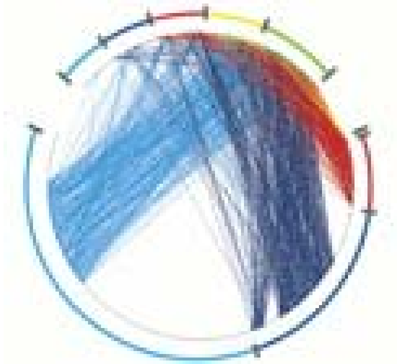
Lecture 17: Social Dynamics & Cooperativity on Networks

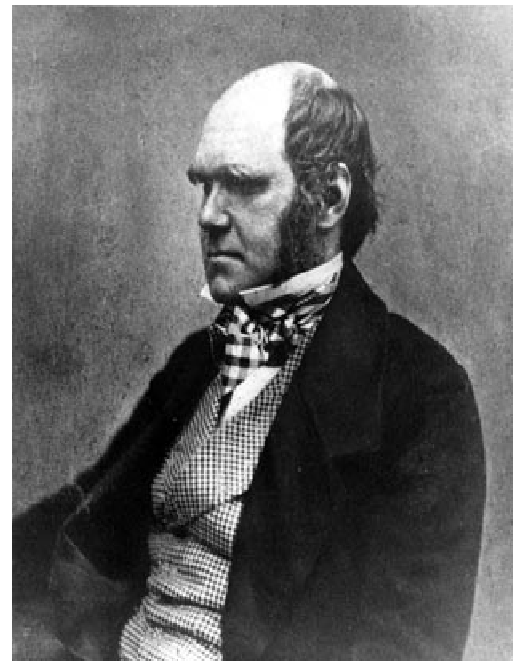


Social dynamics on networks

- We have studied the statistical properties of complex networks so far
- How about the dynamics on the networks
- Such as social dynamics, e.g. various social games
- Prisoner's dilemma (PD) game
- Amount of cooperativity
- Evolution of cooperativity
- Dependence on network structure

The puzzle of emergence of cooperation

He who was ready to sacrifice his life (…), rather than betray his comrades, would often leave no offspring to inherit his noble nature… Therefore, it seems scarcely possible (…) that the number of men gifted with such virtues (…) would be increased by natural selection, that is, by the survival of the fittest.



Charles Darwin
(Descent of Man, 1871)

One of the problems for the next century

E. Pennisi, *Science* **309**, 93 (2005)

“Others with a mathematical bent are applying evolutionary game theory, a modeling approach developed for economics, to quantify cooperation and predict behavioral outcomes under different circumstances.”

The Prisoners' Dilemma (PD)

- A simple game that has become the dominant paradigm for social scientists since it was invented about 1960.
- How the game works -- a simple narrative.
- PD games help to explain why we do dumb things
- Modeling PD:
  - Game theoretic problems: payoffs for each player depend on actions of both
  - Two possible strategies: An agent cooperates when she performs value-increasing promises, and defects when she breaches

Modeling Two-party choice

Player 1

Cooperate		

Modeling Two-party choice

Player 1

Defect		

Modeling Two-party choice

Player 2

	Cooperate	

Modeling Two-party choice

Player 2

		Defect

Modeling Two-party choice: both cooperate

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	Both cooperate	
	Defect		

Modeling Two-party choice: both defect

Player 2

Player 1

	Cooperate	Defect
Cooperate		
Defect		Both defect

Modeling Two-party choice

Player 2

Player 1

	Cooperate	Defect
Cooperate		Player 1 cooperates, Player 2 defects
Defect		

Modeling Two-party choice

Player 2

Player 1

	Cooperate	Defect
Cooperate		
Defect	Player 1 defects, Player 2 cooperates	

Modeling Two-party choice

Player 2

Player 1

	Cooperate	Defect
Cooperate	Both cooperate	Player 1 cooperates, Player 2 defects
Defect	Player 1 defects, Player 2 cooperates	Both defect

Let's examine Joint Cooperation

Player 2

	Cooperate	Defect
Cooperate	Both cooperate	
Defect		

Player 1

Joint Defection

Player 1 \ Player 2	Cooperate	Defect
Cooperate		
Defect		Both defect

Player 1: Sucker's Payoff

Player 2

Player 1

	Cooperate	Defect
Cooperate		Player 1 cooperates, Player 2 defects
Defect		

Player 1: Defector’s Payoff

		Player 2	
		Cooperate	Defect
Player 1	Cooperate		
	Defect	Player 1 defects, Player 2 cooperates	

Let's apply this to promising

Player 2

Player 1

	Cooperate	Defect
Cooperate	Both cooperate	Player 1 gets sucker's payoff
Defect	Player 1--defector's payoff	Both defect

Modeling Promisor Choices

Player 2

Player 1		Cooperate	Defect
	Cooperate	Both promise and perform	Player 2 breaches, Player 1 performs
	Defect	Player 1 performs, player 2 breaches	Both defect: No one performs

