ON THE PERIODICITY OF THE SUMS OF SINES

SV

Problem 1. Let $c(x)$ be a function

$$c(x) = \sum_{i=1}^{N} \sin(\omega_i x) \text{ with } N \in \mathbb{N} \text{ and } x \in \mathbb{R}.$$

It's to be determined, if there exists a value T_0, such that $c(x) = c(x + T_0)$.

Claim 2. A function $c(x) = \sum_{i=1}^{j} \sin(\omega_i x)$ is periodic in T_0, if there exists a value $T_0 \in \mathbb{N}$, such that $T_0 = \gcd(F)^{-1}$ with $F = \{f_i | f_i = \frac{\omega_i}{2\pi}\}$ exists. Herein let gcd be the function of the greatest common divider, so the function, that gives the biggest value ζ of a set Z, such that $\forall z \in Z \exists k \in \mathbb{N} : \frac{z}{\zeta} = k$ is statisfied.

Proof. Lets start with defining the set of all frequencies of the partial waves

$$F := \{f_i | f_i = \frac{\omega_i}{2\pi}\}$$

and define

$$\xi := \gcd(F).$$

Then:

$$\forall f \in F \exists k \in \mathbb{N} : f = k \cdot \xi$$

$$\Leftrightarrow \exists \xi \forall \frac{f}{k} : \xi = \frac{f}{k} \wedge f \in F \wedge k \in \mathbb{N}$$

Combining this with $c(x + T_0)$ yields

$$c(x + T_0) = \sum_{i=1}^{N} \sin(\omega_i x + \omega_i \frac{k_i}{f_i}),$$

what combined with the definition of F can be reduced to

$$c(x + T_0) = \sum_{i=1}^{N} \sin(\omega_i x + 2\pi k_i.$$

The rear inner term can be dropped because $\sin(a + \phi) = \sin(a) \Leftrightarrow \phi \in \mathbb{Z}$ and $k \in \mathbb{N} \wedge \mathbb{N} \subseteq \mathbb{Z}$.

This proves that $c(x + T_0) = c(x) \Leftarrow T_0 = \gcd(F)^{-1}$. $\square$