Software Verification and Z3 Theorem Prover

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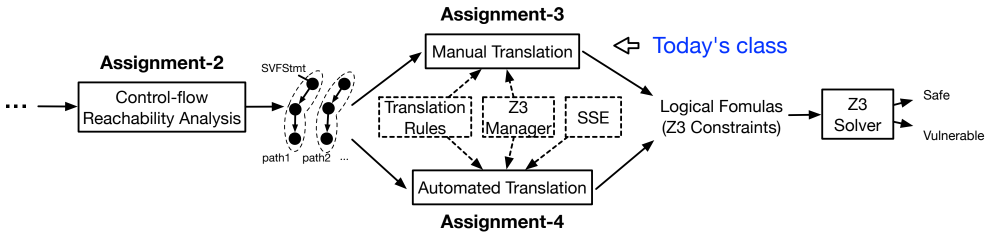
Today’s class



- In this class, we will learn how to manually translate source code into logical formulas (Z3 constraints/expressions).
- We introduce Z3 solver, Z3 constraint format **Z3 manager** APIs.
- Then, we will demonstrate **examples** for **manual translation** from code to Z3 constraints.

Z3 Theorem Prover

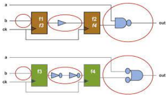
- Z3 is a Satisfiability Modulo Theories (SMT) solver from Microsoft Research[1].
- Targeted at solving problems in software verification and software analysis.
- Main applications are static checking, test case generation, and more ..

Hardware verification

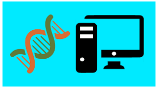
Software analysis/testing

Architecture

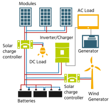
Modeling



Geometrical solving



Biological analysis

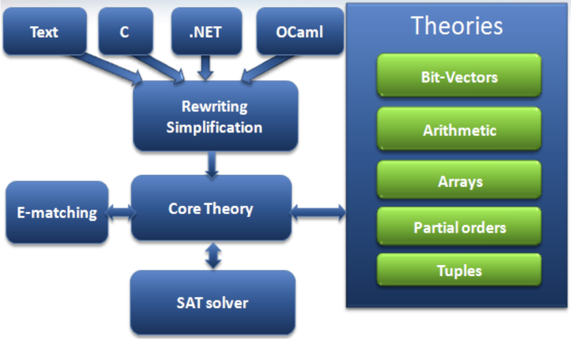


Hybrid system analysis

...

[1] http://research.microsoft.com/projects/z3

Z3 Framework[2]



- Z3 is an effective tool to solve **logical formulas** (Z3 constraints).
- `https://github.com/Z3Prover/z3`.
- Its SMT solver supports theories such as fixed-size bit-vectors, arithmetic, extensional arrays, datatypes, uninterpreted functions, and quantifiers.
- Z3 has official APIs for **C, C++, Python, .NET**, etc.
- **Z3 solver** can find one of the feasible solutions in a set of constraints.

[2] `https://nikolajbjorner.github.io/slides/Z3_System.pdf`

Z3 Learning Materials

- Z3 GitHub repository `https://github.com/z3prover/z3`
- Getting Started with Z3: A Guide `https://jfmc.github.io/z3-play`
- Z3 tutorials `https://github.com/philzook58/z3_tutorial`
- Z3 slides `https://github.com/Z3Prover/z3/wiki/Slides`
- Programming Z3 `http://theory.stanford.edu/~nikolaj/programmingz3.html#sec-logical-interface`

Z3 Solver and Z3 Formulas

Z3 solver accepts a first-order (predicate) logical formula ϕ, and outputs one of the following results.

- `sat` if ϕ is satisfiable
- `unsat` if there is a counterexample which make ϕ unsatisfiable
- `unknown` if ϕ is too complex and can not be solved within a time frame.

Z3 Solver and Z3 Formulas

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- `sat` if ϕ is satisfiable
- `unsat` if there is a counterexample which make ϕ unsatisfiable
- `unknown` if ϕ is too complex and can not be solved within a time frame.

You play around and check the satisfiability of your Z3 constraints/formulas here: `https://jfmc.github.io/z3-play` or `https://compsys-tools.ens-lyon.fr/z3/index.php`

Z3's Logical Formula (Constants, Check-Sat and Evaluation)

The Z3 input format (formula format) is an extension of the SMT-LIB 2.0 standard[3].
A Z3 formula expression (`z3::expr`) has the following keywords:

- `echo` displays a message
- `declare-const` declares a constant of a given type (a.k.a `sort`)
- `declare-fun` declares a function
- `assert` adds a formula into the Z3 internal stack
- `check-sat` determines whether the current formulas on the Z3 stack are satisfiable or not
- `get-model` is used to retrieve an interpretation (one solution) that makes all formulas on the Z3 internal stack true
- `eval` evaluates a variable/expression produced by a model when the formulas is satisfiable.

