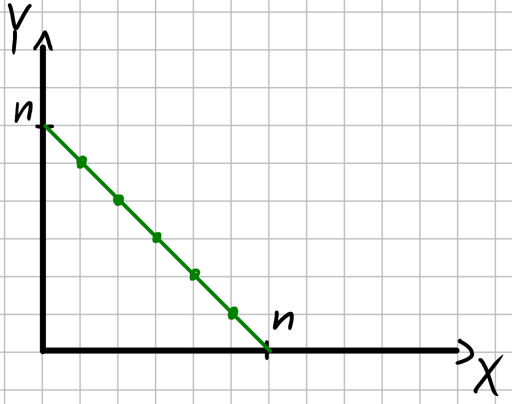
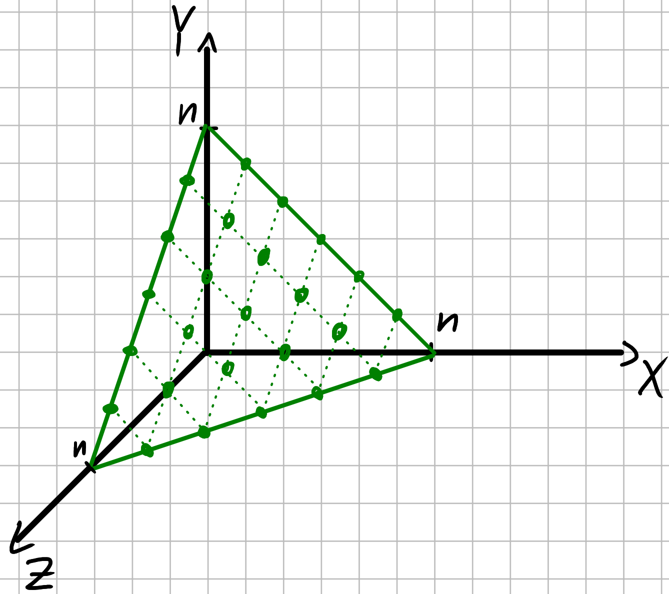
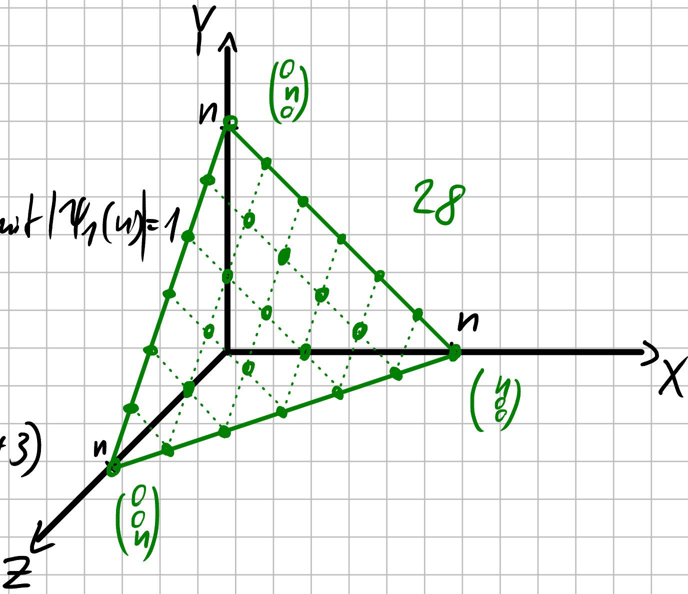




Sei $\Psi_m(k)$ die Menge der geordneten m-Partitionen von k.

