Quantum Machine Learning and its Applications

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Presented By
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24MCMI17
MTech-AI.

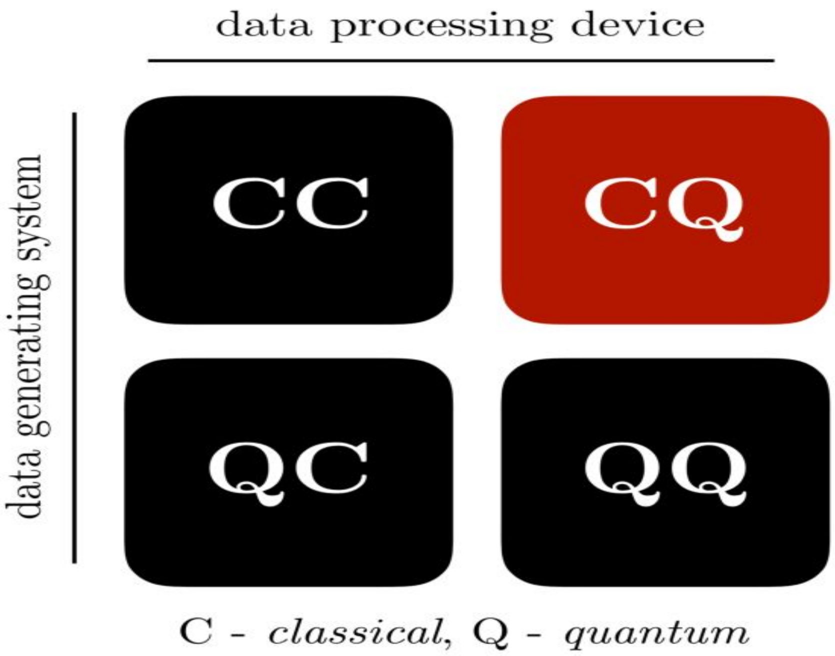
Agenda

- Quantum Machine learning
- Quantum Model
- Introduction to Variational Circuit
- A Simple Model
- Gradient Descent Methods
- Encoding Classical Data into Quantum Data
- Quantum Neural Network
- Barren Plateaus

