Abstract Interpretation for Code Analysis and Verification

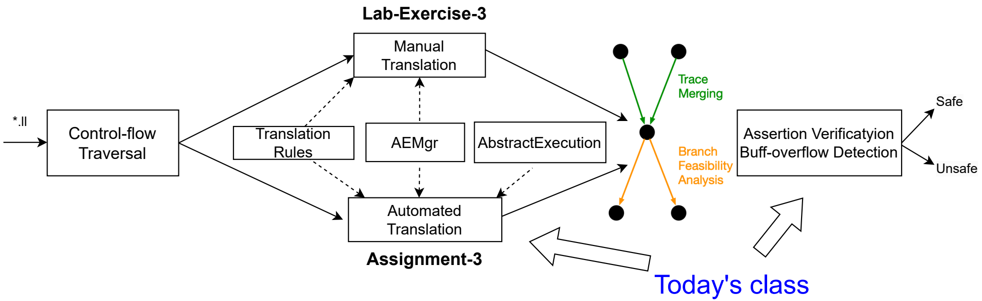
(Week 9)

Yulei Sui

School of Computer Science and Engineering

University of New South Wales, Australia

Today’s class



Topological Order

Analysis Order of Nodes on Control-Flow Graph

❓ How to analyze a program **free of loop**?

✔ Analyze each node **once** adhering to the **topological order** on the acyclic control-flow graph of the program.

Topological Order

Analysis Order of Nodes on Control-Flow Graph

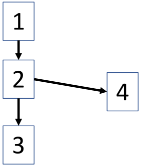
? How to analyze a program **free of loop**?

✔ Analyze each node **once** adhering to the **topological order** on the acyclic control-flow graph of the program.

A **topological order** of a graph $G(V, E)$ is a linear ordering of its nodes such that for every directed edge $a \rightarrow b$, node a always precedes node b in the ordering.

- Must be a **direct acyclic graph** (DAG) and has at least one topo ordering.
- The ordering respects the **direction of edges.**

Example of topological order:



acyclic graph G

1 2 3 4 ✔

1 2 4 3 ✔

1 3 2 4 ✘

Valid/invalid topological order

How About Analyzing Loops?

- **Topological Order** can only be used for directed acyclic graphs (DAGs).
- **Weak Topological Order (WTO)** is a relaxation of the more stringent topological order for graphs with loops.
  - **Cycles Permitted**: allows for cycles within the graph.
  - **Hierarchical Decomposition**: A graph is decomposed into a hierarchical structure where each node or a strongly connected component (SCC) can contain subnodes.
  - **Weak Topological Order or Partial Order**: In a WTO, nodes and SCCs are arranged in a partial order (not enumerating possible infinite loop paths). This order respects the dependencies in a way that allows for iterative analysis.
  - We will practice loop handling using WTO in `Assignment-3`. Function recursions will not be handled in this Assignment.

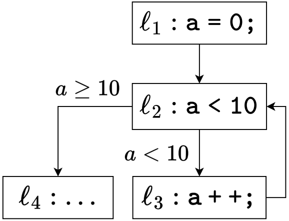
Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

❓ How to analyze a program **containing loops**?

✅ We can analyze a program containing loops adhering to the **weak topological order** (WTO) on its control flow graph.

What is the weak topological order?



Control Flow Graph

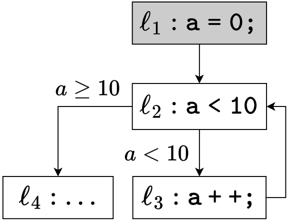
Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

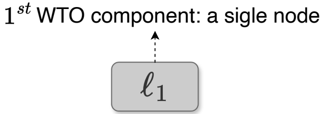
❓ How to analyze a program **containing loops**?

✔ We can analyze a program containing loops adhering to the **weak topological order** (WTO) on its control flow graph.

What is the weak topological order?



Control Flow Graph



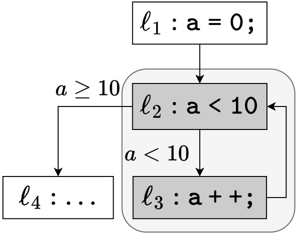
Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

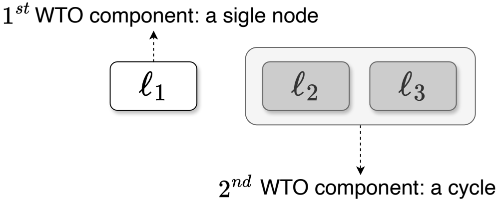
❓ How to analyze a program **containing loops**?

✔ We can analyze a program containing loops adhering to the **weak topological order** (WTO) on its control flow graph.

What is the weak topological order?



Control Flow Graph



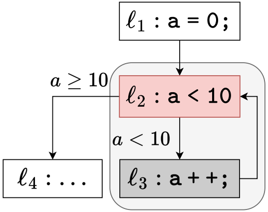
Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

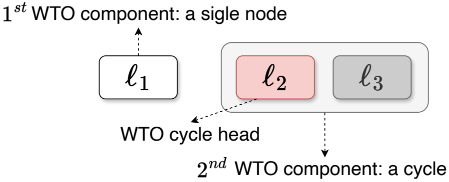
❓ How to analyze a program **containing loops**?

✅ We can analyze a program containing loops adhering to the **weak topological order** (WTO) on its control flow graph.

What is the weak topological order?



Control Flow Graph



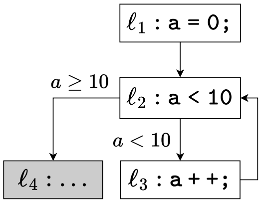
Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

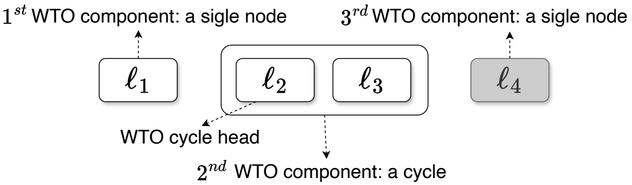
❓ How to analyze a program **containing loops**?

✔ We can analyze a program containing loops adhering to the **weak topological order** (WTO) on its control flow graph.

What is the weak topological order?



Control Flow Graph



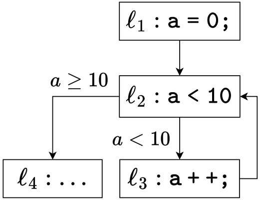
Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

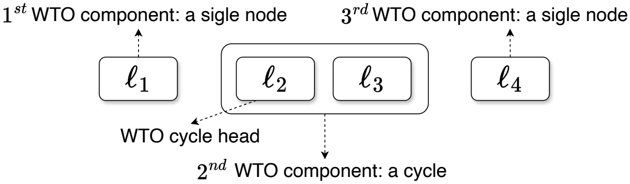
❓ How to analyze a program **containing loops**?

✔ We can analyze a program containing loops adhering to the **weak topological order** (WTO) on its control flow graph.

What is the weak topological order?



Control Flow Graph



Analyze each node following the WTO

Analyze ℓ_1 ⇨ Repeat: analyze ℓ_2 and ℓ_3 ⇨ Analyze ℓ_4

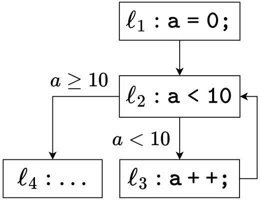
Weak Topological Order

Analysis Order of Nodes on Control-Flow Graph

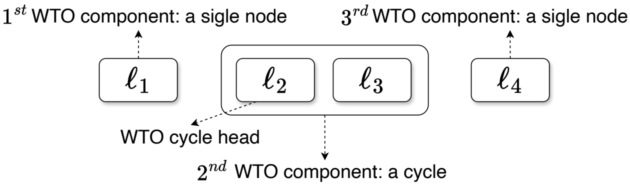
? How to analyze a program **containing loops**?

