ROB6323: Reinforcement learning and optimal control for autonomous systems I

Exercise series 1

For questions requesting a written answer, please provide a detailed explanation and typeset answers (e.g. using LaTeX[1]). Include plots where requested in the answers (or in a Jupyter notebook where relevant). For questions requesting a software implementation, please provide your code in runnable Jupyter Notebook. Include comments explaining how the functions work and how the code should be run if necessary. Code that does not run out of the box will be considered invalid.

Exercise 1 [25 points]

Find all the minimum(s), if they exist, of the functions below. Characterize the type of minimum (global, local, strict, etc) and justify your answers with a mathematical argument.

- $-\mathrm{e}^{-(2x-1)^2}$, where $x \in \mathbb{R}$
- $(1-x)^2 + 50(2y - x^2 + 5)^2$, where $x, y \in \mathbb{R}$
- $10x + x^2 + y - 4y^2$, where $x, y \in \mathbb{R}$
- $x^T \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} x + \begin{bmatrix} -1 & 1 \end{bmatrix} x$, where $x \in \mathbb{R}^2$
- $x^T \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} x + \begin{bmatrix} 10 & 1 \end{bmatrix} x$, where $x \in \mathbb{R}^2$
- $\frac{1}{2} x^T \begin{bmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} x - \begin{bmatrix} 0 & 0 & 4 \end{bmatrix} x$, where $x \in \mathbb{R}^3$

Exercise 2 [25 points]

Consider the following optimization problem

$$\min_{\mathbf{x}} \frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x} \tag{1}$$

$$\text{subject to } \mathbf{A}\mathbf{x} = \mathbf{b} \tag{2}$$

where $\mathbf{Q} \in \mathbb{R}^{n \times n} > 0$ and $\mathbf{A} \in \mathbb{R}^{m \times n}$ is full rank with $m < n$ and $\mathbf{b} \in \mathbb{R}^m$ is an arbitrary vector.

- Write the Lagrangian of the optimization problem as well as the first order optimality conditions (KKT conditions)

[1] 1https://en.wikibooks.org/wiki/LaTeX, NYU provides access to Overleaf to all the community https://www.overleaf.com/edu/nyu

- Solve the KKT system and find the optimal Lagrange multipliers as a function of $\mathbf{Q}$, $\mathbf{A}$ and $\mathbf{b}$.
- Use the above results to compute the minimum of the function below (and the value of $\mathbf{x}$ and of the Lagrange multipliers)

$$\frac{1}{2}\mathbf{x}^T \begin{bmatrix} 100 & 2 & 1 \\ 2 & 10 & 3 \\ 1 & 3 & 1 \end{bmatrix} \mathbf{x} \tag{3}$$

  under the constraint that the sum of the components of the vector $\mathbf{x} \in \mathbb{R}^3$ should be equal to 1. Verify that the constraint is indeed satisfied for your result. (Hint: use python for all your numerical computation.)[2]

Exercise 3 [50 points]

Answer the questions in Jupyter notebook *Exercise_3.ipynb*

[2] in particular the numpy function solve `https://docs.scipy.org/doc/numpy/reference/generated/numpy.linalg.solve.html` is generally more robust and efficient when computing the solution of $Ax = b$ that taking the inverse of A explicitly.