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Taller 4 - Métodos computacionales - Santiago Vargas

$$I = \int_a^b f(x)\, dx \cong \int_a^b P_1(x)\, dx$$

Sea

$$f(x) \approx P_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$\int_a^b \left[\frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b) \right] dx$$

$$= \frac{f(a)}{a-b} \int_a^b x-b\, dx + \frac{f(b)}{b-a} \int_a^b (x-a)\, dx$$

$$= \frac{f(a)}{a-b} \int_a^b x\, dx - \frac{f(a)b}{a-b} \int_a^b 1\, dx + \frac{f(b)}{b-a} \int_a^b (x-a)\, dx$$

$$= \frac{f(a) x^2}{2(a-b)} \Big|_{x=a}^b - \frac{b f(a)}{a-b} \int_a^b 1\, dx + \frac{f(b)}{b-a} \int_a^b (x-a)\, dx$$

$$\frac{f(a) x^2}{2(a-b)} \Big|_{x=a}^b = \frac{b^2 f(a)}{2(a-b)} - \frac{a^2 f(a)}{2(a-b)} = -\frac{1}{2} f(a)(a+b)$$

$$= -\frac{1}{2} f(a)(a+b) - \frac{b f(a)}{a-b} \int_a^b 1\, dx + \frac{f(b)}{b-a} \int_a^b (x-a)\, dx$$

$$= -\frac{1}{2} f(a)(a+b) + \left(-\frac{b f(a) x}{a-b} \right) \Big|_{x=a}^b + \frac{f(b)}{b-a} \int_a^b (x-a)\, dx$$

$$\left(-\frac{b f(a) x}{a-b} \right) \Big|_{x=a}^b = \left(-\frac{bb\, f(a)}{a-b} \right) - \left(-\frac{ab\, f(a)}{a-b} \right) = b f(a)$$

$$= b f(a) - \frac{1}{2} f(a)(a+b) + \frac{f(b)}{b-a} \int_a^b (x-a)\, dx$$

$$= b f(a) - \frac{1}{2} f(a)(a+b) + \frac{f(b)}{b-a} \int_a^b x\, dx - \frac{a f(b)}{b-a} \int_a^b 1\, dx$$

$$= b f(a) - \frac{1}{2} f(a)(a+b) + \frac{f(b) x^2}{2(b-a)} \Big|_{x=a}^{b} - \frac{a f(b)}{b-a} \int_a^b 1 \, dx$$

$$\frac{f(b) x^2}{2(b-a)} \Big|_{x=a}^{b} = \frac{b^2 f(b)}{2(b-a)} - \frac{a^2 f(b)}{2(b-a)} = \frac{1}{2} f(b)(a+b)$$

$$= b f(a) - \frac{1}{2} f(a)(a+b) + \frac{1}{2} f(b)(a+b) + \left(- \frac{a f(b) x}{b-a} \right) \Big|_{x=a}^{b}$$

$$- \frac{a f(b) x}{b-a} \Big|_{x=a}^{b} = \left(- \frac{a b f(b)}{b-a} \right) - \left(- \frac{a a f(b)}{b-a} \right) = -a f(b)$$

$$= -\frac{1}{2} f(a)(a+b) + \frac{1}{2} f(b)(a+b) - a f(b) + b f(a)$$

$$= \frac{1}{2} (b-a)(f(a) + f(b))$$