# Code Verification and Automated Theorem Prover

## (Week 4)

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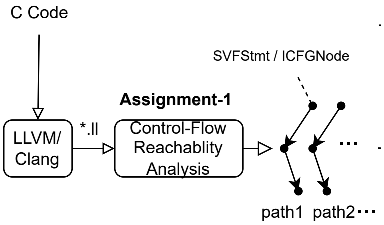
# Today’s class



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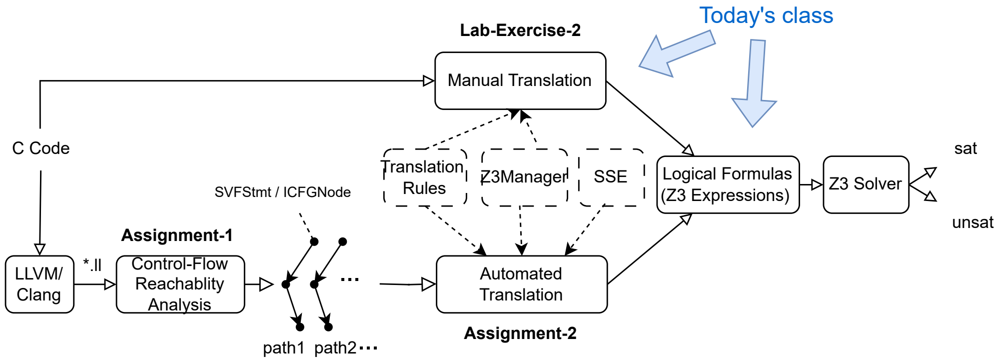


- In Lab-Exercise-2 and Assignment-2, we will conduct code verification to **prove code assertions** on top of reachability analysis (Assignment-1).
- Translating **C statements (Lab-Exercise-2)** and **SVFStmt/ICFGNode (Assignment-2)** to **logical formulas/expressions** and solve them to verify code assertions using automated theorem prover (i.e., Z3)

# Formal Verification For Code

$$\text{Specification} \stackrel{?}{\equiv} \text{Code implementation}$$

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- Proving the correctness of your code given a specification (or spec) using formal methods of mathematics
- Make the connection between specifications and implementations rigid, reliable and secure by translating specification and code into logical formulas.
- The application of theorem proving tools to perform satisfiability checking of logical formulas.

# Specification

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  - Formal specification in a separate file from the source code, written in a specification language and accepted by theorem provers

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- Specifications **embedded in** the source code (This course)
  - `assume(expr)`: an assumed **precondition** of a program that expression `expr` always be true and uses this assumed knowledge to execute the program. `assmue` is often **optional** as many verification scenarios may not have preconditions, including Lab-Exercise-2 and Assignment-2.
  - `assert(expr)`: an expected **postcondition** embedded in the program to check that `expr` always holds for any execution, otherwise the program terminates. We use `svf_assert` in our lab/assignment as an alternative for verification purposes.

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  - `assert(expr)`: an expected **postcondition** embedded in the program to check that `expr` always holds for any execution, otherwise the program terminates. We use `svf_assert` in our lab/assignment as an alternative for verification purposes.
- Hoare logic triple $P\{prog\}Q$, represents a program expressed by a predicate (first-order) logic. It describes that when the **precondition** $P$ is met, executing the **program** *prog* establishes the **postcondition** $Q$.

Hoare logic: `https://en.wikipedia.org/wiki/Hoare_logic`

Formal specifications: `https://www.hillelwayne.com/post/why-dont-people-use-formal-methods`

# Pre-/Post-Conditions and Satisfiability

Prove whether the post-condition (`assert`) holds after executing the program given the pre-condition (`assume`).

```
assume(100 > x > 0);  // P
        if(x > 10) {
            y = x + 1;
        }
        else {
            y = 10;
        }
assert(y >= x + 1);  // Q
```

$\xrightarrow{\text{translate}}$ $\psi(P\{\texttt{prog}\}Q)$ $\xrightarrow{\text{feed into}}$ SAT/SMT Solver

logical formula

Will the assertion hold?

# Assertions as Specifications

- In our lab and assignments, we need to verify whether the assertions (`svf_assert`) as specifications are satisfiable (expected results) or not.
- An assertion is a predicate or an expression that **always should evaluate to true** at that point during code execution.
  - help a programmer **read the code**
  - help the program **detect its own defects**
  - help catch errors earlier and **pinpoint sources of errors**

```
assert(expr);

or

svf_assert(expr);
```

$\xrightarrow{\textit{unfold}}$

```
if(expr is true){
    // continue normal execution
}
else{
    __assert_fail();
    // program failure and terminate the program
}
```

# Satisfiability Solving as Logic Inference

Satisfiability solving of hoare logic triple $P\{prog\}Q$ as a logic inference problem:

- Given $P\{prog\}Q$ represented by a set of constraints (logical formulas) extracted from code, we express $P\{prog\}$ as $KB$ **knowledge base** or **premises**, and $Q$ is the **conclusion.** Revisit our previous example as below:

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- Given $P\{prog\}Q$ represented by a set of constraints (logical formulas) extracted from code, we express $P\{prog\}$ as $KB$ **knowledge base** or **premises**, and $Q$ is the **conclusion.** Revisit our previous example as below:
  - $KB$ : $(100 > \mathtt{x} > 0) \wedge ((\mathtt{x} > \mathtt{10} \wedge \mathtt{y} \equiv \mathtt{x} + \mathtt{1}) \vee (\mathtt{x} \leq \mathtt{10} \wedge \mathtt{y} \equiv \mathtt{10}))$
  - $Q$ : $\mathtt{y} \geq \mathtt{x} + \mathtt{1}$
- $KB \vdash Q$ ?
  - Does KB semantically entail $Q$?
  - If all constraints in $KB$ are true, is the assertion true?
  - Is the specification $Q$ satisfiable given constraints from code?
- Each element (**proposition** or **predicate**) in $KB$ can be seen as a premise and $Q$ is the conclusion.

