Calculus I.

NoteBook #1

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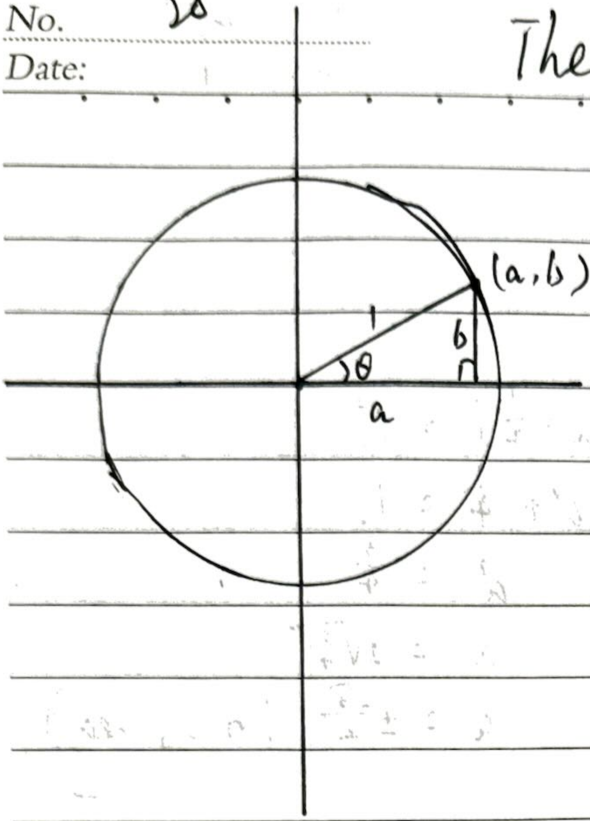
No. 28
Date:

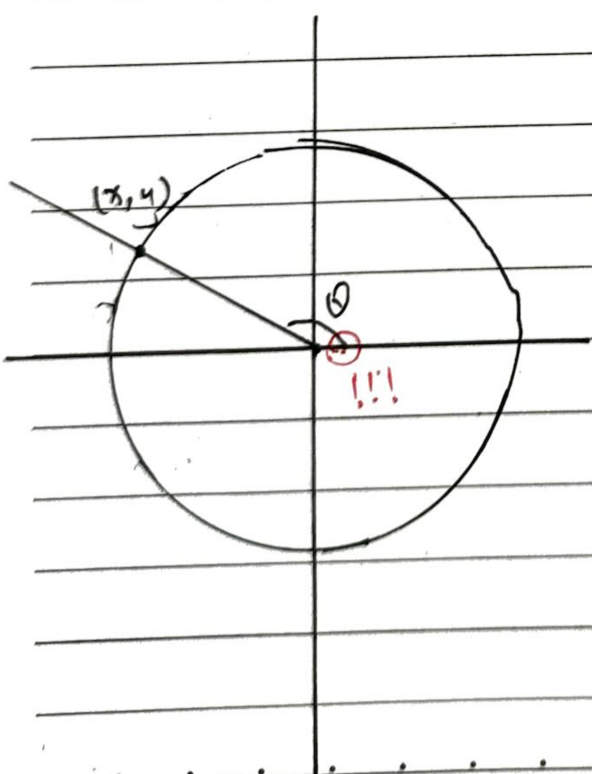
The Unit Circle.



$\cos(\theta) = \frac{adj}{hyp} = a$

$\sin(\theta) = \frac{opp}{hyp} = b.$

$\tan(\theta) = \frac{\sin}{\cos} = \frac{b}{a}.$



$\cos(\theta) = x.$

$\sin(\theta) = y$

$\tan(\theta) = \frac{y}{x}.$

Properties of Trig functions.

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Periodic Property:

$\cos(\theta + 2\pi) = \cos(\theta)$. $\qquad$ $\cos(\theta - 2\pi) = \cos(\theta)$.

$\sin(\theta + 2\pi) = \sin(\theta)$. $\qquad$ $\sin(\theta - 2\pi) = \sin(\theta)$.

