Neighbors of a Pixel

- A pixel *p* at coordinates (x,y) has four *horizontal* and *vertical* neighbors whose coordinates are given by:

(x+1,y), (x-1, y), (x, y+1), (x,y-1)

	(x, y-1)	
(x-1, y)	*p (x,y)*	(x+1, y)
	(x, y+1)	

This set of pixels, called the *4-neighbors* or *p*, is denoted by $N_4(p)$.

Each pixel is one unit distance from (x,y) and some of the neighbors of p lie outside the digital image if (x,y) is on the border of the image.

Neighbors of a Pixel

- The four *diagonal* neighbors of *p* have coordinates:

(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

(x-1, y-1)		(x+1, y-1)
	p (x,y)	
(x-1, y+1)		(x+1, y+1)

and are denoted by $N_D(p)$.

These points, together with the 4-neighbors, are called the 8-neighbors of p, denoted by $N_8(p)$.

(x-1, y-1)	(x, y-1)	(x+1, y-1)
(x-1, y)	*p (x,y)*	(x+1, y)
(x-1, y+1)	(x, y+1)	(x+1, y+1)

As before, some of the points in $N_D(p)$ and $N_8(p)$ fall outside the image if (x,y) is on the border of the image.

Adjacency and Connectivity

- Let *V*: a set of intensity values used to define adjacency and connectivity.
- In a binary image, *V* = {1}, if we are referring to adjacency of pixels with value 1.
- In a gray-scale image, the idea is the same, but *V* typically contains more elements, for example, *V* = {180, 181, 182, …, 200}
- If the possible intensity values 0 – 255, *V* set can be any subset of these 256 values.

Types of Adjacency

1. **4-adjacency:** Two pixels *p* and *q* with values from *V* are 4-adjacent if *q* is in the set $N_4(p)$.
2. **8-adjacency:** Two pixels *p* and *q* with values from *V* are 8-adjacent if *q* is in the set $N_8(p)$.
3. **m-adjacency =(mixed)**

Types of Adjacency

- **m-adjacency:**

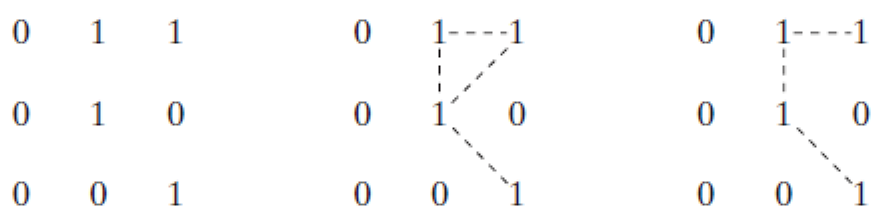
  Two pixels *p* and *q* with values from *V* are m-adjacent if :

  - q is in $N_4(p)$ or
  - q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixel whose values are from V (no intersection)

- **Important Note:** the type of adjacency used must be specified

Types of Adjacency

- Mixed adjacency is a modification of 8-adjacency. It is introduced to eliminate the ambiguities that often arise when 8-adjacency is used.
- For example:



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

Types of Adjacency

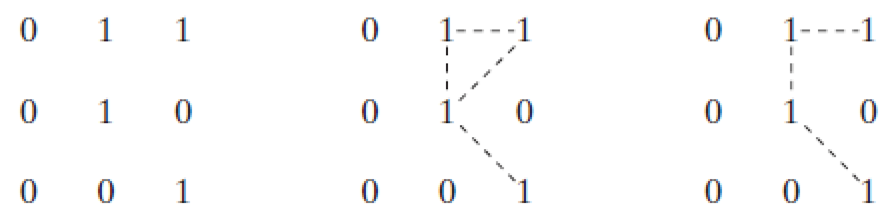
- In this example, we can note that to connect between two pixels (finding a path between two pixels):
  - In 8-adjacency way, you can find multiple paths between two pixels
  - While, in m-adjacency, you can find only one path between two pixels
- So, m-adjacency has eliminated the multiple path connection that has been generated by the 8-adjacency.
- Two subsets *S1* and *S2* are adjacent, if some pixel in *S1* is adjacent to some pixel in *S2*. Adjacent means, either 4-, 8- or m-adjacency.

A Digital Path

- A digital path (or curve) from pixel p with coordinate (x,y) to pixel q with coordinate (s,t) is a sequence of distinct pixels with coordinates (x_0,y_0), (x_1,y_1), …, (x_n, y_n) where $(x_0,y_0) = (x,y)$ and $(x_n, y_n) = (s,t)$ and pixels (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$
- n is the length of the path
- If $(x_0,y_0) = (x_n, y_n)$, the path is closed.
- We can specify 4-, 8- or m-paths depending on the type of adjacency specified.

