SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA
DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING
ACADEMIC YEAR 2019-2020 SEMESTER 1
DIGITAL SIGNAL PROCESSING
TUTORIAL 12

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1. The general form of the transfer function $H(z)$ of a linear-phase FIR filter with a real-valued impulse response is given by

$$H(z) = (1+z^{-1})^{N_1}(1-z^{-1})^{N_2}\prod_{i=1}^{N_3}(1+\alpha_i z^{-1}+z^{-2})\prod_{i=1}^{N_4}(1+\beta_i z^{-1}+\gamma_i z^{-2}+\beta_i z^{-3}+z^{-4})$$

What are the values of the constants $N_1, N_2, N_3,$ and N_4 for the lowest-order Type I, Type II, Type III, and Type IV linear-phase FIR filters, respectively.

2. Design a first-order lowpass IIR digital filter with normalized 3-dB cutoff frequency 0.42 rad/samples.

3. A bandstop IIR digital filter can be generated by a second-order transfer functions given by

$$H_{BS}(z) = \frac{1+\alpha}{2}\frac{1-2\beta z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}, |\alpha|<1, |\beta|<1$$

(a) Determine the squared-magnitude response of the bandstop IIR filter.
(b) Show that the notch frequency ω_0, at which the magnitude response is 0, is given by $\omega_0 = \cos^{-1}\beta$.
(c) Determine the magnitude response at $\omega = 0$ and $\omega = \pi$.
(d) It is known that the maximum magnitude response of the filter is 1. Show that the 3-dB notch bandwidth of the bandstop filter is given by $B_w = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)$.

4. Based on the results obtained in Question 3, design a bandstop filter with notch frequency at $0.35\,\pi$, and a 3-dB notch bandwidth of $0.15\,\pi$.

5. Show that the following M^{th}-order complex coefficient transfer function is that of a causal allpass filter.

$$A_M(z) = \frac{d_M^* + d_{M-1}^* z^{-1} + \cdots + d_1^* z^{-M+1} + z^{-M}}{1 + d_1 z^{-1} + \cdots + d_{M-1} z^{-M+1} + d_M z^{-M}}$$

6. The transfer function of a Type 2 linear phase FIR filter is given by

$$H_1(z) = 2.5(1-1.6z^{-1}+2z^{-2})(1+1.6z^{-1}+z^{-2})(1+z^{-1})(1-0.8z^{-1}+0.5z^{-2})$$

(a) Determine the transfer function $H_2(z)$ of a minimum-phase FIR filter having the same magnitude as that of $H_1(z)$.
(b) Determine the transfer function $H_3(z)$ of a maximum-phase FIR filter having the same magnitude as that of $H_1(z)$.
(c) How many other length-8 FIR filter exist that have the same magnitude response as that of $H_1(z)$?

7. A typical transmission channel is characterized by a causal transfer function

$$H(z) = \frac{(2.2+5z^{-1})(1-3.1z^{-1})}{(1+0.81z^{-1})(1-0.62z^{-1})}$$

In order to correct for the magnitude distortion introduced by the channel on a signal passing through it, we wish to connect a causal stable digital filter characterized by a transfer function $G(z)$ at the receiving end.

Determine $G(z)$.

8. Figure 1 shows a typical closed-loop discrete-time feedback control system in which $G(z)$ is the plant and $C(z)$ is the compensator. If $G(z) = \frac{z^{-2}}{1+1.5z^{-1}+0.5z^{-2}}$ and $C(z) = K$, determine the range of values of K for which the overall structure is stable.

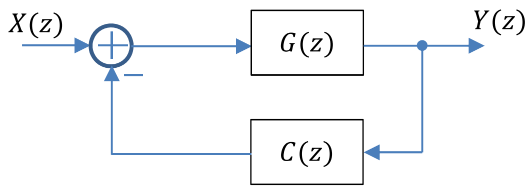

Figure 1

9. In the closed-loop discrete-time feedback control system of Figure 1, the plant transfer function is given by

$$G(z) = \frac{1.2 + 1.8z^{-1}}{1 + 0.7z^{-1} + 0.8z^{-2}}$$

Determine the transfer function $C(z)$ of the compensator so that the overall closed-loop transfer function of the feedback system is

$$H(z) = \frac{z^{-1} + 1.35z^{-2} + 0.9z^{-3} + 0.3375z^{-4}}{0.3 + 0.5z^{-1} + 0.505z^{-2} + 0.375z^{-3} + 0.21z^{-4}}.$$