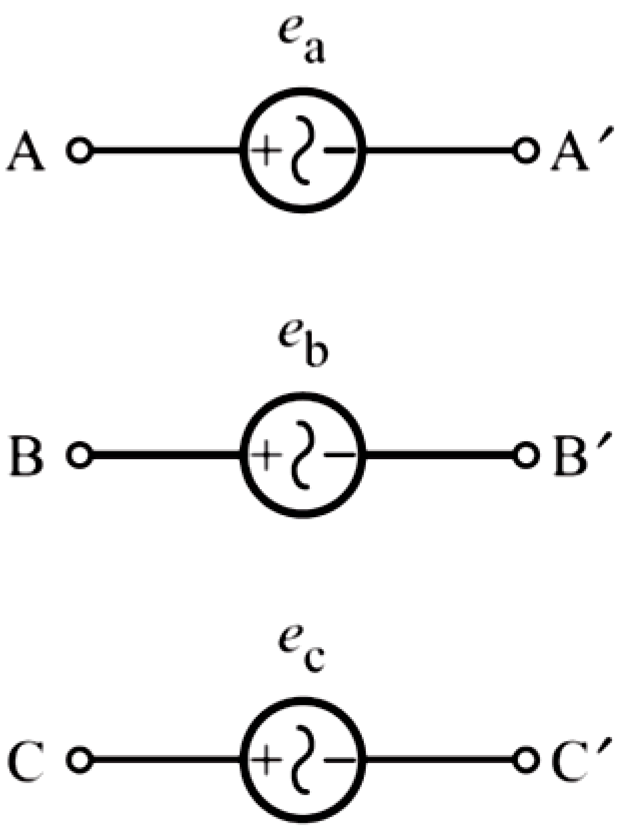
Topics to be Discussed

- Three-Phase System.
  - Advantages.
  - Concept.
  - Generation.
- Unbalanced Three-Phase System.
- Voltages And Currents Relations.
  - (1) Star-Connected System.
  - (2) Delta-Connected System
- Power In Three-phase System.
- Power Measurement.
  - Two-Wattmeter Method.
- Power Factor Measurement.
  - Important Points.

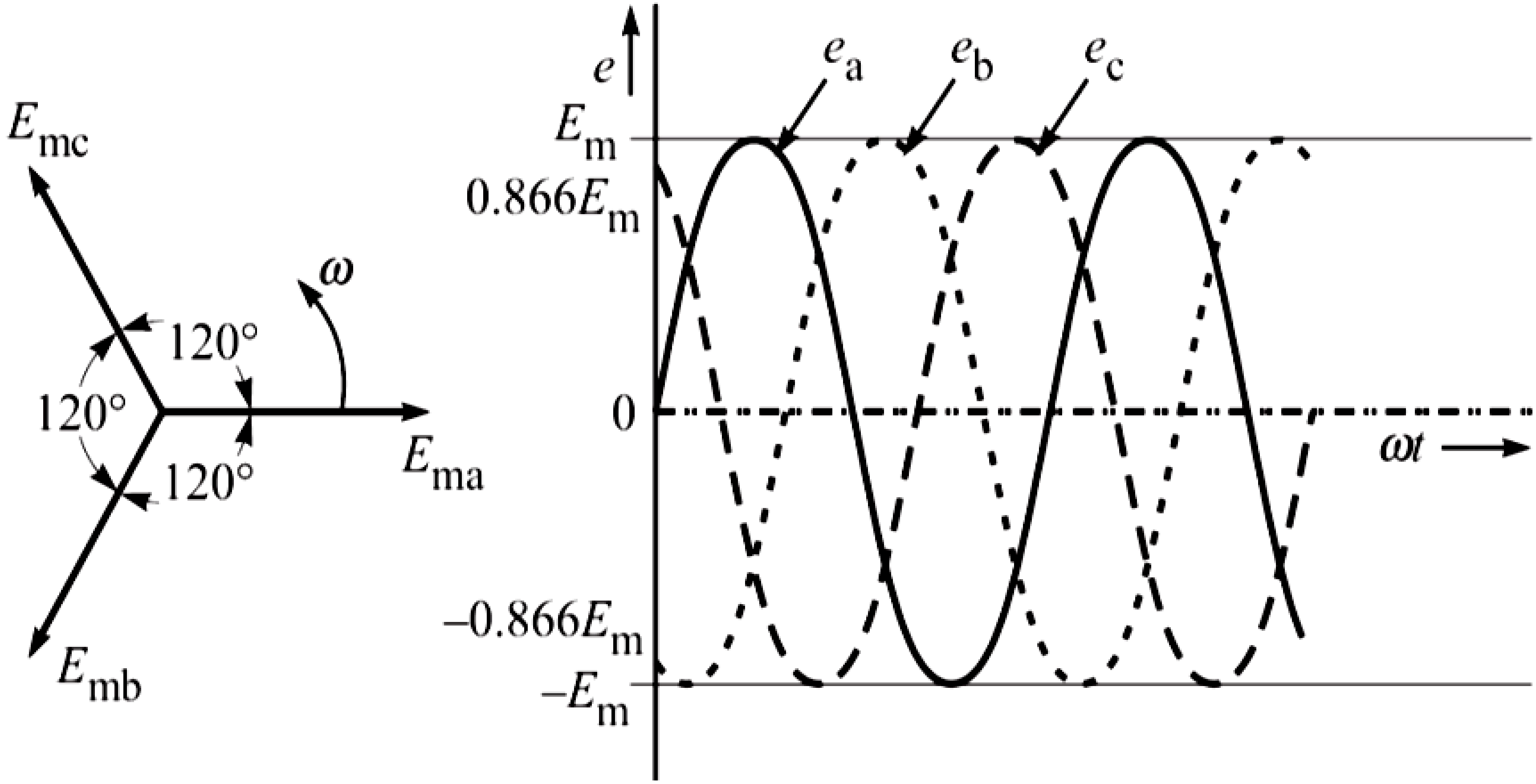
Advantages of Three-Phase System

- Transmission lines require much less conductor material.
- A three-phase machine gives a higher output.
- A three-phase motor develops a uniform (not a pulsating) torque.
- The three-phase induction motors are self-starting.
- Can be used to supply domestic as well as industrial (or commercial) power.
- The voltage regulation is better.

Concept of Three-phase Voltages

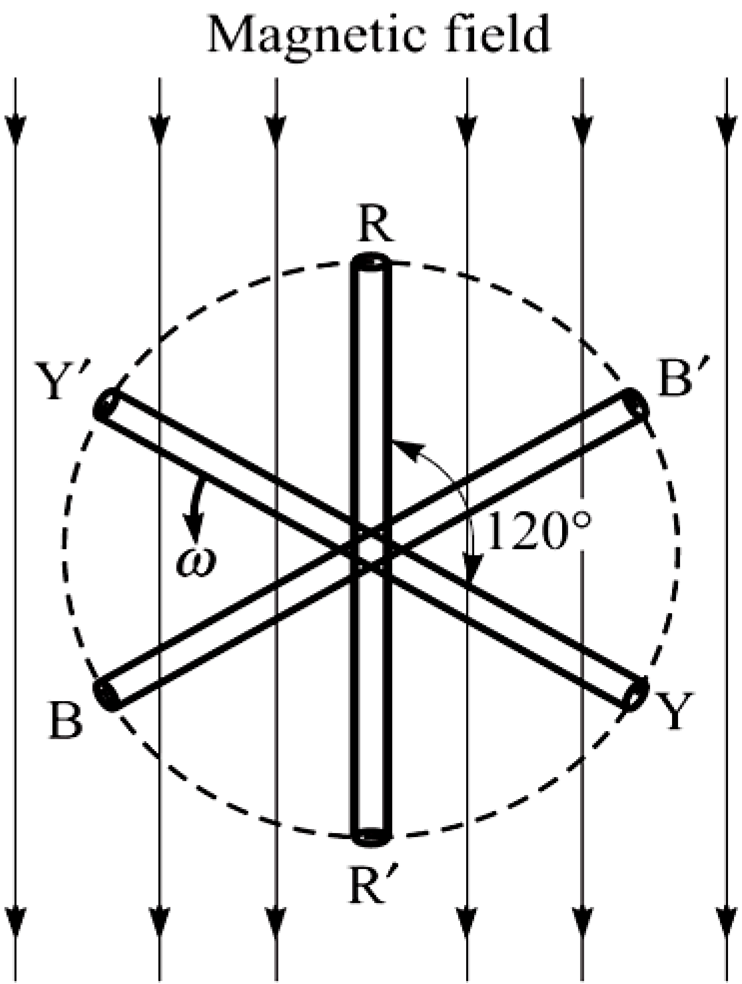


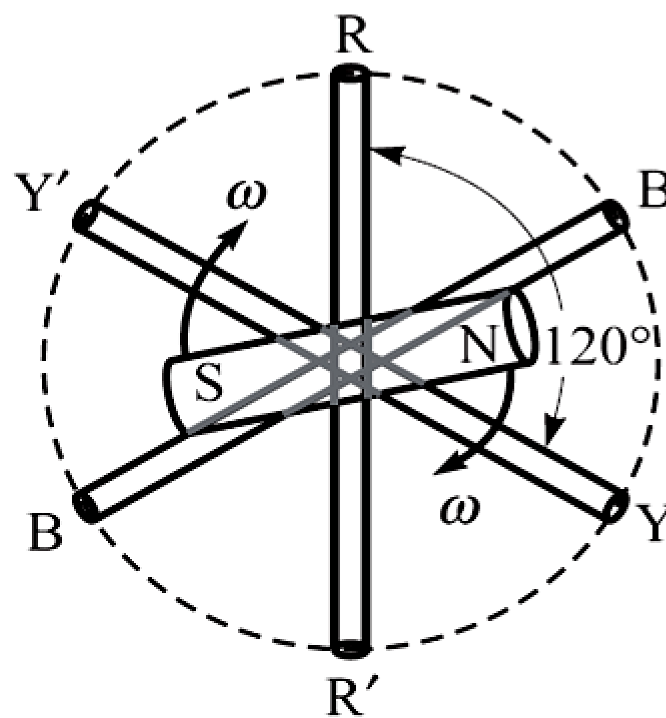
- The ***phase order*** or ***phase sequence*** or ***phase rotation*** is *abc*.



