Quantum Machine Learning and its Applications

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Postulates of Quantum mechanics

Postulate-1: The state of quantum system is described by unit vector.

State of a system : $|\psi\rangle \in$ H (complex linear vector space)

1. Notation : Bra-ket Notation

$$|\psi\rangle = \sum_i \alpha_i |\mathbf{i}\rangle$$

2. Matrix Representation

$$|\psi\rangle = \mathbf{[[v1],[v2],....[vd]]_{dx1}}$$

3. Inner product : $\langle\varphi|\psi\rangle$ or $<\psi|\varphi>$
4. Outer Product: $|\psi\rangle\langle\psi|$

Postulates of Quantum mechanics

Postulate-2: Physical quantities - observables

Linear transformation operators:

1. Unitary operators:

   $U^{\dagger}U=I=UU^{\dagger}$

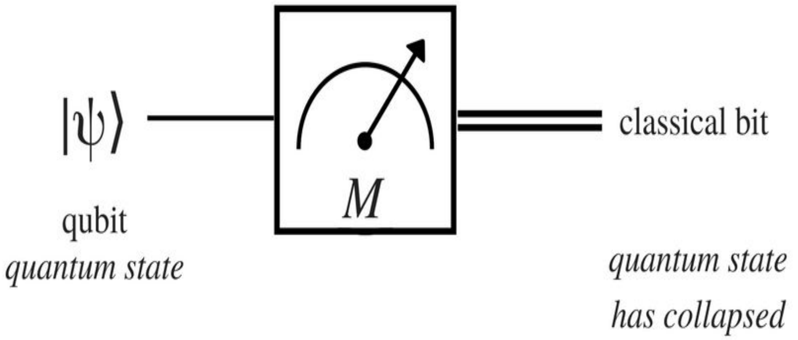
2. Hermitian operators:

   $H=H^{\dagger}$

Measurements

In Quantum Mechanics, Quantum Measurement is the process of extracting the classical Information from the quantum states.

- Collapse of **superposition**
- **Probabilistic** outcome
- Impact on the system
- **Irreversible process**



Postulates of Measurements

The Core Principle: The Measurement Postulate

Quantum measurements are described by a collection {Mm} of measurement operators.

1.The probability that result m occurs is

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle$$

2. The state of the system after the measurement is

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}}$$

3. Completeness Equation $\sum_m M_m^\dagger M_m = I$ **(or)** $1 = \sum_m p(m) = \sum_m \langle\psi|M_m^\dagger M_m|\psi\rangle$

Measurement of a Qubit

A qubit's state $|\psi\rangle$ is a **unit vector** in a 2-dimensional complex vector space called a Hilbert space, C^2 .

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where α,β are complex numbers called amplitudes. The condition $|\alpha|^2 + |\beta|^2 = 1$

These Projectors are: a) $M = P^\dagger$ (Hermitian) b). $P^2 = p$ c). $\sum P_i = I$

Eigenvalues and Eigenvectors of Z:

- Eigenvalue $m_0 = +1$ corresponds to the eigenvector $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. This is often interpreted as "spin-up."
- Eigenvalue $m_1 = -1$ corresponds to the eigenvector $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. This is often interpreted as "spin-down."

The Projectors:

- Projector onto the $|0\rangle$ state: $P_0 = |0\rangle\langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$.
- Projector onto the $|1\rangle$ state: $P_1 = |1\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

Measurement of a Qubit

1. Probabilities:

Probability of measuring 0 (outcome +1 for Z):

$p(0) = |\langle 0|\psi\rangle|^2$

$\langle 0|\psi\rangle = \langle 0|(\alpha|0\rangle + \beta|1\rangle) = \alpha\langle 0|0\rangle + \beta\langle 0|1\rangle = \alpha(1) + \beta(0) = \alpha$

Therefore, $p(0) = |\alpha|^2$.

Probability of measuring 1 (outcome -1 for Z):

$p(1) = |\langle 1|\psi\rangle|^2$

$\langle 1|\psi\rangle = \langle 1|(\alpha|0\rangle + \beta|1\rangle) = \alpha\langle 1|0\rangle + \beta\langle 1|1\rangle = \alpha(0) + \beta(1) = \beta$

Therefore, $p(1) = |\beta|^2$.

