ELL 100 - Introduction to Electrical Engineering

LECTURE 31: MAGNETIC CIRCUITS

INTRODUCTION

- A **magnetic circuit** is made up of one or more **closed loop** paths containing a **magnetic flux** ϕ (= **magnetic** field/**flux density** $\boldsymbol{B}$ × **cross-sectional area** $\boldsymbol{A}$).

- The **flux** is usually **generated** by **permanent magnets** or **electromagnets** and confined to a **path** by **magnetic cores** consisting of **ferromagnetic materials** like **iron**, although there **may be air gaps** or other materials in the path.

- Magnetic circuits are employed to efficiently channel magnetic fields in many **devices** such as **electric motors, generators, transformers, relays, solenoids, loudspeakers, hard disks, MRI machines**.

APPLICATIONS

Motors

Generators

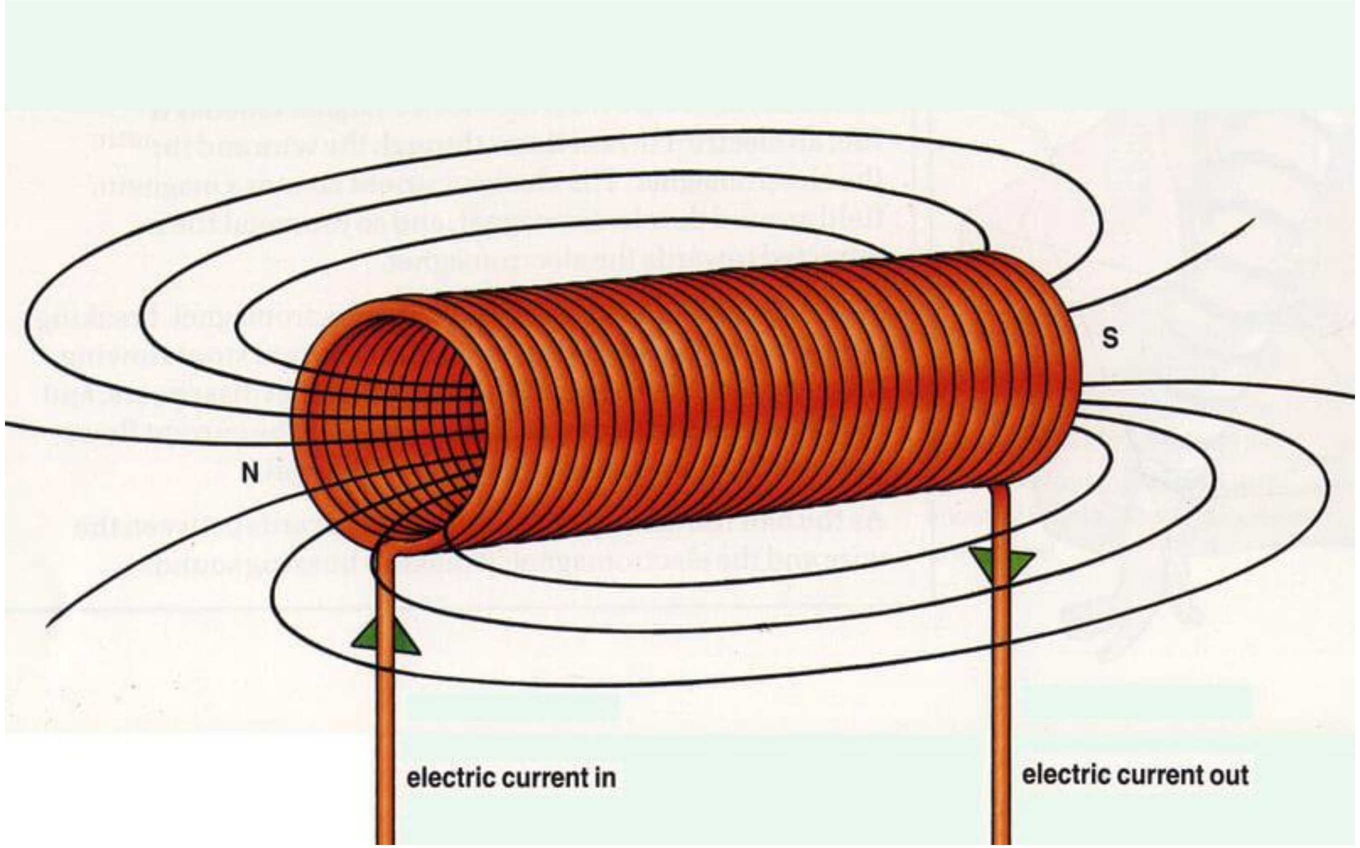
Transformers

Circuit Breakers

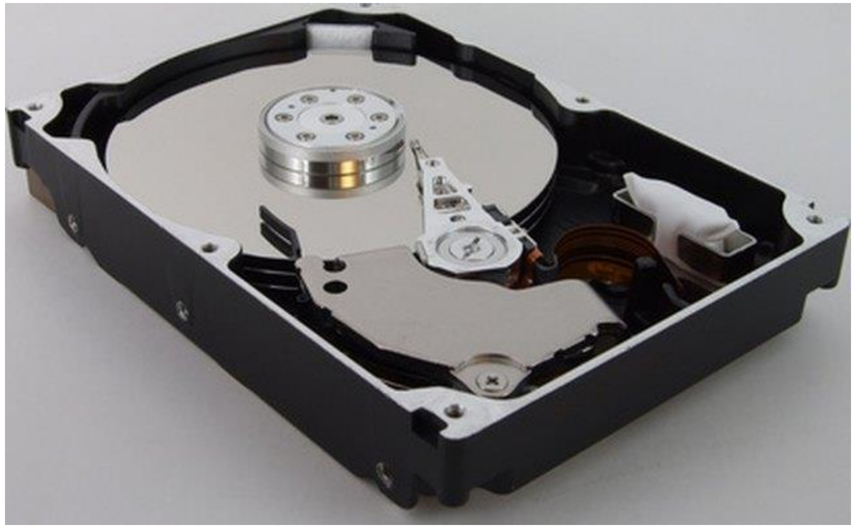
Relay Switches

Applications

Solenoids



Hard Disks

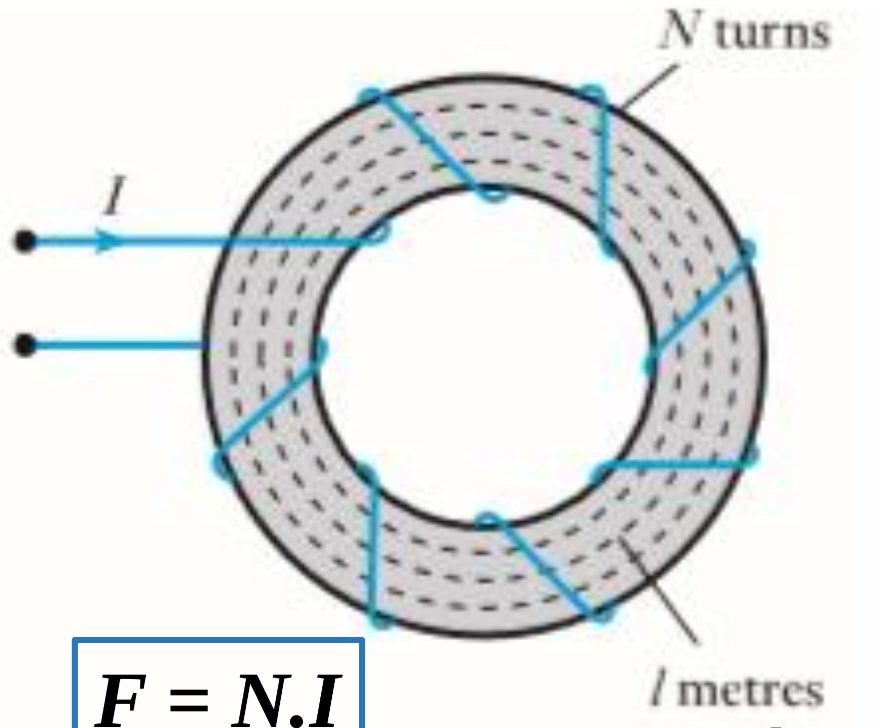


MRI Machines

Magnetomotive force (*mmf*)

- In an **electric circuit**, the **current** is **induced** due to the existence of an **electromotive force** (***emf E***, **battery voltage**). By analogy, we say that in a **magnetic circuit** the **magnetic flux** is **induced** due to the existence of a **magnetomotive force** (***mmf F***) caused by a **current flowing through one or more turns of coil**.

- The value of the ***mmf F*** is **proportional** to the **current** flowing through the coil and to the **number of turns** in the **coil**, and is expressed in units of "**ampere-turns**" or just **amperes** (number of turns is dimensionless).



$$\boxed{F = N.I}$$

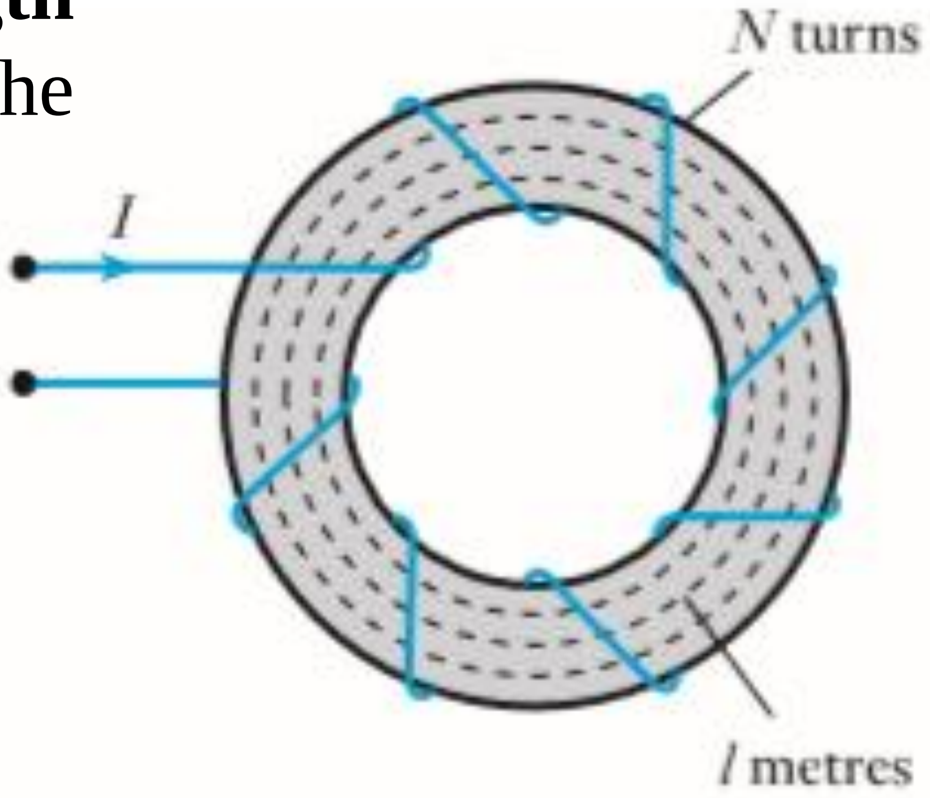
Magnetic field strength/intensity (*H*)

- The **magnetomotive force per unit length** of the magnetic circuit is termed the **magnetic field strength/intensity** (***H***).

$$H = \frac{F}{l}$$

where, $F = NI$ amperes

where l is the length of the magnetic circuit or flux loop



- **Units** of H are **ampere-turns per metre** (At/m) or just **ampere per metre** (A/m)

$$\boxed{H = \frac{NI}{l} \text{ amperes per metre}}$$

Permeability of free space μ_0 (magnetic constant)

The permeability of free space or non-magnetic materials is

$\boxed{\mu_0 = \dfrac{B}{H} \text{ for a vaccum and non-magnetic materials}}$

$\boxed{\mu_0 = 4\pi * 10^{-7} \ H/m}$ (The **units** of $\boldsymbol{\mu_0}$ are H/m **(Henry per meter))**

where $\boldsymbol{B}$ (= ϕ/$\boldsymbol{A}$) is the **magnetic flux density** (units of Tesla, T),

$\boldsymbol{A}$ is the **cross-sectional area** through which the flux passes,

ϕ is the **magnetic flux** (units of Weber (Wb)) and

$\boldsymbol{H}$ is the **magnetic field strength** (units A/m).

SOLVED EXAMPLE

Q. A coil of 200 turns is wound uniformly over a wooden ring having a mean circumference of 600 mm and a uniform cross-sectional area of 500 mm². If the current through the coil is 4.0 A, calculate (a) the magnetic field strength; (b) the flux density; (c) the total flux.

Ans: $N = 200$, $I = 4$ A, $l = 600$ mm $= 0.6$ m, $A = 500$ mm^2 $= 5\times10^{-4}$ m^2

(a) $H = N.I / l = 200 \times 4 / 0.6 = 1333.3$ A/m

(b) $B = \mu_0 H = 4\pi\times10^{-7} \times 1333.3 = 1.6755\times10^{-3}$ T $= 1.6755$ mT

(c) $\phi = B.A = 1.6755\times10^{-3} \times 5\times10^{-4} = 8.376\times10^{-7}$ Wb $= 0.8376$ μWb

SOLVED EXAMPLE

Q. Calculate the magnetomotive force required to produce a flux of 0.015 Wb across an air-gap 2.5 mm long, having an effective area of 200 cm^2.