Dann gibt es zu $X_1 \in \{0,\dots,n\}$ genau $|\Psi_{m-1}(n-X_1)|$ Kombinationen aus $X_2,\dots,X_m$, sodass $(X_1,\dots,X_m) \in \Psi_m(k)$

$$\Rightarrow P(X_1=r) = \frac{|\Psi_{m-1}(n-r)|}{|\Psi_m(n)|}$$



$$|\Psi_2(n)| = \binom{n+1}{1} = n+1 = \sum_{k=0}^{n} |\Psi_1(k)| \quad \text{mit } |\Psi_1(k)| = 1$$

$$|\Psi_3(n)| = \sum_{k=0}^{n} |\Psi_2(k)| = \frac{1}{2}(n+1)(n+2)$$

$$|\Psi_4(n)| = \sum_{k=0}^{n} |\Psi_3(k)| = \frac{1}{6}(n+1)(n+2)\cdot\ldots\cdot(n+3)$$

$$|\Psi_k(n)| = \prod_{i=1}^{k-1} \frac{n+i}{i}$$

$$\Rightarrow P(X_1=r) = \frac{|\Psi_{m-1}(n-r)|}{|\Psi_m(n)|} = \frac{\prod_{i=1}^{m-2} \frac{n-r+i}{i}}{\prod_{i=1}^{m-1} \frac{n+i}{i}} = \frac{\prod_{i=1}^{m-2} \frac{n+i-r}{n+i}}{n+(m-1)}$$

$$= \frac{\prod_{i=1}^{m-2} 1 - \frac{r}{n+i}}{n+(m-1)}$$ (m−1) hingepfuscht weils sonst $\sum_r P(X_1=r) \neq 1$

$$P(X_1 \leq \alpha) = \sum_{r=0}^{\alpha} \frac{m-1}{n+(m-1)} \prod_{i=1}^{m-2} 1 - \frac{r}{n+i}$$

mit $$\prod_{i=1}^{m-2} 1 - \frac{r}{n+i} = \frac{\Gamma(n+1)\Gamma(m+n-r-1)}{\Gamma(m+n-1)\,\Gamma(n-r+1)} = \frac{n!\,(m+n-r-2)!}{(m+n-2)!\,(n-r)!}$$

$$= \frac{n!}{(n-r)!\,r!}\, r!\, \frac{(m+n-r-2)!}{(m+n-2)!} = \binom{n}{r} r!\, \frac{(m+n-2-r)!}{(m+n-2)!}$$

$$\Rightarrow P(X_1 \leq \alpha) = \frac{m-1}{(n+(m-1))(m+n-2)!} \sum_{r=0}^{\alpha} \binom{n}{r} r!\,(m+n-2-r)!$$

$$= \frac{(\alpha+1)!\binom{n}{\alpha+1}(\alpha-m-n+2)(m+n-\alpha-3)! + (m+n-1)(m+n-2)!}{(n+m-1)(m+n-2)!}$$

$$= 1 + \frac{(\alpha+1)!\binom{n}{\alpha+1}(\alpha-m-n+2)(m+n-\alpha-3)!}{(n+m-1)(m+n-2)!}$$

$$= 1 - \frac{n!\,(m+n-\alpha-2)(m+n-\alpha-3)!}{(n-(\alpha+1))!\,(n+m-1)(m+n-2)!}$$

$$P(X_1 = r) = \qquad \stackrel{!}{=} \frac{\binom{M}{x}\binom{N-M}{\vartheta-x}}{\binom{N}{\vartheta}}$$

$$F(x,y) = \sum_{u=1}^{\infty}\sum_{n=0}^{\infty} \psi_u(n)\, x^u y^n = \sum_{n=0}^{\infty}\sum_{u=1}^{\infty} \psi_u(n)\, x^u y^n$$

$$\psi_{u+1}(n) = \sum_{r=0}^{n} \psi_u(r) \quad , \; u \geq 1, \; n \geq 0$$

$$\psi_{u+1}(n)\, x^{u+1} y^n = x \sum_{r=0}^{n} x^u \psi_u(r)\, y^n$$

$$\sum_{u=1}^{\infty}\sum_{n=0}^{\infty} \psi_{u+1}(n) x^{u+1} y^n = x \sum_{u=1}^{\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{n} \psi_u(r)\, x^u y^n$$

$$\left(\sum_{u=1}^{\infty}\sum_{n=0}^{\infty} \psi_u(n)\, x^u y^n - \sum_{n=0}^{\infty} \psi_1(n)\, x y^n\right) = \ldots$$