✔ We can analyze a program containing loops adhering to the **weak topological order** (WTO) on its control flow graph.

What is the weak topological order?



Control Flow Graph



Analyze each node following the WTO

Analyze ℓ_1 ⇨ Repeat: analyze ℓ_2 and ℓ_3 ⇨ Analyze ℓ_4

Perform widening when analyzing the cycle heads (i.e., ℓ_2)

WTO, Widening and Narrowing

Why Weak Topological Order (WTO)?

- Handling cyclic dependencies
- Efficient fixed-point computation

Why Widening?

- Over-approximation
- Prevent non-termination

Why Narrowing?

- Refine precision after widening converges
- The specific conditions or constraints used for narrowing:
  - Loop exit conditions (this course)
  - Type constraints (8-bit integer ranging from [-128, 127])
  - Bounds from arithmetic operations If `x = y + z`, and $y \in [1, 5]$ and $z \in [2, 3]$, then $x \in [3, 8]$. If widening gives [1, 10], narrowing can refine this to [3, 8].
  - User-specification (assertions and guard conditions)

Revisit the Notations and Data Structure

- An **abstract trace** $\sigma \in \mathbb{L} \times \mathcal{V} \rightarrow \mathbb{A}$ represents a list of abstract states before ($\overline{\ell}$) and after ($\underline{\ell}$) each program statement ℓ (`preAbsTrace` and `postAbsTrace`).
- An **abstract state** (`AbstractState` in Lab-3 and Assignment-3) is defined as a map $AS : \mathcal{V} \rightarrow \mathbb{A}$ associating program variables $\mathcal{V}$ with an abstract value in $\mathbb{A}$, approximating the runtime states of program variables.

Revisit the Notations and Data Structure

- An **abstract trace** $\sigma \in \mathbb{L} \times \mathcal{V} \to \mathbb{A}$ represents a list of abstract states before ($\overline{\ell}$) and after ($\underline{\ell}$) each program statement ℓ (`preAbsTrace` and `postAbsTrace`).
- An **abstract state** (`AbstractState` in Lab-3 and Assignment-3) is defined as a map $AS : \mathcal{V} \to \mathbb{A}$ associating program variables $\mathcal{V}$ with an abstract value in $\mathbb{A}$, approximating the runtime states of program variables.
- An **abstract value** can be either an interval or a memory address.

	Notation	Domain	SSE Data Structure
Abstract **trace**	$\mathbb{L} \times \mathcal{V} \to \mathbb{A}$	σ	`preAbsTrace`: **trace** before `ICFGNodes` `postAbsTrace`: **trace** after `ICFGNodes`
Abstract **state** at $L \in \mathbb{L}$	$\mathcal{V} \to \mathbb{A}$	$\sigma_{\overline{\ell}}$ $\sigma_{\underline{\ell}}$	`preAbsTrace[node]`: **state** before `node` ℓ `postAbsTrace[node]`: **state** after `node` ℓ
Abstract **value** of varId at $L \in \mathbb{L}$	$\mathbb{A}$	$\sigma_{\underline{\ell}}$(varId)	`as` $=$ `postAbsTrace[node]` `as[VarID]`: **value** of varId after `node` ℓ

Overall Algorithm of Abstract Interpretation in Assignment-3

Algorithm 1: Analyse from main function

```
Function analyse() // driver function to start the analysis:
    initWTO();
    handleGlobalNode();
    handleFunction(mainFun);
```

Algorithm 2: Handle Function

```
Function handleFunction(fun):
    worklist := [funEntryICFGNode] while worklist ≠ ∅ do
        n := worklist.pop_front() ;
        if n is a cycle head then
            cycle := cycle_head_to_cycle[n] ;
            handleICFGCycle(cycle); ;                    // Assignment-3
            foreach n′ ∈ getNextNodesOfCycle(cycle) do
                worklist.push_back(n′) ;
        else
            if handleICFGNode(n) == false then
                foreach n′ ∈ getNextNodes(n) do
                    worklist.push_back(n′) ;
```

Algorithm 3: Handle ICFG Node

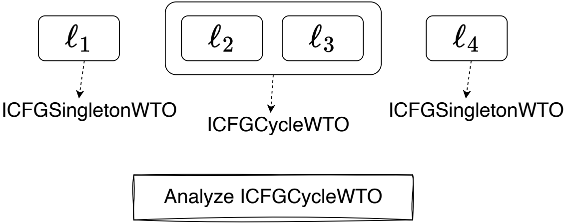
```
Function handleICFGNode(n):
    feasible, as_pre := mergeStatesFromPredecessors(node);
    if !feasible then
        return false;
    as_last := σ_n;
    σ_n := as_pre;
    foreach stmt ∈ n→getSVFStmts() do
        updateAbsState(stmt),; ;                         // Assignment-3
        bufOverflowDetection(stmt); ;                    // Assignment-3

    if n is CallICFGNode then
        // Handle stub function and external call;
        // Skip recursive call;
        // Handle normal call;
    if σ_n ≡ as_last then
        return false; // state not changed
    return true; // state changed
```

Overall Algorithm of Abstract Interpretation in Assignment-3

Algorithm 4: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

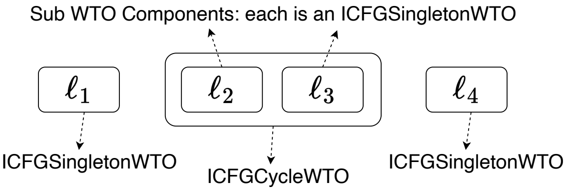
Widening and Narrowing



Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

Widening and Narrowing

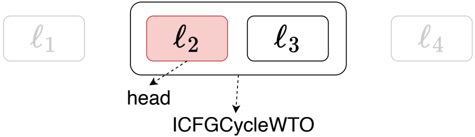


Analyze ICFGCycleWTO

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

Weak Topological Order

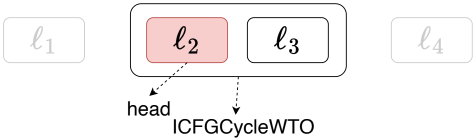


Get ICFGCycleWTO head ℓ_2

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();    // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;    // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;    // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;    // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;    // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;    // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

Weak Topological Order

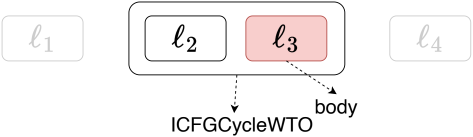


Handle ICFGCycleWTO head ℓ_2

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();    // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;    // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;    // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;    // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;    // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;    // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

Widening and Narrowing



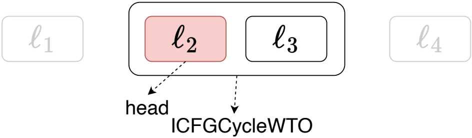
Handle ICFGCycleWTO body ℓ_3

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(h);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

Note: `getIWTOcomponents` returns Cycle WTO body, i.e., ℓ_3

Widening and Narrowing



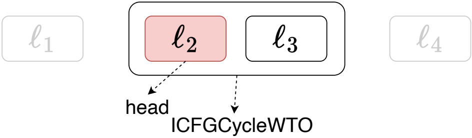
When $cur\_iter \geq Options :: WidenDelay()$

perform widening on ℓ_2

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

Widening and Narrowing

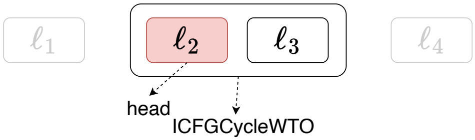


Widening reaches a fixpoint

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

Widening and Narrowing

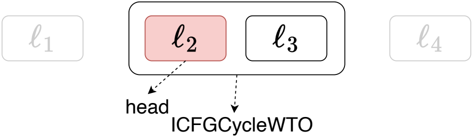


Perform narrowing on ℓ_2

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();    // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;    // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;    // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;    // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;    // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;    // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

Widening and Narrowing



Narrowing reaches a fixpoint, Finish!