Ans: $\phi = 0.015$ Wb, $l = 2.5$ mm $= 2.5\times10^{-3}$ m, $A = 200$ cm^2 $= 2\times10^{-2}$ m^2

$$B = \phi / A = 0.015 / (2\times10^{-2}) = 0.75 \text{ T}$$

$$H = B / \mu_0 = 0.75 / (4\pi\times10^{-7}) = 5.97\times10^{5} \text{ A/m}$$

$$F\ (mmf) = H.l = 5.97\times10^{5} \times 2.5\times10^{-3} = 1.49\times10^{3} \text{ A}$$

Relative Permeability μ_r

- The **ratio of the flux density *B*** produced in a **material to** the flux density produced in **vacuum** (or in a non-magnetic core) for a particular applied magnetic field strength *H*.
- For air and **non-magnetic materials**, μ_r = **1**
- For **ferromagnetic materials**, e.g. some forms of nickel–iron alloys, the relative permeability can be as large as ~100000 i.e. **~10^6**.
- For a material having a relative permeability μ_r,

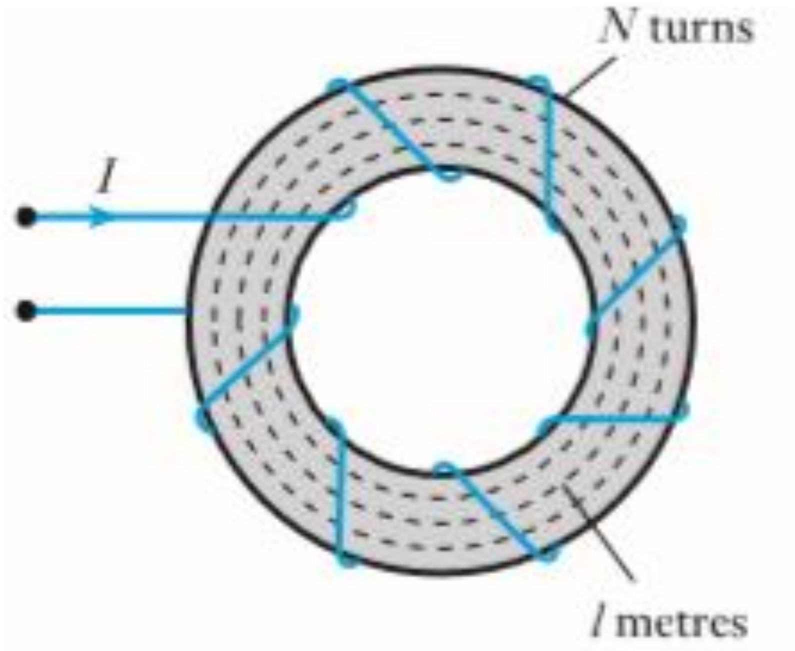
$$\boxed{\begin{array}{l} B = \mu_r \mu_0 = \mu H \\ \text{where, } \mu = \mu_0 \mu_r \text{ is the absolute permeability} \end{array}}$$

Reluctance S

It is the opposition that a magnetic circuit offers to the passage of magnetic flux through it (**ratio of *mmf* applied to the flux induced**).

$$\phi = BA \qquad (1)$$

$$mmf,\ F = Hl \qquad (2)$$



Dividing (1) by (2),

$$\frac{\phi}{F} = \frac{BA}{Hl} = \frac{\mu_0\mu_r HA}{Hl} = \mu_0\mu_r\frac{A}{l}$$

Thus, $\boxed{\boldsymbol{S = F/\phi = l/(\mu A)}}$, where $\mu = \mu_0\mu_r$ (S has units of A/Wb)

The **inverse of reluctance** is known as **permeance** (ease of flux passage)

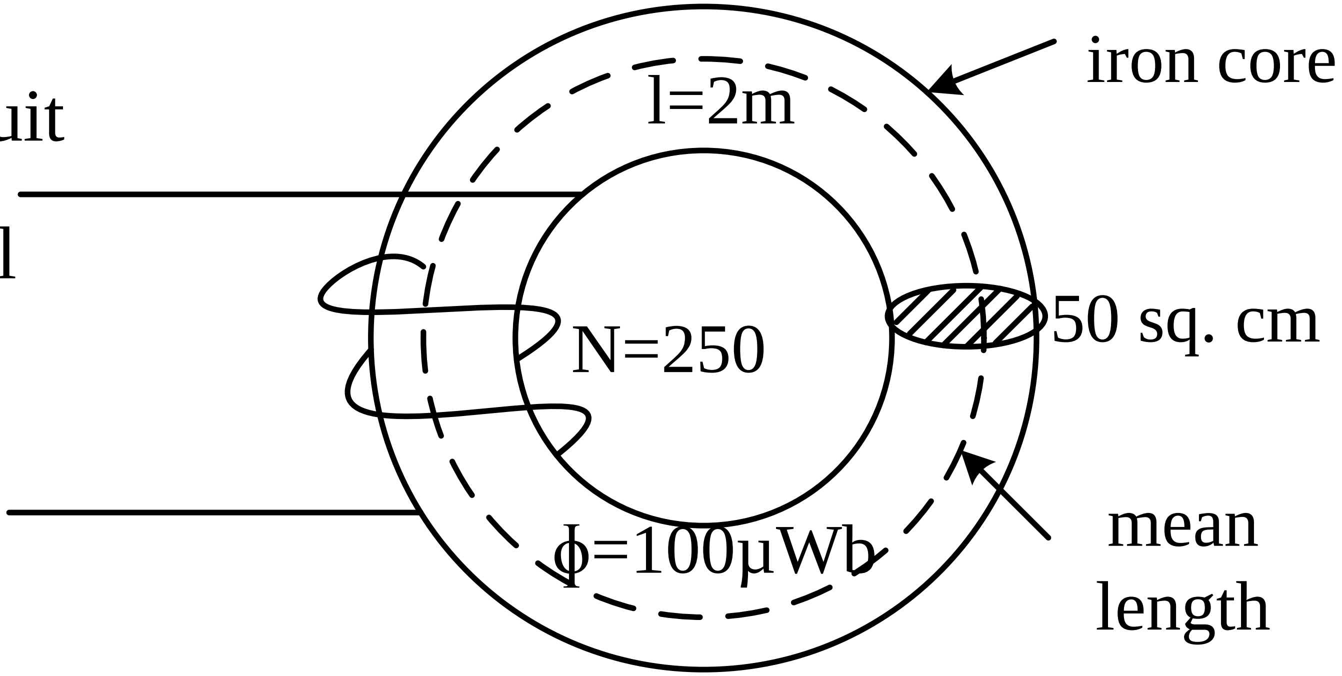
SOLVED EXAMPLE

Q. The simple magnetic circuit shown has a cross-sectional area of 50 cm^2 and mean length of 2 m. The relative permeability of the core is 100. The coil has 250 turns and the flux produced is 100 μWb.

Find:

(a) Reluctance of the magnetic circuit

(b) Current flowing through the coil

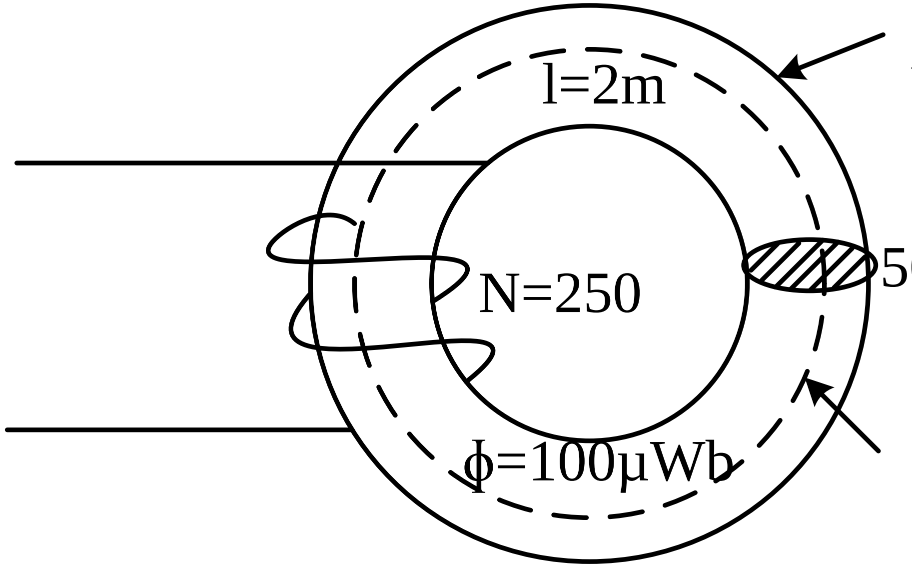


Ans:

(a) Reluctance, $S = \dfrac{l}{\mu A}$

$$\mu = \mu_0 \mu_r = 4\pi * 10^{-7} * 100 = 4\pi * 10^{-5}$$

$$\boxed{S} = \frac{2}{4\pi * 10^{-5} * 50 * 10^{-4}} = \boxed{3.18 * 10^{6}\, A/Wb}$$



(b) Magnetic field strength, $H = \dfrac{mmf}{l} = \dfrac{NI}{l}$

$$I = \frac{Hl}{N}$$

$$H = \frac{B}{\mu} = \frac{\phi}{A\mu} = \frac{100 * 10^{-6}}{50 * 10^{-4} * 4\pi * 10^{-5}} = 159.15\, A/m$$

$$\boxed{I} = \frac{Hl}{N} = \frac{159.15 * 2}{250} = \boxed{1.27\, A}$$

SOLVED EXAMPLE

Q. The air gap in a magnetic circuit is 1.5 mm long and 2500 mm^2 in cross-sectional area. Calculate (a) The reluctance of the air gap (b) The *mmf* required to set up a flux of 800 µWb in the air gap.

Ans: (a) Reluctance, $S = \dfrac{l}{\mu A}$

$$\mu = \mu_0 \mu_r = 4\pi * 10^{-7} * 1 = 4\pi * 10^{-7} \qquad (\mu_r \text{ of air} = 1)$$

$$\boxed{S} = \frac{1.5*10^{-3}}{4\pi*10^{-7}*2500*10^{-6}} = \boxed{4.77*10^{5}\, A/Wb}$$

(b) $H = \dfrac{B}{\mu} = \dfrac{\phi}{A\mu} = \dfrac{800*10^{-6}}{2500*10^{-6}*4\pi*10^{-7}} = 2.55*10^{5}\, A/m$

$$\boxed{mmf} = Hl = 2.55*10^{5}*1.5*10^{-3} \boxed{= 382.5 A}$$

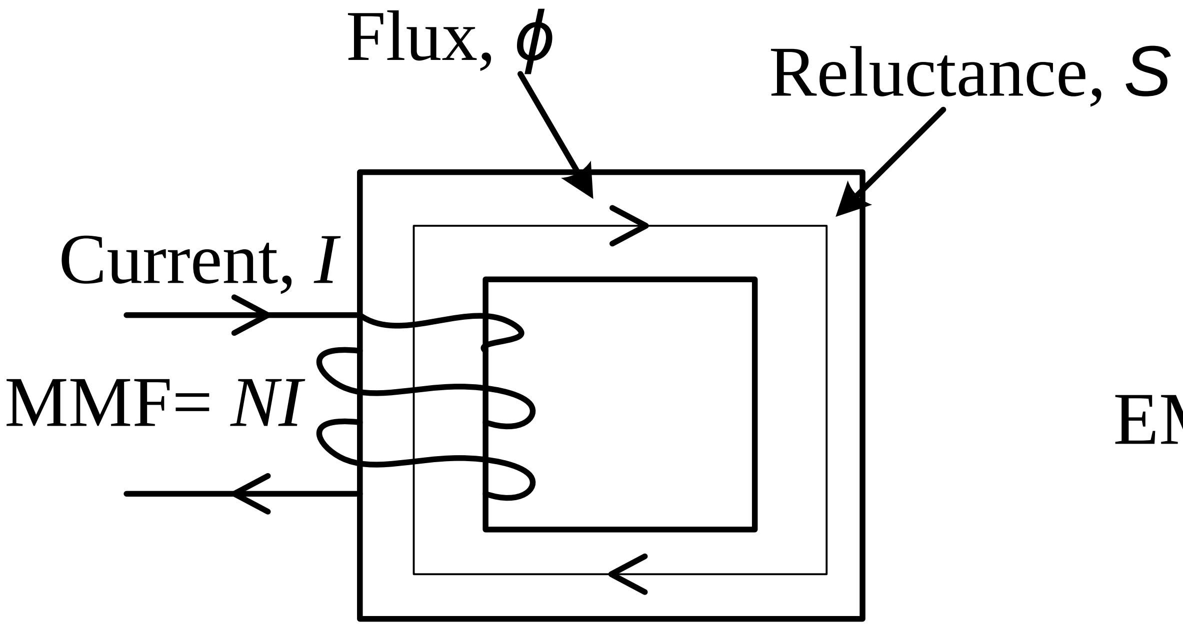
EXERCISES

Q. A mild steel (relative permeability of 400) ring having a cross-sectional area of 500 mm^2 and a mean circumference of 400 mm has a coil of 200 turns wound uniformly around it. Calculate (a) The reluctance of the ring
(b) The current required to produce a flux of 800 μWb in the ring.