$$F(x,y) - \sum_{n=0}^{\infty} \psi_1(n)\, x y^n = x($$

$$G_n(x) = \sum_{k=1}^{\infty} \psi_k(n) x^k$$

$$\psi_{k+1}(n) x^k = x^k \sum_{r=0}^{n} \psi_k(r)$$

$$\frac{1}{x} \sum_{k=1}^{\infty} \psi_k(n) x^k - \psi_1(n) = \sum_{r=0}^{n} \sum_{k=1}^{\infty} x^k \psi_k(r) = \sum_{r=0}^{n} G_r(x)$$

$$\Leftrightarrow \frac{1}{x} G_n(x) - \psi_1(n) = \sum_{r=0}^{n} G_r(x)$$

$$\Leftrightarrow G_n(x) = x\left(\sum_{r=0}^{n} G_r(x) + \psi_1(n)\right) \Leftrightarrow G_n(x) = \frac{x}{1-x}\left(\sum_{r=0}^{n-1} G_r(x) + \psi_1(n)\right)$$

$$\text{mit } G_0(x) = \sum_{k=1}^{\infty} x^k = \frac{1}{1-x} - 1 = \frac{x}{1-x}, \quad \psi_1(n) = 1$$

$$\Rightarrow G_1(x) = \frac{x}{1-x}\left(\frac{x}{1-x} + 1\right) = \frac{x}{(1-x)^2}$$

$$G_2(x) = \frac{x}{1-x}\left(\frac{1}{1-x} + \frac{x}{(1-x)^2}\right) = \frac{x}{(1-x)^3}$$

$$G_n(x) = \frac{x}{(1-x)^{n+1}}$$

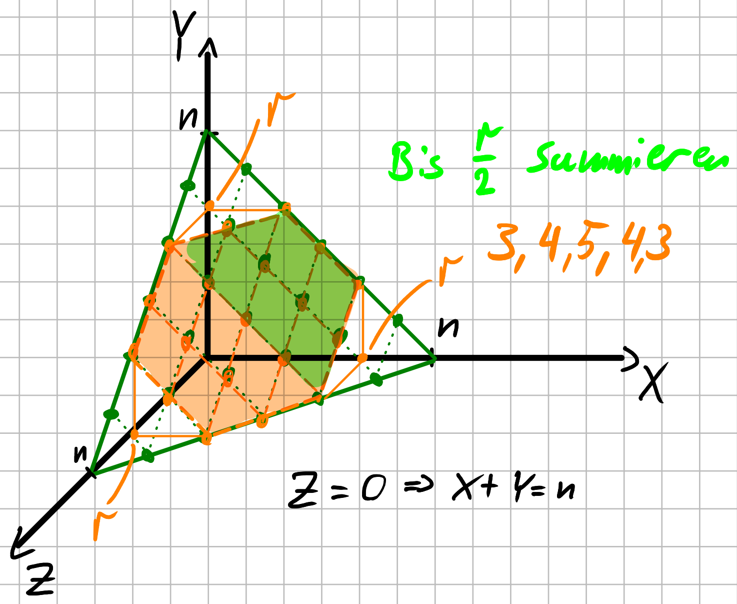
$$\frac{1}{(1-x)} = \sum_{k=0}^{\infty} x^k$$

$$\sum_{k=0}^{\infty} x^k \sum_{r=0}^{\infty} x^r = \sum_{k=0}^{\infty} \sum_{r=0}^{k} x^r x^{k-r} = \sum_{k=0}^{\infty} \sum_{r=0}^{k} x^k = \sum_{k=0}^{\infty} (k+1) x^k$$

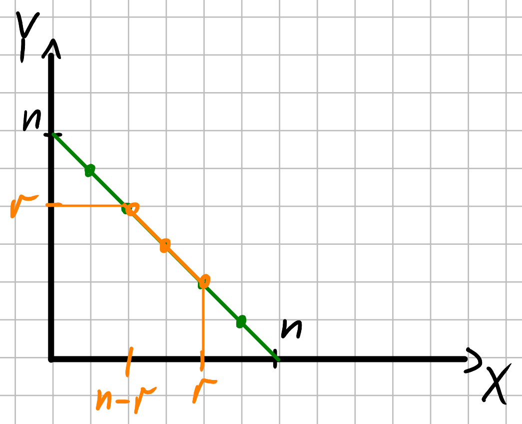
$$\sum_{k=0}^{\infty} k x^{k-1} = \sum_{k=1}^{\infty} k x^{k-1} = \sum_{k=0}^{\infty} (k+1) x^k = Dg(x)$$

