

Thriving in Time's Embrace: Lampreys Adapt Sex Ratios for Mutual Benefits

Summary

Adaptive sex ratio variations have significant and complex feedback effects on the ecosystem. It not only helps species to maintain population during ecological imbalances, but also increases the ecosystem stability. To study the interactions between lampreys and outer environment under the circumstance of sex ratio alteration, we establish **Eco-Adaptive Interaction Model** and analyze the effects of gender-adaptive regulation of resource availability, food chains and food webs, interspecific competition, parasitic relationships on the ecosystem.

We first model the relationship between sex variations within the lamprey population. The model constructs sex-altering ratios through resource-sensitive **Time-Lag Equations** and applies the ratios to lamprey female and male populations. By modifying the logistic model, we obtained an Eco-Adaptive Model in which **lamprey's environmental capacity changes** comprehensively with population size, sex ratio, and resource amount.

To investigate the interaction of lampreys with the ecosystem, we used an **Interspecific Interaction Matrix** to describe their relationships such as competition, predation, and parasitism. Based on the Lotka-Volterra and Leslie models, we established an Eco-Adaptive Interaction Model that considers sex ratio. As a case study, we tested for interspecific mutualism and favoritism and found that the level of intrinsic facilitation of both species was positively correlated with the level of model convergence.

Meanwhile, we established a **Susceptibility-Infection-Recovery differential equation** model for lampreys to parasites and other infectious diseases, and observed that the sex-change survival strategy of lampreys is conducive to the spread of symbiotic parasites.

We observed that lamprey sexual differentiation strategies enhance ecosystem stability through **negative feedback regulation**. For there are significant differences between male and female individuals of lampreys, this differentiation strategy of lampreys can reduce their environmental biomass within a certain period of time, thereby improving resource recovery capabilities.

We examined the ecosystem stability by indicators such as biomass and resources, and obtained stable regression results through multiple ecological disturbance experiments. We used the log-response-ratio Stability (Irr.S) index to show that lampreys **increased the stability of the current ecosystem by 26.4%**.

Keywords: Negative Feedback; Differential Equation; LLR Examination; Interaction Matrix

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1 Introduction

1.1 Background

Lamprey is an ancient and primary animal surviving till now, which is due to its adaptability, derived from its sex ratio variation. Its adaptability sex ratio variation under limitation of resources can benefit the overall ecosystem. While Fisher's principle predicts a 1:1 sex ratio in offspring when raising males and females entail equal costs and benefits, environmental factors can disrupt this balance [1][7]. Insufficient food supply results in a higher proportion of males, reaching 78%. Freshwater lampreys feed only on plankton while sea lampreys suck blood for living. **The weight of a female lamprey is about 4 times as that of a male one**[5]. Lampreys play the role of both predator and parasite in ecosystem. Thus, exploring the relationship between changes in the sex ratio of the lamprey population and local conditions is essential to gain insight of interspecies interactions in ecosystems.

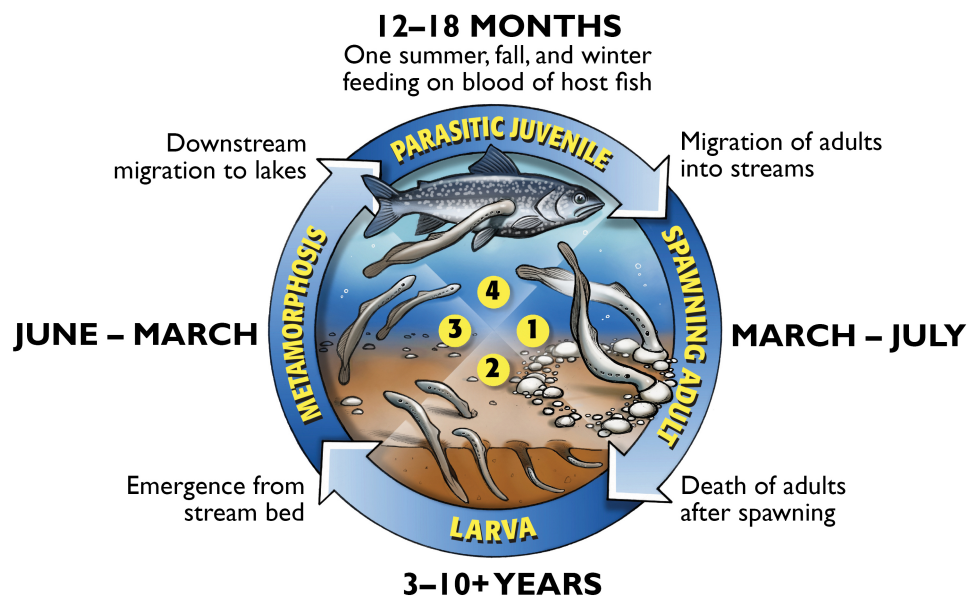


Figure 1: At-sea feeding ecology of parasitic lampreys (Image source: Great Lakes Fishery Commission.)

1.2 Restatement of the problems

Considering the background information and conditions identified in the problem, we need to address the issues below:

- Establish a differential equations model to analyze the impact of adaptive sex ratio variations in lampreys on the overall ecological system.
- Explore the superiority and inferiority of the elastic sex ratio within the population of lampreys.

- Analyze the impact on ecosystem stability based on variations in the sex ratio of lampreys especially when some disturbance happen.
- Examine whether changes and in the sex ratio of lampreys are beneficial to other species in their ecosystem, such as parasites.

1.3 Our Work

In this paper, we established **3** mathematical models progressively and solve **4** problems, which is shown in figure 2.

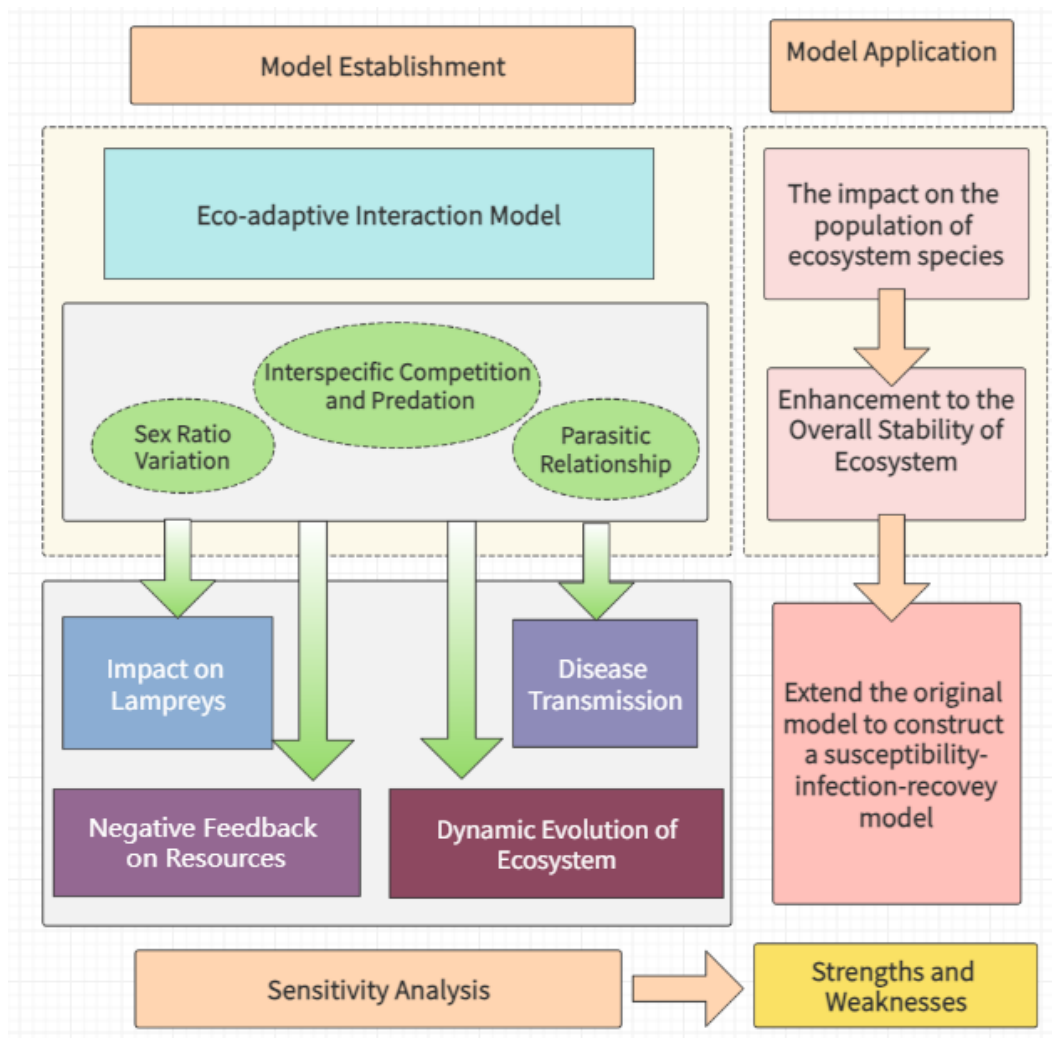


Figure 2: Flow Chart of Our Work

In detail, we have done the following:

