Notes

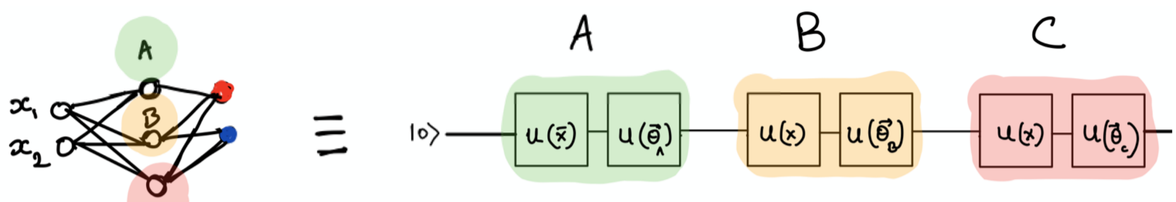
Idea is to use orthogonal quantum neural network with a data re-uploading classifier in combination with RBS-gates.

The idea is to use variational quantum circuits for performing training and inference on orthogonal neural networks.

I will start with a fully connected neural network layer with an orthogonal weight matrix that uses the RBS gate.

Data loading will be done with amplitude encoding and the interesting thing is that the data does not have to be flattened. We can load a matrix where the rows can be loaded in superposition. (I read a paper where they explained this thoroughly). And the amplitudes represent the probability in the vector representation. To load the data onto a single qubit, we can use a unitary operation which is basically a parameterised matrix multiplication representing the rotation on the Bloch sphere. To load for instance a vector (x1,x2) into the qubit, we start with the initial state vector |0>
Apply a unitary operation U(x1,x2,0) in which we will end up in a new point on the Bloch sphere.

The data re-uploading neural network is represented as followed:



In a normal neural network, every neuron receives input from all neuron of the previous layer, while in the data re-uploading the single qubit classifier receives information from the previous processing unit and the input. It processes everything all together and the final output of the computation is a quantum state encoding several repetitions of input uploads and processing parameters. In the initial idea of data re-uploading they are using backpropagation for the weights, but my idea was to use RBS-gates which is a method that replaces the backpropagation where the weights have the same value in all node per layer.

So it will be in the position of the vector theta.

The RBS gate is a two-qubit gate that is parametrisable with an angle theta, which represents the weights. Its matrix representation consists of this angle theta in combination with cosine, sinus and minus sinus.

$$RBS(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad RBS(\theta) : \begin{cases} |01\rangle \mapsto \cos\theta\,|01\rangle - \sin\theta\,|10\rangle \\ |10\rangle \mapsto \sin\theta\,|01\rangle + \cos\theta\,|10\rangle \end{cases}$$

It is basically a rotation in two-dimensional subspace spanned by the basis {|01>, |10>} and with the identity basis {|00> and |11>}
Say for instance we have 2 qubits, one |0> and the other one is state |1>. Qubit state |1> stays the same with the cosine theta amplitude or switches with the other qubit when it is corresponding to the positive or negative sinus theta amplitude.

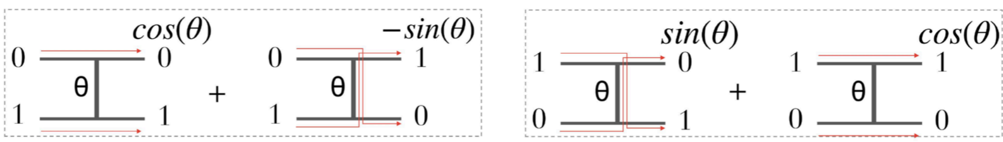
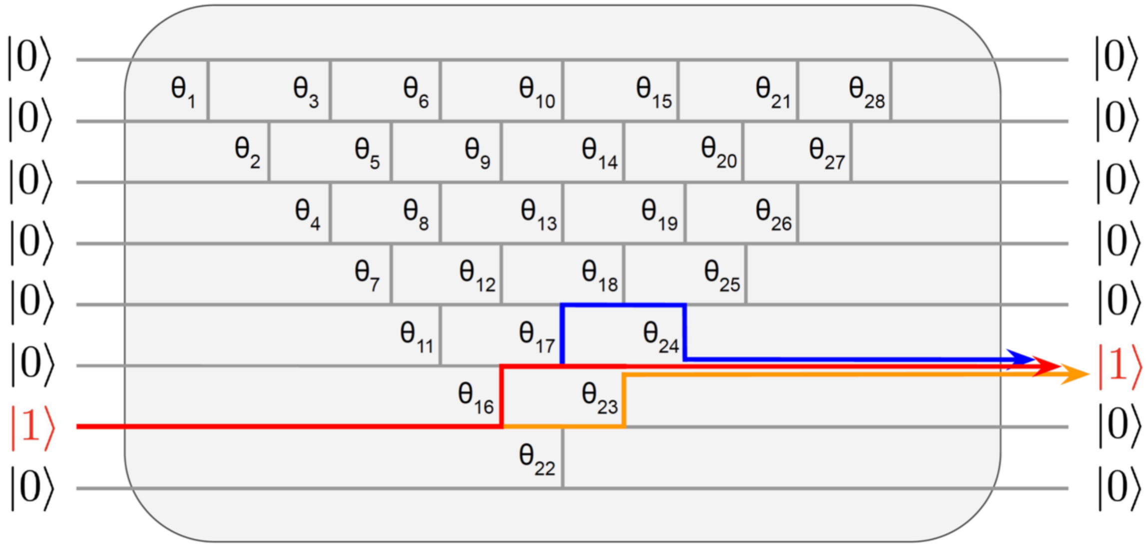