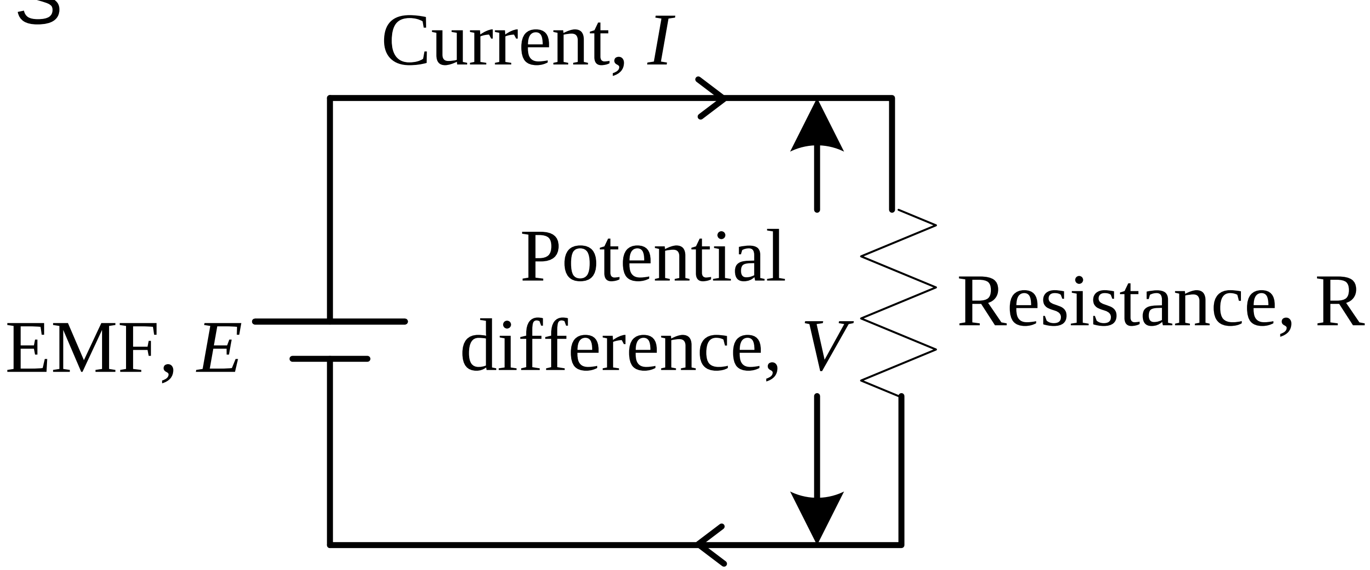
Q. A rectangular shaped core is made of mild steel (relative permeability of 940) plates 15 mm × 20 mm cross-section. The mean length of the magnetic path is 18 cm. The exciting coil has 300 turns with a current of 0.7 A flowing through it. Calculate:

(a) Magneto-motive force (b) Flux density
(c) Reluctance (d) Magnetic flux

MAGNETIC CIRCUIT ANALOGY WITH ELECTRICAL CIRCUIT



Magnetic circuit



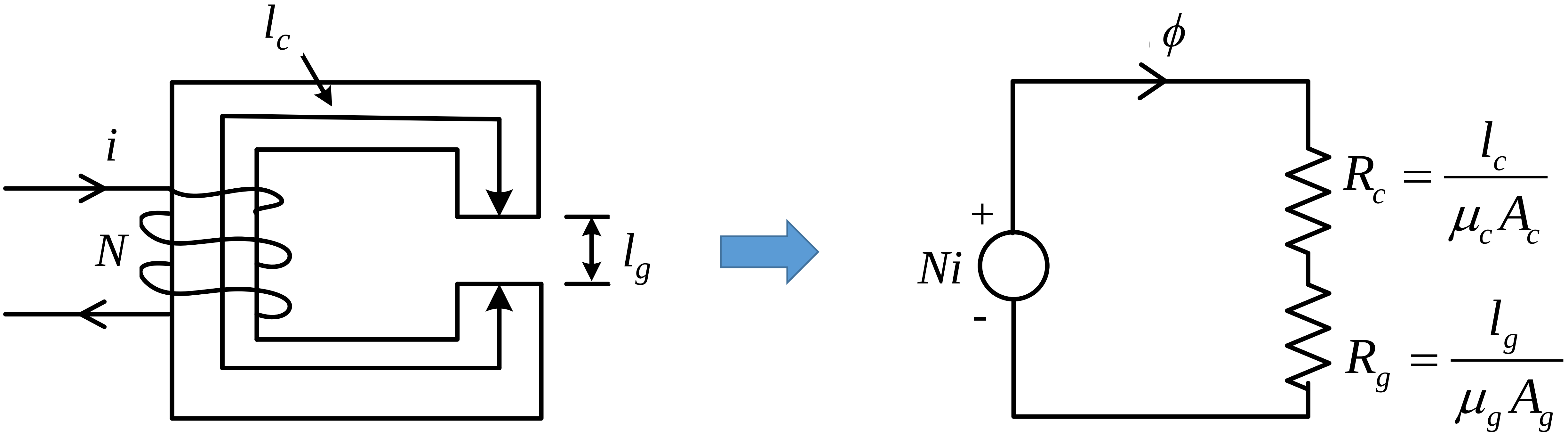
Analogous electrical circuit

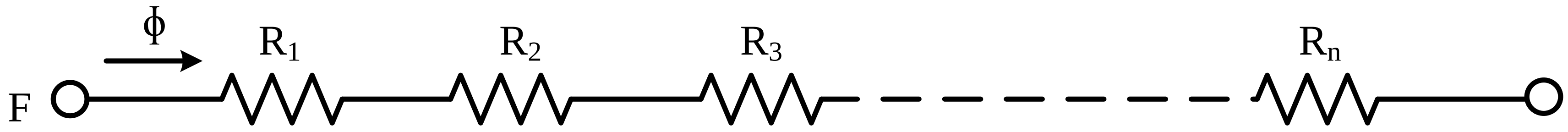
Magnetic Circuit Analogy with Electrical Circuit

S.No	Magnetic circuit quantity	Electrical circuit quantity
1	Magnetic flux density $\boldsymbol{B}$ (T = Wb/m^2)	Current density $\boldsymbol{J}$ (A/m^2)
2	Magnetic flux $\boldsymbol{\phi}$ (Wb)	Current $\boldsymbol{I}$ (A)
3	Magnetic field intensity $\boldsymbol{H}$ (A/m)	Electric field intensity $\boldsymbol{E}$ (V/m)
4	m.m.f. $\boldsymbol{F}$ (A)	e.m.f. $\boldsymbol{E}$ (V)
5	Reluctance $\boldsymbol{S}$ (A/Wb)	Resistance $\boldsymbol{R}$ (Ω = V/A)
6	Permeance (H = Wb/A)	Conductance (S = A/V)
7	Permeability $\boldsymbol{\mu}$ (H/m)	Conductivity $\boldsymbol{\sigma}$ (S/m)

SERIES CONNECTION IN MAGNETIC CIRCUIT

The applied *mmf* is equal to the sum of the *mmf*s dropped across each series element, while the flux through each series element is the same.

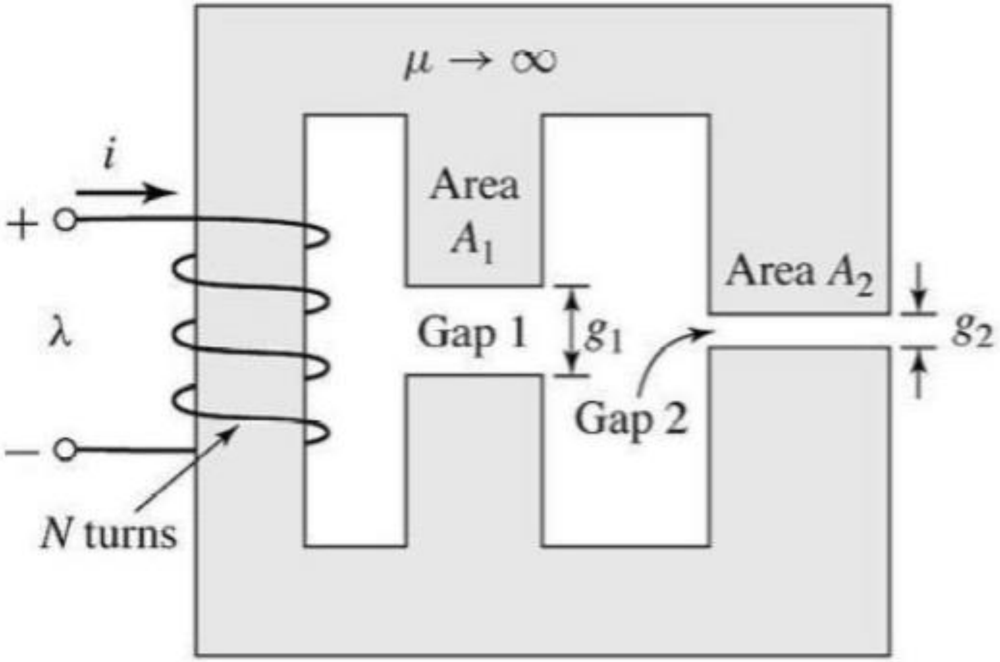


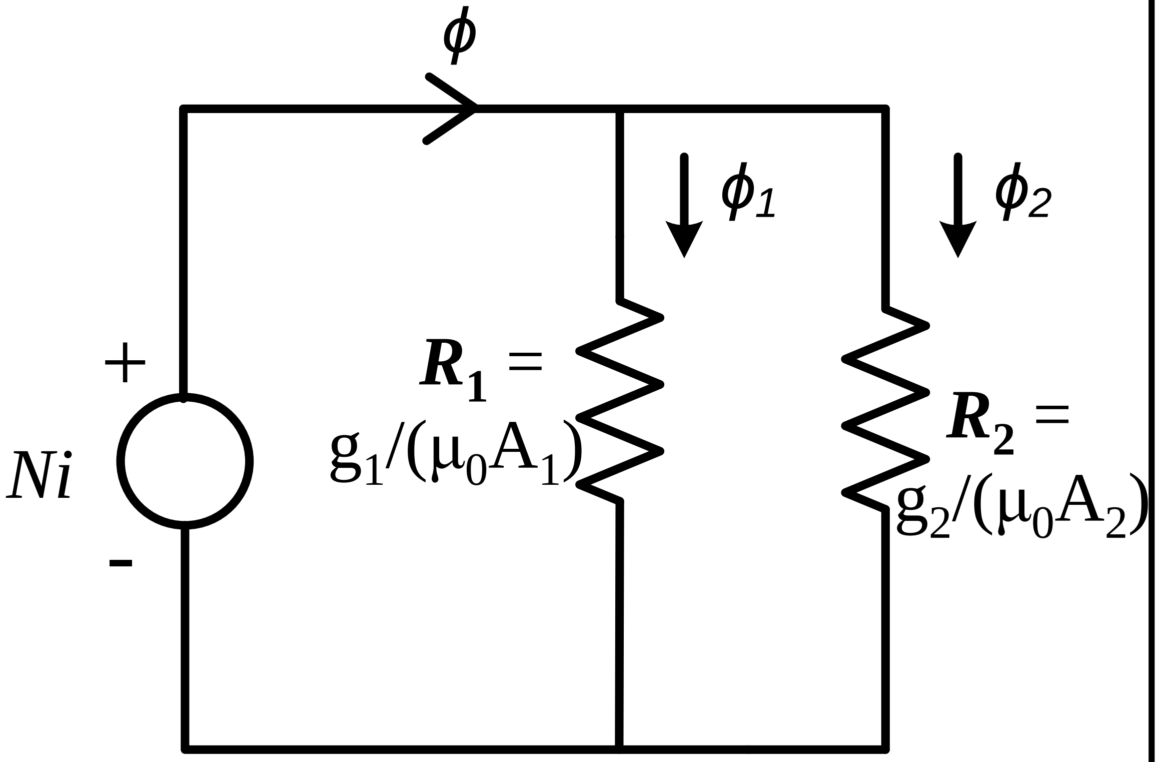


$$\boxed{F = \phi R_{eq} = \phi\left(R_1 + R_2 + R_3 + \ldots\ldots + R_n\right)}$$ (R_i denote reluctances)

Parallel Connection in Magnetic Circuit

The same *mmf* appears across all the reluctances in parallel, while the total flux is the sum of the individual fluxes in each parallel element.





$$\boxed{F = \phi R_{eq} \text{ where } R_{eq} = R_1 \,||\, R_2 = R_1 R_2 / (R_1 + R_2)}$$

SOLVED EXAMPLE

Q. A ring of 30-cm mean diameter is made using a cylindrical iron rod of diameter 2.5 cm. A saw-cut 1-mm wide is made through the ring to create an air-gap. A coil with 500 turns of wire is wound on the ring. Calculate the current required in the exciting coil to produce a flux of 4 mWb in the ring. Assume the relative permeability of iron at this flux density as 800. Neglect any leakage or fringing of the magnetic field.