- Establish a model of population changes of each species in the ecosystem. The model is resource sensitive and takes into account sex differences, changes in the male-to-female ratio

of lampreys and the population of each sex. We use an updated Lotka-Volterra differential equation model to describe the impact of lamprey sexual variation on the entire ecosystem.

- The quantitative effects of interspecific relationships of different organisms (e.g. competition, predation, and parasitism) on ecosystems are discussed. We use an interspecific interaction matrix to describe various forms of interactions between organisms, and their productivity fluctuates periodically, depicting real ecosystems.
- Describe the negative feedback regulatory effects of sexual differentiation on resources in lampreys. Since male lampreys are overall smaller, their non-differentiation into females can reduce resource consumption and increase the production of target resources within a certain period of time, thereby maintaining ecosystem stability in the long term.
- Establish an evaluation index of ecosystem stability. We use log response ratio to model Stability(Irr.S). Further, its value was tested for an ecosystem containing lampreys with variable sex ratios. We conducted multiple perturbation experiments on the original model to test the impact of different perturbations.
- Establish a susceptibility-infection-recovery model of lamprey-transmitted parasites. Based on the living habits of sea lampreys, we studied their strategies for transmitting parasites. The results we obtained show that, sexual differentiation strategy of sea lamprey enhances the transmission of parasites to a certain extent.

2 Assumptions and Justification

To simplify the problems and make it more convenient for us to simulate the situation, we make the following basic assumptions and each of them is properly justified.

- **Assumption 1: The biomass mentioned is the average biomass, and the seasonal impact on the population of lampreys is not significant.**
Justification: The volatility of the data is relatively small, and the seasonal impact is not significant. Obtaining average biomass is easier than obtaining real-time or periodic biomass data.
- **Assumption 2: Neglecting the internal differences between male and female individuals, it is assumed that lampreys of the same sex have similar body weight and size.**
Justification: There are differences in body types within each sex, but to simplify the model and reduce unnecessary complexity, we can ignore individual differences and use average data for simulation.
- **Assumption 3: Assuming that the lifespan of individual lampreys is a relatively fixed value.**
Justification: The lifespan of individual lampreys is relatively stable in the short term. In this case, it is a reasonable approximation to assume that the lifespan of individual lampreys is relatively fixed in the model.

- **Assumption 4: Different species compete for the same resource.**

Justification: By making such assumptions, different resources such as light, land, energy, humus, etc. can be abstracted into a single resource using a computational method.

- **Assumption 5: This model only considers the flow of energy, and does not take into account limiting effects such as material flow.**

Justification: We first hope to establish a more basic model to study the material flow cycle relationship between lampreys and the environment. However, data on other elements such as nitrogen are difficult to obtain, and considering them in the model is not conducive to research and simulation.

- **Assumption 6: The maximum resource carrying capacity of the environment is directly proportional to resources, without considering marginal effects.**

Justification: We believe that the impact of increased resources on population growth is mainly limited by the biological characteristics of the population, so the marginal effects involved are relatively small and can be ignored. Moreover, considering marginal effects may introduce more variables and nonlinear relationships, making the model more complex.

3 Notations

The primary notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbols	Definitions
A	producers in the ecological chain, like angae
L	current biomass of lampreys
B	current biomass of secondary consumers who feed on lampreys, like sea lions
M	parasites on lampreys, like myxoma
γ_i	the natural growth rate of i
K_i	biomass under maximum environmental carrying capacity
α_i	the coefficient related to the growth impact on i
R_i	female differentiation under the resource condition level of i
γ_1	Lagrangian interpolation of R_1 and R_2 , product of γ_0
μ	a weight coefficient decided by R
L_1	current biomass of female lampreys
L_2	current biomass of male lampreys
$\{k_{ij}\}_{i,j}$	the coefficients of competition/predation matrix

4 Eco-Adaptive Interaction Model

4.1 Model Overview

Lampreys have diverse interactions with their environment, and play variable roles in the ecosystem. Considering that lampreys alter their sex ratio to change their species' consumption

of environmental resources, we build a differential equations model of their interaction with the ecosystem. Later, for sea lamprey lives a parasitic life and often has the function of transmitting parasites and other pathogens, we establish an infection model to describe this phenomenon, which is illustrated in figure 3.

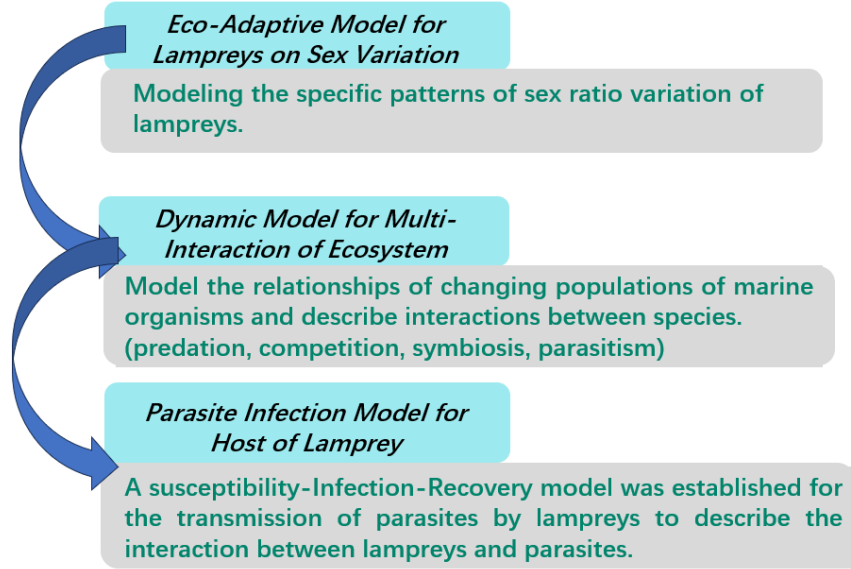


Figure 3: Model Establishment Process

4.2 Eco-Adaptive Model for Lampreys on Sex Variation

In this section, we build a model of the lamprey population under certain resource conditions. Under resource constraints, we define $L(t)$ as the function representing the biomass of lamprey. According to the logistic model, we have,

$$\frac{dL}{dt} = \gamma_1 L \left(1 - \frac{L}{K_1} \right) \quad (1)$$

where r_i is the natural growth rate(birth rate - death rate), K_i is the capacity of biomass of lampreys in the current environment.

However, **Logistic Model** only takes intraspecific competition into account. It ignores sex ratio variation as well as other factors such as sex differentialization, interspecies relations. Thus, it is necessary to adjust the equation to the issues we concern.

4.2.1 Natural Growth for Variable Sex Ratio

According to the research on lamprey sizes[4], sex ratio variation can have a linear relation to the resource they occur. We apply Lagrange's interpolation to the birth rate of lampreys. We give:

$$\gamma_1 = \gamma_0 * R = \gamma_0 \left(\frac{A - K_0}{0 - K_A} R_1 + \frac{A - 0}{K_A - 0} R_2 \right)$$

$$K_1 = K_1(A) = a' A$$

where γ_1 stands for the sex birth rate of lampreys, $R = f(A, t)$ represents the ratio of females among the offspring born at time t . According to Assumption 3, we hold the view that the death rate of lamprey can be estimated by the birth rate before some time T , so we estimate the sex ratio variation,

$$R' = S(k_1 R_t - k_2 R_{t-T}) \quad (2)$$

We get $S = \min(R, 1 - R)$ as the success chance of lamprey breeding. Observing that male lamprey population is usually more than that of female, the equation above can be degraded to $S = R$.

