Quantum Machine Learning and its Applications

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- Classical Bits vs Quantum Bits
- Dirac Notation of Quantum Bit
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Introduction to Quantum ML

Modern Computers

CPU

The invention of the modern computer, followed by the central processing unit (CPU), led to the "digital revolution", transforming industries through process automation and the rise of information technology.

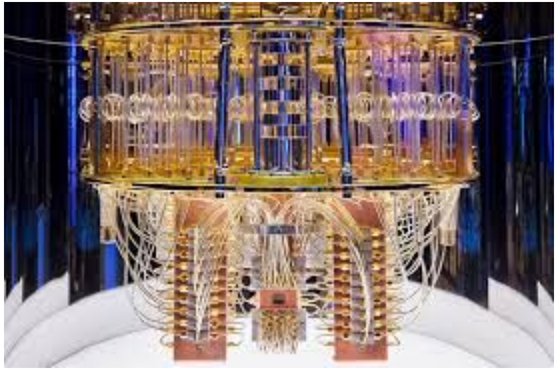
Introduction to Quantum ML

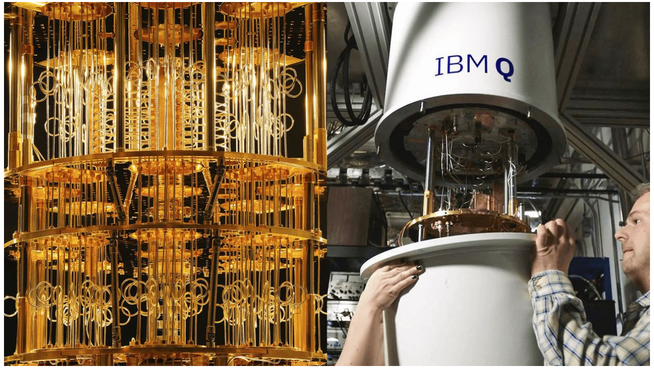
GPU

NVIDIA's GeForce 6600 GPU, released in 2004

More recently, the development of graphical processing units (GPUs) has powered the era of artificial intelligence (AI) and big data, enabling breakthroughs in areas such as intelligent transportation systems, autonomous driving, scientific simulations, and complex data analysis.

Introduction to Quantum ML





Quantum computers

quantum computers (Feynman, 2017), which leverage the unique principles of quantum mechanics such as **superposition** and **entanglement** to process information in ways that classical systems cannot, with the potential to revolutionize diverse aspects of daily life.

Introduction to Quantum ML

Limitations of AI using GPUs over QPUs

- Parallelism vs Quantum superposition
- Handling Complex optimization Landscapes
- Memory Bottlenecks
- Security & Cryptography in AI

The convergence of the computational power offered by quantum machines
and the limitations faced by AI models has led to the rapid emergence of the field: **quantum machine learning (QML)** (Biamonte et al., 2017).

