

Q5: How does Shor's algorithm threaten the security of RSA and Elliptic Curve Cryptography (ECC), and what are the potential consequences for current digital infrastructure?

Soln:

Shor's algorithm, developed by mathematician Peter Shor in 1994, poses a significant threat to widely used cryptographic systems, particularly RSA and Elliptic Curve Cryptography (ECC).

RSA is based on the difficulty of factoring large integers, while ECC depends on the difficulty of solving the elliptic curve discrete logarithm problem. Classical computers are unable to solve these problems efficiently.

However, Shor's algorithm, when run on a sufficiently powerful quantum computer, can break

A powerful quantum computer can solve both of these problems in polynomial time. Specifically, it can factor large integers and compute discrete logarithms exponentially faster than the best-known classical algorithms. This renders both RSA and ECC vulnerable to being broken, meaning that encrypted messages could be decrypted, digital signatures could be forged, and secure communications could be compromised.

The implications for digital infrastructure are profound. RSA and ECC are foundational to securing internet traffic, banking systems, emails, digital certificates, and more. If large scaled quantum computers become practical,

much of the world's digital security would be obsolete.

Data encrypted today could be stored and decrypted in the future - a threat known as "harvest now, decrypt later." This is supposed to spur a global race to develop quantum-resistant (post-quantum) cryptographic algorithms that can withstand quantum attacks.

In summary, Shor's algorithm undermines the

core assumptions that make RSA and ECC secure.

Its potential realization threatens the integrity, confidentiality, and trust of the entire digital ecosystem, necessitating a swift transition to quantum-safe cryptography.

Q2: Discuss the role of quantum key distribution

(QKD) in future cryptographic systems. How does it differ from classical public key encryption?

Soln: Instead of public keys were used.

→ Quantum key Distribution (QKD) is a revolutionary approach to secure communication that uses principles of quantum mechanics to distribute encryption keys between parties in a provably secure manner. Unlike,

classical public-key encryption, which relies

→ QKD is important for protecting communication against future quantum computer attacks.

Difference from classical public key encryption is given below;

Quantum Key Distribution (QKD)

i. Security based on physical laws of quantum mechanics, providing provable unconditional security.

ii. Keys are securely distributed over quantum channels using quantum states like photons.

iii. Any attempts to eavesdrop disturb the quantum state which can be immediately detected.

iv. Produces symmetric keys to be used for encryption/decryption after secure exchange.

v. Unconditional secure against quantum computer attacks.

Classical Public key Encryption

i. Security based on computational hardness assumptions

ii. Keys are exchanged or encrypted mathematically over classical communication channel.

iii. Eavesdropping can not be detected directly, security depends on difficulty of decryption with the private key.

iv. Uses public-private key-pairs for encryption/decryption and digital signature.

v. Vulnerable to quantum algorithm's like Shor's which can break many

not dependent just relies towards

classical crypto systems.

v. Require specialized
quantum hardware.

Implementation with
software and classical
hardware widely deployable

Q3: What is the main differences between

lattice based cryptography and traditional
number theoretical approaches like RSA, particularly
in the context of quantum resistance?

Ans:

Lattice Based Cryptography

1. Based on math
problems about point

in space called lattice

more secure and

Traditional Cryptography

1. Based on hard math
problems like
factoring large number.

- addition

Lattice Based Cryptography

Traditional Cryptography

2. Considered safe against quantum computers, no known quantum attack can easily break it.

3. Keys and ciphertext are usually large in size.

4. Algorithms often use simple math operations like addition and multiplication on vectors

5. Supports advanced functions like homomorphic encryption and identity based encryption.

2. Vulnerable to quantum computers, using Shor's algorithm which can break it faster.

3. Keys are smaller and faster to use.

4. Uses complex operations like modular exponentiation.

5. Mostly supports basic encryption and digital signatures.

Question 4:

Develop a python based PRNG that uses the current system time and a custom seed value. Write a complete program and corresponding output.

Ans: PRNG means Pseudo - Random Number

Generators.

import time

#Simple PRNG using linear congruence

def prng(custom_seed):

$m = 2^{48}$

$A = 25214903917$

$C = 11$

#mix time and seed

state = (custom_seed ^ time.time_ns()) % m

#generate 10 random integers

for i in range(10)

$$\text{state} = (a * \text{state} + c) \% m$$

print(state)

#Example run

custom_seed = 2025

prng(custom_seed)

Example Output:

1 8 7 5 8 5 9 5 0 6 8 8 4 1 7

1 1 3 7 3 9 0 2 7 9 0 4 3 9 8

1 0 5 2 8 1 3 5 1 5 0 6 8 2 9 3

2 9 4 3 3 4 9 0 1 9 6 6 1 2 4 6

Ques 5:

Explain the sieve of Eratosthenes algorithm and use it to find all prime numbers less than 50. How does its time complexity compare to trial division?

Ans: Explain the Sieve of Eratosthenes algorithm and use it to find all prime numbers less than 50

Ans:

→ Sieve Eratosthenes Algorithm is an effective way to find all prime numbers up to a given number n . It works by iteratively marking the multiples of each prime number starting from 2.

How it works:

1. Create a list of numbers from 2 to n .
2. Start with the first prime number 1.
3. Mark all multiples of 2 (4, 6, 8, ...) as not prime.
4. Move to the next unmarked number (3) and mark all multiples of 3 (6, 9, 12, ...).
5. Repeat this for the next unmarked numbers (5, 7, 11, ...) up to n .
6. Remaining unmarked numbers are prime.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

? medium (medium)

Prime less than 50: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37,

41, 43, 47

Time Complexity: medium (medium) to fast

Sieve of Eratosthenes: medium (medium)

Time complexity is approximately $O(n \log \log n)$

which is very efficient for generating all primes up to n .

Trial division: $E = (m, n) \log \log n$

Time complexity is $O(n\sqrt{n})$ - much slower for large n .

→ Sieve of is significantly faster than trial division

when generating a prime number

→ Trial division is better for checking if a single number is prime. not generating many.

Ques 6: State and Explain the necessary and sufficient conditions for a composite number to be a Carmichael number. Then verify whether the numbers $n=561$ and $n=1105$ and $n=1729$ are Carmichael numbers?

Ans: A ~~char~~ Carmichael number is a special type of composite number \cancel{n} that fools Fermat's primality test.

A composite number \cancel{n} is called a Carmichael number if $a^{n-1} \equiv 1 \pmod n$ for all integers such that $\gcd(a, n) = 1$.

