 5,00,000

(c) 4,50,000 (d) 4,00,000

Solution: When we write all 6-digit numbers we will have to write all the numbers from 100000 to 999999, a total of 9 lac numbers in 6 digits without omitting a single number. There will be a complete symmetry and balance in the use of all the digits. However the digit 0 is not going to be used in the leftmost place.

Using this logic we can visualise that when we write 9 lakh, 6-digit numbers, the units place, tens place, hundreds place, thousands place, ten thousands place and lakh place – Each of these places will be written 9 lakh times. Thinking about the units place, we can think as follows: In writing the units place 9 lakh times (once for every number) we will be using the digit 0, 1, 2, 3 … 9 an equal number of times. Hence any particular digit like 6 would get used in the units digit a total number of 90000 times (9 lakh/10). The same logic will continue for tens, hundreds, thousands and ten thousands, i.e., the digit 6 will be used (9 lakh/10) =

90000 times in each of these places. (Note here that we are dividing by 10 because we have to equally distribute 9 lakh digits amongst the ten digits 0, 1, 2, 3,4, 5, 6, 7, 8, 9.)

Finally for the lakh place since "0" is not be used in the lakh place as it is the leftmost digit of the number, the number of times the digit 6 will be used would be 9 lakh/9 = 1 lakh. Hence the next time you are solving the problem of this type, you should solve directly using $\frac{9\,\text{lakh}}{9}+\frac{9\,\text{lakh}}{10}\times 5$ = 5,50,000.

Hence, Option (a) is the correct answer.

BLOCK REVIEW TESTS

Review Test 1

1. Lata has the same number of sisters as she has brothers, but her brother Shyam has twice as many sisters as he has brothers. How many children are there in the family?

   (a) 7 (b) 6

   (c) 5 (d) 3

2. How many times does the digit 6 appear when you count from 11 to 100?

   (a) 9 (b) 10

   (c) 19 (d) 20

3. If m $< n$, then

   (a) $m\,.m < n\,.\,n$

   (b) $m.m > n\,.\,n$

   (c) $m\,.\,n.\,n < n.\,m\,.\,m$

   (d) $m\,.\,m\,.\,m < n.\,n\,.\,n$

4. A square is drawn by joining the midpoints of the side of a given square. A third square is drawn in side the second square in the same way and this process is continued indefinitely. If a side of the first square is 8 cm, the sum of the areas of all the squares (in sq. cm) is

   (a) 128 (b) 120

   (c) 96 (d) None of these

5. Find the least number which when divided by 6, 15, 17 leaves a remainder 1, but when divided by 7 leaves no remainder.

(a) 211 (b) 511

(c) 1022 (d) 86

6. The number of positive integers not greater than 100, which are not divisible by 2, 3 or 5 is

(a) 26 (b) 18

(c) 31 (d) None of these

7. The smallest number which when divided by 4, 6 or 7 leaves a remainder of 2, is

(a) 44 (b) 62

(c) 80 (d) 86

8. An intelligence agency decides on a code of 2 digits selected from 0, 1, 2,….9. But the slip on which the code is hand-written allows confusion between top and bottom, because these are indistinguishable. Thus, for example, the code 91 could be confused with 16. How many codes are there such that there is no possibility of any confusion?

(a) 25 (b) 75

(c) 80 (d) None of these

9. Suppose one wishes to find distinct positive integers x, y such that $(x + y)/xy$ is also a positive integer. Identify the correct alternative.

(a) This is never possible

(b) This is possible and the pair (x, y) satisfying the stated condition is unique.

(c) This is possible and there exist more than one but a finite number of ways of choosing the pair(x, y).

(d) This is possible and the pair (x, y) can be chosen in infinite ways.

10. A young girl counted in the following way on the fingers of her left hand. She started calling the thumb 1, the index finger 2, middle finger 3, ring finger 4, little finger 5, then reversed direction, calling

the ring finger 6, middle finger 7, index finger 8, thumb 9, then back to the index finger for 10, middle finger for 11, and so on. She counted up to 1994. She ended on her.

(a) thumb (b) index finger

(c) middle finger (d) ring finger

11. 139 persons have signed up for an elimination tournament. All players are to be paired up for the first round, but because 139 is an odd number one player gets a bye, which promotes him to the second round, without actually playing in the first round. The pairing continues on the next round, with a bye to any player left over. If the schedule is planned so that a minimum number of matches is required to determine the champion, the number of matches which must be played is

(a) 136 (b) 137

(c) 138 (d) 139

12. The product of all integers from 1 to 100 will have the following numbers of zeros at the end.

(a) 20 (b) 24

(c) 19 (d) 22

13. There are ten 50 paise coins placed on a table. Six of these show tails four show heads. A coin chosen at random and flipped over (not tossed). This operation is performed seven times. One of the coins is then covered. Of the remaining nine coins five show tails and four show heads. The covered coin shows

(a) a head (b) a tail

(c) more likely a head (d) more likely a tail

14. A five digit number is formed using digits 1, 3, 5, 7 and 9 without repeating any one of them. What is the sum of all such possible numbers?

(a) 6666600 (b) 6666660

(c) 6666666 (d) None

15. From each of two given numbers, half the smaller number is subtracted. Of the resulting numbers the larger one is three times as large as the smaller. What is the ratio of the two numbers?

(a) 2:1 (b) 3:1

(c) 3:2 (d) None

16. If the harmonic mean between two positive numbers is to the inverse of their geometric mean as 12:13; then the numbers could be in the ratio

(a) 12:13 (b) 1/12:1/13

(c) 4:9 (d) 2:3

17. Fourth term of an arithmetic progression is 8. What is sum of the first 7 terms of the arithmetic progression?

(a) 7 (b) 64

(c) 56 (d) Cannot be determined

18. It takes the pendulum of a clock 7 seconds to strike 4 o'clock. How much time will it take to strike 11 o'clock?

(a) 18 seconds (b) 20 seconds

(c) 19.25 seconds (d) 23.33 seconds

19. Along a road lie an odd number of stones placed at intervals of 10 m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is

(a) 35 (b) 15

(c) 29 (d) 31

20. What is the smallest number which when increased by 5 is completely divisible by 8, 11 and 24?

(a) 264 (b) 259

(c) 269 (d) None of these

21. Which is the least number that must be subtracted from 1856, so that the remainder when divided by 7, 12 and 16 will leave the same remainder 4.

(a) 137 (b) 1361

(c) 140 (d) 172

22. Two positive integers differ by 4 and sum of their reciprocals is 10/21. Then one of the numbers is

(a) 3 (b) 1

(c) 5 (d) 21

23. $5^6 - 1$ is divisible by

(a) 13 (b) 31

(c) 5 (d) None of these

24. For the product $n(n + 1)(2n + 1)$. n Œ N, which one of the following is necessarily false?

(a) It is always even

(b) Divisible by 3

(c) Always divisible by the sum of the square of first n natural numbers

(d) Never divisible by 237

25. The remainder obtained when a prime number greater than 6 is divided by 6 is

(a) 1 or 3 (b) 1 or 5

(c) 3 or 5 (d) 4 or 5

Review Test 2

Directions for Questions 1 to 4: Four sisters Suvarna, Tara, Uma and Vibha playing a game such that the loser doubles the money of each of the other player. They played four games and each sister lost one game in alphabetical order. At the end of fourth game each sister had ` 32.

1. Who started with the lowest amount?

   (a) Suvarna (b) Tara

   (c) Uma (d) Vibha

2. Who started with the highest amount?

   (a) Suvarna (b) Tara

   (c) Uma (d) Vibha

3. What was the amount with Uma at the end of the second round?

   (a) 36 (b) 72

   (c) 16 (d) None of these

4. How many rupees did Suvarna start with?

   (a) 60 (b) 34

   (c) 66 (d) 28

5. If n is an integer, how many values of n will give an integral value of $(16n^2+7n+6)/n$?

   (a) 2 (b) 3

   (c) 4 (d) None of these

6. A student instead of finding the value of $7/8^{th}$ of a number found the value of $7/18^{th}$ of the number. If his answer differed from the actual one by 770. Find the numbers.

   (a) 1584 (b) 2520

   (c) 1728 (d) 1656

7. P and Q are two integers such that $PQ = 8^2$. Which of the following cannot be the value of $P+Q$?

(a) 20 (b) 65

(c) 16 (d) 35

8. If m and n are integers divisible by 5, which of the following is not necessarily true?

(a) $m - n$ is divisible by 5

(b) $m^2 - n^2$ is divisible by 25

(c) $m + n$ is divisible by 10

(d) None of the above

9. Which of the following is true?

(a) $7^{3^2} = (7^3)^2$ (b) $7^{3^2} > (7^3)^2$

(c) $7^{3^2} < (7^3)^2$ (d) None of these

10. P, Q and R are three consecutive odd numbers in ascending order. If the value of three times P is three less than two times R. find the value of R.

(a) 5 (b) 7

(c) 9 (d) 11

11. ABC is a three-digit number in which $A > 0$. The value of ABC is equal to the sum of the factorials of its three digits. What is the value of B?

(a) 9 (b) 7

(c) 4 (d) 2

12. A, B and C are defined as follows:

$A = (2.000004) \prod [(2.000004)^2 + (4.000008)]$

$B = (3.000003) \prod [(3.000003)^2 + (9.000009)]$

$C = (4.000002) \prod [(4.000002)^2 + (8.000004)]$

Which of the following is true about the value of the above three expressions?

(a) All of them lie between 0.18 and 0.20

(b) A is twice C

(c) C is the smallest

(d) B is the smallest

13. Let $x < 0.50$, $0 < y < 1$, $z > 1$. Given a set of numbers, the middle number, when they arranged in ascending order is called the median. So the median of the numbers x, y and z would be

(a) less than one (b) between 0 and 1

(c) greater than one (d) cannot say

14. let a, b, c d, and e be integers such that $a = 6b = 12c$, and $2b = 9d = 12e$. Then which of the following pairs contains a number that is not an integer?.

(a) $\left(\frac{a}{27}, \frac{b}{e}\right)$ (b) $\left(\frac{a}{36}, \frac{c}{e}\right)$

(c) $\left(\frac{a}{3}, \frac{bd}{9}\right)$ (d) $\left(\frac{a}{7}, \frac{c}{d}\right)$

15. If a, $a + 2$, and $a + 4$ are prime numbers, then the number of possible solutions for a is:

(a) One (b) Two

(c) Three (d) More than three

16. Let x and y be positive integers such that x is prime and y is composite. Then,

(a) $y - x$ cannot be an even integer

(b) xy cannot be an even integer

(c) $(x + y)$ cannot be an even integer

(d) None of the above statement is true

17. Let n (>1) be a composite natural number such that the square root of n is not an integer. Consider the following statements:

A: n has a factor which is greater than 1 and less than square root n

B: n has a factor which is greater than square root n but less than n

Then

(a) Both A and B are false

(b) Both A and B are true

(c) A is false but B is true

(d) A is true and B is false

18. What is the remainder when 4^{96} is divided by 6?

(a) 0 (b) 2

(c) 3 (d) 4

19. What is the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7?

(a) 646 (b) 676

(c) 683 (d) 797

20. The infinite sum $1+\frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4}+------$

(a) $\frac{27}{14}$ (b) $\frac{29}{13}$

(c) $\frac{49}{27}$ (d) $\frac{256}{147}$

21. If the product of n positive real numbers is unity, then their sum is necessarily:

(a) a multiple of n (b) equal to $n+\frac{1}{n}$

(c) never less than n (d) None of these

22. How many three digit positive integer, with digits x, y and z in the hundred's, ten's and unit's place respectively, exist such that $x < y$, $z < y$ and x π 0?

(a) 245 (b) 285

(c) 240 (d) 320

23. How many even integers n, where $100 \leq n \leq 200$, are divisible neither by seven nor by nine?

(a) 40 (b) 37

(c) 39 (d) 38

24. A positive whole number M less than 100 is represented in base 2 notation, base 3 notation, and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the three cases the leading digit in 1. Then M equals:

(a) 31 (b) 63

(c) 75 (d) 91

25. In a certain examinations paper, there are n questions. For $j = 1, 2 \ldots. n$. there are 2^{n-1} students who answered j or more questions wrongly. If the total number of wrong answers is 4095, then the value of n is:

(a) 12 (b) 11

(c) 10 (d) 9

Review Test 3

1. In 1936, I was as old as the number formed by the last two digits of my year of birth. Find the date of birth of my father who is 25 years older to me.

   (a) 1868 (b) 1893

   (c) 1902 (d) 1900

   (e) Can't be determined

2. Find the total number of integral solutions of the equation $(407)x - (ddd)y = 2589$, where '*ddd*' is a three-digit number.

   (a) 0 (b) 1

   (c) 2 (d) 3

   (e) Can't be determined

3. Find the digit at the ten's place of the number $N = 7^{281} \times 3^{264}$.

   (a) 0 (b) 1

   (c) 6 (d) 5

   (e) None of these

4. Raju went to a shop to buy a certain number of pens and pencils. Raju calculated the amount payable to the shopkeeper and offered that amount to him. Raju was surprised when the shopkeeper returned him ` 24 as balance. When he came back home, he realized that the shopkeeper had actually transposed the number of pens with the number of pencils. Which of the following is certainly an invalid statement?

   (a) The number of pencils that Raju wanted to buy was 8 more than the number of pens.

   (b) The number of pens that Raju wanted to buy was 6 less than the number of pencils.

   (c) A pen cost ` 4 more than a pencil.

   (d) None of the above.

5. HCF of 384 and a^5b^2 is 16ab. What is the correct relation between *a* and *b*?

(a) $a = 2b$ (b) $a + b = 3$

(c) $a - b = 3$ (d) $a + b = 5$

6. In ancient India, 0 to 25 years of age was called Brahmawastha and 25 to 50 was called Grahastha. I am in Grahastha and my younger brother is also in Grahastha such as the difference in our ages is 6 years and both of our ages are prime numbers. Also twice my brother's age is 31 more than my age. Find the sum of our ages.

(a) 80 (b) 68

(c) 70 (d) 71

7. Volume of a cube with integral sides is the same as the area of a square with integral sides. Which of these can be the volume of the cube formed by using the square and its replicas as the 6 faces?

(a) 19683 (b) 512

(c) 256 (d) Both (a) and (b)

8. Let A be a two-digit number and B be another two-digit number formed by reversing the digits of A. If A + B + (Product of digits of the number A) = 145, then what is the sum of the digits of A?

(a) 9 (b) 10

(c) 11 (d) 12

9. When a two-digit number N is divided by the sum of its digits, the result is Q. Find the minimum possible value of Q.

(a) 10 (b) 2

(c) 5.5 (d) 1.9

10. A one-digit number, which is the ten's digit of a two digit number *X*, is subtracted from *X* to give *Y* which is the quotient of the division of 999 by the cube of a number. Find the sum of the digits of *X*.

(a) 5 (b) 7

(c) 6 (d) 8

11. After Yuvraj hit 6 sixes in an over, Geoffery Boycott commented that Yuvraj just made 210 runs in the over. Harsha Bhogle was shocked and he asked Geoffery which base system was he using? What must have been Geoffery's answer?

(a) 9 (b) 2

(c) 5 (d) 4

12. Find the ten's digit of the number 7^{2010}.

(a) 0 (b) 1

(c) 2 (d) 4

13. Find the HCF of 481 and the number 'aaa' where 'a' is a number between 1 and 9 (both included).

(a) 73 (b) 1

(c) 27 (d) 37

14. The number of positive integer valued pairs (x, y), satisfying $4x - 17y = 1$ and $x < 1000$ is:

(a) 59 (b) 57

(c) 55 (d) 58

15. Let a, b, c be distinct digits. Consider a two digit number 'ab' and a three digit number 'ccb' both defined under the usual decimal number system. If $(ab)^2 = ccb$ and $ccb > 300$ then the value of b is:

(a) 1 (b) 0

(c) 5 (d) 6

16. The remainder 7^{84} is divided by 342 is:

(a) 0 (b) 1

(c) 49 (d) 341

17. Let x, y and z be distinct integers, x and y are odd and positive, and z is even and positive. Which one of the following statements can't be true?

(a) $(x - z)^2 y$ is even (b) $(x - z)y^2$ is odd

(c) $(x - y)y$ is odd (d) $(x - y)^2 z$ is even

18. A boy starts adding consecutive natural numbers starting with 1. After some time he reaches a total of 1000 when he realizes that he has made the mistake of double counting 1 number. Find the number double counted.

(a) 44 (b) 45

(c) 10 (d) 12

19. In a number system the product of 44 and 11 is 1034. The number 3111 of this system, when converted to decimal number system, becomes:

(a) 406 (b) 1086

(c) 213 (d) 691

20. Ashish is given ` 158 in one rupee denominations. He has been asked to allocate them into a number of bags such that any amount required between Re 1 and ` 158 can be given by handing out a certain number of bags without opening them. What is the minimum number of bags required?

(a) 11 (b) 12

(c) 13 (d) None of these

Review Test 4

1. Find the number of 6-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 once such that the 6-digit number is divisible by its unit digit.

   (a) 648 (b) 528

   (c) 728 (d) 128

2. Which is the highest 3-digit number that divides the number 11111….1(27 times) perfectly without leaving any remainder?

   (a) 111 (b) 333

   (c) 666 (d) 999

3. W1, W2, … , W7 are 7 positive integral values such that by attaching the coefficients of +1, 0 and -1 to each value available and adding the resultant values, any number from 1 to 1093 (both included) could be formed. If W1, W2, … , W7 are in ascending order, then what is the value of W3?

   (a) 10 (b) 9

   (c) 0 (d) 1

4. What is the unit digit of the number $63^{25} + 25^{63}$?

   (a) 3 (b) 5

   (c) 8 (d) 2

5. Find the remainder when $(2222^{5555} + 5555^{2222})$ is divided by 7.

   (a) 1 (b) 0

   (c) 2 (d) 5

6. What is the number of nines used in numbering a 453 page book?

   (a) 86 (b) 87

   (c) 84 (d) 85

7. How many four digit numbers are divisible by 5 but not by 25?

(a) 2000 (b) 8000

(c) 1440 (d) 9999

8. The sum of two integers is 10 and the sum of their reciprocals is 5/12. What is the value of larger of these integers?

(a) 7 (b) 5

(c) 6 (d) 4

9. Saurabh was born in 1989. His elder brother Siddhartha was also born in the 1980's such that the last two digits of his year of birth form a prime number P. Find the remainder when (P^2 + 11) is divided by 5.

(a) 0 (b) 1

(c) 2 (d) 3

10. The HCF of x and y is H. Find the HCF of $(x - y)$ and $(x^3 + y^3)/(x^2 - xy + y^2)$.

(a) H – 1 (b) H^2

(c) H (d) H + 1

11. 4 bells toll together at 9:00 A.M. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours?

(a) 3 (b) 5

(c) 6 (d) 9

12. What power of 210 will exactly divide 142!

(a) 22 (b) 11

(c) 34 (d) 33

13. Find the total numbers between 122 and 442 that are divisible by 3 but not by 9.

(a) 70 (b) 71

(c) 72 (d) 73

14. If 146! is divisible by 6^n, then find the maximum value of n.

(a) 74 (b) 75

(c) 76 (d) 70

15. If we add the square of the digit in the tens place of a positive two-digit number to the product of the digits of that number, we shall get 52, and if we add the square of the digit in the units place to the same product of the digits, we shall get 117. Find the two-digit number.

