

3

Chapter Review

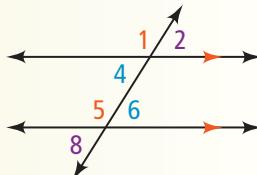
Connecting **BIG** Ideas and Answering the Essential Questions

1 Reasoning and Proof

You can prove that two lines are parallel by using special angle relationships and the relationships of two lines to a third line.

Parallel Lines and Angle Pairs (Lessons 3-2 and 3-3)

$$\begin{aligned}\angle 1 &\cong \angle 5 \\ \angle 4 &\cong \angle 6 \\ m\angle 4 + m\angle 5 &= 180 \\ \angle 2 &\cong \angle 8\end{aligned}$$



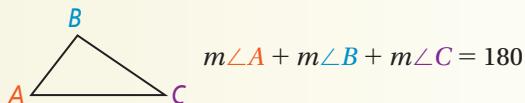
Parallel and Perpendicular Lines (Lesson 3-4)

$$\begin{aligned}a \parallel b \text{ and } b \parallel c &\rightarrow a \parallel c \\ a \perp b \text{ and } a \perp c &\rightarrow b \parallel c\end{aligned}$$

2 Measurement

The sum of the measures of the angles of a triangle is 180.

Parallel Lines and Triangles (Lesson 3-5)



$$m\angle A + m\angle B + m\angle C = 180$$

3 Coordinate Geometry

You can write the equation of a line by using its slope and a point on the line.

Lines in the Coordinate Plane (Lesson 3-7)

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Slopes of Parallel and Perpendicular Lines (Lesson 3-8)

Parallel lines: equal slopes
Perpendicular lines:
product of slopes is -1



Chapter Vocabulary

- alternate exterior angles (p. 142)
- alternate interior angles (p. 142)
- auxiliary line (p. 172)
- corresponding angles (p. 142)
- exterior angle of a polygon (p. 173)
- flow proof (p. 158)
- parallel lines (p. 140)
- parallel planes (p. 140)
- point-slope form (p. 190)
- remote interior angles (p. 173)
- same-side interior angles (p. 142)
- skew lines (p. 140)
- slope (p. 189)
- slope-intercept form (p. 190)
- transversal (p. 141)

Choose the correct term to complete each sentence.

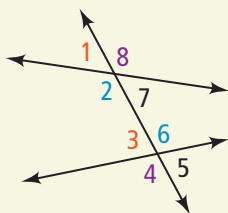
1. A(n) intersects two or more coplanar lines at distinct points.
2. The measure of a(n) of a triangle is equal to the sum of the measures of its two remote interior angles.
3. The linear equation $y - 3 = 4(x + 5)$ is in .
4. When two coplanar lines are cut by a transversal, the angles formed between the two lines and on opposite sides of the transversal are .
5. Noncoplanar lines that do not intersect are .
6. The linear equation $y = 3x - 5$ is in .

3-1 Lines and Angles

Quick Review

A **transversal** is a line that intersects two or more coplanar lines at distinct points.

$\angle 1$ and $\angle 3$ are **corresponding angles**.



$\angle 2$ and $\angle 6$ are **alternate interior angles**.

$\angle 2$ and $\angle 3$ are **same-side interior angles**.

$\angle 4$ and $\angle 8$ are **alternate exterior angles**.

Example

Name two other pairs of corresponding angles in the diagram above.

$\angle 5$ and $\angle 7$

$\angle 2$ and $\angle 4$

Exercises

Identify all numbered angle pairs that form the given type of angle pair. Then name the two lines and transversal that form each pair.

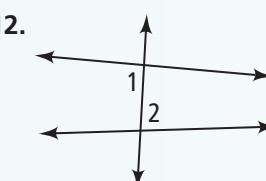
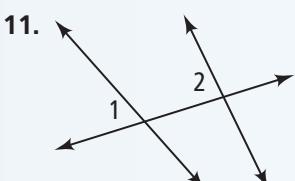
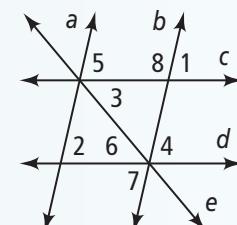
7. alternate interior angles

8. same-side interior angles

9. corresponding angles

10. alternate exterior angles

Classify the angle pair formed by $\angle 1$ and $\angle 2$.