Introduction to Quantum ML

Application of Quantum Machine Learning

1. Drug Discovery and Materials Science

2. Financial Risk Analysis and Fraud Detection

3. Artificial Intelligence

4. Optimization Problems

5. More Faster Algorithms

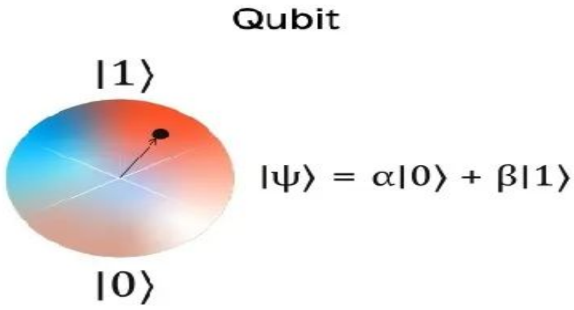
6. Advance Security and Cryptography

7. Scientific Research

Quantum Bits

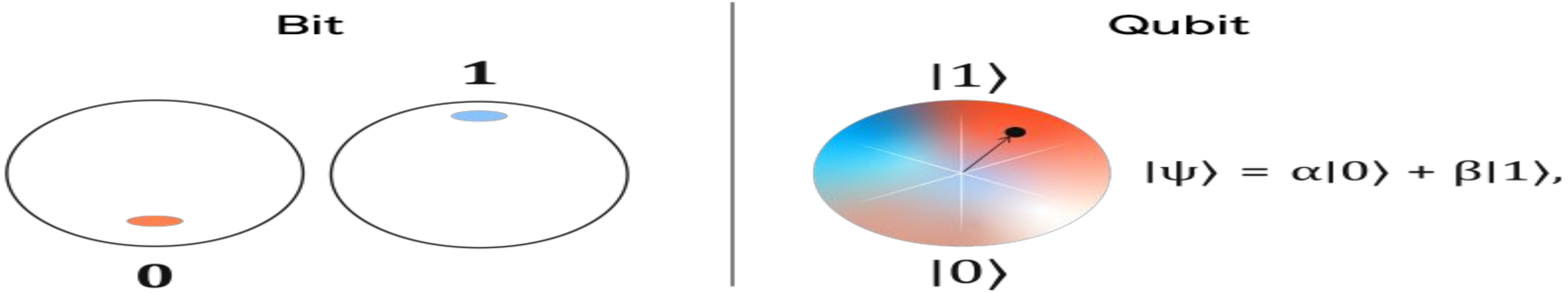
The Quantum Bit, or Qubit for short is the fundamental concept of Quantum computation and quantum information. In Quantum Computing, a Qubit can exist in a **superposition** of both states simultaneously allowing them to process multiple possibilities concurrently..

- A Linear combination of 0 and 1
- Are probabilistic : their value is uncertain until measured.
- Can exhibit **entanglement**, where the states of multiple qubits are co-related.



Classical Bits vs Quantum Bits

1. The bit is the fundamental concept of classical computation and classical information.	1. The Quantum Bit, or Qubit for short is the fundamental concept of Quantum computation and quantum information.
2. Deterministic values	2. Probabilistic values
3. No Superposition	3. Superposition
4. No Entanglement	4. Entanglement
5. No Interference	5. Interference

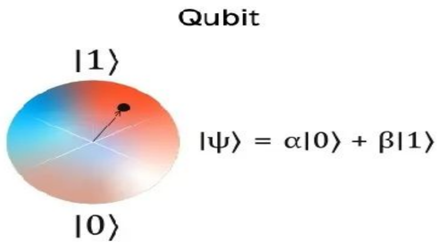


Dirac Notation of QuBit

Dirac notation, also known as bra-ket notation, is a mathematical notation used in quantum mechanics and quantum computing to represent quantum states and operations.

Key Concepts:

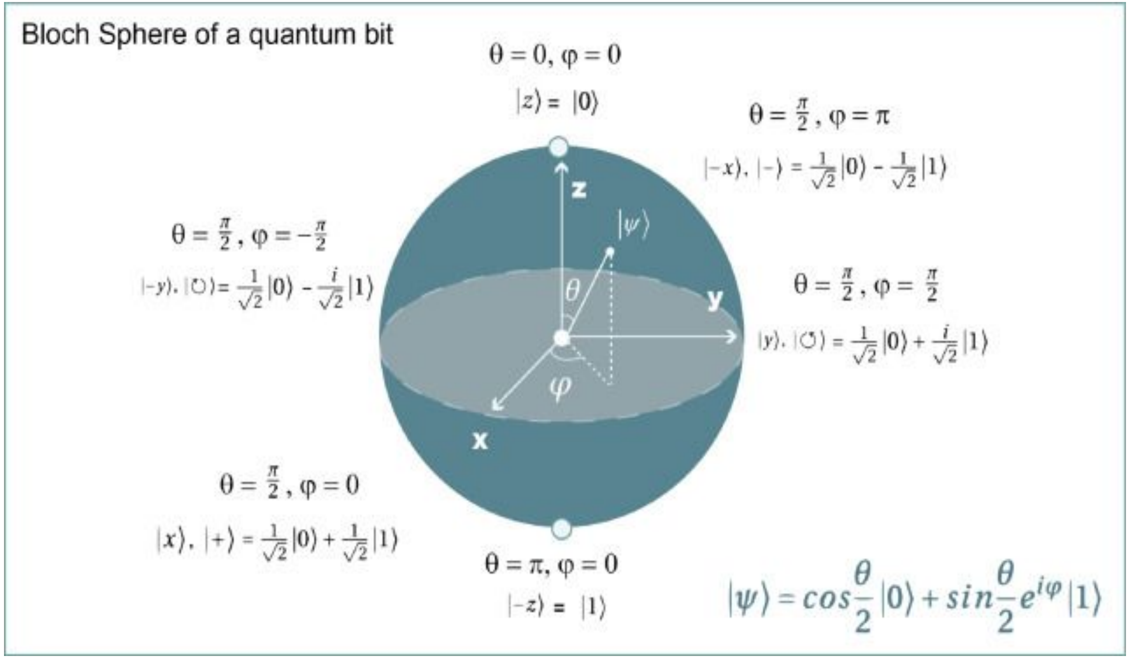
- **Kets ($|\psi\rangle$):** Represent quantum states as column vectors. For example, $|0\rangle$ and $|1\rangle$ represent the basis states of a qubit.
- **Bras ($\langle\psi|$):** Represent the conjugate transpose of a ket, essentially row vectors.
- **Bra-ket ($\langle\psi|\varphi\rangle$):** Represents the inner product of two quantum states, analogous to the dot product.
- **Outer Product ($|\psi\rangle\langle\varphi|$):** Represents a matrix formed by the outer product of two vectors.



