Multi-Information Bottleneck Module for Multi-Factor Portfolio Signal Compression

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Abstract

In this work, we propose a novel Multi-Information Bottleneck (Multi-IB) framework designed to simultaneously compress multiple predictive signals or factor inputs, retaining only the most relevant information for robust portfolio decision-making. The module generalizes the classic Information Bottleneck method to multi-input, multi-output setups and provides explicit control over noise and irrelevant signal interactions. We derive a mathematical trade-off expression, develop a multi-IB optimizer, and outline an empirical case study for validating the framework in factor compression scenarios.

1 Introduction

Modern portfolio management increasingly relies on multiple predictive signals (factors) for asset allocation and risk management. However, these signals often contain redundant or noisy information, increasing the risk of overfitting and poor out-of-sample performance. The Information Bottleneck (IB) method, originally developed in information theory, provides a principled way to extract relevant information while discarding noise.

We extend this idea by developing a Multi-Information Bottleneck (Multi-IB) framework that can handle multiple predictive signals and multiple predictive targets (e.g., future returns, volatility, macroeconomic indicators).

2 Mathematical Formulation

Let

- $X = (X_1, X_2, \ldots, X_m)$ denote the set of m input predictive signals.
- $Y = (Y_1, Y_2, \ldots, Y_n)$ denote the set of n predictive targets.
- T be a compressed representation (bottleneck).

We want to find a stochastic encoder $p(T|X)$ that minimizes the following objective:

$$\mathcal{L} = I(X;T) - \beta I(T;Y),$$

where

- $I(X;T)$ is the mutual information between inputs and bottleneck.
- $I(T;Y)$ is the mutual information between bottleneck and targets.
- $\beta > 0$ controls the trade-off between compression and prediction accuracy.

2.1 Generalization to Multi-Inputs and Multi-Outputs

The classic IB setup considers one X and one Y. We generalize:

$$\mathcal{L}_{\text{multi}} = I(X_1, \ldots, X_m; T) - \beta I(T; Y_1, \ldots, Y_n).$$

We may further introduce weights w_j to emphasize certain targets:

$$\mathcal{L}_{\text{weighted}} = I(X;T) - \beta \sum_{j=1}^{n} w_j I(T; Y_j).$$

This enables fine-grained control in multi-task or multi-objective settings.

3 Algorithm

Algorithm 1 Multi-Information Bottleneck Optimizer

Require: Input signals $X = (X_1, \dots, X_m)$, predictive targets $Y = (Y_1, \dots, Y_n)$, trade-off parameter β, weights w_j.

```
Initialize encoder p(T|X) (e.g., neural network or kernel mapping).
repeat
    Estimate I(X;T) using variational mutual information estimators.
    Estimate I(T;Y_j) for each j.
    Compute objective L_weighted = I(X;T) − β Σ_j w_j I(T;Y_j).
    Update encoder parameters via gradient descent to minimize L_weighted.
until convergence
Output final compressed representation T.
```

4 Implementation Remarks

- **Mutual Information Estimation:** For high-dimensional signals, mutual information is estimated using variational bounds (e.g., Barber-Agakov, MINE).
- **Encoder Choices:** We can implement $p(T|X)$ using deep networks (Multi-layer Perceptrons, Graph Neural Nets) or kernel machines depending on interpretability and computational needs.
- **Noise Control:** Explicit regularization terms (e.g., sparsity or entropy penalties) can help suppress spurious correlations and noise.

5 Empirical Study Outline

We propose to evaluate this module on a multi-factor equity portfolio:

- **Inputs:** Value, momentum, quality, sentiment, and macro signals.
- **Targets:** Forward 1-month return, volatility, and risk regime indicators.
- **Metrics:** Information ratio, realized Sharpe ratio, effective signal dimensionality (entropy-based).
- **Ablation Studies:** Effect of varying β, number of bottleneck dimensions, and factor subset inclusion.

6 Expected Outcomes

- A compact representation T capturing only sufficient predictive information.
- Reduction of signal noise and improved generalization in portfolio optimization.
- Clear trade-off curves between compression and predictive accuracy.

7 Future Work

Possible extensions include dynamic β scheduling, real-time bottleneck adaptation, and integration with online portfolio learning frameworks.

8 Conclusion

We introduced a Multi-Information Bottleneck framework for compressing and integrating multiple predictive signals in finance. The derived mathematical formulation, algorithmic steps, and proposed empirical validation provide a foundation for rigorous study and real-world implementation.

Code and Data Availability

The full implementation, including mutual information estimators and optimizer scripts, will be provided upon request or in the supplementary material.