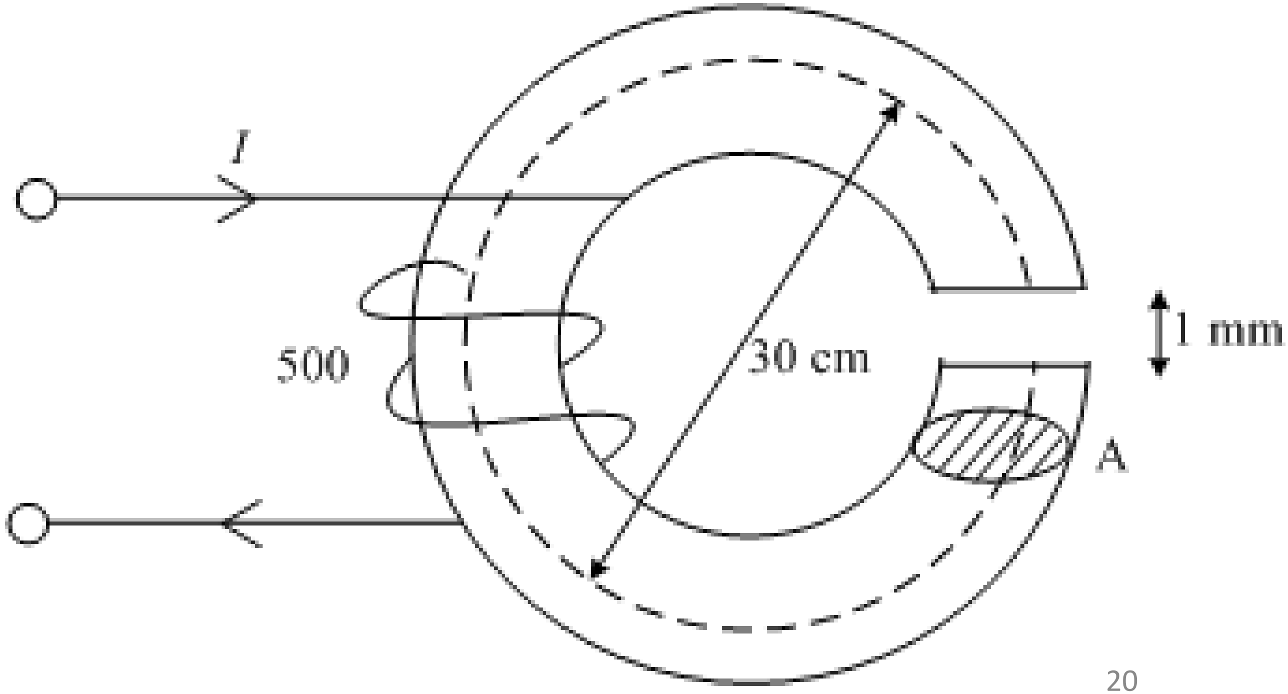
Ans: $A = \pi r^2 = 3.14 \times (0.025\ \text{m})^2$

$= 4.91 \times 10^{-4}\ \text{m}^2$

$\Rightarrow B = \phi / A$

$= (4 \times 10^{-3}\ \text{Wb}) / (4.91 \times 10^{-4}\ \text{m}^2)$

$= 8.15\ \text{Wb/m}^2$



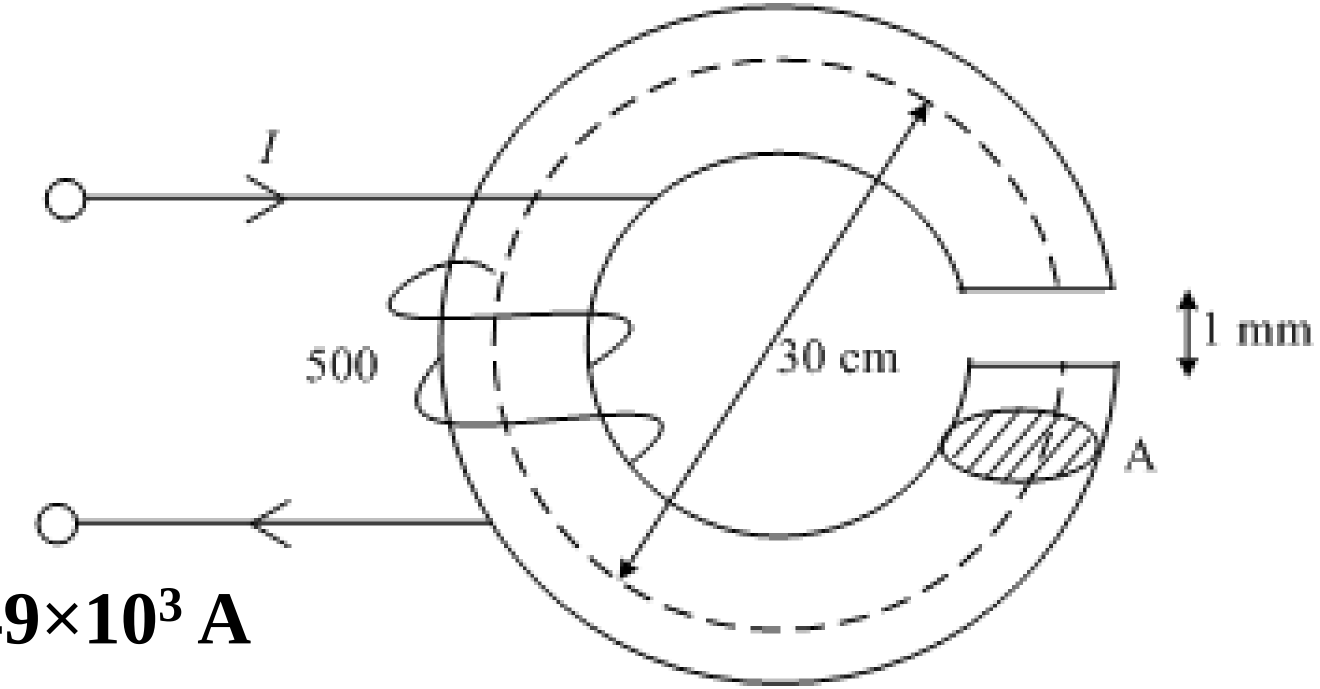
First consider air gap:

$H_1 = B / \mu_0 = 8.15/(4\pi\times10^{-7})$ A/m

$\quad = 6.49\times10^6$ A/m

$l_1 = 1$ mm $= 10^{-3}$ m

$\Rightarrow$ *mmf* across air gap $\boldsymbol{F_1} = H_1\, l_1 = \mathbf{6.49\times10^3}$ **A**



Now consider iron ring:

$H_2 = B / (\mu_r \mu_0) = 8.15/(800\times4\pi\times10^{-7})$ A/m

$\quad = 8.1\times10^3$ A/m

$l_2 \sim 2\pi R = 2\times3.14\times0.15$ m $= 0.94$ m

$\Rightarrow$ *mmf* across iron ring $\boldsymbol{F_2} = H_2\, l_2 = \mathbf{7.63\times10^3}$ **A**

Total *mmf* $\boldsymbol{F = F_1 + F_2}$

$\quad = \mathbf{1.41\times10^4}$ **A**

$\Rightarrow$ $\boxed{\textbf{Current } \boldsymbol{I}}$ $\boldsymbol{= F / N}$

$\quad = (1.41\times10^4)/500$ $\boxed{= \mathbf{28.24\ A}}$

SOLVED EXAMPLE

Q. An iron ring of mean circumference 50-cm has an air gap of 0.1-cm length and a winding of 300 turns. If the relative permeability of iron is 400 when a current of 1 ampere flows through the coil, find the flux density in the air gap.

Ans: Let B be the flux density in the iron ring.
Its value would also be the same in the air gap.

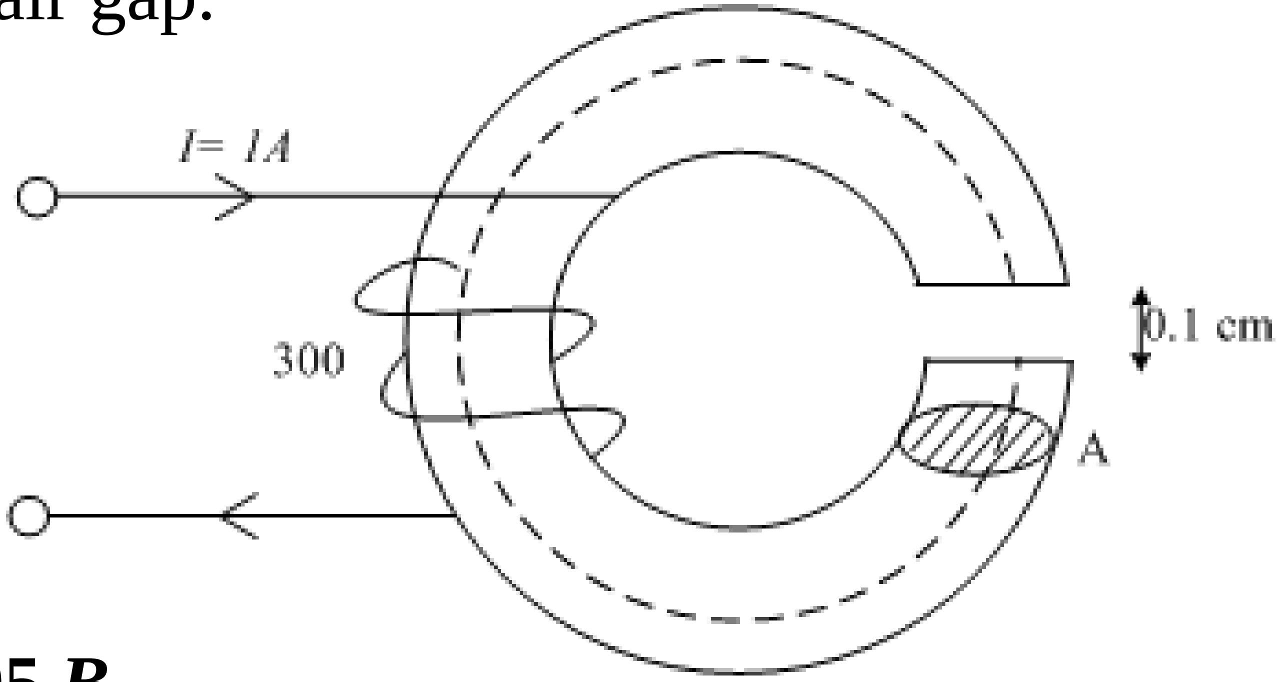
First consider iron ring:

$H_1 = B / (\mu_r \mu_0) = B / (400 \times 4\pi \times 10^{-7})$

$= 1989.44\ B$

$l_1 = 50 \text{ cm} = 0.5 \text{ m}$

$\Rightarrow$ *mmf* across iron ring $\boldsymbol{F_1} = H_1\, l_1 = \mathbf{995}\ \boldsymbol{B}$



Now consider air gap:

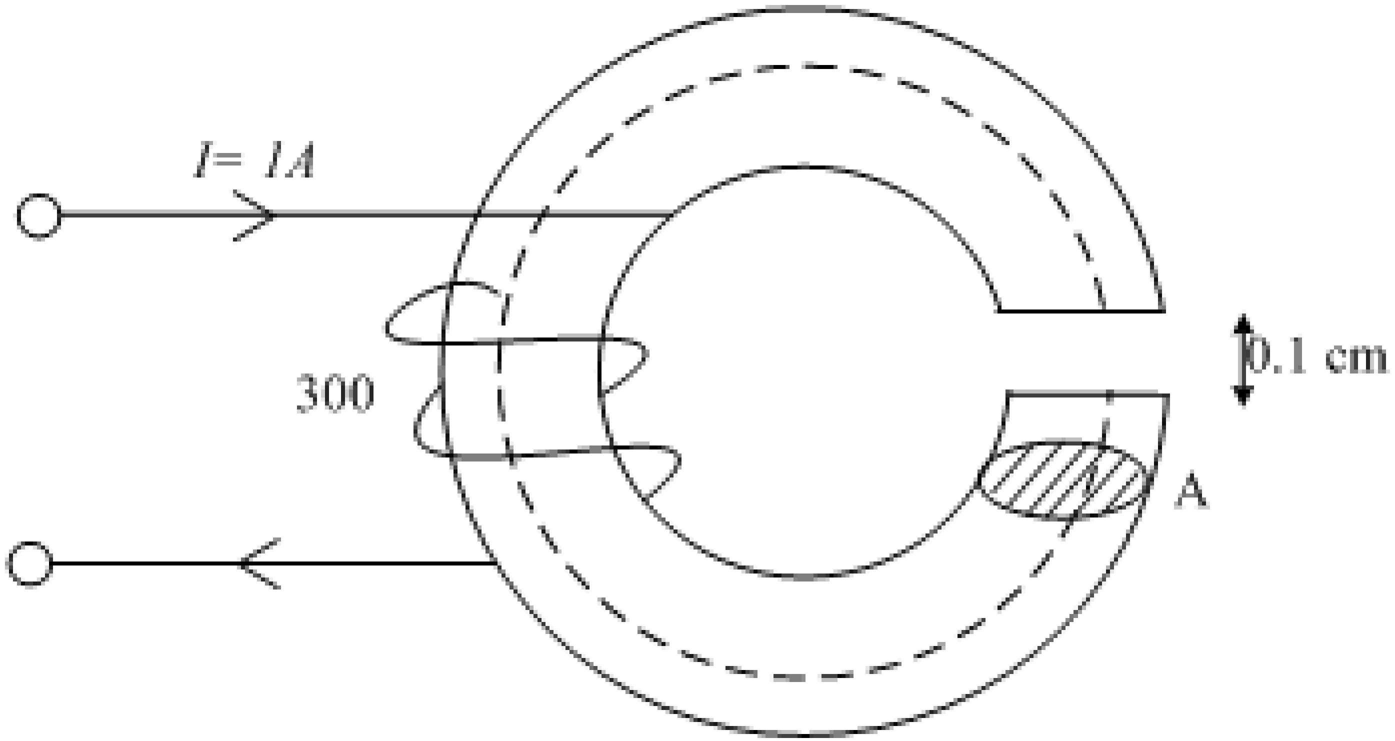
$H_2 = B / \mu_0 = B / (4\pi\times10^{-7})$

$= 7.96\times10^5\ B$

$l_2 = 0.1\text{ cm} = 10^{-3}\text{ m}$

$\Rightarrow$ *mmf* across air gap

$\boldsymbol{F_2} = H_2\, l_2 = \mathbf{796\ B}$



Total *mmf* $\boldsymbol{F = F_1 + F_2}$

$= (995 + 796)B = \mathbf{1791\ B}$

But given total *mmf* $\boldsymbol{F}$ $= N.I = 300\times1 =$ **300**

=> $\boxed{\boldsymbol{B}} = 300/1791 = \boxed{\mathbf{0.1675\ Wb/m^2}}$

EXERCISES

Q. An iron ring of mean circumference 50 cm has an air gap of 1 mm. It is uniformly wound with a coil having 200 turns. If the relative permeability of iron is 300 when a current of 1 A flows through the coil, calculate the flux density in the air gap. Neglect leakage and fringing.

Q. A current of 3 A flows through a coil of 1000 turns uniformly wound on an iron ring having a mean circumference of 40 cm and a cross-sectional area of 4 cm^2. The relative permeability of the iron is 80. Calculate: (a) the magnetomotive force (b) the magnetic field strength (c) the flux density (d) the flux.