# Propositional Logic (Statement Logic)

A **proposition** is a statement that is either true or false. Propositional logic studies the ways statements can interact with each other.

- **Propositional variables** (e.g., $S$) represent propositions or statements in the formal system.
- A **propositional formula** is logical formula with **propositional variables** and **logical connectives** like `and` ($\wedge$) , `or` ($\vee$), `negation` ($\neg$), `implication` ($\rightarrow$)
  - $(S_1 \wedge S_2) \rightarrow Q$. This formula means that if $S_1$ and $S_2$ are both true, then $Q$ is true.
  - $S_1$ and $S_2$ are propositional variables. $\wedge$ and $\rightarrow$ are logical connectives.
- **Logic inference** allows certain logic formulas to be derived. These derived formulas are called **theorems** (or true propositions). The derivation can be interpreted as proof of the proposition represented by the theorem.

https://en.wikipedia.org/wiki/Propositional_calculus

http://discrete.openmathbooks.org/dmoi2/sec_propositional.html

# Predicate Logic (First-Order Logic)

**Predicate logic is propositional logic with predicates and quantification.**

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- **Predicate logic**: is more expressive and further analyzes proposition(s) by representing their entities' properties and relations and to group entities, i.e., additionally covers predicates and quantification.

# Predicate Logic (First-Order Logic)

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- **Propositional logic**: boolean logic which represents statements without reflecting their structures and relations
- **Predicate logic**: is more expressive and further analyzes proposition(s) by representing their entities' properties and relations and to group entities, i.e., additionally covers predicates and quantification.
- A **predicate** *P* takes one or more variables/entities as input and outputs a proposition and has a truth value (either true or false).
  - A statement whose truth value is dependent on variables.
  - For example, in $P(x) : x > 5$, "$x$" is the variable and "$x > 5$" is the predicate. After assigning x with the value 6, $P(x)$ becomes a proposition $6 > 5$.
- A **quantifier** is applied to a set of entities
  - Universal quantifier $\forall$, meaning all, every
  - Existential quantifier $\exists$, meaning some, there exists

https://en.wikipedia.org/wiki/First-order_logic https://www.youtube.com/watch?v=ARywou8HLQk

# Predicate Logic (Natural Language Example)

Consider the two statements

- "Jack got a high distinction"
- "Peter got a high distinction"

In propositional logic, these statements are viewed as being unrelated and the sub-statements/words/entities are not further analyzed.

- **Predicate logic** allows us to define a **predicate** $P$ representing "got a high distinction" which occurs in both sentences.
- $P(x)$ is the **predicate logic statement (formula)** which accepts a name $x$ and output as "$x$ got a high distinction".

# Predicate Logic (Code Example)

Consider these four statements

| | |
|---|---|
| $S_1$: | $x > 20;$ |
| $S_2$: | $x > 10;$ |
| $S_2 \rightarrow Q$: | if(x $>$ 10) y = 15; |
| $Q$: | $y = 15;$ |

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| $S_1$: | $x > 20;$ |
| $S_2$: | $x > 10;$ |
| $S_2 \rightarrow Q$: | if($x > 10$) y = 15; |
| $Q$: | $y = 15;$ |

- In **propositional logic**, each statement (including its variables and constants) is viewed as one proposition. Their relations are not further analyzed.
  - Given propositions $S_1$ and $S_2 \rightarrow Q$ as the knowledge base $KB$. Does the following semantically entail $\{S_1, S_2 \rightarrow Q\} \vdash Q$ or $(S_1 \wedge (S_2 \rightarrow Q)) \rightarrow Q$ hold?

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  - Answer: No! (The relation between $S_1$ and $S_2$ is not captured).

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| $S_1$: | x $>$ 20; |
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| $Q$: | y $=$ 15; |

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  - Given propositions $S_1$ and $S_2 \rightarrow Q$ as the knowledge base $KB$. Does the following semantically entail $\{S_1, S_2 \rightarrow Q\} \vdash Q$ or $(S_1 \wedge (S_2 \rightarrow Q)) \rightarrow Q$ hold?
  - Answer: No! (The relation between $S_1$ and $S_2$ is not captured).
- **Predicate logic** allows us to define **three predicates**: $P_1(x)$ represents x$>$20; $P_2(x)$ represents x$>$10; $Q(y)$ represents y$=$15 for the properties of x, y. Does the following hold using predicate logical for the inference?
  - $\{P_1(x), P_2(x) \rightarrow Q(y)\} \vdash Q(y)$ or $(P_1(x) \wedge P_2(x) \rightarrow Q(y)) \rightarrow Q(y)$

# Predicate Logic (Code Example)

Consider these four statements

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| $S_1$: | x > 20; |
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  - Given propositions $S_1$ and $S_2 \rightarrow Q$ as the knowledge base $KB$. Does the following semantically entail $\{S_1, S_2 \rightarrow Q\} \vdash Q$ or $(S_1 \wedge (S_2 \rightarrow Q)) \rightarrow Q$ hold?
  - Answer: No! (The relation between $S_1$ and $S_2$ is not captured).
- **Predicate logic** allows us to define **three predicates**: $P_1(x)$ represents x > 20; $P_2(x)$ represents x > 10; $Q(y)$ represents y = 15 for the properties of x, y. Does the following hold using predicate logical for the inference?
  - $\{P_1(x), P_2(x) \rightarrow Q(y)\} \vdash Q(y)$ or $(P_1(x) \wedge P_2(x) \rightarrow Q(y)) \rightarrow Q(y)$
  - $\{x > 20, x > 10 \rightarrow y = 15\} \vdash y = 15$
  - Answer: Yes!