$$g(x)^2 = Dg(x) \Rightarrow D(g^2(x)) = 2g(x)\,Dg(x) = 2g^3(x)$$

$$D(g^n(x))$$



$Z = 0 \Rightarrow X + Y = n$



$r - (n-r) = 2r - n + 1$ Punkte

$$P(X_1 = r) = \frac{|\tilde{\Psi}_{m-1}(n-r)|}{|\Psi_m(n)|}$$

$$\tilde{\Psi}^r_k(n) := \sum_{i=0}^{r} \tilde{\Psi}^r_{k-1}(n-i)$$

mit $\tilde{\Psi}^r_2(n) = 2r - n + 1$

$$\Rightarrow \tilde{\Psi}^r_2(n) = \sum_{i=0}^{\tilde{r}} (2r - (n-i) + 1)$$

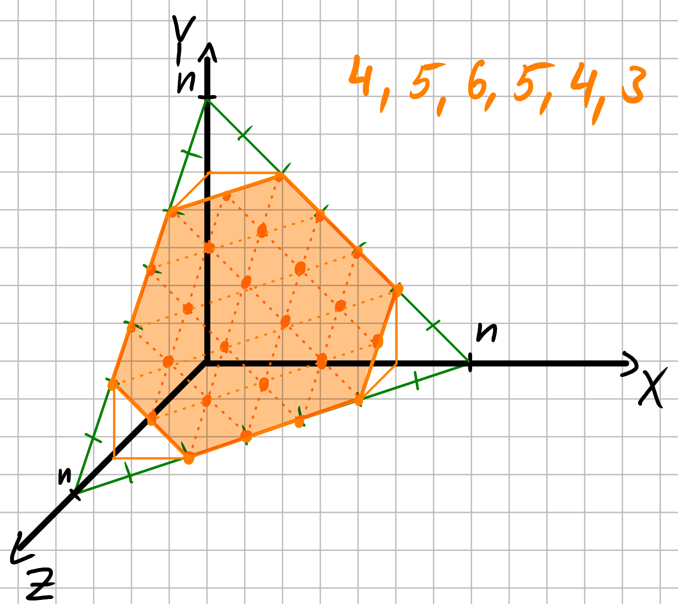
$$\Rightarrow \tilde{\Psi}^r_3(n)$$

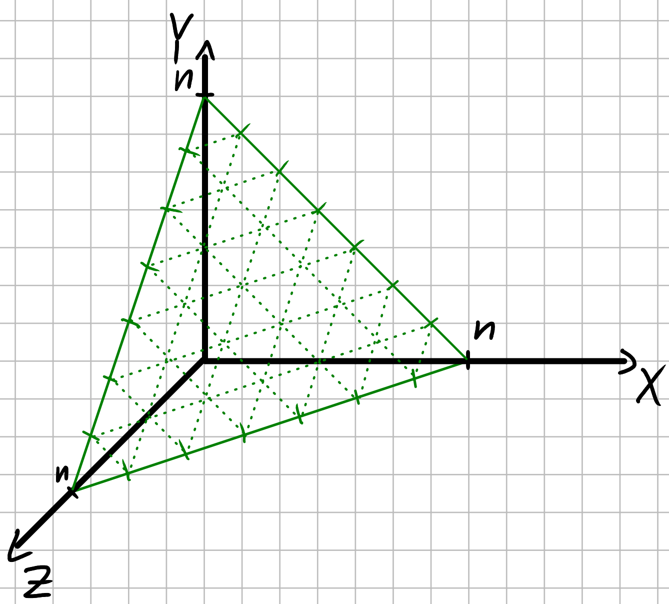
$$|\Psi_2(n)| = \binom{n+1}{1} = n+1 = \sum_{k=0}^{n} |\Psi_1(k)| \quad \text{mit } |\Psi_1(k)| = 1$$