A Digital Path

- Return to the previous example:



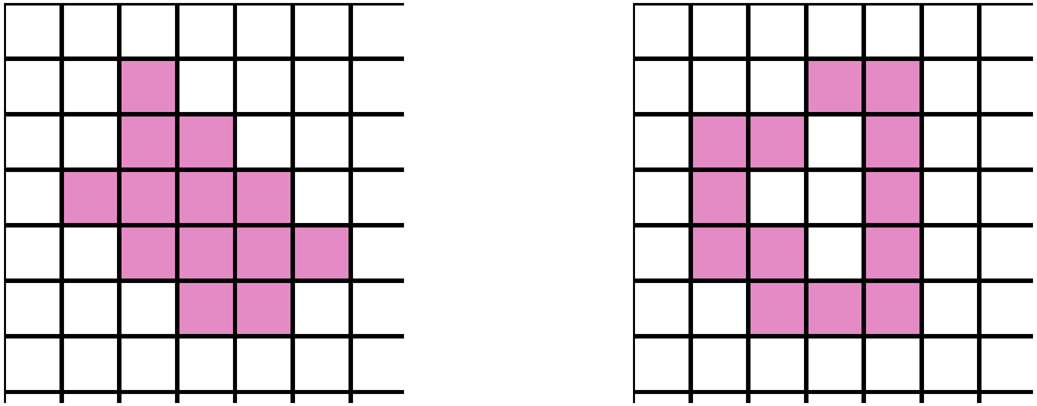
a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

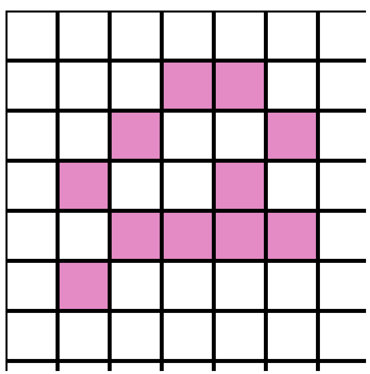
In figure (b) the paths between the top right and bottom right pixels are 8-paths. And the path between the same 2 pixels in figure (c) is m-path

Connectivity

- Let S represent a subset of pixels in an image, two pixels p and q are said to be connected in S if there exists a path between them consisting entirely of pixels in S.
- For any pixel p in S, the set of pixels that are connected to it in S is called a *connected component* of S.



4 Connected



8 Connected

Image Enhancement

Spatial Domain Methods: manipulates the pixel of a given image for enhancement.

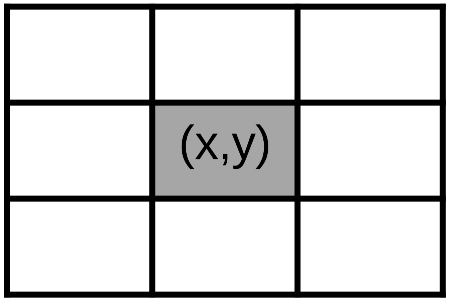
Frequency Domain Methods: manipulates the Fourier transform of a given image for enhancement.

$$g(x,y)=T[f(x,y)]$$

$f(x,y)$: input

$g(x,y)$: output

T : transformation function



3x3 neighborhood (mask)

Image Enhancement in Spatial Domain

Point Processing: enhancement at any point in an image depends only the gray-level at that point.

Mask Processing/Filtering: where the values of the mask coefficients determine the nature of the process.

Point Operations

- Point operations changes a pixel's intensity value according to some function (don't care about pixel's neighbor)

$$a' \leftarrow f(a)$$
$$I'(u,v) \leftarrow f\big(I(u,v)\big)$$

- Also called a **homogeneous operation**
- New pixel intensity **depends on**
  - Pixel's previous intensity *I(u,v)*
  - Mapping function *f*( )
- **Does not depend on**
  - Pixel's location *(u,v)*
  - Intensities of neighboring pixels

Some Homogeneous Point Operations

- Addition (Changes brightness)

  $$f(p) = p + k \qquad \text{E.g.} \quad f_{\text{bright}}(p) = p + 10$$

- Multiplication (Stretches/shrinks image contrast range)

  $$f(p) = k \times p \qquad \text{E.g.} \quad f_{\text{contrast}}(p) = p \times 1.5$$

- Real-valued functions

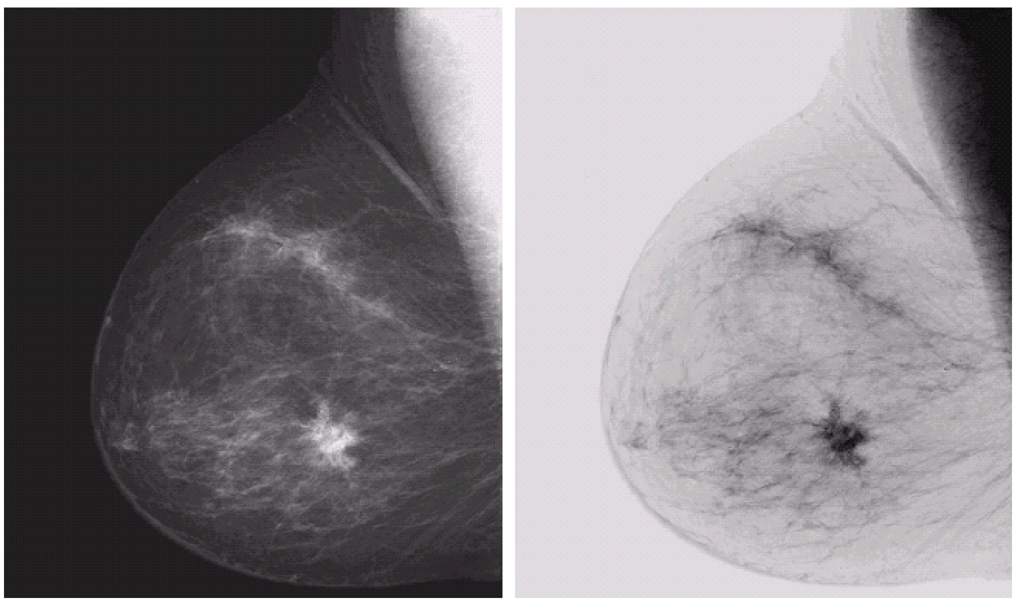
  $$\exp(x), \log(x), (1/x), x^k, \text{etc.}$$

- Quantizing pixel values
- Global thresholding
- Gamma correction

Image Negatives

$$s = (L-1) - r$$

s is the pixel value of the output image and r is the pixel value of the input image.

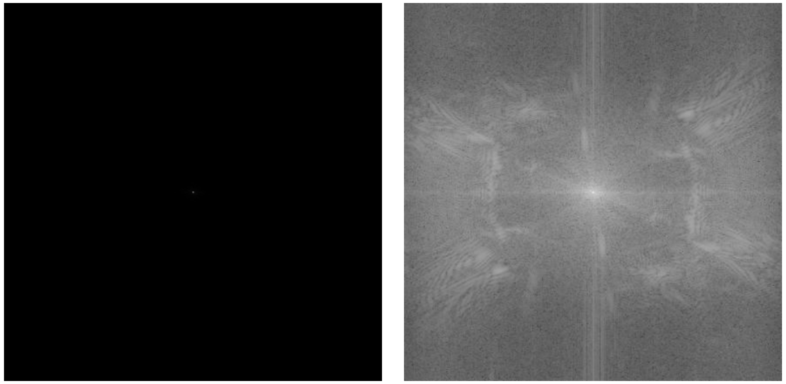


(left) Original digital mammogram. (right) Negative image obtained using the negative transformation

Logarithmic Transformations

$$s = c \log(1 + r)$$

s is the pixel value of the output image and r is the pixel value of the input image.



(left) Fourier spectrum of Barbara's image. (right) Result of applying the log transformation

Thresholding

- Input values below **threshold** a_{th} set to a_0
- Input values above **threshold** a_{th} set to a_1

$$f_{\mathrm{threshold}}(a) = \begin{cases} a_0 & \text{for } a < a_{\mathrm{th}} \\ a_1 & \text{for } a \geq a_{\mathrm{th}} \end{cases}$$

- Converts grayscale image to binary image (binarization) if
  - $a_0 = 0$
  - $a_1 = 1$

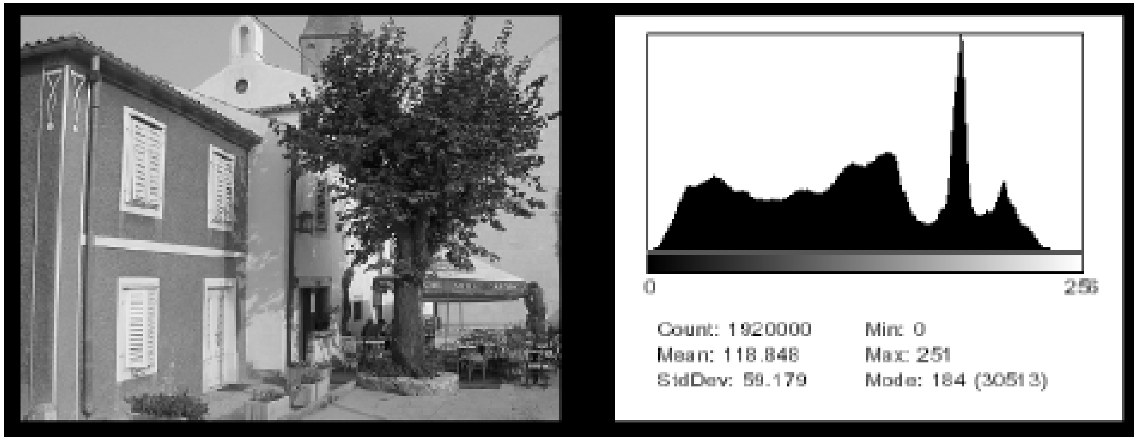
Thresholding Example

Original Image

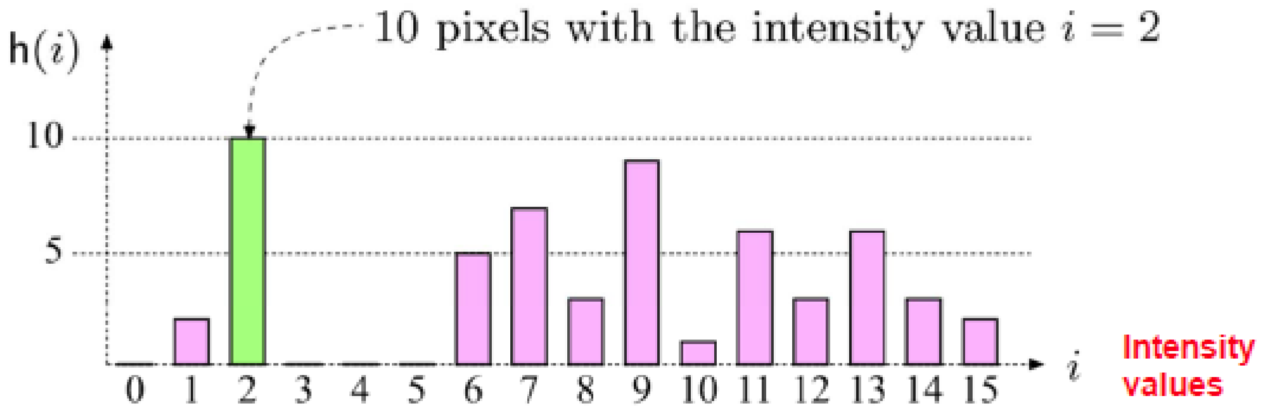
Thresholded Image

Histograms

- Histograms plots how many times (frequency) each intensity value in image occurs
- Example:
  - Image (left) has 256 distinct gray levels (8 bits)
  - Histogram (right) shows frequency (how many times) each gray level occurs



Histograms

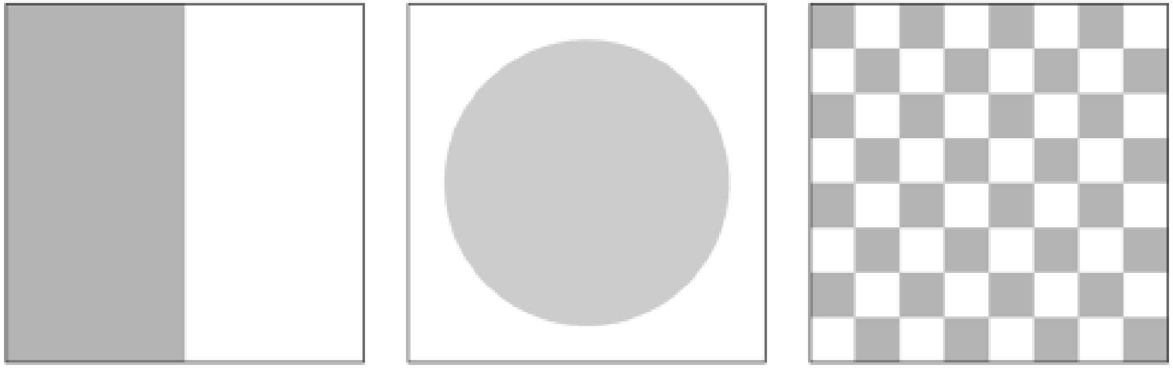


h(i)	0	2	10	0	0	0	5	7	3	9	1	6	3	6	3	2
i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

- E.g. K = 16, 10 pixels have intensity value = 2
- Histograms: only statistical information
- No indication of **location** of pixels

Histograms

- Different images can have **same** histogram
- 3 images below have same histogram



- Half of pixels are gray, half are white
  - Same histogram = same statisics
  - Distribution of intensities could be different
- Can we reconstruct image from histogram? No!

Histograms

- So, a histogram for a grayscale image with intensity values in range

$$I(u,v) \in [0, K-1]$$

would contain exactly K entries

- E.g. 8-bit grayscale image, $K = 2^8 = 256$
- Each histogram entry is defined as:

  h(i) = number of pixels with intensity *I* for all 0 < *i* < *K*.

- E.g: h(255) = number of pixels with intensity = 255
- Formal definition $\mathrm{h}(i) = \mathrm{card}\{(u,v) \mid I(u,v) = i\}$

Number (size of set) of pixels

such that

Histograms

- Histograms help detect image acquisition issues
- Problems with image can be identified on histogram
  - Over and under exposure
  - Brightness
  - Contrast
  - Dynamic Range
- Point operations can be used to alter histogram. E.g
  - Addition
  - Multiplication
  - Exp and Log
  - Intensity Windowing (Contrast Modification)

Image Brightness

- Brightness of a grayscale image is the **average intensity** of all pixels in image

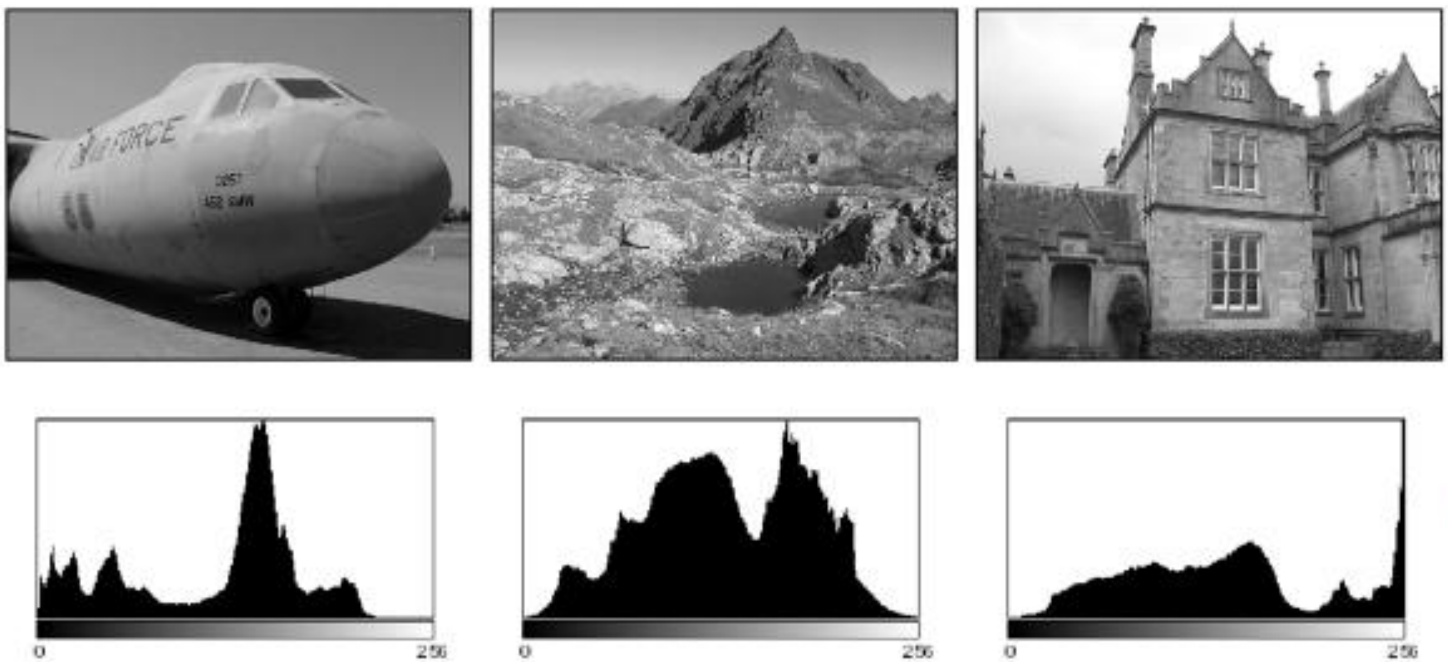
$$B(I) = \frac{1}{wh} \sum_{v=1}^{h} \sum_{u=1}^{w} I(u,v)$$

2. Divide by total number of pixels

1. Sum up all pixel intensities

Detecting Bad Exposure using Histograms

Exposure? Are intensity values spread (good) out or bunched up (bad)



Image

Histogram

(a) (b) (c)

Underexposed

Properly Exposed

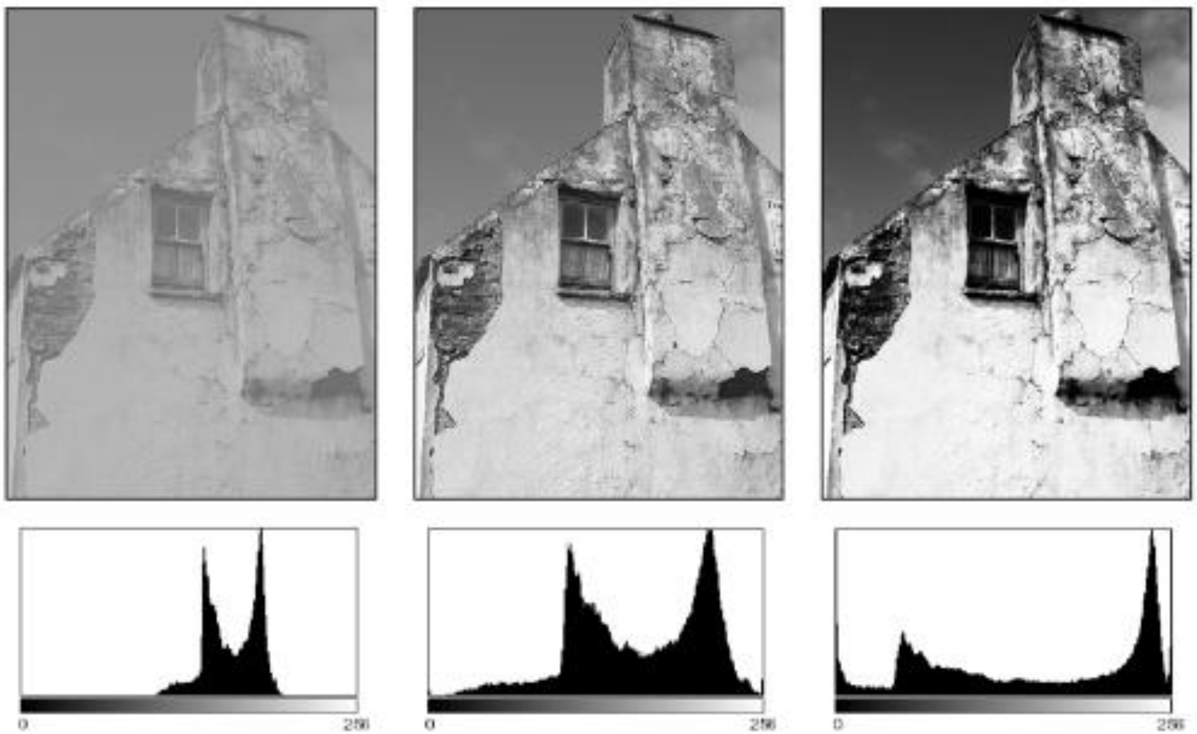
Overexposed

Image Contrast

- The contrast of a grayscale image indicates how easily objects in the image can be distinguished
- **High contrast image:** many distinct intensity values
- **Low contrast:** image uses few intensity values

Histograms and Contrast

Good Contrast? Widely spread intensity values
+ large difference between min and max intensity values



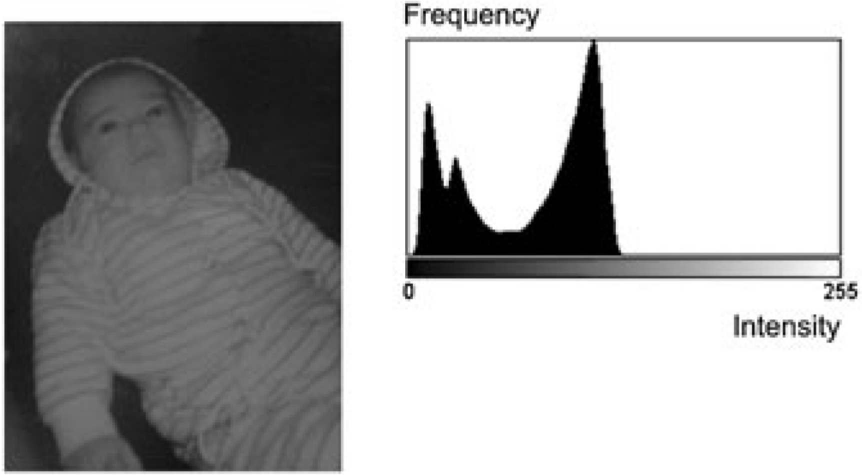
Image

Histogram

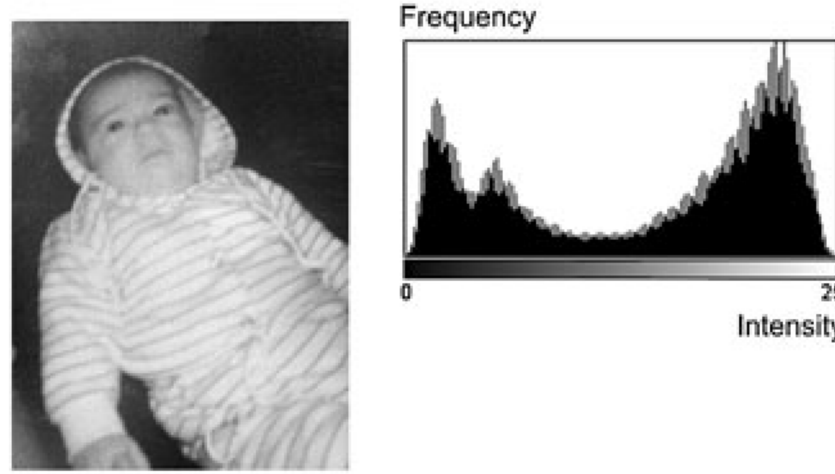
(a) Low contrast

(b) Normal contrast

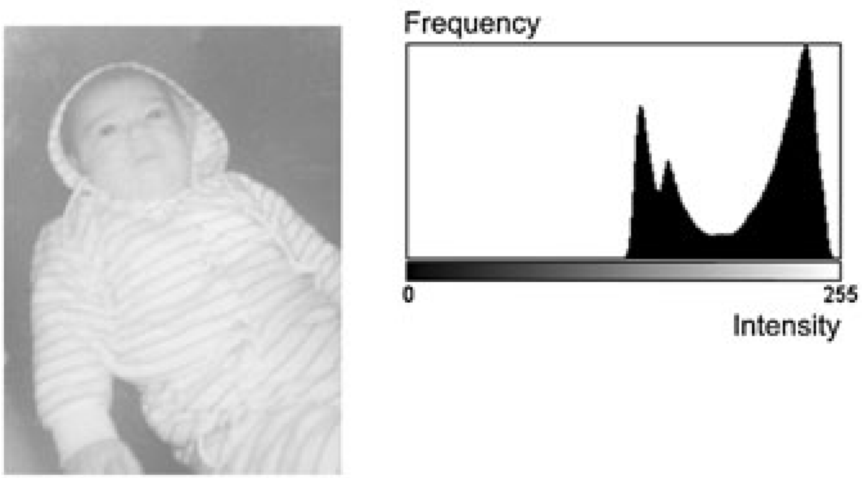
(c) High contrast



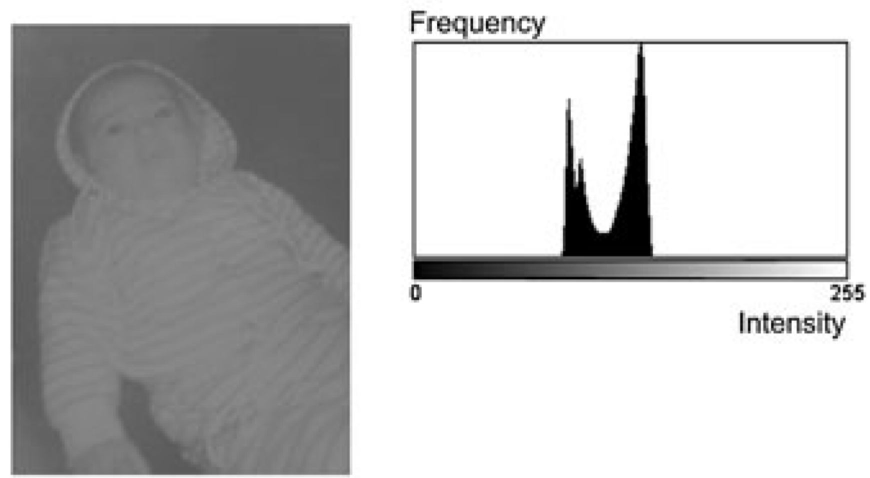
Dark image



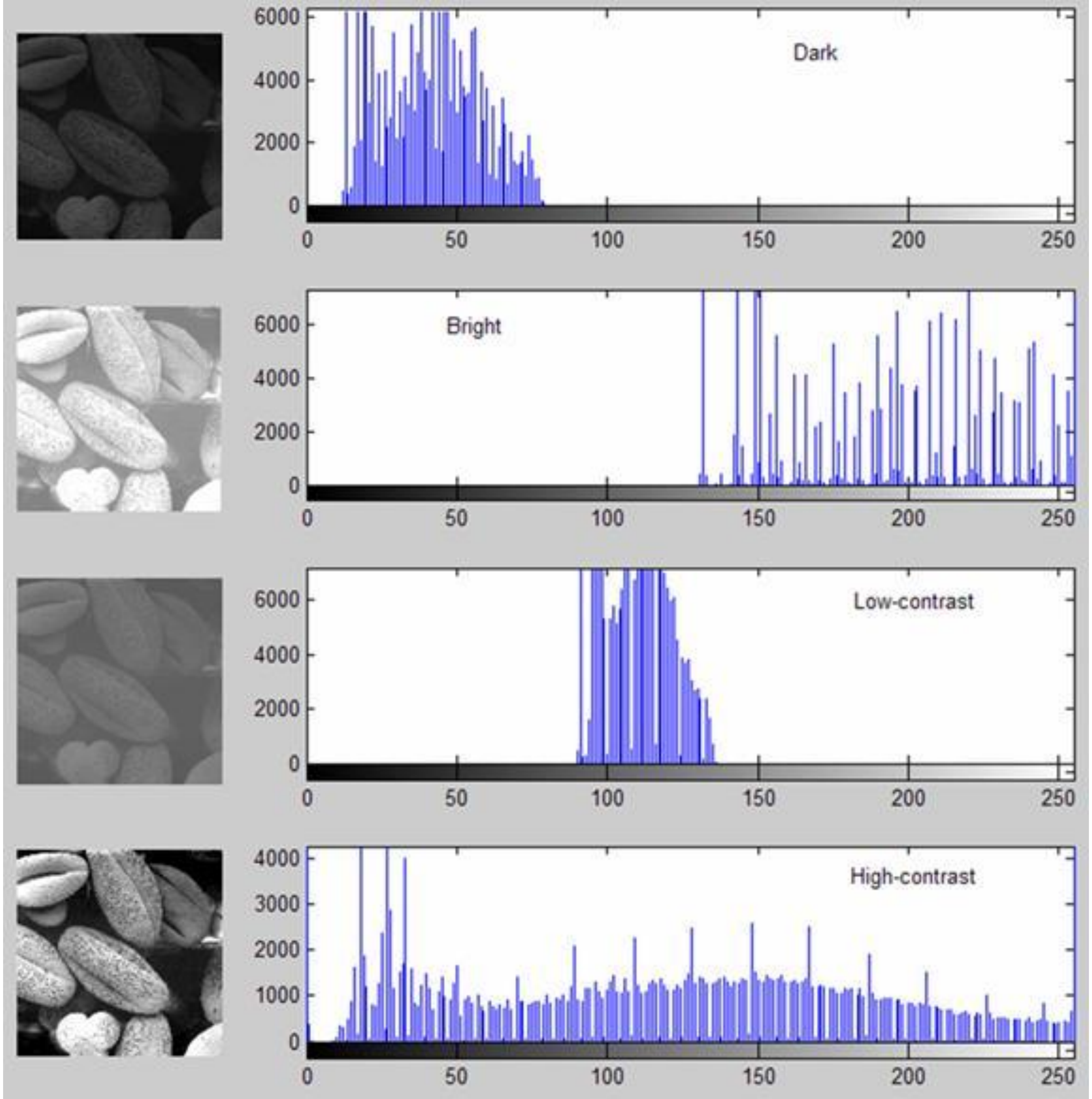
High contrast image



Bright image



Low contrast image



Dark
Bright
Low-contrast
High-contrast

Color Digital Images and RGB Histograms

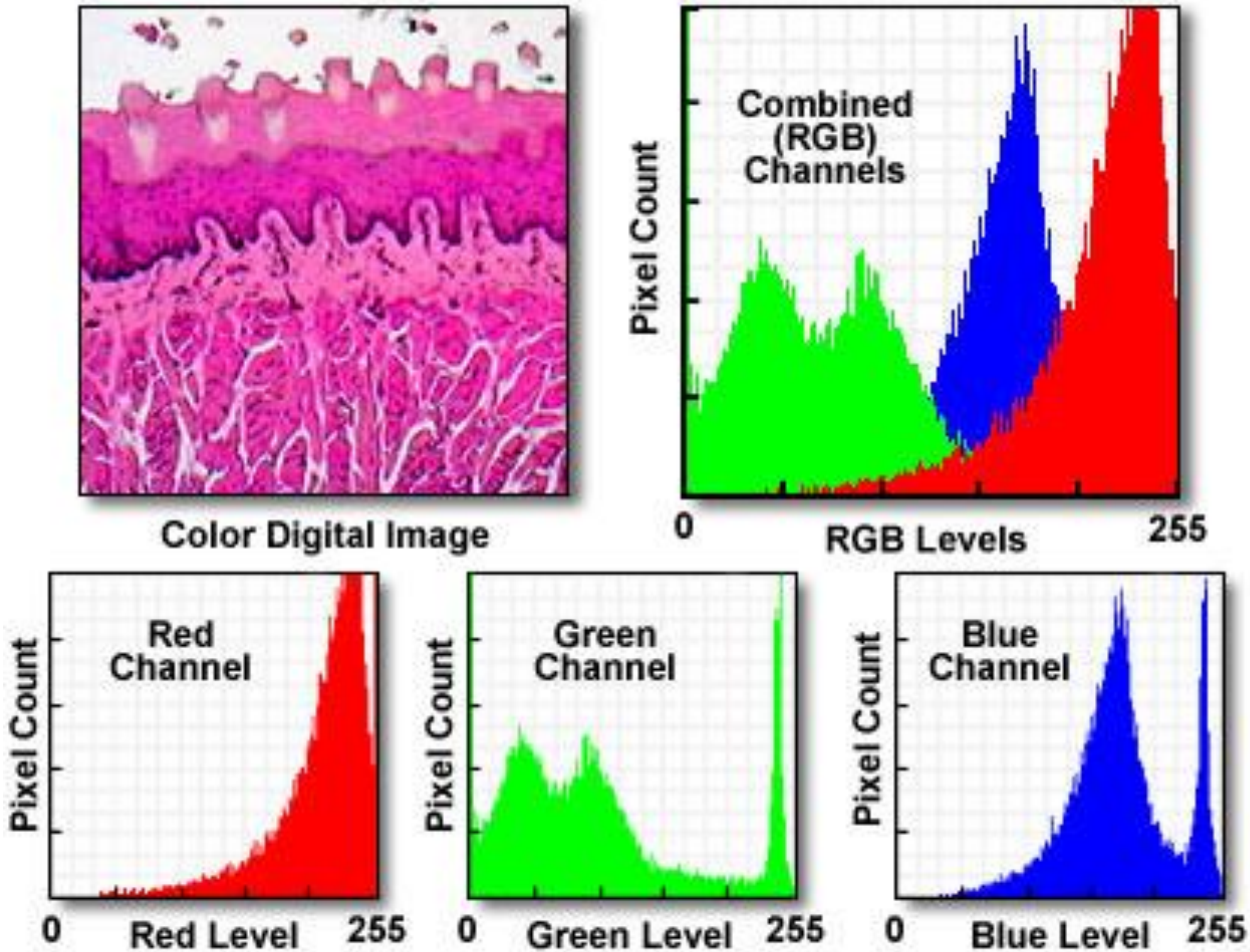


Figure 8

Contrast Equation?

- Many different equations for contrast exist
- Examples:

$$\text{Contrast} = \frac{\text{Change in Luminance}}{\text{Average Luminance}}$$

- Michalson's equation for contrast

$$C_M(I) = \frac{\max(I) - \min(I)}{\max(I) + \min(I)}$$

Contrast Equation?

- These equations work well for simple images with 2 luminances (i.e. uniform foreground and background)
- Does not work well for complex scenes with many luminances or if min and max intensities are small