Although, \cancel{n} is not prime, it still behaves like a prime number under Fermat's test.

A composite Necessary and Sufficient

Conditions (Korselt's criterion)

Let n be a composite number. Then

n is a Carmichael number if and only if:

1. n is square-free (no prime-factor is repeated)

2. For every divisor p of n , it holds that $\frac{p-1}{n-1}$

Verify. The three numbers $b=1$ satisfies $P-1 \mid n-1$

We apply Korselt's criteria to each n

1. $n = 561$

\rightarrow factorization $561 = 3 \times 11 \times 17$ (composite)

\rightarrow Square-free : Yes (distinct prime)

$\rightarrow n-1 = 560$

For $P=3 : P-1=2 \rightarrow 560/2 = 280$ (divides)

for $P=11 : P-1=10$

$\rightarrow 560/10 = 56$ (divides)

for $P=17 : P-1=16$

$\rightarrow 560/16 = 35$ (divides)

All condition satisfied. So 561 is a Carmichael number.

2. $n = 1105$

\rightarrow Factorization $1105 = 5 \times 13 \times 17$ (composite)

\rightarrow Square-free : Yes.

$\rightarrow n-1 = 1104$ with modulus of 4 and 16

$P=5 ; P-1=4 , 1104/4 = 276$ (divides)

$P=13 ; P-1=12 , 1104/12 = 92$ (divides)

$P=17 ; P-1=16 , 1104/16 = 69$ (divides)

All condition satisfied, so, 1105 is a Carmichael number.

3. $n=1729$

\rightarrow Factorization : $1729 = 7 \times 13 \times 19$ (composite)

\rightarrow Square free = Yes

$\rightarrow n-1 = 1728$

$P=7 ; P-1=6 ; 1728/6 = 288$ (divides)

$P=13 ; P-1=12 ; 1728/12 = 144$ (divides)

$P=19 ; P-1=18 ; 1728/18 = 96$ (divides)

All conditions satisfied, so, 1729 is a Carmichael number.

So, All these three numbers are

Carmichael numbers.

Ques 7: Determine whether the following are valid algebraic structures and justify your answer:-

i. Is the set \mathbb{Z}_{11} with operations $(+, \cdot)$ a ring?

ii. Are the sets $(\mathbb{Z}_{37}, +)$ and $(\mathbb{Z}_{35}, \times)$ Abelian groups?

Ans:

(i) Yes, \mathbb{Z}_{11} is a ring.

$\rightarrow \mathbb{Z}_{11}$ is the set of integers modulo 11.

$$\mathbb{Z}_{11} = \{0, 1, 2, -1, \dots, 10\}$$

\rightarrow Addition and multiplication are defined modulo 11.

\rightarrow A ring requires:-

1. $(\mathbb{Z}_{11}, +)$ is an Abelian group (for every a , there is a $-a$ mod 11).
closure, associativity, identity(0), inverse and commutativity are all satisfied.

2. (\mathbb{Z}_{11}, \cdot) is closed and associative.

3. Distributive property holds.

$$a \cdot (b+c) \equiv a \cdot b + a \cdot c \pmod{11}$$

All these properties of ring is satisfied, So, \mathbb{Z}_{11} is a ring.

(ii) a) $(\mathbb{Z}_{37}, +)$ is an Abelian group

→ It consists of integers modulo 37:

$$\mathbb{Z}_{37} = \{0, 1, \dots, 36\}$$

→ Operation: Addition modulo 37.

→ Closure, associativity, identity 0, inverse

($\forall a \in \mathbb{Z}_{37}, \exists -a \text{ mod } 37$), Commutativity

So, $(\mathbb{Z}_{37}, +)$ is an Abelian group.

b) $(\mathbb{Z}_{35}, \times)$

This is the set of units modulo 35 under multiplication.

$$\mathbb{Z}_{35}^*$$

units mod 35

units mod 35

units mod 35

$$(ab)^{-1} = b^{-1}a^{-1} \equiv (ba)^{-1}$$

units mod 35

Ques 8: What is the remainder when -52 is reduced modulo 31?

Step 1: Write the congruence idea

We want r such that $-52 \equiv r \pmod{31}$

and $0 \leq r < 31$

Step 2: Add multiples of 31 until we get a non-negative remainder

$$-52 + 31 = -21 \text{ (still negative)}$$

$$-21 + 31 = 10 \text{ (non negative and less than 31)}$$

Step 3: $-52 \equiv 10 \pmod{31}$

* Ques 9: Determine the multiplicative inverse of 7 mod 26 if it exists. (Use extended Euclidean Algorithm).

Ans:

We want to find the multiplicative inverse of 7 mod 26 - meaning we want an integer x such that:

$$7x \equiv 1 \pmod{26}$$

Step 1: Checks if inverse exists. (Apply Euclidean Alg.)

An inverse exists if $\gcd(7, 26) = 1$

$$26 = 3 \times 7 + 5$$

$$7 = 1 \times 5 + 2$$

$$5 = 2 \times 2 + 1$$

$$2 = 2 \times 1 + 0$$

$\gcd(7, 26) = 1$ means inverse exists.

Step 2: Extended Euclidean Algorithm (Back substitute to find x)

$$1 = 5 - 2 \times 2$$

$$1 = 5 - 2 \times (7 - 1 \times 5)$$

$$\therefore 1 = 5 - 2 \cdot 7 + 2 \cdot 5$$

$$1 = 3 \cdot 5 - 2 \cdot 7$$

$$1 = 3 \cdot (26 - 3 \cdot 7) - 2 \cdot 7$$

$$1 = 3 \cdot 26 - 9 \cdot 7 - 2 \cdot 7$$

$$\therefore 1 = 3 \cdot 26 - 11 \cdot 7$$

Step 3: Identify inverse

$$1 = 3 \cdot 26 - 11 \cdot 7$$

$$\therefore -11 \cdot 7 \equiv 1 \pmod{26}$$

$$\therefore x = -11$$

Now, we reduce $-11 \pmod{28}$. Note: ~~it is -11~~

To convert $-11 \equiv 25 \pmod{28}$ it is to self

So, the multiplicative inverse of $7 \pmod{28}$ is 15 .

Ques 10:

Evaluate $(-8 * 5) \pmod{17}$, and explain how to simplify negative modular multiplication.

