Quantum Machine Learning and its Applications -2

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Basis States

Example

A quantum state of a system with classical states ♣, ♦, ♥, and ♠:

$$\frac{1}{2}\,|♣\rangle - \frac{i}{2}\,|♦\rangle + \frac{1}{\sqrt{2}}\,|♠\rangle = \begin{pmatrix} \frac{1}{2} \\ -\frac{i}{2} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

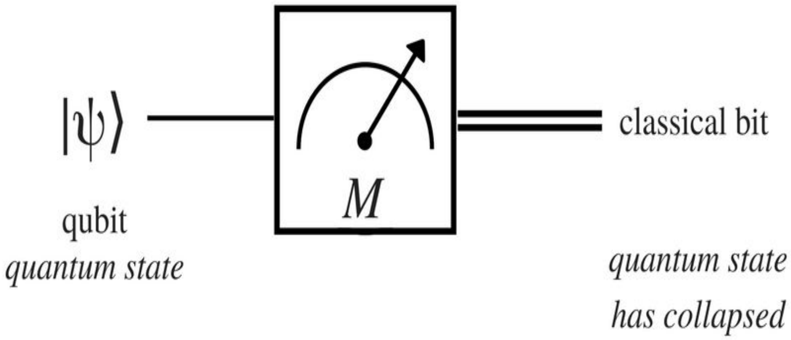
- There are Black Club,Red Diamond, Red Heart, Black Spade.
- States are unique in shape or color.
- These are mutually exclusive states.

Measurements

Measurements represent an interface between quantum and classical information:

- Performing a measurement on a system extracts classical information about its quantum state.
- In general, the system is changed (or destroyed) in the process.

- Collapse of **superposition**
- **Probabilistic** outcome
- Impact on the system
- **Irreversible process**



Measurements

Initially our focus will be on *destructive measurements* — which produce a classical outcome alone. (The post-measurement state of the system is not specified.)

Two ways to describe destructive measurements

1. As *collections of matrices,* one for each measurement outcome.
2. As *channels* whose outputs are always classical states (represented by diagonal density matrices).

Non-destructive measurements will be discussed later in the lesson. (They can always be described as compositions of destructive measurements and channels.)

Types of Measurements

- projective measurements - Destructive Measurement
- Quantum channels
- Partial Measurements
- General Measurements
- **Non-Destructive Measurement**
- **State Tomography**

Projective Measurements

In quantum mechanics, a projective measurement is described by an **observable**, which is a Hermitian operator M.

. A Hermitian operator has two crucial properties that make it suitable for describing measurements:

1. **Real Eigenvalues:** Its eigenvalues (the possible measurement outcomes) are always real numbers. This makes sense, as a measurement in a lab should yield a real number (e.g., +1 for spin-up, -1 for spin-down).
2. **Orthogonal Eigenvectors:** Its eigenvectors form a complete orthonormal basis for the state space. This means any quantum state can be written as a unique combination of these eigenvectors. These eigenvectors are the "definite states" the system can collapse into.

Let's say our observable M has eigenvalues $\{m\}$ and corresponding eigenvectors $\{|m\rangle\}$. The set $\{|m\rangle\}$forms an orthonormal basis, meaning $\langle mi|mj\rangle=\delta ij$ (it's 1 if $i=j$ and 0 otherwise).

Associated with each possible outcome m is a **projector**, Pm defined as: $Pm=|m\rangle\langle m|$

Projective Measurements

The Two Fundamental Rules of Projective Measurement

- **Rule 1: The Probability Rule (Born Rule):** The probability of obtaining the outcome m is given by the square of the magnitude of the projection of $|\psi\rangle$ onto the eigenvector $|m\rangle$.

$$p(m) = \langle\psi|P_m|\psi\rangle = \langle\psi|m\rangle\langle m|\psi\rangle = |\langle m|\psi\rangle|^2$$

- **Rule 2: The State Collapse Rule:** If the outcome m is obtained, the state of the system immediately after the measurement is no longer $|\psi\rangle$. It **collapses** to the corresponding normalized eigenvector $|m\rangle$. The post-measurement state, $|\psi'\rangle$, is:

$$|\psi'\rangle = \frac{P_m|\psi\rangle}{\sqrt{p(m)}} = \frac{|m\rangle\langle m|\psi\rangle}{|\langle m|\psi\rangle|} = \frac{\langle m|\psi\rangle}{|\langle m|\psi\rangle|}|m\rangle$$

.

