Datasheet

Standard Sequences

$\delta[n] = 1$ for $n = 0$ and 0 otherwise. $\mu[n] = 1$ for $n \geq 0$ and 0 otherwise.

Geometric Progression

$\sum_{n=0}^{r} a^n z^{-n} = \frac{1-a^{r+1}z^{-r-1}}{1-az^{-1}}$ provided that $az^{-1} \neq 1$

$\sum_{n=0}^{\infty} a^n z^{-n} = \frac{1}{1-az^{-1}}$ provided that $|az^{-1}| < 1$

Forward and Inverse Transforms

DTFT: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ $\quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$

DFT: $X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi\frac{kn}{N}}$ $\quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi\frac{kn}{N}}$

z: $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$ $\quad x[n] = \frac{1}{2\pi j}\oint_C X(z)z^{n-1}\,dz$

Convolution

DTFT: $v[n] = x[n] \circledast y[n] \triangleq \sum_{r=-\infty}^{\infty} x[r]y[n-r]$ $\Leftrightarrow$ $V(e^{j\omega}) = X(e^{j\omega})Y(e^{j\omega})$

$v[n] = x[n]y[n]$ $\Leftrightarrow$ $V(e^{j\omega}) = X(e^{j\omega}) \circledast Y(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$

DFT: $v[n] = x[n] \text{ Ⓝ } y[n] \triangleq \sum_{r=0}^{N-1} x[r]h[\langle n-r\rangle_N]$ $\Leftrightarrow$ $V[k] = X[k]Y[k]$

$v[n] = x[n]y[n]$ $\Leftrightarrow$ $V[k] = \frac{1}{N}X[k] \text{ Ⓝ } Y[k] = \frac{1}{N}\sum_{r=0}^{N-1} X[r]Y[\langle k-r\rangle_N]$

Order Estimation for FIR Filters: $N \cong \frac{-20\log_{10}(\sqrt{\delta_p\delta_s})-13}{14.6(\omega_s-\omega_p)/2\pi}$

Transformations

	Substitution	Parameters
Bilinear Transformation: $H(s) \leftrightarrow H(z)$	$s = k\left(\frac{1-z^{-1}}{1+z^{-1}}\right),\ k > 0,$ or $z = \frac{k+s}{k-s}$	$\Omega = k\tan\left(\frac{\omega}{2}\right)$ or $\omega = 2\tan^{-1}\frac{\Omega}{k}$
Spectral transformation Lowpass-to-lowpass $H(z) \leftrightarrow H(\hat{z})$	$z^{-1} = \frac{\hat{z}^{-1}-\alpha}{1-\alpha\hat{z}^{-1}}$, or $\hat{z}^{-1} = \frac{z^{-1}+\alpha}{1+\alpha z^{-1}}$	$\alpha = \frac{\sin\left(\frac{\omega-\hat{\omega}}{2}\right)}{\sin\left(\frac{\omega+\hat{\omega}}{2}\right)}$ or $\tan\left(\frac{\omega}{2}\right) = \left(\frac{1+\alpha}{1-\alpha}\right)\tan\left(\frac{\hat{\omega}}{2}\right)$
Spectral transformation Lowpass-to-highpass $H(z) \leftrightarrow H(\hat{z})$	$z^{-1} = -\frac{\hat{z}^{-1}+\alpha}{1+\alpha\hat{z}^{-1}}$, or $\hat{z}^{-1} = -\frac{z^{-1}+\alpha}{1+\alpha z^{-1}}$	$\alpha = -\frac{\cos\left(\frac{\omega-\hat{\omega}}{2}\right)}{\cos\left(\frac{\omega+\hat{\omega}}{2}\right)}$, or $\text{ctan}\left(\frac{\omega}{2}\right) = \left(\frac{-1+\alpha}{1+\alpha}\right)\tan\left(\frac{\hat{\omega}}{2}\right)$