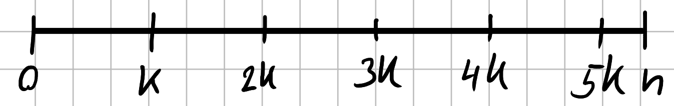
$$|\Psi_3(n)| = \sum_{k=0}^{n} |\Psi_2(k)| = \frac{1}{2}(n+1)(n+2)$$

$$|\Psi_4(n)| = \sum_{k=0}^{n} |\Psi_3(k)| = \frac{1}{6}(n+1)(n+2) \cdot \ldots \cdot (n+3)$$

$$|\Psi_k(n)| = \prod_{i=1}^{k-1} \frac{n+i}{i}$$

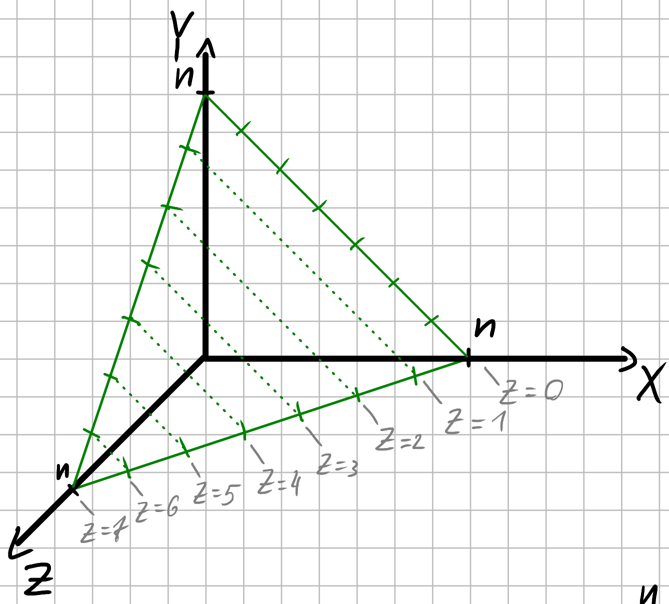






$m=6$ $\quad$ m-Part von n mit $c \leq k$

$$\left[\frac{n}{m}, k\right]$$



$$\psi_3(Z) = \sum_{z=0}^{Z} \psi_2(Z-z)$$

$$\psi_2(n) = n+1 = \sum_{Y=0}^{n} 1$$

$$\psi_k(n) = \sum_{X_n=0}^{n} \psi_{k-1}(n-X_n)$$

$$= \sum_{r=0}^{n} \psi_{k-1}(r)$$

$$\psi_3(n) = \sum_{r=0}^{n} (r+1) = \frac{1}{2}(n+1)(n+2)$$

$$\Rightarrow \psi_4(n) = \sum_{r=0}^{n} \frac{1}{2}(r+1)(r+2) = \frac{1}{6}(n+1)(n+2)(n+3)$$

$$\psi_k(n) = \frac{1}{(k-1)!}\prod_{r=1}^{k-1}(n+r) = \frac{1}{(k-1)!}\,\frac{(n+k-1)!}{n!}$$

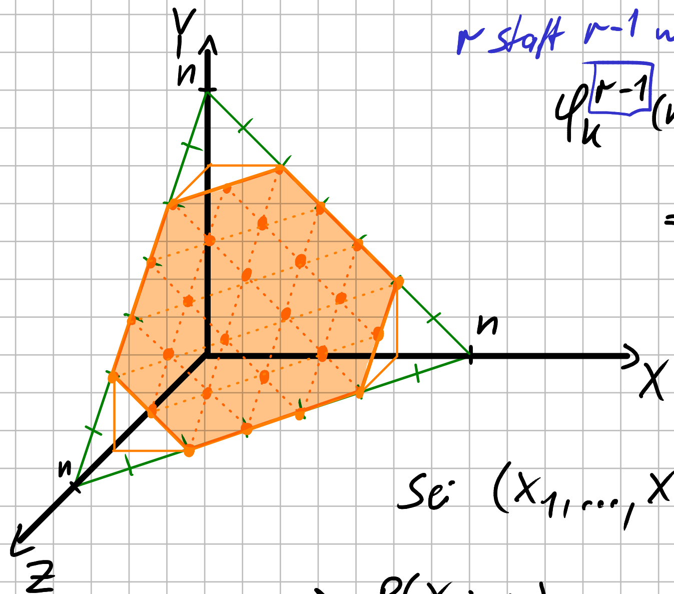
r statt $r-1$ wäre ohne Grenze, als Elem $< r$