Find:

$$\cos(5\pi) = \cos(\pi + 4\pi).$$
$$= \cos(\pi)$$
$$= -1.$$

$$\sin(-420^\circ) = \sin(-60^\circ - 360^\circ).$$
$$= \sin(-60^\circ).$$
$$= -\sin(60^\circ).$$
$$= -\frac{\sqrt{3}}{2}$$

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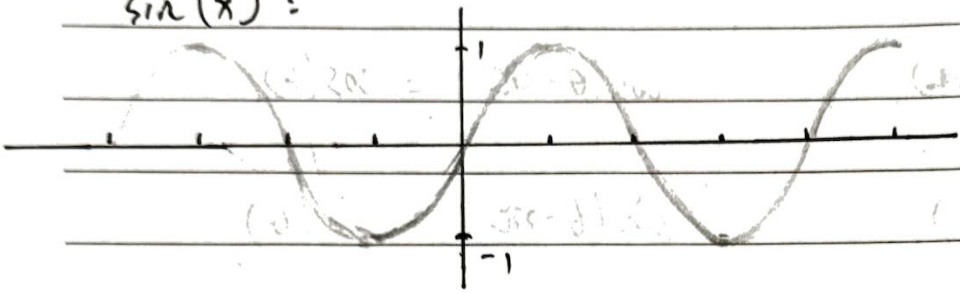
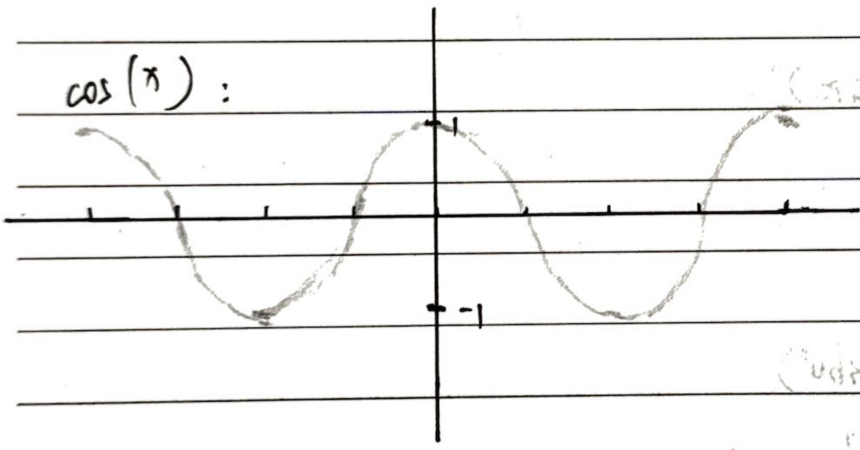
Even, Odd Property:

$\sin(x)$:



odd.

$\cos(x)$:



even.

$\tan(\theta) = \frac{\sin\theta}{\cos\theta}$

$\therefore \lim_{x \to \frac{\pi}{2}} \tan(x) = +\infty.$

$\lim_{x \to -\frac{\pi}{2}} \tan(x) = -\infty.$

odd.

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Pythagorean Property:

$(\sin\theta)^2 + (\cos\theta)^2 = 1$

e.g. $\sin(t) = -\frac{2}{7}$, find $\cos(t)$.

let $\cos(t) = a$.

$\sin^2(t) + a^2 = 1$.

$a^2 = 1-(\frac{2}{7})^2$

$a^2 = \frac{45}{49}$

$a = \pm \frac{3\sqrt{5}}{7}$ (nm, pos).

$\cos(t) = \boxed{-\frac{3\sqrt{5}}{7}}$

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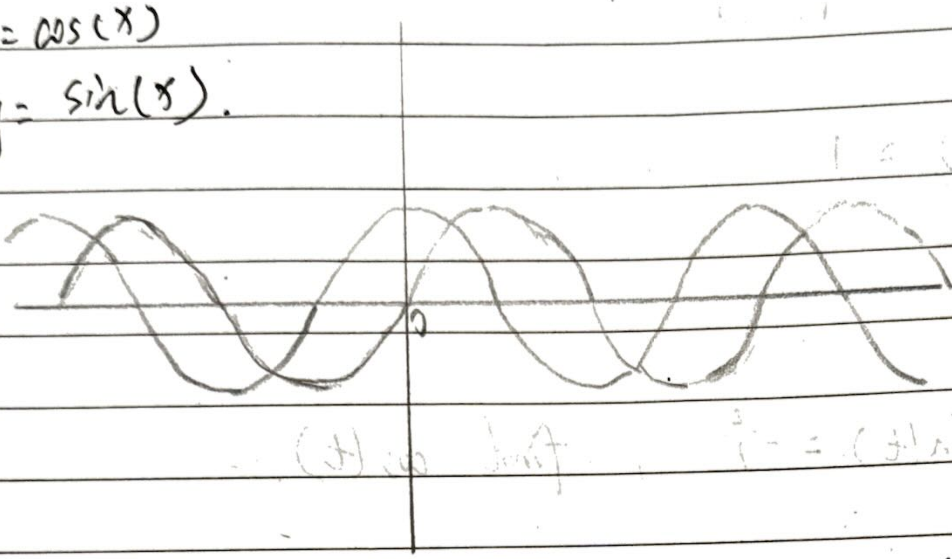
Graphs of Sine and Cosine.