(a) 18 (b) 39

(c) 49 (d) 28

16. Find the smallest natural number n such that $(n + 1)n[(n - 1)!]$ is divisible by 990.

(a) 2 (b) 4

(c) 10 (d) 11

17. If x, y and z are odd, even and odd respectively, then $(x^2 - yz^2 + y^3)$ and $(x^2 + y^2 + z^2)$ are respectively:

(a) Odd & Odd (b) Even & Odd

(c) Odd & Even (d) Odd & Odd

18. A two digit number N has its digits reversed to form another two digit number M. What could be the unit digit of M if product of M and N is 574?

(a) 1 (b) 3

(c) 6 (d) 9

19. For what relation between b and c is the number abcacb divisible by 7, if $b > c$?

(a) $b + c = 7$ (b) $b = c + 7$

(c) $2bc = 7$ (d) $c = 7b$

20. What is the remainder when a^6 is divided by $(a + 1)$?

(a) $a + 1$ (b) a

(c) 0 (d) 1

Review Test 5

1. What is the last digit of 62^43^54^65^76^87?

   (a) 2 (b) 4

   (c) 6 (d) 8

2. $N = 99^3 - 36^3 - 63^3$, how many factors does N have?

   (a) 51 (b) 96

   (c) 128 (d) 192

3. Find the highest power of 2 in 1! + 2! + 3! + 4!..............600!

   (a) 1 (b) 494

   (c) 3. 0 (d) 256

4. 100! is divisible by 160^n...what is the max. integral value of *n*?

   (a) 19 (b) 24

   (c) 26 (d) 28

5. What is the sum of the digits of the decimal form of the product $2^{999} * 5^{1001}$?

   (a) 2 (b) 4

   (c) 5 (d) 7

6. What is the remainder when 1*1 + 11*11 + 111*111 + 1111*1111 +......+ (2001 times 1)*(2001 times 1) is divided by 100 ?

   (a) 99 (b) 22

   (c) 01 (d) 21

7. What is the remainder when 789456123 is divided by 999?

   (a) 123 (b) 369

   (c) 963 (d) 189

8. What is the total number of the factors of 16!

   (a) 2016 (b) 1024

(c) 3780 (d) 5376

9. Find the sum of the first 125 terms of the sequence 1, 2, 1, 3, 2, 1, 4, 3, 2, 1, 5, 4, 3, 2...

(a) 616 (b) 460

(c) 750 (d) 720

10. Umesh purchased a Tata Nano recently, but the faulty car odometer of Tata Nano proceeds from digit 4 to digit 6, always skipping the digit 5, regardless of position. If the odometer now reads 003008 (starting with 000000), how many km has Nano actually travelled?

(a) 2100 (b) 1999

(c) 2194 (d) 2195

11. What is the number of consecutive zeroes in the end of 1000!?

(a) 248 (b) 249

(c) 250 (d) 251

12. Mr. Ramlal lived his entire life during the 1800s. In the last year of his life, Ramlal stated: Once I was x years old in the year x^2. He was born in the year

(a) 1822 (b) 1851

(c) 1853 (d) 1806

13. Find the unit's digit of LCM of $13^{501} - 1$ and $13^{501} + 1$

(a) 2 (b) 4

(c) 5 (d) 8

14. If you were to add all odd numbers between 1 and 2007 (both inclusive), the result would be

(a) A perfect square (b) Divisible by 2008

(c) Multiple of 251 (d) All of the above

15. Find the remainder when $971(30^{99} + 61^{100}) * (1148)^{56}$ is divided by 31

(a) 25 (b) 0

(c) 11 (d) 21

16. What is the remainder when 2^{100} is divided by 101?

(a) 1 (b) 100

(c) 0 (d) 99

17. Find the last two digits of 2^{134}

(a) 04 (b) 84

(c) 24 (d) 64

18. Find the remainder when $(10^3 + 9^3)^{1000}$ by 12^3

(a) 01 (b) 11

(c) 1001 (d) 1727

19. The number of factors of the number 3000 are:

(a) 16 (b) 32

(c) 24 (d) 28

20. If N! has 73 zeroes at the end then find the value of N?

(a) 295 (b) 300

(c) 290 (d) Not possible

ANSWER KEY

Review Test 1

1. (a)	2. (c)	3. (d)	4. (a)
5. (b)	6. (a)	7. (d)	8. (c)
9. (a)	10. (b)	11. (c)	12. (b)
13. (a)	14. (a)	15. (a)	16. (c)
17. (c)	18. (d)	19. (d)	20. (b)
21. (d)	22. (a)	23. (b)	24. (d)
25. (b)			

Review Test 2

1. (d)	2. (a)	3. (b)	4. (c)
5. (c)	6. (a)	7. (d)	8. (c)
9. (b)	10. (c)	11. (c)	12. (d)
13. (b)	14. (d)	15. (a)	16. (d)
17. (b)	18. (d)	19. (b)	20. (c)
21. (c)	22. (c)	23. (c)	24. (d)
25. (a)			

Review Test 3

1. (b)	2. (a)	3. (c)	4. (d)
5. (d)	6. (a)	7. (d)	8. (c)
9. (d)	10. (a)	11. (d)	12. (d)
13. (d)	14. (a)	15. (a)	16. (b)
17. (a)	18. (c)	19. (a)	20. (d)

Review Test 4

1. (a)	2. (d)	3. (b)	4. (c)
5. (b)	6. (d)	7. (c)	8. (c)
9. (a)	10. (c)	11. (b)	12. (a)
13. (b)	14. (d)	15. (c)	16. (c)
17. (c)	18. (a)	19. (b)	20. (d)

Review Test 5

1. (a)	2. (b)	3. (c)	4. (a)
5. (d)	6. (c)	7. (b)	8. (d)
9. (c)	10. (c)	11. (b)	12. (d)
13. (b)	14. (d)	15. (b)	16. (a)
17. (b)	18. (a)	19. (b)	20. (d)

BLOCK II

AVERAGES AND MIXTURES

Chapter 3
Averages

Chapter 4
Alligations

...BACK TO SCHOOL

- **The Relevance of Average**

Average is one of the most important mathematical concepts that we use in our day-to-day life. In fact, even the most non-mathematical individuals regularly utilize the concept of averages on a day-to-day basis.

So, we use averages in all the following and many more instances.

- How a class of students fared in an exam is assessed by looking at the average score.
- What is the average price of items purchased by an individual.
- A person might be interested in knowing his average telephone expenditure, electricity expenditure, petrol expenditure, etc.
- A manager might be interested in finding out the average sales per territory or even the average growth rate month to month.
- Clearly there can be immense application of averages that you might be able to visualise on your own.
- **The Meaning of an Average**

An average is best seen as a representative value which can be used to represent the value of the general term in a group of values.

For instance, suppose that a cricket team had 10 partnerships as follows:

1st wicket 28	2nd wicket 42
3rd wicket 112	4th wicket 52
5th wicket 0	6th wicket 23
7th wicket 41	8th wicket 18

9th wicket 9

10th wicket 15

On adding the ten values above, we get a total of 340—which gives an average of 34 runs per wicket, i.e. the average partnership of the team was 34 runs.

In other words, if we were to replace the value of all the ten partnerships by 34 runs, we would get the same total score. Hence, 34 represents the average partnership value for the team.

Suppose, in a cricket series of 5 matches between 2 countries, you are given that Team A had an average partnership of 58 Runs per wicket while Team B had an average partnership of 34 runs per wicket. What conclusion can you draw about the performance of the two teams, given that both the teams played 5 complete test matches?

Obviously, Team B would have performed much worse than Team A: For that matter, if I tell you that the average daytime high temperature of Lucknow was 18° C for a particular month, you can easily draw some kind of conclusion in your mind about the month we could possibly be talking about.

Thus, you should realize that the beauty of averages lies in the fact that it is one single number that tells you a lot about the group of numbers —hence, it is one number that represents an entire group of numbers.

But one of the key concepts that you need to understand before you move into the chapters of this block is the concept of <u>WEIGHTED AVERAGES.</u>

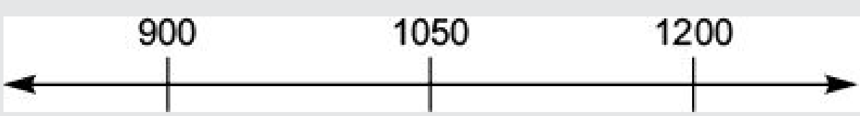
As always, the concept is best explained through a concrete example.

Suppose I had to buy a shirt and a trousers and let us say that the average cost of a shirt was ` 1200 while that of a trousers was ` 900.

In such a case, the average cost of a shirt and a trousers would be given by (1200 + 900)/2 = 1050.

This can be visualised on the number line as:

(midpoint) = answer



As you can easily see in the figure, the average occurs at the midpoint of the two numbers.

Now, let us try to modify the situation:

Suppose, I were to buy 2 trousers and 1 shirt. In such a case I would end up spending (900 + 900 + 1200) = ` 3000 in buying a total of 3 items. What would be my average in this case?

Obviously, 3000/3 = ` 1000!!

Clearly, the average has shifted!!

On the number line we could visualise this as follows:

(Answer) (midpoint)

It is clearly visible that the average has shifted towards 900 (which was the cost price of the trousers—the larger purchased item.)

In a way, this shift is similar to the way a two pan weighing balance shifts on weights being put on it. The balance shifts towards the pan containing the larger weight.

Similarly, in this case, the correct average (1000) is closer to 900 than it is to 1200. This has happened because the number of elements in the group of average 900 is greater than the number of elements in the group having average 1200. Since, this is very similar to the system of weights, we call this as a weighted average situation.

At this stage, you should realize that weighted averages are not solely restricted to two groups. We can also come up with a weighted average situation for three groups (although in such a case the representation of the weighted average on the number line might not be so easily possible.) In fact, it is the number line representation of a weighted average situation that is defined as alligation (when 2 groups are involved).

Pre-assessment Test

1. X's age is $1/10^{th}$ of Y's present age. Y's age will be thrice of Z's age after 10 years. If Z's eighth birthday was celebrated two years ago, then the present age of X must be

   (a) 5 years (b) 10 years

   (c) 15 years (d) 20 years

2. Dravid was twice as old as Rahul 10 years back. How old is Rahul today if Dravid will be 45 years old 15 years hence?

   (a) 20 years (b) 10 years

   (c) 30 years (d) None of these

3. A demographic survey of 100 families in which two parents were present revealed that the average age A, of the oldest child, is 15 years less than ½ the sum of the ages of the two parents. If X represents the age of one parent and Y the age of the other parent, then which of the following is equivalent to A?

   (a) $\frac{X+Y}{2} - 15$ (b) $\frac{X+Y}{2} + 15$

   (c) $\frac{X+Y}{2} - 15$ (d) X + Y – 7.5

4. If 10 years are subtracted from the present age of Randy and the remainder divided by 12, then you would get the present age of his grandson Sandy. If Sandy is 19 years younger to Sundy whose age is 24, then what is the present age of Randy?

   (a) 80 years (b) 70 years

   (c) 60 years (d) None of these

5. Two groups of students, whose average ages are 15 years and 25 years, combine to form a third group whose average age is 23 years. What is the ratio of the number of students in the first group to the number of students in the second group?

(a) 8 : 2 (b) 2 : 8

(c) 4 : 6 (d) None of these

6. A year ago, Mohit was four times his son's age. In six years, his age will be 9 more than twice his son's age. What is the present age of the son?

(a) 10 years (b) 9 years

(c) 20 years (d) None of these

7. In 1952, I was as old as the number formed by the last two digits of my birth year. When I mentioned this interesting coincidence to my grandfather, he surprised me by saying that the same applied to him also.

The difference in our ages is:

(a) 40 years (b) 50 years

(c) 60 years (d) None of these

8. The average age of three boys is 18 years. If their ages are in the ratio 4:5:9, then the age of the youngest boy is

(a) 8 years (b) 9 years

(c) 12 years (d) 16 years

9. "I am eight times as old as you were when I was as old as you are", said a man to his son. Find out their present ages if the sum of their ages is 75 years.

(a) 40 years and 35 years

(b) 56 years and 19 years

(c) 48 years and 27 years

(d) None of these

10. My brother was 3 years of age when my sister was born, while my mother was 26 years of age when I was born. If my sister was 4 years of age when I was born, then what was the age of my father and mother respectively when my brother was born?

(a) 35 years, 33 years (b) 35 years, 29 years

(c) 32 years, 23 years (d) None of these

11. Namrata's father is now four times her age. In five years, he will be three times her age. In how many years, will he be twice her age?

(a) 5 (b) 20

(c) 25 (d) 15

12. A father is twice as old as his daughter. 20 years back, he was seven times as old as the daughter. What are their present ages?

(a) 24, 12 (b) 44, 22

(c) 48, 24 (d) none of these

13. The present ages of three persons are in the proportion of 5:8:7. Eight years ago, the sum of their ages was 76. Find the present age of the youngest person.

(a) 20 (b) 25

(c) 30 (d) None of these

14. The average age of a class is 14.8 years. The average age of the boys in the class is 15.4 years and that of girls is 14.4 years. What is the ratio of boys to girls in the class?

(a) 1 : 2 (b) 3 : 2

(c) 2 : 3 (d) None of these

15. In an organisation, the daily average wages of 20 illiterate employees is decreased from ` 25 to ` 10, thus the average salary of all the literate (educated) and illiterate employees is decreased by ` 10 per day. The number of educated employees working in the organisation are:

(a) 15 (b) 20

(c) 10 (d) 25

16. Mr.Akhilesh Bajpai while going from Lucknow to Jamshedpur covered half the distance by train at the speed of 96 km/hr, half the rest of the distance by his scooter at the speed of 60 km/hr and the

remaining distance at the speed of 40 km/hr by car. The average speed at which he completed his journey is:

(a) 64 km/hr (b) 56 km/hr

(c) 60 km/hr (d) 36 km/hr

17. There are four types of candidates in AMS Learning Systems preparing for the CAT. The number of students of Engineering, Science, Commerce and Humanities is 400, 600, 500 and 300 respectively and the respective percentage of students who qualified the CAT is 80%, 75%, 60% and 50% respectively the overall percentage of successful candidates in our institute is:

(a) 67.77% (b) 66.66%

(c) 68.5% (d) None of these

18. Mr. Jagmohan calculated the average of 10 'Three digit numbers'. But due to mistake he reversed the digits of a number and thus his average increased by 29.7. The difference between the unit digit and hundreds digit of that number is:

(a) 4 (b) 3

(c) 2 (d) can't be determined

Directions for Questions 19 and 20: Answer the questions based on the following Information.

Production pattern for number of units (in cubic feet) per day.

Days	1	2	3	4	5	6	7
Numbers of units	150	180	120	250	160	120	150

For a truck that can carry 2,000 cubic feet, hiring cost per day is `1,000. Storing cost per cubic feet is `5 per day. Any residual material left at the end of the seventh day has to be transferred.

19. If all the units should be sent to the market, then on which days should the trucks be hired to minimize the cost:

(a) 2^{nd} , 4^{th} , 6^{th} , 7^{th} (b) 7^{th}

(c) 2^{nd} , 4^{th} , 5^{th} , 7^{th} (d) None of these

20. If the storage cost is reduced to ` 0.9 per cubic feet per day, then on which day/days, should the truck be hired?

(a) 4^{th} (b) 7^{th}

(c) 4^{th} and 7^{th} (d) None of these

ANSWER KEY

1. (a)	2. (a)	3. (a)	4. (b)
5. (b)	6. (b)	7. (b)	8. (c)
9. (c)	10. (d)	11. (b)	12. (c)
13. (b)	14. (c)	15. (c)	16. (a)
17. (a)	18. (b)	19. (a)	20. (b)

SCORE INTERPRETATION ALGORITHM FOR PRE-ASSESSMENT TEST OF BLOCK II

If You Scored: < 12: (In Unlimited Time)

Step One

Go through the first chapter of the block, viz., Averages. When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

Then move onto the LOD 1 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1. While doing so do not think about the time requirement .Once you finish solving LOD 1, revise the questions and their solution processes.

Also at this stage study the concept of averages from your school text books (Class 8, 9 & 10) and solve all the questions which are available to you in those books.

Step Two

After finishing LOD 1 of Averages, move into LOD 2 and then LOD 3 of this chapter.

Step Three

Go to the chapter of alligations and study the shortcuts provided carefully. Understand the use of the alligation process as clearly as possible.

Then move to the LOD 1 exercise of the same. (Note: The chapter of alligations does not have an LOD2 & LOD 3 exercise)

Step Four

Go to the practice test given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your unlimited time score.

In case the growth in your score is not significant, go back to the theory of each chapter and review each of the questions you have solved for both the chapters.

If You Scored: > 12 (In Unlimited Time)

Follow the same process as above. The only difference is that the school book work is optional – do it only if you feel you need to. However, your concentration during the solving of the two chapters has to be on developing your speed at solving questions on this chapter.

Chapter 3 Averages

THEORY

The average of a number is a measure of the central tendency of a set of numbers. In other words, it is an estimate of where the center point of a set of numbers lies.

The basic formula for the average of n numbers $x_1, x_2, x_3, \ldots x_n$ is

$A_n = (x_1 + x_2 + x_3 + \ldots + x_n)/n$ = (Total of set of n numbers)/n

This also means $A_n \times n$ = total of the set of numbers.

The average is always calculated for a set of numbers.

Concept of weighted average: When we have two or more groups whose individual averages are known, then to find the combined average of all the elements of all the groups we use weighted average. Thus, if we have k groups with averages $A_1, A_2 \ldots A_k$ and having $n_1, n_2 \ldots n_k$ elements then the weighted average is given by the formula:

$$A_w = \frac{n_1 A_1 + n_2 A_2 + n_3 A_3 + \ldots + n_k A_k}{n_1 + n_2 + n_3 + \ldots + n_k}$$

Another meaning of average The average [also known as *arithmetic mean* (AM)] of a set of numbers can also be defined as the number by which we can replace each and every number of the set without changing the total of the set of numbers.

Properties of average (AM) The properties of averages [arithmetic mean] can be elucidated by the following examples:

Example 1: The average of 4 numbers 12, 13, 17 and 18 is:

Solution: Required average = (12 + 13 + 17 + 18)/4 = 60/4 = 15

This means that if each of the 4 numbers of the set were replaced by 15 each, there would be no change in the total.

This is an important way to look at averages. In fact, whenever you come across any situation where the average of a group of '*n*' numbers is given, you should visualise that there are '*n*' numbers, each of whose value is the average of the group. This view is a very important way to visualise averages.

This can be visualised as

12 Æ +3 Æ 15

13 Æ +2 Æ 15

17 Æ –2 Æ 15

18 Æ –3 Æ 15

60 Æ +0 Æ 60

Example 2: In Example 1, visualise addition of a fifth number, which increases the average by 1.

15 + 1 = 16

15 + 1 = 16

15 + 1 = 16

15 + 1 = 16

The +1 appearing 4 times is due to the fifth number, which is able to maintain the average of 16 first and then 'give one' to each of the first 4.

Hence, the fifth number in this case is 20

Example 3: The average always lies above the lowest number of the set and below the highest number of the set.

Example 4: The net deficit due to the numbers below the average always equals the net surplus due to the numbers above the average.