Plugging in payoffs

First number is payoff for Player 1
Second number is payoff for Player 2

Player 2

Player 1

	Cooperate	Defect
Cooperate	3, 3	-1, 4
Defect	4, -1	0, 0

Defection dominates for Player 1

	Cooperate	Defect
Cooperate	3	-1
Defect	4	0

Player 1

Defection dominates for Player 2

Player 2

	Cooperate	Defect
Cooperate	3	→ 4
Defect	-1	→ 0

Defection dominates

		Cooperate	Defect
Player 1	Cooperate	a	c
	Defect	b	d

I am always better off if the opponent cooperates

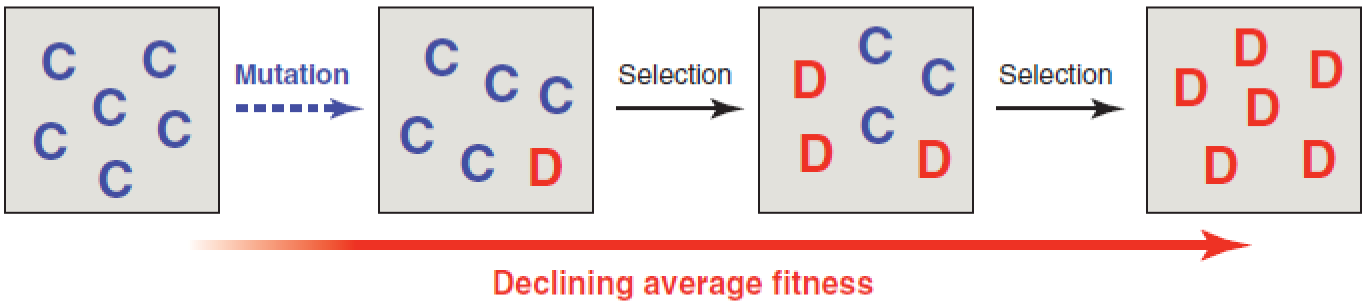
Player 1		Cooperate	Defect
	Cooperate	a	← c
	Defect	b	← d

a+a is greater than any other summation of payoffs

Player 2

	Cooperate	Defect
Cooperate	a+a	c+c
Defect	b+b	d+d

Defection is favored individually



Without any mechanism for the evolution of cooperation, natural selection favors defectors. In a mixed population, defectors, D, have a higher payoff (= fitness) than cooperators, C. Therefore, natural selection continuously reduces the abundance, i, of cooperators until they are extinct. The average fitness of the population also declines under natural selection. The total population size is given by N. If there are i cooperators and N − i defectors, then the fitness of cooperators and defectors, respectively, is given by $f_C = [b(i - 1)/(N - 1)] - c$ and $f_D = bi/(N - 1)$. The average fitness of the population is given by = $(b - c)i/N$.

Source, Nowak Science 2006

The paradox of the PD game

- While cooperation is collectively rational, defection is individually rational.
- The undersupply of cooperation is “the tragedy of the commons.” Garrett Hardin, The Tragedy of the Commons (1968).

PD game on complex networks

- Consider a social network
- Individuals are capable of making rational choices, modeled in terms of a game, associated with well-defined strategies
- Let's restrict the analysis to symmetric two-player games such as PD
- Let's look at the nodes as players
- Each player is a pure strategist, adopting either a cooperative (C) or a defecting (D) strategy

PD game on complex networks

- It is not difficult to show that it is best to defect for rational players to get the highest payoff independently in a single round of the PD
- But mutual cooperation results in a higher income for both of them.
- Therefore, this situation creates the so-called dilemma for selfish players.

PD game on complex networks

- The defector will always have the highest reward *T* (temptation to defect) when playing against the cooperator which will receive the lowest payoff *S* (sucker value)
- If both cooperate, they will receive a payoff *R* (reward for cooperation)
- If both defect, they will receive a payoff *P* (punishment)
- Moreover, these four payoffs satisfy the following inequalities: $T>R>P>S$ and $T+S<2R$

Payoff	*C*	*D*
C	*(R;R)*	*(S; T)*
D	*(T; S)*	*(P; P)*

Calculating the cooperativity

- One of the important issues in this context is to study the fraction of cooperating agents
- We start from an initial configuration with equal number of cooperating (C) and defecting (D) agents that are randomly distributed across the network
- At each generation, each agent plays with its neighbours and payoffs are accrued as dictated by the PD payoff matrix (previous page)
- Accumulated payoffs of all agents are computed by adding up the results of the games with their neighbours

Calculating the cooperativity

- All players update their strategies synchronously by the following rules
  - Each individual i chooses at random a neighbour j and compares its payoff P_i with P_j
  - If $P_i > P_j$, player i keeps the same strategy for the next generation
  - If, $P_i < P_j$, the player i adopts the strategy of its neighbour j for the next round of the game with probability

$$P_{i\rightarrow j} = \frac{P_j - P_i}{b \times maximum\{K_i, K_j\}},$$

  - where K_i is the degree of player i.

Calculating the cooperativity

- We let the system evolve until a stationary state is reached characterized by a stable average level of cooperativity, that is the fraction of C agents in the network #C
- To compute #C we let the dynamics evolve over a transient time windows W_1, and we further evolve the system over time windows of W_2 generations
- In each time window, we compute the average value and the variation of #C
- When the variation is less than or equal to $1/\sqrt{N}$, we stop the simulation and consider the average cooperation obtained in the last time window as the coopeartivity of the network

PD game on complex networks

- each player plays a PD with its neighbors
- Let's represent the players' strategies with two-component vector, taking the value s=(1,0) for C-strategist and s=(0,1) for D-strategist. The total payoff of a certain player x is the sum over all interactions, so the payoff P_x can be written as

$$P_x = \sum_{y \in \Omega_x} s_x A s_y^{\mathrm{T}},$$

- where Ω_x is the set of neighbors of element *x* and *A* is the payoff matrix

$$A = \begin{bmatrix} R & S \\ T & P \end{bmatrix}.$$

PD game on complex networks

- Nowak and May proposed the following simplified version of the payoff matrix

$$A = \begin{bmatrix} 1 & 0 \\ b & 0 \end{bmatrix},$$

- b represents the advantage of defectors over cooperators and $1<b<2$.
- Therefore, we can rescale the game depending on the single parameter b.