3-2 Properties of Parallel Lines

Quick Review

If two parallel lines are cut by a transversal, then

- corresponding angles, alternate interior angles, and alternate exterior angles are congruent
- same-side interior angles are supplementary

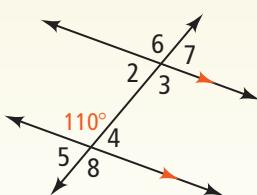
Example

Which other angles measure 110?

$\angle 6$ (corresponding angles)

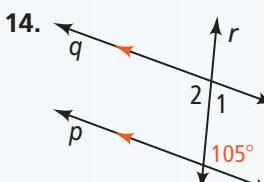
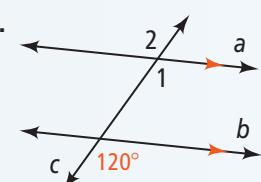
$\angle 3$ (alternate interior angles)

$\angle 8$ (vertical angles)

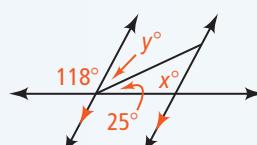


Exercises

Find $m\angle 1$ and $m\angle 2$. Justify your answers.



15. Find the values of x and y in the diagram below.



3-3 Proving Lines Parallel

Quick Review

If two lines and a transversal form

- congruent corresponding angles,
 - congruent alternate interior angles,
 - congruent alternate exterior angles, or
 - supplementary same-side interior angles,
- then the two lines are parallel.

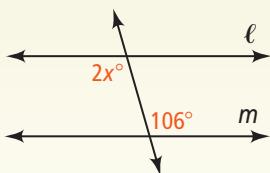
Example

What is the value of x for which $\ell \parallel m$?

The given angles are alternate interior angles. So, $\ell \parallel m$ if the given angles are congruent.

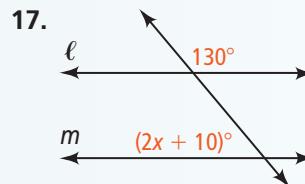
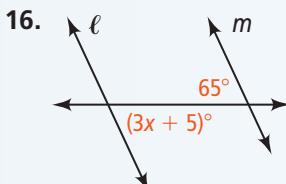
$$2x = 106 \quad \text{Congruent } \triangle \text{ have equal measures.}$$

$$x = 53 \quad \text{Divide each side by 2.}$$



Exercises

Find the value of x for which $\ell \parallel m$.



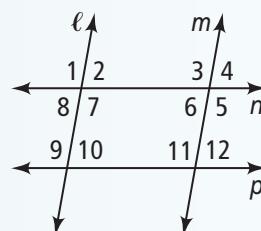
Use the given information to decide which lines, if any, are parallel. Justify your conclusion.

18. $\angle 1 \cong \angle 9$

19. $m\angle 3 + m\angle 6 = 180$

20. $m\angle 2 + m\angle 3 = 180$

21. $\angle 5 \cong \angle 11$



3-4 Parallel and Perpendicular Lines

Quick Review

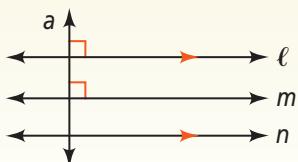
- Two lines \parallel to the same line are \parallel to each other.
- In a plane, two lines \perp to the same line are \parallel .
- In a plane, if one line is \perp to one of two \parallel lines, then it is \perp to both \parallel lines.

Example

What are the pairs of parallel and perpendicular lines in the diagram?

$$\ell \parallel n, \ell \parallel m, \text{ and } m \parallel n.$$

$$a \perp \ell, a \perp m, \text{ and } a \perp n.$$

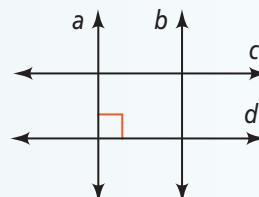


Exercises

Use the diagram at the right to complete each statement.