Example 5: *Ages and averages:* If the average age of a group of persons is x years today then after n years their average age will be $(x + n)$.

Also, n years ago their average age would have been $(x – n)$. This happens due to the fact that for a group of people, 1 year is added to each person's age every year.

Example 6: A man travels at 60 kmph on the journey from A to B and returns at 100 kmph. Find his average speed for the journey.

Solution:

Average speed = (total distance)/(total time)

If we assume distance between 2 points to be d

Then

Average speed = $2d/[(d/60) + (d/100)] = (2 \times 60 \times 100)/ (60 + 100) = (2 \times 60 \times 100)/160 = 75$

Average speed = $(2S_1 \lozenge S_2)/(S_1 + S_2)$ [S_1 and S_2 are speeds]

of going and coming back respectively.

Short Cut The average speed will always come out by the following process:

The ratio of speeds is 60 : 100 = 3 : 5 (say $r_1 : r_2$)

Then, divide the difference of speeds (40 in this case) by $r_1 + r_2$ (3 + 5 = 8, in this case) to get one part. (40/8 = 5, in this case)

The required answer will be three parts away (i.e. r_1 parts away) from the lower speed.

Check out how this works with the following speeds:

$S_1 = 20$ and $S_2 = 40$

Step 1: Ratio of speeds = 20 : 40 = 1 : 2

Step 2: Divide difference of 20 into 3 parts $(r_1 + r_2)$ Æ = 20/3 = 6.66

Required average speed = 20 + 1 × 6.66

Note: This process is essentially based on alligations and we shall see it again in the next chapter.

Exercise for Self-practice

Find the average speed for the above problem if

(1) $S_1 = 20$ $S_2 = 200$

(2) $S_1 = 60$ $S_2 = 120$

(3) $S_1 = 100$ $S_2 = 50$

(4) $S_1 = 60$ $S_2 = 180$

WORKED-OUT PROBLEMS

Problem 3.1 The average of a batsman after 25 innings was 56 runs per innings. If after the 26th inning his average increased by 2 runs, then what was his score in the 26th inning?

Solution *Normal process:*

Runs in 26th inning = Runs total after 26 innings – Runs total after 25 innings

$= 26 \times 58 - 25 \times 56$

For mental calculation use:

$(56 + 2) \times 26 - 56 \times 25$

$= 2 \times 26 + (56 \times 26 - 56 \times 25)$

$= 52 + 56 = 108$

Short Cut Since the average increases by 2 runs per innings it is equivalent to 2 runs being added to each score in the first 25 innings. Now, since these runs can only be added by the runs scored in the 26th inning, the score in the 26th inning must be $25 \times 2 = 50$ runs higher than the average after 26 innings (i.e. new average = 58).

Hence, runs scored in 26th inning = New Average + Old innings × Change in average

$= 58 + 25 \times 2 = 108$

Visualise this as

Average in first 25 innings	Average after 26 innings
56	58
56	58
56	58
…	…

…		…

25 times…		26 times…

Difference in total is two, 25 times and 58 once, that is, $58 + 25 \times 2$.

Problem 3.2 The average age of a class of 30 students and a teacher reduces by 0.5 years if we exclude the teacher.

If the initial average is 14 years, find the age of the class teacher.

Solution *Normal process:*

Age of teacher = Total age of (students + teacher)

–Total age of students

$= 31 \times 14 - 30 \times 13.5$

$= 434 - 405$

= 29 years

Short Cut The teacher after fulfilling the average of 14 (for the group to which he belonged) is also able to give 0.5 years to the age of each of the 30 students. Hence, he has 30 × 0.5 Æ 15 years to give over and above maintaining his own average age of 14 years.

Age of teacher = $14 + 30 \times 0.5 = 29$ years

(*Note:* This problem should be viewed as change of average from 13.5 to 14 when teacher is included.)

Problem 3.3 The average marks of a group of 20 students on a test is reduced by 4 when the topper who scored 90 marks is replaced by a new student. How many marks did the new student have?

Solution *Normal process:*

Let initial average be x.

Then the initial total is $20x$

New average will be $(x - 4)$ and the new total will be $20(x - 4) = 20x - 80$.

The reduction of 80 is created by the replacement.

Hence, the new student has 80 marks less than the student he replaces. Hence, he must have scored 10 marks.

Short Cut The replacement has the effect of reducing the average marks for each of the 20 students by 4. Hence, the replacement must be 20 × 4 = 80 marks below the original.

Hence, answer = 10 marks.

Problem 3.4 The average age of 3 students *A*, *B* and *C* is 48 marks. Another student *D* joins the group and the new average becomes 44 marks. If another student *E*, who has 3 marks more than *D*, joins the group, the average of the 4 students *B*, *C*, *D* and *E* becomes 43 marks. Find how many marks *A* got in the exam.

Solution Solve while reading. The first sentence gives you a total of 144 for *A*, *B* and *C*'s marks. *Second sentence*: When *D* joins the group, the total becomes 44 × 4 = 176. Hence *D* must get 32 marks.

Alternatively, you can reach this point by considering the first 2 statements together as:

D's joining the group reduces the average from 48 to 44 marks (i.e. 4 marks).

This means that to maintain the average of 44 marks, *D* has to take 4 marks from *A*, 4 from *B* and 4 from *C* Æ *A* total of 12 marks. Hence, he must have got 32 marks.

From here:

The first part of the third sentence gives us information about *E* getting 3 marks more than 32 Æ Hence, *E* gets 35 marks.

Now, it is further stated that when *A* is replaced by *E*, the average marks of the students reduces by 1 to 43.

Mathematically this can be shown as

$A + B + C + D = 44 \times 4 = 176$ while, $B + C + D + E$

$= 43 \times 4 = 172$

Subtracting the two equations, we get $A - E = 4$ marks.

Hence, *A* would have got 39 marks.

Alternatively, you can think of this as:

The replacement of *A* with *E* results in the reduction of 1 mark from each of the 4 people who belong to the group. Hence, the difference is 4 marks.

Hence, *A* would get 4 marks more than *E* i.e. A gets 39 marks.

Problem 3.5 The mean temperature of Monday to Wednesday was 27 °C and of Tuesday to Thursday was 24 °C. If the temperature on Thursday was 2/3rd of the temperature on Monday, what was the temperature on Thursday?

Solution From the first sentence, we get that the total from Monday to Wednesday was 81 while from Tuesday to Thursday was 72. The difference is arising out of the replacement of Monday by Thursday.

This can be mathematically written as

Mon + Tue + Wed = 81 (1)

Tue + Wed + Thu = 72 (2)

Hence, Mon – Thu = 9

We have two unknown variables in the above equation. To solve for 2 unknowns, we need a new equation. Looking back at the problem we get the equation:

Thu = (2/3) × Mon

Solving the two equations we get: Thursday = 18 °C.

However, in the exam, you should avoid using equation-solving as much as possible. You should, ideally, be able to reach half way through the solution during the first reading of the question, and then meet the gap through the use of options.

The answer to this problem should be got by the time you finish reading the question for the first time.

Thus suppose we have the equations:

$M - T = 9$ and $T = 2M/3$ or $T/M = 2/3$ and have the options for T as

(a) 12 (b) 15

(c) 18 (d) 27

To check which of these options is the appropriate value, we need to check one by one.

Option (a) gives $T = 12$, then we have $M = 21$. But 12/21 π 2/3. Hence, this is not the correct option.

Option (b) gives T = 15, then M = 24. But again 15/24 π 2/3. Hence, this is not the correct option.

Option (c) gives T = 18, then M = 27. Now 18/27 = 2/3. Hence, this is the correct option.

So we no longer need to check for option (d).

However, if we had checked for option (d) then T = 27, so M = 36. But again 27/36 π 2/3. Hence, this is not the correct option.

In the above, we used 'solving-while-reading' and 'option-based' approaches.

These two approaches are very important and by combining the two, you can reach amazing speeds in solving the question.

You are advised to practice both these approaches while solving questions, which will surely improve your efficiency and speed. You will see that, with practice, you will be able to arrive at the solution to most of the LOD I problems (given later in this chapter) even as you finish reading the questions. And since it is the LOD I level problems that appear in most examinations (like CMAT, Bank PO, MAT, Indo MAT, NMIMS, NIFT, NLS and most other aptitude exams) you will gain a significant advantage in solving these problems.

On LOD II, LOD III and CAT type problems, you will find that using solving-while-reading and option-based approaches together would take you through anywhere between 30–70% of the question by the time you finish reading the question for the first time.

This will give you a tremendous time advantage over the other students appearing in the examination.

Problem 3.6 A person covers half his journey by train at 60 kmph, the remainder half by bus at 30 kmph and the rest by cycle at 10 kmph. Find his average speed during the entire journey.

Solution Recognise that the journey by bus and that by cycle are of equal distance. Hence, we can use the short cut illustrated earlier to solve this part of the problem.

Using the process explained above, we get average speed of the second half of the journey as

$10 + 1 \times 5 = 15$ kmph

Then we employ the same technique for the first part and get

$15 + 1 \times 9 = 24$ kmph (Answer)

Problem 3.7 A school has only 3 classes that contain 10, 20 and 30 students respectively. The pass percentage of these classes are 20% , 30% and 40% respectively. Find the pass percentage of the entire school.

Solution

Using weighted average: $\frac{10 \times 0.2 + 20 \times 0.3 + 30 \times 0.4}{10 + 20 + 30} = \frac{20}{60} = 33.33\%$

Alternatively, we can also use solving-while-reading as

Recognize that the pass percentage would be given by

$$\frac{\text{Passed students}}{\text{Total students}}$$

As soon as you get into the second line of the question get back to the first sentence and get the total number of passed students = 2 + 6 + 12 and you are through with the problem.

LEVEL OF DIFFICULTY (I)

1. The average age of 24 students and the principal is 15 years. When the principal's age is excluded, the average age decreases by 1 year. What is the age of the principal?

   (a) 38 (b) 40
   (c) 39 (d) 37

2. The average weight of 3 men A, B and C is 84 kg. Another man D joins the group and the average now becomes 80 kg. If another man E, whose weight is 3 kg more than that of D, replaces A then the average weight of B, C, D and E becomes 78 kg. The weight of A is

   (a) 70 kg (b) 72 kg
   (c) 79 kg (d) 78 kg

3. The mean temperature of Monday to Wednesday was 37 °C and of Tuesday to Thursday was 34 °C. If the temperature on Thursday was 4/5 that of Monday, the temperature on Thursday was

   (a) 38 °C (b) 36 °C
   (c) 40 °C (d) 39 °C

4. Three years ago, the average age of A, B and C was 27 years and that of B and C 5 years ago was 20 years. A's present age is

   (a) 30 years (b) 35 years
   (c) 40 years (d) 48 years

5. Ajit Tendulkar has a certain average for 9 innings. In the tenth inning, he scores 100 runs thereby increasing his average by 8 runs. His new average is

   (a) 20 (b) 24
   (c) 28 (d) 32

6. The average of the first five multiples of 7 is

(a) 20 (b) 21

(c) 28 (d) 30

7. There are three fractions *A*, *B* and *C*. If $A = \frac{1}{4}$ and *B* = 1/6 and the average of *A*, *B* and *C* is 1/12. What is the value of *C*?

(a) –1/2 (b) – 1/6

(c) –1/3 (d) – 1/4

8. The marks obtained by Hare Rama in Mathematics, English and Biology are respectively 93 out of 100, 78 out of 150 and 177 out of 200. Find his average score in percent.

(a) 87.83 (b) 86.83

(c) 76.33 (d) 77.33

9. The average monthly expenditure of a family was ` 2750 for the first 3 months, ` 3150 for the next three months and ` 6750 for the next three months. Find the average income of the family for the 9 months, if they save ` 650 per month.

(a) 4866.66 (b) 5123.33

(c) 4666.66 (d) 4216.66

10. The average age of a family of 6 members is 22 years. If the age of the youngest member be 7 years, what was the average age of the family at the birth of the youngest member?

(a) 15 (b) 18

(c) 21 (d) 12

11. The average age of 8 persons in a committee is increased by 2 years when two men aged 35 years and 45 years are substituted by two women. Find the average age of the two women.

(a) 48 (b) 45

(c) 51 (d) 42

12. The average temperature for Wednesday, Thursday and Friday was 40 °C. The average for Thursday, Friday and Saturday was 41 °C. If the temperature on Saturday was 42 °C, what was the temperature on Wednesday?

(a) 39 °C (b) 44 °C

(c) 38 °C (d) 41 °C

13. The speed of the train in going from Nagpur to Allahabad is 100 km/hr while when coming back from Allahabad to Nagpur, its speed is 150 km/hr. Find the average speed during the whole journey.

(a) 125 (b) 75

(c) 135 (d) 120

14. The average weight of a class of 29 students is 40 kg. If the weight of the teacher be included, the average rises by 500 gm. What is the weight of the teacher?

(a) 40.5 kg (b) 50.5 kg

(c) 45 kg (d) 55 kg

15. The average of 3 numbers is 17 and that of the first two is 16. Find the third number.

(a) 15 (b) 16

(c) 17 (d) 19

16. The average weight of 19 men in a ship is increased by 3.5 kg when one of the men, who weighs 79 kg, is replaced by a new man. Find the weight of the new man upto 2 decimal places

(a) 105.75 (b) 107.55

(c) 145.50 (d) 140.50

17. The age of Shaurya and Kauravki is in the ratio 2 : 6. After 5 years, the ratio of their ages will become 6 : 8. Find the average of their ages after 10 years.

(a) 12 (b) 13

(c) 17 (d) 24

18. Find the average of the first 97 natural numbers.

(a) 47 (b) 37

(c) 48 (d) 49

19. Find the average of all prime numbers between 30 and 50.

(a) 39.8 (b) 38.8

(c) 37.8 (d) 41.8

20. If we take four numbers, the average of the first three is 16 and that of the last three is 15. If the last number is 18, the first number is

(a) 20 (b) 21

(c) 23 (d) 25

21. The average of 5 consecutive numbers is n. If the next two numbers are also included, the average will.

(a) increase by 1 (b) remain the same

(c) increase by 1.4 (d) increase by 2

22. The average of 50 numbers is 38. If two numbers, namely, 45 and 55 are discarded, the average of the remaining numbers is

(a) 36.5 (b) 37

(c) 37.6 (d) 37.5

23. The average of ten numbers is 7. If each number is multiplied by 12, then the average of the new set of numbers is

(a) 7 (b) 19

(c) 82 (d) 84

24. In a family of 8 males and a few ladies, the average monthly consumption of grain per head is 10.8 kg. If the average monthly consumption per head be 15 kg in the case of males and 6 kg in the case of females, find the number of females in the family.

(a) 8 (b) 7

(c) 9 (d) 15

25. Average marks obtained by a student in 3 papers is 52 and in the fourth paper he obtains 60 marks. Find his new average.

(a) 54 (b) 52

(c) 55 (d) 53.5

26. The average earning of Shambhu Nath Pandey for the initial three months of the calendar year 2002 is ` 1200. If his average earning for the second and third month is ` 1300 find his earning in the first month?

(a) 900 (b) 1100

(c) 1000 (d) 1200

27. In a hotel where rooms are numbered from 101 to 130, each room gives an earning of ` 3000 for the first fifteen days of a month and for the latter half, ` 2000 per room. Find the average earning per room per day over the month. (Assume 30 day month)

(a) 2250 (b) 2500

(c) 2750 (d) 2466.66

28. The average weight of 5 men is decreased by 3 kg when one of them weighing 150 kg is replaced by another person. Find the weight of the new person.

(a) 165 kg (b) 135 kg

(c) 138 kg (d) 162 kg

29. The average age of a group of men is increased by 5 years when a person aged 18 years is replaced by a new person of aged 38 years. How many men are there in the group?

(a) 3 (b) 4

(c) 5 (d) 6

30. The average score of a cricketer in three matches is 22 runs and in two other matches, it is 17 runs. Find the average in all the five matches.

(a) 20 (b) 19.6

(c) 21 (d) 19.5

31. The average of 13 papers is 40. The average of the first 7 papers is 42 and of the last seven papers is 35. Find the marks obtained in the 7th paper.

(a) 23 (b) 38

(c) 19 (d) 39

32. The average age of the Indian cricket team playing the Nagpur test is 30. The average age of 5 of the players is 27 and that of another set of 5 players, totally different from the first five, is 29. If it is the captain who was not included in either of these two groups, then find the age of the captain.

(a) 75 (b) 55

(c) 50 (d) 58

33. Siddhartha has earned an average of 4200 dollars for the first eleven months of the year. If he justifies his staying on in the US on the basis of his ability to earn at least 5000 dollars per month for the entire year, how much should he earn (in dollars) in the last month to achieve his required average for the whole year?

(a) 14,600 (b) 5,800

(c) 12,800 (d) 13,800

34. A bus goes to Ranchi from Patna at the rate of 60 km per hour. Another bus leaves Ranchi for Patna at the same time as the first bus at the rate of 70 km per hour. Find the average speed for the journeys of the two buses combined if it is known that the distance from Ranchi to Patna is 420 kilometers.

(a) 64.615 kmph (b) 64.5 kmph

(c) 63.823 kmph (d) 64.82 kmph

35. A train travels 8 km in the first quarter of an hour, 6 km in the second quarter and 40 km in the third quarter. Find the average speed of the train per hour over the entire journey.

(a) 72 km/h (b) 18 km/h

(c) 77.33 km/h (d) 78.5 km/h

36. The average weight of 6 men is 68.5 kg. If it is known that Ram and Tram weigh 60 kg each, find the average weight of the others.

(a) 72.75 kg (b) 75 kg

(c) 78 kg (d) 80 kg

37. The average score of a class of 40 students is 52. What will be the average score of the rest of the students if the average score of 10 of the students is 61.

(a) 50 (b) 47

(c) 48 (d) 49

38. The average age of 80 students of IIM, Bangalore of the 1995 batch is 22 years. What will be the new average if we include the 20 faculty members whose average age is 37 years?

(a) 32 years (b) 24 years

(c) 25 years (d) 26 years

39. Out of three numbers, the first is twice the second and three times the third. The average of the three numbers is 88. The smallest number is

(a) 72 (b) 36

(c) 42 (d) 48

40. The sum of three numbers is 98. If the ratio between the first and second is 2 : 3 and that between the second and the third is 5 : 8, then the second number is

(a) 30 (b) 20

(c) 58 (d) 48

41. The average height of 30 girls out of a class of 40 is 160 cm and that of the remaining girls is 156 cm. The average height of the whole class is

(a) 158 cm (b) 158.5 cm

(c) 159 cm (d) 157 cm

42. The average weight of 6 persons is increased by 2.5 kg when one of them whose weight is 50 kg is replaced by a new man. The weight of the new man is

(a) 65 kg (b) 75 kg

(c) 76 kg (d) 60 kg

43. The average age of three boys is 15 years. If their ages are in the ratio 3 : 5 : 7, the age of the youngest boy is

(a) 21 years (b) 18 years

(c) 15 years (d) 9 years

44. The average age of A, B, C and D five years ago was 45 years. By including X, the present average age of all the five is 49 years. The present age of X is

(a) 64 years (b) 48 years

(c) 45 years (d) 40 years

45. The average salary of 20 workers in an office is ` 1900 per month. If the manager's salary is added, the average salary becomes ` 2000 per month. What is the manager's annual salary?