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();    // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;    // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;    // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;    // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;    // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;    // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

Abstract Interpretation on SVFIR

Week 9

Yulei Sui

School of Computer Science and Engineering

University of New South Wales, Australia

Abstract Interpretation on Pointer-Free SVFIR

Interval Domain

- For simplicity, let's first consider abstract execution on a pointer-free language.
- This means there are no operations for memory allocation (like $\texttt{p} = \texttt{alloc}_{\texttt{o}}$) or for indirect memory accesses (such as `p = *q` or `*p = q`).
- Here are the pointer-free SVFSTMTs and their C-like forms:

SVFSTMT	**C-Like form**
CONSSTMT	$\ell : \texttt{p} = \texttt{c}$
COPYSTMT	$\ell : \texttt{p} = \texttt{q}$
BINARYSTMT	$\ell : \texttt{r} = \texttt{p} \otimes \texttt{q}$
PHISTMT	$\ell : \texttt{r} = \texttt{phi}(\texttt{p}_1, \texttt{p}_2, \ldots, \texttt{p}_\texttt{n})$
SEQUENCE	$\ell_1 ; \ell_2$
BRANCHSTMT	ℓ_1 : if$(x < c)$ then ℓ_2 else ℓ_3

Abstract Interpretation on Pointer-Free SVFIR

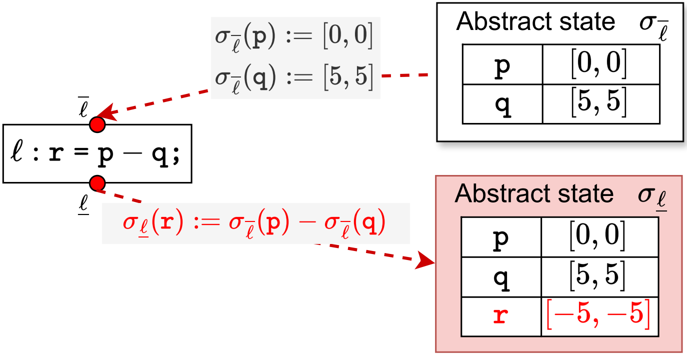
Interval Domain

Let's use the *Interval* abstract domain to update σ based on the following rules for different SVFStmt:

SVFStmt	**C-Like form**	**Abstract Execution Rule**
ConsStmt	$\ell : \mathtt{p} = \mathtt{c}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := [\mathtt{c}, \mathtt{c}]$
CopyStmt	$\ell : \mathtt{p} = \mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$
BinaryStmt	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$
PhiStmt	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p}_1, \mathtt{p}_2, \ldots, \mathtt{p}_n)$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$
Sequence	$\ell_1; \ell_2$	$\forall v \in \mathbb{V}, \sigma_{\overline{\ell_2}}(v) \sqsupseteq \sigma_{\underline{\ell_1}}(v)$
BranchStmt	ℓ_1 : if$(x < c)$ then ℓ_2 else ℓ_3	$\sigma_{\overline{\ell_2}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1] \neq \perp$ $\sigma_{\overline{\ell_3}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty]$, if $\sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty] \neq \perp$

Abstract Interpretation on BINARYSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$

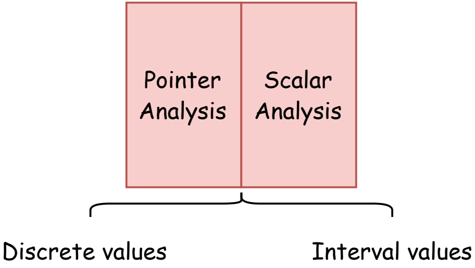


Abstract Interpretation in the Presence of Pointers

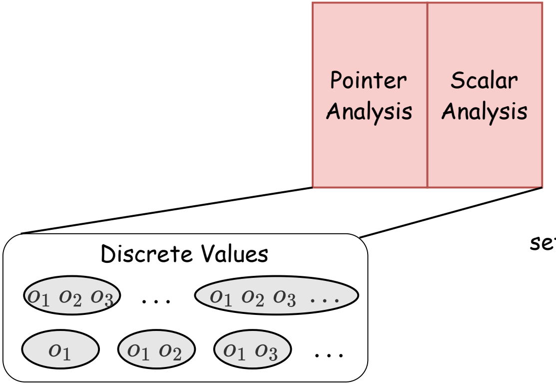
- SVFIR in the presence of pointers contain pointer-related statements including AddrStmt, GepStmt, LoadStmt and StoreStmt.
- Abstract interpretation needs to be performed on **a combined domain of intervals and addresses.**

SVFStmt	**C-Like form**
ConsStmt	$\ell : \mathtt{p} = \mathtt{c}$
CopyStmt	$\ell : \mathtt{p} = \mathtt{q}$
BinaryStmt	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$
PhiStmt	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$
Sequence	$\ell_1 ; \ell_2$
BranchStmt	ℓ_1 : if($x < c$) then ℓ_2 else ℓ_3
AddrStmt	$\ell : \mathtt{p} = \mathtt{alloc}$
GepStmt	$\ell : \mathtt{p} = \&(\mathtt{q} \rightarrow \mathtt{i})$ or $\mathtt{p} = \&\mathtt{q}[\mathtt{i}]$
LoadStmt	$\ell : \mathtt{p} = *\mathtt{q}$
StoreStmt	$\ell : *\mathtt{p} = \mathtt{q}$

Combined Analysis



Combined Analysis Using Discrete Values



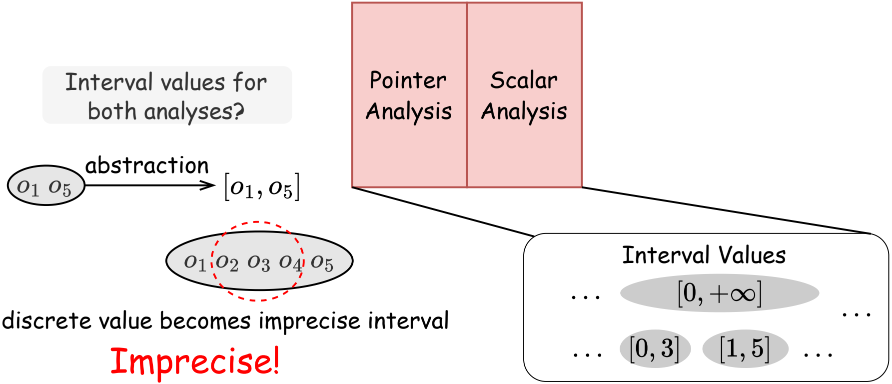
Discrete values for both analyses?

{1, 2, 3, ..., infinite numbers}

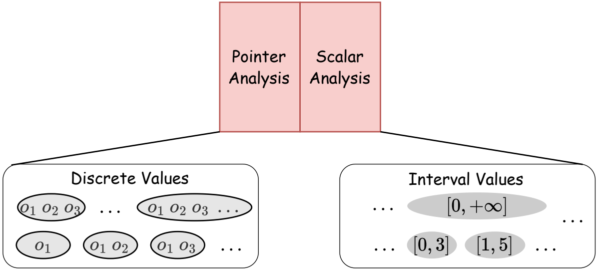
set of all possible values with inputs

Unscalable!

