Discrete Structures

Day 12

Q. Every student at your school either can speak Russian or knows C++.

$$\forall x(P(x) \vee Q(x))$$

$$\forall x(P(x) \oplus Q(x))$$

https://math.stackexchange.com/questions/3598348/problem-from-kenneth-rosens-discrete-mathematics-and-its-applications-section

You go to one of your friend's house and you are asked: "Tea or Coffee". Here this or is an exclusive or.

Q. For each of these sentences, determine whether an inclusive or, or an exclusive or, is intended.

a) Experience with C++ or Java is required. "Inclusive or"

b) Lunch includes soup or salad. "Exclusive or"

c) To enter the country you need a passport or a voter registration card. "Inclusive or"

d) Publish or perish. "Exclusive or"

Distribution of Quantifiers over Conjunction and Disjunction

-

Q1. Let $Q(x, y)$ be the statement "$x + y = x - y$." If the domain for both variables consists of all integers, what are the truth values?

a) $Q(1, 1)$

b) $Q(2, 0)$

c) $\forall y Q(1, y)$

d) $\exists x Q(x, 2)$

e) $\exists x \exists y Q(x, y)$

f) $\forall x \exists y Q(x, y)$

g) $\exists y \forall x Q(x, y)$

h) $\forall y \exists x Q(x, y)$

i) $\forall x \forall y Q(x, y)$

Rules of Inference

An *argument* in propositional logic is a sequence of propositions. All but the final proposition in the argument are called *premises* and the final proposition is called the *conclusion*. An argument is *valid* if the truth of all its premises implies that the conclusion is true.

To deduce new statements from statements we already have, we use rules of inference which are templates for constructing valid arguments. Rules of inference are our basic tools for establishing the truth of statements.

Fallacies: Some common forms of incorrect reasoning, which lead to invalid arguments.

Skip kia //

Rule of Inference	*Tautology*	*Name*
p $p \to q \quad \therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q \quad \therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r \quad \therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p \quad \therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p $q \quad \therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r \quad \therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

Q. “If you have a current password, then you can log onto the network.”

“You have a current password.”

Therefore,

“You can log onto the network.”

Determine whether this is a valid argument.

Let,p: You have a current password.

q: You can log onto the network.

Then, the argument has the form

$p \to q$

p

$\therefore q$

We know that when p and q are propositional variables, the statement $((p \to q) \wedge p) \to q$ is a tautology (Modus ponens)

Hence, the conclusion is valid.

Q. State which rule of inference is the basis of the following argument: "It is below freezing now. Therefore, it is either below freezing or raining now."

Let,

p: It is below freezing now.

q: It is raining now.

Then, the argument has the form

p

$\therefore p \vee q$

This is an argument that uses the addition rule.

Q. State which rule of inference is the basis of the following argument: “It is below freezing and raining now. Therefore, it is below freezing now.”

Let,

p: It is below freezing now.

q: It is raining now.

Then, the argument has the form

$p \wedge q$

$\therefore p$

This argument uses the simplification rule.

Q. Show that the premises “It is not sunny this afternoon and it is colder than yesterday,” “We will go swimming only if it is sunny,” “If we do not go swimming, then we will take a canoe trip,” and “If we take a canoe trip, then we will be home by sunset” lead to the conclusion “We will be home by sunset.”

Let, p: It is sunny this afternoon; q: It is colder than yesterday; r: We will go swimming;

s: We will take a canoe trip; t: We will be home by sunset.

The premises are:

$\neg p \wedge q$: It is not sunny this afternoon and it is colder than yesterday. $r \rightarrow p$: We will go swimming only if it is sunny.

$\neg r \rightarrow s$: If we do not go swimming, then we will take a canoe trip. $s \rightarrow t$: If we take a canoe trip, then we will be home by sunset.

Conclusion: t: We will be home by sunset

We construct an argument to show that our premises lead to the desired conclusion as follows.

Step	Reason
1. $\neg p \wedge q$	Premise
2. $\neg p$	Simplification using (1)
3. $r \rightarrow p$	Premise
4. $\neg r$	Modus tollens using (2) and (3)
5. $\neg r \rightarrow s$	Premise
6. s	Modus ponens using (4) and (5)
7. $s \rightarrow t$	Premise
8. t	Modus ponens using (6) and (7)

Q5. Show that the premises “If you send me an e-mail message, then I will finish writing the program,” “If you do not send me an e-mail message, then I will go to sleep early,” and “If I go to sleep early, then I will wake up feeling refreshed” lead to the conclusion “If I do not finish writing the program, then I will wake up feeling refreshed.”

Q6. Show that the hypotheses “Jasmine is skiing or it is not snowing” and “It is snowing or Bart is playing hockey” imply that “Jasmine is skiing or Bart is playing hockey.”

Q7. Show that the premises $(p \land q) \lor r$ and $r \rightarrow s$ imply the conclusion $p \lor s$.

Q. Show that the premises $(p \wedge q) \vee r$ and $r \rightarrow s$ imply the conclusion $p \vee s$.

Sl. No.	Step	Rule
1	$(p \wedge q) \vee r$	Premise
2	$r \rightarrow s$	Premise
3	$(p \vee r) \wedge (q \vee r)$	Distribution (1)
4	$(p \vee r)$	Simplifcation (3)
5	$\neg r \vee s$	
6	$r \vee p$	Commutative (4)
7	$p \vee s$	Resolution (5) and (6)

Q. Is the following argument valid?

If you do every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics. Therefore, you did every problem in this book.

p: You did every problem in this book.

q: You learned discrete mathematics.

Then this argument is of the form: if $p \rightarrow q$ and q, then p.

$p \rightarrow q$

q

$\therefore$ p

The proposition $((p \rightarrow q) \wedge q) \rightarrow p$ is not a tautology, $((p \rightarrow q) \wedge q) \rightarrow p$ *is F when p is F.*

This type of incorrect reasoning is called the **fallacy of affirming the conclusion**.