(a) ` 24,000 (b) ` 25,200

(c) ` 45,600 (d) None of these

46. If a, b, c, d and e are five consecutive odd numbers, then their average is

(a) $5(a + b)$ (b) $(a\ b\ c\ d\ e)/5$

(c) $5(a + b + c + d + e)$ (d) None of these

47. The average of first five multiples of 3 is

(a) 3 (b) 9

(c) 12 (d) 15

48. The average weight of a class of 40 students is 40 kg. If the weight of the teacher be included, the average weight increases by 500 gm.

The weight of the teacher is

(a) 40.5 kg (b) 60 kg

(c) 62 kg (d) 60.5 kg

49. In a management entrance test, a student scores 2 marks for every correct answer and loses 0.5 marks for every wrong answer. A student attempts all the 100 questions and scores 120 marks. The number of questions he answered correctly was

(a) 50 (b) 45

(c) 60 (d) 68

50. The average age of four children is 8 years, which is increased by 4 years when the age of the father is included. Find the age of the father.

(a) 32 (b) 28

(c) 16 (d) 24

51. The average of the first ten natural numbers is

(a) 5 (b) 5.5

(c) 6.5 (d) 6

52. The average of the first ten whole numbers is

(a) 4.5 (b) 5

(c) 5.5 (d) 4

53. The average of the first ten even numbers is

(a) 18 (b) 22

(c) 9 (d) 11

54. The average of the first ten odd numbers is

(a) 11 (b) 10

(c) 17 (d) 9

55. The average of the first ten prime numbers is

(a) 15.5 (b) 12.5

(c) 10 (d) 12.9

56. The average of the first ten composite numbers is

(a) 12.9 (b) 11

(c) 11.2 (d) 10

57. The average of the first ten prime numbers, which are odd, is

(a) 12.9 (b) 13.8

(c) 17 (d) 15.8

58. The average weight of a class of 30 students is 40 kg. If, however, the weight of the teacher is included, the average becomes 41 kg. The weight of the teacher is

(a) 31 kg (b) 62 kg

(c) 71 kg (d) 70 kg

59. Ram bought 2 toys for ` 5.50 each, 3 toys for ` 3.66 each and 6 toys for ` 1.833 each. The average price per toy is

(a) ` 3 (b) ` 10

(c) ` 5 (d) ` 9

60. 30 oranges and 75 apples were purchased for ` 510. If the price per apple was ` 2, then the average price of oranges was

(a) ` 12 (b) ` 14

(c) ` 10 (d) ` 15

61. The average income of Sambhu and Ganesh is ` 3,000 and that of Arun and Vinay is ` 500. What is the average income of Sambhu, Ganesh, Arun and Vinay?

(a) ` 1750 (b) ` 1850

(c) ` 1000 (d) ` 2500

62. A batsman made an average of 40 runs in 4 innings, but in the fifth inning, he was out on zero. What is the average after fifth inning?

(a) 32 (b) 22

(c) 38 (d) 49

63. The average weight of 40 teachers of a school is 80 kg. If, however, the weight of the principal be included, the average decreases by 1 kg. What is the weight of the principal?

(a) 109 kg (b) 29 kg

(c) 39 kg (d) None of these

64. The average temperature of 1st, 2nd and 3rd December was 24.4 °C. The average temperature of the first two days was 24 °C. The temperature on the 3rd of December was

(a) 20 °C (b) 25 °C

(c) 25.2 °C (d) None of these

65. The average age of Ram and Shyam is 20 years. Their average age 5 years hence will be

(a) 25 years (b) 22 years

(c) 21 years (d) 20 years

66. The average of 20 results is 30 and that of 30 more results is 20. For all the results taken together, the average is

(a) 25 (b) 50

(c) 12 (d) 24

67. The average of 5 consecutive numbers is 18. The highest of these numbers will be

(a) 24 (b) 18

(c) 20 (d) 22

68. The average of 6 students is 11 years. If 2 more students of age 14 and 16 years join, their average will become

(a) 12 years (b) 13 years

(c) 21 years (d) 19 years

69. The average of 8 numbers is 12. If each number is increased by 2, the new average will be

(a) 12 (b) 14

(c) 13 (d) 15

70. Three years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average of the family is the same today. What is the age of the baby?

(a) 1 year (b) 2 years

(c) 6 months (d) 9 months

71. Sambhu’s average daily expenditure is ` 10 during May, ` 14 during June and ` 15 during July. His approximate daily expenditure for the 3 months is

(a) ` 13 approximately

(b) ` 12

(c) ` 12 approximately

(d) ` 10

72. A ship sails out to a mark at the rate of 15 km per hour and sails back at the rate of 20 km/h. What is its average rate of sailing?

(a) 16.85 km (b) 17.14 km

(c) 17.85 km (d) 18 km

73. The average temperature on Monday, Tuesday and Wednesday was 41 °C and on Tuesday, Wednesday and Thursday it was 40 °C. If on Thursday it was exactly 39 °C, then on Monday, the temperature was

(a) 42 °C (b) 46 °C

(c) 23 °C (d) 26 °C

74. The average of 20 results is 30 out of which the first 10 results are having an average of 10. The average of the rest 10 results is

(a) 50 (b) 40

(c) 20 (d) 25

75. A man had seven children. When their average age was 12 years a child aged 6 years died. The average age of the remaining 6 children is

(a) 6 years (b) 13 years

(c) 17 years (d) 15 years

76. The average income of Ram and Shyam is ` 200. The average income of Rahul and Rohit is ` 250. The average income of Ram, Shyam, Rahul and Rohit is

(a) ` 275 (b) ` 225

(c) ` 450 (d) ` 250

77. The average weight of 35 students is 35 kg. If the teacher is also included, the average weight increases to 36 kg. The weight of the teacher is

(a) 36 kg (b) 71 kg

(c) 70 kg (d) 45 kg

78. The average of x, y and z is 45. x is as much more than the average as y is less than the average. Find the value of z.

(a) 45 (b) 25

(c) 35 (d) 15

79. Find the average of four numbers $2\frac{3}{4}$, $5\frac{1}{3}$, $4\frac{1}{6}$, $8\frac{1}{2}$.

(a) $5\frac{3}{16}$ (b) $3\frac{3}{16}$

(c) $16\frac{5}{3}$ (d) $3\frac{16}{5}$

80. The average salary per head of all the workers in a company is ` 95. The average salary of 15 officers is ` 525 and the average salary per head of the rest is ` 85. Find the total number of workers in the workshop.

(a) 660 (b) 580

(c) 650 (d) 460

81. The average age of 8 men is increased by 2 years when one of them whose age is 24 years is replaced by a woman. What is the age of the woman?

(a) 35 years (b) 28 years

(c) 32 years (d) 40 years

82. The average monthly expenditure of Ravi was ` 1100 during the first 3 months, ` 2200 during the next 4 months and ` 4620 during the subsequent five months of the year. If the total saving during the year was ` 2100, find Ravi's average monthly income.

(a) ` 1858 (b) ` 3108.33

(c) ` 3100 (d) None of these

83. Shyam bought 2 articles for ` 5.50 each, and 3 articles for ` 3.50 each, and 3 articles for ` 5.50 each and 5 articles for ` 1.50 each. The average price for one article is

(a) ` 3 (b) ` 3.10

(c) ` 3.50 (d) ` 2

84. In a bag, there are 150 coins of Re. 1, 50 p and 25 p denominations. If the total value of coins is ` 150, then find how many rupees can be constituted by 50 p coins.

(a) 16 (b) 20

(c) 28 (d) None of these

LEVEL OF DIFFICULTY (II)

1. With an average speed of 40 km/h, a train reaches its destination in time. If it goes with an average speed of 35 km/h, it is late by 15 minutes. The length of the total journey is:

   (a) 40 km (b) 70 km

   (c) 30 km (d) 80 km

2. In the month of July of a certain year, the average daily expenditure of an organisation was ` 68. For the first 15 days of the month, the average daily expenditure was ` 85 and for the last 17 days, ` 51. Find the amount spent by the organisation on the 15th of the month.

   (a) ` 42 (b) ` 36

   (c) ` 34 (d) ` 52

3. In 1919, W. Rhodes, the Yorkshire cricketer, scored 891 runs for his county at an average of 34.27; in 1920, he scored 949 runs at an average of 28.75; in 1921, 1329 runs at an average of 42.87 and in 1922, 1101 runs at an average of 36.70. What was his county batting average for the four years?

   (a) 36.23 (b) 37.81

   (c) 35.88 (d) 28.72

4. A train travels with a speed of 20 m/s in the first 10 minutes, goes 8.5 km in the next 10 minutes, 11 km in the next 10, 8.5 km in the next 10 and 6 km in the next 10 minutes. What is the average speed of the train in kilometer per hour for the journey described?

   (a) 42 kmph (b) 35.8 kmph

   (c) 55.2 kmph (d) 46 kmph

5. One-fourth of a certain journey is covered at the rate of 25 km/h, one-third at the rate of 30 km/h and the rest at 50 km/h. Find the average speed for the whole journey.

(a) 600/53 km/h
(b) 1200/53 km/h
(c) 1800/53 km/h
(d) 1600/53

6. Typist *A* can type a sheet in 6 minutes, typist *B* in 7 minutes and typist *C* in 9 minutes. The average number of sheets typed per hour per typist for all three typists is

(a) 265/33
(b) 530/63
(c) 655/93
(d) 530/33

7. Find the average increase rate if increase in the population in the first year is 30% and that in the second year is 40%.

(a) 41
(b) 56
(c) 40
(d) 38

8. The average income of a person for the first 6 days is ` 29, for the next 6 days it is ` 24, for the next 10 days it is ` 32 and for the remaining days of the month it is ` 30. Find the average income per day.

(a) ` 31.64
(b) ` 30.64
(c) ` 29.26
(d) Cannot be determined

9. In hotel Jaysarmin, the rooms are numbered from 101 to 130 on the first floor, 221 to 260 on the second floor and 306 to 345 on the third floor. In the month of June 2012, the room occupancy was 60% on the first floor, 40% on the second floor and 75% on the third floor. If it is also known that the room charges are ` 200, ` 100 and ` 150 on each of the floors, then find the average income per room for the month of June 2012.

(a) ` 151.5
(b) ` 88.18
(c) ` 78.3
(d) ` 65.7

10. A salesman gets a bonus according to the following structure: If he sells articles worth ` $x$ then he gets a bonus of ` (x/100 – 1). In the month of January, his sales value was ` 100, in February it was ` 200, from March to November it was ` 300 for every month and in December it was ` 1200. Apart from this, he also receives a basic

salary of ` 30 per month from his employer. Find his average income per month during the year.

(a) ` 31.25 (b) ` 30.34

(c) ` 32.5 (d) ` 34.5

11. A man covers half of his journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. Find his average speed during the entire journey.

(a) 36 kmph (b) 30 kmph

(c) 24 kmph (d) 18 kmph

12. The average weight of 5 men is decreased by 3 kg when one of them weighing 150 kg is replaced by another person. This new person is again replaced by another person whose weight is 30 kg lower than the person he replaced. What is the overall change in the average due to this dual change?

(a) 6 kg (b) 9 kg

(c) 12 kg (d) 15 kg

13. Find the average weight of four containers, if it is known that the weight of the first container is 100 kg and the total of the second, third and fourth containers' weight is defined by $f(x) = x^2 - 3/4\ (x^2)$ where $x = 100$

(a) 650 kg (b) 900 kg

(c) 750 kg (d) 450 kg

14. There are five boxes in a cargo hold. The weight of the first box is 200 kg and the weight of the second box is 20% higher than the weight of the third box, whose weight is 25% higher than the first box's weight. The fourth box at 350 kg is 30% lighter than the fifth box. Find the difference in the average weight of the four heaviest boxes and the four lightest boxes.

(a) 51.5 kg (b) 75 kg

(c) 37.5 kg (d) 112.5 kg

15. For Question 14, find the difference in the average weight of the heaviest three and the lightest three.

(a) 116.66 kg (b) 125 kg

(c) 150 kg (d) 112.5 kg

16. A batsman makes a score of 270 runs in the 87th inning and thus increases his average by a certain number of runs that is a whole number. Find the possible values of the new average.

(a) 98 (b) 184

(c) 12 (d) All of these

17. 19 persons went to a hotel for a combined dinner party. 13 of them spent ` 79 each on their dinner and the rest spent ` 4 more than the average expenditure of all the 19. What was the total money spent by them?

(a) 1628.4 (b) 1534

(c) 1492 (d) None of these

18. There were 42 students in a hostel. Due to the admission of 13 new students, the expenses of the mess increase by ` 31 per day while the average expenditure per head diminished by ` 3. What was the original expenditure of the mess?

(a) ` 633.23 (b) ` 583.3

(c) ` 623.3 (d) ` 632

19. The average price of 3 precious diamond studded platinum thrones is ` 97610498312 if their prices are in the ratio 4:7:9. The price of the cheapest is:

(a) 5, 65, 66, 298.972 (b) 5, 85, 66, 29, 897.2

(c) 58, 56, 62, 889.72 (d) None of these

20. The average weight of 47 balls is 4 gm. If the weight of the bag (in which the balls are kept) be included, the calculated average weight per ball increases by 0.3 gm. What is the weight of the bag?

(a) 14.8 gm (b) 15.0 gm

(c) 18.6 gm (d) None of these

21. The average of 71 results is 48. If the average of the first 59 results is 46 and that of the last 11 is 52. Find the 60th result.

(a) 132 (b) 122

(c) 134 (d) 128

22. A man covers 1/3rd of his journey by cycle at 50 km/h, the next 1/3 by car at 30 km/h, and the rest by walking at 7 km/h. Find his average speed during the whole journey.

(a) 14.2 kmph (b) 15.3 kmph

(c) 18.2 kmph (d) 12.8 kmph

23. The average age of a group of 14 persons is 27 years and 9 months. Two persons, each 42 years old, left the group. What will be the average age of the remaining persons in the group?

(a) 26.875 years (b) 26.25 years

(c) 25.375 years (d) 25 years

24. In an exam, the average was found to be x marks. After deducting computational error, the average marks of 94 candidates got reduced from 84 to 64. The average thus came down by 18.8 marks. The numbers of candidates who took the exam were:

(a) 100 (b) 90

(c) 110 (d) 105

25. The average salary of the entire staff in an office is ` 3200 per month. The average salary of officers is ` 6800 and that of non-officers is ` 2000. If the number of officers is 5, then find the number of non-officers in the office?

(a) 8 (b) 12

(c) 15 (d) 5

26. A person travels three equal distances at a speed of x km/h, y km/h and z km/h respectively. What will be the average speed during the whole journey?

(a) $xyz/(xy + yz + zx)$ (b) $(xy + yz + zx)/xyz$

(c) $3xyz/(xy + yz + xz)$ (d) None of these

Directions for Questions 27 to 30: Read the following passage and answer the questions that follow.

In a family of five persons *A*, *B*, *C*, *D* and *E*, each and everyone loves one another very much. Their birthdays are in different months and on different dates. *A* remembers that his birthday is between 25th and 30th, of *B* it is between 20th and 25th, of *C* it is between 10th and 20th, of *D* it is between 5th and 10th and of *E* it is between 1st to 5th of the month. The sum of the date of birth is defined as the addition of the date and the month, for example 12th January will be written as 12/1 and will add to a sum of the date of 13. (Between 25th and 30th includes both 25 and 30).

27. What may be the maximum average of their sum of the dates of birth?

(a) 24.6 (b) 15.2

(c) 28 (d) 32

28. What may be the minimum average of their sum of the dates of births?

(a) 24.6 (b) 15.2

(c) 28 (d) 32

29. If it is known that the dates of birth of three of them are even numbers then find maximum average of their sum of the dates of birth.

(a) 24.6 (b) 15.2

(c) 27.6 (d) 28

30. If the date of birth of four of them are prime numbers, then find the maximum average of the sum of their dates of birth.

(a) 27.2 (b) 26.4

(c) 28 (d) None of these

31. The average age of a group of persons going for a picnic is 16.75 years. 20 new persons with an average age of 13.25 years join the group on the spot due to which the average of the group becomes 15 years. Find the number of persons initially going for the picnic.

(a) 24 (b) 20

(c) 15 (d) 18

32. A school has only four classes that contain 10, 20, 30 and 40 students respectively. The pass percentage of these classes are 20%, 30%, 60% and 100% respectively. Find the pass % of the entire school.

(a) 56% (b) 76%

(c) 34% (d) 66%

33. Find the average of $f(x)$, $g(x)$, $h(x)$, $d(x)$ at $x = 10$. $f(x)$ is equal to $x^2 + 2$, $g(x) = 5x^2 – 3$, $h(x) = \log x^2$ and $d(x) = (4/5)x^2$

(a) 170 (b) 170.25

(c) 70.25 (d) 70

34. Find the average of $f(x) – g(x)$, $g(x) – h(x)$, $h(x) – d(x)$, $d(x) – f(x)$

(a) 0 (b) –2.25

(c) 4.5 (d) 2.25

35. $\sum_{r=1}^{n} (n+1)\, r$ where $r = n$.

(a) $\frac{(n-1)(n)(n+1)}{2}$ (b) $\frac{n(n+1)^2}{2}$

(c) $\frac{n(n-1)^2}{2}$ (d) $\frac{n^2}{2}$

36. The average of ‘n’ numbers is z. If the number x is replaced by the number x^1, then the average becomes z^1. Find the relation between n, z, z^1, x and x^1.

(a) $\left[\frac{z^1-2}{x^1-x}=\frac{1}{n}\right]$

(b) $\left[\frac{x^1-x}{z^1}=\frac{1}{n}\right]$

(c) $\left[\frac{z-z^1}{x-x^1}=\frac{1}{n}\right]$

(d) $\left[\frac{x-x^1}{z-z^1}=\frac{1}{n}\right]$

37. The average salary of workers in Mindworkzz is ` 2,000, the average salary of faculty being ` 4,000 and the management trainees being ` 1,250. The total number of workers could be

(a) 450
(b) 300
(c) 110
(d) 500

Directions for Questions 38 to 41: Read the following and answer the questions that follows.

During a criket match, India playing against NZ scored in the following manner:

Partnership	*Runs scored*
1st wicket	112
2nd wicket	58
3rd wicket	72
4th wicket	92
5th wicket	46
6th wicket	23

38. Find the average runs scored by the first four batsmen.

(a) 83.5
(b) 60.5
(c) 66.8
(d) Cannot be determined

39. The maximum average runs scored by the first five batsmen could be

(a) 80.6
(b) 66.8
(c) 76
(d) Cannot be determined.

40. The minimum average runs scored by the last five batsmen to get out could be

(a) 53.6 (b) 44.4

(c) 66.8 (d) 0

41. If the fifth down batsman gets out for a duck, then find the average runs scored by the first six batsmen.

(a) 67.1 (b) 63.3

(c) 48.5 (d) Cannot be determined

42. The weight of a body as calculated by the average of 7 different experiments is 53.735 gm. The average of the first three experiments is 54.005 gm, of the fourth is 0.004 gm greater than the fifth, while the average of the sixth and seventh experiment was 0.010 gm less than the average of the first three. Find the weight of the body obtained by the fourth experiment.