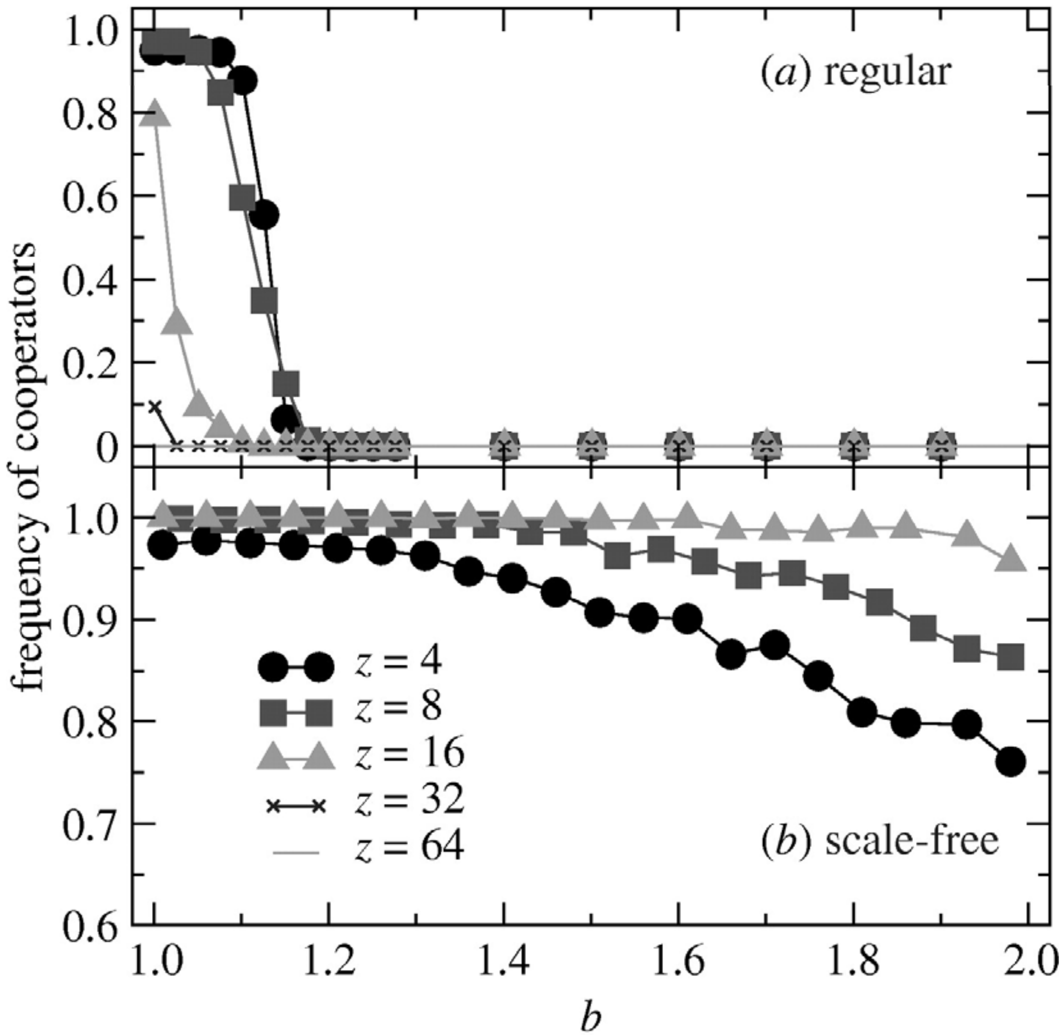
PD game on complex networks

- After this, the player *x* will inspect the payoff collected by its neighbors in the generation, and then update its strategy for the next generation to play by the following rule:
  - It will select one player y randomly from its neighbors
  - Whenever $P_y > P_x$, player x will adopt the strategy of player y with probability given by $W_{sx \leftarrow sy} = (P_y - P_x)/(Dk_>)$, where $k_> = \max\{k_x, k_y\}$ and $D = T - S = b$. k_x and k_y are the degrees of players x and y, respectively
  - The synchronous update is used, where all the players decide their strategies at the same time. All pairs of players x and y who are directly connected on the network model engage in each generation of the PD by using the above update rule

Some simulation results

- We are usually interested in the fraction of cooperating agents #C at the end of the simulation
- The most important issue is the interplay between #C and the structure of the network
- Specific structure may favor cooperativity
- Let's look at some results

