2.3.1 Bending moments

$$M_z(x) = R_{1,y}[x - x_1] - \int_0^x \int_0^x \tilde{q}(x) d\tilde{x} dx + Fsin(\theta)[x - x_{A_1}] + R_{2,y}[x - x_2] - Psin(\theta)[x - x_{A_2}] + R_{3,y}[x - x_3] \tag{2.3}$$

$$M_y(x) = -R_{1,z}[x - x_1] - Fcos(\theta)[x - x_{A_1}] - R_{2,z}[x - x_2] + Pcos(\theta)[x - x_{A_2}] - R_{3,z}[x - x_3] \tag{2.4}$$

2.3.2 Torsion

$$T_x(x) = -\int_0^x \tau(\tilde{x}) d\tilde{x} + R_{1,y}(z_h - \tilde{z})[x - x_1]^0 + R_{2,y}(z_h - \tilde{z})[x - x_2]^0 + R_{3,y}(z_h - \tilde{z})[x - x_3]^0 - Fcos(\theta)(y_p)[x - x_{A_1}]^0 + Fsin(\theta)(0 - \tilde{z})[x - x_{A_1}] + Pcos(\theta)(y_p)[x - x_{A_2}]^0 - Psin(\theta)(0 - \tilde{z})[x - x_{A_2}]^0 \tag{2.5}$$

2.3.3 Shear

$$S_y(x) = R_{1,y}[x - x_1]^0 + Fsin(\theta)[x - x_I]^0 + R_{2,y}[x - x_2]^0 - Psin(\theta)[x - x_{A_2}]^0 + R_{3,y}[x - x_3]^0 - \int_0^x \tilde{q}(x) d\tilde{x} \tag{2.6}$$

$$S_z(x) = R_{1,z}[x - x_1]^0 + Fcos(\theta)[x - x_I]^0 + R_{2,z}[x - x_2]^0 - Pcos(\theta)[x - x_{A_2}]^0 + R_{3,z}[x - x_3]^0 \tag{2.7}$$

2.3.4 Moment curvature

$$v(x) = -\frac{1}{EI_{zz}}\left(\frac{R_{1,y}}{6}[x - x_1]^3 - \int_0^{x_c} \int_0^{x_b} \int_0^{x_a} \int_0^x \tilde{q}(x) d\tilde{x} dx_a dx_b dx_c + \frac{Fsin(\theta)}{6}[x - x_{A_1}]^3 + \frac{R_{2,y}}{6}[x - x_2]^3 - \frac{Psin(\theta)}{6}[x - x_{A_2}]^3 + \frac{R_{3,y}}{6}[x - x_3]^3\right) + C_1 x + C_2 \tag{2.8}$$

$$w(x) = -\frac{1}{EI_{yy}}\left(-\frac{R_{1,z}}{6}[x - x_1]^3 - \frac{F\cos(\theta)}{6}[x - x_{A_1}]^3 - \frac{R_{2,z}}{6}[x - x_2]^3 + \frac{Pcos(\theta)}{6}[x - x_{A_2}]^3 - \frac{R_{3,z}}{6}[x - x_3]^3\right) + C_3 x + C_4 \tag{2.9}$$

2.3.5 Boundary conditions

$v(x_1)$	$d_1 \cos(\theta)$
$v(x_2)$	0
$v(x_3)$	$d_3 \cos(\theta)$
$w(x_1)$	$d_1 \sin(\theta)$
$w(x_2)$	0
$w(x_3)$	$d_3 \sin(\theta)$

2.3.6 Final system

$$M_z(l_a) = 0 : R_{1,y}[l_a - x_1] - \int_0^{l_a}\int_0^{x} \tilde{q}(x)d\tilde{x}dx - Fsin(\theta)[l_a - x_{A_1}] + R_{2,y}[l_a - x_2] - Psin(\theta)[l_a - x_{A_2}] + R_{3,y}[l_a - x_3] = 0 \tag{2.10}$$

$$M_y(l_a) = 0 : -R_{1,z}[l_a - x_1] - Fcos(\theta)[l_a - x_{A_1}] - R_{2,z}[l_a - x_2] + Pcos(\theta)[l_a - x_{A_2}] - R_{3,z}[l_a - x_3] = 0 \tag{2.11}$$

$$T(l_a) = 0 : -\int_0^{l_a} \tau(\tilde{x})d\tilde{x} + R_{1,y}(z_h - \tilde{z}) + R_{2,y}(z_h - \tilde{z}) + R_{3,y}(z_h - \tilde{z}) - Fcos(\theta)(y_p) + Fsin(\theta)(0 - \tilde{z}) + Pcos(\theta)(y_p) - Psin(\theta)(0 - \tilde{z}) = 0 \tag{2.12}$$

$$\sum F_y = 0 : R_{1,y} + Fsin(\theta) + R_{2,y} - Psin(\theta) + R_{3,y} - \int_0^{l_a} \tilde{q}(x)d\tilde{x} = 0 \tag{2.13}$$

$$\sum F_z = 0 : R_{1,z} + Fcos(\theta) + R_{2,z} - Pcos(\theta) + R_{3,z} = 0 \tag{2.14}$$

$$v(x_2) = 0 : -\frac{1}{EI_{zz}}\left(\frac{R_{1,y}}{6}[x_2 - x_1]^3 - \int_0^{x_2}\int_0^{x}\int_0^{x}\int_0^{x} \tilde{q}(x)d\tilde{x}dx_a dx_b dx_c + \frac{Fsin(\theta)}{6}[x_2 - x_{A_1}]^3\right) + C_1x_2 + C_2 = 0 \tag{2.15}$$

