SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING
ACADEMIC YEAR 2019-2020 SEMESTER 1
DIGITAL SIGNAL PROCESSING
TUTORIAL 9

1. Develop an algorithm for the complex multiplication of two complex numbers using only three real multiplications and five real additions.

2. Develop the flow graph for the DIT FFT algorithm from that of Figure on Slide 307 of Lecture Notes 6 (L6) for the case $N = 8$ in which the input is in normal order and output is in the bit reversed order.

3. If M DFT samples of the N-point DFT of a length-N sequence are required with $M < N$, what is the smallest value of M for which the N-point FFT algorithm is computationally more efficient than a direct computation of the M DFT samples? What are the values of M for the following values of N: $N = 32, N = 64,$ and $N = 128$.

4. We know that the total number of complex multiplications $\mathcal{R}(v)$ needed in the implementation of the DIT and DIF FFT algorithms can be made smaller than $N/2 \log_2 N$ if the multiplications by ± 1 etc. can be avoid. Develop an exact expression for $\mathcal{R}(v)$ that includes only non-trivial multiplications by complex twiddle factors, i.e., excluding the multiplications with twiddle factors ± 1 and $\pm j$.

5. We wish to determine the sequence $y[n]$ generated by a linear convolution of a length-40 real sequence $x[n]$ and a length-21 real sequence $h[n]$. To this end, we can follow one of the following methods:
Method # 1. Direct computation of the linear convolution.
Method # 2. Computation of the linear convolution via a single circular convolution.
Method # 3. Computation of the linear convolution using FFT algorithm.
Determine the least number of real multiplications needed in each of the above methods. For the FFT algorithm, do not include in the count multiplications by $\pm 1, \pm j,$ and W_N^0.

6. We have known that the computation of DFT may be expressed in matrix form

$$\boldsymbol{X} = \mathbf{W}_N \boldsymbol{x}$$

where $\boldsymbol{X}$ and $\boldsymbol{x}$ are the DFT vector and input sample vector respectively, and $\mathbf{W}_N$ the constant matrix for DFT computation given on Slide 261 of L6.

(a) Show that the 8-point DIT FFT algorithm shown in Figure on Slide 304 of L6 is equivalent to expressing the DFT matrix as a product of four matrices as indicated below:

$$\mathbf{W}_N = \mathbf{V}_8 \mathbf{V}_4 \mathbf{V}_2 \mathbf{E} \tag{8.1}$$

Determine the matrices given above and show that multiplication by each matrix $\mathbf{V}_k, k = 8, 4, 2,$ requires at most eight complex multiplications.

(Hints: The matrix $\mathbf{E}$ is used for input sequence re-ordering, and $\mathbf{V}_2, \mathbf{V}_4, \mathbf{V}_8$ is used to realize the first, second and third stage butterflies, respectively.)

(b) Since the DFT matrix $\mathbf{W}_N$ is its own transpose, i.e., $\mathbf{W}_N = \mathbf{W}_N^T$, another FFT algorithm is readily obtained by forming the transpose of the right-hand side of Eq. (8.1), resulting in a factorization of $\mathbf{W}_N$ given by

$$\mathbf{W}_N = \mathbf{E}^T \mathbf{V}_2^T \mathbf{V}_4^T \mathbf{V}_8^T$$

Show that the flow graph representation of the above factorization is precisely the 8-point DIF FFT algorithm of Figure on Slide 319 of L6.

7. The butterfly in the following figure was taken from a decimation-in-frequency FFT with N=16, where the input sequence was arranged in normal order. Note that a 16-point FFT will have four stages, indexed m=1, 2, 3, 4. Which of the four stages have butterflies of this form? Justify your answer.