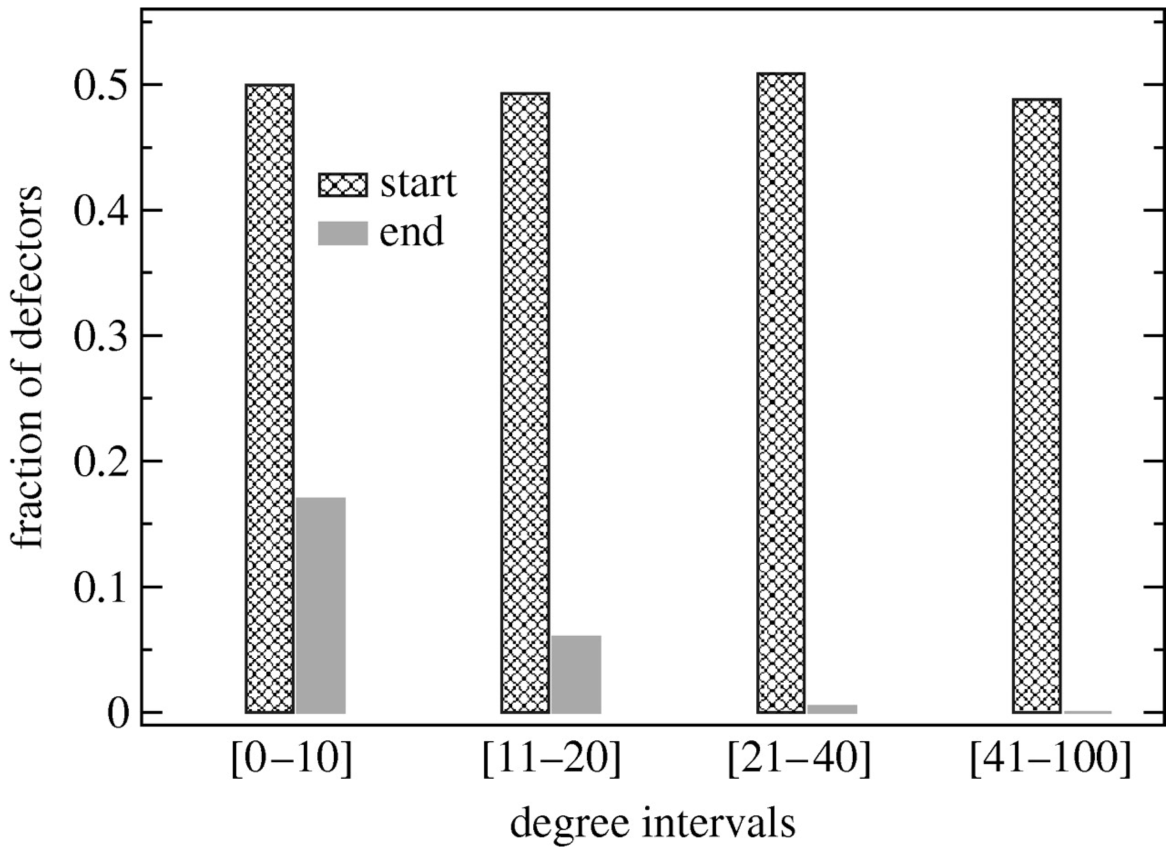
PD game on scale-free networks



Frequency of cooperators on different networks. Results for the PD shown as a function of the cheating advantage b. Results for (a) regular networks with different values of the average connectivity z; (b) scale-free ones and different values of z. In all cases, N=10^4. Cooperation hardly dominates on regular networks, but clearly dominates for all values of b on scale-free ones generated including growth and preferential attachment

