SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING
ACADEMIC YEAR 2019-2020 SEMESTER 1
DIGITAL SIGNAL PROCESSING
TUTORIAL 14

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1. To design a lowpass digital filter with $\omega_p = 0.24\pi, \omega_s = 0.68\pi, \alpha_p = 1\text{dB}$, and $\alpha_s = 24\text{dB}$ using bilinear transformation $s \to k\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$, we have to first design a prototype lowpass analog filter.

(a) If the lowpass analog filter has a passband edge $F_p = 10\text{Hz}$, determine the value of k, and the stopband edge F_s of the analog prototype filter.

(b) Using k =10 in the bilinear transformation, determine F_p and F_s of the analog prototype filter.

2. A chebyshev lowpass analog filter meeting the analog specification of question 1(b) is given by

$$H_a(s) = \frac{15.4035s^{-2}}{1 + 4.3463s^{-1} + 17.2830s^{-2}}$$

Use bilinear transformation to transform the analog filter into the lowpass digital filter.

3. Let $H_{LP}(z)$ be an IIR lowpass transfer function with a zero (pole) at $z = z_k$. Let $H_D(\hat{z})$ denote the lowpass transfer function obtained by lowpass-to-lowpass transformation given by $z^{-1} = \frac{\hat{z}^{-1}-\alpha}{1-\alpha\hat{z}^{-1}}$, which moves the zero (pole) at $z = z_k$ of $H_{LP}(z)$ to a new location $\hat{z} = \hat{z}_k$. Express $\hat{z}_k$ in terms of z_k. If $H_{LP}(z)$ has a zero at $z = -1$, show that $H_D(\hat{z})$ also has a zero at $z = -1$.

4. A second-order lowpass IIR digital filter with a 3-dB cutoff frequency at $\omega_c = 0.55\pi$ has a transfer function

$$G_{LP}(z) = \frac{0.34(1 + z^{-1})^2}{1 + 0.1842z^{-1} + 0.1776z^{-2}}$$

Design a second-order highpass filter $H_{HP}(z)$ with a 3-dB cutoff frequency at $\widehat{\omega}_c = 0.45\pi$ by the lowpass-to-highpass spectral transformation.

5. A third-order elliptic highpass filter with a passband edge at $\omega_p = 0.52\pi$ has a transfer functioin

$$G_{HP}(z) = \frac{0.2397(1 - 1.5858z^{-1} + 1.5858z^{-2} - z^{-3})}{1 + 0.3272z^{-1} + 0.7459z^{-2} + 0.179z^{-3}}.$$

Design a highpass filter $H_{HP}(z)$ with a passband edge at $\widehat{\omega}_p = 0.48\pi$ by transforming the above highpass transfer function using the lowpass-to-lowpass spectral transformation.

6. Let $h_d[n], -\infty < n < \infty$, denote the impulse response samples of an ideal zero-phase lowpass filter with a frequency response $H_d(e^{j\omega})$. It has been shown that the frequency response $H(e^{j\omega})$ of the zero-phase FIR filter $h[n], -M < n < M$, obtained by multiplying $h_d[n]$ with a rectangular window $w_R[n], -M < n < M$, has the least integral-squared error $E_R = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H\left(e^{j\omega}\right) - H_d\left(e^{j\omega}\right)\right|^2 d\omega$. Let E_{Hann} denote the integral-squared error if a length-$(2M + 1)$ Hanning window is used to develop the FIR filter. Determine an expression for the excess error $E_{excess} = E_R - E_{Hann}$.

7. Design causal FIR filters with the smallest length meeting the following specification using the approach based on fixed window function.

(a) Lowpass filter, $\omega_p = 0.65\pi, \omega_s = 0.76\pi, \delta_p = 0.002, \delta_s = 0.004$.

(b) Highpass filter, $\omega_p = 0.58\pi,\ \omega_s = 0.42\pi,\ \delta_p = 0.008,\ \delta_s = 0.01$.

(c) bandpass filter, $\omega_{p1} = 0.4\pi, \omega_{p2} = 0.55\pi, \omega_{s1} = 0.25\pi, \omega_{s2} = 0.75\pi, \delta_p = 0.02, \delta_{s1} = 0.006$, $\delta_{s2} = 0.008$, where δ_{s1} and δ_{s2} are, respectively, the ripple in the lower and upper stopbands.

(d) bandstop filter $\omega_{p1} = 0.33\pi, \omega_{p2} = 0.8\pi, \omega_{s1} = 0.5\pi, \omega_{s2} = 0.7\pi, \delta_{p1} = 0.04, \delta_{p2} = 0.04$, $\delta_s = 0.03$, where δ_{p1} and δ_{p2} are, respectively, the ripple in the lower and upper passbands.

8. A lowpass FIR filter of order $N = 71$ is to be designed with a transition band given by $\omega_s - \omega_p = 0.04\pi$ with minimax criteria. Determine the approximate value of the stopband attenuation α_s in dB and the corresponding stopband ripple δ_s of the designed filter if the filter order is estimated using each of the following formulas: (a) Kaiser's formula, (b) Bellanger's formula. Assume the passband and stopband ripples to be the same.

9. Repeat Problem 8 if the filter is designed using the Kaiser's window-based method.