$y = \cos(x)$

$y = \sin(x)$.



$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$ $\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$.

- Domain $(-\infty, \infty)$.
- Range $[-1, 1]$.
- Even/odd : sin = odd, cos = even.

Period : $\sin(x + 2\pi) = \sin(x)$.

$\cos(x + 2\pi) = \cos(x)$.

Graphs of Sinusoidal Functions

	Middle	amplitude	period	
$y=\sin(x)$	$y=0$	1	2π	
$y=3\sin(2x)$	$y=0$	3	π	$(\frac{1}{2}\cdot 2\pi)$
$y=3\sin(2x)+1$	$y=1$	$\cancel{3}$ 3	π	
$y=3\sin(2(x-\frac{\pi}{4}))$		It's better to extract the factor.		
$=3\sin(2x-\frac{\pi}{2})$	$y=0$	3	π	shift $\cancel{\frac{\pi}{2}}$ $\frac{\pi}{4}$ (right)
$y=3\sin(2(x-\frac{\pi}{4}))-1$	$y=-1$	3	π	shift $\frac{\pi}{4}$ (right)

For the graphs of $y=A\cos(Bx-C)+D$ and $y=A\sin(Bx-C)+D$ with B positive.

- midline : $y=D$.
- amplitude : $|A|$
- period : $2\pi \div B$.
- horizontal shift (right) : $\frac{C}{B}$.

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~~param~~ paramitors of $y = \frac{1}{3}\cos\left(\frac{1}{2}x + 3\right) - 5$.

midline : -5.

amplitude : $\frac{1}{3}$.

period : $2\pi \div \frac{1}{2}$
$= 4\pi$.

horizontal shift (right) : $-3 \div \frac{1}{2} = -6$.

Graphs of Tan, Sec, Cot, Csc

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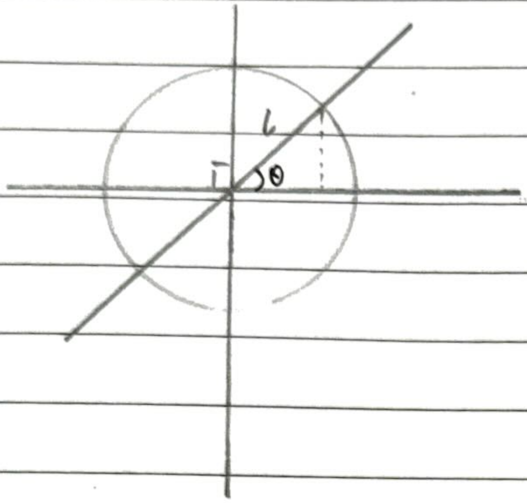
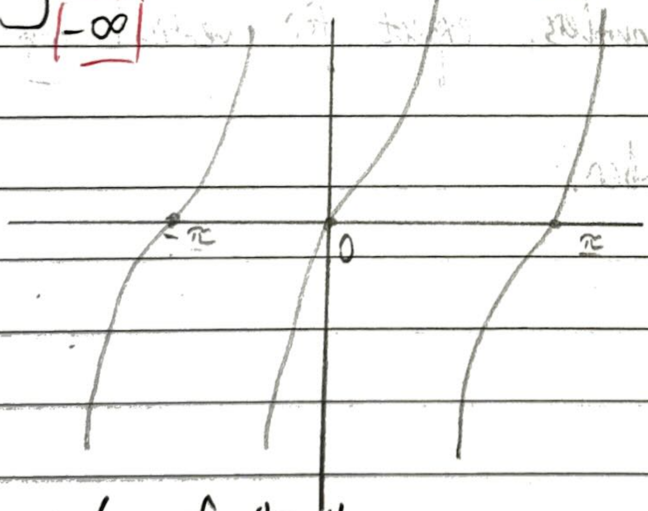


since the line l could be described as a slope value.
And the "slope" is $\frac{\text{rise}}{\text{run}}$, ~~whi~~ wich is $\frac{\sin\theta}{\cos\theta}$, which is $\tan\theta$,
As a conclution, the $\boxed{\text{slope}}$ of line l is $\boxed{\tan\theta}$

As the angle increase towards $\boxed{\frac{\pi}{2}}$, the slope goes to $\boxed{\infty}$, (~~whi with~~ which does behave the same as $\tan\theta$).