# Satisfiability Checking (Revisit Our Example)

Given the predicate formula $\psi(P\{\texttt{prog}\}Q)$, we can verify the correctness of a program against the assertion specification $Q$ by checking $\psi$'s satisfiability (SAT).

```
assume(100 > x > 0);  // P
        if(x > 10) {
            y = x + 1;
        }
        else {
            y = 10;
        }
assert(y >= x + 1);  // Q
```

$\xrightarrow{\text{translate}}$ $\psi(P\{\texttt{prog}\}Q)$ (logical formula) $\xrightarrow{\text{feed into}}$ SAT/SMT Solver

# Satisfiability Checking for Code Verification

- $\psi(P\{\mathtt{prog}\}Q)$ is satisfiable if a program *prog* is correct for there valid inputs.

$$\forall \mathtt{x}\ \forall \mathtt{y}\ \ P(\mathtt{x}) \land S_{prog}(\mathtt{x}, \mathtt{y}) \rightarrow Q(\mathtt{x}, \mathtt{y})$$

  - $P(x)$ is the pre-condition predicate $(\mathtt{100} > \mathtt{x} > \mathtt{0})$ over variables $\mathtt{x}$.
  - $S_{prog}(\mathtt{x}, \mathtt{y})$ is the predicate representing *prog* which accepts $\mathtt{x}$ as its input, and terminates with output $\mathtt{y}$.
  - $Q(\mathtt{x}, \mathtt{y})$ is the post-condition predicate $(\mathtt{y} >= \mathtt{x} + \mathtt{1})$ over variables $\mathtt{x}, \mathtt{y}$.

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  - $Q(\texttt{x}, \texttt{y})$ is the post-condition predicate $(\texttt{y} >= \texttt{x} + \texttt{1})$ over variables $\texttt{x}, \texttt{y}$.
- How to prove correctness for all inputs $\texttt{x}$? Search for counterexample $\texttt{x}$ where $\psi$ does not hold, that is

$$\exists \texttt{x}\ \exists \texttt{y} \quad \neg(P(\texttt{x}) \wedge S_{prog}(\texttt{x}, \texttt{y})) \rightarrow Q(\texttt{x}, \texttt{y}))$$
$$\Rightarrow \quad \exists \texttt{x}\ \exists \texttt{y} \quad P(\texttt{x}) \wedge S_{prog}(\texttt{x}, \texttt{y}) \wedge \neg Q(\texttt{x}, \texttt{y}) \qquad \text{(simplification)}$$

  Note that $P(\texttt{x})$ is always true if a program does not have a pre-condition.

Logic formula simplification: `https://en.wikipedia.org/wiki/Logical_equivalence`

# Satisfiability Checking for Code Verification

Checking whether the logical formula $\psi$ is satisfiable by an SMT solver.

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$$\xrightarrow{\text{translate}} \underbrace{\exists \mathrm{x}\ \exists \mathrm{y}\ \ P(\mathrm{x}) \land S_{prog}(\mathrm{x}, \mathrm{y})) \land \neg Q(\mathrm{x}, \mathrm{y})}_{\text{logical formula } \psi} \xrightarrow{\text{feed into}} \text{SMT Solver}$$

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Unsatisfiable! **couterexample** `x = 10` **found**!

SMT: `https://en.wikipedia.org/wiki/Satisfiability_modulo_theories`

# Translating Code into Logical Formulas

- How to extract $P(\mathtt{x}) \wedge S_{prog}(\mathtt{x}, \mathtt{y})) \wedge \neg Q(\mathtt{x}, \mathtt{y})$ from code?

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- First-order logical formulas
  - The formulas of predicate logic are constructed from **propositional**, **predicate** and **object variables** by using **logical connectives** and **quantifiers** (This class)
- Translation
  - Translating `SVFStmts` of **each program path** (from Assignment-2) into a logical formula $\psi$, and then proving the non-existence of counterexamples (or check `unsat`) for each path.
  - $\forall path \in prog \quad checking(\psi_{path})$

$$\psi_{path_1}: \quad \exists \mathtt{x}\ P(\mathtt{x}) \wedge \big((\mathtt{x} > 10) \wedge (\mathtt{y} \equiv \mathtt{x} + 1)\big) \wedge \neg Q(\mathtt{x}, \mathtt{y}) \quad (\texttt{if}\ \text{branch})$$

$$\psi_{path_2}: \quad \exists \mathtt{x}\ P(\mathtt{x}) \wedge \big((\mathtt{x} \leq 10) \wedge (\mathtt{y} \equiv 10)\big) \wedge \neg Q(\mathtt{x}, \mathtt{y}) \quad (\texttt{else}\ \text{branch})$$

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    $\psi_{path_2}: \exists x(100 > x > 0) \wedge \big((\mathtt{x} \leq 10) \wedge (\mathtt{y} \equiv 10)\big) \wedge \neg(\mathtt{y} \geq \mathtt{x} + 1)$ (`else` branch)
  - $\psi_{path_2}$ : **has a counterexample** `x = 10`!!