(a) 49.353 gm (b) 51.712 gm

(c) 53.072 gm (d) 54.512 gm

43. A man's average expenditure for the first 4 months of the year was ` 251.25. For the next 5 months the average monthly expenditure was ` 26.27 more than what it was during the first 4 months. If the person spent ` 760 in all during the remaining 3 months of the year, find what percentage of his annual income of ` 3000 he saved in the year.

(a) 14% (b) –5.0866%

(c) 12.5% (d) None of these

44. A certain number of trucks were required to transport 60 tons of steel wire from the TISCO factory in Jamshedpur. However, it was found that since each truck could take 0.5 tons of cargo less, another 4 trucks were needed. How many trucks were initially planned to be used?

(a) 10 (b) 15

(c) 20 (d) 25

45. One collective farm got an average harvest of 21 tons of wheat and another collective farm that had 12 acres of land less given to wheat, got 25 tons from a hectare. As a result, the second farm harvested 300 tons of wheat more than the first. How many tons of wheat did each farm harvest?

(a) 3150, 3450

(b) 3250, 3550

(c) 2150, 2450

(d) None of these

LEVEL OF DIFFICULTY (III)

Directions for Questions 1 to 8: Read the following:

There are 3 classes having 20, 25 and 30 students respectively having average marks in an examination as 20, 25 and 30 respectively. If the three classes are represented by *A*, *B* and *C* and you have the following information about the three classes, answer the questions that follow:

A Æ Highest score 22, Lowest score 18

B Æ Highest score 31, Lowest score 23

C Æ Highest score 33, Lowest score 26

If five students are transferred from *A* to *B*.

1. What will happen to the average score of *B*?

   (a) Definitely increase (b) Definitely decrease

   (c) Remain constant (d) Cannot say

2. What will happen to the average score of *A*?

   (a) Definitely increase (b) Definitely decrease

   (c) Remain constant (d) Cannot say

In a transfer of 5 students from *A* to *C*

3. What will happen to the average score of *C*?

   (a) Definitely increase (b) Definitely decrease

   (c) Remain constant (d) Cannot say

4. What will happen to the average score of *A*?

   (a) Definitely increase (b) Definitely decrease

   (c) Remain constant (d) Cannot say

In a transfer of 5 students from *B* to *C* (Questions 5–6)

5. What will happen to the average score of *C*?

(a) Definitely increase (b) Definitely decrease

(c) Remain constant (d) Cannot say

6. Which of these can be said about the average score of *B*?

(a) Increases if *C* decreases

(b) Decreases if *C* increases

(c) Increases if *C* decreases

(d) Decreases if *C* decreases

7. In a transfer of 5 students from *A* to *B*, the maximum possible average achievable for group *B* is

(a) 25 (b) 24.5

(c) 25.5 (d) 24

8. For the above case, the maximum possible average achieved for group *A* will be

(a) 20.66 (b) 21.5

(c) 20.75 (d) 20.5

9. What will be the minimum possible average of Group *A* if 5 students are transferred from *A* to *B*?

(a) 19.55 (b) 21.5

(c) 19.33 (d) 20.5

10. If 5 students are transferred from *B* to *A*, what will be the minimum possible average of *A*?

(a) 20.69 (b) 21

(c) 20.75 (d) 20.6

11. For question 10, what will be the maximum average of *A*?

(a) 23.2 (b) 22.2

(c) 18.75 (d) 19

Directions for Questions 12 to 17: Read the following and answer the questions that follow.

If 5 people are transferred from *A* to *B* and another independent set of 5 people are transferred back from *B* to *A*, then after this operation (Assume that the set transferred from *B* to *A* contains none from the set of students that came to *B* from *A*)

12. What will happen to *B*'s average?

    (a) Increase if *A*'s average decreases

    (b) Decrease always

    (c) Cannot be said

    (d) Decrease if *A*'s average decreases

13. What can be said about *A*'s average?

    (a) Will decrease

    (b) Will always increase if *B*'s average changes

    (c) May increase or decrease

    (d) Will increase only if *B*'s average decreases

14. At the end of the 2 steps mentioned above (in the *direction*) what could be the maximum value of the average of class *B*?

    (a) 25.4 (b) 25

    (c) 24.8 (d) 24.6

15. For question 14, what could be the minimum value of the average of class *B*?

    (a) 22.4 (b) 24.2

    (c) 25 (d) 23

16. What could be the maximum possible average achieved by class *A* at the end of the operation?

    (a) 25.2 (b) 26

    (c) 23.25 (d) 23.75

17. What could be the minimum possible average of class *A* at the end of the operation?

(a) 21.4 (b) 19.2

(c) 28.5 (d) 20.25

Directions for Questions 18 to 23: Read the following and answer the questions that follow.

If 5 people are transferred from *C* to *B*, further, 5 more people are transferred from *B* to *A*, then 5 are transferred from *A* to *B* and finally, 5 more are transferred from *B* to *C*.

18. What is the maximum possible average achieved by class *C*?

(a) 30.833 (b) 30

(c) 29.66 (d) 30.66

19. What is the maximum possible average of class *B*?

(a) 26 (b) 27

(c) 25 (d) 28

20. What is the maximum possible average value attained by class *A*?

(a) 22.75 (b) 23.75

(c) 23.5 (d) 24

21. The minimum possible value of the average of group *C* is:

(a) 26.3 (b) 27.5

(c) 29.6 (d) 28

22. The minimum possible average of group *B* after this set of operation is

(a) 21.6 (b) 21.4

(c) 21.8 (d) 21.2

23. The minimum possible average of group *A* after the set of 3 operation is

(a) 20 (b) 20.3

(c) 20.4 (d) 19.8

24. Which of these will definitely not constitute an operation for getting the minimum possible average value for group *A*:

    (a) Transfer of five 31s from *B* to *A*

    (b) Transfer of five 26s from *C* to *B*

    (c) Transfer of five 22s from *A* to *B*

    (d) Transfer of five 33s from *C* to *B*

25. For getting the lowest possible value of *C*'s average, the sequence of operations could be

    (a) Transfer five 33s from *C* to *B*, five 23s from *B* to *A*, five 18s from *A* to *B*, five 18s from *B* to *C*

    (b) Transfer five 33s from *C* to *B*, 31s from *B* to *A*,

    (c) Both a and b

    (d) None of the above

26. If we set the highest possible average of class *C* as the primary objective and want to achieve the highest possible value for class *B* as the secondary objective, what is the maximum value of class *B*'s average that is attainable?

    (a) 27 (b) 26

    (c) 25 (d) 24

27. For Question 26, if the secondary objective is changed to achieving the minimum possible average value of class *B*'s average, the lowest value of class *B*'s average that could be attained is

    (a) 22.2 (b) 23

    (c) 22.6 (d) 22

28. For question 27, what can be said about class *A*'s average?

    (a) Will be determined automatically at 22.25

    (b) Will have a maximum possible value of 22.25

    (c) Will have a minimum possible value of 22.25

(d) Will be determined automatically at 22.5

29. A team of miners planned to mine 1800 tons of ore during a certain number of days. Due to technical difficulties in one-third of the planned number of days, the team was able to achieve an output of 20 tons of ore less than the planned output. To make up for this, the team overachieved for the rest of the days by 20 tons. The end result was that the team completed the task one day ahead of time. How many tons of ore did the team initially plan to ore per day?

(a) 50 tons (b) 100 tons

(c) 150 tons (d) 200 tons

30. According to a plan, a team of woodcutters decided to harvest 216 m^3 of wheat in several days. In the first three days, the team fulfilled the daily assignment, and then it harvested 8 m^3 of wheat over and above the plan everyday. Therefore, a day before the planned date, they had already harvested 232 m^3 of wheat. How many cubic metres of wheat a day did the team have to cut according to the plan?

(a) 12 (b) 13

(c) 24 (d) 25

31. On an average, two liters of milk and one liter of water are needed to be mixed to make 1 kg of sudha shrikhand of type *A*, and 3 liters of milk and 2 liters of water are needed to be mixed to make 1 kg of sudha shrikhand of type *B*. How many kilograms of each type of shrikhand was manufactured if it is known that 130 liters of milk and 80 liters of water were used?

(a) 20 of type *A* and 30 of type *B*

(b) 30 of type *A* and 20 of type *B*

(c) 15 of type *A* and 30 of type *B*

(d) 30 of type *A* and 15 of type *B*

32. There are 500 seats in Minerva Cinema, Mumbai, placed in similar rows. After the reconstruction of the hall, the total number of seats

became 10% less. The number of rows was reduced by 5 but each row contained 5 seats more than before. How many rows and how many seats in a row were there initially in the hall?

(a) 20 rows and 25 seats

(b) 20 rows and 20 seats

(c) 10 rows and 50 seats

(d) 50 rows and 10 seats

33. One fashion house has to make 810 dresses and another one 900 dresses during the same period of time. In the first house, the order was ready 3 days ahead of time and in the second house, 6 days ahead of time. How many dresses did each fashion house make a day if the second house made 21 dresses more a day than the first?

(a) 54 and 75 (b) 24 and 48

(c) 44 and 68 (d) 04 and 25

34. A shop sold 64 kettles of two different capacities. The smaller kettle cost a rupee less than the larger one. The shop made 100 rupees from the sale of large kettles and 36 rupees from the sale of small ones. How many kettles of either capacity did the shop sell and what was the price of each kettle?

(a) 20 kettles for 2.5 rupees each and 14 kettles for 1.5 rupees each

(b) 40 kettles for 4.5 rupees each and 24 kettles for 2.5 rupees each

(c) 40 kettles for 2.5 rupees each and 24 kettles for 1.5 rupees each

(d) either a or b

35. An enterprise got a bonus and decided to share it in equal parts between the exemplary workers. It turned out, however, that there were 3 more exemplary workers than it had been assumed. In that case, each of them would have got 4 rupees less. The administration had found the possibility to increase the total sum of the bonus by 90 rupees and as a result each exemplary worker got 25 rupees. How many people got the bonus?

(a) 9 (b) 18

(c) 8 (d) 16

Directions for Questions 36 to 39: Read the following and answer the questions that follows.

In the island of Hoola Boola Moola, the inhabitants have a strange process of calculating their average incomes and expenditures. According to an old legend prevalent on that island, the average monthly income had to be calculated on the basis of 14 months in a calendar year while the average monthly expenditure was to be calculated on the basis of 9 months per year. This would lead to people having an underestimation of their savings since there would be an underestimation of the income and an overestimation of the expenditure per month.

36. If the minister for economic affairs decided to reverse the process of calculation of average income and average expenditure, what will happen to the estimated savings of a person living on Hoola Boola Moola island?

    (a) It will increase

    (b) It will decrease

    (c) It will remain constant

    (d) Will depend on the value

37. If it is known that Mr. Magoo Hoola Boola estimates his savings at 10 Moolahs and if it is further known that his actual expenditure is 288 Moolahs in an year (Moolahs, for those who are not aware, is the official currency of Hoola Boola Moola), then what will happen to his estimated savings if he suddenly calculates on the basis of a 12 month calendar year?

    (a) Will increase by 5 (b) Will increase by 15

    (c) Will increase by 10 (d) Will triple

38. Mr. Boogie Woogie comes back from the USA to Hoola Boola Moola and convinces his community comprising 546 families to start calculating the average income and average expenditure on the basis of 12 months per calendar year. Now if it is known that the

average estimated income on the island is (according to the old system) 87 Moolahs per month, then what will be the change in the average estimated savings for the island of Hoola Boola Moola. (Assume that there is no other change).

(a) 251.60 Moolahs (b) 565.5 Moolahs

(c) 625.5 Moolahs (d) Cannot be determined

39. Mr. Boogle Woogle comes back from the USSR and convinces his community comprising 273 families to start calculating the average income on the basis of 12 months per calendar year. Now if it is known that the average estimated income in his community is (according to the old system) 87 Moolahs per month, then what will be the change in the average estimated savings for the island of Hoola Boola Moola. (Assume that there is no other change).

(a) 251.60 Moolahs (b) 282.75 Moolahs

(c) 312.75 Moolahs (d) Cannot be determined

Directions for Questions 40 to 44: Read the following and answer the questions that follows.

The Indian cricket team has to score 360 runs on the last day of a test match in 90 overs, to win the test match. This is the target set by the opposing captain Brian Lara after he declared his innings closed at the overnight score of 411 for 7.

The Indian team coach has the following information about the batting rates (in terms of runs per over) of the different batsmen:

Assume that the run rate of a partnership is the weighted average of the individual batting rates of the batsmen involved in the partnership (on the basis of the ratio of the strike each batsman gets, i.e. the run rate of a partnership is defined as the weighted average of the run rates of the two batsmen involved weighted by the ratio of the number of balls faced by each batsman).

Since decimal fractions of runs are not possible for any batsman, assume that the estimated runs scored by a batsman in an inning (on the basis of his run rate and the number of overs faced by him) is rounded off to the next higher integer immediately above the estimated value of the runs scored during the innings.

For example, if a batsman scores at an average of 3 runs per over for 2.1666 overs, then he will be estimated to have scored 2.1666 × 3 = 6.5 runs in his innings, but since this is not possible, the actual number of runs scored by the batsman will be taken as 7 (the next higher integer above 6.5).

Runs scored per over in different batting styles

Name of Batsman	*Defensive*	*Normal*	*Aggressive*
Das	3	4	5
Dasgupta	2	3	4
Dravid	2	3	4
Tendulkar	4	6	8
Laxman	4	5	6
Sehwag	4	5	6
Ganguly	3	4	5
Kumble	2	3	4
Harbhajan	3	4	5
Srinath	3	4	5
Yohannan	2	3	4

Also, this rounding off can take place only once for one innings of a batsman.

Assume no extras unless otherwise stated.

Assume that the strike is equally shared unless otherwise stated.

40. If the first wicket pair of Das and Dasgupta bats for 22 overs and during this partnership Das has started batting normally and turned aggressive after 15 overs while Dasgupta started off defensively but shifted gears to bat normally after batting for 20 overs, find the expected score after 22 overs.

    (a) 65 (b) 71

(c) 82 (d) 58

41. Of the first-wicket partnership between Das and Dasgupta as per the previous question, the ratio of the number of runs scored by Das to those scored by Dasgupta is:

(a) 46 : 25 (b) 96 : 46

(c) 41 : 32 (d) Cannot be determined

42. The latest time by which Tendulkar can come to bat and still win the game, assuming that the run rate at the time of his walking the wicket is into 2.5 runs per over, is (assuming he shares strike equally with his partner and that he gets the maximum possible support at the other end from his batting partner and both play till the last ball).

(a) After 50 overs (b) After 55 overs

(c) After 60 overs (d) Cannot be determined

43. For question 42, where Tendulkar batted aggressively and assuming that it is the Tendulkar-Laxman pair that wins the game for India (after Tendulkar walks into bat with the current run rate at 2.5 per over, and at the latest possible time for him to win the game with maximum possible support from the opposite end), what will be Tendulkar's score for the innings (assume equal strike)?

(a) 105 (b) 120

(c) 135 (d) None of these

44. For questions 42 and 43, if it was Laxman who batted with Tendulkar for his entire innings, then how many runs would Laxman score in the innings?

(a) 105 (b) 75

(c) 90 (d) Cannot be determined

Directions for Questions 45 to 49: Read the following and answer the questions that follow.

If Sachin Tendulkar walks into bat after the fall of the fifth wicket and has to share partnerships with Ganguly, Kumble, Harbhajan, Srinath and Yohannan, who have batted normally, defensively, defensively, defensively and defensively respectively while Tendulkar has batted normally, aggressively, aggressively, aggressively and aggressively respectively in each of the five partnerships that lasted for 12, 10, 8, 5 and 10 overs respectively, sharing strike equally with Ganguly and keeping two-thirds of the strike in his other four partnerships, then answer the following questions:

45. How many runs did Sachin score during his innings?

(a) 128 (b) 212

(c) 176 (d) None of these

46. The highest partnership that Tendulkar shared in was worth:

(a) 60 (b) 61

(c) 62 (d) 58

47. The above partnership was shared with:

(a) Ganguly (b) Yohannan

(c) Kumble (d) All three

48. If India proceeded to win the match based on the runs scored by these last five partnerships (assuming the last wicket pair remained unbeaten), what could be the maximum score at which Tendulkar could have come into bat:

(a) 103 for 5 (b) 97 for 5

(c) 100 for 5 (d) 104 for 5

49. For Question 48, what could be the minimum score at which Tendulkar could have come to bat:

(a) 103 for 5 (b) 97 for 5

(c) 104 for 5 (d) 98 for 5

ANSWER KEY

Level of Difficulty (I)

1. (c)	2. (c)	3. (b)	4. (c)
5. (c)	6. (b)	7. (b)	8. (d)
9. (a)	10. (b)	11. (a)	12. (a)
13. (d)	14. (d)	15. (d)	16. (c)
17. (a)	18. (d)	19. (a)	20. (b)
21. (a)	22. (d)	23. (d)	24. (b)
25. (a)	26. (c)	27. (b)	28. (b)
29. (b)	30. (a)	31. (c)	32. (c)
33. (d)	34. (a)	35. (a)	36. (a)
37. (d)	38. (c)	39. (d)	40. (a)
41. (c)	42. (a)	43. (d)	44. (c)
45. (d)	46. (d)	47. (b)	48. (d)
49. (d)	50. (b)	51. (b)	52. (a)
53. (d)	54. (b)	55. (d)	56. (c)
57. (d)	58. (c)	59. (a)	60. (a)
61. (a)	62. (a)	63. (c)	64. (c)
65. (a)	66. (d)	67. (c)	68. (a)
69. (b)	70. (b)	71. (a)	72. (b)
73. (a)	74. (a)	75. (b)	76. (b)
77. (b)	78. (a)	79. (a)	80. (a)
81. (d)	82. (b)	83. (c)	84. (d)

Level of Difficulty (II)

1. (b)	2. (c)	3. (c)	4. (c)
5. (c)	6. (b)	7. (a)	8. (d)
9. (a)	10. (c)	11. (c)	12. (a)
13. (a)	14. (b)	15. (a)	16. (d)
17. (d)	18. (a)	19. (d)	20. (d)
21. (b)	22. (b)	23. (c)	24. (a)
25. (c)	26. (c)	27. (c)	28. (b)
29. (d)	30. (a)	31. (b)	32. (d)
33. (b)	34. (a)	35. (b)	36. (c)
37. (c)	38. (d)	39. (a)	40. (d)
41. (d)	42. (c)	43. (b)	44. (c)

45. (a)

Level of Difficulty (III)

1. (b)	2. (d)	3. (b)	4. (d)
5. (d)	6. (b)	7. (b)	8. (a)
9. (c)	10. (d)	11. (b)	12. (b)
13. (b)	14. (c)	15. (a)	16. (c)
17. (d)	18. (a)	19. (b)	20. (b)
21. (b)	22. (b)	23. (a)	24. (a)
25. (a)	26. (d)	27. (c)	28. (a)
29. (b)	30. (c)	31. (a)	32. (a)
33. (a)	34. (c)	35. (b)	36. (a)
37. (b)	38. (d)	39. (b)	40. (b)
41. (b)	42. (c)	43. (b)	44. (d)
45. (b)	46. (b)	47. (b)	48. (d)
49. (b)			

Solutions and Shortcuts

Level of Difficulty (I)

1. $P = 25 \times 15 - 24 \times 14 = 375 - 336 = 39$
2. D's weight $= 4 \times 80 - 3 \times 84 = 320 - 252 = 68$. E's weight $= 68 + 3 = 71$.