22. If $b \perp c$ and $b \perp d$, then $c \underline{\quad} d$.

23. If $c \parallel d$, then $\underline{\quad} \perp c$.



24. **Maps** Morris Avenue intersects both 1st Street and 3rd Street at right angles. 3rd Street is parallel to 5th Street. How are 1st Street and 5th Street related? Explain.

3-5 Parallel Lines and Triangles

Quick Review

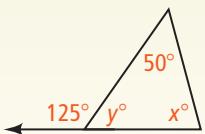
The sum of the measures of the angles of a triangle is 180.
The measure of each **exterior angle** of a triangle equals the sum of the measures of its two **remote interior angles**.

Example

What are the values of x and y ?

$$x + 50 = 125 \quad \text{Exterior Angle Theorem}$$

$$x = 75 \quad \text{Simplify.}$$



$$x + y + 50 = 180 \quad \text{Triangle Angle-Sum Theorem}$$

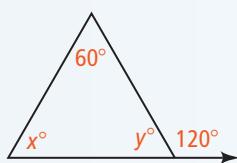
$$75 + y + 50 = 180 \quad \text{Substitute } 75 \text{ for } x.$$

$$y = 55 \quad \text{Simplify.}$$

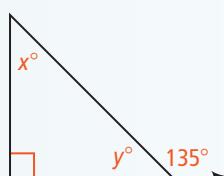
Exercises

Find the values of the variables.

25.



26.



The measures of the three angles of a triangle are given.
Find the value of x .

27. $x, 2x, 3x$

28. $x + 10, x - 20, x + 25$

29. $20x + 10, 30x - 2, 7x + 1$

3-6 Constructing Parallel and Perpendicular Lines

Quick Review

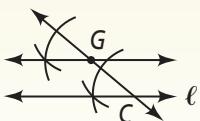
You can use a compass and a straightedge to construct

- a line parallel to a given line through a point not on the line
- a line perpendicular to a given line through a point on the line, or through a point not on the line

Example

Which step of the parallel lines construction guarantees the lines are parallel?

The parallel lines construction involves constructing a pair of congruent angles. Since the congruent angles are corresponding angles, the lines are parallel.



Exercises

30. Draw a line m and point Q not on m . Construct a line perpendicular to m through Q .

Use the segments below.



31. Construct a rectangle with side lengths a and b .

32. Construct a rectangle with side lengths a and $2b$.

33. Construct a quadrilateral with one pair of parallel opposite sides, each side of length $2a$.

3-7 Equations of Lines in the Coordinate Plane

Quick Review

Slope-intercept form is $y = mx + b$, where m is the slope and b is the y -intercept.

Point-slope form is $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

Example

What is an equation of the line with slope -5 and y -intercept 6 ?

Use slope-intercept form: $y = -5x + 6$.

Example

What is an equation of the line through $(-2, 8)$ with slope 3 ?

Use point-slope form: $y - 8 = 3(x + 2)$.

Exercises

Find the slope of the line passing through the points.

34. $(6, -2), (1, 3)$ 35. $(-7, 2), (-7, -5)$

36. Name the slope and y -intercept of $y = 2x - 1$.
Then graph the line.

37. Name the slope of and a point on $y - 3 = -2(x + 5)$.
Then graph the line.

Write an equation of the line.

38. slope $-\frac{1}{2}$, y -intercept 12

39. slope 3 , passes through $(1, -9)$

40. passes through $(4, 2)$ and $(3, -2)$

3-8 Slopes of Parallel and Perpendicular Lines

Quick Review

Parallel lines have the same slopes.

The product of the slopes of two perpendicular lines is -1 .

Example

What is an equation of the line perpendicular to $y = 2x - 5$ that contains $(1, -3)$?

Step 1 Identify the slope of $y = 2x - 5$.

The slope of the given line is 2 .

Step 2 Find the slope of a line perpendicular to $y = 2x - 5$.

The slope is $-\frac{1}{2}$, because $2\left(-\frac{1}{2}\right) = -1$.

Step 3 Use point-slope form to write $y + 3 = -\frac{1}{2}(x - 1)$.