Combined Analysis Using Interval Values



Abstract Interpretation Over a Combined Domain

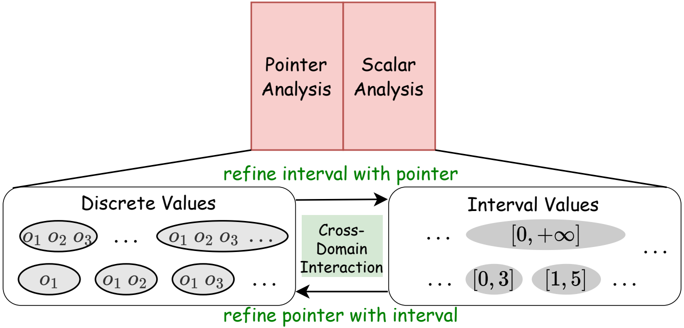


p = malloc(m*sizeof(int)); // p points to an array of size m
q = malloc(n*sizeof(int)); // q points to an array of size n

m = r[i];

- The discrete values for points-to set of `p`, `q` depend on interval values of `m` and `n`.
- The interval value of `m` depends on the pointer aliasing between `p`, `q` and `&r[i]`.
- Cyclic dependency between two domains requiring a bi-directional refinement. (variables highlighted in blue and red denote the discrete values and interval values dependent),

Abstract Interpretation Over a Combined Domain

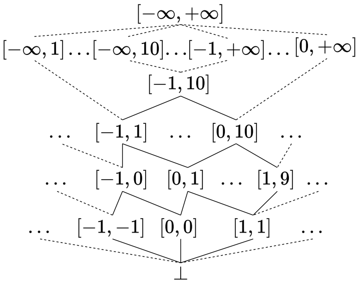


We require **a combination of interval and memory address domains** to precisely and efficiently perform abstract execution on SVFIR in the presence of pointers.

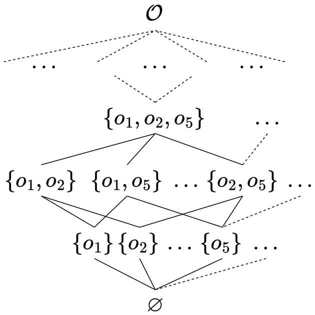
Precise Sparse Abstract Execution via Cross-Domain Interaction, ICSE 2024

Abstract Interpretation over Interval and MemAddress Domains

A Combined Domain of Intervals and Discrete Memory Addresses



Interval domain for scalar variables



MemAddress domain for discrete memory address values

SVF Program Variables (SVFVar)

Program Variables	Domain	Meanings
`SVFVar`	$\mathbb{V} = \mathbb{P} \cup \mathbb{O}$	Program Variables
`ValVar`	$\mathbb{P}$	Top-level variables (scalars and pointers)
`ObjVar`	$\mathbb{O} = \mathbb{S} \cup \mathbb{G} \cup \mathbb{H} \cup \mathbb{C}$	Memory Objects (constant data, stack, heap, global) (function objects are considered as global objects)
`FIObjVar`	$o \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H})$	A single (base) memory object
`GepObjVar`	$o_i \in (\mathbb{S} \cup \mathbb{G} \cup \mathbb{H}) \times \mathbb{P}$	i-th subfield/element of an (aggregate) object
`ConstantData`	$\mathbb{C}$	Constant data (e.g., numbers and strings)
Program Statement	$\ell \in \mathbb{L}$	Statements labels

Abstract Trace for The Combined Domain

- For top-level variables $\mathbb{P}$, we use $\sigma \in \mathbb{L} \times \mathbb{P} \to \mathit{Interval} \times \mathit{MemAddress}$ to track the memory addresses or interval values of these variables.
- For memory objects $\mathbb{O}$, we use $\delta \in \mathbb{L} \times \mathbb{O} \to \mathit{Interval} \times \mathit{MemAddress}$ to track their abstract values

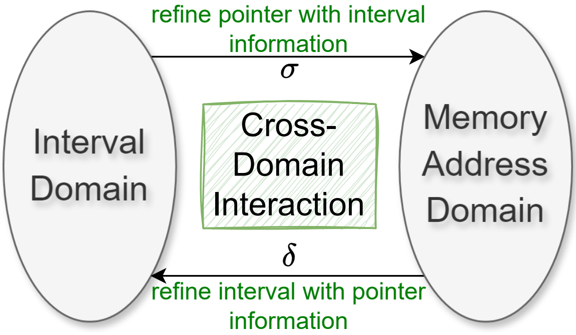
	Notation	Domain	Data Structure Implementation
Abstract trace	σ	$\mathbb{L} \times \mathbb{P} \to \mathit{Interval} \times \mathit{MemAddress}$	*preAbsTrace*, *postAbsTrace*
	δ	$\mathbb{L} \times \mathbb{O} \to \mathit{Interval} \times \mathit{MemAddress}$	
Abstract state	σ_L	$\mathbb{P} \to \mathit{Interval} \times \mathit{MemAddress}$	*AbstractState.varToAbsVal*
	δ_L	$\mathbb{O} \to \mathit{Interval} \times \mathit{MemAddress}$	*AbstractState.addrToAbsVal*
Abstract value	$\sigma_L(p)$	$\mathit{Interval} \times \mathit{MemAddress}$	*AbstractValue*
	$\delta_L(o)$		

- *Interval* is used for tracking the interval value of **scalar variables** $\mathbb{P}$.
- *MemAddress* is used for tracking the memory addresses of **memory address variables** $\mathbb{O}$.

Implementation of Abstract Trace and State in Assignment-3

- For a program point L, *AEState* consists of:
  - Top-level variable, *varToAbsVal* : $\sigma_L \in \mathbb{P} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$
  - Memory object, *addrToAbsVal* : $\delta_L \in \mathbb{O} \rightarrow \mathit{Interval} \times \mathit{MemAddress}$
- The abstract trace has two maps, *preAbsTrace* and *postAbsTrace*, which maintains abstract states before and after each `ICFGNode` respectively.
  - For an `ICFGNode` ℓ, *preAbsTrace*(ℓ) retrieves the abstract state $\langle \sigma_{\overline{\ell}}, \delta_{\overline{\ell}} \rangle$, and *postAbsTrace*($\ell$) represents $\langle \sigma_{\underline{\ell}}, \delta_{\underline{\ell}} \rangle$.
  - For each abstract state $\langle \sigma_{\overline{\ell}}, \delta_{\overline{\ell}} \rangle$ we use `as[varId]` to operate $\sigma_{\underline{\ell}}$ and use `storeValue` and `loadValue` to operate $\delta_{\underline{\ell}}$.
  - Each variable's `AbstractValue` (e.g., `as[VarId]`) is initialized as $\perp$ in an `AbstractState` before assigned a new value. An uninitialized variable is assigned with $\top$ for over-approximation.
  - Each `AbstractValue` (e.g., `as[VarId]`) is a 2-element tuple consisting of an interval `as[VarId].getInterval()` and an address set `as[Varid].getAddrs()`.
  - Print out `SVFVars` and their `AbstractValues` in an `AbstractState` by invoking `as.printAbstractState()`