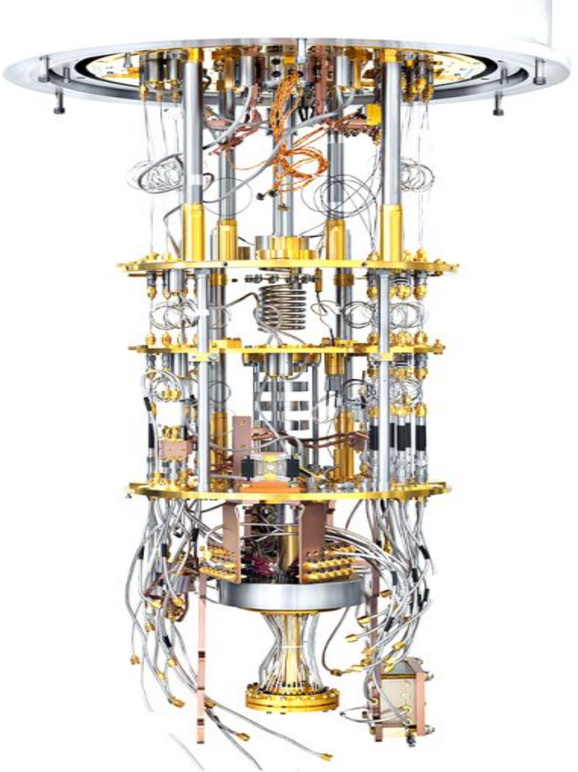
Quantum Model

$$f(x;\theta) \longrightarrow |f(x;\theta)\rangle$$

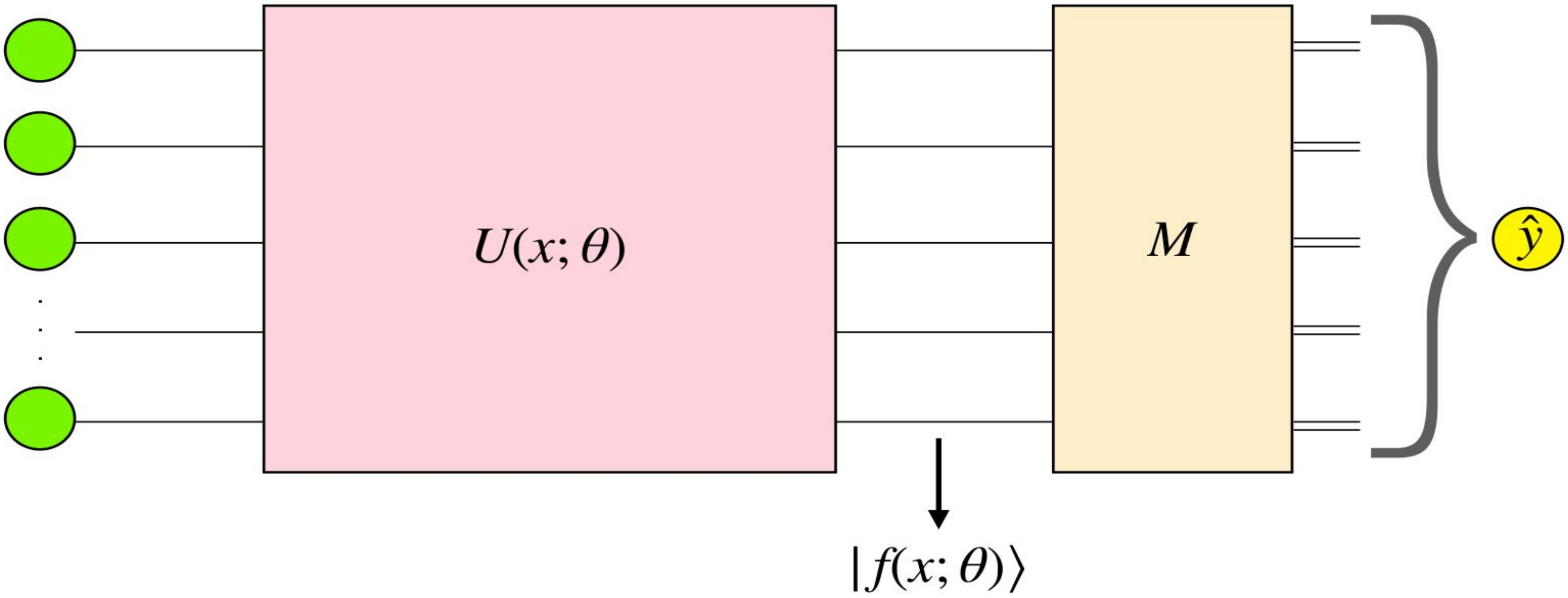
Quantum Machine Learning



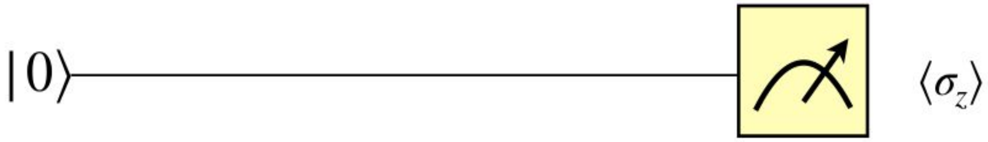
Source: Maria Schuld and Francesco Petruccione. *Supervised learning with quantum computers*. Vol. 17. Springer, 2018.

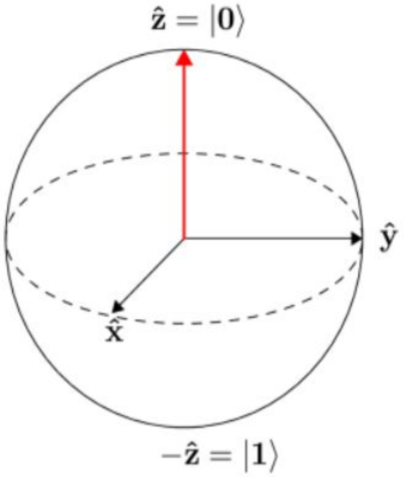


Quantum Machine Learning



A simple Example

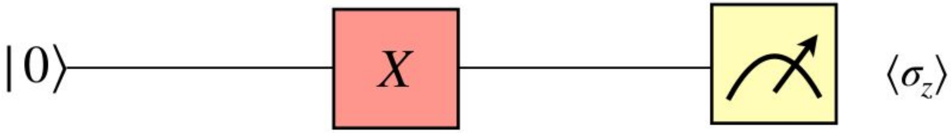


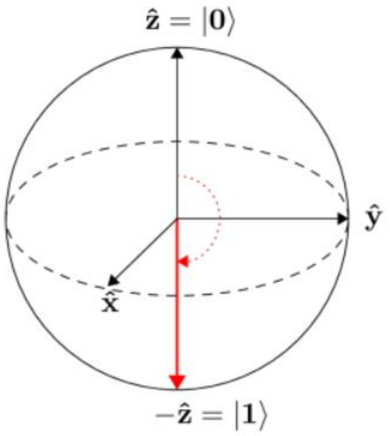


$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Calculating the expectation value: $\langle 0 | \sigma_z | 0 \rangle = [1 \quad 0] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$

A simple Example

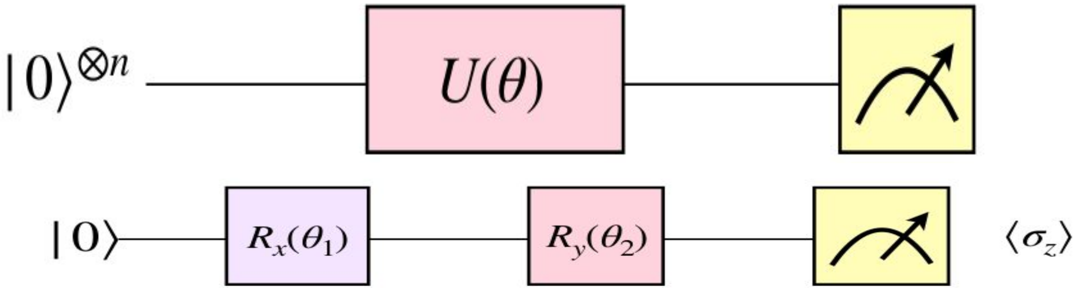




$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Calculating the expectation value: $\langle 1 \mid \sigma_z \mid 1 \rangle = [0 \quad 1] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -1$

A simple Example



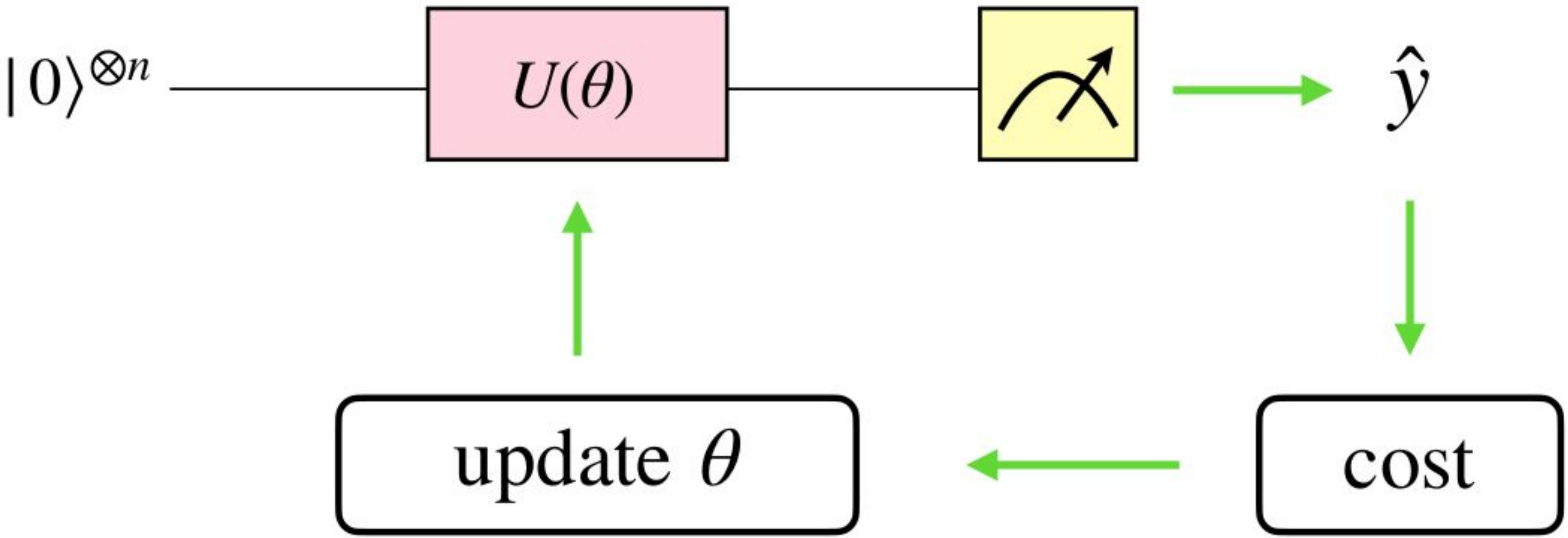
$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad R_x(\theta_1) = \begin{bmatrix} \cos\frac{\theta_1}{2} & -i\,\sin\frac{\theta_1}{2} \\ -i\,\sin\frac{\theta_1}{2} & \cos\frac{\theta_1}{2} \end{bmatrix} \qquad R_y(\theta_2) = \begin{bmatrix} \cos\frac{\theta_2}{2} & -\sin\frac{\theta_2}{2} \\ \sin\frac{\theta_2}{2} & \cos\frac{\theta_2}{2} \end{bmatrix} \qquad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