As the angle decrease towards $\boxed{-\frac{\pi}{2}}$, the slope goes to $\boxed{-\infty}$



$\tan\theta$ has period of "π".

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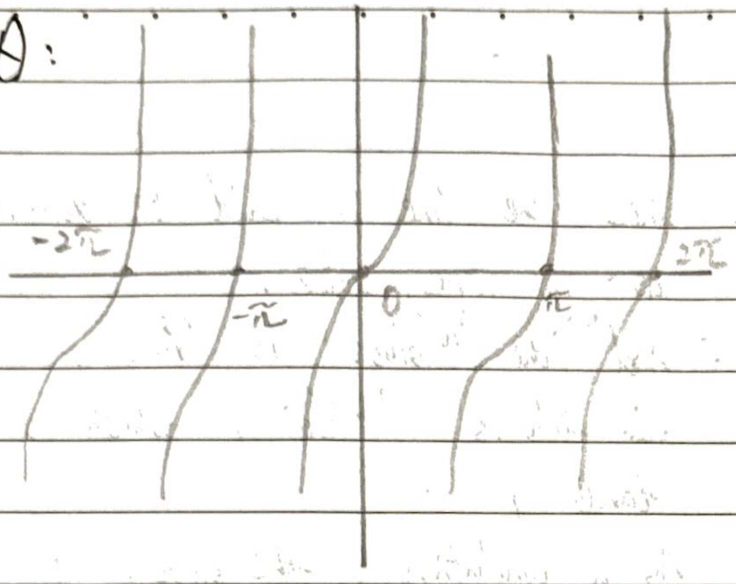
tan θ :



consider this graph of $y = \tan(x)$.

x-intercepts: $k\pi$ where $k \in \mathbb{Z}$

vertical asympotetes : $k\frac{\pi}{2}$ where $k \in \mathbb{Z}$.

domain : All real numbers. expect for vertical asympotetes

range : All real number.

period: π.

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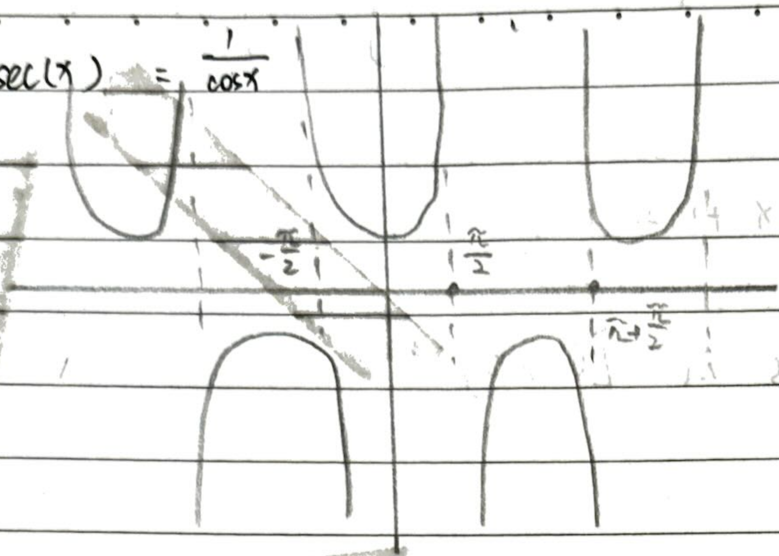
$y = \sec(x) = \frac{1}{\cos x}$



x-intercepts: No.

vertical asymptotes: $\frac{3}{2}k\pi$, where $k \in \mathbb{Z}$.

domain: all real numbers expect for vertical asymptotes.

range: $[-1, -\infty)$, $[1, \infty)$.

period: 2π.

