# Discrete Structures

Day 7

Q. Let $p$, $q$, and $r$ be the propositions

$p$ : Grizzly bears have been seen in the area.

$q$ : Hiking is safe on the trail.

$r$ : Berries are ripe along the trail.

Write these propositions using $p$, $q$, and $r$ and logical connectives (including negations).

**a)** Berries are ripe along the trail, but grizzly bears have not been seen in the area.

**b)** Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

**c)** If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

**d)** It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

**e )** Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

$p$ : Grizzly bears have been seen in the area.

$q$ : Hiking is safe on the trail.

$r$ : Berries are ripe along the trail.

Write these propositions using $p$, $q$, and $r$ and logical connectives (including negations).

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

$$r \wedge \neg p$$

**b)** Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

$$\neg p \wedge q \wedge r$$

**c)** If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

$$r \rightarrow (q \leftrightarrow \neg p)$$

**d)** It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

$$\neg\, q \wedge \neg p \wedge r$$

**e )** Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

$$p \wedge r \rightarrow \neg q$$

Q. Let $p$, $q$, and $r$ be the propositions

$p$ :You get an A on the final exam.

$q$ :You do every exercise in this book.

$r$ :You get an A in this class.

and:
but/nevertheless/however/even so/inspite/yet/though/still/nonetheless/anyhow/anyway

Write these propositions using $p$, $q$, and $r$ and logical connectives (including negations).

**a)** You get an A in this class, but you do not do every exercise in this book.

**b)** You get an A on the final, you do every exercise in this book, and you get an A in this class.

**c)** To get anA in this class, it is necessary for you to get an A on the final.

**d)** You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

**e)** Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

**f )** You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

- $p$ :You get an A on the final exam.
  $q$ :You do every exercise in this book.
  $r$ :You get an A in this class.

Write these propositions using $p$, $q$, and $r$ and logical connectives (including negations).

a) You get an A in this class, but you do not do every exercise in this book.

$$p \wedge \neg q$$

**b)** You get an A on the final, you do every exercise in this book, and you get an A in this class.

$$p \wedge q \wedge r$$

**c)** To get an A in this class, it is necessary for you to get an A on the final.

$$r \rightarrow p$$

**d)** You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

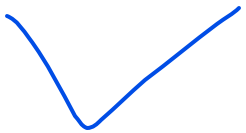
$$p \wedge \neg q \wedge r$$

**e)** Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

$$p \wedge q \rightarrow r$$

**f )** You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

$$r \leftrightarrow (q \vee r)$$



Q. Write each of these statements in the form "if *p*, then *q*"

**a)** It snows whenever the wind blows from the northeast.

**b)** The apple trees will bloom if it stays warm for a week.

**c)** That the Pistons win the championship implies that they beat the Lakers.

**d)** It is necessary to walk 8 miles to get to the top of Long's Peak.

**e)** To get tenure as a professor, it is sufficient to be world famous.

**f)** If you drive more than 400 miles, you will need to buy gasoline.

**g)** Your guarantee is good only if you bought your CD player less than 90 days ago.

**h)** Jan will go swimming unless the water is too cold.

Write each of these statements in the form "if $p$, then $q$"

a) It snows whenever the wind blows from the northeast. $q$ *whenever* $p$

If the wind blows from the northeast, then it snows

**b)** The apple trees will bloom if it stays warm for a week. $q$ *if* $p$

If it stays warm for a week, then the apple trees will bloom

**c)** That the Pistons win the championship implies that they beat the Lakers. $p$ *implies* $q$

If the Pistons win the championship, then they beat the Lakers.

**d)** It is necessary to walk 8 miles to get to the top of Long's Peak.

What is necessary to get to the top of Long's Peak : to walk 8 miles (conclusion/consequence)

If you get to the top of Long's Peak, then must have walked 8 miles.

**e)** To get tenure as a professor, it is sufficient to be world famous.

What is sufficient to get tenure as a professor: to be world famous (hypothesis/antecedent/premise)

If you are world famous, then you will get tenure as a professor.

**f )** If you drive more than 400 miles, you will need to buy gasoline.

If you drive more than 400 miles, then you will need to buy gasoline.

**g)** Your guarantee is good only if you bought your CD player less than 90 days ago. $p$ *only if* $q$

If your guarantee is good, then you have bought your CD player less than 90 days ago.

**h)** Jan will go swimming unless the water is too cold. $q$ *unless not* $p$

If the water is not too cold, then Jan will go swimming.

# Translating English Sentences

- Translate the English sentences into expressions involving propositional variables and logical connectives
- Once we have translated sentences from English into logical expressions we can analyze these logical expressions to determine their truth values, we can manipulate them, and we can use rules of inference (will be taught later) to reason about them.

Q. Translated into a logical expression

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

- "You can access the Internet from campus": a
- "You are a computer science major": c
- "You are a freshman": f

$$a \rightarrow (c \vee \neg f)$$

Q. “You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.”

r: You can ride the roller coaster.

u: you are under 4 feet tall.

o: you are older than 16 years old.

$$(u \wedge \neg o) \rightarrow \neg r$$

Q. The automated reply cannot be sent when the file system is full

a: The automated reply can be sent.

f: The file system is full.

$$f \rightarrow \neg a$$

System specifications should be **consistent**, that is, they should not contain conflicting requirements that could be used to derive a contradiction.

Q. Determine whether these system specifications are consistent:

"The diagnostic message is stored in the buffer or it is retransmitted."

"The diagnostic message is not stored in the buffer."

"If the diagnostic message is stored in the buffer, then it is retransmitted."

p: The diagnostic message is stored in the buffer .

q: The diagnostic message is retransmitted.

$p \vee q$

$\neg p$

$p \rightarrow q$

| p | q | | | |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | T | F | F |
| F | T | T | T | T |
| F | F | F | T | T |

# Logic Puzzles: Puzzles that can be solved using logical reasoning are known as **logic puzzles**.

- An island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people *A* and *B*. What are *A* and *B* if *A* says "*B* is a knight" and *B* says "The two of us are opposite types?"
- Let
  - $p$ : A is a knight. and $\neg p$: A is knave
  - $q$: B is a knight. and $\neg q$: B is knave

## Assume A is a knight.

If *A* is a knight, then he is telling the truth when he says that *B* is a knight.

However, if *B* is a knight, then *B*'s statement that *A* and *B* are of opposite types contradicts. We can conclude that *A* is not a knight, that is, that *p* is false.

## Assume A is a knave.

If *A* is a knave, then everything A says is false. *A*'s statement that *B* is a knight is a lie. Means that *q* is false and *B* is also a knave. If *B* is a knave, then *B*'s statement that *A* and *B* are opposite types is a lie which is consistent with both *A* and *B* being knaves.

- We can conclude that both *A* and *B* are knaves.