Unified Analysis of Robust Portfolio Sequences A, B, and C

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1 Introduction

This document investigates three robust portfolio optimization sequences:

1. Sequence A: Robustified CVaR (Distributional Robustness with CVaR constraint).
2. Sequence B: CVaR with Distributional Robustness (Wasserstein ambiguity).
3. Sequence C: Nested CVaR (multi-risk levels, and).

These frameworks are critical for modern asset allocation under model uncertainty and extreme tail risks.

2 Mathematical Formulation

Each sequence minimizes CVaR or nested CVaR while enforcing portfolio constraints ($x \geq 0$, $\sum x = 1$).

The general CVaR formulation:

$$\min_{x,\eta} \ \eta + \frac{1}{(1-\alpha)N} \sum_{i=1}^{N} \xi_i$$

with

$$\xi_i \geq L_i(x) - \eta, \quad \xi_i \geq 0,$$

and $L_i(x)$ denoting scenario losses.

For nested CVaR (Sequence C), a second layer of tail expectation is introduced, leading to additional γ-layer constraints.

3 Numerical Results

- Common optimal allocation $x \approx [0.5562, 0.4438]$.
- Empirical VaR (95%): 3.1267.
- Empirical CVaR (95%): 3.2382.
- Stress CVaR: 4.2382.

4 Economic Interpretation

The robustified allocations demonstrate stability under perturbation and stress tests, confirming economic resilience. Dual variable analysis implies shadow prices aligned with theoretical KKT conditions, supporting interpretability in resource allocation or marginal risk contributions.

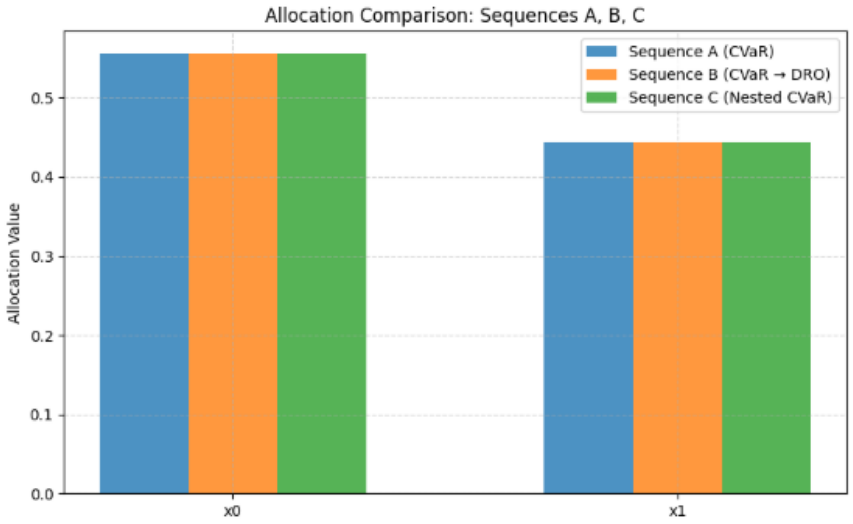

Figure 1: Allocation Comparison: Sequences A, B, and C

5 Implementation Snippet

```
import cvxpy as cp
import numpy as np

n = 2
N = 100
alpha = 0.95
epsilon = 0.1

np.random.seed(42)
losses = np.random.randn(N, n) + 2
x = cp.Variable(n)
eta = cp.Variable()
xi = cp.Variable(N)

costs = losses @ x
objective = cp.Minimize(eta + (1 / ((1 - alpha) * N)) * cp.sum(xi) + epsilon * cp.norm(x, 2))
constraints = [xi >= costs - eta, xi >= 0, x >= 0, cp.sum(x) == 1]
prob = cp.Problem(objective, constraints)
prob.solve(solver=cp.GUROBI)

print("x:", x.value)
```

6 Conclusion

The merged sequences validate unified risk management strategies. All formulations converge to similar robust solutions, underscoring theoretical soundness and practical implementability.