Drawbacks of Modus Ponen

$P \rightarrow Q$

$P(x) \rightarrow Q(x)$

$Q(x) \rightarrow S(x)$

$S(A)$ is valid

$\neg P(x) \rightarrow Q(x)$

$P(x) \rightarrow \neg Q(x)$

$\neg P(x) \rightarrow \neg Q(x)$

Not a sound mechanism

PREDICATE CALCULUS

Resolution in FOL

Basic Steps

- Convert the set of rules and facts into clause form (conjunction of clauses/disjunction of clauses)
- Insert the negation of the goal as another clause
- Use resolution to deduce a refutation

CNF
DNF

If a refutation is obtained, then the goal can be deduced from the set of facts and rules.

Resolution

- $C1 \wedge C2 \wedge \ldots\ldots \wedge Ck \rightarrow G$
- $\neg(C1 \wedge C2 \wedge \ldots\ldots \wedge Ck) \vee G$ is valid
- $C1 \wedge C2 \wedge \ldots\ldots \wedge Ck \wedge \neg G$ is unsatisfiable

All men are mamals

All mamals drink milk

Tom is a man

To Prove:
Tom Drinks milk
drinkmilk(Tom)

$\forall x : man(x) \Rightarrow mamal(x)$

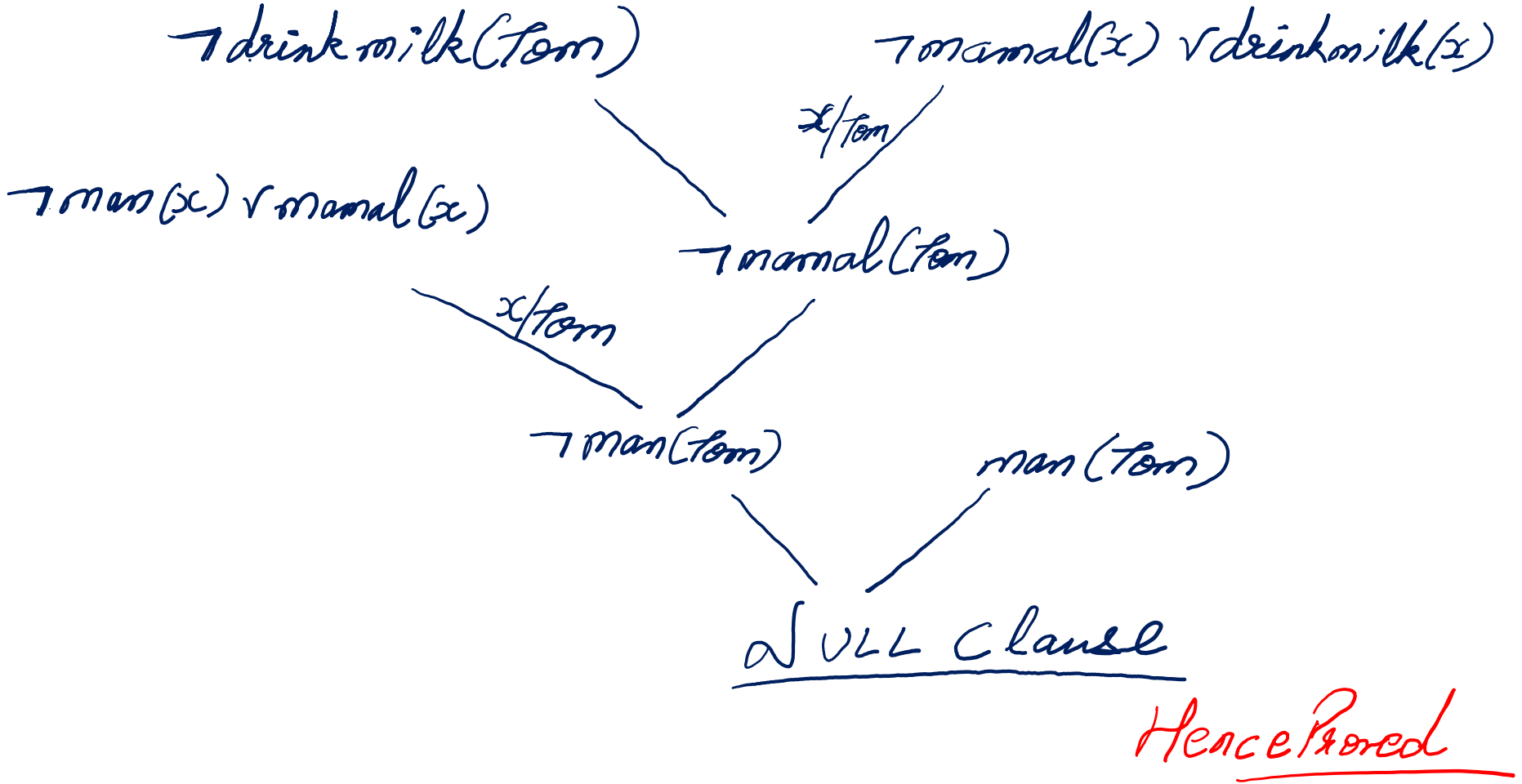
① $\neg man(x) \vee mamal(x)$

$\forall x : mamal(x) \rightarrow drinkmilk(x)$

② $\neg mamal(x) \vee drinkmilk(x)$

③ $man(Tom)$

④ $\neg drinkmilk(Tom)$



Hence Proved

Examples

- Harry, Ron and Draco are students of the Hogwarts school for wizards
- Every student is either wicked or is a good Quiditch player, or both
- No Quiditch player likes rain and all wicked students like potions
- Draco dislikes whatever Harry likes and likes whatever Harry dislikes
- Draco likes rain and potions
- Is there a Student who is good in Quiditch but not in potions?

Rules and Facts

- Harry, Ron and Draco are students of the Hogwarts school for wizards

1. Student (Harry)
2. Student (Ron)
3. Student (Draco)

Facts and Rules

- Every student is either wicked or is a good Quiditch player, or both

$\forall x: student(x) \rightarrow wicked(x) \vee quidditch(x)$

(4) $\neg student(x) \vee wicked(x) \vee quidditch(x)$

Facts and Rules

- (No Quiditch player likes rain) and (all wicked students like potions)