Qubit Example

The System: A single qubit in a general superposition. $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$ where α and β are complex numbers and $|\alpha|2+|\beta|2=1$.

Eigenvalues and Eigenvectors of Z:

- Eigenvalue $m_0 = +1$ corresponds to the eigenvector $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. This is often interpreted as "spin-up."
- Eigenvalue $m_1 = -1$ corresponds to the eigenvector $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. This is often interpreted as "spin-down."

The Projectors:

- Projector onto the $|0\rangle$ state: $P_0 = |0\rangle\langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$.
- Projector onto the $|1\rangle$ state: $P_1 = |1\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

1. **Calculating Probabilities:**

Probability of measuring 0 (outcome +1 for Z):

$p(0) = |\langle 0|\psi\rangle|^2$

$\langle 0|\psi\rangle = \langle 0|(\alpha|0\rangle + \beta|1\rangle) = \alpha\langle 0|0\rangle + \beta\langle 0|1\rangle = \alpha(1) + \beta(0) = \alpha$

Therefore, $p(0) = |\alpha|^2$.

Probability of measuring 1 (outcome -1 for Z):

$p(1) = |\langle 1|\psi\rangle|^2$

$\langle 1|\psi\rangle = \langle 1|(\alpha|0\rangle + \beta|1\rangle) = \alpha\langle 1|0\rangle + \beta\langle 1|1\rangle = \alpha(0) + \beta(1) = \beta$

Therefore, $p(1) = |\beta|^2$.

Qubit Example

2. Determine the Post-Measurement State (State Collapse)

If the outcome is 0:

The qubit collapses to the state $|0\rangle$. Formally:

$$|\psi'\rangle = \frac{P_0|\psi\rangle}{\sqrt{p(0)}} = \frac{(|0\rangle\langle 0|)(\alpha|0\rangle+\beta|1\rangle)}{|\alpha|} = \frac{\alpha|0\rangle\langle 0|0\rangle+\beta|0\rangle\langle 0|1\rangle}{|\alpha|} = \frac{\alpha|0\rangle}{|\alpha|}$$

This state is just $|0\rangle$ with a phase factor. The qubit is now definitively in the state **|0⟩**.

If the outcome is 1:

The qubit collapses to the state $|1\rangle$. Formally:

$$|\psi'\rangle = \frac{P_1|\psi\rangle}{\sqrt{p(1)}} = \frac{(|1\rangle\langle 1|)(\alpha|0\rangle+\beta|1\rangle)}{|\beta|} = \frac{\beta|1\rangle}{|\beta|}$$

The qubit is now definitively in the state **|1⟩**.

Quantum channels

A quantum channel (also known as a quantum operation or a CPTP map) is a mathematical object that describes the most general physical evolution of a quantum state.

- **Input:** The state of the system before the process, described by a density matrix ρin.
- **Process:** The channel, denoted by E.
- **Output:** The state of the system after the process, ρout=E(ρin)

The evolution of the state is then given by:. $\rho_{out} = \mathcal{E}(\rho_{in}) = \sum_i K_i \rho_{in} K_i^\dagger$

This framework is guaranteed to be "Completely Positive and Trace-Preserving" (CPTP).

Interpretation: The Physical Meaning of the Result

$$\rho_{in} = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \xrightarrow{\text{Z-measurement channel}} \rho_{out} = \begin{pmatrix} \rho_{00} & 0 \\ 0 & \rho_{11} \end{pmatrix}$$

- It preserves the diagonal elements.

- It completely destroys the off-diagonal elements (the coherences).

