Reasoning in Predicate Logic

- Reasoning through Modus Ponen
- Reasoning through Refutation (Resolution)

$A \rightarrow B$ is valid

A is valid

$\therefore B$ is valid

1. All man are mamal

$\forall x: man(x) \longrightarrow mamal(x)$

2. All mamals drink milk

$\forall x: mamal(x) \rightarrow drinkmilk(x)$

3. Tom is a man

man (Tom)

To Prove

Tom drinks milk

drink milk (Tom)

$\frac{A \rightarrow B \quad A}{\therefore B}$

From (1) and (3), we get

man (x) → mamal(x)

man (Tom) (x/Tom)

4. mamal (Tom)

From (2) and (4), on unifying x with Tom, we get

mamal (Tom) → drinkmilk (Tom)

mamal (Tom)

drinkmilk (Tom)

Hence Proved

1. John likes all kinds of food

$\forall x: food(x) \rightarrow likes(x, John)$

2. Apple is a food

$food(Apple)$

3. Pizza is a food

$food(Pizza)$

4. Sue eats anything that Bill eats

$\forall x: eats(Bill, x) \rightarrow eats(Sue, x)$

5. Anything anyone eats and is not killed by is a food

$\forall x \forall y: eats(x, y) \wedge \neg killed(x) \rightarrow food(y)$

6. Bill eats peanuts and is still alive

$$Eats(Bill, Peanuts) \wedge \neg killed(Bill)$$

To Prove

John likes peanuts $likes(peanuts, John)$

On Substituting x with Bill and y with Peanuts in (5) and (6), we get

$$Eats(Bill, Peanuts) \wedge \neg Killed(Bill) \rightarrow food(Peanuts)$$

$$Eats(Bill, Peanuts) \wedge \neg Killed(Bill)$$

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(7) $food(Peanuts)$

On Substituting x with peanuts in (1) and (7), we get

$$food(Peanuts) \rightarrow likes(Peanuts, John)$$

$food(Peanuts)$

$likes(Peanuts, John)$

Hence Proved

1. Members of a club are Joe, Sally and Ellen
2. Joe is married to Sue
3. Bill is brother of Ellen
4. Spouse of every married person is also the member of the club
5. The last meeting of the club was at Joe's house

To Prove

Sue is a member of the club and last meeting was held at Sue's house

1. Member(Joe, club)
2. Member(Sally, club)
3. Member(Bill, club)
4. Member(Ellen, club)
5. married(Joe, Sue)
6. married(Sue, Joe)
7. brother(Bill, Ellen)
8. $\forall x \exists y$: married(x, y) $\wedge$ member(x, club) $\rightarrow$ member(y, club)
9. $\forall x \exists y$: married(x, y) $\wedge$ house(x) $\rightarrow$ house(y)
10. house(Joe)

To Prove
a. member(Sue, club)
b. ~~house(Joe) $\rightarrow$~~ house(Sue)

From ①, ⑤ and ⑧, on unifying x with Joe and y with Sue, we get

married (Joe, Sue) ∧ member (Joe, Club) → member (Sue, Club)

member (Joe, club)

married (Joe, Sue)

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member (Sue, Club)

Hence Proved (a)

From ⑤, ⑨ and ⑩, on unifying x with Joe and y with Sue, we get

married (Joe, Sue) ∧ house (Joe) → house (Sue)

married (Joe, Sue)

house (Joe)

∴ house (Sue)

Hence Proved (b)

$$\frac{A \to B \quad A}{\therefore B}$$