4.2.2 Model Evolution for Male and Female Lampreys

We noticed a huge difference of figure between female and male lampreys[5]. Thus it is necessary to model the population or biomass for both male and female lampreys separately. It can be written as,

$$\begin{cases} \frac{dL_1}{dt} = \gamma_0 R' L_1 \left(1 - \frac{[\mu * (L_1 + L_2)]}{K_1} \right) \\ \frac{dL_2}{dt} = \gamma_0 (1 - R') L_1 \left(1 - \frac{[\mu * (L_1 + L_2)]}{K_1} \right), R \in \left[0, \frac{1}{2} \right] \end{cases} \quad (3)$$

where L_1 represents the overall population of female lampreys, and similarly L_2 represents that of male's.

We introduce μ to bridge the gap between population and biomass, which is in direct proportion to the rate of the actual lamprey biomass by the biomass assuming that the population of lamprey is filled of female ones. We get the average female lamprey biomass over male ones is equations4. Therefore, μ can be determined by:

$$\begin{cases} \mu = C_0 R + (1 - R) \\ C_0 = 4 \end{cases} \quad (4)$$

4.3 Dynamic System for Multi-Interaction of Ecosystem

4.3.1 Modified Lotka-Volterra Equation

First, we assume there are 3 specific species in an simplified ecosystem, which are algae, lamprey and sea lions. To reflex their relations, we show out the food chain on the left of figure 4 and build their differential equations, also called dynamical systems, formed by **predation relations**. This is our **Modified Lotka-Volterra Equation**.

As is shown below, the first and third equations of the dynamic system(1) of equations are the classical Lotka-Volterra equations for the dynamic systems of the prey population, like algae, and the dynamics of the number of predator populations, like sea lions, whereas in the second equation we describe the dynamics of the predator population, lampreys, where γ_1 is the rate of growth of the predator population, but it is computed by means of a function related to the prey population A , which is an extension of the original model.

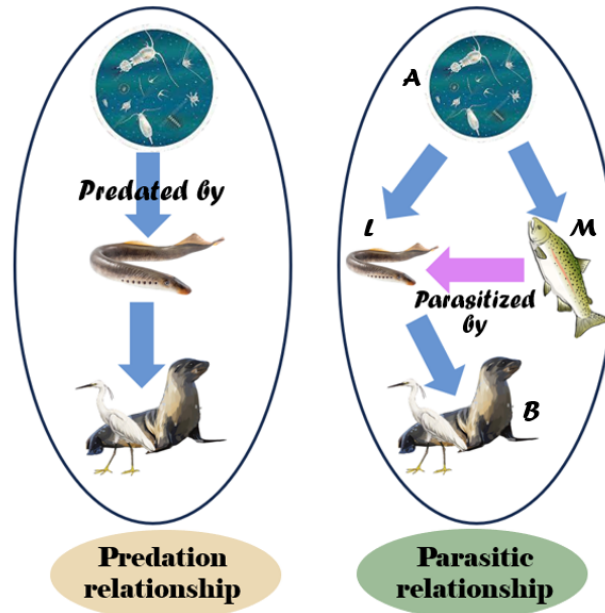


Figure 4: Food Chain and the One Mixed with Parasitic Relation. We test the right food net in Section 5.

$$\begin{cases} \frac{dA}{dt} = \gamma_A A \left(1 - \frac{A}{K_A} - \alpha_A L \right) \\ \frac{dL}{dt} = \gamma_1 L \left(1 - \frac{L}{K_1} - \alpha_1 B \right) \\ \frac{dB}{dt} = \gamma_2 B (\alpha_2 L - \alpha_3 B - 1) \end{cases} \quad (5)$$

in which A represents producers in the ecological chain, such as some plankton. L represents primary consumers, in this case we mean lampreys which can feed on plankton. B represents secondary consumers, in this case we refer to those animals that can feed on lampreys, for example egrets and sea lions.

4.3.2 Integrated with Sex Ratio Alteration condition

Second, expected to show the sex ratio alteration effect to refine the model, we combine equations with (5) together. We get the dynamic system as follows:

$$\left\{ \begin{array}{l} \frac{dA}{dt} = \gamma_A A \left(1 - \frac{A}{K_A} - \alpha_A [\mu * (L_1 + L_2)] \right) \\ \frac{dL_1}{dt} = \gamma_0 R' L_1 \left(1 - \frac{[\mu * (L_1 + L_2)]}{K_1} - \alpha_1 B \right) \\ \frac{dL_2}{dt} = \gamma_0 (1 - R') L_1 \left(1 - \frac{[\mu * (L_1 + L_2)]}{K_1} - \alpha_1 B \right), R \in \left[0, \frac{1}{2} \right] \\ \frac{dB}{dt} = \gamma_2 B (\alpha_2 ([\mu * (L_1 + L_2)]) - \alpha_3 B - 1) \end{array} \right. \quad (6)$$

4.3.3 Integrated with Parasitic Coupled System Model

Third, to consider parasitic behavior of adult lampreys, we include another type of species like salmon, which lampreys can parasitic on, shown up rightly in figure 4. Therefore, involved with **Parasitic Coupled System Model** between lampreys and salmon.

We get a parasitic model(7) from paper [6]. in which p refers to the number of parasitic organisms, h refers to the number of hosts, and K refers to the levels of immobilization facilitation. How we determine the inter-species relationship is illustrated in the figure 5.

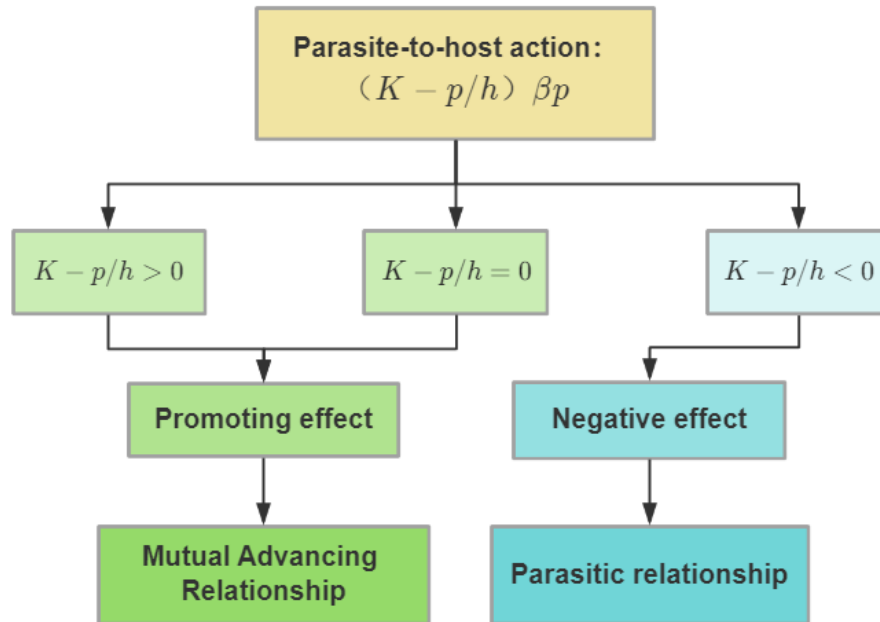


Figure 5: Parasite to host relationship

- (i) The densities of hosts and parasites, denoted by $-a_1$ and $-a_2 \frac{p}{h}$ respectively, are limited by their densities. Increased density may reduce the actual population growth rate.