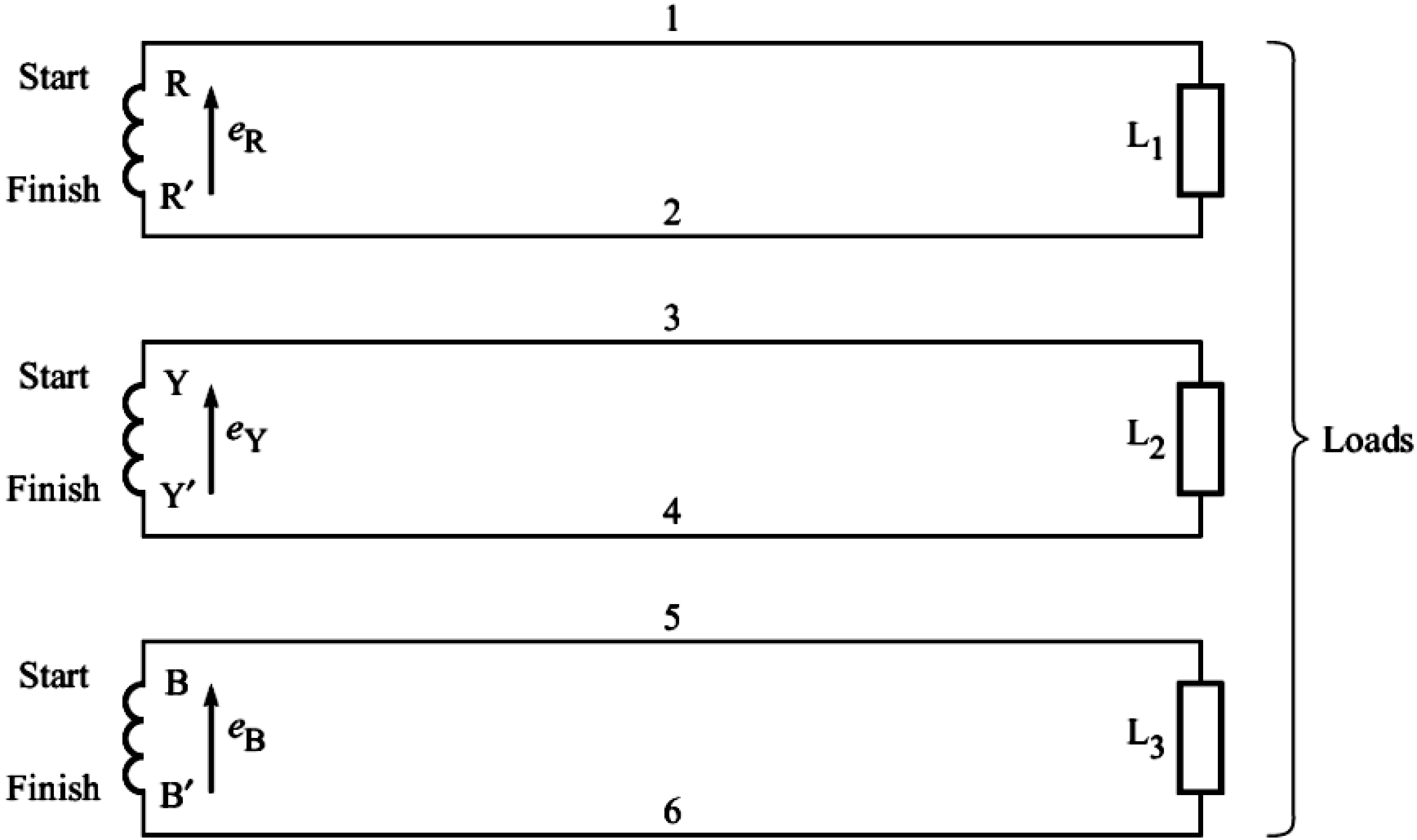
E_{mc}
120°
120°
120°
ω
E_{ma}
E_{mb}
e
e_a
e_b
e_c
E_m
$0.866E_m$
0
ωt
$-0.866E_m$
$-E_m$

Generation of Three-phase Voltages





- Whether the coils rotate anti-clockwise or the magnet on the rotor rotates clockwise, the effect is the same.
- But the latter is safer and easier to make external connections to stationary coils.



Three windings connected to three loads using six line conductors

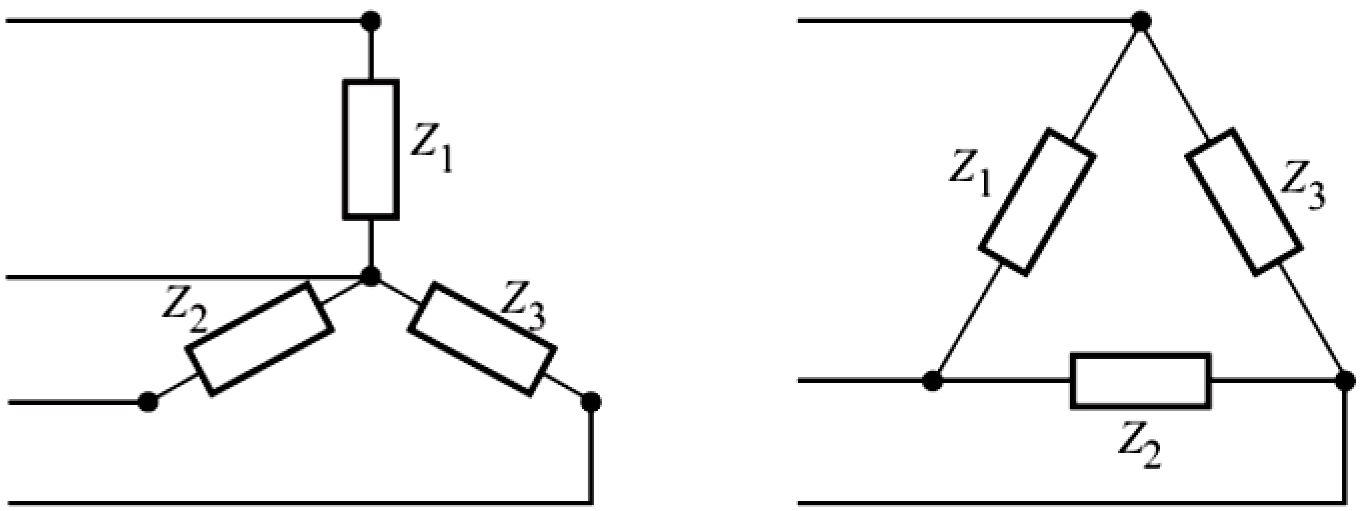
- Thus, the terminals on the periphery appear in the order : R, B', Y, R', B, Y'.
- The three emfs generated e_R, e_Y and e_B connected to three respective loads L_1, L_2 and L_3.
- This necessitates the use of six line conductors.
- Obviously, it is cumbersome and expensive.
- Let us now consider how it may be simplified.

Three-phase Loads

There are *two* kinds of three-phase systems :