Q. A coil of wire wound up like a solenoid consists of 50 turns and carries a current of 5 A. If the length of the solenoid is 20 cm, calculate: (a) magnetic field intensity and (b) flux density within the solenoid coil.

Exercises

Q. A coil having 200 turns and carrying a current of 3 A is wound uniformly over a toroidal ring with a mean circumference of 20 cm and cross-section of 5 cm^2. The material of the ring has μ_r = 1000. Calculate: (i) magnetic field strength (ii) flux density (iii) total flux.

Q. The mean length of a magnetic circuit is 40 cm with cross-sectional area 8 cm^2 and relative permeability of the core material being 200. When a magnetizing coil carries a current of 6 A, the magnetic flux produced in the core is 3 mWb. Find: (i) reluctance of the magnetic circuit (ii) flux density in the core (iii) magnetic field intensity (iv) number of turns of the coil.

Exercises

Q. A magnetic circuit has cross-sectional area of 50 cm^2 and mean length of 2 m. The relative permeability of core material is 80. Calculate the reluctance of the magnetic circuit if the magnetizing coil has 150 turns and the core flux is 80 µWb. What is the value of the current flowing in the coil?

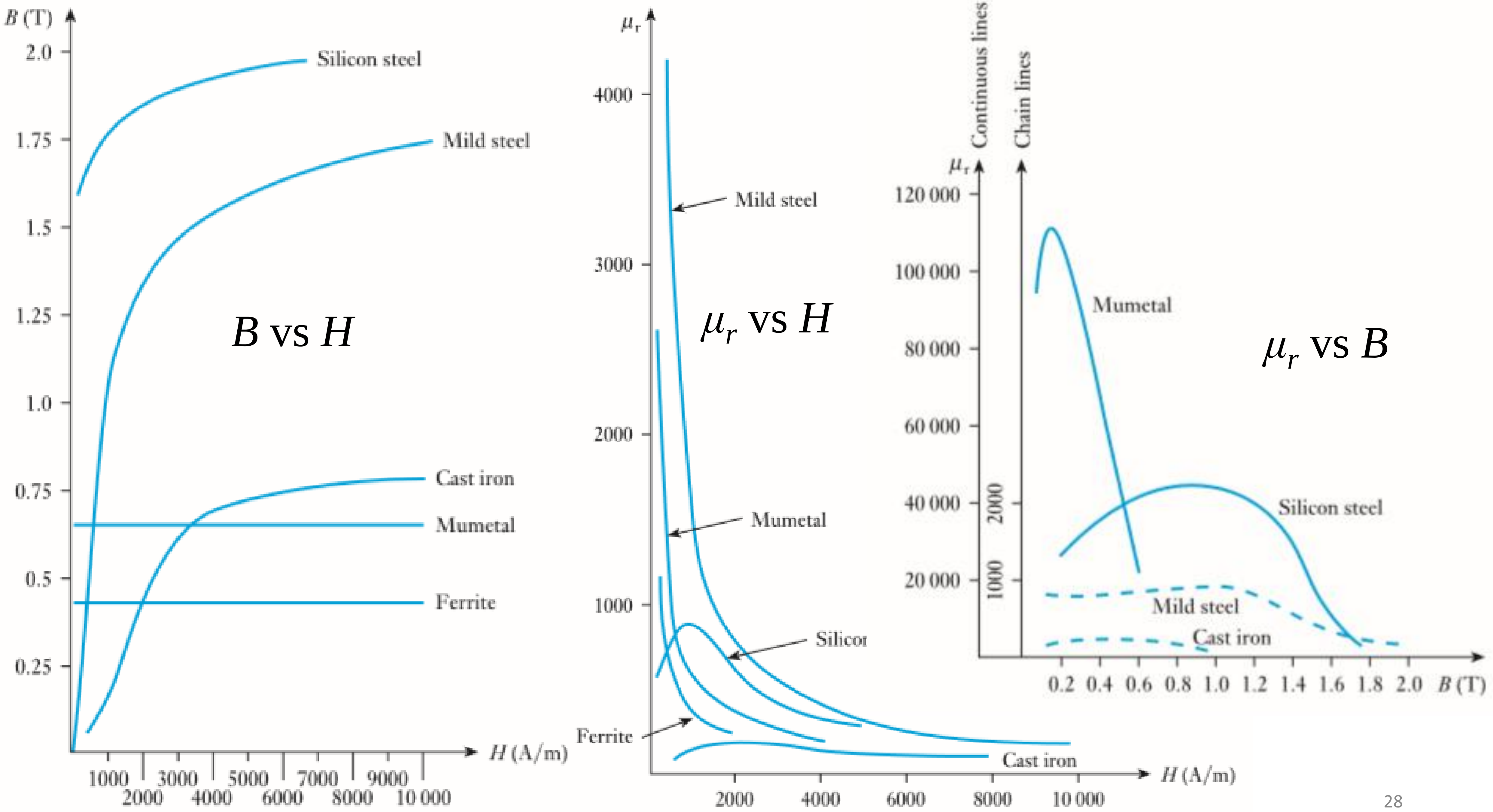
Q. A mild steel ring having a cross-sectional area of 5 cm^2 and a mean circumference of 40 cm has a coil of 200 turns wound uniformly around it. Calculate (i) the reluctance of the ring (ii) the current required to produce a flux of 800 µWb in the ring. Assume relative permeability of mild steel to be 380 at the flux density developed in the core.

***B-H* CURVE**

- The ***B-H*** or **magnetization curve** gives the **relation** between **flux density *B*** and **field intensity *H***.

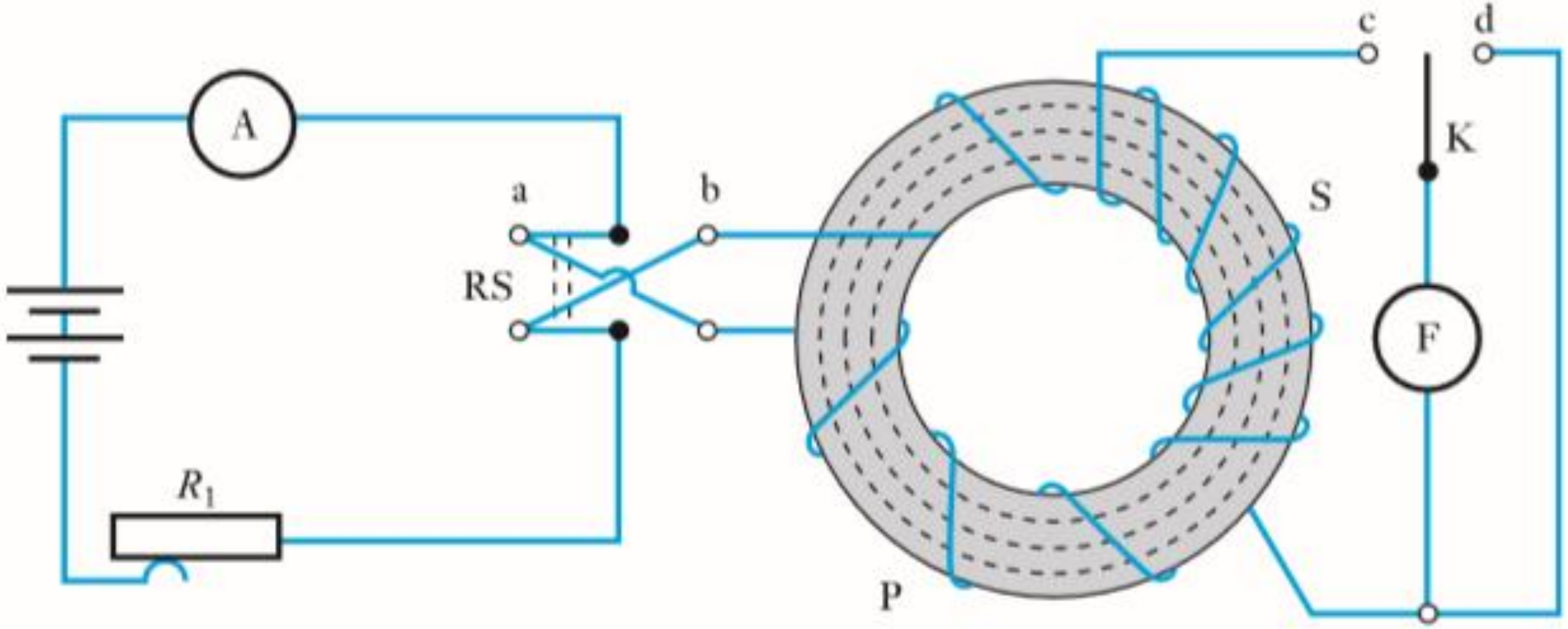
- It is **not a straight line** (as naively expected from the relation $B = \mu H$) and is actually **non-linear** as the permeability $\boldsymbol{\mu}$ typically **depends** on the applied field strength ***H***.

- The complete ***B-H* curve** is usually described as a **hysteresis loop**. The **area** contained **within** a hysteresis **loop** indicates the **energy required** to perform the '**magnetize - demagnetize**' process.


B vs H
B (T)
2.0
1.75
1.5
1.25
1.0
0.75
0.5
0.25
Silicon steel
Mild steel
Cast iron
Mumetal
Ferrite
H (A/m)
1000
2000
3000
4000
5000
6000
7000
8000
9000
10 000
μr vs H
μr
4000
3000
2000
1000
Mild steel
Mumetal
Silicon
Ferrite
Cast iron
H (A/m)
2000
4000
6000
8000
10 000
μr vs B
Continuous lines
Chain lines
μr
120 000
100 000
80 000
60 000
40 000
20 000
2000
1000
Mumetal
Silicon steel
Mild steel
Cast iron
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
B (T)

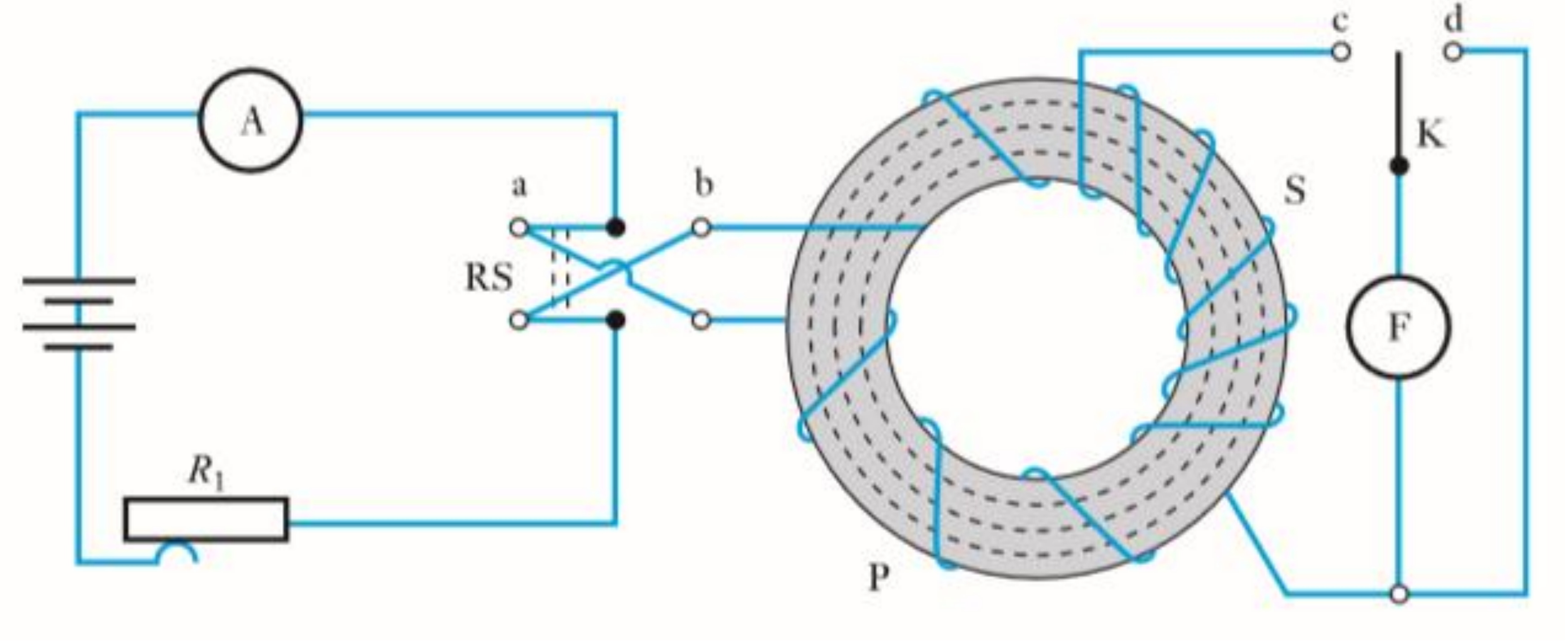
Experimental Determination of *B-H* Curve

By means of a **fluxmeter F** to measure the induced flux ϕ (and thus flux density B), for a given applied field intensity H (through applied current I)



Fluxmeter is a special type of permanent-magnet moving-coil **instrument** that gives a **deflection** of its needle **proportional** to the **flux** measured through the **coil connected** to it.

