



# Computational Assemblies: Analysis, Design, and Fabrication

Peng Song

Ziqi Wang

Marco Livesu

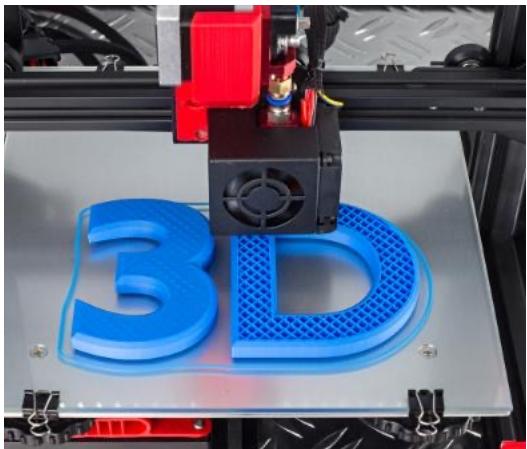


# Timetable

		Peng	Ziqi	Marco
Introduction	~20 mins	X		
Computational analysis of assemblies	~50 mins	X		
Computational design of assemblies	~50 mins		X	
Computational fabrication of assemblies	~50 mins			X
Q & A	~10 mins	X	X	X

# What Is This Part About?

- Shape decomposition as a mean to **overcome the limitations** of digital fabrication hardware



3D PRINTING



CNC MILLING



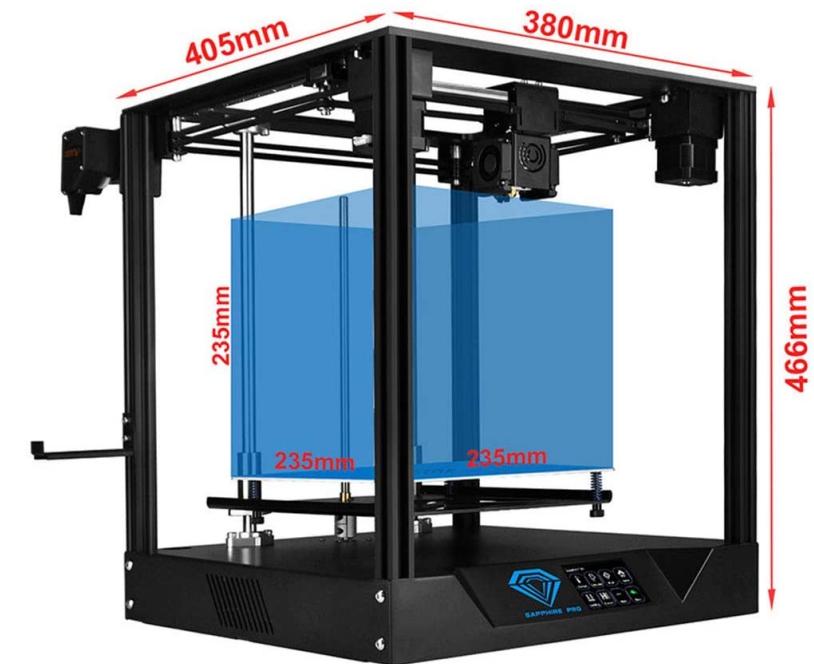
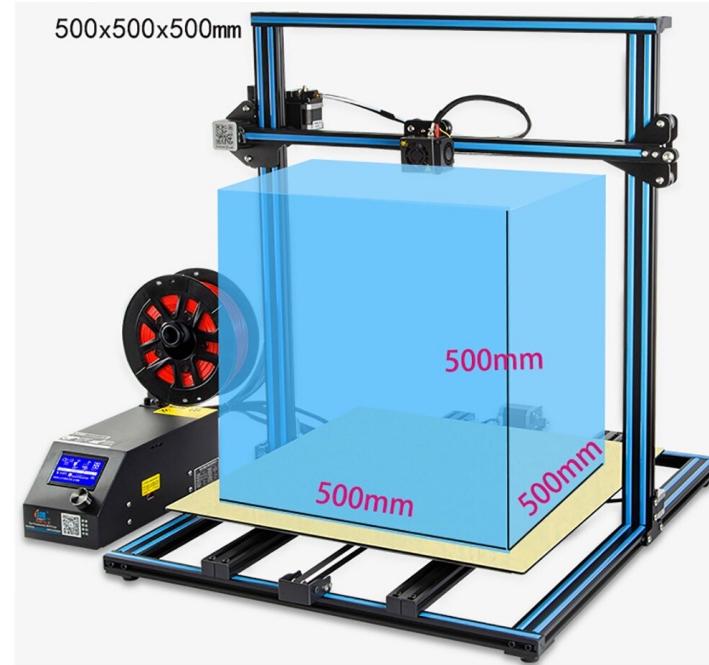
LASER CUTTING



CASTING/MOLDING

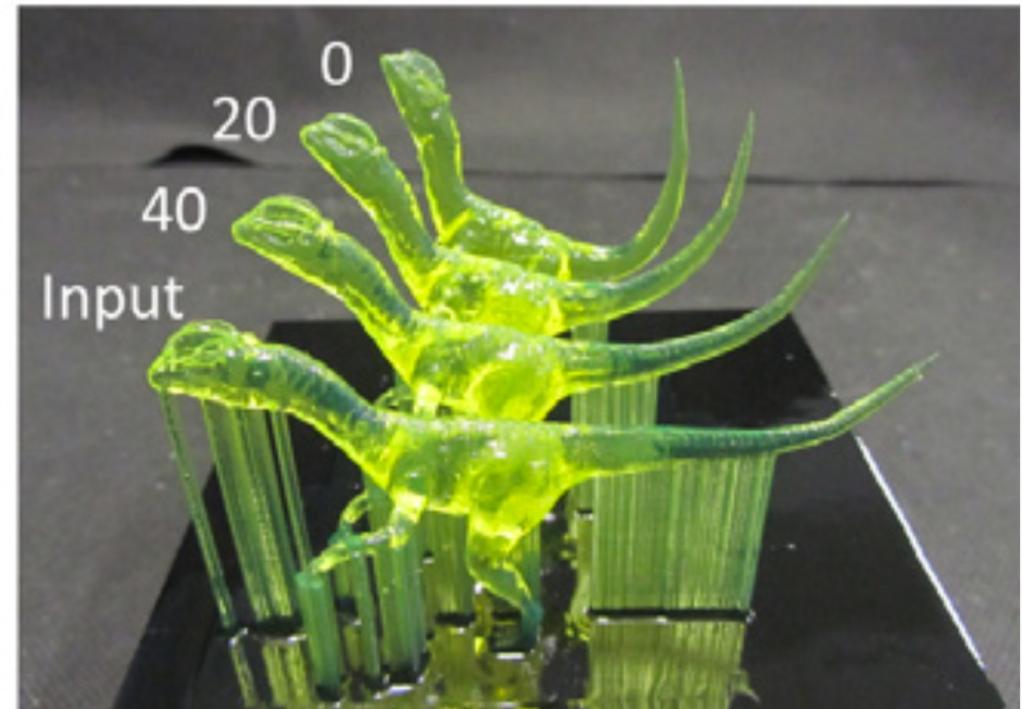
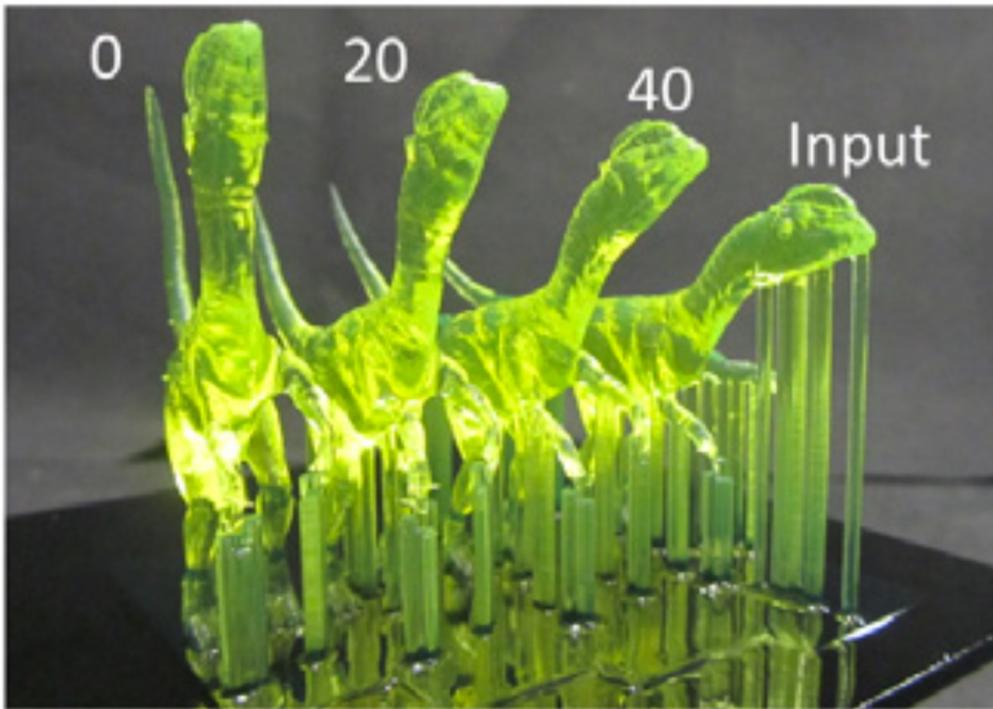
# Remark #1

- Unlike previous parts, here assemblies **\*are not\*** an artistic/functional choice. We split shapes **because we are obliged to!**
- Algorithms must be designed to address **hardware dependent** constraints (e.g. size)
- Constraints can be either **hard** or **soft**



## Remark #2

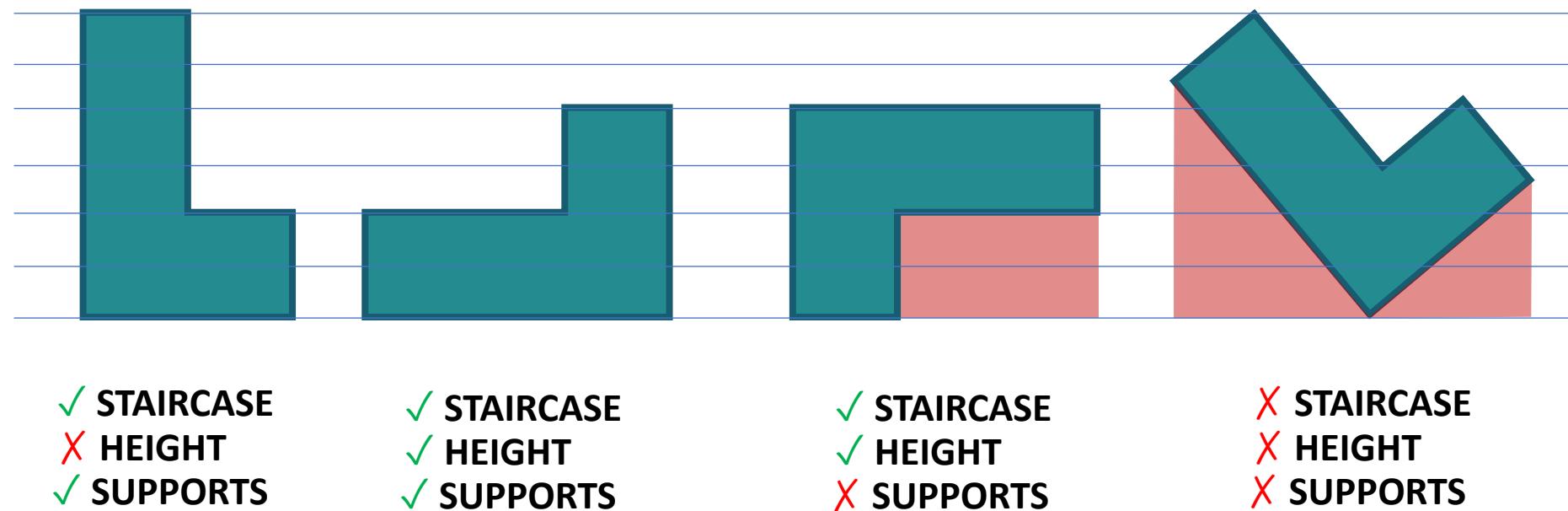
- Splitting is just one possible way to address manufacturing constraints
- Mesh **deformation** is a valid alternative



[Hu et al., CAD 2015]

# Remarks

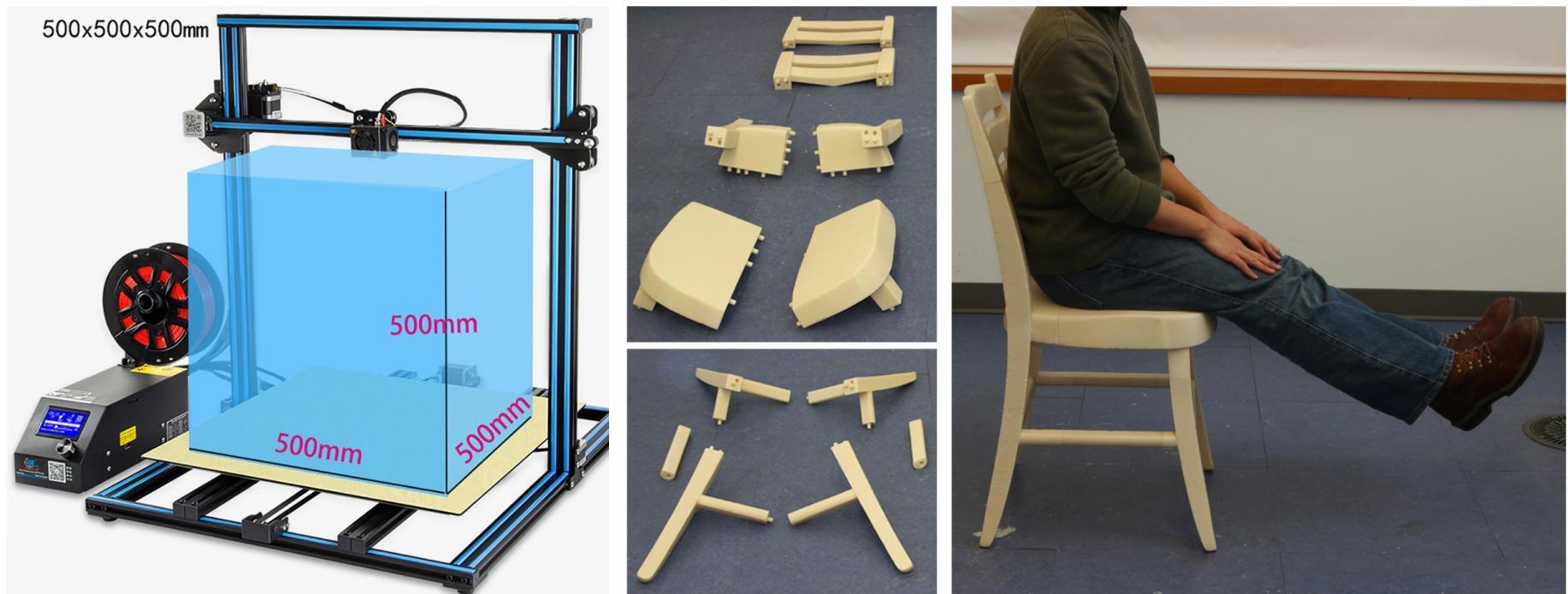
- Splitting is just one possible way to address manufacturing constraints
- Mesh **deformation** is a valid alternative
- Also **re-orientation** may be a valid alternative



[Livesu et al., EG STAR 2017]

# Shape Decomposition for Manufacturing

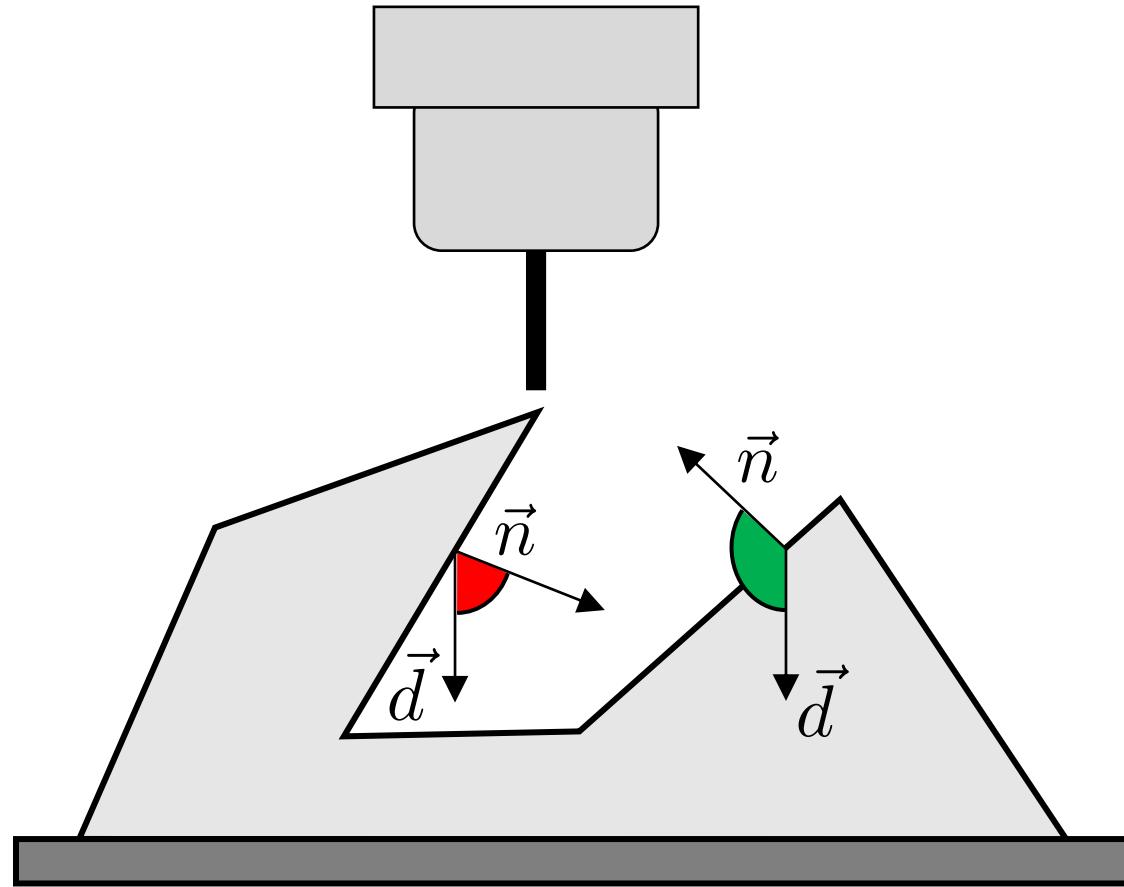
– size



[Luo et al., SIGGRAPH 2012]

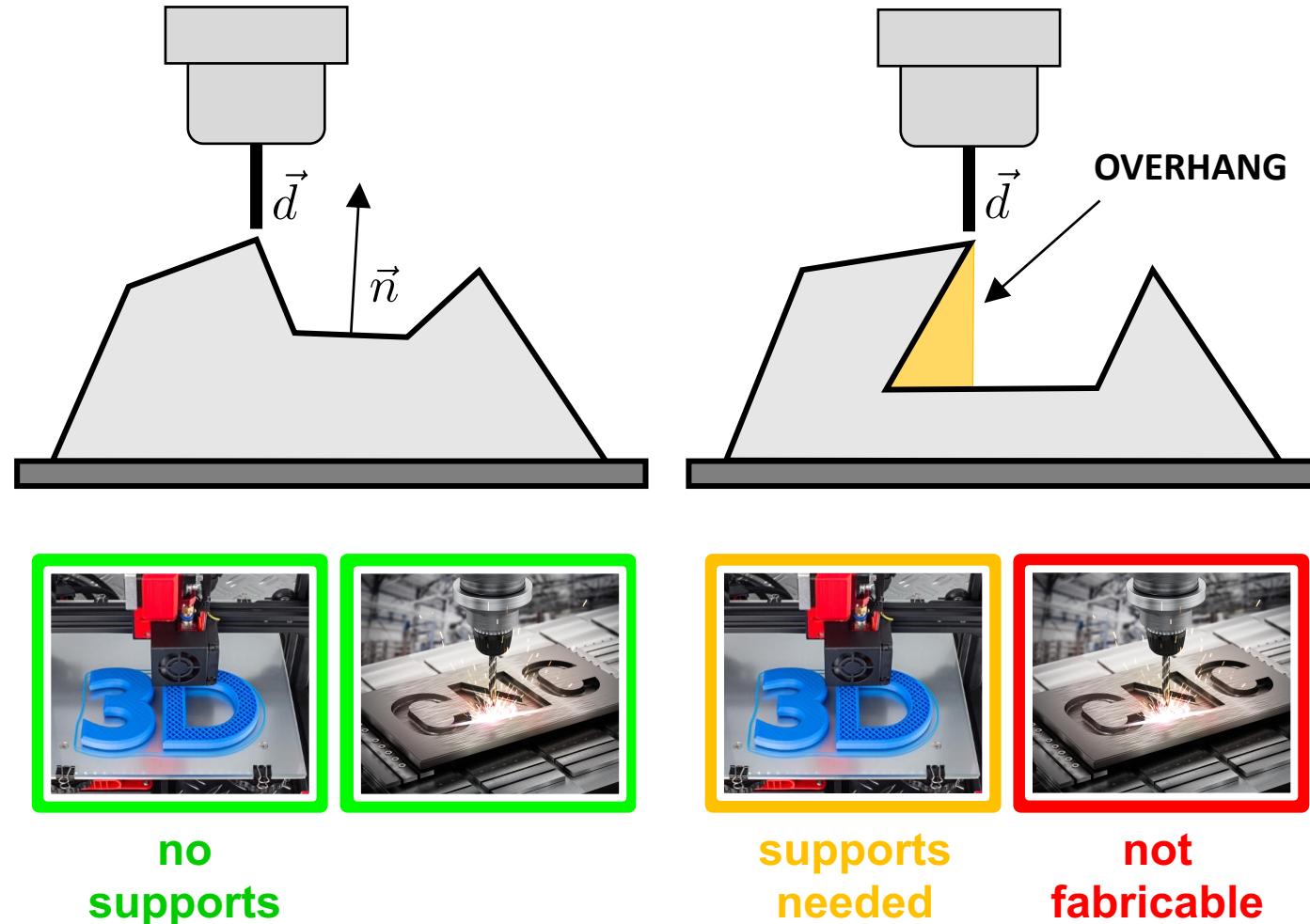
# Shape Decomposition for Manufacturing

- size
- **geometry**



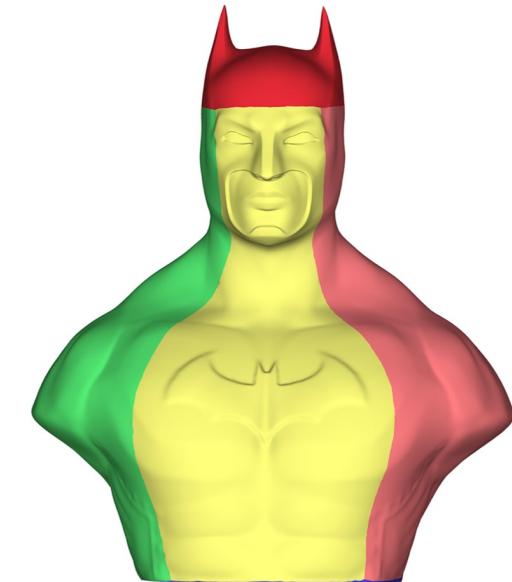
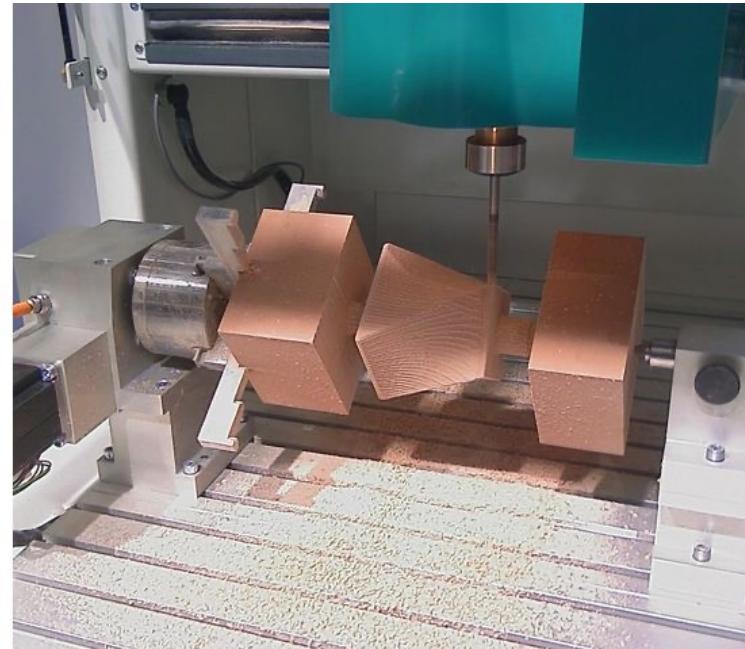
# Shape Decomposition for Manufacturing