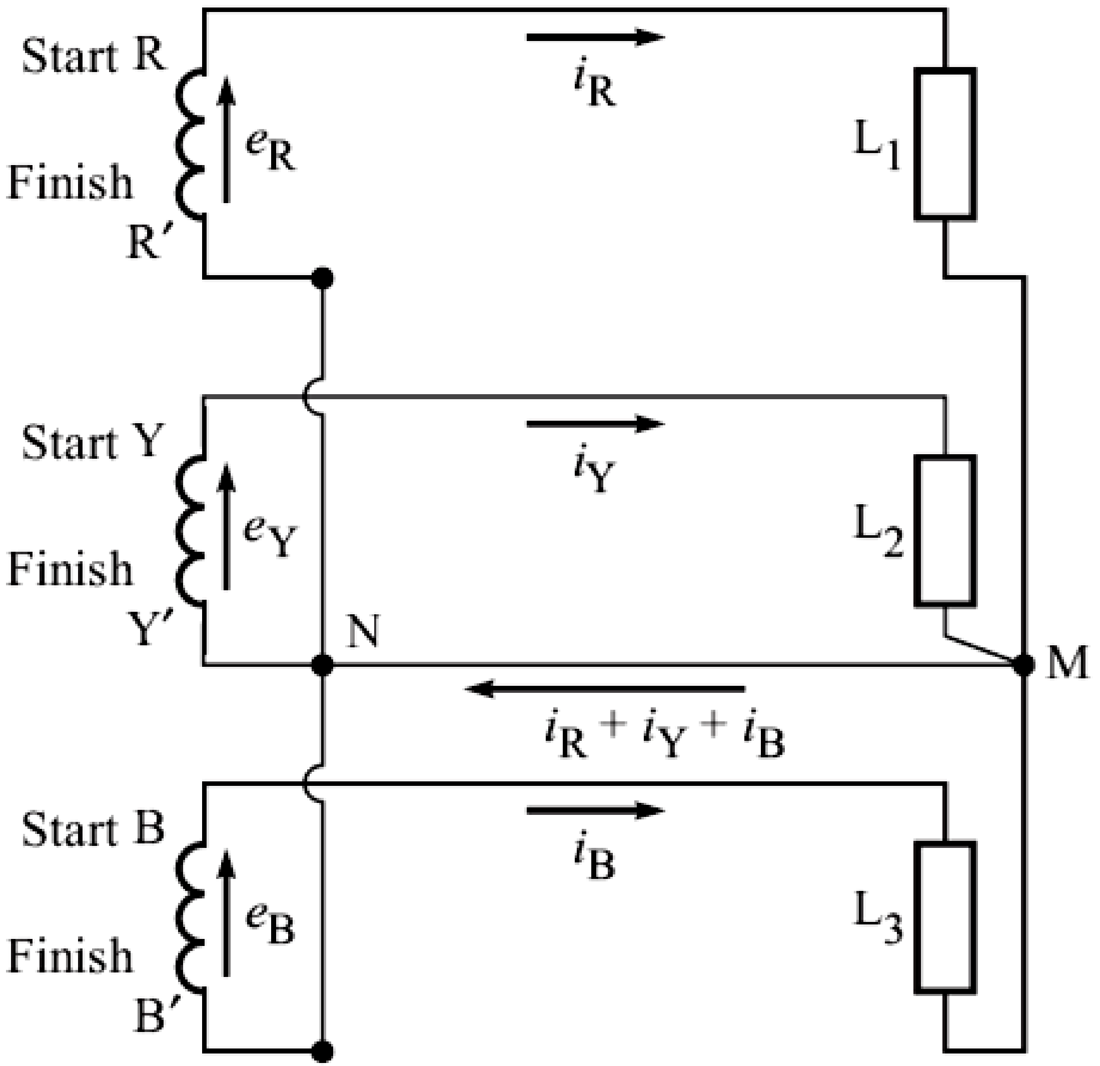
(i) Star or wye (Y) connection, and

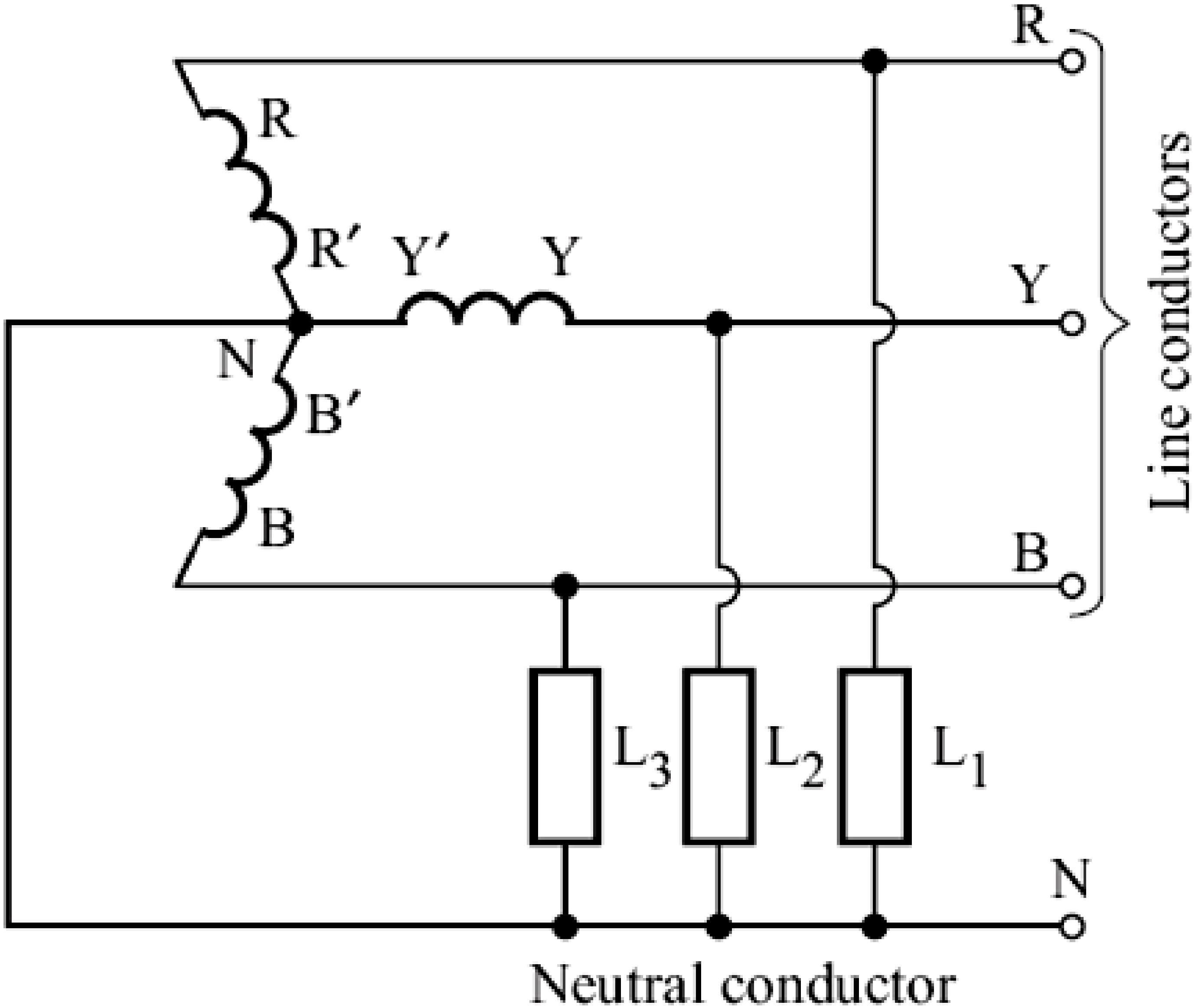
(ii)Delta (Δ) or mesh connection



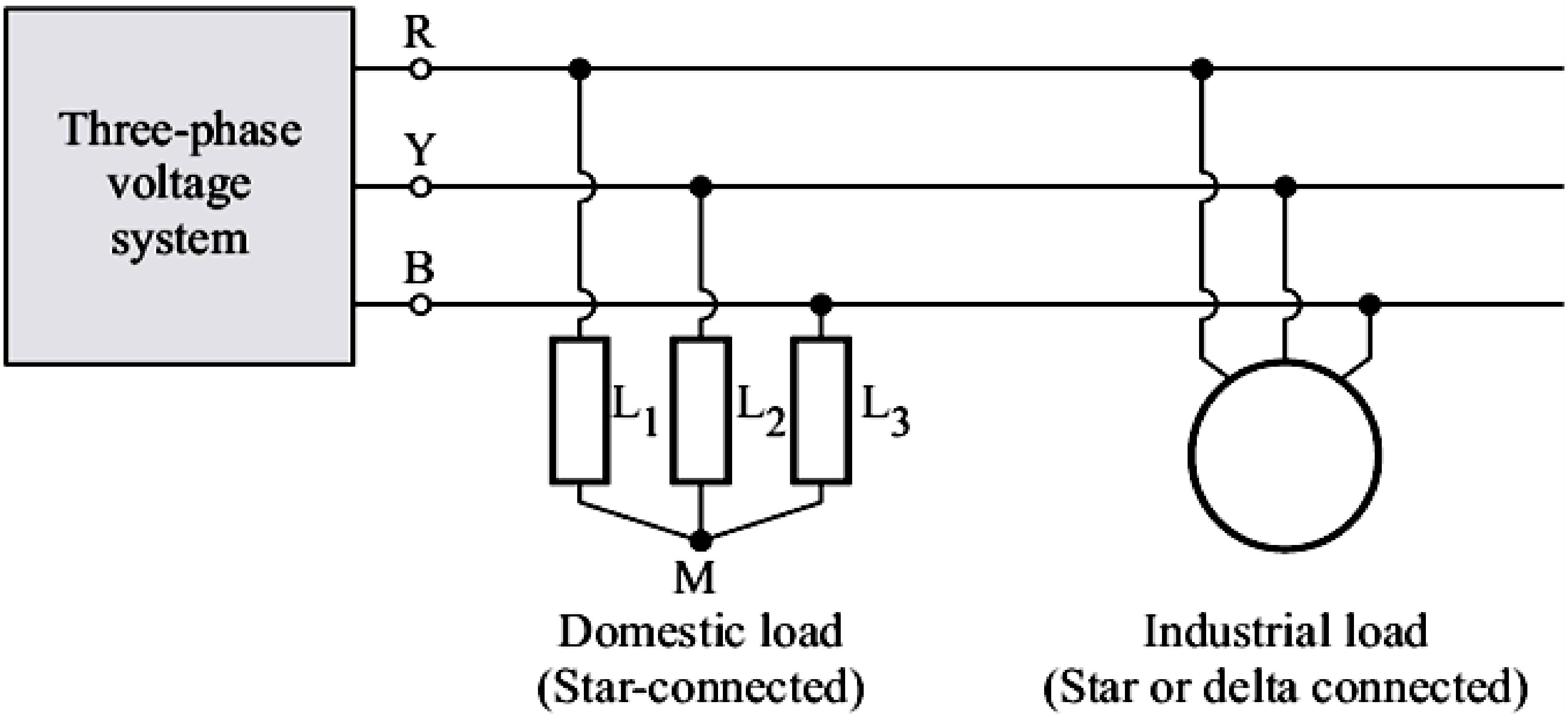
If all the three impedances are equal, the load is said to be a *balanced load.*