- (ii) When the ratio $\frac{p}{h}$ of parasitoid to host population approaches 0, the facilitation effect of parasitoids on hosts is maximized, which is represented by the intrinsic facilitation level K .
- (iii) Parasite impact on the host depends on their density. If the ratio $\frac{p}{h}$ remains below the level of K , parasites facilitate host population size; if it exceeds K , parasites inhibit the host.
- (iv) The growth rate of the host population is inversely proportional to $\frac{p}{h}$.

$$\begin{cases} \frac{dh}{dt} = h \left(r_1 - \alpha_1 h + \left(K - \frac{p}{h} \right) \beta p \right) \equiv f_1(h, p) \\ \frac{dp}{dt} = p \left(r_2 - \alpha_2 \frac{p}{h} \right) \equiv f_2(h, p) \end{cases} \quad (7)$$

Obviously model (3) is a nonlinear system, and in order to study the dynamic evolutionary behavior of the parasites and hosts within the system, it needs to be analyzed for stability with the help of nonlinear dynamical system theory.

Now we expect to simplifying the format of dynamic systems by writing in matrix and vector form. Here it is:

Let $\vec{X} = (\vec{A}, L, B, M)$. The system of equations listed above can be simplified with **Interspecific In- teraction Matrix Equations** :

$$\frac{d\mathbf{X}}{dt} = [(\mathbf{1} + \mathbf{X}) + \alpha_0 + \mathbf{f}] \odot (\mathbf{H}_0 \mathbf{X}) \quad (8)$$

or:

$$\frac{d\ln \mathbf{X}}{dt} = [(\mathbf{H}\mathbf{X}) + \alpha_0 + \mathbf{f}] \odot \vec{\gamma} \quad (9)$$

where \odot stands for Hamilton vector multiplication, and in which:

$$\begin{cases} \alpha_0 = (1, 1, 1, 1)^T \\ \vec{f} = \left(0, \gamma_L \left(K - \frac{M}{L} \right) \beta M, 0, \alpha_2 \frac{M^2}{L} \right)^T \\ \vec{\gamma} = (\gamma_A, \gamma_1, \gamma_2, 1)^T \end{cases}$$

The matrices involved are:

$$\mathbf{H}_0 = \begin{bmatrix} \gamma_A & 0 & 0 & 0 \\ 0 & \gamma_1 & 0 & 0 \\ 0 & 0 & \gamma_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} -\frac{1}{K_A} & -k_{12} & -k_{13} & 0 \\ k_{21} & -\frac{1}{K_1} & -k_{23} & 0 \\ -k_{31} & -k_{32} & -\frac{1}{K_2} & 0 \\ 0 & 0 & 0 & \alpha_M \end{bmatrix}$$

And, we would combine equations (8) with Sex Ratio Alteration condition, which means we have to substitute the second equation in (8) with the following equations (8), then we get **Eco-Adaptive Interaction Model** successfully.

$$\begin{cases} \frac{dL_1}{dt} = \gamma_L R' L_1 \left(1 - k_{21} A - \frac{[\mu * (L_1 + L_2)]}{K_1} - k_{23} B \right) - h \left(K - \frac{M}{L} \right) \beta M \\ \frac{dL_2}{dt} = \gamma_L R' L_1 \left(1 - k_{21} A - \frac{[\mu * (L_1 + L_2)]}{K_1} - k_{23} B \right) - h \left(K - \frac{M}{L} \right) \beta M \end{cases} \quad (10)$$

Speaking from a biological perspective, we address the dynamic nature of the climate and the uncertainties surrounding drought cycles and severity by incorporating random variables into our modeling approach. Climate change is viewed as a stochastic process, and the simulation of climate fluctuations relies on randomly generated numbers from a random number generator. Additionally, the duration and severity of droughts follow a specific probability distribution.

In this study, we employ an exponential distribution model to simulate the likelihood of drought events. The exponential distribution characterizes the probability distribution of time intervals between events in a Poisson point process—a process where events occur continuously and independently at a constant average rate. The parameter λ in the exponential distribution represents the frequency of drought events. A higher λ corresponds to increased drought event frequency, while a lower λ indicates lower frequency and magnitude of drought events. Therefore, the adjustment of λ allows us to modulate the frequency of drought events in our modeling approach. To evaluate stability of an ecosystem, we give the following Index which indicate diversity and sensitiveness of the certain ecosystem.

4.3.4 Shannon Index

The Shannon Index[9], also known as Shannon entropy or Shannon-Wiener index, is a measure of biodiversity or uncertainty in information theory. In ecology, it is commonly used to quantify the diversity of species in a community. The Shannon Index (H') is calculated using the formula:

$$H' = - \sum_{i=1}^S p_i \cdot \log_2(p_i)$$

4.3.5 Long-Term Stability Response(S) and LRR

In ecological studies, understanding the long-term stability responses of a system is crucial for assessing its resilience and dynamics over extended periods. In this context, two key metrics often employed are denoted as S and $lrr.S$. [9]

We determined long-term temporal stability, S , as $\frac{\mu}{\sigma}$, where μ is the average productivity of a plot across all years and σ is the temporal standard deviation in the productivity of that plot across all years. We calculated long-term stability responses as the natural logarithm of the ratio (log response ratio or lrr) of the long-term stability within each treatment plot divided by the average

long-term stability in the reference plots($lrr.S$), where $lrr.S = \ln(S)$.

$$lrr.S(t) = \frac{1}{T} \sum_{i=t-T}^{t-1} \log(lrr.S(i))$$

4.4 Parasite Infection Model for Host of Lamprey

We further discuss the impact on the ecosystem of sex ratio variation of lampreys. Sea lampreys are generally considered to be parasites that feed on the blood of their hosts[8]. In the process, the lampreys may help other small parasites enter their hosts, helping to spread the disease. For this consideration, we model this dynamic process of spreading the disease. We introduce S as the susceptible population of host by lampreys. Similarly, I as the infected and R as the recovered. To describe the population of parasites, we hold X . We introduce our **Susceptible-Infected-Recovered Relations Dynamic System**[6] as follows,

$$\begin{cases} \frac{dS}{dt} = \alpha \left(S + \eta \left(\frac{X}{I} \right) I + R \right) \left(1 - \frac{S + I + R}{K} \right) \\ \quad - \frac{\beta \left(\frac{X}{I} \right) SI}{S + I + R} \\ \frac{dI}{dt} = \frac{\beta \left(\frac{X}{I} \right) SI}{S + I + R} - \left(d + \delta \left(\frac{X}{I} \right) \right) I \\ \frac{dR}{dt} = \frac{\lambda Y I}{X} I \end{cases} \quad (11)$$

where X is the population of parasites. We will apply our model in the following section, and we will figure out the interaction between lampreys and the components of a specific ecosystem.

5 Application of Eco-Adaptive Interaction Model

5.1 Effect of the Alterable Sex Ratio on the Larger Environment

What is always present in the process of maintaining individual survival and population continuity is the interaction between organisms and the environment. When environmental resource conditions are poor, lampreys differentiate more into smaller, less resource-intensive males during their juvenile years. Thus the continuation of its population under that environmental condition can be maintained. Meanwhile, in order to study the effect of this sex ratio change of lamprey on the surrounding environment, we use the dynamic system with non-sex ratio variation and the one with it constructed in Section 3 to simulate the effect of changing sex ratio of lamprey on the environment in predatory and parasitic relationships, respectively.

First, before experiment with our two models, we would like to test our models first. On the one hand, We just experiment with Modified Lotka-Volterra Equation to verify whether it has stable point and its convergence. See figure 6. Therefore we can get out that our model is well defined and constructed after initial test. On the other hand, for our the dynamic system with non-sex ratio variation, we expect to set the sex ratio to a specific point first and do convergence analysis.

