

Métodos computacionales - Taller 4 - Santiago Vargas

Demostración 2

$$\text{Sea } e(x) = \frac{f''(\xi)}{2} (x-a)(x-b)$$

$$E = \int_a^b e(x) dx = \int_a^b \frac{f''(\xi)}{2} (x-a)(x-b) dx$$

$$\frac{f''(\xi)}{2} \int_a^b (x-a)(x-b) dx \quad (1)$$

Se resuelve la siguiente integral:

$$\int_a^b (x-a)(x-b) dx \quad (2)$$

$$= \int_a^b (ab - ax - bx + x^2) dx$$

$$= \int_a^b x^2 dx + (-a-b) \int_b^a x dx + ab \int_b^a 1 dx$$

$$\frac{x^3}{3} \Big|_b^a + (-a-b) \int_b^a x dx + ab \int_b^a 1 dx$$

$$= \frac{1}{3} (a^3 - b^3) + (-a-b) \int_b^a x dx + ab \int_b^a 1 dx$$

$$= \frac{1}{3} (a^3 - b^3) + \frac{1}{2} x^2 (-a-b) \Big|_{x=b}^a + ab \int_b^a 1 dx$$

$$\frac{1}{2} x^2 (-a-b) \Big|_{x=b}^a = \left(\frac{1}{2} a^2 (-a-b) \right) - \left(\frac{1}{2} b^2 (-a-b) \right) = -\frac{1}{2} (a-b)(a+b)^2$$

$$= -\frac{1}{2}(a-b)(a+b)^2 + \frac{1}{3}(a^3-b^3) + ab \int_b^a 1 dx$$

$$= -\frac{1}{2}(a-b)(a+b)^2 + \frac{1}{3}(a^3-b^3) + abx \Big|_{x=b}^a$$

$$abx \Big|_{x=b}^a = aab - abb = ab(a-b)$$

$$= \frac{1}{3}(a^3-b^3) - \frac{1}{2}(a-b)(a+b)^2 + ab(a-b)$$

$$= -\frac{1}{6}(a-b)^3$$

Reemplazando (2) en (1)

$$\int_a^b \left(\frac{f''(\xi)}{2} (x-a)(x-b) \right) dx$$

$$\frac{f''(\xi)}{2} \int_a^b (x-a)(x-b) dx$$

$$\left(\frac{f''(\xi)}{2} \right) \left(-\frac{1}{6}(a-b)^3 \right) = -\frac{1}{12} f''(\xi) (a-b)^3$$

$$= -\frac{(a-b)^3}{12} f''(\xi)$$

Sea $(a-b) = h$:

$$= -\frac{h^3}{12} f''(\xi)$$