Abstract Trace for The Combined Domain



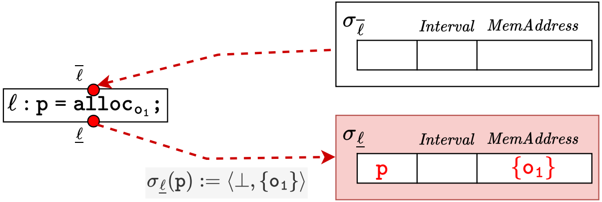
Abstract Execution Rules on SVFIR in the Presence of Pointers

Now let's use the *Interval* $\times$ *MemAddress* abstract domain to update σ and δ based on the following rules for different SVFSTMT:

SVFSTMT	C-Like form	Abstract Execution Rule
CONSSTMT	$\ell : \mathtt{p} = \mathtt{c}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle [\mathtt{c}, \mathtt{c}], \bot \rangle$
COPYSTMT	$\ell : \mathtt{p} = \mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$
BINARYSTMT	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$
CMPSTMT	$\ell : \mathtt{r} = \mathtt{p} \odot \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\odot} \sigma_{\overline{\ell}}(q)$
PHISTMT	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$
BRANCHSTMT	$\ell_1 : \text{if}(x < c) \text{ then } \ell_2 \text{ else } \ell_3$	$\sigma_{\overline{\ell_2}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1], \text{ if } \sigma_{\underline{\ell_1}}(x) \sqcap [-\infty, c-1] \neq \bot$ $\sigma_{\overline{\ell_3}}(x) := \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty], \text{ if } \sigma_{\underline{\ell_1}}(x) \sqcap [c, +\infty] \neq \bot$
SEQUENCE	$\ell_1; \ell_2$	$\delta_{\overline{\ell_2}} \sqsupseteq \delta_{\underline{\ell_1}}, \sigma_{\overline{\ell_2}} \sqsupseteq \sigma_{\underline{\ell_1}}$
ADDRSTMT	$\ell : \mathtt{p} = \mathtt{alloc}_{\mathtt{o_i}}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle \bot, \{o_i\} \rangle$
GEPSTMT	$\ell : \mathtt{p} = \&(\mathtt{q} \rightarrow \mathtt{i})$ or $\mathtt{p} = \&\mathtt{q}[\mathtt{i}]$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{\mathtt{o} \in \gamma(\sigma_{\overline{\ell}}(\mathtt{q}))} \bigsqcup_{j \in \gamma(\sigma_{\overline{\ell}}(\mathtt{i}))} \langle \bot, \{\mathtt{o.fld}_j\} \rangle$
LOADSTMT	$\ell : \mathtt{p} = *\mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \bigsqcup_{o \in \{o \mid o \in \sigma_{\overline{\ell}}(q)\}} \delta_{\overline{\ell}}(o)$
STORESTMT	$\ell : *\mathtt{p} = \mathtt{q}$	$\delta_{\underline{\ell}} := (\{o \mapsto \sigma_{\overline{\ell}}(\mathtt{q}) \mid o \in \gamma(\sigma_{\overline{\ell}}(\mathtt{p}))\} \sqcup \delta_{\underline{\ell}})$

Abstract Interpretation on ADDRSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
ADDRSTMT	$\ell : \texttt{p} = \texttt{alloc}_{\texttt{o}_i}$	$\sigma_{\underline{\ell}}(\texttt{p}) := \langle \bot, \{o_i\} \rangle$



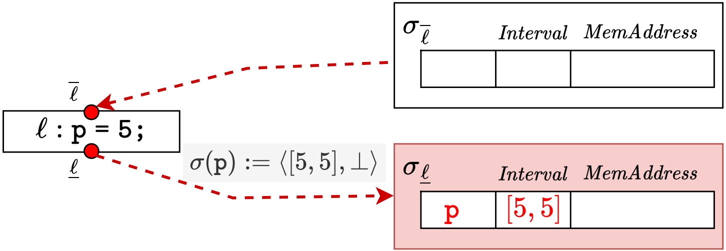
Algorithm 13: Abstract Execution Rule for ADDRSTMT

Function *updateStateOnAddr(addr)*:

```
node = addr→getICFGNode();
as = getAbsStateFromTrace(node);
initObjVar(as, SVFUtil :: cast⟨ObjVar⟩(addr→getRHSVar()));
as[addr→getLHSVarID()] = as[addr→getRHSVarID()];
```

Abstract Interpretation on CONSSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
CONSSTMT	$\ell : \mathtt{p} = \mathtt{c}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \langle[\mathtt{c}, \mathtt{c}], \perp\rangle$

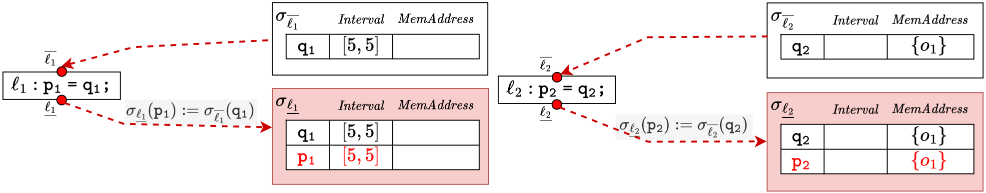


Algorithm 14: Abstract Execution Rule for CONSSTMT

Function *updateStateOnAddr(addr)*:
```
    node = addr → getICFGNode();
    as = getAbsStateFromTrace(node);
    initObjVar(as, SVFUtil :: cast⟨ObjVar⟩(addr → getRHSVar()));
    as[addr → getLHSVarID()] = as[addr → getRHSVarID()];
```

Abstract Interpretation on CopyStmt

SVFStmt	C-Like form	Abstract Execution Rule
CopyStmt	$\ell : \mathtt{p} = \mathtt{q}$	$\sigma_{\underline{\ell}}(\mathtt{p}) := \sigma_{\overline{\ell}}(\mathtt{q})$



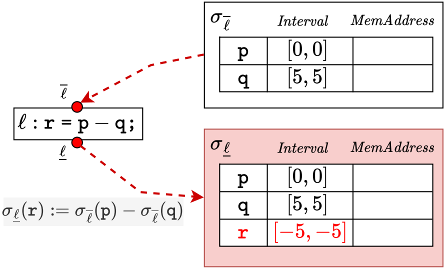
Algorithm 15: Abstract Execution Rule for CopyStmt

Function *updateStateOnCopy(copy)*:

    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state at $\underline{\ell}$;
    // Assign RHS's abstract value to LHS;

Abstract Interpretation on BinaryStmt

SVFStmt	C-Like form	Abstract Execution Rule
BinaryStmt	$\ell : \mathtt{r} = \mathtt{p} \otimes \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$



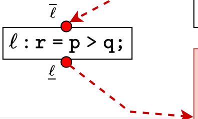
Algorithm 16: Abstract Execution Rule for BinaryStmt

```
Function updateStateOnBinary(binary):
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state at ℓ;
    // Assign the results after the binary
    operation of the two operands op0 and op1;
```

Operands `op0` and `op1` are assumed to be properly initialized (no uninitialized variables or randomization).