# Translating Code into Logical Formulas

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  - $\psi_{path_2}$ : **has a counterexample** `x = 10`!!
  - **Manual translation** of C statements to logic expressions via Z3 theorem prover APIs (`Z3Mgr.h/cpp`) (Lab-Exercise-2)
  - **Automatic translation** of SVFIR to logic expressions during control-flow reachability analysis (Assignment-2)

# Closed-World Programs: Checking SAT vs. Proving Non-Existence of Counterexamples

- If the program operates in a **closed-world** (value initialisations are fixed and there are no inputs from externals such as main's arguments, like the tests in **Exercise-2**), there is no need to find the existence of invalid inputs (counterexamples).
- For the above closed-world programs, **logical errors** can be verified against assertions, rather than finding the **existence of counterexamples.** Checking satisfiability is essentially **the same** as checking the non-existence of counterexamples.
- The testing programs in the **Exercise 2** are all closed-world programs except branch examples. Hence **checking sat** of $P(\mathtt{x}) \wedge S_{prog}(\mathtt{x}, \mathtt{y}) \wedge Q(\mathtt{x}, \mathtt{y})$ or **proving unsat** of $P(\mathtt{x}) \wedge S_{prog}(\mathtt{x}, \mathtt{y}) \wedge \neg Q(\mathtt{x}, \mathtt{y})$ are **both good** for validating your z3 expression translations.

# Theorem Prover Tools

- **Interactive theorem provers** (proof assistants)
  - Formal proofs by human-machine collaboration via expressive specification languages; may not work directly on source code.
  - For example, ACL2, Coq, Isabelle, and HOL provers.
- **Automated theorem provers**
  - Proof automation (but less expressive than interactive provers); can work on real-world source code.
  - For example, Z3 and CVC.

Theorem prover tools: `https://en.wikipedia.org/wiki/Theorem_prover`

# Automated Theorem Provers

A prover/solver checks if a formula $\psi(P\{\texttt{prog}\}Q)$ is satisfiable (SAT).

- If yes, the solver returns a **model** *m*, a valuation of $\texttt{x}, \texttt{y}, \texttt{z}$ of $\texttt{prog}$ that satisfies $\psi$ (i.e., *m* makes $\psi$ true).
- Otherwise, the solver returns unsatisfiable (UNSAT)

**SAT** vs. **SMT** solvers

- **SAT** solvers accept **propositional logic** (Boolean) formulas, typically in the conjunctive normal form (CNF).
- **SMT** (satisfiability modulo theories) solvers generalize the Boolean satisfiability problem (SAT), and accept both propositional logic and more expressive **predicate logic** formulas.
  - Z3 Automated Theorem Prover, a cross-platform satisfiability modulo theories (SMT) solver developed by Microsoft (This course).

Z3: `https://github.com/Z3Prover/z3/wiki#background`

# Code to Logic Expressions with Z3 Theorem Prover

## (Week 5)

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# Z3 Theorem Prover

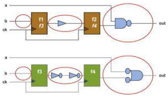
- Z3 is a Satisfiability Modulo Theories (SMT) solver from Microsoft Research.
- Targeted at solving problems in software verification and software analysis.
- Main applications are static checking, test case generation, and more ..

Hardware verification

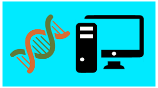
Software analysis/testing

Architecture

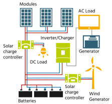
Modeling



Geometrical solving



Biological analysis

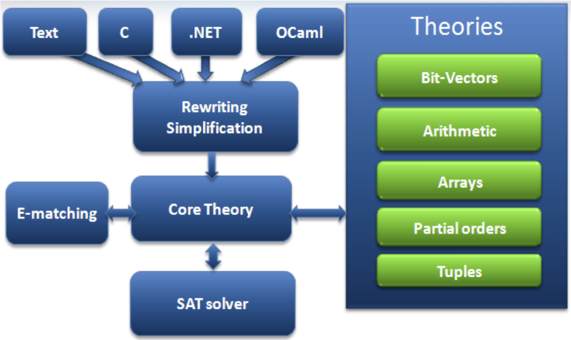



Hybrid system analysis

...

https://www.microsoft.com/en-us/research/project/z3-3/

# Z3 Framework




- Z3 is an effective tool to solve **logical formulas** (Z3 expressions/constraints).
- Z3 GitHub `https://github.com/Z3Prover/z3`.
- Z3 tutorials `https://github.com/philzook58/z3_tutorial`
- Z3 slides `https://github.com/Z3Prover/z3/wiki/Slides`
- Its SMT solver supports theories such as fixed-size bit-vectors, arithmetic, extensional arrays, datatypes, uninterpreted functions, and quantifiers.
- Z3 has official APIs for **C, C++, Python, .NET**, etc.
- **Z3 solver** can find one of the feasible solutions in a set of constraints.

# Z3 Solver and Z3 Formulas

Z3 solver accepts a first-order (predicate) logical formula $\psi$, and outputs one of the following results.

- `sat` if $\psi$ is satisfiable
- `unsat` if there is a counterexample which make $\psi$ unsatisfiable
- `unknown` if $\psi$ is too complex and can not be solved within a time frame.

You play around and check the satisfiability of your Z3 constraints/formulas here:
`https://jfmc.github.io/z3-play` or
`https://compsys-tools.ens-lyon.fr/z3/index.php`

# Z3 Playground (`https://jfmc.github.io/z3-play`)

## jfmc's Z3 Playground

Acknowled

[See the Z3 repository for the original `rise4fun` documents]

# Getting Started with Z3: A Guide

Be sure to follow along with the examples by clicking the "edit" link in the corner. See what the tool says, try your own formulas, and experiment!