   Now, we know that $A + B + C + D = 4 \times 80 = 320$ and $B + C + D + E = 78 \times 4 = 312$. Hence, A's weight is 8 kg more than E's weight. $A = 71 + 8 = 79$.
3. Monday + Tuesday + Wednesday = 3 × 37 = 111;

   Tuesday + Wednesday + Thursday = 3 × 34 = 102. Thus, Monday – Thursday = 9 and

   Thursday = 4 × Monday/5 Æ Thursday = 36 and Monday = 45.
4. Today's total age of A, B and $C = 30 \times 3 = 90$.

   Today's total age for B and $C = 25 \times 2 = 50$.

   C's age $= 90 - 50 = 40$.

5. 9x + 100 = 10(x + 8) Æ x = 20 (average after 9 innings). Hence, new average = 20 + 8 = 28.
6. 7 × 3 = 21.
7. 1/4 + 1/6 + C = 3 × 1/12 Æ C = –1/6.
8. His total score is 93 + 78 + 177 = 348 out of 450. % score = 77.33
9. Average income over 9 months = [3 × (2750 + 650) + 3 × (3150 + 650) + 3 × (6750+650)]/9

   = [3 × 3400 + 3 × 3800 + 3 × 7400]/9 = 4866.66
10. Today's total age = 6 × 22 = 132 years. Total age of the family excluding the youngest member (for the remaining 5 people) = 132 – 7 = 125. Average age of the other 5 people in the family = 25 years.

    7 years ago their average age = 25 – 7 =18 years.
11. If the average age of 8 people has gone up by 2 years it means the total age has gone up by 16 years. Thus the total age of the two women would be: 35 + 45 + 16 = 96. Hence, their average age = 48.
12. W + T + F = 120; T + F + S = 123 Æ S – W = 3. Hence temperature on Wednesday = 42 – 3 = 39.
13. The average speed can be calculated by assuming a distance of 300 km (LCM of 100 and 150). Then time taken @ 100 kmph = 3 hours and time taken @ 150kmph = 2 hours. Average speed = Total distance/total time = 600/5 = 120 kmph.
14. Teacher's weight = 40.5 × 30 – 40 × 29 = 1215 – 1160 = 55.
15. 3 × 17 – 2 × 16 = 51 – 32 = 19.
16. The weight of the new man would be 19 × 3.5 kgs more than the weight of the man he replaces. New man's weight = 79 + 19 × 3.5 = 145.5 kgs.
17. Let their current ages be x and 3x (ratio of 2:6). Then their ages after 5 years would be x + 5 and 3x + 5. Now it is given that (x + 5)/(3x + 5) = ¾ Æ x = 1 and hence their current ages are 1 years and 3 years respectively. After 10 years their average age would be 12 years.
18. The average would be given by the average of the first and last numbers (since the series 1, 2, 3, 4…97 is an Arithmetic

Progression).

Hence, the average = (1 + 97)/2 = 49

19. We need the average of the numbers: 31, 37, 41, 43 and 47

Average = Total/number of numbers Æ 199/5 = 39.8

20. Let the numbers be a, b, c and d respectively. $a + b + c = 16 \times 3 = 48$ and $b + c + d = 15 \times 3 = 45$.

Also, since $d = 18$, we have $b + c = 45 – 18 = 27$. Hence, $a = 48 – (b + c)$ Æ $a = 21$.

21. If the numbers are $a + 1$, $a + 2$, $a + 3$, $a + 4$ and $a + 5$ the average would be $a + 3$. If we take 7 numbers as:

$a + 1$, $a + 2$, $a + 3$, $a + 4$, $a + 5$, $a + 6$ and $a + 7$ their average would be a+4. Hence, the average increases by 1.

22. Total of 48 numbers = 50 × 38 – 45 – 55 = 1800. Average of 48 numbers = 1800/48 = 37.5.

23. When we multiply each number by 12, the average would also get multiplied by 12. Hence, the new average = 7 × 12 = 84.

24. Let the number of ladies be n. Then we have $8 \times 15 + n \times 6 = (8 + n) \times (10.8)$ Æ $120 + 6n = 86.4 + 10.8n$ Æ $4.8n = 33.6$ Æ $n = 7$.

25. (3 × 52 + 60)/4 = 216/4 = 54.

26. 1200 × 3 – 1300 × 2 = 1000.

27. (2000 × 15 + 3000 × 15)/30 = (2000 + 3000)/2 = 2500.

28. The decrease in weight would be 15 kgs (5 people’s average weight drops by 3 kgs). Hence, the new person’s weight = 150 × 15 = 135.

29. When a person aged 18 years is replaced by a person aged 38 years, the total age of the group goes up by 20 years. Since this leads to an increase in the average by 5 years, it means that there are 20/5 = 4 persons in the group.

30. (22 × 3 + 17 × 2)/5 = 100/5 = 20.

31. Let the number of marks in the 7^{th} paper be M. Then the total of the first seven papers = 7 × 42 while the total of the last 7 (i.e. 7^{th} to 13^{th} papers) would be 7 × 35.

Total of 1st 7 + total of 7th to 13th = total of all 13 + marks in the 7th paper Æ

7 × 42 + 7 × 35 = 13 × 40 + *M*

539 = 520 + *M* Æ *M* = 19.

(**Note:** We write this equation since marks in the seventh paper is counted in both the first 7 and the last 7)

32. Let the captain's age by *C*. Then: 11 × 30 = 27 × 5 + 29 × 5 + *C* Æ 330 = 135 + 145 + *C* Æ *C* = 50.
33. His earning in the 12th month should be: 5000 × 12 – 4200 × 11 = 60000 – 46200 = 13800.
34. Total distance by total time = 840/13 = 64.615.
35. In three quarters of an hour the train has traveled 54 km. Thus, in a full hour the train would have traveled 1/3rd more (as it gets 1/3rd time more). Thus, the speed of the train = 54 + 1 × 54/3 = 54 + 18 = 72.
36. Total weight of all 6 = 68.5 × 6. Total weight of Ram and Tram = 60 × 2 =120. Average weight of the 4 people excluding Ram and Tram = (68.5 × 6 – 120)/4 = 72.75 kg.
37. 10 × 61 + 30 × *A* = 40 × 52 Æ *A* = (2080 – 610)/30 = 1470/30 = 49.
38. (80 × 22 + 20 × 37)/100 = 2500/100 = 25
39. If we take the first number as 6n, the second number would be 3*n* and the third would be 2*n*. Sum of the three numbers = 6*n* + 3*n* + 2*n* = 11*n* = 88 × 3 Æ *n* = 24. The smallest number would be 2*n* = 48.
40. The ratio between the first , second and third would be: 10:15:24. Since their total is 98, the numbers would be 20, 30 and 48 respectively. The second number is 30.
41. (30 × 160 + 10 × 156)/40 = 159. (note this question can also be solved using the alligation method explained in the next chapter.
42. The total weight of the six people goes up by 15 kgs (when the average for 6 persons goes up by 2.5 kg). Thus, the new person must be 15 kgs more than the person who he replaces.Hence, the new person's weight = 50 + 15 = 65 kg.

43. Total age = 3 × 15 = 45. Individual ages being in the ratio 3:5:7 their ages would be 9, 15 and 21 years respectively. The youngest boy would be 9 years.

44. 50 × 4 + *X* = 49 × 5 Æ *X* = 45.

45. (20 × 1900 + *M*) = 21 × 2000 Æ *M* = 4000. Hence, the salary is ` 4000 per month which also means ` 48,000 per year.

46. 5 consecutive odd numbers would always be in an Arithmetic progression and their average would be the middle number. The average would be ‘c’ in this case.

47. The average of 3, 6, 9, 12 and 15 would be 9.

48. 40 × 40 + *T* = 41 × 40.5 Æ *T* = 1660.5 – 1600 = 60.5 kgs.

49. If the number of questions correct is *N*, then the number of wrong answers is 100 – *N*. Using this we get:

    N × 2 – (100 – *N*) × 0.5 = 120 Æ 2.5 *N* = 170 Æ *N* = 68.

50. Required age of the father will be given by the equation: 5 × 12 = 4 × 8 + *F* Æ *F* = 28.

51. Required average = (1 + 2 + 3 + … + 10)/10 = 55/10 = 5.5. Alternately you could use the formula for sum of the first *n* natural numbers as *n*(*n* + 1)/2 with *n* as 10. Then average = Sum/10 = 10 × 11/2 × 10 = 5.5

52. Required average = (0 + 1 + 2 + … + 9)/10 = 45/10 = 4.5

53. Required average = (2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20)/10 = 110/10 = 11. Alternately you could use the formula for sum of the first n even natural numbers as *n*(*n* + 1) with *n* as 10. Then average = Sum/10 = 10 × 11/10 = 11.

54. The sum of the first *n* odd numbers = n^2. In this case *n* = 10 Æ Sum = 10^2 = 100. Required average = 100/10 = 10.

55. Required average = (2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 + 29)/10 = 129/10 = 12.9

56. Required average = (4 + 6 + 8 + 9 + 10 + 12 + 14 + 15 + 16 + 18)/10 = 112/10 = 11.2

57. Required average = (3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 + 29 + 31)/10 = 158/10 = 15.8

58. Teacher’s weight = 31 × 41 – 30 × 40 = 1271 – 1200 = 71.

59. Required average = (2 × 5.5 + 3 × 3.666 + 6 × 1.8333)/11 = (11 + 11 + 11)/11 = 3

60. 30 × *P* + 75 × 2 = 510 Æ *P* = (510 – 150)/30 = 12

61. Required average = (2 × 3000 + 2 × 500)/4 = 7000/4 = 1750.

62. Required average = Total runs/ total innings = (40 × 4 + 0)/5= 160/5 = 32.

63. Principal’s weight = 41 × 79 – 40 × 80 = 3239 – 3200 = 39.

64. Temperature on 3rd December = 24.4 × 3 – 24 × 2 = 73.2 – 48 = 25.2

65. Average age 5 years hence would be 5 years more than the current average age. Hence, 20 + 5 = 25.

66. Required average = (20 × 30 + 30 × 20)/50 = 1200/50 = 24.

67. The numbers would form an *AP* with common difference 1 and the middle term (also the 3rd term) as 18. Thus, the numbers would be 16, 17, 18, 19 and 20. The highest of these numbers would be 20.

68. Required average = (6 × 11 + 14 + 16)/8 = 96/8 = 12

69. The new average would also go up by 2. Hence, 12 + 2 = 14.

70. Total age 3 years ago for 5 people = 17 × 5 = 85. Today, the family’s total age = 17 × 6 = 102. The age of the 5 older people would be 85 + 3 × 5 = 100. Hence, the baby’s age is 2 years.

71. Required average = (10 × 31 + 14 × 30 + 15 × 31)/92 = (310 + 420 + 465)/92 = 12.98 (which is closest to 13.

72. Assume a distance of 60 km. In such a case, the Required average = Total distance/Total time = (60 + 60)/(4 + 3) = 120/7 = 17.14

73. (Mon + Tue + wed) = 41 × 3 = 123. (Tue + Wed + Thu) = 40 × 3 = 120.

    Mon – Thu = 123 – 120 = 3. Since Thursday’s temperature is given as 39, Monday’s temperature would be 39 + 3 = 42.

74. Required average = (30 × 20 – 10 × 10)/10 = 500/10 = 50.

75. Total age of 7 children = 12 × 7 = 84 years. When the 6 year old child dies, the total age of the remaining 6 children would be 84 – 6

= 78. Required average = 78/6 = 13 years.

76. Required average = (2 × 200 + 2 × 250)/4 = 900/4 = 225.

77. Teacher's weight = 36 × 36 – 35 × 35 = 1296 – 1225 = 71 kgs.

78. The statement '*x* is as much more than the average as *y* is less than the average signifies that the numbers *x*, y, z form an Arithmetic Progression with *z* as the middle term. *z*'s value would then be equal to the average of the three numbers. This average is given as 45. Hence, the correct answer is *z* = 45.

79. The sum of the given 4 numbers is 20.75. The required average = 20.75/4. Option (a) is correct.

80. Let the number of non officer workers in the company be *W*. Then we will have the following equation: (15 × 525 + *W* × 85) = (15 + *W*) × 95 Æ 10 *W* = 15 × 430 Æ *W* = 645. Thus, the total number of workers in the company would be 645 + 15 = 660.

81. The woman's age would be 8 × 2 = 16 years more than the age of the man she replaces. Age of the woman = 24 + 2 × 8 = 40 years.

82. Required average income = (Total expenditure + total savings]/12

    = [(1100 × 3 + 2200 × 4 + 4620 × 5) + 2100]/12 = 37300/12 = 3108.333

83. Required average = (2 × 5.5 + 3 × 3.5 + 3 × 5.5 + 5 × 1.5)/13 = 45.5/13 = 3.5

84. For 150 coins to be of a value of ` 150, using only 25 paise, 50 paise and 1 Re coins, we cannot have any coins lower than ther value of 1 `. Thus, the number of 50 paise coins would be 0. Option (d) is correct.

Level of Difficulty (II)

1. The train needs to travel 15 minutes extra @35 kmph. Hence, it is behind by 8.75 kms. The rate of losing distance is 5 kmph. Hence, the train must have travelled for 8.75/5 = 1 hour 45 minutes. @ 40 kmph Æ 70 km.

   Alternatively, you can also see that 12.5% drop in speed results in 14.28% increase in time. Hence, total time required is 105 minutes @ 40 kmph Æ 70 kilometers.

Alternatively, solve through options.

2. Standard question requiring good calculation speed. Obviously, the 15th day is being double counted. Calculations can be reduced by thinking as:

   Surplus in first 15 days – Deficit in last 17 days = 255 – 289 Æ Net deficit of 34. This means that the average is reducing by 34 due to the double counting of the 15th day. This can only mean that the 15th day's expenditure is ` 68 – 34 = 34.

   (Lengthy calculations would have yielded the following calculations:

   85*15 + 51*17 – 68*31 = 34)

3. Find out the number of innings in each year. Then the answer will be given by:

   $\frac{\text{Total runs in 4 years}}{\text{Total innings in 4 years}}$ (4270/119 = 35.88)

4. Find the total distance covered in each segment of 10 minutes. You will get total distance = 46 kilometers in 50 mins.

5. Assume that the distance is 120 km. Hence, 30 km is covered @ 25 kmph, 40 @ 30 kmph and so on.

   Then average speed is 120/total time

6. In three hours the total number of sheets typed will be: 60/6 + 60/7 + 60/9 = 10 + 8.57 + 6.66.

   Hence the number of sheets/hour is 25.23/3 = 8.41 is equivalent to 530/63.

7. 100 Æ 130 Æ 182. Hence, 82/2 = 41.

8. You do not know the number of days in the month. Hence, the question cannot be answered.

9. The number of rooms is 18 + 16 + 30 on the three floors respectively.

   Total revenues are: 18*200 + 16*100 + 30*150 = 9700 required average = 9700/110 = 88.18.

Note here that if you could visualize here that since the number of rooms is 110 the decimal values cannot be .3 or .7 which effectively means that options 3 and 4 are rejected.

10. Replace x with the sales value to calculate the bonus in a month.
11. Use the same process as Q. No. 5 above.
12. The weight of the second man is 135 and that of the third is 105. Hence, net result is a drop of 45 for 5 people. Hence, 9 kg is the drop.
13. Put $x = 100$ to get the weight of the containers. Use these weights to find average weight as 2600/4 = 650.
14. The weight of the boxes are 1st box Æ 200, 3rd box Æ 250 kg, 2nd box Æ 300 kg, 4th box Æ 350 and 5th box Æ 500 kg. Hence difference between the heavier 4 and the lighter 4 is 300. Hence, difference in the averages is 75.
15. Difference between heaviest three and lightest three totals is: (350 + 500) – (300 + 200) = 350

    Difference in average weights is 350/3 = 116.66.
16. Part of the runs scored in the 87th innings will go towards increasing the average of the first 86 innings to the new average and the remaining part of the runs will go towards maintaining the new average for the 87th innings. The only constraint in this problem is that there is an increase in the average by a whole number of runs. This is possible for all three options.
17. Assume x is the average expenditure of 19 people. Then, $19x = 13*79 + 6(x + 4)$.
18. 42 A + 31 = 55 (A – 3) Æ 13 A = 196 Æ A = 196/13 = 15.07. Total expenditure original = 15.07 × 42 = 633.23
19. The total price of the three stones would be 97610498312 × 3 = 292831494936. Since, this price is divided into the three stones in the ratio of 4 : 7 : 9, the price of the cheapest one would be = (4 × 2928314936/20) = 58566298987.2
20. The average weight per ball is asked. Hence, the bag does not have to be counted as the 48th item.

21. 71*48 = 59*46 + x + 11*52 Æ x = 72.

Alternately, this can be solved by using the concepts of surpluses and deficits as:

2*59 (deficit) – 4 * 11(surplus) + 48 (average to be maintained by the 60th number) = 118-44 + 48 = 122.

22. Solve through the same process as the Q. No. 5 of this chapter.

23. (14 * 333 – 2 * 504)/12.

24. $\dfrac{(84-64)\times 94}{18.8}$.

25. Use alligation to solve. 20———32———68. Thus, 5 corresponds to 12, hence for 36 the answer will be 15.

26. Let the equal distances be '*d*' each. Then 3d/(d/x + d/y + d/z) = 3xyz/(x + y + z).

27–30. You have to take between 25th and 30th to mean that both these dates are also included.

27. The maximum average will occur when the maximum possible values are used. Thus:

A should have been born on 30th, *B* on 25th, *C* on 20th, *D* on 10th and *E* on 5th. Further, the months of births in random order will have to be between August to December to maximize the average.

Hence the total will be 30 + 25 + 20 + 10 + 5 + 12 + 11 + 10 + 9 + 8 = 140. Hence average is 28.

28. The minimum average will be when we have 1 + 5 + 10 + 20 + 25 + 1 + 2 + 3 + 4 + 5 = 76. Hence, average is 15.2.

29. This does not change anything. Hence the answer is the same as Q. 27.

30. The prime dates must be 29th, 23rd, 19th and 5th. Hence, the maximum possible average will reduce by 4/5 = 0.8. Hence, answer will be 27.2.

31. Solve using alligation. Since 15 is the mid-point of 13.25 and 16.75, the ratio is 1:1 and hence there are 20 people who were going for the picnic initially.

32. The number of pass candidates are 2 + 6 + 18 + 40 = 66 out of a total of 100. Hence, 66%.

33&34. Put x = 10 in the given equations and find the average of the resultant values.

35. Solve through options.

36. $nz–x + x^1 = nz^1$ Æ Simplify to get Option (c) correct.

37. By alligation the ratio is 3:8. Hence, only 110 is possible.

Q38-41:

38. You don't know who got out when. Hence, cannot be determined.

39. Since possibilities are asked about, you will have to consider all possibilities. Assume, the sixth and seventh batsmen have scored zero. Only then will the possibility of the first 5 batsmen scoring the highest possible average arise. In this case the maximum possible average for the first 5 batsmen could be 403/5 = 80.6.

40. Again it is possible that only the first batsman has scored runs.

41. We cannot find out the number of runs scored by the 7th batsman. Hence answer is (d).

42. You can take 53 as the base to reduce your calculations. Otherwise the question will become highly calculation intensive.