State after operations: $|\psi\rangle = R_y(\theta_2)\ R_x(\theta_1)\ |0\rangle$

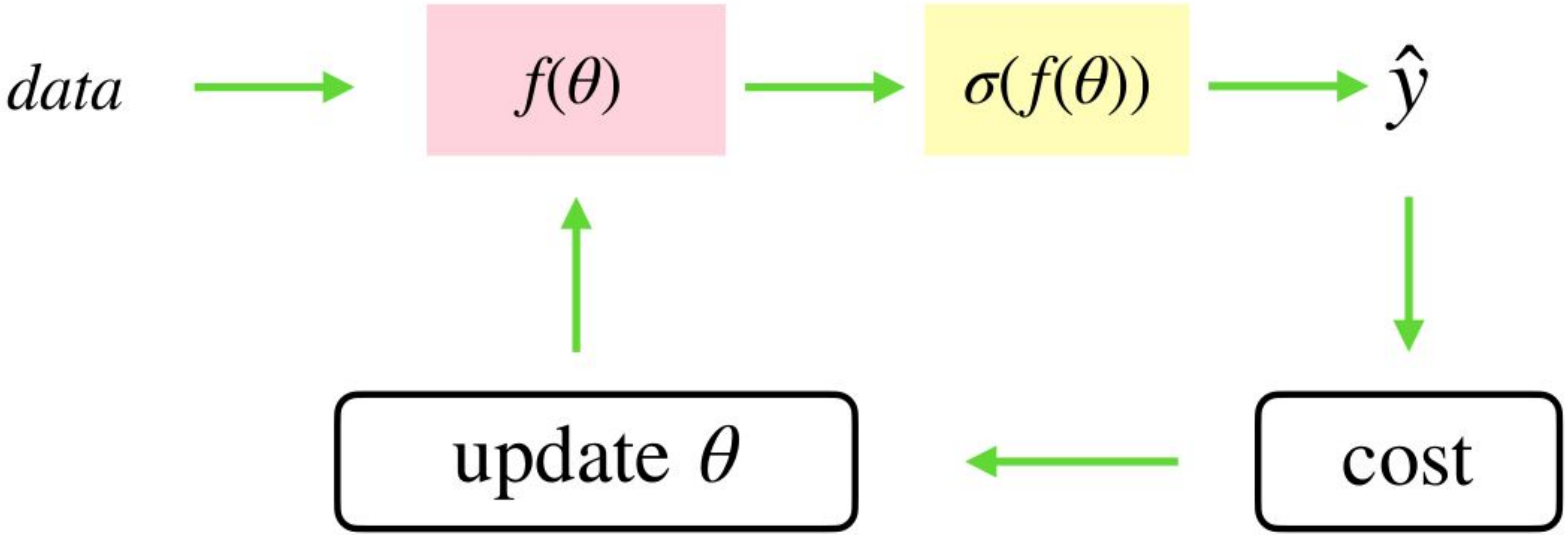
Calculating the expectation value: $\langle \psi \mid \sigma_z \mid \psi \rangle = \langle 0 \mid R_x(\theta_1)^{\dagger} Ry(\theta_2)^{\dagger} \sigma_z\ R_y(\theta_2) R_x(\theta_1) \mid 0 \rangle = \cos(\theta_1)\ \cos(\theta_2)$

How do we choose the parameters?

A simple Example



A simple Example



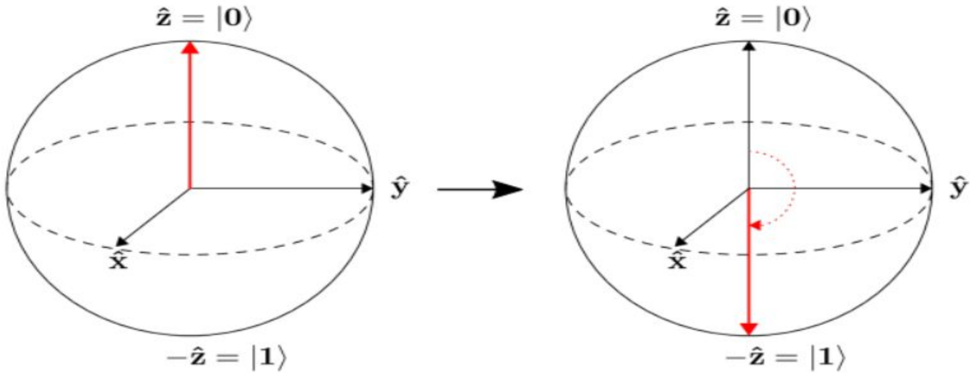
A simple Example

Define a cost function which we want to minimise:

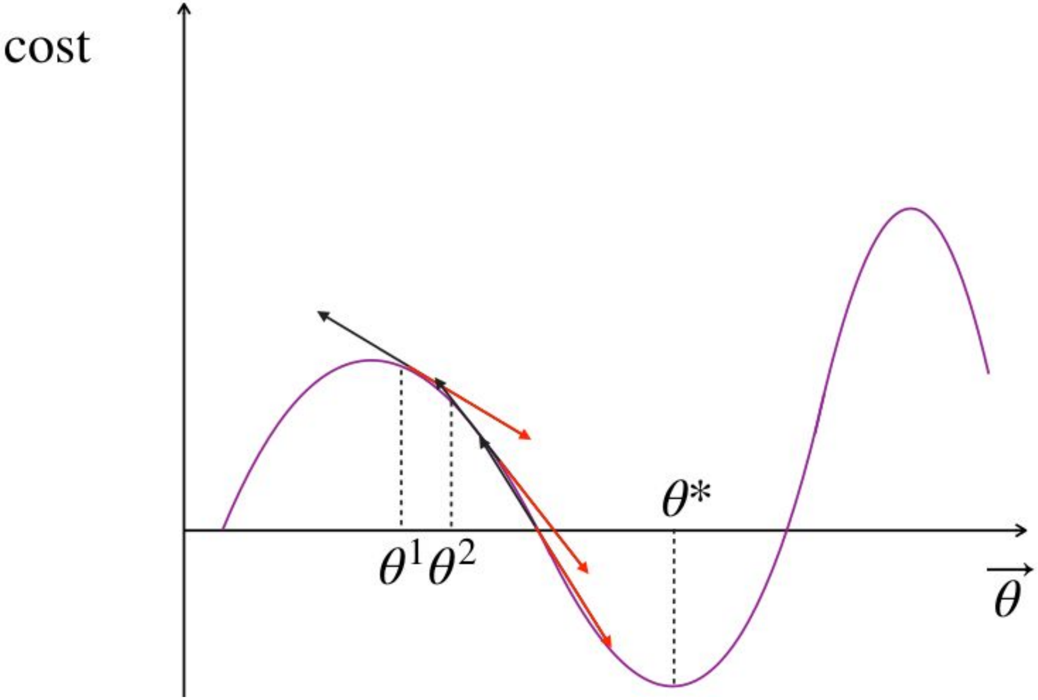
$$\langle \psi \mid \sigma_z \mid \psi \rangle = \langle 0 \mid R_x(\theta_1)^\dagger Ry(\theta_2)^\dagger \sigma_z \; R_y(\theta_2) R_x(\theta_1) \mid 0 \rangle = \cos(\theta_1)\,\cos(\theta_2)$$

$$\begin{bmatrix}1 & 0\end{bmatrix} \begin{bmatrix}1 & 0 \\ 0 & -1\end{bmatrix} \begin{bmatrix}1 \\ 0\end{bmatrix} = 1$$

$$\begin{bmatrix}0 & 1\end{bmatrix} \begin{bmatrix}1 & 0 \\ 0 & -1\end{bmatrix} \begin{bmatrix}0 \\ 1\end{bmatrix} = -1$$



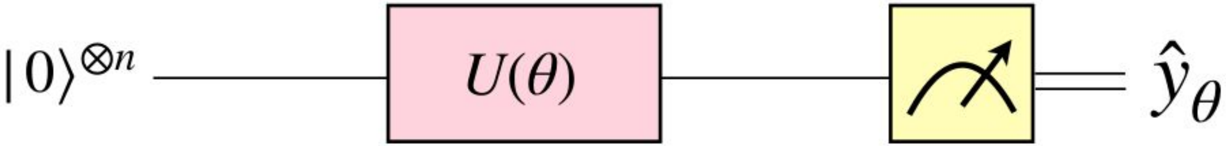
Gradient Descent



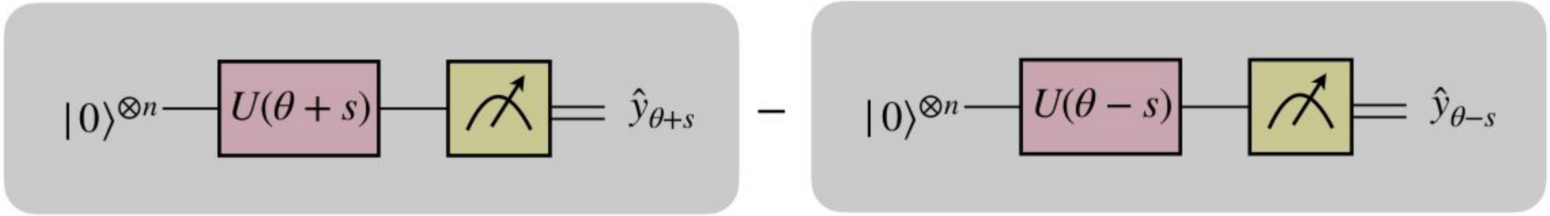
$$\Theta_{n+1} = \Theta_n - \alpha \frac{\partial}{\partial \Theta_n} J(\Theta_n)$$

$\Theta \rightarrow$ Parameter Vector
$J \rightarrow$ Cost Function
$\alpha \rightarrow$ Slope Parameter