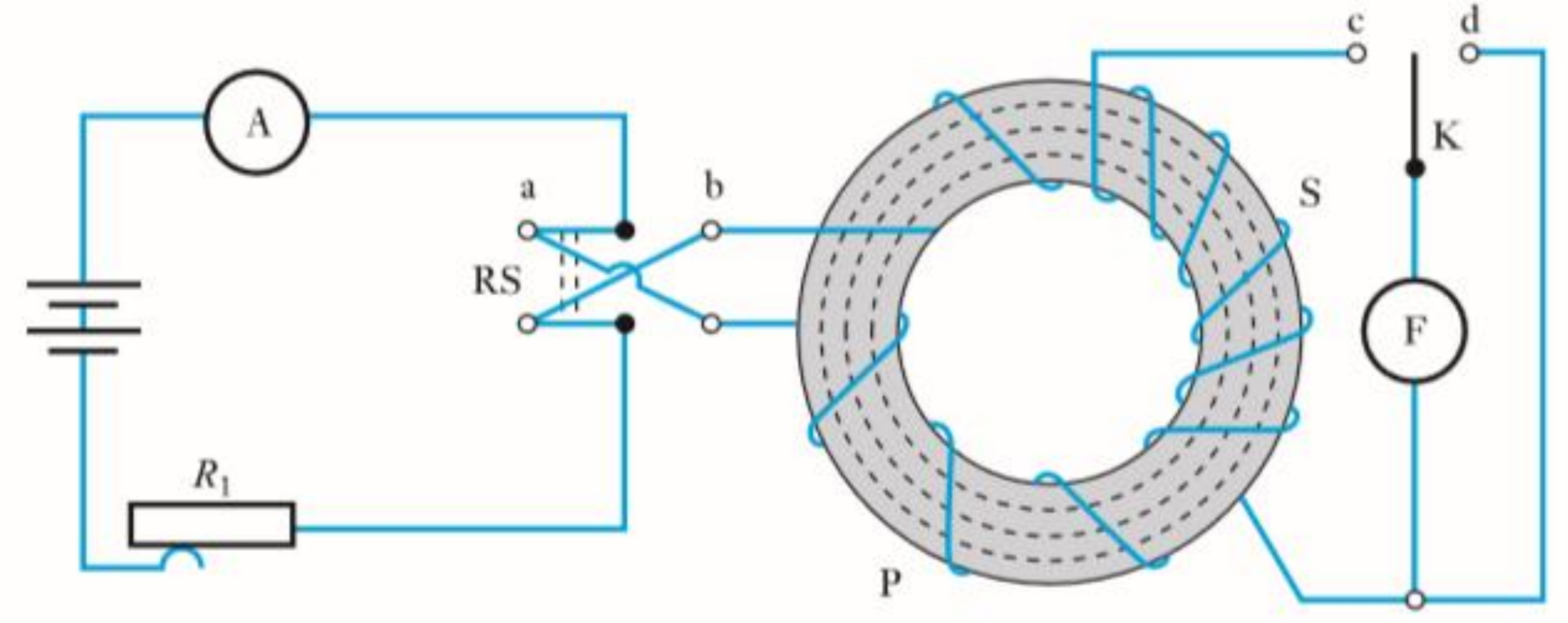
Experimental Determination of *B-H* Curve



Input coil P is connected to a battery through a **reversing switch RS**, an **ammeter A** (to **measure** input **current**) and a **variable resistor R_1** (to **vary** the **input current I** and hence the **applied field intensity $H \propto I$**).

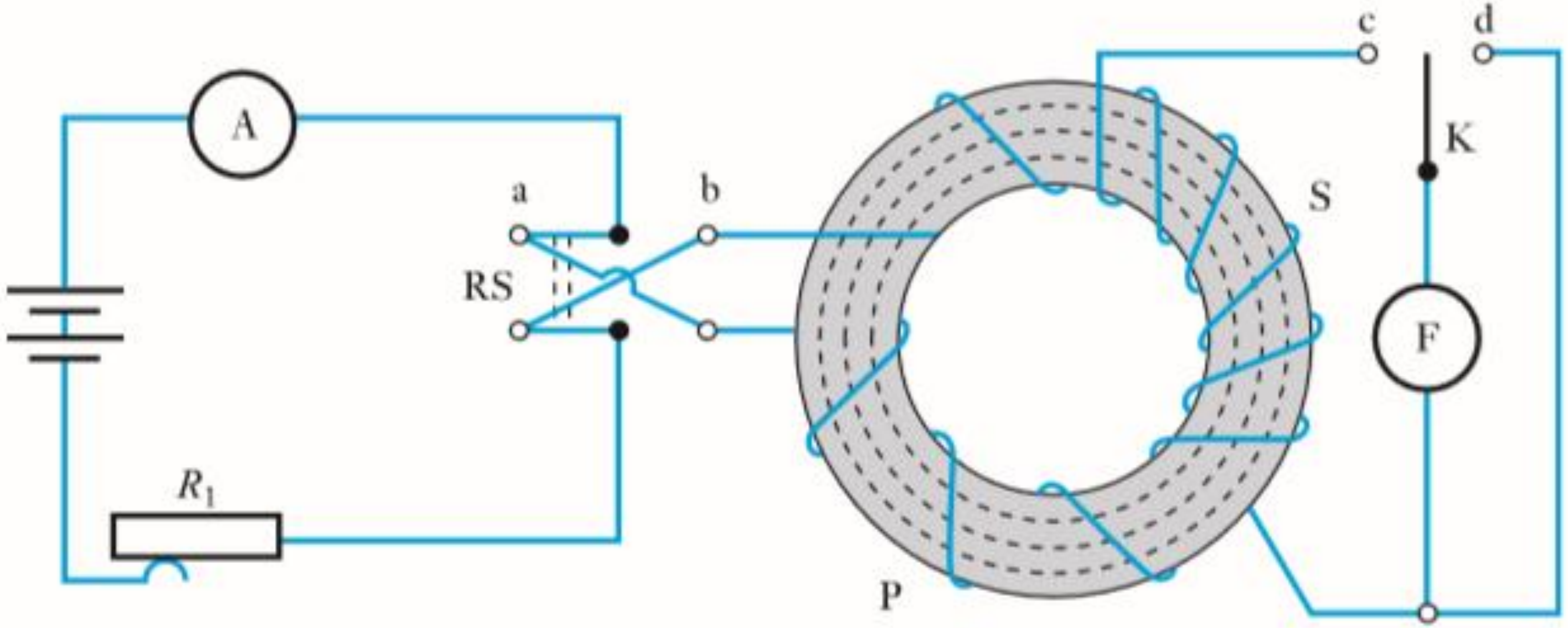
Output coil S is connected through a **two-way switch K** to fluxmeter F.

Experimental Determination of *B-H* Curve



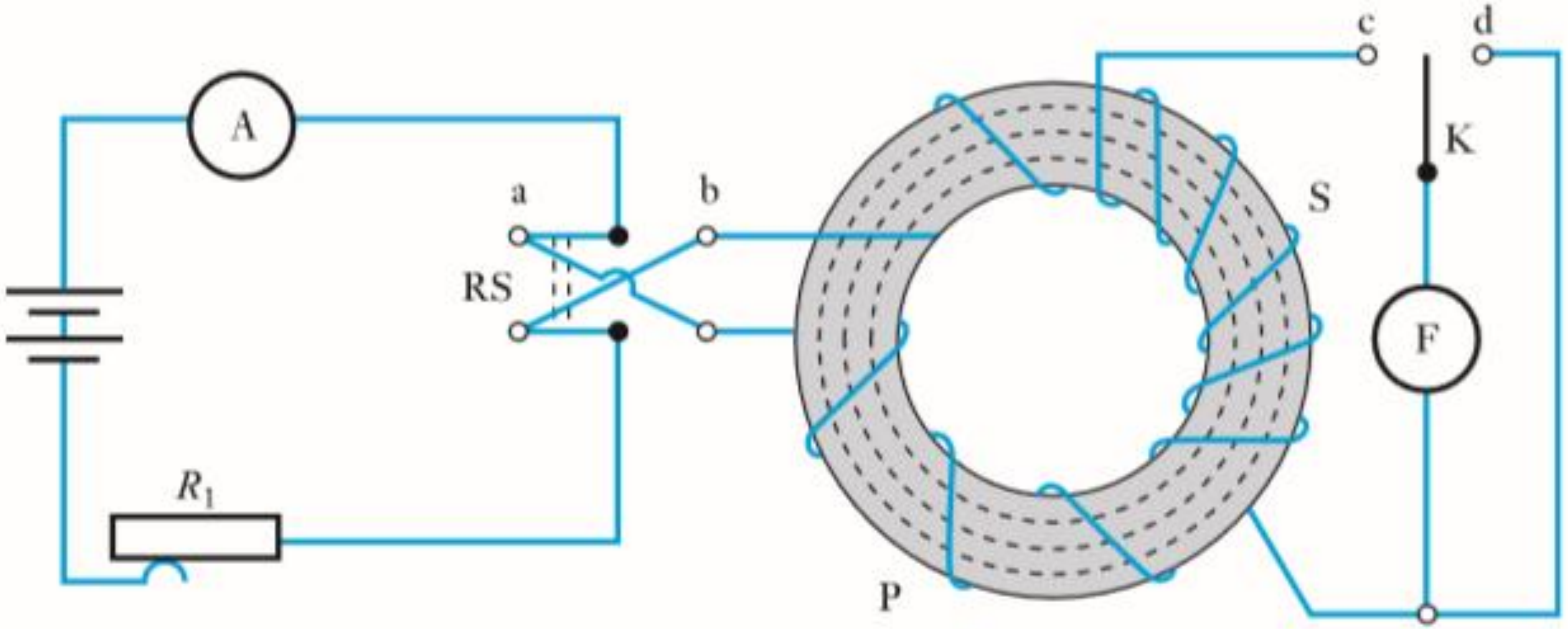
The **current** through input **coil P** is adjusted to a **desired value** by means of $\mathbf{R_1}$ and **switch RS** is then **reversed several times** to bring the core into a ‘**cyclic**’ steady-state condition, i.e. the **flux** in the ring **reverses** from a certain value in **one direction** to the same value in the **reverse direction**. During this cyclic operation, **switch K is at position ‘d’**, thereby **short-circuiting** the **fluxmeter**.

Experimental Determination of *B-H* Curve



Now, with switch **RS initially** at ‘**up**’ position, switch **K** is **moved** over to **‘c’**, the **current** through P is **reversed** by moving **RS quickly** over to ‘**down**’ position. As the **flux** in the ring changes suddenly due to the **above process**, the **e.m.f. induced in S** (Lenz’s law) sends a **current** through the **fluxmeter** and produces a **deflection** that is **proportional** to the **change of flux-linkages** in output **coil S**. This fluxmeter deflection $\boldsymbol{\theta}$ is noted.

Experimental Determination of *B-H* Curve



If N_P is the **number of turns** on coil **P**, l the mean **circumference** of the **ring**, and I is the **input current** through **P**, the applied magnetic field strength is

$$\boxed{H = \frac{IN_p}{l}} \qquad (1)$$

- If '$\boldsymbol{\theta}$' is the **fluxmeter deflection** when current through coil P is reversed and '$\boldsymbol{c}$' is the **fluxmeter constant** (flux change per unit deflection),

$$\boxed{\text{Change of flux linkages with the coil } S = c\theta} \qquad (2)$$

- If the **flux** in the ring **changes** from $\boldsymbol{\phi}$ **to** $\boldsymbol{-\phi}$ when the current through input coil P is reversed, and if $\boldsymbol{N_s}$ is the **number of turns** of output coil **S**, the change of flux linkages (∝ emf induced) with coil S is

$$\boxed{\text{Change of flux*number of turns on } S = 2\phi N_s} \qquad (3)$$

- Equating (2) and (3), we get $2\phi N_s = c\theta$ => $\boxed{\phi = \dfrac{c\theta}{2N_s}}$

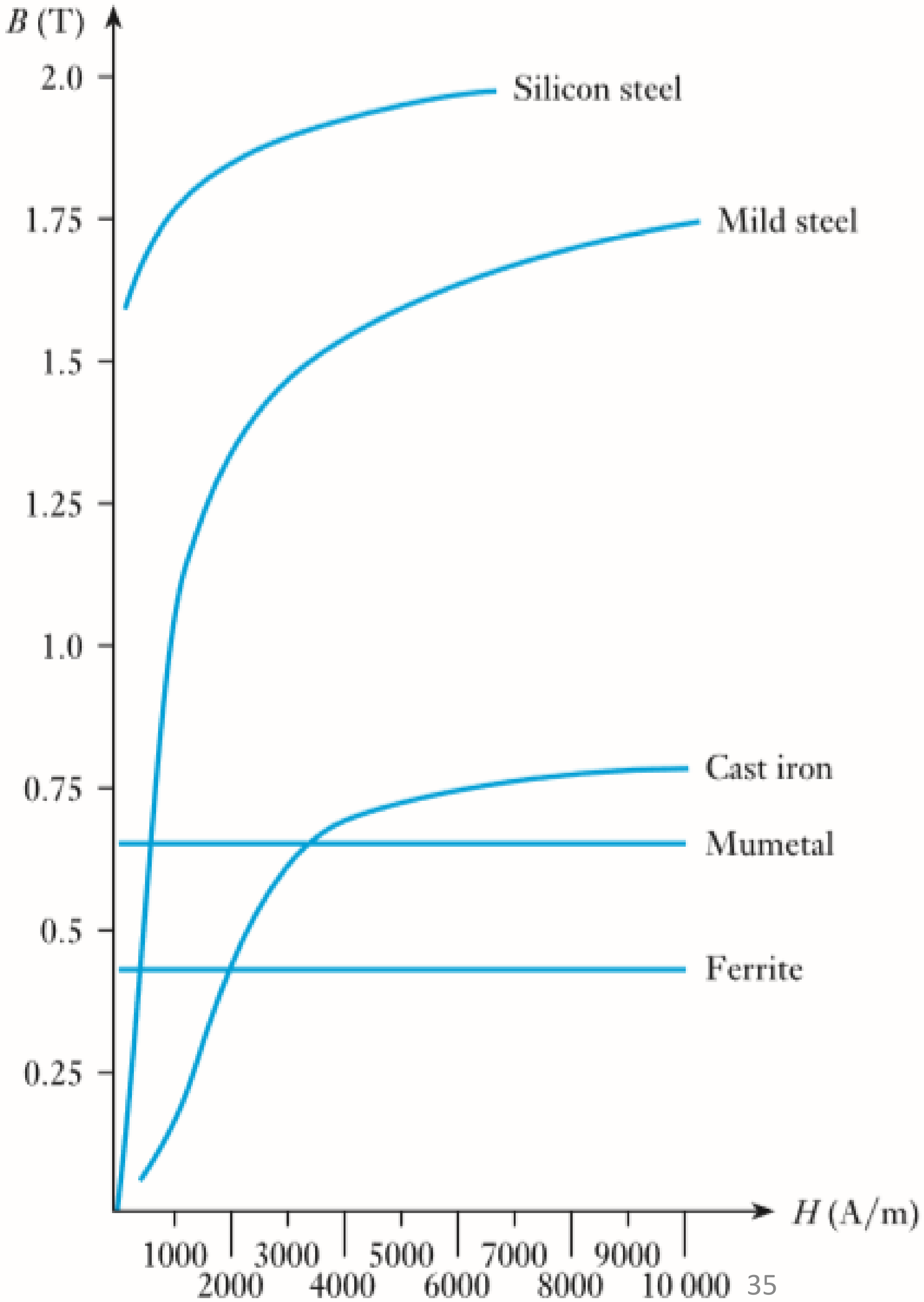
- If $\boldsymbol{A}$ is the **cross-sectional area** of the ring,

Flux density $\boxed{B = \dfrac{\phi}{A} = \dfrac{c\theta}{2AN_s}}$

Experimental Determination of *B-H* Curve

The test is performed with different values of the current I (hence different values of applied field strength H); and the corresponding values of flux density B are determined (by measuring the fluxmeter deflection θ).

