Q) A uniform magnetic field passes through a circular coil whose normal is parallel to the magnetic field. The coil's area is 10^{-2} m^2 and it has a resistance of 1 m Ω. B varies with time as show. Plot the current in the coil.

A) $\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt}$

$\varepsilon = IR$; $I = \frac{\varepsilon}{R}$

$$I = -\frac{A}{R}\frac{dB}{dt}$$

$$\frac{dB}{dt} = \frac{0.01-0}{11-0} = \frac{0.01}{11}$$

$$I = \frac{-10^{-2}}{10^{-3}} \times \frac{0.01}{11} = \frac{-10^{-2} \times 10^{3} \times 10^{-2}}{11}$$

$$= \frac{-10^{-2+3-2}}{11}$$

$$= \frac{-10^{-1}}{11} = -\frac{1}{11 \times 10} = -\frac{1}{110} = -9.09 \times 10^{-3}$$

Self induced emf & Self inductance :

The induced emf, e, in a coil is proportional to the rate of the change of the magnetic flux passing through it due to its own current. The emf is termed as "Self induced emf."

* The induced emf e is proportional to the rate of change of current through coil and this proportionality constant is callend the self inductance, L.

$$e_1 = -L\frac{di}{dt} \qquad e_1 = -N\frac{d\phi}{dt} \qquad L\frac{i}{t} = N\frac{\phi}{t}$$

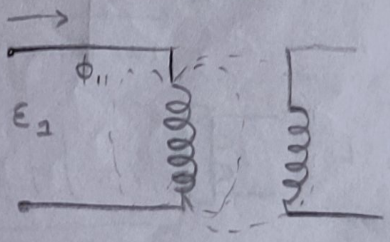
$$L\frac{di}{dt} = N\frac{d\phi}{dt} \qquad \boxed{L = \frac{N\phi}{i}}$$

* The negative sign is used to indicate that EMF is opposing the cause producing it.

Self & Mutual Inductance:

* The induced emf e_2 is proportional to the rate of change of current through coil 1, and this proportionality constant M is called mutual inductance.

$$e_2 = M\frac{di_1}{dt}$$



$$e_2 = N_2\frac{d\phi_{12}}{dt}$$

$$M\frac{di_1}{dt} = N_2\frac{d\phi_{12}}{dt}$$

$$\therefore M = N_2\frac{d\phi_{12}}{dt_1}$$

$$\phi_2 = \phi_1 + \phi_{12}$$

$$\phi_2 = \phi_{21} + \phi_{22}$$

μ_r is constant ; $\frac{d\phi_{12}}{di_1}$ is congtant :

$$\boxed{M = N_2\frac{\phi_{12}}{I_1}} \qquad \boxed{M_{21} = N_2\frac{\phi_{21}}{i_1}}$$

Unit : Henry (H)

$$\boxed{M_{12} = N_1\frac{\phi_{12}}{i_2}}$$

Coupling Coefficient:

Self-Inductances; L_1 & L_2 are;

$$\boxed{L_1 = \frac{N_1 \Phi_1}{I_1} \quad \text{and} \quad L_2 = \frac{N_2 \Phi_2}{I_2}}$$

$$L_1 L_2 = \frac{M^2}{k^2}$$

$$\frac{N_1 \Phi_1}{I_1} \cdot \frac{N_2 \Phi_2}{I_2} = \frac{N_1 \Phi_1 N_2 \Phi_2}{I_1 I_2}$$

Mutual Inductances: M

$$\boxed{M = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_1 \Phi_{21}}{I_2}}$$

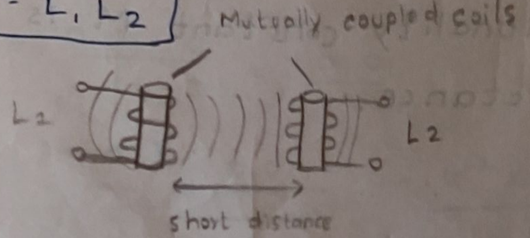
where; $\boxed{\Phi_{12} = k\Phi_1 \; ; \; \Phi_{21} = k\Phi_2}$

