# SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
## DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING
## ACADEMIC YEAR 2019-2020 SEMESTER 1
## DIGITAL SIGNAL PROCESSING
## TUTORIAL 8

1. (a) Consider a length-$N$ sequence $x[n], 0 \leq n \leq N-1$, with an $N$-point DFT $X[k], 0 \leq k \leq N-1$. Define a sequence $y[n]$ of length $LN$, $0 \leq n \leq NL-1$, given by

$$y[n] = \begin{cases} x[n/L], n = 0, L, 2L, \dots, (N-1)L, \\ 0, \quad \text{otherwise}, \end{cases} \tag{8.1}$$

where $L$ is a positive integer. Express the $NL$-point DFT $Y[k]$ of $y[n]$ in terms of $X[k]$.

(b) The 5-point DFT $X[k]$ of a length-5 sequence $x[n]$ is shown in Figure 8.1. Sketch the 20-point DFT $Y[k]$ of a length-20 sequence $y[n]$ generated using (8.1).

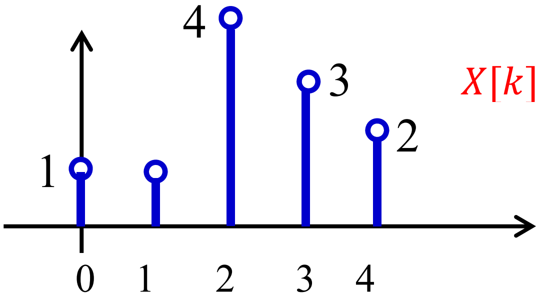


Figure 8.1

2. Consider the sequence $\{x[n]\} = \{2, -5, 6, -3, 4, -4, 0, -7, 8\}, -5 \leq n \leq 3$.

(a) Let $\{y[n]\}$ denote the sequence obtained by a left circular shift of $\{x[n]\}$ by 12 sample periods. Determine the value of the sample $y[-3]$.

(b) Let $\{z[n]\}$ denote the sequence obtained by a right circular shift of $\{x[n]\}$ by 15 sample periods. Determine the value of the sample $z[2]$.

3. Let $\{x[n]\} = \{-3, 2, -1, 4\}$, and $\{h[n]\} = \{1, 3, 2, -2\}$ be two length-4 sequences defined for $0 \leq n \leq 3$. Determine $y[n] = x[n]$ 4 $h[n]$

4. (a) Let $g[n]$ and $h[n]$ be two sequences of length 6 each. If $y_L[n]$ and $y_C[n]$ denote the linear and 6-point circular convolutions of $g[n]$ and $h[n]$, respectively, develop a method to determine $y_C[n]$ in terms of $y_L[n]$.

(b) Consider the two length-6 sequences, $\{g[n]\} = \{3, -5, 2, 6, -1, 4\}$, and $\{h[n]\} = \{-2, 4, 7, -5, 4, 3\}$. Determine the $y_L[n]$ obtained by a linear convolution of $g[n]$ and $h[n]$. Using the method developed in Part (a), determine the sequence $y_C[n]$ given by the circular convolution of $g[n]$ and $h[n]$ from $y_L[n]$.

5. Denote $X[k], 0 \leq k \leq N-1$, the $N$-point DFT of sequence $x[n]$, with $N$ even. Define two length-$\left(\frac{N}{2}\right)$ sequences given by: $g[n] = \frac{1}{2}(x[2n] + x[2n+1])$ and $h[n] = \frac{1}{2}(x[2n] - x[2n+1]), 0 \leq n \leq \frac{N}{2} - 1$. If $G[k]$ and $H[k], 0 \leq k \leq N/2 - 1$ are the $\left(\frac{N}{2}\right)$-point DFT of $g[n]$ and $h[n]$, respectively, determine $X[k]$ in terms of $G[k]$ and $X[k]$.

6. Let $x[n], 0 \leq n \leq N-1$, be a length-$N$ sequence with an $N$-point DFT given by $X[k], 0 \leq k \leq N-1$.
(a) If $x[n]$ is a symmetric sequence satisfying the condition $x[n] = x[\langle N-1-n \rangle_N]$, show that $X[N/2] = 0$ for $N$ even.
(b) If $x[n]$ is an antisymmetric sequence satisfying the condition $x[n] = -x[\langle N-1-n \rangle_N]$, show that $X[0] = 0$.
(c) If $x[n]$ is a sequence satisfying the condition $x[n] = -x[\langle n+M \rangle_N]$ with $N = 2M$, show that $X[2l] = 0$ for $l = 0, 1, \dots, M-1$.

7. Let $x[n] = \{2, 1, 2\}, 0 \leq n \leq 2$ and $w[n] = \{-4, 0, -3, 2\}, 0 \leq n \leq 3$. If $w[n] = x[n] \circledast y[n]$, determine $y[n]$ using a DFT-based method.