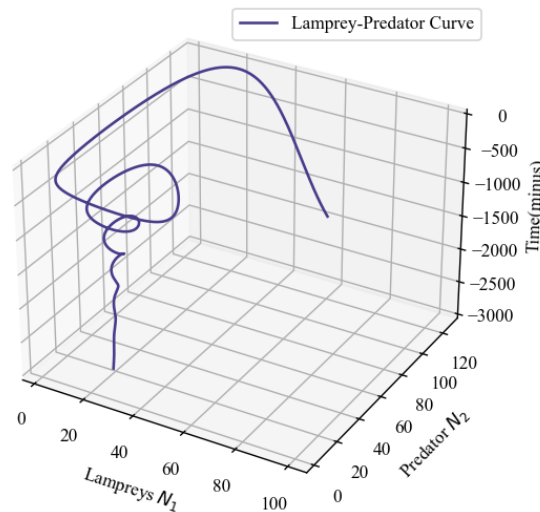
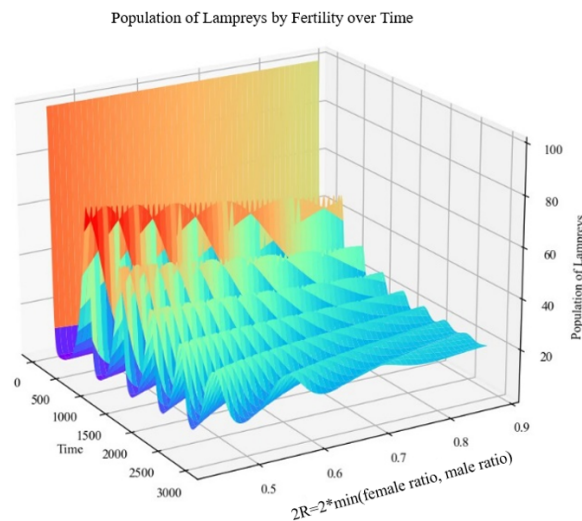


Figure 6: stable point and convergence

Then we can get figure 7. At this time, the fertility rate is supposed to equal to $\min\{\text{female rate, male rate}\}$, but in all cases, female rate is smaller, so we take $2 \times \text{female rate}$ into account. As we can see, $2R$ go up, the frequency of population as a function of time slows down, which means the population becomes more stable as female ratio: male ratio verge to 1. And the most important thing is that our Modified Lotka-Volterra Equation integrated with Sex ratio variation and parameters are well defined and established."

Figure 7: population of lampreys by $2 \times \text{female ratio}$

In the following, we will show the outcome of Modified Lotka-Volterra Equation with non-sex ratio variation and Eco-Adaptive Interaction Model.

From the two figures, we can see that after sufficient time, the results of both models tend to

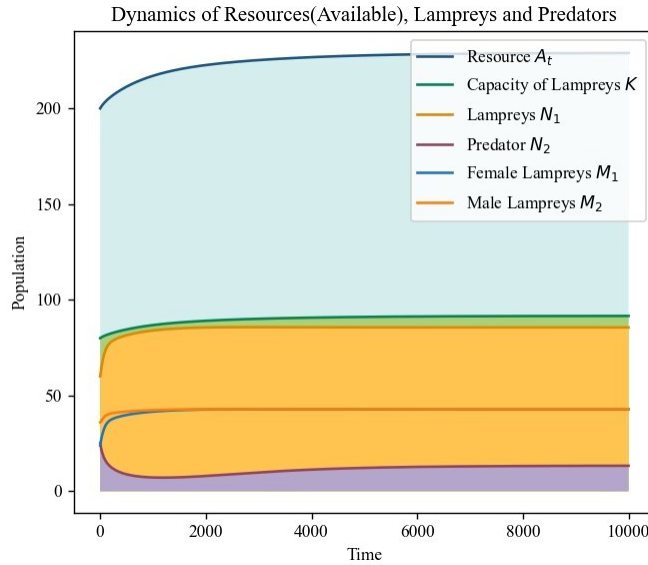


Figure 8: Modified Lotka-Volterra Equation result

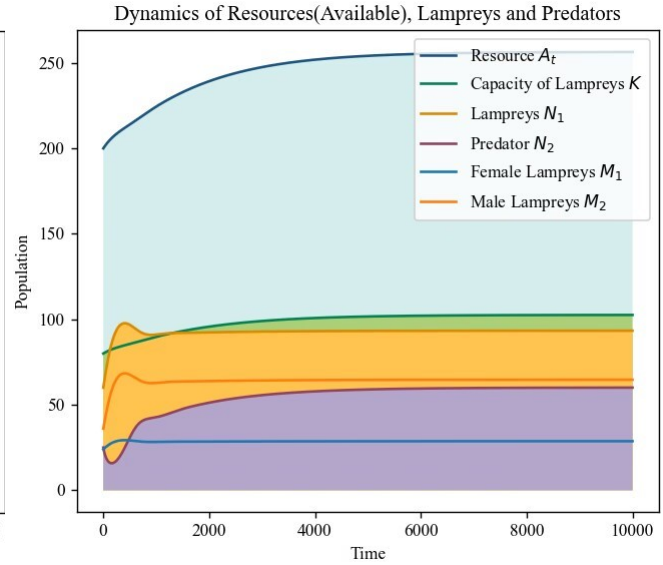


Figure 9: Modified Lotka-Volterra Equation integrated with sex ratio variation result

stabilize. We are considering analyzing the situation where $t > 4000$. Compared with the one with non-sex ratio variation, Take the ability of lampreys to change their gender with the environment into consideration, we can see a higher Resource Availability and a corresponding increase in Capacity of lampreys in the resulting in a stable proportion of females to the overall population. The results are shown in the table 2 below.

That means that considering only the predator-prey relationship (in food chain and food web) and the ability of lampreys to change their sex over time, the proportion of females in lampreys will converge to a constant value. Therefore, in order to obtain better Resources Availability, the best method to adapt to the environment is to find the stability sex ratio that is suitable for the environment.

Afterwards, we introduced the species parasitized by adult lampreys and the parasitic-host conjugate relationship between them, in order to consider the Eco-Adaptive Interaction Model based on the parasitic habits of lampreys. We can see from Figure(10) that the larger effect on host by parasites, the higher the population of lampreys is.

5.2 Superiority and Inferiority Analysis

By controlling the variable of fecundity, we successfully modeled the effects of lamprey populations on the regeneration of environmental resources.

Table 2: equations with sex ratio variation VS equations without it

	the one with non-sex ratio variation	the one with sex ratio variation
Resource Availability	233	257
Capacity of lampreys	83	103

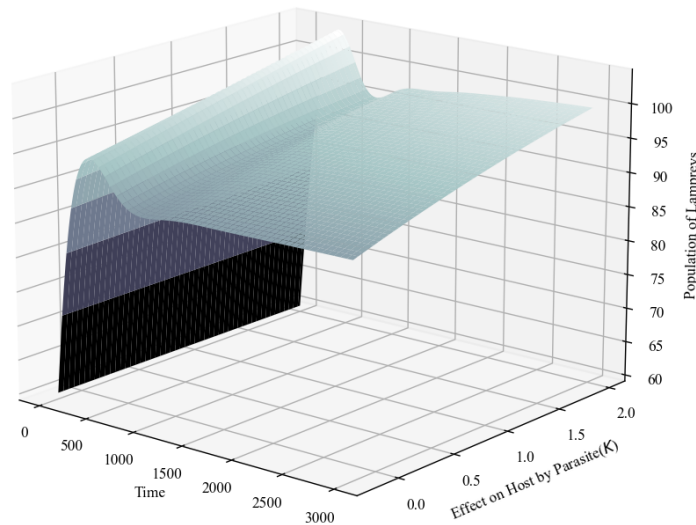


Figure 10: Eco-Adaptive Interaction Model

To exclude that it is some unwitnessed parameters' small changes subject to positive distribution that causes the outcome, We will do hypothesis testing that is a way to determine whether the hypothesis is valid in a specific probability.

Table 3: lampreys' number

	Mean	Variance
Sex-ratio Variable	93.309	0.001
Sex-ratio Non-variable	85.573	6.735

For lampreys' number, we can tell from the table 3 that compared to the dynamic system with non-sex ratio variation, Eco-Adaptive Interaction Model sample has a significantly large mean and remarkable smaller variance. Besides, in the Shapiro-Wilk test, the value of statistic acquired is close to 0.834. And since the p-value obtained (6.195×10^{-6}) is much less than 0.05, we can disprove the original hypothesis and conclude that there is a significant difference, that is 7.736, between the observed dataset and the normal distribution at the 0.05 level of significance.