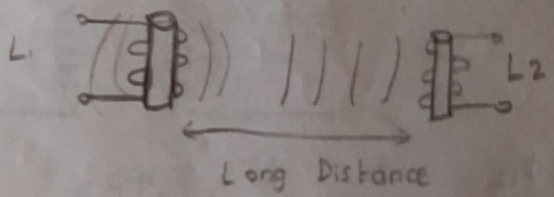
and k is the coupling coefficient

$$\boxed{L_1 L_2 = \frac{M^2}{k^2}} \quad \text{or} \quad \boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

If $k = 1$

$$\boxed{M^2 = L_1 L_2}$$





$$L_1 = \frac{\mu_0 \mu_r N_1^2 A}{l}$$

$$L_2 = \frac{\mu_0 \mu_r N_2^2 A}{l}$$

Example: Two inductors whose self-Inductances are given as 75 mH and 55 mH, are positioned next to each other on a common magnetic core so that 75 % of the lines of flux from first coil are cutting second coil. Calculate the total mutual inductance.

$$M = k\sqrt{L_1 L_2}$$

$$= 0.75\sqrt{75\,mH \times 55\,mH} = 48.2\ mH$$

Eg: If $\phi_1 = 100\ Wb$; $\phi_2 = 50\ Wb$; $\phi_{12} = 50\ Wb$

A) WKT : $\phi_{12} = k\,\phi_1$

$$k = \frac{\phi_{12}}{\phi_1} = \frac{5\cancel{0}}{10\cancel{0}} = 0.5$$

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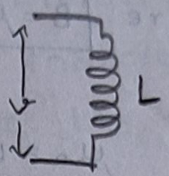
$V = L\dfrac{dI}{dt}$; $V = N\dfrac{d\phi}{dt}$



$$\boxed{L = N\frac{d\phi}{dI}}$$

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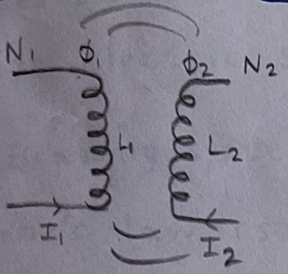
$$\boxed{L = N\frac{\phi}{I}} \Rightarrow \boxed{L_1 = N_1\frac{\phi_1}{I_1}}$$



$$\boxed{L_2 = N_2\frac{\phi_2}{I_2}}$$

$\phi_{12} = k\,\phi_1$

when they come closer ;

$$V_2 = N_2\frac{d\phi_{12}}{dt} \rightarrow (1)$$

$$V_2 = M\frac{dI_2}{dt} \Rightarrow M\frac{dI_1}{dt} \rightarrow (2)$$

From (1) & (2) ;

$$N_2\frac{d\phi_{12}}{dt} = M\frac{dI_1}{dt}$$

$$\boxed{M = N_2\frac{d\phi_2}{I_2}} \rightarrow (3)$$

$$\boxed{M = N_2\frac{\phi_{12}}{I_2} = N_2\frac{\phi_{21}}{I_1}}$$

$$M = N_2 \frac{k\phi_1}{I_2} = N_1 \frac{k\phi_2}{I_2}$$

$$L_1 . L_2 = \frac{N_1 \phi_1}{I_1} \frac{N_2 \phi_2}{I_2}$$

$$L_1 L_2 = \frac{M^2}{k^2} \Rightarrow ; \; k^2 = \frac{M^2}{L_1 L_2}$$

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

$\phi = BA$; where $\boxed{B = \mu H}$

$\boxed{\mu = \mu_o \mu_r}$

$\mu_r = \frac{\mu}{\mu_o}$ $\qquad \mu_o = 4\pi \times 10^{-7}$ H/m

$$\boxed{B = \mu_o \mu_r H}$$

WKT: $H = \frac{NI}{l}$. A/m

$$\boxed{B = \frac{\mu_o \mu_r N I}{l}}$$

$$L_1 = N_1 \frac{\phi}{I_1} = \frac{N_1 B_1 A_1}{I_1}$$

$$L_1 = \frac{N_1 \mu_o \mu_r N_1 \cancel{I_1} A}{\cancel{I_1} \times l}$$

$$\therefore \boxed{L_1 = \frac{\mu_o \mu_r N_1^2 A}{l}}$$

Similarly:

$$\boxed{L_2 = \frac{\mu_o \mu_r N_2^2 A}{l}}$$

$$\boxed{M = \frac{N_1 N_2 \mu_o \mu_r A}{l}}$$