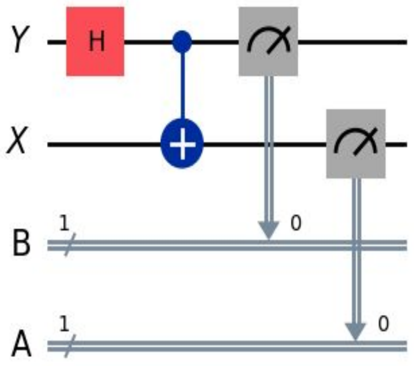
Partial Measurement

A **partial measurement** is a measurement performed on only a *subset* of a multi-qubit system. The core question it answers is: *If I measure one qubit, what happens to the other qubits it might be entangled with?*

Imagine you have a two-qubit system. Instead of measuring both qubits to get an outcome like `01` or `11`, you decide to measure only the *first* qubit.

- **The Measurement:** You perform a standard projective measurement on the first qubit. You will get a classical outcome, say `0`.

- **The Consequence:** This act of measurement not only collapses the first qubit into the definite state $|0\rangle$ but it can also *instantly change* the state of the second qubit, even if it's light-years away.



General Measurement

- a general measurement refers to a broad class of measurement operations that can be described by Positive Operator-Valued Measures (POVMs).
- Unlike simpler projective measurements, POVMs don't necessarily collapse the quantum state into an eigenstate of the measurement operator.
- Instead, they provide probabilities for different outcomes and can be used to describe more complex measurement scenarios.
- a POVM is described by a set of positive operators $\{Em\}$ where the only constraint is that they sum to the identity: $\sum mEm=I$. The probability of outcome m is $p(m)=Tr(\rho Em)$

Non Destructive Measurement

Question: Can we measure a system without completely destroying its state?

Naimark's Theorem (The Theory): Yes! Any generalized measurement can be realized by coupling the system to a helper qubit (an **ancilla**) and then performing a standard projective measurement on the ancilla.

Goal: We want to measure the "Z-observable" of a system qubit (i.e., find out if it's in the $|0\rangle$ or $|1\rangle$ part of its superposition) without destroying the qubit.

Detailed Example: Using a CNOT Gate for an NDM

System Qubit (S): In a general superposition $|\psi\rangle_S = \alpha|0\rangle_S + \beta|1\rangle_S$.

Ancilla Qubit (A): Prepared in the state $|0\rangle_A$.

Initial Combined State: $|\Psi_{in}\rangle = (\alpha|0\rangle_S + \beta|1\rangle_S) \otimes |0\rangle_A = \alpha|00\rangle_{SA} + \beta|10\rangle_{SA}$.

$$|\Psi_{out}\rangle = \text{CNOT}_{S\to A}|\Psi_{in}\rangle = \text{CNOT}_{S\to A}(\alpha|00\rangle + \beta|10\rangle) = \alpha|00\rangle_{SA} + \beta|11\rangle_{SA}$$

Case A - We measure 0 on the ancilla: In our state $\alpha|00\rangle+\beta|11\rangle$, only the first term has the ancilla as $|0\rangle$. So, $p(0)A=|\alpha|2$ and The post-measurement state $\frac{\text{Proj}_{0,A}|\Psi_{out}\rangle}{\sqrt{p(0)_A}} = \frac{\alpha|00\rangle}{|\alpha|} = |00\rangle_{SA}$

The system qubit is now definitively in the state $|0\rangle S$. Similarly Case B - when we measure 1 on the ancilla.

State Tomography

The Task: You are given a source of many identical qubits in a completely **unknown** state ρ. Reconstruct ρ.

The Analogy: CT Scan

A CT scanner takes many 2D X-ray slices from different angles to reconstruct a 3D model.

Quantum state tomography is the exact same idea:

- The **unknown state** ρ is the 3D object.
- The **measurements in different bases (X, Y, Z)** are the 2D slices from different angles.
- The **many identical copies** of the state are needed because each measurement "slice" destroys the copy it was performed on.
- The **reconstruction algorithm** is the computer that stitches the measurement statistics together to produce the final density matrix.