## Introduction

Z3 is a state-of-the art theorem prover from Microsoft Research. It can be used to check the satisfiability of logical formulas over one or more theories. Z3 offers a compelling match for software analysis and verification tools, since several common software constructs map directly into supported theories.

The main objective of the tutorial is to introduce the reader on how to use Z3 effectively for logical modeling and solving. The tutorial provides some general background on logical modeling, but we have to defer a full introduction to first-order logic and decision procedures to text-books.

Z3 is a low level tool. It is best used as a component in the context of other tools that require solving logical formulas. Consequently, Z3 exposes a number of API facilities to make it convenient for tools to map into Z3, but there are no stand-alone editors or user-centric facilities for interacting with Z3. The language syntax used in the front ends favor simplicity in contrast to linguistic convenience.

## Basic Commands

The Z3 input format is an extension of the one defined by the SMT-LIB 2.0 standard. A Z3 script is a sequence of commands. The **help** command displays a list of all available commands. The command **echo** displays a message. Internally, Z3 maintains a **stack** of user provided formulas and declarations. We say these are the **assertions** provided by the user. The command **declare-const** declares a constant of a given type (aka sort). The command **declare-fun** declares a function. In the following example, we declared a function that receives an integer and a boolean, and returns an integer.

✎ Try it

```
(echo "starting Z3...")
(declare-const a Int)
(declare-fun f (Int Bool) Int)
```

```
; You can edit this code!
; Click here and start typing.

```

▶ Run

# Z3's Logical Formula (Constants, Check-Sat and Evaluation)

The Z3 input format (formula format) is an extension of the SMT-LIB 2.0 standard. A Z3 formula expression (`z3::expr`) has the following keywords:

- **`echo`** displays a message
- **`declare-const`** declares a constant of a given type (a.k.a `sort`)
- **`declare-fun`** declares a function
- **`assert`** adds a formula into the Z3 internal stack
- **`check-sat`** determines whether the current formulas on the Z3 stack are satisfiable or not
- **`get-model`** is used to retrieve an interpretation (one solution) that makes all formulas on the Z3 internal stack true
- **`eval`** evaluates a variable/expression produced by a model when the formulas is satisfiable.

SMT-LIB 2.0: `https://homepage.cs.uiowa.edu/~tinelli/papers/BarST-SMT-10.pdf`

# Constants, Check-Sat and Evaluation (Example)

$$\psi : (\mathtt{x} > \mathtt{10}) \ \wedge \ (\mathtt{y} \equiv \mathtt{x} + \mathtt{1})$$

How to represent this formula in Z3 and feed it into Z3's solver?

# Constants, Check-Sat and Evaluation (Example)

$$\psi : (x > 10) \ \wedge \ (y \equiv x + 1)$$

How to represent this formula in Z3 and feed it into Z3's solver?

```
(echo "starting Z3...")
(declare-const x Int) ;/// Declare an Int type variable "x"
(declare-const y Int) ;/// Declare an Int type variable "y"
(assert (> x 10))  ;/// Add the first part (x>10) of the conjunction into the solver
(assert (= y (+ x 1))) ;/// Add the second part (y==x+1) of the conjunction
(check-sat) ;/// Check whether added formulas are satisfiable.
(eval x) ;/// Evaluate the value of x when the formula is satisfiable
(eval y) ;/// Evaluate the value of y when the formula is satisfiable
```

# Constants, Check-Sat and Evaluation (Example)

$$\psi : (x > 10) \land (y \equiv x + 1)$$

How to represent this formula in Z3 and feed it into Z3's solver?

```
(echo "starting Z3...")
(declare-const x Int) ;/// Declare an Int type variable "x"
(declare-const y Int) ;/// Declare an Int type variable "y"
(assert (> x 10))  ;/// Add the first part (x>10) of the conjunction into the solver
(assert (= y (+ x 1))) ;/// Add the second part (y==x+1) of the conjunction
(check-sat) ;/// Check whether added formulas are satisfiable.
(eval x) ;/// Evaluate the value of x when the formula is satisfiable
(eval y) ;/// Evaluate the value of y when the formula is satisfiable
```

Outputs of Z3's solver:

```
starting Z3...
sat /// (check-sat) result
11 /// the value of x as one satisfiable solution
12 /// the value of y as one satisfiable solution
```

# Z3's Logical Formula (Uninterpreted Function)

The basic building blocks of SMT formulas are constants and uninterpreted functions.

- An uninterpreted function **has no other property** (no priori interpretation) **than its signature** (i.e., function name and arguments).
- An uninterpreted functions in first-order logic have **no side-effects** (e.g., can not change argument values and never return different values for the same input)
- **Constants** in Z3 can also be seen as **functions that take no arguments.**
- **The details and characteristics** of uninterpreted functions are **ignored.** This can **generalize and simplify** theorems and proofs.

# Uninterpreted Function (Example)

```
(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
(check-sat)
```

# Uninterpreted Function (Example)

```
(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
(check-sat)
```

Outputs of Z3's solver:

```
sat
```

The solver returns `sat`, because `f` is an uninterpreted function (i.e., all that is known about `f` is its signature), so it is possible that `f(10) = 1`.

