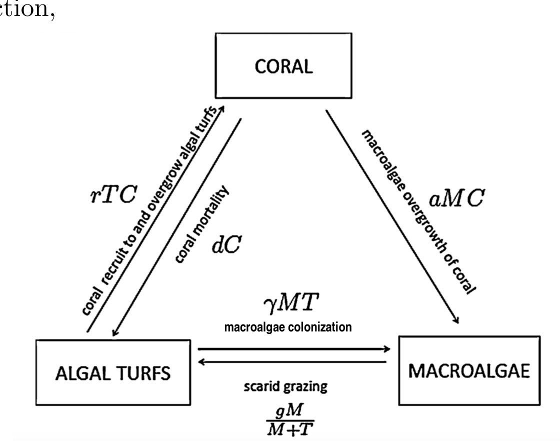
Coral reef ecosystems support a plethora of diverse organisms in the colorful calcium carbonate accretions of the stony coral reef builder and provide a source of rare biomedical and industrial substrates. Of the many organisms in this ecosystem, the growth of coral reef organisms is observed to be dependent upon a variety of species of algae.

This complex interaction between coral reefs and algae exist with three morphological types of algae being seen as key: microalgae, macroalgae, and algal turf. Microalgae serve as primary producers of O_2, simple carbohydrates, amino acids, and $CaCO_3$ for the coral in a symbiotic relationship where the microalgae are provided a habitat (Dietrich, 1985).

Macroalgae overgrow upon the coral leading to a state of decreased coral reef covering. Despite consumption of dead coral giving scarids their common name, parrotfishes, the main influence scarids have on coral reefs is realized by the grazing of scarids upon these macroalgae. After macroalgae are grazed, algal turf and coral organisms exist in a dynamic equilibrium until macroalgae grow back after a time delay of 6 to 12 months. Our model identifies two types of organisms belonging to the coral reef ecosystem as the most relevant organisms relating to the long term stability of a coral reef: macroalgae and algal turf. The interactions between these organisms can be described in the following directed graph, where the direction denotes which organism is benefited by the interaction,



We began by studying the following delay differential equation (DDE) model:

$$\begin{cases} \frac{dM}{dt} = aMC - \frac{gM(t-\tau)}{1-C(t-\tau)} + \gamma M(1-M-C), \\ \frac{dC}{dt} = rC(1-M-C) - dC - aMC, \end{cases}$$

where $\tau \geq 0$ is a fixed time delay observed in the growth of algal turf upon the grazing of macroalgae; r is the rate corals overgrow upon algal turfs; d is the mortality rate of corals; a is the rate that macroalgae overgrow upon corals; γ is the rate that macroalgae spread over algal turfs, and; g is the indiscriminate grazing rate of scarid, and we also assume $a < d < \gamma < r < 2\gamma$ and $0 < g < \gamma$ [**?**].

Notice that algal turf (i.e. the third element of the ecosystem being considered) is given by $1 - C - M$, as we assume that $M + C + T = 1$, given that $\frac{dM}{dt} + \frac{dC}{dt} + \frac{dT}{dt} = 0$. By analyzing the nullcline of this system, three equilibrium points are discovered: (N) $(0, 0)$; (B) $(0, 1 - \frac{g}{\gamma})$, and; (D) $(1 - \frac{d}{r}, 0)$, and in Li et. al, 2014, these equilibrium points were discovered to be unstable, stable, and stable, respectively.

We extended the DDE model to a Stochastic DDE Model (SDDE) by accounting for noise in scarid grazing of macroalgae by adding a noise term in the derivative of M,

$$\begin{cases} dM = (aMC - \frac{gM(t-\tau)}{1-C(t-\tau)+\gamma MT})dt + \beta M(1-M)dW, \\ dC = (rTC - dC - aMC)dt. \end{cases}$$

We denote the initial condition with θ in radians, such that $C = \sin(\theta)$ and $M = \cos(\theta)$. The impact of noise on the stability of the equilibrium points was studied using Monte Carlo simulations with varying values for $g \in [0.2, 0.8]$, $\tau \in [0.5, 1]$, $\beta \in [0, 1]$, and $\theta \in (0, \frac{\pi}{2}]$.