Countable Vs Uncountable

Infinite Sets

R : Set of all real numbers has infinite elements (uncountable)

I : Set of all integers has infinite elements (Countable)

An **infinite set** is said to be **countable** if there is a 1-1 correspondence b/n the elements of this set and the set of positive integers.

Uncountable Infinite sets

R: set of all real numbers

Q = [0,1]

There are infinite set of numbers between 0 to 1 and this infinite set is bugger then the infinite set of integers

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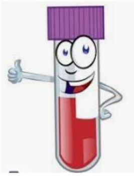
Experiments and Sample Spaces

Certainty with in uncertainty



Experiment: Going to the mall

Outcome: infected, Not infected

An Experiment or trail is any procedure that can be repeated infinite times and has a well defined set of outcomes

The set of all possible outcomes of an experiment is called the **sample space**. The elements in a sample space are **mutually exclusive** and **collectively exhaustive**

The outcome in every trail is uncertain but the set of outcomes is certain.

Experiments Involving Coin Tosses

Certainty with in uncertainty

1 Coin	{H, T}	2
2 Coin	{HH, HT, TH, TT}	4
3 Coin	{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}	8

Experiments Involving Fair Dice

Certainty with in uncertainty

	Ω	$I\Omega I$
1 Dice	{1, 2, 3, 4, 5, 6}	6
2 Dice		36
N dice		6^n

Experiments Involving Cards

Certainty with in uncertainty

With Repetition	Ω	$I\Omega I$
1 card	{52}	52
2 cards		52^2
N Cards		52^n

Without Repetition		
1 card	{52}	52
2 cards		52 * 51
N Cards		**52* 51* …..**

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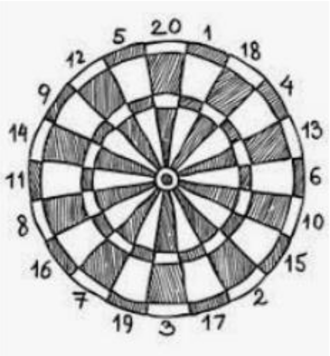
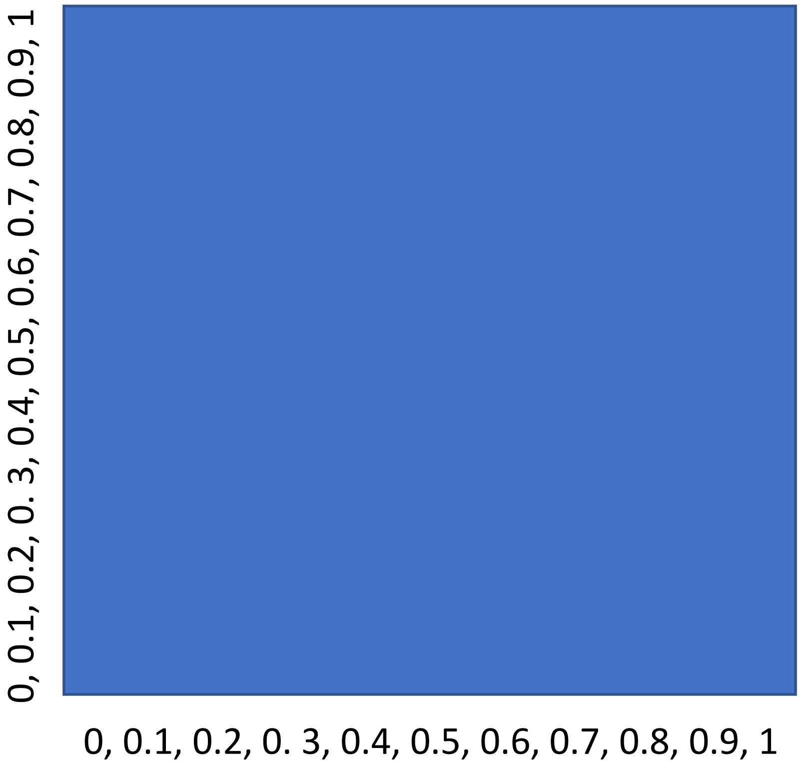
Experiments: Continuous outcomes



Certainty with in uncertainty

Dart board of square 1 mts by 1 mts



$$\Omega = \{(x, y) s.t\ 0 < x, y < 1\}$$

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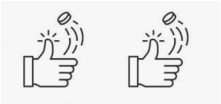
Events of an Experiment

Certainty with in uncertainty



An event is a set of outcomes of **an** experiment. This set is a subset of sample set.

Ω = { HT, TH, HH, TT, }

A = {HT, HH } A is an even that the first toss results in an head

Event of both tosses resulting in tails B = {TT }

3 52 card

Event that there are exactly 2 aces in a hand of 3 cards

$$|C| = \binom{4}{2} * \binom{48}{1} = 288$$

2+1=3

We say an even has occurred if the outcome of the experiment lies in the set A.

Events of an Experiment

Union of events

A = { (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) } Event that the first die shows a 2

B = { (1,4), (2,4), (3,4), (4,4), (5,4), (6,4) } Event that the second die shows a 4

D = A ∩B = { 2,4} Event that first die shows 2 and the second die shows a 4

E = A^C Event that first die does not shows 2

Events of an Experiment

Multiple events

A = The hand contains ace of spades

B = The hand contains ace of Clubs

C = The hand contains ace of hearts

A ∪ B ∪ C   hand contains atleast 1 ace

A ∩ B ∩ C   hand contains all aces

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Events of an Experiment

Disjoint events

$2\ events\ A\ and\ B\ are\ said\ to\ be\ disjoints\ if\ they\ can\ not\ occur\ simultaneously.$

i.e, A ∩ B = ∅

$simple\ example$ = A and A^C

Not necessary that the disjoints events should be a complement always.

A = event of first die showing 1 and B = event of first die showing 2, they can not occur together and hence are disjoint events.

The events A1, A2, A3, ……, An are said to be mutually disjoint or pairwise disjoint, if

$A_i \cap A_j$ = ∅ ∀ i, s.t i≠ j

A = {HH}

B = {TT}

C = {HT, TH} here A ∩ B = ∅, B ∩ C = ∅ and A ∩ C = ∅

In addition if A ∪ B ∪ C = **Ω**

Then, they are said to partition the sample space

The events A1, A2, A3, ……, An are mutually Disjoint and A1 ∪ A2 ∪ A3 ∪ An = Ω then A1, A2, A3, ……, An are said to partition the sample space.

Axioms of Probability

Recap

Experiments

Sample spaces

Events

What is the chance of an event?

Goal: Assign a number to each event such that this number reflects the chance the experiment resulting in that event.

Axioms of Probability

The probability function

P(A) = ? **Where: P is Probability function and A is an event.**

What are the conditions that such a probability function must satisfy ?

(Axioms of Probability)

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Axioms of Probability

The Axioms of probability:

Axiom 1 **(Non negativity)** — $P(A) \geq 0 \ \forall A$

Axiom 2 **(Normalisation)** — $P(\Omega) = 1$

Axiom 3 — If the events A1, A2, A3, ……, An are mutually disjoint then $P(A1 \cup A2 \cup A3 \cup An) = \sum_i^n P(A_i)$

(finite additivity)