Statistics and Hypothesis Testing

Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug can produce a satisfactory result than not.

Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario and list out the three conditions that this distribution follows.

Answer 1(a).

The Type of probability Distribution that would accurately portray the scenario is **Binomial Distribution** because the following conditions are satisfied which are required for Binomial Distribution:

1. The Number of trials to be performed are fixed i.e 10 in the above case.
2. Each trial result in only two possible outcomes. One is known as Success and the other Failure. In the above case also either the Drug will pass the Quality check (Success) or fail (Failure).
3. The probability of success is same for every Trial.
4. The Trials are independent i.e the outcome of one trial does not affect the other trials.

Hence Binomial Distribution will be used.

b.) Calculate the required probability.

Answer 1(b).

Given: Number of trials(n): 10

The drug performs 4 times more likely to produce results, Thus probability of success is not equal to the probability of failure.

Let us assume the probability of success be = x

Then the probability of failure = 4x

Applying x+4x=1

x = 0.2

4x=0.8

Thus, now using Binomial Theorem and considering getting an unsatisfactory result as success. We get:

n = 10

p=0.2

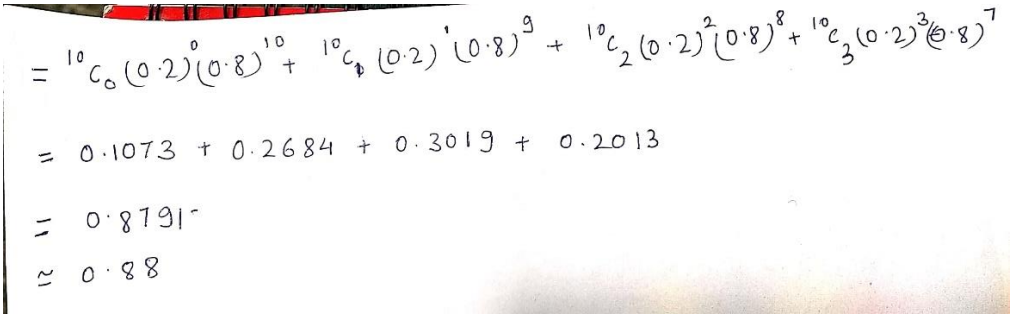
r=3

P(X<=3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)

Using the formula for Binomial Distribution

$$P(X = r) = {}^{n}C_{r}(p)^{r}(1-p)^{n-r}$$

Where **n** is **no. of trials**, **p** is **probability of success** and **r** is **no. of successes after n trials**.



So, the required probability is 0.88.

<u>Question 2:</u>

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.) Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.

b.) Find the required range.

Answer 2(a)

The main methodology which will be used is Central Limit Theorem. We can not perform the test in all 80000 new products for the time of effect. So, a sample of 100 was taken. As per CLT

Sampling Distribution should follow below properties:

The sampling distribution mean = Population Mean

Sampling Distribution's Standard Deviation (Standard Error) = $\boldsymbol{\sigma/\sqrt{n}}$ **,** where **σ** is the population's deviation and n are sample size.

When Sample size is > 30, sampling Distribution follows normal Distribution.

Answer 2(b)

To solve the above problem and get the required range we will need to get the margin of Error.

Using the formula given below we will get the Confidence interval i.e the range within which the mean of the population will lie.

$$\text{Confidence interval} = (\bar{X} - \frac{Z^* S}{\sqrt{n}}, \bar{X} + \frac{Z^* S}{\sqrt{n}}),$$

Where $\overline{X}$ is Sample mean = 207

Standard deviation S = 65

Sample size = n

For the Confidence level of 95% Z = 1.96

We get the confidence interval as = $\left\{\left(207 - \frac{1.96*65}{\sqrt{100}}\right), \left(207 + \frac{1.96*65}{\sqrt{100}}\right)\right\}$

$$= \left((207 - 12.74), (207 + 12.74)\right)$$

Hence the Required Range is (194.26,219.74).

Question 3:

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of α and β come out to be 0.05 and 0.45 respectively.

Answer 3(a).

Here it is given that the painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job.

Hence the Null Hypothesis will mean time of effect is less than or equal to 200

H0: μ =>200

The Alternate Hypothesis will mean time of effect is more than 200.

H1: μ < 200

We will proceed by following below methods:

Critical Value method:

Given in the statement:

- Significance level is 5%, as it is an upper tailed test then α = 0.05. The cumulative probability would be 1-0.05 = 0.95.
- With this we will calculate the z-critical score. Zc = 1.645
- Sampling Distribution μ_x = μ = 200
- Sample Standard deviation: **σ =** 65 seconds.
- Sampling Standard deviation would be:

. σ_x= **σ/**$\sqrt{\boldsymbol{n}} = \frac{65}{\sqrt{100}}$ = 6.5

Upper critical value = μ + (Zc * σ_x)

UCV = 200 + (1.645 * 6.5)

= 200 + 10.6925 = 210.6925

Given that sample mean is 207 which is less then 210.6925, we will Fail to reject the null Hypothesis.

P -Value Method:

- Significance level is 5%, as it is an upper tailed test then α = 0.05. The cumulative probability would be 1-0.05 = 0.95.
- Sampling Distribution μ_x = μ = 200
- Sample Standard deviation: **σ =** 65 seconds.
- Sampling Standard deviation would be:

. σ_x= **σ/**$\sqrt{n} = \frac{65}{\sqrt{100}}$ = 6.5

- Sample mean X = $\bar{2}$07 seconds

Z score = $(X - \mu)/\ \sigma_x$ = (207 – 200)/6.5 = 1.07

Getting the values from the Z table Z value for 1.07 corresponds to 0.8599

So, p-value will be = 1-0.8599 = 0.1401

As it is upper tailed test, p-value will be 0.1401 which is greater than significance level (0.05).

Hence, we will fail to reject the Null Hypothesis.

Answer 3(b)

Consider there is Drug test as per which Null hypothesis says the drug is not harmful. Type -I error would result to the rejection of null hypothesis and would mean the drug is harmful even when it is not harmful. In such cases we can use values of α and β to be 0.15 each as in such a case it would give a false positive but that can be retested once again. The harmful drug can be prevented by coming into the market. Hence the value of α and β should be high i.e 0.15 each as we are able to control the risk posed by higher value of α.

Consider Car manufacture unit which manufactures the Engine of the card themselves. It is expected to work fine and without failure. But it turned out that later its engine failed and resulted in more economical loss to the company. This turns out to be Type-II error and with our given set of values we would prefer α = 0.05 and β = 0.45.

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new customers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer 4: <u>Why A/B Testing:</u>

A/B Testing is simply a way for the companies to test how much a particular variable affects Audience reaction. A/B checks for User response for a certain new feature launched by a company. AB testing is a basically online marketing tactic where different versions of a product are used to see which one users or consumers prefer. With online products, such as a webpage, email campaign or advertisement, A/B testing can be carried out with relative ease and yield quick results due to the instant and detailed analysis that testers receive.

<u>How A/B testing is used:</u>

- As a team is already divided, we will further analyze the responses for few days and will record the responses in Boolean format i.e 0 or 1. If the response is positive it is 1 else 0.
- This way we will get the data points for different responses.
- We will count the frequency of the these 0s and 1s.
- We can use XLSTATs add-in excel so that we can analyze this with two sample mean analysis.
- This will give use the Zc value, p- value and Hypothesized mean which are required for our analysis.
- We will analyze the null hypothesis based on the results. We can use null hypothesis as the one Tag line is more effective than other one.

Submitted by:

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