A single qubit state $|\psi\rangle$ can be written as a linear combination of the basis states $|0\rangle$ and $|1\rangle$: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are complex numbers. The probabilities of measuring $|0\rangle$ or $|1\rangle$ are $|\alpha|^2$ and $|\beta|^2$, respectively.

Bloch Sphere Representation of Qubit

The Bloch Sphere is a powerful geometric tool used in quantum computing to represent the state of a single qubit. It provides a visual way to understand quantum states, ***superposition***, and phase — abstract concepts that are often difficult to grasp.



Link:https://www.sharetechnote.com/html/QC/QuantumComputing_BlochSphere.html

Quantum Gates

Quantum gates are the fundamental building blocks of quantum circuits

Types of Quantum Gates:

Single-Qubit Gates:

These gates operate on a single qubit and are the most basic type. Examples include:

- **Hadamard gate (H)**
- **Pauli gates (X, Y, Z)**
- **Phase gate (S, T)**

Multi-Qubit Gates:

These gates act on two or more qubits and are crucial for creating entanglement and performing more complex computations. Examples include:

- **CNOT gate**
- **SWAP gate**
- **Controlled Z gate**
- **Toffoli gate**

Quantum Gates

Quantum gates are Hermitian, unitary and square to Identity

Single-Qubit Gates

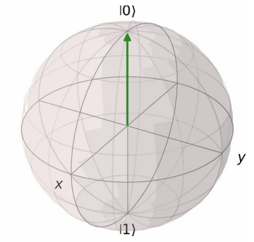
Single qubit inputs are $|0\rangle$ to $|1\rangle$ and can be represented by matrix forms as

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \text{ and } |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

Single Qubit Gates are X- gate, Y-gate, Z- gate, H –gate, S-gate, T-gate.

1. X – Gate or Quantum Not Gate: bit flip

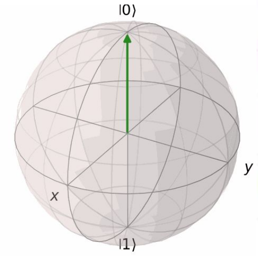
$$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$



X -gate	
Input	Output
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|1\rangle + \beta\|0\rangle$

2. Y – Gate : Both bit flip and phase flip

$$Y = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$$

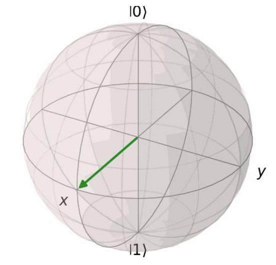


Y -gate	
Input	Output
$\|0\rangle$	$i\|1\rangle$
$\|1\rangle$	$-i\|0\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$i\alpha\|1\rangle - i\beta\|0\rangle$

Quantum Gates

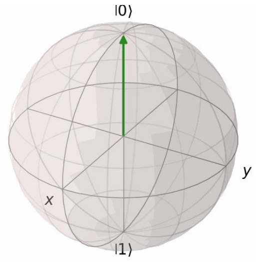
3. z – Gate: phase flip only when input in |1>

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



Z-gate	
Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$-\|1\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|0\rangle - \beta\|1\rangle$