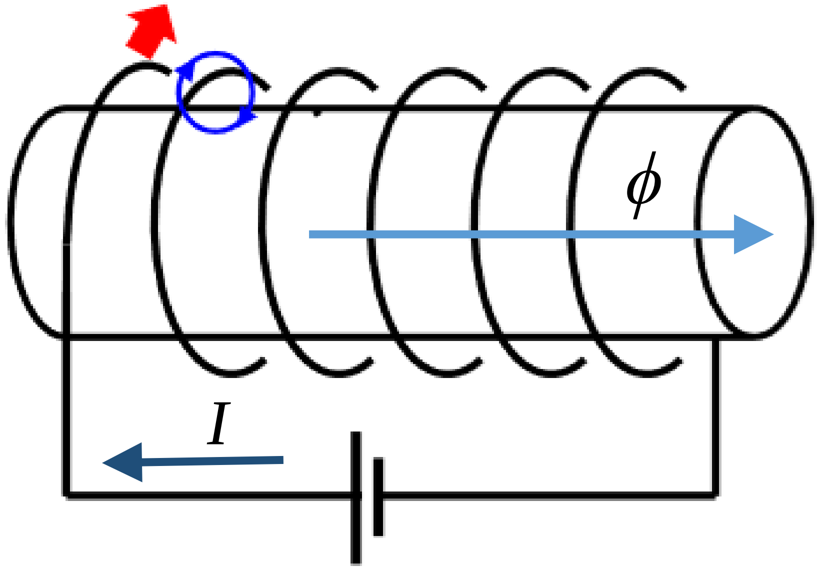
The B-H data when plotted yield graphs like the ones shown.

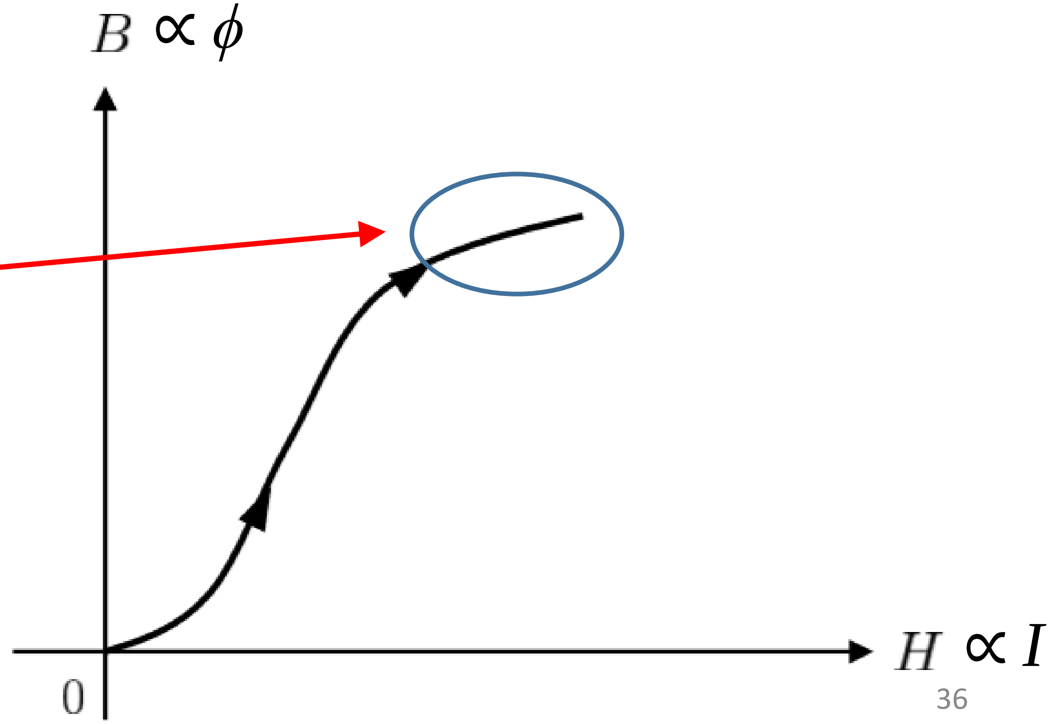


Magnetization and Hysteresis

- A demagnetized magnetic material has $B = 0$ when $H = 0$. As the applied magnetic field strength H is increased, the typical B-H relationship observed is of the type shown below.

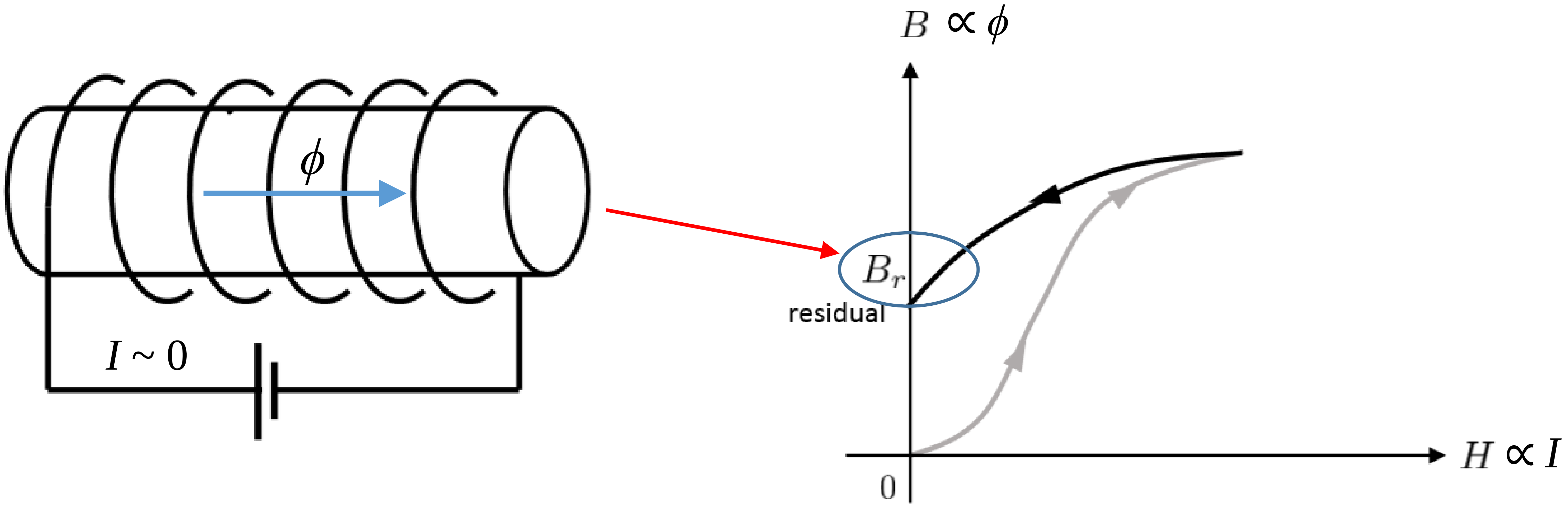
- For **large** values of $\boldsymbol{H}$, the induced flux density $\boldsymbol{B}$ in the **material** starts **saturating**.





Magnetization and Hysteresis

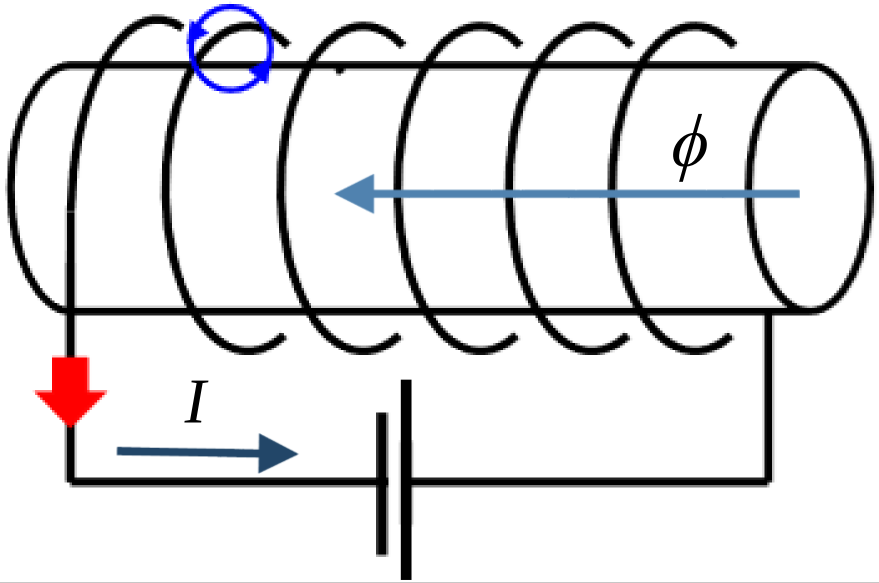
➢ Now if H is reduced back to 0 (by ramping the current back to zero), the induced flux density B does not follow the same path. There is **residual magnetism (B_r)** leftover in the material even at $\boldsymbol{H}$ **= 0**.

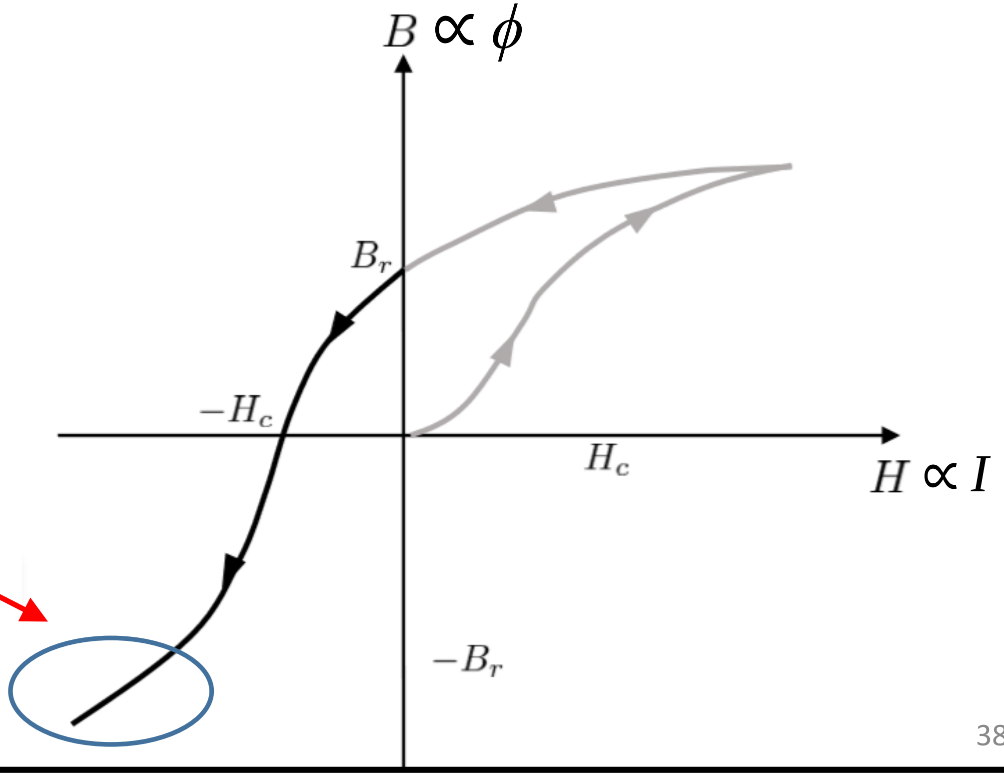


Magnetization and Hysteresis

- As H is further reduced to negative values (by reversing the direction of applied current), ***B* returns to zero** only when $\boldsymbol{H = -H_c}$. The value of $\boldsymbol{H_c}$ is called "**coercive force/field**".

- As $\boldsymbol{H}$ is made even **more negative**, the flux density $\boldsymbol{B}$ in the material **saturates** in the **negative** direction.

