Geometry

Common Core

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PEARSON

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Acknowledgments appear on page 1039, which constitute an extension of this copyright page.

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ISBN-13: 978-0-13-328115-6
ISBN-10: 0-13-328115-9
4 5 6 7 8 9 10 V057 18 17 16 15 14

Contents *in Brief*

Welcome to ***Pearson Geometry Common Core Edition*** student book. Throughout this textbook, you will find content that has been developed to cover many of the Standards for Mathematical Content and all of the Standards for Mathematical Practice from the Common Core State Standards for Mathematics. The End-of-Course Assessment provides students with practice with all of the Standards for Mathematical Content listed on pages xx to xxiii.

Series *Authors*

Randall I. Charles, Ph.D., is Professor Emeritus in the Department of Mathematics and Computer Science at San Jose State University, San Jose, California. He began his career as a high school mathematics teacher, and he was a mathematics supervisor for five years. Dr. Charles has been a member of several NCTM committees and is the former Vice President of the National Council of Supervisors of Mathematics. Much of his writing and research has been in the area of problem solving. He has authored more than 90 mathematics textbooks for kindergarten through college.

Dan Kennedy, Ph.D., is a classroom teacher and the Lupton Distinguished Professor of Mathematics at the Baylor School in Chattanooga, Tennessee. A frequent speaker at professional meetings on the subject of mathematics education reform, Dr. Kennedy has conducted more than 50 workshops and institutes for high school teachers. He is coauthor of textbooks in calculus and precalculus, and from 1990 to 1994, he chaired the College Board's AP Calculus Development Committee. He is a 1992 Tandy Technology Scholar and a 1995 Presidential Award winner.

Basia Hall currently serves as Manager of Instructional Programs for the Houston Independent School District. With 33 years of teaching experience, Ms. Hall has served as a department chair, instructional supervisor, school improvement facilitator, and professional development trainer. She has developed curricula for Algebra 1, Geometry, and Algebra 2 and co-developed the Texas state mathematics standards. A 1992 Presidential Awardee, Ms. Hall is past president of the Texas Association of Supervisors of Mathematics and is a state representative for the National Council of Supervisors of Mathematics (NCSM).

Consulting *Authors*

Stuart J. Murphy is a visual learning author and consultant. He is a champion of developing visual learning skills and using related strategies to help children become more successful students. He is the author of *MathStart*, a series of children's books that presents mathematical concepts in the context of stories, and *I See I Learn*, a Pre-Kindergarten and Kindergarten learning initiative that focuses on social and emotional skills. A graduate of the Rhode Island School of Design, he has worked extensively in educational publishing and has been on the authorship teams of a number of elementary and high school mathematics programs. He is a frequent presenter at meetings of the National Council of Teachers of Mathematics, the International Reading Association, and other professional organizations.

Grant Wiggins, Ed.D., is the President of Authentic Education in Hopewell, New Jersey. He earned his B.A. from St. John's College in Annapolis and his Ed.D. from Harvard University. Dr. Wiggins consults with schools, districts, and state education departments on a variety of reform matters; organizes conferences and workshops; and develops print materials and Web resources on curricular change. He is perhaps best known for being the coauthor, with Jay McTighe, of *Understanding by Design*[1] and *The Understanding by Design Handbook*, the award-winning and highly successful materials on curriculum published by ASCD. His work has been supported by the Pew Charitable Trusts, the Geraldine R. Dodge Foundation, and the National Science Foundation.

[1] ASCD, publisher of "The Understanding by Design Handbook" coauthored by Grant Wiggins and registered owner of the trademark "Understanding by Design," has not authorized or sponsored this work and is in no way affiliated with Pearson or its products.

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Geometry

Laurie E. Bass is a classroom teacher at the 9–12 division of the Ethical Culture Fieldston School in Riverdale, New York. A classroom teacher for more than 30 years, Ms. Bass has a wide base of teaching experience, ranging from Grade 6 through Advanced Placement Calculus. She was the recipient of a 2000 Honorable Mention for the Radio Shack National Teacher Awards. She has been a contributing writer for a number of publications, including software-based activities for the Algebra 1 classroom. Among her areas of special interest are cooperative learning for high school students and geometry exploration on the computer. Ms. Bass is a frequent presenter at local, regional, and national conferences.

Art Johnson, Ed.D., is a professor of mathematics education at Boston University. He is a mathematics educator with 32 years of public school teaching experience, a frequent speaker and workshop leader, and the recipient of a number of awards: the Tandy Prize for Teaching Excellence, the Presidential Award for Excellence in Mathematics Teaching, and New Hampshire Teacher of the Year. He was also profiled by the Disney Corporation in the American Teacher of the Year Program. Dr. Johnson has contributed 18 articles to NCTM journals and has authored over 50 books on various aspects of mathematics.

Algebra 1 and Algebra 2

Allan E. Bellman, Ph.D., is an Associate Professor of Mathematics Education at the University of Mississippi. He previously taught at the University of California, Davis for 12 years and in public school in Montgomery County, Maryland for 31. He has been an instructor for both the Woodrow Wilson National Fellowship Foundation and the Texas Instruments' T^3 program. Dr. Bellman has expertise in the use of technology in education and assessment-driven instruction, and speaks frequently on these topics. He was a 1992 Tandy Technology Scholar and has twice been listed in Who's Who Among America's Teachers.

Sadie Chavis Bragg, Ed.D., is Senior Vice President of Academic Affairs and professor of mathematics at the Borough of Manhattan Community College of the City University of New York. She is a past president of the American Mathematical Association of Two-Year Colleges (AMATYC). In recognition of her service to the field of mathematics locally, statewide, nationally, and internationally, she was awarded AMATYC's most prestigious award, the Mathematics Excellence Award for 2010. Dr. Bragg has coauthored more than 50 mathematics textbooks for kindergarten through college.

William G. Handlin, Sr., is a classroom teacher and Department Chair of Mathematics and former Department Chair of Technology Applications at Spring Woods High School in Houston, Texas. Awarded Life Membership in the Texas Congress of Parents and Teachers for his contributions to the well-being of children, Mr. Handlin is also a frequent workshop and seminar leader in professional meetings.

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From the **Authors**

Welcome

Math is a powerful tool with far-reaching applications throughout your life. We have designed a unique and engaging program that will enable you to tap into the power of mathematics and mathematical reasoning. This award-winning program has been developed to align fully to the Common Core State Standards.

Developing mathematical understanding and problem-solving abilities is an ongoing process—a journey both inside and outside the classroom. This course is designed to help you make sense of the mathematics you encounter in and out of class each day and to help you develop mathematical proficiency.

You will learn important mathematical principles. You will also learn how the principles are connected to one another and to what you already know. You will learn to solve problems and learn the reasoning that lies behind your solutions. You will also develop the key mathematical practices of the Common Core State Standards.

Each chapter begins with the "big ideas" of the chapter and some essential questions that you will learn to answer. Through this question-and-answer process you will develop your ability to analyze problems independently and solve them in different applications.

Your skills and confidence will increase through practice and review. Work the problems so you understand the concepts and methods presented and the thinking behind them. Then do the exercises. Ask yourself how new concepts relate to old ones. Make the connections!

Everyone needs help sometimes. You will find that this program has built-in opportunities, both in this text and online, to get help whenever you need it.

This course will also help you succeed on the tests you take in class and on other tests like the SAT, ACT, and state exams. The practice exercises in each lesson will prepare you for the format and content of such tests. No surprises!

The problem-solving and reasoning habits and skills you develop in this program will serve you in all your studies and in your daily life. They will prepare you for future success not only as a student, but also as a member of a changing technological society.

Best wishes,

Randy Charles

Allan E. Bellman

Sadie C. Bragg

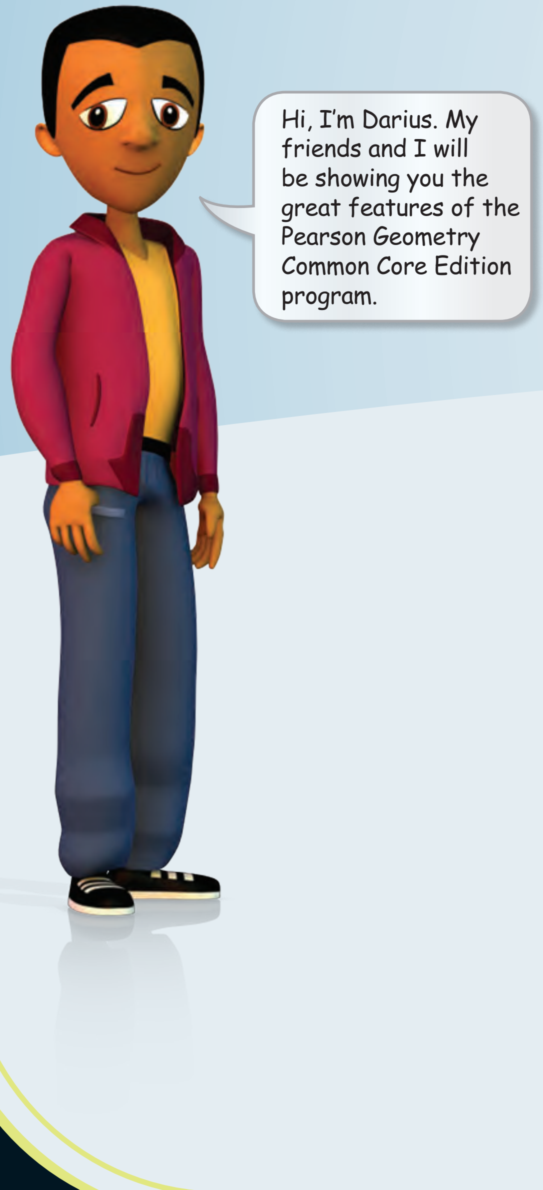
William G. Handlin

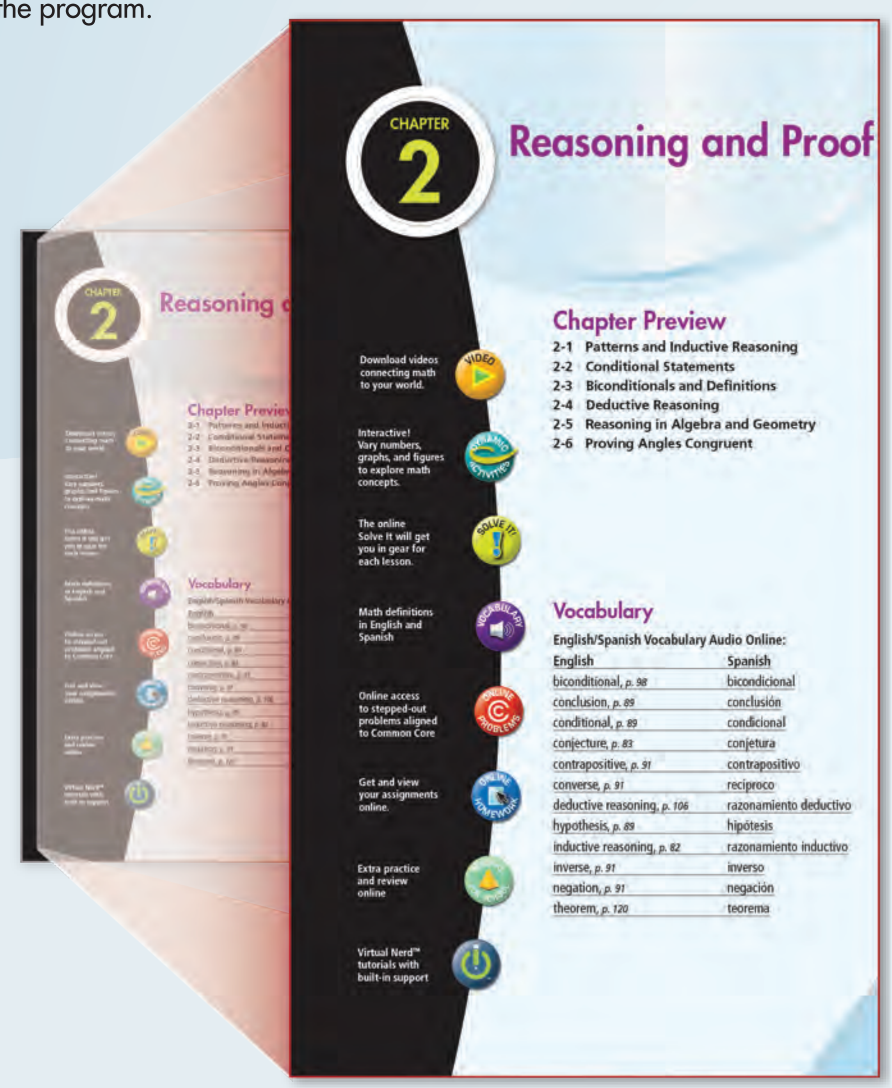
Laurie E. Bass

Stuart J. Murphy

Power*Geometry.com*

Welcome to Geometry. *Pearson Geometry Common Core Edition* is part of a blended digital and print environment for the study of high school mathematics. Take some time to look through the features of our mathematics program, starting with **PowerGeometry.com,** the site of the digital features of the program.




CHAPTER 2

Reasoning and Proof

Download videos connecting math to your world. VIDEO

Interactive! Vary numbers, graphs, and figures to explore math concepts. DYNAMIC ACTIVITIES

The online Solve It will get you in gear for each lesson. SOLVE IT!

Math definitions in English and Spanish. VOCABULARY

Online access to stepped-out problems aligned to Common Core. ONLINE PROBLEMS

Get and view your assignments online. ONLINE HOMEWORK

Extra practice and review online

Virtual Nerd™ tutorials with built-in support

Chapter Preview

2-1 Patterns and Inductive Reasoning
2-2 Conditional Statements
2-3 Biconditionals and Definitions
2-4 Deductive Reasoning
2-5 Reasoning in Algebra and Geometry
2-6 Proving Angles Congruent

Vocabulary

English/Spanish Vocabulary Audio Online:

English	Spanish
biconditional, *p. 98*	bicondicional
conclusion, *p. 89*	conclusión
conditional, *p. 89*	condicional
conjecture, *p. 83*	conjetura
contrapositive, *p. 91*	contrapositivo
converse, *p. 91*	recíproco
deductive reasoning, *p. 106*	razonamiento deductivo
hypothesis, *p. 89*	hipótesis
inductive reasoning, *p. 82*	razonamiento inductivo
inverse, *p. 91*	inverso
negation, *p. 91*	negación
theorem, *p. 120*	teorema

On each **chapter opener,** you will be find a listing of the online features of the program. Look for these buttons throughout the lessons.

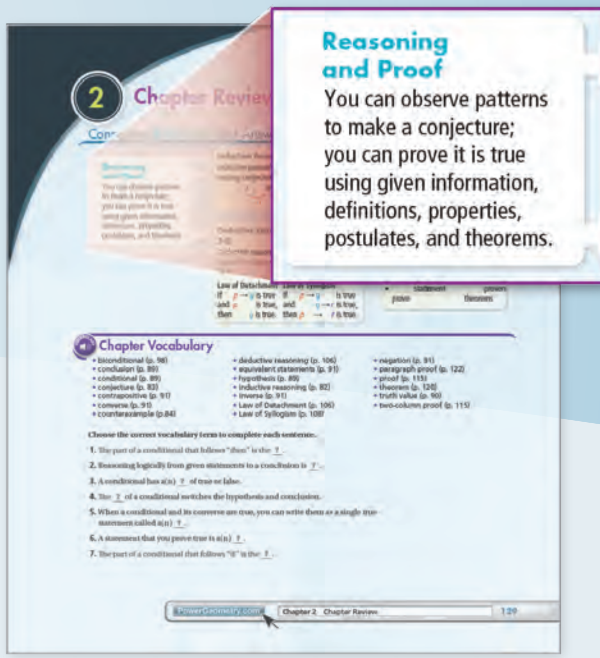
Big *Ideas*

We start with **Big Ideas.** Each chapter is organized around Big Ideas that convey the key mathematics concepts you will be studying in the program. Take a look at the Big Ideas on pages xxiv and xxv.

Common Core State Standards

The Common Core State Standards have a similar organizing structure. They begin with **Conceptual Categories,** such as Algebra or Functions. Within each category are **domains** and **clusters.**

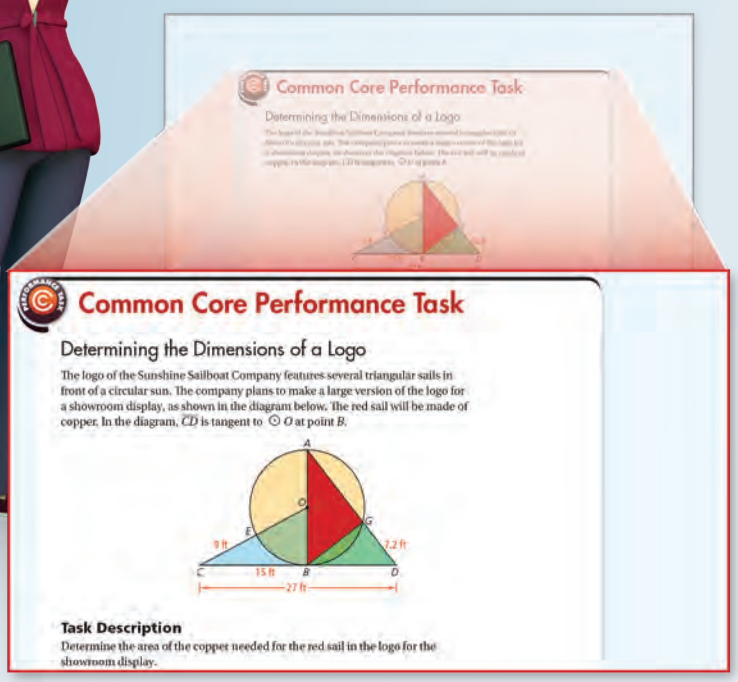
The **Big Ideas** are organizing ideas for all of the lessons in the program. At the beginning of each chapter, we'll tell you which Big Ideas you'll be studying. We'll also present an **Essential Question** for each Big Idea.



In the **Chapter Review** at the end of the chapter, you'll find an answer to the Essential Question for each Big Idea. We'll also remind you of the lesson(s) where you studied the concepts that support the Big Ideas.

Exploring *Concepts*

The lessons offer many opportunities to explore concepts in different contexts and through different media.



Common Core Performance Task

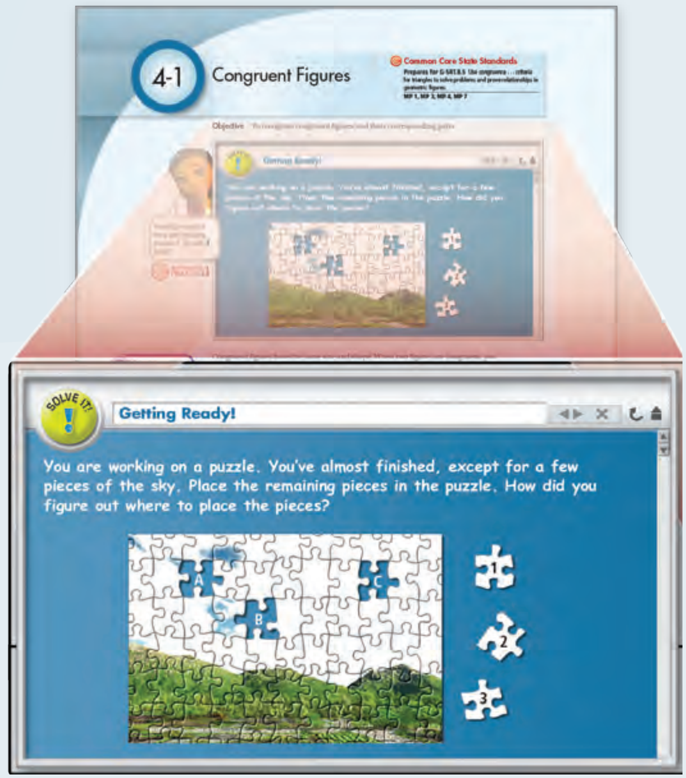
Determining the Dimensions of a Logo

The logo of the Sunshine Sailboat Company features several triangular sails in front of a circular sun. The company plans to make a large version of the logo for a showroom display, as shown in the diagram below. The red sail will be made of copper. In the diagram, $\overline{CD}$ is tangent to $\odot O$ at point B.

Task Description

Determine the area of the copper needed for the red sail in the logo for the showroom display.

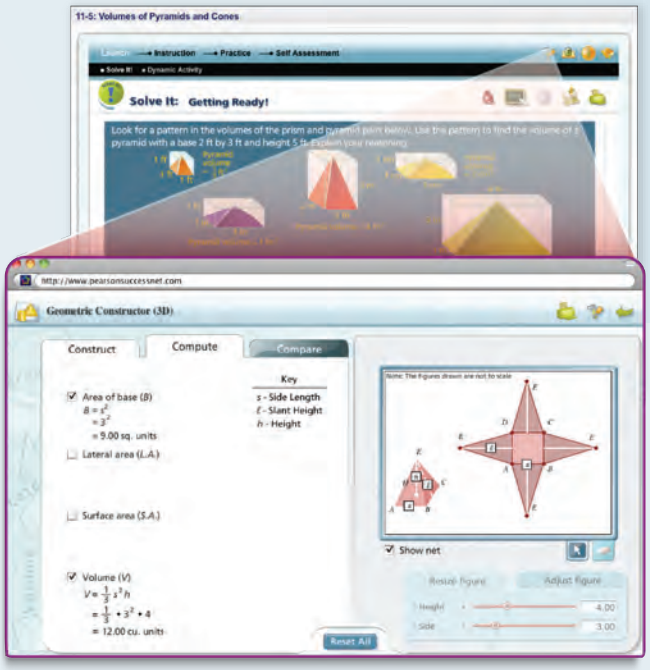
For each chapter, there is **Common Core Performance Task** that you will work on throughout the chapter. See pages xii and xiii for more information.



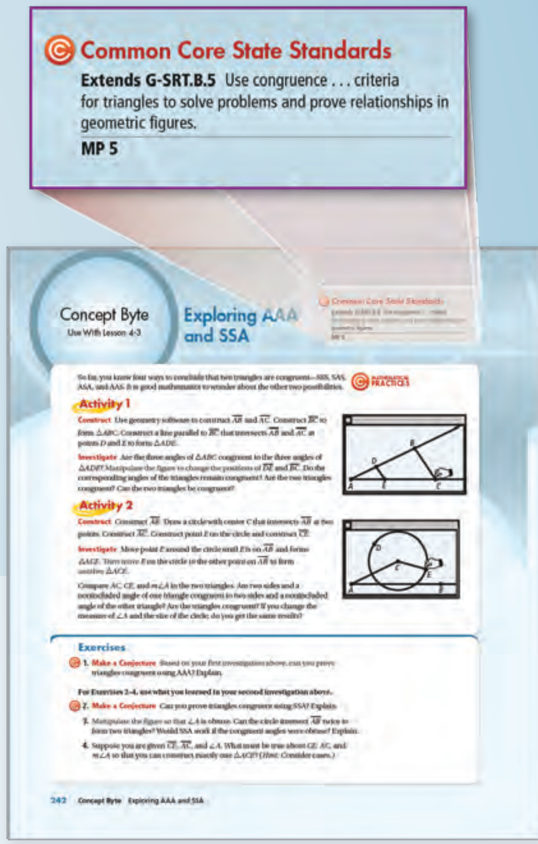
Here's another cool feature. Each lesson opens with a **Solve It,** a problem that helps you connect what you know to an important concept in the lesson. Do you notice how the Solve It frame looks like it comes from a computer? That's because all of the Solve Its can be found at **PowerGeometry.com.**

Developing Mathematical Proficiency

The **Standards for Mathematical Practice** describe processes, practices, and habits of mind of mathematically proficient students. Many of the features in *Pearson Geometry Common Core Edition* help you become more proficient in math.



Want to do some more exploring? Try the **Math Tools** at **PowerGeometry.com.** Click on this icon to access these tools: Graphing Utility, Number Line, Algebra Tiles, and 2D and 3D Geometric Constructor. With the Math Tools, you can continue to explore the concepts presented in the lesson.

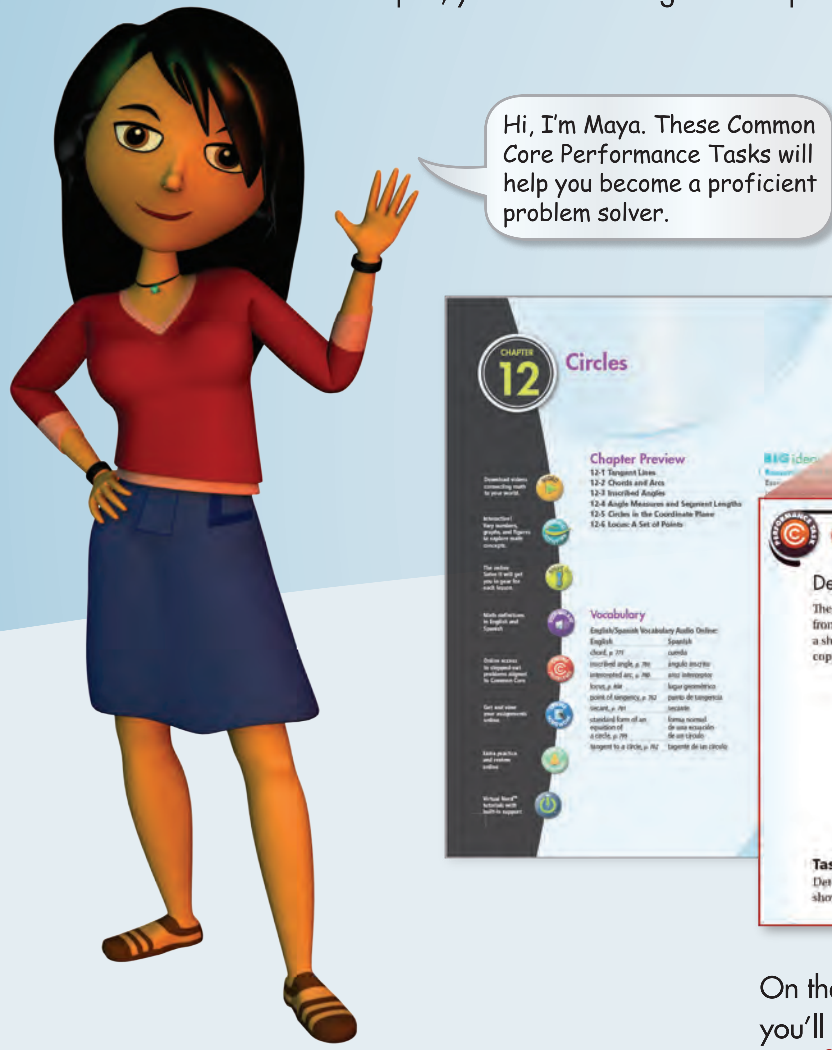


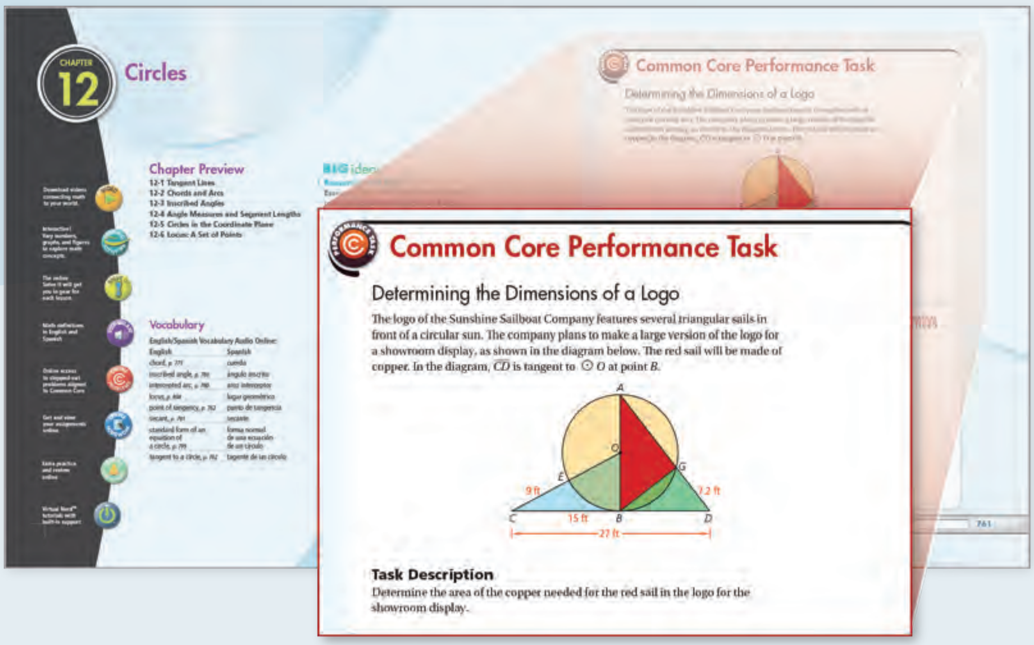
Try a **Concept Byte!** In a Concept Byte, you might explore technology, do a hands-on activity, or try a challenging extension.

The text in the top right corner of the first page of a lesson or Concept Byte tells you the **Standards for Mathematical Content** and the **Standards for Mathematical Practice** that you will be studying.

Solving *Problems*

Pearson Geometry Common Core Edition includes many opportunities to build on and strengthen your problem-solving abilities. In each chapter, you'll work through a multi-part Performance Task.

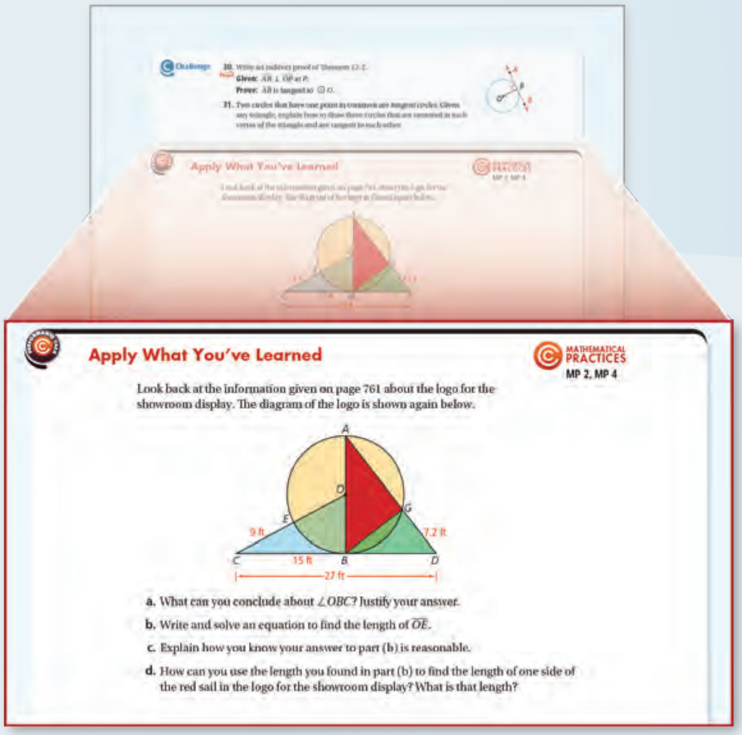




On the **Chapter Opener,** you'll be introduced to the chapter **Performance Task.** You'll start to make sense of the problem and think about solution plans.

Developing Proficiency with Problem Solving

Proficient Problem Solvers make sense of problem situations, develop workable solution plans, model the problem situation with mathematics, and communicate their thinking clearly.



Apply What You've Learned

MATHEMATICAL PRACTICES
MP 2, MP 4

Look back at the information given on page 761 about the logo for the showroom display. The diagram of the logo is shown again below.

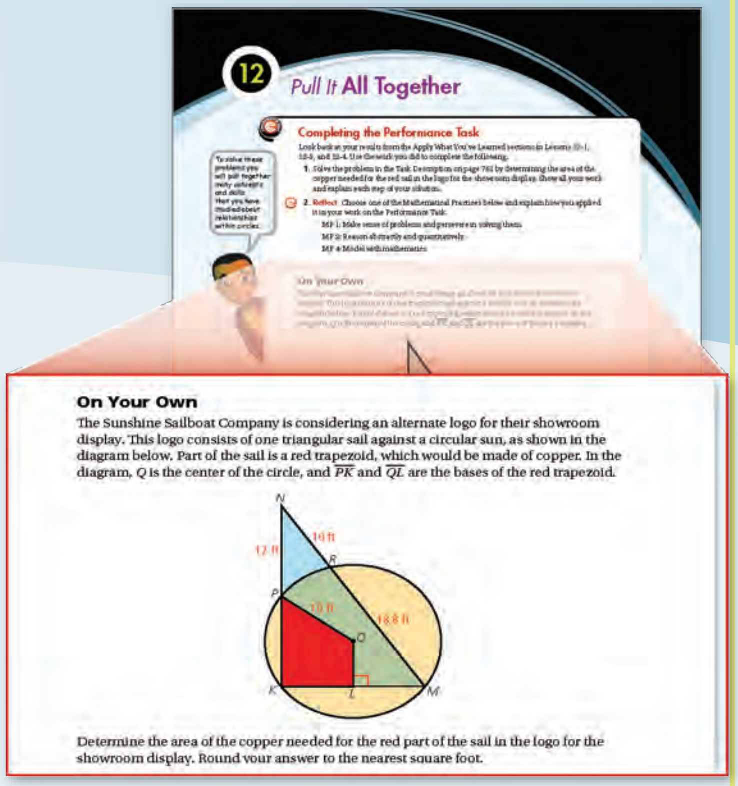
a. What can you conclude about $\angle OBC$? Justify your answer.

b. Write and solve an equation to find the length of $\overline{OE}$.

c. Explain how you know your answer to part (b) is reasonable.

d. How can you use the length you found in part (b) to find the length of one side of the red sail in the logo for the showroom display? What is that length?