$$\varphi_k^{\boxed{r-1}}(n) = \psi_k(n) - k\,\psi_k(n-r)$$

$$= \frac{(n+k-1)!}{n!\,(k-1)!} - \boxed{k\,\frac{(n-r+k-2)!}{(n-(r+1))!\,(k-1)!}}$$

Für $n < r-1$

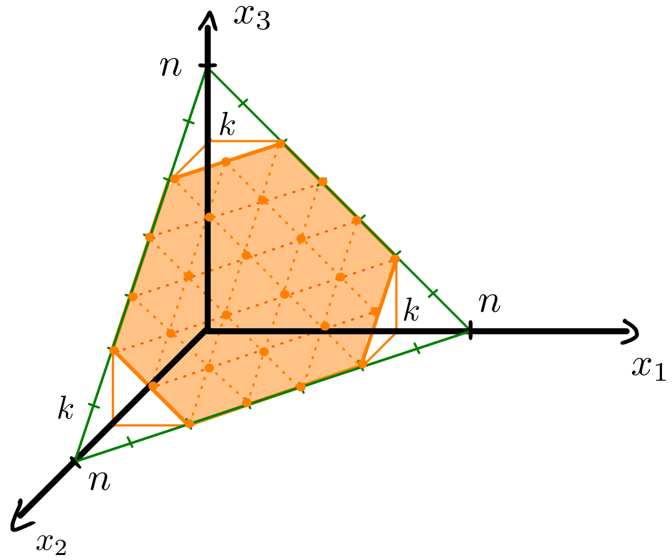
$=$ Anzahl an k-Part von n mit Elementen $\leq r$



Sei $(x_1, \dots, x_k)$ k-Part von n mit $x_j \leq r$

$$\Rightarrow P(x_j = m) =$$

$$\ln P(x_j=m) = \ln \varphi_{k-1}^{r}(n-m) - \ln \varphi_k^{r}(n)$$



x_3
n
k
k
n
x_1
k
n
x_2

$$\frac{\varphi_{k-1}^{r-1}(n-m)}{\varphi_k^{r-1}(n)} = \frac{1}{(k-2)!}\left(\frac{(n-m+k-2)!}{(n-m)!} - (k-1)\frac{(n-m-r+k-2)!}{(n-m-r)!}\right)$$

$$\cdot (k-1)!\left(\frac{(n+k-1)!}{n!} - k\,\frac{(n-r+k-1)!}{(n-r)!}\right)^{-1}$$

$$= (k-1)\ \frac{(n-m+k-2)!(n-m-r)! - (k-1)(n-m-r+k-2)!(n-m)!}{(n-m)!(n-m-r)!}$$

$$\cdot\left(\frac{(n-r)!(n+k-1)! - k\,(n-r+k-1)!\,n!}{n!(n-r)!}\right)^{-1}$$

$$= (k-1)\ \frac{(n-m+k-2)!(n-m-r)! - (k-1)(n-m-r+k-2)!(n-m)!}{(n-r)!(n+k-1)! - k\,(n-r+k-1)!\,n!}\ \frac{n!\,(n-r)!}{(n-m)!\,(n-m-r)!}$$

$\ln P(X_j=m) =$

$$P(X_j=m) = \frac{\varphi_{k-1}^r(n-m)}{\sigma_k^r(n)} \qquad \text{mit } \sigma_k^r(n) = \sum_{m=0}^{n} \varphi_{k-1}^r(n-m)$$

$$\varphi_k^r(n) = \psi_k(n) - k\,\psi_k(n-r) \qquad \psi_k(n) = \sum_{r=0}^{n} \psi_{k-1}(r) = \sum_{X_n=0}^{n} \psi_{k-1}(n-X_n)$$