43. 251.25*4 + 277.52 * 5 + 760 = 3152.6

44. Solve using options. 20 is the only possible value.

45. Check through options to solve.

Level of Difficulty (III)

1. Definitely decrease, since the highest marks in Class A is less than the lowest marks in Class B.

2. Cannot say since there is no indication of the values of the numbers which are transferred.

3. It will definitely decrease since the highest possible transfer is lower than the lowest value in C.

4. The effect on A will depend on the profile of the people who are transferred. Hence, anything can happen.

5. Cannot say since there is a possibility that the numbers transferred are such that the average can either increase, decrease or remain constant.
6. If *C* increases, then the average of *C* goes up from 30. For this to happen it is definite that the average of *B* should drop.
7. The maximum possible average for *B* will occur if all the 5 transferees from *A* have 22 marks.
8. The average of Group *A* after the transfer in Q. 7 above is:

   (400 – 18*5)/15 = 310/15 = 20.66
9. (400 – 22*5)/15 = 19.33
10. 400 + 23*5 = 515. Average = 515/25 = 20.6
11. 400 + 31*5 = 555. Average = 555/25 = 22.2
12. Will always decrease since the net value transferred from *B* to *A* will be higher than the net value transferred from *A* to *B*.
13. Since the lowest score in Class *B* is 23 which is more than the highest score of any student in Class *A*. Hence, *A*'s average will always increase.
14. The maximum possible value for *B* will happen when the *A* to *B* transfer has the maximum possible value and the reverse transfer has the minimum possible value.
15. For the minimum possible value of *B* we will need the *A* to *B* transfer to be the lowest possible value while the *B* to *A* transfer must have the highest possible value. Thus, *A* to *B* transfer Æ 18* 5 while *B* to *A* transfer will be 31*5. Hence answer is 22.4.
16. The maximum value for *A* will happen in the case of Q. 15. Then the increment for group *A* is:

   31*5 – 18*5 = 5*(31 – 18) = 65.

   Thus maximum possible value is 465/20 = 23.25.
17. Minimum possible average will happen for the transfer we saw in Q. 14. Thus the answer will be 405/20 = 20.25.
18. The maximum possible value for *C* will be achieved when the transfer from *C* is of five 26's and the transfer back from *B* is of five

31’s. Hence, difference is totals will be +25. Hence, max. average = (900 + 25)/30 = 30.833.

[Note here that 900 has come by 30*30]

19. For the maximum possible value of Class *B* the following set of operations will have to hold:

    Five 33’s are transferred from *C* to *B*, whatever goes from *B* to *A* comes back from *A* to *B*, then five 23’s are transferred from *B* to *C*. This leaves us with:

    Increase of 50 marks Æ average increases by 2 to 27.

20. A will attain maximum value if five 33’s come to *A* from *C* through *B* and five 18’s leave *A*. In such a case the net result is going to be a change of +75. Thus the average will go up by 75/20 = 3.75 to 23.75.

21–23. Will be solved by the same pattern as the above questions.

24. Only option *A* will give us the required situation since the transfer of five 31’s increases the value of the average of group *A*.

25–28. Will be solved by the same pattern as above questions.

29–35. These are standard questions using the concept of averages. Hence, analyse each and every sentence by itself and link the interpretations. If you are getting stuck, the only reason is that you have not used the information in the questions fully.

36. Monthly estimates of income is reduced as the denominator is increased from 12 to 14 at the same time the monthly estimate of expenditure is increased as the denominator is reduced from 12 to 9. Hence, the savings will be underestimated.

37–39. Use the averages formulae and common sense to answer.

40–49. The questions are commonsensical with a lot of calculations and assumptions involved. You have to solve these using all the information provided.

40. Das’s score = 15*2 + 7*2.5 = 47.5 Æ 48.

    Dasgupta’s score = 20*1 + 2*1.5 = 23

41. From the above the answer is 48:23 = 96:46.

42–44.By maximum possible support from the other end, you have to assume that he has Laxman or Sehwag batting aggressively for the entire tenure at the crease. Strike has to be shared equally.

42. Through options, After 60 overs, score would be 150. Then Tendulkar can score @ 4 runs per over (sharing the strike and batting aggressively) and get maximum support @ 3 runs per over. Thus in 30 overs left the target will be achieved.

43. Tendulkar's score for the innings will be 30*4 = 120.

44. We do not know when Laxman would have come into bat. Hence this cannot be determined.

45–49.Build in each of the conditions in the problem to form a table like:

Partnership	Partner	Overs faced	Tendulkar's score	Partner's score
6th wicket	Ganguly	12	6 overs × 6	6 overs × 4
7th wicket	and so on			
8th wicket				
9th wicket				
10th wicket				

Chapter 4 Alligations

INTRODUCTION

The chapter of alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together. I have often seen students having a lot of difficulty in solving questions on alligation. Please remember that all problems on alligation can be solved through the weighted average method. Hence, the student is advised to revert to the weighted average formula in case of any confusion.

The use of the techniques of this chapter for solving weighted average problems will help you in saving valuable time wherever a direct question based on the mixing of two groups is asked. Besides, in the case of questions that use the concept of the weighted average as a part of the problem, you will gain a significant edge if you are able to use the techniques illustrated here.

THEORY

In the chapter on Averages, we had seen the use of the weighted average formula. To recollect, the weighted average is used when a number of smaller groups are mixed together to form one larger group.

If the average of the measured quantity was

A_1 for group 1 containing n_1 elements

A_2 for group	2	containing	n_2	elements
A_3 for group	3	containing	n_3	elements
A_k for group	k	containing	n_k	elements

We say that the weighted average, Aw is given by:

$Aw = (n_1A_1 + n_2\, A_2 + n_3\, A_3 + \ldots\ldots + n_kA_k)/\ (n_1 + n_2 + n_3 \ldots + n_k)$

That is, the weighted average

$$= \frac{\text{Sum total of all groups}}{\text{Total number of elements in all groups together}}$$

In the case of the situation where just two groups are being mixed, we can write this as:

$Aw = (n_1A_1 + n_2\, A_2\,)/(n_1 + n_2)$

Rewriting this equation we get: $(n_1 + n_2)\, Aw = n_1A_1 + n_2A_2$

$n_1(Aw - A_1) = n_2\, (A_2 - Aw)$

or $n_1/n_2 = (A_2 - Aw)/(Aw - A_1)$ Æ The alligation equation.

The Alligation Situation

Two groups of elements are mixed together to form a third group containing the elements of both the groups.

If the average of the first group is A_1 and the number of elements is n_1 and the average of the second group is A_2 and the number of elements is n_2, then to find the average of the new group formed, we can use either the weighted average equation or the alligation equation.

As a convenient convention, we take $A_1 < A_2$. Then, by the principal of averages, we get $A_1 < Aw < A_2$.

Illustration 1

Two varieties of rice at ` 10 per kg and ` 12 per kg are mixed together in the ratio 1 : 2. Find the average price of the resulting mixture.

Solution $1/2 = (12 – Aw)/(Aw – 10)$ Æ $Aw – 10 = 24 – 2Aw$

fi $3Aw = 34$ fi $Aw = 11.33$ `/kg.

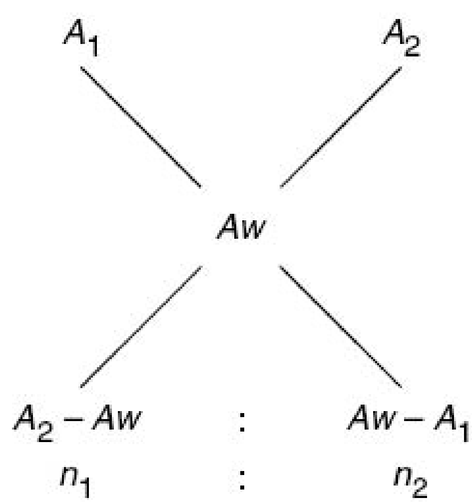
Illustration 2

On combining two groups of students having 30 and 40 marks respectively in an exam, the resultant group has an average score of 34. Find the ratio of the number of students in the first group to the number of students in the second group.

Solution $n_1/n_2 = (40 – 34)/(34 – 30) = 6/4 = 3/2$

Graphical Representation of Alligation

The formula illustrated above can be represented by the following cross diagram:



[Note that the cross method yields nothing but the alligation equation. Hence, the cross method is nothing but a graphical representation of the alligation equation.]

As we have seen, there are five variables embedded inside the alligation equation. These being:

the three averagesÆ A_1, A_2 and Aw

and the two weightsÆ n_1 and n_2

Based on the problem situation, one of the following cases may occur with respect to the knowns and the unknown, in the problem.

Case	*Known*	*Unknown*
I	(a) A_1, A_2, Aw	(a) $n_1 : n_2$
	(b) A_1, A_2, Aw, n_1	(b) n_2 and $n_1 : n_2$
II	A_1, A_2, n_1, n_2	Aw
III	A_1, Aw, n_1, n_2	A_2

Now, let us try to evaluate the effectiveness of the cross method for each of the three cases illustrated above:

Case 1: A_1, A_2, Aw are known; may be one of n_1 or n_2 is known.

To find: $n_1 : n_2$ and n_2 if n_1 is known OR n_1 if n_2 is known.

Let us illustrate through an example:

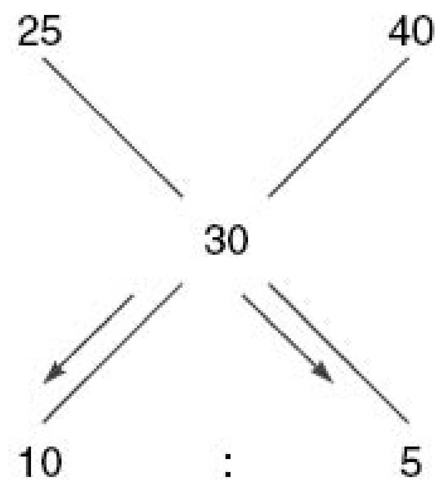
Illustration 3

On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find

(a) The ratio of students in the classes

(b) The number of students in the first class if the second class had 30 students.

Solution



(a) Hence, solution is 2 : 1.

(b) If the ratio is 2 : 1 and the second class has 30 students, then the first class has 60 students.

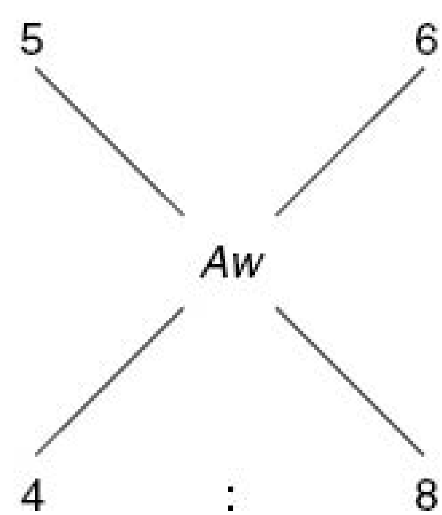
Note: The cross method becomes pretty effective in this situation when all the three averages are known and the ratio is to be found out.

Case 2: A_1, A_2, n_1 and n_2 are known, Aw is unknown.

Illustration 4

4 kg of rice at ` 5 per kg is mixed with 8 kg of rice at ` 6 per kg. Find the average price of the mixture.

Solution



= (6 – *Aw)* : (*Aw* – 5)

fi(6 – *Aw*)/(*Aw* – 5) = 4/8 Æ12 – 2 *Aw* = *Aw* – 5

3 *Aw* = 17

\ *Aw* = 5.66 `/kg. (Answer)

Task for student: Solve through the alligation formula approach and through the weighted average approach to get the solution. Notice, the amount of time required in doing the same.

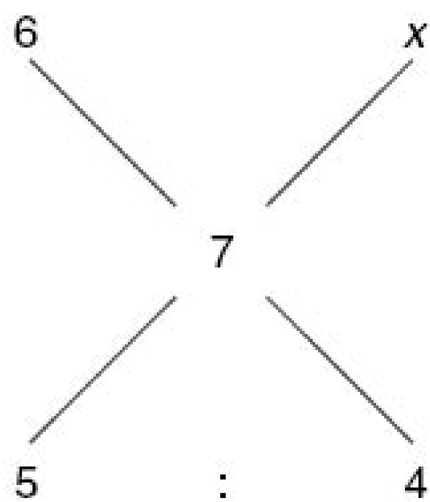
Note: The cross method becomes quite cumbersome in this case, as this method results in the formula being written. Hence, there seems to be no logic in using the cross method in this case.

Case 3: A_1, *Aw*, n_1 and n_2 are known; A_2 is unknown.

Illustration 5

5 kg of rice at ` 6 per kg is mixed with 4 kg of rice to get a mixture costing ` 7 per kg. Find the price of the costlier rice.

Solution Using the cross method:



= (*x* – 7) : 1

\ (*x* – 7)/1 = 5/4 Æ 4*x* – 28 = 5

\ *x* = ` 8.25.

Task for student: Solve through the alligation formula approach and through the weighted average approach to get the solution. Notice the amount of time required in doing the same.

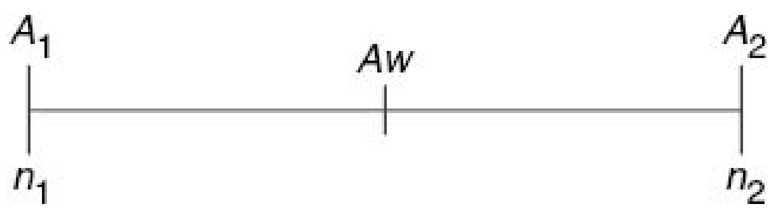
Note: The cross method becomes quite cumbersome in this case since this method results in the formula being written. Hence, there seems to be no logic in using the cross method in this case.

The above problems can be dealt quite effectively by using the straight line approach, which is explained below.

The Straight Line Approach

As we have seen, the cross method becomes quite cumbersome in Case 2 and Case 3. We will now proceed to modify the cross method so that the question can be solved graphically in all the three cases.

Consider the following diagram, which results from closing the cross like a pair of scissors. Then the positions of A_1, A_2, Aw, n_1 and n_2 are as shown.



Visualise this as a fragment of the number line with points A_1, Aw and A_2 in that order from left to right.

Then,

(a) n_2 is responsible for the distance between A_1 and Aw or n_2 corresponds to $Aw - A_1$

(b) n_1 is responsible for the distance between Aw and A_2. or n_1 corresponds to $A_2 - Aw$

(c) $(n_1 + n_2)$ is responsible for the distance between A_1 and A_2. or $(n_1 + n_2)$ corresponds to $A_2 - A_1$.

The processes for the 3 cases illustrated above can then be illustrated below:

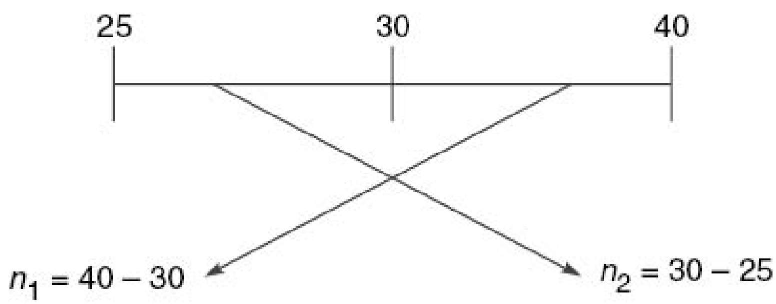
Illustration 6

On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find

(a) the ratio in which the classes were mixed.

(b) the number of students in the first class if the second class had 30 students.

Solution



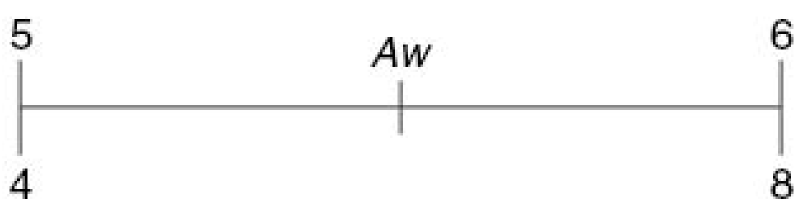
Hence, ratio is 2 : 1, and the second class has 60 students.

Case 2 A_1, A_2, n_1 and n_2 are known; *Aw* is unknown.

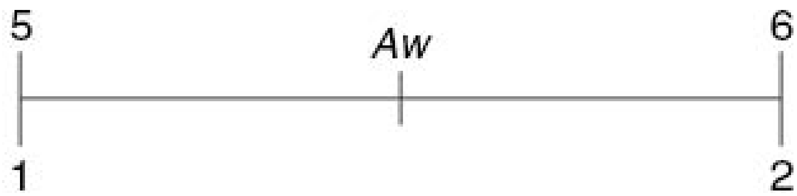
Illustration 7

4 kg of rice at ` 5 per kg is mixed with 8 kg of rice at ` 6 per kg. Find the average price of the mixture.

Solution



is the same as



Then, by unitary method:

$n_1 + n_2$ corresponds to $A_2 – A_1$

Æ 1 + 2 corresponds to 6 – 5

That is, 3 corresponds to 1

\ n_2 will correspond to $\frac{(A_2 - A_1) \times n_2}{(n_1 + n_2)}$

In this case (1/3) × 2 = 0.66.

Hence, the required answer is 5.66.

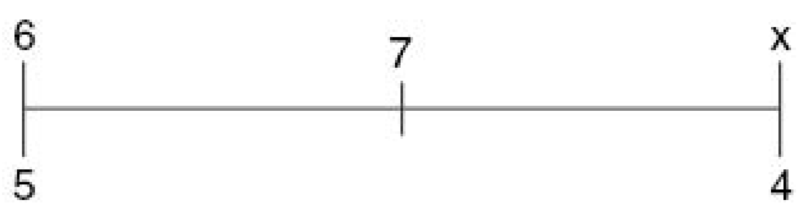
Note: In this case, the problem associated with the cross method is overcome and the solution becomes graphical.

Case 3: A_1, Aw, n_1 and n_2 are known; A_2 is unknown.

Illustration 8

5 kg of rice at ` 6 per kg is mixed with 4 kg of rice to get a mixture costing ` 7 per kg. Find the price of the costlier rice.

Using straight line method:



4 corresponds to 7 – 6 and 5 corresponds to $x – 7$.

The thought process should go like:

4 Æ1

\ 5 Æ1.25

Hence, $x – 7 = 1.25$

and $x = 8.25$

SOME TYPICAL SITUATIONS WHERE ALLIGATIONS CAN BE USED

Given below are typical alligation situations, which students should be able to recognize. This will help them improve upon the time required in solving questions. Although in this chapter we have illustrated problems based on alligation at level 1 only, alligation is used in more complex problems where the weighted average is an intermediate step in the solution process.

The following situations should help the student identify alligation problems better as well as spot the way A_1, A_2, n_1 and n_2 and Aw are mentioned in a problem.

In each of the following problems the following magnitudes represent these variables:

$A_1 = 20, A_2 = 30, n_1 = 40, n_2 = 60$

Each of these problems will yield an answer of 26 as the value of *Aw*.