Throughout the chapter, you will **Apply What You've Learned** to solve problems that relate to the Performance Task. You'll be asked to reason quantitatively and model with mathematics.



On Your Own

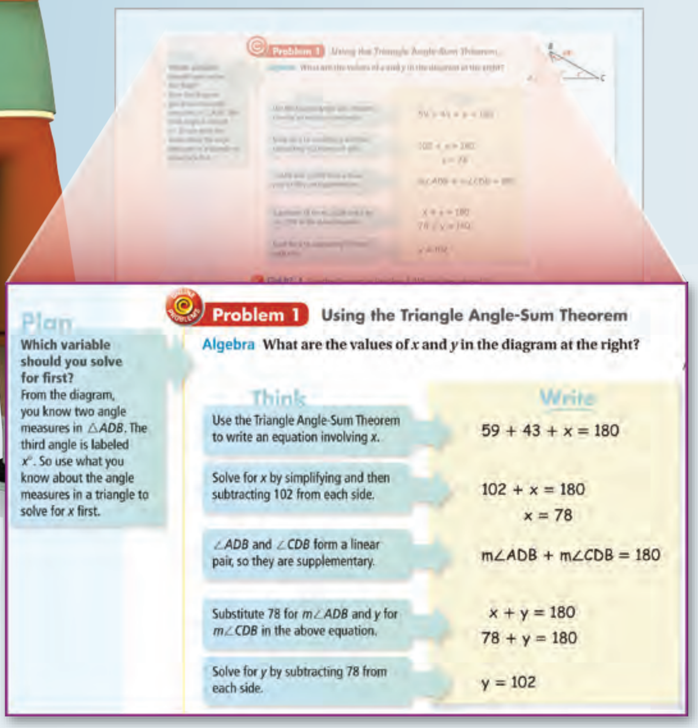
The Sunshine Sailboat Company is considering an alternate logo for their showroom display. This logo consists of one triangular sail against a circular sun, as shown in the diagram below. Part of the sail is a red trapezoid, which would be made of copper. In the diagram, Q is the center of the circle, and $\overline{PK}$ and $\overline{QL}$ are the bases of the red trapezoid.

Determine the area of the copper needed for the red part of the sail in the logo for the showroom display. Round your answer to the nearest square foot.

In the **Pull It All Together** at the end of the chapter, you will use the concepts and skills presented throughout the chapter to solve the Performance Task. Then you'll have another Task to solve **On Your Own.**

Thinking *Mathematically*

Mathematical reasoning is the key to making sense of math and solving problems. Throughout the program you'll learn strategies to develop mathematical reasoning habits.



Problem 1 Using the Triangle Angle-Sum Theorem

Algebra What are the values of x and y in the diagram at the right?

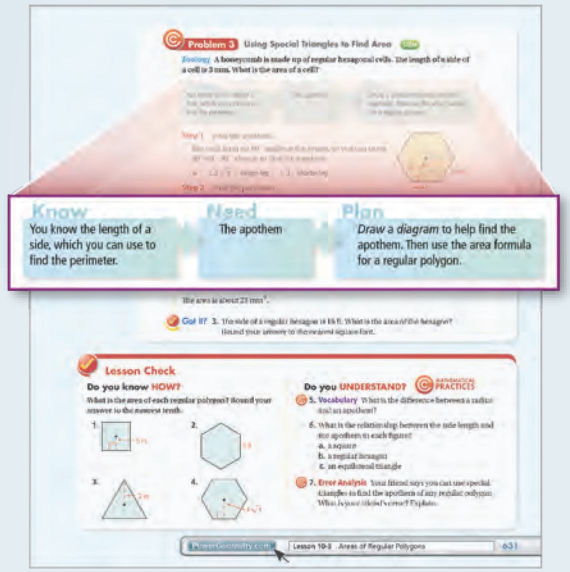
Plan

Which variable should you solve for first?
From the diagram, you know two angle measures in $\triangle ADB$. The third angle is labeled $x°$. So use what you know about the angle measures in a triangle to solve for x first.

Think	Write
Use the Triangle Angle-Sum Theorem to write an equation involving x.	$59 + 43 + x = 180$
Solve for x by simplifying and then subtracting 102 from each side.	$102 + x = 180$ $x = 78$
$\angle ADB$ and $\angle CDB$ form a linear pair, so they are supplementary.	$m\angle ADB + m\angle CDB = 180$
Substitute 78 for $m\angle ADB$ and y for $m\angle CDB$ in the above equation.	$x + y = 180$ $78 + y = 180$
Solve for y by subtracting 78 from each side.	$y = 102$

The worked-out problems include call-outs that reveal the strategies and reasoning behind the solution. Look for the boxes labeled **Plan** and **Think.**

The **Think-Write** problems model the thinking behind each step of a solution.

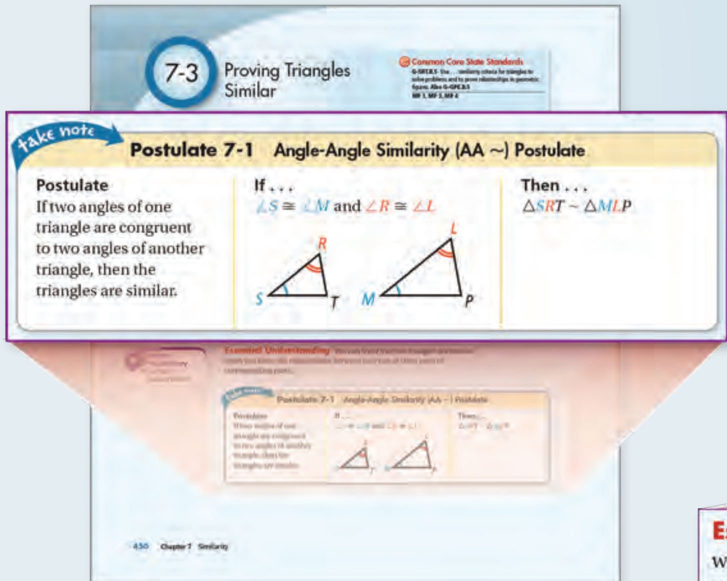


Know	Need	Plan
You know the length of a side, which you can use to find the perimeter.	The apothem	*Draw a diagram* to help find the apothem. Then use the area formula for a regular polygon.

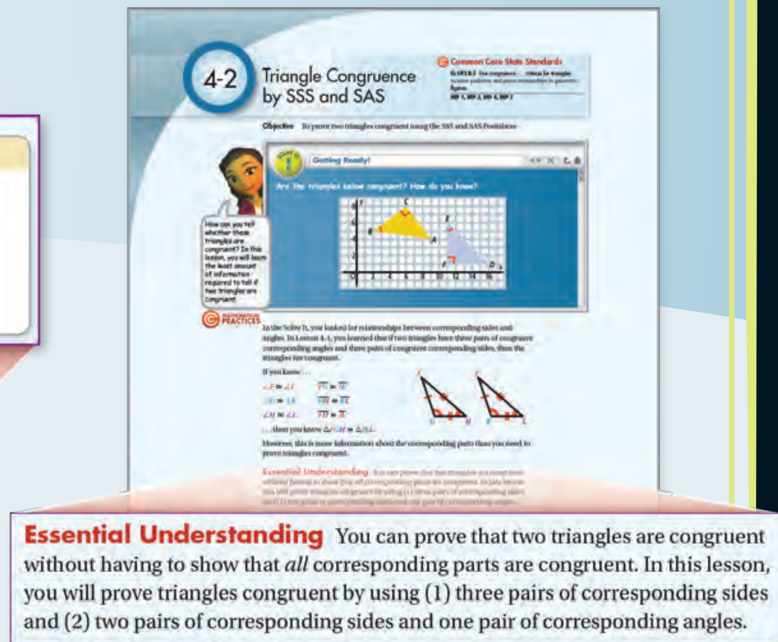
Other worked-out problems model a problem-solving plan that includes the steps of stating what you **Know,** identifying what you **Need,** and developing a **Plan.**

Standards for Mathematical Practice

The **Standards for Mathematical Practice** emphasize sense-making, reasoning, and critical reasoning. Many features in *Pearson Geometry Common Core Edition* provide opportunities for you to develop these skills and dispositions.



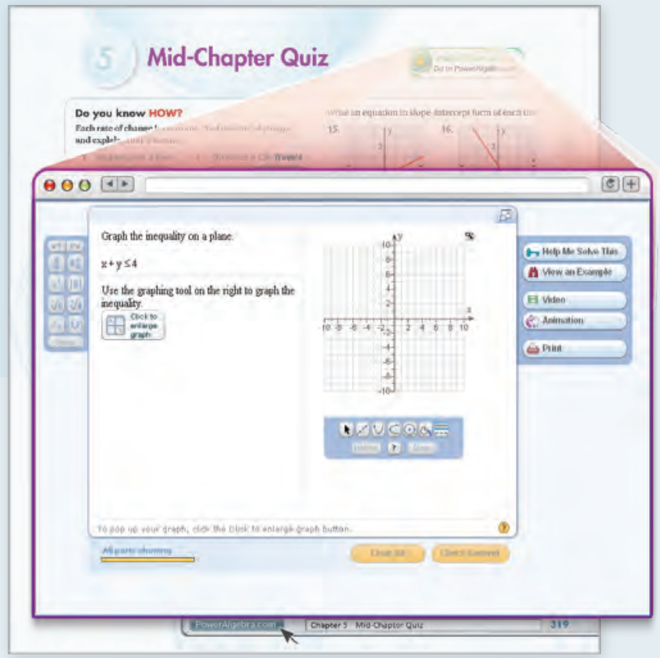
A **Take Note** box highlights important concepts in a lesson. You can use these boxes to review concepts throughout the year.



Part of thinking mathematically is making sense of the concepts that are being presented. The **Essential Understandings** help you build a framework for the Big Ideas.

Practice *Makes Perfect*

Ask any professional and you'll be told that the one requirement for becoming an expert is practice, practice, practice. *Pearson Geometry Common Core Edition* offers rich and varied exercises to help you become proficient with the mathematics.



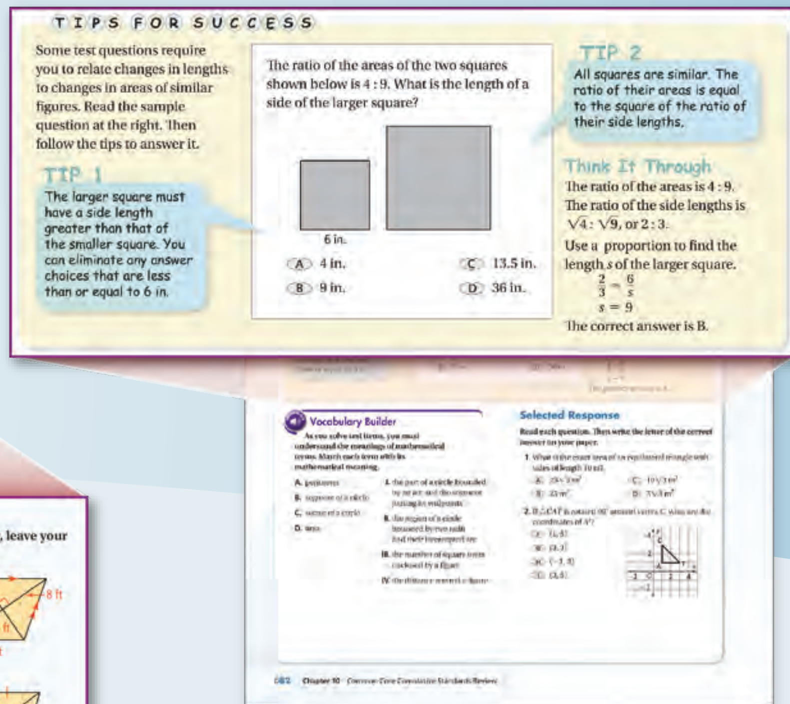
Want more practice? Look for this icon in your book. Check out all of the opportunities in **MathXL® for School.** Your teacher can assign you some practice exercises or you can choose some on your own. And you'll know right away if you got the right answer!

Acing *the Test*

Doing well on tests, whether they are chapter tests or state assessments, depends on a deep understanding of math concepts, fluency with calculations and computations, and strong problem-solving abilities.

Assessing the Common Core State Standards

All of these opportunities for practice help you prepare for assessments throughout the year, including the assessments to measure your proficiency with the Common Core State Standards.



TIPS FOR SUCCESS

Some test questions require you to relate changes in lengths to changes in areas of similar figures. Read the sample question at the right. Then follow the tips to answer it.

The ratio of the areas of the two squares shown below is 4 : 9. What is the length of a side of the larger square?

6 in.

A 4 in.
B 9 in.
C 13.5 in.
D 36 in.

TIP 1

The larger square must have a side length greater than that of the smaller square. You can eliminate any answer choices that are less than or equal to 6 in.

TIP 2

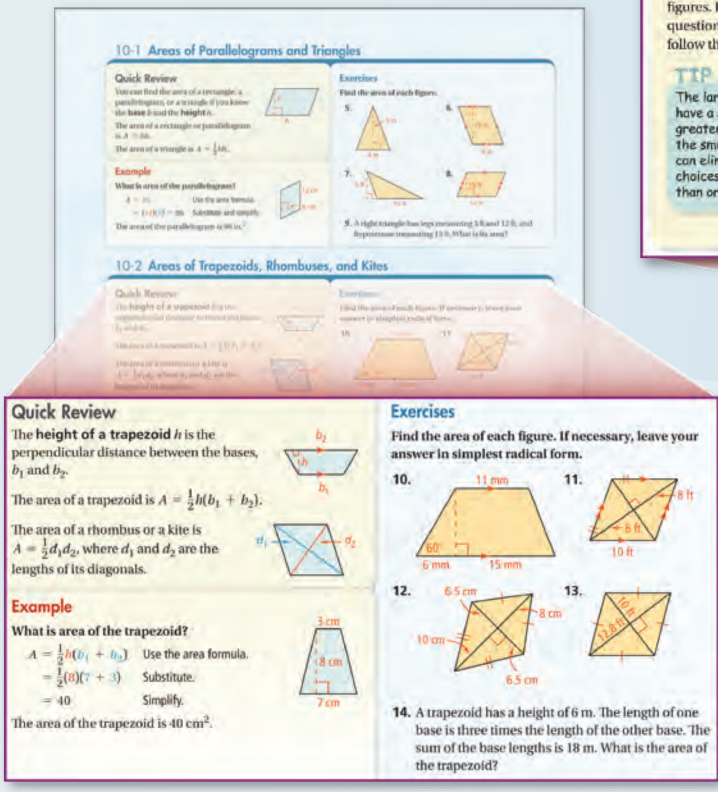
All squares are similar. The ratio of their areas is equal to the square of the ratio of their side lengths.

Think It Through

The ratio of the areas is 4 : 9.
The ratio of the side lengths is $\sqrt{4} : \sqrt{9}$, or 2 : 3.
Use a proportion to find the length s of the larger square.

$$\frac{2}{3} = \frac{6}{s}$$
$$s = 9$$

The correct answer is B.



10-1 Areas of Parallelograms and Triangles

10-2 Areas of Trapezoids, Rhombuses, and Kites

Quick Review

The **height of a trapezoid** h is the perpendicular distance between the bases, b_1 and b_2.

The area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.

The area of a rhombus or a kite is $A = \frac{1}{2}d_1d_2$, where d_1 and d_2 are the lengths of its diagonals.

Example

What is area of the trapezoid?

$A = \frac{1}{2}h(b_1 + b_2)$ Use the area formula.

$= \frac{1}{2}(8)(7 + 3)$ Substitute.

$= 40$ Simplify.

The area of the trapezoid is 40 cm^2.

Exercises

Find the area of each figure. If necessary, leave your answer in simplest radical form.

10. 11. 12. 13.

14. A trapezoid has a height of 6 m. The length of one base is three times the length of the other base. The sum of the base lengths is 18 m. What is the area of the trapezoid?

At the end of the chapter, you'll find a **Quick Review** of the concepts in the chapter and a few examples and exercises so you can check your skill at solving problems related to the concepts.

In the Cumulative Standards Review at the end of the chapter, you'll also find **Tips for Success** to strengthen your test-taking skills. We include problems of all different formats and types so you can feel comfortable with any test item on your state assessment.

Standards *for Mathematical Practice*

The **Common Core State Standards** are made of two separate, but equally important sets of standards:

- **Standards** *for Mathematical Content*
- **Standards** *for Mathematical Practice*

The **Math Content Standards** are grade-specific, while the **Math Practices Standards** are the same from Kindergarten through High School. The **Math Practices** describe qualities and habits of mind that strong mathematical thinkers exhibit.

The eight **Standards for Mathematical Practice,** numbered 1 through 8, can be put into the four groups shown on this page and the next. Included with the statement of each standard for each group is a description of what the Math Practice means for you.

Making Sense of and Solving Problems

1. Make sense of problems and persevere in solving them.

When you make sense of problems, you can explain the meaning of the problem, and you are able to find an entry point to its solution and plan a solution pathway. You can look at a problem and analyze givens, constraints, relationships, and goals. You can think of similar problems or can break the problem into easier-to-solve problems. You are able to track your progress as you work through the solution and check your answer using a different method. As you work through your solution, you frequently check whether the results you are getting make sense.

6. Attend to precision.

You attend to precision when you communicate clearly and precisely the approach you used to solve a problem, and you also understand the approaches that your classmates used. You identify the meaning of symbols that you use and, you always specify units of measure, and you include labels on the axes of graphs. Your answers are expressed with the appropriate degree of accuracy. You are able to give clear, concise definitions of math terms.

Reasoning and Communicating

2. Reason abstractly and quantitatively.

As a strong math thinker and problem solver, you are able to make sense of quantities in problem situations. You can both represent a problem situation using symbols or equations and explain what the symbols or equations represent in relationship to the problem situation. As you represent a situation symbolically or mathematically, you can explain the meaning of the quantities.

3. Construct viable arguments and critique the reasoning of others.

You are able to communicate clearly and convincingly about your solutions to problems. You can build sound mathematical arguments, drawing on definitions, assumptions, or established solutions. You can develop and explore conjectures about mathematical situations. You make use of examples and counterexamples to support your arguments and justify your conclusions. You respond clearly and logically to the positions and conclusions of your classmates, and are able to compare two arguments, identifying any flaws in logic or reasoning that the arguments may contain. You can ask useful questions to clarify or improve the argument of a classmate.

Representing and Connecting

4. Model with mathematics.

As a strong math thinker, you are able to use mathematics to represent a problem situation and can make connections between a real-world problem situation and mathematics. You see the applicability of mathematics to everyday problems. You can explain how geometry can be used to solve a carpentry problem or algebra to solve a proportional relationship problem. You can define and map relationships among quantities in a problem, using appropriate tools to do so. You are able to analyze the relationships and draw conclusions.

5. Use appropriate tools strategically.

As you develop models to match a given problem situation, you are able to strategize about which tools would be most helpful to use to solve the problem. You consider all tools, from paper and pencil to protractors and rulers, to calculators and software applications. You can articulate the appropriateness of different tools and recognize which would best serve your needs for a given problem. You are especially insightful about technology tools and use them in ways that deepen or extend your understanding of concepts. You also make use of mental tools, such as estimation, to determine the reasonableness of a solution.

Seeing Structure and Generalizing

7. Look for and make use of structure.

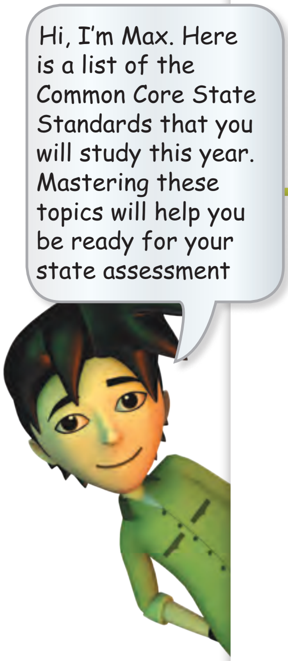
You are able to go beyond simply solving problems, to see the structure of the mathematics in these problems, and to generalize mathematical principles from this structure. You are able to see complicated expressions or equations as single objects, or a being composed of many parts.

8. Look for and express regularity in repeated reasoning.

You notice when calculations are repeated and can uncover both general methods and shortcuts for solving similar problems. You continually evaluate the reasonableness of your solutions as you solve problems arising in daily life.

Standards for *Mathematical Content*

Geometry



Number and Quantity

Quantities

Reason quantitatively and use units to solve problems

N-Q.A.1♦ Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Geometry

Congruence

Experiment with transformations in the plane

G-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions

G-CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

G-CO.C.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

G-CO.C.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

G-CO.C.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and its converse, rectangles are parallelograms with congruent diagonals.*

Make geometric constructions

G-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

G-CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

♦ These standards are not part of the PARCC Model Curriculum Framework for Geometry.

Similarity, Right Triangles, and Trigonometry

Understand similarity in terms of similarity transformations

G-SRT.A.1a Verify experimentally the properties of dilations given by a center and a scale factor: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

G-SRT.A.1b Verify experimentally the properties of dilations given by a center and a scale factor: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

G-SRT.B.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally and its converse; the Pythagorean Theorem proved using triangle similarity*.

G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles

G-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles

G-SRT.D.9◆ (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT.D.10◆ (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.D.11◆ (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles

Understand and apply theorems about circles

G-C.A.1 Prove that all circles are similar.

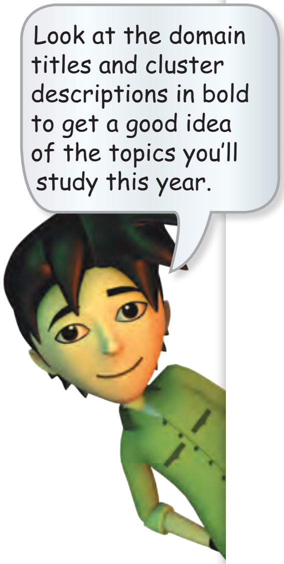
G-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle*.

G-C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G-C.A.4◆ (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles

G-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.



Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section

G-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G-GPE.A.2◆ Derive the equation of a parabola given a focus and directrix.

Use coordinates to prove simple geometric theorems algebraically

G-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically.

G-GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G-GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems

G-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three-dimensional objects

G-GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry

Apply geometric concepts in modeling situations

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

G-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Statistics and Probability

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data

S-CP.A.1◆ Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S-CP.A.2◆ Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S-CP.A.3◆ Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

S-CP.A.4◆ Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

S-CP.A.5◆ Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events in a uniform probability model

S-CP.B.6◆ Find the conditional probability of *A* given *B* as the fraction of *B's* outcomes that also belong to *A*, and interpret the answer in terms of the model.

S-CP.B.7◆ Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

S-CP.B.8◆ (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.

S-CP.B.9◆ (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions

S-MD.B.6◆ (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

S-MD.B.7◆ (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

BIGideas

These Big Ideas are the organizing ideas for the study of important areas of mathematics: algebra, geometry, and statistics.

Algebra

Properties

- In the transition from arithmetic to algebra, attention shifts from arithmetic operations (addition, subtraction, multiplication, and division) to use of the *properties* of these operations.
- All of the facts of arithmetic and algebra follow from certain properties.

Variable

- Quantities are used to form expressions, equations, and inequalities.
- An expression refers to a quantity but does not make a statement about it. An equation (or an inequality) is a statement about the quantities it mentions.
- Using variables in place of numbers in equations (or inequalities) allows the statement of relationships among numbers that are unknown or unspecified.

Equivalence

- A single quantity may be represented by many different expressions.
- The facts about a quantity may be expressed by many different equations (or inequalities).

Solving Equations & Inequalities

- Solving an equation is the process of rewriting the equation to make what it says about its variable(s) as simple as possible.
- Properties of numbers and equality can be used to transform an equation (or inequality) into equivalent, simpler equations (or inequalities) in order to find solutions.
- Useful information about equations and inequalities (including solutions) can be found by analyzing graphs or tables.
- The numbers and types of solutions vary predictably, based on the type of equation.

Proportionality

- Two quantities are *proportional* if they have the same ratio in each instance where they are measured together.
- Two quantities are *inversely proportional* if they have the same product in each instance where they are measured together.

Function

- A function is a relationship between variables in which each value of the input variable is associated with a unique value of the output variable.
- Functions can be represented in a variety of ways, such as graphs, tables, equations, or words. Each representation is particularly useful in certain situations.
- Some important families of functions are developed through transformations of the simplest form of the function.
- New functions can be made from other functions by applying arithmetic operations or by applying one function to the output of another.

Modeling

- Many real-world mathematical problems can be represented algebraically. These representations can lead to algebraic solutions.
- A function that models a real-world situation can be used to make estimates or predictions about future occurrences.

Statistics and Probability

Data Collection and Analysis

- Sampling techniques are used to gather data from real-world situations. If the data are representative of the larger population, inferences can be made about that population.
- Biased sampling techniques yield data unlikely to be representative of the larger population.
- Sets of numerical data are described using measures of central tendency and dispersion.

Data Representation

- The most appropriate data representations depend on the type of data—quantitative or qualitative, and univariate or bivariate.
- Line plots, box plots, and histograms are different ways to show distribution of data over a possible range of values.

Probability

- Probability expresses the likelihood that a particular event will occur.
- Data can be used to calculate an experimental probability, and mathematical properties can be used to determine a theoretical probability.
- Either experimental or theoretical probability can be used to make predictions or decisions about future events.
- Various counting methods can be used to develop theoretical probabilities.

Geometry

Visualization

- Visualization can help you see the relationships between two figures and connect properties of real objects with two-dimensional drawings of these objects.

Transformations

- Transformations are mathematical functions that model relationships with figures.
- Transformations may be described geometrically or by coordinates.
- Symmetries of figures may be defined and classified by transformations.

Measurement

- Some attributes of geometric figures, such as length, area, volume, and angle measure, are measurable. Units are used to describe these attributes.

Reasoning & Proof

- Definitions establish meanings and remove possible misunderstanding.
- Other truths are more complex and difficult to see. It is often possible to verify complex truths by reasoning from simpler ones using deductive reasoning.

Similarity

- Two geometric figures are similar when corresponding lengths are proportional and corresponding angles are congruent.
- Areas of similar figures are proportional to the squares of their corresponding lengths.
- Volumes of similar figures are proportional to the cubes of their corresponding lengths.

Coordinate Geometry

- A coordinate system on a line is a number line on which points are labeled, corresponding to the real numbers.
- A coordinate system in a plane is formed by two perpendicular number lines, called the *x*- and *y*-axes, and the quadrants they form. The coordinate plane can be used to graph many functions.
- It is possible to verify some complex truths using deductive reasoning in combination with the distance, midpoint, and slope formulas.

1

Tools of Geometry

Chapters 1 & 2

Numbers

Quantities

Reason quantitatively and use units to solve problems

Geometry

Congruence

Experiment with transformations in the plane
Prove geometric theorems
Make geometric constructions

2 Reasoning and Proof

Visual See It!

Reasoning Try It!

Practice Do It!

3

Parallel and Perpendicular Lines

Chapters 3 & 4

Geometry

Congruence

Experiment with transformations in the plane

Prove geometric theorems

Make geometric constructions

Similarity, Right Triangles, and Trigonometry

Prove theorems involving similarity

Geometry continued

Expressing Geometric Properties with Equations

Use coordinates to prove simple geometric theorems algebraically

Modeling with Geometry

Apply geometric concepts in modeling situations

4 Congruent Triangles

Visual See It!

Reasoning Try It!

Practice Do It!

COMMON CORE COMMON CORE

5 Relationships Within Triangles

Chapters 5 & 6

Geometry

Congruence

Prove geometric theorems

Make geometric constructions

Similarity, Right Triangles, and Trigonometry

Prove theorems involving similarity

Geometry continued

Circles

Understand and apply theorems about circles

Expressing Geometric Properties with Equations

Use coordinates to prove simple geometric theorems algebraically

6 Polygons and Quadrilaterals

Visual See It!

Reasoning Try It!

Practice Do It!

7

Similarity

Chapters 7 & 8

Geometry

Similarity, Right Triangles, and Trigonometry

Prove theorems involving similarity

Define trigonometric ratios and solve problems involving right triangles

Apply trigonometry to general triangles

Geometry continued

Expressing Geometric Properties with Equations

Use coordinates to prove simple geometric theorems algebraically

Modeling with Geometry

Apply geometric concepts in modeling situations

8 Right Triangles and Trigonometry

Visual See It!

Reasoning Try It!

Practice Do It!

9 Transformations

Chapters 9 & 10

Geometry

Congruence

Experiment with transformations in the plane

Understand congruence in terms of rigid motions

Make geometric constructions

Similarity, Right Triangles, and Trigonometry

Understand similarity in terms of similarity transformations

Apply trigonometry to general triangles

Geometry continued

Circles

Understand and apply theorems about circles

Find arc lengths and areas of sectors of circles

Expressing Geometric Properties with Equations

Use coordinates to prove simple geometric theorems algebraically

Modeling with Geometry

Apply geometric concepts in modeling situations

10 Area

11 Surface Area and Volume

Chapters 11 & 12

Geometry

Circles

Understand and apply theorems about circles

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section

Geometry continued

Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems

Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry

Apply geometric concepts in modeling situations

12 Circles

Visual See It!

Reasoning Try It!

Practice Do It!

COMMON CORE COMMON CORE COMMON CORE

13 Probability

Chapter 13

Probability

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data

Use the rules of probability to compute probabilities of compound events in a uniform probability model

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions

Entry-Level Assessment

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. What is the solution to $5a - 15 + 9a = 3a + 29$?

- A $a = \frac{14}{11}$
- B $a = 4$
- C $a = 7$
- D $a = 44$

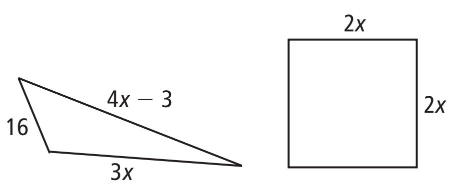
2. A bag contains 4 blue marbles, 6 green marbles, and 2 red marbles. You select one ball at random from the bag. What is $P(\text{red})$?

- F $\frac{1}{6}$
- G $\frac{1}{5}$
- H $\frac{1}{2}$
- I $\frac{5}{6}$

3. You select one green marble from the full bag in Exercise 2. What is the probability that the next marble you select will be blue?

- A $\frac{1}{3}$
- B $\frac{1}{5}$
- C $\frac{4}{7}$
- D $\frac{4}{11}$

4. In the diagram below, the perimeter of the triangle is equal to the perimeter of the square. What is the length of a side of the square?



- F 7
- G 13
- H 26
- I 52

5. What is $5\frac{3}{4}$ written as a decimal?

- A 3.75
- B 5.25
- C 5.75
- D 20.3

6. Maria gave one half of her jelly beans to Carole. Carole gave one third of those to Austin. Austin gave one fourth of those to Tony. If Tony received two jelly beans, how many did Maria start with?

- F 8
- G 24
- H 48
- I 96

7. What is the ratio $0.6 : 2.4$ written in simplest form?

- A $1 : 4$
- B $3 : 4$
- C $4 : 1$
- D $6 : 24$

8. What is the slope of the line through $(-4, 2)$ and $(5, 8)$?

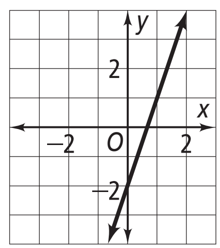
- F $\frac{1}{6}$
- G $\frac{2}{3}$
- H $\frac{3}{2}$
- I 6

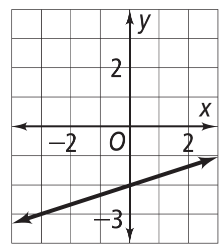
9. What is the solution to the system of equations?

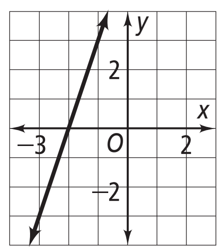
$$y = x - 2$$
$$2x + 2y = 4$$

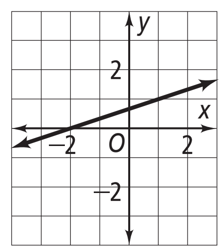
- A $(2, 0)$
- B $(0, -2)$
- C $(-2, 0)$
- D $(0, 2)$

10. Which is the graph of a line with a slope of 3 and a y-intercept of -2?

F


H


G


I


11. Which of the following is equivalent to $(-21)^2$?

- A -441
- B -42
- C 42
- D 441

12. What is the simplified form of $\sqrt{45a^5}$?

- F $3a^2\sqrt{5a}$
- G $a^2\sqrt{45a}$
- H $5a\sqrt{3a^2}$
- I $9a^2\sqrt{5a}$

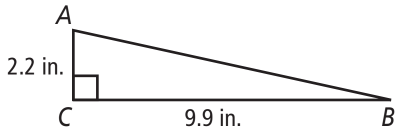
13. What is the next term in the pattern?

$\frac{1}{2}, \frac{1}{4}, \frac{1}{16}, \ldots$

Ⓐ $\frac{1}{20}$ Ⓒ $\frac{1}{64}$

Ⓑ $\frac{1}{32}$ Ⓓ $\frac{1}{256}$

14. What is the area of $\triangle ABC$, to the nearest tenth?



Ⓕ 10.1 in.^{2} Ⓗ 21.8 in.^{2}

Ⓖ 10.9 in.^{2} Ⓘ 217.8 in.^{2}

15. What is the value of the expression $-x(y - 8)^2$ for $x = -2$ and $y = 5$?

Ⓐ -18 Ⓒ 6

Ⓑ -6 Ⓓ 18

16. An athletic club has 248 members. Of these, 164 lift weights and 208 perform cardiovascular exercises regularly. All members do at least one of these activities. How many members do both?