- size
- **geometry**
  - 3d printing
  - 3 axis milling



# Shape Decomposition for Manufacturing

- size
- **geometry**
  - 3d printing
  - 3 axis milling
  - **4 axis milling**



[Nuvoli et al., EG 2021]

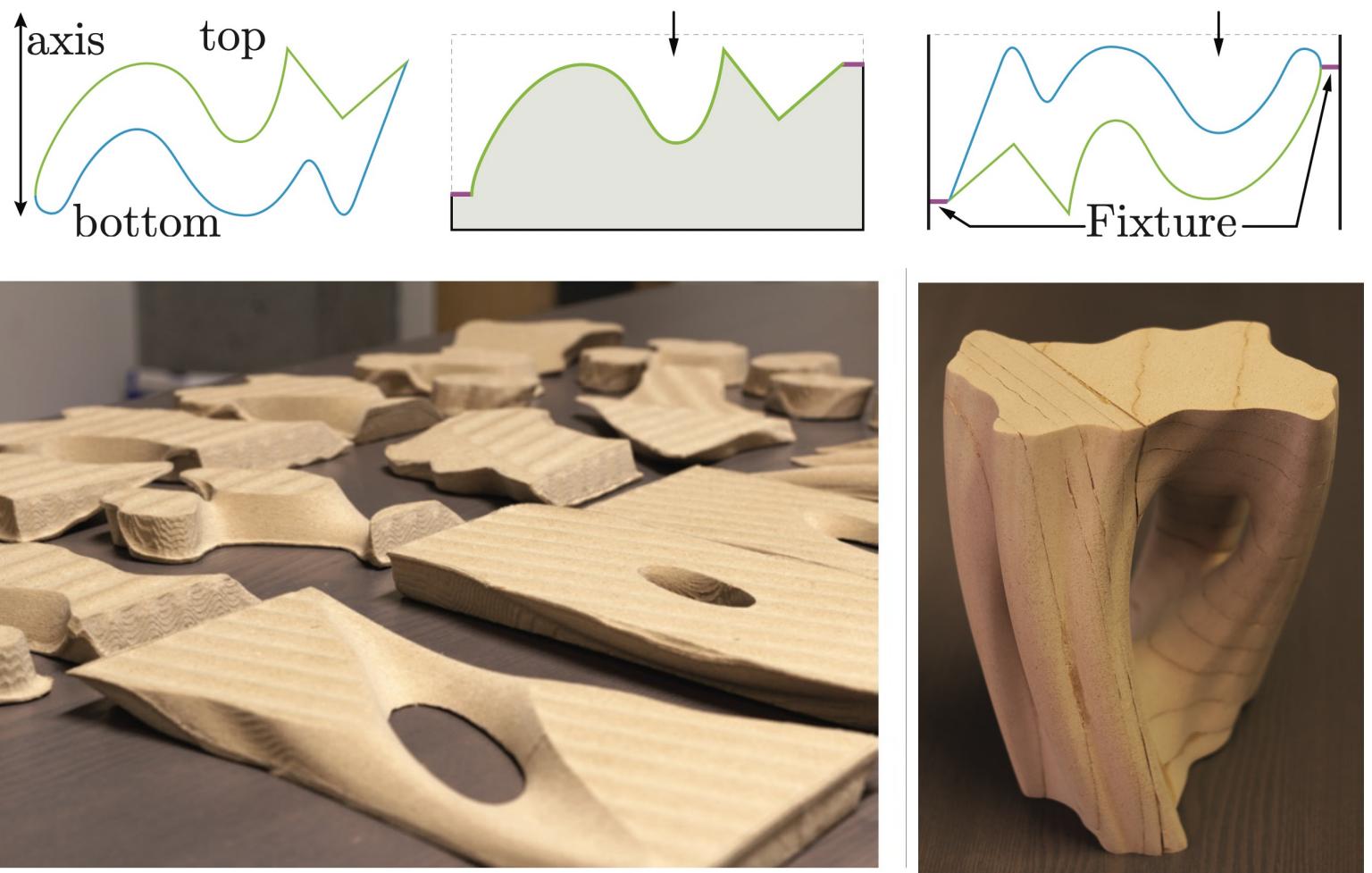


# Shape Decomposition for Manufacturing

- size

- **geometry**

- 3d printing
- 3 axis milling
- 4 axis milling
- DHF



[Yang et al., SIGGRAPH Asia 2020]

# Shape Decomposition for Manufacturing

– size

– **geometry**

- 3d printing
- 3 axis milling
- 4 axis milling
- DHF
- **laser cutting**



[Schüller et al., SIGGRAPH 2018]

# Shape Decomposition for Manufacturing

- size
- **geometry**

- 3d printing
- 3 axis milling
- 4 axis milling
- DHF
- laser cutting
- **rigid molding**



[Alderighi et al., SIGGRAPH 2021]

# Shape Decomposition for Manufacturing

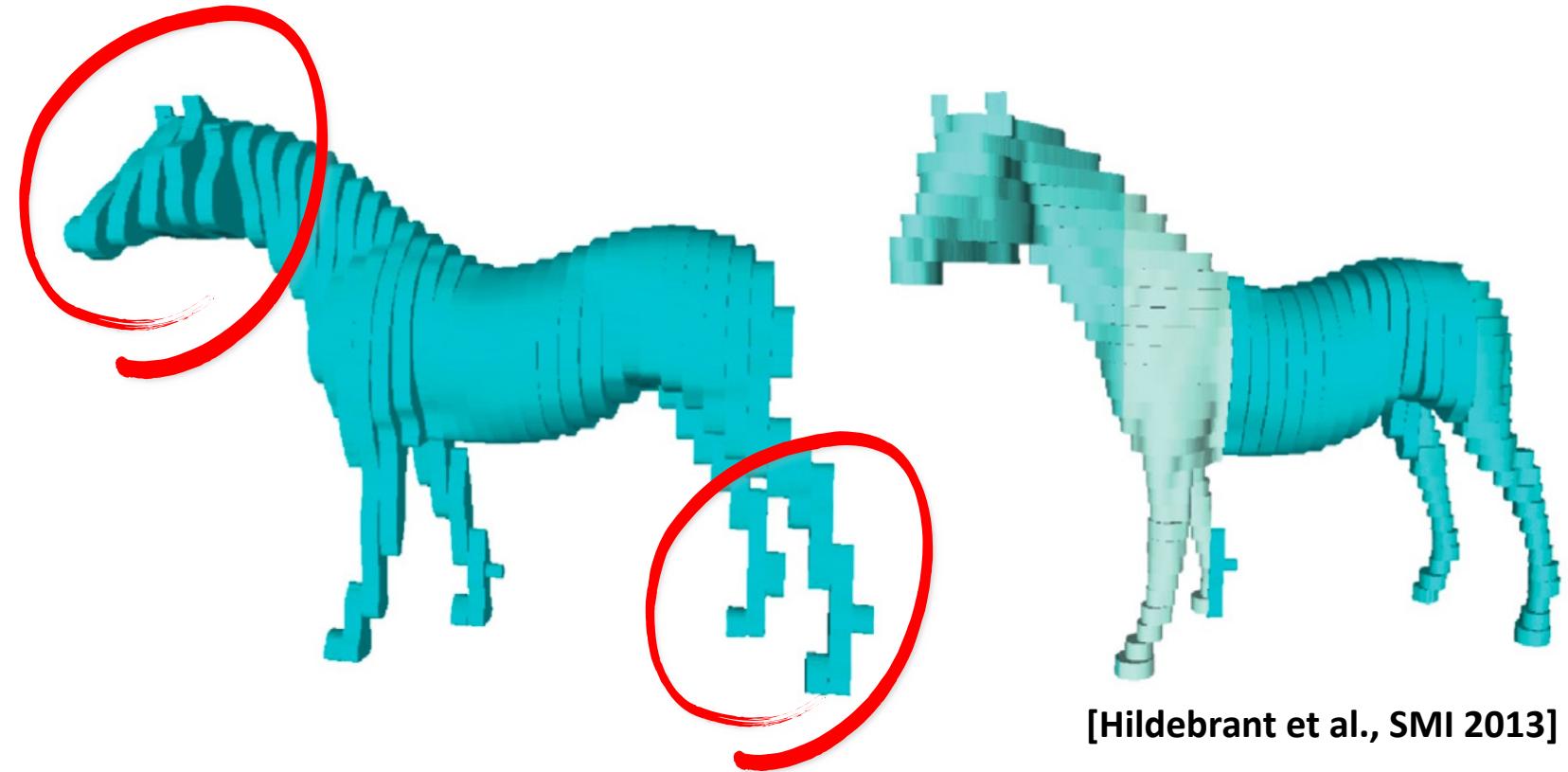
- size
- **geometry**
  - 3d printing
  - 3 axis milling
  - 4 axis milling
  - DHF
  - laser cutting
  - rigid molding
  - **flexible molding**



[Alderighi et al., SIGGRAPH 2019]

# Shape Decomposition for Manufacturing

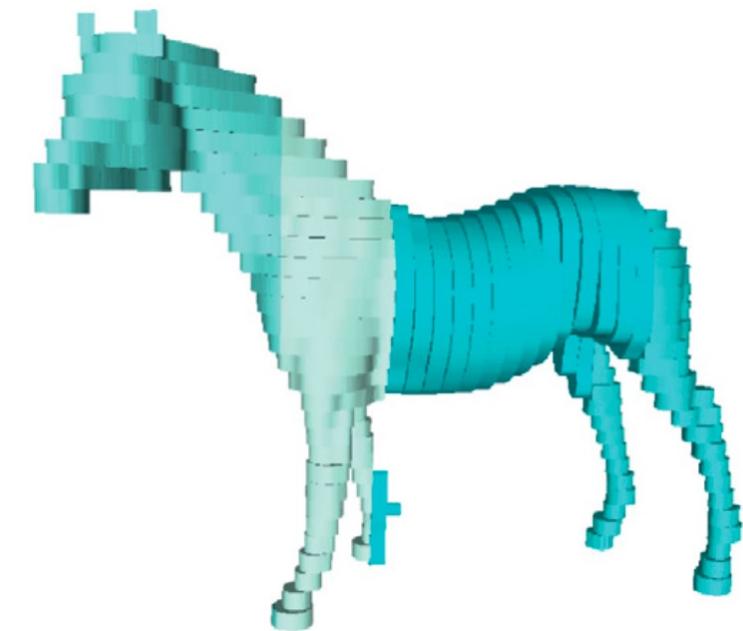
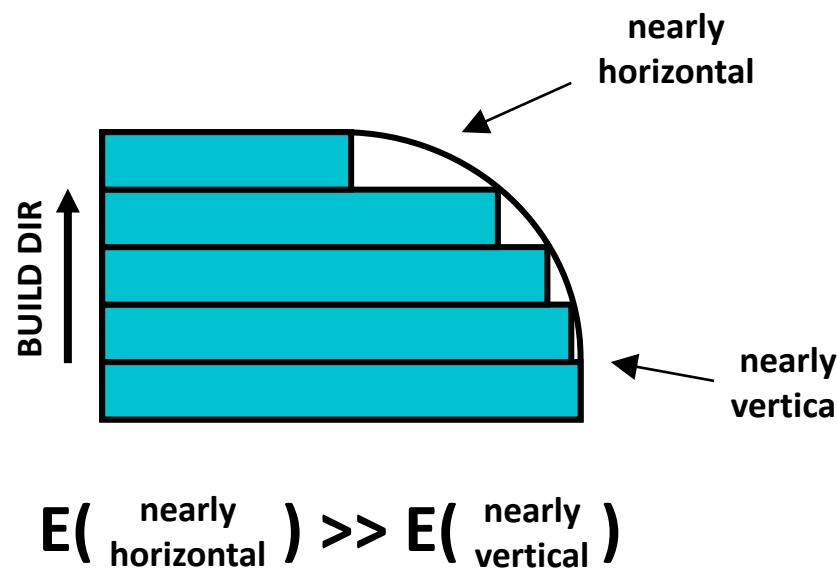
- size
- geometry
- **staircase effect**



[Hildebrant et al., SMI 2013]

# Shape Decomposition for Manufacturing

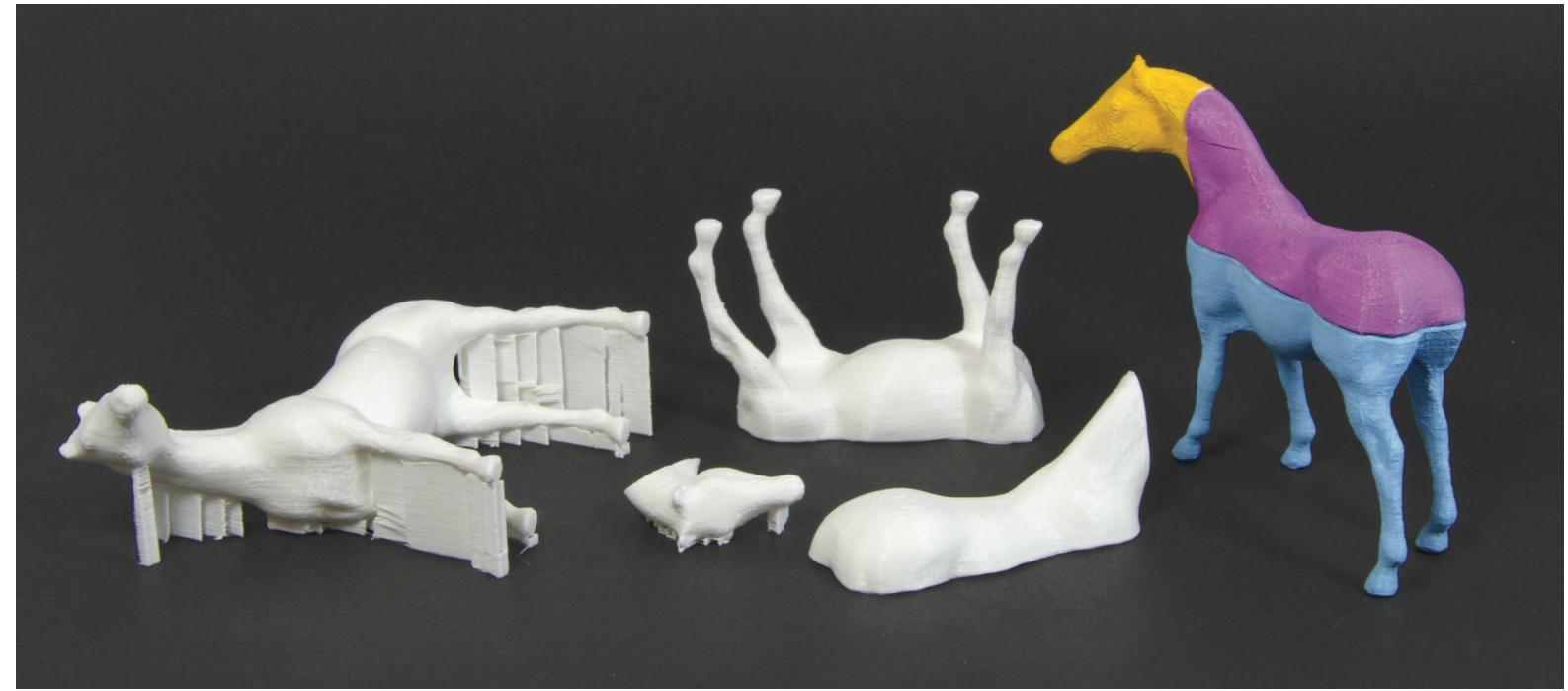
- size
- geometry
- **staircase effect**



[Hildebrant et al., SMI 2013]

# Shape Decomposition for Manufacturing

- size
- geometry
- staircase effect
- **material consumption**

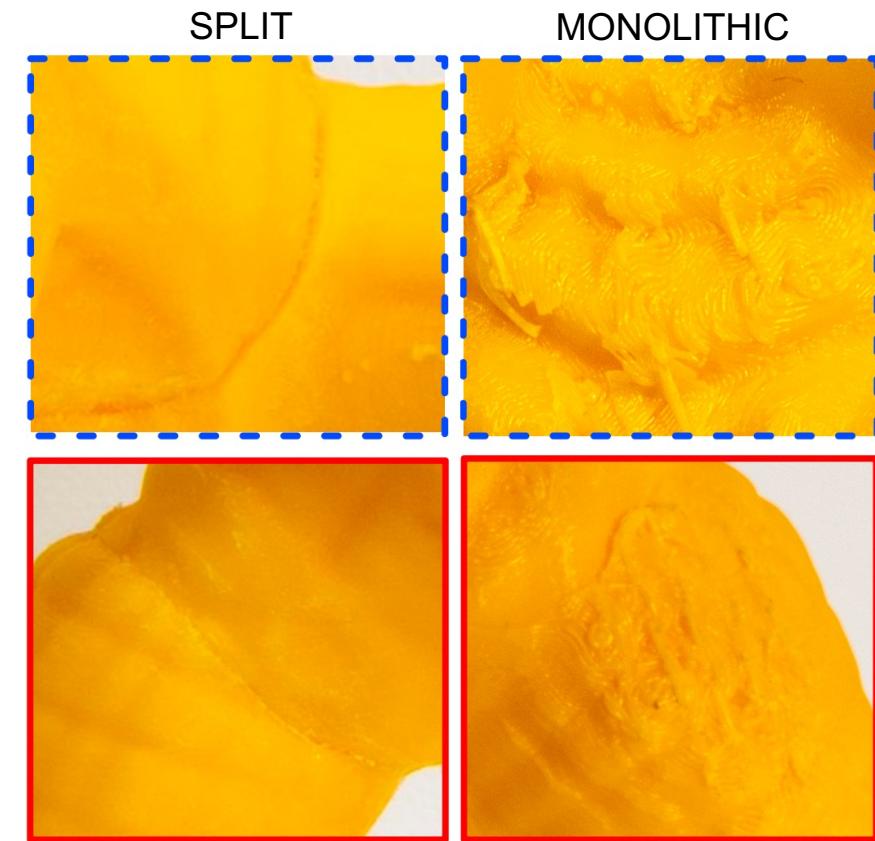


non  
pyramidal

pyramidal  
decomposition

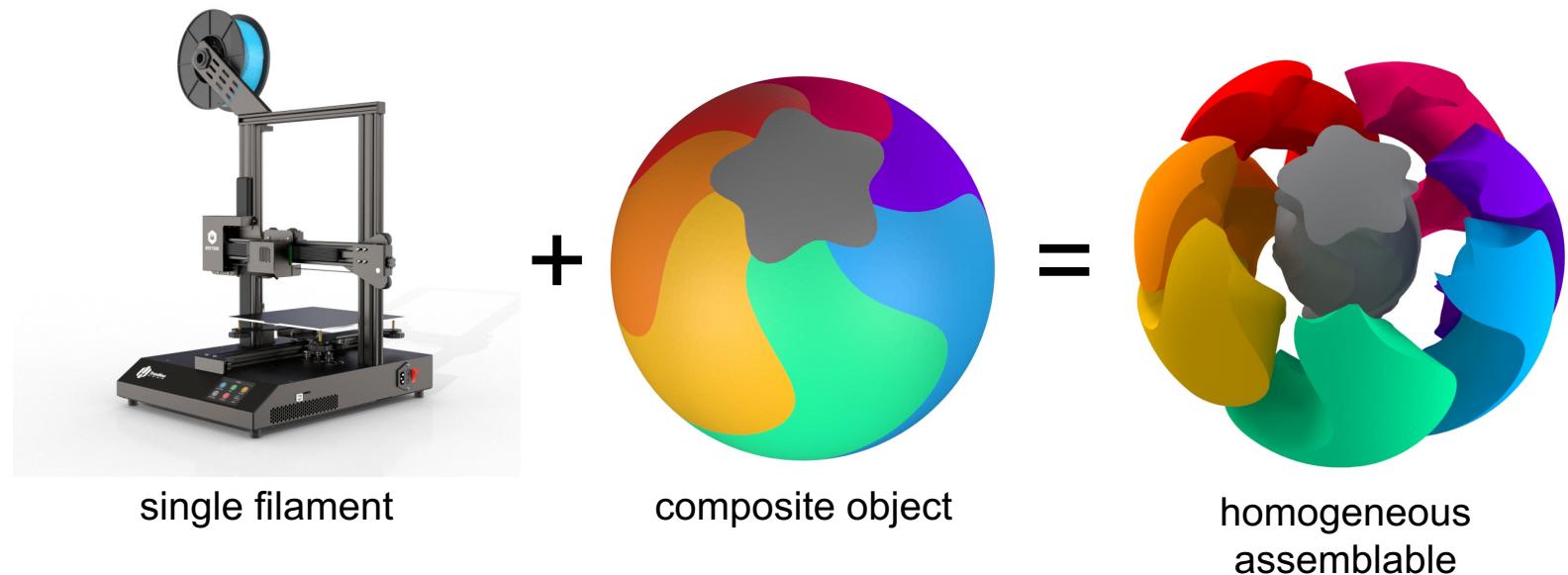
# Shape Decomposition for Manufacturing

- size
- geometry
- staircase effect
- material consumption
- **support artifacts**



# Shape Decomposition for Manufacturing

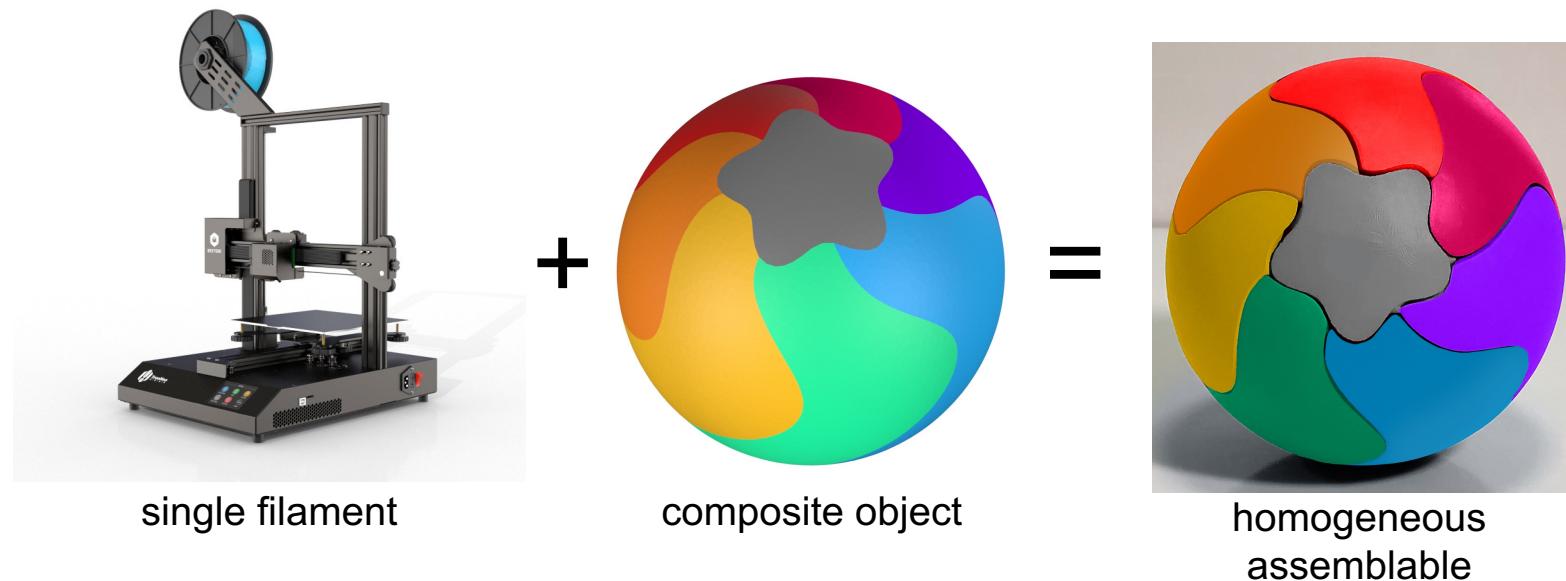
- size
- geometry
- staircase effect
- material consumption
- support artifacts
- **material/color**



[Araújo et al., SIGGRAPH 2019]