$$\Rightarrow\ \sigma_k^r(n) = \sum_{m=0}^{n} \psi_{k-1}(n-m) - (k-1)\,\psi_{k-1}(n-m-r)$$

$$= \psi_k(n) - (k-1)\sum_{m=0}^{n}\psi_{k-1}(m-r) \qquad \text{mit } \psi_{k-1}(m-r) = 0 \text{ für } r>m$$

$$= \psi_k(n) - (k-1)\sum_{m=r}^{n}\psi_{k-1}(m-r) = \psi_k(n) - (k-1)\sum_{m=0}^{n-r}\psi_{k-1}(m)$$

$$\psi_k(n) - k\,\psi_k(n-r)$$

$$\varphi_k^{r-1}(n) = \psi_k(n) - k\,\psi_k(n-r) \quad , \quad \varphi_k(n) = \psi_k(n) - k\,\psi_k(n-(r+1))$$

$$\frac{(n+k-1)!}{n!(k-1)!} - k\,\frac{(n-r+k-2)!}{(n-(r+1))!(k-1)!} \qquad \boxed{n-m-r-1<0} \qquad \boxed{n-m-r+k-3<0}$$

$m \in [0, r]$

$$T_2 f(x,a) = f(a) + \nabla_f(a)^T (x-a) + \frac{1}{2}(x-a)^T H_f(a)(x-a)$$

$$f(x,y) = e^x - e^y$$

$$\nabla_f(x,y) = \begin{pmatrix} e^x \\ -e^y \end{pmatrix} \qquad H_f(x,y) = \begin{pmatrix} e^x & 0 \\ 0 & -e^y \end{pmatrix}$$

$$T_2 f(x,y,u,v) = e^u - e^v + e^u(x-u) - e^v(y-v) + \frac{1}{2}\begin{pmatrix} x-u \\ y-v \end{pmatrix}^T \begin{pmatrix} e^u(x-u) \\ -e^v(y-v) \end{pmatrix}$$

$$= e^u\left(1+x-u+\frac{1}{2}(x-u)^2\right) - e^v\left(1+y-v+\frac{1}{2}(y-v)^2\right)$$

Wähle $u,v = \begin{cases} \frac{a-b}{2}, & a \geq b \\ \frac{b-a}{2}, & b > a \end{cases}$

$$\underset{a\geq b}{\Rightarrow} T_2 f(a,b,u,v) = \exp\left(\frac{a-b}{2}\right)\left(1+a-\frac{a-b}{2}+\frac{1}{2}\left(a-\frac{a-b}{2}\right)^2 - \left(1+b-\frac{a-b}{2}+\frac{1}{2}\left(b-\frac{a-b}{2}\right)^2\right)\right)$$

$$= \exp\left(\frac{a-b}{2}\right)\left(a-b+\frac{1}{2}\left(\frac{a}{2}+\frac{b}{2}\right)^2 - \frac{1}{2}\left(\frac{3}{2}b-\frac{a}{2}\right)^2\right)$$

$$= \exp\left(\frac{a-b}{2}\right)\left(a-b+b(a-b)\right)$$

$$= \exp\left(\frac{a-b}{2}\right)(a-b)(1+b)$$

$$\underset{b>a}{\Rightarrow} T_2 f(a,b,u,v) = \exp\left(\frac{b-a}{2}\right)\left(1+a-\frac{b-a}{2}+\frac{1}{2}\left(a-\frac{b-a}{2}\right)^2 - \left(1+b-\frac{b-a}{2}+\frac{1}{2}\left(b-\frac{b-a}{2}\right)^2\right)\right)$$

$$= \exp\left(\frac{b-a}{2}\right)\left(a-b+\frac{1}{2}\left(\left(\frac{3}{2}a-\frac{b}{2}\right)^2 - \left(\frac{b}{2}+\frac{a}{2}\right)^2\right)\right)$$

$$= \exp\left(\frac{b-a}{2}\right)\left(a-b+a(a-b)\right)$$

$$= \exp\left(\frac{b-a}{2}\right)(a-b)(1+a)$$

Insgesamt also $T_2 f(a,b,u,v) = \exp\left(\frac{|b-a|}{2}\right)(a-b)(1+\max\{a,b\})$