SIVANAGA SURYA VAMSI POPURI

ASU ID: 1217319207

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MULTIVARIABLE CONTROL DESIGN OF A CAR-LIKE ROBOT USING DIFFERENTIAL DRIVE DYNAMICS

Project report for the course EEE588 – Design of Multivariable Control System

Under the guidance of Dr Armando Rodriguez

OVERVIEW

- The goal of this project is to understand what is going on in the process of designing Multiple Input-Multiple Output Controllers (MIMO Controllers), in the process of controlling a car-like robot using differential drive techniques.
- More Specifically, the Controllers being used here are the H_Infinity, LQG/LTRO and the LQG.
- The process of designing the controllers, the various constraints taken into consideration, and the results obtained for various scenarios are illustrated in detail.
- All simulations were done using MATLAB.

CONTENTS INSIDE

The Robot's Model

The model of the robot has been hugely inspired by the differential dynamics as mentioned in [1]. However, the robot being taken into consideration here is a car like robot, which is controlled using the differential drive model.

The assumptions made here are that controls are given only to the rear wheel i.e. the front wheels can rotate, but no motor control given there.

This implies that the center of gravity of the robot is not very close to the line that joins the rear wheels, unlike the differential drive robot, in which the center of gravity is at the mid-point of the line joining the left and right rear wheels.

From [1], the mathematical model can be expressed as follows:

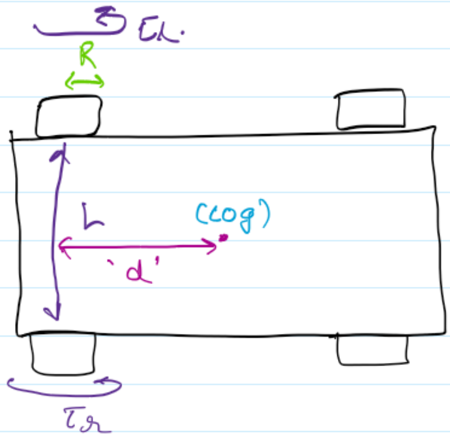
Kinematic Model (without considering the Wheel Motor Dynamics)

The nominal model is non-linear:

$$\left[m + \frac{2I_\omega}{R^2}\right]\dot{v} - m_c d\omega^2 = \frac{1}{R}(\tau_r + \tau_l)$$

$$\left[I + \frac{2L^2}{R^2} I_\omega\right]\dot{\omega} + m_c d\omega v = \frac{L}{R}(\tau_r - \tau_l).$$

where m = mass of Robot + wheels, (Kg)
I_w = wheel + motor inertia ($kg\,m^2$)
R = wheel Radius (rear) (m)
L = dist b/w both rear wheels) (m)
d = dist mid point of 'L' to CoG. (m)
τ_r & τ_l → left & right wheel Torques.



Actuator Dynamics:

$$e_a \text{ (both wheels)} = L_a \frac{di_a}{dt} + R_a i_a + e_b.$$

$$e_b = K_g K_b \omega_{r,l}$$

$$\tau \text{ (both wheels)} = K_t K_g \omega_{r,l}$$

$$I_w \dot{\omega}_{r,l} + \beta \omega = \frac{\tau}{K_g^2}.$$

→ K_g = motor gear ratio.

K_t = Torque constant mNm/A
β = speed damping const (mNms)

K_b = back EMF Constant. (mV/rad/s)

NOMINAL PLANT INPUT AND OUTPUTS

The input controls are the left and right rear wheel torques (**Newton-Meters**).

The outputs are the robot's linear and angular velocity about the center of gravity (**in m/s and rad/s respectively**).

Issues with the System

The system's nominal model is non-linear by default. This is dependent on the term 'd' in the equation.
Also, the value of 'd' determines the stability of the system. If d >=0, the system is stable. Unstable for d < 0 [1] and [2]. (Will be analyzed.)

NOTE THAT IF d = 0, THE SYSTEM BECOMES FIRST ORDER, AND THE ROBOT GETS APPROXIMATED TO A DIFFERENTIAL DRIVE ROBOT, WHICH IS NOT OUR FOCUS IN THE PROJECT.

LINEARIZED MODEL

Linearization is done using the concept of equilibrium, Taylor Expansion and Jacobian Matrices. The process of deriving the linear model is shown below [3].

Lineariz^n:

$$\left[m + \frac{2 I_w}{R^2}\right]\dot{v} - m_c d\omega^2 = \frac{1}{R}(\tau_r + \tau_l)$$

$$\left[I + \frac{2L^2}{R^2} I_w\right]\dot{\omega} + m_c d v\omega = \frac{L}{R}(\tau_R - \tau_l).$$

(I = total inertia)

Let $\frac{1}{m+\frac{2I_w}{R^2}} = p_1$, $\quad \frac{1}{I+\frac{2L^2}{R^2}I_w} = p_2$

$$\Rightarrow \dot{v} = p_1(m_c d\omega^2) + \frac{p_1}{R}(\tau_r + \tau_l)$$

$$\dot{\omega} = p_2(m_c d v\omega) + \frac{p_2 L}{R}(\tau_R - \tau_l)$$

now, Taking derivative:

$$\begin{bmatrix} \delta\dot{v} \\ \delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{v}}{\partial v} & \frac{\partial \dot{v}}{\partial \omega} \\ \frac{\partial \dot{\omega}}{\partial v} & \frac{\partial \dot{\omega}}{\partial \omega} \end{bmatrix} \begin{bmatrix} \delta v \\ \delta \omega \end{bmatrix} + \begin{bmatrix} \frac{\partial \dot{v}}{\partial \tau_r} & \frac{\partial \dot{v}}{\partial \tau_l} \\ \frac{\partial \dot{\omega}}{\partial \tau_r} & \frac{\partial \dot{\omega}}{\partial \tau_l} \end{bmatrix} \begin{bmatrix} \delta \tau_r \\ \delta \tau_l \end{bmatrix}$$

on applying the eq^ns, we get:

$$\boxed{\begin{bmatrix} \delta\dot{v} \\ \delta\dot{\omega} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 2p_1 m_c d\omega \\ 2p_2 m_c d\omega & 2p_2 m_c dv \end{bmatrix}}_{A} \begin{bmatrix} \delta v \\ \delta \omega \end{bmatrix} + \underbrace{\begin{bmatrix} p_1/R & p_1/R \\ p_2 L/R & -p_2 L/R \end{bmatrix}}_{B} \begin{bmatrix} \delta \tau_r \\ \delta \tau_l \end{bmatrix}}$$

↳ Linearized model.

now, $y_1 = v$, $y_2 = \omega$ $\Rightarrow$ $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} v \\ \omega \end{bmatrix}$

& , $D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Equilibrium points can be chosen as desired by the designing party.

Therefore, as can be seen, this is a ***TITO system***.

Note: The above information refers to the NOMINAL PLANT.

DYNAMIC MODEL OF THE ACTUAL ROBOT PLANT (CONSIDERING THE ACTUATOR DYNAMICS)

Taking into account the work of [1] and [2], it can be seen that a stable, observable state-space representation of the actual plant of the robot considering the actuator dynamics is given as follows.

$$A_s = \begin{bmatrix} \frac{-2\beta K_g^2}{mR^2 + I_w} & 2p_1 m_c w_0 d^2 & \frac{K_t K_g}{L_a R m_0} & \frac{K_t K_g}{L_a R M_0} \\ -p_2 m_c w_0 d & p_2\left(-m_c v_0 d - \frac{d^2 K_g^2 \beta}{2R^2}\right) & \frac{p_2 K_t K_g d}{2 L_a R} & -\frac{p_2 K_t K_g d}{2 L_a R} \\ -\frac{K_B K_g}{R} & -\frac{K_B K_g d}{2R} & -\frac{R_a}{L_a} & 0 \\ -\frac{K_B K_g}{R} & \frac{K_B K_g d}{2R} & 0 & -\frac{R_a}{L_a} \end{bmatrix}$$

$$B_s = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \text{state vector } x = \begin{bmatrix} V \\ \omega \\ i_{aR} \\ i_{aL} \end{bmatrix}$$

The stability, controllability and observability of the above system will be analyzed in MATLAB.

CHOOSING THE PARAMETER VALUES FOR THE PLANT

The parameters have been chosen in a hypothetical manner!

The robot’s physical properties (Such as mass, wheel radius, etc.) were decided based on the documentation provided by the Turtlebot 3 Robot’s official website [6].

The robot’s electrical properties and Inertial parameters were chosen based on the robots used at the METS lab at the Arizona State University [2], [5].

The following lists out the parameters used and their values along with the units.

Physical Parameters (Based on Turtlebot 3 Waffle).

M = 1.8 kg (Overall mass)
L = 1.53 m (Distance between rear wheels)
R = 0.15 m (Wheel Radius)
Mc = 1.6 kg (Mass without motors and rear wheels)
d = 0.5 (Note that analysis will be done for different values of d.)

Electrical and Inertial Parameters (Based on METS Robots)

Iw = 0.0728 mu * kg m^2 (Wheel + Motor Inertia)
I = 0.0979 kg m^2 (Total Inertia)
La = 2.3 mH (Armature Inductance)
Ra = 13.7 ohms (Armature Resistance)
Kb = 9.5 mV / rad / sec (Back EMF Constant)
Kt = 9.5 mNm / A (Torque Constant)
Kg = 9.68 (Motor-Wheel Gear Ratio)
Beta = 3.29 mu * Nm * sec (Speed Damping Constant)

Equilibrium velocities (linear and angular)
V_0 = 0.5 rad / sec
W_0 = 0.5 rad / sec

THE OBJECTIVE OF THE STUDY

We know that the robot being considered here is a car-like robot, but a differential model is being used for the sake of analysis. A differential drive model has been implemented once for a car-like robot for a navigation purpose in [8] (Which was my undergraduate final year project!).

<u>The parameter that decides the nature of the robot is the distance 'd'.</u>

WHY?

Because 'd' tells us whether the robot's physical structure resembles a differential drive or a car-like four-wheel robot.

If $d = 0$, it's approximately a differential drive robot.
If $d > 0$, it's approximately a car-like robot.

This study focuses on changing the value of d for d = 0.5, and 1 visualize the frequency and step response plots for various controllers taken into account.

Also, the case when $d < 0$ is also investigated, which theoretically is supposed to imply instability.

All simulations and Setups were done on MATLAB, and the next few sections discuss the results and validations obtained from this study.

RESULTS AND ANALYSIS

The results are displayed as follows. The Sensitivity, Complementary Sensitivity, Singular Values, Freq response, step response and control input response are all given for the following cases:

Case 1: d = 0.5 (LQR, H_inf, LQG/LTRO)
Case 2: d = 1 (LQR, H_inf, LQG/LTRO)
Case 3: d = -0.5 (LQR, H_inf, LQG/LTRO)

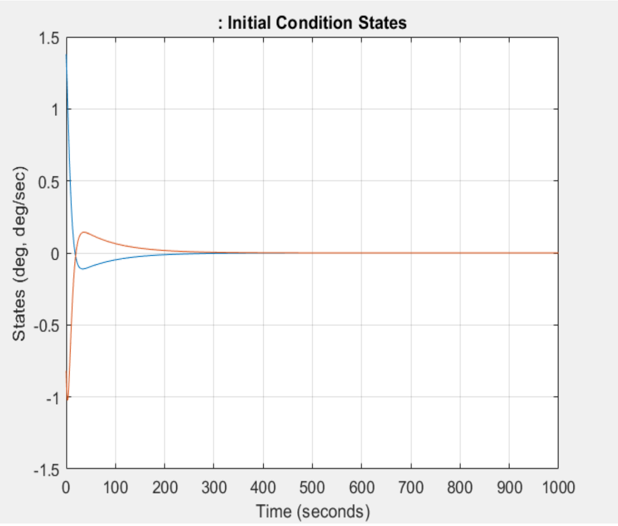
So, embrace yourself for some stuff!

Case 1: d = 0.5 (in meters)

Nominal Plant:

```
Plant =

  From input 1 to output...
                 321.99 (s+0.004514) (s+0.462)
   1:  ----------------------------------------------
       (s+0.01355) (s+5957) (s^2 + 0.3538s + 0.03419)

                 360.7 (s+0.004514) (s-0.3158)
   2:  ----------------------------------------------
       (s+0.01355) (s+5957) (s^2 + 0.3538s + 0.03419)

  From input 2 to output...
                 321.99 (s+0.004514) (s+0.2455)
   1:  ----------------------------------------------
       (s+0.01355) (s+5957) (s^2 + 0.3538s + 0.03419)

                 -360.7 (s+0.3158) (s+0.004514)
   2:  ----------------------------------------------
       (s+0.01355) (s+5957) (s^2 + 0.3538s + 0.03419)
```

The system is Controllable and Observable. (Full Rank)

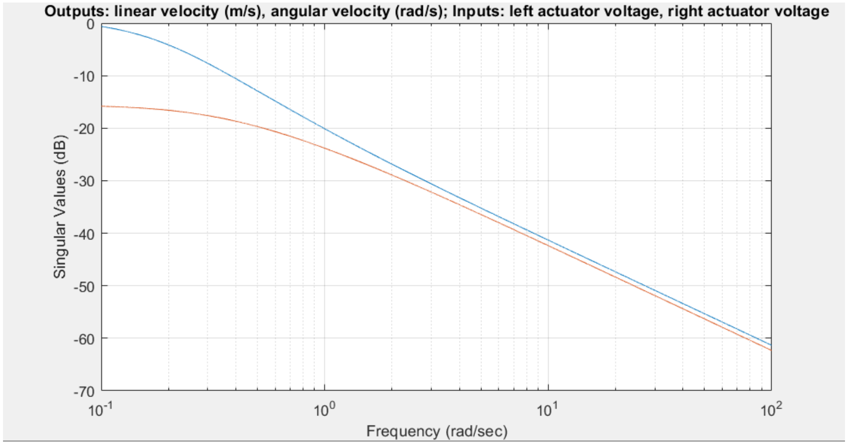


→ **Initial Conditions**

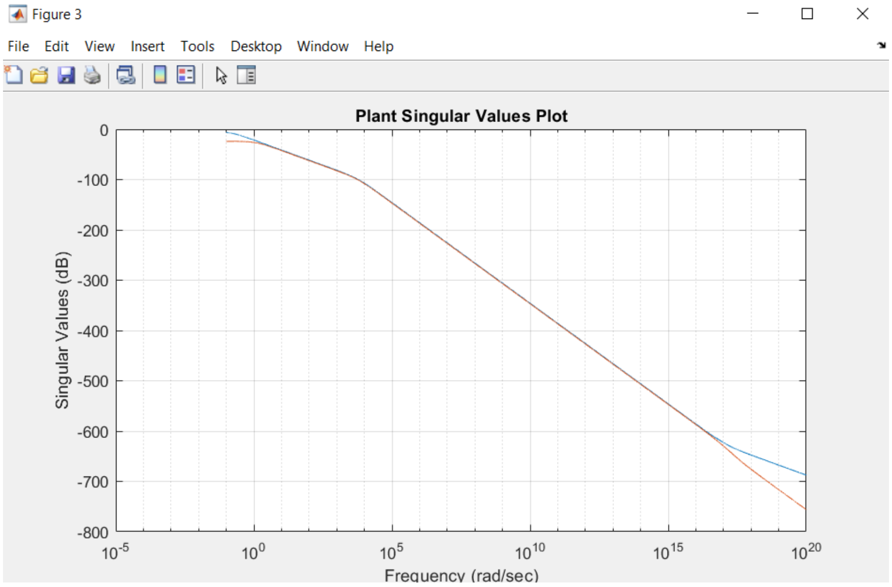
Eigen values:

```
eig_val =

   1.0e+03 *

  Columns 1 through 5

  -0.0002 + 0.0001i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0002 - 0.0001i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -5.9565 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

  Column 6

   0.0000 + 0.0000i
   0.0000 + 0.0000i
   0.0000 + 0.0000i
   0.0000 + 0.0000i
   0.0000 + 0.0000i
  -5.9565 + 0.0000i
```

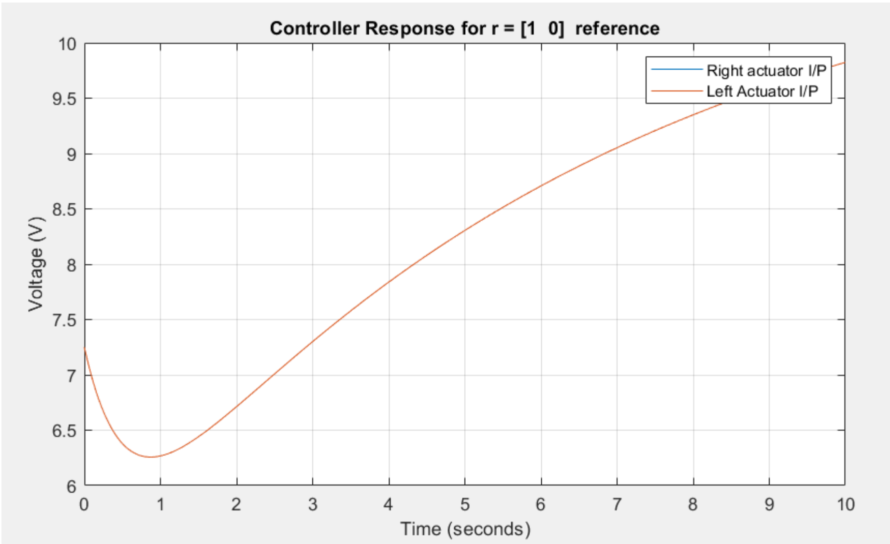
Linear and Angular Velocity Singular values:

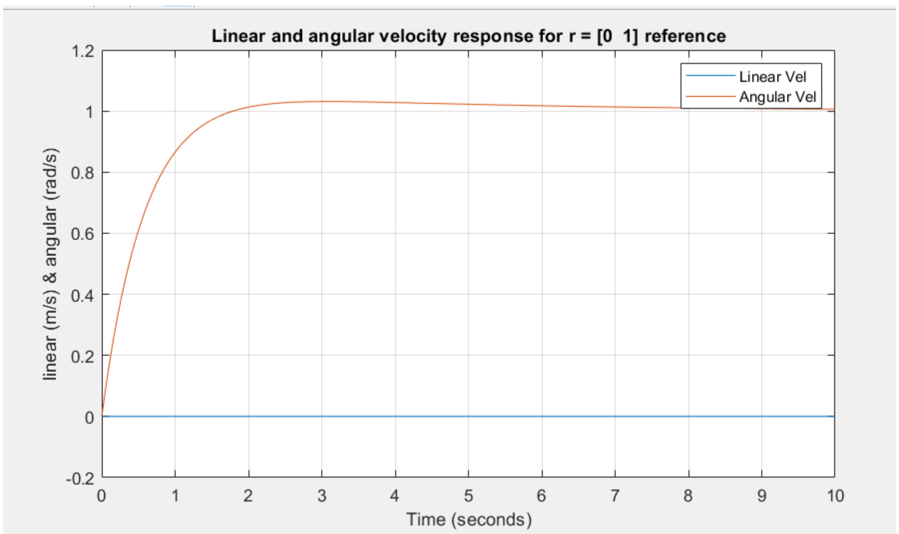


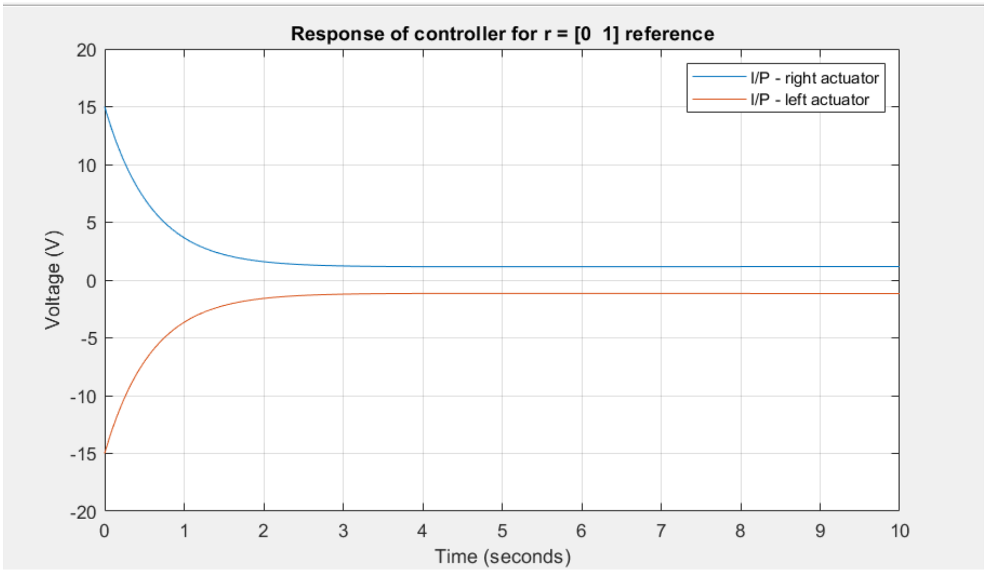
Plant Singular Values

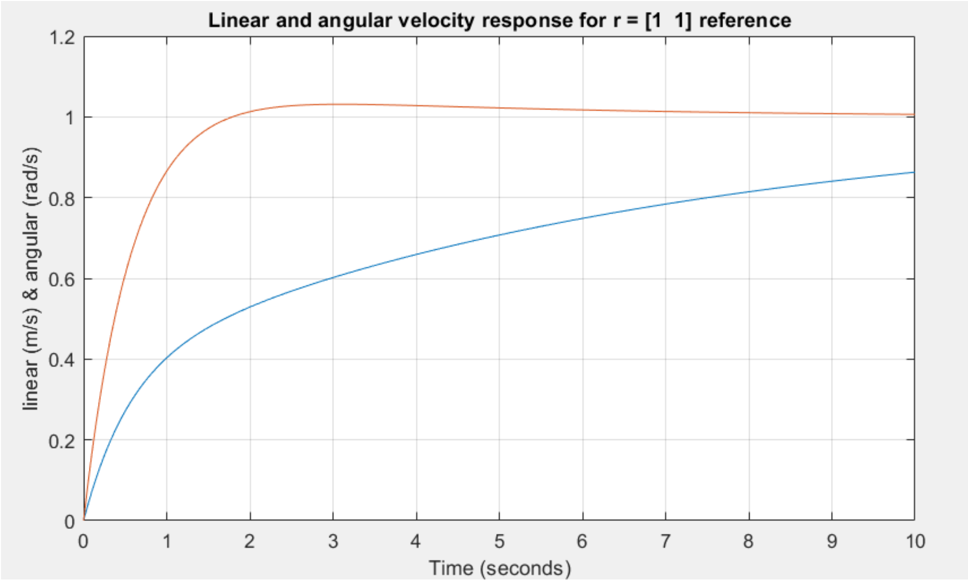


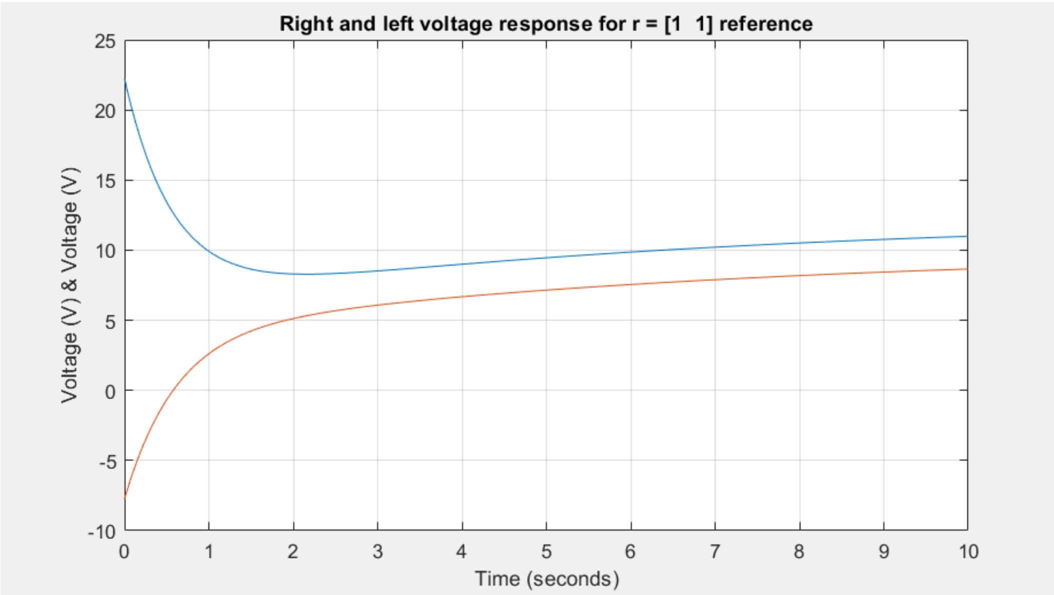
LQR CONTROLLER:

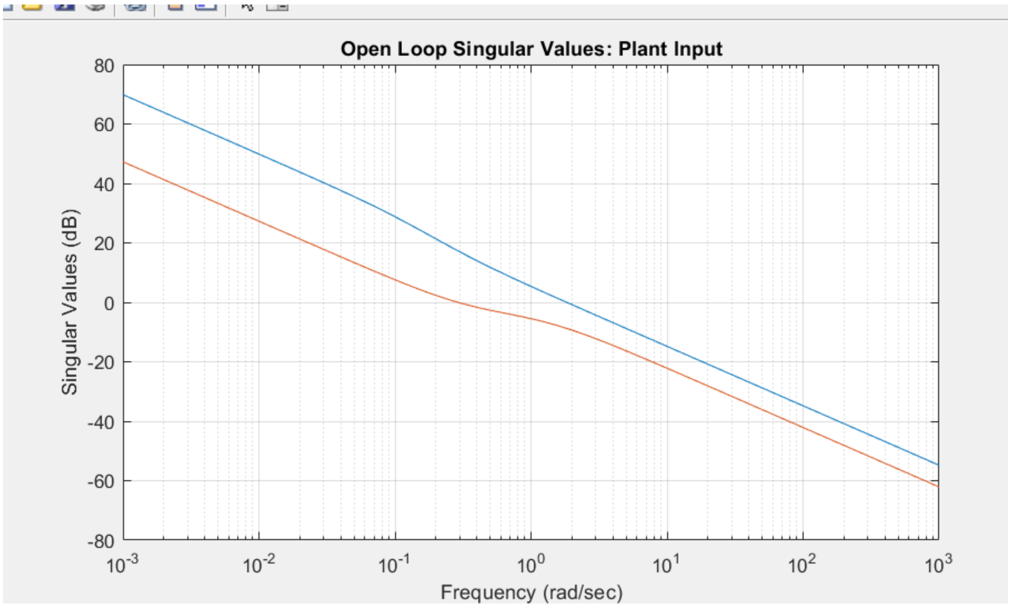


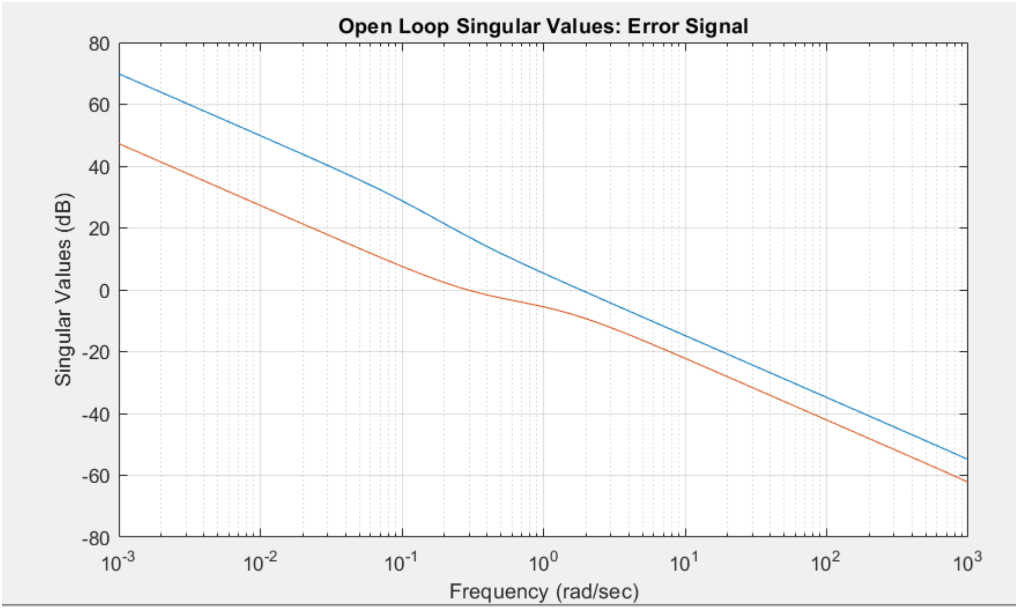


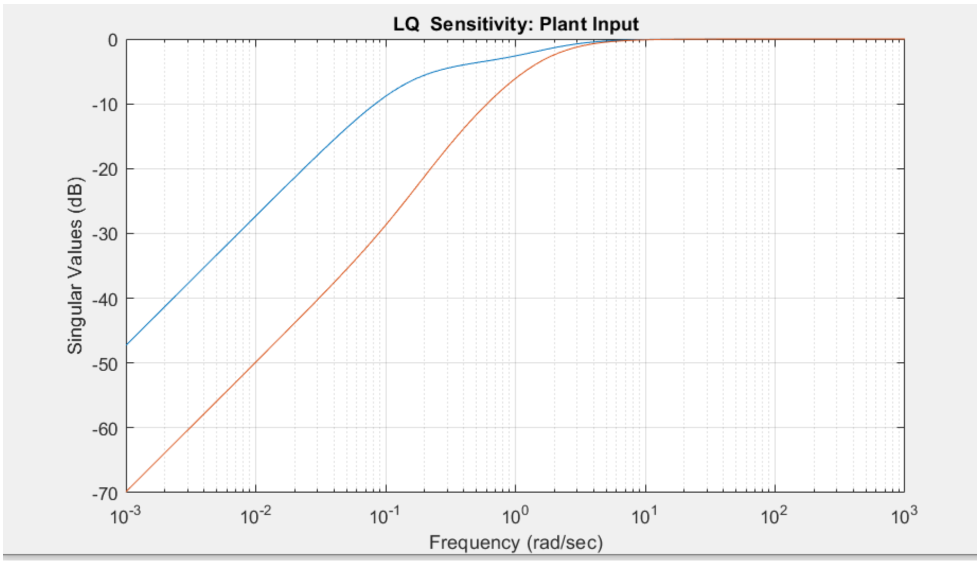

Response of controller for r = [0 1] reference
I/P - right actuator
I/P - left actuator
Voltage (V)
Time (seconds)


Linear and angular velocity response for r = [1 1] reference
linear (m/s) & angular (rad/s)
Time (seconds)


Right and left voltage response for r = [1 1] reference
Voltage (V) & Voltage (V)
Time (seconds)


Open Loop Singular Values: Plant Input
Singular Values (dB)
Frequency (rad/sec)


Open Loop Singular Values: Error Signal
Singular Values (dB)
Frequency (rad/sec)


LQ Sensitivity: Plant Input
Singular Values (dB)
Frequency (rad/sec)

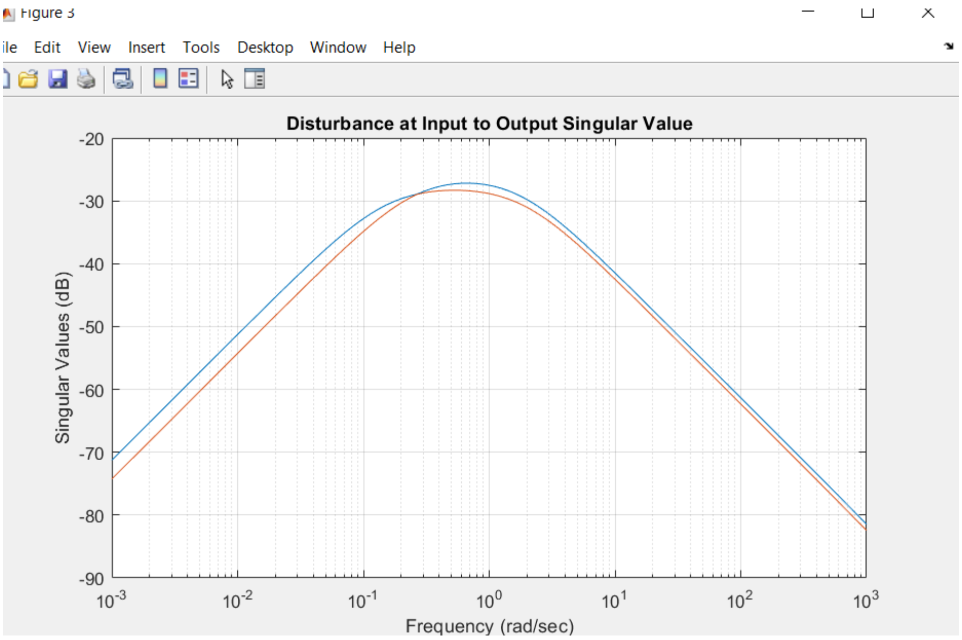
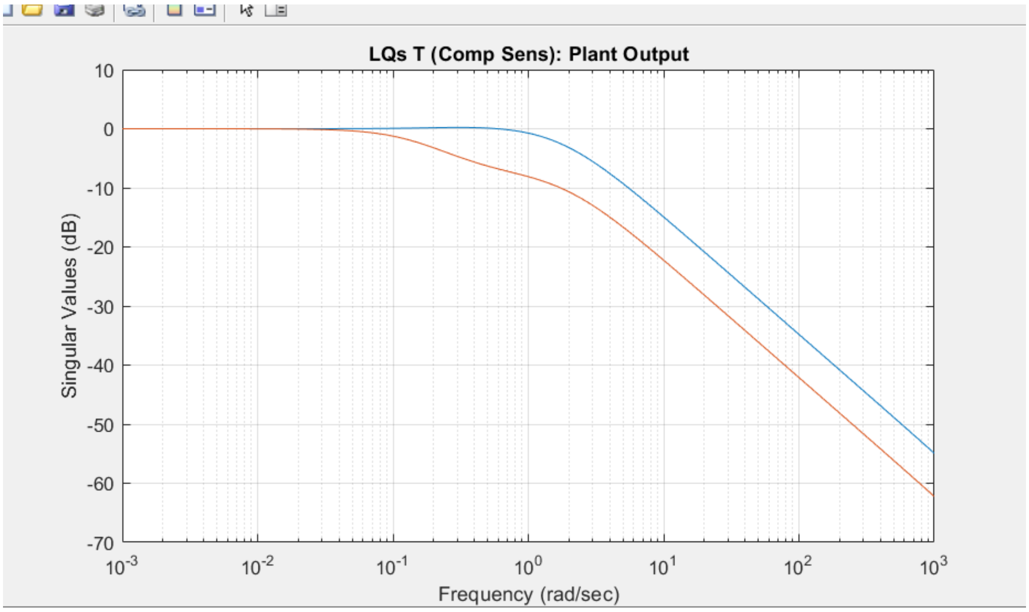
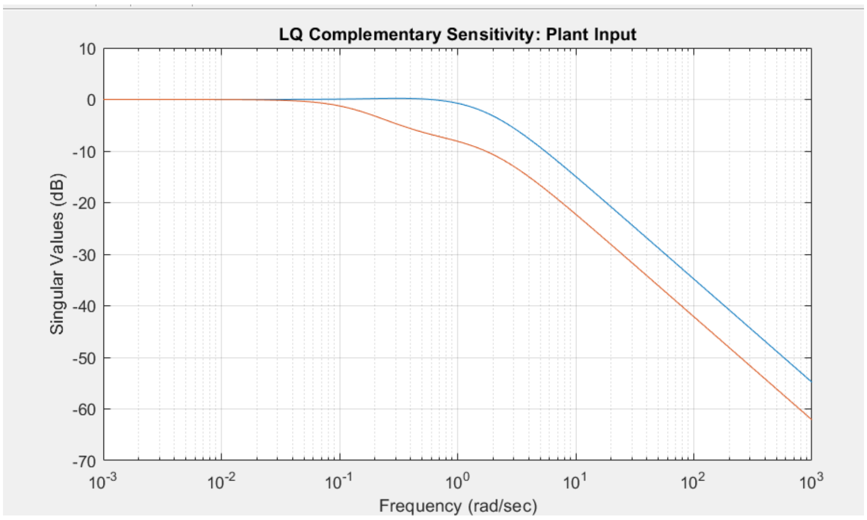
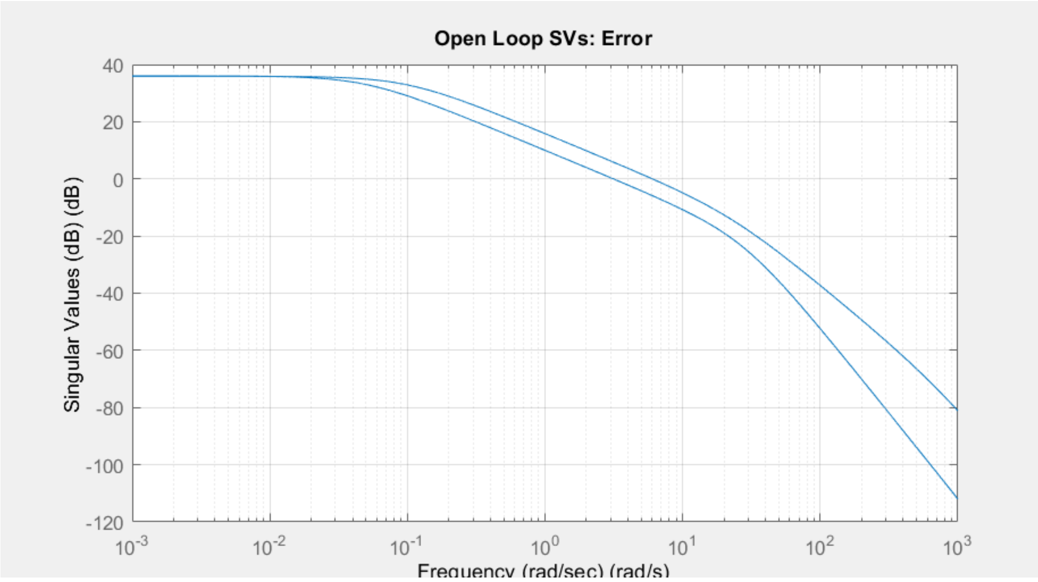


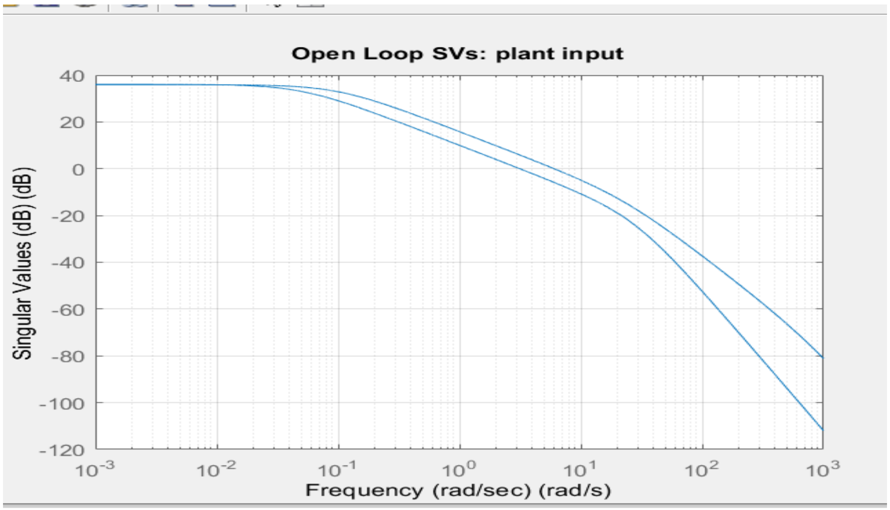
Figure 3
File Edit View Insert Tools Desktop Window Help
Disturbance at Input to Output Singular Value
Singular Values (dB)
Frequency (rad/sec)