# Shape Decomposition for Manufacturing

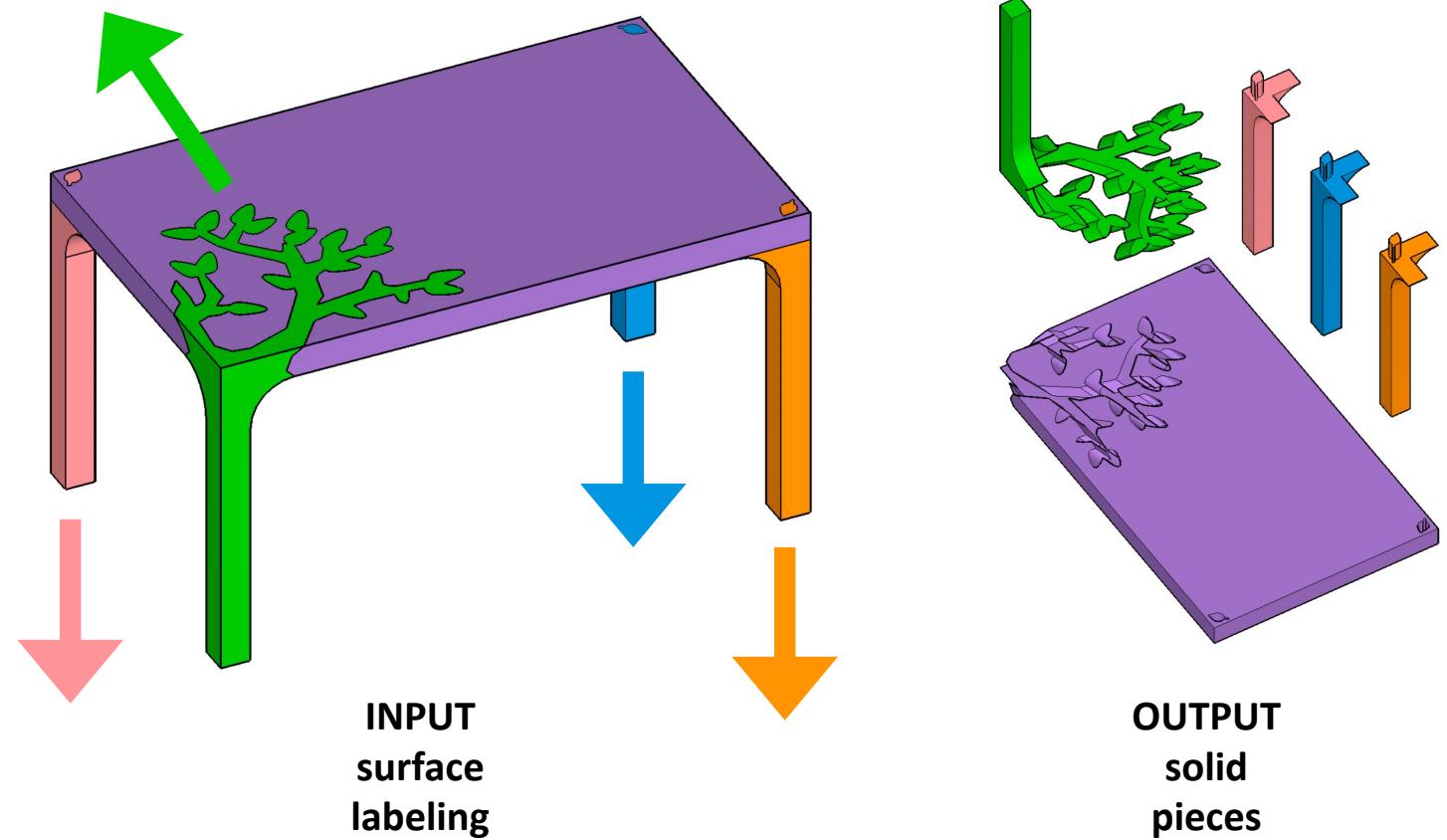
- size
- geometry
- staircase effect
- material consumption
- support artifacts
- **material/color**



[Araújo et al., SIGGRAPH 2019]

# Shape Decomposition for Manufacturing

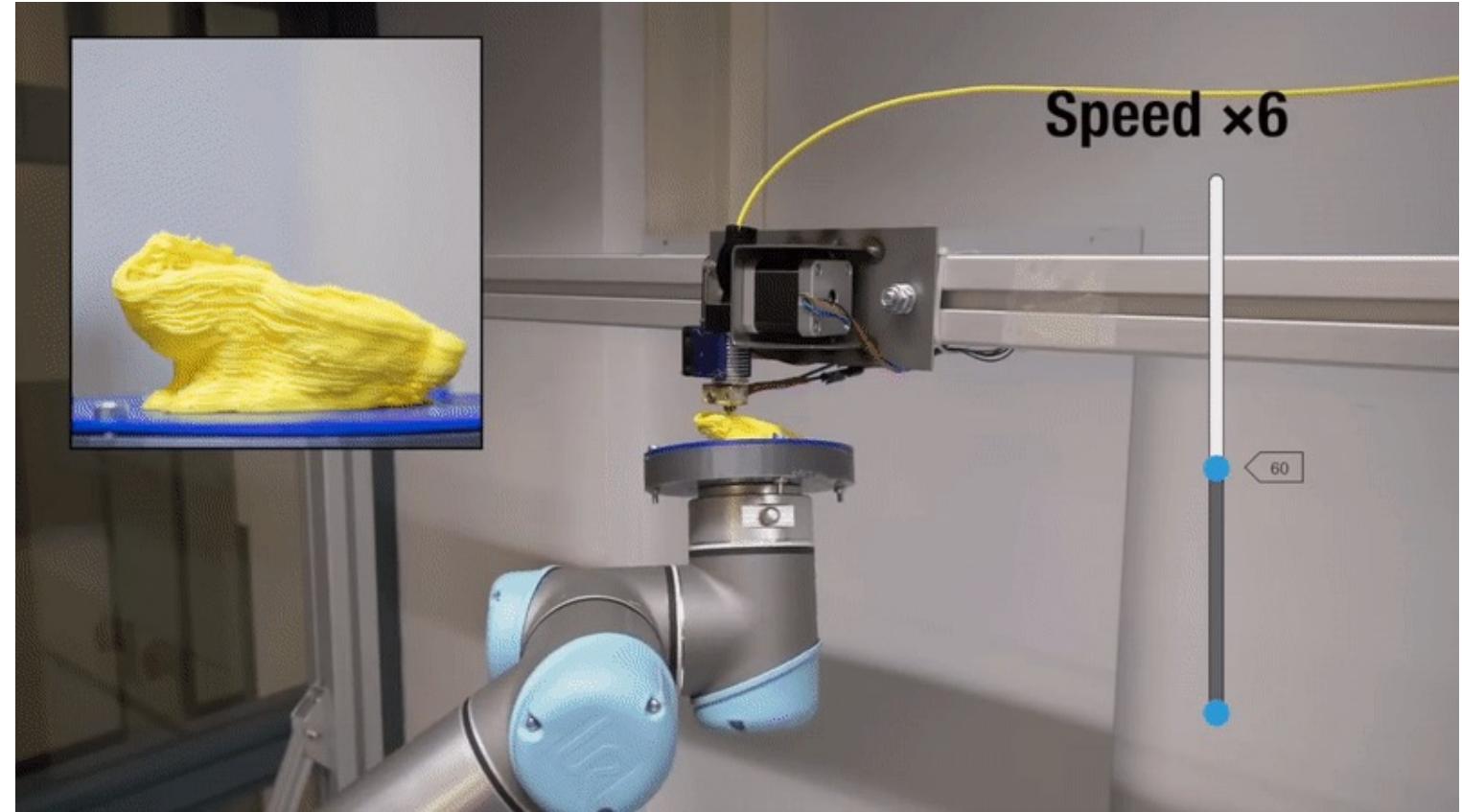
- size
- geometry
- staircase effect
- material consumption
- support artifacts
- material/color
- **assemblability**



[Yao et al., ACM TOG 2017]

# Shape Decomposition for Manufacturing

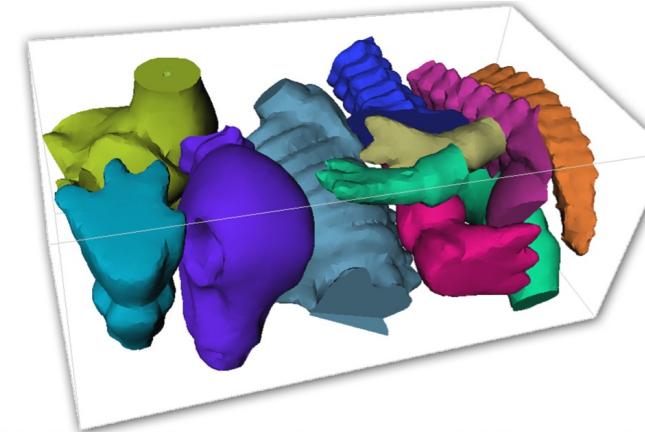
- size
- geometry
- staircase effect
- material consumption
- support artifacts
- material/color
- assemblability
- collision avoidance**



[Dai et al., SIGGRAPH 2018]

# Shape Decomposition for Manufacturing

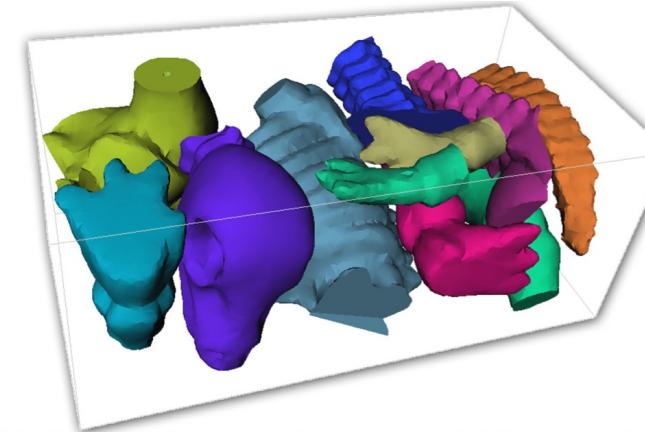
- size
- geometry
- staircase effect
- material consumption
- support artifacts
- material/color
- assemblability
- collision avoidance
- **packing**



[Attene, EG 2015]

# Shape Decomposition for Manufacturing

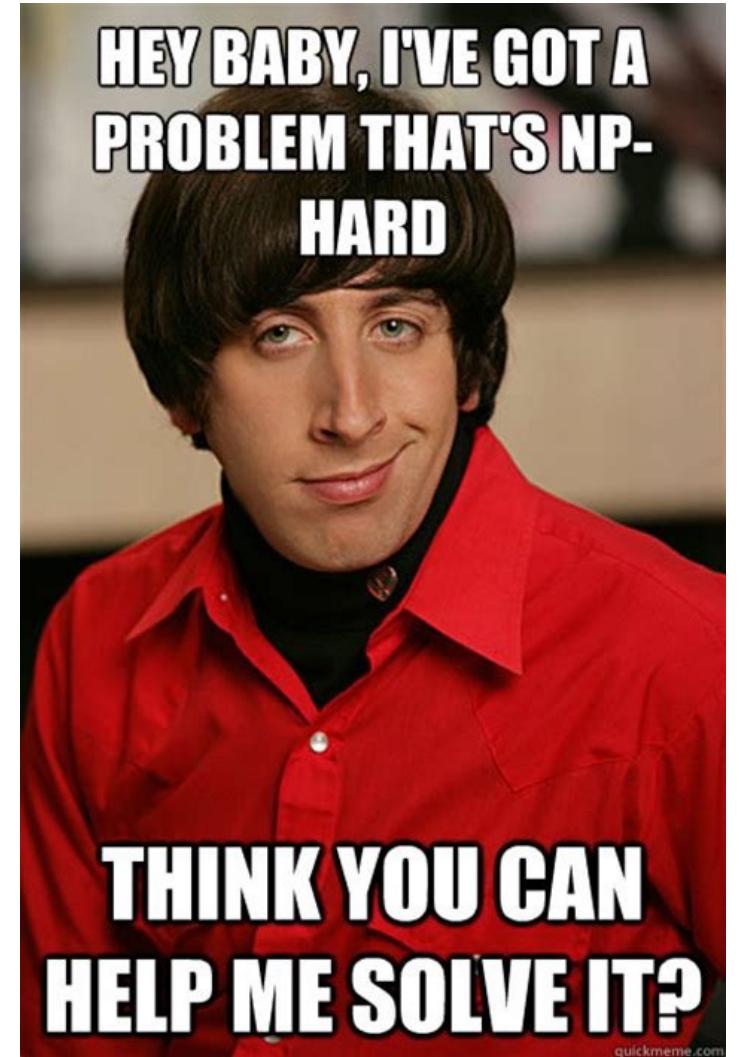
- size
- geometry
- staircase effect
- material consumption
- support artifacts
- material/color
- assemblability
- collision avoidance
- packing
- ... and many others!



[Attene, EG 2015]

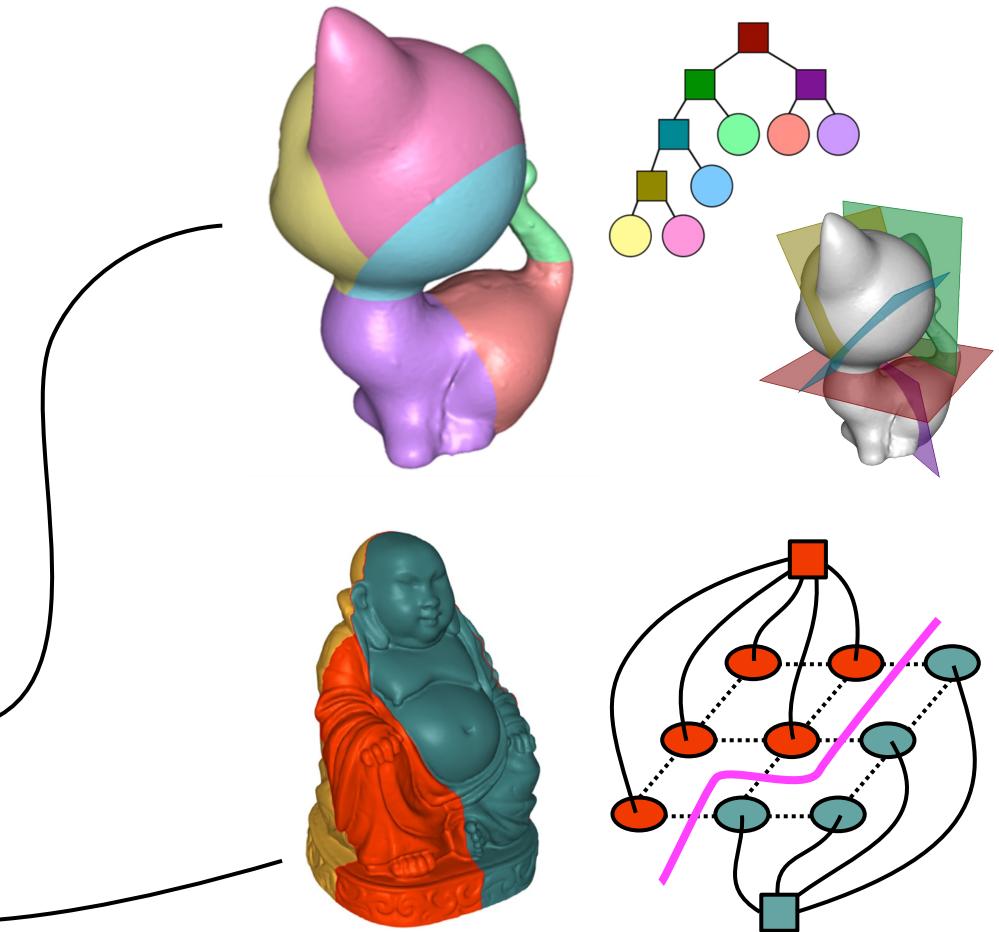
# How Hard it is?

- Computing a **decomposition** is **easy**
- Computing a **constrained decomposition** is **still easy**
  - why? I can split into many tiny pieces!
- Computing a **constrained decomposition** that is **minimal** is **NP-Hard**
  - search space grows exponentially
  - no hope to find the global optimum
  - just strive for a "good" local minimum
  - it's all about heuristics



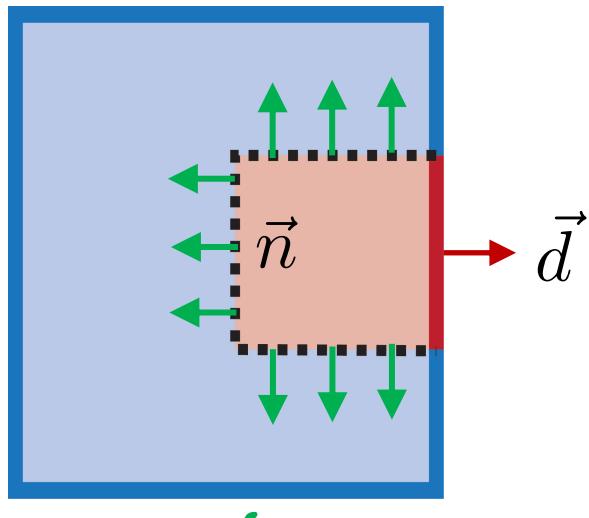
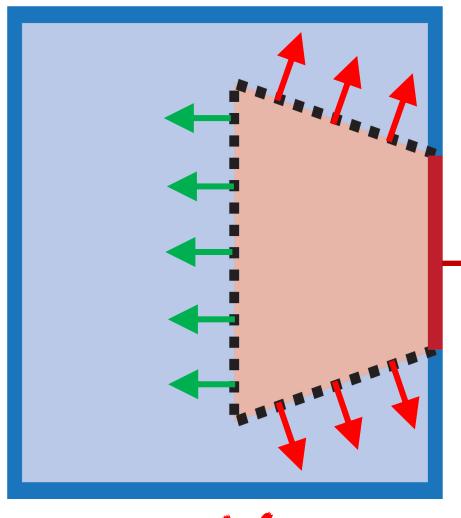
# Recurring Approaches

- Despite this variety of goals, manufacturing paradigms and fabrication hardware, all methods
- Aim to control the **same aspects**
  - part size
  - local surface orientation
  - assemblability
- Mostly exploit **similar techniques**
  - Binary Space Partitions
  - Graph Labeling
  - Mesh Booleans



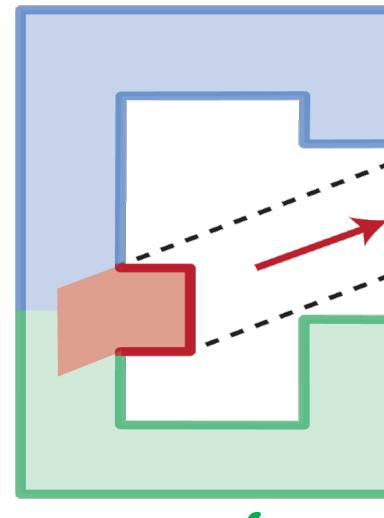
# Assemblability – Rigid Bodies

- Parts must fulfill both **local** and **global** requirements



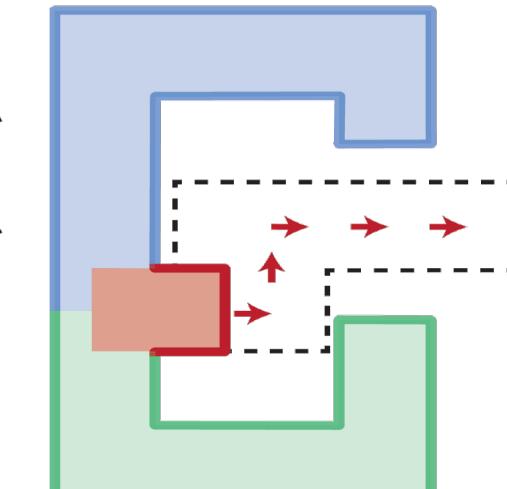
INTERFACES MUST BE **HEIGHT FIELDS**  
W.R.T. THE EXTRACTION DIRECTION

$$\vec{d} \cdot \vec{n} \leq 0$$



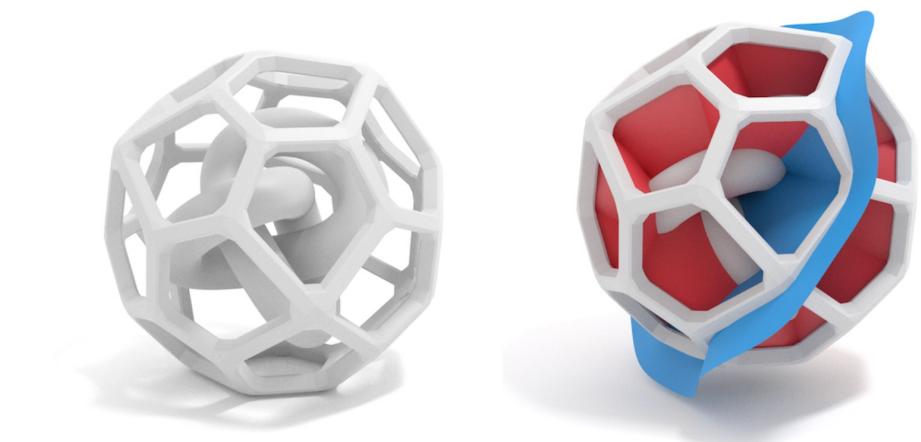
THERE MUST EXIST A **CLEARED**  
EXTRACTION PATH

HUGE SEARCH SPACE!



# Assemblability – Soft Bodies

- Can be extracted even if they **violate** height fieldness
  - full FEM simulation is overly expensive
    - simulating contact and frictional forces is hard!

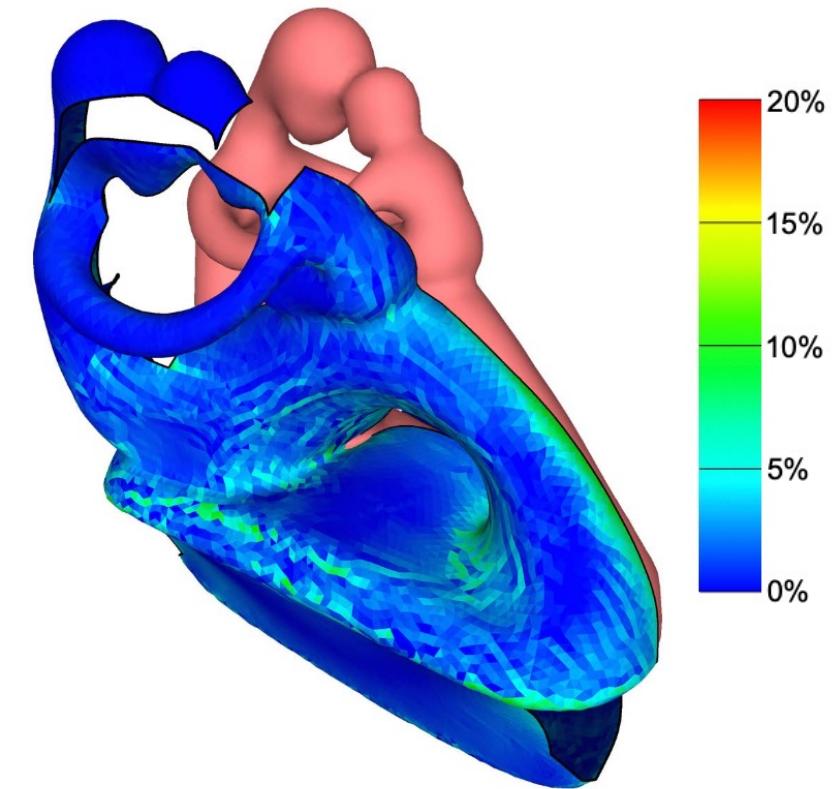


[Alderighi et al., SIGGRAPH 2019]



# Assemblability of Deformable Bodies

- Can be extracted even if they **violate** height fieldness
  - full FEM simulation is overly expensive
    - simulating contact and frictional forces is hard!
- **FlexMolds approach**
  - projective dynamics on a thin shell
  - measure per triangle deformation during extraction

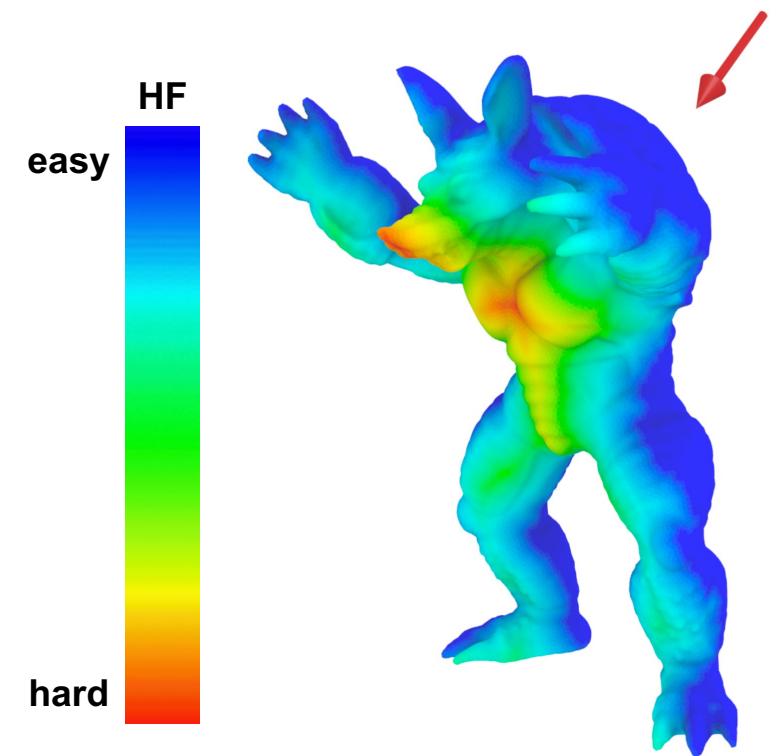


# Assemblability of Deformable Bodies

- Can be extracted even if they **violate** height fieldness
  - full FEM simulation is overly expensive
    - simulating contact and frictional forces is hard!