Ⓕ 40 Ⓗ 84

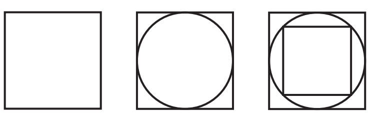
Ⓖ 44 Ⓘ 124

17. What is the solution to $y - 7 > 3 + 2y$?

Ⓐ $y < -10$ Ⓒ $y > -\frac{10}{3}$

Ⓑ $y > 4$ Ⓓ $y < -4$

18. What is the next figure in the sequence below?



Ⓕ a circle inside a square

Ⓖ a square inside a circle inside a square

Ⓗ a circle inside a square inside a circle inside a square

Ⓘ a square inside a circle inside a square inside a circle

19. What is the ratio $18b^2$ to $45b$ written in simplest form?

Ⓐ 18 to 45 Ⓒ b to 2.5

Ⓑ $2b^2$ to $5b$ Ⓓ $2b$ to 5

20. A farmer leans a 12-ft ladder against a barn. The base of the ladder is 3 ft from the barn. To the nearest tenth, how high on the barn does the ladder reach?

Ⓕ 9.2 ft Ⓗ 11.6 ft

Ⓖ 10.8 ft Ⓘ 13.4 ft

21. A map has a scale of 1 in. : 25 mi. Two cities are 175 mi apart. How far apart are they on the map?

Ⓐ 3 in. Ⓒ 6 in.

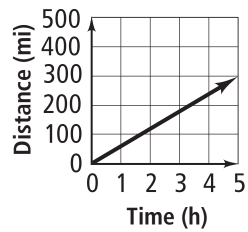
Ⓑ 5 in. Ⓓ 7 in.

22. What is the equation of the line that is parallel to the line $y = 5x + 2$ and passes through the point $(1, -3)$?

Ⓕ $y = -5x + 2$ Ⓗ $y = \frac{1}{5}x - 8$

Ⓖ $y = 5x + 8$ Ⓘ $y = 5x - 8$

23. The graph below shows the distance and time of your car trip. What does the slope of the line mean?



Ⓐ You traveled 0.017 mi/h.

Ⓑ You traveled for 5 h.

Ⓒ You traveled 60 mi/h.

Ⓓ You traveled 300 mi.

24. You are building a rectangular dog pen with an area of 90 ft^2. You want the length of the pen to be 3 ft longer than twice its width. Which equation can you use to find the width w of the pen?

Ⓕ $90 = w(w + 3)$ Ⓗ $90 = 2w(w + 3)$

Ⓖ $90 = w(2w + 3)$ Ⓘ $90 = (2 + w)(w + 3)$

25. The formula for the surface area of a sphere is $A = 4\pi r^2$. What is the formula solved for r?

Ⓐ $r = \frac{A}{2\sqrt{\pi}}$ Ⓒ $r = \frac{1}{2}\sqrt{\frac{A}{\pi}}$

Ⓑ $r = \frac{A}{2\pi}$ Ⓓ $r = 2\sqrt{\frac{A}{\pi}}$

CHAPTER 1

Get Ready!

Skills Handbook, p. 889

Squaring Numbers

Simplify.

1. 3^2 **2.** 4^2 **3.** 11^2

Skills Handbook, p. 890

Simplifying Expressions

Simplify each expression. Use 3.14 for π.

4. $2 \cdot 7.5 + 2 \cdot 11$ **5.** $\pi(5)^2$ **6.** $\sqrt{5^2 + 12^2}$

Skills Handbook, p. 890

Evaluating Expressions

Evaluate the following expressions for $a = 4$ and $b = -2$.

7. $\frac{a+b}{2}$ **8.** $\frac{a-7}{3-b}$ **9.** $\sqrt{(7-a)^2 + (2-b)^2}$

Skills Handbook, p. 892

Finding Absolute Value

Simplify each absolute value expression.

10. $|-8|$ **11.** $|2-6|$ **12.** $|-5-(-8)|$

Skills Handbook, p. 894

Solving Equations

Algebra Solve each equation.

13. $2x + 7 = 13$ **14.** $5x - 12 = 2x + 6$ **15.** $2(x+3) - 1 = 7x$

VOCABULARY

Looking Ahead Vocabulary

16. A child can *construct* models of buildings by stacking and arranging colored blocks. What might the term *construction* mean in geometry?

17. The *Mid*-Autumn Festival, celebrated in China, falls exactly in the middle of autumn, according to the Chinese lunar calendar. What would you expect a *midpoint* to be in geometry?

18. Artists often use long streaks to show *rays* of light coming from the sun. A ray is also a geometric figure. What do you think the properties of a *ray* are?

19. You and your friend work with each other. In other words, you and your friend are *co*-workers. What might the term *collinear* mean in geometry?

CHAPTER 1

Tools of Geometry

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Chapter Preview

1-1 Nets and Drawings for Visualizing Geometry
1-2 Points, Lines, and Planes
1-3 Measuring Segments
1-4 Measuring Angles
1-5 Exploring Angle Pairs
1-6 Basic Constructions
1-7 Midpoint and Distance in the Coordinate Plane
1-8 Perimeter, Circumference, and Area

BIG ideas

1 **Visualization**
Essential Question How can you represent a three-dimensional figure with a two-dimensional drawing?

2 **Reasoning**
Essential Question What are the building blocks of geometry?

3 **Measurement**
Essential Question How can you describe the attributes of a segment or angle?

Vocabulary

English/Spanish Vocabulary Audio Online:

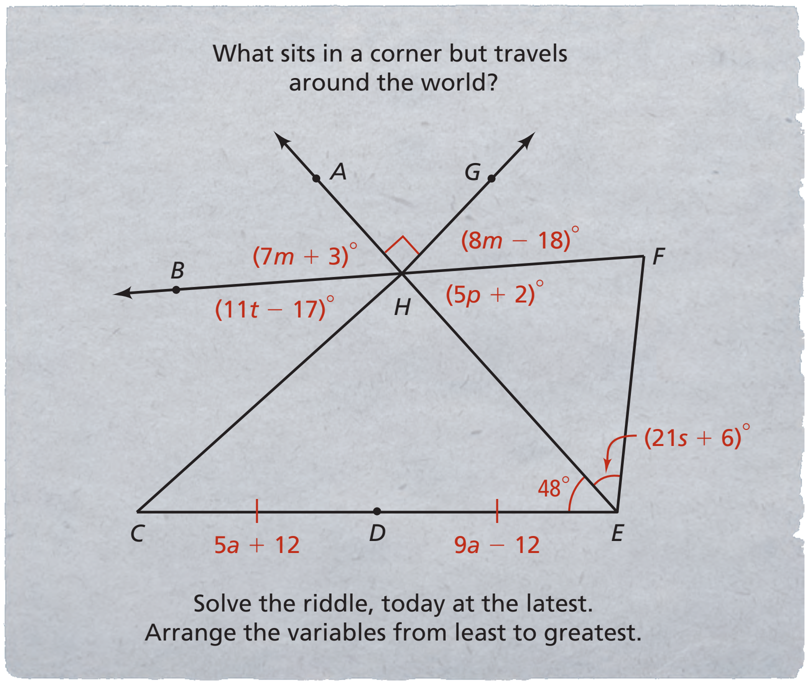
English	Spanish
angle bisector, *p. 37*	bisectriz de un ángulo
congruent segments, *p. 22*	segmentos congruentes
construction, *p. 43*	construcción
isometric drawing, *p. 5*	dibujo isométrico
linear pair, *p. 36*	par lineal
net, *p. 4*	plantilla
orthographic drawing, *p. 6*	dibujo ortográfico
perpendicular bisector, *p. 44*	mediatriz
postulate, *p. 13*	postulado
segment bisector, *p. 22*	bisectriz de un segmento
supplementary angles, *p. 34*	ángulos suplementarios
vertical angles, *p. 34*	ángulos verticales

DOMAINS
- Congruence

Common Core Performance Task

Solving a Riddle

While browsing in an antique store, Cameron found a page that came from an old book of riddles.



Task Description

Find the value of each variable and answer the riddle.

Connecting the Task to the Math Practices

MATHEMATICAL PRACTICES

As you complete the task, you'll apply several Standards for Mathematical Practice.

- You'll make sense of the problem as you analyze the information in a complex diagram. (MP 1)
- You'll attend to precision as you write and solve an equation. (MP 6)
- You'll explain relationships between angles in the diagram. (MP 3)

1-1 Nets and Drawings for Visualizing Geometry

Mathematics Florida Standards

Prepares for MAFS.912.G-CO.1.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

MP 3, MP 4, MP 7

Common Core State Standards

Prepares for G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, and distance along a line, and distance around a circular arc.

MP 3, MP 4, MP 7

Objective To make nets and drawings of three-dimensional figures

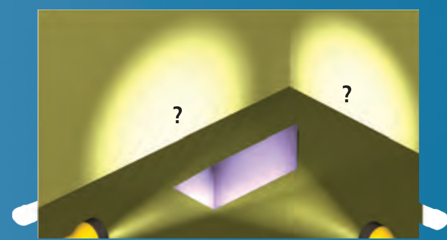
Try to visualize what the figure might look like from different perspectives.

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

When you shine a flashlight on an object, you can see a shadow on the opposite wall. What shape would you expect the shadows in the diagram to have? Explain your reasoning.



Lesson Vocabulary
- net
- isometric drawing
- orthographic drawing

In the Solve It, you had to "see" the projection of one side of an object onto a flat surface. Visualizing figures is a key skill that you will develop in geometry.

Essential Understanding You can represent a three-dimensional object with a two-dimensional figure using special drawing techniques.

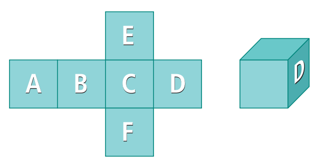
A **net** is a two-dimensional diagram that you can fold to form a three-dimensional figure. A net shows all of the surfaces of a figure in one view.

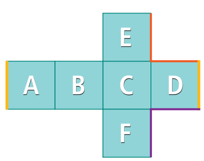
Think

How can you see the 3-D figure?
Visualize folding the net at the seams so that the edges join together. Track the letter positions by seeing one surface move in relation to another.

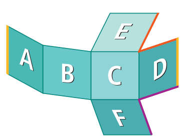
Problem 1 Identifying a Solid From a Net

The net at the right folds into the cube shown beside it. Which letters will be on the top and front of the cube?





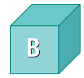
A, C, E, and F all share an edge with D when you fold the net, but only two of those sides are visible in the cube shown.



A wraps around and joins with D to become the back of the cube. B becomes the left side. F folds back to become the bottom.

E folds down to become the top of the cube. C becomes the front.

Got It? **1.** The net in Problem 1 folds into the cube shown at the right. Which letters will be on the top and right side of the cube?



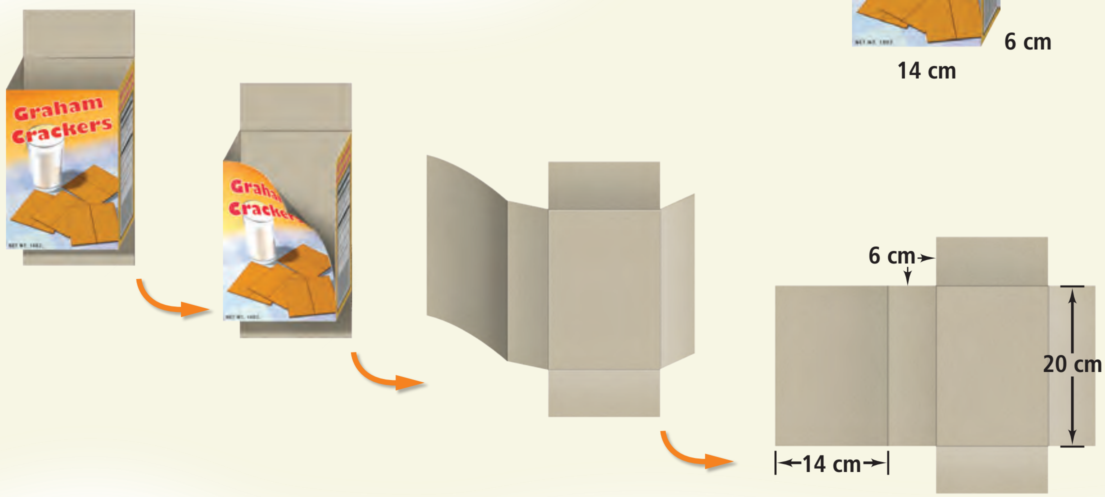
Packaging designers use nets to design boxes and other containers like the box in Problem 2.

Problem 2 Drawing a Net From a Solid

Think

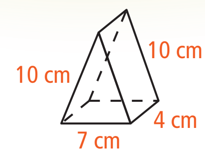
How can you see the net?
Visualize opening the top and bottom flaps of the box. Separate one of the side seams. Then unfold and flatten the box completely.

Package Design **What is a net for the graham cracker box to the right? Label the net with its dimensions.**



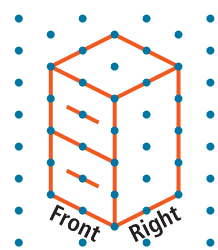
Got It? **2. a.** What is a net for the figure at the right? Label the net with its dimensions.

b. Reasoning Is there another possible net for the figure in part (a)? If so, draw it.



An **isometric drawing** shows a corner view of a three-dimensional figure. It allows you to see the top, front, and side of the figure. You can draw an isometric drawing on isometric dot paper. The simple drawing of a file cabinet at the right is an isometric drawing.

A net shows a three-dimensional figure as a folded-out flat surface. An isometric drawing shows a three-dimensional figure using slanted lines to represent depth.



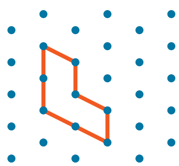
Problem 3 Isometric Drawing

Plan

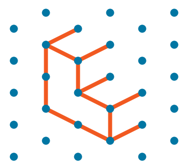
Is there more than one way to make an isometric drawing?
Yes. You can start with any edge of the structure. Use that edge as a reference to draw the other edges.

What is an isometric drawing of the cube structure at the right?

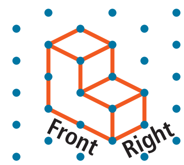
Step 1
Draw the front edges.



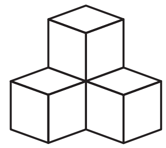
Step 2
Draw the right edges.



Step 3
Draw the back edges.



Got It? **3.** What is an isometric drawing of this cube structure?



An **orthographic drawing** is another way to represent a three-dimensional figure. An orthographic drawing shows three separate views: a top view, a front view, and a right-side view.

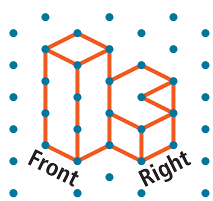
Although an orthographic drawing may take more time to analyze, it provides unique information about the shape of a structure.

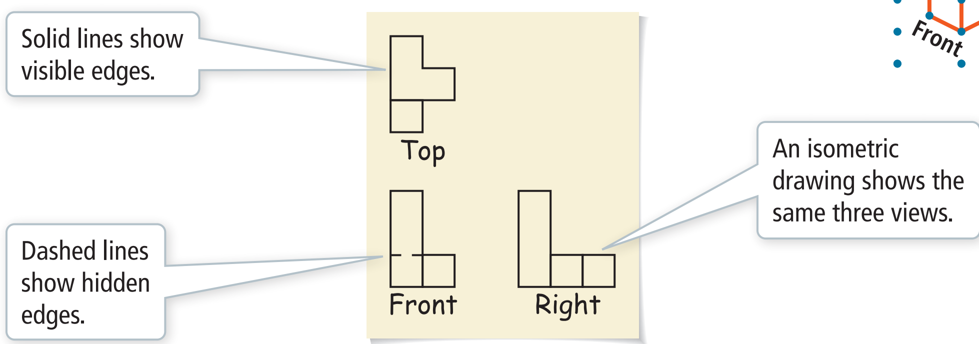
Problem 4 Orthographic Drawing

Plan

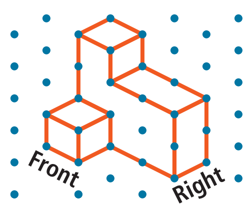
How can you determine the three views?
Rotate the structure in your head so that you can "see" each of the three sides straight on.

What is the orthographic drawing for the isometric drawing at the right?





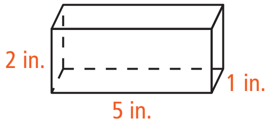
Got It? **4.** What is the orthographic drawing for this isometric drawing?



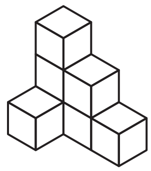
Lesson Check

Do you know HOW?

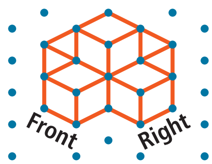
1. What is a net for the figure below? Label the net with its dimensions.



2. What is an isometric drawing of the cube structure?



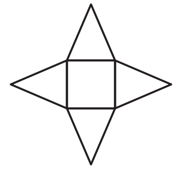
3. What is the orthographic drawing of the isometric drawing at the right? Assume there are no hidden cubes.



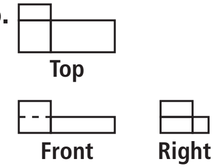
Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Vocabulary Tell whether each drawing is *isometric*, *orthographic*, a *net*, or *none*.

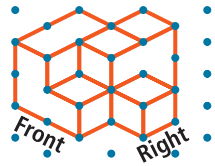
a.



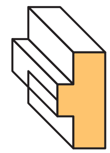
b.



c.



d.



5. Compare and Contrast What are the differences and similarities between an isometric drawing and an orthographic drawing? Explain.

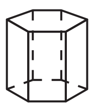
Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Match each three-dimensional figure with its net.

See Problem 1.

6.



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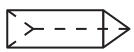

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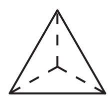
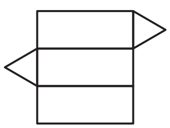
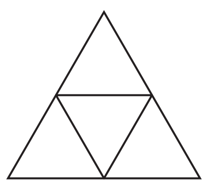


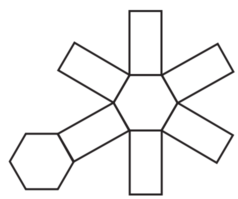
A.



B.



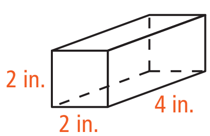
C.



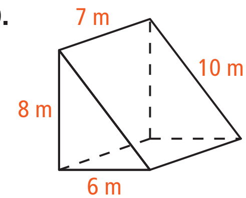
Draw a net for each figure. Label the net with its dimensions.

See Problem 2.

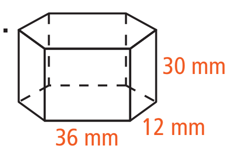
9.



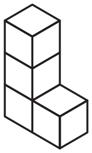
10.

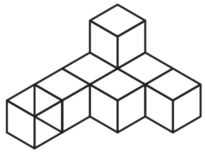


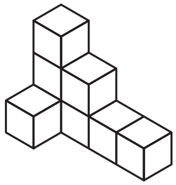
11.

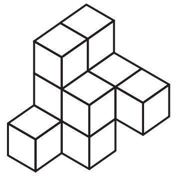


Make an isometric drawing of each cube structure on isometric dot paper. See Problem 3.

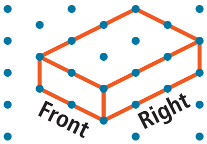
12. 

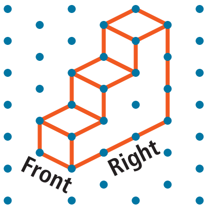
13. 

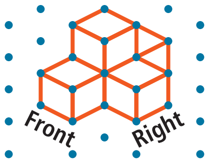
14. 

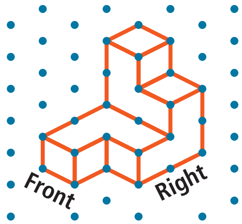
15. 

For each isometric drawing, make an orthographic drawing. Assume there are no hidden cubes. See Problem 4.

16. 

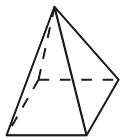
17. 

18. 

19. 

B Apply

20. Multiple Representations There are eight different nets for the solid shown at the right. Draw as many of them as you can. (*Hint*: Two nets are the same if you can rotate or flip one to match the other.)



21. a. Open-Ended Make an isometric drawing of a structure that you can build using 8 cubes.

b. Make an orthographic drawing of this structure.

22. Think About a Plan Draw a net of the can at the right.

- What shape are the top and bottom of the can?
- If you uncurl the body of the can, what shape do you get?

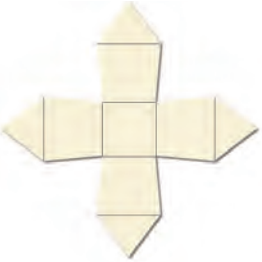
23. History In 1525, German printmaker Albrecht Dürer first used the word *net* to describe a printed pattern that folds up into a three-dimensional shape. Why do you think he chose to use the word *net*?

Manufacturing **Match the package with its net.**

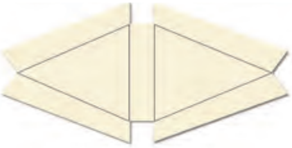
24. 

25. 

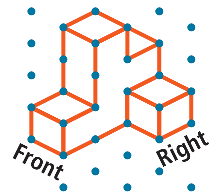
26. 

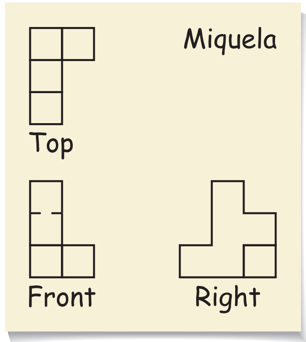
A. 

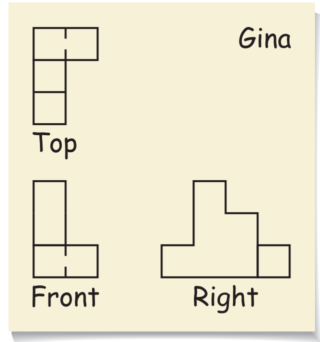
B. 

C. 

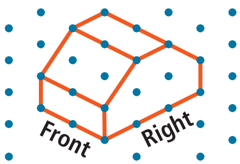
27. **Error Analysis** Miquela and Gina drew orthographic drawings for the cube structure at the right. Who is correct?

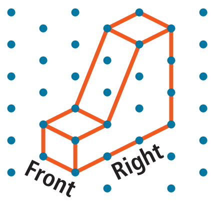


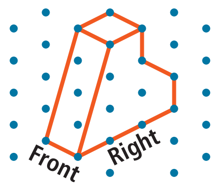




Make an orthographic drawing for each isometric drawing.

28. 

29. 

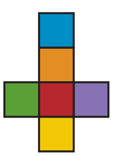
30. 

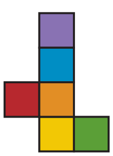
31. **Fort** Use the diagram of the fort at the right.
   a. Make an isometric drawing of the fort.
   b. Make an orthographic drawing of the fort.

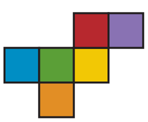
32. **Aerial Photography** Another perspective in aerial photography is the "bird's-eye view," which shows an object from directly overhead. What type of drawing that you have studied in this lesson is a bird's-eye view?

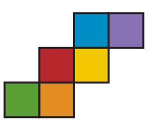
33. **Writing** Photographs of buildings are typically not taken from a bird's-eye view. Describe a situation in which you would want a photo showing a bird's-eye view.

Visualization Think about how each net can be folded to form a cube. What is the color of the face that will be opposite the red face?

34. 

35. 

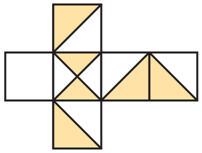
36. 

37. 

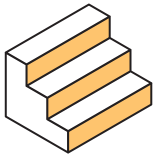
38. **Multiple Representations** There are 11 different nets for a cube. Four of them are shown above.
   a. Draw the other seven nets.
   b. **Writing** Suppose you want to make 100 cubes for an art project. Which of the 11 nets would you use? Explain why.

Challenge

39. The net at the right folds into a cube. Sketch the cube so that its front face is shaded as shown below.



40. **Architecture** What does the net of the staircase shown look like? Draw the net. (*Hint*: Visualize stretching the stairs out flat.)



41. A hexomino is a two-dimensional figure formed with six squares. Each square shares at least one side with another square. The 11 nets of a cube that you found in Exercise 38 are hexominoes. Draw as many of the remaining 24 hexominoes as you can.

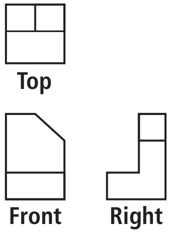
42. **Visualization** Use the orthographic drawing at the right.

a. Make an isometric drawing of the structure.

b. Make an isometric drawing of the structure from part (a) after it has been turned on its base 90° counterclockwise.

c. Make an orthographic drawing of the structure from part (b).

d. Turn the structure from part (a) 180°. Repeat parts (b) and (c).

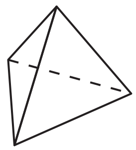


Standardized Test Prep

SAT/ACT

43. How many possible nets does the solid at the right have?

(A) 1 (B) 2 (C) 3 (D) 4



44. Solve $10a - 5b = 25$ for b.

(F) $b = 10a + 25$ (G) $b = 10a - 25$ (H) $b = 2a + 5$ (I) $b = 2a - 5$

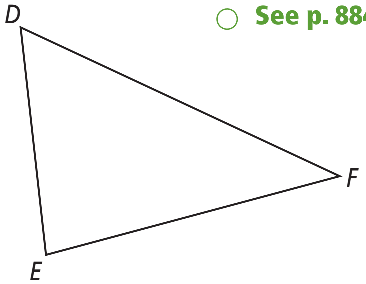
Short Response

45. Graph the equation $x + 2y = -3$. Label the x- and y-intercepts.

Mixed Review

For Exercises 46 and 47, use the diagram at the right.

See p. 884.



46. Measure DE and EF to the nearest millimeter.

47. Measure each angle to the nearest degree.

48. Draw a triangle that has sides of length 6 cm and 5 cm with a 90° angle between those two sides.

Get Ready! **To prepare for Lesson 1-2, do Exercises 49–51.**

Coordinate Geometry **Graph the points on the coordinate plane.**

See p. 893.

49. (0, 0), (2, 2), (0, 3) **50.** (1, 2), (−4, 3), (−5, 0) **51.** (−4, −5), (0, −1), (3, −2)

1-2 Points, Lines, and Planes

Common Core State Standards
G-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment . . .
MP 1, MP 3, MP 4, MP 6

Mathematics Florida Standards
MAFS.912.G-CO.1.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment . . .
MP 1, MP 3, MP 4, MP 6

Objective To understand basic terms and postulates of geometry

SOLVE IT! Getting Ready!

Make the figure at the right with a pencil and a piece of paper. Is the figure possible with a straight arrow and a solid board? Explain.

MATHEMATICAL PRACTICES In this lesson, you will learn basic geometric facts to help you justify your answer to the Solve It.

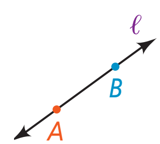
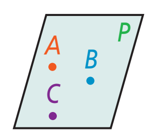
Essential Understanding Geometry is a mathematical system built on accepted facts, basic terms, and definitions.

Lesson Vocabulary
- point
- line
- plane
- collinear points
- coplanar
- space
- segment
- ray
- opposite rays
- postulate
- axiom
- intersection

In geometry, some words such as *point*, *line*, and *plane* are undefined. Undefined terms are the basic ideas that you can use to build the definitions of all other figures in geometry. Although you cannot define undefined terms, it is important to have a general description of their meanings.

take note

Key Concept Undefined Terms

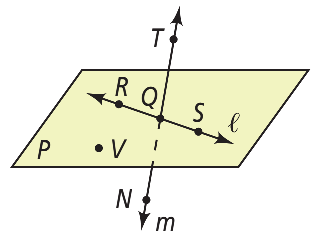
Term Description	How to Name It	Diagram
A **point** indicates a location and has no size.	You can represent a point by a dot and name it by a capital letter, such as A.	
A **line** is represented by a straight path that extends in two opposite directions without end and has no thickness. A line contains infinitely many points.	You can name a line by any two points on the line, such as $\overleftrightarrow{AB}$ (read "line AB") or $\overleftrightarrow{BA}$, or by a single lowercase letter, such as line ℓ.	
A **plane** is represented by a flat surface that extends without end and has no thickness. A plane contains infinitely many lines.	You can name a plane by a capital letter, such as plane P, or by at least three points in the plane that do not all lie on the same line, such as plane ABC.	

Points that lie on the same line are **collinear points.** Points and lines that lie in the same plane are **coplanar.** All the points of a line are coplanar.

Think

Why can figures have more than one name?
Lines and planes are made up of many points. You can choose any two points on a line and any three or more noncollinear points in a plane for the name.

Problem 1 Naming Points, Lines, and Planes



A What are two other ways to name $\overleftrightarrow{QT}$?

Two other ways to name $\overleftrightarrow{QT}$ are $\overleftrightarrow{TQ}$ and line m.

B What are two other ways to name plane *P*?

Two other ways to name plane *P* are plane *RQV* and plane *RSV*.

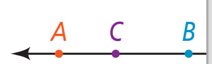
C What are the names of three collinear points? What are the names of four coplanar points?

Points *R, Q,* and *S* are collinear. Points *R, Q, S,* and *V* are coplanar.

Got It? **1. a.** What are two other ways to name $\overleftrightarrow{RS}$?
b. What are two more ways to name plane *P*?
c. What are the names of three other collinear points?
d. What are two points that are *not* coplanar with points *R*, *S*, and *V*?

The terms *point*, *line*, and *plane* are not defined because their definitions would require terms that also need defining. You can, however, use undefined terms to define other terms. A geometric figure is a set of points. **Space** is the set of all points in three dimensions. Similarly, the definitions for *segment* and *ray* are based on points and lines.

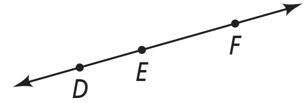
take note Key Concept Defined Terms

Definition	How to Name It	Diagram
A **segment** is part of a line that consists of two endpoints and all points between them.	You can name a segment by its two endpoints, such as $\overline{AB}$ (read "segment *AB*") or $\overline{BA}$.	
A **ray** is part of a line that consists of one endpoint and all the points of the line on one side of the endpoint.	You can name a ray by its endpoint and another point on the ray, such as $\overrightarrow{AB}$ (read "ray *AB*"). The order of points indicates the ray's direction.	
Opposite rays are two rays that share the same endpoint and form a line.	You can name opposite rays by their shared endpoint and any other point on each ray, such as $\overrightarrow{CA}$ and $\overrightarrow{CB}$.	

Plan

How do you make sure you name all the rays?
Each point on the line is an endpoint for a ray. At each point, follow the line both left and right to see if you can find a second point to name the ray.

Problem 2 Naming Segments and Rays



A What are the names of the segments in the figure at the right?

The three segments are $\overline{DE}$ or $\overline{ED}$, $\overline{EF}$ or $\overline{FE}$, and $\overline{DF}$ or $\overline{FD}$.

B What are the names of the rays in the figure?

The four rays are $\overrightarrow{DE}$ or $\overrightarrow{DF}$, $\overrightarrow{ED}$, $\overrightarrow{EF}$, and $\overrightarrow{FD}$ or $\overrightarrow{FE}$.

C Which of the rays in part (B) are opposite rays?

The opposite rays are $\overrightarrow{ED}$ and $\overrightarrow{EF}$.

Got It? 2. **Reasoning** $\overrightarrow{EF}$ and $\overrightarrow{FE}$ form a line. Are they opposite rays? Explain.

A **postulate** or **axiom** is an accepted statement of fact. Postulates, like undefined terms, are basic building blocks of the logical system in geometry. You will use logical reasoning to prove general concepts in this book.

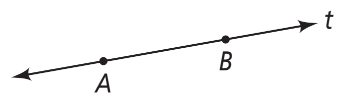
You have used some of the following geometry postulates in algebra. For example, you used Postulate 1-1 when you graphed equations such as $y = 2x + 8$. You graphed two points and drew the line through the points.

take note

Postulate 1-1

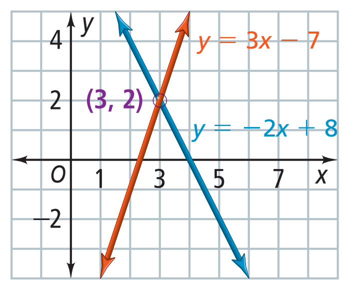
Through any two points there is exactly one line.

Line t passes through points A and B. Line t is the only line that passes through both points.



When you have two or more geometric figures, their **intersection** is the set of points the figures have in common.

In algebra, one way to solve a system of two equations is to graph them. The graphs of the two lines $y = -2x + 8$ and $y = 3x - 7$ intersect in a single point (3, 2). So the solution is (3, 2). This illustrates Postulate 1-2.

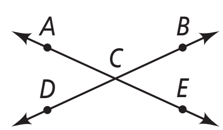


take note

Postulate 1-2

If two distinct lines intersect, then they intersect in exactly one point.

$\overleftrightarrow{AE}$ and $\overleftrightarrow{DB}$ intersect in point C.



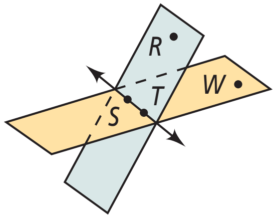
There is a similar postulate about the intersection of planes.

take note

Postulate 1-3

If two distinct planes intersect, then they intersect in exactly one line.

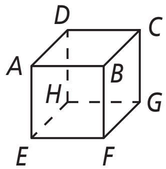
Plane RST and plane WST intersect in $\overleftrightarrow{ST}$.



When you know two points that two planes have in common, Postulates 1-1 and 1-3 tell you that the line through those points is the intersection of the planes.

Problem 3 Finding the Intersection of Two Planes

Each surface of the box at the right represents part of a plane. What is the intersection of plane *ADC* and plane *BFG*?