Ans: Step 1: $\cancel{-8 * 5}$ Multiply $\therefore -8 * 5 = -40$

Step 2: Reduce modulo 17

$-40 \pmod{17}$

To get a positive remainder, we can add 17 repeatedly until the result is in the range $0 \leq r < 17$.

$$-40 + 17 = -23 \text{ (still negative)}$$

$$-23 + 17 = -6 \text{ (still negative)}$$

$$-6 + 17 = 11 \text{ (positive)}$$

$$\therefore (-8 * 5) \pmod{17} = 11$$

(Ans)

Ques 11: State and prove Bézout's Theorem.

Use it to find the multiplicative inverse of 97 modulo 385.

Ans: Bézout's Theorem also called Bézout's Identity.

Let a and b be integers, not both zero. Then there exist integers x and y such that:

$$ax + by = \gcd(a, b)$$

Moreover, the gcd of ~~a, b~~ a and b is the smallest positive integer that can be expressed as a linear combination of a and b .

If $\gcd(a, m) = 1$, then Bézout theorem tells us there exists an x such that:

$$ax + by = my = 1$$

$$\Rightarrow ax \equiv 1 \pmod{m}$$

that x is the multiplicative inverse of a mod m .

Use Bézout theorem to find the inverse of $97 \text{ mod } 385$
we have to solve, $97x \equiv 1 \pmod{385}$

$$\Rightarrow 97x + 385y = 1$$

We solve this by using Extended Euclidean Algorithm

Step 1. Apply Euclidean Algorithm

$$\gcd(385, 97) : 385 = 3 \cdot 97 + 94$$

$$97 = 1 \cdot 94 + 3$$

$$94 = 31 \cdot 3 + 1$$

unit 3 module repeat until $3 \cdot 1 + 0$

$\therefore \gcd(385, 97) = 1 \rightarrow \text{inverse exists.}$

Step 2: Extended Euclidean Algorithm (Back-Substitution)

$$1 = 94 - 31 \cdot 3$$

$$\Rightarrow 1 = 94 - 31 \cdot (97 - 1 \cdot 94)$$

$$\Rightarrow 1 = 94 - 31 \cdot 97 + 31 \cdot 94$$

$$\Rightarrow 1 = 32 \cdot 94 - 31 \cdot 97, \quad u + d, p = 0$$

$$\Rightarrow 1 = 32 \cdot (385 - 3 \cdot 97) - 31 \cdot 97, \quad u + d, p = 0$$

$$= 32 \cdot 385 - 96 \cdot 97 - 31 \cdot 97, \quad u + d, p = 0$$

$$1 = 32 \cdot 385 - 127 \cdot 97$$

$$97 \cdot (-127) + 385 \cdot 32 = 1$$

$\therefore -127$ is the reverse of $97 \pmod{385}$

We want positive inverse, so reduce

$$-127 \pmod{385} = 385 - 127 = 258$$

(Ans)

Ques 12:

Using Bézout's identity, prove that the equation $ax + by = \gcd(a, b)$ has integer solutions. Find x such that $43x = 1 \pmod{240}$.

Ans:

Let a, b be integers, not both zero, and let $d = \gcd(a, b)$. Use Euclidian algorithm to compute d .

$$a = q_1 b + r_1, \quad 0 \leq r_1 < b$$

$$b = q_2 r_1 + r_2 \quad (r_2 < b)$$

$$r_1 = q_3 r_2 + r_3$$

$$r_2 = q_4 r_3 + r_4$$

$$r_{k-2} = q_k r_{k-1} + r_k$$

$$r_{k-1} = q_{k+1} r_k + 0$$

$$25 + 3P \cdot 3 = 3P$$

$$81 + 25 \cdot 3 = 3P$$

$$4 + 25 \cdot 3 = 3P$$

So, $r_k = d$. Each remainder r_i is an integer linear combination of a and b because $r_1 = a - q_1 b$ and therefore $r_{i+1} = r_{i-1} - q_{i+1} r_i$. By back substitution we express r_k as $r_k = xa + yb$ for some integers x, y .

Hence d is an integer linear combination of a and b . Since any common divisor of a and b divides every linear combination, d is the smallest positive such combination and we have Bézout's identity:

$$ax + by = \gcd(a, b)$$

→ Find x with $43x \equiv 1 \pmod{240}$

We need an integer x with $43x + 240y = 1$. Use the Euclidean algorithm.

$$25 = 43 \cdot 0 + 25$$

$$25 = 43 \cdot 1 + 12$$

$$12 = 43 \cdot 0 + 12$$

$$12 = 43 \cdot 1 + 1$$

$$1 = 43 \cdot 1 - 43 \cdot 1 = 1$$

$$25 = 43 \cdot 0 + 25$$

$$25 = 43 \cdot 1 + 12$$

$$12 = 43 \cdot 0 + 12$$

$$12 = 43 \cdot 1 - 43 \cdot 1 = 1$$

$$240 = 5 \cdot 43 + 25$$

$$43 = 1 \cdot 25 + 18$$

$$25 = 1 \cdot 18 + 7$$

$$18 = 2 \cdot 7 + 4$$

$$7 = 1 \cdot 4 + 3$$

$$4 = 1 \cdot 3 + 1$$

$$3 = 3 \cdot 1 + 0$$

Back-substitute to express 1 as a combination

of 43 and 240

$$1 = 4 - 1 \cdot 3$$

$$= 4 - 1 \cdot (7 - 1 \cdot 4)$$

$$= 4 - 1 \cdot 7 + 1 \cdot 4 = 2 \cdot 4 - 1 \cdot 7$$

$$= 2 \cdot 4 \cdot (18 - 2 \cdot 7) - 1 \cdot 7$$

$$= 2 \cdot 18 - 4 \cdot 7 - 1 \cdot 7$$

$$= 2 \cdot 18 - 5 \cdot 7$$

$$= 2 \cdot 18 - 5 \cdot (25 - 1 \cdot 18)$$

$$= 2 \cdot 18 - 5 \cdot 25 + 5 \cdot 18$$

$$= 7 \cdot 18 - 5 \cdot 25 \quad : 1 = 67.43 -$$

$$= 7 \cdot (43 - 1 \cdot 25) - 5 \cdot 25$$

$$= 7 \cdot 43 - 1 \cdot 25$$

$$= 7 \cdot 43 - 1 \cdot (240 - 5 \cdot 43)$$

$$12.240$$

\therefore So, $67 \cdot 25 + (-12) \cdot 240 \equiv 1$, Thus $x = 67$ is a solution, it gives the multiplicative inverse of 43 modulo 240.

