Kolmogorov Complexity Constraint Module for Portfolio Optimization

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Abstract

We introduce a novel regularization framework for portfolio optimization that explicitly penalizes allocation strategies based on their Kolmogorov complexity (KC). By approximating KC through model sparsity and parametric simplicity, we encourage simpler, more interpretable, and generalizable portfolio solutions. This approach is grounded in the Minimum Description Length (MDL) principle and aims to mitigate overfitting while promoting robust out-of-sample performance.

1 Introduction

Kolmogorov complexity, defined as the length of the shortest program that outputs a given object, provides a theoretical foundation for measuring model simplicity. In financial contexts, simpler models are more likely to generalize well and be interpretable by human analysts. Inspired by the MDL principle, we propose integrating a KC penalty into portfolio optimization objectives.

2 Mathematical Formulation

2.1 Classical Objective

Let $\mathbf{w}$ denote the portfolio weight vector. A standard mean-variance objective might be:

$$\min_{\mathbf{w}} \quad \mathbf{w}^\top \Sigma \mathbf{w} - \lambda \mathbf{w}^\top \mu, \tag{1}$$

where Σ is the covariance matrix, μ is the expected return vector, and λ controls the risk-return trade-off.

2.2 Kolmogorov Complexity Penalty

Exact KC is uncomputable; however, we approximate it via a surrogate penalty encouraging sparsity and low parameter magnitude:

$$\Omega_{\mathrm{KC}}(\mathbf{w}) = \gamma\|\mathbf{w}\|_0 + \eta\|\mathbf{w}\|_1, \tag{2}$$

where $\|\mathbf{w}\|_0$ is the cardinality (number of non-zero elements) and $\|\mathbf{w}\|_1$ is the L1 norm. γ and η are hyperparameters controlling sparsity and smallness.

2.3 Combined Objective

Our final optimization problem becomes:

$$\min_{\mathbf{w}} \quad \mathbf{w}^\top \Sigma \mathbf{w} - \lambda \mathbf{w}^\top \mu + \Omega_{\mathrm{KC}}(\mathbf{w}). \tag{3}$$

This formulation directly penalizes complex solutions, leading to simpler, interpretable portfolios.

3 Algorithm

Algorithm 1 Portfolio Optimization with KC Penalty

```
Input: Σ, μ, λ, γ, η
Initialize w (e.g., equal weights)
while not converged do
    Compute gradient: 2Σw − λμ + γ∂‖w‖_0 + η · sign(w)
    Update weights via proximal gradient or iterative shrinkage
end while
Output: Optimized w
```

4 Python Implementation

```
import numpy as np

def kc_penalty(w, gamma=1e-2, eta=1e-2):
    l0 = np.count_nonzero(w)
    l1 = np.sum(np.abs(w))
    return gamma * l0 + eta * l1

def objective(w, Sigma, mu, lam, gamma, eta):
    risk = w.T @ Sigma @ w
    ret = w.T @ mu
    penalty = kc_penalty(w, gamma, eta)
    return risk - lam * ret + penalty

def grad_objective(w, Sigma, mu, lam, eta):
    grad_risk = 2 * Sigma @ w
    grad_ret = lam * mu
    grad_l1 = eta * np.sign(w)
    return grad_risk - grad_ret + grad_l1

def optimize_weights(Sigma, mu, lam=0.1, gamma=1e-2, eta=1e-2, lr=1e-2,
    w = np.ones_like(mu) / len(mu)
    for _ in range(epochs):
        grad = grad_objective(w, Sigma, mu, lam, eta)
        w -= lr * grad
        # Proximal operator for L0 approximation (threshold small eleme
        w[np.abs(w) < gamma] = 0
        # Projection (optional): ensure sum to 1
        w = np.maximum(w, 0)
        w /= np.sum(w + 1e-8)
    return w
```

5 Numerical Experiments

5.1 Setup

We test our method on synthetic covariance matrices and random expected returns. Out-of-sample returns are evaluated to compare generalization against

classical mean-variance.

5.2 Results

Preliminary results show that our KC-penalized portfolios are sparser, have lower turnover, and maintain comparable or improved Sharpe ratios. The penalty successfully reduces parameter complexity.

6 Discussion and Future Work

While KC is theoretically non-computable, our surrogate formulation offers a practical approach. Future work includes tighter approximations using compression-based metrics and extensions to nonlinear strategies (e.g., deep factor models).

7 Conclusion

This work introduces a Kolmogorov complexity-inspired regularization for portfolio optimization, promoting sparse and interpretable allocations. Our theoretical framework and empirical findings demonstrate potential for robust financial model design.

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References

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