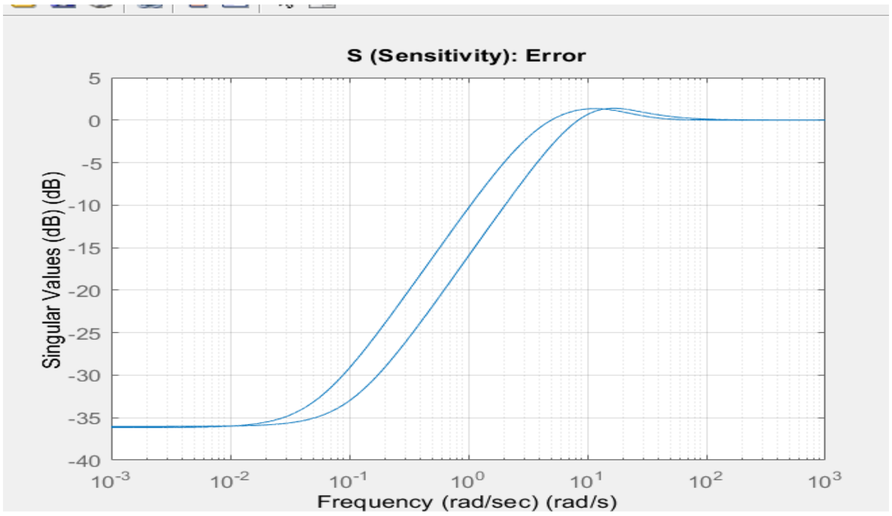

LQs T (Comp Sens): Plant Output
Singular Values (dB)
Frequency (rad/sec)

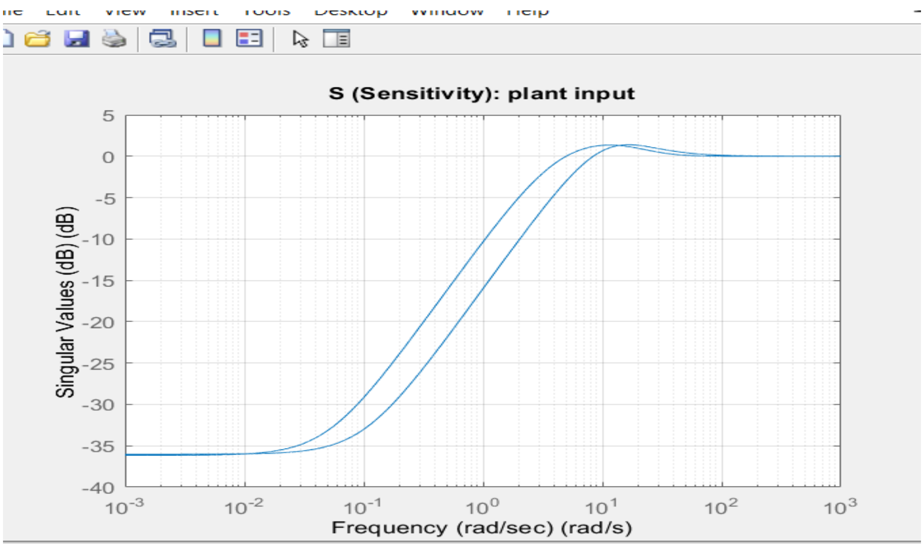

LQ Complementary Sensitivity: Plant Input
Singular Values (dB)
Frequency (rad/sec)

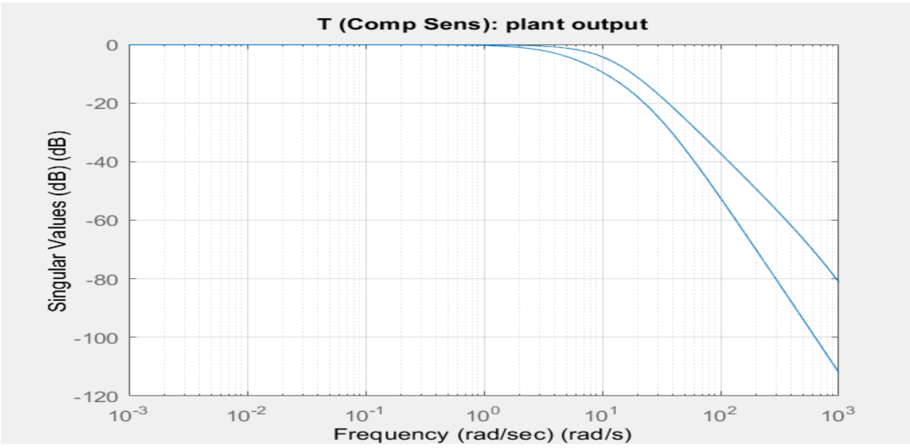
H_infinity Controller

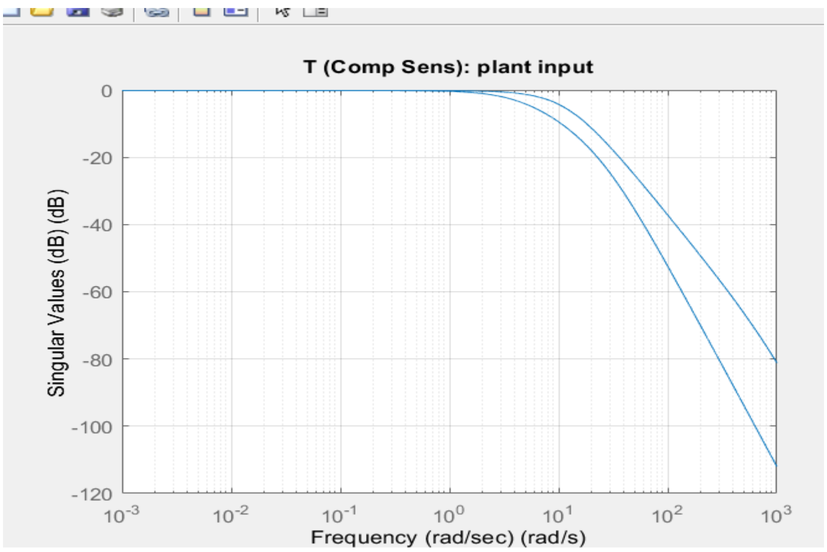


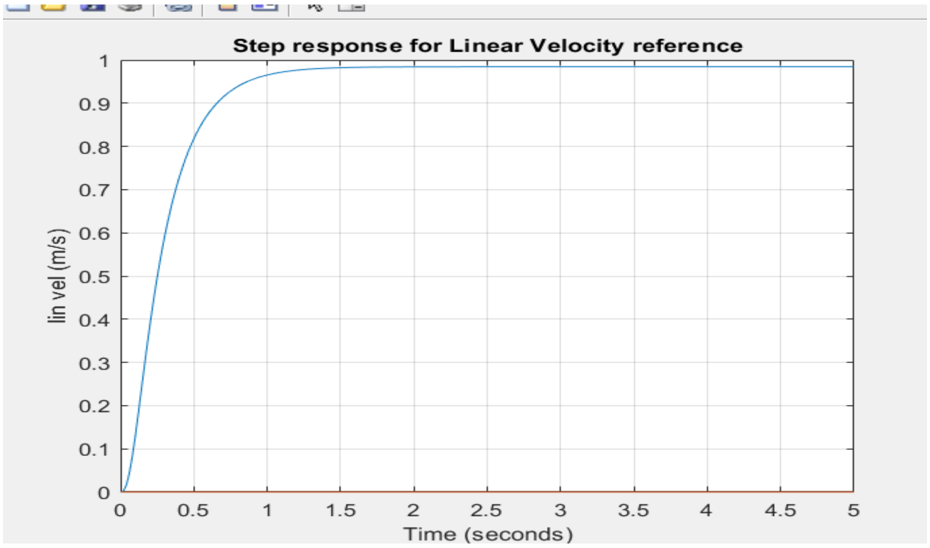




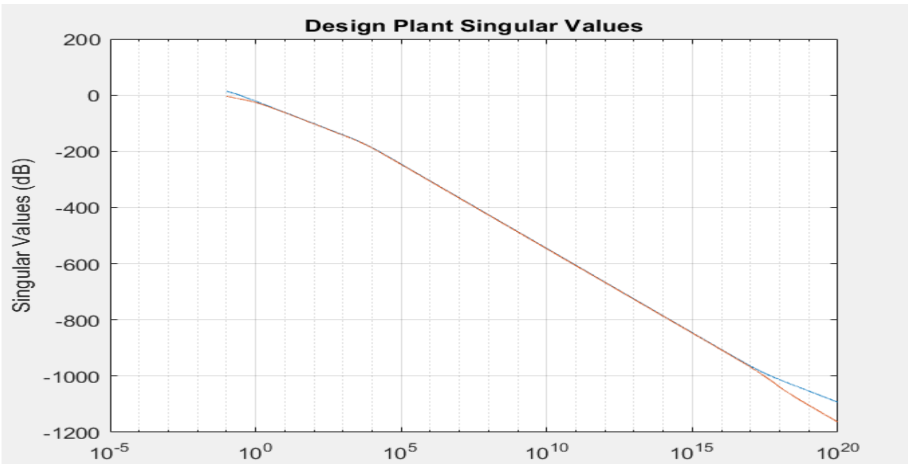

S (Sensitivity): plant input
Singular Values (dB) (dB)
Frequency (rad/sec) (rad/s)

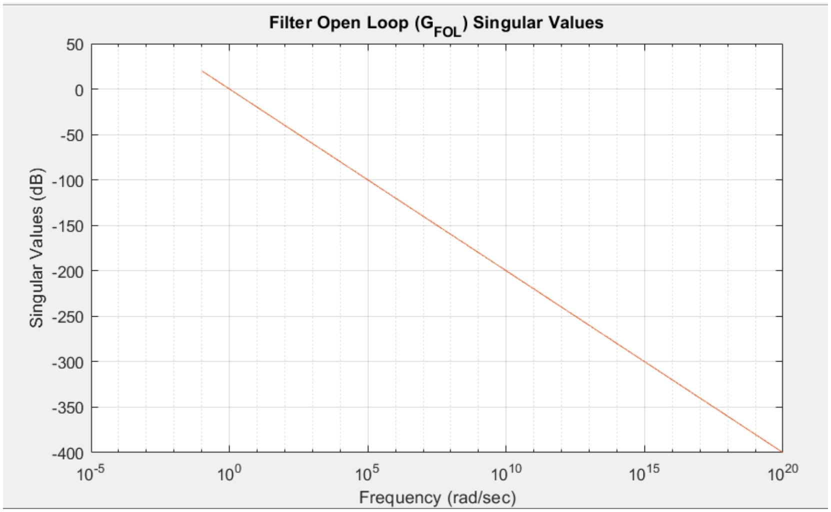

T (Comp Sens): plant output
Singular Values (dB) (dB)
Frequency (rad/sec) (rad/s)

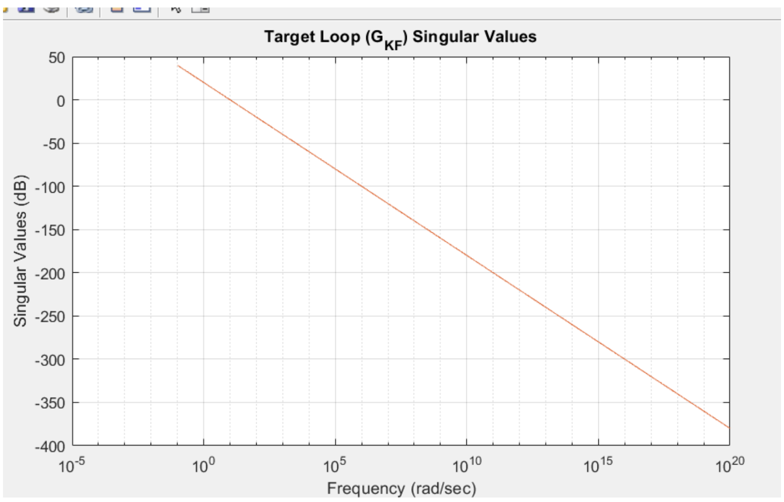

T (Comp Sens): plant input
Singular Values (dB) (dB)
Frequency (rad/sec) (rad/s)

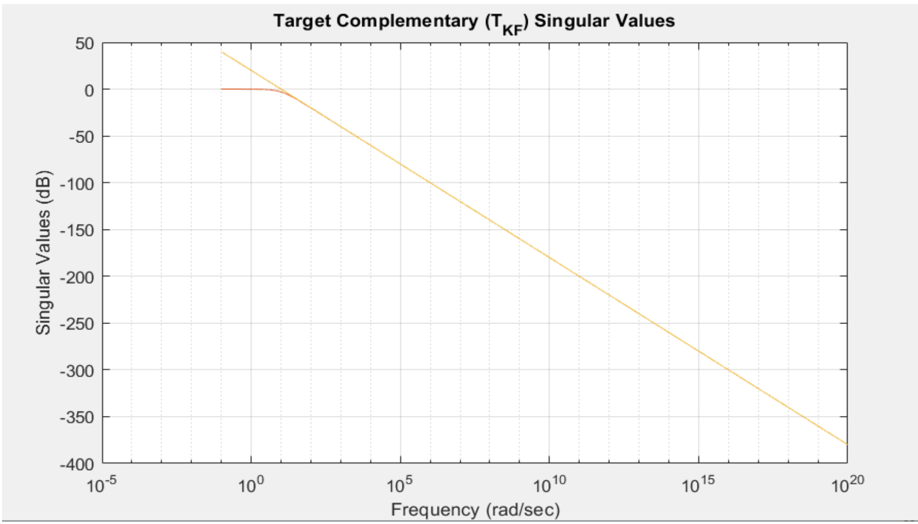


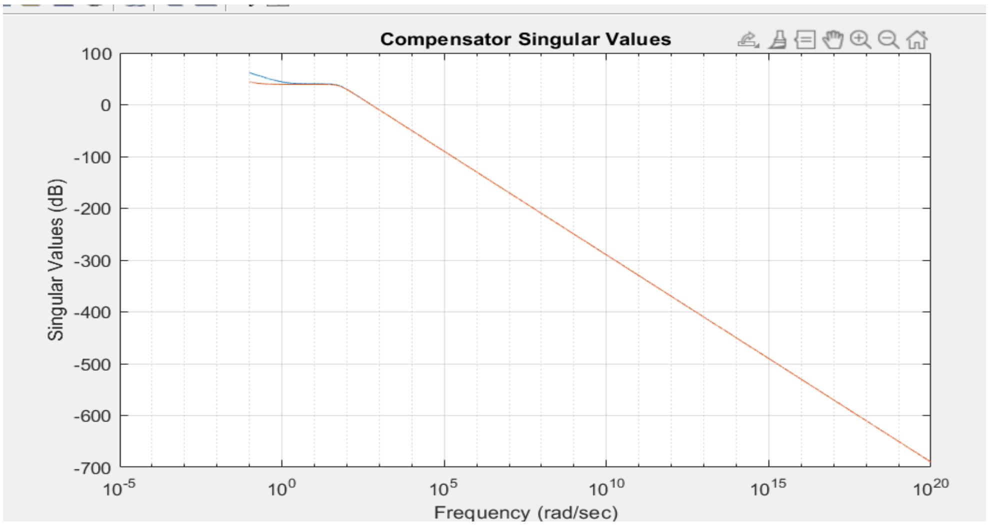
LQG / LTRO CONTROLLER

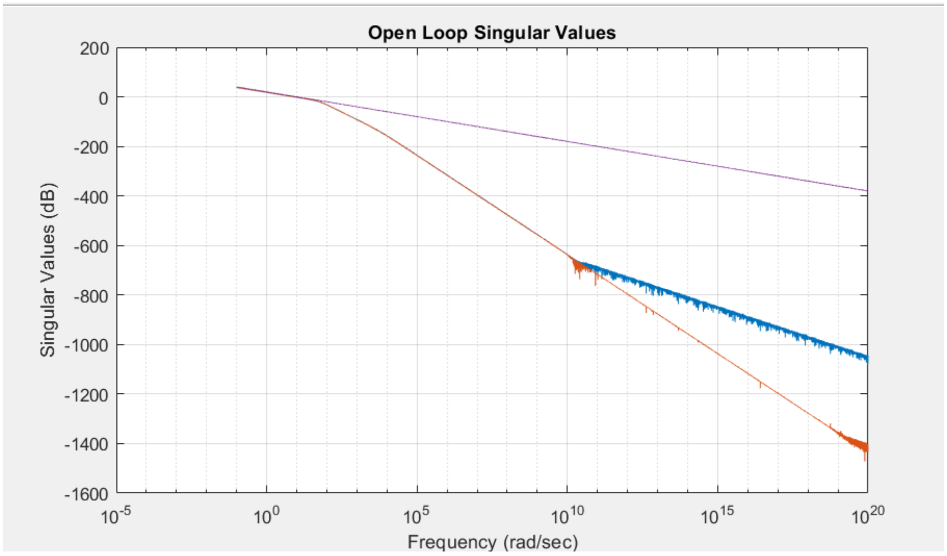


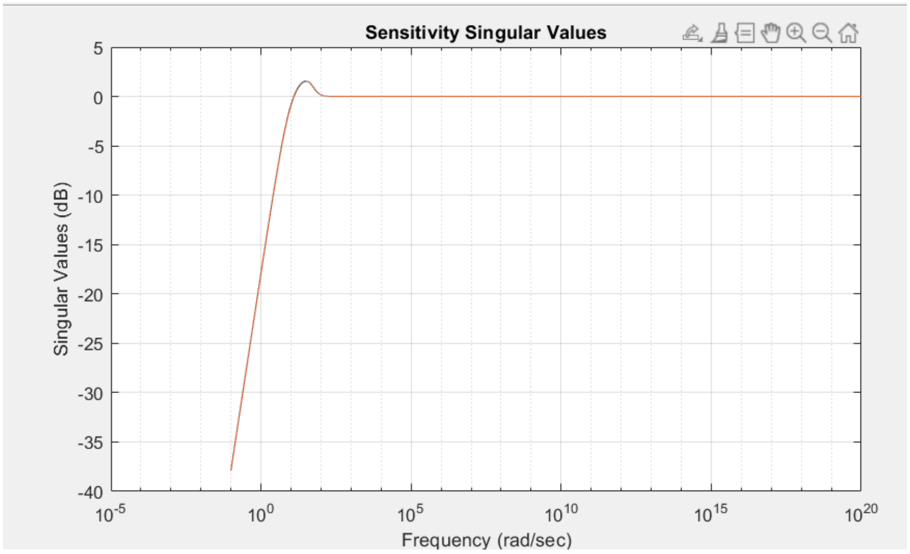

Filter Open Loop (G_{FOL}) Singular Values
Singular Values (dB)
Frequency (rad/sec)
50
0
-50
-100
-150
-200
-250
-300
-350
-400
10^{-5}
10^{0}
10^{5}
10^{10}
10^{15}
10^{20}


Target Loop (G_{KF}) Singular Values
Singular Values (dB)
Frequency (rad/sec)
50
0
-50
-100
-150
-200
-250
-300
-350
-400
10^{-5}
10^{0}
10^{5}
10^{10}
10^{15}
10^{20}


Target Complementary (T_{KF}) Singular Values
Singular Values (dB)
Frequency (rad/sec)
50
0
-50
-100
-150
-200
-250
-300
-350
-400
10^{-5}
10^{0}
10^{5}
10^{10}
10^{15}
10^{20}


Compensator Singular Values
Singular Values (dB)
Frequency (rad/sec)


Open Loop Singular Values
Singular Values (dB)
Frequency (rad/sec)


Sensitivity Singular Values
Singular Values (dB)
Frequency (rad/sec)

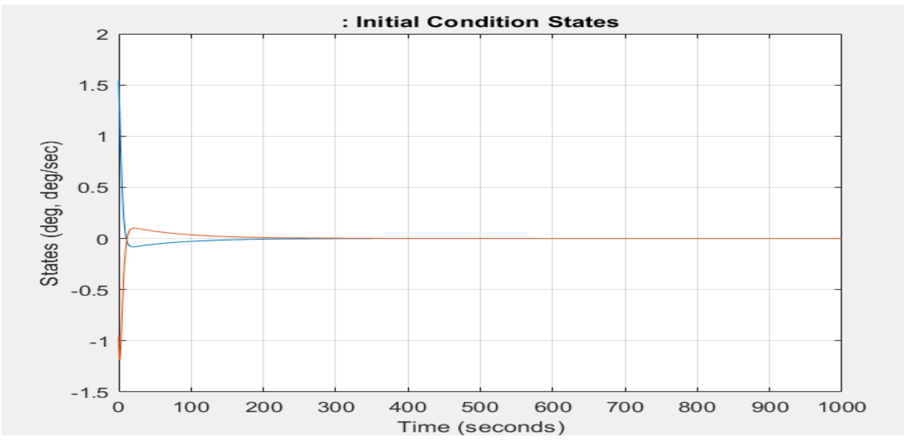
Case 2: d = 1 (in meters)

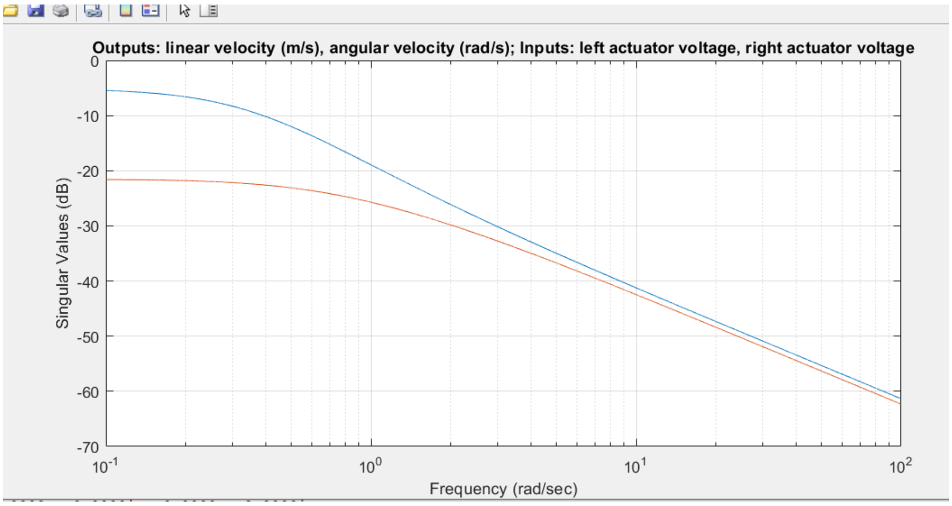
Nominal Plant

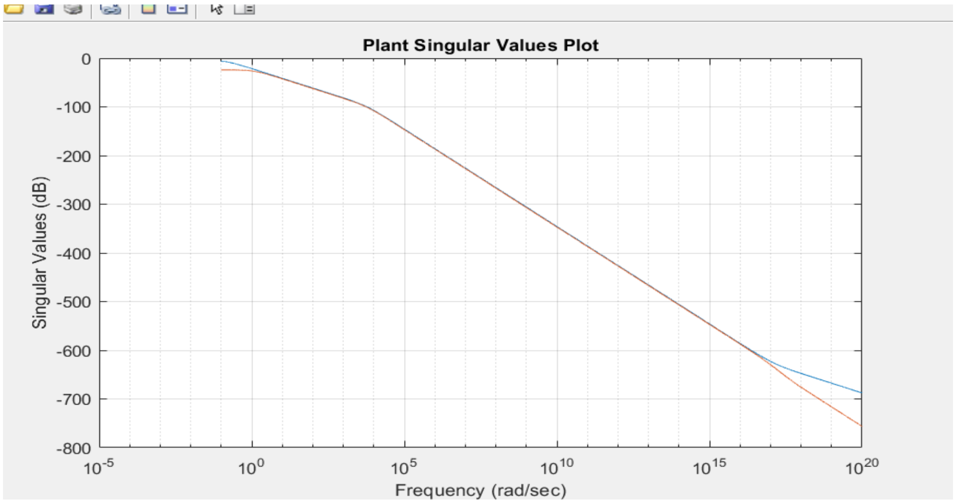
```
lant =

 From input 1 to output...
              321.99 (s+0.004514) (s+0.9241)
  1:  ---------------------------------------------
      (s+0.01355) (s+5957) (s^2 + 0.7076s + 0.1368)

               360.7 (s+0.004514) (s-0.6316)
  2:  ---------------------------------------------
      (s+0.01355) (s+5957) (s^2 + 0.7076s + 0.1368)

 From input 2 to output...
               321.99 (s+0.491) (s+0.004514)
  1:  ---------------------------------------------
      (s+0.01355) (s+5957) (s^2 + 0.7076s + 0.1368)

              -360.7 (s+0.6316) (s+0.004514)
  2:  ---------------------------------------------
      (s+0.01355) (s+5957) (s^2 + 0.7076s + 0.1368)

ontinuous-time zero/pole/gain model.
```

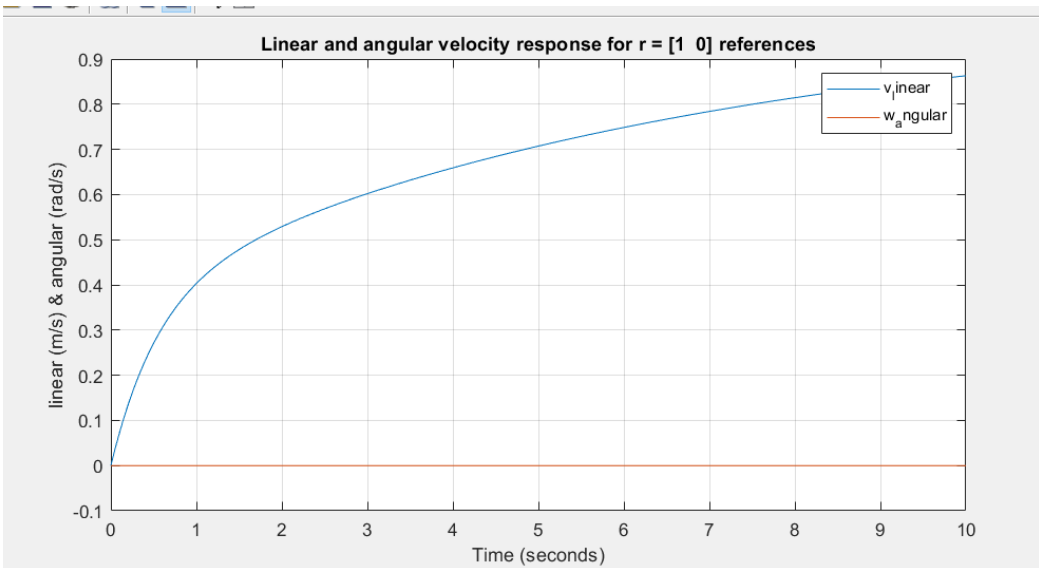
Eigen Values:

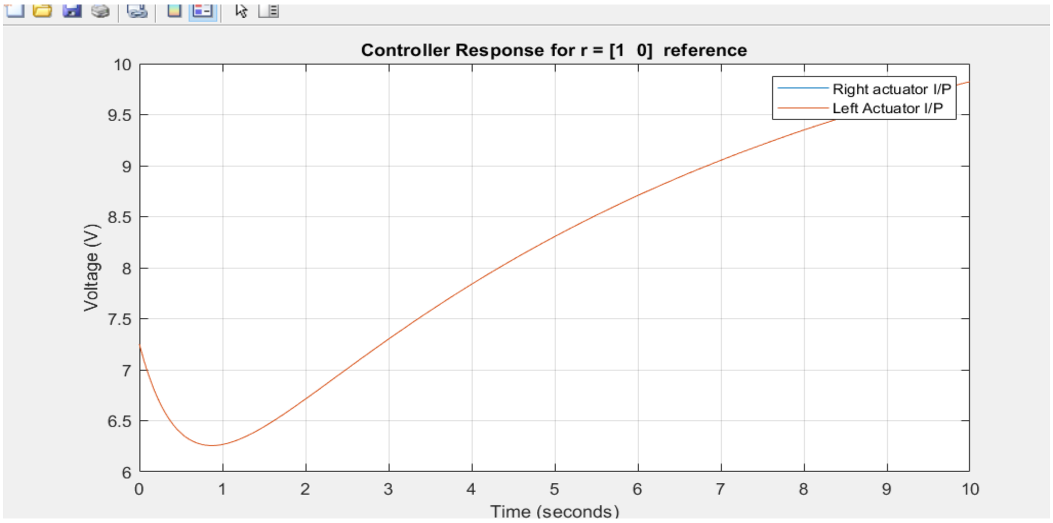
```
eig_val =

  1.0e+03 *

Columns 1 through 5

-0.0004 + 0.0001i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
 0.0000 + 0.0000i  -0.0004 - 0.0001i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
 0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
 0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i
 0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -5.9565 + 0.0000i
 0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

Column 6

 0.0000 + 0.0000i
 0.0000 + 0.0000i
 0.0000 + 0.0000i
 0.0000 + 0.0000i
 0.0000 + 0.0000i
-5.9565 + 0.0000i
```

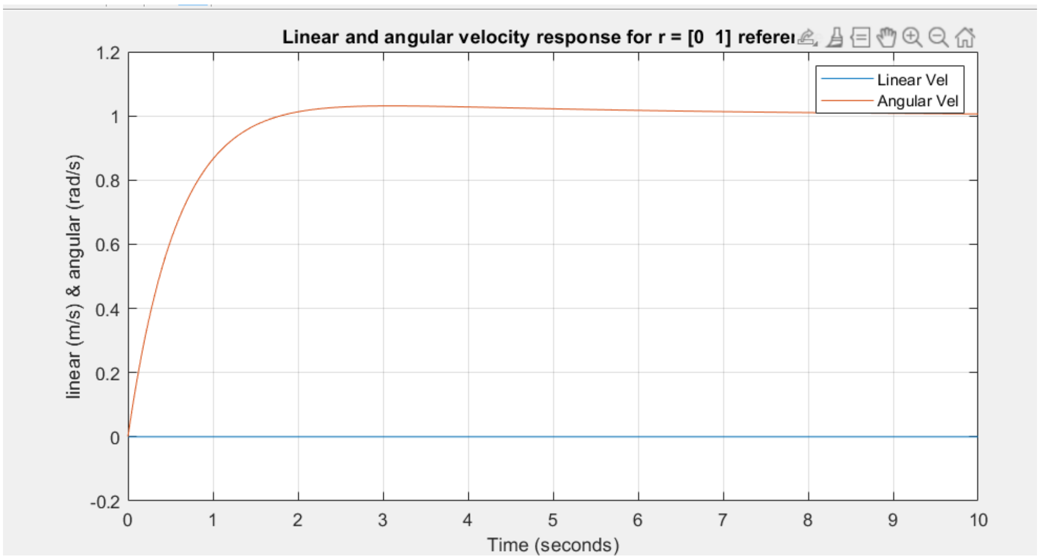


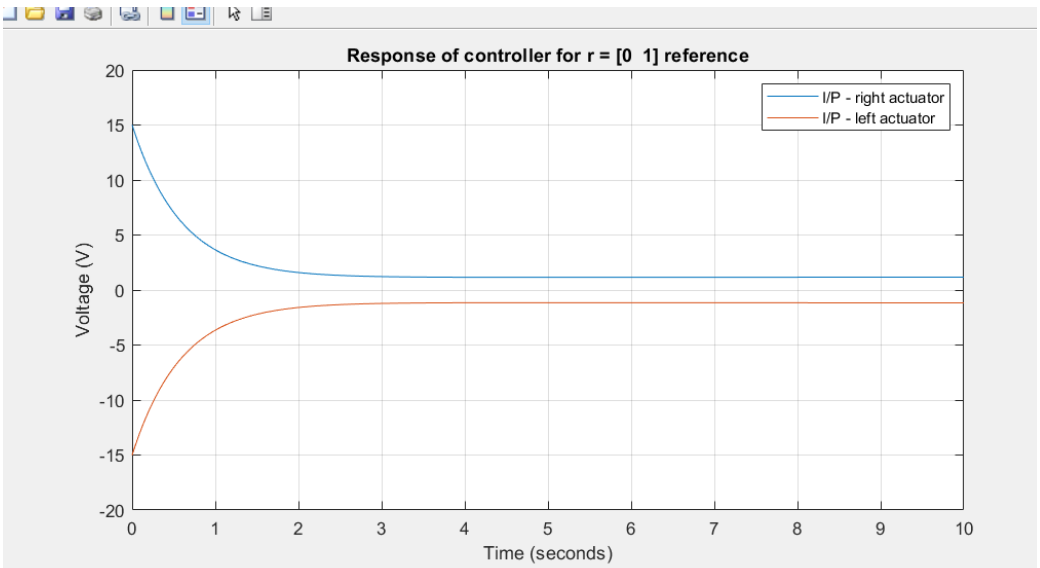

Outputs: linear velocity (m/s), angular velocity (rad/s); Inputs: left actuator voltage, right actuator voltage
Singular Values (dB)
Frequency (rad/sec)

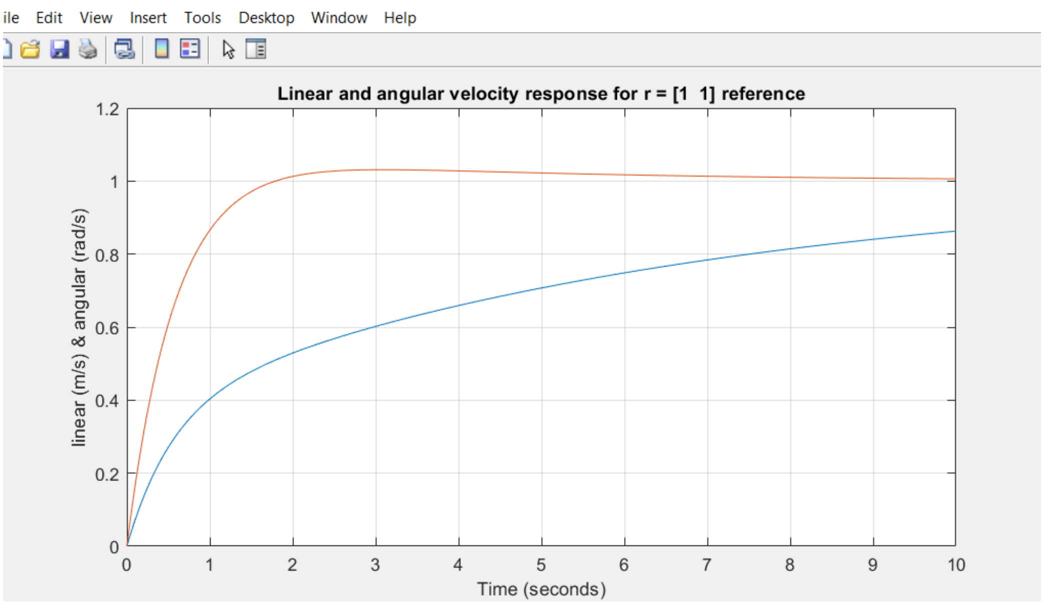

Plant Singular Values Plot
Singular Values (dB)
Frequency (rad/sec)

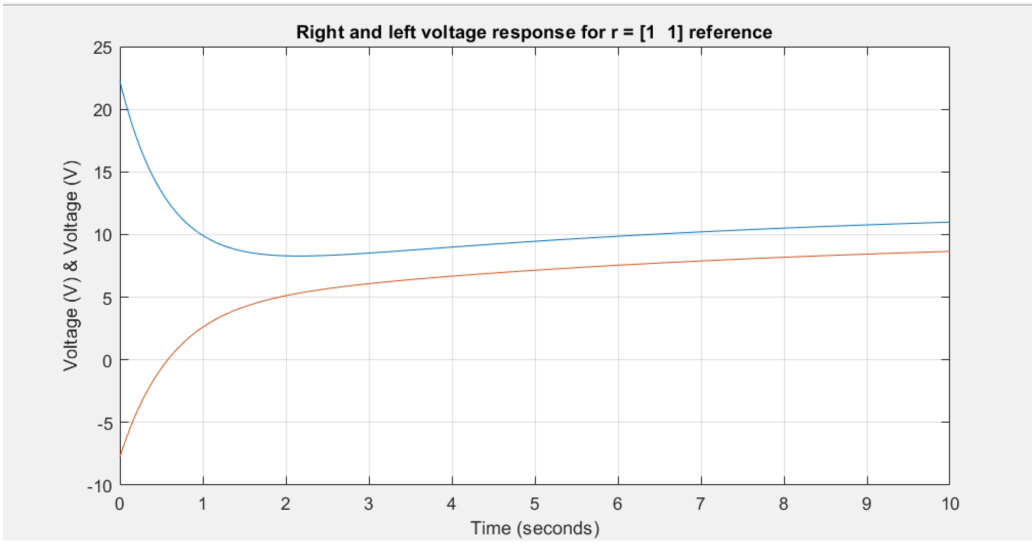

Linear and angular velocity response for r = [1 0] references
v_linear
w_angular
linear (m/s) & angular (rad/s)
Time (seconds)

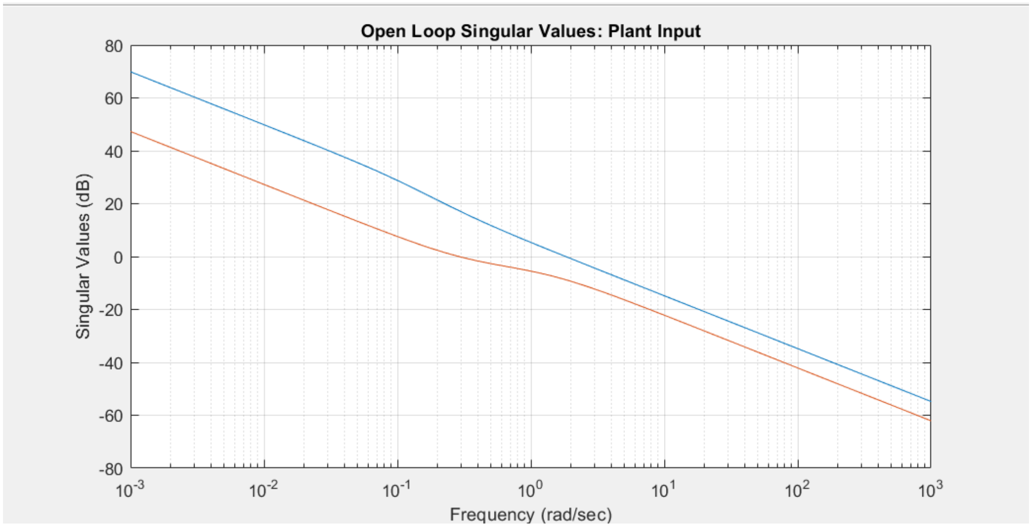

Controller Response for r = [1 0] reference
Right actuator I/P
Left Actuator I/P
Voltage (V)
Time (seconds)
10
9.5
9
8.5
8
7.5
7
6.5
6
0
1
2
3
4
5
6
7
8
9
10

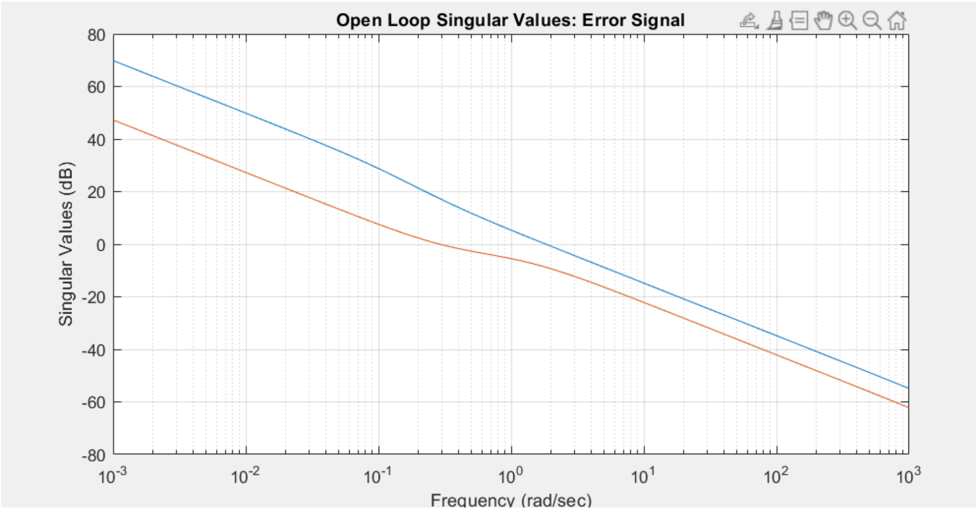

Linear and angular velocity response for r = [0 1] referei
Linear Vel
Angular Vel
linear (m/s) & angular (rad/s)
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
6
7
8
9
10

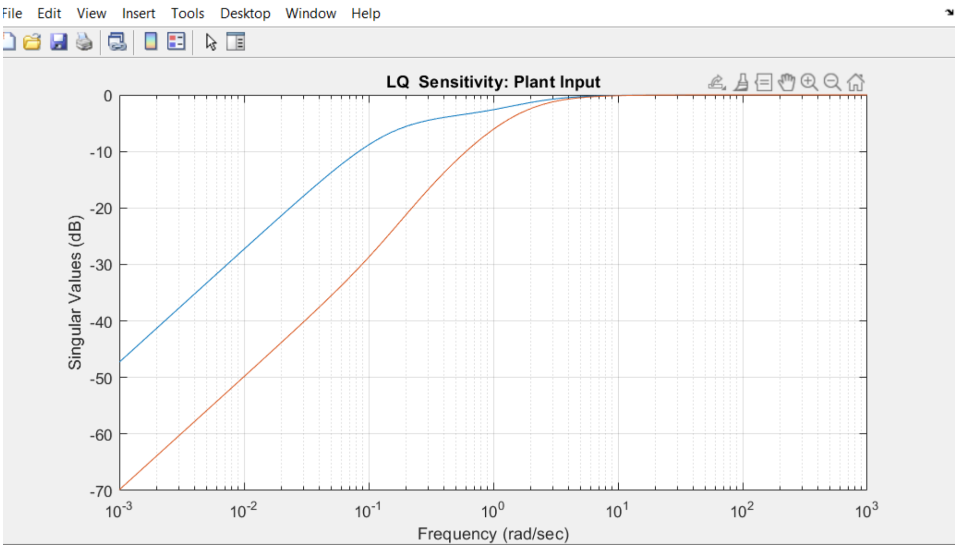

Response of controller for r = [0 1] reference
I/P - right actuator
I/P - left actuator
Voltage (V)
Time (seconds)
20
15
10
5
0
-5
-10
-15
-20
0
1
2
3
4
5
6
7
8
9
10

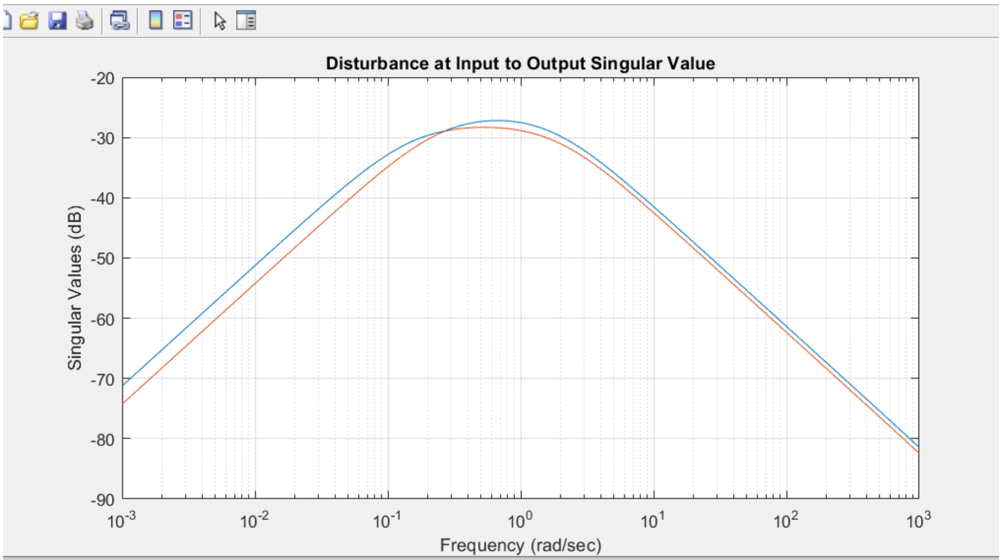

ile Edit View Insert Tools Desktop Window Help
Linear and angular velocity response for r = [1 1] reference
linear (m/s) & angular (rad/s)
Time (seconds)

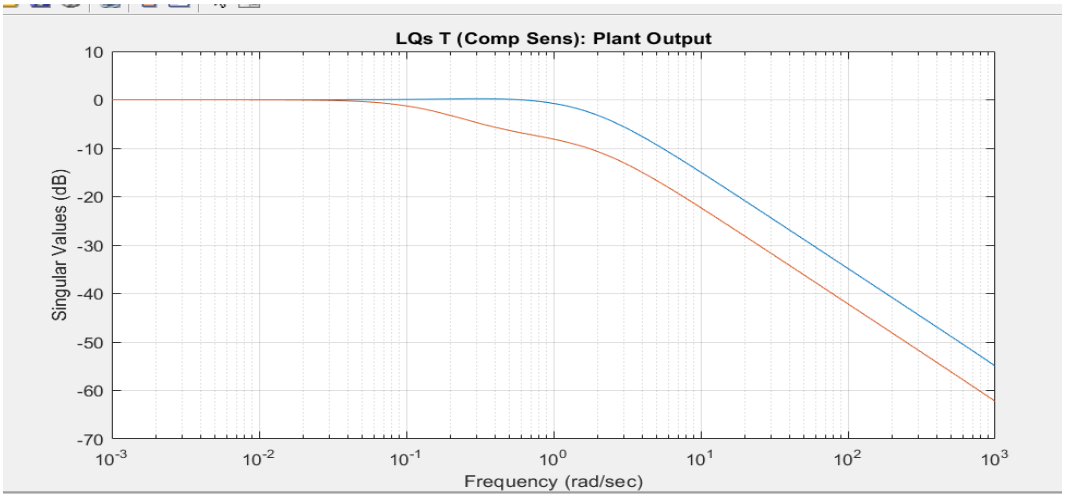

Right and left voltage response for r = [1 1] reference
Voltage (V) & Voltage (V)
Time (seconds)

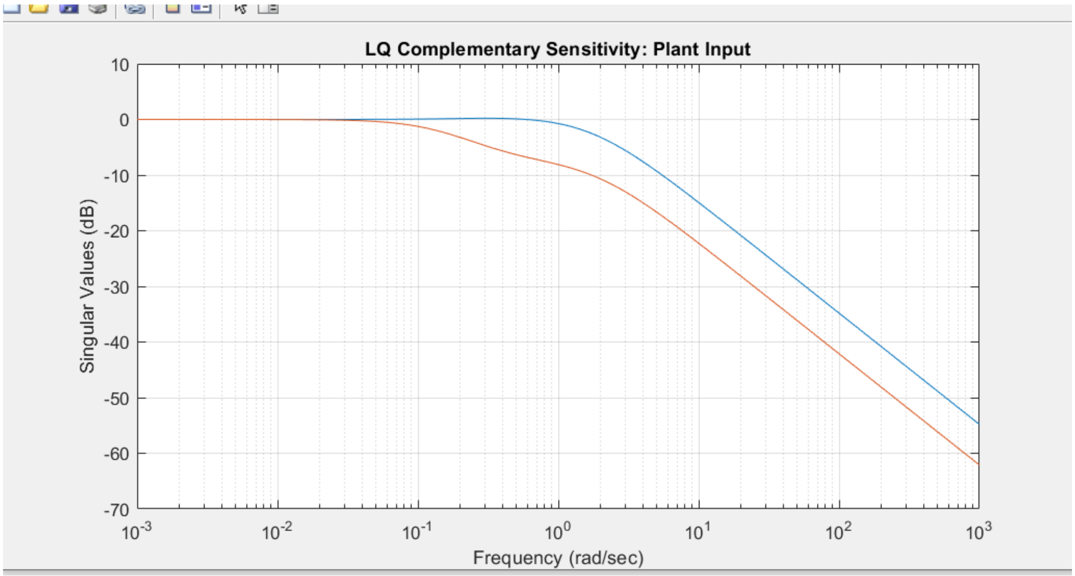

Open Loop Singular Values: Plant Input
Singular Values (dB)
Frequency (rad/sec)


Open Loop Singular Values: Error Signal
Singular Values (dB)
Frequency (rad/sec)

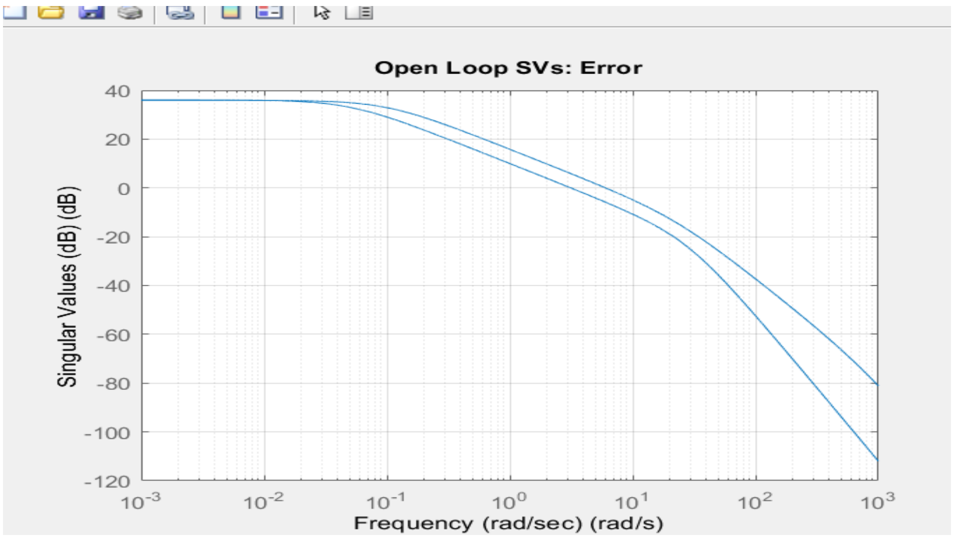

File Edit View Insert Tools Desktop Window Help
LQ Sensitivity: Plant Input
Singular Values (dB)
Frequency (rad/sec)

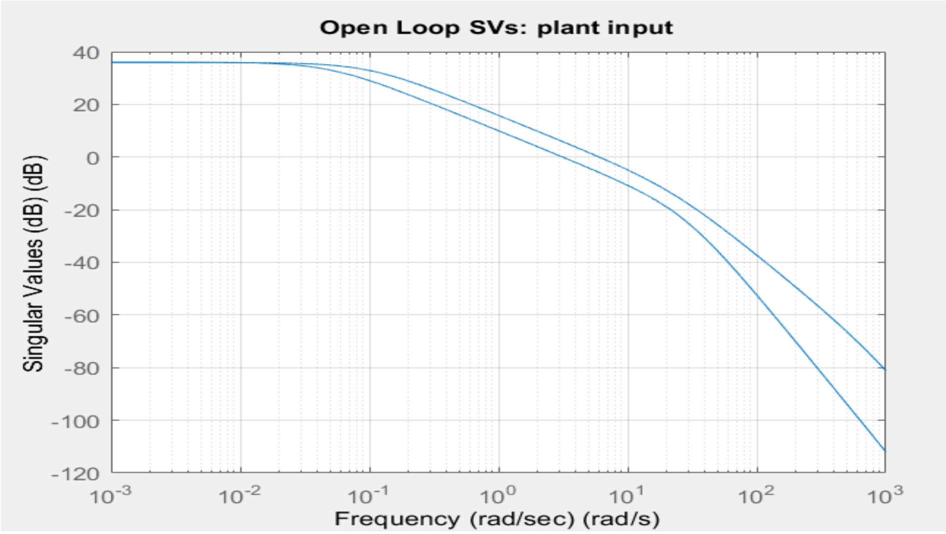

Disturbance at Input to Output Singular Value
Singular Values (dB)
Frequency (rad/sec)

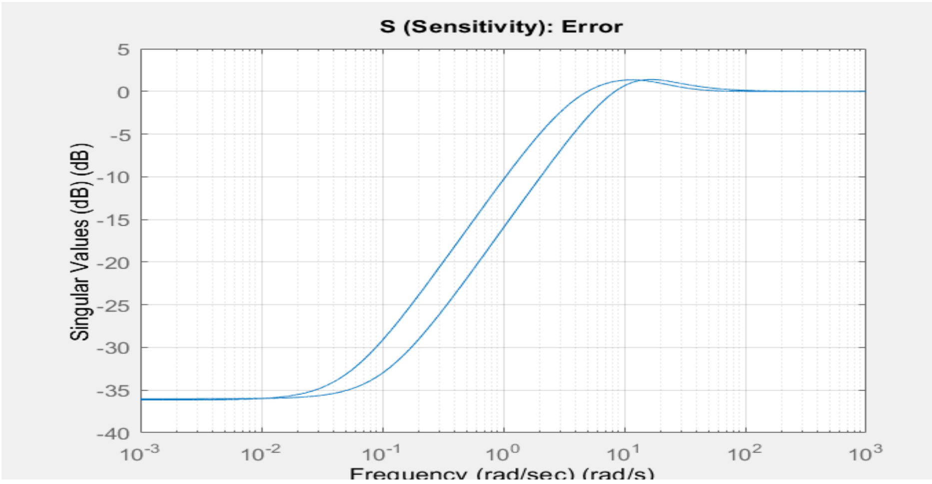


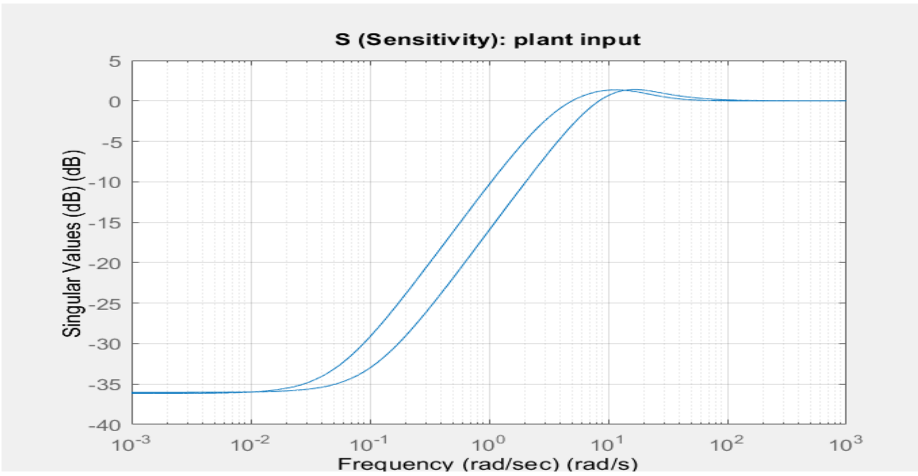


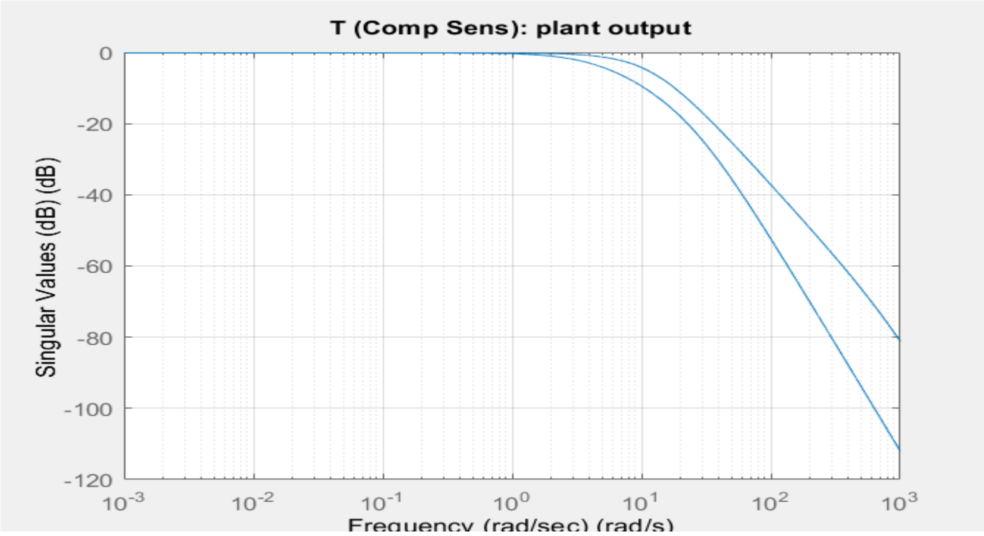
H INFINITY CONTROLLER

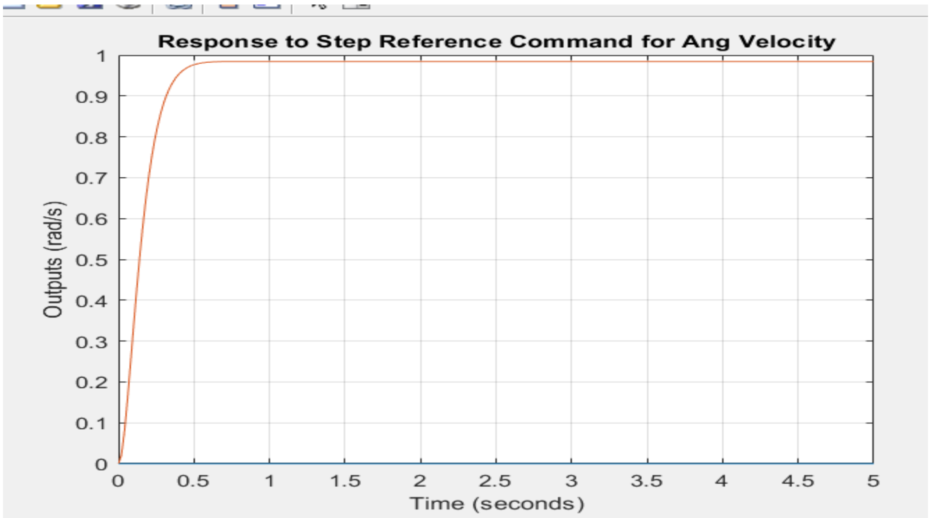


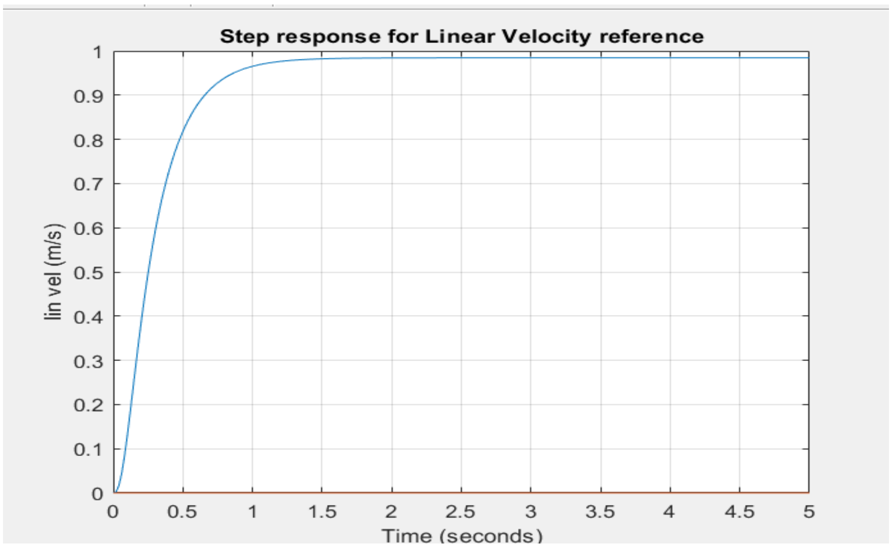

Open Loop SVs: plant input
Singular Values (dB) (dB)
Frequency (rad/sec) (rad/s)


S (Sensitivity): Error
Singular Values (dB) (dB)
Frequency (rad/sec) (rad/s)

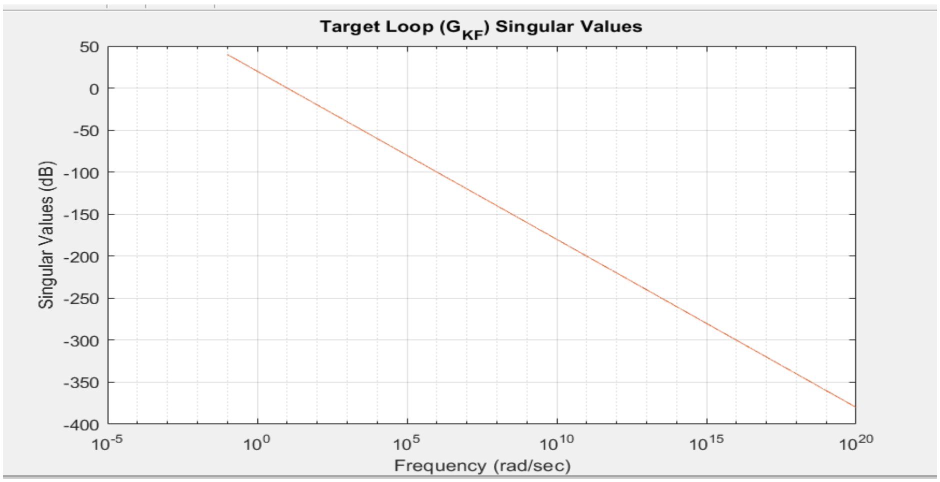

S (Sensitivity): plant input
Singular Values (dB) (dB)
Frequency (rad/sec) (rad/s)