Star (Y) Connected Three-phase System




R
R
R′
Y′
Y
Y
N
B′
B
B
Line conductors
L3
L2
L1
N
Neutral conductor

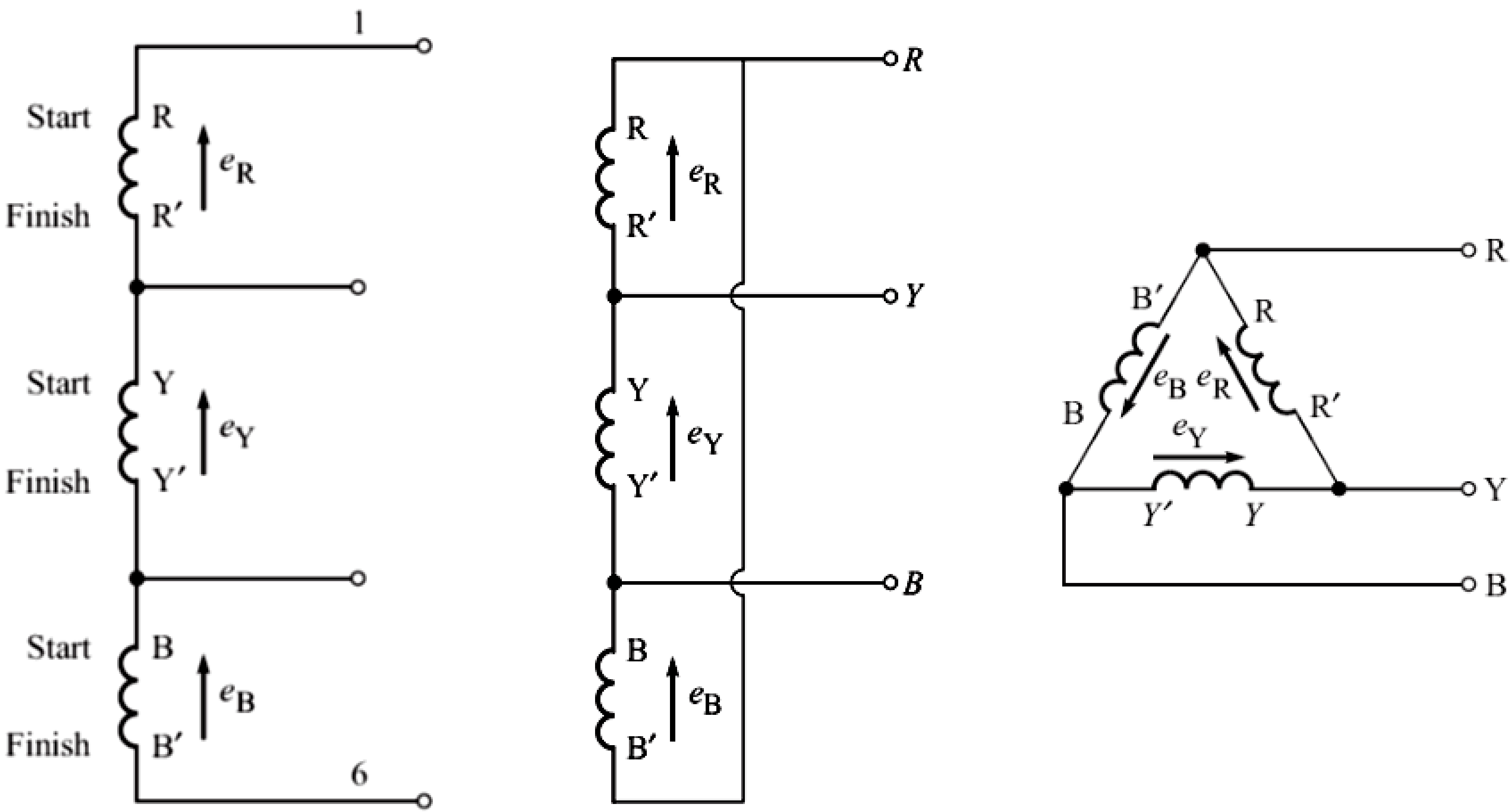
Unbalanced Three-Phase System



The common point M is called ***local neutral point***.

- Practically, it may not be possible always to make this star-connected domestic load balanced.
- However, as per KCL, the sum of the three line currents must still be zero.
- Hence, the voltages across the three loads get adjusted, resulting in ***neutral shift*** or ***floating neutral***.
- This situation is definitely undesirable.
- Therefore, we use ***Four-Wire Three-Phase Voltage System***.

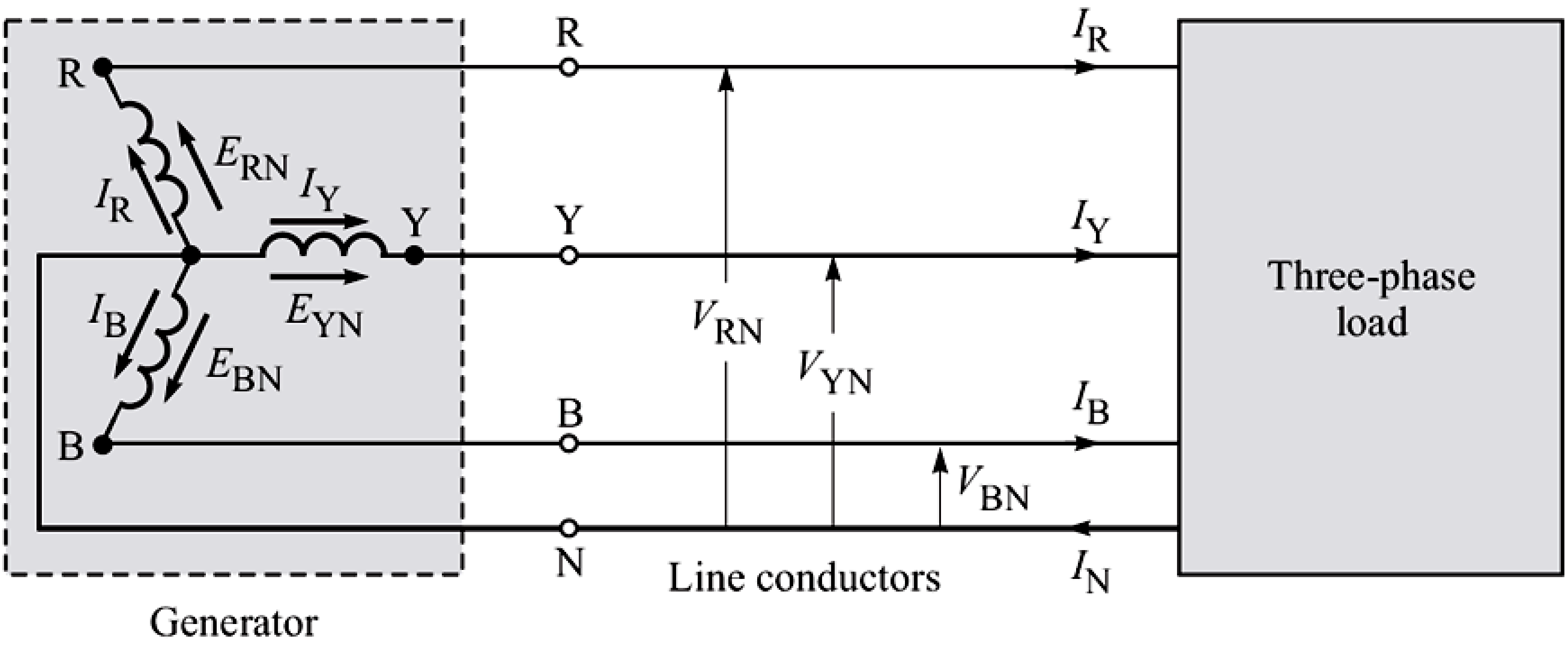
Delta (Δ) Connected Three-phase System



Note that the 'finish' of one phase is connected to the 'start' of another phase.

Voltages And Currents Relations in 3-φ Systems

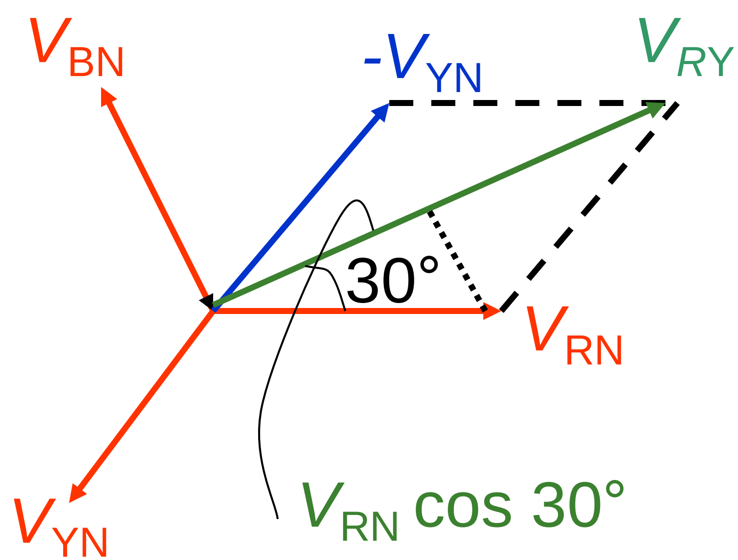
(1) Star-Connected System



- In a three-phase system, there are two sets of voltages :
  - the set of ***phase voltages***, and
  - the other is the set of ***line voltages***.
- $\boldsymbol{V}_{RN}$, $\boldsymbol{V}_{YN}$ and $\boldsymbol{V}_{BN}$ denote the set of three phase voltages.
- The term ‘line voltage’ is used to denote the voltage between two lines.
- $\boldsymbol{V}_{RY}$ represents line voltage between the lines R and Y.

$$\mathbf{V}_{\mathrm{RY}} = \mathbf{V}_{\mathrm{RNY}} = \mathbf{V}_{\mathrm{RN}} + \mathbf{V}_{\mathrm{NY}}$$

$$= \mathbf{V}_{\mathrm{RN}} - \mathbf{V}_{\mathrm{YN}} = \mathbf{V}_{\mathrm{RN}} + (-\mathbf{V}_{\mathrm{YN}})$$