- **FlexMolds approach**
  - projective dynamics on a thin shell
  - measure per triangle deformation during extraction

- **MetaMolds approach**
  - fully geometric
  - geodesic distance from closest HF region
  - works remarkably well in practice!



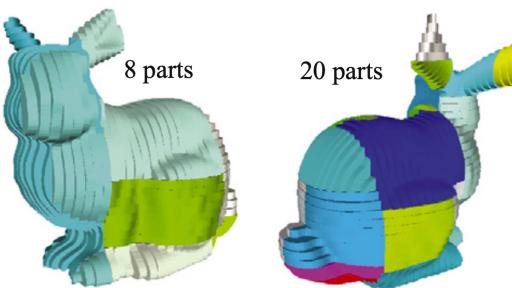
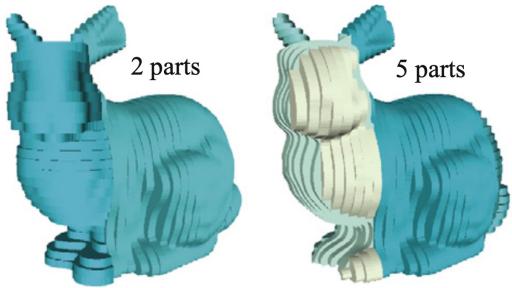
# Binary Space Partitions

Size



[Luo et al., SIGGRAPH 2012]

Staircase Effect

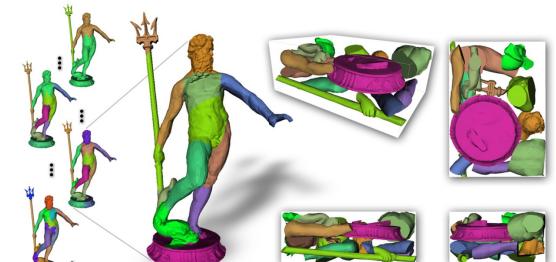


[Hildebrand et al., SMI 2013]



[Chen et al., SIGGRAPH Asia 2015]

Packing

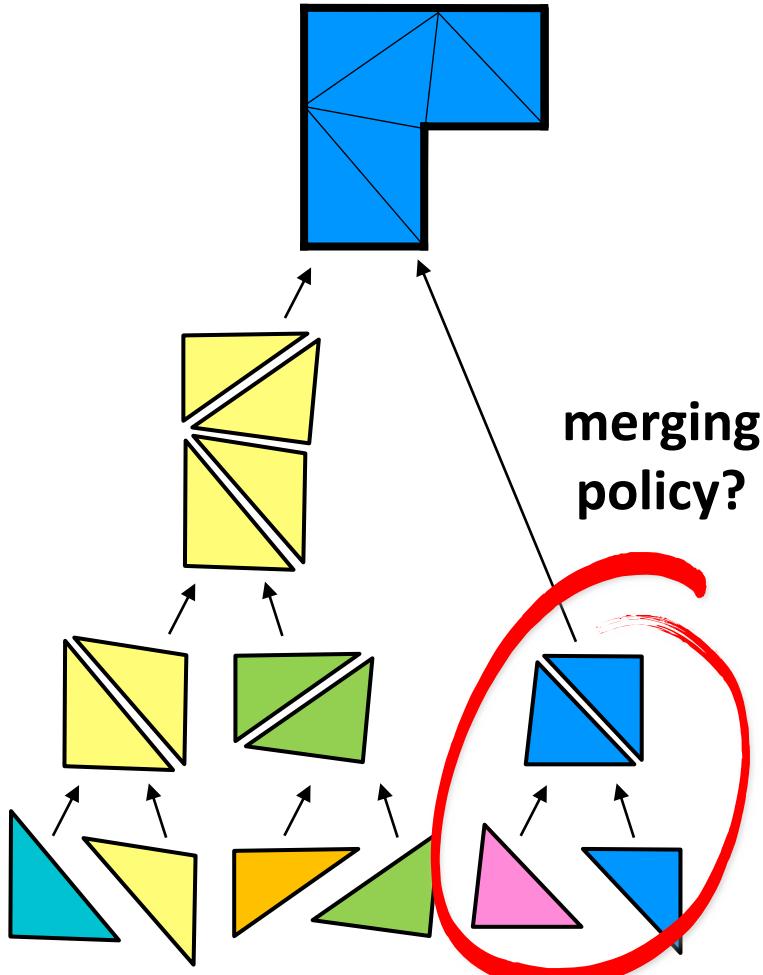


[Attene, EG 2015]

Top Down  
(with beam search)

Bottom Up  
(greedy)

# Top Down vs Bottom Up



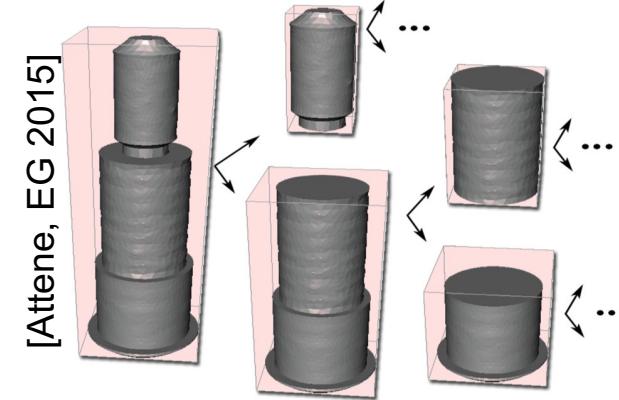
## Shapes in a Box:

- **discretize** domain (tetrahedralization)
- minimize absolute aboxiness

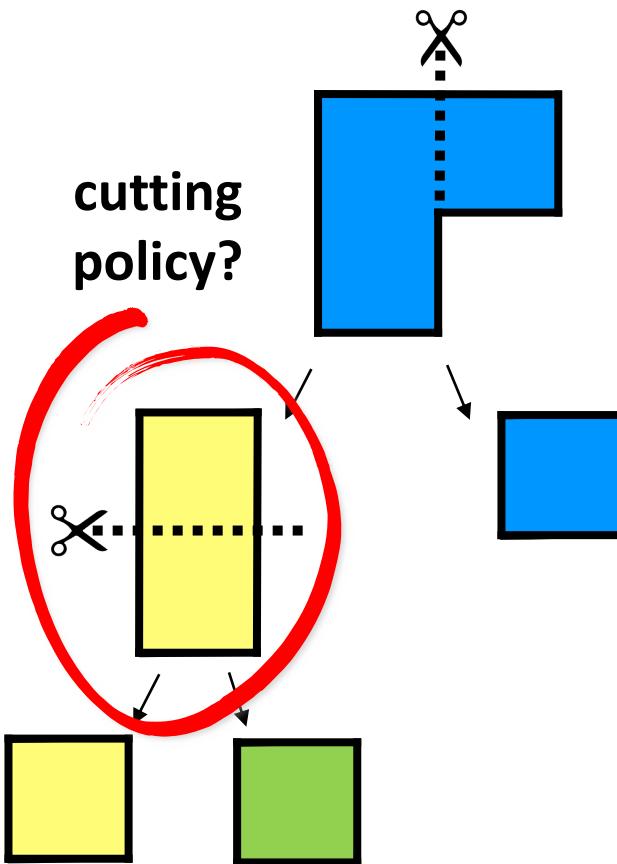
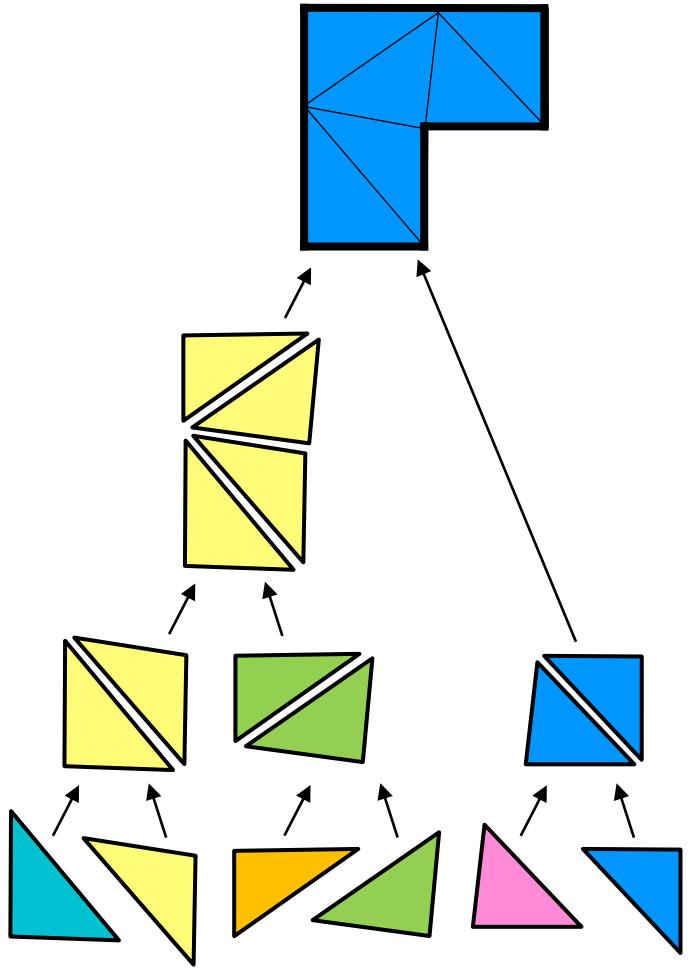
$$A(\mathcal{O}) = VOL(OBB(\mathcal{O})) - VOL(\mathcal{O})$$

cluster  
of  
tetrahedra

minimal  
oriented  
bbox

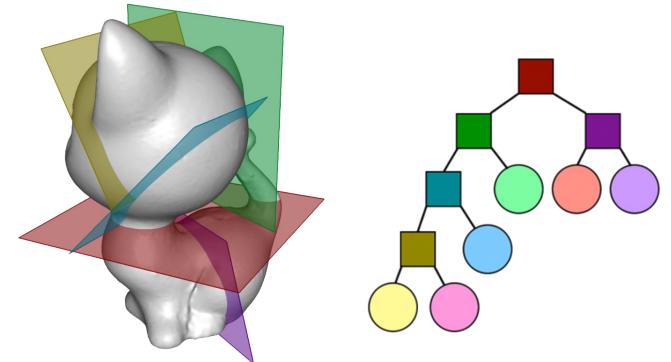


# Top Down vs Bottom Up



Chopper:

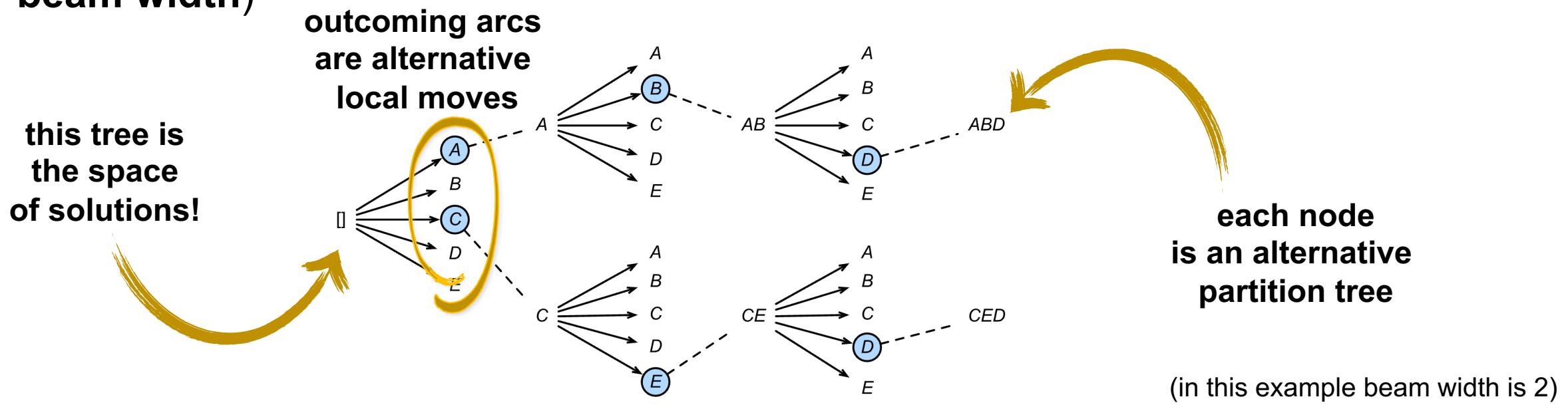
- **discretize** set of planar cuts
- minimize split metric based on
  - # of parts
  - connectors
  - structure/fragility
  - aesthetics (hide seams)
  - symmetry



[Luo et al., SIGGRAPH 2012]

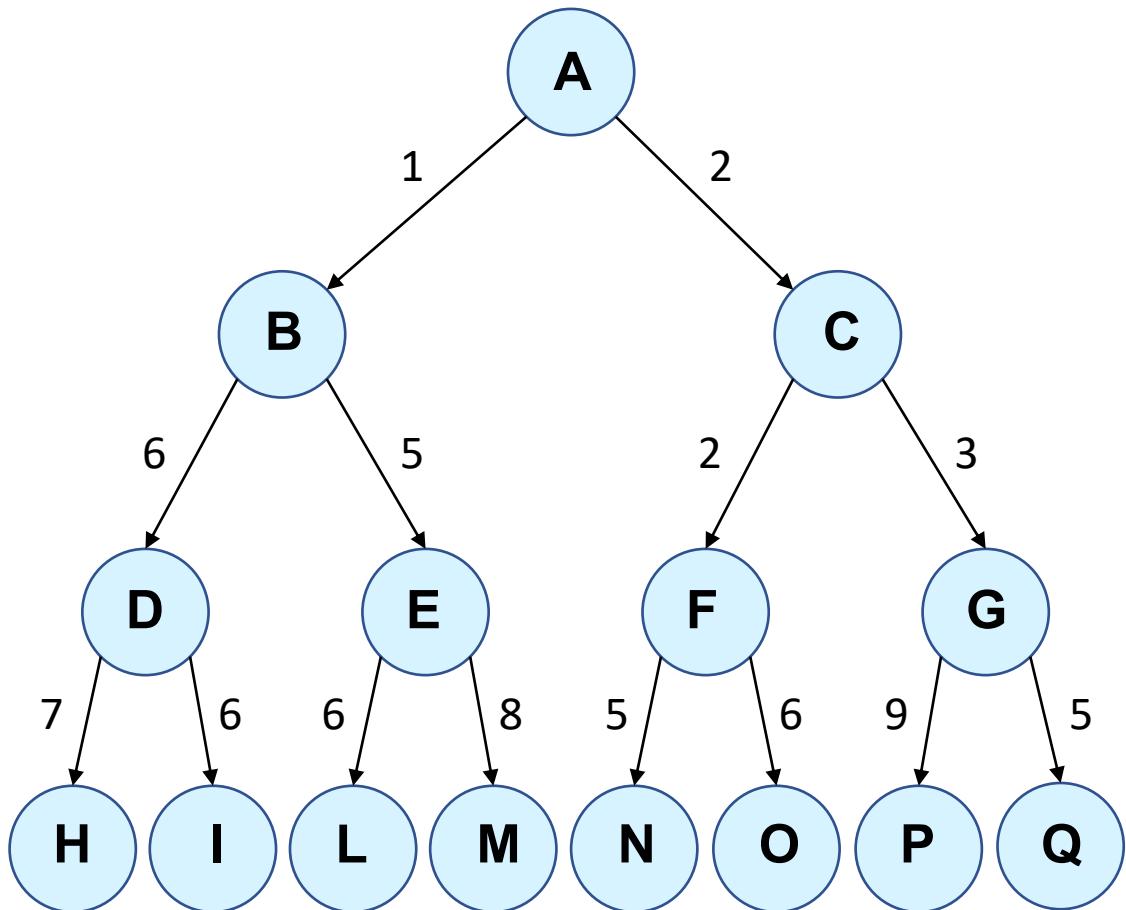
# Exploring the Space of Solutions

- **Greedy:** at each step pick the best move
  - easy to get stuck at local minima!
- **Beam search** allows to explore a wider portion of the feasible space
  - assumption: partial solutions can be ranked
  - idea: at each level, continue exploring only the N best partial solutions (N is called **beam width**)

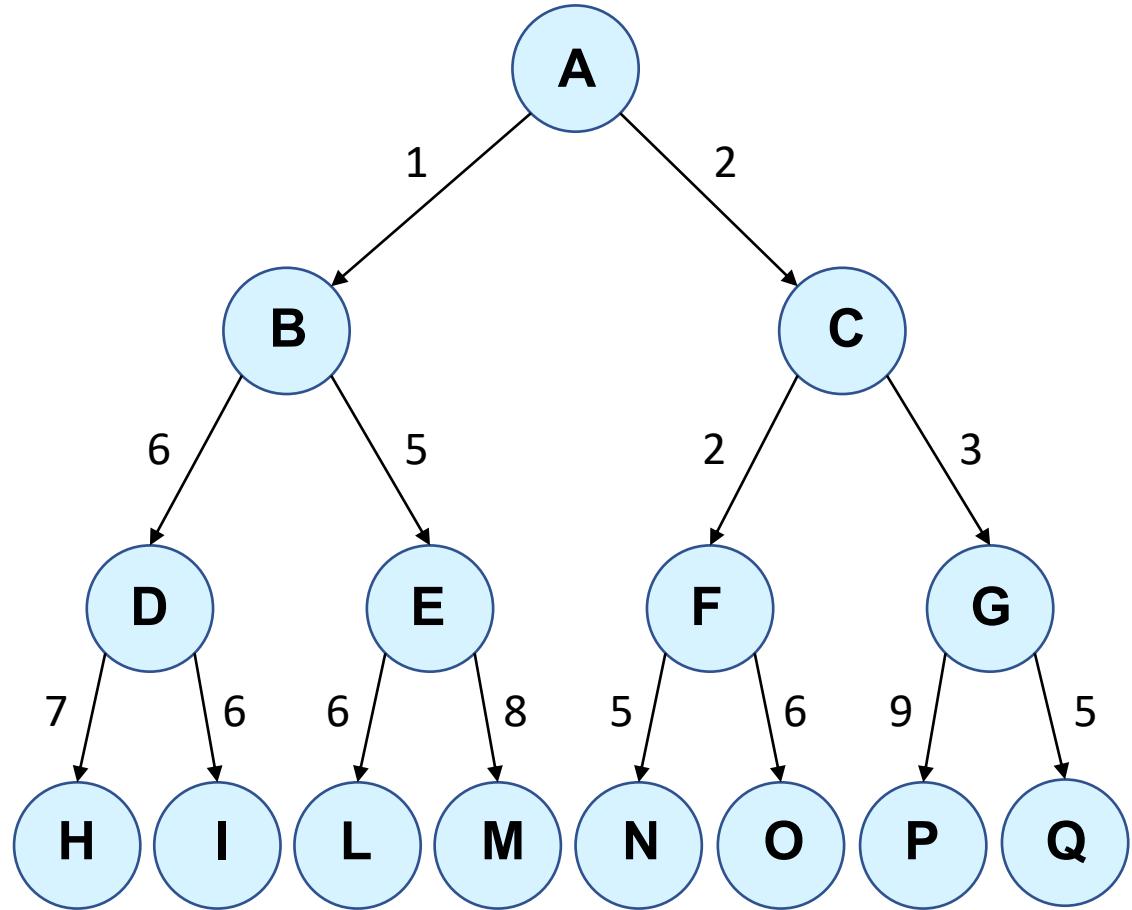


# Beam Search vs Greedy

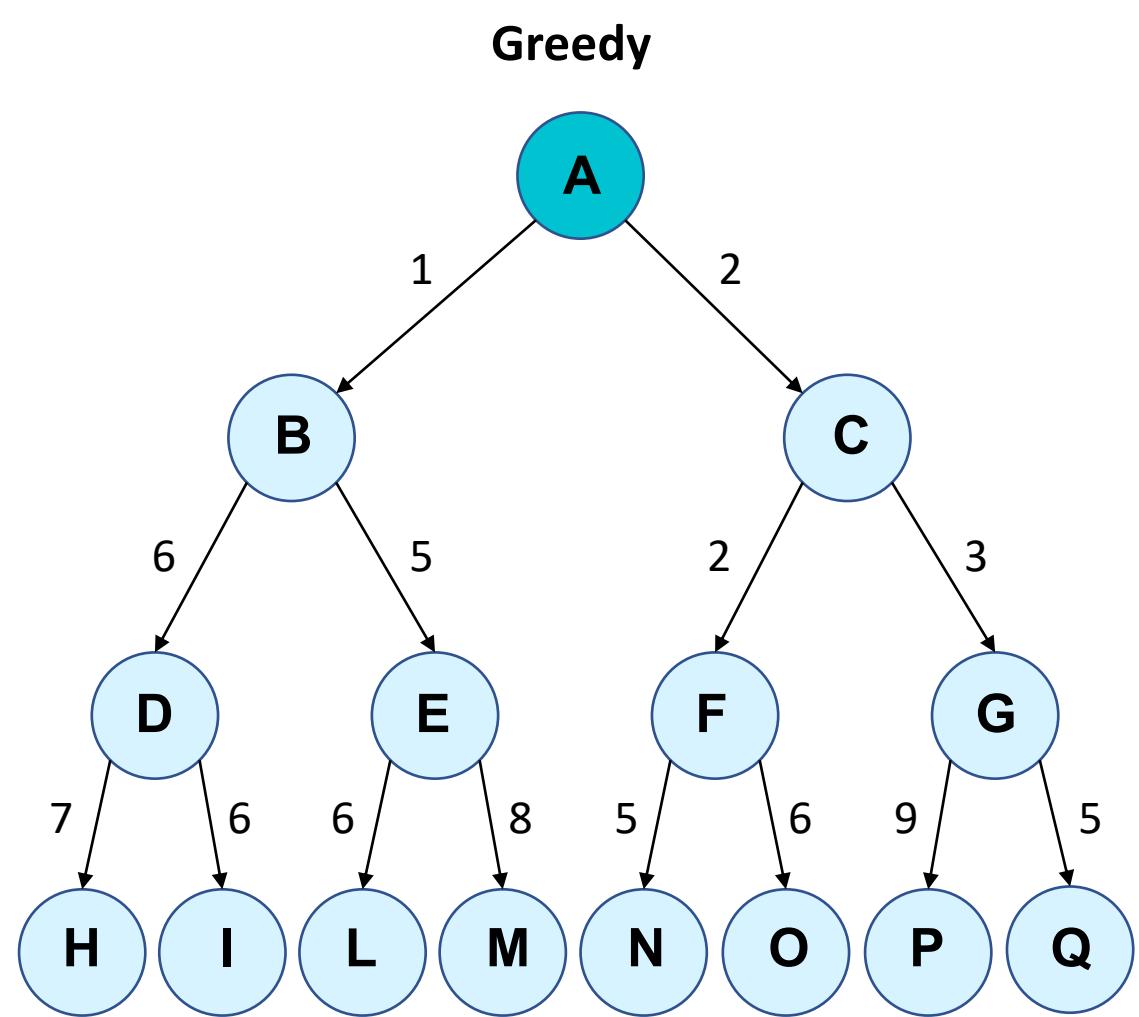
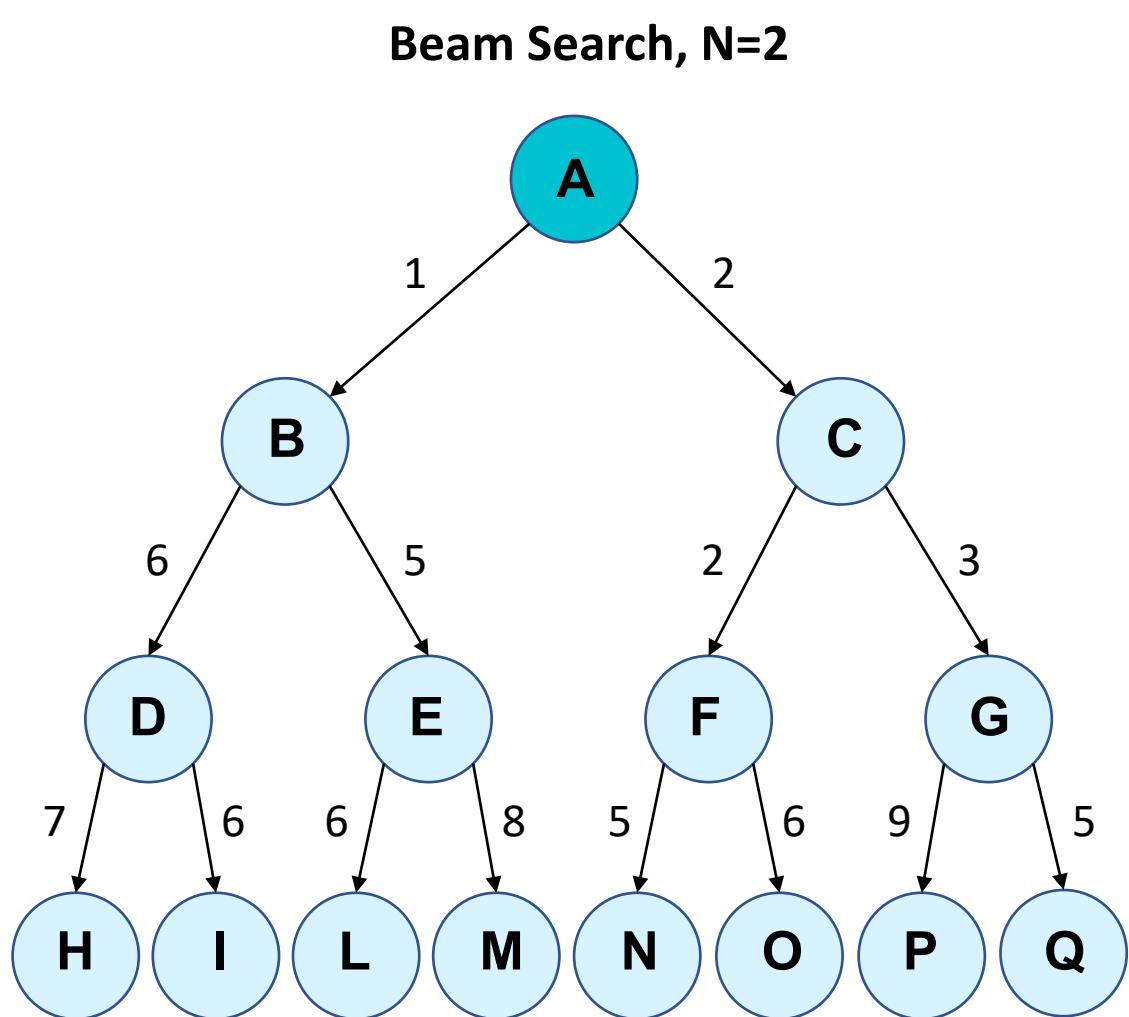
Beam Search, N=2