Know

Plane *ADC* and plane *BFG*

Need

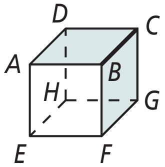
The intersection of the two planes

Plan

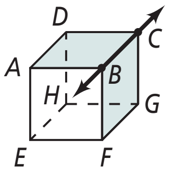
Find the points that the planes have in common.

Think

Is the intersection a segment?
No. The intersection of the sides of the box is a segment, but planes continue without end. The intersection is a line.



Focus on plane *ADC* and plane *BFG* to see where they intersect.



You can see that both planes contain point *B* and point *C*.

The planes intersect in $\overleftrightarrow{BC}$.

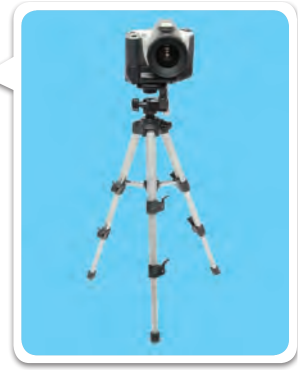
Got It? **3. a.** What are the names of two planes that intersect in $\overleftrightarrow{BF}$?

b. Reasoning Why do you only need to find two common points to name the intersection of two distinct planes?

When you name a plane from a figure like the box in Problem 3, list the corner points in consecutive order. For example, plane *ADCB* and plane *ABCD* are also names for the plane on the top of the box. Plane *ACBD* is not.

Photographers use three-legged tripods to make sure that a camera is steady. The feet of the tripod all touch the floor at the same time. You can think of the feet as points and the floor as a plane. As long as the feet do not all lie in one line, they will lie in exactly one plane.

This illustrates Postulate 1-4.

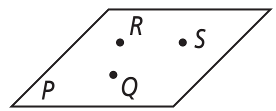


take note

Postulate 1-4

Through any three noncollinear points there is exactly one plane.

Points *Q*, *R*, and *S* are noncollinear. Plane *P* is the only plane that contains them.

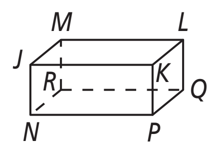


Problem 4 Using Postulate 1-4

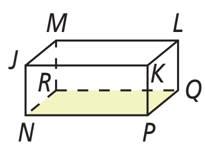
Plan

How can you find the plane?
Try to draw all the lines that contain two of the three given points. You will begin to see a plane form.

Use the figure at the right.

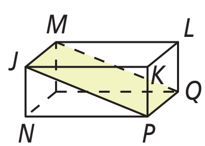


A What plane contains points *N, P,* and *Q*? Shade the plane.



The plane on the bottom of the figure contains points *N*, *P*, and *Q*.

B What plane contains points *J, M,* and *Q*? Shade the plane.



The plane that passes at a slant through the figure contains points *J*, *M*, and *Q*.

Got It? 4. a. What plane contains points *L*, *M*, and *N*? Copy the figure in Problem 4 and shade the plane.

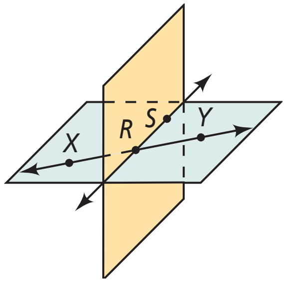
b. Reasoning What is the name of a line that is coplanar with $\overleftrightarrow{JK}$ and $\overleftrightarrow{KL}$?

Lesson Check

Do you know HOW?

Use the figure at the right.

1. What are two other names for $\overleftrightarrow{XY}$?
2. What are the opposite rays?
3. What is the intersection of the two planes?



Do you UNDERSTAND? MATHEMATICAL PRACTICES

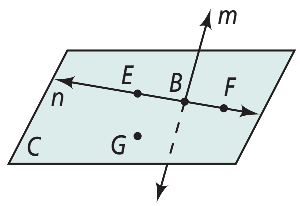
4. **Vocabulary** A segment has endpoints R and S. What are two names for the segment?
5. Are $\overrightarrow{AB}$ and $\overrightarrow{BA}$ the same ray? Explain.
6. **Reasoning** Why do you use two arrowheads when drawing or naming a line such as $\overleftrightarrow{EF}$?
7. **Compare and Contrast** How is naming a ray similar to naming a line? How is it different?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

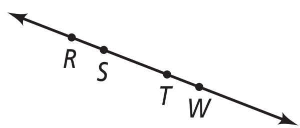
Use the figure at the right for Exercises 8–11. See Problem 1.

8. What are two other ways to name $\overleftrightarrow{EF}$?
9. What are two other ways to name plane C?
10. Name three collinear points.
11. Name four coplanar points.



Use the figure at the right for Exercises 12–14. See Problem 2.

12. Name the segments in the figure.
13. Name the rays in the figure.
14. **a.** Name the pair of opposite rays with endpoint T.
    b. Name another pair of opposite rays.



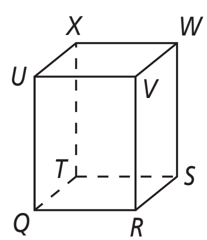
Use the figure at the right for Exercises 15–26.

Name the intersection of each pair of planes. See Problem 3.

15. planes QRS and RSW
16. planes UXV and WVS
17. planes XWV and UVR
18. planes TXW and TQU

Name two planes that intersect in the given line.

19. $\overleftrightarrow{QU}$
20. $\overleftrightarrow{TS}$
21. $\overleftrightarrow{XT}$
22. $\overleftrightarrow{VW}$

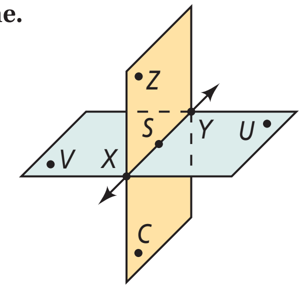


Copy the figure. Shade the plane that contains the given points. See Problem 4.

23. R, V, W
24. U, V, W
25. U, X, S
26. T, U, V

B Apply

Postulate 1-4 states that any three noncollinear points lie in exactly one plane. Find the plane that contains the first three points listed. Then determine whether the fourth point is in that plane. Write *coplanar* or *noncoplanar* to describe the points.



27. Z, S, Y, C

28. S, U, V, Y

29. X, Y, Z, U

30. X, S, V, U

31. X, Z, S, V

32. S, V, C, Y

If possible, draw a figure to fit each description. Otherwise, write *not possible*.

33. four points that are collinear

34. two points that are noncollinear

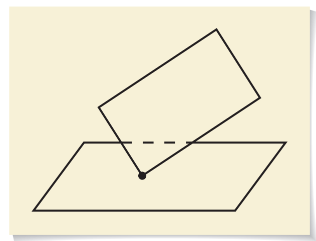
35. three points that are noncollinear

36. three points that are noncoplanar

37. Open-Ended Draw a figure with points B, C, D, E, F, and G that shows $\overleftrightarrow{CD}$, $\overleftrightarrow{BG}$, and $\overleftrightarrow{EF}$, with one of the points on all three lines.

38. Think About a Plan Your friend drew the diagram at the right to prove to you that two planes can intersect in exactly one point. Describe your friend's error.

- How do you describe a plane?
- What does it mean for two planes to intersect each other?
- Can you define an endpoint of a plane?



39. Reasoning If one ray contains another ray, are they the same ray? Explain.

For Exercises 40–45, determine whether each statement is *always, sometimes,* or *never* true.

40. $\overleftrightarrow{TQ}$ and $\overleftrightarrow{QT}$ are the same line.

41. $\overrightarrow{JK}$ and $\overrightarrow{JL}$ are the same ray.

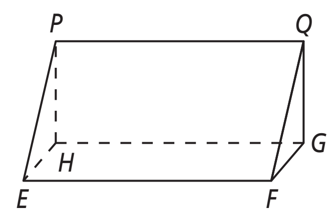
42. Intersecting lines are coplanar.

43. Four points are coplanar.

44. A plane containing two points of a line contains the entire line.

45. Two distinct lines intersect in more than one point.

46. Use the diagram at the right. How many planes contain each line and point?



a. $\overleftrightarrow{EF}$ and point G

b. $\overleftrightarrow{PH}$ and point E

c. $\overleftrightarrow{FG}$ and point P

d. $\overleftrightarrow{EP}$ and point G

e. Reasoning What do you think is true of a line and a point not on the line? Explain. (*Hint*: Use two of the postulates you learned in this lesson.)

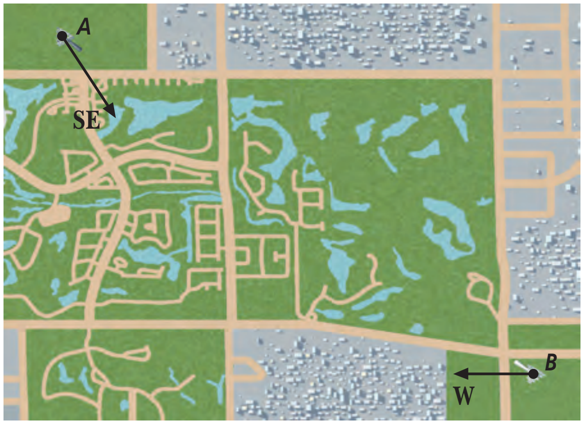
In Exercises 47–49, sketch a figure for the given information. Then state the postulate that your figure illustrates.

47. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect in point C.

48. The noncollinear points A, B, and C are all contained in plane N.

49. Planes LNP and MVK intersect in $\overleftrightarrow{NM}$.

50. Telecommunications A cell phone tower at point A receives a cell phone signal from the southeast. A cell phone tower at point B receives a signal from the same cell phone from due west. Trace the diagram at the right and find the location of the cell phone. Describe how Postulates 1-1 and 1-2 help you locate the phone.



51. Estimation You can represent the hands on a clock at 6:00 as opposite rays. Estimate the other 11 times on a clock that you can represent as opposite rays.

52. Open-Ended What are some basic words in English that are difficult to define?

Coordinate Geometry **Graph the points and state whether they are collinear.**

53. (1, 1), (4, 4), (−3, −3)

54. (2, 4), (4, 6), (0, 2)

55. (0, 0), (−5, 1), (6, −2)

56. (0, 0), (8, 10), (4, 6)

57. (0, 0), (0, 3), (0, −10)

58. (−2, −6), (1, −2), (4, 1)

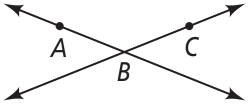
Challenge

59. How many planes contain the same three collinear points? Explain.

60. How many planes contain a given line? Explain.

61. a. Writing Suppose two points are in plane P. Explain why the line containing the points is also in plane P.

b. Reasoning Suppose two lines intersect. How many planes do you think contain both lines? Use the diagram at the right and your answer to part (a) to explain your answer.

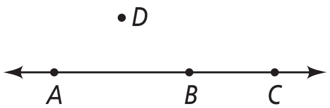


Probability **Suppose you pick points at random from *A, B, C,* and *D* shown below. Find the probability that the number of points given meets the condition stated.**

62. 2 points, collinear

63. 3 points, collinear

64. 3 points, coplanar



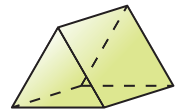
Standardized Test Prep

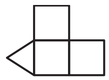
SAT/ACT

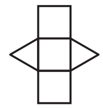
65. Which geometric term is undefined?

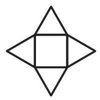
(A) segment (C) ray
(B) collinear (D) plane

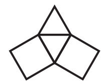
66. Which diagram is a net of the figure shown at the right?



(F) 

(H) 

(G) 

(I) 

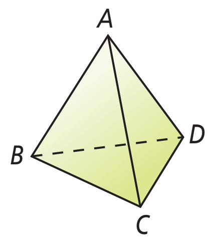
67. You want to cut a block of cheese into four pieces. What is the least number of cuts you need to make?

(A) 2 (B) 3 (C) 4 (D) 5

Short Response

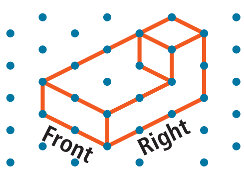
68. The figure at the right is called a tetrahedron.

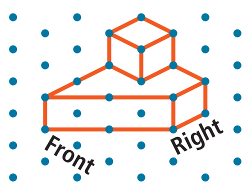
a. Name all the planes that form the surfaces of the tetrahedron.
b. Name all the lines that intersect at D.

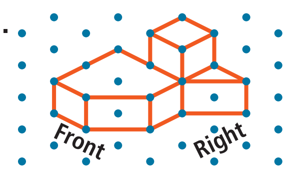


Mixed Review

Make an orthographic drawing for each figure. Assume there are no hidden cubes. See Lesson 1-1.

69. 

70. 

71. 

Simplify each ratio. See p. 891.

72. 30 to 12 **73.** $\frac{15x}{35x}$ **74.** $\frac{n^2 + n}{4n}$

Get Ready! **To prepare for Lesson 1-3, do Exercises 75–80.**

Simplify each absolute value expression. See p. 892.

75. $|-6|$ **76.** $|3.5|$ **77.** $|7 - 10|$

Algebra **Solve each equation.** See p. 894.

78. $x + 2x - 6 = 6$ **79.** $3x + 9 + 5x = 81$ **80.** $w - 2 = -4 + 7w$

1-3 Measuring Segments

MAFS Florida Standards

MAFS.912.G-CO.1.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment . . .
Also MAFS.912.G-GPE.2.6

MP 2, MP 3, MP 4, MP 6

Objective To find and compare lengths of segments

SOLVE IT! Getting Ready!

On a freshwater fishing trip, you catch the fish below. By law, you must release any fish between 15 and 19 in. long. You need to measure your fish, but the front of the ruler on the boat is worn away. Can you keep your fish? Explain how you found your answer.

Analyze the problem to figure out what you know and what you need to find next.

MATHEMATICAL PRACTICES In the Solve It, you measured the length of an object indirectly.

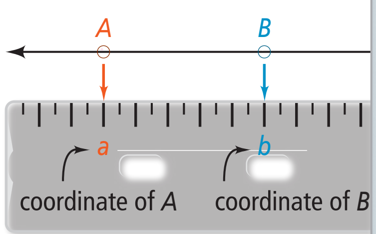
Lesson Vocabulary
- coordinate
- distance
- congruent segments
- midpoint
- segment bisector

Essential Understanding You can use number operations to find and compare the lengths of segments.

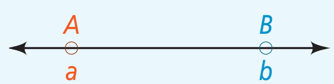
take note

Postulate 1-5 Ruler Postulate

Every point on a line can be paired with a real number. This makes a one-to-one correspondence between the points on the line and the real numbers. The real number that corresponds to a point is called the **coordinate** of the point.



The Ruler Postulate allows you to measure lengths of segments using a given unit and to find distances between points on a number line. Consider $\overleftrightarrow{AB}$ at the right. The **distance** between points A and B is the absolute value of the difference of their coordinates, or $|a - b|$. This value is also AB, or the length of $\overline{AB}$.



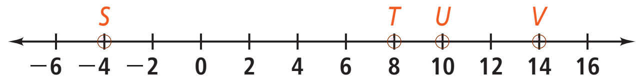
$$AB = |a - b|$$

Think

What are you trying to find?
ST represents the length of $\overline{ST}$, so you are trying to find the distance between points *S* and *T*.

Problem 1 Measuring Segment Lengths

What is ST?



The coordinate of S is -4.
Ruler Postulate
The coordinate of T is 8.

$ST = |-4 - 8|$ Definition of distance

$= |-12|$ Subtract.

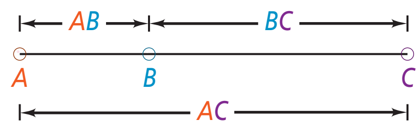
$= 12$ Find the absolute value.

Got It? **1.** What are UV and SV on the number line above?

take note

Postulate 1-6 Segment Addition Postulate

If three points A, B, and C are collinear and B is between A and C, then $AB + BC = AC$.



Problem 2 Using the Segment Addition Postulate

Algebra **If $EG = 59$, what are EF and FG?**

Know
$EG = 59$
$EF = 8x - 14$
$FG = 4x + 1$

Need
EF and FG

Plan
Use the Segment Addition Postulate to write an equation.

$EF + FG = EG$ Segment Addition Postulate

$(8x - 14) + (4x + 1) = 59$ Substitute.

$12x - 13 = 59$ Combine like terms.

$12x = 72$ Add 13 to each side.

$x = 6$ Divide each side by 12.

Use the value of x to find EF and FG.

$EF = 8x - 14 = 8(6) - 14 = 48 - 14 = 34$

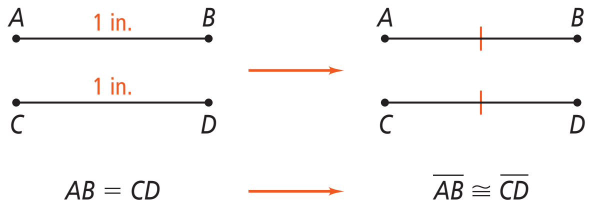
$FG = 4x + 1 = 4(6) + 1 = 24 + 1 = 25$

Substitute 6 for x.

Got It? **2.** In the diagram, $JL = 120$. What are JK and KL?

When numerical expressions have the same value, you say that they are equal (=). Similarly, if two segments have the same length, then the segments are **congruent (≅) segments.**

This means that if $AB = CD$, then $\overline{AB} \cong \overline{CD}$. You can also say that if $\overline{AB} \cong \overline{CD}$, then $AB = CD$.



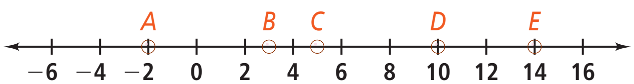
As illustrated above, you can mark segments alike to show that they are congruent. If there is more than one set of congruent segments, you can indicate each set with the same number of marks.

Problem 3 Comparing Segment Lengths

Plan

How do you know if segments are congruent?
Congruent segments have the same length. So find and compare the lengths of $\overline{AC}$ and $\overline{BD}$.

Are $\overline{AC}$ and $\overline{BD}$ congruent?



$AC = |-2 - 5| = |-7| = 7$

$BD = |3 - 10| = |-7| = 7$

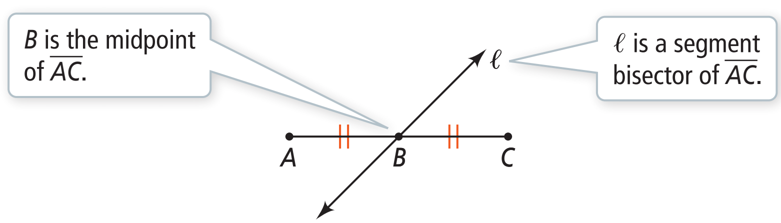
Definition of distance

Yes. $AC = BD$, so $\overline{AC} \cong \overline{BD}$.

Got It? **3. a.** Use the diagram above. Is $\overline{AB}$ congruent to $\overline{DE}$?

b. Reasoning To find AC in Problem 3, suppose you subtract -2 from 5. Do you get the same result? Why?

The **midpoint** of a segment is a point that divides the segment into two congruent segments. A point, line, ray, or other segment that intersects a segment at its midpoint is said to *bisect* the segment. That point, line, ray, or segment is called a **segment bisector.**

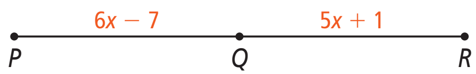


Problem 4 Using the Midpoint

Plan

How can you use algebra to solve the problem?
The lengths of the congruent segments are given as algebraic expressions. You can set the expressions equal to each other.

Algebra **Q is the midpoint of $\overline{PR}$. What are PQ, QR, and PR?**



Step 1 Find x.

$PQ = QR$	Definition of midpoint
$6x - 7 = 5x + 1$	Substitute.
$x - 7 = 1$	Subtract $5x$ from each side.
$x = 8$	Add 7 to each side.

Step 2 Find PQ and QR.

$PQ = 6x - 7$		$QR = 5x + 1$
$= 6(8) - 7$	Substitute 8 for x.	$= 5(8) + 1$
$= 41$	Simplify.	$= 41$

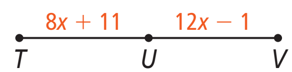
Step 3 Find PR.

$PR = PQ + QR$	Segment Addition Postulate
$= 41 + 41$	Substitute.
$= 82$	Simplify.

PQ and QR are both 41. PR is 82.

Got It? **4. a. Reasoning** Is it necessary to substitute 8 for x in the expression for QR in order to find QR? Explain.

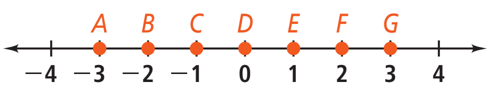
b. U is the midpoint of $\overline{TV}$. What are TU, UV, and TV?



Lesson Check

Do you know HOW?

Name each of the following.



1. The point on $\overrightarrow{DA}$ that is 2 units from D

2. Two points that are 3 units from D

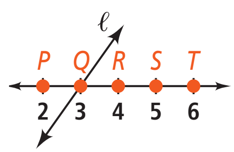
3. The coordinate of the midpoint of $\overline{AG}$

4. A segment congruent to $\overline{AC}$

Do you UNDERSTAND?

MATHEMATICAL PRACTICES

5. Vocabulary Name two segment bisectors of $\overline{PR}$.



6. Compare and Contrast Describe the difference between saying that two segments are *congruent* and saying that two segments have *equal length*. When would you use each phrase?

7. Error Analysis You and your friend live 5 mi apart. He says that it is 5 mi from his house to your house and −5 mi from your house to his house. What is the error in his argument?

Practice and Problem-Solving Exercises

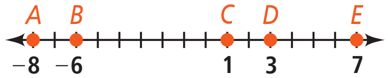
MATHEMATICAL PRACTICES

A Practice

Find the length of each segment. ○ **See Problem 1.**

8. $\overline{AB}$ **9.** $\overline{BD}$

10. $\overline{AD}$ **11.** $\overline{CE}$



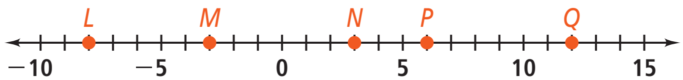
Use the number line at the right for Exercises 12–14. ○ **See Problem 2.**

12. If $RS = 15$ and $ST = 9$, then $RT = ■$.

13. If $ST = 15$ and $RT = 40$, then $RS = ■$.

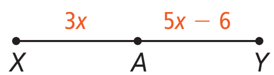
14. **Algebra** $RS = 8y + 4$, $ST = 4y + 8$, and $RT = 15y - 9$.
   a. What is the value of y?
   b. Find RS, ST, and RT.

Use the number line below for Exercises 15–18. Tell whether the segments are congruent. ○ **See Problem 3.**



15. $\overline{LN}$ and $\overline{MQ}$ **16.** $\overline{MP}$ and $\overline{NQ}$ **17.** $\overline{MN}$ and $\overline{PQ}$ **18.** $\overline{LP}$ and $\overline{MQ}$

19. **Algebra** A is the midpoint of $\overline{XY}$. ○ **See Problem 4.**
   a. Find XA.
   b. Find AY and XY.



Algebra **For Exercises 20–22, use the figure below. Find the value of PT.**

20. $PT = 5x + 3$ and $TQ = 7x - 9$

21. $PT = 4x - 6$ and $TQ = 3x + 4$

22. $PT = 7x - 24$ and $TQ = 6x - 2$

B Apply

On a number line, the coordinates of X, Y, Z, and W are -7, -3, 1, and 5, respectively. Find the lengths of the two segments. Then tell whether they are congruent.

23. $\overline{XY}$ and $\overline{ZW}$ **24.** $\overline{ZX}$ and $\overline{WY}$ **25.** $\overline{YZ}$ and $\overline{XW}$

Suppose the coordinate of A is 0, $AR = 5$, and $AT = 7$. What are the possible coordinates of the midpoint of the given segment?

26. $\overline{AR}$ **27.** $\overline{AT}$ **28.** $\overline{RT}$

29. Suppose point E has a coordinate of 3 and $EG = 5$. What are the possible coordinates of point G?

Visualization **Without using your ruler, sketch a segment with the given length. Use your ruler to see how well your sketch approximates the length provided.**

30. 3 cm **31.** 3 in. **32.** 6 in. **33.** 10 cm **34.** 65 mm

35. **Think About a Plan** The numbers labeled on the map of Florida are mile markers. Assume that Route 10 between Quincy and Jacksonville is straight.

Suppose you drive at an average speed of 55 mi/h. How long will it take to get from Live Oak to Jacksonville?

- How can you use mile markers to find distances between points?
- How do average speed, distance, and time all relate to each other?

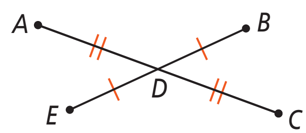
36. On a number line, A is at -2 and B is at 4. What is the coordinate of C, which is $\frac{2}{3}$ of the way from A to B?

Error Analysis **Use the highway sign for Exercises 37 and 38.**

37. A driver reads the highway sign and says, "It's 145 miles from Mitchell to Watertown." What error did the driver make? Explain.

38. Your friend reads the highway sign and says, "It's 71 miles to Watertown." Is your friend correct? Explain.

Algebra **Use the diagram at the right for Exercises 39 and 40.**



39. If $AD = 12$ and $AC = 4y - 36$, find the value of y. Then find AC and DC.

40. If $ED = x + 4$ and $DB = 3x - 8$, find ED, DB, and EB.

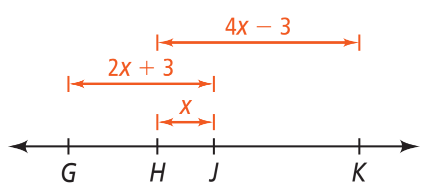
41. **Writing** Suppose you know PQ and QR. Can you use the Segment Addition Postulate to find PR? Explain.

Challenge

42. C is the midpoint of $\overline{AB}$, D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{ED}$, G is the midpoint of $\overline{EF}$, and H is the midpoint of $\overline{DB}$. If $DC = 16$, what is GH?

43. **a.** **Algebra** Use the diagram at the right. What algebraic expression represents GK?

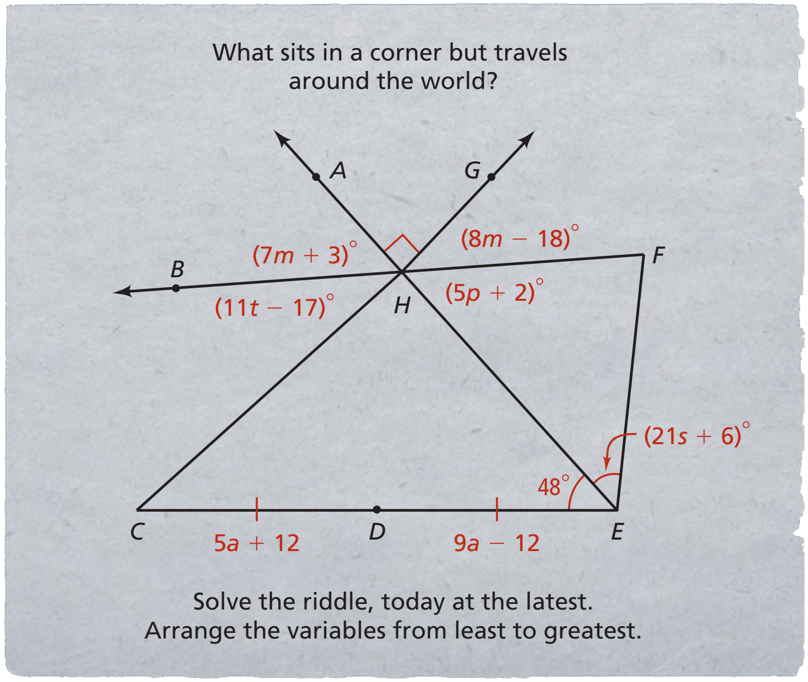
b. If $GK = 30$, what are GH and JK?



Apply What You've Learned

MATHEMATICAL PRACTICES
MP 1

Look back at the information on page 3 about the riddle Cameron found in an antique store. The page from the old riddle book is shown again below.



a. What relationship between two segments can you state based on information in the diagram? How does the diagram show this relationship?

b. Write and solve an equation to find the value of the variable a.

c. How can you be sure you solved the equation in part (b) correctly?

1-4 Measuring Angles

Common Core State Standards

G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment

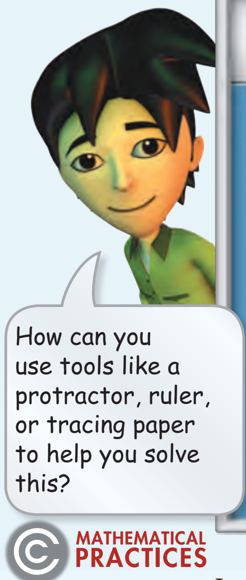
MP 1, MP 3, MP 6

Mathematics Florida Standards

MAFS.912.G-CO.1.1 Know precise definition of angle, circle, perpendicular line, parallel line, and line segment

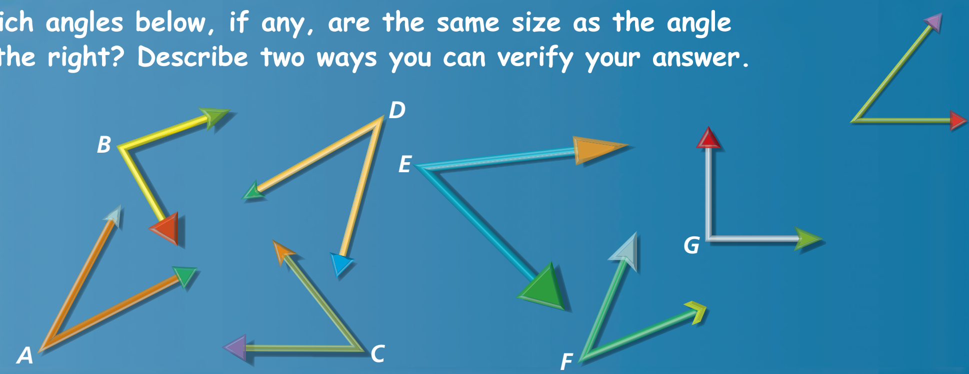
MP 1, MP 3, MP 6

Objective To find and compare the measures of angles



SOLVE IT! Getting Ready!

Which angles below, if any, are the same size as the angle at the right? Describe two ways you can verify your answer.



In this lesson, you will learn to describe and measure angles like the ones in the Solve It.

Lesson Vocabulary
- angle
- sides of an angle
- vertex of an angle
- measure of an angle
- acute angle
- right angle
- obtuse angle
- straight angle
- congruent angles

Essential Understanding You can use number operations to find and compare the measures of angles.

take note

Key Concept Angle

Definition

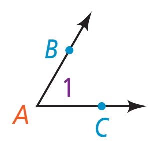
An **angle** is formed by two rays with the same endpoint.

The rays are the **sides** of the angle. The endpoint is the **vertex** of the angle.

How to Name It

You can name an angle by
- its vertex, $\angle A$
- a point on each ray and the vertex, $\angle BAC$ or $\angle CAB$
- a number, $\angle 1$

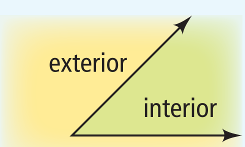
Diagram



The sides of the angle are $\overrightarrow{AB}$ and $\overrightarrow{AC}$. The vertex is A.

When you name angles using three points, the vertex must go in the middle.

The *interior* of an angle is the region containing all of the points between the two sides of the angle. The *exterior* of an angle is the region containing all of the points outside of the angle.



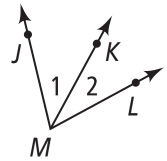
Think

What rays form $\angle 1$?

$\overrightarrow{MJ}$ and $\overrightarrow{MK}$ form $\angle 1$.

Problem 1 Naming Angles

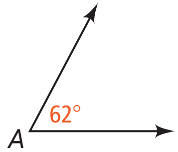
What are two other names for $\angle 1$?



$\angle JMK$ and $\angle KMJ$ are also names for $\angle 1$.

Got It? **1. a.** What are two other names for $\angle KML$?

b. Reasoning Would it be correct to name any of the angles $\angle M$? Explain.



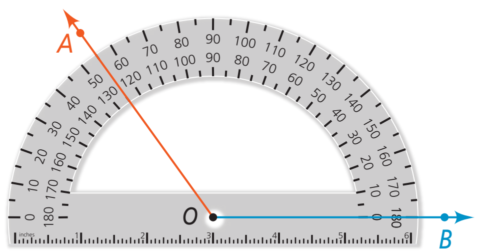
One way to measure the size of an angle is in degrees. To indicate the measure of an angle, write a lowercase m in front of the angle symbol. In the diagram, the measure of $\angle A$ is 62. You write this as $m\angle A = 62$. In this book, you will work only with degree measures.

A circle has 360°, so 1 degree is $\frac{1}{360}$ of a circle. A protractor forms half a circle and measures angles from 0° to 180°.

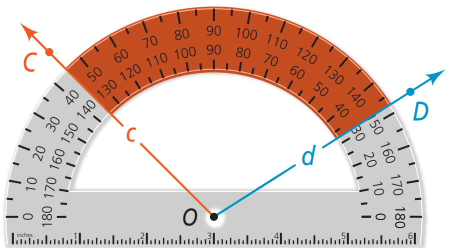
take note

Postulate 1-7 Protractor Postulate

Consider $\overrightarrow{OB}$ and a point A on one side of $\overrightarrow{OB}$. Every ray of the form $\overrightarrow{OA}$ can be paired one to one with a real number from 0 to 180.



The Protractor Postulate allows you to find the measure of an angle. Consider the diagram below. The **measure** of $\angle COD$ is the absolute value of the difference of the real numbers paired with $\overrightarrow{OC}$ and $\overrightarrow{OD}$. That is, if $\overrightarrow{OC}$ corresponds with c, and $\overrightarrow{OD}$ corresponds with d, then $m\angle COD = |c - d|$.



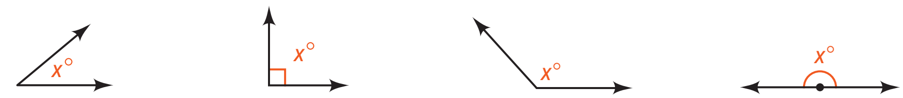
Notice that the Protractor Postulate and the calculation of an angle measure are very similar to the Ruler Postulate and the calculation of a segment length.