Abstract Interpretation on CmpStmt

SVFStmt	C-Like form	Abstract Execution Rule
CmpStmt	$\ell : \mathtt{r} = \mathtt{p} \odot \mathtt{q}$	$\sigma_{\underline{\ell}}(r) := \sigma_{\overline{\ell}}(p) \hat{\otimes} \sigma_{\overline{\ell}}(q)$



$\sigma_{\underline{\ell}}(\mathtt{r}) := \sigma_{\overline{\ell}}(\mathtt{p}) > \sigma_{\overline{\ell}}(\mathtt{q})$

$\sigma_{\overline{\ell}}$	*Interval*	*MemAddress*
p	$[10, 10]$	
q	$[5, 5]$	

$\sigma_{\underline{\ell}}$	*Interval*	*MemAddress*
p	$[10, 10]$	
q	$[5, 5]$	
r	$[1, 1]$	

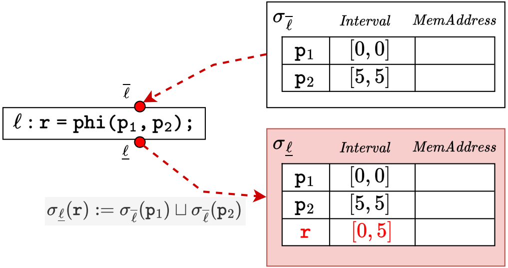
Algorithm 17: Abstract Execution Rule for CmpStmt

```
Function updateStateOnCmp(cmp):
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state at ℓ̲;
    // Assign the results after the
    comparison operation of the two operands;
```

Operands `op0` and `op1` are assumed to be properly initialized (no uninitialized variables or randomization).

Abstract Interpretation on PhiStmt

SVFStmt	C-Like form	Abstract Execution Rule
PhiStmt	$\ell : \mathtt{r} = \mathtt{phi}(\mathtt{p_1}, \mathtt{p_2}, \ldots, \mathtt{p_n})$	$\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$



Algorithm 18: Abstract Execution Rule for PhiStmt

Function *updateStateOnPhi(phi)*:
  // Retrieve ICFGNode ℓ;
  // Retrieve the abstract state at $\underline{\ell}$;
  // Join the abstract values of all n operands retrieved from $\overline{\ell}$ or from the ICFGNode where each operand is defined.;
  // Assign the joined values to the result operand.;
  // $\sigma_{\underline{\ell}}(r) := \bigsqcup_{i=1}^{n} \sigma_{\overline{\ell}}(p_i)$

Abstract Interpretation on GEPSTMT

SVFSTMT	C-Like form	Abstract Execution Rule
GEPSTMT	$\ell : \texttt{p} = \&(\texttt{q} \rightarrow \texttt{i})$ or $\texttt{p} = \&\texttt{q}[\texttt{i}]$	$\sigma_{\underline{\ell}}(\texttt{p}) := \bigsqcup_{\texttt{o} \in \gamma(\sigma_{\overline{\ell}}(\texttt{q}))} \bigsqcup_{j \in \gamma(\sigma_{\overline{\ell}}(\texttt{i}))} \langle \bot, \{\texttt{o.fld}_j\} \rangle$

$\overline{\ell}$

ℓ : p = &(q[i]);

$\underline{\ell}$

$\sigma_{\underline{\ell}}(\texttt{p}) := \langle \bot, \{\texttt{o}_1.\texttt{fld}_1\} \rangle \sqcup \langle \bot, \{\texttt{o}_1.\texttt{fld}_2\} \rangle$

$\sigma_{\overline{\ell}}$

	Interval	*MemAddress*
q		$\{\texttt{o}_1\}$
i	$[1, 2]$	

$\sigma_{\underline{\ell}}$

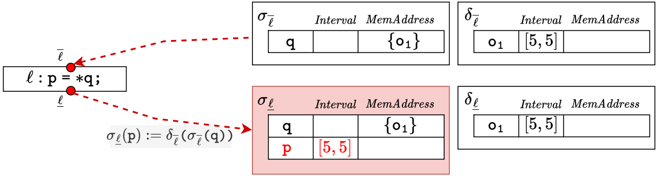
	Interval	*MemAddress*
q		$\{\texttt{o}_1\}$
i	$[1, 2]$	
p		$\{\texttt{o}_1.\texttt{fld}_1, \texttt{o}_1.\texttt{fld}_2\}$

Algorithm 19: Abstract Execution Rule for GEPSTMT

```
Function updateStateOnGep(gep):
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state as at ℓ;
    // Retrieve the field index or array index
    i given as.getElementIndex(gep);
    // Retrieve the memory address value via
    as.getGepObjAddrs(rhs, i) and assign it to
    LHS
```

Abstract Interpretation on LoadStmt

SVFStmt	C-Like form	Abstract Execution Rule
LoadStmt	$\ell : \texttt{p} = *\texttt{q}$	$\sigma_{\underline{\ell}}(\texttt{p}) := \bigsqcup_{o \in \{o \mid o \in \sigma_{\overline{\ell}}(q)\}} \delta_{\overline{\ell}}(o)$



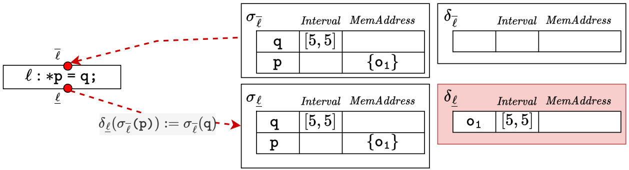
Algorithm 20: Abstract Execution Rule for LoadStmt

Function *updateStateOnLoad(load)*:

```
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state as at ℓ;
    // Load the value from RHS via
    as.loadValue(rhs) and assign it to LHS;
```

Abstract Interpretation on STORESTMT

SVFSTMT	C-Like form	Abstract Execution Rule
STORESTMT	$\ell : *\mathrm{p} = \mathrm{q}$	$\delta_{\underline{\ell}} := (\{o \mapsto \sigma_{\overline{\ell}}(\mathrm{q}) \mid o \in \gamma(\sigma_{\overline{\ell}}(\mathrm{p}))\} \sqcup \delta_{\underline{\ell}})$



Algorithm 21: Abstract Execution Rule for STORESTMT

```
Function updateStateOnStore(store):
    // Retrieve ICFGNode ℓ;
    // Retrieve the abstract state as at ℓ;
    // Store RHS value to LHS via as.storeValue;
```

An Example: Abstract Trace σ for Top-level Variables

```c
extern void assert(int);

int main(){
    int a = 0;
    while(a < 10) {
        a++;
    }
    assert(a == 10);
    return 0;
}
```

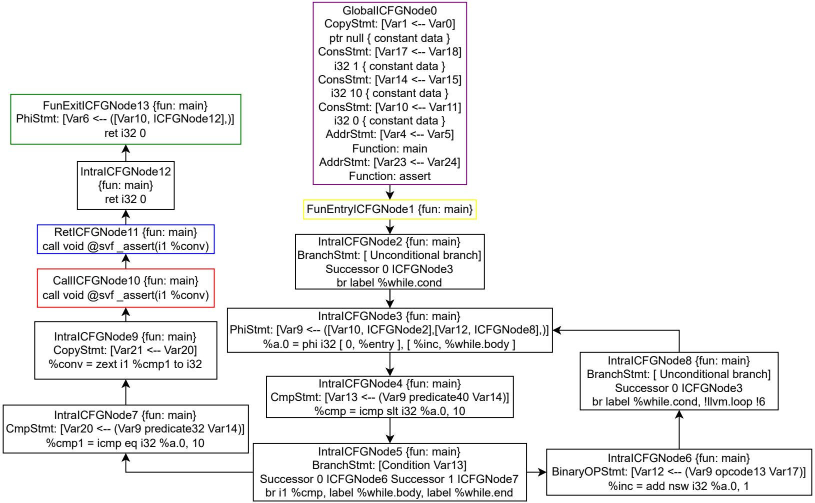
Source Code

Compile to LLVM IR ⇨

```llvm
define dso_local i32 @main() {
entry:
  br label %while.cond
while.cond:
  %a.0 = phi i32 [ 0, %entry ], [ %inc, %while.body ]
  %cmp = icmp slt i32 %a.0, 10
  br i1 %cmp, label %while.body, label %while.end
while.body:
  %inc = add nsw i32 %a.0, 1
  br label %while.cond,
while.end:
  %cmp1 = icmp eq i32 %a.0, 10
  %conv = zext i1 %cmp1 to i32
  call void @assert(i32 noundef %conv)
  ret i32 0
}
```