Gradient-based methods



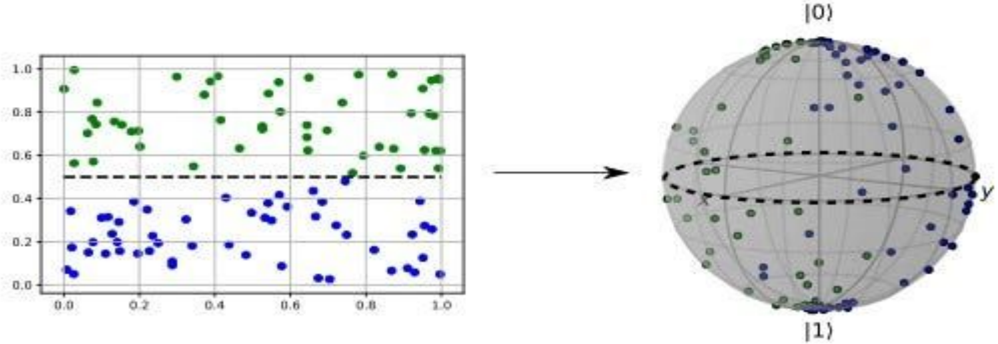
Gradient =



Encoding Classical Data

Types of Encoding classical data into Quantum states

- Basis encoding
- Amplitude encoding
- Angle encoding
- Feature Map



Basic Encoding

For example,

$$x^1 = (0,\ 1)^{\mathrm{T}}$$
$$x^2 = (1,\ 1)^{\mathrm{T}}$$

Creating a superposition of states using 2 qubits:

$$\cancel{|00\rangle} + \frac{1}{\sqrt{2}}|01\rangle + \cancel{|10\rangle} + \frac{1}{\sqrt{2}}|11\rangle$$

$$(0, \quad \frac{1}{\sqrt{2}}, \quad 0, \quad \frac{1}{\sqrt{2}})^{\mathrm{T}}$$

Amplitude Encoding

For example,

$$x^1 = (0.888, \ -1.25)^{\mathrm{T}}$$
$$x^2 = (-0.23, \ 0.992)^{\mathrm{T}}$$

$$\downarrow$$

$$\frac{1}{\sqrt{4}}(0.888, \ -1.25, \ -0.23, \ 0.992)^{\mathrm{T}}$$

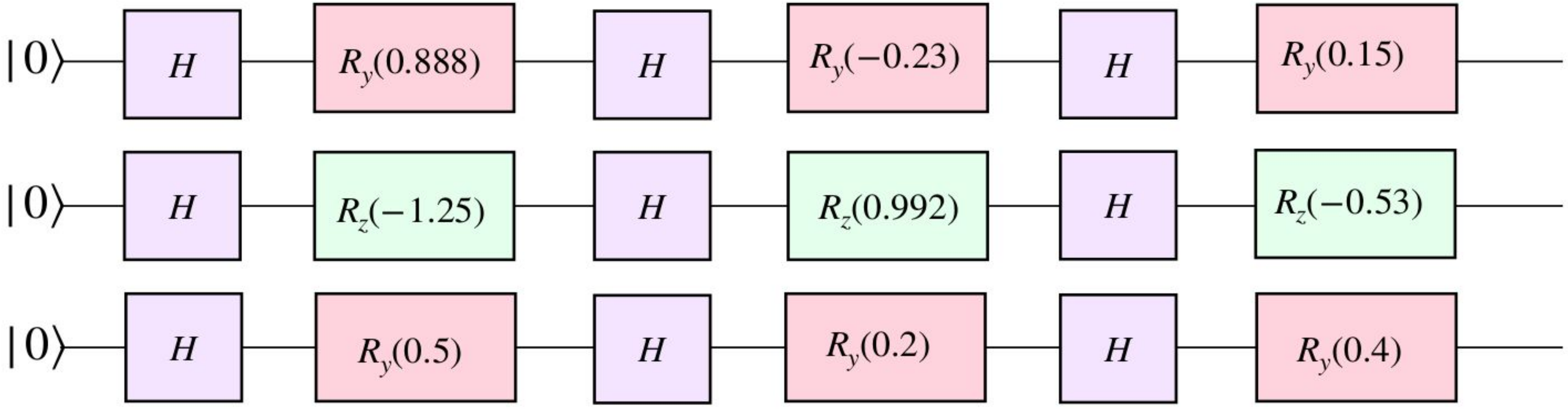
$$|00\rangle \qquad |01\rangle \qquad |10\rangle \qquad |11\rangle$$

$$|D\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=1}^{2^n} x_i \, |\, i\rangle \longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.888 \\ -1.25 \\ -0.23 \\ 0.992 \end{bmatrix} \qquad 2^n = 4, \ n = 2$$

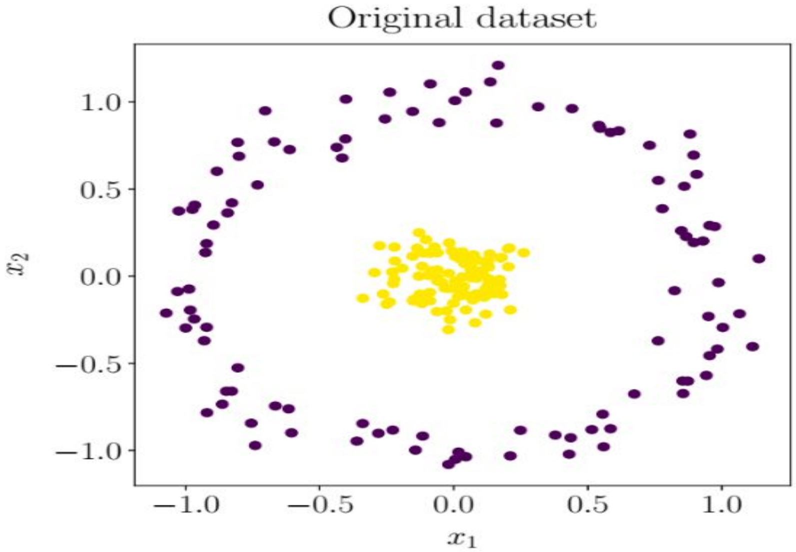
Angle Encoding

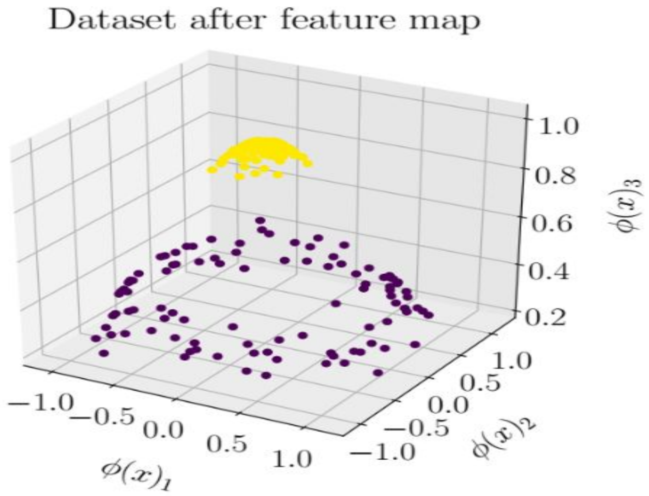
For example,

$$x^1 = \begin{bmatrix} 0.888 \\ -1.25 \\ 0.5 \end{bmatrix} \qquad x^2 = \begin{bmatrix} -0.23 \\ 0.992 \\ 0.2 \end{bmatrix} \qquad x^3 = \begin{bmatrix} 0.15 \\ -0.53 \\ 0.4 \end{bmatrix}$$



