First order Logic
First order Predicate Logic
Predicate Calculus
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KNOWLEDGE REPRESENTATION

First Order Logic

Introduction

P Q R

int(J) int(P) int(S)

int(X)
Generalized

- Also known as Predicate Logic
- Can capture more sentences then Prepositional Logic
- Mostly used to define rules in Logic Programming
- Generalizes Prepositional Logic

Prolog

intelligent(Tom) $\wedge$ hardworking(Tom) $\rightarrow$ scoregoodmarks(Tom)

Example

- Tom is intelligent and Tom is hard working then Tom will score good marks
- Jerry is intelligent and Jerry is hard working then Jerry will score good marks
- Ram is intelligent and Ram is hard working then Ram will score good marks

intelligent(x) $\wedge$ hardworking(x) $\rightarrow$ scoregoodmarks(x)

Example

- X is intelligent and X is hard working then X will score good marks
- All students who are hardworking and are intelligent will score good marks
- For all X such that X is hardworking and X is intelligent then X will scores good marks

Problem of Infinite Model

- In general prepositional logic can deal with only finite number of models
- If there are only three students Tom, Jerry and Ram, Then

  T : Tom is intelligent and Tom is hard Working

  J : Jerry is intelligent and Jerry is hard Working

  R : Ram is intelligent and Ram is hard Working

All student are intelligent and hardworking $\boxed{\leftrightarrow}$ $\underline{T \land J \land R}$

Solution - FOL

~~mortal(man)~~

- First Order Logic or FOL allows us to overcome this problem
- We can generalize the statements
- Have infinite values which can be substituted to generalized variables and can fit the model

Terms

$\forall x : man(x) \rightarrow mortal(x)$

Predicates

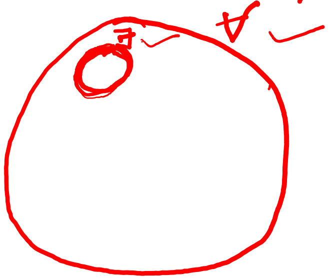
  - All men are mortals
  - Some birds cannot fly
  - At least one planet has life on it

$\exists x : bird(x) \rightarrow \neg fly(x)$

$\exists x : planet(x) \rightarrow haslife(x)$

Syntax of FOL

- Syntax of First Order Logic can be defined in terms of
  - Terms — Variables, Constants, functions
  - Predicates — Takes Terms as I/P and return T/F as o/p
  - Quantifiers —
    - Universal — $\forall$
    - Existential — $\exists$



Terms

- Denotes some object other than true or false

  Tom is intelligent

  All men are mortal

  Terms

  Constants -- 5, Ram, Tom etc.

  Variables – some symbol which can hold some value in that particular set

  n $\in$ N, X $\in$ name of student

Terms - Functions

Functions – functional terms which can table values from W1 to Wn and return W

plus(3,4) = 7

Functional Term

Constant Terms

Predicates

- Are like functions except that there return values are True or False
- Example
  - gt(x,y) is true iff x>y
  - gt is a predicate which takes two arguments of type number

Types of Predicates

- A predicate with no variable is a preposition (It Rains ---- Raining) Sun shine
- A predicate with one variable is known as a property male(X), Female(Y), Mortal(X)
- Is true if and only iff x is male|female|mortal

Formulation of Predicates

- Let P(x,y,......) and Q(x,y,.........) be two predicates
- Then so are

$$P(x) \vee Q(y)$$

$$P \wedge Q$$

$$P \rightarrow Q$$

$$\neg P$$

Predicate Examples

- If x is a man than x is mortal

$$man(x) \rightarrow mortal(x)$$

$$\neg man(x) \vee mortal(x)$$

$P \rightarrow Q$

$\neg P \vee Q$

- If x is a natural no. then n is either even or odd

$$natural(x) \rightarrow even(x) \vee odd(x)$$

$\neg natural(x) \vee even(x) \vee odd(x)$

Quantifiers

- There are two basic qunatifiers
- Universal quantifier - $\forall$ - for all
- Existantial quantifier - $\exists$ - there exists

Exercise

In absence of any Quantifier — Assume it of universal type

1. All dogs are faithfull
2. All birds cannot fly

   some
3. Mammals drink milk

   Man is mortal

   Man is a mamal

   Tom is man
4. At least one planet has life on it

$\forall x: dog(x) \rightarrow faithful(x)$

$\exists x: bird(x) \rightarrow \neg fly(x)$

$\forall x: mamal(x) \rightarrow drinkmilk(x)$

$\forall x: man(x) \rightarrow mortal(x)$

$\forall x: man(x) \rightarrow mamal(x)$

$man(Tom)$

$\exists x: planet(x) \rightarrow haslife(x)$

Sentences

- A predicate is a sentence
- If S and S' are sentences and x is a variable then
  - S, $\neg S$, $\exists x\ S$, $\forall x\ S$, $S \wedge S'$, $S \vee S'$, $S \rightarrow S'$ are sentences
- Nothing else is a sentences

Example

- Birthday(x,y) – x celebrates birthday on date y

Everyday somebody celebrates her birthday

$\forall y\ \exists x$: Birthday(x,y)

For all dates there exists a person who celebrates his/her birthday on that date

i.e. everyday somebody celebrates his/her birthday

Example

- Brother(x,y) – x is y's Brother
- Likes(x,y) – x likes y

$\forall x\ \forall y\ Brother(x,y) \rightarrow Likes(x,y)$

Everyone likes all his/her brothers

Let m(x) represent mother of x then everyone loves his/her mother can be written as

$\forall x\ Loves(x, m(x))$

x's mother

Example

- Any number is the successor of its number
- Succ(x), Pred(x)
- Equal(x,y)

y

x

y

- $\forall$x equal(x, Succ(Pred(x)))

Alternate Representation

- Previous sentence can also be written as
- $\forall x\ (Succ(Pred(x)) = x)$

Not Allowed in Predicates (Intially)

FOL with Equality

- We are allowed to use the equality sign (=) between two functions
- This is just for representational ease
- We modify the definition of sentence to include equality as

  term = term is also a sentence

Exercise

- Some dogs bark
- All dogs have four legs
- No dogs fly
- Fathers are parents

$\exists x : dog(x) \rightarrow bark(x)$

$\forall x : dog(x) \rightarrow haslegs(x, 4)$

$\forall x : dog(x) \rightarrow \neg fly(x)$

$\forall x : father(x) \rightarrow parent(x)$