Table 4: Resource Availability/algae's scale

	Mean	Variance
Sex-ratio Variable	255.791	0.379
Sex-ratio Non-variable	228.57	0.126

For the similarity, we repeat the Shapiro-Wilk test on the Resource Availability/algae's number and obtained the statistic (0.784) and p-value (3.879×10^{-7}) that again proves that the sample is not from a normal distribution.

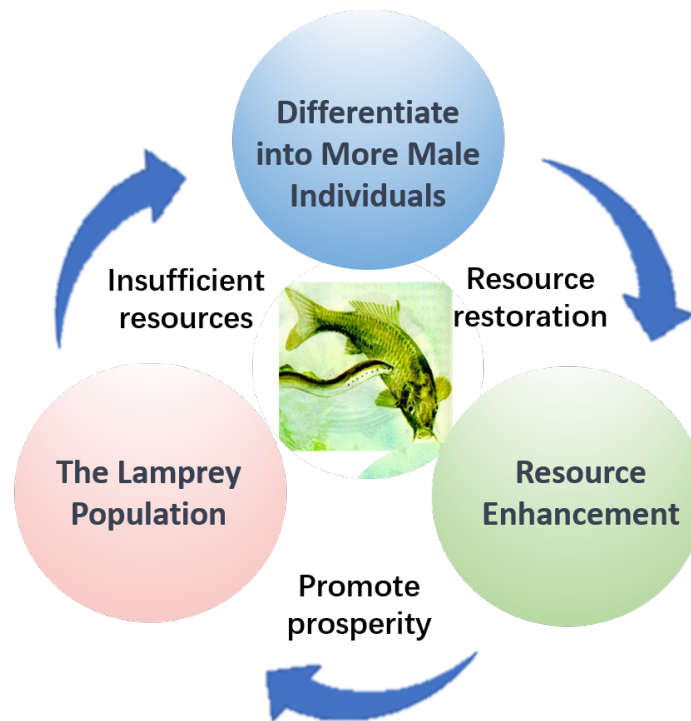


Figure 11: Feedback diagram between lampreys and resources

In conclusion, we can say that there is a huge advantage at the 0.05 level of significance, which is 7.736 more in population number super lower oscillation, and more resource availability. This is because ecosystems become more stable is not because of added parameters, nor because of chance, or called noise.

5.2.1 Superiority of Adaptive Sex Ratio Variation

Based on the model results, we can conclude that lampreys adjust their sex ratio according to environmental resource abundance. This adaptation is based on the differing resource consumption levels between sexes, enhancing the population's ability to adapt. The feedback circle is depicted in figure 11.

- **Ensuring population continuity:** In times of scarce food, the proportion of smaller, less resource-intensive males increases. Such adjustments **alleviate survival competition pressure** in its populations during resource scarcity.
- **Reduced environmental pressure:** This characteristic is also conducive to **the resource recovery**, improving conditions for future lamprey generations. Moreover, this adaptive sex ratio adjustment enhances the population's overall adaptability, enabling better survival and reproduction across diverse environments.
- **Fewer Intraspecific Competition:** Adjusting the sex ratio in response to resource availability can help reduce intraspecific competition within the population. This may lead to more efficient resource utilization and increased survival rates.

5.2.2 Inferiority of Adaptive Sex Ratio Variation

This resiliency in sex ratios can also have some drawbacks, as mentioned in figure 12.

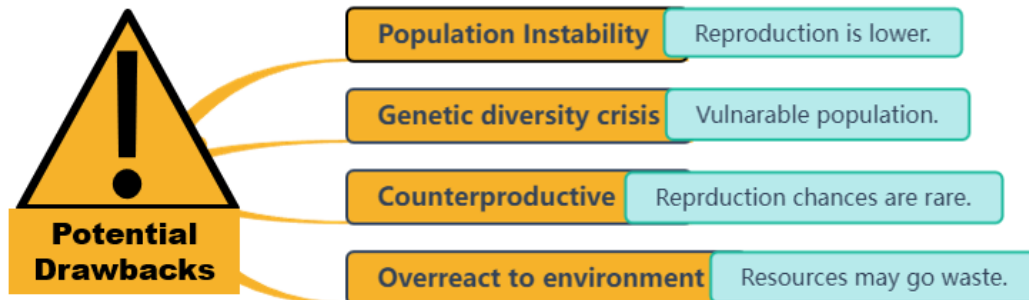


Figure 12: Drawbacks of adaptive sex ratio

- **Causing population instability:** In resource-poor environments, fewer females may lower the population's reproductive rate, affecting long-term reproductive success.
- **Genetic diversity crisis:** Genetically, sex transition in specific conditions may **reduce genetic diversity** within the lampreys population, particularly when resources are scarce. A rise in male numbers may diminish the gene pool's diversity, impacting the population's adaptability and viability.
- **Counterproductive:** An increase in male numbers in resource-scarce conditions may **spur competition for resources like reproduction chances**. Conversely, in resource-rich environments, a decline in male proportion is likely to lower reproductive rates.
- **Environmental Uncertainty:** There may be some uncertainty in predicting the regulation of sex transition by environmental change. If the rate or magnitude of environmental change exceeds the adaptive capacity of lamprey populations, **excessive losses or wasted resources** may result.

5.3 The Impact of Alterable Sex Ratio on the Eco-stabilization

5.3.1 Analysis with Shannon Index

We introduced two oscillations to simulate two events that cause negative impact on the environment, such as flooding. The purpose is to explore the effects of ecologically negative events on overall organisms, especially resources. Where dA denotes the differentiation on the resources and its value obeys $N(1.0, 0.82)$ normal distribution. τ denotes the duration of the disaster, which obeys $P(t, \lambda=0.02)$.

The change curve of population number and each index ,after we introduce the disturbance with the probability obeying exponential distribution,is shown in Figure 13 .

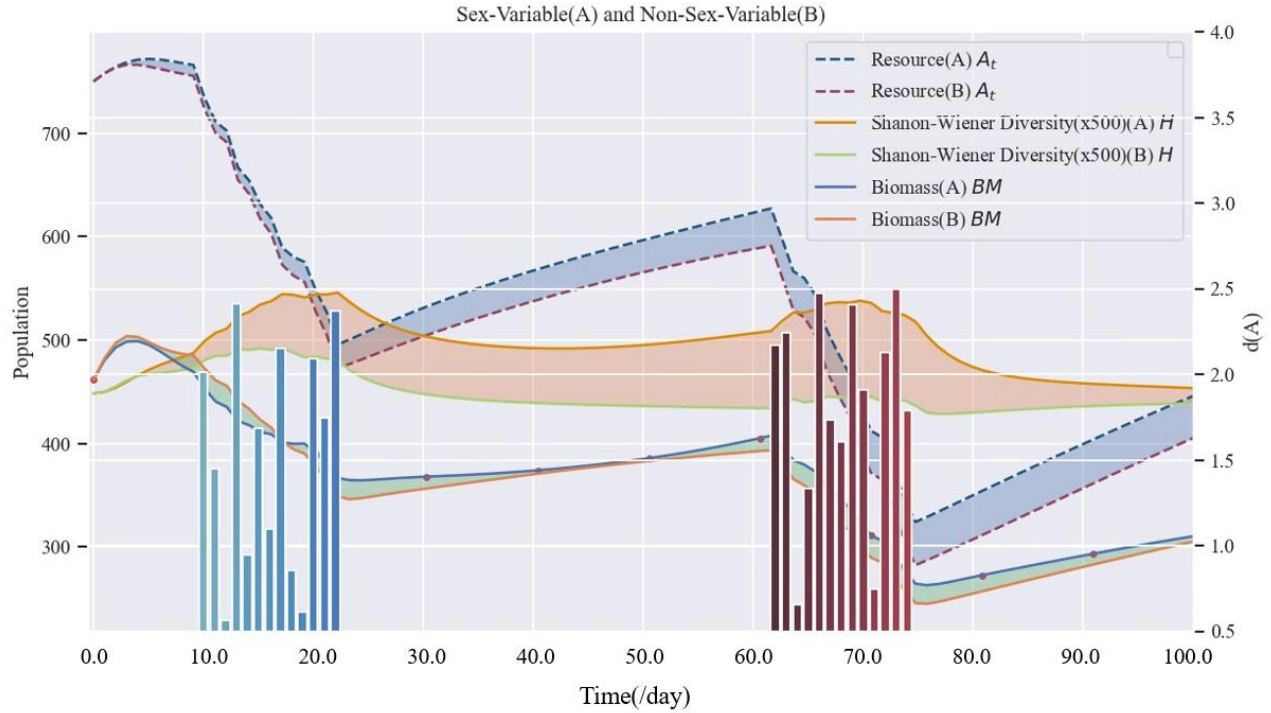


Figure 13: The population varies over time as disturbance happens occasionally with both Sex ratio change and that doesn't change.

