

B.Tech II Year I Semester (R15) Supplementary Examinations June 2017

**DISCRETE MATHEMATICS**

(Common to CSE &amp; IT)

Time: 3 hours

Max. Marks: 70

**PART – A**

(Compulsory Question)

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- 1 Answer the following: (10 X 02 = 20 Marks)
- What is conjunction? Construct the truth table.
  - Show that the formula  $Q \cup (P \cap \sim Q) \cup (\sim P \cap Q)$  is a tautology.
  - Define functions.
  - Let  $\lfloor \sqrt{x} \rfloor$  be the greatest integer  $\leq \sqrt{x}$ . Show that  $\lfloor \sqrt{x} \rfloor$  is a primitive recursive.
  - Prove that the minimum weight of the nonzero code words in a group code is equal to its minimum distance.
  - Prove that Boolean intensity  $(a \cap b) \cup (a \cap b') = a$ .
  - Define planar graph.
  - Mention the importance of graph coloring.
  - Prove that a tree with  $n$  – vertices has precisely  $n - 1$  edges.
  - A label identifier for a computer program consists of one letter followed by three digits. If repetitions are allowed, how many distinct label identifiers are possible.

**PART – B**

(Answer all five units, 5 X 10 = 50 Marks)

**UNIT – I**

- 2 (a) Construct the truth table for  $(P \vee Q) \vee \sim P$ .  
 (b) Demonstrate that  $R$  is a valid inference from the premises  $P \rightarrow Q, Q \rightarrow R$  and  $P$ .

**OR**

- 3 Obtain the principal disjunctive normal form of:

- $\sim P \vee Q$ .
- $(P \cap Q) \vee (\sim P \cap R) \vee (Q \cap R)$ .

**UNIT – II**

- 4 (a) Let  $Z$  be the set of integers and let  $R$  be the relation called congruence modulo 3 defined by:  $R = \{ \langle x, y \rangle / x \in Z \cap y \in Z \cap (x - y) \text{ is divisible by } 3 \}$ , determine the equivalence classes generated by the elements of  $z$ .  
 (b) Let  $X = \{2, 3, 6, 12, 24, 36\}$  and the relation  $\leq$  be such that  $x \leq y$  if  $x$  divides  $y$ . Draw the Hasse diagram of  $\langle X, \leq \rangle$ .

**OR**

- 5 Let  $F_x$  be the set of all one to one, onto mappings from  $X$  onto  $X = \{1, 2, 3, 4\}$ . Find all the elements of  $F_x$  and find the inverse of each element.

**UNIT – III**

- 6 (a) Prove that a subset  $S \neq \phi$  of  $G$  is a subgroup of  $\langle G, * \rangle$ . If for any pair of elements  $a, b \in S, a * b^{-1} \in S$ .  
 (b) Show that every cyclic group of order  $n$  is isomorphic to the group  $\langle Z_n, t_n \rangle$ .

**OR**

- 7 (a) Let  $\langle L, \leq \rangle$  be a lattice in which  $*$  and  $\oplus$  denote the operations of meet and join respectively. For any  $a, b \in L$   $a \leq b = a \Leftrightarrow a \oplus b = b$   
 (b) In a bounded lattice  $\langle L, *, \oplus, 0, 1 \rangle$ , an element  $b \in L$  is called a complement of an element  $a \in L$  if  $a * b = 0$  and  $a \oplus b = 1$ .

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**UNIT – IV**

8 Explain the merge sort with an example and algorithm.

**OR**

9 (a) Explain the weighted trees and prefix codes thoroughly.

(b) What is spanning tree? Illustrate with one example.

**UNIT – V**

10 In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns car, dog, pun or byte occurs?

**OR**

11 Use generating functions to determine how many four elements subsets of  $S = \{1, 2, 3, \dots, 15\}$  contains no consecutive integers.

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