4. Hadamard gate (H): creates superposition

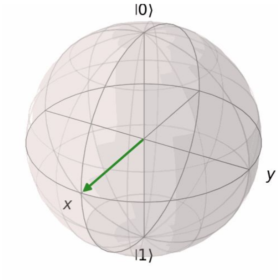


$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Input	Output
$\|0\rangle$	$\frac{\|0\rangle + \|1\rangle}{\sqrt{2}}$
$\|1\rangle$	$\frac{\|0\rangle - \|1\rangle}{\sqrt{2}}$
$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\left(\frac{\|0\rangle + \|1\rangle}{\sqrt{2}}\right) + \beta\left(\frac{\|0\rangle - \|1\rangle}{\sqrt{2}}\right)$

Quantum Gates

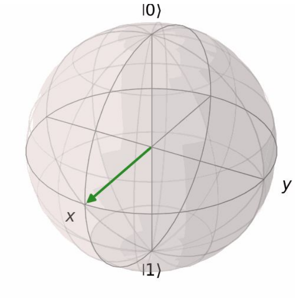
5. Phase Gate (S Gate): The Phase gate or S gate is a gate that transfers $|0\rangle$ into $|0\rangle$ and $|1\rangle$ into $i|1\rangle$.



$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$i\|1\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|0\rangle + i\,\beta\|1\rangle$

6. T- Gate : It is a single qubit gate and it is also called π/8 gate.



$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$e^{i\pi/4}\|1\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|0\rangle + e^{i\pi/4}\,\beta\|1\rangle$

Quantum Gates

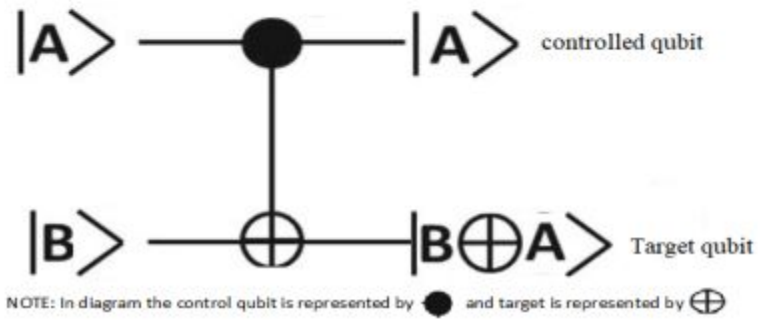
Multiple Gates: Multiple qubit consists of control gate and target gate. The action of gate as follows;

i) The Target qubit is altered only when the control qubit is $|1\rangle$, and

ii) The control qubit remains unaltered during the transformations

Controlled Gate (CNOT):

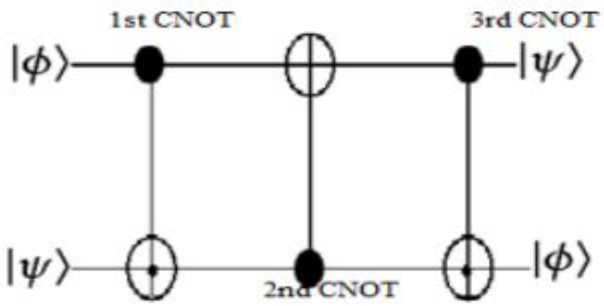
$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



Input	Output
$|00\rangle$	$|00\rangle$
$|01\rangle$	$|01\rangle$
$|10\rangle$	$|11\rangle$
$|11\rangle$	$|10\rangle$

Quantum Gates

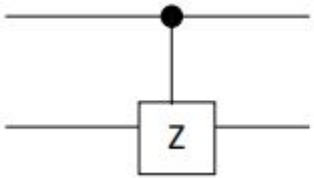
Swap Gate:



$$\text{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|10\rangle$
$\|10\rangle$	$\|01\rangle$
$\|11\rangle$	$\|11\rangle$

Controlled Z Gate:



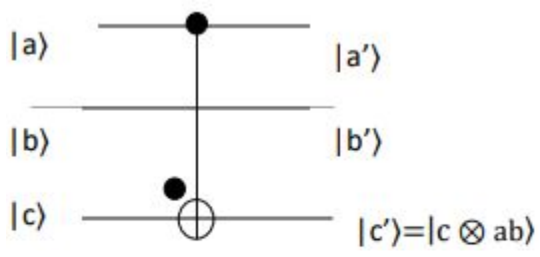
$$\text{Controlled Z gate} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|10\rangle$
$\|11\rangle$	$-\|11\rangle$