1. A man buys 40 kg of rice at ` 20/kg and 60 kg of rice at ` 30/kg. Find his average price. (26/kg)
2. Pradeep mixes two mixtures of milk and water. He mixes 40 litres of the first containing 20% water and 60 litres of the second containing 30% water. Find the percentage of water in the final mixture. (26%)
3. Two classes are combined to form a larger class. The first class having 40 students scored an average of 20 marks on a test while the second having 60 students scored an average of 30 marks on the same test. What was the average score of the combined class on the test. (26 marks)
4. A trader earns a profit of 20% on 40% of his goods sold, while he earns a profit of 30% on 60% of his goods sold. Find his percentage profit on the whole. (26%)
5. A car travels at 20 km/h for 40 minutes and at 30 km/h for 60 minutes. Find the average speed of the car for the journey. (26 km/hr)

6. 40% of the revenues of a school came from the junior classes while 60% of the revenues of the school came from the senior classes. If the school raises its fees by 20% for the junior classes and by 30% for the senior classes, find the percentage increase in the revenues of the school. (26%)

Some Keys to spot A_1, A_2 and Aw and differentiate these from n_1 and n_2

1. Normally, there are 3 averages mentioned in the problem, while there are only 2 quantities. This isn't foolproof though, since at times the question might confuse the student by giving 3 values for quantities representing n_1, n_2 and $n_1 + n_2$ respectively.
2. A_1, A_2 and Aw are always rate units, while n_1 and n_2 are quantity units.
3. The denominator of the average unit corresponds to the quantity unit (i.e. unit for n_1 and n_2).
4. All percentage values represent the average values.

A Typical Problem

A typical problem related to the topic of alligation goes as follows:

4 litres of wine are drawn from a cask containing 40 litres of wine. It is replaced by water. The process is repeated 3 times

(a) What is the final quantity of wine left in the cask.

(b) What is the ratio of wine to water finally.

If we try to chart out the process, we get: Out of 40 litres of wine, 4 are drawn out.

This leaves 36 litres wine and 4 litres water. (Ratio of 9 : 1)

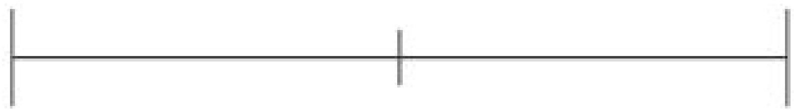
Now, when 4 litres are drawn out of this mixture, we will get 3.6 litres of wine and 0.4 litres of water (as the ratio is 9 : 1). Thus at the end of the second step we get: 32.4 litres of wine and 7.6 litres of water. Further, the process is repeated, drawing out 3.24 litres wine and 0.76 litres water leaving 29.16 litres of wine and 10.84 litres of water.

This gives the final values and the ratio required.

A closer look at the process will yield that we can get the amount of wine left by:

$40 \times 36/40 \times 36/40 \times 36/40 = 40 \times (36/40)^3$

fi $40 \times (1 - 4/40)^3$



This yields the formula:

Wine left : Capacity × (1 – fraction of wine withdrawn)^n for n operations.

Thus, you could have multiplied:

$40 \times (0.9)^3$ to get the answer

That is, reduce 40 by 10% successively thrice to get the required answer.

Thus, the thought process could be:

40 – 10% Æ 36 – 10% Æ 32.4 – 10% Æ 29.16

LEVEL OF DIFFICULTY (I)

1. If 5 kg of salt costing ` 5/kg and 3 kg of salt costing ` 4/kg are mixed, find the average cost of the mixture per kilogram.

   (a) ` 4.5 (b) ` 4.625

   (c) ` 4.75 (d) ` 4.125

2. Two types of oils having the rates of ` 4/kg and ` 5/kg respectively are mixed in order to produce a mixture having the rate of ` 4.60/kg. What should be the amount of the second type of oil if the amount of the first type of oil in the mixture is 40 kg?

   (a) 75 kg (b) 50 kg

   (c) 60 kg (d) 40 kg

3. How many kilograms of sugar worth ` 3.60 per kg should be mixed with 8 kg of sugar worth ` 4.20 per kg, such that by selling the mixture at ` 4.40 per kg, there may be a gain of 10%?

   (a) 6 kg (b) 3 kg

   (c) 2 kg (d) 4 kg

4. A mixture of 125 gallons of wine and water contains 20% water. How much water must be added to the mixture in order to increase the percentage of water to 25% of the new mixture?

   (a) 10 gals (b) 8.5 gals

   (c) 8 gals (d) 8.33 gals

5. Ravi lends ` 3600 on simple interest to Harsh for a period of 5 years. He lends a part of the amount at 4% interest and the rest at 6% and receives ` 960 as the amount of interest. How much money did he lend on 4% interest rate?

   (a) ` 2800 (b) ` 2100

   (c) ` 2400 (d) ` 1200

6. 400 students took a mock exam in Delhi. 60% of the boys and 80% of the girls cleared the cut off in the examination. If the total percentage of students qualifying is 65%, how many girls appeared in the examination?

(a) 100 (b) 120

(c) 150 (d) 300

7. A man purchased a cow and a calf for ` 1300. He sold the calf at a profit of 20% and the cow at a profit of 25%. In this way, his total profit was $23\frac{1}{13}$ %. Find the cost price of the cow.

(a) ` 1100 (b) ` 600

(c) ` 500 (d) ` 800

8. The average salary per head of all employees of a company is ` 600. The average salary of 120 officers is ` 4000. If the average salary per head of the rest of the employees is ` 560, find the total number of workers in the company.

(a) 10200 (b) 10320

(c) 10500 (d) 10680

9. A dishonest milkman purchased milk at ` 10 per litre and mixed 5 litres of water in it. By selling the mixture at the rate of ` 10 per litre he earns a profit of 25%. The quantity of the amount of the mixture that he had was:

(a) 15 litres (b) 20 litres

(c) 25 litres (d) 30 litres

10. A cistern contains 50 litres of water. 5 litres of water is taken out of it and replaced by wine. The process is repeated again. Find the proportion of wine and water in the resulting mixture.

(a) 1 : 4 (b) 41 : 50

(c) 19 : 81 (d) 81 : 19

11. A container has a capacity of 20 gallons and is full of spirit. 4 gallons of spirit is drawn out and the container is again filled with

water. This process is repeated 5 times. Find out how much spirit is left in the resulting mixture finally?

(a) $6\frac{257}{525}$ gallons (b) $6\frac{346}{625}$ gallons

(c) 6.5 gallons (d) 6.25 gallons

12. A vessel is full of refined oil. 1/4 of the refined oil is taken out and the vessel is filled with mustard oil. If the process is repeated 4 times and 10 litres of refined oil is finally left in the vessel, what is the capacity of the vessel?

(a) 33 litres (b) $\frac{2460}{81}$ litres

(c) $\frac{2560}{81}$ litres (d) 30 litres

13. In what ratio should two qualities of coffee powder having the rates of ` 47 per kg and ` 32 per kg be mixed in order to get a mixture that would have a rate of ` 37 per kg?

(a) 1 : 2 (b) 2 : 1

(c) 1 : 3 (d) 3 : 1

14. A thief steals four gallons of liquid soap kept in a train compartment's bathroom from a container that is full of liquid soap. He then fills it with water to avoid detection. Unable to resist the temptation he steals 4 gallons of the mixture again, and fills it with water. When the liquid soap is checked at a station it is found that the ratio of the liquid soap now left in the container to that of the water in it is 36 : 13. What was the initial amount of the liquid soap in the container if it is known that the liquid soap is neither used nor augmented by anybody else during the entire period?

(a) 7 gallons (b) 14 gallons

(c) 21 gallons (d) 28 gallons

15. In what ratio should water be mixed with soda costing ` 12 per litre so as to make a profit of 25% by selling the diluted liquid at ` 13.75 per litre?

(a) 10 : 1 (b) 11 : 1

(c) 1 : 11 (d) 12 : 1

16. A sum of ` 36.90 is made up of 90 coins that are either 20 paise coins or 50 paise coins. Find out how many 20 paise coins are there in the total amount.

(a) 47 (b) 43

(c) 27 (d) 63

17. A dishonest grocer professes to sell pure butter at cost price, but he mixes it with adulterated fat and thereby gains 25%. Find the percentage of adulterated fat in the mixture assuming that adulterated fat is freely available.

(a) 20% (b) 25%

(c) 33.33% (d) 40%

18. A mixture of 70 litres of alcohol and water contains 10% of water. How much water must be added to the above mixture to make the water 12.5% of the resulting mixture?

(a) 1 litre (b) 1.5 litre

(c) 2 litres (d) 2.5 litres

19. A mixture of 20 litres of brandy and water contains 10% water. How much water should be added to it to increase the percentage of water to 25%?

(a) 2 litres (b) 3 litres

(c) 2.5 litres (d) 4 litres

20. A merchant purchased two qualities of pulses at the rate of ` 200 per quintal and ` 260 per quintal. In 52 quintals of the second quality, how much pulse of the first quality should be mixed so that by

selling the resulting mixture at ` 300 per quintal, he gains a profit of 25%?

(a) 100 quintals (b) 104 quintals

(c) 26 quintals (d) None of these

21. A man buys milk at ` 8.5 per litre and dilutes it with water. He sells the mixture at the same rate and thus gains 11.11%. Find the quantity of water mixed by him in every litre of milk.

(a) 0.111 litres (b) 0.909 litres

(c) 0.1 litre (d) 0.125 litres

22. There are two mixtures of honey and water, the quantity of honey in them being 25% and 75% of the mixture. If 2 gallons of the first are mixed with three gallons of the second, what will be the ratio of honey to water in the new mixture?

(a) 11 : 2 (b) 11 : 9

(c) 9 : 11 (d) 2 : 11

23. There are two kinds of alloys of tin and copper. The first alloy contains tin and copper such that 93.33% of it is tin. In the second alloy there is 86.66% tin. What weight of the first alloy should be mixed with some weight of the second alloy so as to make a 50 kg mass containing 90% of tin?

(a) 15 kg (b) 30 kg

(c) 20 kg (d) 25 kg

24. Two containers of equal capacity are full of a mixture of oil and water. In the first, the ratio of oil to water is 4 : 7 and in the second it is 7 : 11. Now both the mixtures are mixed in a bigger container. What is the resulting ratio of oil to water?

(a) 149 : 247 (b) 247 : 149

(c) 143 : 241 (d) 241 : 143

25. Two vessels contain spirit and water mixed respectively in the ratio of 1 : 3 and 3 : 5. Find the ratio in which these are to be mixed to get a new mixture in which the ratio of spirit to water is 1 : 2.

(a) 2 : 1 (b) 3 : 1

(c) 1 : 2 (d) 1 : 3

26. The price of a pen and a pencil is ` 35. The pen was sold at a 20% profit and the pencil at a 10% loss. If in the transaction a man gains ` 4, how much is cost price of the pen?

(a) ` 10 (b) ` 25

(c) ` 20 (d) None of these

27. A person purchased a cupboard and a cot for ` 18,000. He sold the cupboard at a profit of 20% and the cot at a profit of 30%. If his total profit was 25.833%, find the cost price of the cupboard.

(a) ` 10,500 (b) ` 12,000

(c) ` 7500 (d) ` 10,000

28. A vessel is full of a mixture of kerosene and petrol in which there is 18% kerosene. Eight litres are drawn off and then the vessel is filled with petrol. If the kerosene is now 15%, how much does the vessel hold?

(a) 40 litres (b) 32 litres

(c) 36 litres (d) 48 litres

29. Two solutions of 90% and 97% purity are mixed resulting in 21 litres of mixture of 94% purity. How much is the quantity of the first solution in the resulting mixture?

(a) 15 litres (b) 12 litres

(c) 9 litres (d) 6 litres

30. In the Singapore zoo, there are deers and there are ducks. If the heads are counted, there are 180, while the legs are 448. What will be the number of deers in the zoo?

(a) 136 (b) 68

(c) 44 (d) 22

31. A bonus of ` 9,85,000 was divided among 300 workers of a factory. Each male worker gets 5000 rupees and each female worker gets

2500 rupees. Find the number of male workers in the factory.

(a) 253 (b) 47

(c) 94 (d) 206

32. What will be the ratio of petrol and kerosene in the final solution formed by mixing petrol and kerosene that are present in three vessels of equal capacity in the ratios 4 : 1, 5 : 2 and 6 : 1 respectively?

(a) 166 : 22 (b) 83 : 22

(c) 83 : 44 (d) None of these

33. A mixture worth ` 3.25 a kg is formed by mixing two types of flour, one costing ` 3.10 per kg while the other ` 3.60 per kg. In what proportion must they have been mixed?

(a) 3 : 7 (b) 7 : 10

(c) 10 : 3 (d) 7 : 3

34. A 20 percent gain is made by selling the mixture of two types of ghee at ` 480 per kg. If the type costing 610 per kg was mixed with 126 kg of the other, how many kilograms of the former was mixed?

(a) 138 kg (b) 34.5 kg

(c) 69 kg (d) Cannot be determined

35. In what proportion must water be mixed with milk so as to gain 20% by selling the mixture at the cost price of the milk? (Assume that water is freely available)

(a) 1 : 4 (b) 1 : 5

(c) 1 : 6 (d) 1 : 12

36. A bartender stole champagne from a bottle that contained 50% of spirit and he replaced what he had stolen with champagne having 20% spirit. The bottle then contained only 25% spirit. How much of the bottle did he steal?

(a) 80% (b) 83.33%

(c) 85.71% (d) 88.88%

37. A bag contains a total of 105 coins of ` 1, 50 p and 25 p denominations. Find the total number of coins of Re 1 if there are a total of 50.5 rupees in the bag and it is known that the number of 25 paise coins are 133.33% more than the number of 1 rupee coins.

(a) 56 (b) 25

(c) 24 (d) None of these

38. A man possessing ` 6800, lent a part of it at 10% simple interest and the remaining at 7.5% simple interest. His total income after $3\frac{1}{2}$ years was ` 1904. Find the sum lent at 10% rates.

(a) ` 1260 (b) ` 1700

(c) ` 1360 (d) None of these

39. If a man decides to travel 80 kilometres in 8 hours partly by foot and partly on a bicycle, his speed on foot being 8 km/h and that on bicycle being 16 km/h, what distance would he travel on foot?

(a) 20 km (b) 30 km

(c) 48 km (d) 60 km

40. Two vessels contain a mixture of spirit and water. In the first vessel the ratio of spirit to water is 8 : 3 and in the second vessel the ratio is 5 : 1. A 35 litre cask is filled from these vessels so as to contain a mixture of spirit and water in the ratio of 4 : 1. How many litres are taken from the first vessel?

(a) 11 litres (b) 22 litres

(c) 16.5 litres (d) 17.5 litres

ANSWER KEY

Level of Difficulty (I)

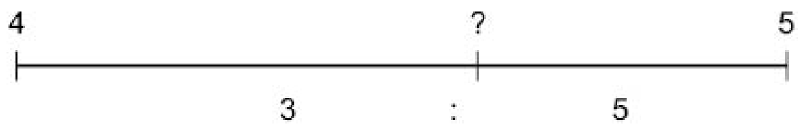
1. (b)	2. (c)	3. (d)	4. (d)
5. (d)	6. (a)	7. (d)	8. (b)
9. (c)	10. (c)	11. (b)	12. (c)

13. (a)	14. (d)	15. (c)	16. (c)
17. (a)	18. (c)	19. (d)	20. (c)
21. (a)	22. (b)	23. (d)	24. (a)
25. (c)	26. (b)	27. (c)	28. (d)
29. (c)	30. (c)	31. (c)	32. (b)
33. (d)	34. (d)	35. (b)	36. (b)
37. (c)	38. (c)	39. (c)	40. (a)

Solutions and Shortcuts

Level of Difficulty (I)

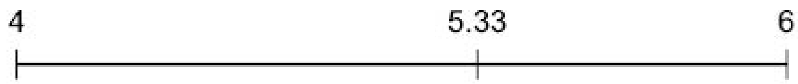
1. Solving the following alligation figure:



The answer would be 4.625/kg.

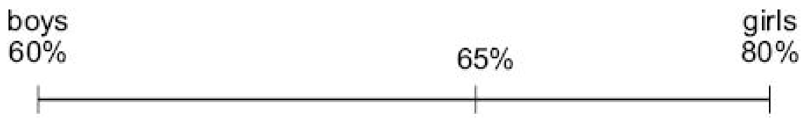
2. Mixing ` 4/kg and ` 5/kg to get ` 4.6 per kg we get that the ratio of mixing is 2:3. If the first oil is 40 kg, the second would be 60 kg.
3. Since by selling at ` 4.40 we want a profit of 10%, it means that the average cost required is ` 4 per kg. Mixing sugar worth ` 3.6/kg and `4.2/kg to get ` 4/kg means a mixture ratio of 1:2. Thus, to 8 kg of the second variety we need to add 4 kg of the first variety to get the required cost price.
4. In 125 gallons we have 25 gallons water and 100 gallons wine. To increase the percentage of water to 25%, we need to reduce the percentage of wine to 75%. This means that 100 gallons of wine = 75% of the new mixture. Thus the total mixture = 133.33 gallons. Thus, we need to mx 133.33 – 125 = 8.33 gallons of water in order to make the water equivalent to 25% of the mixture.
5. Since, Ravi earns ` 960 in 5 years, it means that he earns an interest of 960/5 = ` 192 per year. On an investment of 3600, an annual interest of 192 represents an average interest rate of 5.33%.

   Then using the alligation figure below:



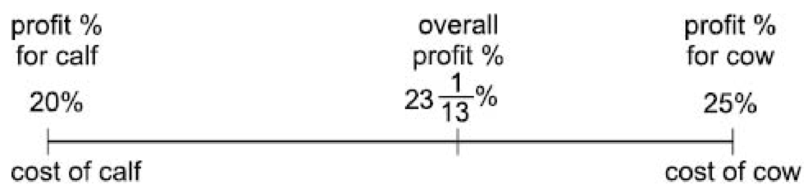
We get the ratio of investments as 1:2. Hence, he lent 1 × 3600/3 = 1200 at 4% per annum.

6. The ratio of boys and girls appearing for the exam can be seen to be 3:1 using the following alligation figure.



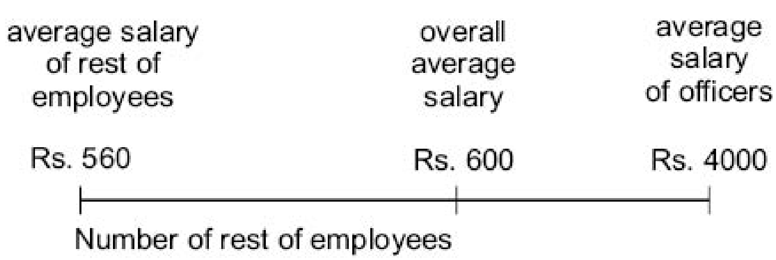
This means that out of 400 students, there must have been 100 girls who appeared in the exam.

7. The ratio of the cost of the cow and the calf would be 40:25 or 8:5 as can be seen from the following alligation figure:



Thus, the cost of the cow would be `800.

8.



From the figure it is clear that the ratio of the number of officers to the number of other employees would be 40:3400. Since there are 120 officers, there would be 3400 × 3 = 10200 workers in the company. Thus the total number of employees would be 10200 + 120 = 10320.

9. The cost price of the mixture would have been ` 8 per liter for him to get a profit of 25% by selling at ` 10 per liter. The ratio of mixing would have been 1:4 (water is to milk) as can be seen in the figure: