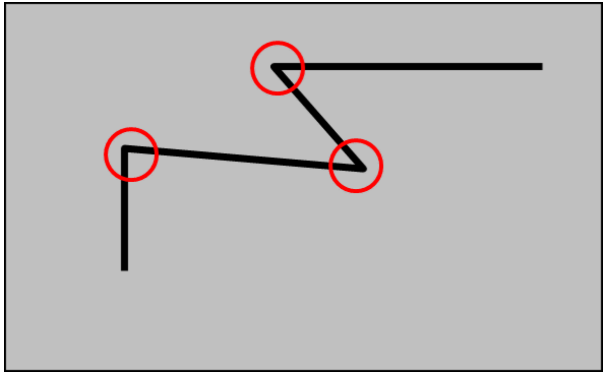
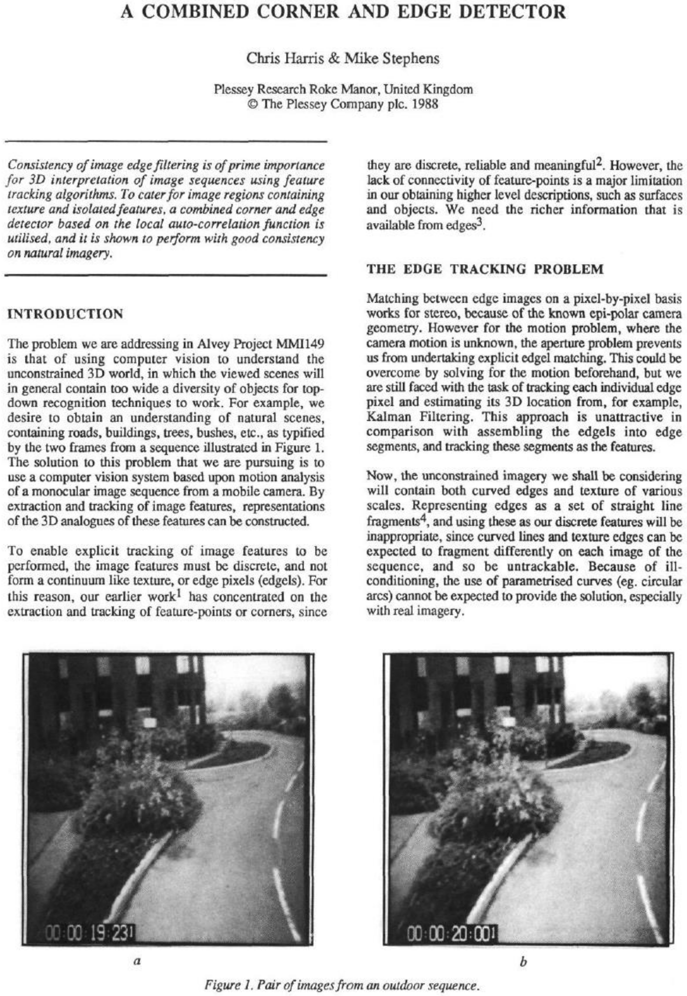
- History
  - C. Harris, M. Stephens. “A Combined Corner and Edge Detector”. 1988
  - Extract corners and infer features of an image





A COMBINED CORNER AND EDGE DETECTOR

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Consistency of image edge filtering is of prime importance for 3D interpretation of image sequences using feature tracking algorithms. To cater for image regions containing texture and isolated features, a combined corner and edge detector based on the local auto-correlation function is utilised, and it is shown to perform with good consistency on natural imagery.

INTRODUCTION

The problem we are addressing in Alvey Project MMI149 is that of using computer vision to understand the unconstrained 3D world, in which the viewed scenes will in general contain too wide a diversity of objects for top-down recognition techniques to work. For example, we desire to obtain an understanding of natural scenes, containing roads, buildings, trees, bushes, etc., as typified by the two frames from a sequence illustrated in Figure 1. The solution to this problem that we are pursuing is to use a computer vision system based upon motion analysis of a monocular image sequence from a mobile camera. By extraction and tracking of image features, representations of the 3D analogues of these features can be constructed.

To enable explicit tracking of image features to be performed, the image features must be discrete, and not form a continuum like texture, or edge pixels (edgels). For this reason, our earlier work[1] has concentrated on the extraction and tracking of feature-points or corners, since they are discrete, reliable and meaningful[2]. However, the lack of connectivity of feature-points is a major limitation in our obtaining higher level descriptions, such as surfaces and objects. We need the richer information that is available from edges[3].

THE EDGE TRACKING PROBLEM

Matching between edge images on a pixel-by-pixel basis works for stereo, because of the known epi-polar camera geometry. However for the motion problem, where the camera motion is unknown, the aperture problem prevents us from undertaking explicit edgel matching. This could be overcome by solving for the motion beforehand, but we are still faced with the task of tracking each individual edge pixel and estimating its 3D location from, for example, Kalman Filtering. This approach is unattractive in comparison with assembling the edgels into edge segments, and tracking these segments as the features.

Now, the unconstrained imagery we shall be considering will contain both curved edges and texture of various scales. Representing edges as a set of straight line fragments[4], and using these as our discrete features will be inappropriate, since curved lines and texture edges can be expected to fragment differently on each image of the sequence, and so be untrackable. Because of ill-conditioning, the use of parametrised curves (eg. circular arcs) cannot be expected to provide the solution, especially with real imagery.

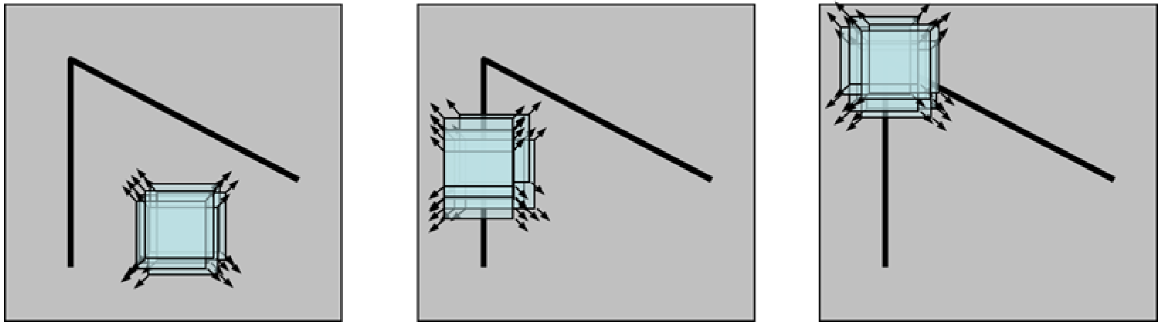
a b

Figure 1. Pair of images from an outdoor sequence.

- Edge in an image is a region where a sudden change in image pixel intensity occurs
- A corner is a point whose local neighborhood stands in two dominant and different edge directions
- Corner points are invariant to translation, rotation and illumination and can be used as key feature points to represent an image.
- Corners contain the most important features in restoring image information, and they can be used to minimize the amount of processed data for motion tracking, image stitching, building 2D mosaics, stereo vision, image representation and other related computer vision areas.

INTRODUCTION

- The Basic Idea
  - Find small patches of an image that generates a large variation when moved around



"flat" region:
no change in
all directions

"edge":
no change
along the edge
direction

"corner":
significant
change in all
directions

INTRODUCTION

❖ Harris Detector: Mathematics

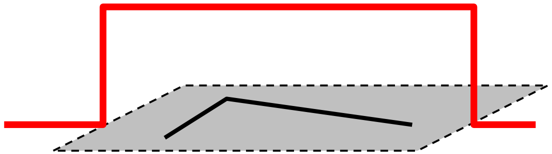
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

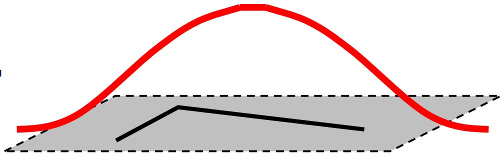
Actual Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

INTRODUCTION

❖ Harris Detector: Mathematics

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

E is the difference between the original and the moved window.

u & ***v*** is the window's displacement in the ***x*** and ***y*** direction

w(x, y) is the window at position ***(x, y)***

I(x, y) is the intensity of the image at a position ***(x, y)***

I(x+u, y+v) is the intensity of the moved window

INTRODUCTION

- Harris Detector: Mathematics
  - Appling Taylor series expansion and approximation

$$\sum_{x,y}[I(x+u,y+v)-I(x,y)]^2 \approx \sum_{x,y}[I(x,y)+uI_x+vI_y-I(x,y)]^2$$

$$\approx \sum_{x,y} u^2{I_x}^2+2uvI_xI_y+v^2{I_y}^2 \approx [u \quad v]\left(\sum\begin{bmatrix}{I_x}^2 & I_xI_y \\ I_xI_y & {I_y}^2\end{bmatrix}\right)\begin{bmatrix}u \\ v\end{bmatrix}$$

$$f(x+u,y+v) = f(x,y) + uf_x(x,y) + vf_y(x,y) +$$

First partial derivatives

$$\frac{1}{2!}\left[u^2 f_{xx}(x,y) + uv f_{xy} x,y + v^2 f_{yy}(x,y)\right] +$$

Second partial derivatives

$$\frac{1}{3!}\left[u^3 f_{xxx}(x,y) + u^2 v f_{xxy}(x,y) + uv^2 f_{xyy}(x,y) + v^3 f_{yyy}(x,y)\right]$$

Third partial derivatives

$+ \ \ldots$ (Higher order terms)

First order approx

$$f(x+u,y+v) \approx f(x,y) + uf_x(x,y) + vf_y(x,y)$$

INTRODUCTION

❖ Harris Detector: Mathematics

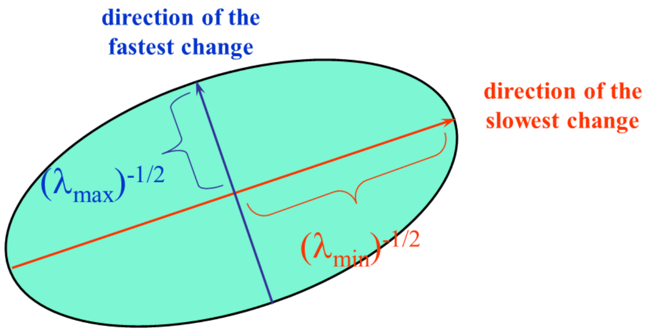
$$E(u,v) \cong [u,v] \;\; M \;\; \begin{bmatrix} u \\ v \end{bmatrix}$$

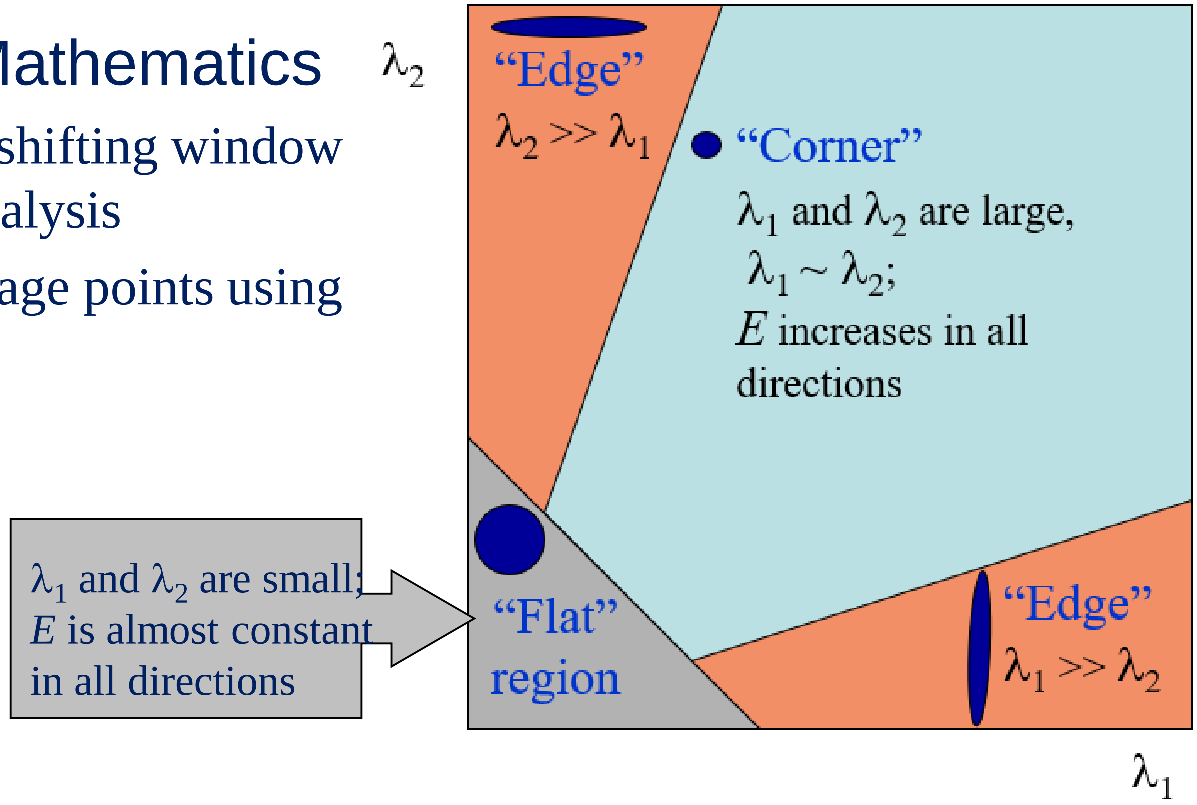
where M is a 2×2 matrix computed from image derivatives

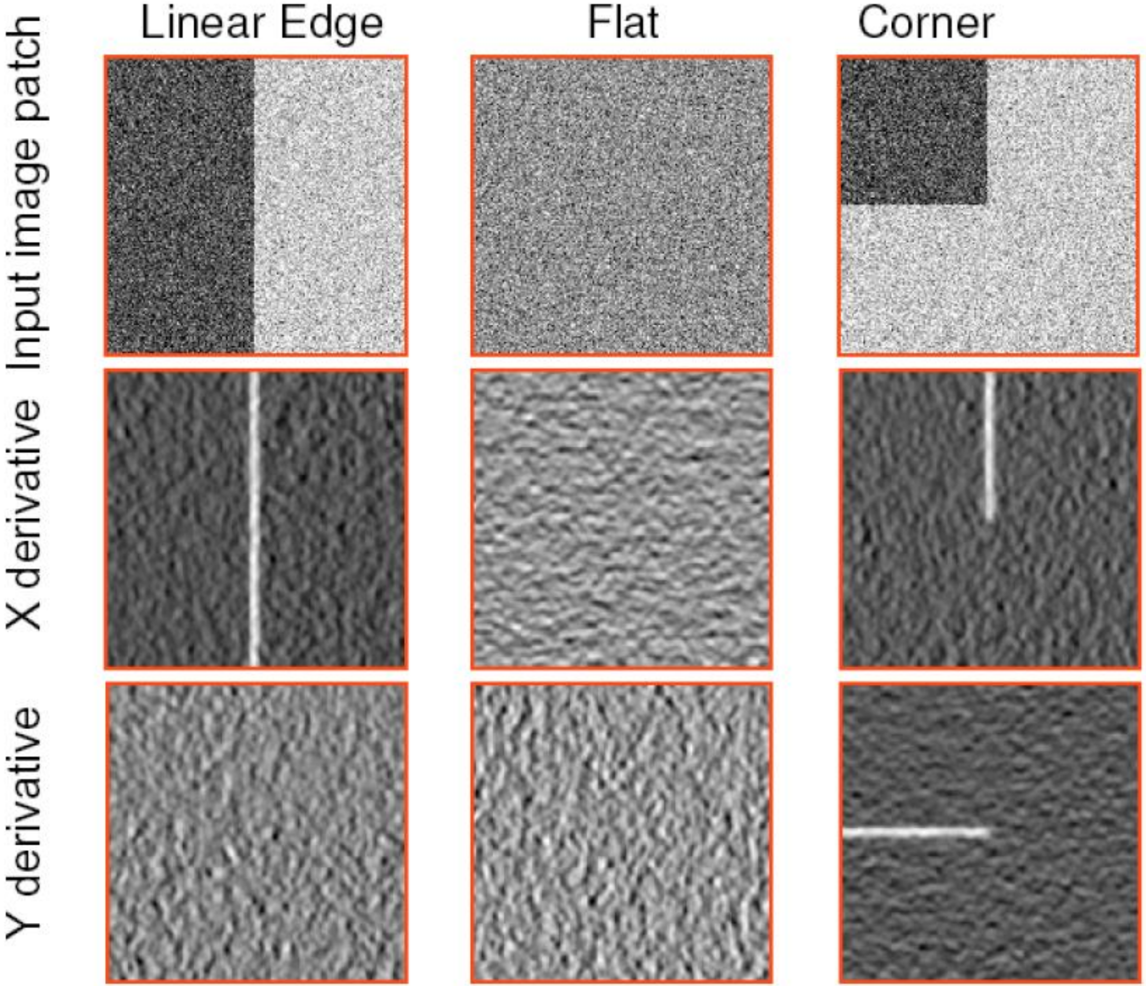
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

INTRODUCTION

- Harris Detector: Mathematics
  - Intensity change in shifting window using eigenvalue analysis
  - Classification of image points using eigenvalues of *M*

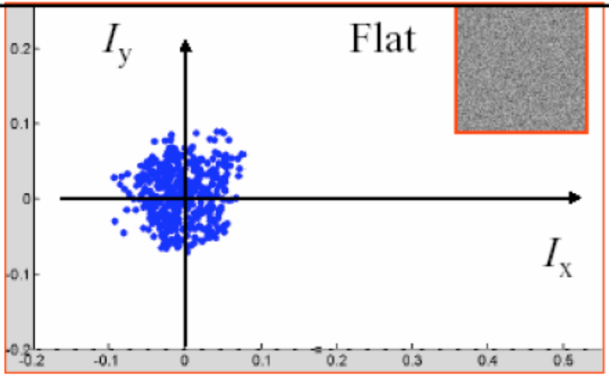


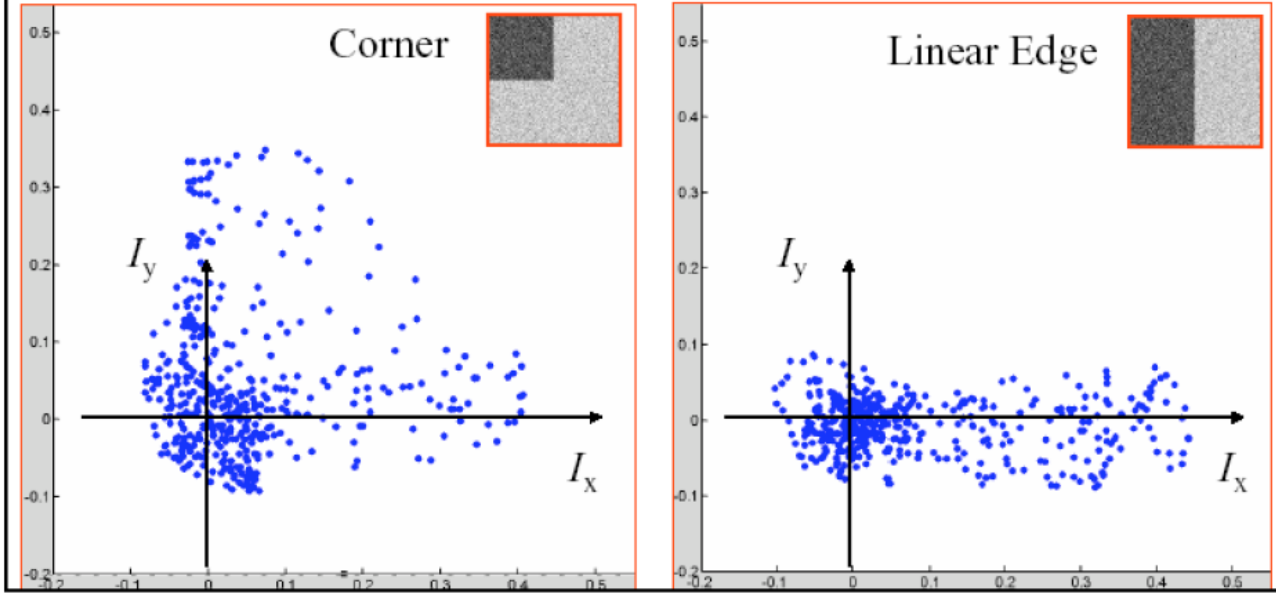




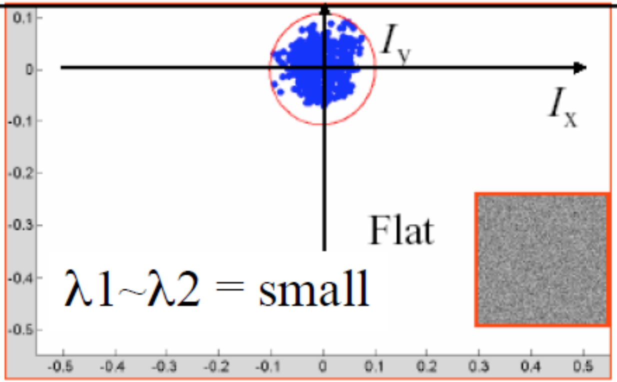
Linear Edge
Flat
Corner
Input image patch
X derivative
Y derivative

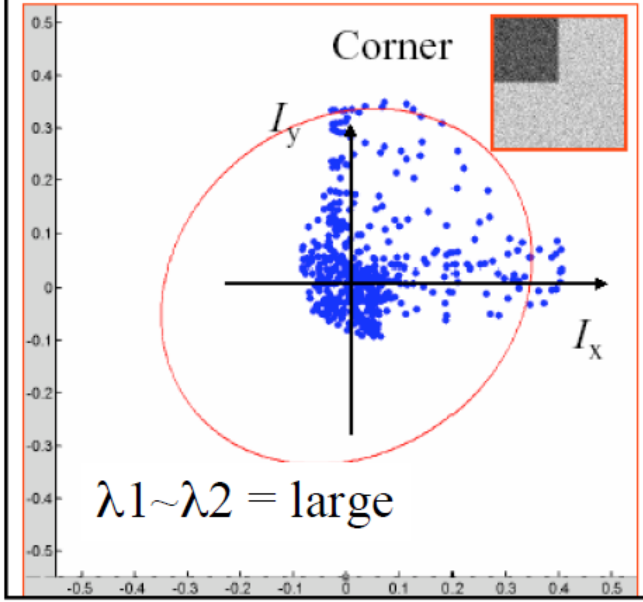
The distribution of the x and y derivatives is very different for all three types of patches

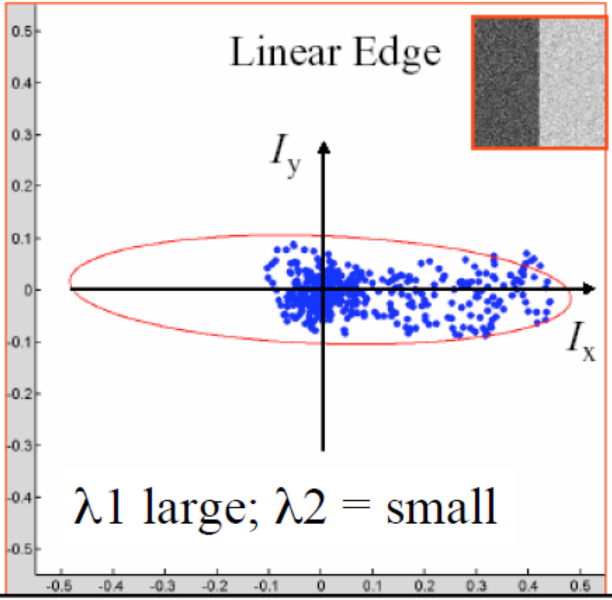




The distribution of x and y derivatives can be characterized by the shape and size of the principal component ellipse







INTRODUCTION

- Harris Detector: Mathematics
  - Finding the eigenvalues is computationally expensive
  - An approximation is used instead of direct eigen value computation with Harris response function, R

$$R = \det M - k\left(\operatorname{trace} M\right)^2$$

where

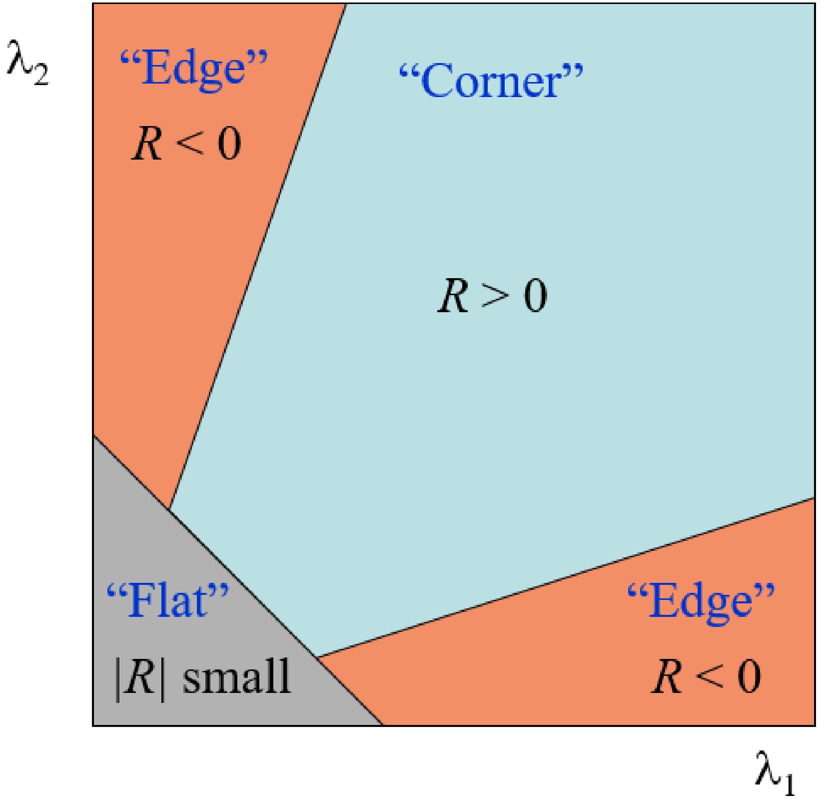
$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

(k – empirical constant, k = 0.04-0.06)

INTRODUCTION

❖ Harris Detector: Mathematics

- ❖ R depends only on eigenvalues of M
- ❖ R is large for a corner
- ❖ R is negative with large magnitude for an edge
- ❖ |R| is small for a flat region



$$M = \sum_{(x,y)\in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \sum_{(x,y)\in W} I_x^2 & \sum_{(x,y)\in W} I_x I_y \\ \sum_{(x,y)\in W} I_x I_y & \sum_{(x,y)\in W} I_y^2 \end{bmatrix}$$

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \quad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x2} = I_x . I_x \quad I_{y2} = I_y . I_y \quad I_{xy} = I_x . I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x2} = G_{\sigma\prime} * I_{x2} \quad S_{y2} = G_{\sigma\prime} * I_{y2} \quad S_{xy} = G_{\sigma\prime} * I_{xy}$$

4. Define at each pixel (x, y) the matrix

$$H(x,y) = \begin{bmatrix} S_{x2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y2}(x,y) \end{bmatrix}$$

5. Compute the response of the detector at each pixel

$$R = Det(H) - k(Trace(H))^2$$

6. Threshold on value of R. Compute nonmax suppression.

INTRODUCTION

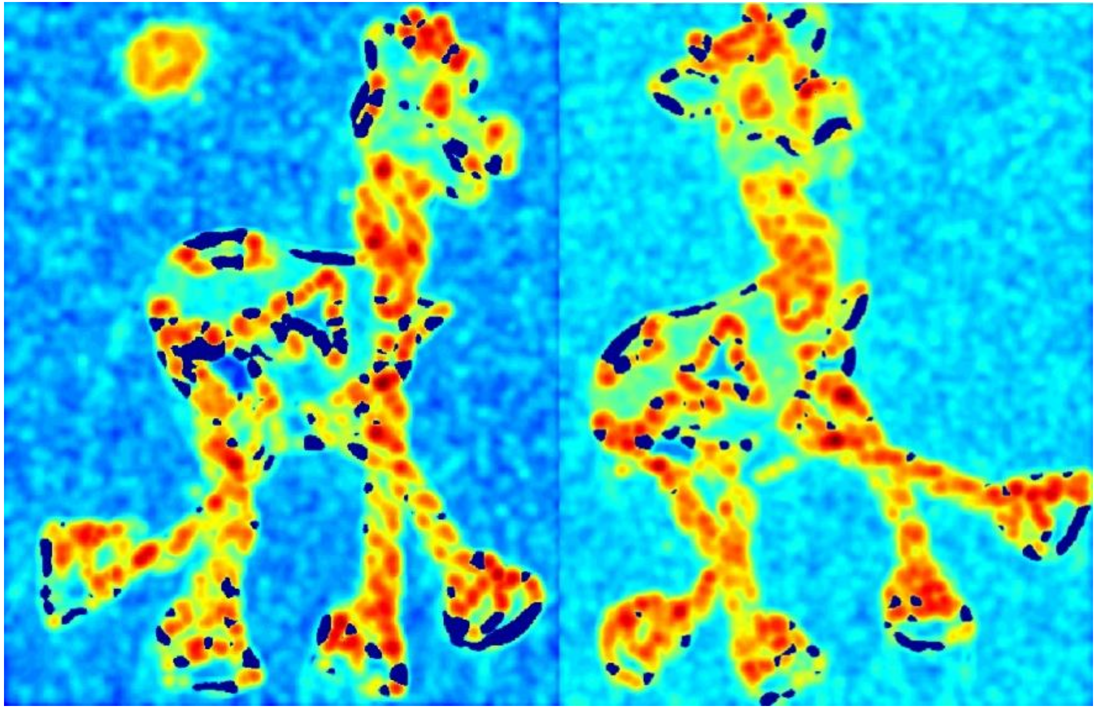
- Harris Detector: Workflow

Sample Test Images

INTRODUCTION

❖ Harris Detector: Workflow

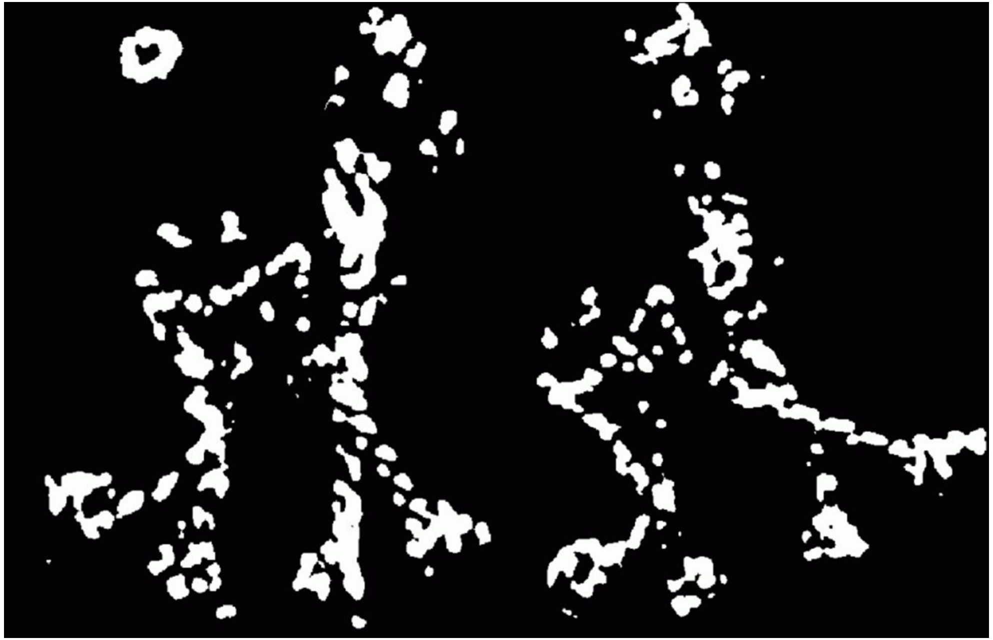
Compute corner response R



INTRODUCTION

❖ Harris Detector: Workflow

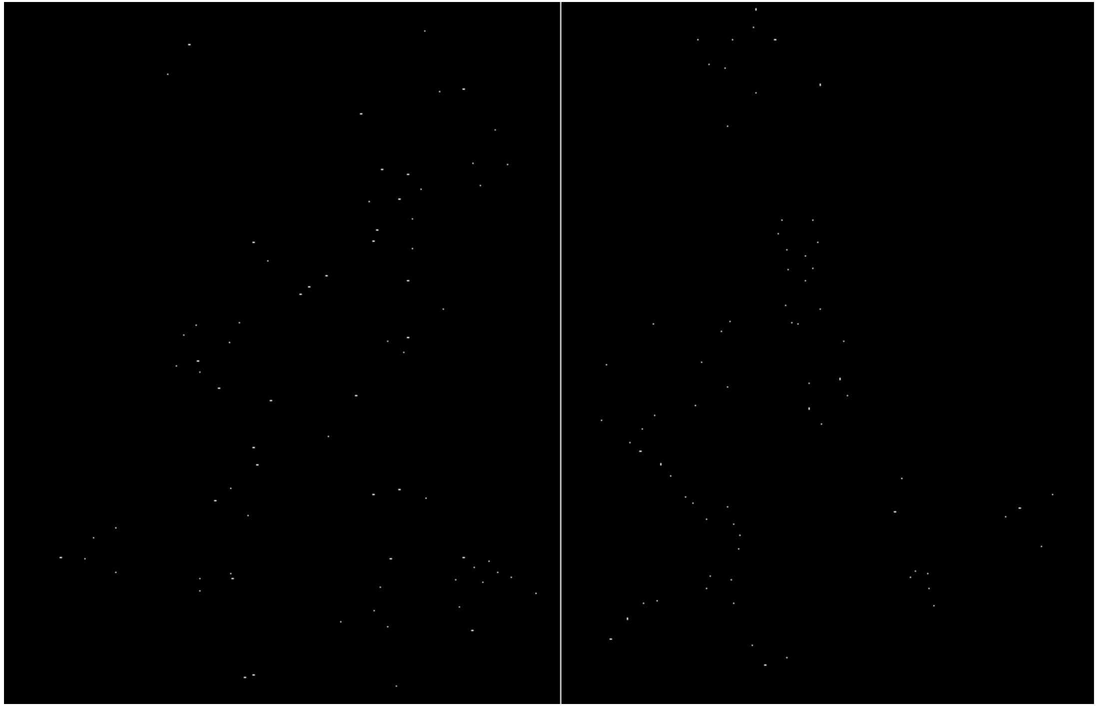
Find points with large corner response: R > threshold



INTRODUCTION

❖ Harris Detector: Workflow

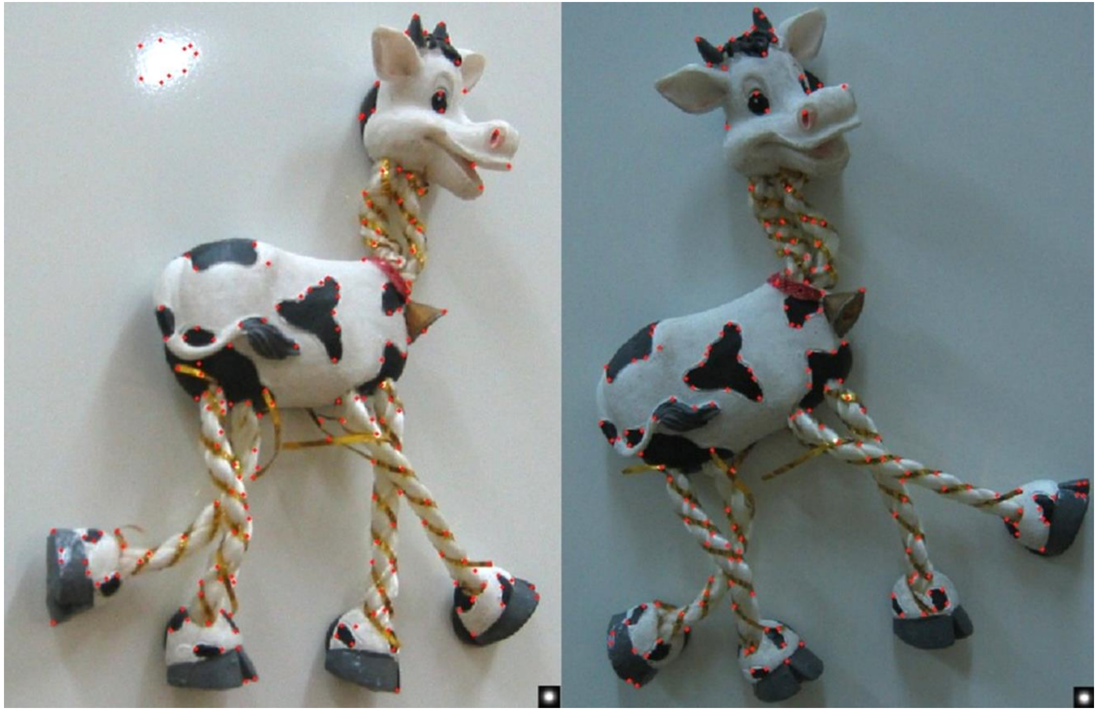
Take only the points of local maxima of R



INTRODUCTION

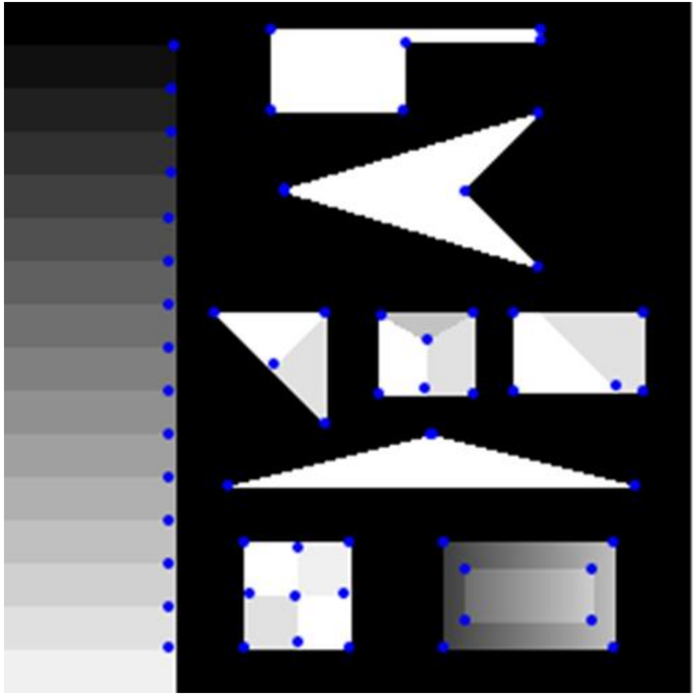
❖ Harris Detector: Workflow

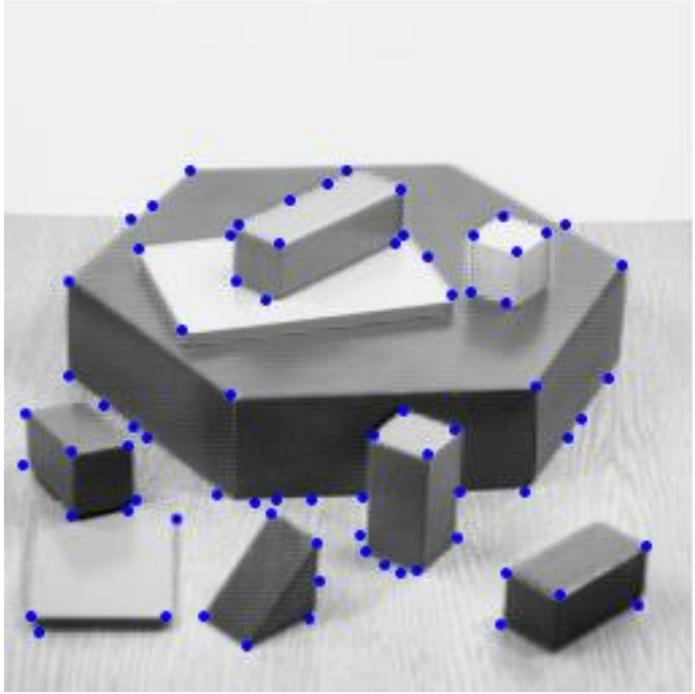
Marked corner points



INTRODUCTION

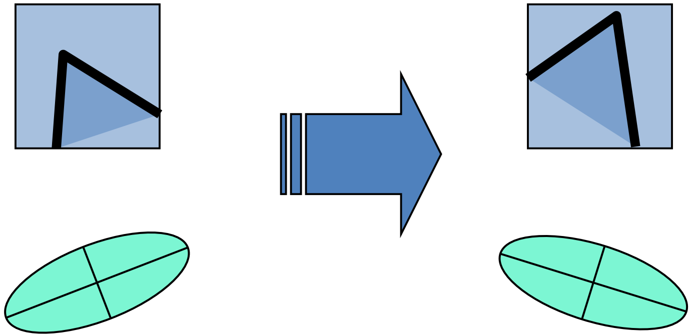
- ❖ Harris Detector: Examples





INTRODUCTION

- Harris Detector: Properties
  - Rotation invariance



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

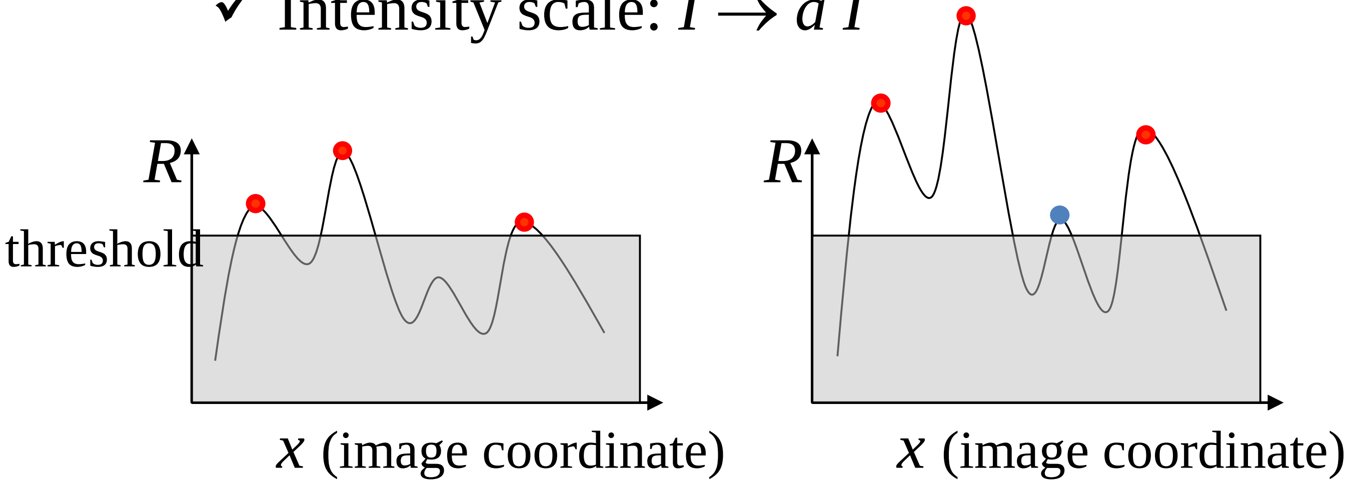
Corner response R is invariant to image rotation

INTRODUCTION

❖ Harris Detector: Properties

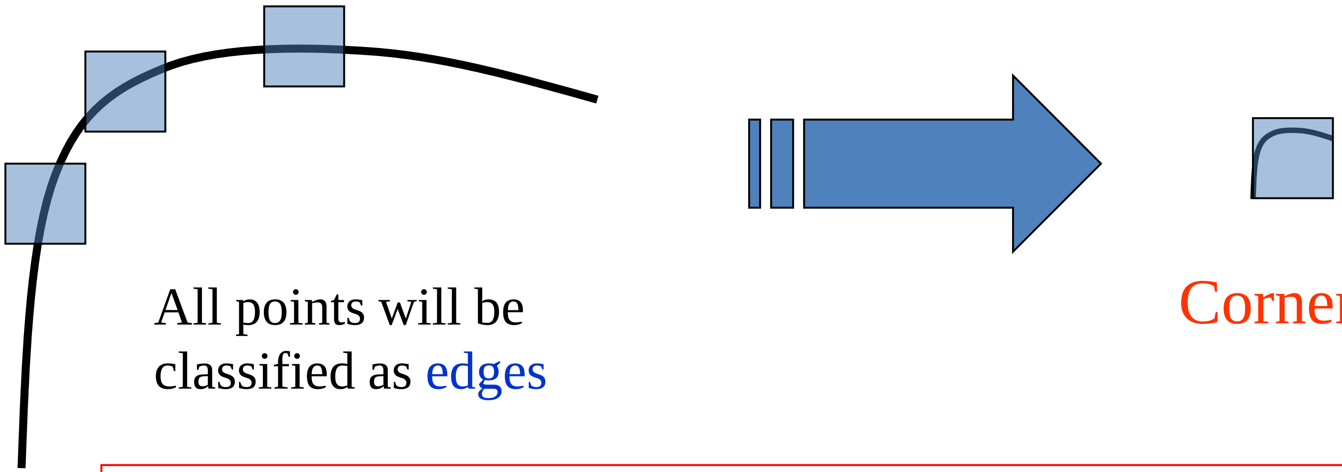
❖ Partial invariance to affine intensity variation

✓ Intensity scale: $I \rightarrow a\, I$



INTRODUCTION

- Harris Detector: Properties
  - Variant to image scale changes



All points will be classified as edges

Corner !

Corner response R is variant to image scale changes

Thank You !