Least Squares
and
Nonlinear Least Squares

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Least squares

- Imagine you plan to solve the problem Ax=y
- but you have many datapoints so that the dimensions of A are mxn
- where m>n
- In python we might make an example matrix A like this, m = 3, n = 2
  - A = np.array([[2,3],[1,5],[4,1]],float)
- and y has m values
  - y = np.array([1,2,3],float)
- this problem is over constrained

Least squares solution

- You can show that the solution to this problem can be properly written

$$Ax = b$$

$$A^T Ax = A^T b$$

$$x = \left(A^T A\right)^{-1} A^T b$$

Non-linear least squares

- By analogy we can write the non linear least squares problem as
- beta is the parameter vector that describes f for any input x
- J, the Jacobian describes the gradient of the error in y with each βi

$$Consider\ \ y=f(x,\beta)$$

$$J\Delta\beta = \Delta y, \quad \text{which is overconstrained}$$

$$J^T J\Delta\beta = J^T \Delta y$$

J is the Jacobian

$\Delta\beta$ is an update to parameter vector β

Δy is the error, $\Delta y=y-f(x,\beta)$, in this iteration