Higher Order Encoding





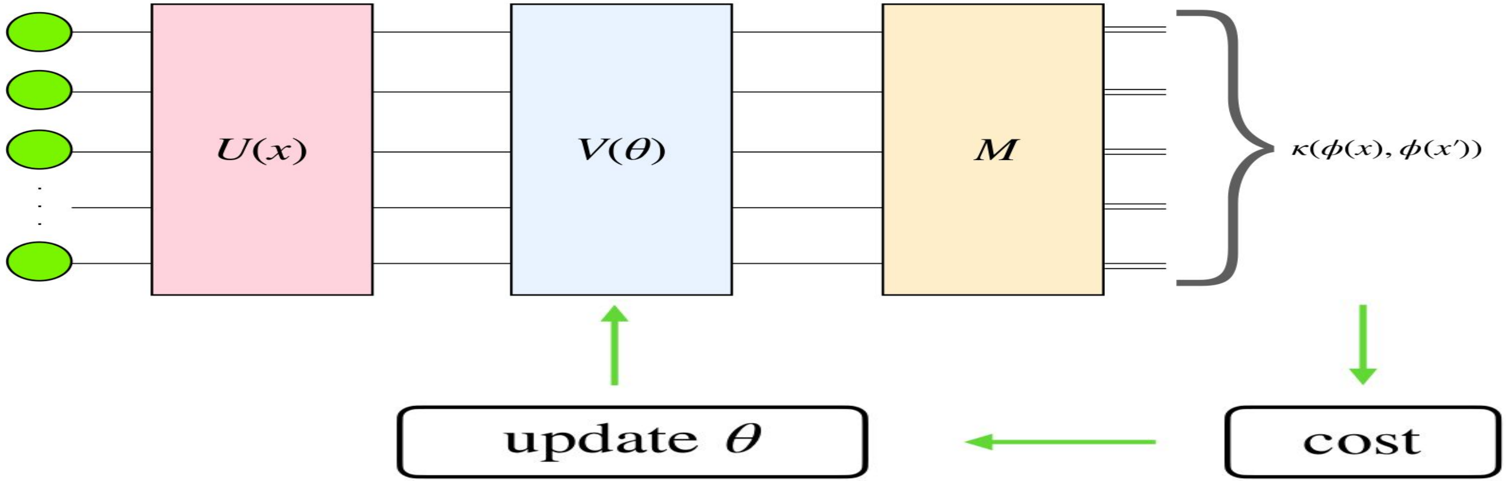
$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\phi(x) = \begin{bmatrix} \phi(x_1) \\ \phi(x_2) \\ \phi(x_3) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_1 * x_2 \end{bmatrix}$$

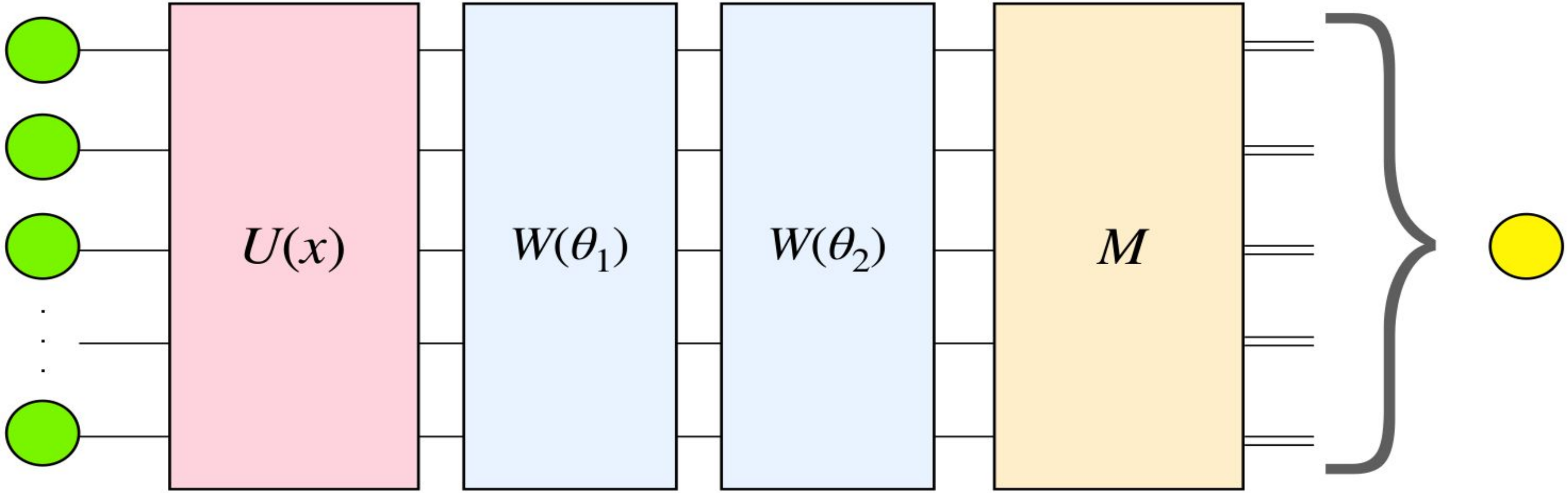
Quantum kernels

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \longrightarrow |\phi(x)\rangle = \begin{bmatrix} \phi(x_1) \\ \phi(x_2) \\ \vdots \end{bmatrix}$$