Greedy

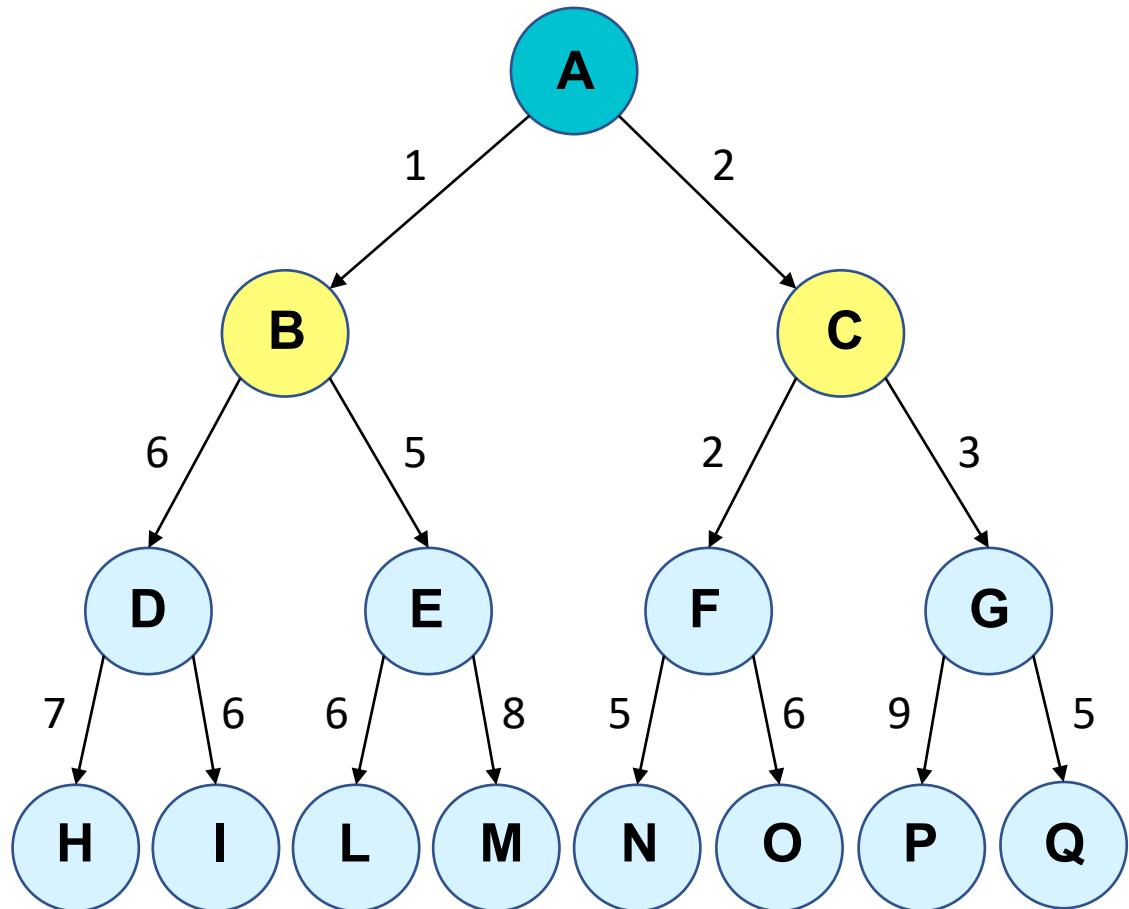


# Beam Search vs Greedy

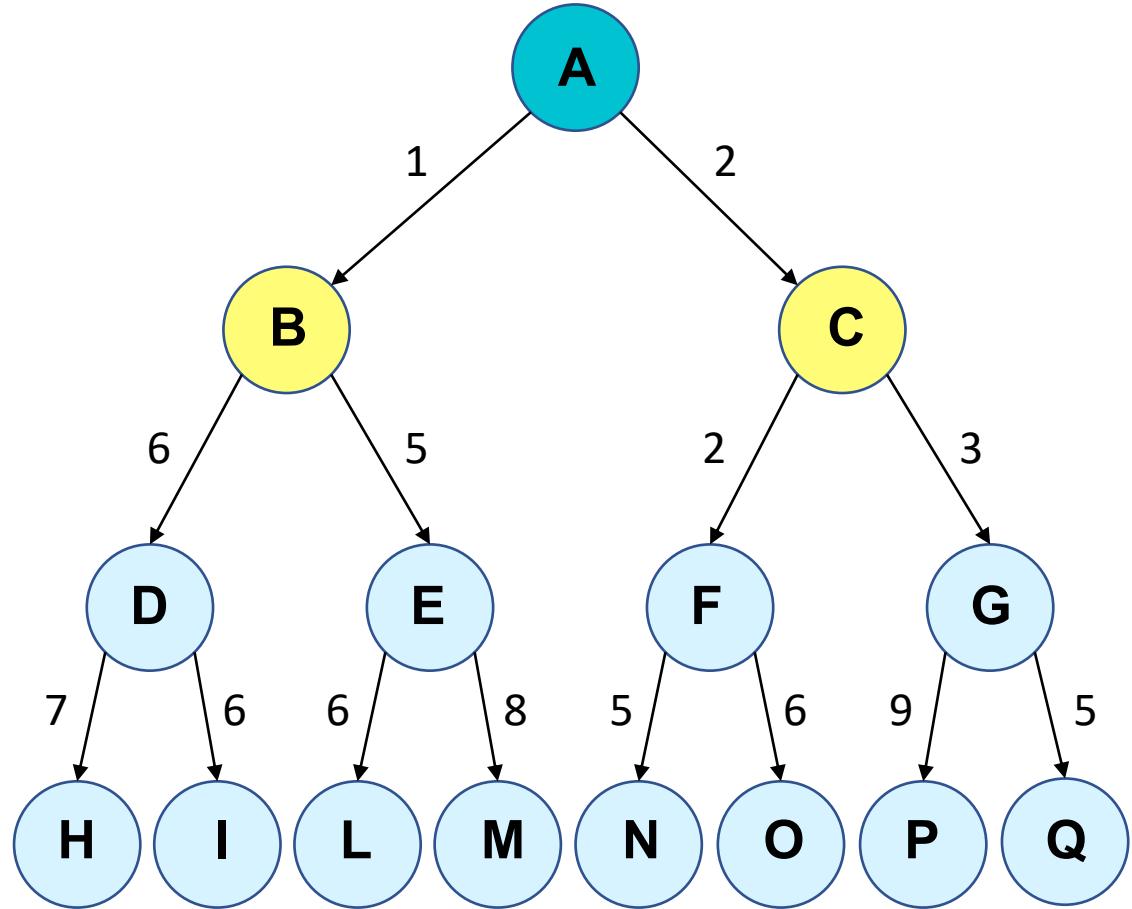


# Beam Search vs Greedy

Beam Search, N=2

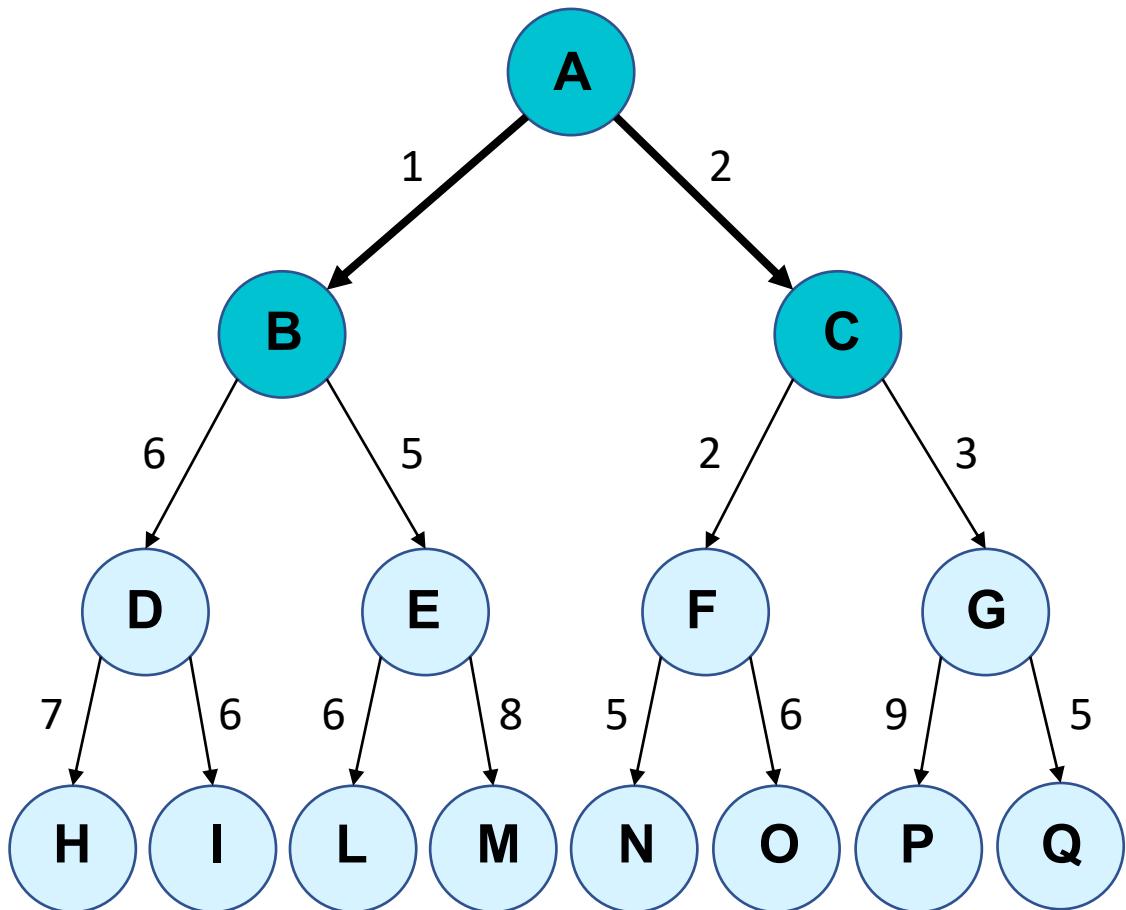


Greedy

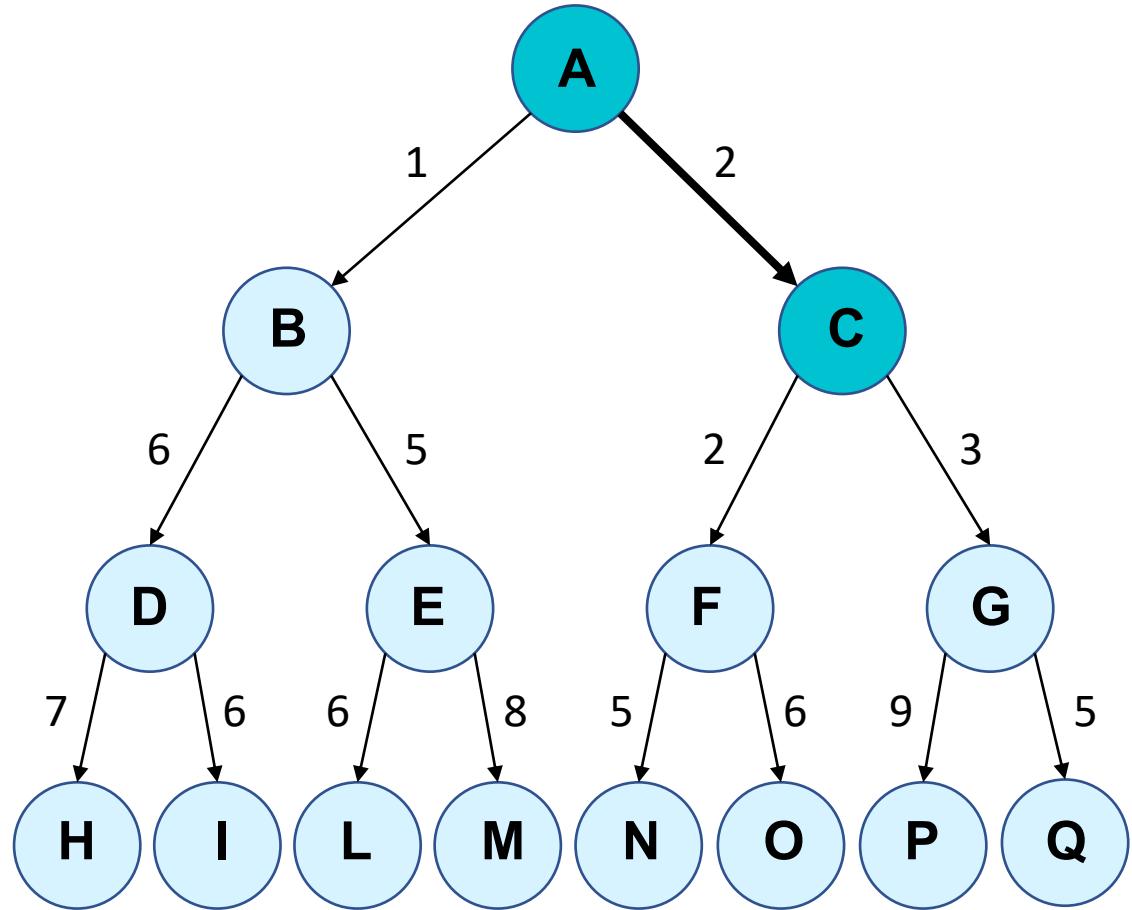


# Beam Search vs Greedy

Beam Search, N=2

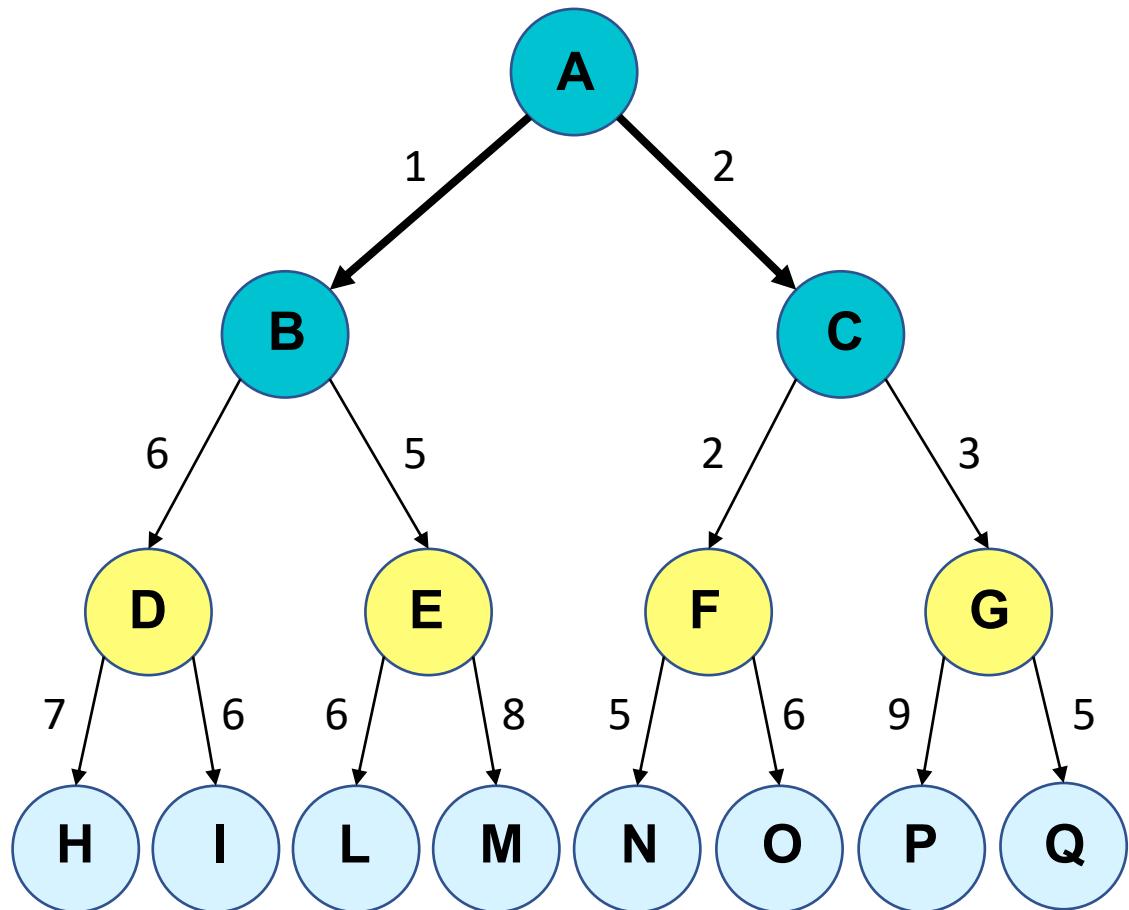


Greedy

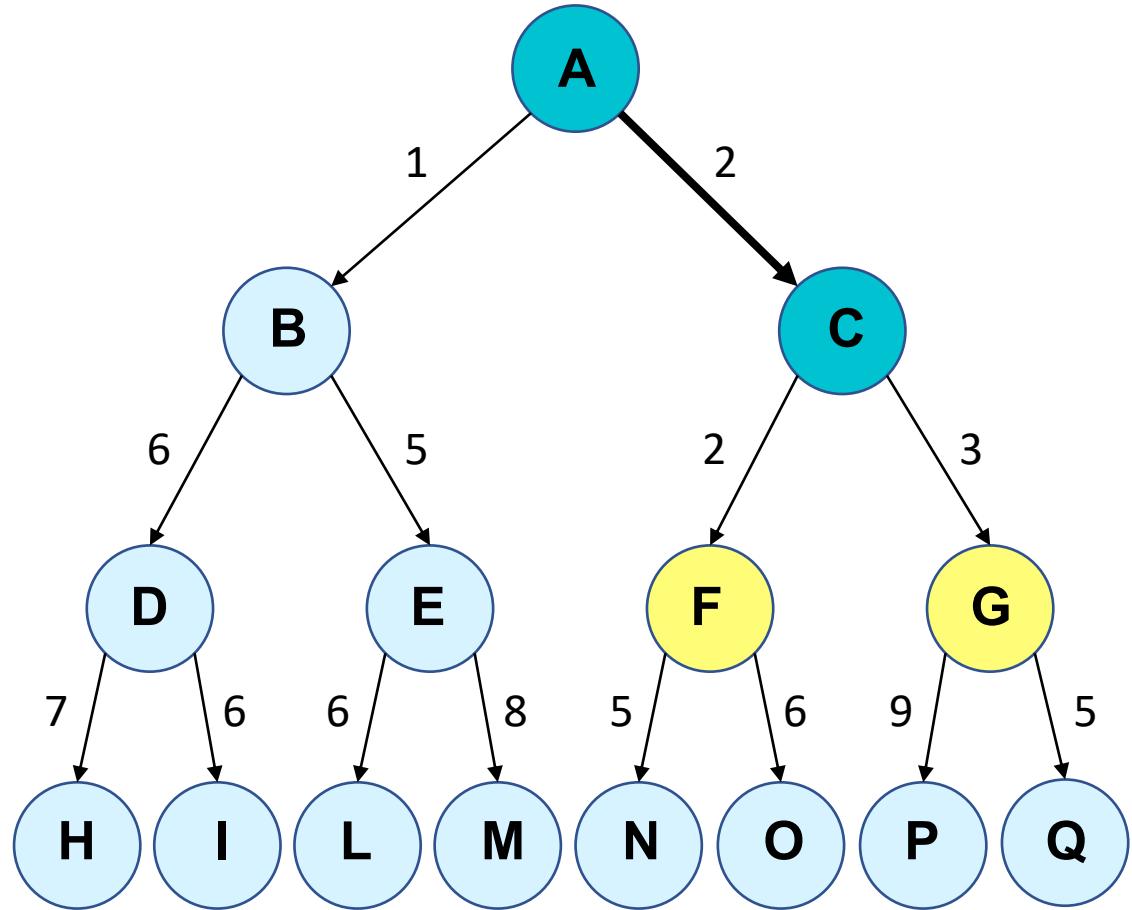


# Beam Search vs Greedy

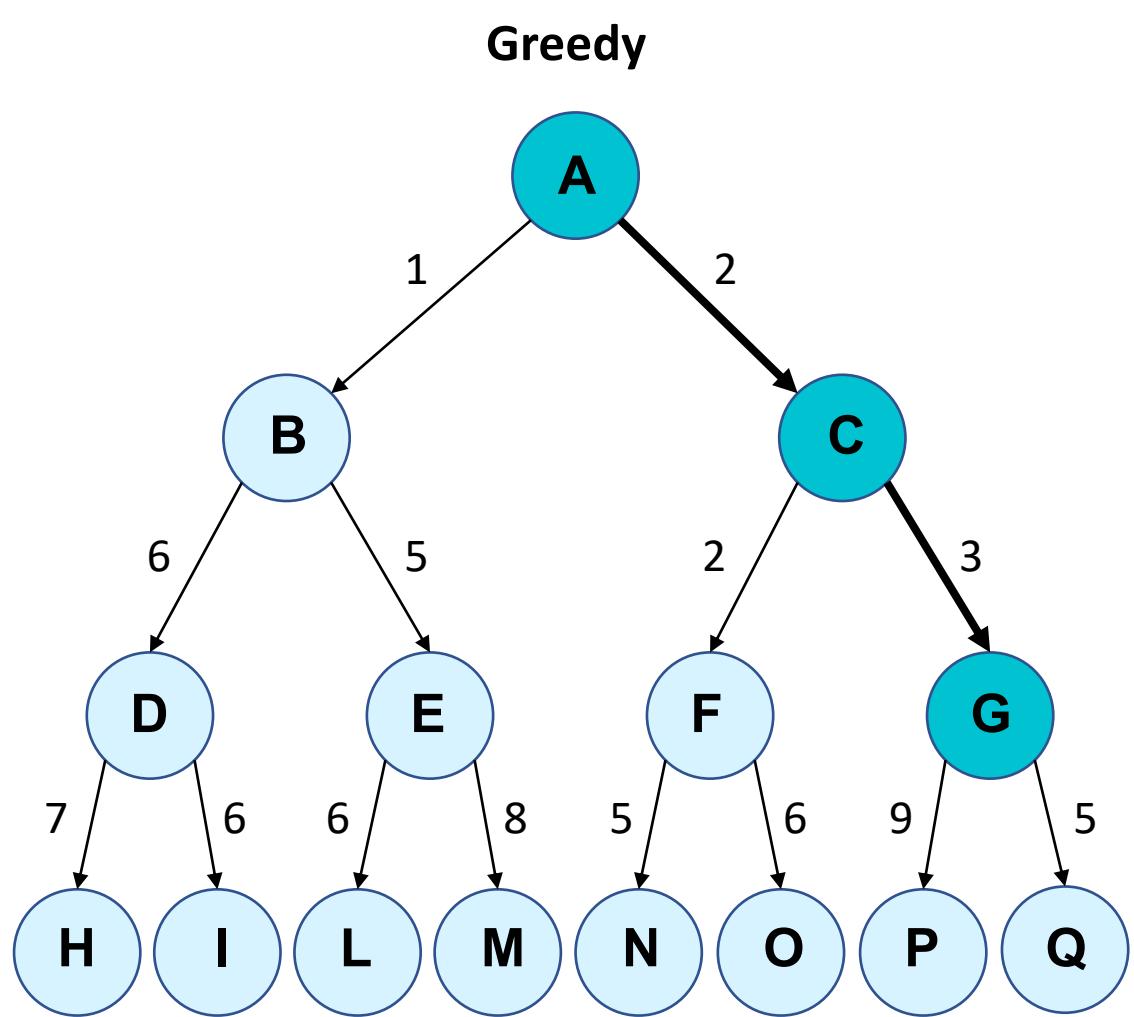