Quantum Gates

Toffoli Gate:

$$\text{Toffoli Gate} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

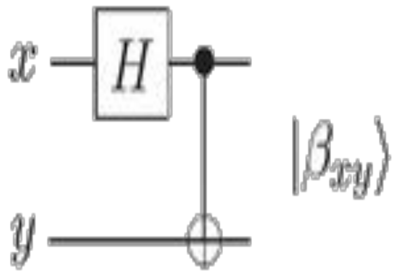


Input to the gate $\|abc\rangle$	Output of the gate $\|a'b'c'\rangle$
$\|100\rangle$	$\|100\rangle$
$\|001\rangle$	$\|001\rangle$
$\|010\rangle$	$\|010\rangle$
$\|011\rangle$	$\|011\rangle$
$\|100\rangle$	$\|100\rangle$
$\|101\rangle$	$\|101\rangle$
$\|110\rangle$	$\|111\rangle$
$\|111\rangle$	$\|110\rangle$

Bell States

Each Bell state is a superposition of two computational basis states in the four-dimensional Hilbert space.

In	Out
$\|00\rangle$	$(\|00\rangle + \|11\rangle)/\sqrt{2} \equiv \|\beta_{00}\rangle$
$\|01\rangle$	$(\|01\rangle + \|10\rangle)/\sqrt{2} \equiv \|\beta_{01}\rangle$
$\|10\rangle$	$(\|00\rangle - \|11\rangle)/\sqrt{2} \equiv \|\beta_{10}\rangle$
$\|11\rangle$	$(\|01\rangle - \|10\rangle)/\sqrt{2} \equiv \|\beta_{11}\rangle$



Quantum Information

A *quantum state* of a system is represented by a *column vector* whose indices are placed in correspondence with the classical states of that system:

- The entries are complex numbers.
- The sum of the absolute values squared of the entries must equal 1.

Definition

The *Euclidean norm* for vectors with complex number entries is defined like this:

$$v = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} \implies \|v\| = \sqrt{\sum_{k=1}^{n} |\alpha_k|^2}$$

Quantum state vectors are therefore *unit vectors* with respect to this norm.

Quantum Information

Examples of qubit states

- Standard basis states: $|0\rangle$ and $|1\rangle$
- Plus/minus states:

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \quad \text{and} \quad |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

- A state without a special name:

$$\frac{1+2i}{3}|0\rangle - \frac{2}{3}|1\rangle$$

Example

A quantum state of a system with classical states ♣, ♦, ♥, and ♠:

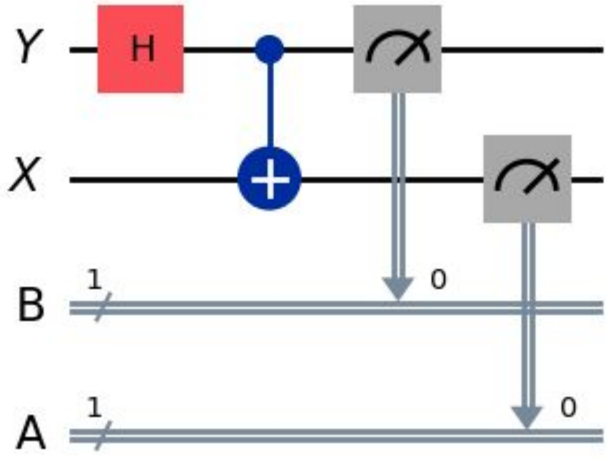
$$\frac{1}{2}|♣\rangle - \frac{i}{2}|♦\rangle + \frac{1}{\sqrt{2}}|♠\rangle = \begin{pmatrix} \frac{1}{2} \\ -\frac{i}{2} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Quantum Circuits

A quantum circuit is a model for quantum computation, analogous to classical circuits, where computations are sequences of quantum gates, measurements, and qubit initializations.