Santos F et al. Proc. R. Soc. B 2006;273:51-55

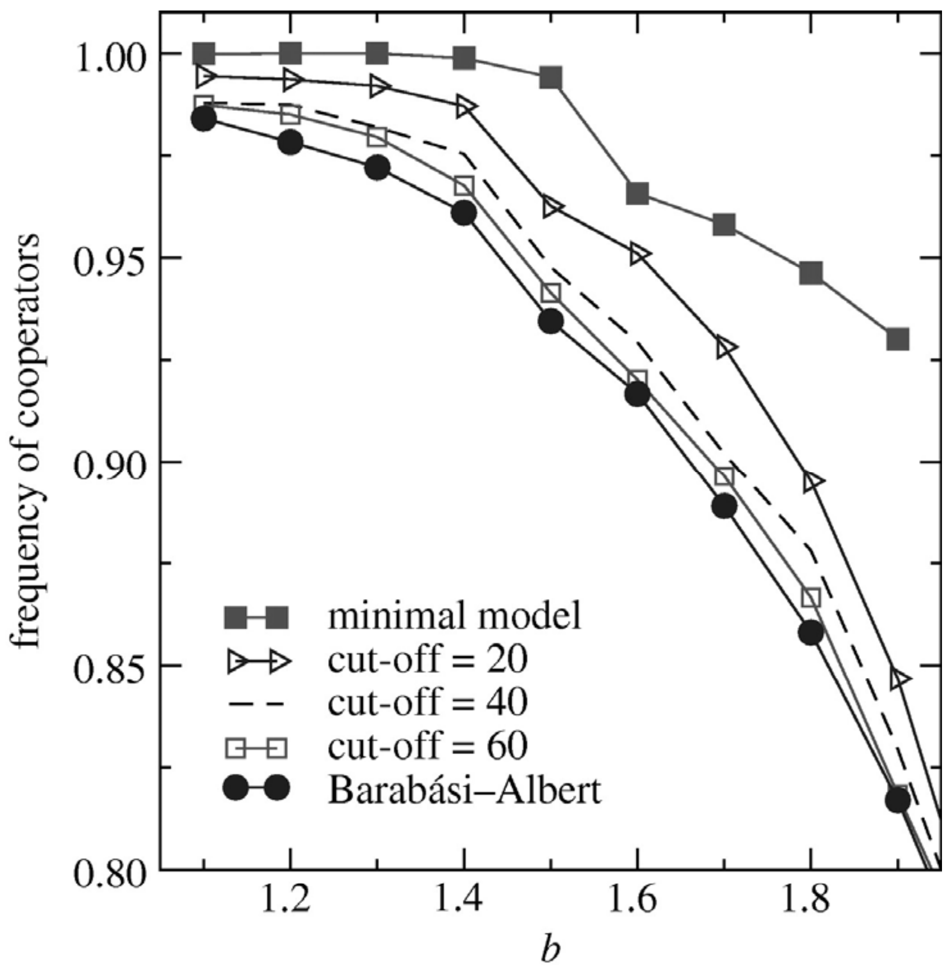
PD game on scale-free networks



Evolution of defectors by degree. The figure provides a typical scenario for the change in the distribution of defectors by degree, occurring as a result of evolution under natural selection. The cross-hatched bars show the fraction of vertices initially occupied by defectors (≈50%), for each degree-range specified by the intervals shown. Evolution leads to a stationary regime, with a distribution of defectors given by the solid bars. Clearly, defectors are efficiently wiped out from those vertices with largest connectivity, managing to survive as moderately connected individuals (results obtained for N=10^3, z=4 and b=1.7).

Santos F et al. Proc. R. Soc. B 2006;273:51-55

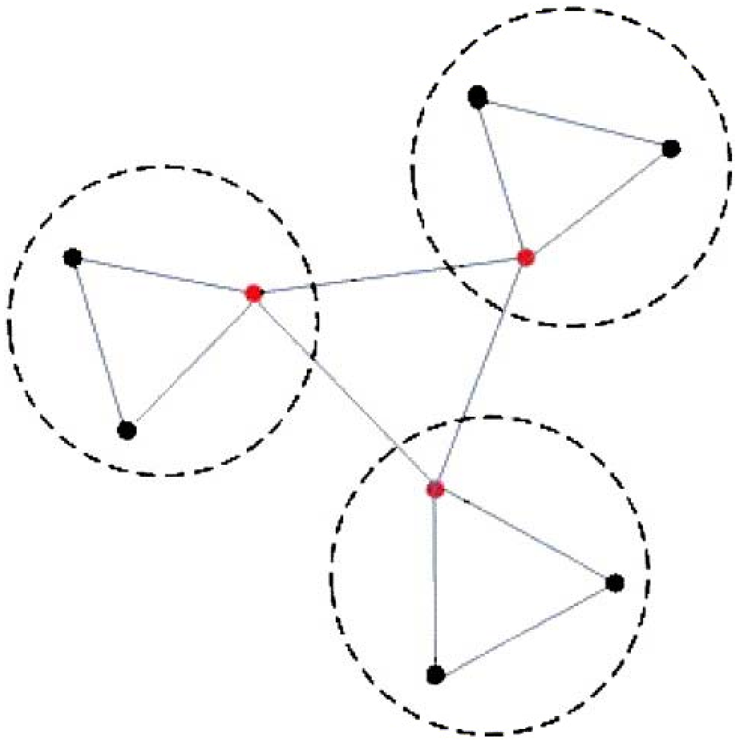
PD game on scale-free networks



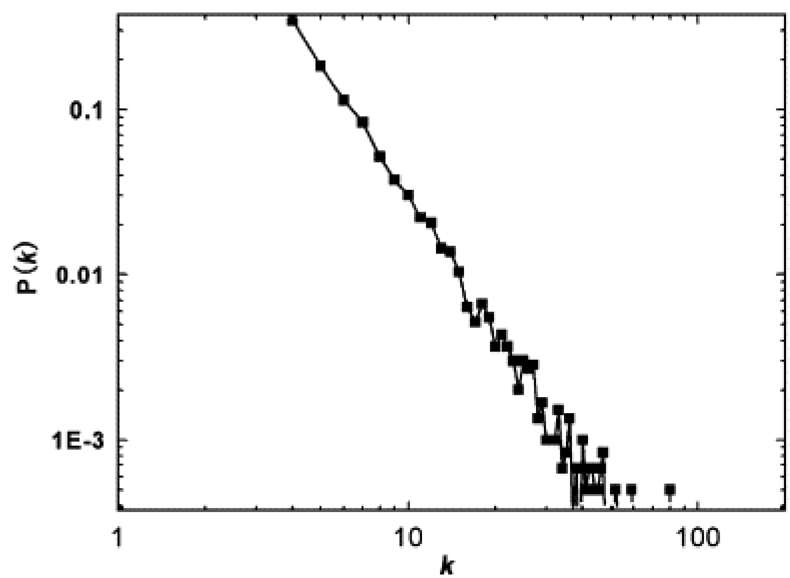
Results for the evolution of cooperation in networks exhibiting SF and truncated SF degree distributions. In all cases, the size is $N=10^4$ and the average connectivity is $z=4$. The results for the BA model (solid circles) are compared with those obtained with the minimal model of Dorogotsev *et al*. (2001; solid squares) and the truncated BA model, imposing cut-offs of 20, 40, and 60 for the maximum connectivity. As one continues to reduce the cut-off for maximum vertex connectivity, a sudden collapse of cooperation takes place, the behaviour resembling closely that obtained for the evolution of cooperation on regular networks.