Exercises

Determine whether \overleftrightarrow{AB} and \overleftrightarrow{CD} are parallel, perpendicular, or neither.

41. $A(-1, -4), B(2, 11), C(1, 1), D(4, 10)$

42. $A(2, 8), B(-1, -2), C(3, 7), D(0, -3)$

43. $A(-3, 3), B(0, 2), C(1, 3), D(-2, -6)$

44. $A(-1, 3), B(4, 8), C(-6, 0), D(2, 8)$

45. Write an equation of the line parallel to $y = 8x - 1$ that contains $(-6, 2)$.

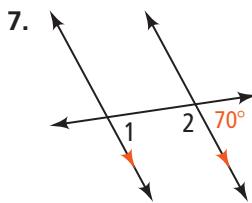
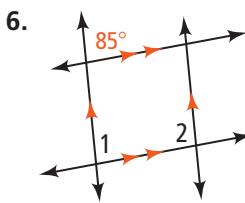
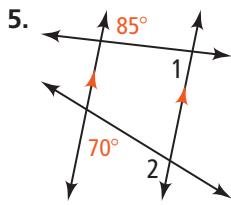
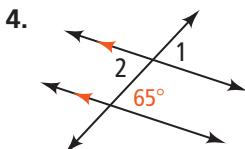
46. Write an equation of the line perpendicular to $y = \frac{1}{6}x + 4$ that contains $(3, -3)$.


Do you know HOW?

Find the measure of the third angle of a triangle given the measures of two angles.

1. 57 and 101
2. 72 and 72
3. x and 20

Find $m\angle 1$ and $m\angle 2$. Justify each answer.



8. Draw a line m and a point T not on the line. Construct the line through T perpendicular to m .
9. Draw any $\angle ABC$. Then construct line m through A so that $m \parallel \overleftrightarrow{BC}$.
10. The measures of the angles of a triangle are $2x$, $x + 24$, and $x - 4$. Find the value of x . Then find the measures of the angles.

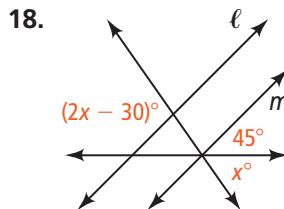
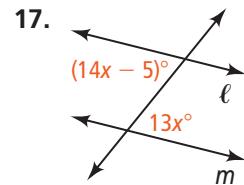
Determine whether the following are *parallel lines*, *skew lines*, or *neither*.

11. opposite sides of a rectangular picture frame
12. the center line of a soccer field and a sideline of the field
13. the path of an airplane flying north at 15,000 ft and the path of an airplane flying west at 10,000 ft

Use the given information to write an equation of each line.

14. slope -5 , y -intercept -2
15. slope $\frac{1}{2}$, passes through $(4, -1)$
16. passes through $(1, 5)$ and $(3, 11)$

Algebra Find the value of x for which $\ell \parallel m$.



Graph each pair of lines. Tell whether they are *parallel*, *perpendicular*, or *neither*.

19. $y = 4x + 7$ and $y = -\frac{1}{4}x - 3$
20. $y = 3x - 4$ and $y = 3x + 1$
21. $y = x + 5$ and $y = -5x - 1$

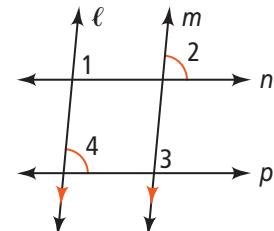
Do you UNDERSTAND?

- © 22. **Developing Proof** Provide the reason for each step.

Given: $\ell \parallel m$, $\angle 2 \cong \angle 4$

Prove: $n \parallel p$

Statements	Reasons
1) $\ell \parallel m$	1) a. ?
2) $\angle 1 \cong \angle 2$	2) b. ?
3) $\angle 2 \cong \angle 4$	3) c. ?
4) $\angle 1 \cong \angle 4$	4) d. ?
5) $n \parallel p$	5) e. ?



- © 23. **Reasoning** Suppose a line intersecting two planes A and B forms a right angle at exactly one point in each plane. What must be true about planes A and B ? (Hint: Draw a picture.)