Key Components:

- **Qubits:** Quantum bits, the fundamental units of quantum information, analogous to classical bits.
- **Quantum Gates:** Operations that act on qubits, like rotations or entanglement operations. Examples include Hadamard gates (H), Pauli-X gates (X), CNOT gates, etc.
- **Quantum Wires:** Represent the flow of qubits through the circuit, connecting gates and measurements.
- **Measurements:** The process of obtaining classical information from qubits. This collapses the quantum state and provides a classical bit as output.
- **Initializations:** Setting qubits to a known initial state (e.g., |0⟩).



https://learning.quantum.ibm.com/course/basics-of-quantum-information/quantum-circuits

Quantum Algorithms

- a quantum algorithm is a step-by-step procedure, where each of the steps can be performed on a quantum computer.
- quantum algorithm is generally uses some essential feature of quantum computation such as quantum superposition or quantum entanglement.

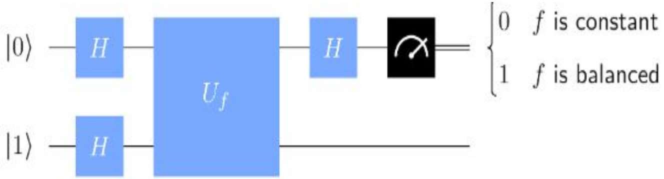
1. **Deutsch jozsa Algorithm:**a function $f{:}\Sigma\rightarrow\Sigma$ from one bit to one bit. There are 4 such functions:

a	$f_1(a)$
0	0
1	0

a	$f_2(a)$
0	0
1	1

a	$f_3(a)$
0	1
1	0

a	$f_4(a)$
0	1
1	1



The first and last of these functions are *constant* and the middle two are *balanced*

2. **Shor's Algorithm** - is a quantum computing algorithm designed for efficiently factoring large numbers
3. **Grover's Algorithm** - is a quantum algorithm that provides a quadratic speedup for **searching unsorted databases** or unstructured lists compared to classical algorithms. (**Classical algorithms** for searching unsorted data require, on average, **N/2 steps** for a list of N items. Grover's algorithm can find the solution in approximately **√N steps**, offering a significant speedup).

https://learning.quantum.ibm.com/course/fundamentals-of-quantum-algorithms/grovers-algorithm