You can classify angles according to their measures.

take note

Key Concept Types of Angles

acute angle	right angle	obtuse angle	straight angle

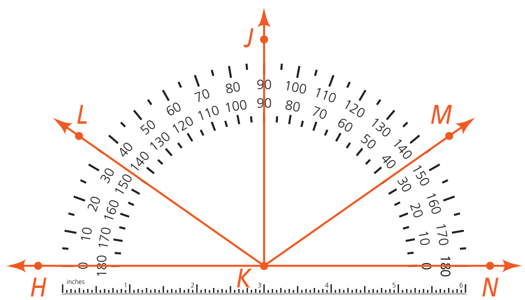


$0 < x < 90$	$x = 90$	$90 < x < 180$	$x = 180$

The symbol ⌝ in the diagram above indicates a right angle.

Problem 2 Measuring and Classifying Angles

What are the measures of $\angle LKN$, $\angle JKL$, and $\angle JKN$? Classify each angle as *acute, right, obtuse,* or *straight.*



Think

Do the classifications make sense?
Yes. In each case, the classification agrees with what you see in the diagram.

Use the definition of the measure of an angle to calculate each measure.

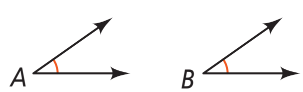
$m\angle LKN = |145 - 0| = 145$; $\angle LKN$ is obtuse.

$m\angle JKL = |90 - 145| = |-55| = 55$; $\angle JKL$ is acute.

$m\angle JKN = |90 - 0| = 90$; $\angle JKN$ is right.

Got It? 2. What are the measures of $\angle LKH$, $\angle HKN$, and $\angle MKH$? Classify each angle as *acute*, *right*, *obtuse*, or *straight*.

Angles with the same measure are **congruent angles.** This means that if $m\angle A = m\angle B$, then $\angle A \cong \angle B$. You can also say that if $\angle A \cong \angle B$, then $m\angle A = m\angle B$.



$m\angle A = m\angle B$
$\angle A \cong \angle B$

You can mark angles with arcs to show that they are congruent. If there is more than one set of congruent angles, each set is marked with the same number of arcs.

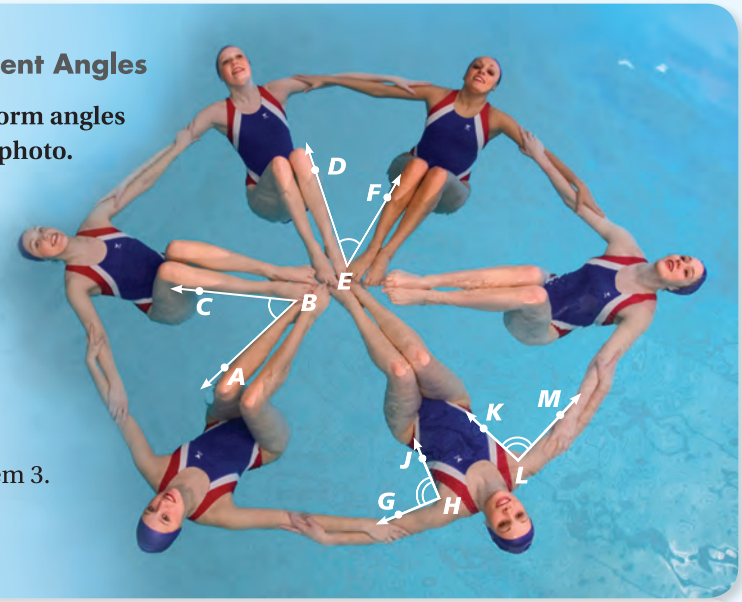
Think

Look at the diagram. What do the angle marks tell you?
The angle marks tell you which angles are congruent.

Problem 3 Using Congruent Angles

Sports **Synchronized swimmers form angles with their bodies, as shown in the photo. If $m\angle GHJ = 90$, what is $m\angle KLM$?**

$\angle GHJ \cong \angle KLM$ because they both have two arcs.
So, $m\angle GHJ = m\angle KLM = 90$.



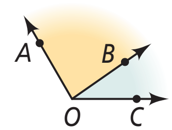
Got It? **3.** Use the photo in Problem 3. If $m\angle ABC = 49$, what is $m\angle DEF$?

The Angle Addition Postulate is similar to the Segment Addition Postulate.

take note

Postulate 1-8 Angle Addition Postulate

If point B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.

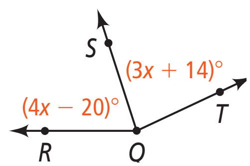


Plan

How can you use the expressions in the diagram?
The algebraic expressions represent the measures of the smaller angles, so they add up to the measure of the larger angle.

Problem 4 Using the Angle Addition Postulate

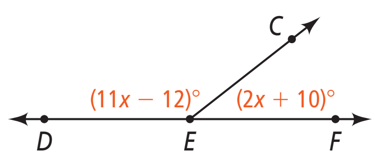
Algebra **If $m\angle RQT = 155$, what are $m\angle RQS$ and $m\angle TQS$?**



$m\angle RQS + m\angle TQS = m\angle RQT$	Angle Addition Postulate
$(4x - 20) + (3x + 14) = 155$	Substitute.
$7x - 6 = 155$	Combine like terms.
$7x = 161$	Add 6 to each side.
$x = 23$	Divide each side by 7.

$m\angle RQS = 4x - 20 = 4(23) - 20 = 92 - 20 = 72$
$m\angle TQS = 3x + 14 = 3(23) + 14 = 69 + 14 = 83$
Substitute 23 for x.

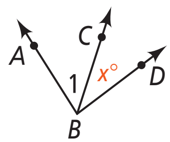
Got It? **4.** $\angle DEF$ is a straight angle. What are $m\angle DEC$ and $m\angle CEF$?



Lesson Check

Do you know HOW?

Use the diagram for Exercises 1–3.



1. What are two other names for $\angle 1$?

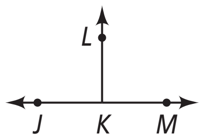
2. **Algebra** If $m\angle ABD = 85$, what is an expression to represent $m\angle ABC$?

3. Classify $\angle ABC$.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** How many sides can two distinct, congruent angles share? Explain.

5. **Error Analysis** Your classmate concludes from the diagram below that $\angle JKL \cong \angle LKM$. Is your classmate correct? Explain.

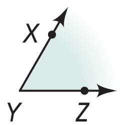


Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

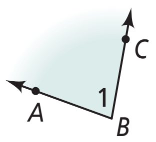
A Practice

Name each shaded angle in three different ways. See Problem 1.

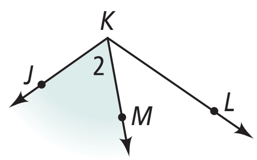
6.



7.



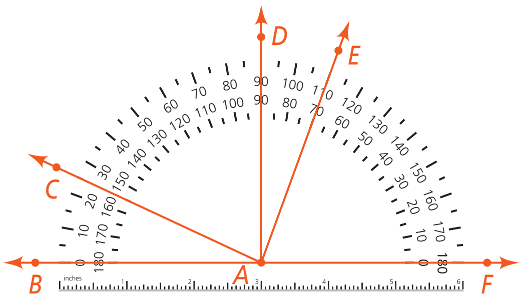
8.



Use the diagram below. Find the measure of each angle. Then classify the angle as *acute*, *right*, *obtuse*, or *straight*. See Problem 2.

9. $\angle EAF$ **10.** $\angle DAF$ **11.** $\angle BAE$

12. $\angle BAC$ **13.** $\angle CAE$ **14.** $\angle DAE$



Draw a figure that fits each description.

15. an obtuse angle, $\angle RST$

16. an acute angle, $\angle GHJ$

17. a straight angle, $\angle KLM$

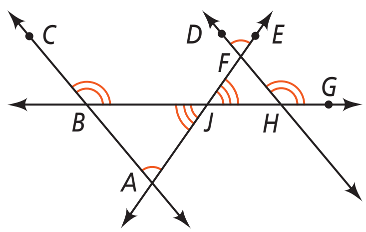
Use the diagram below. Complete each statement. See Problem 3.

18. $\angle CBJ \cong$ ■

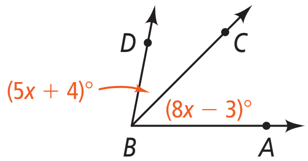
19. $\angle FJH \cong$ ■

20. If $m\angle EFD = 75$, then $m\angle JAB =$ ■.

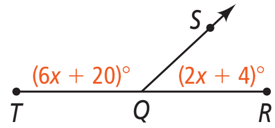
21. If $m\angle GHF = 130$, then $m\angle JBC =$ ■.



22. If $m\angle ABD = 79$, what are $m\angle ABC$ and $m\angle DBC$?



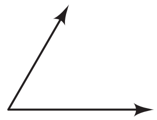
23. $\angle RQT$ is a straight angle. What are $m\angle RQS$ and $m\angle TQS$?

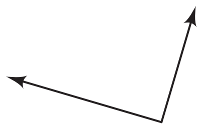


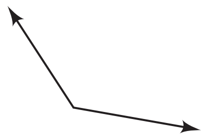
See Problem 4.

B Apply

Use a protractor. Measure and classify each angle.

24. 

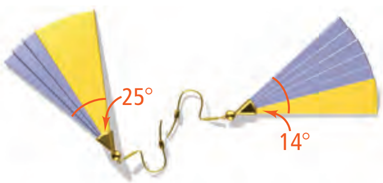
25. 

26. 

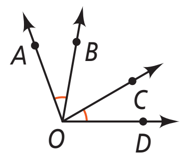
27. 

28. Think About a Plan A pair of earrings has blue wedges that are all the same size. One earring has a 25° yellow wedge. The other has a 14° yellow wedge. Find the angle measure of a blue wedge.

- How do the angle measures of the earrings relate?
- How can you use algebra to solve the problem?



Algebra Use the diagram at the right for Exercises 29 and 30. Solve for x. Find the angle measures to check your work.



29. $m\angle AOB = 4x - 2$, $m\angle BOC = 5x + 10$, $m\angle COD = 2x + 14$

30. $m\angle AOB = 28$, $m\angle BOC = 3x - 2$, $m\angle AOD = 6x$

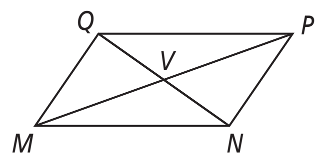
31. If $m\angle MQV = 90$, which expression can you use to find $m\angle VQP$?

Ⓐ $m\angle MQP - 90$

Ⓑ $90 - m\angle MQV$

Ⓒ $m\angle MQP + 90$

Ⓓ $90 + m\angle VQP$



32. Literature According to legend, King Arthur and his knights sat around the Round Table to discuss matters of the kingdom. The photo shows a round table on display at Winchester Castle, in England. From the center of the table, each section has the same degree measure. If King Arthur occupied two of these sections, what is the total degree measure of his section?

C Challenge

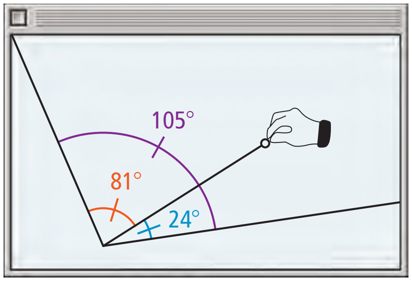
Time Find the angle measure of the hands of a clock at each time.

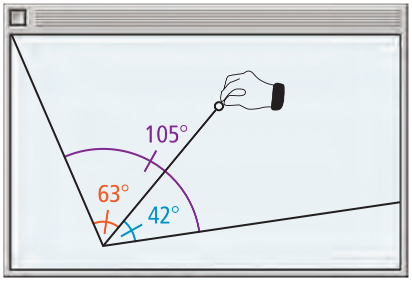
33. 6:00 **34.** 7:00 **35.** 11:00

36. 4:40 **37.** 5:20 **38.** 2:15

39. Open-Ended Sketch a right angle with vertex V. Name it $\angle 1$. Then sketch a 135° angle that shares a side with $\angle 1$. Name it $\angle PVB$. Is there more than one way to sketch $\angle PVB$? If so, sketch all the different possibilities. (*Hint:* Two angles are the same if you can rotate or flip one to match the other.)

40. Technology Your classmate constructs an angle. Then he constructs a ray from the vertex of the angle to a point in the interior of the angle. He measures all the angles formed. Then he moves the interior ray as shown below. What postulate do the two pictures support?





Apply What You've Learned

MATHEMATICAL PRACTICES
MP 6

Look back at the diagram on page 3 for the riddle Cameron found in an antique store. Choose from the following words and equations to complete the sentences below.

right	congruent	obtuse
$21s + 6 = 48$	$21s + 6 + 48 = 90$	$21s + 6 + 48 = 180$
$s = 6$	$s = 2$	$s \approx 1.7$

a. In the riddle's diagram, $\angle HEF$ and $\angle HED$ are __?__ angles.

b. The equation that correctly relates $m\angle HEF$ and $m\angle HED$ is __?__.

c. The solution of the equation from part (b) is __?__.

1-5 Exploring Angle Pairs

Mathematics Florida Standards

Prepares for MAFS.912.G-CO.1.1 Know precise definitions of angle, circle, perpendicular line, parallel line . . .

MP 1, MP 3, MP 4, MP 6

MP 1, MP 3, MP 4, MP 6

Objective To identify special angle pairs and use their relationships to find angle measures

SOLVE IT! Getting Ready!

The five game pieces at the right form a square to fit back in the box. Two of the shapes are already in place. Where do the remaining pieces go? How do you know? Make a sketch of the completed puzzle.

It might help if you make a sketch of the pieces and cut them out.

MATHEMATICAL PRACTICES

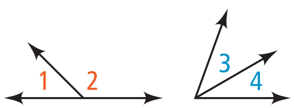
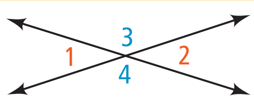
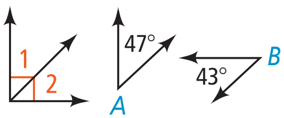
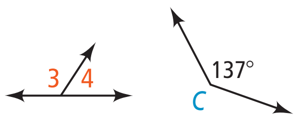
In this lesson, you will learn how to describe different kinds of angle pairs.

Essential Understanding Special angle pairs can help you identify geometric relationships. You can use these angle pairs to find angle measures.

Lesson Vocabulary
- adjacent angles
- vertical angles
- complementary angles
- supplementary angles
- linear pair
- angle bisector

take note

Key Concept Types of Angle Pairs

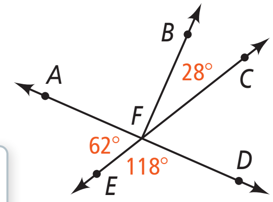
Definition	Example	
Adjacent angles are two coplanar angles with a common side, a common vertex, and no common interior points.	$\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$	
Vertical angles are two angles whose sides are opposite rays.	$\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$	
Complementary angles are two angles whose measures have a sum of 90. Each angle is called the *complement* of the other.	$\angle 1$ and $\angle 2$, $\angle A$ and $\angle B$	
Supplementary angles are two angles whose measures have a sum of 180. Each angle is called the *supplement* of the other.	$\angle 3$ and $\angle 4$, $\angle B$ and $\angle C$	

Plan

What should you look for in the diagram?
For part (A), check whether the angle pair matches every part of the definition of adjacent angles.

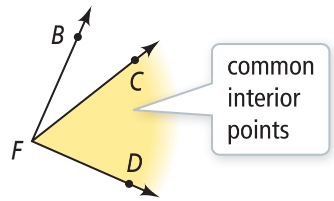
Problem 1 Identifying Angle Pairs

Use the diagram at the right. Is the statement true? Explain.



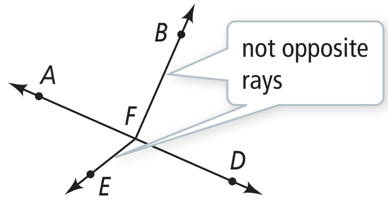
A **$\angle BFD$ and $\angle CFD$ are adjacent angles.**

No. They have a common side ($\overrightarrow{FD}$) and a common vertex (F), but they also have common interior points. So $\angle BFD$ and $\angle CFD$ are not adjacent.



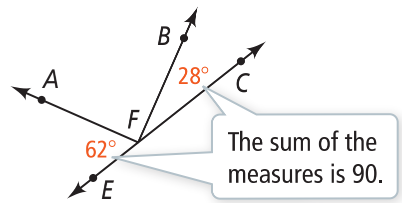
B **$\angle AFB$ and $\angle EFD$ are vertical angles.**

No. $\overrightarrow{FA}$ and $\overrightarrow{FD}$ are opposite rays, but $\overrightarrow{FE}$ and $\overrightarrow{FB}$ are not. So $\angle AFB$ and $\angle EFD$ are not vertical angles.



C **$\angle AFE$ and $\angle BFC$ are complementary.**

Yes. $m\angle AFE + m\angle BFC = 62 + 28 = 90$. The sum of the angle measures is 90, so $\angle AFE$ and $\angle BFC$ are complementary.



Got It? **1.** Use the diagram in Problem 1. Is the statement true? Explain.

a. $\angle AFE$ and $\angle CFD$ are vertical angles.
b. $\angle BFC$ and $\angle DFE$ are supplementary.
c. $\angle BFD$ and $\angle AFB$ are adjacent angles.

take note

Concept Summary Finding Information From a Diagram

There are some relationships you can assume to be true from a diagram that has no marks or measures. There are other relationships you cannot assume directly. For example, you *can* conclude the following from an unmarked diagram.

- Angles are adjacent.
- Angles are adjacent and supplementary.
- Angles are vertical angles.

You *cannot* conclude the following from an unmarked diagram.

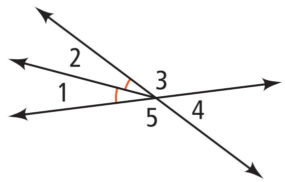
- Angles or segments are congruent.
- An angle is a right angle.
- Angles are complementary.

Think

How can you get information from a diagram?
Look for relationships between angles. For example, look for congruent angles and adjacent angles.

Problem 2 Making Conclusions From a Diagram

What can you conclude from the information in the diagram?

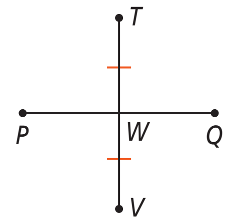


- $\angle 1 \cong \angle 2$ by the markings.
- $\angle 3$ and $\angle 5$ are vertical angles.
- $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, $\angle 4$ and $\angle 5$, and $\angle 5$ and $\angle 1$ are adjacent angles.
- $\angle 3$ and $\angle 4$, and $\angle 4$ and $\angle 5$ are adjacent supplementary angles. So, $m\angle 3 + m\angle 4 = 180$ and $m\angle 4 + m\angle 5 = 180$ by the definition of supplementary angles.

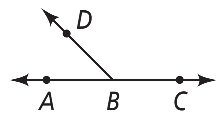
Got It? **2.** Can you make each conclusion from the information in the diagram? Explain.

a. $\overline{TW} \cong \overline{WV}$ **b.** $\overline{PW} \cong \overline{WQ}$

c. $\angle TWQ$ is a right angle. **d.** $\overline{TV}$ bisects $\overline{PQ}$.



A **linear pair** is a pair of adjacent angles whose noncommon sides are opposite rays. The angles of a linear pair form a straight angle.



take note

Postulate 1-9 Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.

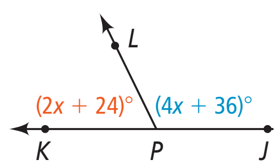
Problem 3 Finding Missing Angle Measures

Algebra **$\angle KPL$ and $\angle JPL$ are a linear pair, $m\angle KPL = 2x + 24$, and $m\angle JPL = 4x + 36$. What are the measures of $\angle KPL$ and $\angle JPL$?**

Know	Need	Plan
$\angle KPL$ and $\angle JPL$ are supplementary.	$m\angle KPL$ and $m\angle JPL$	*Draw a diagram.* Use the definition of supplementary angles to write and solve an equation.

Step 1

$m\angle KPL + m\angle JPL = 180$	Def. of supplementary angles
$(2x + 24) + (4x + 36) = 180$	Substitute.
$6x + 60 = 180$	Combine like terms.
$6x = 120$	Subtract 60 from each side.
$x = 20$	Divide each side by 6.



Step 2 Evaluate the original expressions for $x = 20$.

$m\angle KPL = 2x + 24 = 2 \cdot 20 + 24 = 40 + 24 = 64$

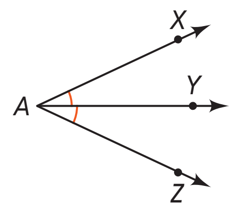
$m\angle JPL = 4x + 36 = 4 \cdot 20 + 36 = 80 + 36 = 116$

Substitute 20 for x.

Got It? 3. a. **Reasoning** How can you check your results in Problem 3?

b. $\angle ADB$ and $\angle BDC$ are a linear pair. $m\angle ADB = 3x + 14$ and $m\angle BDC = 5x - 2$. What are $m\angle ADB$ and $m\angle BDC$?

An **angle bisector** is a ray that divides an angle into two congruent angles. Its endpoint is at the angle vertex. Within the ray, a segment with the same endpoint is also an angle bisector. The ray or segment bisects the angle. In the diagram, $\overrightarrow{AY}$ is the angle bisector of $\angle XAZ$, so $\angle XAY \cong \angle YAZ$.



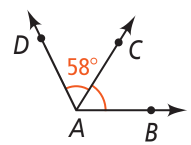
Problem 4 Using an Angle Bisector to Find Angle Measures

Multiple Choice $\overrightarrow{AC}$ **bisects** $\angle DAB$**. If** $m\angle DAC = 58$**, what is** $m\angle DAB$**?**

(A) 29 (B) 58 (C) 87 (D) 116

Plan

Draw a diagram to help you visualize what you are given and what you need to find.



Draw a diagram.

$m\angle CAB = m\angle DAC$	Definition of angle bisector
$= 58$	Substitute.
$m\angle DAB = m\angle CAB + m\angle DAC$	Angle Addition Postulate
$= 58 + 58$	Substitute.
$= 116$	Simplify.

The measure of $\angle DAB$ is 116. The correct choice is D.

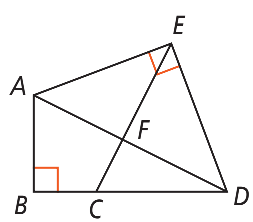
Got It? 4. $\overrightarrow{KM}$ bisects $\angle JKL$. If $m\angle JKL = 72$, what is $m\angle JKM$?

Lesson Check

Do you know HOW?

Name a pair of the following types of angle pairs.

1. vertical angles
2. complementary angles
3. linear pair

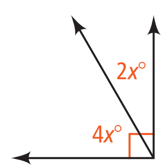


4. $\overrightarrow{PB}$ bisects $\angle RPT$ so that $m\angle RPB = x + 2$ and $m\angle TPB = 2x - 6$. What is $m\angle RPT$?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** How does the term *linear pair* describe how the angle pair looks?

6. **Error Analysis** Your friend calculated the value of x below. What is her error?



~~4x + 2x = 180~~
~~6x = 180~~
~~x = 30~~

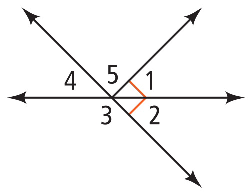
Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Use the diagram at the right. Is each statement true? Explain.

See Problem 1.



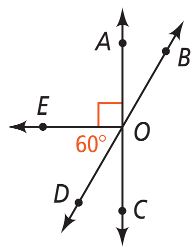
7. $\angle 1$ and $\angle 5$ are adjacent angles.

8. $\angle 3$ and $\angle 5$ are vertical angles.

9. $\angle 3$ and $\angle 4$ are complementary.

10. $\angle 1$ and $\angle 2$ are supplementary.

Name an angle or angles in the diagram described by each of the following.



11. supplementary to $\angle AOD$

12. adjacent and congruent to $\angle AOE$

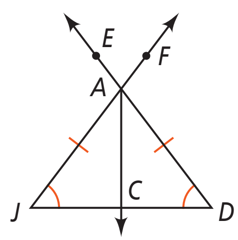
13. supplementary to $\angle EOA$

14. complementary to $\angle EOD$

15. a pair of vertical angles

For Exercises 16–23, can you make each conclusion from the information in the diagram? Explain.

See Problem 2.



16. $\angle J \cong \angle D$

17. $\angle JAC \cong \angle DAC$

18. $m\angle JCA = m\angle DCA$

19. $m\angle JCA + m\angle ACD = 180$

20. $\overline{AJ} \cong \overline{AD}$

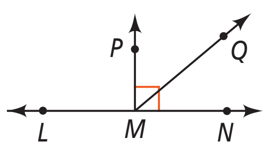
21. C is the midpoint of $\overline{JD}$.

22. $\angle JAE$ and $\angle EAF$ are adjacent and supplementary.

23. $\angle EAF$ and $\angle JAD$ are vertical angles.

See Problem 3.

24. Name two pairs of angles that form a linear pair in the diagram at the right.



25. $\angle EFG$ and $\angle GFH$ are a linear pair, $m\angle EFG = 2n + 21$, and $m\angle GFH = 4n + 15$. What are $m\angle EFG$ and $m\angle GFH$?

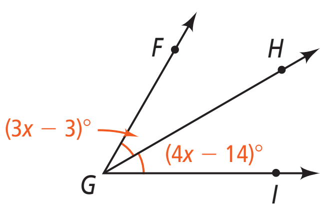
See Problem 4.

26. **Algebra** In the diagram, $\overrightarrow{GH}$ bisects $\angle FGI$.

a. Solve for x and find $m\angle FGH$.

b. Find $m\angle HGI$.

c. Find $m\angle FGI$.



B Apply

Algebra $\overrightarrow{BD}$ **bisects** $\angle ABC$**. Solve for** x **and find** $m\angle ABC$**.**

27. $m\angle ABD = 5x,\ m\angle DBC = 3x + 10$

28. $m\angle ABC = 4x - 12,\ m\angle ABD = 24$

29. $m\angle ABD = 4x - 16,\ m\angle CBD = 2x + 6$

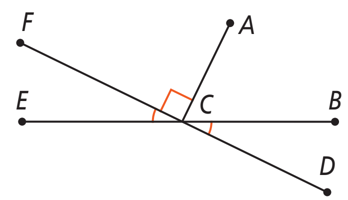
30. $m\angle ABD = 3x + 20,\ m\angle CBD = 6x - 16$

Algebra **Find the measure of each angle in the angle pair described.**

31. Think About a Plan The measure of one angle is twice the measure of its supplement.
- How many angles are there? What is their relationship?
- How can you use algebra, such as using the variable x, to help you?

32. The measure of one angle is 20 less than the measure of its complement.

In the diagram at the right, $m\angle ACB = 65$**. Find each of the following.**



33. $m\angle ACD$

34. $m\angle BCD$

35. $m\angle ECD$

36. $m\angle ACE$

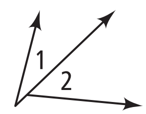
37. Algebra $\angle RQS$ and $\angle TQS$ are a linear pair where $m\angle RQS = 2x + 4$ and $m\angle TQS = 6x + 20$.

a. Solve for x.

b. Find $m\angle RQS$ and $m\angle TQS$.

c. Show how you can check your answer.

38. Writing In the diagram at the right, are $\angle 1$ and $\angle 2$ adjacent? Justify your reasoning.



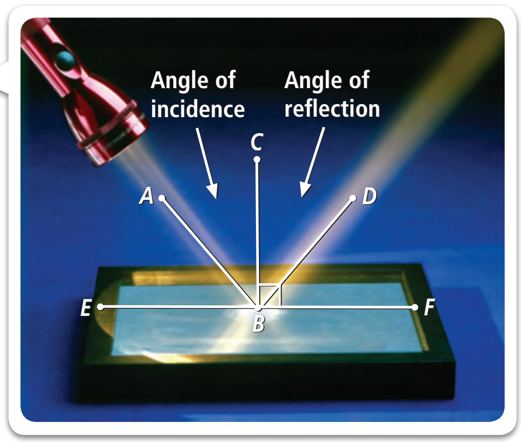
39. Reasoning When $\overrightarrow{BX}$ bisects $\angle ABC$, $\angle ABX \cong \angle CBX$. One student claims there is always a related equation $m\angle ABX = \frac{1}{2}m\angle ABC$. Another student claims the related equation is $2m\angle ABX = m\angle ABC$. Who is correct? Explain.

40. Optics A beam of light and a mirror can be used to study the behavior of light. Light that strikes the mirror is reflected so that the angle of reflection and the angle of incidence are congruent. In the diagram, $\angle ABC$ has a measure of 41.

a. Name the angle of reflection and find its measure.

b. Find $m\angle ABD$.

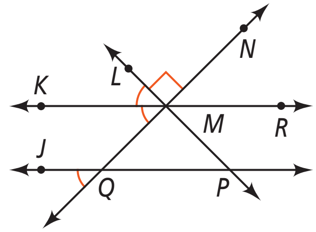
c. Find $m\angle ABE$ and $m\angle DBF$.



41. Reasoning Describe all situations where vertical angles are also supplementary.

Challenge

Name all of the angle(s) in the diagram described by the following.



42. supplementary to $\angle JQM$

43. adjacent and congruent to $\angle KMQ$

44. a linear pair with $\angle LMQ$

45. complementary to $\angle NMR$

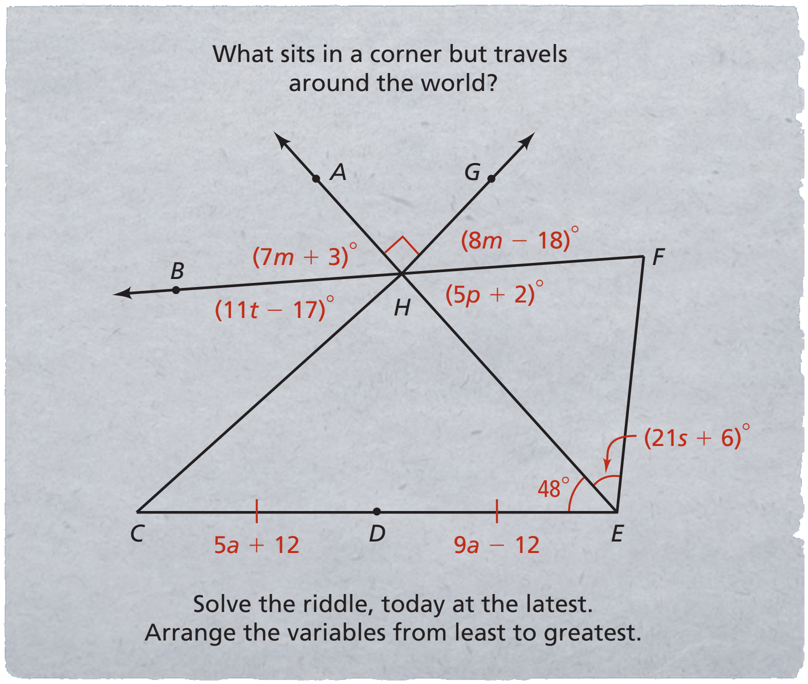
46. Coordinate Geometry The x- and y-axes of the coordinate plane form four right angles. The interior of each of the right angles is a quadrant of the coordinate plane. What is the equation for the line that contains the angle bisector of Quadrants I and III?

47. $\overrightarrow{XC}$ bisects $\angle AXB$, $\overrightarrow{XD}$ bisects $\angle AXC$, $\overrightarrow{XE}$ bisects $\angle AXD$, $\overrightarrow{XF}$ bisects $\angle EXD$, $\overrightarrow{XG}$ bisects $\angle EXF$, and $\overrightarrow{XH}$ bisects $\angle DXB$. If $m\angle DXC = 16$, find $m\angle GXH$.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 3

Look back at the information on page 3 about the riddle Cameron found in an antique store. The page from the old riddle book is shown again below.



a. Name a pair of adjacent complementary angles in the diagram. Explain how you know they are complementary.

b. Name a pair of nonadjacent complementary angles in the diagram.

c. In the Apply What You've Learned sections in Lessons 1-3 and 1-4, you found the values of the variables a and s. Which variable's value can you find next? Find the value of this variable.

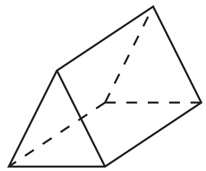
1 Mid-Chapter Quiz

MathXL® for School
Go to PowerGeometry.com

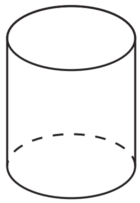
Do you know HOW?

Draw a net for each figure.

1.



2.



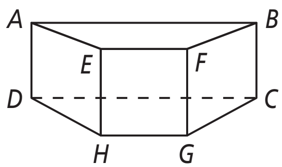
Determine whether the given points are coplanar. If *yes*, name the plane. If *no*, explain.

3. A, E, F, and B

4. D, C, E, and F

5. H, G, F, and B

6. A, E, B, and C

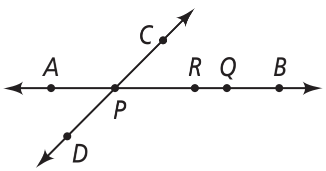


7. Use the figure from Exercises 3–6. Name the intersection of each pair of planes.

a. plane $AEFB$ and plane $CBFG$

b. plane $EFGH$ and plane $AEHD$

Use the figure below for Exercises 8–15.



8. Give two other names for $\overleftrightarrow{AB}$.

9. Give two other names for $\overrightarrow{PR}$.

10. Give two other names for $\angle CPR$.

11. Name three collinear points.

12. Name two opposite rays.

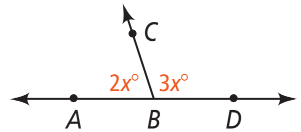
13. Name three segments.