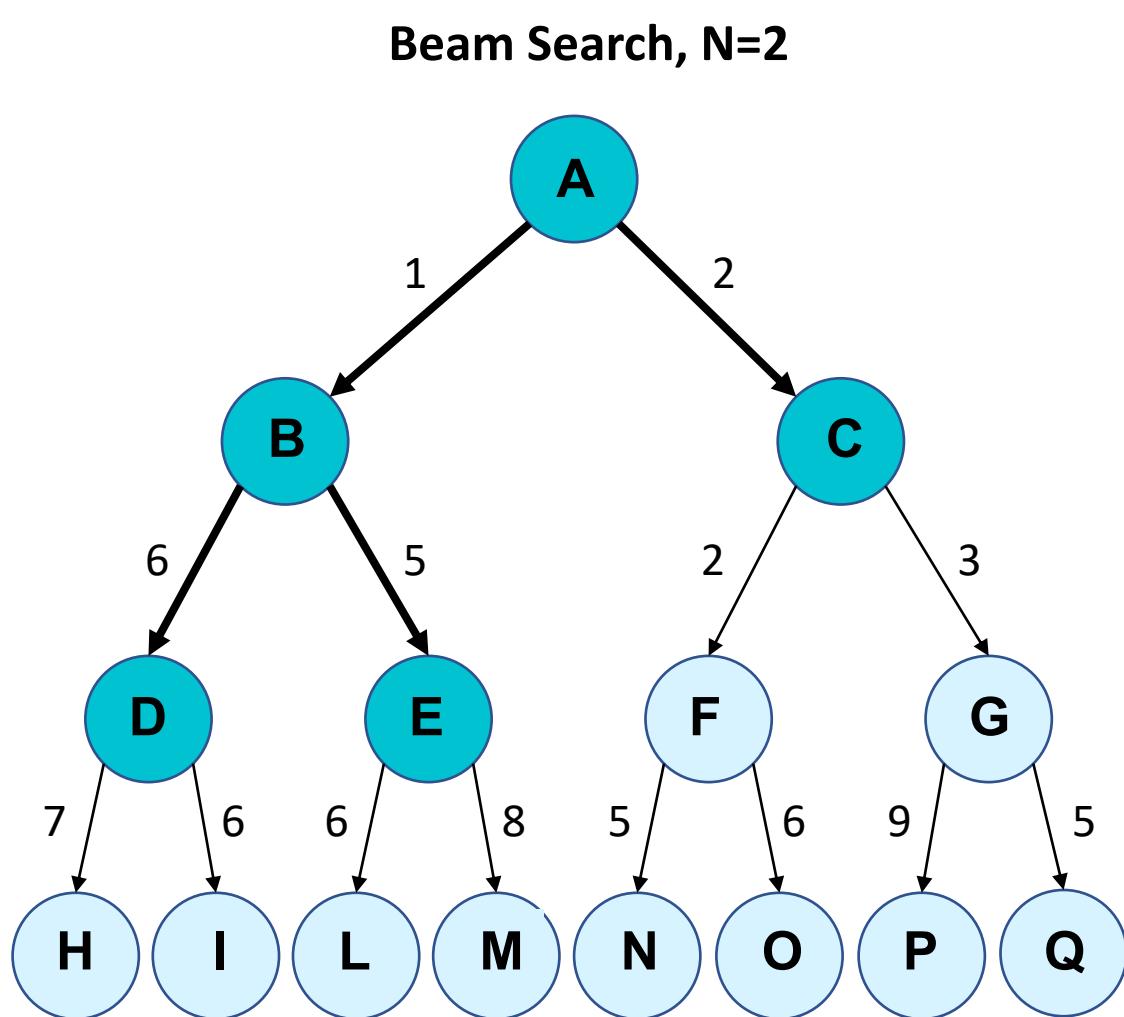
Beam Search, N=2



Greedy

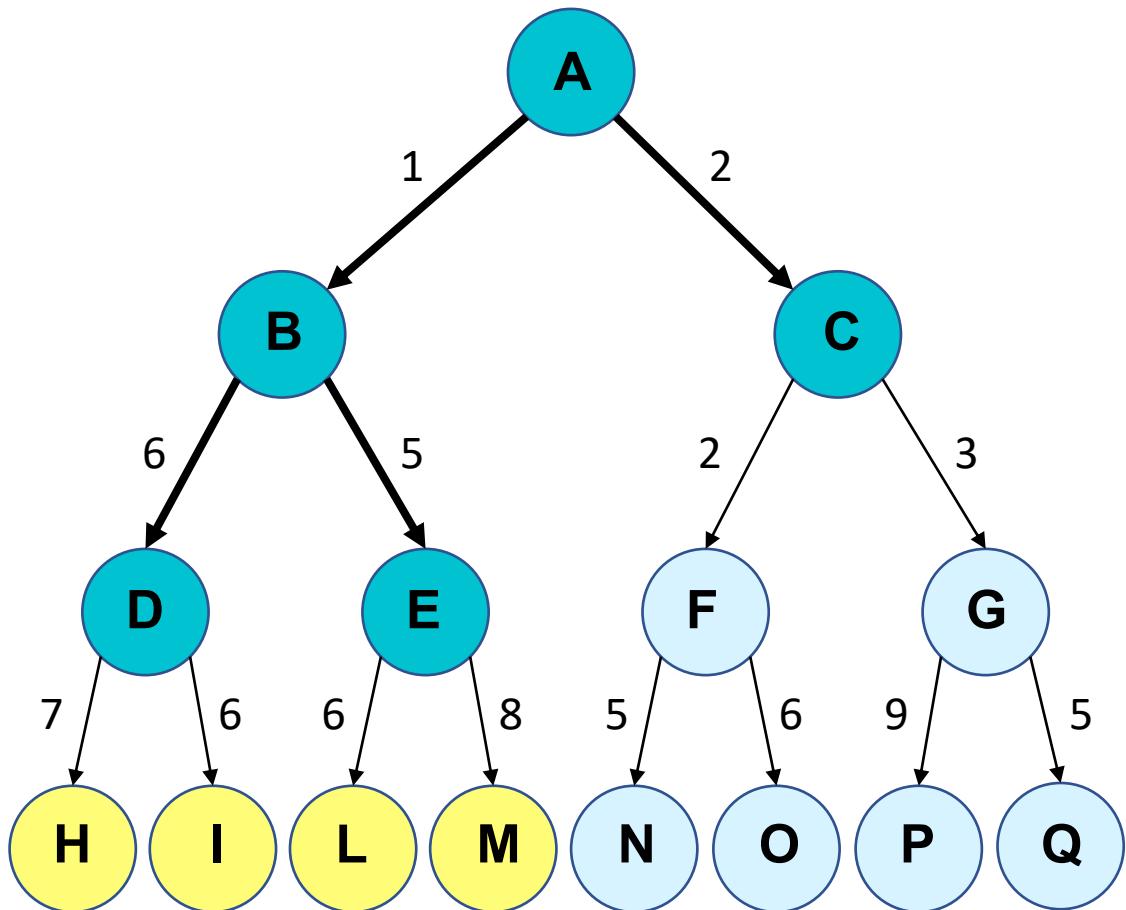


# Beam Search vs Greedy

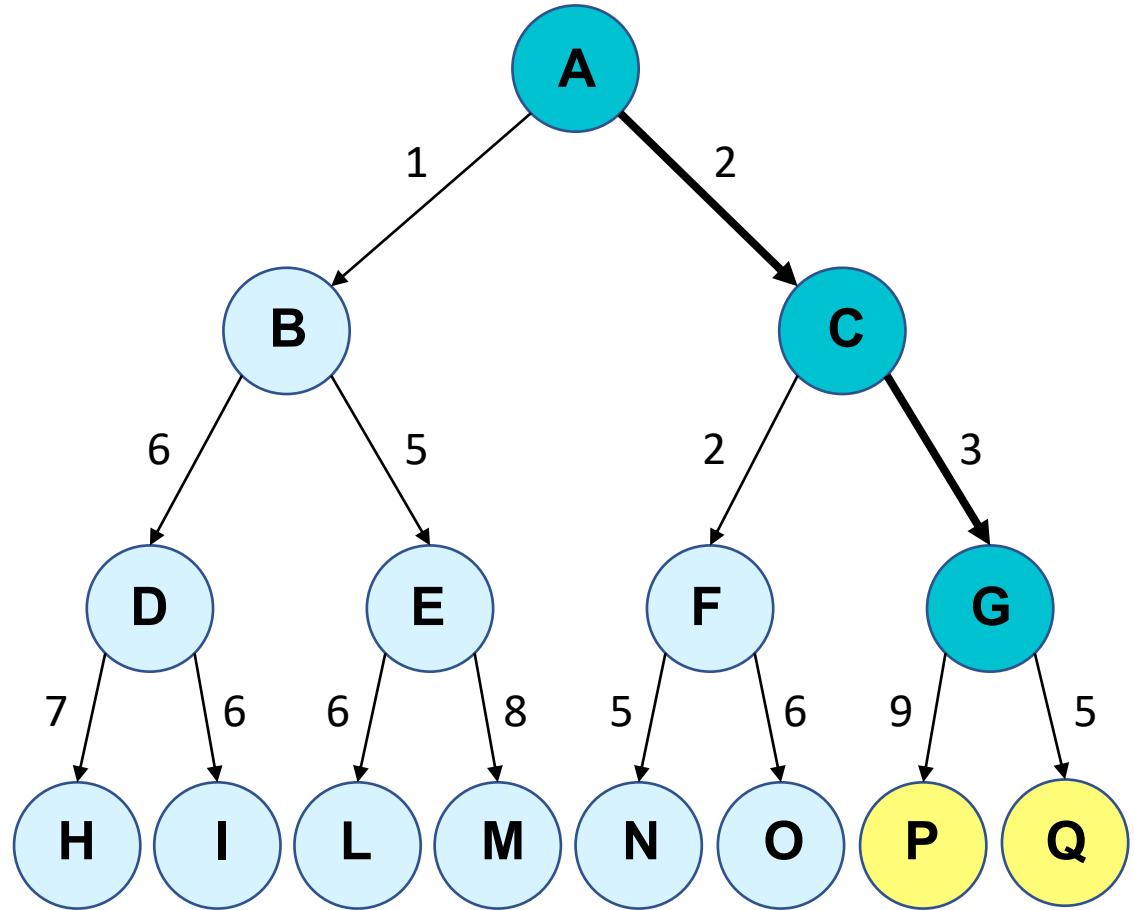


# Beam Search vs Greedy

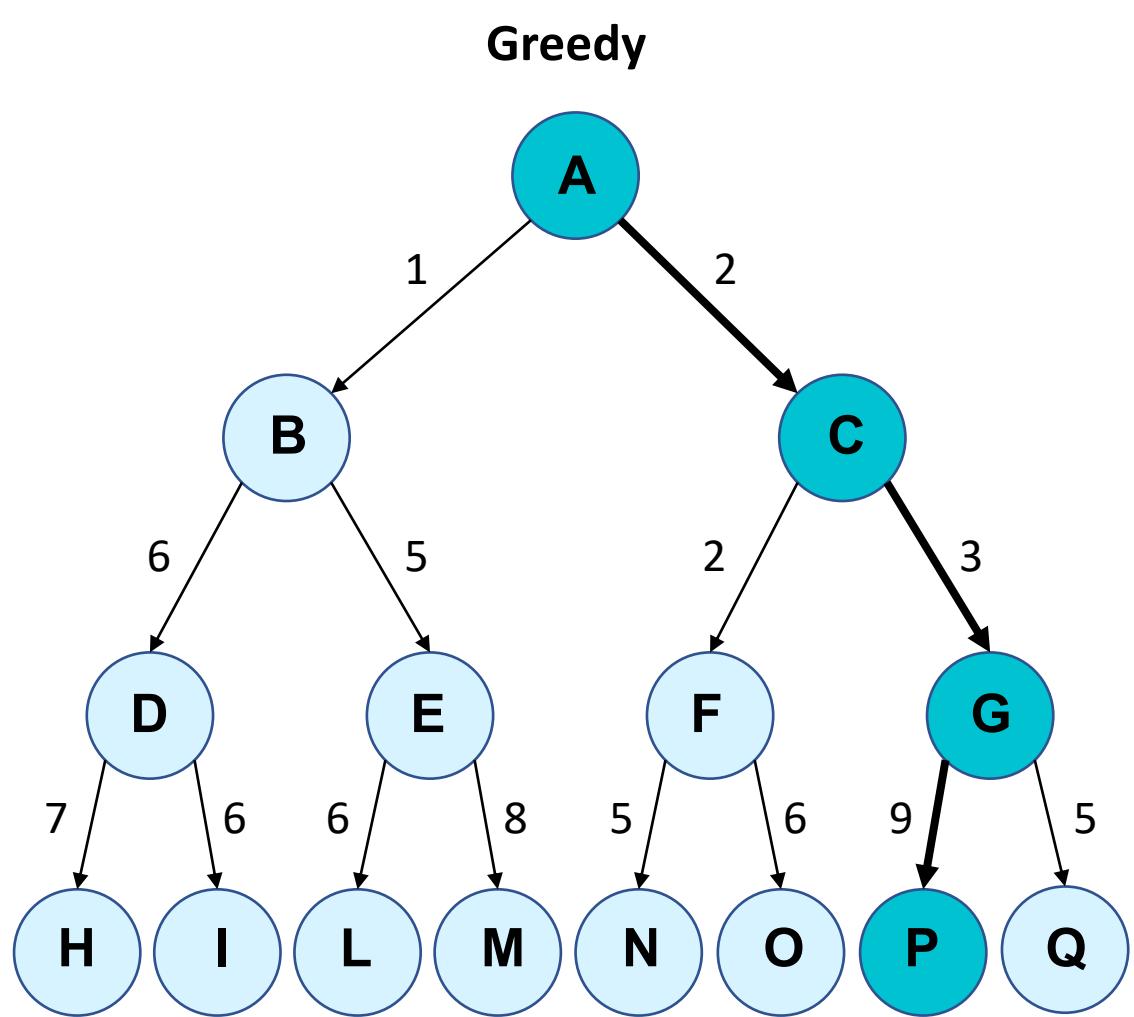
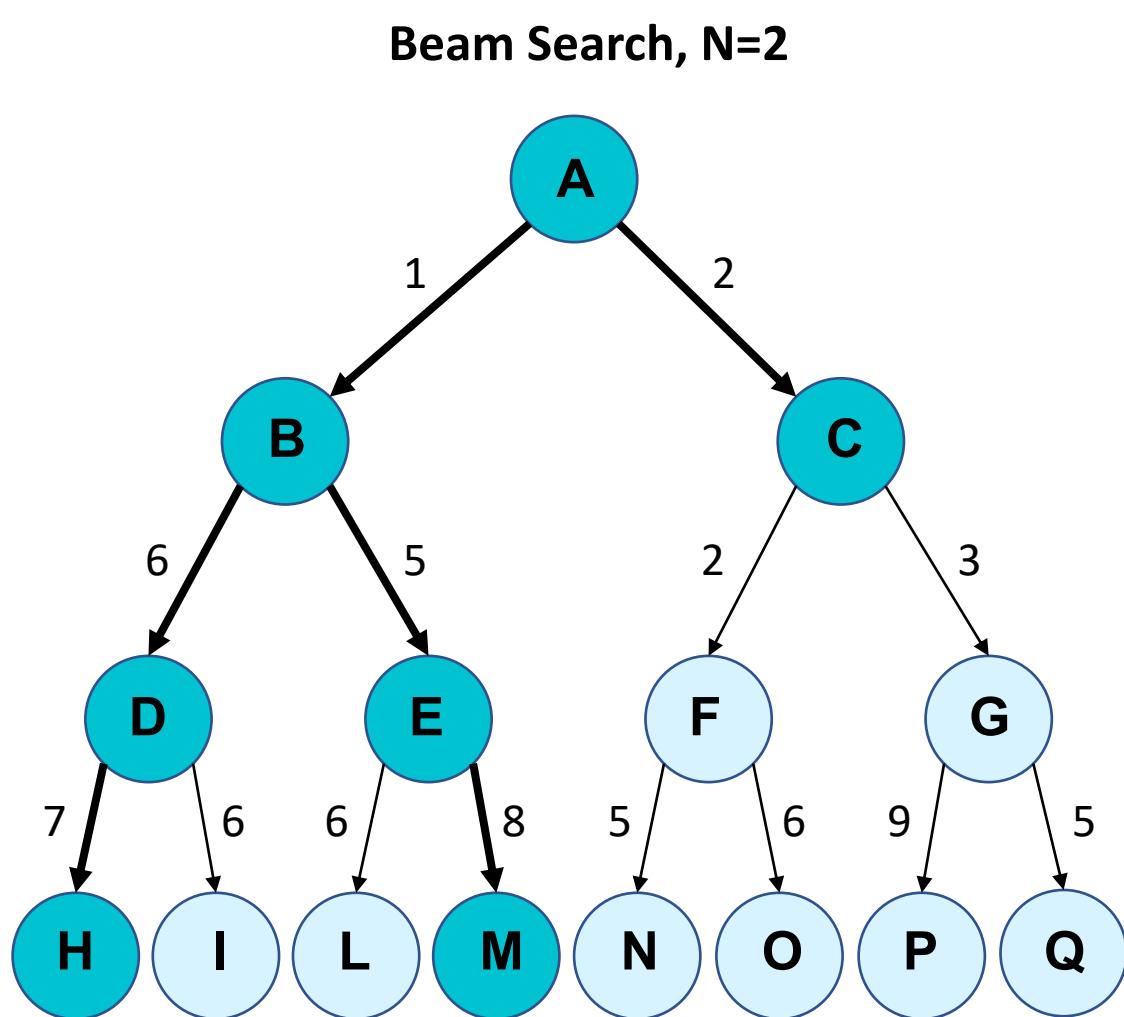
Beam Search, N=2



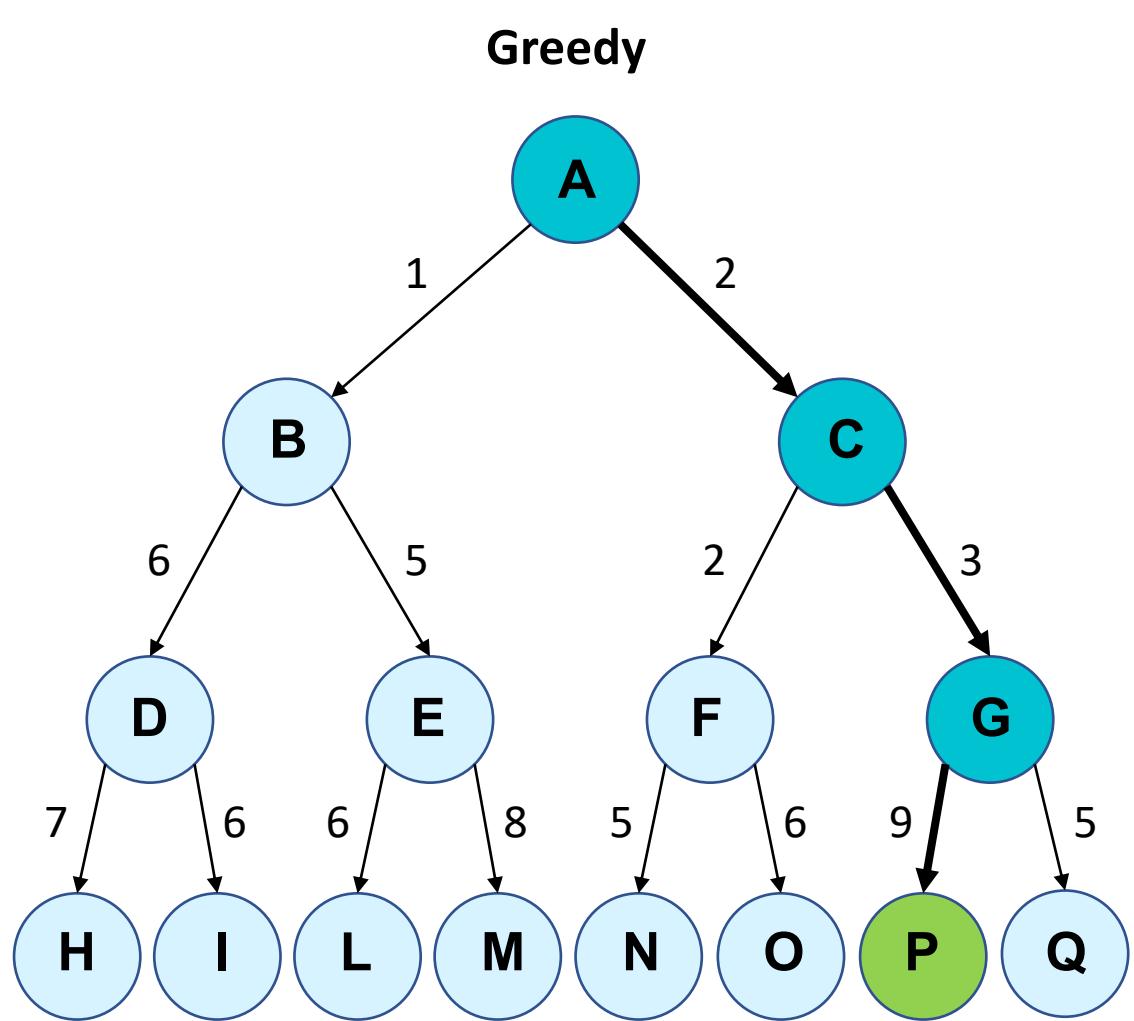
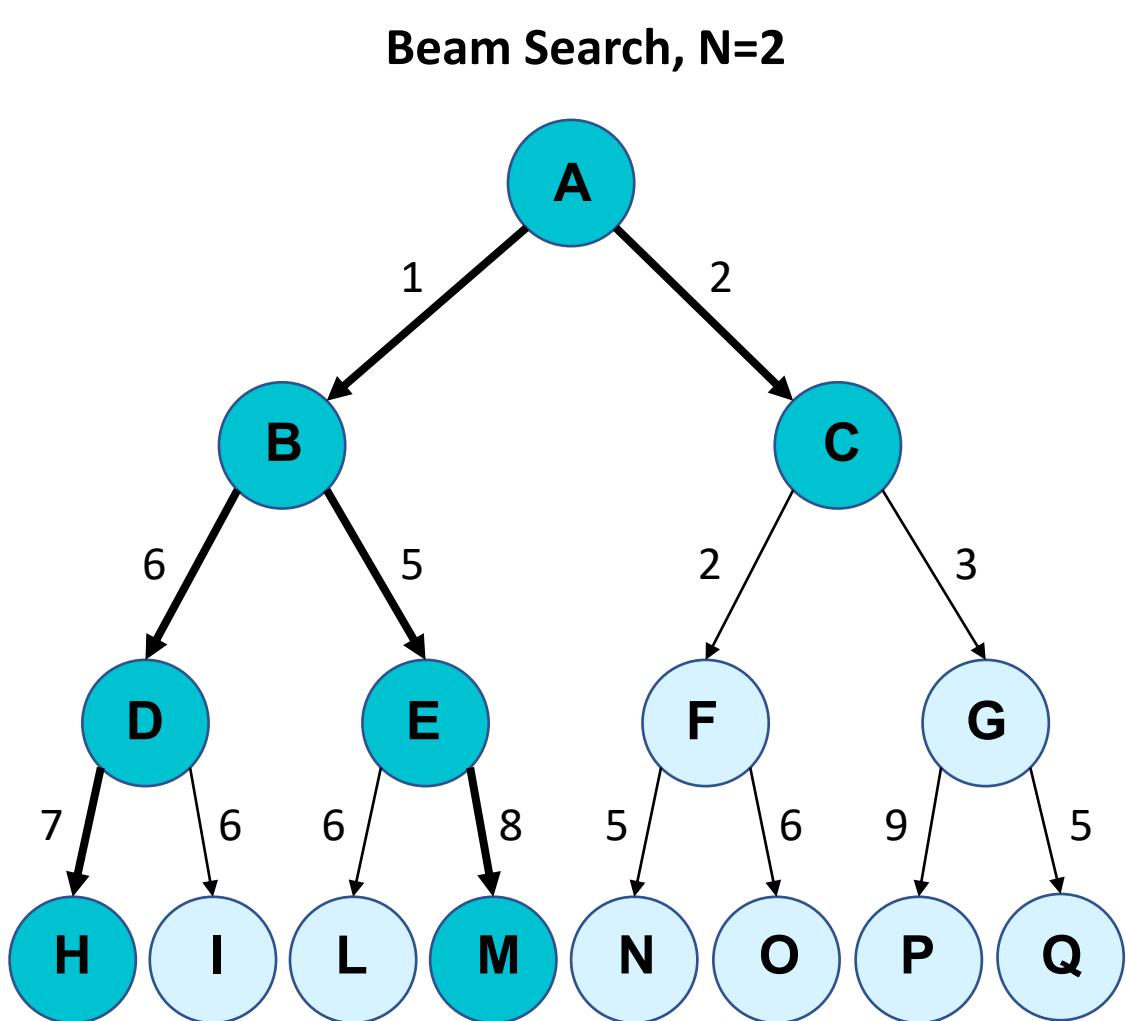
Greedy



