10.10.24

Unit - 2

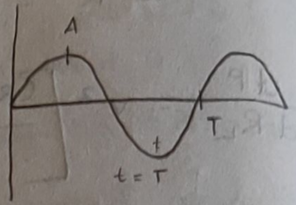
AC CIRCUITS

RLC

$V_i = V_m \sin \omega t$

$\boxed{\omega t = \theta}$

$\boxed{\omega = \frac{\theta}{t}}$



$\theta = \frac{t}{4}$

$V_i \Rightarrow$ instantaneous voltage

$V_m \rightarrow$ Maximum voltage

Angular Frequency ; $\boxed{2\pi f = \omega}$

$\boxed{f = \frac{\omega}{2\pi}}$

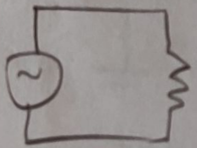
* Input → Voltage
* Output → Current

$V_{AC} = V_{RMS}$

$V_{RMS} = \frac{V_m}{\sqrt{2}}$

$\boxed{V_{RMS} = 0.707\, V_m}$

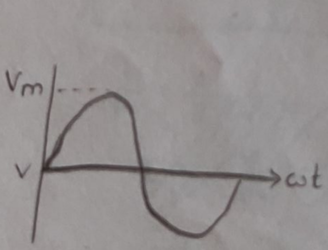
I = ? ; V = ?



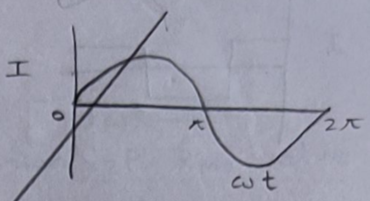
$P = V_{RMS} I_{RMS}$

$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}$

$\boxed{P = \frac{1}{2} V_m I_m}$



* If we use multimeter to measure the voltage, it will be an V_{RMS} value.



$I_{RMS} = \sqrt{\dfrac{I_1^2+I_2^2+I_3^2 \dots I_{10}^2}{10}}$

$\int I_{RMS} = \sqrt{\int_0^{2\pi} \dfrac{I^2 d\theta}{2\pi - 0}}$

$I_{RMS} = \left[\dfrac{I^2}{2\pi}\theta\right]_0^{2\pi} = \dfrac{I^2}{\cancel{2\pi}}(\cancel{2\pi}) - \dfrac{I^2}{2\pi}(0)$

$= \sqrt{I^2}$

$= I$

$= I_m \sin \omega t$

$\sin^2 \omega t = \dfrac{(1-\cos 2\omega t)}{2}$

$= \sqrt{\int \dfrac{I_m^2 \sin^2 \omega t}{2\pi}}$

$= \omega t = \theta$

$= \sqrt{\int_0^{2\pi} \dfrac{I_m^2 \sin^2\theta}{2\pi} d\theta}$

$= \sqrt{\dfrac{I_m^2}{2\pi}} \cdot \sqrt{\int_0^{2\pi} \sin^2\theta \, d\theta} = \dfrac{I_m}{\sqrt{2\pi}} \int \left[\dfrac{1-\cos 2\theta}{2}\right]$

Average current over entire-cycle is zero.

$I_{avg} = \dfrac{I_1 + I_2 + \dots + I_{10}}{10}$

$\int I_{avg} = \int_0^{\pi} \dfrac{I \, d\theta}{\pi}$

$= \left[\dfrac{I}{\pi}\theta\right]_0^{\pi} = \dfrac{I}{\cancel{\pi}}(\cancel{\pi}) - 0$

$= I$

$= I_m \sin \omega t$

$\int I_{avg} = \int_0^{\pi} \dfrac{I_m \sin\theta \, d\theta}{\pi}$

$= \dfrac{I_m}{\pi} \int_0^{\pi} \sin\theta \, d\theta$

$= \dfrac{I_m}{\pi} \left[-\cos\theta\right]_0^{\pi}$

$= \dfrac{I_m}{\pi} \left[-\cos\pi + \cos 0\right]$

$= \dfrac{I_m}{\pi} \left[-(-1) + (1)\right]$

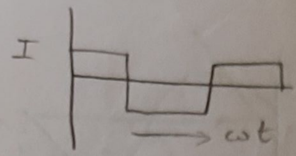
$= \dfrac{I_m}{\pi} [1+1] = \dfrac{2 I_m}{\pi}$

$\therefore \boxed{I_{avg} = 0.637 \, I_m}$

For sinusoidal wav

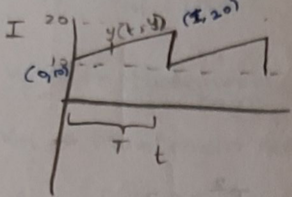
$I_{avg} = 0.637 \, I_m$

$I_{RMS} = 0.707 \, I_m$



For square wave;

$$\boxed{I_m = I_{avg} = I_{RMS}}$$



$$\frac{y-10}{t} = \frac{10}{T}$$

$$\boxed{y = t\frac{10}{T} + 10}$$

$$2\frac{10t}{T} \times 10$$

$$\frac{20t}{T}$$

$$I = t\frac{10}{T} + 10$$

$$I_{RMS} = \sqrt{\frac{\int_0^T \left(t\frac{10}{T}+10\right)^2 dt}{T}}$$

$$= \frac{1}{\sqrt{T}}\sqrt{\int_0^T \left(\frac{100t^2}{T^2} + 100 + \frac{200t}{T}\right) dt}$$