# Uninterpreted Function (Example)

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(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
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```

Outputs of Z3's solver:

```
sat
```

The solver returns `sat`, because `f` is an uninterpreted function (i.e., all that is known about `f` is its signature), so it is possible that `f(10) = 1`.

```
(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
(assert (= (f 10) 2))    ;/// f(10) = 2
(check-sat)
```

# Uninterpreted Function (Example)

```
(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
(check-sat)
```

Outputs of Z3's solver:

```
sat
```

The solver returns `sat`, because `f` is an uninterpreted function (i.e., all that is known about `f` is its signature), so it is possible that `f(10) = 1`.

```
(declare-fun f (Int) Int)    ;/// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    ;/// f(10) = 1
(assert (= (f 10) 2))    ;/// f(10) = 2
(check-sat)
```

Outputs of Z3's solver:

```
unsat
```

The solver returns `unsat`, because `f`, as an uninterpreted function, can never return different values for the same input.

# Uninterpreted Function (Example)

$$\psi : \mathtt{f}(\mathtt{x}) \equiv \mathtt{f}(\mathtt{y}) \ \wedge \ \mathtt{x} != \mathtt{y}$$

```
(declare-const x Int)
(declare-const y Int)
(declare-fun f (Int) Int) ;/// Function f accepts an Int argument and returns a Int
(assert (= (f x) (f y)))
(assert (not (= x y)))
(check-sat)
```

# Uninterpreted Function (Example)

$$\psi : \mathtt{f}(\mathtt{x}) \equiv \mathtt{f}(\mathtt{y}) \ \wedge \ \mathtt{x!} = \mathtt{y}$$

```
(declare-const x Int)
(declare-const y Int)
(declare-fun f (Int) Int) ;/// Function f accepts an Int argument and returns a Int
(assert (= (f x) (f y)))
(assert (not (= x y)))
(check-sat)
```

Outputs of Z3's solver:

```
sat
```

An uninterpreted function can have different inputs and return the same output. For example, `f` can always return `1` regardless the value of the input argument.

# Constants as Uninterpreted Function (Example)

$$\psi : (x > 10) \ \wedge \ (y \equiv x + 1)$$

```
(declare-fun x () Int) ;/// "x" and "y" as an uninterpreted functions
(declare-fun y () Int) ;/// Accepts no argument and return an Int
(assert (> x 10))
(assert (= y (+ x 1)))
(check-sat)
(get-model)
```

Outputs of Z3's solver:

```
sat
(
  (define-fun x () Int
    11)                     ;/// x is evaluated to be 11 for this model
  (define-fun y () Int
    12)                     ;/// y is evaluated to be 11 for this model
)
```

`(declare-const x Int)` can be seen as the syntax sugar for `(declare-fun x () Int)`.

# Z3's Logical Formula (Arithmetic)

- Z3 supports majority of commonly used arithmetic operators, such as +, -, $*$, /, $<<$, $>>$, $<$, $>$, &, $|$ (The ones listed in SVFIR)
- Types of any two operands should be the same otherwise a type conversion is needed.
- Never mix types in arithmetic, and always be explicit.

```
(declare-const a Int)
(declare-const b Float32)
(assert (= a (+ b 1)))
(check-sat)
```

Outputs of Z3's solver:

```
Error: (error "line 3 column 19: Sort mismatch at argument #1 for function
(declare-fun + (Int Int) Int) supplied sort is (_ FloatingPoint 8 24)")
```

# Z3's Logical Formula (`if-then-else` Expression)

- `ite(b, x, y)` represents a conditional expression, where `b` is the condition, `ite` returns `x` if `b` is evaluated true, otherwise `y` is returned
- Used for comparison or branches

```
(ite (and (= x!1 11) (= x!2 false)) 21 0)
```

The above Z3 formula evaluates (returns) 21 when x!1 is equal to 11, and x!2 is equal to false. Otherwise, it returns 0.

# Z3's Logical Formula (Arrays)

Formulating a program of a mathematical theory of computation McCarthy proposed a basic theory of arrays as characterized by the **select-store** axioms.

- `(select a i)`: returns the value stored at position `i` of the array `a`;
- `(store a i v)`: returns a new array identical to `a`, but on position `i` it contains the value `v`.
- Z3 assumes that arrays are extensional over select. Z3 also enforces that if two arrays agree on all reads, then the arrays are equal.

# Z3's Logical Formula (Arrays)

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- `(store a i v)`: returns a new array identical to `a`, but on position `i` it contains the value `v`.
- Z3 assumes that arrays are extensional over select. Z3 also enforces that if two arrays agree on all reads, then the arrays are equal.

The following formulas store `y` to the `x`-th position of array `a` and then load the value at `a`'s `x`-th position to `z`

```
(declare-const x Int)
(declare-const y Bool)
(declare-const z Bool)
(declare-const a (Array Int Bool))  ;/// an array of Bools with Int as the indices
(assert (= (store a x y) a))    ;/// a[x] == y
(assert (= (select a x) z))     ;/// z == a[x]
```

# Z3's Logical Formula (Scopes)

Z3 maintains a global stack of declarations and assertions via **push** and **pop**

- **push**: creates a new scope by saving the current stack size.
- **pop**: removes any assertion or declaration performed between it and the matching push.

The `check-sat` command always operates on the current global stack.