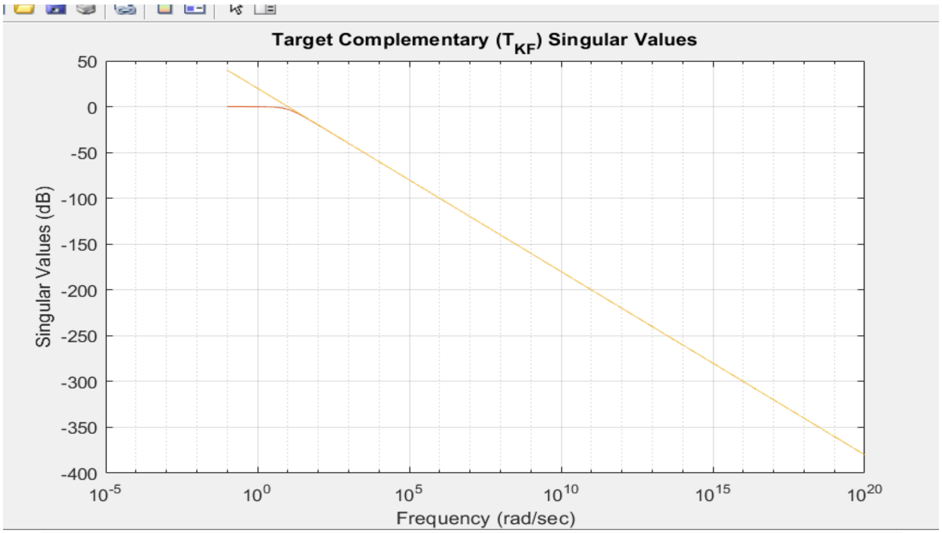

T (Comp Sens): plant output
Singular Values (dB) (dB)
Frequency (rad/sec) (rad/s)

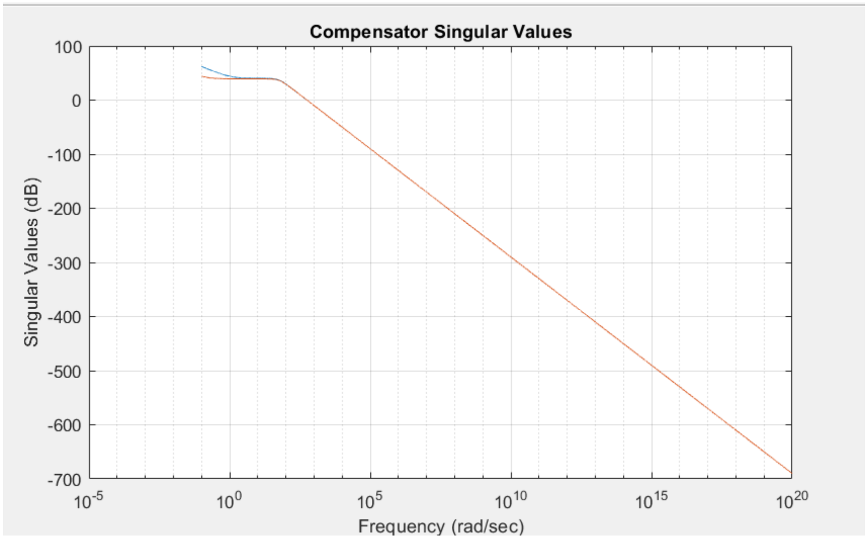

Response to Step Reference Command for Ang Velocity
Outputs (rad/s)
Time (seconds)

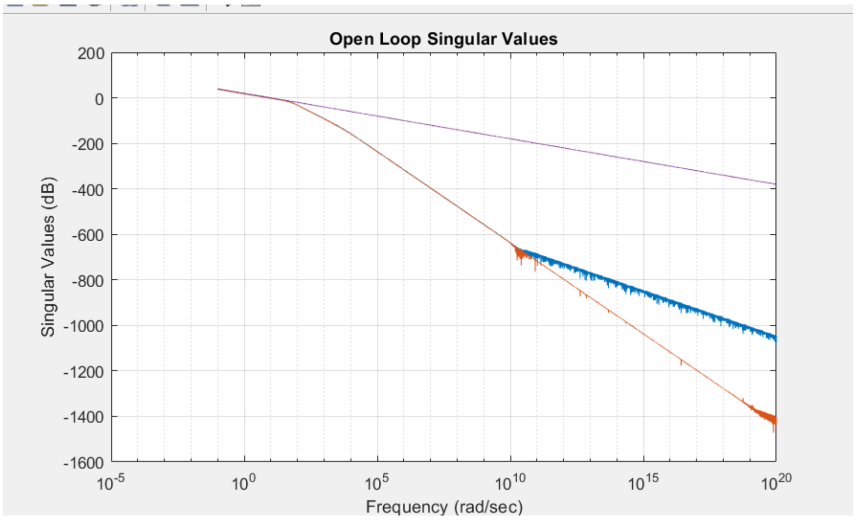

Step response for Linear Velocity reference
lin vel (m/s)
Time (seconds)

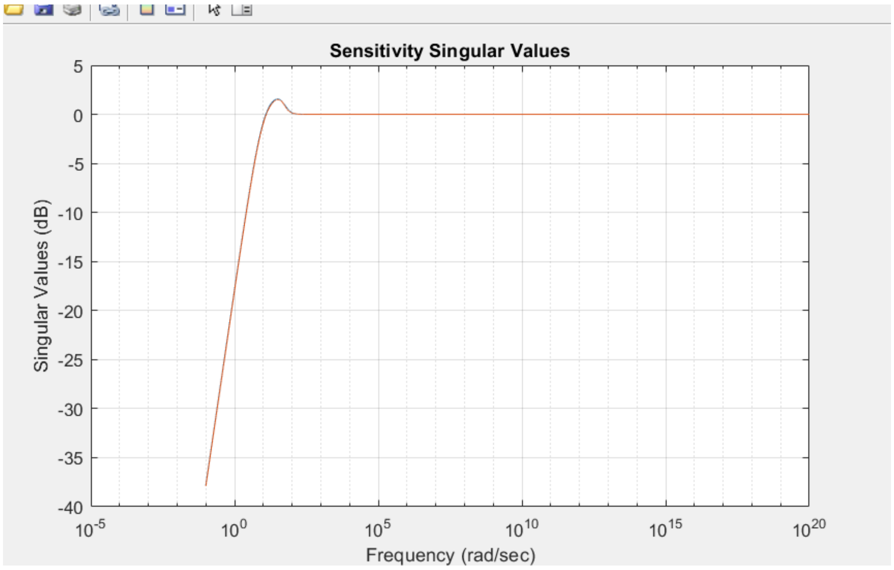
LQG / LTRO Controller











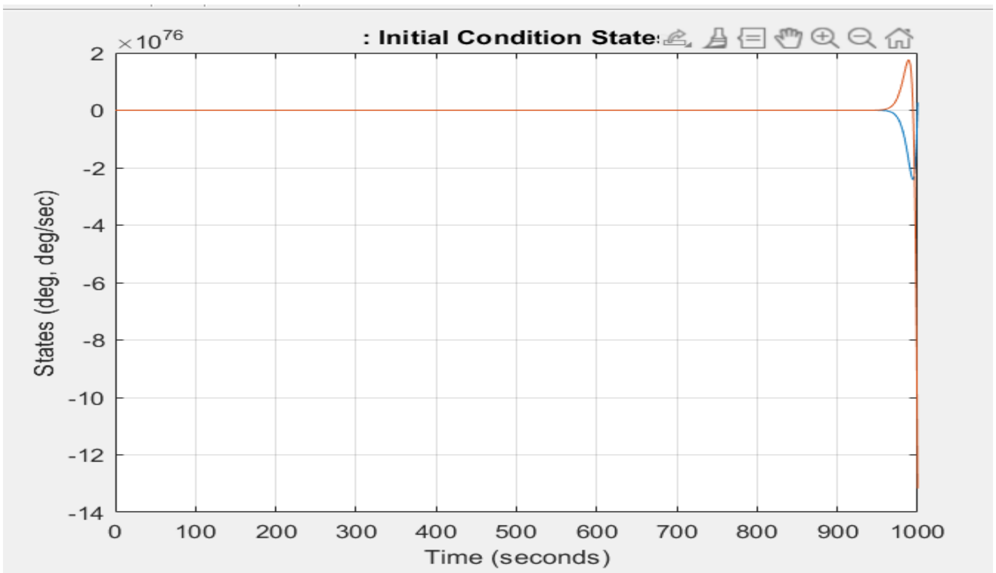
Case 3: d = -0.5 (in meters) – THE UNSTABLE CASE

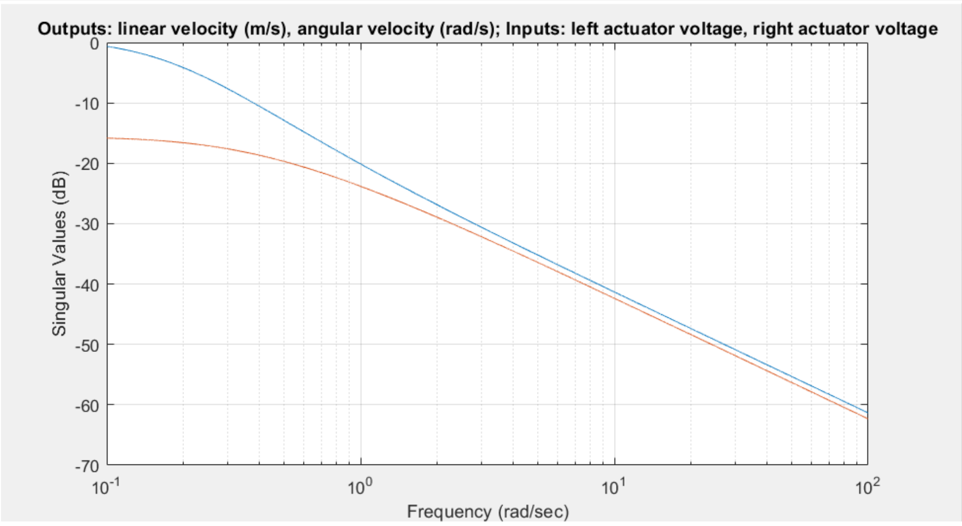
Nominal Plant

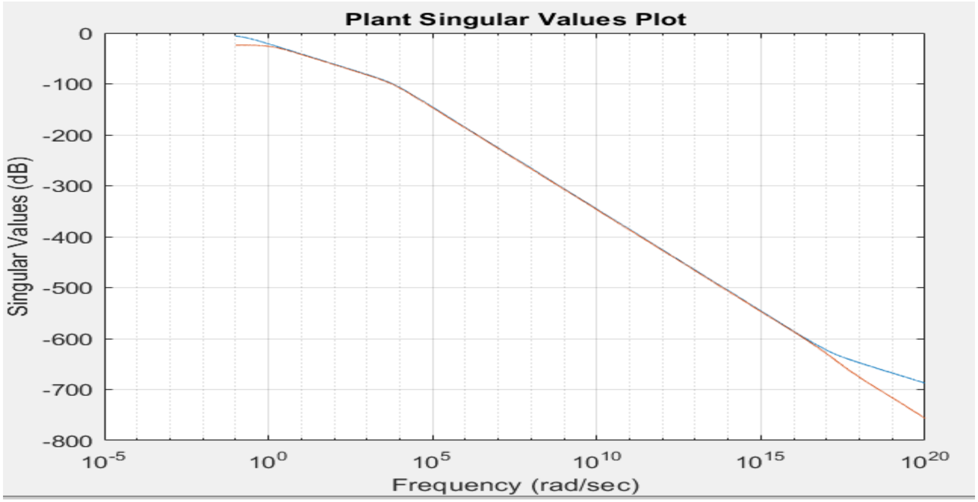
```
Plant =

  From input 1 to output...
                321.99 (s+0.004514) (s-0.462)
   1:  ---------------------------------------------
       (s+0.01355) (s+5957) (s^2 - 0.3538s + 0.03419)

                360.7 (s+0.004514) (s+0.3158)
   2:  ---------------------------------------------
       (s+0.01355) (s+5957) (s^2 - 0.3538s + 0.03419)

  From input 2 to output...
                321.99 (s+0.004514) (s-0.2455)
   1:  ---------------------------------------------
       (s+0.01355) (s+5957) (s^2 - 0.3538s + 0.03419)

                -360.7 (s-0.3158) (s+0.004514)
   2:  ---------------------------------------------
       (s+0.01355) (s+5957) (s^2 - 0.3538s + 0.03419)

Continuous-time zero/pole/gain model.
```

Eigen Values (Normal Modes)

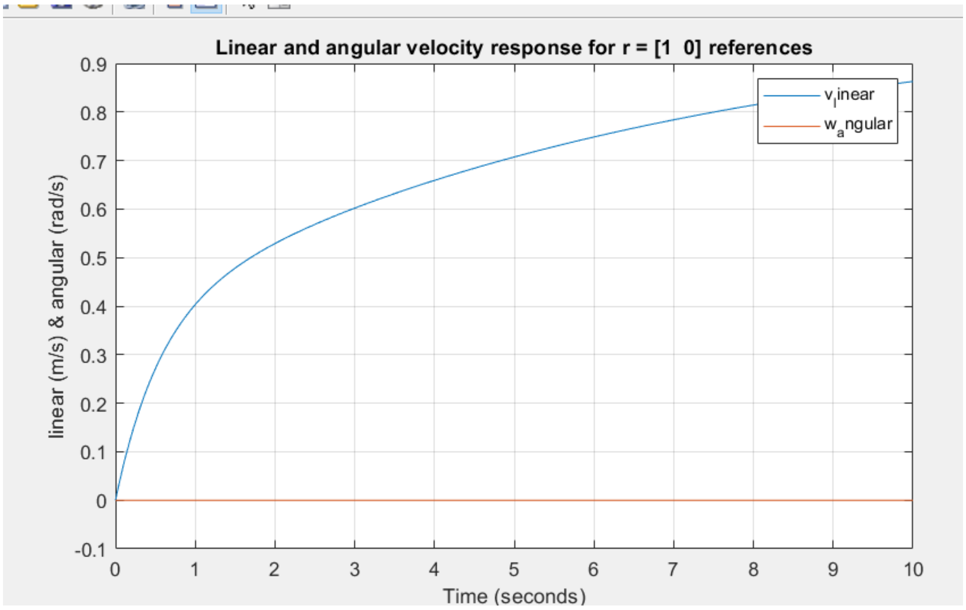
```
eig_val =

   1.0e+03 *

   0.0002 + 0.0001i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0002 - 0.0001i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -5.9565 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -5.9565 + 0.0000i
```



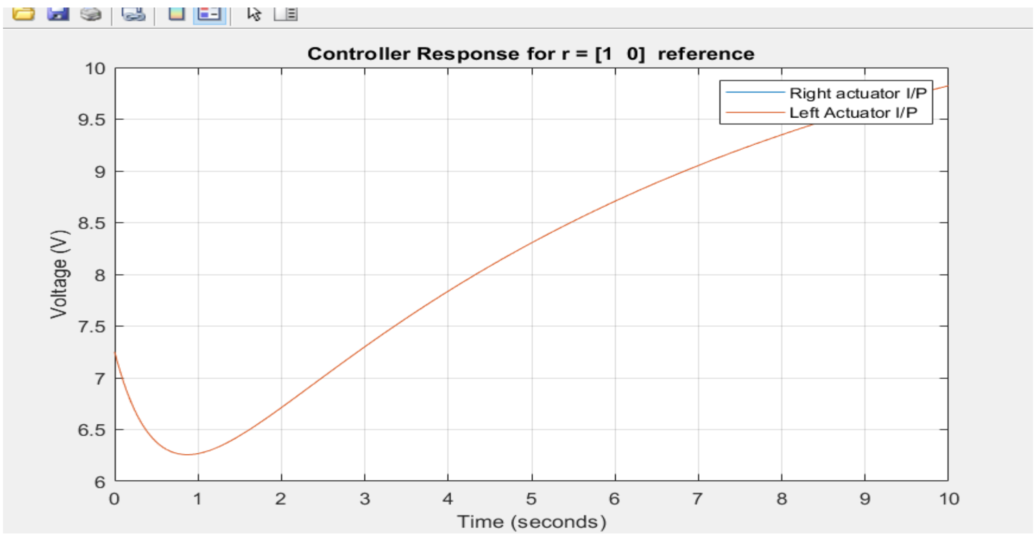




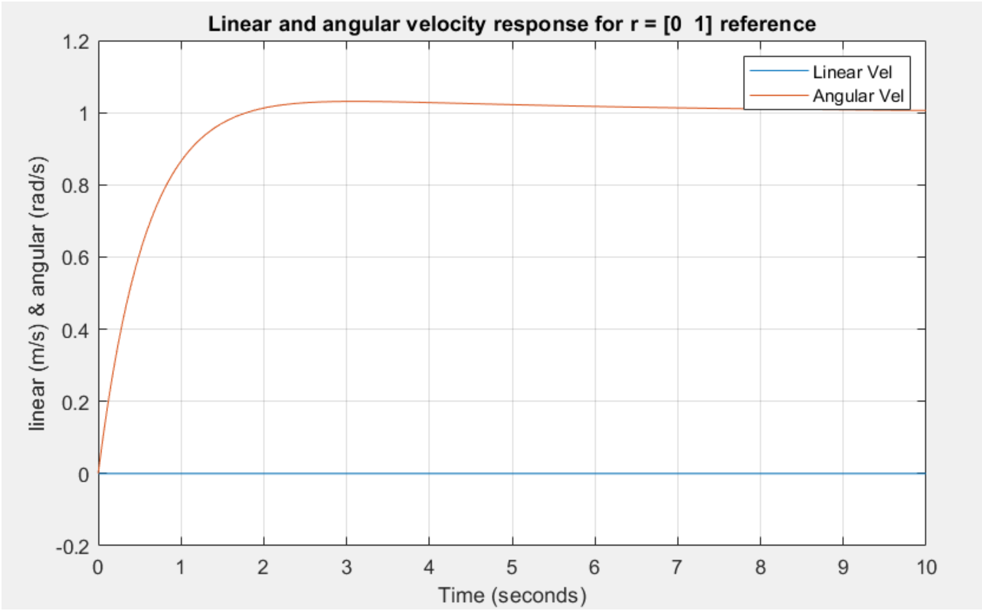
Plant Singular Values Plot
Singular Values (dB)
Frequency (rad/sec)

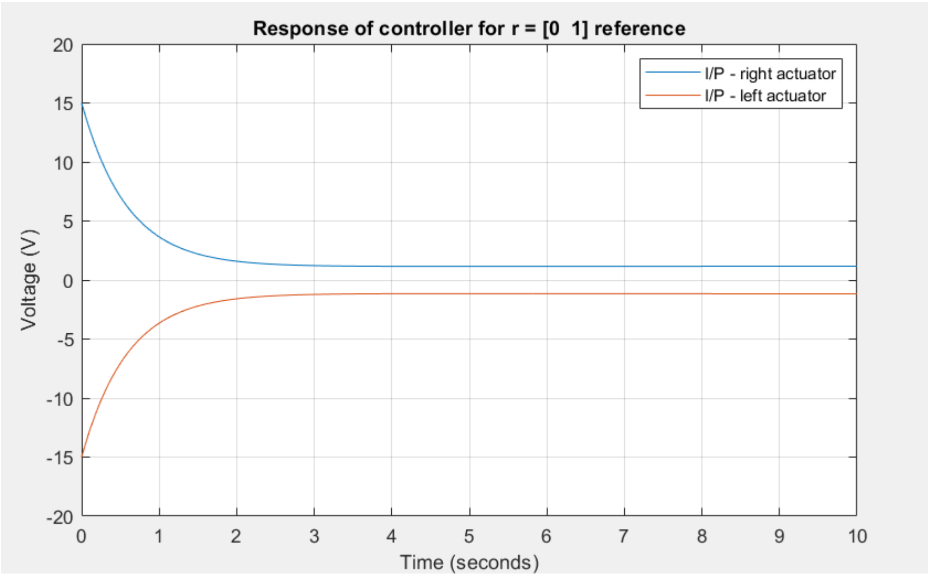


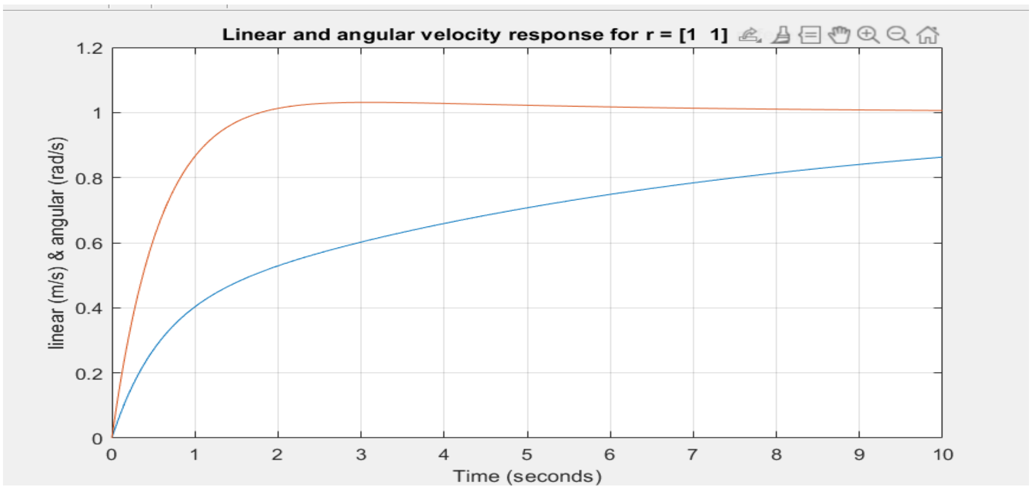
Linear and angular velocity response for r = [1 0] references
linear (m/s) & angular (rad/s)
Time (seconds)
v_linear
w_angular