14. Name two angles that form a linear pair.

15. Name a pair of vertical angles.

16. a. Algebra Find the value of x in the diagram below.

b. Classify $\angle ABC$ and $\angle CBD$ as *acute*, *right*, or *obtuse*.



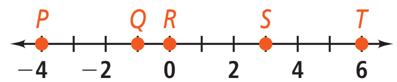
Find the length of each segment.

17. $\overline{PQ}$

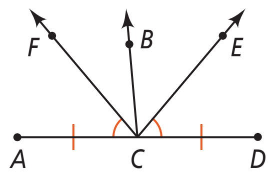
18. $\overline{RS}$

19. $\overline{ST}$

20. $\overline{QT}$



Use the figure below for Exercises 21–23.



21. Algebra If $AC = 4x + 5$ and $DC = 3x + 8$, find AD.

22. If $m\angle FCD = 130$ and $m\angle BCD = 95$, find $m\angle FCB$.

23. If $m\angle FCA = 50$, find $m\angle FCE$.

Do you UNDERSTAND?

24. Error Analysis Suppose $PQ = QR$. Your friend says that Q is always the midpoint of $\overline{PR}$. Is he correct? Explain.

25. Reasoning Determine whether the following situation is possible. Explain your reasoning. Include a sketch.

Collinear points C, F, and G lie in plane M. $\overleftrightarrow{AB}$ intersects plane M at C. $\overleftrightarrow{AB}$ and $\overleftrightarrow{GF}$ do not intersect.

Concept Byte

Use With Lesson 1-6

ACTIVITY

Compass Designs

Common Core State Standards

Prepares for G-CO.D.12 Make formal geometric constructions with a variety of tools and methods (. . . dynamic geometric software . . .)

MP 5

Mathematics Florida Standards

Prepares for MAFS.912.G-CO.4.12 Make formal geometric constructions with a variety of tools and methods (. . . dynamic geometric software . . .)

MP 5

In Lesson 1-6, you will use a compass to construct geometric figures. You can construct figures to show geometric relationships, to suggest new relationships, or simply to make interesting geometric designs.

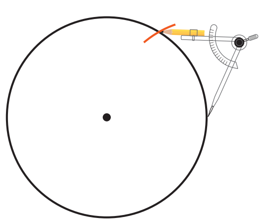
Activity

Step 1 Open your compass to about 2 in. Make a circle and mark the point at the center of the circle. Keep the opening of your compass fixed. Place the compass point on the circle. With the pencil end, make a small arc to intersect the circle.

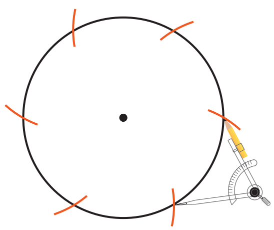
Step 2 Place the compass point on the circle at the arc. Mark another arc. Continue around the circle this way to draw four more arcs—six in all.

Step 3 Place your compass point on an arc you marked on the circle. Place the pencil end at the next arc. Draw a large arc that passes through the circle's center and continues to another point on the circle.

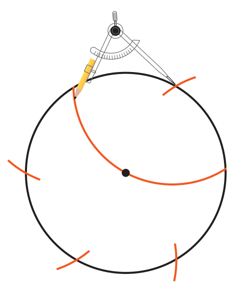
Step 4 Draw six large arcs in this manner, each centered at one of the six points marked on the circle. You may choose to color your design.



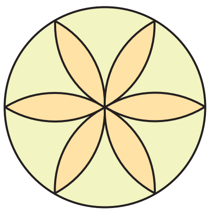
Step 1



Step 2



Step 3



Step 4

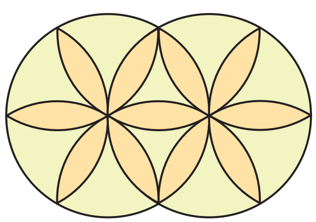
Exercises

1. In Step 2, did your sixth mark on the circle land precisely on the point where you first placed your compass on the circle?

a. Survey the class to find out how many did.

b. Explain why your sixth mark may not have landed on your starting point.

2. Extend your design by using one of the six points on the circle as the center for a new circle. Repeat Steps 1–4 with this circle. Repeat several times to make interlocking circles



1-6 Basic Constructions

Common Core State Standards

G-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge . . .). **Also G-CO.A.1**

MP 1, MP 3, MP 5, MP 7

Mathematics Florida Standards

MAFS.912.G-CO.4.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge . . .). **Also MAFS.912.G-CO.1.1**

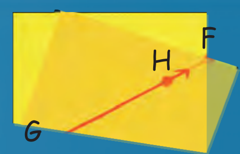
MP 1, MP 3, MP 5, MP 7

Objective To make basic constructions using a straightedge and a compass

Think about how you might compare angles without measuring them.

SOLVE IT! Getting Ready!

Draw $\angle FGH$. Fold your paper so that $\overline{GH}$ lies on top of $\overline{GF}$. Unfold the paper. Label point J on the fold line in the interior of $\angle FGH$. How is $\overrightarrow{GJ}$ related to $\angle FGH$? How do you know?



MATHEMATICAL PRACTICES In this lesson, you will learn another way to construct figures like the one above.

Lesson Vocabulary
- straightedge
- compass
- construction
- perpendicular lines
- perpendicular bisector

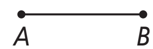
Essential Understanding You can use special geometric tools to make a figure that is congruent to an original figure without measuring. This method is more accurate than sketching and drawing.

A **straightedge** is a ruler with no markings on it. A **compass** is a geometric tool used to draw circles and parts of circles called *arcs*. A **construction** is a geometric figure drawn using a straightedge and a compass.

Problem 1 Constructing Congruent Segments

Construct a segment congruent to a given segment.

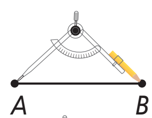
Given: $\overline{AB}$



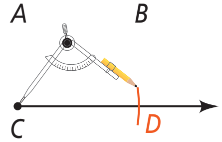
Construct: $\overline{CD}$ **so that** $\overline{CD} \cong \overline{AB}$

Step 1 Draw a ray with endpoint C.

Step 2 Open the compass to the length of $\overline{AB}$.



Step 3 With the same compass setting, put the compass point on point C. Draw an arc that intersects the ray. Label the point of intersection D.



$\overline{CD} \cong \overline{AB}$

Think

Why must the compass setting stay the same?

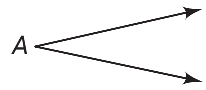
Using the same compass setting keeps segments congruent. It guarantees that the lengths of $\overline{AB}$ and $\overline{CD}$ are exactly the same.

Got It? 1. Use a straightedge to draw $\overline{XY}$. Then construct $\overline{RS}$ so that $RS = 2XY$.

Problem 2 Constructing Congruent Angles

Construct an angle congruent to a given angle.

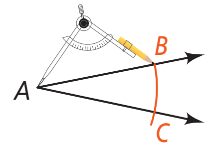
Given: $\angle A$



Construct: $\angle S$ **so that** $\angle S \cong \angle A$

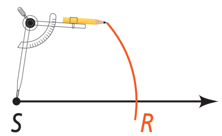
Step 1
Draw a ray with endpoint S.

Step 2
With the compass point on vertex A, draw an arc that intersects the sides of $\angle A$. Label the points of intersection B and C.

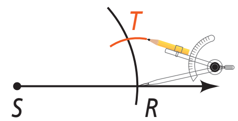


Think
Why do you need points like *B* and *C*?
B and *C* are reference points on the original angle. You can construct a congruent angle by locating corresponding points *R* and *T* on your new angle.

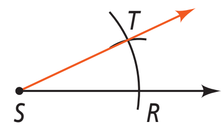
Step 3
With the same compass setting, put the compass point on point S. Draw an arc and label its point of intersection with the ray as R.



Step 4
Open the compass to the length BC. Keeping the same compass setting, put the compass point on R. Draw an arc to locate point T.

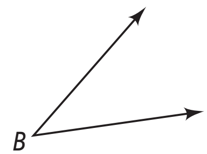


Step 5
Draw $\overrightarrow{ST}$.

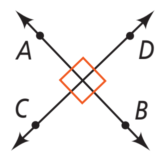


$\angle S \cong \angle A$

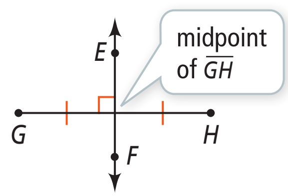
Got It? 2. a. Construct $\angle F$ so that $m\angle F = 2m\angle B$.
b. Reasoning How is constructing a congruent angle similar to constructing a congruent segment?



Perpendicular lines are two lines that intersect to form right angles. The symbol $\perp$ means "is perpendicular to." In the diagram at the right, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$.



A **perpendicular bisector** of a segment is a line, segment, or ray that is perpendicular to the segment at its midpoint. In the diagram at the right, $\overleftrightarrow{EF}$ is the perpendicular bisector of $\overline{GH}$. The perpendicular bisector bisects the segment into two congruent segments. The construction in Problem 3 will show you how this works. You will justify the steps for this construction in Chapter 4, as well as for the other constructions in this lesson.



Problem 3 Constructing the Perpendicular Bisector

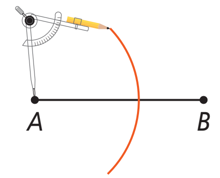
Construct the perpendicular bisector of a segment.

Given: $\overline{AB}$

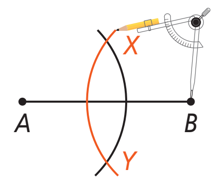
Construct: $\overleftrightarrow{XY}$ so that $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$

Think
Why must the compass opening be greater than $\frac{1}{2}AB$?
If the opening is less than $\frac{1}{2}AB$, the two arcs will not intersect in Step 2.

Step 1
Put the compass point on point A and draw a long arc as shown. Be sure the opening is greater than $\frac{1}{2}AB$.

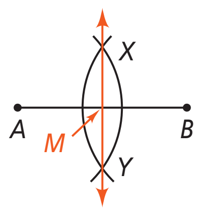


Step 2
With the same compass setting, put the compass point on point B and draw another long arc. Label the points where the two arcs intersect as X and Y.



Step 3
Draw $\overleftrightarrow{XY}$. Label the point of intersection of $\overline{AB}$ and $\overleftrightarrow{XY}$ as M, the midpoint of $\overline{AB}$.

$\overleftrightarrow{XY} \perp \overline{AB}$ at midpoint M, so $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$.

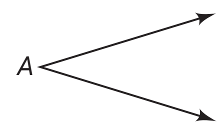


Got It? **3.** Draw $\overline{ST}$. Construct its perpendicular bisector.

Problem 4 Constructing the Angle Bisector

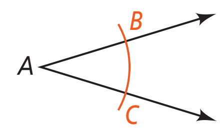
Construct the bisector of an angle.

Given: $\angle A$



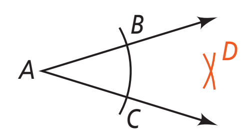
Construct: $\overrightarrow{AD}$, the bisector of $\angle A$

Step 1
Put the compass point on vertex A. Draw an arc that intersects the sides of $\angle A$. Label the points of intersection B and C.



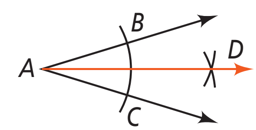
Think
Why must the arcs intersect?
The arcs need to intersect so that you have a point through which to draw a ray.

Step 2
Put the compass point on point C and draw an arc. With the same compass setting, draw an arc using point B. Be sure the arcs intersect. Label the point where the two arcs intersect as D.



Step 3
Draw $\overrightarrow{AD}$.

$\overrightarrow{AD}$ is the bisector of $\angle CAB$.



Got It? **4.** Draw obtuse $\angle XYZ$. Then construct its bisector $\overrightarrow{YP}$.

Lesson Check

Do you know HOW?

For Exercises 1 and 2, draw $\overline{PQ}$. Use your drawing as the original figure for each construction.

1. Construct a segment congruent to $\overline{PQ}$.
2. Construct the perpendicular bisector of $\overline{PQ}$.
3. Draw an obtuse $\angle JKL$. Construct its bisector.

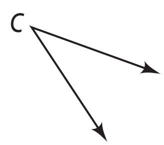
Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** What two tools do you use to make constructions?
5. **Compare and Contrast** Describe the difference in accuracy between sketching a figure, drawing a figure with a ruler and protractor, and constructing a figure. Explain.
6. **Error Analysis** Your friend constructs $\overleftrightarrow{XY}$ so that it is perpendicular to and contains the midpoint of $\overline{AB}$. He claims that $\overline{AB}$ is the perpendicular bisector of $\overleftrightarrow{XY}$. What is his error?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

For Exercises 7–14, draw a diagram similar to the given one. Then do the construction. Check your work with a ruler or a protractor.



7. Construct $\overline{XY}$ congruent to $\overline{AB}$. See Problem 1.
8. Construct $\overline{VW}$ so that $VW = 2AB$.
9. Construct $\overline{DE}$ so that $DE = TR + PS$.
10. Construct $\overline{QJ}$ so that $QJ = TR - PS$.
11. Construct $\angle D$ so that $\angle D \cong \angle C$. See Problem 2.
12. Construct $\angle F$ so that $m\angle F = 2m\angle C$.
13. Construct the perpendicular bisector of $\overline{AB}$. See Problem 3.
14. Construct the perpendicular bisector of $\overline{TR}$.
15. Draw acute $\angle PQR$. Then construct its bisector. See Problem 4.
16. Draw obtuse $\angle XQZ$. Then construct its bisector.

B Apply

Sketch the figure described. Explain how to construct it. Then do the construction.

17. $\overleftrightarrow{XY} \perp \overleftrightarrow{YZ}$
18. $\overrightarrow{ST}$ bisects right $\angle PSQ$.
19. **Compare and Contrast** How is constructing an angle bisector similar to constructing a perpendicular bisector?

20. **Think About a Plan** Draw an $\angle A$. Construct an angle whose measure is $\frac{1}{4}m\angle A$.
- How is the angle you need to construct related to the angle bisector of $\angle A$?
- How can you use previous constructions to help you?

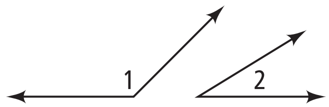
21. Answer the questions about a segment in a plane. Explain each answer.
   a. How many midpoints does the segment have?
   b. How many bisectors does it have?
   c. How many lines in the plane are its perpendicular bisectors?
   d. How many lines in space are its perpendicular bisectors?

For Exercises 22–24, copy ∠1 and ∠2. Construct each angle described.

22. $\angle B$; $m\angle B = m\angle 1 + m\angle 2$

23. $\angle C$; $m\angle C = m\angle 1 - m\angle 2$

24. $\angle D$; $m\angle D = 2m\angle 2$



25. **Writing** Explain how to do each construction with a compass and straightedge.
   a. Draw a segment $\overline{PQ}$. Construct the midpoint of $\overline{PQ}$.
   b. Divide $\overline{PQ}$ into four congruent segments.

26. **a.** Draw a large triangle with three acute angles. Construct the bisectors of the three angles. What appears to be true about the three angle bisectors?
   b. Repeat the constructions with a triangle that has one obtuse angle.
   c. **Make a Conjecture** What appears to be true about the three angle bisectors of any triangle?

Use a ruler to draw segments of 2 cm, 4 cm, and 5 cm. Then construct each triangle with the given side measures, if possible. If it is not possible, explain why not.

27. 4 cm, 4 cm, and 5 cm

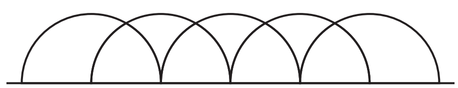
28. 2 cm, 5 cm, and 5 cm

29. 2 cm, 2 cm, and 5 cm

30. 2 cm, 2 cm, and 4 cm

31. **a.** Draw a segment, $\overline{XY}$. Construct a triangle with sides congruent to $\overline{XY}$.
   b. Measure the angles of the triangle.
   c. **Writing** Describe how to construct a 60° angle using what you know. Then describe how to construct a 30° angle.

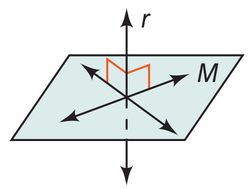
32. Which steps best describe how to construct the pattern at the right?



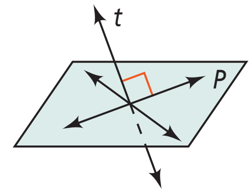
   (A) Use a straightedge to draw the segment and then a compass to draw five half circles.
   (B) Use a straightedge to draw the segment and then a compass to draw six half circles.
   (C) Use a compass to draw five half circles and then a straightedge to join their ends.
   (D) Use a compass to draw six half circles and then a straightedge to join their ends.

Challenge

33. Study the figures. Complete the definition of a line perpendicular to a plane: A line is perpendicular to a plane if it is __?__ to every line in the plane that __?__.



Line $r \perp$ plane M.



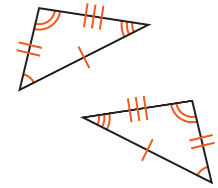
Line t is not $\perp$ plane P.

34. a. Use your compass to draw a circle. Locate three points A, B, and C on the circle.

b. Draw $\overline{AB}$ and $\overline{BC}$. Then construct the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$.

c. **Reasoning** Label the intersection of the two perpendicular bisectors as point O. What do you think is true about point O?

35. Two triangles are *congruent* if each side and each angle of one triangle is congruent to a side or angle of the other triangle. In Chapter 4, you will learn that if each side of one triangle is congruent to a side of the other triangle, then you can conclude that the triangles are congruent without finding the angles. Explain how you can use congruent triangles to justify the angle bisector construction.



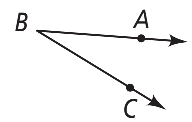
Standardized Test Prep

SAT/ACT

36. What must you do to construct the midpoint of a segment?

(A) Measure half its length.
(B) Construct an angle bisector.
(C) Measure twice its length.
(D) Construct a perpendicular bisector.

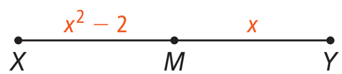
37. Given the diagram at the right, what is NOT a reasonable name for the angle?



(F) $\angle ABC$
(G) $\angle B$
(H) $\angle CBA$
(I) $\angle ACB$

Short Response

38. M is the midpoint of $\overline{XY}$. Find the value of x. Show your work.



Mixed Review

39. $\angle DEF$ is the supplement of $\angle DEG$ with $m\angle DEG = 64$. What is $m\angle DEF$?

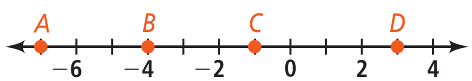
See Lesson 1-5.

40. $m\angle TUV = 100$ and $m\angle VUW = 80$. Are $\angle TUV$ and $\angle VUW$ a linear pair? Explain.

Find the length of each segment.

See Lesson 1-3.

41. $\overline{AC}$
42. $\overline{AD}$
43. $\overline{CD}$
44. $\overline{BC}$



Get Ready! **To prepare for Lesson 1-7, do Exercises 45–47.**

Algebra Evaluate each expression for $a = 6$ and $b = -8$.

See p. 890.

45. $(a - b)^2$
46. $\sqrt{a^2 + b^2}$
47. $\frac{a + b}{2}$

Concept Byte

Use With Lesson 1-6

TECHNOLOGY

Exploring Constructions

Mathematics Florida Standards

MAFS.912.G-CO.4.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge . . .).

MP 5

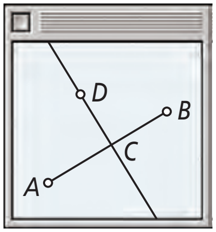
MATHEMATICAL PRACTICES

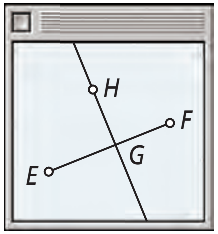
You can use Draw tools or Construct tools in geometry software to make points, lines, and planes. A figure made by Draw has no constraints. When you manipulate, or try to change, a figure made by Draw, it moves or changes size freely. A figure made by Construct is related to an existing object. When you manipulate the existing object, the constructed object moves or resizes accordingly.

In this Activity, you will explore the difference between Draw and Construct.

Activity

Draw $\overline{AB}$ and Construct the perpendicular bisector $\overleftrightarrow{DC}$. Then Draw $\overline{EF}$ and Construct G, any point on $\overline{EF}$. Draw $\overleftrightarrow{HG}$.





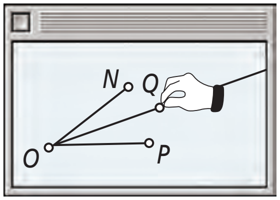
1. Find EG, GF, and $m\angle HGF$. Try to drag G so that $EG = GF$. Try to drag H so that $m\angle HGF = 90$. Were you able to draw the perpendicular bisector of $\overline{EF}$? Explain.

2. Drag A and B. Observe AC, CB, and $m\angle DCB$. Is $\overleftrightarrow{DC}$ always the perpendicular bisector of $\overline{AB}$ no matter how you manipulate the figure?

3. Drag E and F. Observe EG, GF, and $m\angle HGF$. How is the relationship between $\overline{EF}$ and $\overleftrightarrow{HG}$ different from the relationship between $\overline{AB}$ and $\overleftrightarrow{DC}$?

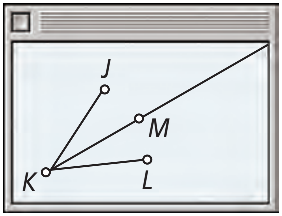
4. Write a description of the general difference between Draw and Construct. Then use your description to explain why the relationship between $\overline{EF}$ and $\overleftrightarrow{HG}$ differs from the relationship between $\overline{AB}$ and $\overleftrightarrow{DC}$.

Exercises



5. a. Draw $\angle NOP$. Draw $\overrightarrow{OQ}$ in the interior of $\angle NOP$. Drag Q until $m\angle NOQ = m\angle QOP$.

b. Manipulate the figure and observe the different angle measures. Is $\overrightarrow{OQ}$ always the angle bisector of $\angle NOP$?



6. a. Draw $\angle JKL$.

b. Construct its angle bisector, $\overrightarrow{KM}$.

c. Manipulate the figure and observe the different angle measures. Is $\overrightarrow{KM}$ always the angle bisector of $\angle JKL$?

d. How can you manipulate the figure on the screen so that it shows a right angle? Justify your answer.

1-7 Midpoint and Distance in the Coordinate Plane

Mathematics Florida Standards

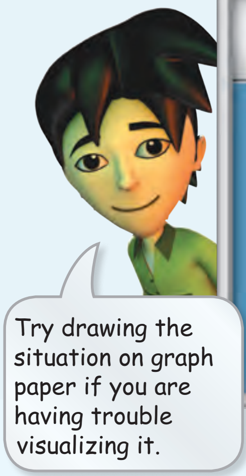
Prepares for MAFS.912.G-GPE.2.4 Use coordinates to prove simple geometric theorems . . .

Prepares for MAFS.912.G-GPE.2.7 Use coordinates to compute perimeters . . . and areas . . .

Also MAFS.912.G-GPE.2.6

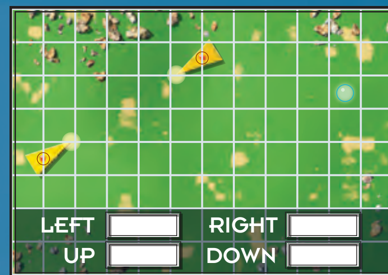
MP 1, MP 3, MP 4

Objectives To find the midpoint of a segment
To find the distance between two points in the coordinate plane



SOLVE IT! **Getting Ready!**

In a video game, two ancient structures shoot light beams toward each other to form a time portal. The portal forms exactly halfway between the two structures. Your character is on the grid shown as a blue dot. How do you direct your character to the portal? Explain how you found your answer.

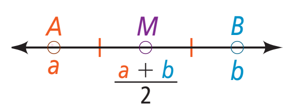
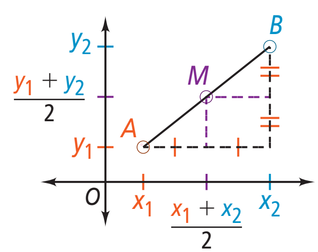


MATHEMATICAL PRACTICES In this lesson, you will learn how to find midpoints and distance on a grid like the one in the Solve It.

Essential Understanding You can use formulas to find the midpoint and length of any segment in the coordinate plane.

take note

Key Concept Midpoint Formulas

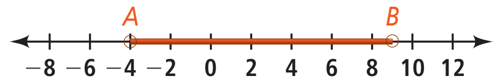
Description	Formula	Diagram
On a Number Line The coordinate of the midpoint is the *average* or *mean* of the coordinates of the endpoints.	The coordinate of the midpoint M of $\overline{AB}$ is $\frac{a+b}{2}$.	
In the Coordinate Plane The coordinates of the midpoint are the average of the x-coordinates and the average of the y-coordinates of the endpoints.	Given $\overline{AB}$ where $A(x_1, y_1)$ and $B(x_2, y_2)$, the coordinates of the midpoint of $\overline{AB}$ are $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.	

Think

Which Midpoint Formula do you use?
If the endpoints are real numbers, use the formula for the number line. If they are ordered pairs, use the formula for the coordinate plane.

Problem 1 Finding the Midpoint

A **$\overline{AB}$ has endpoints at -4 and 9. What is the coordinate of its midpoint?**

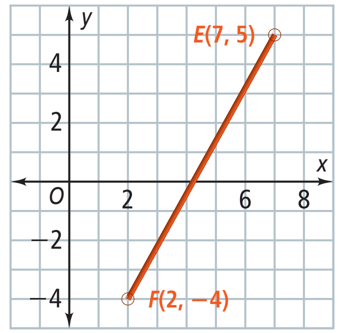


Let $a = -4$ and $b = 9$.

$$M = \frac{a+b}{2} = \frac{-4+9}{2} = \frac{5}{2} = 2.5$$

The coordinate of the midpoint of $\overline{AB}$ is 2.5.

B **$\overline{EF}$ has endpoints $E(7, 5)$ and $F(2, -4)$. What are the coordinates of its midpoint M?**



Let $E(7, 5)$ be (x_1, y_1) and $F(2, -4)$ be (x_2, y_2).

$$x\text{-coordinate of } M = \frac{x_1+x_2}{2} = \frac{7+2}{2} = \frac{9}{2} = 4.5$$

$$y\text{-coordinate of } M = \frac{y_1+y_2}{2} = \frac{5+(-4)}{2} = \frac{1}{2} = 0.5$$

The coordinates of the midpoint of $\overline{EF}$ are $M(4.5, 0.5)$.

Got It? **1. a.** $\overline{JK}$ has endpoints at -12 and 4 on a number line. What is the coordinate of its midpoint?

b. What is the midpoint of $\overline{RS}$ with endpoints $R(5, -10)$ and $S(3, 6)$?

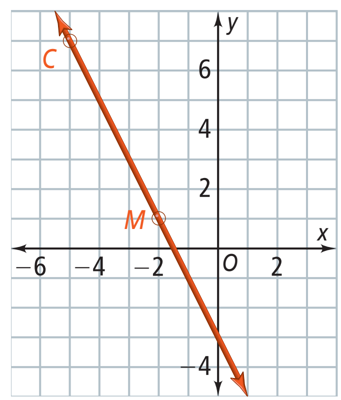
When you know the midpoint and an endpoint of a segment, you can use the Midpoint Formula to find the other endpoint.

Plan

How can you find the coordinates of *D*?
Use the Midpoint Formula to set up an equation. Split that equation into two equations: one for the *x*-coordinate and one for the *y*-coordinate.

Problem 2 Finding an Endpoint

The midpoint of $\overline{CD}$ is $M(-2, 1)$. One endpoint is $C(-5, 7)$. What are the coordinates of the other endpoint D?



Let $M(-2, 1)$ be (x, y) and $C(-5, 7)$ be (x_1, y_1). Let the coordinates of D be (x_2, y_2).

$$(-2, 1) = \left(\frac{-5+x_2}{2}, \frac{7+y_2}{2}\right)$$

x		*y*
$-2 = \frac{-5+x_2}{2}$	Use the Midpoint Formula.	$1 = \frac{7+y_2}{2}$
$-4 = -5 + x_2$	Multiply each side by 2.	$2 = 7 + y_2$
$1 = x_2$	Simplify.	$-5 = y_2$

The coordinates of D are $(1, -5)$.

Got It? **2.** The midpoint of $\overline{AB}$ has coordinates $(4, -9)$. Endpoint A has coordinates $(-3, -5)$. What are the coordinates of B?

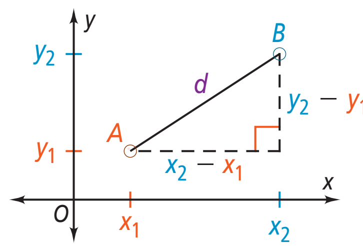
In Lesson 1-3, you learned how to find the distance between two points on a number line. To find the distance between two points in a coordinate plane, you can use the Distance Formula.

take note

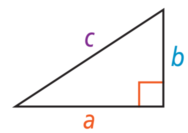
Key Concept Distance Formula

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



The Distance Formula is based on the *Pythagorean Theorem*, which you will study later in this book. When you use the Distance Formula, you are really finding the length of a side of a right triangle. You will verify the Distance Formula in Chapter 8.



$a^2 + b^2 = c^2$

Problem 3 Finding Distance

GRIDDED RESPONSE

What is the distance between $U(-7, 5)$ and $V(4, -3)$? Round to the nearest tenth.

Let $U(-7, 5)$ be (x_1, y_1) and $V(4, -3)$ be (x_2, y_2).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Use the Distance Formula.

$= \sqrt{(4 - (-7))^2 + (-3 - 5)^2}$ Substitute.

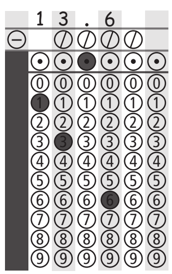
$= \sqrt{(11)^2 + (-8)^2}$ Simplify within the parentheses.

$= \sqrt{121 + 64}$ Simplify.

$= \sqrt{185}$

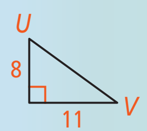
185 √ 13.60147051 Use a calculator.

To the nearest tenth, $UV = 13.6$.



Think

What part of a right triangle is $\overline{UV}$?
$\overline{UV}$ is the hypotenuse of a right triangle with legs of length 11 and 8.

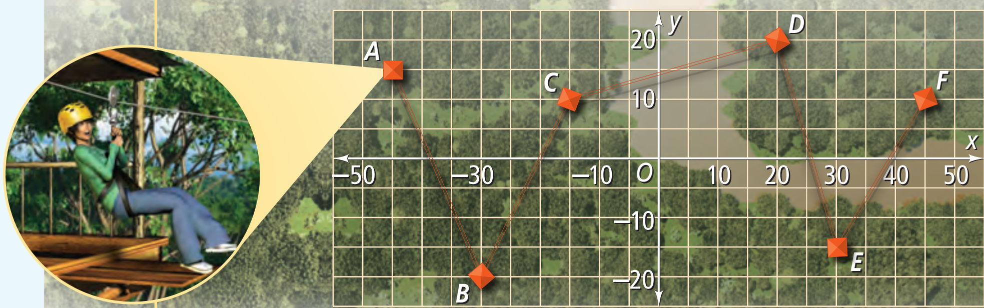


Got It? 3. a. $\overline{SR}$ has endpoints $S(-2, 14)$ and $R(3, -1)$. What is SR to the nearest tenth?

b. Reasoning In Problem 3, suppose you let $V(4, -3)$ be (x_1, y_1) and $U(-7, 5)$ be (x_2, y_2). Do you get the same result? Why?

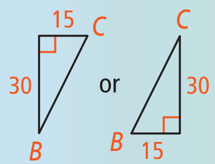
Problem 4 Finding Distance

Recreation **On a zip-line course, you are harnessed to a cable that travels through the treetops. You start at Platform *A* and zip to each of the other platforms. How far do you travel from Platform *B* to Platform *C*? Each grid unit represents 5 m.**



Think

Where's the right triangle?
The lengths of the legs of the right triangle are 15 and 30. There are two possibilities:



Let Platform $B(-30, -20)$ be (x_1, y_1) and Platform $C(-15, 10)$ be (x_2, y_2).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Use the Distance Formula.

$= \sqrt{(-15 - (-30))^2 + (10 - (-20))^2}$ Substitute.

$= \sqrt{15^2 + 30^2} = \sqrt{225 + 900} = \sqrt{1125}$ Simplify.

$\sqrt{\ }$ 1125 ENTER 33.54101966 Use a calculator.

You travel about 33.5 m from Platform *B* to Platform *C*.

Got It? **4.** How far do you travel from Platform *D* to Platform *E*?

Lesson Check

Do you know HOW?

1. $\overline{RS}$ has endpoints $R(2, 4)$ and $S(-1, 7)$. What are the coordinates of its midpoint M?

2. The midpoint of $\overline{BC}$ is $(5, -2)$. One endpoint is $B(3, 4)$. What are the coordinates of endpoint C?

3. What is the distance between points $K(-9, 8)$ and $L(-6, 0)$?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Reasoning** How does the Distance Formula ensure that the distance between two different points is positive?

5. **Error Analysis** Your friend calculates the distance between points $Q(1, 5)$ and $R(3, 8)$. What is his error?

$$\begin{aligned} d &= \sqrt{(1-8)^2 + (5-3)^2} \\ &= \sqrt{(-7)^2 + 2^2} \\ &= \sqrt{49 + 4} \\ &= \sqrt{53} \approx 7.3 \end{aligned}$$

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Find the coordinate of the midpoint of the segment with the given endpoints. See Problem 1.

6. 2 and 4 **7.** -9 and 6 **8.** 2 and -5 **9.** -8 and -12

Find the coordinates of the midpoint of $\overline{HX}$.