LLVM IR

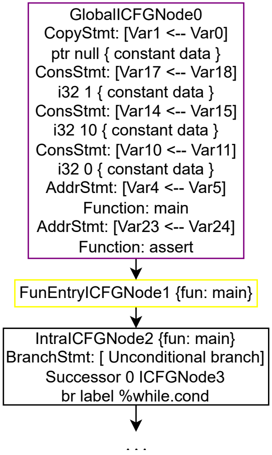
An Example: Abstract Trace σ for Top-level Variables



ICFG

An Example: Abstract Trace σ for Top-level Variables

Before Entering Loop



ICFG

Algorithm 22: Abstract execution guided by WTO

Function `handleStatement`(ℓ)**:**
  $tmpAS := preAbsTrace[\ell]$;
  if ℓ *is* ConsStmt *or* AddrStmt **then**
    `updateStateOnAddr`(ℓ);
  else if ℓ *is* CopyStmt **then**
    `updateStateOnCopy`(ℓ);
  $\ldots$;

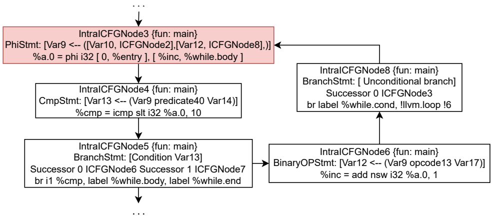
$postAbsTrace[ICFGNode0].varToAbsVal$:

`SVFVar`	`AbstractValue`$\langle Interval, MemAddress\rangle$
$Var0$	$\langle\bot, \{0x7f00\}\rangle$
$Var1$	$\langle\bot, \{0x7f00\}\rangle$
$Var18$	$\langle[1, 1], \bot\rangle$
$Var17$	$\langle[1, 1], \bot\rangle$
$Var14$	$\langle[10, 10], \bot\rangle$
$Var15$	$\langle[10, 10], \bot\rangle$
$Var10$	$\langle[0, 0], \bot\rangle$
$Var11$	$\langle[0, 0], \bot\rangle$
	...

Print out the table via `as.printAbstractState()`. The `AbstractValue` can **either be an interval or addresses**, but not both!

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 0)



ICFG

postAbsTrace[*ICFGNode3*].*varToAbsVal* :

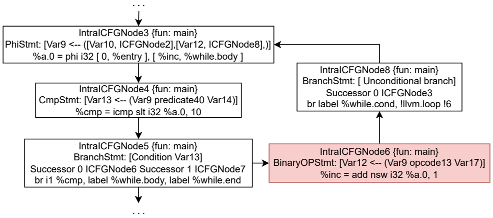
SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
	...
Var10	$\langle [0,0], \bot \rangle$
Var9	$\langle [0,0], \bot \rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 0)



ICFG

$postAbsTrace[ICFGNode6].varToAbsVal$:

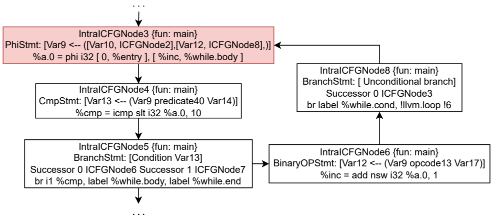
SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
	...
Var10	$\langle [0, 0], \perp \rangle$
Var9	$\langle [0, 0], \perp \rangle$
Var12	$\langle [1, 1], \perp \rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 1)



ICFG

postAbsTrace[*ICFGNode3*].*varToAbsVal* :

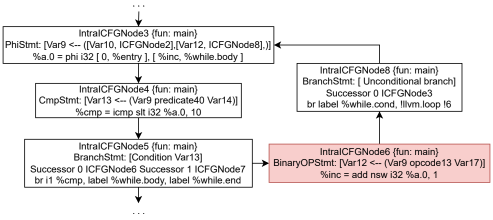
SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
...	
Var9	$\langle [0,1], \perp \rangle$
Var12	$\langle [1,1], \perp \rangle$
...	

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Widen Delay Phase (cur_iter is 1)



ICFG

postAbsTrace[*ICFGNode6*].*varToAbsVal* :

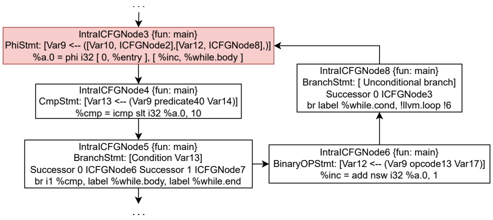
SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
	. . .
Var9	$\langle [0,1], \perp \rangle$
Var12	$\langle [1,2], \perp \rangle$
	. . .

Algorithm 12: Handle ICFG Cycle

Function `handleICFGCycle` *(cycle)*:
  ℓ := `cycle.getHead().getICFGNode()`; // cycle head ICFGNode ℓ
  `increasing` := `true`;
  `i` := 0; // analysis iteration for the loop
  while *true* **do**
    $\mathtt{as}_{\mathtt{pre}} := \sigma_\ell$; // abstract state in the last iteration
    `handleICFGNode`(ℓ);
    $\mathtt{as}_{\mathtt{cur}} := \sigma_\ell$; // abstract state in the current iteration
    if `i` $\geq$ `Options.WidenDelay()` **then**
      if `increasing` **then**
        $\sigma_\ell := \mathtt{as}_{\mathtt{pre}} \nabla \mathtt{as}_{\mathtt{cur}}$; // widening
        if $\sigma_\ell \equiv \mathtt{as}_{\mathtt{pre}}$ **then**
          `increasing` := `false`;
          continue;
      else
        $\sigma_\ell := \mathtt{as}_{\mathtt{pre}} \Delta \mathtt{as}_{\mathtt{cur}}$; // narrowing
        if $\sigma_\ell \equiv \mathtt{as}_{\mathtt{pre}}$ **then**
          break;
    // analyze remaining cycle components after two fixed-points
    foreach `comp` $\in$ `cycle.getWTOComponents()` **do**
      if `comp` *is Singleton* **then**
        `handleICFGNode(comp.getICFGNode())`
      else if `comp` *is Cycle* **then**
        `handleICFGCycle(comp)`;
    `i++`;
  return;

An Example: Abstract Trace σ for Top-level Variables

Widen Phase (cur_iter is 2)



ICFG

$postAbsTrace[ICFGNode3].varToAbsVal$:

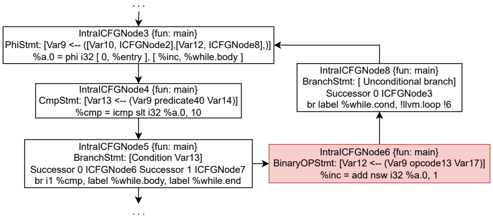
SVFVar	$\langle Interval, MemAddress\rangle$
	...
Var9	$\langle[0,+\infty],\perp\rangle$
Var12	$\langle[1,+\infty],\perp\rangle$
	...