Santos F et al. Proc. R. Soc. B 2006;273:51-55

PD game on community networks



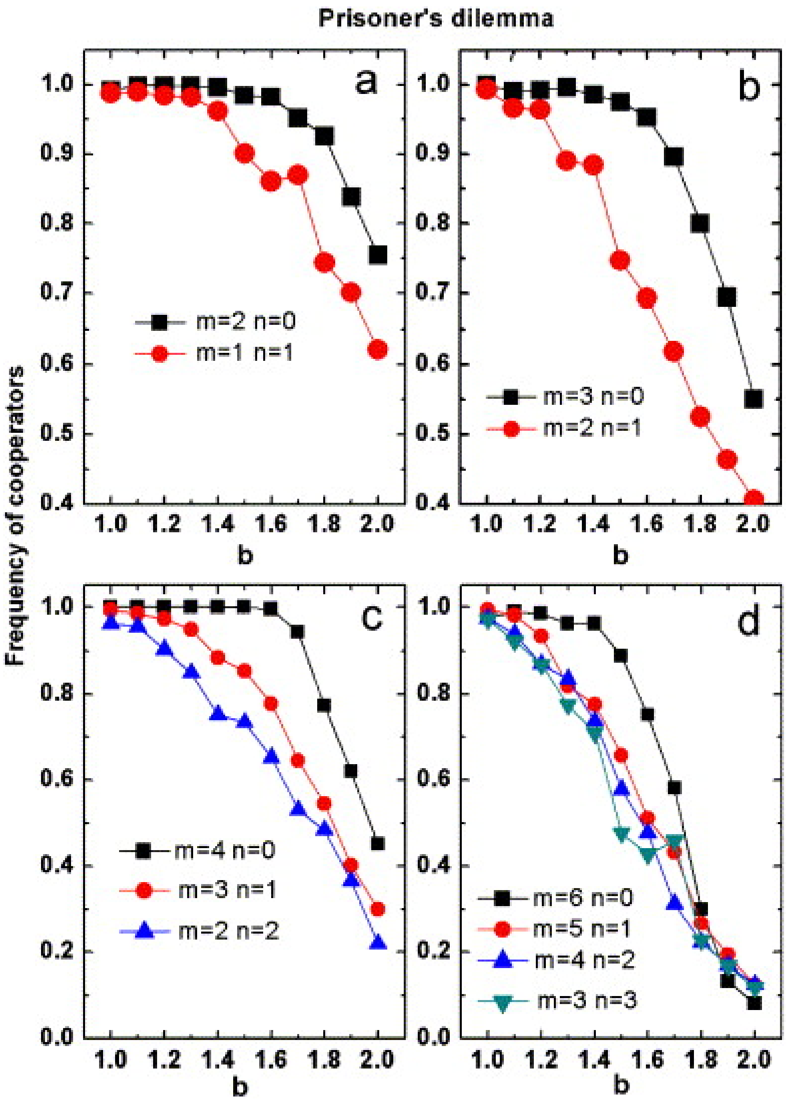
A model network with M = 3 communities and m_0 = 3. The red dots (n) are chosen to connect to each other between every two different communities



The degree-distribution of a network with N = 6000, M = 3, n =1 and m_0 = 3

Source, Chen et al, Physica A, 2007

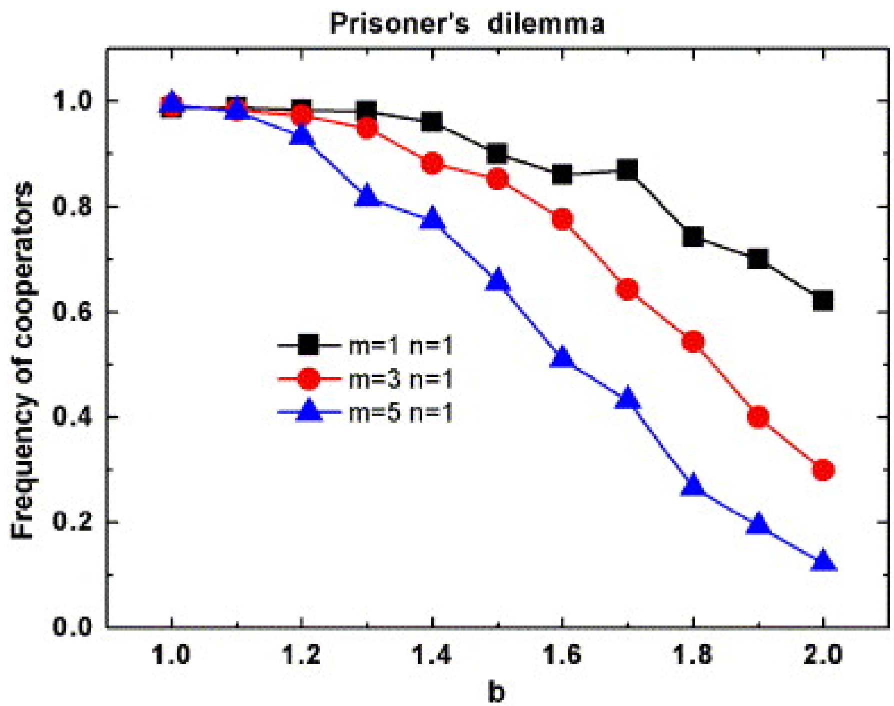
PD game on community networks



Frequency of cooperators for the PD as a function of the parameter b for different values of the average degree a, 4; b, 6; c, 8; d, 12. The colored lines in each subgraph correspond to different m and n for a fixed value of the average degree