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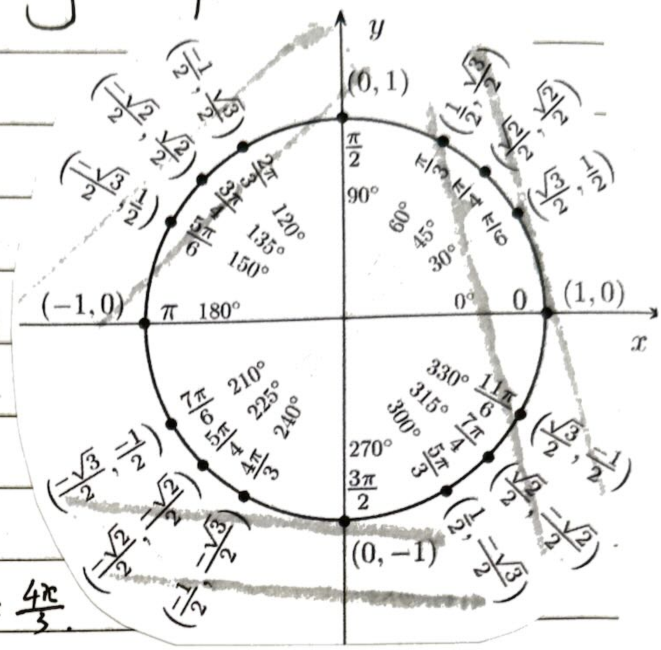
Solving Basic Trig Equations



For the equation $2\cos x + 1 = 0$.

a) Find the solutions in $[0, 2\pi)$

$$2\cos(x) = -1$$
$$\cos(x) = -\frac{1}{2}$$

from the special values: $x = \frac{2\pi}{3}$ or $x = \frac{4\pi}{3}$.

b) Find general formula for All ~~sool~~ solutions

since x has value of $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$.

so ALL solutions are

$\cancel{\frac{2\pi}{3} \pm}$ $\frac{2\pi}{3} + 2k\pi$

$\frac{4\pi}{3} + 2k\pi$ (where $k \in \mathbb{Z}$).

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E For equation $2\tan(x) = \sqrt{3} - \tan(x)$.

a) Find the solutions in interval $[0, 2\pi)$

$$3\tan(x) = \sqrt{3}$$

$$\tan(x) = \frac{\sqrt{3}}{3}$$

$$\frac{\sin(x)}{\cos(x)} = \frac{\sqrt{3}}{3}$$

$$\frac{\sin(\frac{\pi}{6})}{\cos(\frac{\pi}{6})} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}. \quad \text{or} \quad x = \frac{7\pi}{6}.$$

b) For all solutions:

$$x = \begin{cases} \frac{\pi}{6} + k\pi \\ \cancel{\frac{7\pi}{6} + k\pi}. \end{cases} \quad (k \in \mathbb{Z})$$

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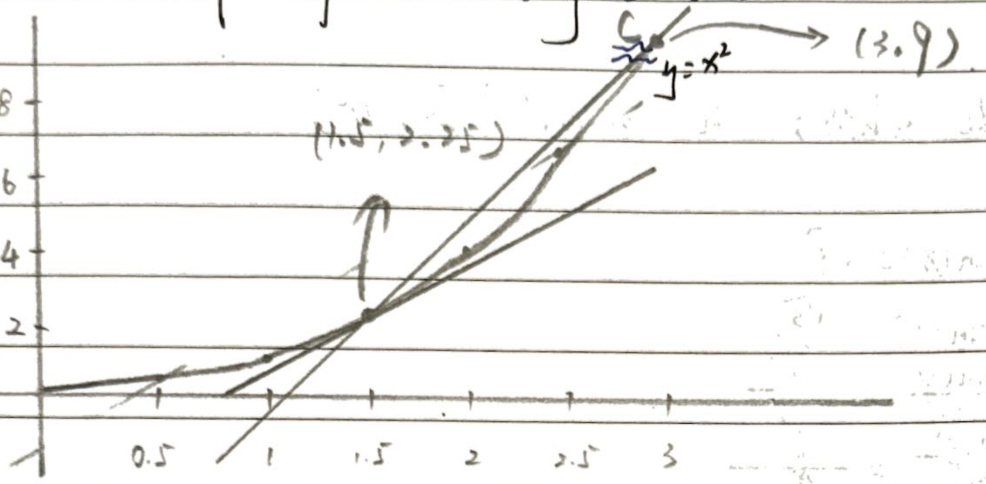
Derivatives and Rate of Change.