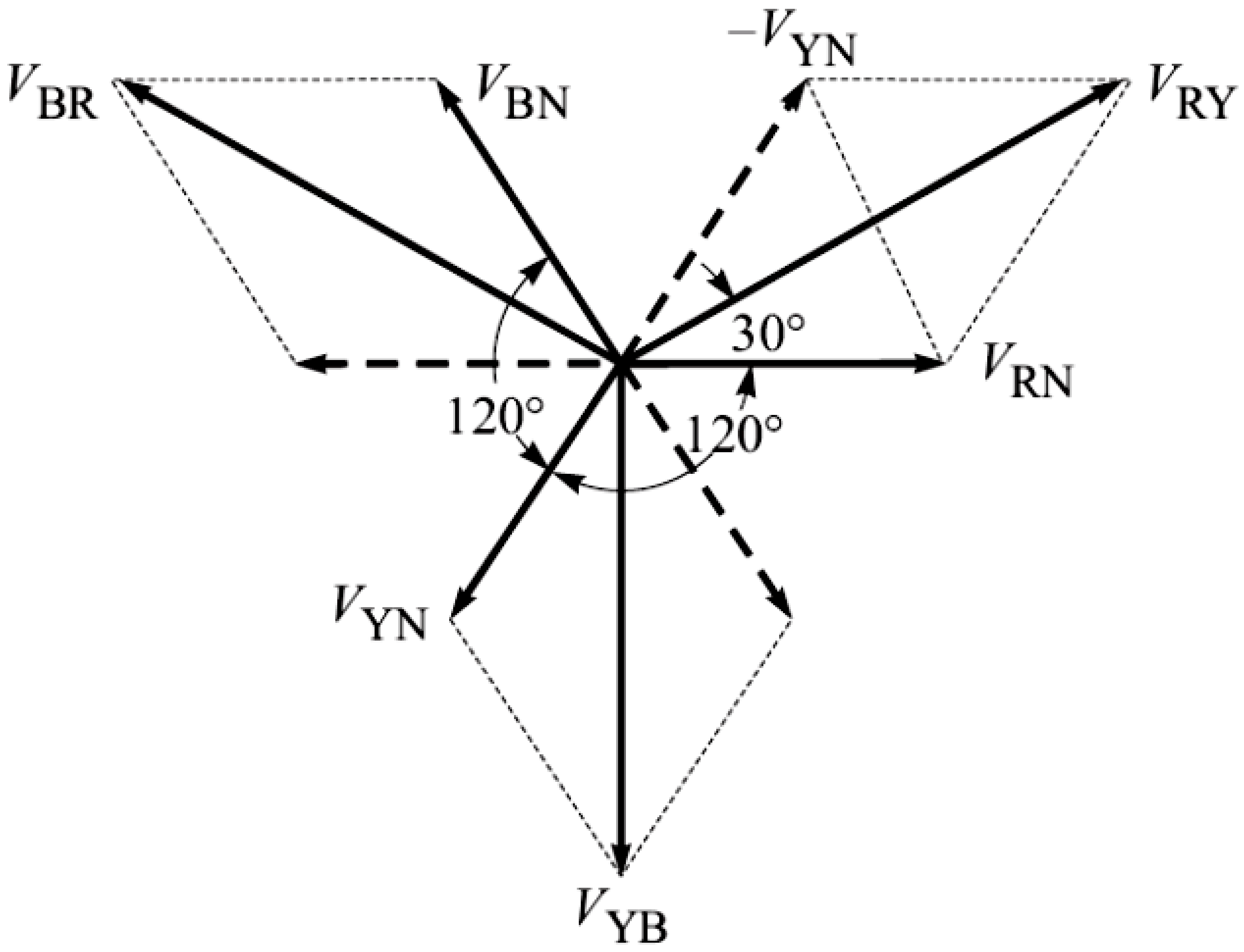
$$V_{\mathrm{RY}} = 2(V_{\mathrm{RN}} \cos 30^{\mathrm{o}})$$

or $$V_{\mathrm{L}} = 2V_{\mathrm{ph}} \left(\sqrt{3}/2\right) = \sqrt{3}V_{\mathrm{ph}}$$

$$\boxed{V_{\mathrm{L}} = \sqrt{3}V_{\mathrm{ph}}}$$

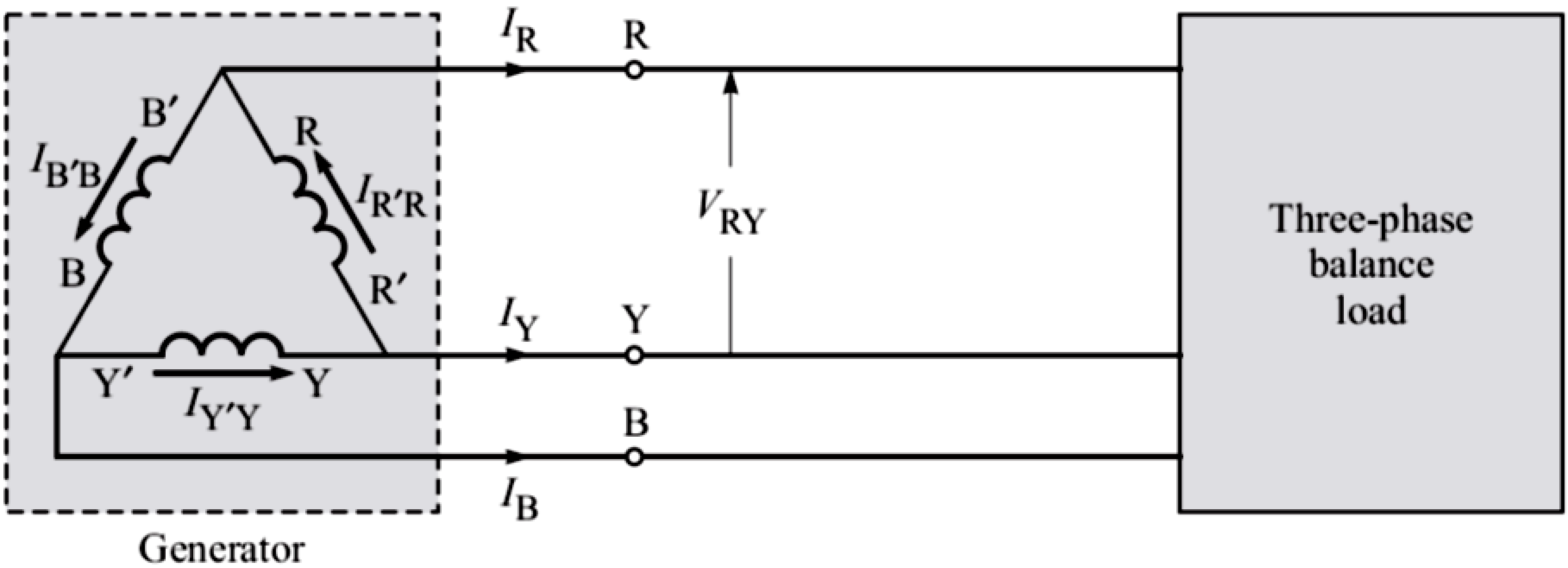
and $$\boxed{I_{\mathrm{L}} = I_{\mathrm{ph}}}$$

Click



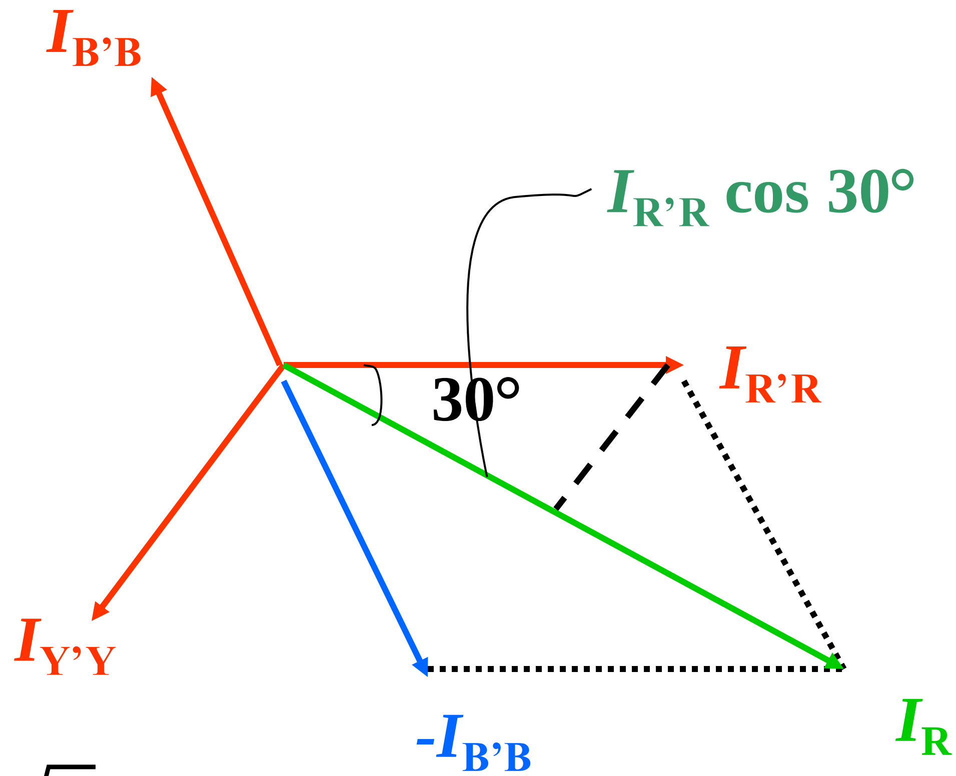
Star-connected System

(2) Delta-Connected System



$$\left|\mathbf{I}_{R'R}\right| = \left|\mathbf{I}_{Y'Y}\right| = \left|\mathbf{I}_{B'B}\right| = I_{ph}\,(\text{say})$$

$$\mathbf{I}_{\mathrm{R}} = \mathbf{I}_{\mathrm{R'R}} - \mathbf{I}_{\mathrm{B'B}}$$



$$I_{\mathrm{R}} = 2(I_{\mathrm{ph}} \cos 30^{\mathrm{o}}) = \sqrt{3} I_{\mathrm{ph}}$$