Reconstruct: Use these expectation values to build the density matrix:

$$\rho = \frac{1}{2}(I + \langle X \rangle X + \langle Y \rangle Y + \langle Z \rangle Z)$$

Where Pauli Operators are: $r_x = \langle X \rangle, r_y = \langle Y \rangle, r_z = \langle Z \rangle.$

Inner Product

When we use the Dirac notation, a ket is a column vector, and its corresponding bra is a row vector:

$$|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} \qquad \langle\psi| = \begin{pmatrix} \overline{\alpha_1} & \cdots & \overline{\alpha_n} \end{pmatrix}$$

Suppose that we have two kets:

$$|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} \quad \text{and} \quad |\phi\rangle = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix}$$

Inner Product

We then have

$$\langle\psi|\phi\rangle = \begin{pmatrix}\overline{\alpha_1} & \cdots & \overline{\alpha_n}\end{pmatrix}\begin{pmatrix}\beta_1 \\ \vdots \\ \beta_n\end{pmatrix} = \overline{\alpha_1}\beta_1 + \cdots + \overline{\alpha_n}\beta_n$$

This is the *inner product* of $|\psi\rangle$ and $|\phi\rangle$.

Inner Product

Alternatively, suppose that we have two column vectors expressed like this:

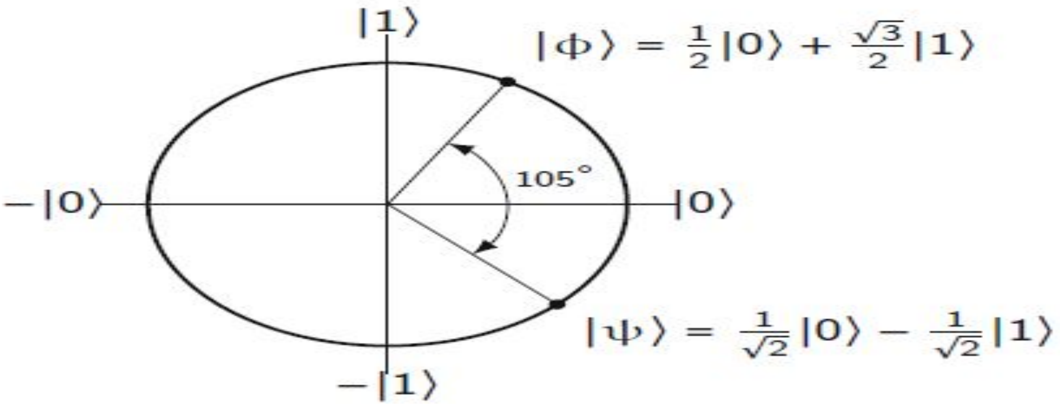
$$|\psi\rangle = \sum_{a\in\Sigma} \alpha_a |a\rangle \quad \text{and} \quad |\phi\rangle = \sum_{b\in\Sigma} \beta_b |b\rangle$$

Then the inner product of these vectors is as follows:

$$\begin{aligned}
\langle\psi|\phi\rangle &= \Big(\sum_{a\in\Sigma} \overline{\alpha_a}\langle a|\Big)\Big(\sum_{b\in\Sigma} \beta_b|b\rangle\Big) \\
&= \sum_{a\in\Sigma}\sum_{b\in\Sigma} \overline{\alpha_a}\beta_b\langle a|b\rangle \\
&= \sum_{a\in\Sigma} \overline{\alpha_a}\beta_a
\end{aligned}$$

