Exercises 2.

1. Find the Dehn function of the semigroup $\langle x, y \mid yx = 1 \rangle$.

2. Prove that free groups of rank ≥ 2 are not abelian.

3. Let $Fr(2)$ be a free group on free generators x, y. Prove that the subgroup of $Fr(2)$ generated by elements $X = \{x^n y x^n, n = 0, 1, 2, \ldots\}$ is a free group on the set of free generators X.

4. An element g of a free group $Fr(n)$ in ~~reg~~ reduced form $g = x_{i_1}^{\varepsilon_1} \cdots x_{i_k}^{\varepsilon_k}$, $\varepsilon_i = \pm 1$, is called <u>cyclically reduced</u> if

$$x_{i_1}^{\varepsilon_1} \cdot x_{i_k}^{\varepsilon_k} \neq 1.$$

The element g can be represented as

$$g = x_{i_1}^{\varepsilon_1} \cdots x_{i_s}^{\varepsilon_s} \sigma(g) x_{i_s}^{-\varepsilon_s} \cdots x_{i_1}^{-\varepsilon_1}, \quad \sigma(g) \text{ is}$$

cyclically reduced.

Prove that two elements $g_1, g_2 \in F_2(n)$ are conjugate if and only if $\sigma(g_1) = vw$, $\sigma(g_2) = wv$ for some elements v, w.

5. Prove that a free group does not contain nonidentical elements of finite order.