Reducing modulo 240, the inverse is,

$$(1-9) \cdot 0 \quad S.O. L.O.$$

$$43^{-1} \equiv 67 \pmod{240}$$

Ques 13: Prove Fermat's Little Theorem and explain how it is used to test for primality. Is 561 a prime number based on this test? Evaluate $5^{123} \pmod{175}$ using Fermat's Little Theorem. Show all steps.

Ans:

If p is prime and a is an integer with $\gcd(a, p) = 1$ then $a^{p-1} \equiv 1 \pmod{p}$

Proof (standard multiplicative-residue proof): Consider the nonzero residue classes modulo $p: \{1, 2, \dots, p-1\}$. Multiplying each element by a (with $\gcd(a, p) = 1$) permutes this set modulo p . So $a \cdot 1, a \cdot 2, \dots, a \cdot (p-1)$ is congruent modulo p to some permutation of $1, 2, \dots, p-1$. Taking the product of the terms in both lists gives

$$a^{p-1} \cdot (1 \cdot 2 \cdots (p-1)) \equiv 1 \cdot 2 \cdots (p-1) \pmod{p}$$

Since $1 \cdot 2 \cdots (p-1)$ is not divisible by p , we may cancel it in the field $\mathbb{Z}/p\mathbb{Z}$, yielding $a^{p-1} \equiv 1 \pmod{p}$.

Using Fermat's theorem as a primality test

- Pick an integer a with $1 < a < n$.
- Compute $a^{n-1} \pmod{n}$.

• If $a^{n-1} \not\equiv 1 \pmod{n}$, then n is definitely composite.

• If $a^{n-1} \equiv 1 \pmod{n}$, then n is probably prime.

→ Is 561 prime based on this test?

$$561 = 3 \times 11 \times 17$$

For any a with $\gcd(a, 561) = 1$, by Fermat's theorem,

$$a^2 \equiv 1 \pmod{3}, \quad a^{10} \equiv 1 \pmod{11}, \quad a^{16} \equiv 1 \pmod{17}.$$

$\text{lcm}(2, 10, 16) = 80$ and 560 is a multiple of each of

2, 10, 16, we get $a^{560} \equiv 1 \pmod{\text{each of } 3, 11, 17}$.

By Chinese Remainder Theorem this implies

$$a^{560} \equiv 1 \pmod{561}$$

for every a with $\gcd(a, 561) \neq 1$, the Fermat test will say 'probably prime' for 561, but 561 is actually composite - so, 561 is a Carmichael number.

$\rightarrow 175 \neq 25 \cdot 7$. Because $\gcd(5, 175) = 5 \neq 1$, Fermat's theorem does not apply directly to modulo 175.

1. Modulo 25

For $n \geq 2$, $5^n = 25 \equiv 0 \pmod{25}$. Hence for any exponent ≥ 2 ,

$$5^{123} \equiv 0 \pmod{25}$$

2. Modulo 7

$\gcd(5, 7) = 1$, so we can use Fermat: $5^6 \equiv 1 \pmod{7}$.

Compute exponent reduction: $123 \equiv 3 \pmod{6}$. Thus

$$5^{123} \equiv 5^3 \pmod{7}$$

$$5^2 = 25 \equiv 4 \pmod{7} \quad \text{so } 5^3 \equiv 5 \cdot 4 = 20 \equiv 6 \pmod{7}$$

$$5^{123} \equiv 6 \pmod{7}$$

3. Combine by CRT

$$x \equiv 0 \pmod{25}, \quad x \equiv 6 \pmod{7}$$

$x = 25k$. Then $25k \equiv 6 \pmod{7}$. Since $25 \equiv 4 \pmod{7}$.

$$4k \equiv 6 \pmod{7}$$

Multiply both sides by the inverse of 4 modulo 7.

Since $25 \equiv 4$ the inverse is $4^{-1} \equiv 2 \pmod{7}$

$$k = 5 + 7t \text{ and } x = 25k = 25(5 + 7t) = 125 + 175t$$

The last residue modulo 175 is

$$x \equiv 125 \pmod{175}$$

$$\text{Ans: } 5^{123} \equiv 125 \pmod{175}$$

Ques 14. State and prove the Chinese Remainder Theorem
Then solve the following system of congruence.

$$x \equiv 2 \pmod{3}, x \equiv 3 \pmod{5}, x \equiv 2 \pmod{7}$$

Ans:

State: Let m_1, m_2, \dots, m_k be pairwise coprime positive integers such that $\gcd(m_i, m_j) = 1$ for $i \neq j$

for any integers a_1, \dots, a_k the system

$$x \equiv a_i \pmod{m_i} \quad \text{for } i=1, 2, \dots, k$$

has a unique solution modulo $m_1 m_2 \dots m_k$.

Proof:

Let $M = \prod_{i=1}^k m_i$ and for each i put $M_i = \frac{M}{m_i}$

Since $\gcd(M_i, m_i) = 1$, there exists an inverse y_i with

$$M_i y_i \equiv 1 \pmod{m_i}$$

$$\text{Then } n = \sum_{i=1}^k (a_i M_i y_i)$$

Satisfies $x \equiv a_i \pmod{m_i}$ for every i (because all terms with $j \neq i$ vanish modulo m_i and $M_j y_j \equiv 1 \pmod{m_j}$)
So, solution exists.

$$\begin{cases} x \equiv 2 \pmod{3} \\ x \equiv 3 \pmod{5} \\ x \equiv 2 \pmod{7} \end{cases}$$

Hence $m_1 = 3, m_2 = 5, m_3 = 7$ are pairwise coprime

$$\text{and } M = 3 \times 5 \times 7 = 105$$

$$i \quad m_i \quad a_i \quad M_i = \frac{M}{m_i} \quad y_i \equiv M_i^{-1} \pmod{m_i} \quad a_i M_i y_i$$

$$1 \quad 3 \quad 2 \quad 35 \quad 35 \equiv 2 \pmod{3} \quad 2 \times 35 \times 2$$

Substitution basis $y_1 = 2$ $= 140$

$$2 \quad 5 \quad 3 \quad 21 \quad y_2 = 1 \quad 3 \times 21 \times 1 \\ = 63$$

$$3 \quad 7 \quad 2 \quad 15 \quad y_3 = 1 \quad 30$$

Now, $140 + 63 + 30 = 233$

$$\Rightarrow 233 \pmod{105} \\ = 23$$

(Ans)

Ques 15:

Briefly explain the CIA triad in information security. How does each component contribute to building a secure system?