Common Core Cumulative Standards Review



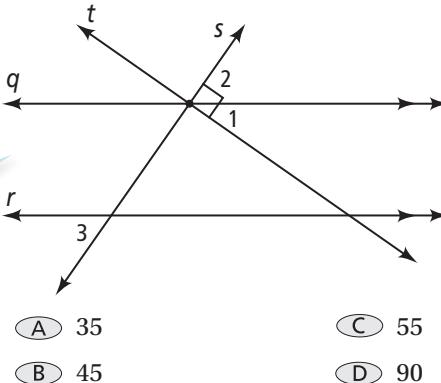
TIPS FOR SUCCESS

Some test questions ask you to analyze a diagram. Read the sample question at the right. Then follow the tips to answer it.

TIP 2

Use the diagram to find the relationship between $\angle 2$ and $\angle 3$.

In the diagram, $q \parallel r$ and $s \perp t$. If $m\angle 1 = 35$, what is $m\angle 3$?



TIP 1

Use $m\angle 1$ and what you know about the measures of angles formed by perpendicular lines to find $m\angle 2$.

Think It Through

Since $s \perp t$, the angle formed by $\angle 1$ and $\angle 2$ is a right angle. So $m\angle 1 + m\angle 2 = 90$. Substitute 35 for $m\angle 1$ and solve for $m\angle 2$.

$$m\angle 2 = 90 - 35 = 55$$

$\angle 2$ and $\angle 3$ are alternate exterior angles. Since $q \parallel r$, $\angle 2 \cong \angle 3$. So $m\angle 3 = 55$. The correct answer is C.



Vocabulary Builder

As you solve test items, you must understand the meanings of mathematical terms. Match each term with its mathematical meaning.

- | | |
|-------------------------|--|
| A. transversal | I. two angles whose measures have sum 90 |
| B. complementary angles | II. a point that divides a segment into two congruent segments |
| C. conditional | III. an <i>if-then</i> statement |
| D. midpoint | IV. a line that intersects two coplanar lines at two distinct points |
| E. supplementary angles | V. two angles whose measures have sum 180 |

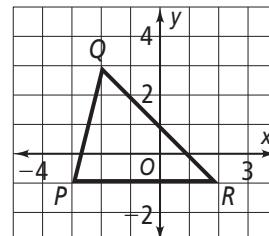
Selected Response

Read each question. Then write the letter of the correct answer on your paper.

1. Which expression describes the area of a square that has side lengths $7n^3$?

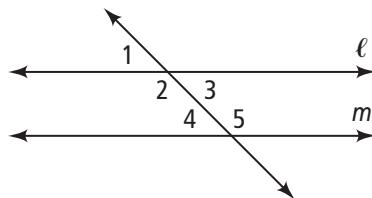
- (A) $14n^6$ (C) $49n^6$
 (B) $14n^9$ (D) $49n^9$

2. What is the area of $\triangle PQR$?



- (F) 10 units² (H) 20 units²
 (G) 15 units² (I) 25 units²

3. Which condition(s) will allow you to prove that $\ell \parallel m$?



- I. $\angle 1 \cong 4$
- II. $\angle 2 \cong \angle 5$
- III. $\angle 3 \cong \angle 4$
- IV. $m\angle 2 + m\angle 4 = 180$

- A III only C II and III only
 B I and IV only D I, II, III, and IV

4. The length of your rectangular vegetable garden is 15 times its width. You used 160 ft of fencing to surround the garden. How much area do you have for planting?

- F 150 ft² H 800 ft²
 G 375 ft² I 1600 ft²

5. Which of the following angle relationships can you use to prove that two lines are parallel?

- A supplementary corresponding angles
 B congruent alternate interior angles
 C congruent vertical angles
 D congruent same-side interior angles

6. Which point lies farthest from the origin?

- F (0, -7) H (-4, -3)
 G (-3, 8) I (5, 1)

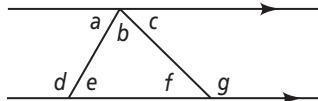
7. $\angle A$ and $\angle B$ are supplementary vertical angles. What is $m\angle B$?