$$\kappa(\phi(x), \phi(x')) = \langle \phi(x) | \phi(x') \rangle$$

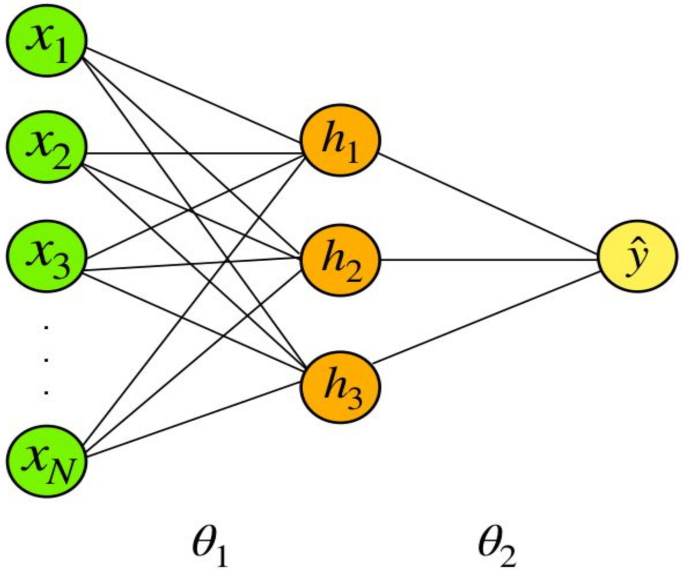


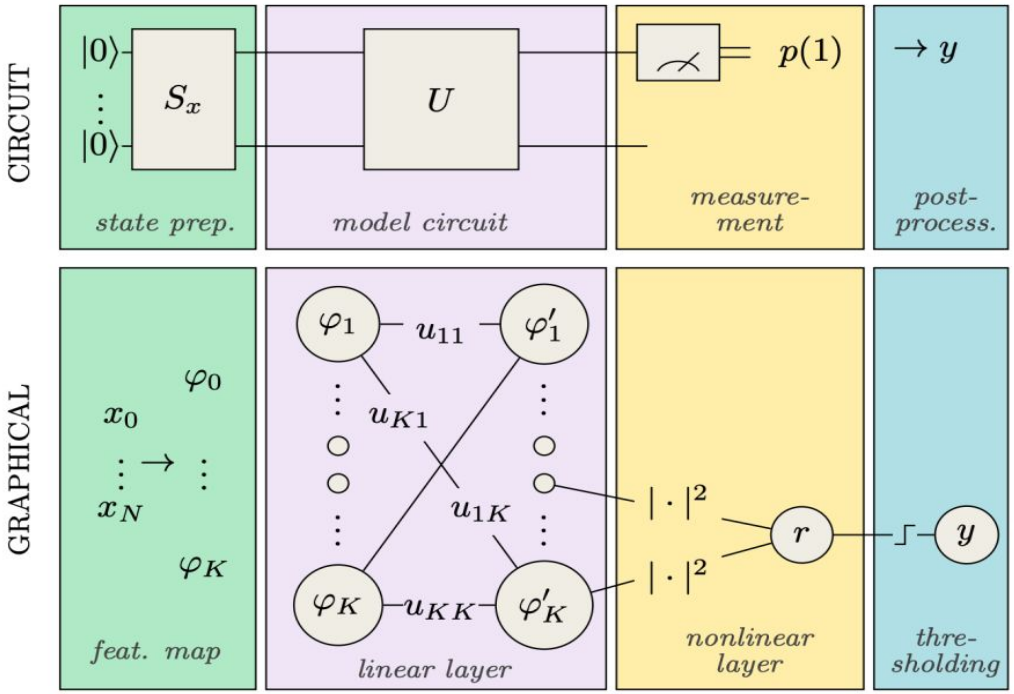
Quantum Neural Network



$$\hat{y} = \sigma(\theta_2(\theta_1 x))$$

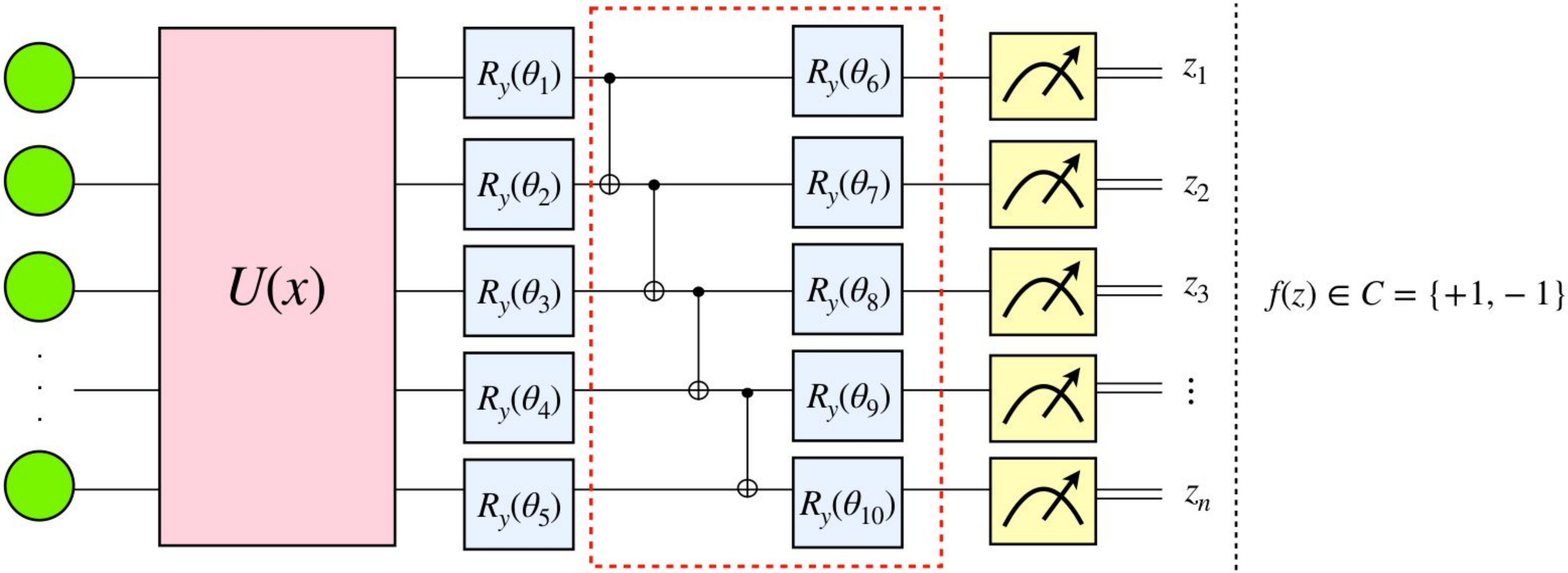
Quantum Neural Network



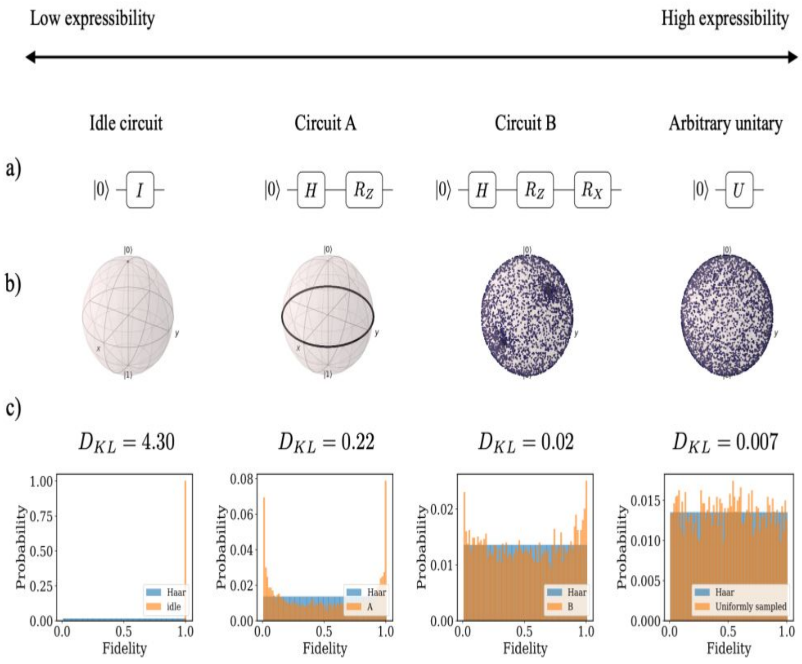


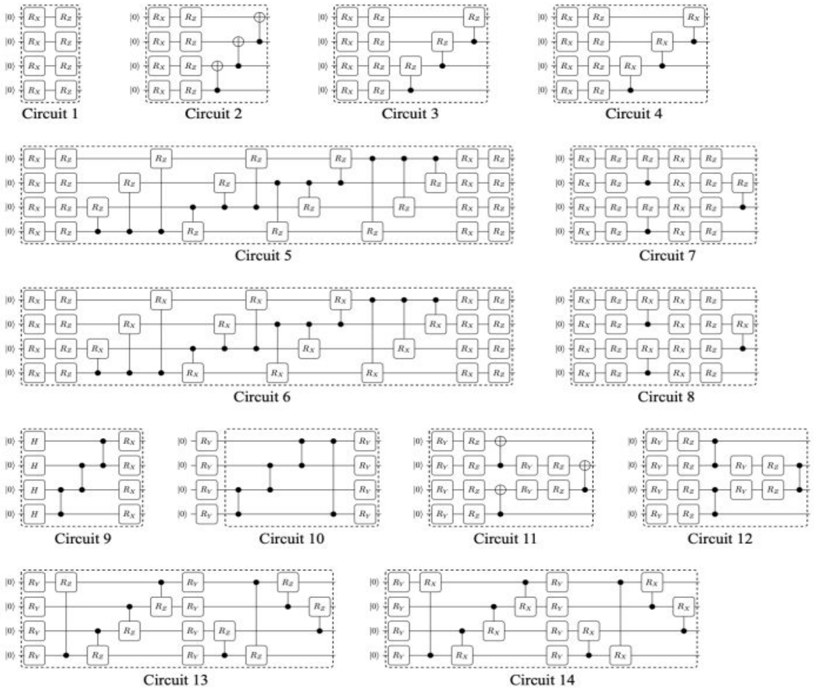
Source:Schuld, Maria, et al. "Circuit-centric quantum classifiers." Physical Review A 101.3 (2020): 032308.

Quantum Variational Classifier



Variational forms





Source: Sim, Sukin, Peter D. Johnson, and Alán Aspuru-Guzik. "Expressibility and Entangling Capability of Parameterized Quantum Circuits for Hybrid Quantum-Classical Algorithms." Advanced Quantum Technologies 2.12 (2019): 1900070.