Looking at the graph Figure 13, we can see that the blue and red bars represent two perturbations that affect the level of resource availability and the level of environmental diversity (the Shannon index) of both systems and reduce them. For resource availability, Shannon index, and biomass, the Sex-Variable curve is higher than the Non-Sex-Variable curve. **Therefore, we can obtain through qualitative analysis that given the changes in the sex ratios of lampreys, the stability of the ecosystem will be improved.**

5.3.2 Analysis with long-term temporal stability(S) and LRR Index

First, after experiencing the $i(i=1,2)$ disturbance, we can calculate the long-term temporal stability(S) when the sex ratio does not change and when it changes in the same time period, which need to be experimented a lot of times to get a relatively stable bar chart.

It is easy to say that the Y one (with Sex ratio variation) is more stable than X one because it always has higher value of long-term temporal stability(S). See figure 14.

Second, calculate the value of $lrr.S$, and then use them as the horizontal axis and the vertical axis respectively to make the corresponding scatter plot. From these points we can construct a linear regression, and we could see Figure 15.

From Figure 15, we can tell that $\frac{S_{sexChange}}{S_{nonSexChange}} = e^{lrr.S(y)-lrr.S(x)} \approx 1.2636$. Therefore we can tell that in terms of multiples of S, thus we conclude that ecosystem with sex-ratio variable lampreys is **26.36% more stable** than that with non-sex-variable lampreys.

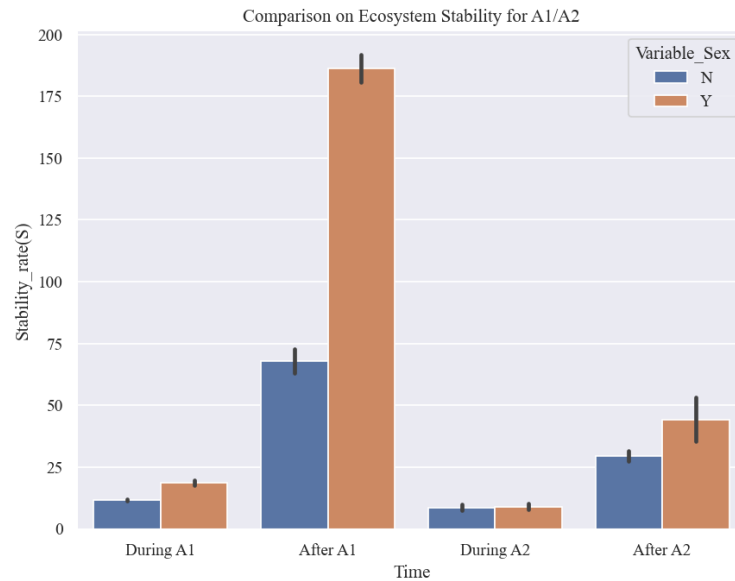


Figure 14: Long-term temporal stability(S)

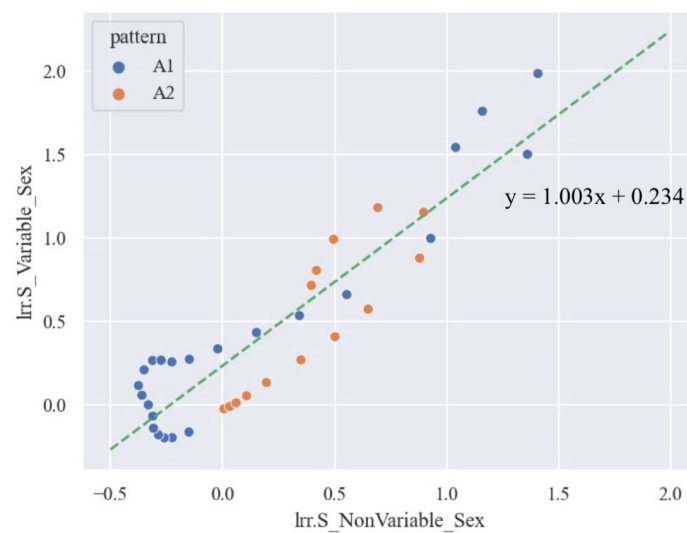


Figure 15: A linear regression for LRR. X axis stands for non-sex ratio variation. Y axis stand for sex ratio variation.

The conclusion obtained is that the variable sex ratio of lamprey contributes to ecosystem stability. Specifically it contributes to both biomass stability and biodiversity stability in the ecosystem.

5.4 Analysis effects of variable sex ratios of lampreys on parasites

Here, we consider the interspecies relationship based on the Eco-Adaptive Interaction Model as shown in Figure16. The parasite is in symbiotic relationship with lampreys and lives on the surface of the lampreys body or inside the lampreys body, and considering the parasitic habit of the lampreys, e.g., parasitizing the salmon, we consider that the parasite carries a contagious disease, and this disease can infect the salmon's decline in health and even die in a near future, instead of immediately, but not to infect the lampreys.

We can use (11) to analysis get the corresponding outcome. See Figure17.

First after a period of time, the numbers of each population in both the Eco-ADdaptive Dynamic Model (denoted E) and the dynamical system with non-sex ratio variation (denoted D) stabilize

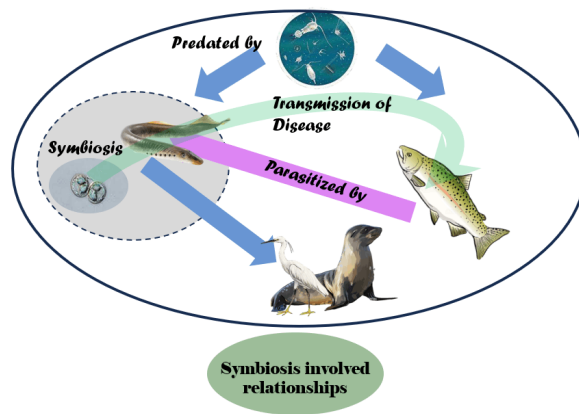


Figure 16: Eco-Adaptive Interaction Model integrated with parasites

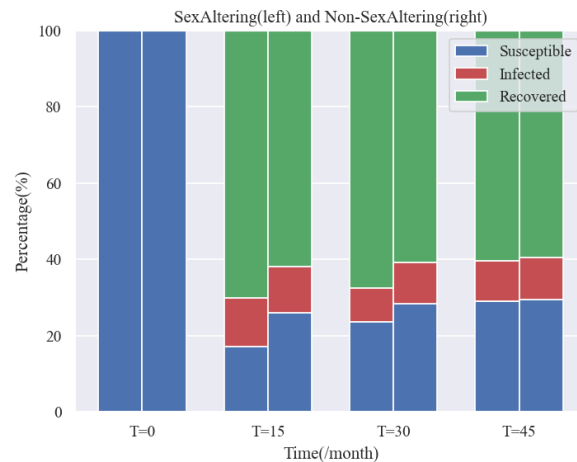


Figure 17: the proportion of each type of salmon

so that we only need to consider the percentage of disease states when we consider the effect of the infection model here. Then, we know from the figure that the number of infected people are basically unchanged, but compared with D E corresponding to the susceptibility ratio and resistance ratio increased slowly, which means that E reagent makes the parasite slow down the damage to the host salmons while maintaining the population size, thus can avoid the host dying too fast leading to the narrowing of the survival space, and get more chances of survival and survival time.

6 Sensitivity Analysis

When establishing the differential equation for the model, we added white noise to simulate the effect of unpredictable factors on the effect caused on the ecological system. The variance of white noise needs to be within a certain range.