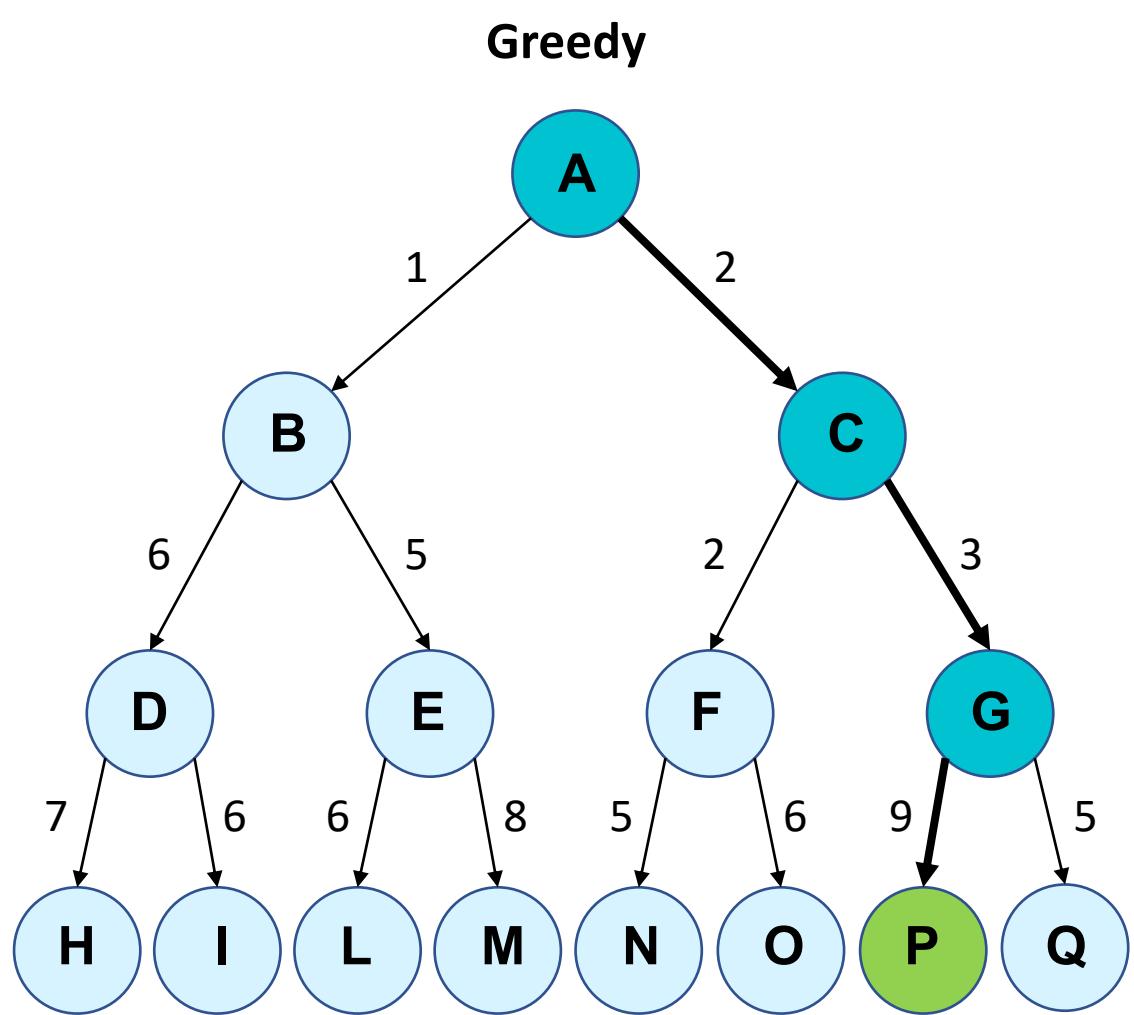
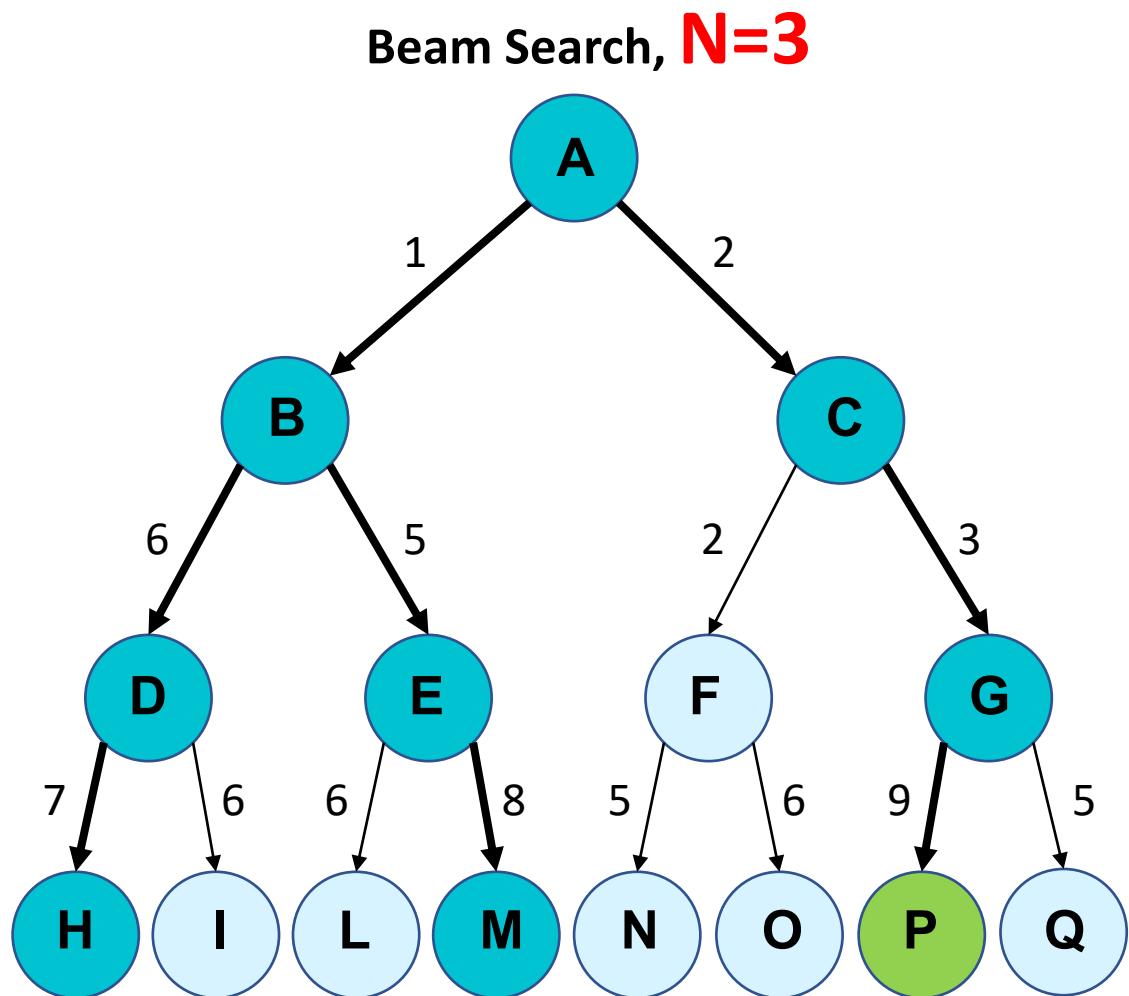
# Beam Search vs Greedy



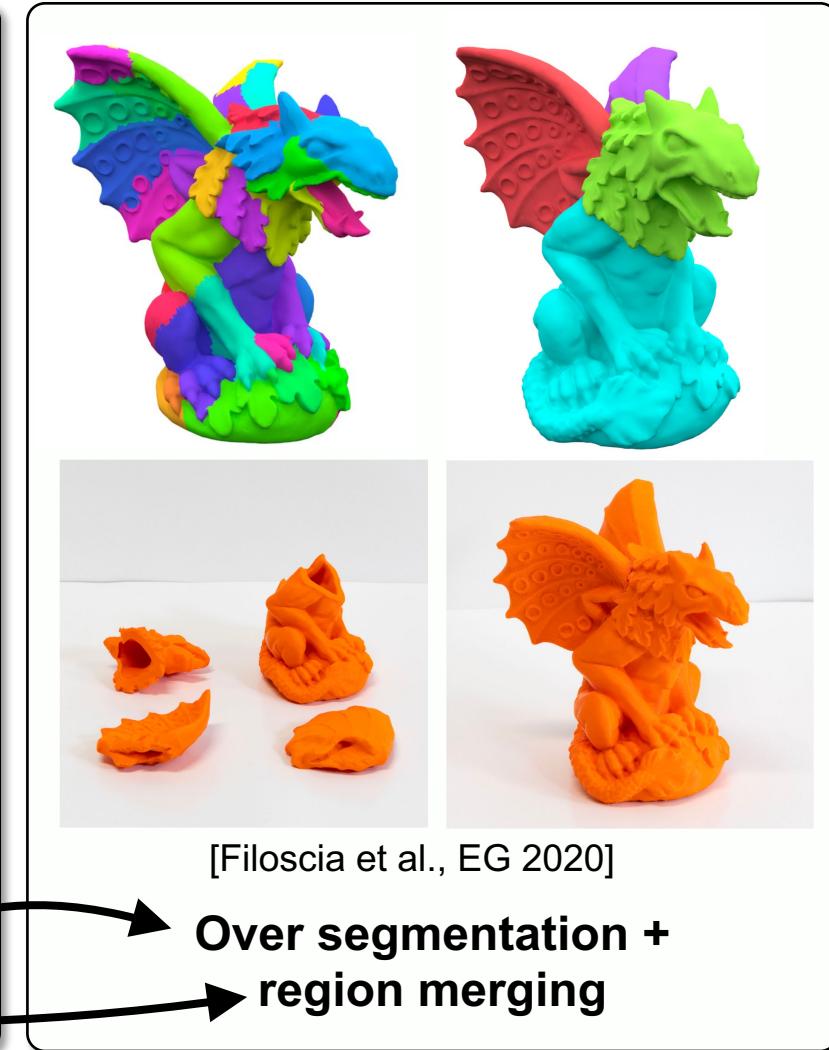
# Beam Search vs Greedy



# Beam Search vs Greedy



# Labeling



# Graph Cuts

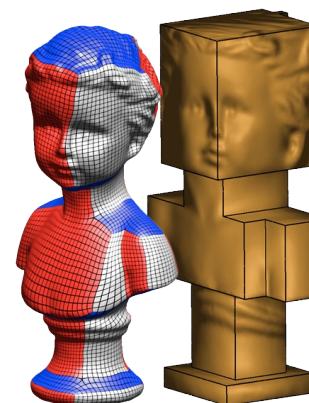
- Solves a multi-labeling problem on a generic graph  $G(N,A)$  by minimizing

$$L = \arg \min \sum_{i \in N} C_i(l) + \sum_{ij \in A} C_{ij}(l_i, l_j)$$

**DATA TERM**  
cost of assigning  
label  $l$  to node  $i$

**SMOOTH TERM**  
cost of assigning  
labels  $l_i, l_j$  to  
adjacent nodes  $i, j$

- The problem is NP-Complete
  - finds a local minimum
  - depends on initialization and processing order
  - heavily used in Vision/Graphics
  - it works remarkably well in practice!



[PolyCut, SIG Asia 2013]

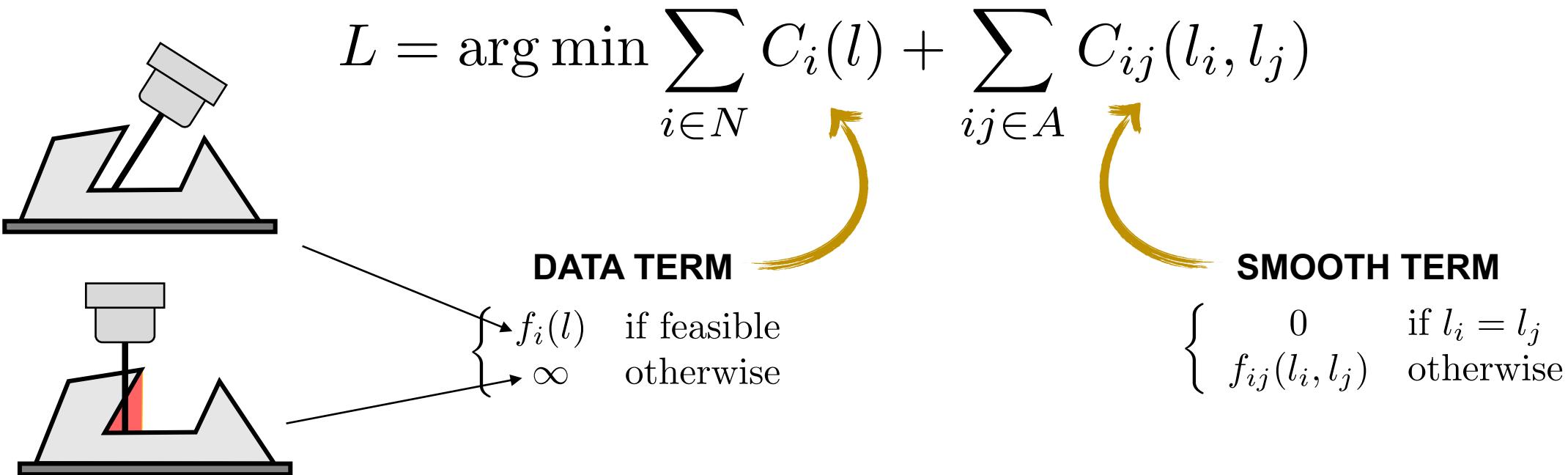


[GrabCut, SIGGRAPH 2004]



# Graph Cuts for Digital Manufacturing

- The **graph** is the dual mesh
  - one node per triangle / tetrahedron / voxel
- The **labels** are candidate machining / extraction directions



# Graph Cuts in Digital Manufacturing

- The **graph** is the dual mesh
  - one node per triangle / tetrahedron / voxel
- The **labels** are candidate machining / extraction directions

**Surface2Volume**

G: dual tetmesh

L: extraction directions



[Araújo et al., SIGGRAPH 2019]

**HF Decomp**

G: dual trimesh

L: HF directions

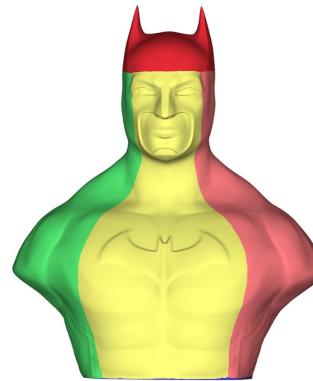


[Herholz et al., EG 2015]

**4 Axis Milling**

G: dual trimesh

L: milling directions



[Nuvoli et al., EG 2021]

**Rigid Molding**

G: dual tetmesh

L: molding directions



[Alderighi et al., SIG Asia 2021]

**DHF Slicer**

G: dual trimesh

L: DHF directions

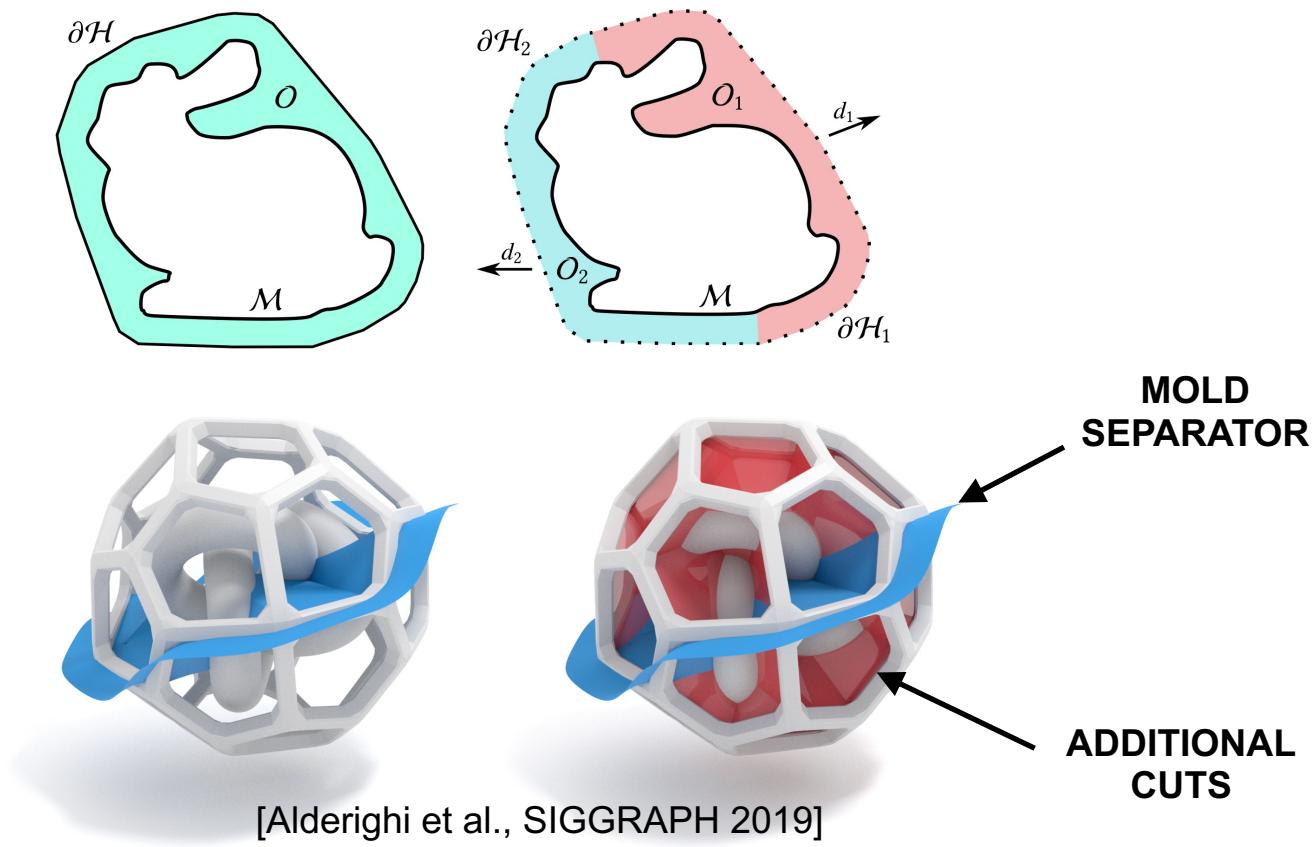


[Yang et al., SIG Asia 2020]

# Greedy Labeling for Volume-Aware molding

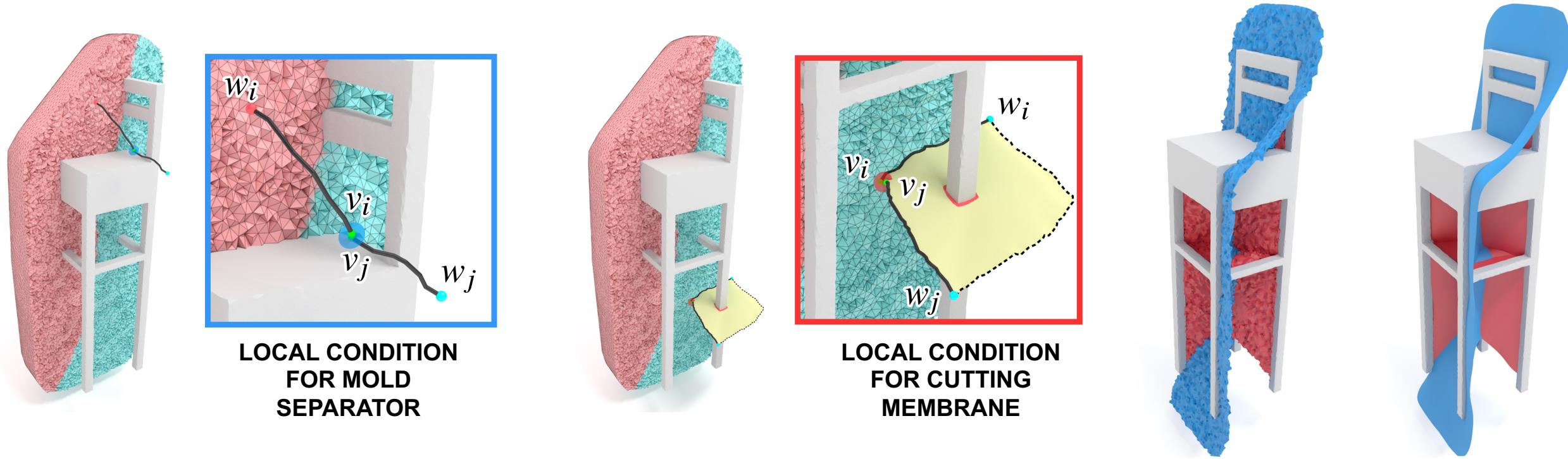
- Greedy labeling does not typically work as well as graph cuts, but when you have the **right idea**, it becomes extremely powerful!

- bipartition the **outer shell** finding two mold directions that maximize visual coverage
- propagate the labeling **inside the volume**, also defining additional cuts



# Greedy Labeling for Volume-Aware molding

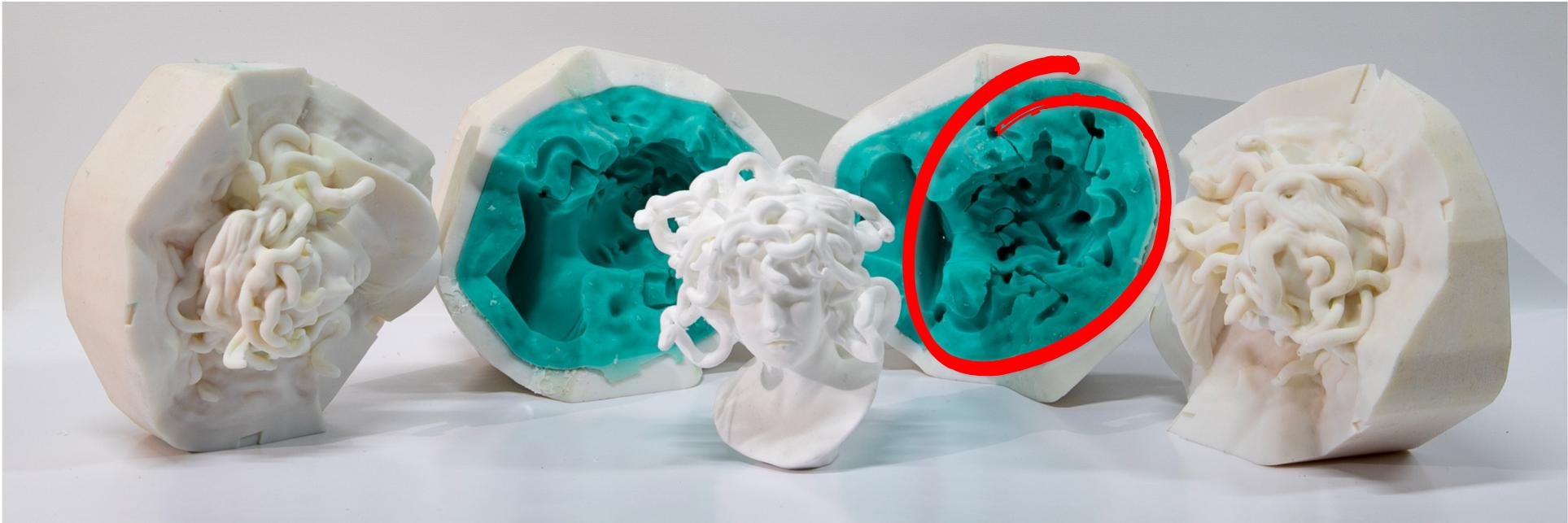
- Greedy labeling does not typically work as well as graph cuts, but when you have the **right idea**, it becomes extremely powerful!