- A 45 C 135
 B 90 D 180

8. Which types of angles can an obtuse triangle have?

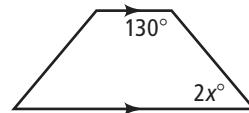
- I. a right angle
 - II. two acute angles
 - III. an obtuse angle
 - IV. two vertical angles
- F I and II H III and IV
 G II and III I I and IV

9. Given the diagram below, which expression could be used to find the sum of the angles in the triangle?



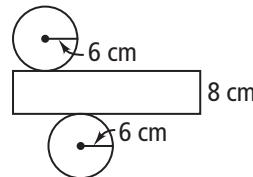
- A $f + g + c$
 B $a + b + c$
 C $a + b + e$
 D $d + e + g$

10. What is the value of x in the figure?



- F 20 H 45
 G 25 I 50

11. The net for a cylindrical container that holds a stack of DVDs is shown below. What is the total area of the net?



- A 226 cm² C 528 cm²
 B 302 cm² D 582 cm²

12. What are the coordinates of the midpoint of a segment with endpoints $(-1, 2)$ and $(5, 6)$?

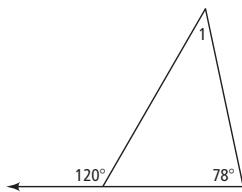
- F (2, 4) H (6, 4)
 G (4, 8) I (3, 4)

13. What is the measure of any exterior angle of an equiangular triangle?

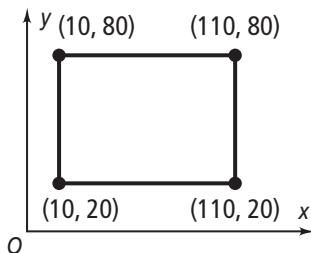
- A 30 C 90
 B 60 D 120

Constructed Response

14. What is the measure of $\angle 1$?



15. Two angles of an isosceles triangle have measures 54.5 and 71. What is the measure of the third angle?
16. In the coordinate plane, \overleftrightarrow{AB} contains $(-2, -4)$ and $(6, 8)$. \overleftrightarrow{CD} contains $(6, y)$ and $(12, 10)$. For what value of y are the lines parallel?
17. A circular wading pool has a diameter of 10 ft. What is the circumference of the wading pool in feet? Use 3.14 for π .
18. What is the measure of the complement of a 56° angle?
19. A new athletic field is being constructed, as shown below. The given coordinates are in terms of yards. What is the area of the field in square yards?



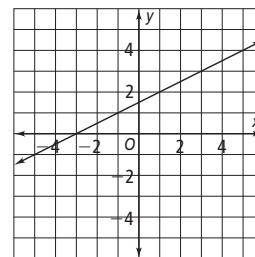
20. Is your friend's argument for the following situation valid? Explain.

Given: If you buy a one-year membership at the gym, then you get one month free. You got a free month at the gym.

Your friend's conclusion: You bought a one-year membership.

21. Draw \overline{MN} . Then construct the perpendicular bisector of \overline{MN} .

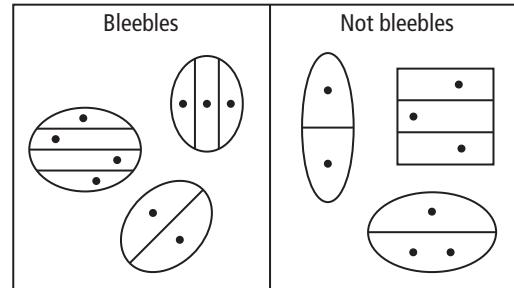
22. What is the equation for a line that passes through the point $(3, -3)$ and is parallel to the line shown in the graph below?



Extended Response

23. \overline{CD} has endpoints $C(5, 7)$ and $D(10, -5)$. What are the coordinates of the midpoint of \overline{CD} ? What is CD ? Show your work.

24. Examples and nonexamples of *bleebles* are shown.



- a. Is the figure at the right a *bleeble*? Explain your reasoning.
b. What is a definition for *bleeble*?

