SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING
ACADEMIC YEAR 2019-2020 SEMESTER 1
DIGITAL SIGNAL PROCESSING
TUTORIAL 6

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1. Consider an LTI system discrete-time system with an impulse response $h[n] = (0.4)^n \mu[n]$. Determine the frequency response $H(e^{j\omega})$ of the system and evaluate its value at $\omega = \pm\frac{\pi}{4}$.

2. (a) Design a length-5 FIR bandpass filter with an anti-symmetric impulse response $h[n]$, i.e., $h[n] = -h[4-n], 0 \le n \le 4$, satisfying the following magnitude response values: $\left|H\left(e^{j\frac{\pi}{4}}\right)\right| = 0.5$ and $\left|H\left(e^{j\frac{\pi}{2}}\right)\right| = 1$.

(b) Determine the exact expression for the frequency response of the filter designed.
(c) Find the phase delay and group delay of the filter.

3. (a) Design a length-4 FIR bandpass filter with a symmetric impulse response $h[n]$, i.e., $h[n] = h[3-n], 0 \le n \le 3$, satisfying the following magnitude response values: $\left|H\left(e^{j\frac{\pi}{4}}\right)\right| = 1$ and $\left|H\left(e^{j\frac{\pi}{2}}\right)\right| = 0.5$.

(b) Determine the exact expression for the frequency response of the filter designed.
(c) Find the phase delay and group delay of the filter.

4. Consider the two LTI causal digital filters with impulse responses given by

$$h_A[n] = 0.3\delta[n] - \delta[n-1] + 0.3\delta[n-2], \qquad h_B[n] = 0.3\delta[n] + \delta[n-1] + 0.3\delta[n-2]$$

(a) Sketch the magnitude and phase response of the two filters and compare their characteristics.
(b) Let $h_A[n]$ be the impulse response of a causal digital filter with a frequency response $H_A(e^{j\omega})$. Define another digital filter whose impulse response $h_C[n]$ is given by

$$h_C[n] = (-1)^n h_A[n], \qquad \text{for all } n.$$

What is the relation between the frequency response $H_C(e^{j\omega})$ of this filter and the frequency response $H_A(e^{j\omega})$ of the prototype filter?

5. An LTI causal discrete-time system is characterized by the difference equation

$$y[n] = d_3x[n] + d_2x[n-1] + d_1x[n-2] + x[n-3] - d_1y[n-1] - d_2y[n-2] - d_3y[n-3],$$

where $y[n]$ and $x[n]$ are, respectively, the output and input sequences. Determine the expression for frequency response and show that it has a unity magnitude response for all values of ω.

6. Show that the group delay $\tau(\omega)$ of an LTI discrete-time system characterized by a frequency response $H(e^{j\omega})$ can be expressed as

$$\tau(\omega) = \mathrm{Re}\left\{ j\frac{dH(e^{j\omega})}{d\omega} \Big/ H(e^{j\omega}) \right\}$$

7. Show that the sequence $u[n] = z^n$, where z is a complex constant, is an eigenfunction of an LTI discrete-time system. Is the sequence $v[n] = z^n\mu[n]$ with z a complex constant also an eigenfunction of an LTI discrete-time system? Justify your answer.