Find the slope of the tangent line:



$\frac{\text{rise}}{\text{run}}$ t $= \frac{\Delta y}{\Delta x} = \frac{f(x) - f(1.5)}{3-1.5} = 4.5$

so. 4.5 is the slope of the segent line.

But how could we approach the value of the slope of the tangent line.

We can select a "smaller" "c" value.
We can keep doing this ~~press~~ prosice until point "c" is very close to the ~~point of change~~ tangent point.

~~The~~

The ~~older~~ definition:

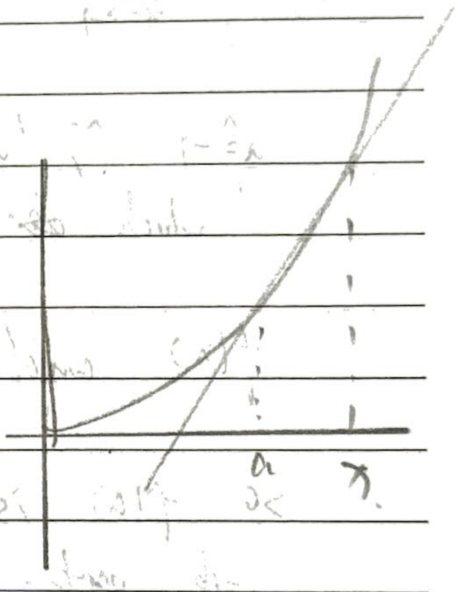
$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

(Make ~~sa~~ sure that the limit exist and is the same from both side as ~~x~~ $x \to a$).

Another version:

let "h" be the "run" $(x - a)$.

so, we get $f'(a) = \lim_{x \to a} \frac{f(a+h) - f(a)}{h}$.



which is then equivelent to:

$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

The following expressions represent the derivative of some function at some value of a

For each example, find the function and the value of a.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad ①$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad ②.$$

1): $\lim_{x \to -1} \frac{(x+5)^2 - 16}{x+1}$

from formula ①, we can see, a got to be -1 which does satisfi $x - a = x + 1$

$f(x)$ could be $(x+5)^2$ ~~+ 16~~.

so $f(a)$ is $(-1+5)^2 = 16$ which corsponds to the -16 part.

so $a = -1$, $f(a) = (a+5)^2$.

2) : $\lim_{h\to 0} \frac{3^{2+h}-9}{h}$

from formula ② :

$a=2$, $f(a)=3^x$

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Computing Derivatives from the Definition.

Find the derivative of $f(x) = \frac{1}{\sqrt{3-x}}$ at $x=-1$

from the defination:

$$f'(-1) = \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{3+1-h}} - \frac{1}{\sqrt{4}}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{4-h}} - \frac{1}{2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2}{2\sqrt{4-h}} - \frac{\sqrt{4-h}}{2\sqrt{4-h}}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2-\sqrt{4-h}}{2\sqrt{4-h}}}{h}$$

$$= \lim_{h\to 0} \frac{2-\sqrt{4-h}}{2h\sqrt{4-h}}$$

$$= \lim_{h\to 0} \frac{2-\sqrt{4-h}}{2h\sqrt{4-h}} \cdot \frac{2+\sqrt{4-h}}{2+\sqrt{4-h}}$$

$$= \lim_{h\to 0} \frac{4-4+h}{4h\sqrt{4-h}+2h(4-h)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(4\sqrt{4-h}+2(4-h))}$$

$$= \lim_{h\to 0} \frac{1}{4\sqrt{4-h}+(4-h)\cdot 2}$$

$$= \frac{1}{4 \cdot 2 + 4 \times 2}$$

$$= \frac{1}{16}.$$

So the answer is $\frac{1}{16}$!!!

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Find the equation of the tangent line to

$y = x^3 - 3x$ at $x = 2$.

let $f(x) = x^3 - 3x$.

so : $f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

$= \lim_{h \to 0} \frac{(2+h)^3 - 3(2+h) - f(2)}{h}$

$= \lim_{h \to 0} \frac{2^3 + 3 \times 2^2 h + 3 \times 2 h^2 + h^3 - 3 \times 2 - h - 2}{h}$

$= \lim_{h \to 0} \frac{8 - 8 + 12h + 6h^2 + h^3 - 3h}{h}$

$= \lim_{h \to 0} \frac{(12 - 3 + 6h + h^2) \cdot \cancel{h}}{\cancel{h}}$

$= \lim_{h \to 0} 12 - 3 + 6h + h^2$

$= 12 - 3$

$= 9$.

