SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING
ACADEMIC YEAR 2019-2020 SEMESTER 1
DIGITAL SIGNAL PROCESSING
TUTORIAL 7

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1. Determine the N-point DFTs of the following length-N sequences defined for $0 \leq n \leq N-1$.

(a) $x_a[n] = \sin\left(\frac{2\pi n}{N}\right)$, (b) $x_b[n] = \sin^2\left(\frac{2\pi n}{N}\right)$, (c) $x_c[n] = \sin^3\left(\frac{2\pi n}{N}\right)$

2. Let $x[n]$ be a length-N sequence with $X[k]$ donating its N-point DFT. We represent the DFT operation as $X[k] = \mathcal{F}\{x[n]\}$. Determine the sequence $y[n]$ obtained by applying the DFT operation 4 times to $x[n]$, i.e.,

$$y[n] = \mathcal{F}\left\{\mathcal{F}\left\{\mathcal{F}\{\mathcal{F}\{x[n]\}\}\right\}\right\}$$

3. Let $x[n], 0 \leq n \leq N-1$, be a length-N sequence with an N-point DFT given by $X[k], 0 \leq k \leq N-1$. Determine the 2N-point DFT of each of the following length-2N sequence in terms of $X[k]$.

(a) $g[n] = \begin{cases} x[n], & 0 \leq n \leq N-1 \\ 0, & N \leq n \leq 2N-1 \end{cases}$ (b) $h[n] = \begin{cases} 0, & 0 \leq n \leq N-1 \\ x[n-N], & N \leq n \leq 2N-1 \end{cases}$

4. Let $x[n], 0 \leq n \leq N-1$, be a length-N sequence with an N-point DFT given by $X[k], 0 \leq k \leq N-1$. Define a length-3N sequence $y[n]$ given by

$$y[n] = \begin{cases} x[n], & 0 \leq n \leq N-1 \\ 0, & N \leq n \leq 3N-1 \end{cases}$$

with $Y[k], 0 \leq k \leq 3N-1,$ denoting its 3N-point DFT. Let $W[l] = Y[3l+2], 0 \leq l \leq N-1,$ with $w[n], 0 \leq n \leq N-1,$ denoting its N-point IDFT. Express $w[n]$ in terms of $x[n]$.

5. Consider a rational discrete-time Fourier transform $X(e^{j\omega})$ with real coefficients of the form of

$$X(e^{j\omega}) = \frac{P(e^{j\omega})}{D(e^{j\omega})} = \frac{p_0 + p_1 e^{-j\omega} + \cdots + p_{M-1} e^{-j\omega(M-1)}}{d_0 + d_1 e^{-j\omega} + \cdots + d_{N-1} e^{-j\omega(N-1)}}$$

Let $P[k]$ denote the M-point DFT of the numerator coefficients $\{p_i\}$ and $D[k]$ denote the N-point DFT of the denominator coefficients $\{d_i\}$. Determine the exact expressions of the DTFT $X(e^{j\omega})$ for $M = N = 4$, if the 4-point DFTs of its numerator and denominator coefficients are given by

$$P[k] = \{3.5, -0.5 - j9.5, 2.5, -0.5 + j9.5\}, D[k] = \{17, 7.4 + j12, 17.8, 7.4 - j12\}.$$

6. Let $X(e^{j\omega})$ denote the DTFT of the length-9 sequence $\{x[n]\} = \{1, -3, 4, -5, 7, -5, 4, -3, 1\}$.

(a) For the DFT sequence $X_1[k]$, obtained by sampling $X(e^{j\omega})$ at uniform intervals of $\pi/6$ starting from $\omega = 0$, determine the IDFT $x_1[n]$ of $X_1[k]$ without computing $X(e^{j\omega})$ and $X_1[k]$. Can you recover $x[n]$ from $x_1[n]$?

(b) For the DFT sequence $X_2[k]$, obtained by sampling $X(e^{j\omega})$ at uniform intervals of $\pi/4$ starting from $\omega = 0$, determine the IDFT $x_2[n]$ of $X_2[k]$ without computing $X(e^{j\omega})$ and $X_2[k]$. Can you recover $x[n]$ from $x_2[n]$?