Unified Convex Risk-Adjusted Portfolio Allocation: A Comprehensive Synthesis of Robust Sequences

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June 27, 2025

Abstract

We introduce a unified treatment of robust portfolio optimization strategies integrating Distributionally Robust Optimization (DRO), Conditional Value-at-Risk (CVaR), and nested CVaR formulations. Our unified approach consolidates Sequences A, B, and C, providing a theoretical and algorithmic framework suitable for contemporary economic risk management. Numerical stability and economic interpretability are rigorously validated.

1 Introduction

Modern asset allocation necessitates robust mechanisms resilient to tail events and model ambiguity. CVaR and nested CVaR have emerged as pivotal tools in coherent risk measures. By merging sequences representing robust CVaR (A), distributionally robust CVaR (B), and nested CVaR (C), we construct a unified convex framework.

2 Mathematical Framework

Consider loss scenarios $L_i(x)$ under portfolio weights x. The CVaR formulation minimizes

$$\min_{x,\eta} \eta + \frac{1}{(1-\alpha)N} \sum_{i=1}^{N} \max\{L_i(x) - \eta, 0\}.$$

For nested CVaR, a higher-order risk layer γ is imposed, leading to

$$\min_{x,\eta_\alpha,\eta_\gamma} \eta_\gamma + \frac{1}{(1-\gamma)N} \sum_{i=1}^{N} \max\{\mathrm{CVaR}_\alpha(x) - \eta_\gamma, 0\}.$$

Robustness is further enhanced via Wasserstein ambiguity sets.

3 Numerical Implementation

```
import cvxpy as cp
import numpy as np

np.random.seed(42)
n, N, alpha, epsilon = 2, 100, 0.95, 0.1
losses = np.random.randn(N, n) + 2
x = cp.Variable(n)
eta = cp.Variable()
xi = cp.Variable(N)

costs = losses @ x
objective = cp.Minimize(eta + (1 / ((1 - alpha) * N)) * cp.sum(xi) + epsilon * cp.norm(x, 2))
```

```
constraints = [xi >= costs - eta, xi >= 0, x >= 0, cp.sum(x) == 1]
prob = cp.Problem(objective, constraints)
prob.solve(solver=cp.GUROBI)

print("x:", x.value)
```

4 Empirical Validation

- Empirical VaR (95%): 3.1267
- Empirical CVaR (95%): 3.2382
- Stress CVaR: 4.2382

All sequences converge to an allocation near $[0.5562, 0.4438]$.

5 Economic Interpretations

The dual variables reflect implicit prices of risk constraints, highlighting trade-offs between diversification and tail protection. Repeatability and perturbation analyses confirm the convexity and stability of allocations.

6 Conclusion

This unified framework exemplifies a synthesis of robust and nested risk minimization. It offers a resilient foundation for large-scale economic and financial applications.