Discrete Structures

Day 11

Nested Quantifiers

Assume that the domain for the variables x and y consists of all real numbers. The statement

$$\forall x \forall y (x + y = y + x)$$

says that $x + y = y + x$ for all real numbers x and y. This is the commutative law for addition of real numbers.

$$\forall x \exists y (x + y = 0)$$

says that for every real number x there is a real number y such that $x + y = 0$. This states that every real number has an additive inverse.

Translate into English the statement

$\forall x \forall y((x > 0) \land (y < 0) \rightarrow (xy < 0))$, where the domain for both variables consists of all real numbers.

For every real number x and for every real number y, if $x > 0$ and $y < 0$, then $xy < 0$.

This can be stated more succinctly as "The product of positive real number and a negative real number is always a negative real number."

Quantifications of Two Variables.

Statement	*When True?*	*When False?*
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair x, y.	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y.
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y.

The Order of Quantifiers

Q. Let $Q(x, y)$ denote "$x + y = 0$." What are the truth values of the quantifications $\exists y \forall x Q(x, y)$ and $\forall x \exists y Q(x, y)$, where the domain for all variables consists of all real numbers?

$\exists y \forall x Q(x, y)$: "There is a real number y such that for every real number x, $Q(x, y)$."

- No matter what value of y is chosen, there is only one value of x for which $x + y = 0$. The statement $\exists y \forall x Q(x, y)$ is false.

$\forall x \exists y Q(x, y)$: "For every real number x there is a real number y such that $Q(x, y)$."

- Given a real number x, there is a real number y such that $x + y = 0$; namely, $y = -x$. Hence, the statement $\forall x \exists y Q(x, y)$ is true.

Q. Let $Q(x, y, z)$ be the statement "$x + y = z$." What are the truth values of the statements $\forall x \forall y \exists z Q(x, y, z)$ and $\exists z \forall x \forall y Q(x, y, z)$, where the domain of all variables consists of all real numbers?

- $\forall x \forall y \exists z Q(x, y, z)$: "For all real numbers x and for all real numbers y there is a real number z such that $x + y = z$," is true.

- The order of the quantification here is important

- $\exists z \forall x \forall y Q(x, y, z)$: "There is a real number z such that for all real numbers x and for all real numbers y such that $x + y = z$," is false.

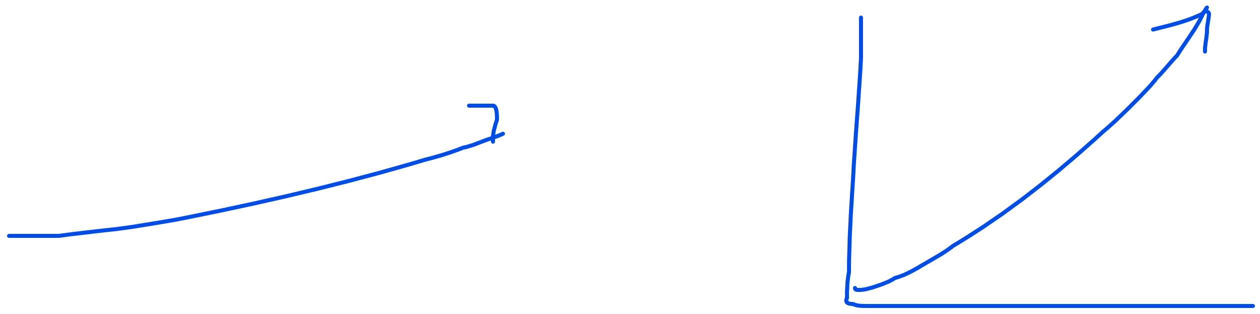
Translating Mathematical Statements into Statements Involving Nested Quantifiers

Q. Translate the statement "The sum of two positive integers is always positive" into a logical expression.

- "For all positive integers x and y, $x + y$ is positive."

  $\forall x \forall y((x > 0) \land (y > 0) \rightarrow (x + y > 0))$, where the domain for both variables consists of all integers.

  $\forall x \forall y(x + y > 0)$, where the domain for both variables consists of all positive integers best of Luck



Translating from Nested Quantifiers into English

Q. Translate the statement $\forall x(C(x) \vee \exists y(C(y) \wedge F(x, y)))$ into English, where

C(x): "*x* has a computer,"

F(x, y): "*x* and *y* are friends,"

and the domain for both *x* and *y* consists of all students in your school.

- For every student *x* in your school, *x* has a computer or there is a student *y* such that *y* has a computer and *x* and *y* are friends.

CSE- 1912031

Sec - A

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Translating English Sentences into Logical Expressions

Q. Express the statement “If a person is female and is a parent, then this person is someone’s mother” as a logical expression involving predicates, quantifiers with a domain consisting of all people, and logical connectives.

“For every person *x*, if person *x* is female and person *x* is a parent, then there exists a person *y* such that person *x* is the mother of person *y*.”

- *F(x):* “*x* is female,”
- *P(x):* “*x* is a parent,” and
- *M(x, y):* “*x* is the mother of *y*.”

$$\forall x((F(x) \wedge P(x)) \rightarrow \exists y M(x, y)).$$

$$\forall x \exists y((F(x) \wedge P(x)) \rightarrow M(x, y)).$$

Negating Nested Quantifiers

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Q2. Let *F(x, y)* be the statement "*x* can fool *y*," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody can fool Fred.

b) Evelyn can fool everybody.

c) Everybody can fool somebody.

d) There is no one who can fool everybody.

e) Everyone can be fooled by somebody.

f) No one can fool both Fred and Jerry.

g) Nancy can fool exactly two people.

h) There is exactly one person whom everybody can fool.

i) No one can fool himself or herself.

j) There is someone who can fool exactly one person besides himself or herself.

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