$\forall x: quiditch(x) \rightarrow \neg likes(x, rain)$

5. $\neg quiditch(x) \vee likes(x, rain)$

$\forall x: wicked(x) \rightarrow likes(x, potion)$

6. $\neg wicked(x) \vee likes(x, potion)$

Facts and Rules

- (Draco dislikes whatever Harry likes) and (likes whatever Harry dislikes)

$\forall x: likes(Harry, x) \rightarrow \neg likes(Draco, x)$

7. $\neg likes(Harry, x) \vee \neg likes(Draco, x)$

$\forall x: \neg likes(Harry, x) \rightarrow likes(Draco, x)$

8. $likes(Harry, x) \vee likes(Draco, x)$

Facts and Rules

- Draco likes rain and potions

9. likes (Draco, rain)
10. likes (Draco, potion)

To Prove

- Is there a Student who is good in Quiditch but not in potions?

$quiditch(x) \wedge \neg likes(x, potion)$

negation

11. $\neg quiditch(x) \vee likes(x, potion)$

1. Student(Harry)
2. Student(Ron)
3. Student(Draco)
4. $\neg$student(x) $\vee$ wicked(x) $\vee$ quiditch(x)
5. $\neg$quiditch(x) $\vee$ $\neg$likes(x,rain)
6. $\neg$wicked(x) $\vee$ likes(x,potion)
7. $\neg$likes(harry,x) $\vee$ $\neg$likes(draco,x)
8. likes(harry,x) $\vee$ likes(draco,x)
9. likes(draco,rain)
10. likes(draco,potion)
11. $\neg$quiditch(x) $\vee$ likes(x,potion)


¬quiditch(x) ∨ likes(x, potion)
¬student(x) ∨ wicked(x) ∨ quiditch(x)
student(Harry)
¬student(x) ∨ wicked(x) ∨ likes(x, potion)
x/Harry
¬wicked(x) ∨ likes(x, potion)
Wicked(Harry) ∨ likes(Harry, potion)
x/Harry
¬likes(Harry, x) ∨ ¬likes(Draco, x)
likes(Harry, potion)
x/potion
¬likes(Draco, potion)
likes(Draco, potion)
NULL clause

Solution

- Unify 11 with 4

12. ¬student(x) ∨ wicked(x) ∨ likes(x,potion)

- Unify 12 with 1

13. wicked(harry) ∨ likes(harry,potion)

- Unify 13 with 6

14. likes(harry,potion)

- Unify 14 with 7

15. ¬likes(draco,potion)

- Unify 15 with 10
- **NULL CLAUSE**

Exercise

- The law says that it is a crime for a Gaul to sell potion formulas to hostile nations.
- The country Rome, an enemy of Gaul, has acquired some potion formulas, and all of its formulas were sold to it by Druid Traitorix
- Traitorix is a Gaul.
- Is Traitorix a criminal?