Ans:

The CIA triad is a fundamental model for designing and evaluating security policies consist of:-

Confidentiality:

- (i) Definition: Ensures that information is accessible only to authorized individuals
- (ii) Goal: Prevent unauthorized access or disclosure
- (iii) Example: Using encryption and access control.
- (iv) Contribution to Security: Protects sensitive data from hackers, eavesdroppers and unauthorized access (users).

Integrity:

- (i) Definition: Ensures data is accurate, complete and unaltered except by authorized person
- (ii) Goal: Prevent unauthorized modification
- (iii) Example: Digital signature, checksum and hashing

- (iv) Contribution to Security: Maintain trust in data by ensuring it's not tampered with

are corrupted

Availability:

(i) Definition: Ensures authorized users have reliable and timely access to information and resources.

(ii) Goal: Prevent disruption of services.

(iii) Example: Redundant systems, backups and DDOS protection

(iv) Contribution to security: Ensures that data and systems are accessible when needed, even during failure or attacks.

Ques 16: How does steganography differ from cryptography in the context of information security, and what are common techniques used for hiding data in digital media?

Ans. Cryptography: It is the process of converting information into an unreadable format (ciphertext) to protect its content from unauthorized access. The presence of the message is visible but its meaning is hidden.

Steganography: It is the practice of hiding the very existence of information by embedding it inside another medium.

Cryptography	Steganography
① Protect the content of a message.	① Hide the existence of a message.
② Message is visible but unreadable	② Message is invisible to casual observers
③ Message is known to exist	③ Message is hard to detect

Common steganography techniques in digital media:-

1. Least Significant bit (LSB) insertion: Change the smallest bits in image pixels to hide data.
2. Masking and filtering: Hide data in important parts of an image (like watermark). So, it's harder to remove.
3. Transform domain techniques: Hide data in frequency parts of media instead of directly in pixels.
4. Meta data Hiding: Put secret data into unused file information fields (metadata) without changing the visible content.

Ques 17: what are the key difference between phishing, malware and denial of service (DoS) attack in terms of their method and impact of system security?

Ans: Here is the difference between phishing, Malware and denial-of-service (DoS) attack :-

Attack type	Method	Impact on Security
Phishing	Tricking users(usually via fake emails, websites or messages) into revealing sensitive information like Password, bank details, or credit card numbers.	Compromise Confidentiality by stealing personal or log in data
Malware	Installing malicious software(virus, worm, Trojan, ransomware, spyware), on a system often via downloads, email attachments or infected sites	Can harm confidentiality, integrity and availability by stealing data, modifying files or disabling systems.
Denial of Service (DoS)	Flooding a network, server or website with excessive traffic to overload and make it unavailable to users.	Targets availability by disrupting normal access to services.

Ques-18 : Explain how legal frameworks such as General Data Protection Regulation (GDPR) help mitigate cyber attacks and protect user privacy :

Ans:

The ~~GDPR~~ is a legal framework in the European Union designed to strengthen data privacy, data security and user rights. It helps mitigate cyber attacks and protect user privacy in several ways:

1. Data Protection by Design and by Default.

→ Security must be built into systems from the start.

2. Data Minimization and Purpose Limitation:

Collect and store only the necessary data.

3. Security and Breach Notification: Use strong

safeguards and report breaches within 72 hours.

4. Accountability & Penalties: Show compliance

or face heavy fines.

5. User's Rights and Transparency:

Users can access, delete or transfer their data any time.

How GDPR helps against Cyber attacks.

Cybersecurity Threat

Data breaches

Encryption, access control and breach notification requirements

Phishing/identity theft

Data minimization reduces

Insider threats

Strict accountability and audit trials

Ransomware

Backup and recovery plans enforced through security obligations

Unauthorized tracking

User consent and clear data usage policies.

Ques-19: Explain the basic working of the DES

Algorithm using a single 64 bit plaintext block and a 56 bit key show how the initial permutation, round function and final permutation contribute to the encryption process.

Ans: DES basic working.

→ Using 64 bit plaintext and 56 bit key.

Step 1: Input block and key

→ plaintext 64 bits

→ key 64 bits input but only 56 bits are used.
(8 bits for parity)

Step - 2: Initial permutation (IP)

→ The 64 bit plaintext undergoes initial permutation (IP) using a fix table.

→ The bits are re-arranged according to the IP table.

Output: Two 32 bit halves, L₀ and R₀.

Step 3: 16 Round of feistel structure

Each Round has:

→ Expansion (E)

→ Key mixing (XOR)

→ Substitution (S_n box)

→ Permutation (P)

Steps Per Round:

a. Expansion (E-box)

$R(n-1)$ 32 bits → expand to 48 bits using expansion table

b. key mixing

→ The 48 bit expanded $R(n-1)$ is XORed with 48 bit Subkey k_{n-1}

→ Subkeys are generated from the 56 bit main key one for each round

→ final output

c. Substitution (S-box)

- The 32-bit result is divided into 8 blocks of 4 bits
- Each block goes through an S-box (Substitution box), outputting 4 bit each.
- Final result : 32 bits.

d. Permutation (P-box)

- The 32-bit output from S-box is permuted again using a fixed table.
- Increases confusion and diffusion

e. Feistel Swap

→ New values: $L(n) = R(n-1)$

$$R(n) = L(n-1) \text{ XOR } f(R(n-1), k_n)$$

f. Repeat for 16 Rounds

- Each round uses a different 48 bit subkeys generated from the original key using PC-1 shifts and PC-2 tables.

5. Final Permutation (FP):

After Round 16, R_{16} and L_{16} are swapped, and then the final permutation / inverse of FP is applied.

