जय महाकाल

Discrete Structures

Day 10

Q. Use predicates and quantifiers to express the system specifications

"Every mail message larger than one megabyte will be compressed"

- *S(m, 1):* Mail message *m* is larger than *1* megabytes, where the variable m has

the domain of all mail messages.

C(m): Mail message *m* will be compressed.

$$\forall m(S(m, 1) \rightarrow C(m))$$

"If a user is active, at least one network link will be available."

A(u): User *u* is active.

- *S(n, available):* Network link *n* is in *available*, where *n* has the domain of all network links.

$$\exists uA(u) \rightarrow \exists nS(n, \text{available})$$

Q. Consider these statements. The first two are called *premises* and the third is called the *conclusion*. The entire set is called an *argument*.

"All lions are fierce."

"Some lions do not drink coffee."

"Some fierce creatures do not drink coffee."

P(x): x is a lion.

Q(x): x is fierce.

R(x): x drinks coffee.

$\forall x(P(x) \rightarrow Q(x)).$

$\exists x(P(x) \land \neg R(x)).$

$\exists x(Q(x) \land \neg R(x)).$

Q. Consider these statements, of which the first three are premises and the fourth is a valid conclusion.

"All hummingbirds are richly colored."

"No large birds live on honey."

"Birds that do not live on honey are dull in color."

"Hummingbirds are small."

P(x): x is a hummingbird.

Q(x): x is large.

R(x): x lives on honey.

S(x): x is richly colored.

hello

$\forall x(P(x) \to S(x))$. "small" is the same as "not large" and that $\neg\exists x(Q(x) \wedge R(x))$. "dull in color" is the same as "not richly colored."

$\forall x(\neg R(x) \to \neg S(x))$.

$\forall x(P(x) \to \neg Q(x))$.

- hello

SSB

Q. Translate these statements into English, where *C(x)* is "*x* is a comedian" and *F(x)* is "*x* is funny" and the domain consists of all people.

a) $\forall x(C(x) \rightarrow F(x))$ **b)** $\forall x(C(x) \wedge F(x))$

c) $\exists x(C(x) \rightarrow F(x))$ **d)** $\exists x(C(x) \wedge F(x))$

a) For every person x, If x is a comedian then x is funny. Or Every comedian is funny.

b) For every person x, x is a comedian and x is funny. Or Every person is a funny comedian.

c) There exists a person such that if she or he is a comedian, then she or he is funny.

d) There exists a person such that she or he is a comedian and she or he is funny. Or Some comedians are funny.

Q. Translate these statements into English, where *R(x)* is "*x* is a rabbit" and *H(x)* is "*x* hops" and the domain consists of all animals.

a) $\forall x(R(x) \rightarrow H(x))$ **b)** $\forall x(R(x) \wedge H(x))$

c) $\exists x(R(x) \rightarrow H(x))$ **d)** $\exists x(R(x) \wedge H(x))$

Q. Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows C++.

$$\exists x(P(x) \land Q(x))$$

b) There is a student at your school who can speak Russian but who doesn't know C++.

$$\exists x(P(x) \land \neg Q(x))$$

c) Every student at your school either can speak Russian or knows C++.

$$\forall x(P(x) \lor Q(x))$$

d) No student at your school can speak Russian or knows C++.

$$\forall x \neg (P(x) \lor Q(x)) \text{ or } \neg \exists x (P(x) \lor Q(x))$$

They need to cover before ex...

Q. Let *C(x)* be the statement "*x* has a cat," let *D(x)* be the statement "*x* has a dog," and let *F(x)* be the statement "*x* has a ferret." Express each of these statements in terms of *C(x)*, *D(x)*, *F(x)*, quantifiers, and logical connectives. Let the domain consist of all students in your class.

a) A student in your class has a cat, a dog, and a ferret.

b) All students in your class have a cat, a dog, or a ferret.

c) Some student in your class has a cat and a ferret, but not a dog.

d) No student in your class has a cat, a dog, and a ferret.

e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

Q. Translate each of these statements into logical expression using predicates, quantifiers, and logical connectives.

a) No one is perfect.

b) Not everyone is perfect.

c) All your friends are perfect.

d) At least one of your friends is perfect.

e) Everyone is your friend and is perfect.

f) Not everybody is your friend or someone is not perfect.

Let *P(x): x* is perfect.

F(x): x is your friend; and let the domain be all people.

a) $\forall x \neg P(x)$ **b)** $\neg \forall x\, P(x)$ **c)** $\forall x(F(x) \rightarrow P(x))$

d) $\exists x(F(x) \wedge P(x))$ **e)** $\forall x(F(x) \wedge P(x))$ or $(\forall x\, F(x)) \wedge (\forall x\, P(x))$

f) $(\neg \forall x\, F(x)) \vee (\exists x \neg P(x))$

Q. Express each of these statements using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase “It is not the case that.”)

a) Some old dogs can learn new tricks.

b) No rabbit knows calculus.

c) Every bird can fly.

d) There is no dog that can talk.

e) There is no one in this class who knows French and Russian.

a) Let $T(x)$: x can learn new tricks, where domain be old dogs.

Some old dogs can learn new tricks: $\exists x\, T(x)$

Negation of $\exists x\, T(x)$ is $\forall x \neg T(x)$: “No old dogs can learn new tricks.”

b) Let $C(x)$: x knows calculus, where domain be rabbits.

No rabbit knows calculus: $\forall x \neg C(x)$

Negation of $\forall x \neg C(x)$ is $\exists x\, C(x)$: “There is a rabbit that knows calculus.”

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d) There is no dog that can talk.

Negation: There is a dog that can talk.

Let T(x): x can talk.

There is no dog that can talk:

$(\forall x \neg)T(x)$

Negating:

$\neg(\forall x \neg)T(x)$: $\exists x\, T(x)$.

There is a dog that can talk.