Facts and Rules

- The law says that it is a crime for a Gaul to sell potion formulas to hostile nations
- $\forall x\ \forall y\ \forall z\ \text{Gaul}(x) \wedge \text{Potion}(y) \wedge \text{Hostile}(z) \wedge \text{Sells}(x,y,z) \rightarrow \text{Criminal}(x)$

1. $\neg\text{Gaul}(x) \vee \neg\text{Potion}(y) \vee \neg\text{Hostile}(z) \vee \neg\text{Sells}(x,y,z) \vee \text{Criminal}(x)$

Facts and Rules

- The country Rome, an enemy of Gaul, has acquired some potion formulas, and all of its formulas were sold to it by Druid Traitorix
2. $\exists y$ Potion(y) $\wedge$ Owns(Rome,y)
- $\forall y$ Potion(y) $\wedge$ Owns(Rome,y) $\rightarrow$ Sells (Traitorix, y, Rome)
3. $\neg$Potion(y) $\vee \neg$Owns(Rome,y) $\vee$ Sells (Traitorix, y, Rome)
4. Hostile(Rome)

Facts and Rules

- Traitorix is a Gaul

5. Gaul(Traitorix)
6. Potion(y)

To Prove

- Is Traitorix a criminal?
- **Criminal(Traitorix)**

- **Negation**

7. **¬Criminal(Traitorix)**

Knowledge Base

1. $\neg Gaul(x) \vee \neg Potion(y) \vee \neg Hostile(z) \vee \neg Sells(x,y,z) \vee Criminal(x)$
2. $Potion(y) \wedge Owns(Rome,y)$
3. $\neg Potion(y) \vee \neg Owns(Rome,y) \vee Sells\ (Traitorix,\ y,\ Rome)$
4. $Hostile(Rome)$
5. $Gaul(Traitorix)$
6. $Potion(y)$
7. **$\neg Criminal(Traitorix)$**

Solution

- Unify 7 with 1
8. ¬Gaul(Traitorix) ∨ ¬Potion(y) ∨ ¬Hostile(z) ∨ ¬Sells(Traitorix,y,z)
- Unify 8 with 6
9. ¬Gaul(Traitorix) ∨ ¬Hostile(z) ∨ ¬Sells(Traitorix,y,z)
- Unify 9 with 5
10. ¬Hostile(z) ∨ ¬Sells(Traitorix,y,z)
- Unify 10 with 4
11. ¬Sells(Traitorix,y,Rome)
- Unify 11 with 3
12. ¬Potion(y) ∨ ¬Owns(Rome,y)
- Unify 12 with 2
- **NULL CLAUSE**

Exercise

- John likes all kinds of food
- Apple is food
- Chicken is food
- Anything anyone eats and is not killed by is food
- Bill eats peanuts and is still alive
- Sue eats everything that Bill eats
- Prove John likes peanuts

Knowledge Base

1. $\neg food(x) \vee likes(x,John)$
2. $food(Apple)$
3. $food(Chicken)$
4. $\neg eats(Bill,x) \vee eats(Sue,x)$
5. $\neg eats(x,y) \vee killed(x,y) \vee food(y)$
6. $Eats(Bill, Peanuts) \wedge \neg killed(Bill, Peanuts)$
7. $\neg likes(Peanuts, John)$

Solution

- Unify 7 with 1

8. ¬food(Peanuts)

- Unify 8 with 5

9. ¬eats(x,Peanuts) ∨ killed(x,Peanuts)

- Unify 9 with 6
- **NULL CLAUSE**

Exercise

- Members of XYZ club are Joe, Sally, Bill and Ellen
- Joe is married to Sue
- Bill is brother of Ellen
- Spouse of every married person is also the member of the club
- The last meeting of the club was at Joe's house
- Prove that last meeting of the club was at Sue's house (Sue is member of the club and the meeting was held at her house)

Knowledge Base

1. Member(Joe)
2. Member(Sally)
3. Member(Bill)
4. Member(Ellen)
5. Married(Joe,Sue)
6. Married(Sue,Joe)
7. Brother(Bill,Ellen)
8. $\neg$married(x,y) $\vee$ $\neg$member(x) $\vee$ member(y)
9. $\neg$married(x,y) $\vee$ $\neg$house(x) $\vee$ house(y)
10. house(Joe)
11. $\neg$Member(Sue)
12. $\neg$House(Sue)

Solution

- Unify 11 with 8

12. $\neg married(x,Sue) \vee \neg member(x)$

- Unify 12 with 5

13. $\neg member(Joe)$

- Unify 13 with 1
- **NULL CLAUSE**