Ques 20:

Given that,

$$R_0 = \text{0xF0 F0 F0 F0}$$

$$K_1 = \text{0X0F0F0F0F0F0}$$

$$L_0 = \text{0x = AAAAAAAA}$$

Step-1: First Round function (only NOR)

$$f(R_0, K_1) = R_0 \oplus K_1$$

$$= 0X F0 F0 F0 F0 \oplus 0X0F0F0F0F0$$

$$= 0X FFFFFFFF$$

Step-2: Compute L_1 and R_1

$$L_1 = R_0 \rightarrow 0XFOFOFOFO$$

$$R_1 = L_0 \oplus P(R_0, k_1) = 0XA\overset{A}{AAA}AAA \oplus 0XFFFFFFFFFF$$
$$= 0X5555555555$$

$$L_1 = 0XFOFOFOFO \quad \text{and} \quad R_1 = 0X5555555555$$

Ques 2:

Ans: $0X23 \rightarrow R_0 \text{ w=2}, \text{ col=3} \rightarrow D_4$

$0XA7 \rightarrow R_{w=1}, \text{ col=7} \rightarrow 63$

$0X4E \rightarrow R_{w=4}, \text{ col=10} \rightarrow 2E$

$0X19 \rightarrow R_{w=1}, \text{ col=9} \rightarrow C_6$

Output word $\{D_4, 63, 2E, C_6\}$

When no value found given, N/A (not available)

$0X0$

Ans: 23. Given that,

$$C_01 = [0x01, 0x02, 0x03, 0x04]$$

$$= [00000001, 00000002, 00000003, 00000000]$$

Mix columns Matrix.

$$M_2 = \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix}$$

Step 1: First Row of output

$$= 0x02 \oplus 0x01 \oplus 0x03 \cdot 0x02 \oplus 0x01 \cdot 0x03$$

$$\oplus 0x01 \cdot 0x04$$

$$= 0x02 \oplus 0x06 \oplus 0x05 \oplus 0x04$$

$$= 00000010 \oplus 00000010 \oplus 00000011 \oplus$$

$$00000100$$

$$= 00000011 \quad \text{using result below on next}$$

$$= 0x03$$

Step-2: Second Row Output

$$\begin{aligned} & 0x01 \cdot 0x01 \oplus 0x02 \cdot 0x02 \oplus 0x03 \cdot 0x03 \oplus 0x01 \cdot 0x04 \\ &= 0x01 \oplus 0x04 \oplus (0x03 \cdot 0x02 \oplus 0x03 \cdot 0x01) \oplus \\ &\quad 0x03 \oplus 0x04 \quad \text{Note: } 0x01 \cdot 0x04 \\ &= 0x01 \oplus 0x04 \oplus (0x06 \oplus 0x03) \oplus 0x04 \\ &= 0x01 \oplus 0x04 \oplus 0x05 \oplus 0x04 \\ &= 0x00000001 \oplus 00000000 \oplus 00000101 \oplus \\ &= 00000100 \quad = 0x04 \quad \text{Ans} \end{aligned}$$

Step 3: Third Row output

$$\begin{aligned} & 0x01 \cdot 0x01 \oplus 0x01 \cdot 0x02 \oplus 0x02 \cdot 0x03 \oplus 0x03 \cdot 0x04 \\ &= 0x01 \oplus 0x02 \oplus 0x03 \oplus ((0x02 + 0x01) \cdot 0x04) \\ &= 0x01 \oplus 0x02 \oplus 0x03 \oplus (0x02 \cdot 0x04 \oplus 0x01 \cdot 0x04) \\ &= 0x01 \oplus 0x02 \oplus 0x03 \oplus 0x08 \oplus 0x04 \\ &= 00000001 \oplus 00000010 \oplus 00000011 \oplus 00001000 - \\ &\quad 00000100 \\ &= 00001001 \quad = 0x09 \end{aligned}$$

Step 4: Fourth Row output

$$= (0x03, 0x01) \oplus (0x01, 0x02) \oplus (0x01, 0x03)$$

$$\oplus (0x02, 0x04)$$

$$(10x0, 20x0 \oplus 30x0, 40x0) \oplus$$

$$= 0x03 \oplus 0x02 \oplus 0x03 \oplus 0x08$$

$$= 0x0011 \oplus 0x0010 \oplus 0x0011 \oplus 0x1006$$

$$= \cancel{0x1007} \oplus 0x1010 \oplus 0x0000 \oplus 10x0 =$$

$$= 0x0A$$

$$10100000 \oplus 00100000 \oplus 10000000 =$$

$$\text{Output} = [0x03, 0x04, 0x09, 0x0A]$$

$$P0X0 = 00100000$$

24

AES-OFB:-

using with brief: egfr

is a stream cipher mode of AES

encription $(0x01, 0x02, 0x03, 0x04) \oplus P0X0 \oplus S0X0 \oplus 10X0 =$

→ It turns the block cipher (AES) into a

synchronous stream cipher using feedback

$P0X0 \oplus S0X0 \oplus 10X0 \oplus 00100000 \oplus 01000000 \oplus 10000000 =$

00100000

$P0X0 = 10010000 =$

- Working Procedure:
- ① Initialization Vector (IV) is chosen randomly and shared with the receiver.
 - ② IV is encrypted using AES and a security key → This generate the first output block.
 - ③ This output block is XORed with the plaintext block to produce the ciphertext.
 - ④ The same output block is used as the input for AES in the next round (not the ciphertext).
 - ⑤ This repeats for every blocks.

Next-Input = AES(Previous_output)

- ⑥ Description uses the same output stream to XOR with the ciphertext → gives the original plaintext

Synchronization between encryption and decryption

decryption:

→ Both sender and Receiver uses the same IV and key and perform the same AES encryption

→ Since the same sequence of output blocks is generated at both ends, they remain synchronized.

→ Even if plaintext/cipher text is lost, they will still stay in sync as long as IV is the same.

Ans 25:

CBC (Cipher Block Chaining) and CFB (cipher Feedback) modes both causes error propagation.

→ A single bit errors in the cipher text will affect multiple blocks in decryption.

In CBC Mode:

→ Each cipher text block is used to decrypt the next plaintext.

→ So, if one cipher text block is corrupted

① The current plaintext block become garbled

② The next block is affected due to XOR with corrupted block

Effect of 1 error affects 2 blocks.

In CFB Mode:

→ Each cipher text block is fed back into the encryption process.

→ If a cipher text block is corrupted

① The corresponding plaintext block become

wrong

→ The next one is also affected

Impact on integrity:

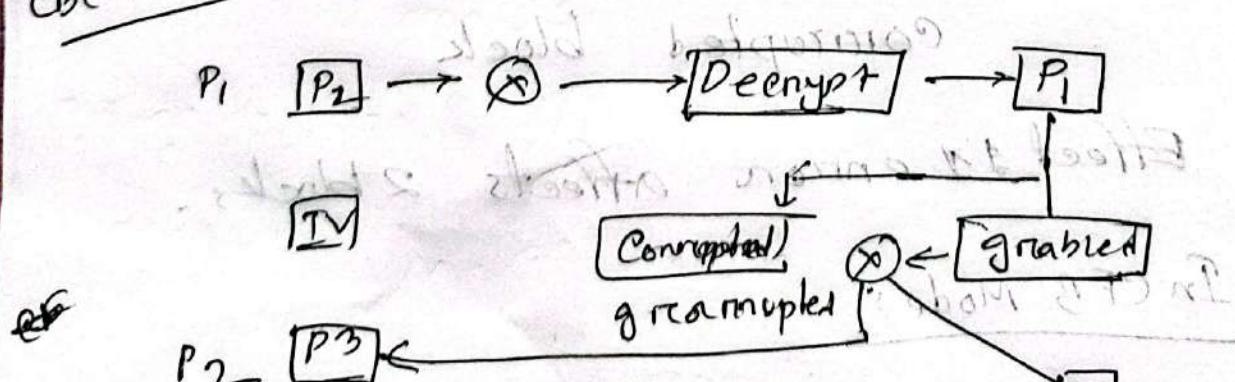
→ Loss of integrity means wrong plaintext is recovered