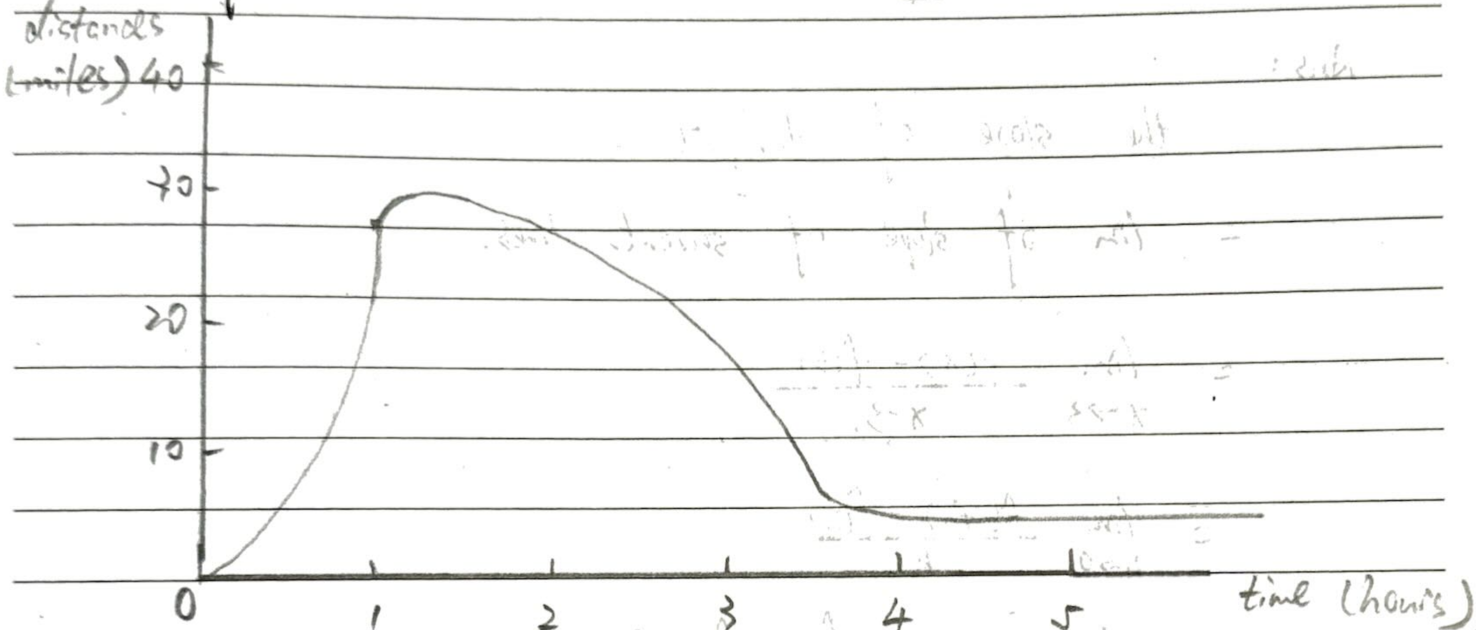
$y = 9(x-2) + 2$

$y = 9x - 18 + 2$

$y = 9x - 16$.

Interpreting Derivatives.

The graph of $y = f(x)$ represents my distance from campus on a bike ride heading due north.



- Interpret the slope of secant line through points $(3, f(3))$ and $(4, f(4))$.

Ans: $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{\text{change in distance}}{\text{change in time}}$.

$= \cancel{a}$ velocity.

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- Interpret the slope of tangent line at $x=3$.

Ans:

the slope of tangent =

= lim of slope of secant lines.

$$= \lim_{x \to 3} \frac{f(x) - f(3)}{x-3}.$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

= derivative of ~~f~~ $f(x)$.

which is the instantaneous velocity at $x=3$.

The Derivative as a function.

for function $f(x) = \frac{1}{x}$, find the derivative $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x - x - h}{x^2 + xh}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{h(x^2 + xh)}$$

$$= -\frac{1}{x^2}$$