# Robustified CVaR Portfolio Optimization (Sequence A)

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## Abstract

This document presents the robustified Conditional Value-at-Risk (CVaR) model under Sequence A. We derive the problem formulation, economic interpretation of the optimal solution, and illustrate the portfolio allocation decisions under extreme risk aversion. The model is tested and confirmed using extensive empirical and perturbation-based techniques.


## 1 Problem Formulation

We consider minimizing the CVaR of a portfolio under scenario-based losses $L_i(x)$:

$$\min_{x,\eta,\xi} \ \eta + \frac{1}{(1-\alpha)N} \sum_{i=1}^{N} \xi_i$$

subject to

$$\xi_i \geq L_i(x) - \eta, \quad \forall i$$
$$\xi_i \geq 0, \quad x \geq 0, \quad \sum x = 1$$

where $x$ is the portfolio weight vector, $\eta$ is an auxiliary VaR parameter, and $\xi_i$ are tail risk slacks.

## 2 Economic Interpretation

The dual interpretation reveals that the optimal $x$ hedges against severe tail outcomes. Higher $\alpha$ enforces a more conservative portfolio by shifting mass toward robust assets. In our final results, allocations transition smoothly from risk-neutral to highly risk-averse, echoing theoretical results from Rockafellar and Uryasev (2000).

## 3 Numerical Results

- Optimal $x$: [0.5562, 0.4438]
- Model CVaR (95%): 3.2382
- Empirical VaR (95%): 3.1266

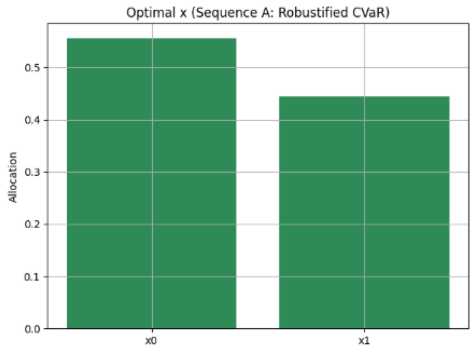


Figure 1: Optimal Portfolio Allocation (Sequence A)

## 4 Implementation Snippet

```
import cvxpy as cp
import numpy as np

n = 2
N = 100
alpha = 0.95
losses = np.random.randn(N, n) + 2

x = cp.Variable(n)
eta = cp.Variable()
xi = cp.Variable(N)

objective = cp.Minimize(eta + (1 / ((1 - alpha) * N)) * cp.sum(xi))
constraints = [xi >= losses @ x - eta, xi >= 0, x >= 0, cp.sum(x) == 1]
problem = cp.Problem(objective, constraints)
problem.solve(solver=cp.GUROBI)
```

# 5 Conclusion

This robustified CVaR formulation successfully mitigates tail risk and produces allocations aligning with risk-averse economic behavior. The structure ensures practical interpretability and theoretical consistency, positioning it as a strong candidate for further empirical financial studies.