10. $H(0, 0)$, $X(8, 4)$ **11.** $H(-1, 3)$, $X(7, -1)$ **12.** $H(13, 8)$, $X(-6, -6)$

13. $H(7, 10)$, $X(5, -8)$ **14.** $H(-6.3, 5.2)$, $X(1.8, -1)$ **15.** $H\left(5\frac{1}{2}, -4\frac{3}{4}\right)$, $X\left(2\frac{1}{4}, -1\frac{1}{4}\right)$

The coordinates of point T are given. The midpoint of $\overline{ST}$ is $(5, -8)$. Find the coordinates of point S. See Problem 2.

16. $T(0, 4)$ **17.** $T(5, -15)$ **18.** $T(10, 18)$

19. $T(-2, 8)$ **20.** $T(1, 12)$ **21.** $T(4.5, -2.5)$

Find the distance between each pair of points. If necessary, round to the nearest tenth. See Problem 3.

22. $J(2, -1)$, $K(2, 5)$ **23.** $L(10, 14)$, $M(-8, 14)$ **24.** $N(-1, -11)$, $P(-1, -3)$

25. $A(0, 3)$, $B(0, 12)$ **26.** $C(12, 6)$, $D(-8, 18)$ **27.** $E(6, -2)$, $F(-2, 4)$

28. $Q(12, -12)$, $T(5, 12)$ **29.** $R(0, 5)$, $S(12, 3)$ **30.** $X(-3, -4)$, $Y(5, 5)$

Maps **For Exercises 31–35, use the map below. Find the distance between the cities to the nearest tenth.** See Problem 4.

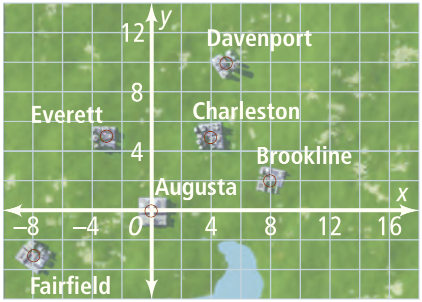
31. Augusta and Brookline

32. Brookline and Charleston

33. Brookline and Davenport

34. Everett and Fairfield

35. List the cities in the order of least to greatest distance from Augusta.



B Apply

Find (a) PQ to the nearest tenth and (b) the coordinates of the midpoint of $\overline{PQ}$.

36. $P(3, 2)$, $Q(6, 6)$ **37.** $P(0, -2)$, $Q(3, 3)$ **38.** $P(-4, -2)$, $Q(1, 3)$

39. $P(-5, 2)$, $Q(0, 4)$ **40.** $P(-3, -1)$, $Q(5, -7)$ **41.** $P(-5, -3)$, $Q(-3, -5)$

42. $P(-4, -5)$, $Q(-1, 1)$ **43.** $P(2, 3)$, $Q(4, -2)$ **44.** $P(4, 2)$, $Q(3, 0)$

45. Think About a Plan An airplane at $T(80, 20)$ needs to fly to both $U(20, 60)$ and $V(110, 85)$. What is the shortest possible distance for the trip? Explain.
- What type of information do you need to find the shortest distance?
- How can you use a diagram to help you?

46. Reasoning The endpoints of $\overline{AB}$ are $A(-2, -3)$ and $B(3, 2)$. Point C lies on AB and is $\frac{2}{5}$ of the way from A to B. What are the coordinates of Point C? Explain how you found your answer.

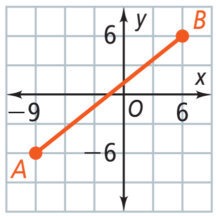
47. Do you use the Midpoint Formula or the Distance Formula to find the following?

a. Given points K and P, find the distance from K to the midpoint of $\overline{KP}$.

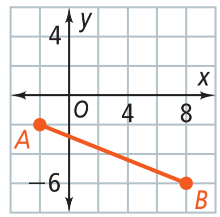
b. Given point K and the midpoint of $\overline{KP}$, find KP.

For each graph, find (a) AB to the nearest tenth and (b) the coordinates of the midpoint of $\overline{AB}$.

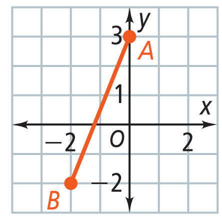
48.



49.



50.

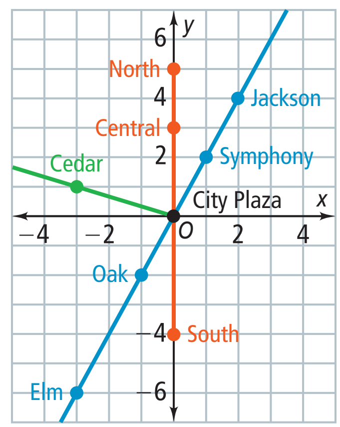


51. Coordinate Geometry Graph the points $A(2, 1)$, $B(6, -1)$, $C(8, 7)$, and $D(4, 9)$. Draw parallelogram $ABCD$, and diagonals $\overline{AC}$ and $\overline{BD}$.

a. Find the midpoints of $\overline{AC}$ and $\overline{BD}$.

b. What appears to be true about the diagonals of a parallelogram?

Travel **The units of the subway map at the right are in miles. Suppose the routes between stations are straight. Find the distance you would travel between each pair of stations to the nearest tenth of a mile.**



52. Oak Station and Jackson Station

53. Central Station and South Station

54. Elm Station and Symphony Station

55. Cedar Station and City Plaza Station

56. Maple Station is located 6 mi west and 2 mi north of City Plaza. What is the distance between Cedar Station and Maple Station?

57. Open-Ended Point $H(2, 2)$ is the midpoint of many segments.

a. Find the coordinates of the endpoints of four noncollinear segments that have point H as their midpoint.

b. You know that a segment with midpoint H has length 8. How many possible noncollinear segments match this description? Explain.

Challenge

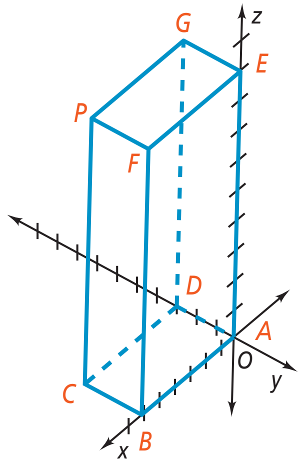
58. Points $P(-4, 6)$, $Q(2, 4)$, and R are collinear. One of the points is the midpoint of the segment formed by the other two points.

a. What are the possible coordinates of R?

b. Reasoning $RQ = \sqrt{160}$. Does this information affect your answer to part (a)? Explain.

Geometry in 3 Dimensions You can use three coordinates (x, y, z) to locate points in three dimensions.

59. Point P has coordinates $(6, -3, 9)$ as shown at the right. Give the coordinates of points A, B, C, D, E, F, and G.



Distance in 3 Dimensions In a three-dimensional coordinate system, you can find the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) with this extension of the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Find the distance between each pair of points to the nearest tenth.

60. $P(2, 3, 4)$, $Q(-2, 4, 9)$ **61.** $T(0, 12, 15)$, $V(-8, 20, 12)$

Standardized Test Prep

SAT/ACT

62. A segment has endpoints $(14, -8)$ and $(4, 12)$. What are the coordinates of its midpoint?

(A) $(9, 10)$ (B) $(-5, 10)$ (C) $(5, -10)$ (D) $(9, 2)$

63. Which of these is the first step in constructing a congruent segment?

(F) Draw a ray. (H) Label two points.

(G) Find the midpoint. (I) Measure the segment.

Short Response

64. The midpoint of $\overline{RS}$ is $N(-4, 1)$. One endpoint is $S(0, -7)$.

a. What are the coordinates of R?

b. What is the length of $\overline{RS}$ to the nearest tenth of a unit?

Mixed Review

Use a straightedge and a compass. See Lesson 1-6.

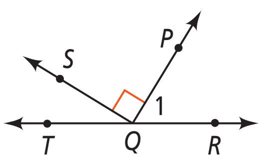
65. Draw $\overline{AB}$. Construct $\overline{PQ}$ so that $PQ = 2AB$.

66. Draw an acute $\angle RTS$. Construct the bisector of $\angle RTS$.

Use the diagram at the right. See Lesson 1-4.

67. Name $\angle 1$ two other ways.

68. If $m\angle PQR = 60$, what is $m\angle RQS$?



Get Ready! **To prepare for Lesson 1-8, do Exercises 69–72.**

Complete each statement. Use the conversion table on page 837. See p. 886.

69. 130 in. = ■ ft **70.** 14 yd = ■ in. **71.** 27 ft = ■ yd **72.** 2 mi = ■ ft

Concept Byte

For Use With Lesson 1-7

Partitioning a Line Segment

Common Core State Standards

G-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Mathematics Florida Standards

MAFS.912.G-GPE.2.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

MP 8

The *directed line segment* from A to B starts at point A and ends at point B.

Suppose point P partitions the directed line segment from A to B in the ratio 2 to 3. You can think of $\overline{AB}$ as being divided into 5 congruent parts so that the length of $\overline{AP}$ is the length of 2 parts and PB is the length of 3 parts.

Example

$\overline{LM}$ is the directed line segment from $L(-4, 1)$ to $M(5, -5)$. What are the coordinates of the point that partitions the segment in the ratio 2 to 1?

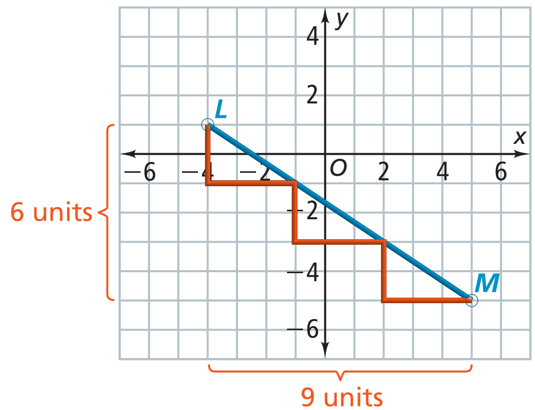
Step 1 Graph $\overline{LM}$ in the coordinate plane.

Step 2 As you move from L to M, you move down 6 units and right 9 units. Divide each of these distances into three equal parts.

vertical distance: $6 \div 3 = 2$

horizontal distance: $9 \div 3 = 3$

Step 3 Beginning at L, move down 2 units and right 3 units to the point at $(-1, -1)$. Repeat this process two more times to reach the points at $(2, -3)$ and $M(5, -5)$.



The points at $(-1, -1)$ and $(2, -3)$ divide $\overline{LM}$ into three congruent parts. Let P be the point with coordinates $(2, -3)$. Then the ratio LP to PM is 2 to 1. So, the coordinates of the point that partitions the directed line segment $\overline{LM}$ in the ratio 2 to 1 are $(2, -3)$.

Exercises

1. **Reasoning** In the Example, how can you show that the points at $(-1, -1)$ and $(2, -3)$ divide $\overline{LM}$ into three congruent parts?

2. $\overline{RS}$ is the directed line segment from $R(-2, -3)$ to $S(8, 2)$. What are the coordinates of the point that partitions the segment in the ratio 2 to 3?

3. Point C lies on the directed line segment from $A(5, 16)$ to $B(-1, 2)$ and partitions the segment in the ratio 1 to 2. What are the coordinates of C?

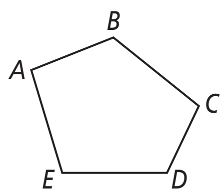
4. The endpoints of $\overline{XY}$ are $X(2, -6)$ and $Y(-6, 2)$. What are the coordinates of point P on $\overline{XY}$ such that XP is $\frac{3}{4}$ of the distance from X to Y?

Classifying Polygons

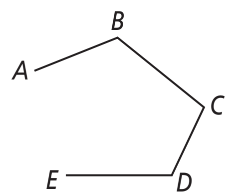
Mathematics Florida Standards

Prepares for MAFS.912.G-MG.1.1 Use geometric shapes, their measures, and their properties to describe objects.

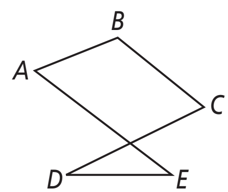
In geometry, a figure that lies in a plane is called a *plane figure*. A **polygon** is a closed plane figure formed by three or more segments. Each segment intersects exactly two other segments at their endpoints. No two segments with a common endpoint are collinear. Each segment is called a *side*. Each endpoint of a side is a *vertex*.



ABCDE is a polygon.



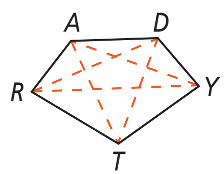
ABCDE is not a polygon.



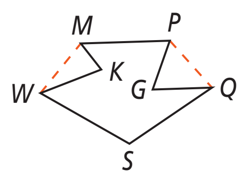
ABCDE is not a polygon.

You can classify a polygon by its number of sides: triangle (3 sides), quadrilateral (4 sides), pentagon (5 sides), hexagon (6 sides), heptagon (7 sides), octagon (8 sides), nonagon (9 sides), and decagon (10 sides). A polygon with n sides is called an n-gon.

You can also classify a polygon as concave or convex, using the diagonals of the polygon. A **diagonal** is a segment that connects two nonconsecutive vertices.



A **convex polygon** has no diagonal with points outside the polygon.

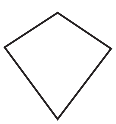


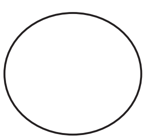
A **concave polygon** has at least one diagonal with points outside the polygon.

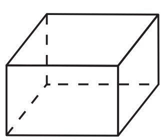
In this textbook, a polygon is convex unless otherwise stated.

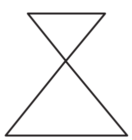
Exercises

Is the figure a polygon? If not, explain why.

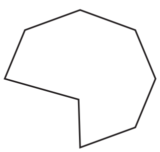
1. 

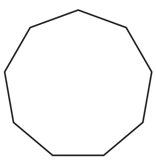
2. 

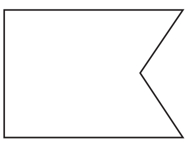
3. 

4. 

Classify the polygon by its number of sides. Tell whether the polygon is *convex* or *concave*.

5. 

6. 

7. 

1-8 Perimeter, Circumference, and Area

Mathematics Florida Standards

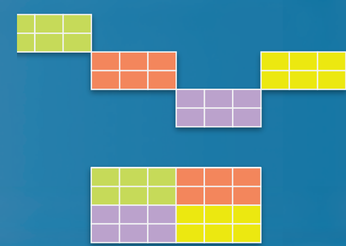
MAFS.912.N-Q.1.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas . . .

MP 1, MP 3, MP 4, MP 7

Objectives To find the perimeter or circumference of basic shapes
To find the area of basic shapes

SOLVE IT! Getting Ready!

You and your friend have two choices for a wall decoration. You say the decoration on the top will use more wall space. Your friend says the two decorations will use the same amount of wall space. Who is correct? Explain.



Think about what "wall space" means.

MATHEMATICAL PRACTICES In the Solve It, you considered various ideas of what it means to take up space on a flat surface.

Essential Understanding Perimeter and area are two different ways of measuring geometric figures.

The **perimeter** P of a polygon is the sum of the lengths of its sides. The **area** A of a polygon is the number of square units it encloses. For figures such as squares, rectangles, triangles, and circles, you can use formulas for perimeter (or *circumference* C for circles) and area.

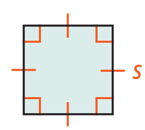
Lesson Vocabulary
- perimeter
- area

take note

Key Concept Perimeter, Circumference, and Area

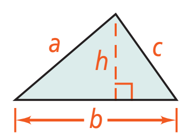
Square
side length s

$P = 4s$
$A = s^2$



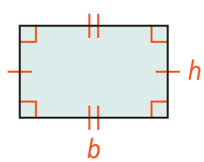
Triangle
side lengths a, b, and c, base b, and height h

$P = a + b + c$
$A = \frac{1}{2}bh$



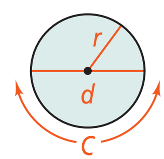
Rectangle
base b and height h

$P = 2b + 2h$, or $2(b + h)$
$A = bh$



Circle
radius r and diameter d

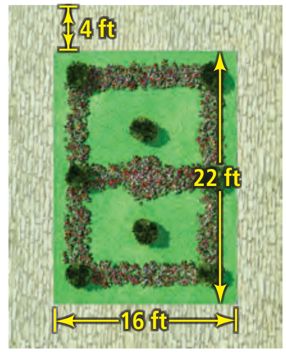
$C = \pi d$, or $C = 2\pi r$
$A = \pi r^2$



The units of measurement for perimeter and circumference include inches, feet, yards, miles, centimeters, and meters. When measuring area, use square units such as square inches (in.^{2}), square feet (ft^2), square yards (yd^2), square miles (mi^2), square centimeters (cm^2), and square meters (m^2).

Problem 1 Finding the Perimeter of a Rectangle

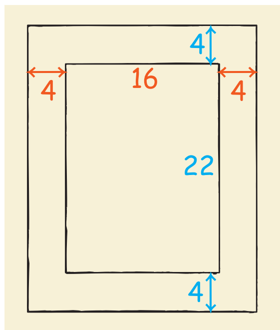
Landscaping **The botany club members are designing a rectangular garden for the courtyard of your school. They plan to place edging on the outside of the path. How much edging material will they need?**



Plan

Why should you *draw a diagram*?
A diagram can help you see the larger rectangle formed by the garden and the path, and which lengths to add together.

Step 1 Find the dimensions of the garden, including the path.



For a rectangle, "length" and "width" are sometimes used in place of "base" and "height."

Width of the garden and path

$= 4 + 16 + 4 = 24$

Length of the garden and path

$= 4 + 22 + 4 = 30$

Step 2 Find the perimeter of the garden including the path.

$P = 2b + 2h$	Use the formula for the perimeter of a rectangle.
$= 2(24) + 2(30)$	Substitute 24 for b and 30 for h.
$= 48 + 60$	Simplify.
$= 108$	

You will need 108 ft of edging material.

Got It? **1.** You want to frame a picture that is 5 in. by 7 in. with a 1-in.-wide frame.
a. What is the perimeter of the picture?
b. What is the perimeter of the outside edge of the frame?

You can name a circle with the symbol $\odot$. For example, the circle with center A is written $\odot A$.

The formulas for a circle involve the special number *pi* (π). Pi is the ratio of any circle's circumference to its diameter. Since π is an irrational number,

$$\pi = 3.1415926\ldots,$$

you cannot write it as a terminating decimal. For an approximate answer, you can use 3.14 or $\frac{22}{7}$ for π. You can also use the π key on your calculator to get a rounded decimal for π. For an exact answer, leave the result in terms of π.

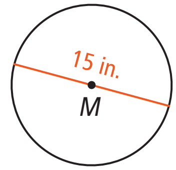
Plan

Which formula should you use?
You want to find circumference. Since you know the diameter in part (A), it would be easier to use the circumference formula that involves diameter.

Problem 2 Finding Circumference

What is the circumference of the circle in terms of π? What is the circumference of the circle to the nearest tenth?

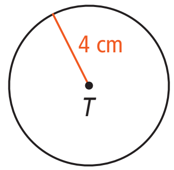
A $\odot M$



$C = \pi d$	Use the formula for circumference of a circle.
$= \pi(15)$	This is the exact answer.
≈ 47.1238898	Use a calculator.

The circumference of $\odot M$ is 15π in., or about 47.1 in.

B $\odot T$



$C = 2\pi r$	Use the formula for circumference of a circle.
$= 2\pi(4)$	This is the exact answer.
$= 8\pi$	Simplify.
≈ 25.13274123	Use a calculator.

The circumference of $\odot T$ is 8π cm, or about 25.1 cm.

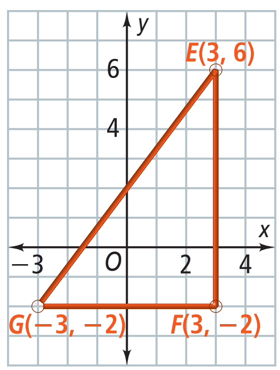
Got It? **2. a.** What is the circumference of a circle with radius 24 m in terms of π?
b. What is the circumference of a circle with diameter 24 m to the nearest tenth?

Plan

What do you need?
To find the perimeter of a figure, you need its side lengths. Use what you know about length on a number line and in the coordinate plane.

Problem 3 Finding Perimeter in the Coordinate Plane

Coordinate Geometry **What is the perimeter of $\triangle EFG$?**



Step 1 Find the length of each side.

$EF = |6 - (-2)| = 8$
$FG = |3 - (-3)| = 6$ Use the Ruler Postulate.

$EG = \sqrt{(3-(-3))^2 + (6-(-2))^2}$ Use the Distance Formula.

$= \sqrt{6^2 + 8^2}$ Simplify within the parentheses.

$= \sqrt{36 + 64}$ Simplify.

$= \sqrt{100}$

$= 10$

Step 2 Add the side lengths to find the perimeter.

$EF + FG + EG = 8 + 6 + 10 = 24$

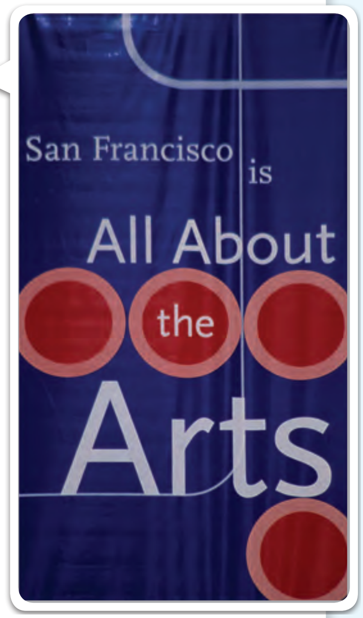
The perimeter of $\triangle EFG$ is 24 units.

Got It? **3.** Graph quadrilateral $JKLM$ with vertices $J(-3, -3)$, $K(1, -3)$, $L(1, 4)$, and $M(-3, 1)$. What is the perimeter of $JKLM$?

To find area, you should use the same unit for both dimensions.

Problem 4 Finding Area of a Rectangle

Banners **You want to make a rectangular banner similar to the one at the right. The banner shown is $2\frac{1}{2}$ ft wide and 5 ft high. To the nearest square yard, how much material do you need?**



Step 1 Convert the dimensions of the banner to yards. Use the conversion factor $\frac{1\text{ yd}}{3\text{ ft}}$.

Width: $\frac{5}{2}\,\cancel{\text{ft}} \cdot \frac{1\text{ yd}}{3\,\cancel{\text{ft}}} = \frac{5}{6}\text{ yd}$ $\qquad 2\frac{1}{2}\text{ ft} = \frac{5}{2}\text{ ft}$

Height: $5\,\cancel{\text{ft}} \cdot \frac{1\text{ yd}}{3\,\cancel{\text{ft}}} = \frac{5}{3}\text{ yd}$

Step 2 Find the area of the banner.

$A = bh$ Use the formula for area of a rectangle.

$= \frac{5}{6} \cdot \frac{5}{3}$ Substitute $\frac{5}{6}$ for b and $\frac{5}{3}$ for h.

$= \frac{25}{18}$

The area of the banner is $\frac{25}{18}$, or $1\frac{7}{18}$ square yards (yd^2). You need 2 yd^2 of material.

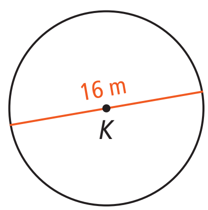
Think

How can you check your conversion?
Yards are longer than feet, so the number you get in yards should be less than the given number in feet. Since $\frac{5}{6} < 2\frac{1}{2}$, the conversion checks.

Got It? **4.** You are designing a poster that will be 3 yd wide and 8 ft high. How much paper do you need to make the poster? Give your answer in square feet.

Problem 5 Finding Area of a Circle

What is the area of $\odot K$ in terms of π?



Step 1 Find the radius of $\odot K$.

$r = \frac{16}{2}$, or 8 The radius is half the diameter.

Step 2 Use the radius to find the area.

$A = \pi r^2$ Use the formula for area of a circle.

$= \pi(8)^2$ Substitute 8 for r.

$= 64\pi$ Simplify.

The area of $\odot K$ is $64\pi\ \text{m}^2$.

Plan

What are you given?
In circle problems, make it a habit to note whether you are given the radius or the diameter. In this case, you are given the diameter.

Got It? **5.** The diameter of a circle is 14 ft.

a. What is the area of the circle in terms of π?

b. What is the area of the circle using an approximation of π?

c. **Reasoning** Which approximation of π did you use in part (b)? Why?

The following postulate is useful in finding areas of figures with irregular shapes.

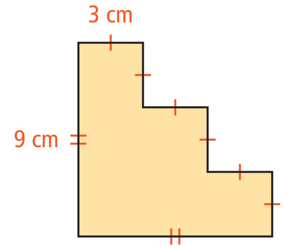
take note

Postulate 1-10 Area Addition Postulate

The area of a region is the sum of the areas of its nonoverlapping parts.

Problem 6 Finding Area of an Irregular Shape

Multiple Choice What is the area of the figure at the right? All angles are right angles.



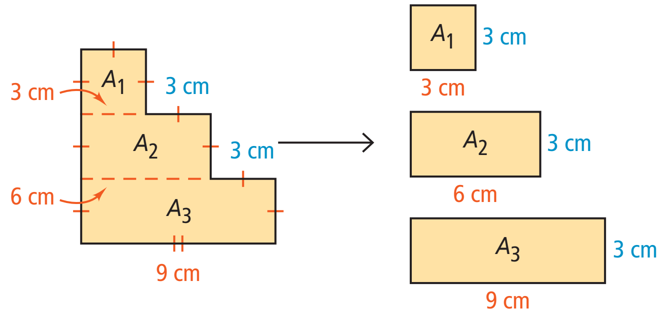
Ⓐ 27 cm^2

Ⓑ 36 cm^2

Ⓒ 45 cm^2

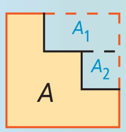
Ⓓ 54 cm^2

Step 1 Separate the figure into rectangles.



Think

What is another way to find the area?
Extend the figure to form a square. Then subtract the areas of basic shapes from the area of the square.



$A = A_{\text{square}} - A_1 - A_2$

Step 2 Find A_1, A_2, and A_3.

Area $= bh$	Use the formula for the area of a rectangle.
$A_1 = 3 \cdot 3 = 9$	Substitute for the base and height.
$A_2 = 6 \cdot 3 = 18$	
$A_3 = 9 \cdot 3 = 27$	

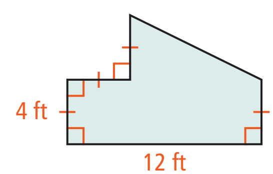
Step 3 Find the total area of the figure.

Total Area $= A_1 + A_2 + A_3$	Use the Area Addition Postulate.
$= 9 + 18 + 27$	
$= 54$	

The area of the figure is 54 cm^2. The correct choice is D.

Got It? 6. a. Reasoning What is another way to separate the figure in Problem 6?

b. What is the area of the figure at the right?



Lesson Check

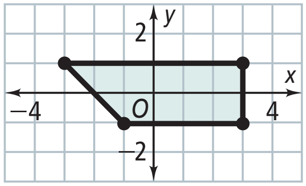
Do you know HOW?

1. What is the perimeter and area of a rectangle with base 3 in. and height 7 in.?

2. What is the circumference and area of each circle to the nearest tenth?

a. $r = 9$ in. **b.** $d = 7.3$ m

3. What is the perimeter and area of the figure at the right?



Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Writing Describe a real-world situation in which you would need to find a perimeter. Then describe a situation in which you would need to find an area.

5. Compare and Contrast Your friend can't remember whether $2\pi r$ computes the circumference or the area of a circle. How would you help your friend? Explain.

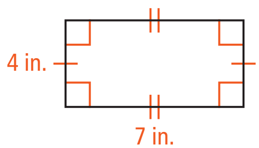
6. Error Analysis A classmate finds the area of a circle with radius 30 in. to be 900 in.^{2}. What error did your classmate make?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

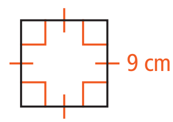
A Practice

Find the perimeter of each figure. See Problem 1.

7.



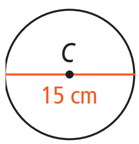
8.



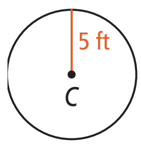
9. Fencing A garden that is 5 ft by 6 ft has a walkway 2 ft wide around it. What is the amount of fencing needed to surround the walkway?

Find the circumference of $\odot C$ in terms of π. See Problem 2.

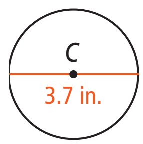
10.



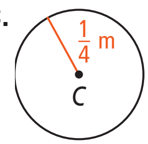
11.



12.



13.



Coordinate Geometry **Graph each figure in the coordinate plane. Find each perimeter.** See Problem 3.

14. $X(0, 2)$, $Y(4, -1)$, $Z(-2, -1)$

15. $A(-4, -1)$, $B(4, 5)$, $C(4, -2)$

16. $L(0, 1)$, $M(3, 5)$, $N(5, 5)$, $P(5, 1)$

17. $S(-5, 3)$, $T(7, -2)$, $U(7, -6)$, $V(-5, -6)$

Find the area of each rectangle with the given base and height. See Problem 4.

18. 4 ft, 4 in.

19. 30 in., 4 yd

20. 2 ft 3 in., 6 in.

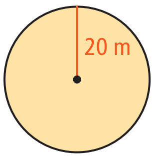
21. 40 cm, 2 m

22. Roads What is the area of a section of pavement that is 20 ft wide and 100 yd long? Give your answer in square feet.

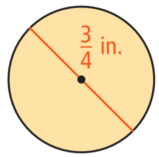
Find the area of each circle in terms of π.

See Problem 5.

23.



24.



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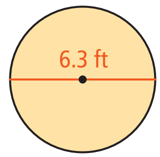

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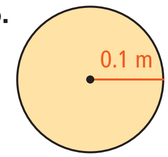


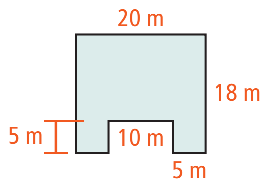
Find the area of each circle using an approximation of π. If necessary, round to the nearest tenth.

27. $r = 7$ ft **28.** $d = 8.3$ m **29.** $d = 24$ cm **30.** $r = 12$ in.

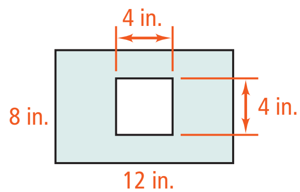
Find the area of the shaded region. All angles are right angles.

See Problem 6.

31.



32.



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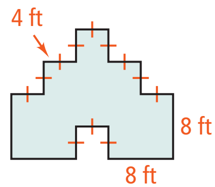


B Apply

Home Maintenance To determine how much of each item to buy, tell whether you need to know area or perimeter. Explain your choice.

34. wallpaper for a bedroom

35. crown molding for a ceiling

36. fencing for a backyard

37. paint for a basement floor

38. Think About a Plan A light-year unit describes the distance that one photon of light travels in one year. The Milky Way galaxy has a diameter of about 100,000 light-years. The distance to Earth from the center of the Milky Way galaxy is about 30,000 light-years. How many more light-years does a star on the outermost edge of the Milky Way travel in one full revolution around the galaxy compared to Earth?

- What do you know about the shape of each orbital path?
- Are you looking for circumference or area?
- How do you compare the paths using algebraic expressions?

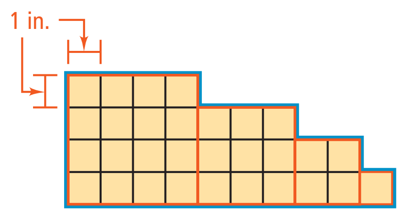
39. a. What is the area of a square with sides 12 in. long? 1 ft long?

b. How many square inches are in a square foot?

40. a. Count squares at the right to find the area of the polygon outlined in blue.

b. Use a formula to find the area of each square outlined in red.

c. Writing How does the sum of your results in part (b) compare to your result in part (a)? Which postulate does this support?



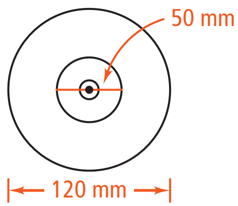
41. The area of an 11-cm-wide rectangle is 176 cm^2. What is its length?

42. A square and a rectangle have equal areas. The rectangle is 64 cm by 81 cm. What is the perimeter of the square?

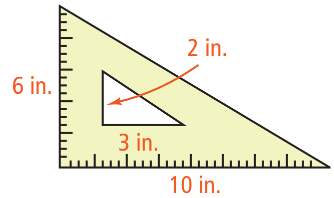
43. A rectangle has perimeter 40 cm and base 12 cm. What is its area?

Find the area of each shaded figure.

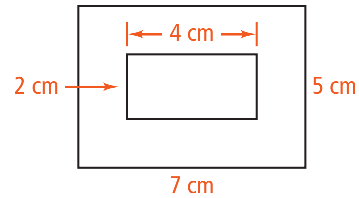
44. compact disc



45. drafting triangle



46. picture frame

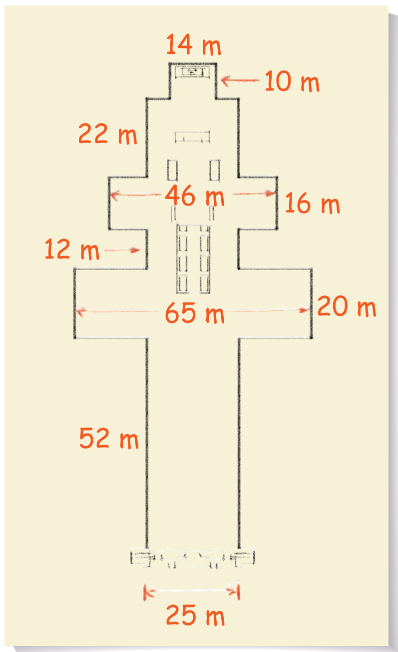


47. a. Reasoning Can you use the formula for the perimeter of a rectangle to find the perimeter of any square? Explain.

b. Can you use the formula for the perimeter of a square to find the perimeter of any rectangle? Explain.

c. Use the formula for the perimeter of a square to write a formula for the area of a square in terms of its perimeter.