# Z3's Logical Formula (Scopes)

Z3 maintains a global stack of declarations and assertions via **push** and **pop**

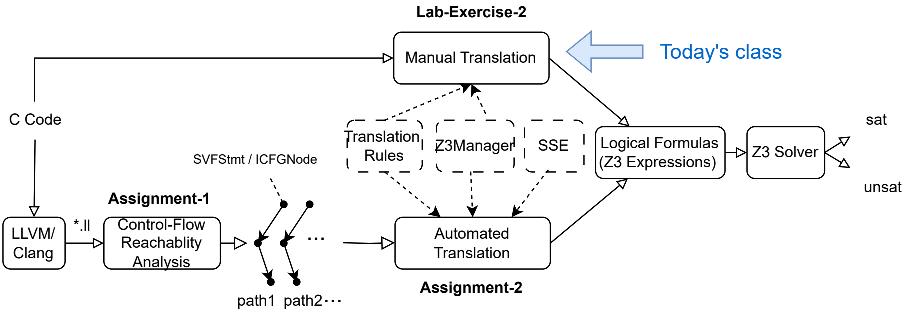
- **push**: creates a new scope by saving the current stack size.
- **pop**: removes any assertion or declaration performed between it and the matching push.

The `check-sat` command always operates on the current global stack.

```
(declare-const x Int)
(declare-const a (Array Int Int))  /// an array of Ints
(push)
(assert (= (store a 1 10) a))      ;/// a[1] == 10
(assert (= (select a 1) x))       ;/// x == a[1]
(assert (= x 20))                 ;/// x == 20
(check-sat)
(pop)                             ;/// remove the three assertions
(assert (= x 10))                 ;/// x == 10
(check-sat)
```

What is the output of the solver?

# Today’s class




- We introduce Z3 solver, Z3 constraint format **Z3Mgr** APIs used for lab/assignment in this course.
- We learn how to manually translate C source code into logical formulas (Z3 constraints/expressions).
- Then, we will demonstrate **examples** for **manual translation** from code to Z3 constraints.

# Translating Code to Z3 Formulas

We provide `Z3Mgr` and its subclass `Z3Examples` (wrapper classes to manipulate Z3 APIs) to generate Z3 formulas or so-called `z3::expr`.

| **Z3Examples API** | **Meanings** |
|---|---|
| `z3::expr getZ3Expr(std::string);` | Create a z3 expr given a string name |
| `z3::expr getZ3Expr(u32_t);` | Create a z3 expr given an integer |
| `z3::expr getCtx().int_val(u32_t);` | Create a z3 expr given an integer |
| `z3::expr getMemObjAddress(std::string);` | Create a **memory object** in program |
| `z3::expr getGepObjAddress(z3::expr, u32_t);` | Create a **field object** with an **offset** of an aggregate |
| `void addToSolver(z3::expr);` | **Add** a Z3 expression/formula to the solver |
| `void resetSolver();` | Clean all formulas in the the solver |
| `solver.check();` | **Check satisfiability** of an z3 formula |
| `z3::expr getEvalExpr(z3::expr);` | **Evaluate an expression** to a value based on a model. |
| `void printExprValues();` | **Print** the values of **all expressions** in the solver. |

More details, refer to

`https://github.com/SVF-tools/Teaching-Software-Verification/wiki/SVF-APIs`

# Z3Mgr::getEvalExpr

```
z3::expr Z3Mgr::getEvalExpr(z3::expr e) {
    z3::check_result res = solver.check();
    assert(res != z3::unsat && "unsatisfied constraints!");
    z3::model m = solver.get_model();
    return m.eval(e);
}
```

The `Z3Mgr::getEvalExpr` method checks if the constraints added to the Z3 solver are satisfiable. If they are, it retrieves the model that satisfies these constraints and evaluates the given complex expression `e` within this model, returning the evaluated result as one of the following:

- Boolean Expression: `is_true()` or `is_false()`
- Integer Expression: `is_numeral()`, `get_numeral_int64()`
- Real Expression: `get_numeral_double()`
- String Expression: `get_numeral_string()`

# APIs for Lab-Exercise-2 vs APIs for Assignment-2

| **Lab-Exercise-2** (`Z3Examples` & `Z3Mgr`) | **Assignment-2** (`Z3SSEMgr` & `Z3Mgr`) |
|---|---|
| `Z3Examples::getZ3Expr(u32_t val)`<br>Get the z3 expression from a constant integer | `Z3Mgr::getZ3Expr(u32_t id)`<br>Get the z3 expression from an SVFVar ID |
| `Z3Examples::getMemObjAddress(string name)`<br>Get the memory object address from a string name | `Z3SSEMgr::getMemObjAddress(u32_t id)`<br>Get the memory object address from SVFVar ID |
| `Z3Examples::getGepObjAddress`<br>Get object address from a pointer and an offset | `Z3SSEMgr::getGepObjAddress`<br>Get object address from a pointer and an offset |
| `Z3Examples::addToSolver(z3::expr e)`<br>Add expr `e` to solver | `Z3SSEMgr::addToSolver(z3::expr e)`<br>Add expr `e` to solver |

| **Shared APIs** |
|---|
| `Z3Mgr::printZ3Exprs()`: Print all z3 expressions |
| `Z3Mgr::printExprValues()`: Print all expressions' values after evaluation |
| `Z3Mgr::getVirtualMemAddress(u32_t id)` and `Z3Mgr::isVirtualMemAddress(u32_t id)`<br>The `id` of an object (`ObjVar`) in SVFIR will be marked using an `AddressMask` (0x7f000000)<br>to mimic the virtual memory address (note that this is not a physical runtime address but an abstract address) |
| `getInternalID(u32_t)` will unmarsk a virtual address to get its original `ObjVar`'s `id`. |
| `Z3Mgr::storeValue(expr loc, expr value)`: stores a `value` to address `loc`. |
| `Z3Mgr::loadValue(expr loc)`: loads a value from address `loc`. |