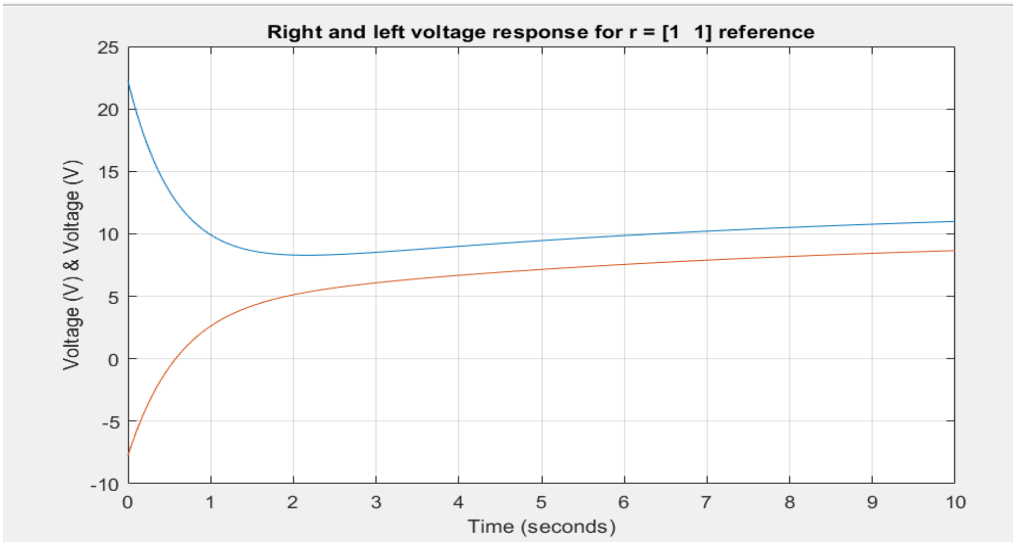


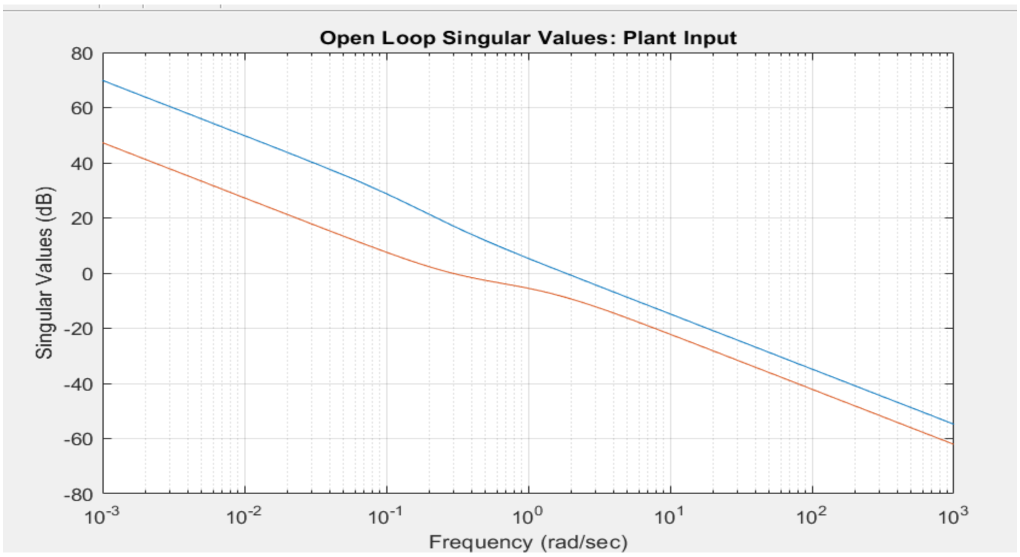
Controller Response for r = [1 0] reference
Voltage (V)
Time (seconds)
Right actuator I/P
Left Actuator I/P

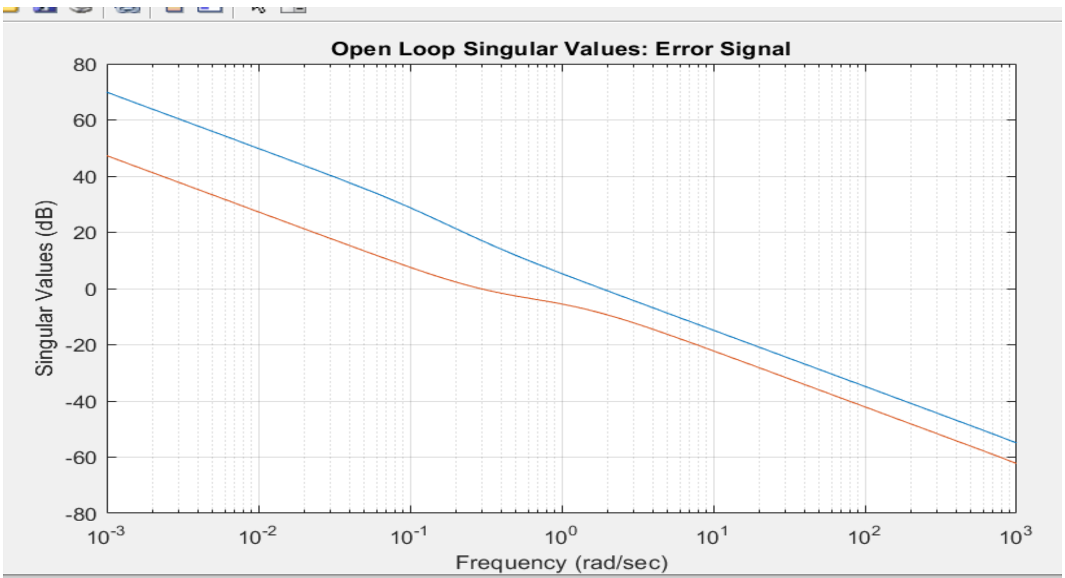

Linear and angular velocity response for r = [0 1] reference
Linear Vel
Angular Vel
linear (m/s) & angular (rad/s)
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
6
7
8
9
10

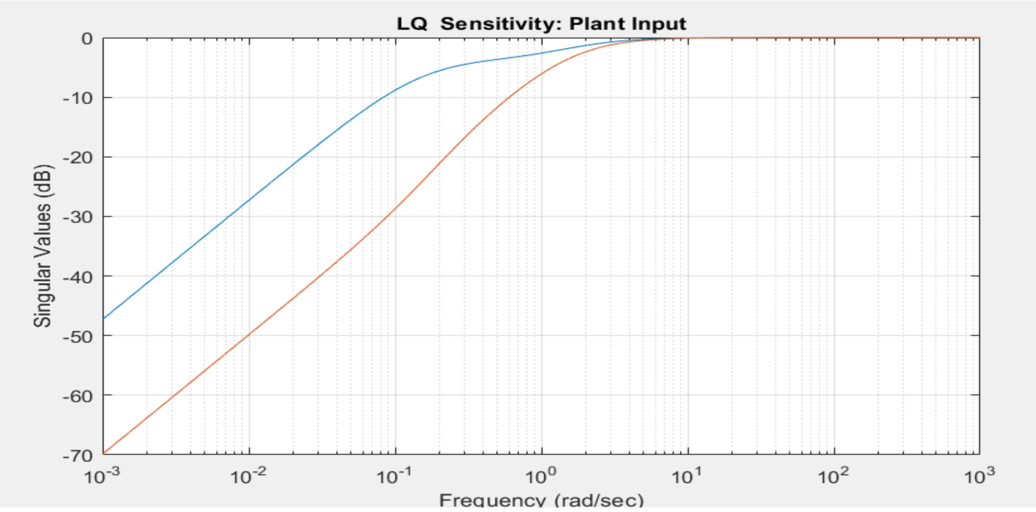

Response of controller for r = [0 1] reference
I/P - right actuator
I/P - left actuator
Voltage (V)
Time (seconds)
20
15
10
5
0
-5
-10
-15
-20
0
1
2
3
4
5
6
7
8
9
10

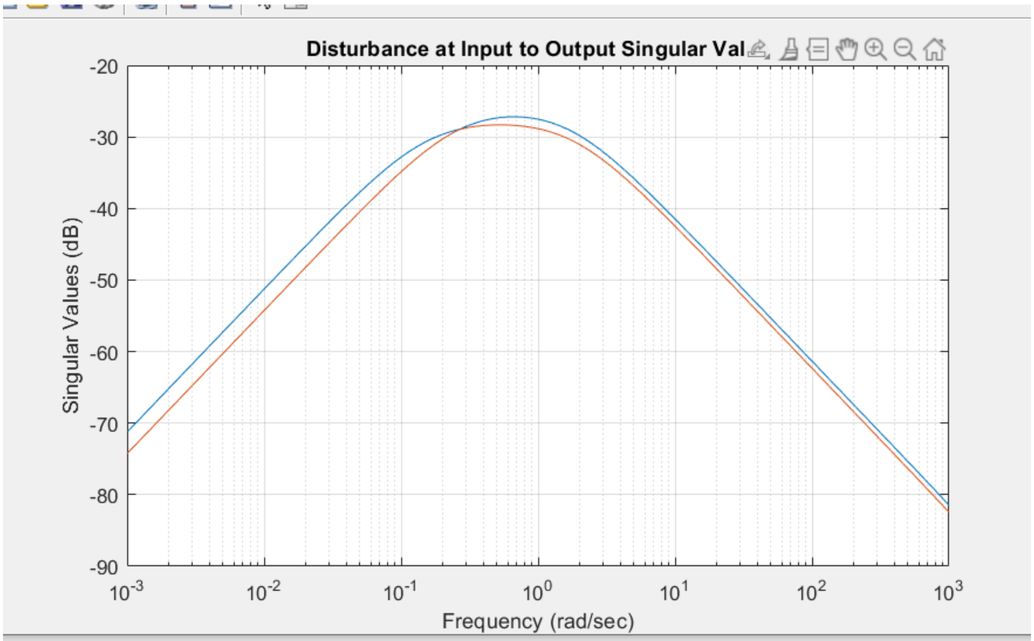

Linear and angular velocity response for r = [1 1]
linear (m/s) & angular (rad/s)
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10

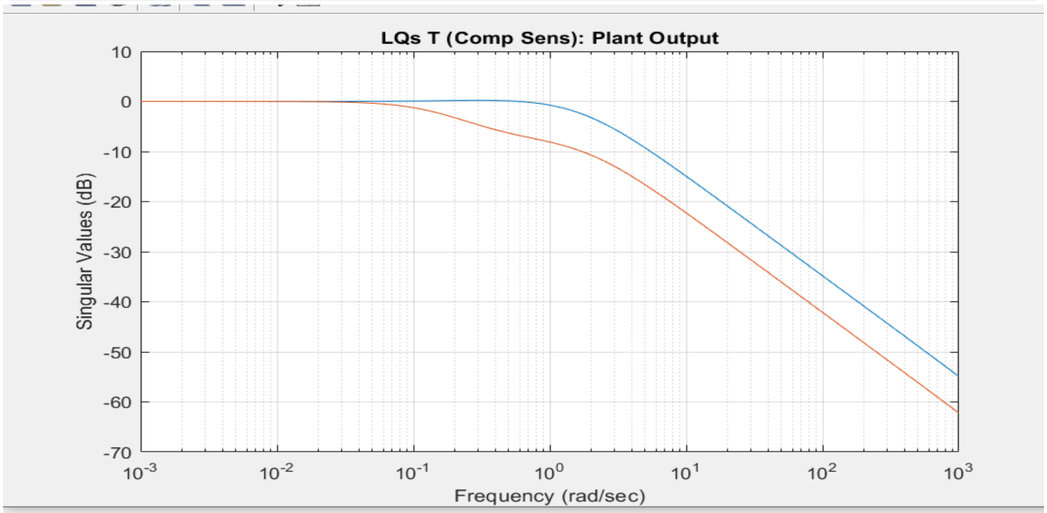

Right and left voltage response for r = [1 1] reference
Voltage (V) & Voltage (V)
Time (seconds)
25
20
15
10
5
0
-5
-10
0
1
2
3
4
5
6
7
8
9
10

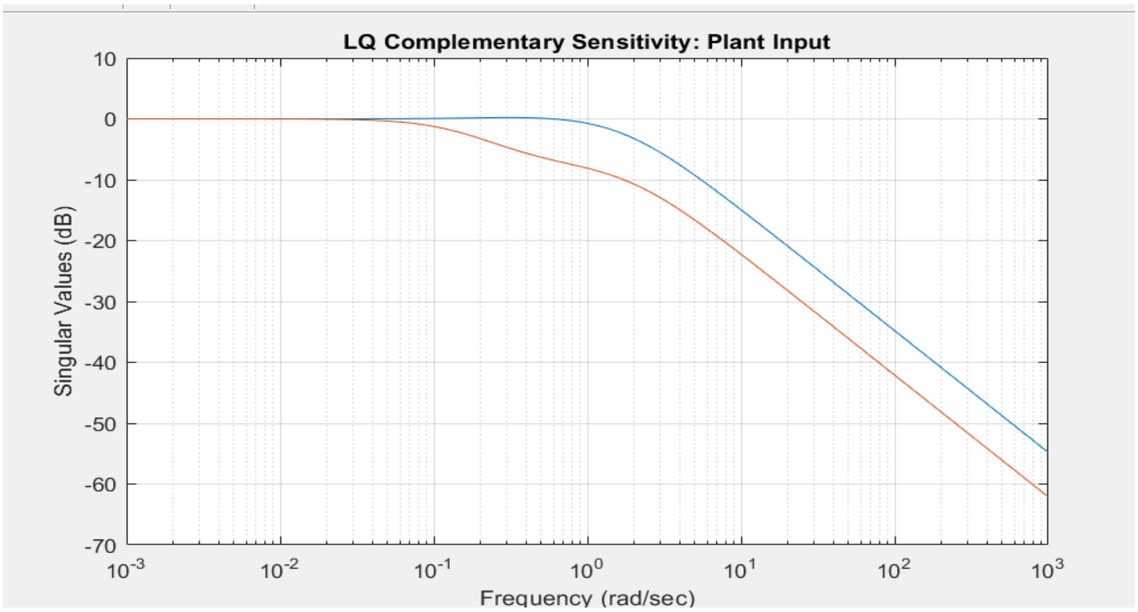

Open Loop Singular Values: Plant Input
Singular Values (dB)
Frequency (rad/sec)
80
60
40
20
0
-20
-40
-60
-80
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}


Open Loop Singular Values: Error Signal
Singular Values (dB)
Frequency (rad/sec)
80
60
40
20
0
-20
-40
-60
-80
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

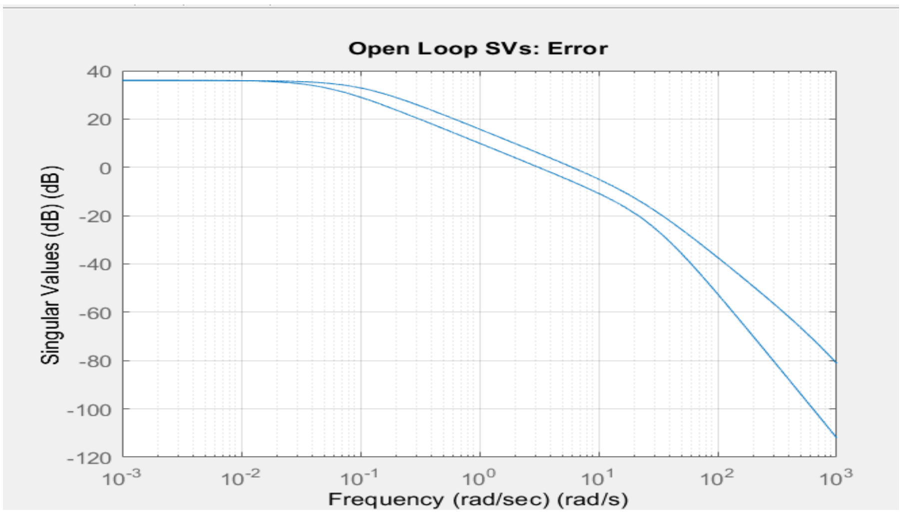

LQ Sensitivity: Plant Input
Singular Values (dB)
Frequency (rad/sec)

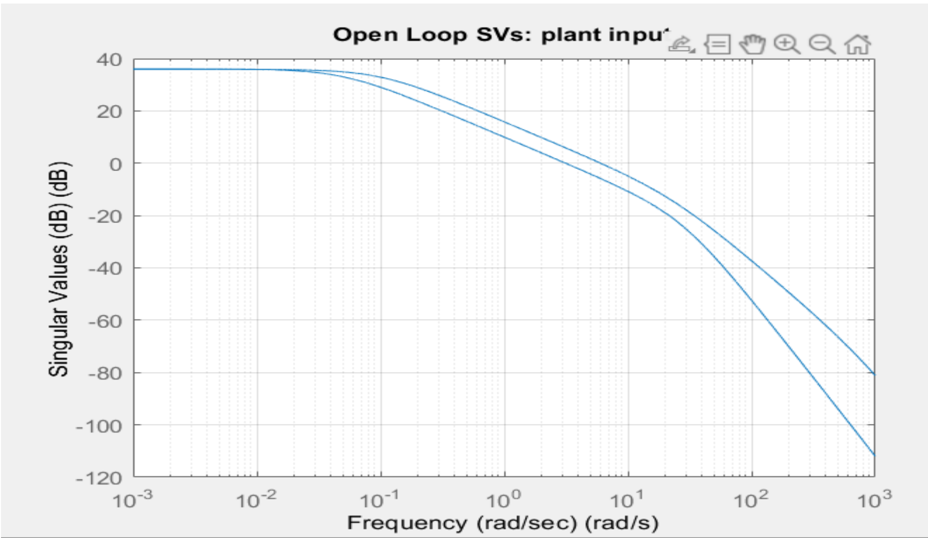

Disturbance at Input to Output Singular Val
Singular Values (dB)
Frequency (rad/sec)

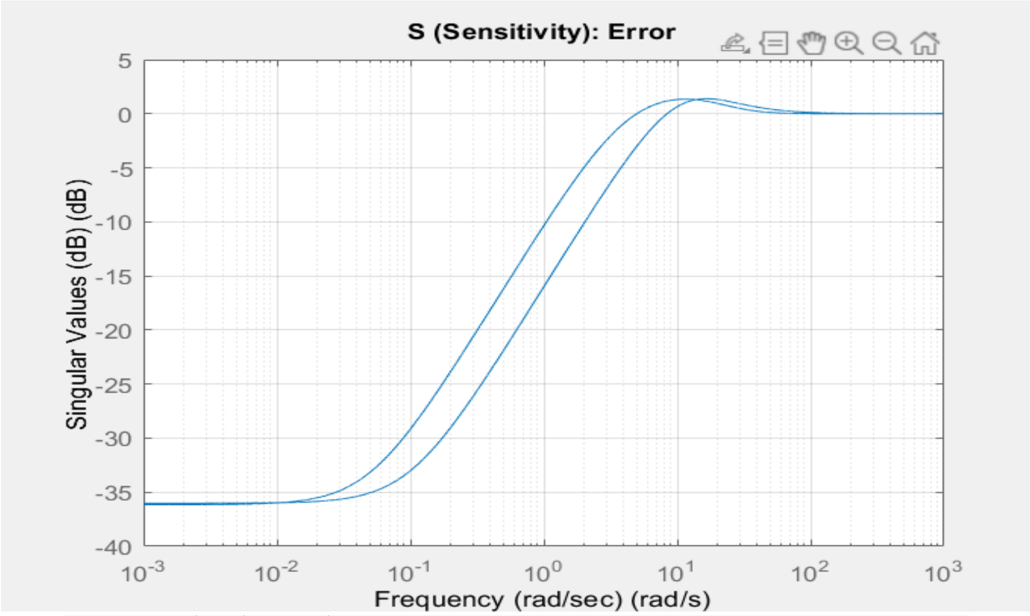

LQs T (Comp Sens): Plant Output
Singular Values (dB)
Frequency (rad/sec)