48. Estimation On an art trip to England, a student sketches the floor plan of the main body of Salisbury Cathedral. The shape of the floor plan is called the building's "footprint." The student estimates the dimensions of the cathedral on her sketch at the right. Use the student's lengths to estimate the area of Salisbury Cathedral's footprint.



49. Coordinate Geometry The endpoints of a diameter of a circle are $A(2, 1)$ and $B(5, 5)$. Find the area of the circle in terms of π.

50. Algebra A rectangle has a base of x units. The area is $(4x^2 - 2x)$ square units. What is the height of the rectangle in terms of x?

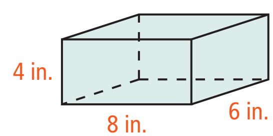
(A) $(4 - x)$ units
(B) $(x - 2)$ units
(C) $(4x^3 - 2x^2)$ units
(D) $(4x - 2)$ units

Coordinate Geometry Graph each rectangle in the coordinate plane. Find its perimeter and area.

51. $A(-3, 2)$, $B(-2, 2)$, $C(-2, -2)$, $D(-3, -2)$

52. $A(-2, -6)$, $B(-2, -3)$, $C(3, -3)$, $D(3, -6)$

53. The surface area of a three-dimensional figure is the sum of the areas of all of its surfaces. You can find the surface area by finding the area of a net for the figure.

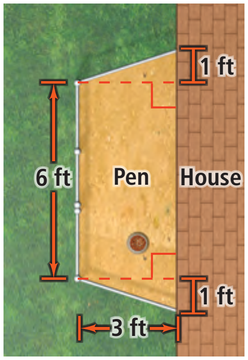


a. Draw a net for the solid shown. Label the dimensions.

b. What is the area of the net? What is the surface area of the solid?

54. Coordinate Geometry On graph paper, draw polygon $ABCDEFG$ with vertices $A(1, 1)$, $B(10, 1)$, $C(10, 8)$, $D(7, 5)$, $E(4, 5)$, $F(4, 8)$, and $G(1, 8)$. Find the perimeter and the area of the polygon.

55. Pet Care You want to adopt a puppy from your local animal shelter. First, you plan to build an outdoor playpen along the side of your house, as shown on the right. You want to lay down special dog grass for the pen's floor. If dog grass costs \$1.70 per square foot, how much will you spend?



56. A rectangular garden has an 8-ft walkway around it. How many more feet is the outer perimeter of the walkway than the perimeter of the garden?

Challenge

Algebra **Find the area of each figure.**

57. a rectangle with side lengths $\frac{2a}{5b}$ units and $\frac{3b}{8}$ units

58. a square with perimeter $10n$ units

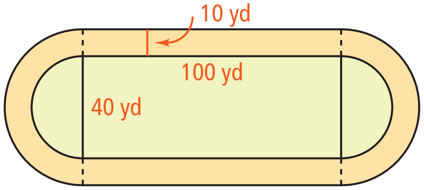
59. a triangle with base $(5x - 2y)$ units and height $(4x + 3y)$ units

Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

60. An athletic field is a 100 yd-by-40 yd rectangle with a semicircle at each of the short sides. A running track 10 yd wide surrounds the field. Find the perimeter of the outside of the running track to the nearest tenth of a yard.



61. A square garden has a 4-ft walkway around it. The garden has a perimeter of 260 ft. What is the area of the walkway in square feet?

62. $A(4, -1)$ and $B(-2, 3)$ are points in a coordinate plane. M is the midpoint of $\overline{AB}$. What is the length of $\overline{MB}$ to the nearest tenth of a unit?

63. Find CD to the nearest tenth if point C is at $(12, -8)$ and point D is at $(5, 19)$.

Mixed Review

Find (a) AB to the nearest tenth and (b) the midpoint coordinates of $\overline{AB}$. See Lesson 1-7.

64. $A(4, 1), B(7, 9)$ **65.** $A(0, 3), B(-3, 8)$ **66.** $A(-1, 1), B(-4, -5)$

$\overleftrightarrow{BG}$ is the perpendicular bisector of $\overline{WR}$ at point K. See Lesson 1-6.

67. What is $m\angle BKR$?

68. Name two congruent segments.

Get Ready! **To prepare for Lesson 2-1, do Exercise 69.**

69. a. Copy and extend this list to show the first 10 perfect squares. See p. 889.

$$1^2 = 1,\ 2^2 = 4,\ 3^2 = 9,\ 4^2 = 16, \ldots$$

b. Which do you think describes the square of any odd number?

It is odd. It is even.

Concept Byte

Use With Lesson 1-8

TECHNOLOGY

Comparing Perimeters and Areas

Mathematics Florida State Standards

Prepares for MAFS.912.G-MG.1.2 Apply concepts of density based on area and volume in modeling situations.

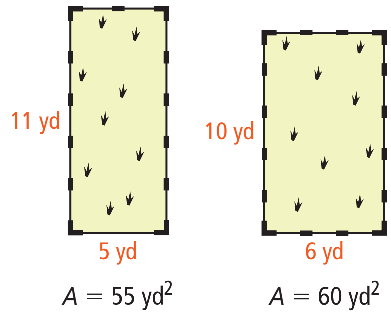
MP 5

You can use a graphing calculator or spreadsheet software to find maximum and minimum values. These values help you solve real-world problems where you want to minimize or maximize a quantity such as cost or time. In this Activity, you will find minimum and maximum values for area and perimeter problems.

MATHEMATICAL PRACTICES

Activity

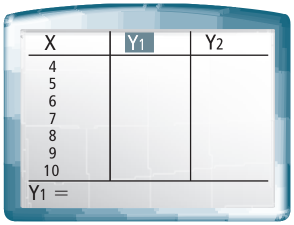
You have 32 yd of fencing. You want to make a rectangular horse pen with maximum area.



1. Draw some possible rectangular pens and find their areas. Use the examples at the right as models.

2. You plan to use all of your fencing. Let X represent the base of the pen. What is the height of the pen in terms of X? What is the area of the pen in terms of X?

3. Make a graphing calculator table to find area. Again, let X represent the base. For Y_1, enter the expression you wrote for the height in Question 2. For Y_2, enter the expression you wrote for the area in Question 2. Set the table so that X starts at 4 and changes by 1. Scroll down the table.
a. What value of X gives you the maximum area?
b. What is the maximum area?



4. Use your calculator to graph Y_2. Describe the shape of the graph. Trace on the graph to find the coordinates of the highest point. What is the relationship, if any, between the coordinates of the highest point on the graph and your answers to Question 3? Explain.

Exercises

5. For a fixed perimeter, what rectangular shape will result in a maximum area?

6. Consider that the pen is not limited to polygon shapes. What is the area of a circular pen with circumference 32 yd? How does this result compare to the maximum area you found in the Activity?

7. You plan to make a rectangular garden with an area of 900 ft^2. You want to use a minimum amount of fencing to keep the cost low.
a. List some possible dimensions for the garden. Find the perimeter of each.
b. Make a graphing calculator table. Use integer values of the base b, and the corresponding values of the height h, to find values for P, the perimeter. What dimensions will give you a garden with the minimum perimeter?

1 *Pull It* All Together

Completing the Performance Task

Look back at your results from the Apply What You've Learned sections in Lessons 1-3, 1-4, and 1-5. Use the work you did to complete the following.

1. Solve the problem in the Task Description on page 3 by finding the value of each variable and answering the riddle. Show all your work and explain each step of your solution.

2. Reflect Choose one of the Mathematical Practices below and explain how you applied it in your work on the Performance Task.

MP 1: Make sense of problems and persevere in solving them.

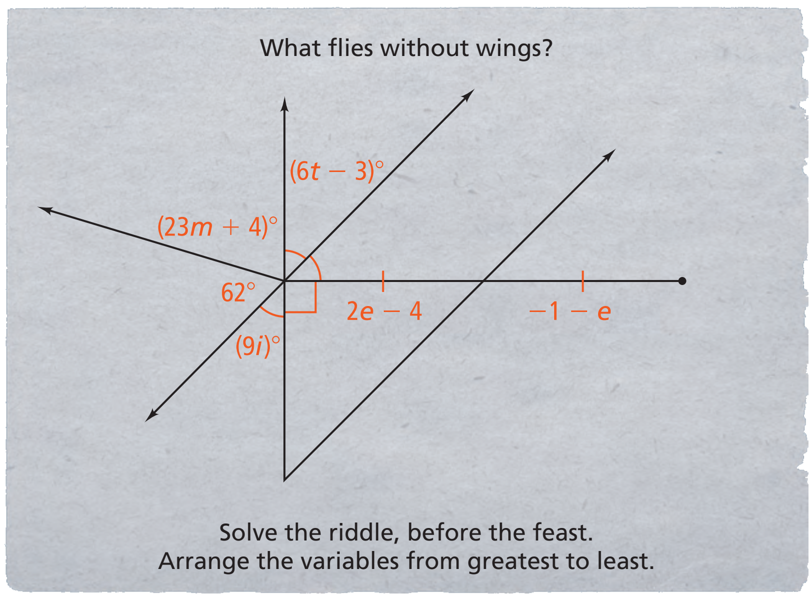
MP 3: Construct viable arguments and critique the reasoning of others.

MP 6: Attend to precision.

To solve these problems, you will pull together many concepts and skills that you learned in this chapter. They are the basic tools used to study geometry.

On Your Own

Cameron found another page from the old riddle book, as shown below.



a. Find the value of each variable.

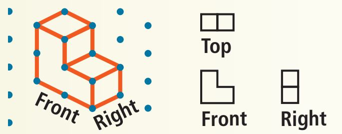
b. What is the answer to the riddle?

1 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

1 Visualization
You can represent a 3-D figure with a 2-D drawing by visualizing the surfaces of the figure and how they relate to each other.

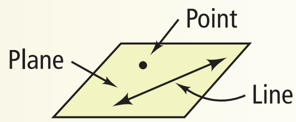
Lesson 1-1 Nets and Drawings



Isometric Orthographic

2 Reasoning
Geometry is a mathematical system built on basic terms, definitions, and assumptions called postulates.

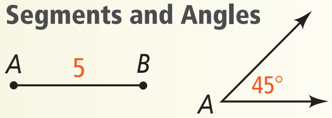
Lesson 1-2 Points, Lines, and Planes



Lesson 1-5 Exploring Angle Pairs
You can conclude these relationships from an unmarked diagram.
- Angles are adjacent.
- Angles are adjacent supplementary.
- Angles are vertical angles.

3 Measurement
You can describe the attributes of a segment or angle by using unit amounts.

Lessons 1-3 and 1-4 Segments and Angles



$AB = 5$ $\quad m\angle A = 45$

Lesson 1-7 Midpoint and Distance

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Chapter Vocabulary

- acute, right, obtuse, straight angles (p. 29)
- adjacent angles (p. 34)
- angle bisector (p. 37)
- collinear points, coplanar (p. 12)
- complementary angles (p. 34)
- congruent angles (p. 29)
- congruent segments (p. 22)
- construction (p. 43)
- isometric drawing (p. 5)
- linear pair (p. 36)
- measure of an angle (p. 28)
- net (p. 4)
- orthographic drawing (p. 6)
- perpendicular bisector (p. 44)
- perpendicular lines (p. 44)
- point, line, plane (p. 11)
- postulate, axiom (p. 13)
- ray, opposite rays (p. 12)
- segment (p. 12)
- segment bisector (p. 22)
- space (p. 12)
- supplementary angles (p. 34)
- vertex of an angle (p. 27)
- vertical angles (p. 34)

Choose the correct term to complete each sentence.

1. A ray that divides an angle into two congruent angles is a(n) __?__.

2. __?__ are two lines that intersect to form right angles.

3. A(n) __?__ is a two-dimensional diagram that you can fold to form a 3-D figure.

4. __?__ are two angles with measures that have a sum of 90.

1-1 Nets and Drawings for Visualizing Geometry

Quick Review

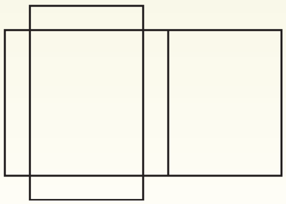
A **net** is a two-dimensional pattern that you can fold to form a three-dimensional figure. A net shows all surfaces of a figure in one view.

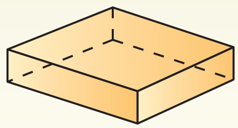
An **isometric drawing** shows a corner view of a three-dimensional object. It allows you to see the top, front, and side of the object in one view.

An **orthographic drawing** shows three separate views of a three-dimensional object: a top view, a front view, and a right-side view.

Example

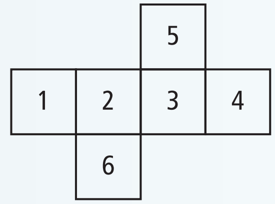
Draw a net for the solid at the right.



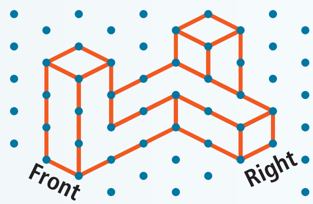


Exercises

5. The net below is for a number cube. What are the three sums of the numbers on opposite surfaces of the cube?



6. Make an orthographic drawing for the isometric drawing at the right. Assume there are no hidden cubes.



1-2 Points, Lines, and Planes

Quick Review

A **point** indicates a location and has no size.

A **line** is represented by a straight path that extends in two opposite directions without end and has no thickness.

A **plane** is represented by a flat surface that extends without end and has no thickness.

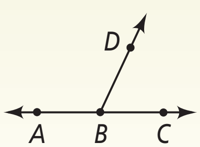
Points that lie on the same line are **collinear points**.

Points and lines in the same plane are **coplanar**.

Segments and **rays** are parts of lines.

Example

Name all the segments and rays in the figure.

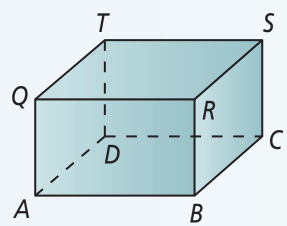


Segments: $\overline{AB}$, $\overline{AC}$, $\overline{BC}$, and $\overline{BD}$

Rays: $\overrightarrow{BA}$, $\overrightarrow{CA}$ or $\overrightarrow{CB}$, $\overrightarrow{AC}$ or $\overrightarrow{AB}$, $\overrightarrow{BC}$, and $\overrightarrow{BD}$

Exercises

Use the figure below for Exercises 7–9.



7. Name two intersecting lines.

8. Name the intersection of planes *QRBA* and *TSRQ*.

9. Name three noncollinear points.

Determine whether the statement is *true* or *false*. Explain your reasoning.

10. Two points are always collinear.

11. $\overrightarrow{LM}$ and $\overrightarrow{ML}$ are the same ray.

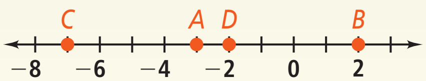
1-3 Measuring Segments

Quick Review

The **distance** between two points is the length of the segment connecting those points. Segments with the same length are **congruent segments**. A **midpoint** of a segment divides the segment into two congruent segments.

Example

Are $\overline{AB}$ and $\overline{CD}$ congruent?



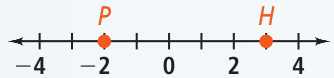
$AB = |-3 - 2| = |-5| = 5$

$CD = |-7 - (-2)| = |-5| = 5$

$AB = CD$, so $\overline{AB} \cong \overline{CD}$.

Exercises

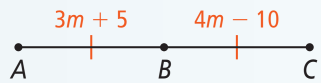
For Exercises 12 and 13, use the number line below.



12. Find two possible coordinates of Q such that $PQ = 5$.

13. Use the number line above. Find the coordinate of the midpoint of $\overline{PH}$.

14. Find the value of m.



15. If $XZ = 50$, what are XY and YZ?

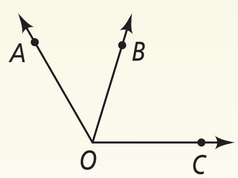
1-4 Measuring Angles

Quick Review

Two rays with the same endpoint form an **angle**. The endpoint is the **vertex** of the angle. You can classify angles as acute, right, obtuse, or straight. Angles with the same measure are **congruent angles**.

Example

If $m\angle AOB = 47$ and $m\angle BOC = 73$, find $m\angle AOC$.

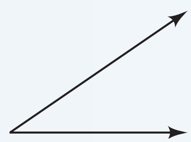


$$\begin{aligned} m\angle AOC &= m\angle AOB + m\angle BOC \\ &= 47 + 73 \\ &= 120 \end{aligned}$$

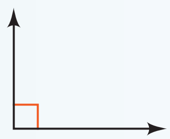
Exercises

Classify each angle as *acute, right, obtuse,* or *straight*.

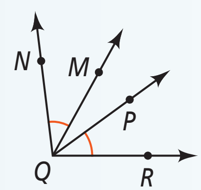
16.



17.



Use the diagram below for Exercises 18 and 19.



18. If $m\angle MQR = 61$ and $m\angle MQP = 25$, find $m\angle PQR$.

19. If $m\angle NQM = 2x + 8$ and $m\angle PQR = x + 22$, find the value of x.

1-5 Exploring Angle Pairs

Quick Review

Some pairs of angles have special names.

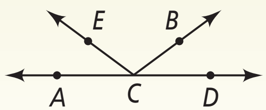
- **Adjacent angles:** coplanar angles with a common side, a common vertex, and no common interior points
- **Vertical angles:** sides are opposite rays
- **Complementary angles:** measures have a sum of 90
- **Supplementary angles:** measures have a sum of 180
- **Linear pair:** adjacent angles with noncommon sides as opposite rays

Angles of a linear pair are supplementary.

Example

Are $\angle ACE$ and $\angle BCD$ vertical angles? Explain.

No. They have only one set of sides with opposite rays.



Exercises

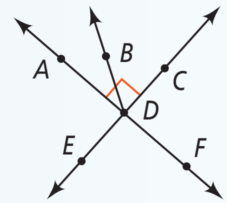
Name a pair of each of the following.

20. complementary angles

21. supplementary angles

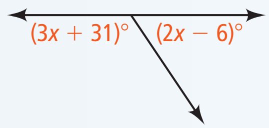
22. vertical angles

23. linear pair

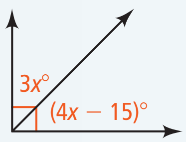


Find the value of *x*.

24.



25.



1-6 Basic Constructions

Quick Review

Construction is the process of making geometric figures using a **compass** and a **straightedge**. Four basic constructions involve congruent segments, congruent angles, and bisectors of segments and angles.

Example

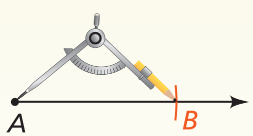
Construct $\overline{AB}$ congruent to $\overline{EF}$.

Step 1

Draw a ray with endpoint *A*.

Step 2

Open the compass to the length of $\overline{EF}$. Keep that compass setting and put the compass point on point *A*. Draw an arc that intersects the ray. Label the point of intersection *B*.



Exercises

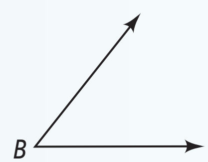
26. Use a protractor to draw a 73° angle. Then construct an angle congruent to it.

27. Use a protractor to draw a 60° angle. Then construct the bisector of the angle.

28. Sketch $\overline{LM}$ on paper. Construct a line segment congruent to $\overline{LM}$. Then construct the perpendicular bisector of your line segment.

29. **a.** Sketch $\angle B$ on paper. Construct an angle congruent to $\angle B$.

b. Construct the bisector of your angle from part (a).



1-7 Midpoint and Distance in the Coordinate Plane

Quick Review

You can find the coordinates of the midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ using the **Midpoint Formula.**

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

You can find the distance d between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ using the **Distance Formula.**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example

$\overline{GH}$ has endpoints $G(-11, 6)$ and $H(3, 4)$. What are the coordinates of its midpoint M?

x-coordinate $= \frac{-11 + 3}{2} = -4$

y-coordinate $= \frac{6 + 4}{2} = 5$

The coordinates of the midpoint of $\overline{GH}$ are $M(-4, 5)$.

Exercises

Find the distance between the points to the nearest tenth.

30. $A(-1, 5)$, $B(0, 4)$

31. $C(-1, -1)$, $D(6, 2)$

32. $E(-7, 0)$, $F(5, 8)$

$\overline{AB}$ has endpoints $A(-3, 2)$ and $B(3, -2)$.

33. Find the coordinates of the midpoint of $\overline{AB}$.

34. Find AB to the nearest tenth.

M is the midpoint of $\overline{JK}$. Find the coordinates of K.

35. $J(-8, 4)$, $M(-1, 1)$

36. $J(9, -5)$, $M(5, -2)$

37. $J(0, 11)$, $M(-3, 2)$

1-8 Perimeter, Circumference, and Area

Quick Review

The perimeter P of a polygon is the sum of the lengths of its sides. Circles have a circumference C. The area A of a polygon or a circle is the number of square units it encloses.

Square: $P = 4s$; $A = s^2$

Rectangle: $P = 2b + 2h$; $A = bh$

Triangle: $P = a + b + c$; $A = \frac{1}{2}bh$

Circle: $C = \pi d$ or $C = 2\pi r$; $A = \pi r^2$

Example

Find the perimeter and area of a rectangle with $b = 12$ m and $h = 8$ m.

$P = 2b + 2h$
$= 2(12) + 2(8)$
$= 40$

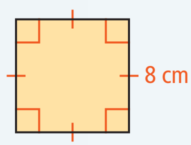
$A = bh$
$= 12 \cdot 8$
$= 96$

The perimeter is 40 m and the area is 96 m^2.

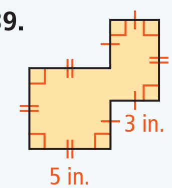
Exercises

Find the perimeter and area of each figure.

38.



39.



Find the circumference and the area for each circle in terms of π.

40. $r = 3$ in.

41. $d = 15$ m

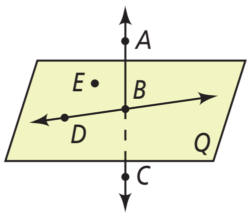
1 Chapter Test

MathXL® for School
Go to PowerGeometry.com

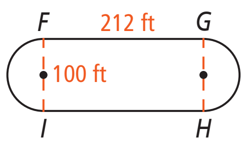
Do you know HOW?

1. Draw a net for a cube.
2. Draw an obtuse $\angle ABC$. Use a compass and a straightedge to bisect the angle.

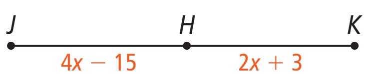
Use the figure for Exercises 3–6.



3. Name three collinear points.
4. Name four coplanar points.
5. What is the intersection of $\overleftrightarrow{AC}$ and plane Q?
6. How many planes contain the given line and point?
   a. $\overleftrightarrow{DB}$ and point A
   b. $\overleftrightarrow{BD}$ and point E
   c. $\overleftrightarrow{AC}$ and point D
   d. $\overleftrightarrow{EB}$ and point C
7. The running track at the right is a rectangle with a half circle on each end. $\overline{FI}$ and $\overline{GH}$ are diameters. Find the area inside the track to the nearest tenth.

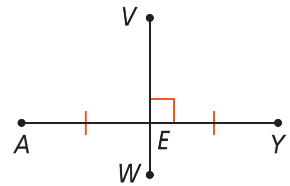


8. **Algebra** $M(x, y)$ is the midpoint of $\overline{CD}$ with endpoints $C(5, 9)$ and $D(17, 29)$.
   a. Find the values of x and y.
   b. Show that $MC = MD$.
9. **Algebra** If $JK = 48$, find the value of x.



10. To the nearest tenth, find the perimeter of $\triangle ABC$ with vertices $A(-2, -2)$, $B(0, 5)$, and $C(3, -1)$.

Use the figure to complete each statement.

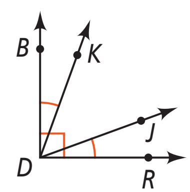


11. $\overline{VW}$ is the $\underline{\ ?\ }$ of $\overline{AY}$.
12. If $EY = 3.5$, then $AY = \underline{\ ?\ }$.
13. $AE = \frac{1}{2}\underline{\ ?\ }$
14. $\underline{\ ?\ }$ is the midpoint of $\underline{\ ?\ }$.

For the given dimensions, find the area of each figure. If necessary, round to the nearest hundredth.

15. rectangle with base 4 m and height 2 cm
16. square with side length 3.5 in.
17. circle with diameter 9 cm

Algebra **Find the value of the variable.**



18. $m\angle BDK = 3x + 4$, $m\angle JDR = 5x - 10$
19. $m\angle BDJ = 7y + 2$, $m\angle JDR = 2y + 7$

Do you UNDERSTAND?

Determine whether each statement is *always*, *sometimes*, or *never* true.

20. $\overrightarrow{LJ}$ and $\overrightarrow{TJ}$ are opposite rays.
21. Angles that form a linear pair are supplementary.
22. The intersection of two planes is a point.
23. Complementary angles are congruent.
24. **Writing** Explain why it is useful to have more than one way to name an angle.
25. You have 30 yd^2 of carpet. You want to install carpeting in a room that is 20 ft long and 15 ft wide. Do you have enough carpet? Explain.

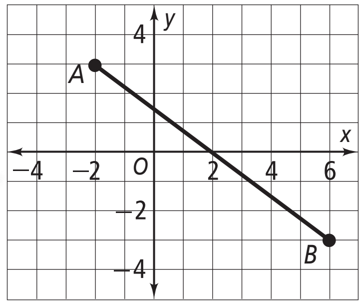
Common Core Cumulative Standards Review

ASSESSMENT

TIPS FOR SUCCESS

Some questions ask you to find a distance using coordinate geometry. Read the sample question at the right. Then follow the tips to answer it.

What is the distance from the midpoint of $\overline{AB}$ to endpoint B?



(A) $\sqrt{10}$ (C) 10

(B) 5 (D) 100

TIP 1

The midpoint divides the segment into two congruent segments that are each half of the total length.

TIP 2

Use the Distance Formula to find the length of the segment.

Think It Through

Find AB using the Distance Formula.

$$AB = \sqrt{(-2-6)^2 + [3-(-3)]^2}$$
$$= \sqrt{100}$$
$$= 10$$

The distance from the midpoint of $\overline{AB}$ to endpoint B is $\frac{1}{2}AB$, or 5. The correct answer is B.

Vocabulary Builder

As you solve test items, you must understand the meanings of mathematical terms. Match each term with its mathematical meaning.

A. segment

B. angle bisector

C. construction

D. net

E. congruent angles

I. angles with the same measure

II. a two-dimensional diagram of a three-dimensional figure

III. the part of a line consisting of two endpoints and all points between

IV. a ray that divides an angle into two congruent angles

V. a geometric figure made using a straightedge and compass

Selected Response

Read each question. Then write the letter of the correct answer on your paper.

1. Points A, B, C, D, and E are collinear. A is to the right of B, E is to the right of D, and B is to the left of C. Which of the following is NOT a possible arrangement of the points from left to right?

(A) D, B, A, E, C (C) B, D, E, C, A

(B) D, B, A, C, E (D) B, A, E, C, D

2. A square and a rectangle have equal area. If the rectangle is 36 cm by 25 cm, what is the perimeter of the square?

(F) 30 cm (H) 120 cm

(G) 60 cm (I) 900 cm

3. Which construction requires drawing only one arc with a compass?

(A) constructing congruent segments

(B) constructing congruent angles

(C) constructing the perpendicular bisector

(D) constructing the angle bisector

4. Rick paints the four walls in a room that is 12 ft long and 10 ft wide. The ceiling in the room is 8 ft from the floor. The doorway is 3 ft by 7 ft, and the window is 6 ft by 5 ft. If Rick does NOT paint the doorway or window, what is the approximate area that he paints?

(F) 301 ft^2 (H) 331 ft^2

(G) 322 ft^2 (I) 352 ft^2

5. If $\angle A$ and $\angle B$ are supplementary angles, what angle relationship between $\angle A$ and $\angle B$ CANNOT be true?

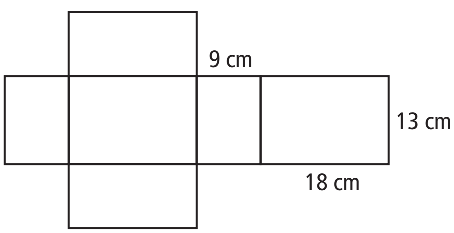
(A) $\angle A$ and $\angle B$ are right angles.

(B) $\angle A$ and $\angle B$ are adjacent angles.

(C) $\angle A$ and $\angle B$ are complementary angles.

(D) $\angle A$ and $\angle B$ are congruent angles.

6. A net for a small rectangular gift box is shown below. What is the total area of the net?



(F) 468 cm^2 (H) 1026 cm^2

(G) 782 cm^2 (I) 2106 cm^2

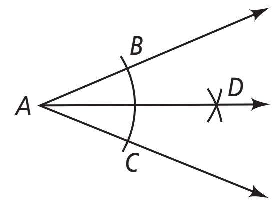
7. The measure of an angle is 12 less than twice the measure of its supplement. What is the measure of the angle?

(A) 28 (C) 64

(B) 34 (D) 116

8. **Given:** $\angle A$

What is the second step in constructing the angle bisector of $\angle A$?



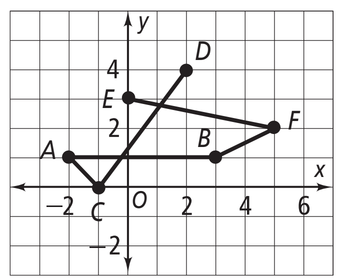
(F) Draw $\overrightarrow{AD}$.

(G) From points B and C, use the same compass setting to draw arcs that intersect at D.

(H) Draw a line segment connecting points B and C.

(I) From point A, draw an arc that intersects the sides of the angle at points B and C.

9. What is the distance, to the nearest tenth, from point D to point E through points C, A, B, and F?



(A) 14.0 (C) 28.2

(B) 18.7 (D) 34.4

10. Which postulate most closely resembles the Angle Addition Postulate?

(F) Ruler Postulate

(G) Protractor Postulate

(H) Segment Addition Postulate

(I) Area Addition Postulate

11. What is the length of the segment with endpoints $A(1, 7)$ and $B(-3, -1)$?

(A) $\sqrt{40}$ (C) $\sqrt{80}$

(B) 8 (D) 40

Constructed Response

12. The measure of an angle is 78 less than the measure of its complement. What is the measure of the angle?

13. The face of a circular game token has an area of 10π cm^2. What is the diameter of the game token? Round to the nearest hundredth of a centimeter.

14. The measure of an angle is one third the measure of its supplement. What is the measure of the angle?

15. Bill's bike wheels have a 26-in. diameter. The odometer on his bike counts the number of times a wheel rotates during a trip. If the odometer counts 200 rotations during the trip from Bill's house to school, how many feet does Bill travel? Round to the nearest foot.

16. Y is the midpoint of $\overline{XZ}$. What is the value of b?

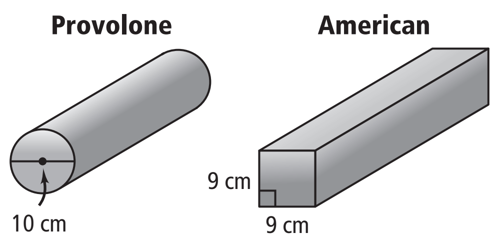
17. A rectangular garden has dimensions 6 ft by 17 ft. A second rectangular garden has dimensions 4 yd by 9 yd. What is the area in square feet of the larger of the two gardens?

18. The sum of the measures of a complement and a supplement of an angle is 200. What is the measure of the angle?

19. What is the area in square units of a rectangle with vertices $(-2, 5)$, $(3, 5)$, $(3, -1)$, and $(-2, -1)$?

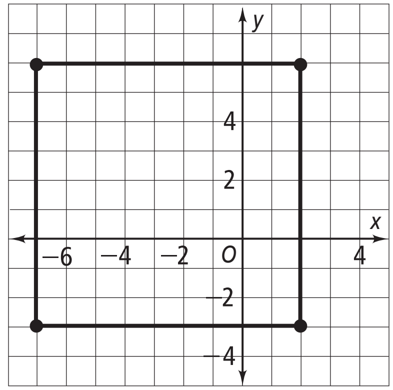
20. $\overline{AB}$ has endpoints $A(-4, 5)$ and $B(3, 5)$. What is the x-coordinate of a point C such that B is the midpoint of $\overline{AC}$?

21. The two blocks of cheese shown below are cut into slices of equal thickness. If the cheese sells at the same cost per slice, which type of cheese slice is the better deal? Explain your reasoning.



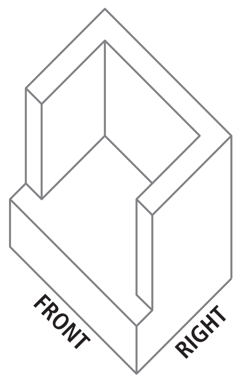
22. The midpoint of $\overline{GI}$ is $(2, -1)$. One endpoint is $G(-1, -3)$. What are the coordinates of endpoint I?

23. Copy the graph below. Connect the midpoints of the sides of the square consecutively. What is the perimeter of the new square? Show your work.



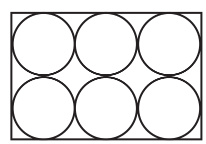
Extended Response

24. Make an orthographic drawing of the three-dimensional drawing. How might an orthographic drawing of the figure be more useful than a net?



25. A packaging company wants to fit 6 energy-drink cans in a cardboard box, as shown below. The bottom of each can is a circle with an area of 9π cm^2.

a. What is the total area taken up by the bottoms of the cans? Round to the nearest hundredth.



b. Will the cans fit in a box with length 16 cm and height 12 cm? Explain.