→ Need for Message Authentication Code (MAC)

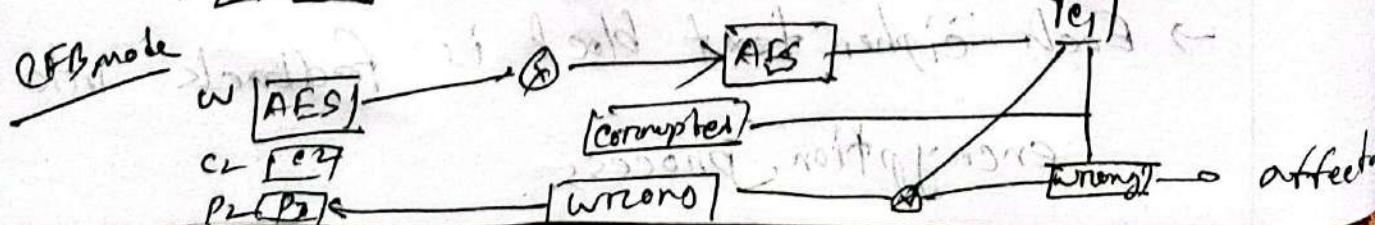
or Authentication Encryption to ensure connection

Error Propagation in AES

CBC Mode



CFB mode



Impact:

Single bit errors in ciphertext causes multiple bit errors in decrypted outputs

(CBC/OFB) propagating error

Ans 2 G

Recommended Mode \rightarrow AES - CTR (Counter Mode)

Justification:

i) Support Parallel Processing:

Each block is encrypted independently using a counter value; allowing simultaneous encryption/decryption

ii) No block dependency: CTR does not rely on previous ciphertext blocks, like CBC, so it's faster and more efficient.

iii) Maintain data confidentiality: CTR hides repeating patterns in plaintext, unlike ECB which

leaks structures.

- ④ Ideal for large files: Works best for big files where speed and efficiency matter.
- ⑤ Error does not propagate: A single bit error affects only the corresponding block, making recovery easier.
- ⑥ Flexible counter based design

Ans 27:

Using the RSA encryption formula.

$$C = m^e \bmod n$$

$$C = 1^5 \bmod 14$$

$$C = 1 \bmod 14$$

Ciphertext $c = 1$

Decryption: $M = C^d \bmod n$

$$= 1^{11} \bmod 14$$

$$= 1 \bmod 14$$

Ans:28 Given that, $H(m) = 5$

$$d = 3$$

$$n = 33$$

Digital Signature Formula: $S = (H(m))^d \text{ mod } n$

$$= 5^3 \text{ mod } 33$$

$$= 125 \text{ mod } 33$$

$$S = 26$$

∴ Digital signature = 26

Ans:29: Given that, $m = 181$

Prime module $p = 17$

Base (generator) $g = 3$

Aleya's private key = 4

Babol's private key = 5

Step 1: Aleya's public key,

$$A = g^a \text{ mod } p$$

$$A = 3^4 \text{ mod } 17$$

$$\therefore A = 13 \therefore \text{Aleya's public key} = 13$$

Step 2: Badol's public key.

$$B = g^b \bmod p$$

$$= 3^5 \bmod 17$$

$$= 243 \bmod 17$$

$$= 5$$

∴ Badol's public key = 5

Ans 30: Given,

Hash = (Sum of ASCII values of characters in M)

$$\bmod 100$$

Message "AB" student

$$\text{ASCII of A} = 65$$

$$\text{ASCII of B} = 66$$

$$\text{Sum} = 65 + 66 = 131$$

$$\text{Hash : } 131 \bmod 100 = 31$$

message "BA"

$$\text{Sum} = 66 + 65 = 131$$

$$\text{Hash : } 131 \bmod 100 = 31 = A$$

This is a collision if two different input produce the same hash output. It shows that weak hash functions are not collision-resistant and can easily lead to security vulnerabilities.

Ans 31

Given, Message $M = 15$

Secret Key $K = 7$

Formula, $MAC = (M+K) \text{ mod } 17$

Original MAC: 5

$$\begin{aligned} MAC &= (15+7) \text{ mod } 17 \\ &= 22 \text{ mod } 17 \\ &= 5 \end{aligned}$$

\therefore original MAC = 5

If attacker change message to 10. Then the MAC will be,

$$\begin{aligned} MAC &= (10+7) \text{ mod } 17 \\ &= 0 \end{aligned}$$

If they guess, probability of connection

$$\text{guess} = \frac{1}{12}$$

which is very low.

If attacker changes the message then
the formula of $\text{MAC}_{\text{new}} = (10 + k) \bmod 12$.

where k is known

$$\text{Now, } 10 + k \not\equiv 5 \pmod{12}$$

$$\Rightarrow \text{ basis } k \equiv -5 \pmod{12}$$

$$\Rightarrow k = 12$$

which is not correct secret key.

$$\text{if basis } (-5 + 12) = 7 \pmod{12}$$

$$0 =$$

Ans 32: TLS = Transport Layer Security used in HTTPS and secure communication.

Main idea: The TLS handshake is how two devices (like browser and server) agree on encryption key and start a secure session. It uses asymmetric cryptography (Public/Private keys) first, then switches to symmetric cryptography for speed.

Step 1: Client Hello

→ The client sends:

- i. Supported TLS version
- ii. Supported cipher suites (Encryption algorithm)
- iii. Random numbers (Client Random)
- iv. Optional extensions

Step 2: Server Hello

→ The server responds with:

- b). chosen TLS version
- ii. Selected cipher suites
- iii. Random number (Server Random)
- iv. Digital certificates.

Step 3: Server Authentication.