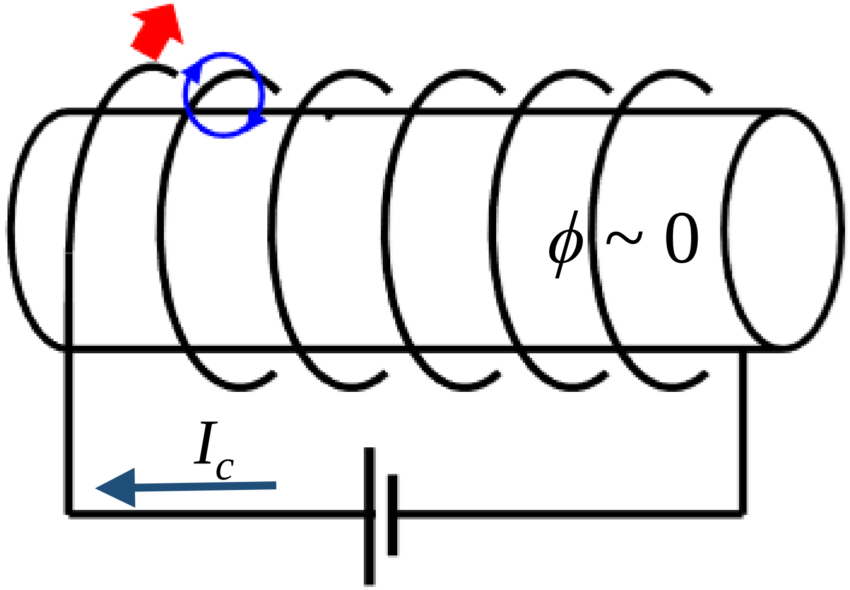


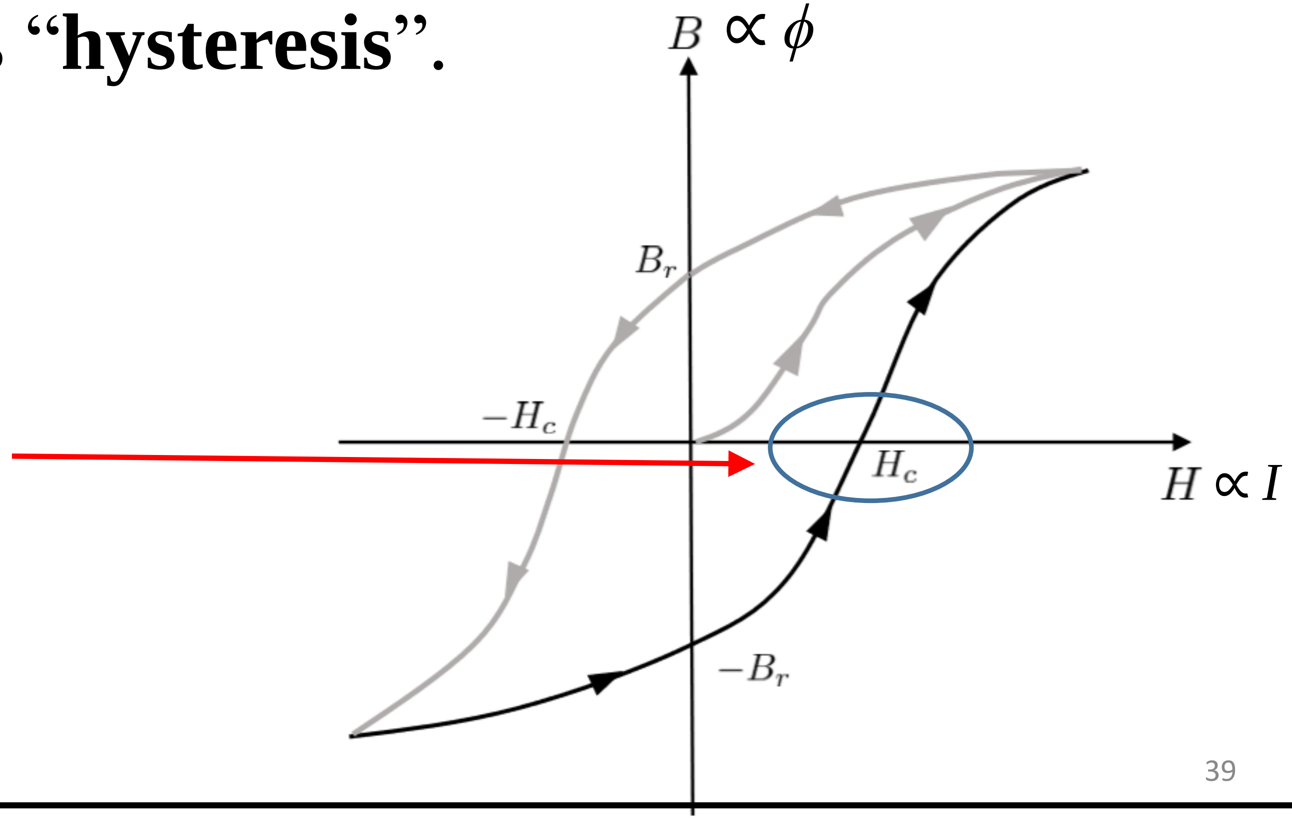


Magnetization and Hysteresis

Now as $\boldsymbol{H}$ is **increased** again **towards positive** values, a **residual magnetism** $\boldsymbol{-B_r}$ remains at $\boldsymbol{H = 0}$. Then $\boldsymbol{B = 0}$ is again reached at $\boldsymbol{H = +H_c}$, and finally positive saturation occurs, but arrived via a **different path**.

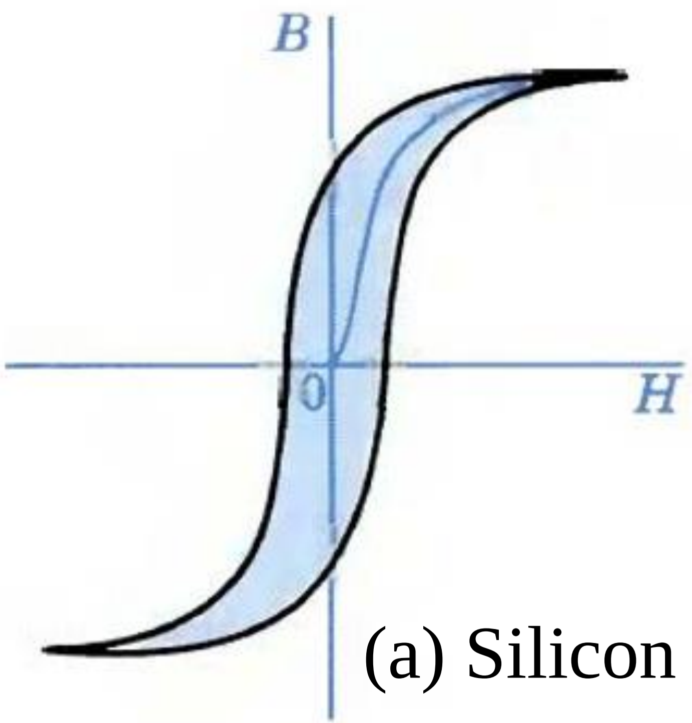
This phenomenon of **non-conformity** (i.e. non-overlapping) of '**increase**' and '**decrease**' curves, is called as "**hysteresis**".



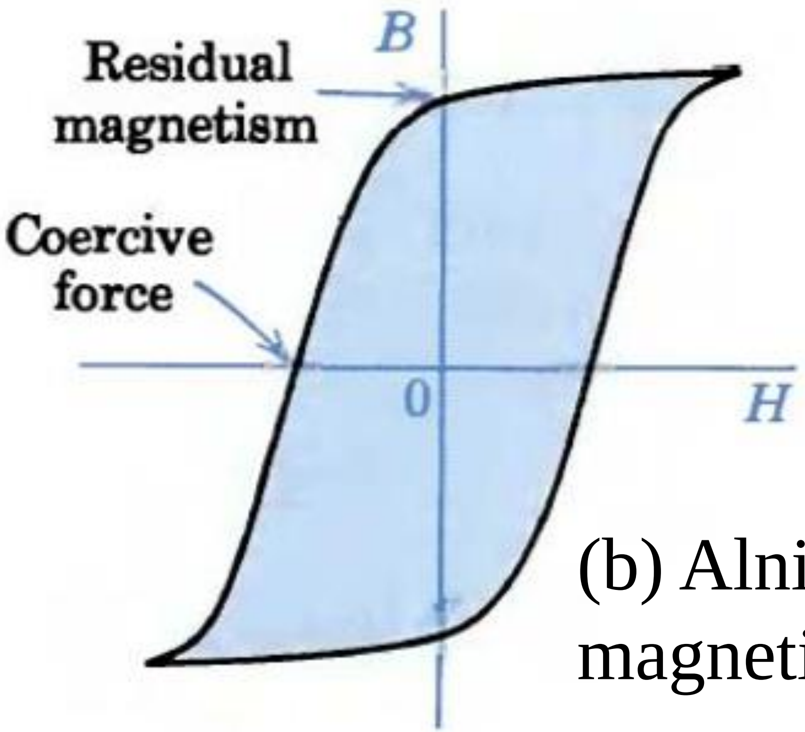


Hysteresis Loop

- Thus, when a magnetic material is magnetized (flux induced), the original state is not returned to when applied magnetizing force/field is removed.
- If the magnetizing force is due to an applied AC current, a hysteresis loop results in every cycle/time period.



(a) Silicon steel



(b) Alnico (ferro-magnetic alloy)

Hysteresis Loss

- The **hysteresis** caused by **cyclic magnetization-demagnetization** leads to some magnetizing **energy** to be **lost**.
- The **area** of the ***B-H* loop** gives the amount of **power lost** as heat in the cyclic magnetization-demagnetization process (less area => less loss)
- Empirical formula for power loss due to hysteresis: $\boxed{P_h = K_h f B_m^n}$

The constants $\boldsymbol{K_h}$ and $\boldsymbol{n}$ **depend** on core **material**.

$\boldsymbol{f}$ is the **frequency** of **AC** magnetizing current.

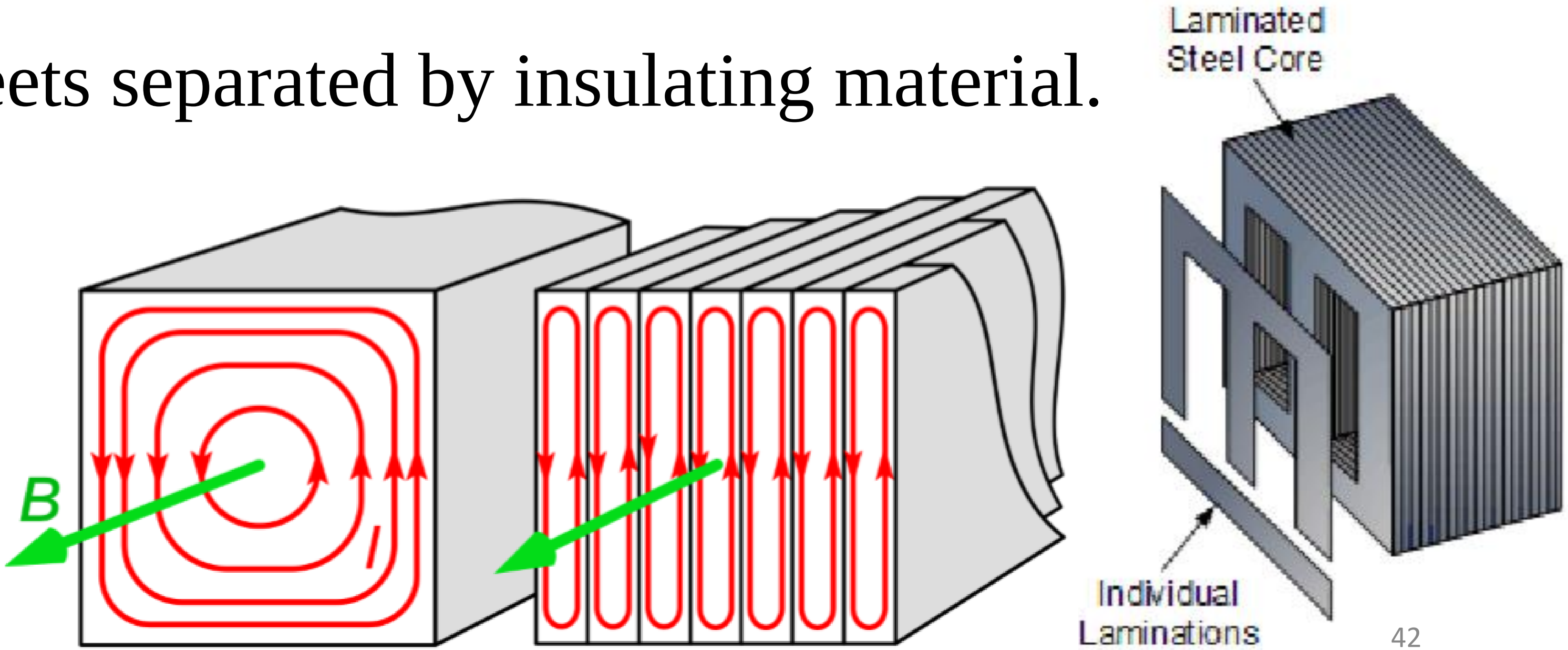
Typically $\boldsymbol{n}$ **~ 1.6 - 2** (called the "**Steinmetz Exponent**")

$\boldsymbol{B_m}$ is the **maximum**/saturated induced **flux density** per cycle

Eddy Current Loss

- If the magnetic core is solid, there can also be Eddy current loss.
- Localized Eddy currents excited by AC magnetic flux induced voltages (Lenz's law) result in I^2R losses.
- Loss can be reduced by using laminated core designs.
- Core in the form of sheets separated by insulating material.
- Loss: $\boxed{P_e = K_e\,(fB_m)^2}$

(K_e lower for laminated as compared to bulk material)



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- Adrian Waygood, "An Introduction to Electrical Science," Abingdon, Oxon ; New York, NY : Routledge, 2018.