[3] `https://homepage.cs.uiowa.edu/~tinelli/papers/BarST-SMT-10.pdf`

Constants, Check-Sat and Evaluation (Example)

$$\phi : (\mathtt{x} > \mathtt{10}) \ \land \ (\mathtt{y} \equiv \mathtt{x} + \mathtt{1})$$

How to represent this formula in Z3 and feed it into Z3's solver?

Constants, Check-Sat and Evaluation (Example)

$$\phi : (x > 10) \ \land \ (y \equiv x + 1)$$

How to represent this formula in Z3 and feed it into Z3's solver?

```
(echo "starting Z3...")
(declare-const x Int) /// Declare an Int type variable "x"
(declare-const y Int) /// Declare an Int type variable "y"
(assert (> x 10))  /// Add the first part (x>10) of the conjunction into the solver
(assert (= y (+ x 1))) /// Add the second part (y==x+1) of the conjunction
(check-sat) /// Check whether added formulas are satisfiable.
(eval x) /// Evaluate the value of x when the formula is satisfiable
(eval y) /// Evaluate the value of y when the formula is satisfiable
```

Constants, Check-Sat and Evaluation (Example)

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How to represent this formula in Z3 and feed it into Z3's solver?

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(echo "starting Z3...")
(declare-const x Int) /// Declare an Int type variable "x"
(declare-const y Int) /// Declare an Int type variable "y"
(assert (> x 10))  /// Add the first part (x>10) of the conjunction into the solver
(assert (= y (+ x 1))) /// Add the second part (y==x+1) of the conjunction
(check-sat) /// Check whether added formulas are satisfiable.
(eval x) /// Evaluate the value of x when the formula is satisfiable
(eval y) /// Evaluate the value of y when the formula is satisfiable
```

Outputs of Z3's solver:

```
starting Z3...
sat /// (check-sat) result
11 /// the value of x as one satisfiable solution
12 /// the value of y as one satisfiable solution
```

Z3's Logical Formula (Uninterpreted Function)

The basic building blocks of SMT formulas are constants and uninterpreted functions.

- An uninterpreted function **has no other property** (no priori interpretation) **than its signature** (i.e., function name and arguments).
- An uninterpreted functions in first-order logic have **no side-effects** (e.g., can not change argument values and never return different values for the same input)
- **Constants** in Z3 can also be seen as **functions that take no arguments.**
- **The details and characteristics** of uninterpreted functions are **ignored.** This can **generalize and simplify** theorems and proofs.

Uninterpreted Function (Example)

```
(declare-fun f (Int) Int)   /// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))   /// f(10) = 1
(check-sat)
```

Uninterpreted Function (Example)

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(assert (= (f 10) 1))   /// f(10) = 1
(check-sat)
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Outputs of Z3's solver:

```
sat
```

The solver returns `sat`, because `f` is an uninterpreted function (i.e., all that is known about `f` is its signature), so it is possible that `f(10) = 1`.

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```
(declare-fun f (Int) Int)   /// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))   /// f(10) = 1
(assert (= (f 10) 2))   /// f(10) = 2
(check-sat)
```

Uninterpreted Function (Example)

```
(declare-fun f (Int) Int)    /// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    /// f(10) = 1
(check-sat)
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Outputs of Z3's solver:

```
sat
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```
(declare-fun f (Int) Int)    /// Function f accepts an Int argument and returns a Int
(assert (= (f 10) 1))    /// f(10) = 1
(assert (= (f 10) 2))    /// f(10) = 2
(check-sat)
```

Outputs of Z3's solver:

```
unsat
```

The solver returns `unsat`, because `f`, as an uninterpreted function, can never return different values for the same input.

Uninterpreted Function (Example)

$$\phi : \mathtt{f(x)} \equiv \mathtt{f(y)} \ \wedge \ \mathtt{x! = y}$$

```
(declare-const x Int)
(declare-const y Int)
(declare-fun f (Int) Int) /// Function f accepts an Int argument and returns a Int
(assert (= (f x) (f y)))
(assert (not (= x y)))
(check-sat)
```

Uninterpreted Function (Example)

$$\phi : \mathtt{f}(\mathtt{x}) \equiv \mathtt{f}(\mathtt{y}) \ \wedge \ \mathtt{x}! = \mathtt{y}$$

```
(declare-const x Int)
(declare-const y Int)
(declare-fun f (Int) Int) /// Function f accepts an Int argument and returns a Int
(assert (= (f x) (f y)))
(assert (not (= x y)))
(check-sat)
```

Outputs of Z3's solver:

```
sat
```

An uninterpreted function can have different inputs and return the same output. For example, `f` can always return `1` regardless the value of the input argument.

Constants as Uninterpreted Function (Example)

$$\phi : (x > 10) \ \land \ (y \equiv x + 1)$$

```
(declare-fun x () Int) /// "x" and "y" as an uninterpreted functions
(declare-fun y () Int) /// Accepts no argument and return an Int
(assert (> x 10))
(assert (= y (+ x 1)))
(check-sat)
(get-model)
```

Outputs of Z3's solver:

```
sat
(
  (define-fun x () Int
    11)                     /// x is evaluated to be 11 for this model
  (define-fun y () Int
    12)                     /// y is evaluated to be 11 for this model
)
```

`(declare-const x Int)` can be seen as the syntax sugar for `(declare-fun x () Int)`.

Z3's Logical Formula (Arithmetic)

- Z3 supports majority of commonly used arithmetic operators, such as +, -, $*$, /, $<<$, $>>$, $<$, $>$, &, $|$ (The ones listed in SVFIR)
- Types of any two operands should be the same otherwise a type conversion is needed.
- Never mix types in arithmetic, and always be explicit.

```
(declare-const a Int)
(declare-const b Float32)
(assert (= a (+ b 1)))
(check-sat)
```

Outputs of Z3's solver:

```
Error: (error "line 3 column 19: Sort mismatch at argument #1 for function
(declare-fun + (Int Int) Int) supplied sort is (_ FloatingPoint 8 24)")
```

Z3's Logical Formula (`if-then-else` Expression)

- `ite(b, x, y)` represents a conditional expression, where `b` is the condition, `ite` returns `x` if `b` is evaluated true, otherwise `y` is returned
- Used for comparison or branches

```
(ite (and (= x!1 11) (= x!2 false)) 21 0)
```

The above Z3 formula evaluates (returns) 21 when x!1 is equal to 11, and x!2 is equal to false. Otherwise, it returns 0.

Z3's Logical Formula (Arrays)

Formulating a program of a mathematical theory of computation McCarthy proposed a basic theory of arrays as characterized by the **select-store** axioms.

- `(select a i)`: returns the value stored at position `i` of the array `a`;
- `(store a i v)`: returns a new array identical to `a`, but on position `i` it contains the value `v`.
- Z3 assumes that arrays are extensional over select. Z3 also enforces that if two arrays agree on all reads, then the arrays are equal.

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- Z3 assumes that arrays are extensional over select. Z3 also enforces that if two arrays agree on all reads, then the arrays are equal.

The following formulas store `y` to the `x`-th position of array `a` and then load the value at `a`'s `x`-th position to `z`

```
(declare-const x Int)
(declare-const y Bool)
(declare-const z Bool)
(declare-const a (Array Int Bool))  /// an array of Bools with Int as the indices
(assert (= (store a x y) a))    /// a[x] == y
(assert (= (select a x) z))     /// z == a[x]
```

Z3's Logical Formula (Scopes)

Z3 maintains a global stack of declarations and assertions via **push** and **pop**

- **push**: creates a new scope by saving the current stack size.
- **pop**: removes any assertion or declaration performed between it and the matching push.

The `check-sat` command always operates on the current global stack.

Z3's Logical Formula (Scopes)

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- **push**: creates a new scope by saving the current stack size.
- **pop**: removes any assertion or declaration performed between it and the matching push.

The `check-sat` command always operates on the current global stack.