# Translation Rules

`expr p = getZ3Expr("p")` `expr q = getZ3Expr("q")` `expr r = getZ3Expr("r")` `expr x = getZ3Expr("x")`

| SVFStmt | C-Like form | Operations |
|---|---|---|
| `AddrStmt` (constant)<br>`AddrStmt` (mem allocation) | `p = c`<br>`p = alloc` | `addToSolver(p == c);`<br>`addToSolver(p == getMemObjAddress("alloc");)` |
| `CopyStmt` | `p = q` | `addToSolver(p == q);` |
| `LoadStmt` | $p = *q$ | `addToSolver(p == loadValue(q));` |
| `StoreStmt` | $*p = q$ | `storeValue(p, q);` |
| `GepStmt` | $p = \&(q \rightarrow i)$ or $p = \&q[i]$ | `addToSolver(p == getGepObjAddress(q,i));` |
| `PhiStmt` | $r = phi(\ell_1 : p,\ \ell_2 : q)$ | if(executed from $l_1$) `addToSolver(p==r);`<br>if(executed from $l_2$) `addToSolver(q==r);` |
| `BranchStmt` | `if(x) r = p else r = q` | `addToSolver(r == ite(x, p, q));` |
| `UnaryOPStmt` | $\neg p$ | `addToSolver(!p);` |
| `BinaryOPStmt` | $r = p \otimes q$ BinaryOPStmt::OpCode | `addToSolver(r ==` $p \otimes q$`);` |
| `CmpStmt` | $r = p \odot q$ CmpStmt::Predicate | `addToSolver(r == ite(`$p \odot q$`, true, false));` |
| `CallPE/RetPE` | $r = f(\ldots, q, \ldots)$ $f(\ldots, p, \ldots)\{\ldots$ return $z\}$ | |
| `CallPE` | `p = q` | `solver.push(); addToSolver(p == q);` |
| `RetPE` | `p = r` | `expr ret = getEvalExpr(r); solver.pop();`<br>`addToSolver(p == ret);` |

# Translating Code to Z3 Formulas (Scalar Example)

The target program code needs to be in **SSA form** (e.g., SVFIR).

- Top-level variables can only be defined once
  - a = 1; a = 2; $\Longrightarrow$ a1 = 1; a2 = 2;
- Memory objects can only be modified/read through top-level pointers at `StoreStmt` and `LoadStmt`.
  - `p = &a; *p = r;` The value of `a` can only be modified/read via dereferencing `p`.

# Translating Code to Z3 Formulas (Scalar Example)

The target program code needs to be in **SSA form** (e.g., SVFIR).

- Top-level variables can only be defined once
  - a = 1; a = 2; $\Longrightarrow$ a1 = 1; a2 = 2;
- Memory objects can only be modified/read through top-level pointers at `StoreStmt` and `LoadStmt`.
  - `p = &a; *p = r;` The value of `a` can only be modified/read via dereferencing `p`.

C code:

```c
int main() {

 int a;
 int b;
 a = 0;
 b = a + 1;
 assert(b>0);

}
```

$\rightarrow$

Translator:

```
// int a;
expr a = getZ3Expr("a");
// int b;
expr b = getZ3Expr("b");
// a = 0;
addToSolver(a==getZ3Expr(0));
// b = a+1;
addToSolver(b==(a+getZ3Expr(1)));
/// validation code for assert(b > 0);
addToSolver(b>getZ3Expr(0));
solver.check();
```

$\rightarrow$

Z3 Formulas:

```
(declare-fun a () Int)
(declare-fun b () Int)
(assert (= a 0))
(assert (= b (+ a 1)))
(assert (> b 0))
(check-sat)
```

$\rightarrow$ Z3 solver

C code Translator Z3 Formulas

# Translating Code to Z3 Formulas (Memory Operation Example)

- Each memory object has a unique ID and allocated with a **virtual memory address**
- In our modeling, the virtual address starts from **0x7f...... + ID** (i.e., 2130706432 + ID in decimal)
- Memory operations will be through store and load values from `loc2ValMap`, an Z3 array.

C code

```c
int main() {
 int* p;
 int x;

 p = malloc(..);
 *p = 5;
 x = *p;
 assert(x==5);
}
```

→

Translator

```
//  int** p;
expr p = getZ3Expr("p");
//  int x;
expr x = getZ3Expr("x");
// p = malloc(..);
expr m = getMemObjAddress("malloc1");
addToSolver(p == m);
// *p = 5;
storeValue(p, getZ3Expr(5));
// x = *p;
addToSolver(x == loadValue(p));
/// validation code for assert(x==5);
addToSolver(x == getZ3Expr(5));
solver.check();
```

→

Z3 Formulas

```
(declare-fun p () Int)
(declare-fun loc2ValMap ()
    (Array Int Int))
(declare-fun x () Int)
(assert (= p 2130706435))
(assert (= x (select
    (store loc2ValMap 2130706435 5)
    2130706435)))
(assert (= x 5))
(check-sat)
```

# What's next?

- (1) Understand Z3 formula format in the slides
- (2) Understand `Z3Mgr` class in the GitHub Repository of Software-Security-Analysis
- (3) Start working on the Quiz-2 on WebCMS
- (4) Start working on Lab-Exercise-2
  - Remember to **`git pull`** or **`docker pull`** to get the latest code template.
  - You will implement a manual translation from code to Z3 formulas using `Z3Mgr` and `Z3Examples` in for code assertion verification.