Barren plateaus

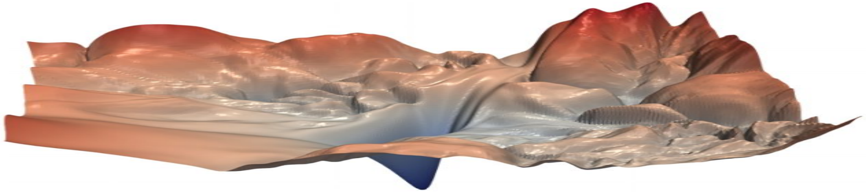
$$\theta_{\text{new}} = \theta - \eta \frac{\partial C(\hat{y}, y)}{\partial \theta}$$

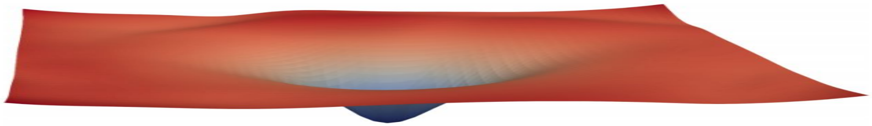
Methods to try avoid barren plateaus

- Introduce structure into the circuit
- Use a local cost function
- Higher order optimisation methods
- Loss landscapes

$$\frac{\partial C}{\partial \theta} = \begin{bmatrix} \frac{\partial C}{\partial \theta_1} \\ \frac{\partial C}{\partial \theta_2} \\ \vdots \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 C}{\partial \theta_1^2} & \frac{\partial^2 C}{\partial \theta_1 \theta_2} & \cdots \\ \frac{\partial^2 C}{\partial \theta_2 \theta_1} & & \\ \vdots & & \end{bmatrix}$$





Thank You