$$w(x_2) = 0 : -\frac{1}{EI_{yy}}\left(-\frac{R_{1,z}}{6}[x_2 - x_1]^3 - \frac{Fcos(\theta)}{6}[x_2 - x_{A_1}]^3\right) + C_3x_2 + C_4 = 0 \tag{2.16}$$

$$w(x_1) : C_3x_1 + C_4 = -d_1 sin(\theta) \tag{2.17}$$

$$w(x_3) : -\frac{1}{EI_{yy}}\left(-\frac{R_{1,z}}{6}[x_3 - x_1]^3 - \frac{Fcos(\theta)}{6}[x_3 - x_{A_1}]^3 - \frac{R_{2,z}}{6}[x_3 - x_2]^3 + \frac{Pcos(\theta)}{6}[x_3 - x_{A_2}]^3\right) + C_3x_3 + C_4 = -d_3 sin(\theta) \tag{2.18}$$

$$v(x_1) : -\frac{1}{EI_{zz}}\left(-\int_0^{x_1}\int_0^{x}\int_0^{x}\int_0^{x} \tilde{q}(x)d\tilde{x}dx_a dx_b dx_c\right) + C_1x_1 + C_2 = d_1 cos(\theta) \tag{2.19}$$

$$v(x_3) : -\frac{1}{EI_{zz}}\left(\frac{R_{1,y}}{6}[x_3 - x_1]^3 - \int_0^{x_3}\int_0^{x}\int_0^{x}\int_0^{x} \tilde{q}(x)d\tilde{x}dx_a dx_b dx_c + \frac{Fsin(\theta)}{6}[x_3 - x_{A_1}]^3 + \frac{R_{2,y}}{6}[x_3 - x_2]^3 - \frac{Psin(\theta)}{6}[x_3 - x_{A_2}]^3\right) + C_1x_3 + C_2 = d_3 cos(\theta) \tag{2.20}$$

2.3.7 System of Equations

Unknown variable vector X:

$$\text{X} = \begin{bmatrix} R_{1,y} \\ R_{2,y} \\ R_{3,y} \\ R_{1,z} \\ R_{2,z} \\ R_{3,z} \\ F \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} \tag{2.21}$$

Coefficient Matrix A

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$$\mathcal{A} = \begin{pmatrix}
(l_a - x_1) & (l_a - x_2) & (l_a - x_3) & 0 & 0 & 0 & sin(\theta)(l_a - x_{A1}) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(l_a - x_1) & -(l_a - x_2) & -(l_a - x_3) & -cos(\theta)(l_a - x_{A1}) & 0 & 0 & 0 & 0 \\
(z_h - \tilde{z}) & (z_h - \tilde{z}) & (z_h - \tilde{z}) & 0 & 0 & 0 & (sin(\theta)(0 - \tilde{z}) - cos(\theta)) & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & sin(\theta) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & cos(\theta) & 0 & 0 & 0 & 0 \\
-\frac{(x_2 - x_1)^3}{EI_{zz}} & 0 & 0 & 0 & 0 & 0 & -\frac{sin(\theta)(x_2 - x_{A1})^3}{6EI_{zz}} & x_2 & 1 & 0 & 0 \\
0 & 0 & 0 & \frac{(x_2 - x_1)^3}{EI_{yy}} & 0 & 0 & \frac{cos(\theta)(x_2 - x_{A1})^3}{6EI_{yy}} & 0 & 0 & x_2 & x_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1 & 1 \\
0 & 0 & 0 & \frac{(x_3 - x_1)^3}{6EI_{yy}} & \frac{(x_3 - x_2)^3}{6EI_{yy}} & 0 & \frac{cos(\theta)(x_3 - x_A 1)^3}{6EI_{yy}} & 0 & 0 & x_3 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1 & 1 & 0 & 0 \\
-\frac{(x_3 - x_1)^3}{6EI_z z} & -\frac{(x_3 - x_2)^3}{6EI_z z} & 0 & 0 & 0 & 0 & -\frac{sin(\theta)(x_3 - x_{A1}}{6EI_z z} & x_3 & 1 & 0 & 0
\end{pmatrix}$$

Resultant vector f

$$F = \begin{bmatrix}
\int_{l_a}^{0} \int_{x}^{0} \tilde{q}(x) d\tilde{x} + Psin(\theta)(l_a - x_{A1}) \\
-Pcos(\theta)(l_a - x_{A1}) \\
\int_0^{l_a} \tau(\tilde{x}) d\tilde{x} - Pcos(\theta) y_p + Psin(\theta)(z_h - \tilde{z}) \\
Psin(\theta) + \int_0^{l_a} \tilde{q}(x) d\tilde{x} \\
Pcos(\theta) \\
-\frac{1}{EI_{zz}} \int_0^{x_2} \int_0^{x} \int_0^{x} \int_0^{x} \tilde{q}(x) dxdxdxdx \\
0 \\
-d_1 sin(\theta) \\
-\frac{Pcos(\theta)(x_3 - x_{A2})^3}{6EI_{yy}} - d_3 sin(\theta) \\
-\frac{1}{EI_{zz}} \int_0^{x_1} \int_0^{x} \int_0^{x} \int_0^{x} \tilde{q}(x) dxdxdxdx + d_1 cos(\theta) \\
-\frac{1}{EI_{zz}} \int_0^{x_3} \int_0^{x} \int_0^{x} \int_0^{x} \tilde{q}(x) dxdxdxdx - \frac{Psin(\theta)}{6EI_{zz}} + d_3 cos(\theta)
\end{bmatrix} \tag{2.22}$$

Now we can solve for the reaction forces by using the following system:

$$Ax = F \tag{2.23}$$