→ The client verifies:

- i. Certificate's validity (via CA)
- ii. Domain name matches with tent
- iii. Certificate hasn't expired or been revoked

Step 4: Key exchange

two common methods:

(1) RSA key exchange (older)

→ Client generates a Pre-master Secret
(random numbers)

→ Encrypts it with the server's

public Key from the certificate.

- Sends it to the server
- Only the server can decrypt it with its private key.

- (b) Diffe-Hellman:
- Both client and server exchange ephemeral public keys.
 - Using asymmetric math, both sides compute the same secret number.

Step 5: Session key Derivation

- Both parties now have
 - i. Client Random
 - ii. Server Random
 - iii. Pre-Master Secret
- They feed these into a key derivation function (KDF)
 - i. Symmetric encryption key
 - ii. Message authentication keys.

Step 6: Finished Messages

→ Both client and server send finished messages.

Ans 33

SSH (Secure Shell) has three main layers

1. Transport Layer Protocol

→ Handles encryption, server authentication and integrity

→ Negotiates algorithms and establishes a secure channel.

2. User Authentication Layer

→ Verifies two clients' identity (Passwords)

public key exchange

3. Connection Layer

→ Manages multiple logical channels over

the secure connection.

fast adaptation exchange

fast reinitialization message

responses backlog

maximum backlog size versus free funds that

Ans 34:

Step 1: Client Hello.

→ The client sends

① Supported TLS version

② Supported encryption algorithm

Step 2: Server Hello

→ The server response with

① Choose TLS version

② Selected cipher suite

Step 3: Server certificate

→ The server sends digital certificates

→ check CA signatures

Step 4: Server key exchange

→ If needed the server sends extra key exchange parameters

Step 5: Client certificate

→ If mutual authentication, the client also

sends its certificates.

Step 6: Key exchange and pre-master secret

Step 7: Change cipher key

Step 8: Finished message.

Ans 34.

The general form of an elliptic curve equation over a finite field is

$$y^2 \equiv x^3 + ax + b \pmod{p}$$

where,

→ a and b are constants in the field \mathbb{F}_p .

→ p is a prime number (for finite fields)

→ The discriminant condition $4a^3 + 27b^2 \neq 0$

(mod p) ensures the curve has no

other singularities.

singularities (no cusps or self-intersections)

Why it is used in Cryptography?

- ① Strong Security per key size: Elliptic curve cryptography (Ecc) provides the same security as traditional Algorithms (like RSA) but with a smaller keys, making it faster and more efficient.
- ② Hard mathematical Problems: The security relies on the Elliptic curve Discrete algorithms problems (ECDLP) which is extremely hard to solve.
- ③ Efficient Computation: Suitable for devices with slow processing power, like IoT devices and mobile systems.
- ④ Widely Adopted: Use in TLS, digital signatures (ECDSA) and key exchange (ECDH).

Ans 36:

Elliptic curve cryptography (ECC) achieves

the same level security as RSA with much smaller key sizes because it is based on a harder underlying mathematical problem

Key Reasons:

→ RSA security is based on the difficulty of integer factorization

→ ECC security is based on the Elliptic Curve Discrete logarithmic Problem (ECDLP)

Given two points p and $q = kp$ on an elliptic curve, it is extremely hard to find k .

Security level Comparison

RSA key size

ECDK key size

Approx Security

1024 bits

160 bits

~ 80 bit security

2048 bits

224 bits

~ 112 bit security

3072 bits

256 bits

~ 128 bit security

Ans 37

Give, elliptic curve

$$\textcircled{1} \quad y^2 \equiv x^3 + 2x + 3 \pmod{97}$$

point $P(3, 6)$

Step 1: check Left Hand Side (LHS)

$$y^2 \equiv 6^2 \pmod{97}$$

$$= 36 \pmod{97}$$

$$= 36$$

Step 2: Checks Right Hand Side (RHS)

$$= x^3 + 2x + 3$$

$$= 3^3 + 2 \cdot 3 + 3$$

$$= 27 + 6 + 3$$

$$= 36 \pmod{97}$$

$\therefore L.H.S = R.H.S$, so, point $(3, 6)$ lies on the curve.

Ans 38: Given that

$$p=23, g=5, h=8$$

message $m=10$.

Random $k=6$

Formula, $c_1 = g^k \bmod p$

$$c_2 = m \cdot k^6 \bmod p$$

Step 1.

$$c_1 = 5^6 \bmod 23$$

$$\stackrel{\text{(use base)}}{=} (5^2)^3 \bmod 23 \quad \text{--- ①}$$

$$\therefore 5^2 = 25 \bmod 23 \quad \text{--- ②}$$

from ① $c_1 = 2^3 \bmod 23$

$$\stackrel{\text{(use ②)}}{=} 8 \bmod 23$$

$$c_1 = 8$$

Step 2: $c_2 = m \cdot k^6 \bmod p = 10 \cdot 8^6 \bmod 23$

$$= 10 \times (8^2)^3 \bmod 23 \Rightarrow 10 \times 36 \bmod 23$$

$$\stackrel{\text{(use ②)}}{=} 36 \bmod 23 \stackrel{\text{(use ②)}}{=} (8^3)^2 \bmod 23$$

$$\stackrel{\text{(use ②)}}{=} 512 \bmod 23 \stackrel{\text{(use ②)}}{=} 6$$

$$\therefore c_2 = 15 \stackrel{\text{(use ②)}}{=} 360 \bmod 23 \stackrel{\text{(use ②)}}{=} 15$$

$$\therefore (c_1, c_2) = (8, 15)$$

Ans 39

Light weight cryptography is designed to use low memory, low processing power and minimal energy, making it suitable for resource-constrained IoT devices.

Importance:

- IoT devices have limited CPU and battery can not handle heavy algorithm like RSA easily.
- Ensures data confidentiality, integrity and authentication without overloading the device.

Example:

Present: A block cipher with 64 bit block size and 80/128 bit key, optimized for low hardware and low energy.

Usage

Ans 40:

Common IoT specifies attacks and mitigation.

1. Firmware attack
 - Attacker replaces legitimate firmware with malicious code

Impact: Can control the device remotely or steal data

Mitigation: Secure boot, firmware, signing and version verification before updates.

2. Physical Tampering

→ Direct manipulation of IoT hardware (opening device, probing chips)

Impact: Can extract encryption keys or modify circuit

Mitigation: Tamper-evident seals, hardware, encryption, protective casting.

3. IoT Botnets (Mirai)

→ Malware infects IoT devices and uses them in large-scale DDoS attacks

Impact: Disrupts services, cause internet outages