Source, Chen et al, Physica A, 2007

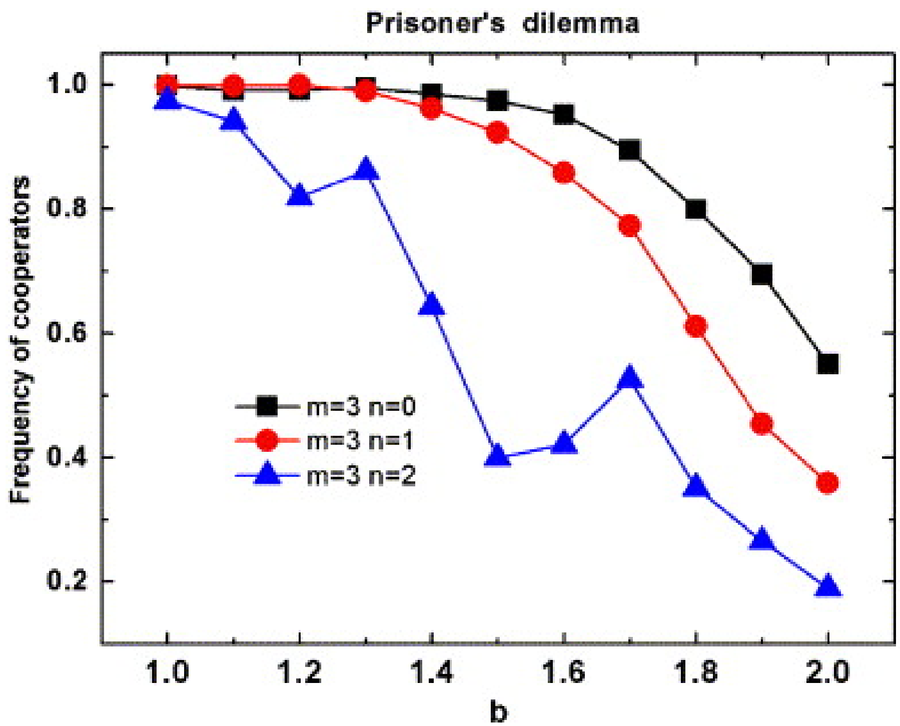
PD game on community networks



Frequency of cooperators in the PD as a function of the parameter b for different values of m, given a fixed value of n = 1

Source, Chen et al, Physica A, 2007

PD game on community networks



Frequency of cooperators in the PD as a function of the parameter b for different values of n, give fixed values of m =3 and m_0 = 3

