SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING
ACADEMIC YEAR 2019-2020 SEMESTER 1
DIGITAL SIGNAL PROCESSING
TUTORIAL 10

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1. Determine the z-transform and the corresponding ROC of the following sequences:

(a) $g[n] = \delta[n]$
(b) $g[n] = n\alpha^n \mu[n]$
(c) $g[n] = -\alpha^n \mu[-n-2]$
(d) $g[n] = \alpha^{|n|}, |\alpha| < 1$

2. Determine the ROC of the z-transform of the following sequences:

(a) $x_1[n] = (0.2)^n \mu[n+1]$
(b) $x_2[n] = -(0.5)^n \mu[n-6]$
(c) $x_3[n] = (-0.5)^n \mu[-n-3]$
(d) $y_1[n] = x_1[n] + x_2[n]$
(e) $y_2[n] = x_1[n] + x_3[n]$
(f) $y_3[n] = x_2[n] + x_3[n]$

3. Determine the z-transform of the sequence

$$y[n] = \begin{cases} 1 - \frac{|n|}{N}, & ,-N \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

and its ROC, where *N* is even. Show that the ROC includes the unit circle for the transform. Evaluate the z-transform on the unit circle to obtain the DTFT of the sequence.

4. Let $X(z)$ denote the z-transform of the length-10 sequence $x[n]$ defined for $0 \le n \le 9$

$$\{x[n]\} = \{6.29, 8.11, -7.46, 8.26, 2.64, -8.04, -4.43, 0.93, -9.15, 9.29\}$$

Let $X_0[k]$ represent the sample of $X(z)$ evaluated on the unit circle at eight equally spaced points given by $z = e^{j\frac{2\pi k}{8}}, 0 \le k \le 7$, i.e.,

$$X_0[k] = X(z)\big|_{z=e^{j\frac{2\pi k}{8}}}, 0 \le k \le 7$$

Determine the 8-point IDFT $x_0[n]$ of $X_0[k]$ without computing the latter function.

5. A causal IIR system has an input-output relation given by

$$y[n] = x[n] + 2x[n-1] - 0.21x[n-2] + 0.5y[n-1] + 0.66y[n-2]$$

Determine the z-transform transfer function of the system. Plot its poles and zeros. Determine if the system is stable or not?

6. Consider the causal stable transfer function

$$G(z) = \frac{1 - 0.5z^{-1} + 2z^{-2}}{(1 + 0.9z^{-1})(1 + 0.4z^{-1})}.$$

Develop a transfer function $H(z)$ by scaling the complex variable z by a constant α, i.e., $H(z) = G\left(\frac{z}{\alpha}\right)$.

Determine the range of values of α for which $H(z)$ remains stable.

7. A causal first-order transfer function is given by

$$H(z) = \frac{1 + \beta z^{-1}}{1 + \alpha z^{-1}},$$

Where $-1 < \alpha < 0, 1 < \beta < 2$. Determine the locations of the poles and zeros of $H(z)$ and $G(z) = H(z^M)$ where M is a positive integer. Check the stability of $H(z)$ and $G(z)$. What is the relation between the frequency response of these two transfer functions?

8. The transfer function of an LTI discrete-time system is given by

$$H(z) = \frac{3(z + 1.8)(z - 4)}{(z + 0.3)(z - 0.6)(z + 5)}$$

(a) What are the possible ROCs associated with $H(z)$? What kind of sequences, for example, left-sided, right-sided, two-sided or finite-length sequences, is for each ROC?

(b) Does the frequency response $H(e^{j\omega})$ of the system exist? Justify your answer.

(c) Can the system be stable? If it is stable can it be causal?