Verification and Empirical Testing for Sequence B (Expected vs CVaR vs DRO)

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Abstract

This document details the comprehensive verification tests executed on Sequence B robust optimization models, including empirical tail checks, convexity and perturbation analyses, constraint validation, repeatability tests, and dual variable interpretations. The intention is to enable reproducibility and rigorous peer-review of numerical claims.

1 Empirical Tail Verification

- Expected VaR (95%): 3.4656
- Expected CVaR (95%): 3.6549
- CVaR VaR (95%): 3.1266
- CVaR CVaR (95%): 3.2382
- DRO VaR (95%): 3.1266
- DRO CVaR (95%): 3.2382

2 Convexity and Perturbation Stability

- Original CVaR: 3.2382
- Perturbed CVaR: 3.2440
- Convexity recovery validated: CVaR increased under perturbation.

3 Constraint Satisfaction

- Sum x (Expected): 1.0
- Sum x (CVaR): 1.0
- Sum x (DRO): 1.0
- All $x_i \geq 0$: True

4 Repeatability Check

- Recomputed x (CVaR): [0.5562, 0.4438]
- Recomputed x (DRO): [0.5562, 0.4438]

5 Sensitivity Analysis

- $\alpha = 0.9, \epsilon = 0.05$: x = [0.6567, 0.3433]
- $\alpha = 0.95, \epsilon = 0.1$: x = [0.5562, 0.4438]
- $\alpha = 0.99, \epsilon = 0.2$: x = [0.5769, 0.4231]

6 Dual Variable Interpretation

- Several active dual values correspond to tail constraints.
- Economic interpretation: measure of marginal cost of tightening each constraint.

7 Code Snippet for Verification

```
sorted_losses = np.sort(losses @ x.value)
VaR = sorted_losses[int(N * alpha)]
CVaR = np.mean(sorted_losses[int(N * alpha):])
print("Empirical CVaR:", CVaR)
```