```
(declare-const x Int)
(declare-const a (Array Int Int))  /// an array of Ints
(push)
(assert (= (store a 1 10) a))     /// a[1] == 10
(assert (= (select a 1) x))      /// x == a[1]
(assert (= x 20))                /// x == 20
(check-sat)
(pop) ; remove the three assertions
(assert (= x 20))                /// x == 10
(check-sat)
```

What is the output of the solver?

Translating Code to Z3 Formulas

We provide a Z3Mgr class (a wrapper class to manipulate Z3 APIs) to generate Z3 formulas or so-called `z3::expr`.

API	Meanings
`z3::expr getZ3Expr(std::string);`	Create a **variable** given a string name
`z3::expr getZ3Expr(int);`	Create a **variable** given an integer
`z3::expr getMemObjAddress(std::string);`	Create a **memory object** in program
`z3::expr getGepObjAddress(z3::expr, u32_t);`	Create a **field object** with an **offset** of an aggregate
`void addToSolver(z3::expr);`	**Add** a Z3 expression/formula to the solver
`void resetSolver();`	Clean all formulas in the the solver
`solver.check();`	**Check satisfiability** of an z3 formula
`z3::expr getEvalExpr(z3::expr);`	**Evaluate an expression** based on a model
`void printExprValues();`	**Print** the values of **all expressions** in the solver.

More details, refer to

`https://github.com/SVF-tools/Teaching-Software-Verification/wiki/SVF-APIs`

Translation Rules

expr p = getZ3Expr("p")   expr q = getZ3Expr("q")   expr r = getZ3Expr("r")

SVFStmt	C-Like form	Operations
AddrStmt (constant) AddrStmt (mem allocation)	p = c p = alloc	addToSolver(p == c); addToSolver(p == getMemObjAddress("alloc");)
CopyStmt	p = q	addToSolver(p == q);
LoadStmt	$p = *q$	addToSolver(p == loadValue(q));
StoreStmt	$*p = q$	storeValue(p, q);
GepStmt	$p = \&(q \rightarrow i)$ or $p = \&q[i]$	addToSolver(p == getGepObjAddress(q,i));
PhiStmt	$r = phi(l_1 : p,\ l_2 : q)$	if(executed from l_1) addToSolver(p==r); if(executed from l_2) addToSolver(q==r);

BranchStmt	if (p) l_1 else l_2	expr cond = getEvalExpr(p);
		if(cond.is_false()) execute l_2
		else execute l_1 addToSolver(cond = true);

UnaryOPStmt	$\neg p$	addToSolver(!p);
BinaryOPStmt	$r = p \otimes q$	addToSolver(r == p $\otimes$ q);
CmpStmt	$r = p \odot q$	addToSolver(r == ite(p $\odot$ q, true, false));
CallPE/RetPE	$r = f(\ldots, q, \ldots)$ $f(\ldots, p, \ldots)\{\ldots$ return $z\}$	
CallPE	p = q	solver.push(); addToSolver(p == q);
RetPE	p = r	expr ret = getZ3Expr(r); solver.pop(); addToSolver(p == ret);

Translating Code to Z3 Formulas (Scalar Example)

The target program code needs to be in **SSA form** (e.g., SVFIR).

- Top-level variables can only be defined once
  - a = 1; a = 2; $\Longrightarrow$ a1 = 1; a2 = 2;
- Memory objects can only be modified/read through top-level pointers at `StoreStmt` and `LoadStmt`.
  - `p = &a; *p = r;` The value of `a` can only be modified/read via dereferencing `p`.

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  - `p = &a; *p = r;` The value of `a` can only be modified/read via dereferencing `p`.

```c
int main() {

 int a;
 int b;
 a = 0;
 b = a + 1;
 assert(b>0);

}
```

C code

→

```cpp
// int a;
expr a = getZ3Expr("a");
// a = 0;
addToSolver(a == 0);
// int b;
expr b = getZ3Expr("b");
// b = a+1;
addToSolver(b == (a + 1));
// assert(b > 0);
addToSolver(b > 0);
solver.check();
```

Translator

→

```
(declare-fun a () Int)
(declare-fun b () Int)
(assert (= a 0))
(assert (= b (+ a 1)))
(assert (> b 0))
(check-sat)
```

Z3 Formulas

→ Z3's SMT solver

Translating Code to Z3 Formulas (Memory Operation Example)

- Each memory object has a unique ID and allocated with a **virtual memory address**
- In our modeling, the virtual address starts from **0x7f...... + ID** (i.e., 2130706432 + ID in decimal)
- Memory operations will be through store and load values from `loc2ValMap`, an Z3 array.

```c
int main() {
 int* p;
 int x;

 p = malloc1(..);
 *p = 5;
 x = *p;
 assert(x==5);
}
```

C code

→

```cpp
//  int** p;
expr p = getZ3Expr("p");
//  int x;
expr x = getZ3Expr("x");
// p = malloc(..);
expr m = getMemObjAddress("malloc1");
addToSolver(p == m);
// *p = 5;
storeValue(p, getZ3Expr(5));
// x = *p;
addToSolver(x == loadValue(p));
// assert(x==5);
addToSolver(x == getZ3Expr(5));
solver.check();
```

Translator

→

```
(declare-fun p () Int)
(declare-fun loc2ValMap ()
    (Array Int Int))
(declare-fun x () Int)
(assert (= p 2130706435))
(assert (= x (select
    (store loc2ValMap 2130706435 5)
    2130706435)))
(assert (= x 5))
(model-add p () Int 2130706435)
(check-sat)
```

Z3 Formulas

What's next?

- (1) Understand Z3 formula format in the slides
- (2) Understand `Z3Mgr` class in the GitHub Repository of Teaching-Software-Verification
- (3) Read through Assignment-3.pdf on Canvas to understand some examples for manual code verification.
- (4) Finish the quizzes of Assignment 3 on Canvas, and implement a manual translation from code to Z3 formulas using `Z3Mgr` i.e., coding task in Assignment 3.