Algorithm 12: Handle ICFG Cycle

Function `handleICFGCycle` *(cycle)*:
- ℓ := `cycle.getHead().getICFGNode()`; // cycle head ICFGNode ℓ
- `increasing` := `true`;
- `i` := 0; // analysis iteration for the loop
- **while** *true* **do**
  - $as_{pre} := \sigma_\ell$; // abstract state in the last iteration
  - `handleICFGNode(`ℓ`)`;
  - $as_{cur} := \sigma_\ell$; // abstract state in the current iteration
  - **if** `i` $\geq$ `Options.WidenDelay()` **then**
    - **if** `increasing` **then**
      - $\sigma_\ell := as_{pre} \nabla as_{cur}$; // widening
      - **if** $\sigma_\ell \equiv as_{pre}$ **then**
        - `increasing` := `false`;
        - **continue**;
    - **else**
      - $\sigma_\ell := as_{pre} \Delta as_{cur}$; // narrowing
      - **if** $\sigma_\ell \equiv as_{pre}$ **then**
        - **break**;
  - // analyze remaining cycle components after two fixed-points
  - **foreach** `comp` $\in$ `cycle.getWTOComponents()` **do**
    - **if** `comp` *is Singleton* **then**
      - `handleICFGNode(comp.getICFGNode())`
    - **else if** `comp` *is Cycle* **then**
      - `handleICFGCycle(comp)`;
  - `i++`;
- **return**;

An Example: Abstract Trace σ for Top-level Variables

Widen Phase (cur_iter is 2)



ICFG

$postAbsTrace[ICFGNode6].varToAbsVal$:

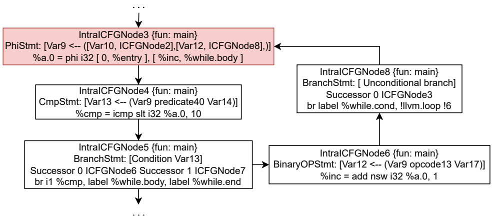
SVFVar	AbstractValue⟨*Interval*, *MemAddress*⟩
	...
Var9	$\langle [0, 9], \perp \rangle$
Var12	$\langle [1, 10], \perp \rangle$
	...

Algorithm 12: Handle ICFG Cycle

Function `handleICFGCycle` *(cycle)*:
- ℓ := `cycle.getHead().getICFGNode()`; // cycle head ICFGNode ℓ
- `increasing` := `true`;
- `i` := 0; // analysis iteration for the loop
- **while** *true* **do**
  - $as_{pre} := \sigma_\ell$; // abstract state in the last iteration
  - `handleICFGNode(`ℓ`)`;
  - $as_{cur} := \sigma_\ell$; // abstract state in the current iteration
  - **if** `i` $\geq$ `Options.WidenDelay()` **then**
    - **if** `increasing` **then**
      - $\sigma_\ell := as_{pre} \nabla as_{cur}$; // widening
      - **if** $\sigma_\ell \equiv as_{pre}$ **then**
        - `increasing` := `false`;
        - **continue**;
    - **else**
      - $\sigma_\ell := as_{pre} \Delta as_{cur}$; // narrowing
      - **if** $\sigma_\ell \equiv as_{pre}$ **then**
        - **break**;
  - // analyze remaining cycle components after two fixed-points
  - **foreach** `comp` $\in$ `cycle.getWTOComponents()` **do**
    - **if** `comp` *is Singleton* **then**
      - `handleICFGNode(comp.getICFGNode())`
    - **else if** `comp` *is Cycle* **then**
      - `handleICFGCycle(comp)`;
  - `i++`;
- **return**;

An Example: Abstract Trace σ for Top-level Variables

Widen Phase Fixed Point



ICFG

postAbsTrace[ICFGNode3].varToAbsVal :

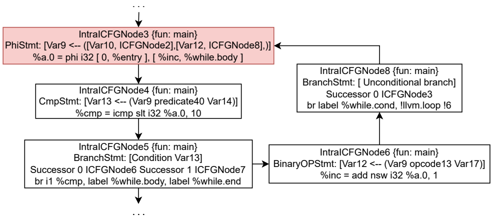
SVFVar	$\langle Interval, MemAddress \rangle$
	...
Var9	$\langle [0, +\infty], \perp \rangle$
Var12	$\langle [1, +\infty], \perp \rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle(cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Narrow Phase



ICFG

postAbsTrace[ICFGNode3].varToAbsVal :

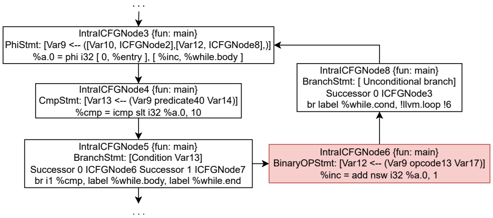
SVFVar	⟨*Interval*, *MemAddress*⟩
	...
Var9	$\langle [0, 10], \bot \rangle$
Var12	$\langle [1, 10], \bot \rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Narrow Phase



ICFG

postAbsTrace[ICFGNode6].varToAbsVal :

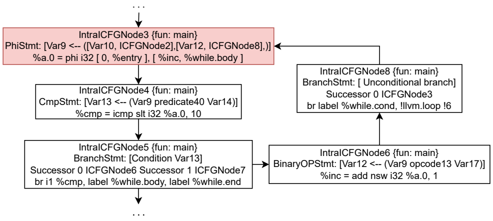
SVFVar	⟨*Interval*, *MemAddress*⟩
	...
Var9	$\langle [0, 9], \bot \rangle$
Var12	$\langle [1, 10], \bot \rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

Narrow Phase Fixed Point



ICFG

postAbsTrace[ICFGNode3].varToAbsVal :

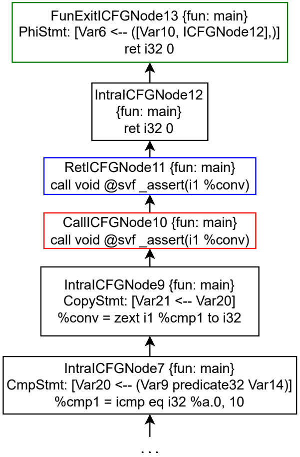
`SVFVar`	$\langle Interval, MemAddress\rangle$
	...
Var9	$\langle[0, 10], \perp\rangle$
Var12	$\langle[1, 10], \perp\rangle$
	...

Algorithm 12: Handle ICFG Cycle

```
Function handleICFGCycle (cycle):
    ℓ := cycle.getHead().getICFGNode();   // cycle head ICFGNode ℓ
    increasing := true;
    i := 0;   // analysis iteration for the loop
    while true do
        as_pre := σ_ℓ;   // abstract state in the last iteration
        handleICFGNode(ℓ);
        as_cur := σ_ℓ;   // abstract state in the current iteration
        if i ≥ Options.WidenDelay() then
            if increasing then
                σ_ℓ := as_pre ∇ as_cur;   // widening
                if σ_ℓ ≡ as_pre then
                    increasing := false;
                    continue;
            else
                σ_ℓ := as_pre Δ as_cur;   // narrowing
                if σ_ℓ ≡ as_pre then
                    break;
        // analyze remaining cycle components after two fixed-points
        foreach comp ∈ cycle.getWTOComponents() do
            if comp is Singleton then
                handleICFGNode(comp.getICFGNode())
            else if comp is Cycle then
                handleICFGCycle(comp);
        i++;
    return;
```

An Example: Abstract Trace σ for Top-level Variables

After Exiting Loop



ICFG

Algorithm 13: Abstract execution guided by WTO

Function `handleStatement(`ℓ`)`**:**
  tmpAS := *preAbsTrace*[ℓ];
  if ℓ *is* CmpStmt **then**
    `updateStateOnCmp(`ℓ`);`
  ...;

postAbsTrace[*ICFGNode7*].*varToAbsVal* :

SVFVar	$\langle$*Interval*, *MemAddress*$\rangle$
...	
Var9	$\langle[10, 10], \perp\rangle$
Var20	$\langle[1, 1], \perp\rangle$
...	