Inner Product

Example

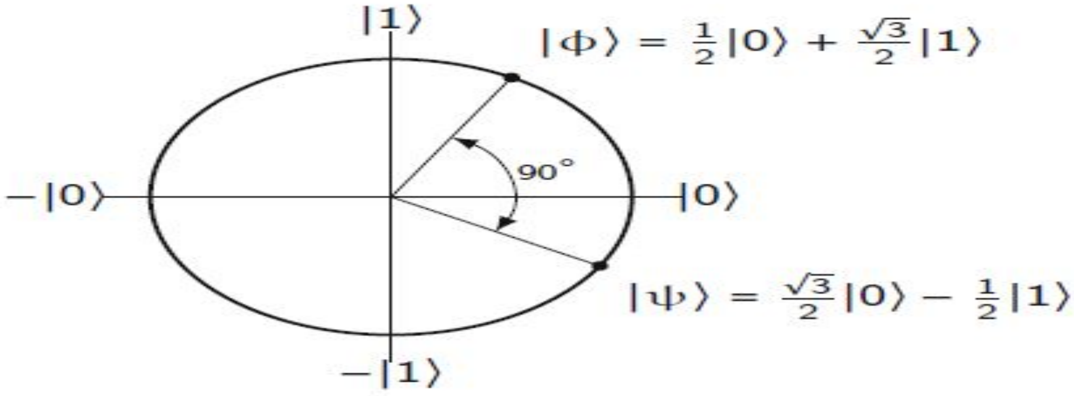


The inner product of these two vectors is

$$\langle\psi|\phi\rangle = \frac{1-\sqrt{3}}{2\sqrt{2}} = \cos(105^\circ) \approx -0.2588$$

Inner Product

Example



The inner product of these two vectors is

$$\langle\psi|\phi\rangle = 0 = \cos(90^\circ)$$

Inner Product

Relationship to the Euclidean norm

The inner product of any vector

$$|\psi\rangle = \sum_{a\in\Sigma} \alpha_a |a\rangle$$

with itself is

$$\langle\psi|\psi\rangle = \sum_{a\in\Sigma} \overline{\alpha_a}\alpha_a = \sum_{a\in\Sigma} |\alpha_a|^2 = \| |\psi\rangle \|^2$$

That is, the Euclidean norm of a vector $|\psi\rangle$ is given by

$$\| |\psi\rangle \| = \sqrt{\langle\psi|\psi\rangle}$$

Inner Product

Conjugate symmetry

For any two vectors

$$|\psi\rangle = \sum_{a\in\Sigma} \alpha_a |a\rangle \quad \text{and} \quad |\phi\rangle = \sum_{b\in\Sigma} \beta_b |b\rangle$$

we have

$$\langle\psi|\phi\rangle = \sum_{a\in\Sigma} \overline{\alpha_a}\beta_a \quad \text{and} \quad \langle\phi|\psi\rangle = \sum_{a\in\Sigma} \overline{\beta_a}\alpha_a$$

and therefore

$$\overline{\langle\psi|\phi\rangle} = \langle\phi|\psi\rangle$$

Inner Product

Linearity in the second argument

Suppose that $|\psi\rangle$, $|\phi_1\rangle$, and $|\phi_2\rangle$ are vectors and α_1 and α_2 are complex numbers. If we define a new vector

$$|\phi\rangle = \alpha_1|\phi_1\rangle + \alpha_2|\phi_2\rangle$$

then

$$\langle\psi|\phi\rangle = \langle\psi|\big(\alpha_1|\phi_1\rangle + \alpha_2|\phi_2\rangle\big) = \alpha_1\langle\psi|\phi_1\rangle + \alpha_2\langle\psi|\phi_2\rangle$$

Inner Product

Conjugate linearity in the first argument

Suppose that $|\psi_1\rangle$, $|\psi_2\rangle$, and $|\phi\rangle$ are vectors and β_1 and β_2 are complex numbers. If we define a new vector

$$|\psi\rangle = \beta_1|\psi_1\rangle + \beta_2|\psi_2\rangle$$

then

$$\langle\psi|\phi\rangle = \left(\overline{\beta_1}\langle\psi_1| + \overline{\beta_2}\langle\psi_2|\right)|\phi\rangle = \overline{\beta_1}\langle\psi_1|\phi\rangle + \overline{\beta_2}\langle\psi_2|\phi\rangle$$

Inner Product

Two vectors $|\psi\rangle$ and $|\phi\rangle$ are *orthogonal* if their inner product is zero:

$$\langle\psi|\phi\rangle = 0$$

An *orthogonal set* $\{|\psi_1\rangle, \ldots, |\psi_m\rangle\}$ is one where all pairs pairs are orthogonal:

$$\langle\psi_j|\psi_k\rangle = 0 \qquad \text{(for all } j \neq k)$$

An *orthonormal set* $\{|\psi_1\rangle, \ldots, |\psi_m\rangle\}$ is an orthogonal set of unit vectors:

$$\langle\psi_j|\psi_k\rangle = \begin{cases} 1 & j = k \\ 0 & j \neq k \end{cases} \qquad \text{(for all } j \neq k)$$

An *orthonormal basis* $\{|\psi_1\rangle, \ldots, |\psi_m\rangle\}$ is an orthonormal set that forms a basis (of a given space).