[Alderighi et al., SIGGRAPH 2019]

# Greedy Labeling for Volume-Aware molding

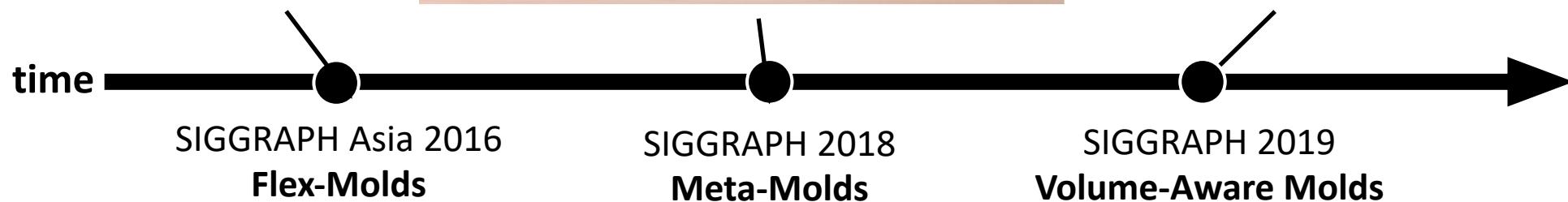
- Greedy labeling does not typically work as well as graph cuts, but when you have the **right idea**, it becomes extremely powerful!



[Alderighi et al., SIGGRAPH 2019]

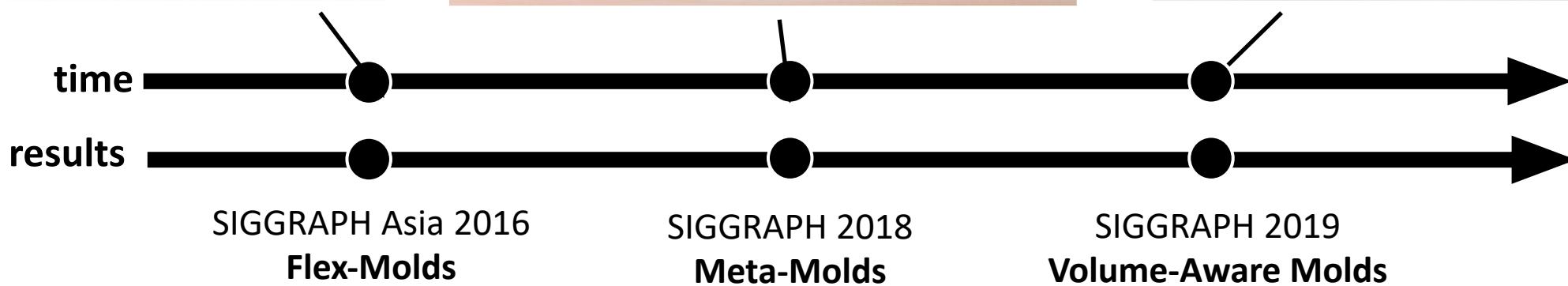
# Greedy Labeling for Volume-Aware molding

- Greedy labeling does not typically work as well as graph cuts, but when you have the **right idea**, it becomes extremely powerful!



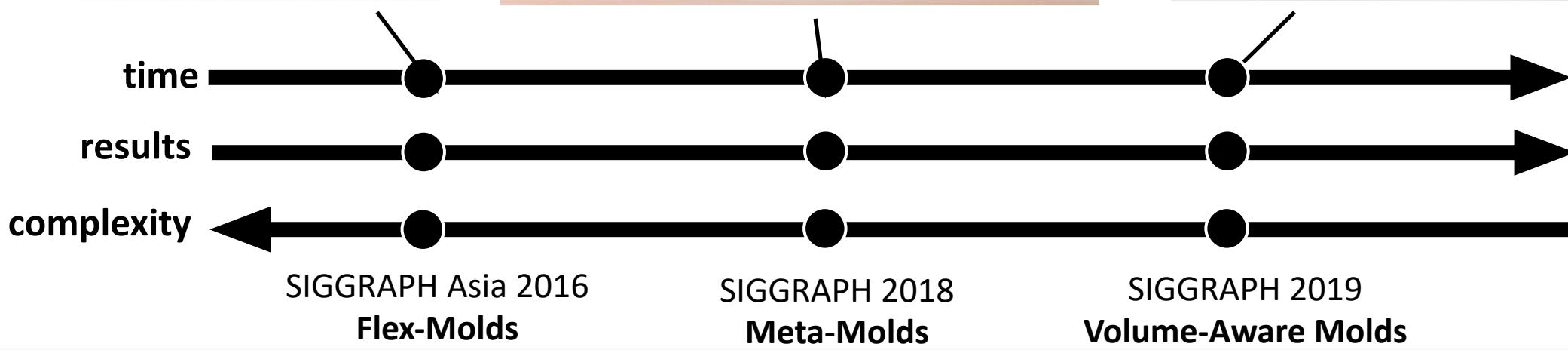
# Greedy Labeling for Volume-Aware molding

- Greedy labeling does not typically work as well as graph cuts, but when you have the **right idea**, it becomes extremely powerful!



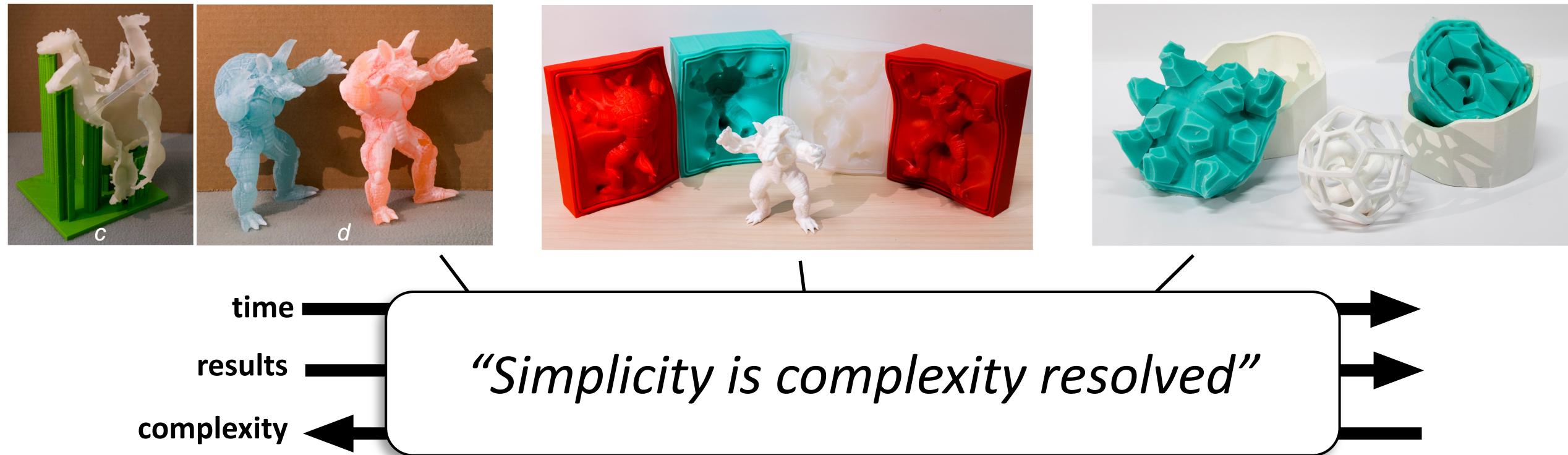
# Greedy Labeling for Volume-Aware molding

- Greedy labeling does not typically work as well as graph cuts, but when you have the **right idea**, it becomes extremely powerful!



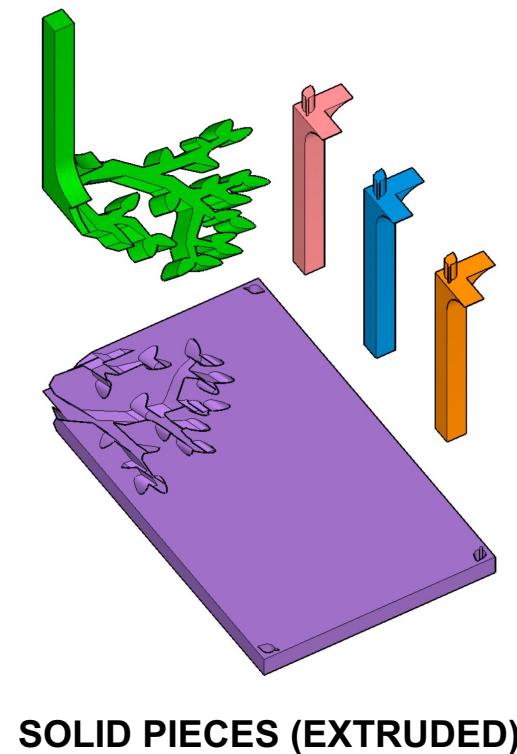
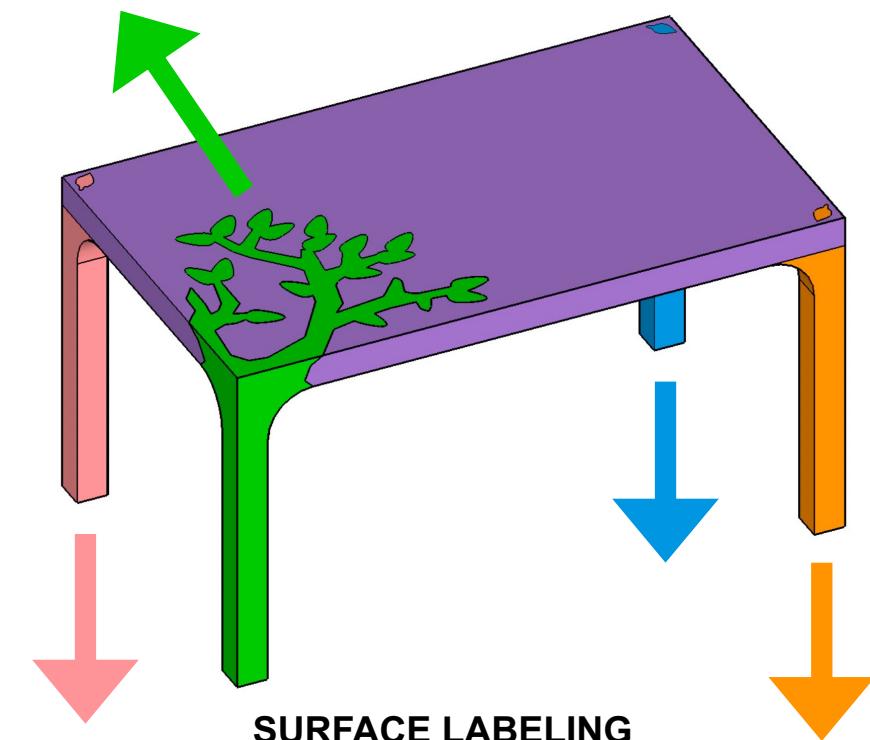
# Greedy Labeling for Volume-Aware molding

- Greedy labeling does not typically work as well as graph cuts, but when you have the **right idea**, it becomes extremely powerful!



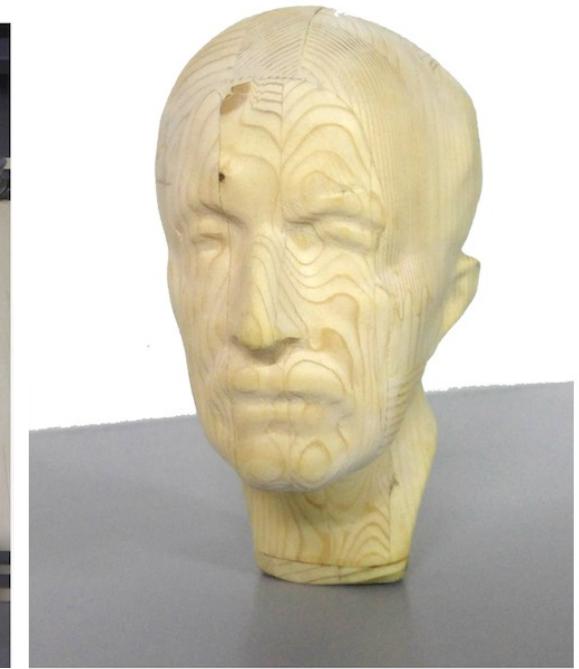
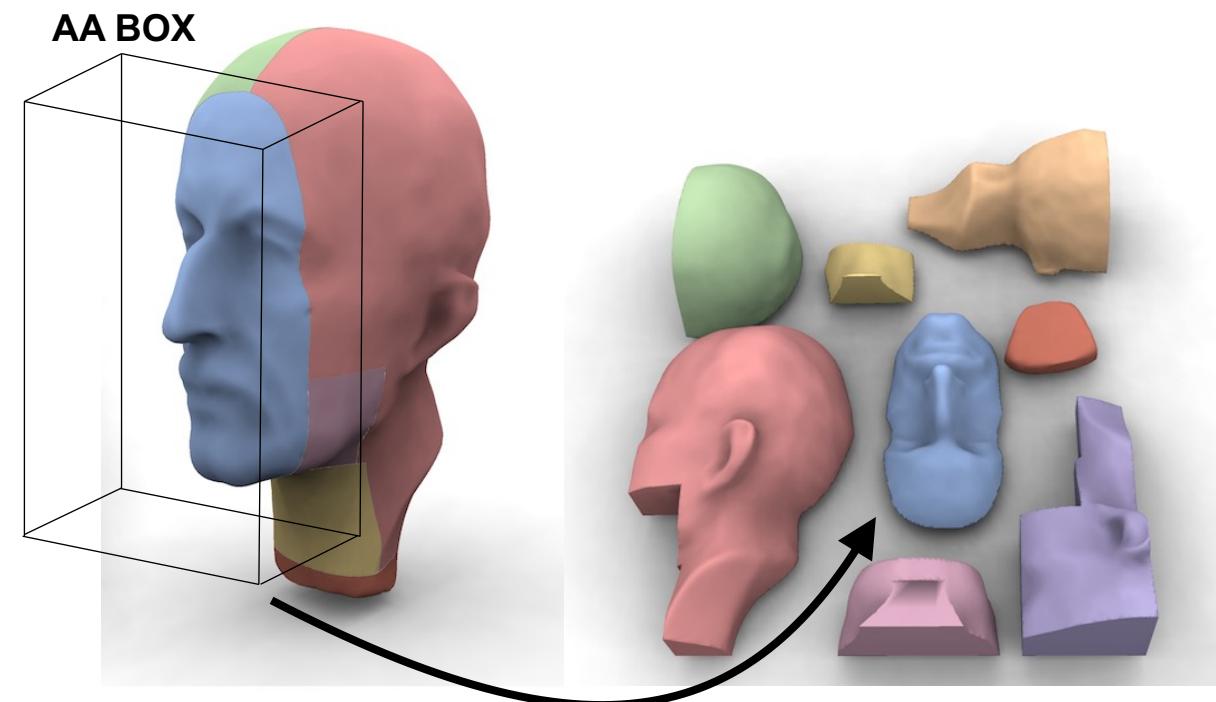
# Surface Mesh Booleans

- **Extrusion** of surface patches along a feasible extraction direction



# Surface Mesh Booleans

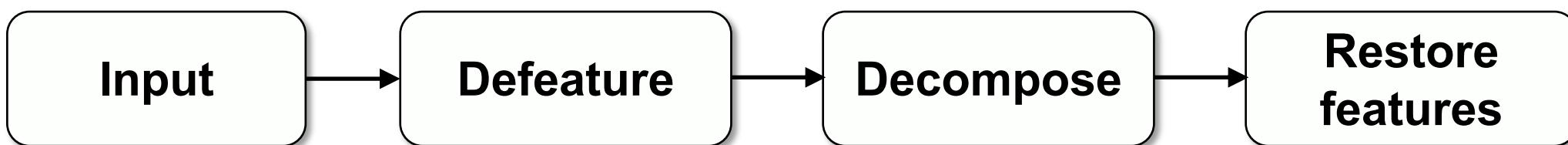
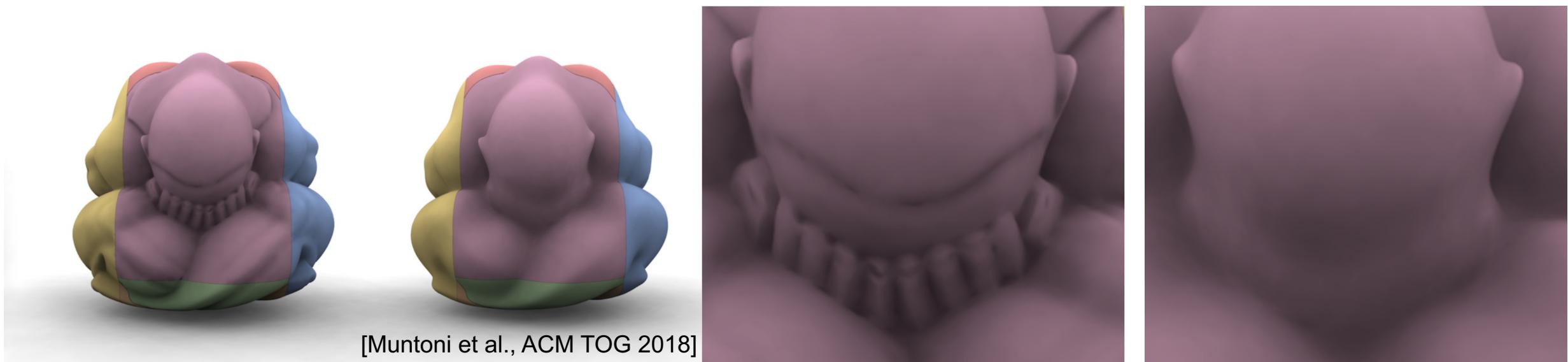
- **Intersection** with an axis aligned box bounding a height field



[Muntoni et al., ACM TOG 2018]

# Defeaturing

- Removing small scale features in pre-processing helps reducing part count



\*without violating constraints

# Defeaturing

- Removing small scale features in pre-processing helps reducing part count

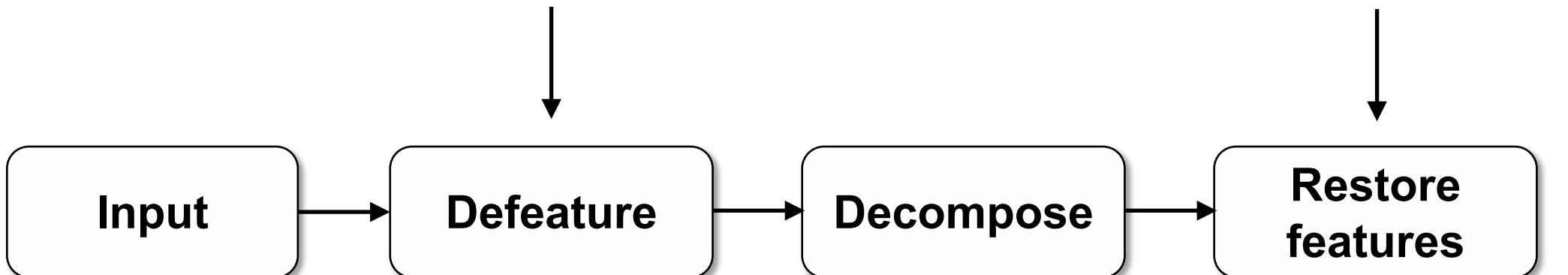
i) Store differential coordinates

$$\delta_i = \frac{1}{|N_i|} \sum_{v_j \in N_i} (v_i - v_j)$$

$$\min \sum_i \|\Delta v_i - \delta_i\|^2$$

subject to  
fabrication constraints

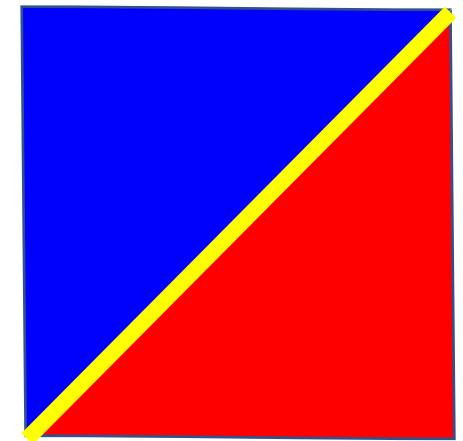
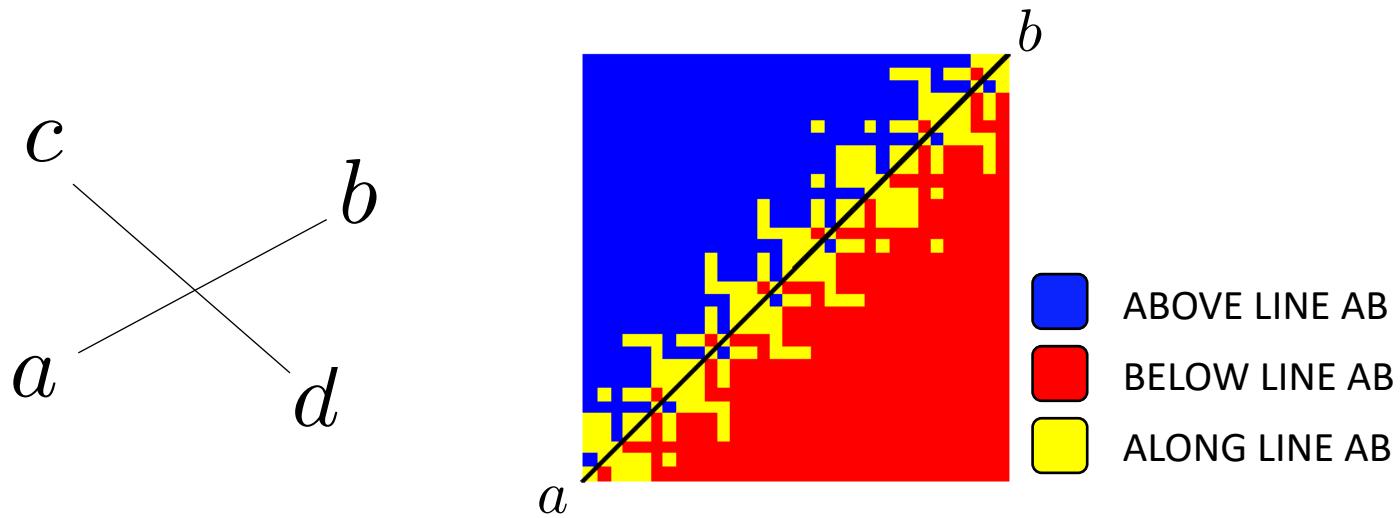
ii) Apply Smoothing



\*without violating constraints

# Mesh Booleans: Robustness

- Mesh booleans and planar cuts require finding intersections between mesh elements. This computation is **not robust** in floating point!



Indirect Predicates [Attene 2020]

Predicates Construction Kit [Levy 2015]

Shewchuk Predicates [Shewchuk 1997]

GMP/ CGAL

- Rational numbers or exact predicates are robust, but **slow!**
- Labeling (with interface smoothing) is **float friendly**

# To Conclude

---

- Shape Decomposition for Fabrication comes in **many flavours** and is useful for a variety of different things
- It all boils down to control **two basic ingredients**
  - local surface orientation
  - part size (either static or in motion, for assemblability)
- Always a **hard** problem, but we have good heuristics
  - greedy is not a bad word
  - when the idea is good, greedy algorithms become fast, easy to code and powerful!