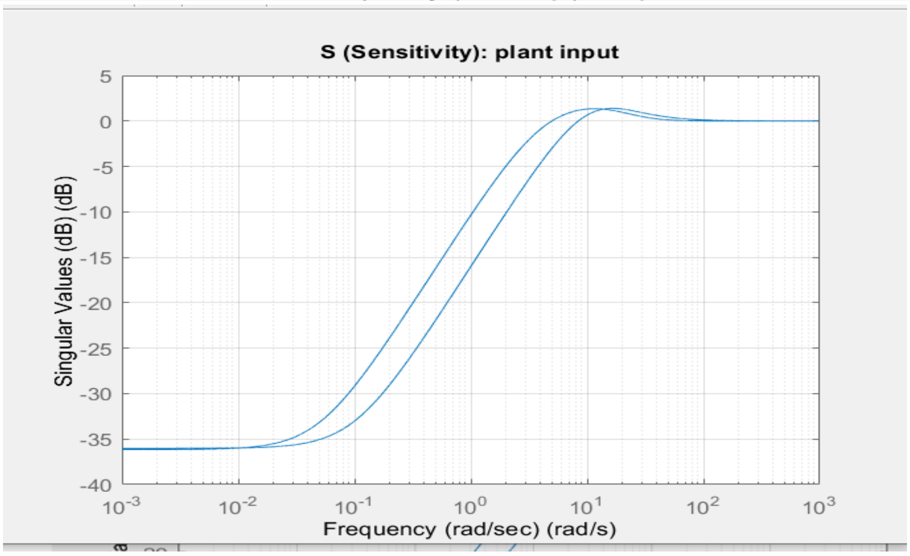


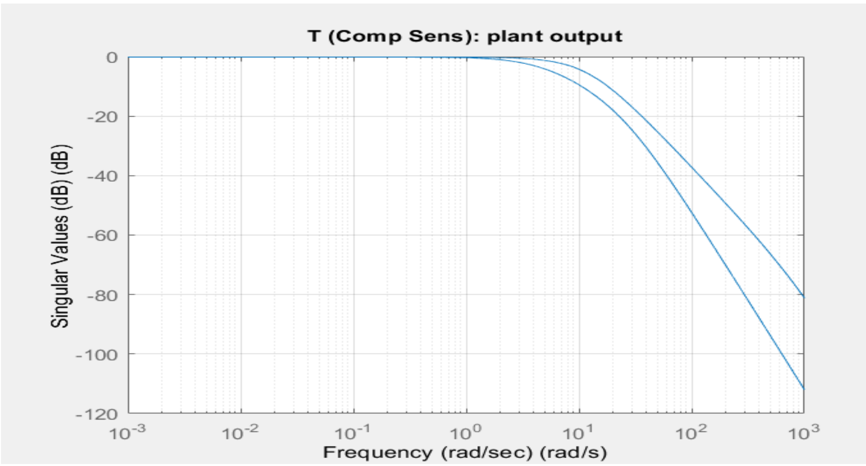
H Infinity Controller

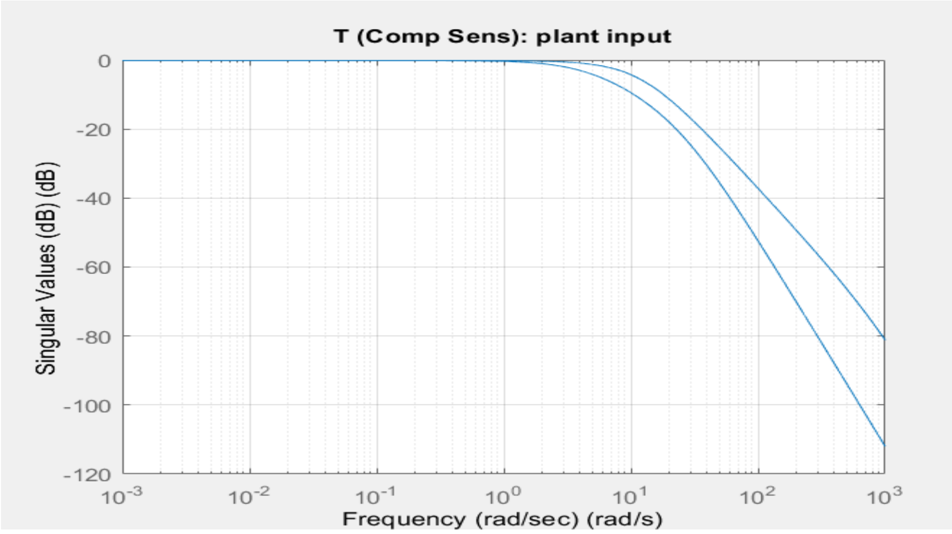


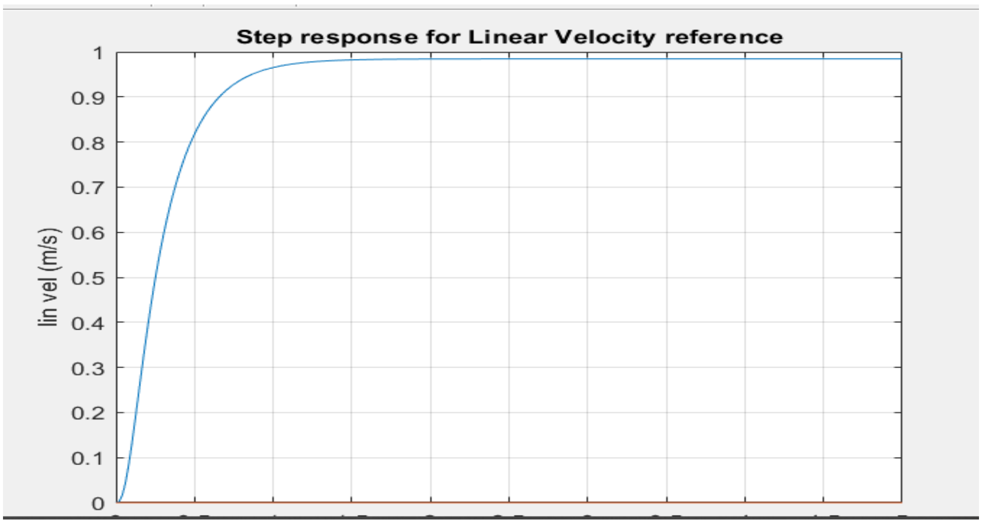


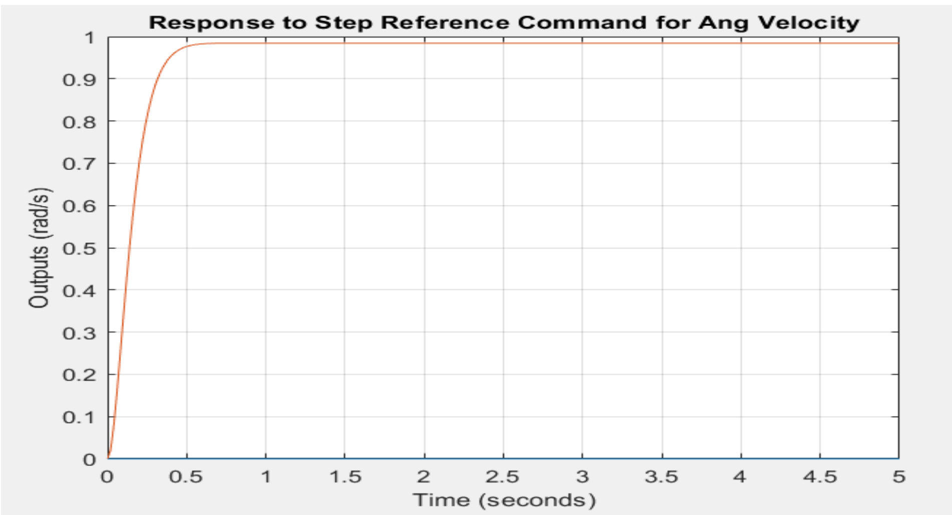

S (Sensitivity): Error
Singular Values (dB) (dB)
5
0
-5
-10
-15
-20
-25
-30
-35
-40
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec) (rad/s)


S (Sensitivity): plant input
Singular Values (dB) (dB)
5
0
-5
-10
-15
-20
-25
-30
-35
-40
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec) (rad/s)

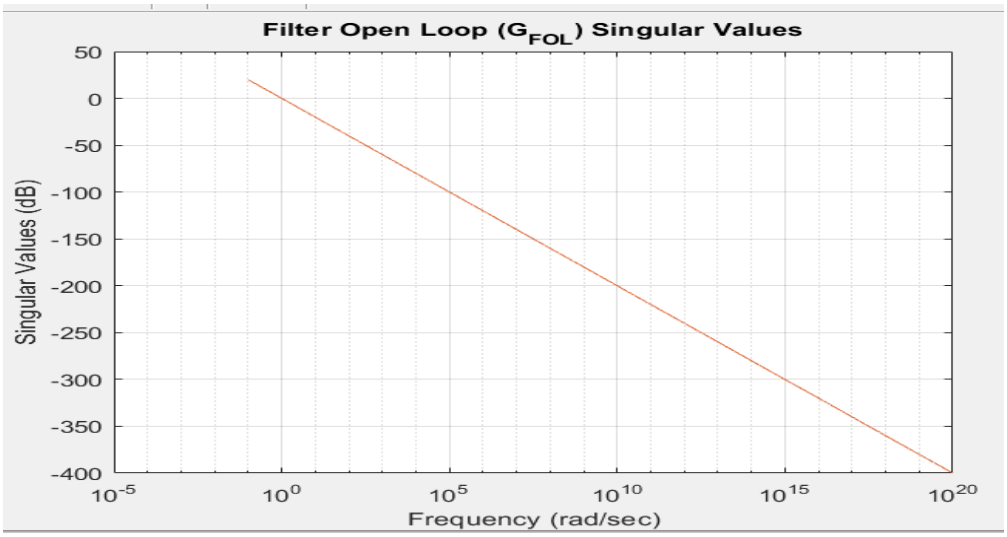

T (Comp Sens): plant output
Singular Values (dB) (dB)
0
-20
-40
-60
-80
-100
-120
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec) (rad/s)

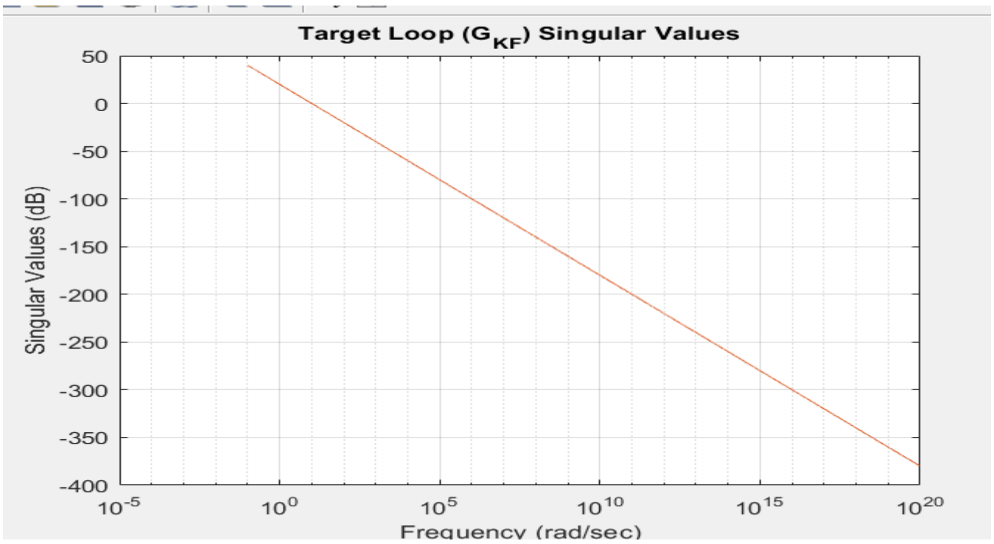

T (Comp Sens): plant input
Singular Values (dB) (dB)
Frequency (rad/sec) (rad/s)

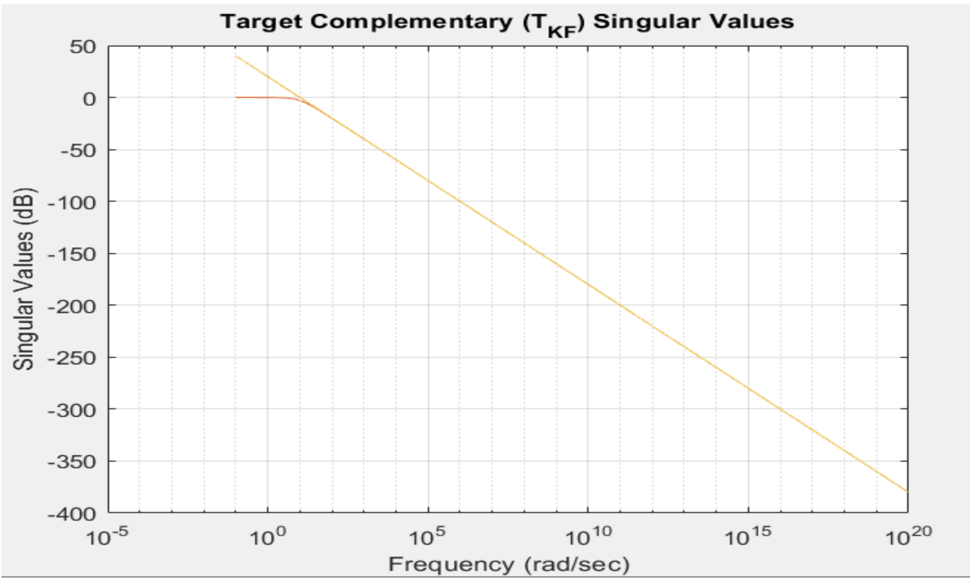

Step response for Linear Velocity reference
lin vel (m/s)

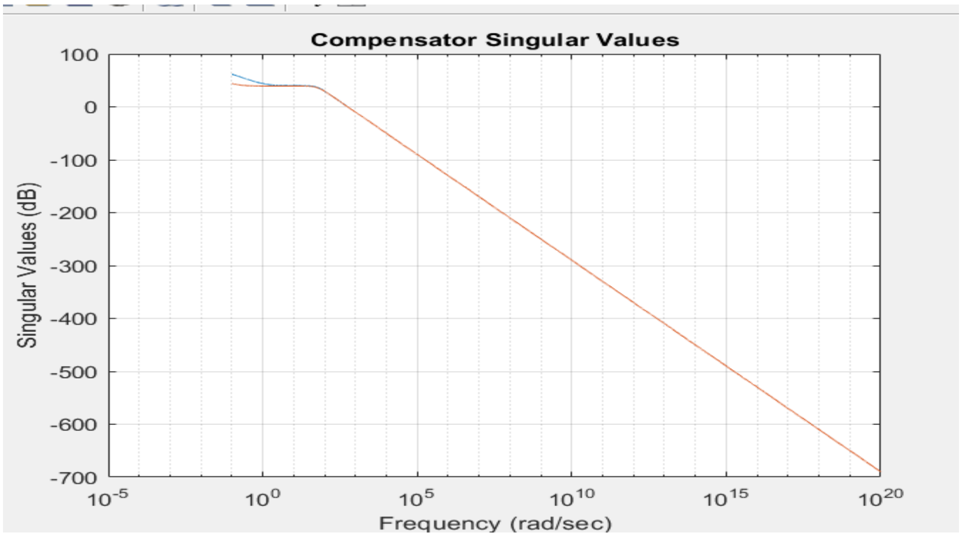

Response to Step Reference Command for Ang Velocity
Outputs (rad/s)
Time (seconds)

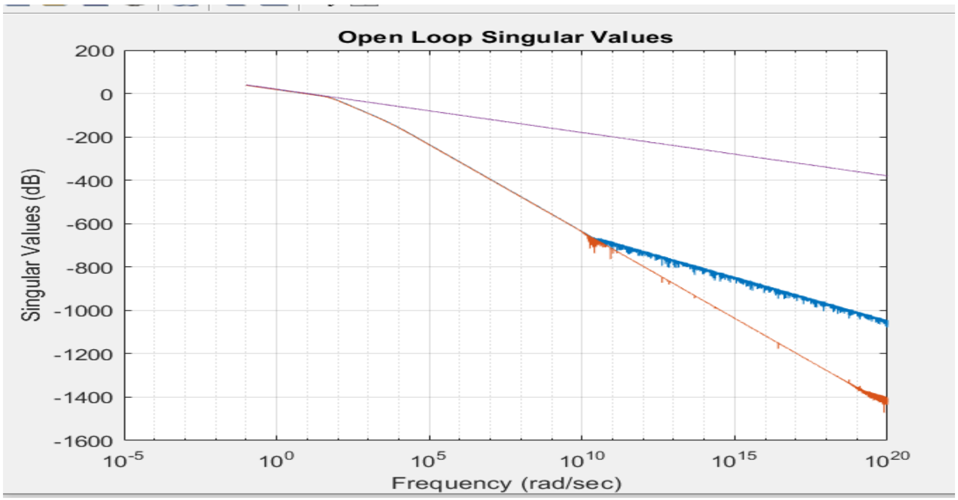
LQG / LTRO Controller

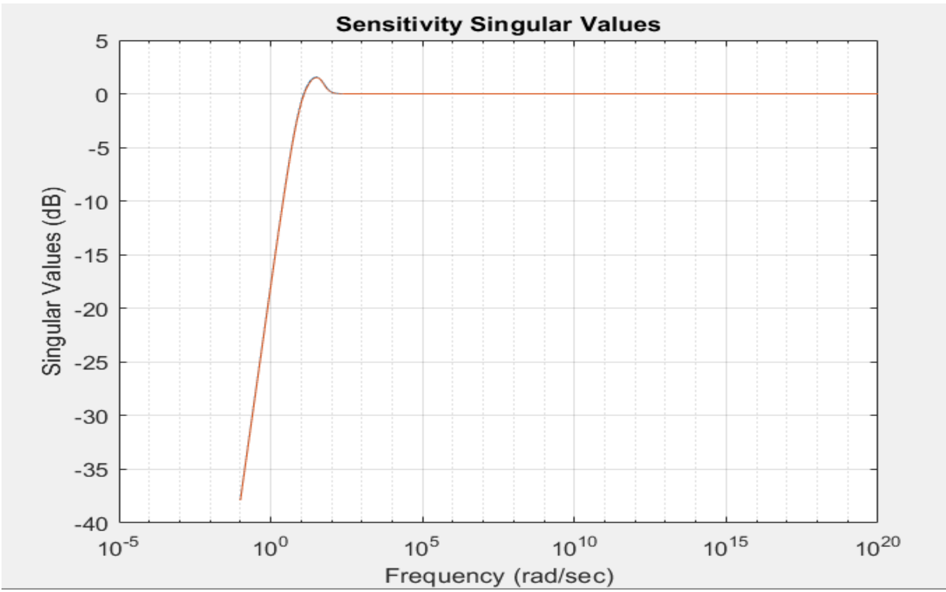







Compensator Singular Values
Singular Values (dB)
Frequency (rad/sec)


Open Loop Singular Values
Singular Values (dB)
Frequency (rad/sec)


Sensitivity Singular Values
Singular Values (dB)
Frequency (rad/sec)

OBSERVATIONS and INFERENCES

1. Three cases of the robot's center of gravity were analyzed for the task of velocity and orientation control.
2. If d > 0, the nominal plant was stable, whereas if d < 0 the nominal plant was unstable (had RHP poles, shown by the eigen values).
3. The controllers were able to stabilize the plant (after considering the dynamic characteristics), despite the instabilities in the plant itself.

CONSTRAINTS

1. The velocity of the robot was restricted to around 1 m/s due to voltage limits as given in [2]
2. The system was non-linear, and had to be linearized about an equilibrium point for easy design of the three controllers.

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