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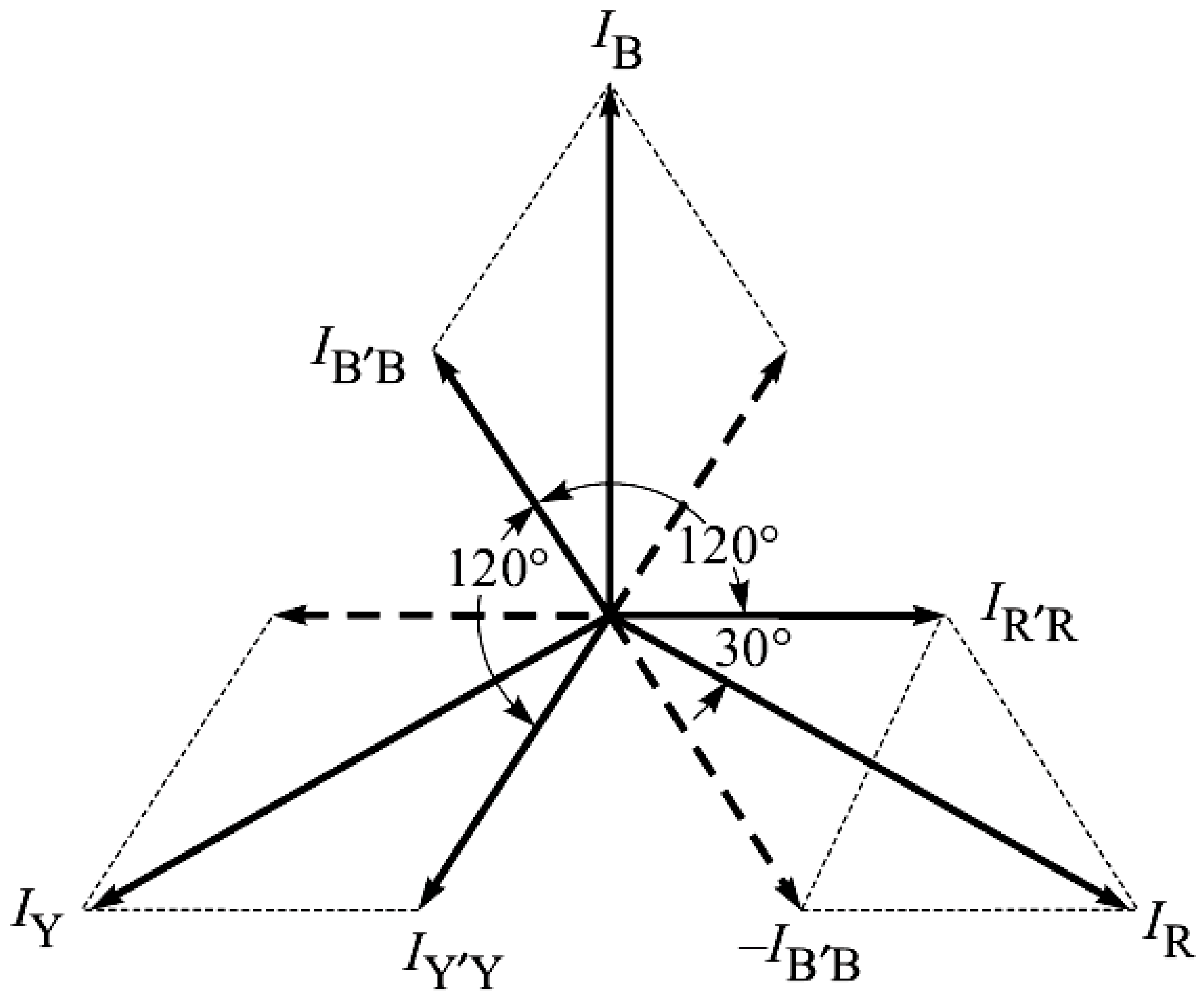
$$\boxed{I_{\mathrm{L}} = \sqrt{3} I_{\mathrm{ph}}}$$ and $$\boxed{V_{\mathrm{L}} = V_{\mathrm{ph}}}$$



Delta-connected System

Important Points about Three-Phase Systems

1. It is normal practice to specify the values of the line voltages and line currents.
2. The current in any phase can be determined by dividing the phase voltage by its impedance.

Example 1

- A 400-V, 3-ϕ supply is connected across a balanced load of three impedances each consisting of a 32-Ω resistance and 24-Ω inductive reactance. Determine the current drawn from the power mains, if the three impedances are

  (*a*) Y-connected, and

  (*b*) Δ-connected.

Click

Solution : **Z** = $R + jX = (32 + j24)\ \Omega$.

Click

$$\therefore \qquad Z = \sqrt{R^2 + X^2} = \sqrt{32^2 + 24^2} = 40\ \Omega$$

(a) Y-connection :

$$V_{\mathrm{ph}} = \frac{V_{\mathrm{L}}}{\sqrt{3}} = \frac{400}{\sqrt{3}}\,\mathrm{V} \quad \Rightarrow \quad I_{\mathrm{ph}} = \frac{V_{\mathrm{ph}}}{Z} = \frac{400/\sqrt{3}}{40} = \frac{10}{\sqrt{3}}\ \mathrm{A}$$

$$\therefore \qquad I_{\mathrm{L}} = I_{\mathrm{ph}} = \frac{10}{\sqrt{3}} = \mathbf{5.78\ A}$$

Click

(b) For Δ-connection :

$$V_{\mathrm{ph}} = V_{\mathrm{L}} = 400\,\mathrm{V} \quad \Rightarrow \quad I_{\mathrm{ph}} = \frac{V_{\mathrm{ph}}}{Z} = \frac{400}{40} = 10\ \mathrm{A}$$

$$\therefore \qquad I_{\mathrm{L}} = \sqrt{3} I_{\mathrm{ph}} = \sqrt{3} \times 10 = \mathbf{17.32\ A}$$

Next

Power In Three-phase System With A Balanced Load

Consider one phase only, $P_1 = V_{\text{ph}} I_{\text{ph}} \cos\phi$

Click

Hence, the total power consumed,

$$P = 3P_1 = 3V_{\text{ph}} I_{\text{ph}} \cos\phi$$

For a *star-connected system,* $V_{\text{L}} = \sqrt{3}\, V_{\text{ph}}$ and $I_{\text{L}} = I_{\text{ph}}$

Click

$$P = 3(V_{\text{L}} / \sqrt{3}) I_{\text{L}} \cos\phi \boxed{= \sqrt{3}\, V_{\text{L}} I_{\text{L}} \cos\phi}$$

For a *delta-connected system,* $V_{\text{L}} = V_{\text{ph}}$ and $I_{\text{L}} = \sqrt{3}$

Click

$$P = 3V_{\text{L}} (I_{\text{L}} / \sqrt{3}) \cos\phi \boxed{= \sqrt{3}\, V_{\text{L}} I_{\text{L}} \cos\phi}$$

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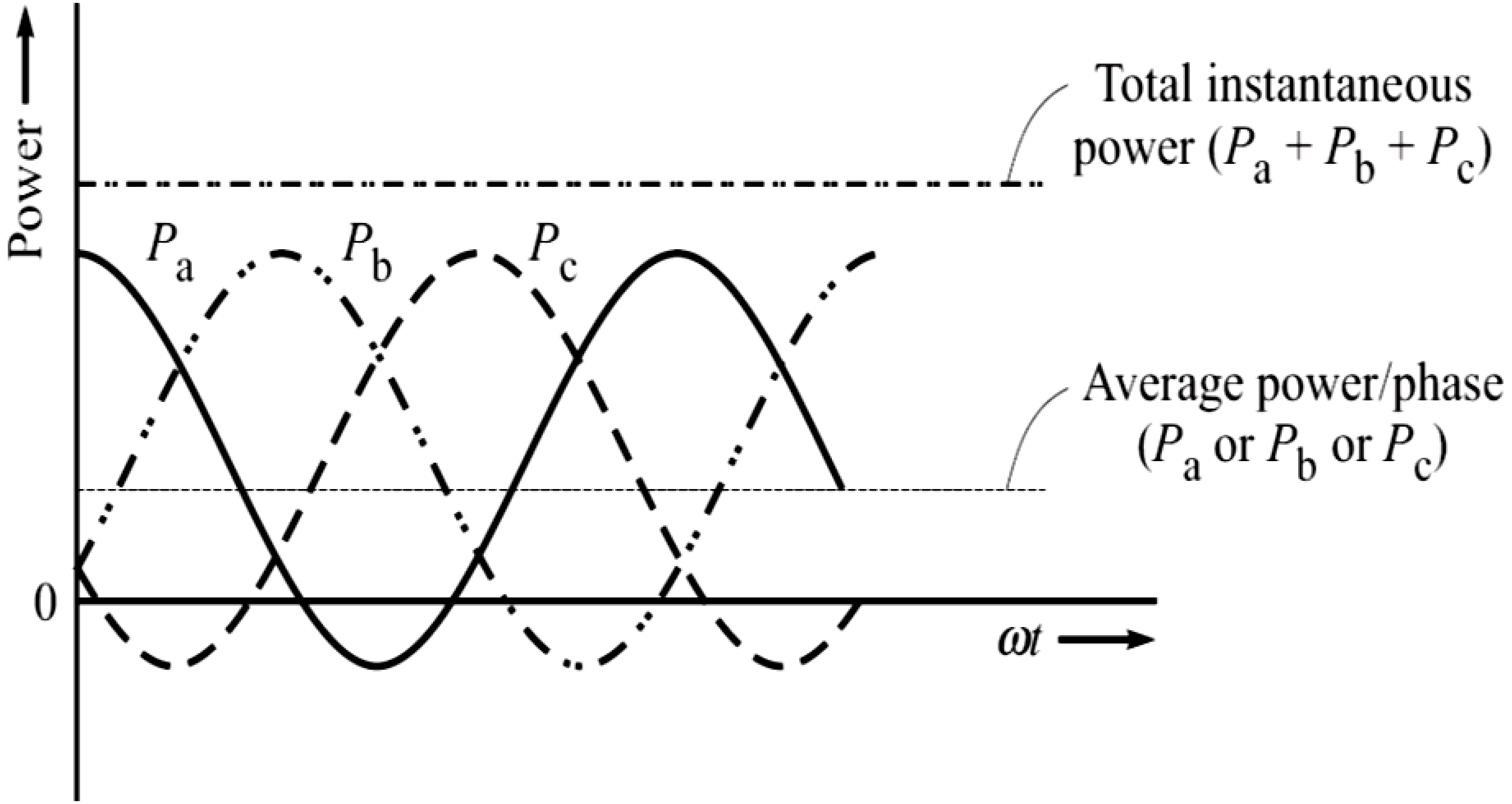
Thus, for *any balanced load*,

$$\boxed{P = \sqrt{3}\, V_{\text{L}} I_{\text{L}} \cos\phi}$$


Power
Total instantaneous power ($P_a + P_b + P_c$)
P_a
P_b
P_c
Average power/phase (P_a or P_b or P_c)
0
ωt

- In three-phase system with balanced load (such as a three-phase motor), there is no variation of power at all.
- This is the reason why for driving heavy mechanical loads we prefer a three-phase motor rather than a single-phase motor.

Example 2

- A 400-V, 3-ϕ supply is connected to a balanced network of three impedances each consisting of a 20-Ω resistance and a 15-Ω inductive reactance.
- If the three impedances are (*a*) star-connected, and (*b*) delta-connected, in each case determine
  - (*i*) the line current,
  - (*ii*) the power factor, and
  - (*iii*) the total power in kW.