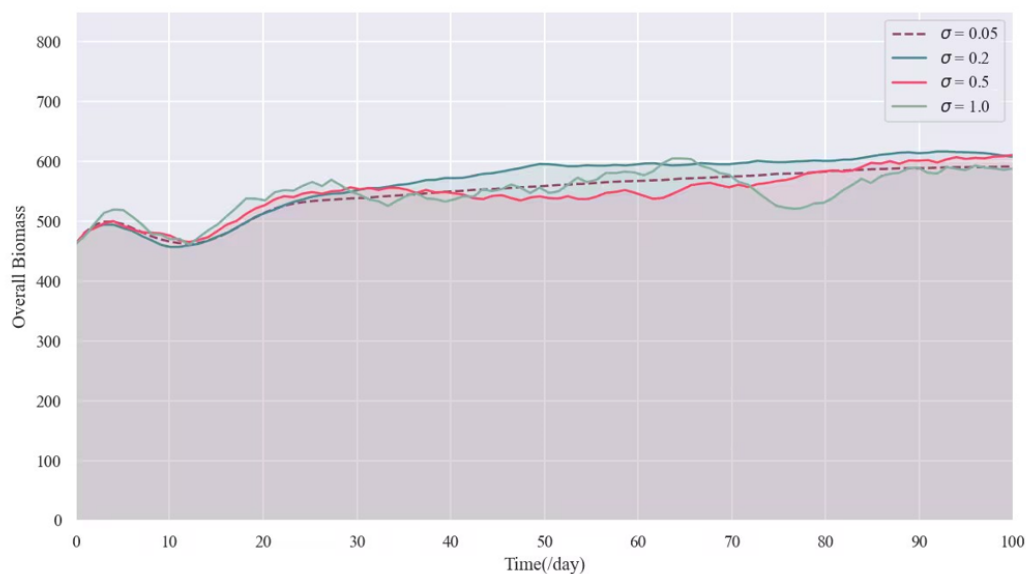


Figure 18: Sensitivity analysis under different white noise

We added the white noise that corresponds to a normal distribution $N(0, \sigma^2)$. Just as illustrated in figure 18, the variance of white noise has little impact on the overall ecosystem. Some accidental factors may cause a negative impact on the overall species in the ecosystem. That is why a more complicated model is needed to obtain better results.

7 Conclusion

- Through simulation, we can observe that after incorporating the effect of lampreys' adaptive sex ratio variation into the model, there is a higher resource availability and corresponding increase in the capacity of lampreys in the model. This leads to a stable proportion of females in the entire population.
- This variable sex ratio has a beneficial impact on the lamprey population. More differentiation into smaller-sized males can reduce resource consumption over a certain period of time and increase the production of target resources within specific time intervals, thereby maintaining

long-term stability of the ecosystem. However, it also has negative effects on the long-term stability and genetic diversity of the lamprey population.

- Through multiple disturbance experiments on the model, we obtained results showing that considering adaptive gender ratio changes, both the biomass and biodiversity stability of the ecological environment are better. The overall stability has improved by 26.4%.
- We established a susceptibility-infection-recovery model of lamprey-transmitted parasites and studied their strategies for parasite transmission. The results show that the gender differentiation strategy of sea lampreys enhances the transmission of parasites to a certain extent.

8 Strengths and Weaknesses

8.1 Strengths

- The model introduces sex ratio dynamics into the classic Lotka-Volterra model, allowing for the exploration of the effects of changing sex ratios on lamprey populations and the wider ecosystem.
- The model comprehensively considers the two states of the lamprey from larval to adult state, autotrophic life and parasitic life, thus assessing the ecological impacts of the lamprey population in both states.
- The model has the flexibility to simulate the population dynamics of lamprey as well as changes in sex ratio by adjusting the parameters in the model.

8.2 Weaknesses

- The model is oversimplified and only considers the flow of energy and resources, ignoring the limiting effects of other material elements such as nitrogen.
- The ecological flow dynamics model that only considers the population size of the species is considered, while ignoring the effects of migration of populations in different areas.
- Our model does not well represent the life history variation of lamprey. For example, lamprey larvae feed on submerged plants, while their adults can live in ecological niches such as scavenging, predation, and parasitism.

8.3 Directions for Improvement

- For weakness 1, this equation can be subsequently considered in the context of the various elemental constraints incorporated into this equation.
- For weaknesses 2 and 3, improved Lotka-Volterra models can be outlined for areas with different manifestations of ecological conditions that are characterized separately. The time-series modeling of in-migration and out-migration can also be incorporated into the calculations.

The model can be extended to become an ecological niche differentiation model, which can more accurately characterize the changes in their populations.

AI cite use [AI: 1]

References

- [1] Johnson, Nicholas S et al. "Field study suggests that sex determination in sea lamprey is directly influenced by larval growth rate." *Proceedings. Biological sciences* vol. 284,1851 (2017): 20170262.
- [2] Hardisty, M. W. "Sex composition of lamprey populations." *Nature* 191.4793 (1961): 1116-1117.
- [3] <https://fishbase.mnhn.fr/summary/Petromyzon-marinus.html>
- [4] Silva S, Servia M J, Vieira-Lanero R, et al. Downstream migration and hematophagous feeding of newly metamorphosed sea lampreys (*Petromyzon marinus* Linnaeus, 1758)[J]. *Hydrobiologia*, 2013, 700: 277-286.
- [5] Dawn P. Toledo.Introduced Species Summary Project Sea lamprey(*Petromyzon marinus*)[EB/OL].(2001-10-29)[2024-2-5].
- [6] Tadiri, Christina P., et al. "A data-validated host-parasite model for infectious disease outbreaks." *Frontiers in Ecology and Evolution* 7 (2019): 307.
- [7] Quintella, B. R., et al. "At-sea feeding ecology of parasitic lampreys." *Journal of Great Lakes Research* 47 (2021): S72-S89.
- [8] L Jones, Michael, et al. "An operating model for the integrated pest management of Great Lakes sea lampreys." *The Open Fish Science Journal* 2.1 (2009).
- [9] Hautier, Yann, et al. "Anthropogenic environmental changes affect ecosystem stability via biodiversity." *Science* 348.6232 (2015): 336-340.

Report on Use of AI

1. OpenAI Enie (Nov 5, 2023 version, Ernie 4.0)

Query1: <Can an ecosystem with variable sex ratios in the lamprey population offer advantages to others in the ecosystem, such as parasites?>

Output: <The sex ratio in a population can have various ecological implications, and it is possible that variable sex ratios in lamprey populations could influence interactions within an ecosystem. However, it's important to note that the specifics of such interactions can be complex and context-dependent.

Lampreys are parasitic or hematophagous fish known for attaching themselves to other fish to feed on their blood and tissues. If there are variations in the sex ratios of lampreys within an ecosystem, it could potentially affect their overall abundance and behavior, which may, in turn, have cascading effects on other species, including parasites.

Here are a few hypothetical scenarios to consider:

Population Dynamics: If the sex ratio of lampreys influences their population size, it could affect the availability of hosts for parasitic lampreys. Higher or lower lamprey abundance might impact the populations of their host fish, which could indirectly affect other species in the ecosystem.

Host-Parasite Relationships: Variable sex ratios could influence the success of lamprey parasitism on host fish. For instance, if there are more female lampreys, they might be more successful in finding suitable hosts, potentially impacting the health of the host populations.

Competition with Other Species: The presence of lampreys and their variable sex ratios could also affect other species that share the same ecological niche or resources. This could include competition with other parasites for hosts or interactions with predators and competitors.

Adaptations by Parasites: Parasites in the ecosystem may evolve strategies based on the variations in lamprey sex ratios. For example, if there are more female lampreys, parasites might evolve to exploit the specific vulnerabilities associated with female lamprey behavior or physiology.

It's crucial to consider the specific ecological context, the characteristics of the lamprey species involved, and the overall dynamics of the ecosystem when assessing the potential advantages or disadvantages for other species, including parasites. Ecological relationships are intricate and can be influenced by a multitude of factors. Research in this area is ongoing, and scientists continue to explore the complexities of interspecies interactions within ecosystems.>

I hope this helps!