Figure 1: Representation of the quantum mapping from Eq.(1) on two qubits.

The RBS gate can be implemented using 4 Hadamard gates, 2 y rotation gates and 2 controlled-z gates.

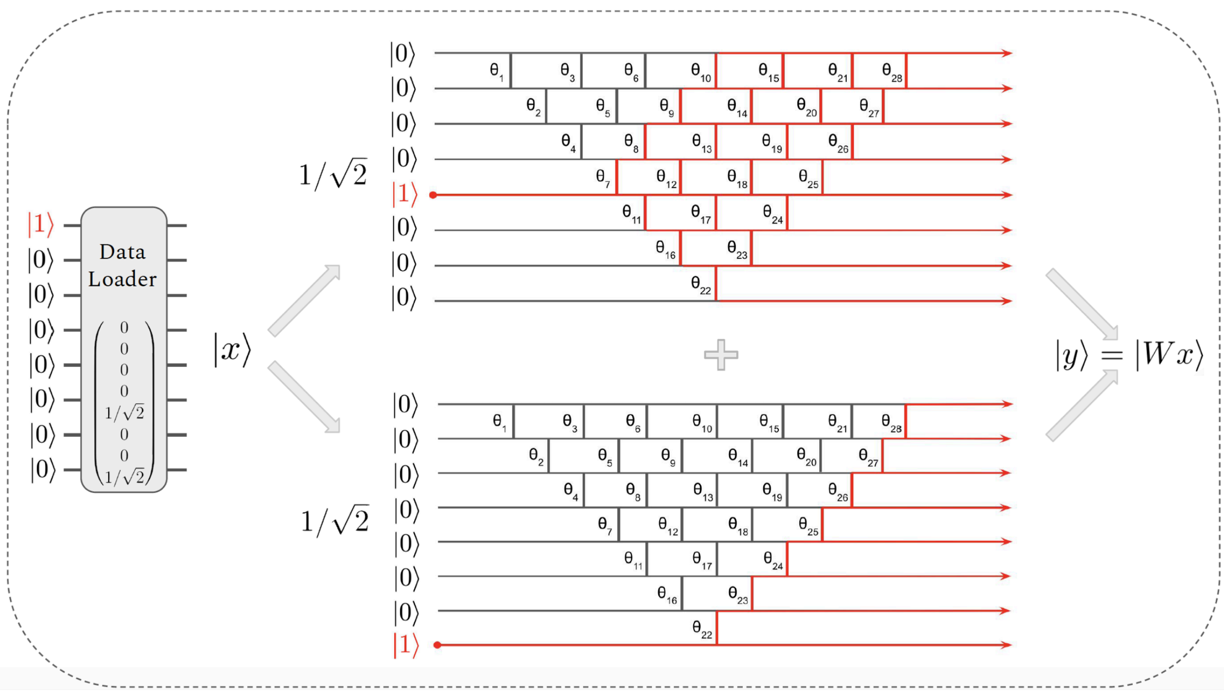
The rbs gates preserve the number of ones and zeros in any basis state

The training will be done classically, but the intuition comes from the quantum circuit. The inference on the orthogonal neural networks can happen both classically or by applying this quantum circuit.



An example of a feed forward in a quantum circuit of a 8x8 fully connected orthogonal layer with the RBS gates, where there are 3 different possible paths

Example with loaded data



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