Data vs Algorithms

Quantum machine learning

CC: Classical ML (e.g. Deep learning)
CQ: ML on classical data using a quantum computer/algorithm
QC: Classical machine learning on quantum data
QQ: Quantum data + Quantum algorithm

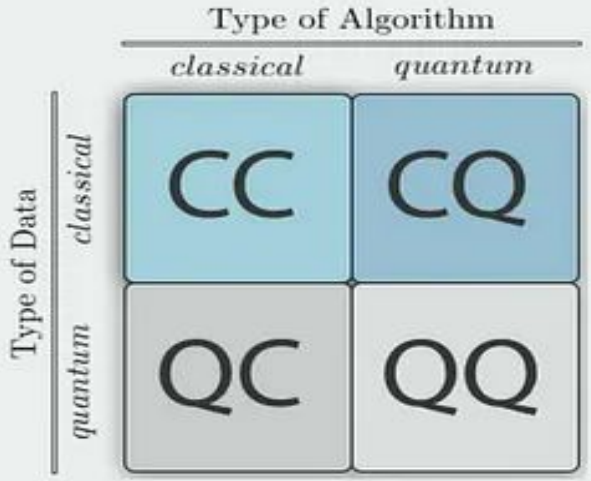


Image: Maria Schuld

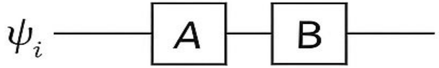
Data vs Algorithms

Quantum machine learning explores two primary approaches:

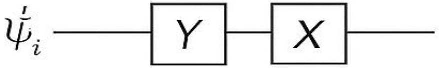
1. **Classical Data with Quantum Algorithms (CQ)**: In this scenario, classical data, such as images of cats and dogs, is processed using quantum algorithms. This approach is known as CQ and holds promise in handling classical data more efficiently through quantum computation.
2. **Quantum Data with Quantum Systems (QQ):** Quantum machine learning can also work with native quantum data or systems. Quantum algorithms are applied directly to data or quantum states, and this branch of quantum machine learning, labeled QQ, explores the possibilities of leveraging quantum-native information.

Quantum Gates in Series

1. Let A & B be two single-qubit quantum gates



$\rightarrow$ $\boxed{BA}$ $\rightarrow$ $BA\,|\psi|i$



$$XY|0\rangle = iZ|0\rangle = i|0\rangle$$

$$XY|1\rangle = iZ|1\rangle = -i|1\rangle$$

Note:

Identity gate $|I|$ = No Operation Gate (NOP Gate)

$\psi_i = y_i$ ——— i

$\psi_i =$ ——— (bare line)

Quantum Gates in parallel

Concept: Applying gates to **different qubits** in the same time-step.

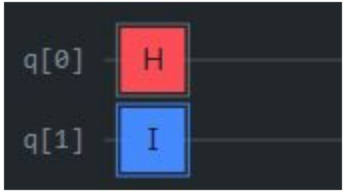
System State: The state of a multi-qubit system is described by the **tensor product** of individual qubit states (e.g., $|0\rangle \otimes |0\rangle = |00\rangle$).

Mathematical Rule: The combined gate is the **tensor product (⊗)** of the individual gates.