2. Post-Measurement state:

If the outcome is 0:

The qubit collapses to the state $|0\rangle$. Formally:

$|\psi'\rangle = \frac{P_0|\psi\rangle}{\sqrt{p(0)}} = \frac{(|0\rangle\langle 0|)(\alpha|0\rangle+\beta|1\rangle)}{|\alpha|} = \frac{\alpha|0\rangle\langle 0|0\rangle+\beta|0\rangle\langle 0|1\rangle}{|\alpha|} = \frac{\alpha|0\rangle}{|\alpha|}$

This state is just $|0\rangle$ with a phase factor. The qubit is now definitively in the state **|0⟩**.

If the outcome is 1:

The qubit collapses to the state $|1\rangle$. Formally:

$|\psi'\rangle = \frac{P_1|\psi\rangle}{\sqrt{p(1)}} = \frac{(|1\rangle\langle 1|)(\alpha|0\rangle+\beta|1\rangle)}{|\beta|} = \frac{\beta|1\rangle}{|\beta|}$

The qubit is now definitively in the state **|1⟩**.

Projective Measurements

A projective measurement is described by an observable, M, a Hermitian operator on the state space of the system being observed.

$P_m = |\psi_m\rangle\langle\psi_m|$ $\qquad$ $P_1 + P_2 + \ldots\ldots\ldots\ldots + P_m = I$

the probability of getting result m is given that

$$p(m) = \langle\psi|P_m|\psi\rangle$$

Given that outcome m occurred, the state of the quantum system immediately after the measurement is

$$\frac{P_m|\psi\rangle}{\sqrt{p(m)}}.$$

In addition to Measurements postulates. It also satisfying the completeness relation $\sum_m P^\dagger{}_m P_m = I$, also satisfy the conditions that

P_m are orthogonal projectors, that is, the M_m are Hermitian, and $P_m P_{m'} = \delta_{m,m'} P_m$.

POVM

Suppose a measurement described by measurement operators M_m is performed upon a quantum system in the state $|\psi>$. Then the probability of outcome m is given by $p(m) = <\psi|M^\dagger{}_m M_m|\psi>$. Suppose we define

$$\mathbf{E_m = M^\dagger{}_m M_m.}$$

E_m is a positive operator such that m $\sum E_m = I$ and $p(m) = <\psi|E_m|\psi>$. The complete set $\{E_m\}$ is known as a POVM.

As an example of a POVM, consider a projective measurement described by measurement operators Pm, where the Pm are projectors such that $P_m P_{m'} = \delta_{mm'} P_m$ and $\sum_m P_m = I$. In this instance (and only this instance) all the POVM elements are the same as the measurement operators themselves, since $E_m = P^\dagger{}_m P_m = P_m$.

Quantum channels

The evolution of the state is then given by: $\rho_{out} = \mathcal{E}(\rho_{in}) = \sum_i K_i \rho_{in} K_i^\dagger$

This framework is guaranteed to be "Completely Positive and Trace-Preserving" (CPTP).

Interpretation: The Physical Meaning of the Result

$$\rho_{in} = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \xrightarrow{\text{Z-measurement channel}} \rho_{out} = \begin{pmatrix} \rho_{00} & 0 \\ 0 & \rho_{11} \end{pmatrix}$$

- It preserves the diagonal elements.
- It completely destroys the off-diagonal elements (the coherences).

State Tomography

Quantum state tomography is :

- The **unknown state** ρ is the 3D object.
- The **measurements in different bases (X, Y, Z)** are the 2D slices from different angles.
- The **many identical copies** of the state are needed because each measurement "slice" destroys the copy it was performed on.
- The **reconstruction algorithm** is the computer that stitches the measurement statistics together to produce the final density matrix.

Reconstruct: Use these expectation values to build the density matrix: $\rho = \frac{1}{2}(I + \langle X \rangle X + \langle Y \rangle Y + \langle Z \rangle Z)$
Where Pauli Operators are: $r_x = \langle X \rangle, r_y = \langle Y \rangle, r_z = \langle Z \rangle.$

$$\rho \equiv \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

- The probability of obtaining result m is P(m) = tr(M_mρ).
- The Post-Measurement state is

$$|\psi_i^m\rangle = \frac{M_m|\psi_i\rangle}{\sqrt{\langle\psi_i|M_m^\dagger M_m|\psi_i\rangle}}$$

Role of Unitary Matrices in QML

1: Quantum Gates

2: The Data Encoding Circuit (Feature Map)

3: The Trainable Model Circuit

4: The Entire Algorithm as a Single Unitary

Role of Hermitian Matrices in QML

1: Predictions

2: Describing the State of the System (Density Matrices)

3: Model Generator: $U=e^{-i\theta H}$.

4: Feature Mapping (Encoder)

Thank You