(*a*) For *star-connected load :* $V_L = \sqrt{3}\, V_{ph}$ and $I_L = I_{ph}$

Click

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

Click

and $Z_{ph} = \sqrt{20^2 + 15^2} = 25\ \Omega$

Click

(*i*) $I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{25} = \mathbf{9.24\ A}$

Click

(*ii*) $\cos\phi = \frac{R_{ph}}{Z_{ph}} = \frac{20}{25} = \mathbf{0.8\ (lagging)}$

(*iii*) $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 9.24 \times 0.8 = \mathbf{5.12\ kW}$

(*b*) For *delta-connected load :* $V_L = V_{ph}$ and $I_L = \sqrt{3}\, I_{ph}$

Click

$$\therefore \qquad V_L = V_{ph} = 400\ \text{V}$$

Click

(*i*) $$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{25} = 16\ \text{A}$$

$$\therefore \qquad I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 16 = \mathbf{27.71\ A}$$

(*ii*) The power factor is same as above,

$$pf = \mathbf{0.8\ (lagging)}$$

Click

(*iii*) $$P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 27.71 \times 0.8 = \mathbf{15.36\ kW}$$

Click

Note that power consumed has become 3 times.

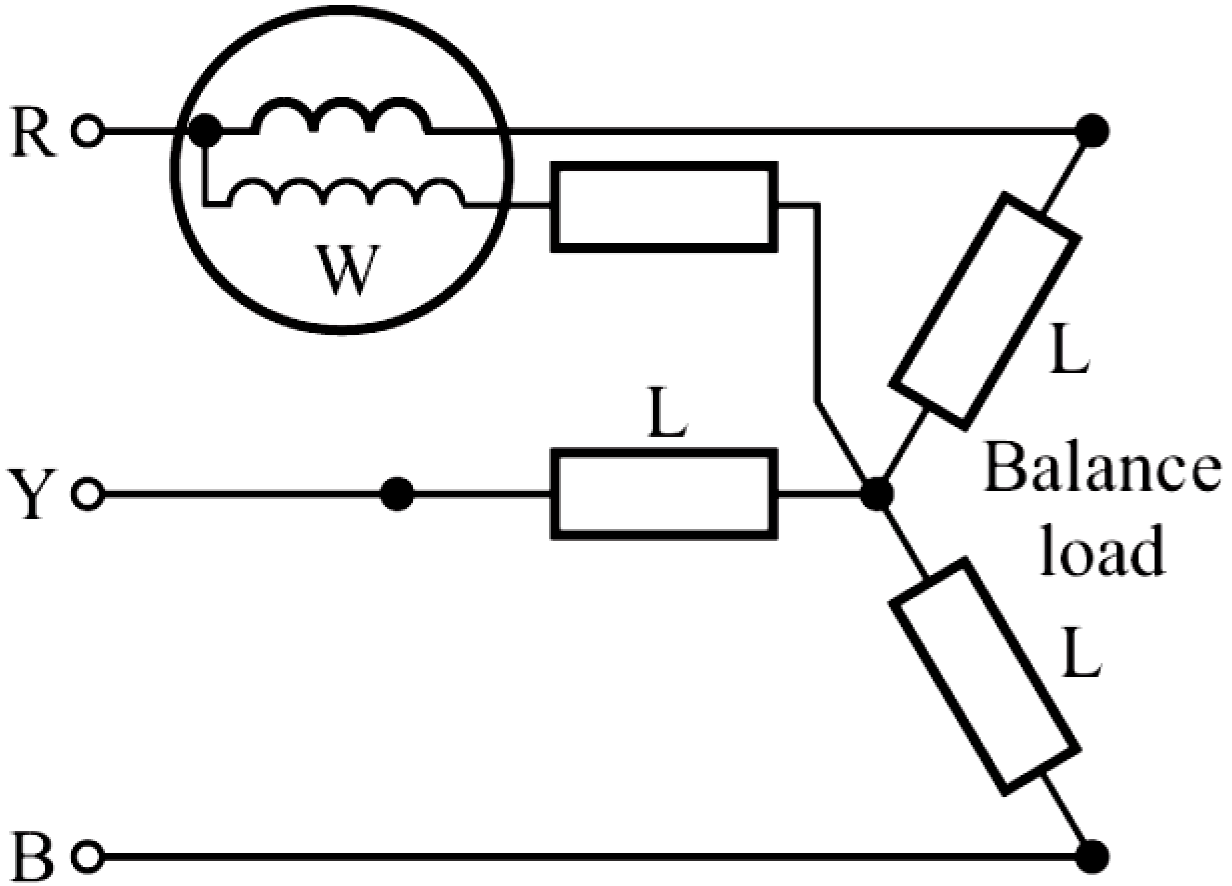
No.	Star-Connected System	Delta-Connected System
1.	Similar ends are joined together.	Dissimilar ends are joined.
2.	$V_L = \sqrt{3}\, V_{ph}$ and $I_L = I_{ph}$	$V_L = V_{ph}$ and $I_L = \sqrt{3}\, I_{ph}$
3.	Neutral wire available.	Neutral wire not available.
4.	4-wire, 3-ϕ system possible.	4-wire, 3-ϕ system not possible.
5.	Both domestic and industrial loads can be handled.	Only industrial loads can be handled.
6.	By earthing the neutral wire, relays and protective devices can be provided in alternators for safety.	Due to absence of neutral wire, it is not possible.

Measurement of Power

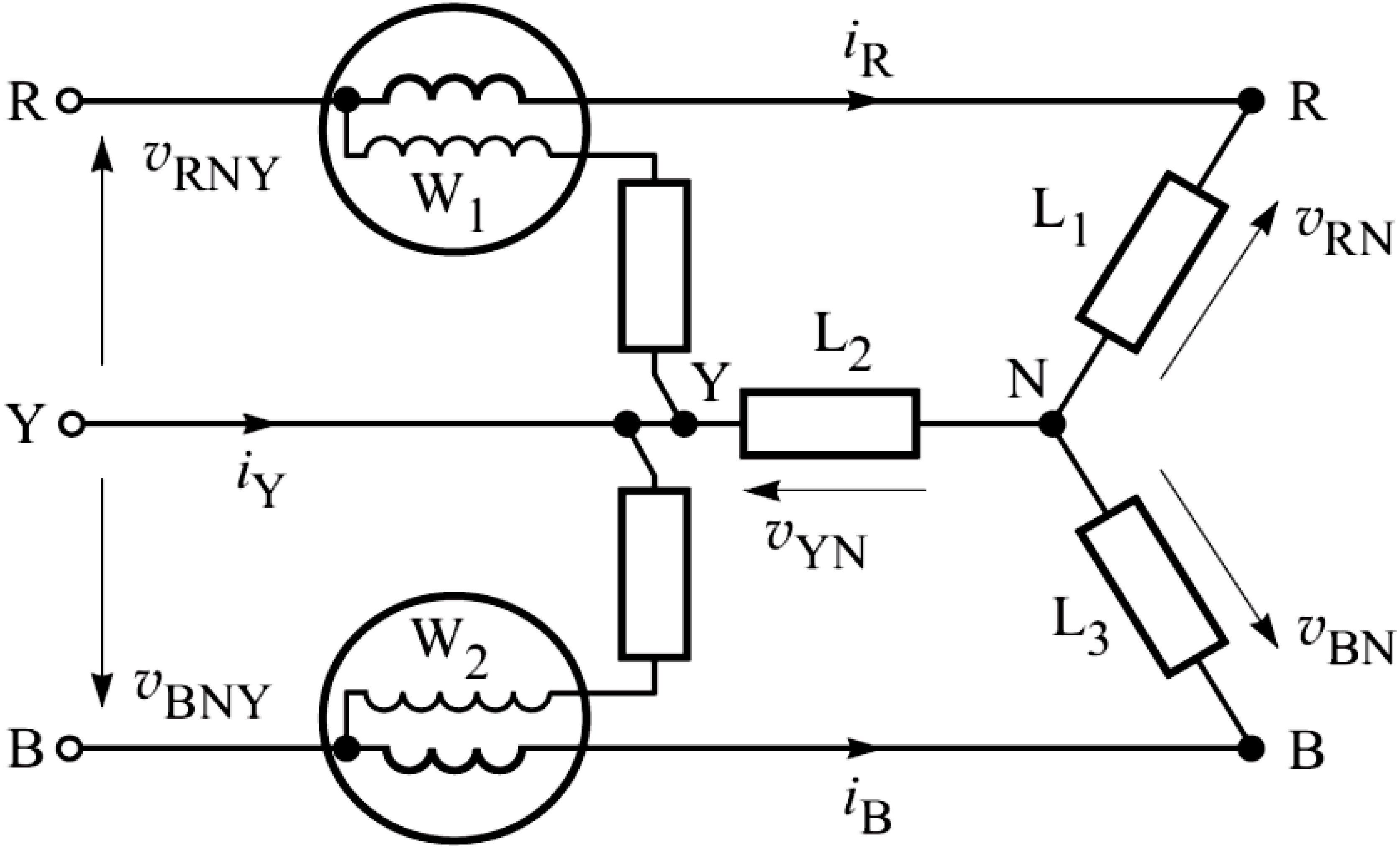
(i) Three-Wattmeter Method : This is simplest and straight forward method.

(ii)Two-Wattmeter Method : This can be used for any balanced or unbalanced load, star- or delta-connected.

(iii)One-Wattmeter Method : This can be used only for a star-connected balanced load.