Source, Chen et al, Physica A, 2007

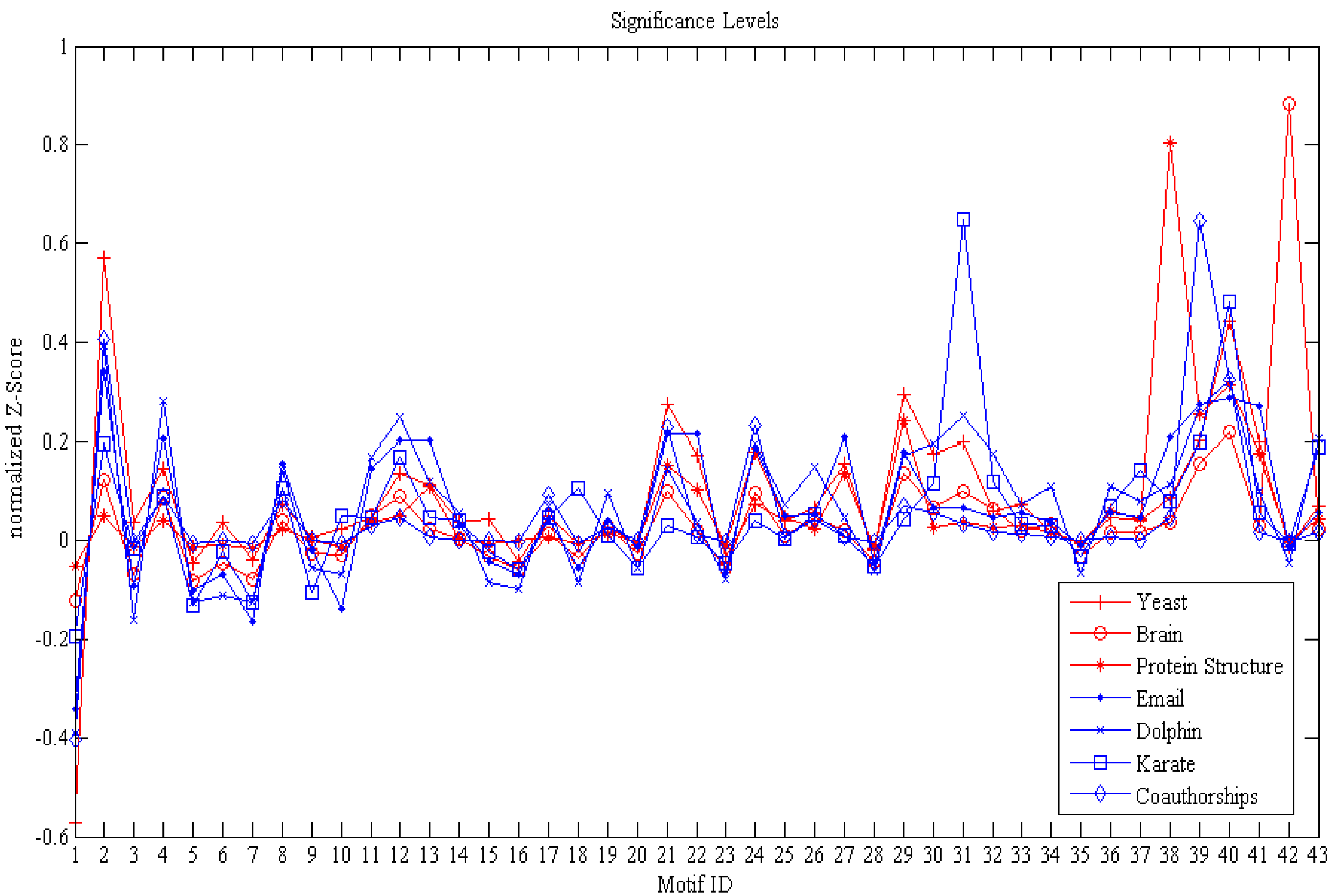
Motifs and cooperativity

A number of real-world networks with the following properties are considered

Type	Real-world networks	N	<k>	Std (k)	APL	C
Biological	Yeast Protein interaction	1458	2.68	3.45	6.71	0.08
	Human Brain	82	8.86	4.22	2.66	0.57
	Protein Structure	95	4.48	1.45	6.22	0.40
Social	Email communication	1163	9.62	9.34	3.60	0.22
	Dolphins' social interaction	62	5.12	2.96	3.3	0.26
	Zachary Karate Club	34	4.59	3.88	2.34	0.57
	Net-science Coauthorships	1589	3.45	3.47	6.02	0.64

Source, Salehi et al, Physica A, 2010

Motifs and cooperativity



The significant level of network motifs, normalized Z-scores, in the networks

Source, Salehi et al, Physica A, 2010

Motifs and cooperativity

Similarity between the significance levels of motifs in the networks as measured by Spearman correlation. The corresponding P-values are presented in parenthesis.

	Yeast	Brain	Pr.Str.	Email	Dolphin	Karate	Coauth.
Yeast	-	0.78 (1.2E-09)	0.85 (1.2E-12)	0.92 (1.0E-17)	0.81 (7.3E-11)	0.72 (7.5E-08)	0.87 (8.9E-14)
Brain	0.78 (1.2E-09)	-	0.74 (1.6E-08)	0.80 (1.4E-10)	0.89 (1.4E-15)	0.70 (2.2E-07)	0.84 (2.8E-12)
Protein Structure	0.85 (1.2E-12)	0.74 (1.6E-08)	-	0.93 (4.2E-19)	0.78 (1.5E-09)	0.62 (1.1E-05)	0.78 (8.0E-10)
Email	0.92 (1.0E-17)	0.80 (1.4E-10)	0.93 (4.2E-19)	-	0.83 (6.3E-12)	0.67 (1.3E-06)	0.86 (1.8E-13)
Dolphin	0.81 (7.2E-11)	0.89 (1.4E-15)	0.78 (1.5E-09)	0.83 (6.3E-12)	-	0.82 (3.3E-11)	0.89 (3.5E-15)
Karate	0.72 (7.4E-08)	0.70 (2.2E-07)	0.62 (1.1E-05)	0.67 (1.3E-06)	0.82 (3.3E-11)	-	0.69 (4.1E-07)
Coauthorship	0.87 (8.9E-14)	0.84 (2.8E-12)	0.78 (8.1E-10)	0.86 (1.8E-13)	0.89 (3.5E-15)	0.69 (4.1E-07)	-

Source, Salehi et al, Physica A, 2010

Motifs and cooperativity

Size	ID	Motif	⟨C#⟩	ID	Motif	⟨C#⟩	ID	Motif	⟨C#⟩
3	1		0.370	2		0.180			
4	3		0.325	4		0.293	5		0.340
	6		0.404	7		0.452	8		0.414
5	9		0.358	10		0.396	11		0.412
	12		0.382	13		0.280	14		0.260
	15		0.390	16		0.358	17		0.357
6	18		0.412	19		0.317	20		0.378
	21		0.262	22		0.288	23		0.372
	24		0.288	25		0.298	26		0.342
	27		0.257	28		0.342	29		0.288
	30		0.345	31		0.377	32		0.237
	33		0.240	34		0.358	35		0.303
	36		0.230	37		0.158	38		0.207
	39		0.240	40		0.222	41		0.168
	42		0.175	43		0.185			

Cooperativity #C in various subgraph structures

Source, Salehi et al, Physica A, 2010

Motifs and cooperativity

The Spearman rank correlation between the #C of motifs and their Z-score in the networks.

Type	Real-world networks	Spearman Correlation	
		r	***P-value***
Biological	Yeast Protein interaction	-0.4564	0.0024
	Human Brain	-0.4369	0.0038
	Protein Structure	-0.6085	1.91E-05
Social	Email communication	-0.5367	2.48E-04
	Dolphins' social interaction	-0.4074	0.0074
	Zachary Karate Club	-0.3466	0.0246
	Net-science Coauthorships	-0.3514	0.0225

Source, Salehi et al, Physica A, 2010

Readings

- Hisashi Ohtsuki, Christoph Hauert, Erez Lieberman, and Martin A. Nowak, A simple rule for the evolution of cooperation on graphs and social networks, Nature, 441: 502-505, 2006.
- F. C. Santos and J. M. Pacheco, Scale-Free Networks Provide a Unifying Framework for the Emergence of Cooperation, Physical Review Letters, 95: 098104, 2005.