Research Papers

1. Training a Quantum Neural Network

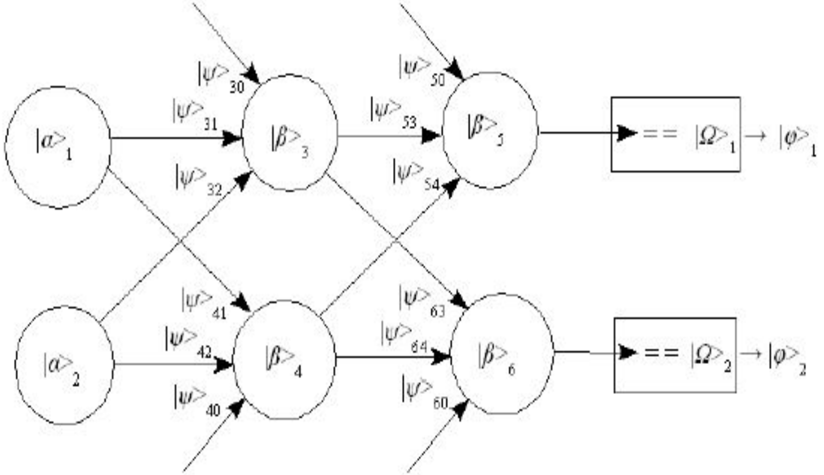


Figure 1: Simple QNN to compute XOR function

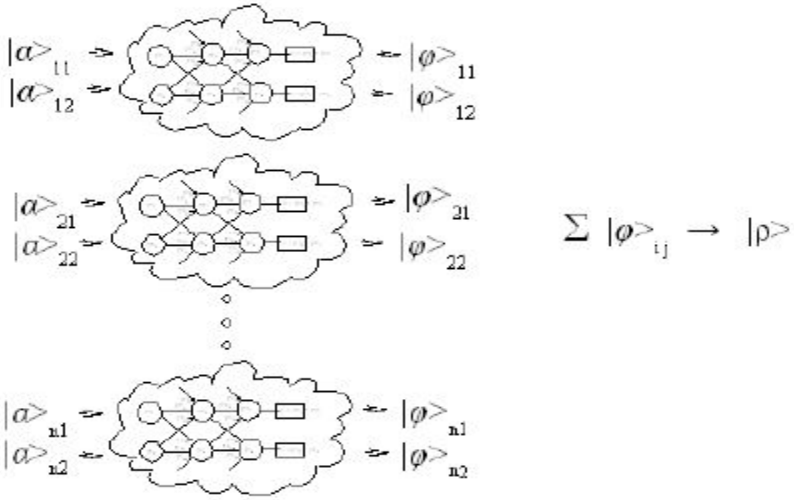
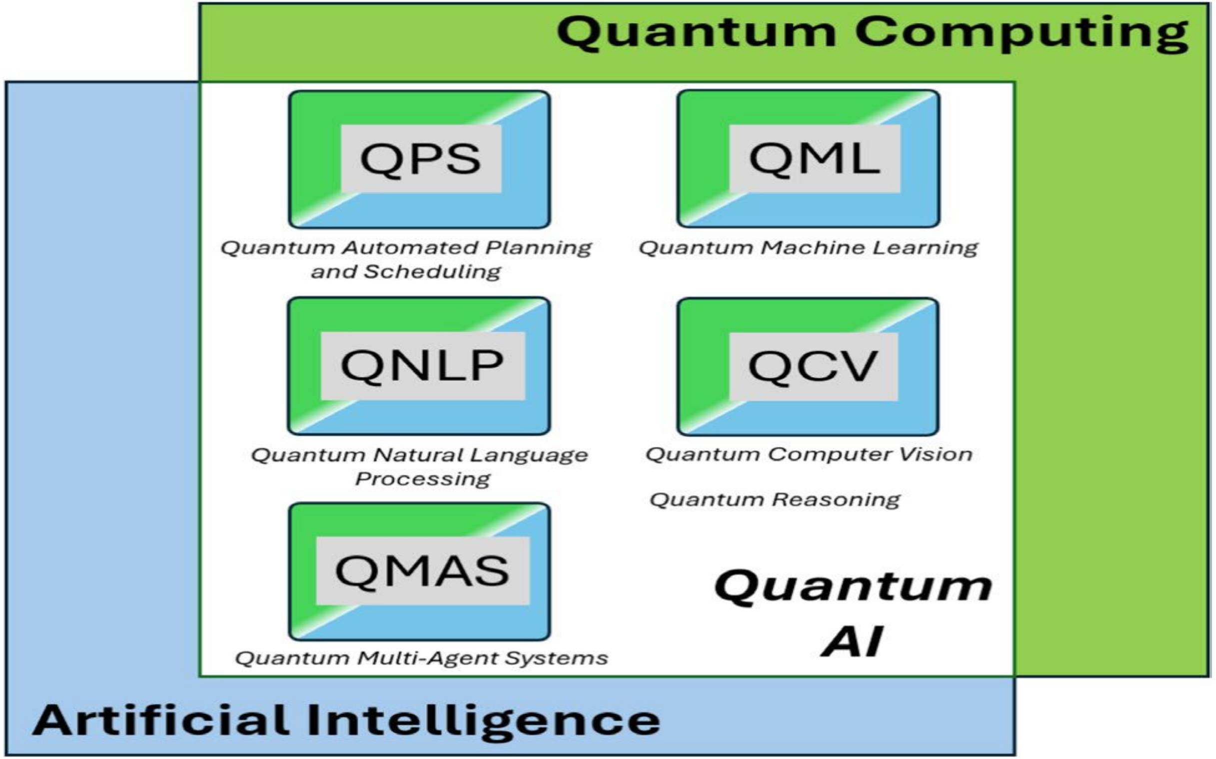


Figure 2: QNN Training

- Using Quantum Search to Learn Network Weights.
- After an average of 58 epochs, the algorithm was able to cover the training data with the standard deviation for this method was much higher, but still reasonable at 64.836.
- After an average of 74 epochs, the randomized search algorithm was able to achieve 95% accuracy on the training set. Backpropagation classified 98% of the training set correctly after around 300 epochs.

Research Papers

2. Quantum Artificial Intelligence



Future Plan

1. **Exploring Research Papers**

   A. Quantum Artificial Intelligence

   B. Quantum Machine Learning

   C. A Leap among Quantum Computing and Quantum Neural Networks

2. **Implementation of quantum gates and circuits using qiskit.**