R
W
L
Y
L
Balance
load
L
B

Two-Wattmeter Method



The sum of the wattmeter readings gives the average value of the total power absorbed by the three phases

Proof : Total instantaneous power

$$= i_{\mathrm{R}} v_{\mathrm{RN}} + i_{\mathrm{Y}} v_{\mathrm{YN}} + i_{\mathrm{B}} v_{\mathrm{BN}}.$$

The instantaneous power measured by W_1,

$$p_1 = i_{\mathrm{R}} (v_{\mathrm{RN}} - v_{\mathrm{YN}})$$

The instantaneous power measured by W_2,

$$p_2 = i_{\mathrm{B}} (v_{\mathrm{BN}} - v_{\mathrm{YN}})$$

$$\therefore \quad p_1 + p_2 = i_{\mathrm{R}}(v_{\mathrm{RN}} - v_{\mathrm{YN}}) + i_{\mathrm{B}}(v_{\mathrm{BN}} - v_{\mathrm{YN}})$$

$$= i_{\mathrm{R}} v_{\mathrm{RN}} + i_{\mathrm{B}} v_{\mathrm{BN}} - (i_{\mathrm{R}} + i_{\mathrm{B}}) v_{\mathrm{YN}}$$

By KCL, $i_{\mathrm{R}} + i_{\mathrm{Y}} + i_{\mathrm{B}} = 0 \Rightarrow (i_{\mathrm{R}} + i_{\mathrm{B}}) = -i_{\mathrm{Y}}$

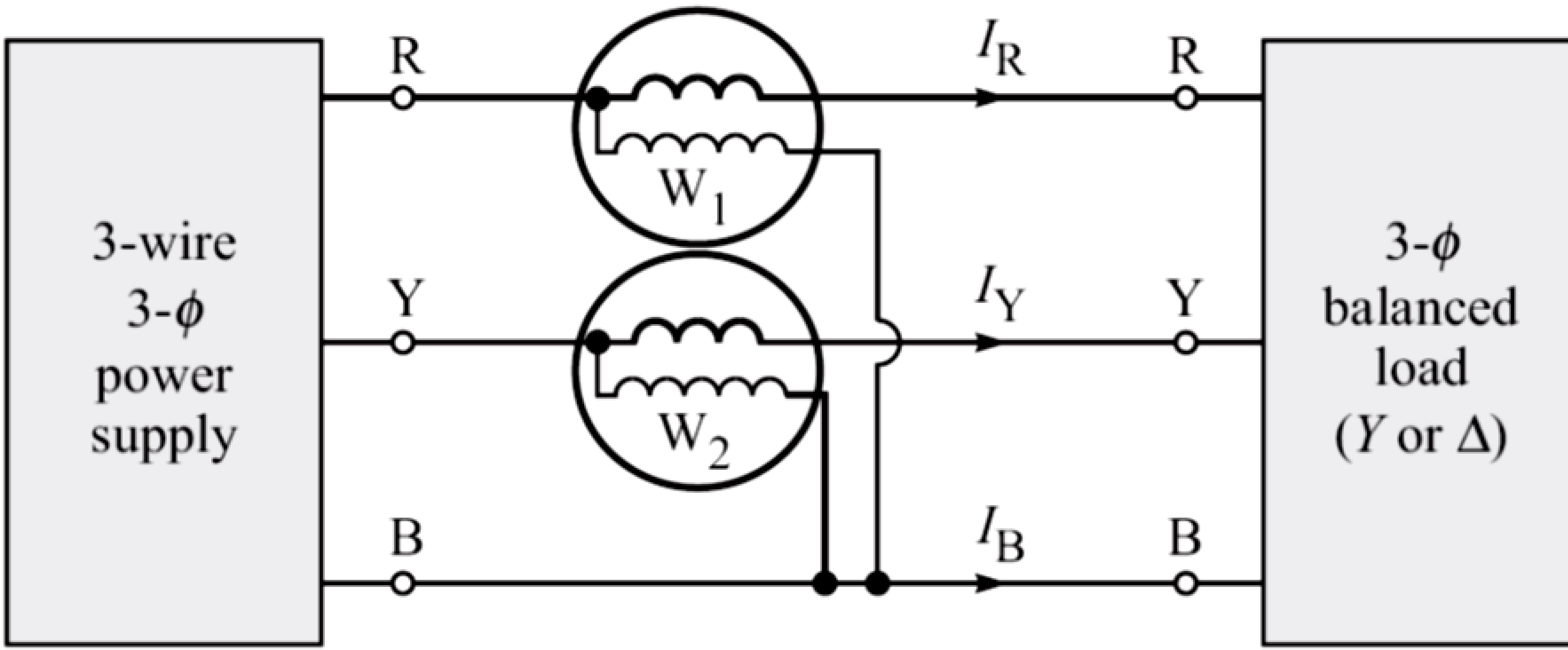
$$\therefore \quad p_1 + p_2 = i_{\mathrm{R}} v_{\mathrm{RN}} + i_{\mathrm{B}} v_{\mathrm{BN}} + i_{\mathrm{Y}} v_{\mathrm{YN}}$$

$$= \text{total instantaneous power}$$

Since we did not assume a balanced load or a sinusoidal waveform, it follows that the sum of the two wattmeter readings gives the total power under all conditions.

Power Factor Measurement by Two-Wattmeter Method

Concept of 'power factor' is meaningful only if the load is balanced.

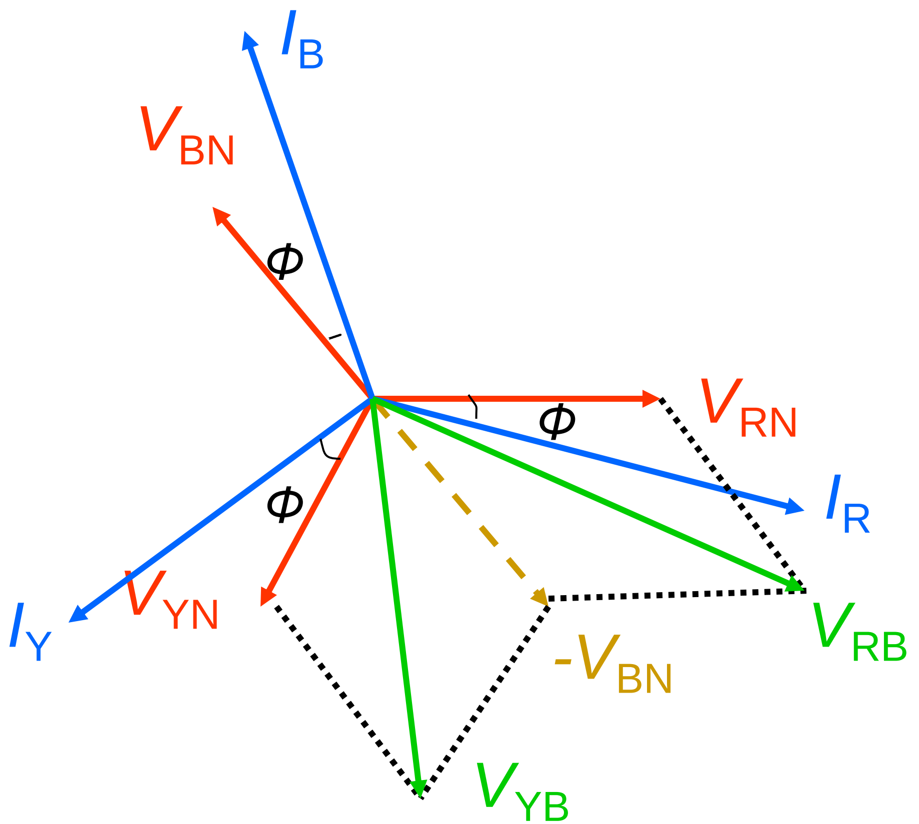


Since the load is balanced,

$$I_R = I_Y = I_B = I_L \text{ (say)}$$

$$\text{and} \quad V_{RN} = V_{YN} = V_{BN} = V_{ph} \text{ (say)}$$

Since $\mathbf{V}_{RB} = \mathbf{V}_{RN} - \mathbf{V}_{BN}$, and $\mathbf{V}_{YB} = \mathbf{V}_{YN} - \mathbf{V}_{BN}$, we can determine the line voltages V_{RB} and V_{YB} by phasor method,



I_B
V_{BN}
Φ
Φ
V_{RN}
I_R
Φ
I_Y
V_{YN}
V_{RB}
$-V_{BN}$
V_{YB}

- It is seen that the line voltage V_{RB} lags the phase voltage V_{RN} by 30° and V_{YB} leads V_{YN} by 30°.
- Thus, the phase angle between V_{RB} and I_R is $(30° - \phi)$.
- Similarly, the phase angle between V_{YB} and I_Y is $(30° + \phi)$.
- Therefore, the readings of the two wattmeters are

$$P_1 = V_{\mathrm{RB}} I_{\mathrm{R}} \cos(30° - \phi) = V_{\mathrm{L}} I_{\mathrm{L}} \cos(30° - \phi)$$

Click

$$P_2 = V_{\mathrm{YB}} I_{\mathrm{Y}} \cos(30° + \phi) = V_{\mathrm{L}} I_{\mathrm{L}} \cos(30° + \phi)$$

Click

$$\therefore \quad \frac{P_1}{P_2} = \frac{\cos(30° - \phi)}{\cos(30° + \phi)}$$

By applying componendo and dividendo,

Click

$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{\cos(30° - \phi) - \cos(30° + \phi)}{\cos(30° - \phi) + \cos(30° + \phi)}$$

$$= \frac{2\sin 30° \sin\phi}{2\cos 30° \cos\phi} = \tan 30° \tan\phi$$

Click

$$\therefore \quad \boxed{\tan\phi = \sqrt{3}\left[\frac{P_1 - P_2}{P_1 + P_2}\right]}$$

Calculate the phase angle ϕ, and then determine the power factor.

Important Points :

- If $\Phi = 0°$, $P_1 = P_2$.
- If $\Phi < 60°$, both P_1 and P_2 are positive; and

$$P = P_1 + P_2$$

- If $\Phi = 60°$, P_2 is zero.
- If $\Phi > 60°$, (i.e., if $pf < 0.5$), P_2 is negative.

In such case, the connection of either the current coil or the potential coil has to be reversed to make positive deflection.

The value P_2 should then be taken as negative while calculating the power factor or the total power.

Example 3

- Two-wattmeter method was used to determine the input power to a three-phase motor. The readings were 5.2 kW and -1.7 kW, and the line voltage was 415 V. Calculate

  (*a*) the total power,

  (*b*) the power factor, and

  (*c*) the line current.

Solution :

Click

(*a*) The total power,

$$P = P_1 + P_2 = 5.2\ \text{kW} - 1.7\ \text{kW} = \mathbf{3.5\ kW}$$

Click

(*b*) $$\tan\phi = \sqrt{3}\left[\frac{P_1 - P_2}{P_1 + P_2}\right] = \sqrt{3}\left[\frac{5.2 - (-1.7)}{5.2 + (-1.7)}\right] = 3.41$$

$$\therefore \quad \phi = \tan^{-1} 3.41 = 73°39'$$

$$\therefore \quad pf = \cos\phi = \cos 73°39' = \mathbf{0.281}$$

Click

(*c*) $$3500 = \sqrt{3} \times 415 \times I_L \times 0.281$$

$$\Rightarrow \quad I_L = \mathbf{17.3\ A}$$

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Review

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