$$= \frac{1}{\sqrt{T}}\sqrt{\left[\frac{100t^3}{3T^2} + 100t + \frac{200t^2}{2T}\right]_0^T}$$

$$= \frac{1}{\sqrt{T}}\sqrt{\left[\frac{100T^{\cancel{3}}}{3\cancel{T^2}} + 100T + \frac{200T^{\cancel{2}}}{2\cancel{T}}\right]}$$

$$= \frac{1}{\sqrt{T}}\sqrt{\frac{100T}{3} + 100T + 100T}$$

$$= \frac{1}{\sqrt{T}}\sqrt{\frac{100T}{3} + 200T}$$

$$= \frac{1}{\sqrt{T}}\sqrt{\frac{700T}{3}} = \frac{1}{\sqrt{3\cancel{T}}}\sqrt{700\cancel{T}}$$

$$= \frac{1}{\sqrt{3}} \times \sqrt{700}$$

$$\boxed{I_{RMS} = 15.27\ A}$$

DEFINITIONS :

i) Form Factor : (K_F) :

* It is defined as the ration of Rms value by Avg Value.

$$\boxed{K_F = \frac{RMS}{Avg}}$$

$$K_F = \frac{I_{RMS}}{I_{avg}} = \frac{0.707\,\cancel{I_m}}{0.637\,\cancel{I_m}} = 1.11$$

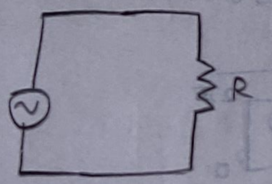
$\therefore$ Form Factor value for a sinusoidal wave = 1.11.

ii) Peak Factor / Crest Factor / Amplitude :

* It is defined as the ratio of maximum value by rms value.

For sinusoidal wave;

$$= \frac{I_m}{I_{RMS}} = \frac{\cancel{I_m}}{\frac{\cancel{I_m}}{\sqrt{2}}} = \sqrt{2} = 1.414$$



$$V_t = V_m \sin\omega t \qquad i = I_m \sin\omega t$$

$$i = \frac{V_m}{R}\sin\omega t$$

$$I_m = \frac{V_m}{R}$$

$$P_i = V_i I_i$$

$$= V_m \sin\omega t \,.\, I_m \sin\omega t$$

$$P = V_{RMS} I_{RMS} = \frac{V_m}{\sqrt{2}}\sin\omega t \; \frac{I_m}{\sqrt{2}}\sin\omega t$$

$$= \frac{V_m I_m}{2}\int_0^{2\pi} \sin^2 \omega t \, dt$$

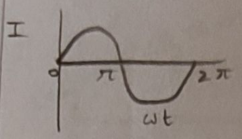
$$= \frac{V_m I_m}{2}\int_0^{2\pi}\left(\frac{1 - \cos 2\omega t}{2}\right)$$

3.

$$P = \frac{V_m I_m}{2}\left(\frac{1}{2}\right)\int_0^{2\pi}(1-\cos 2\omega t)\,dt$$

$$\frac{V_m I_m}{4}\left[t\right]_0^{2\pi} -$$

$$I_{RMS} = \sqrt{\frac{I_1^2 + I_2^2 + I_3^2 + I_4^2 + \dots + I_{10}^2}{10}}$$



$$\int I_{RMS} = \sqrt{\int_0^{2\pi}\frac{I^2}{2\pi}\,d\theta}$$

$I = I_m \sin\omega t$

$\omega t = \theta$

$$\therefore \int I_{RMS} = \sqrt{\int_0^{2\pi}\frac{I_m^2 \sin^2\theta\,d\theta}{2\pi}}$$

$$= \frac{I_m}{\sqrt{2\pi}}\sqrt{\int_0^{2\pi}\sin^2\theta\,d\theta}$$

$$\boxed{\sin^2\theta = \frac{1-\cos 2\theta}{2}}$$

$$\therefore \int I_{RMS} = \frac{I_m}{\sqrt{2\pi}}\sqrt{\int_0^{2\pi}\frac{1-\cos 2\theta}{2}\,d\theta}$$

$$= \frac{I_m}{\sqrt{2\pi}.\sqrt{2}}\sqrt{\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \frac{I_m}{2\sqrt{\pi}}\sqrt{2\pi - \frac{\sin 4\pi}{2}}$$ $\sin 4\pi$

$$= \frac{I_m}{2\sqrt{\pi}} \times \sqrt{2\pi}$$

$$= \frac{I_m}{\sqrt{2}\times\cancel{\sqrt{2}}\times\cancel{\sqrt{\pi}}} \times \cancel{\sqrt{2}}\times\cancel{\sqrt{\pi}}$$

$$\therefore \boxed{I_{RMS} = \frac{I_m}{\sqrt{2}}}$$

$$\boxed{I_{RMS} = 0.707\, I_m}$$