Example:

q_0: $|\psi_0\rangle$ ---[U_1]---

q_1: $|\psi_1\rangle$ ---[U_2]---



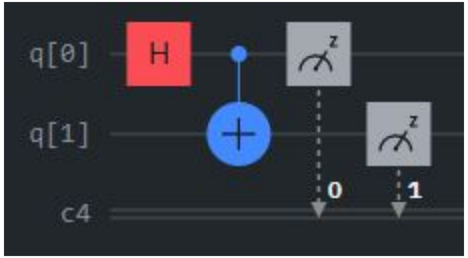
$$U\_out = (U_1 \otimes U_2) \quad |\psi_0 \ \psi_1\rangle = (H\otimes I)|\psi_0 \ \psi_1\rangle$$

q[0]=0,and q[1]=1 then ψ out = (H⊗I)|01⟩ = 1/√2[[1,1],[1,-1]]⊗[[1,0],[0,1]] = U

U = [[1,0,1,0],[0,1,0,1],[1,0,-1,0],[0,1,0,-1]] |01⟩

Quantum Gates in series & parallel

- **Goal:** Create the $|\Phi^+\rangle$ Bell State: $1/\sqrt{2}\ (|00\rangle + |11\rangle)$.
- **Circuit:** H on q_0, then a CNOT gate. U_total = CNOT * $(H \otimes I)$.
- **The CNOT Matrix:** Flips the target qubit if the control is $|1\rangle$.

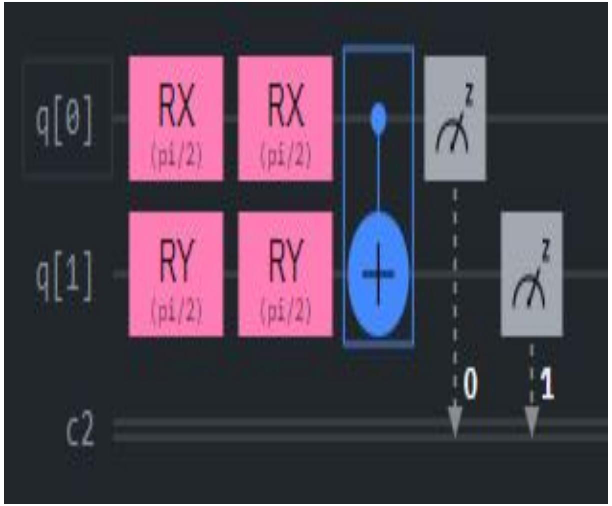


U_total = CNOT * $(H \otimes I)$ $|q_0\ q_1\rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} = 1/\sqrt{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix} |q_0\ q_1\rangle$$

Parameterized Quantum circuit

- A **Quantum Neural Network** is essentially a **Parameterized Quantum Circuit (PQC)**.
- The design of this circuit—the arrangement of gates—is its **architecture**.
- **Gates in Series (Depth):**
    - Create "layers" in the QNN.
    - Deeper circuits can perform more complex transformations, increasing the model's **expressibility**.
    - Think of them as sequential feature extraction steps.
- **Gates in Parallel (Width & Entanglement):**
    - Single-qubit gates in parallel process multiple features simultaneously.
    - **Entangling gates** (like CNOT) are crucial. They create complex correlations between features that are very difficult for classical networks to learn.
    - This ability to efficiently create high-dimensional correlations is a potential source of **quantum advantage**.



Introduction to IBM Qiskit

Qiskit (*Quantum Information Software Kit*) is an open-source, Python-based, high-performance software stack for quantum computing, originally developed by IBM Research and first released in 2017. It provides tools for creating quantum programs (by defining quantum circuits and operations) and executing them on quantum computers or classical simulators. The name "Qiskit" refers broadly to a collection of quantum software tools. It is centered around the core Qiskit SDK, and combined with a suite of tools and services for quantum computation, like the Qiskit Runtime service that enables optimized computations through the cloud. Qiskit allows users to write quantum circuits and execute them on real quantum processors (such as superconducting qubit systems) or on various other compatible quantum devices.

Why Qiskit?

- **Build Circuits:** Visually and programmatically create quantum circuits.
- **Simulate:** Test your circuits on powerful classical simulators.
- **Execute on Real Hardware:** Run your code on IBM's cloud-based quantum computers for free.
- **Rich Ecosystem:** Includes tools for specific applications like chemistry, optimization, and machine learning.

Thank You