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Sample spaces and Events

Introduction to Descriptive statistics

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Descriptive Statistics

- ✓ Different types of data
- ✓ Different types of plots
- ✓ Measure of centrality and Spread

covered previous

Probability Theory

- ✓ Counting, Sample Specs, events, axioms
- ✓ Discrete and continuous RVs
- ✓ Bernoulli, Uniform, Normal dist
- ✓ Sampling strategies

Inferential Statistics

- ✓ Interval Estimators
- ✓ Hypothesis testing (z-test, t-test)
- ✓ ANOVA, Chi-square test
- ✓ Linear Regression

Counting and Probability Theory

- What are sets and some of their properties
- What are experiments, sample spaces, outcomes and events ?
- What are the axioms of probability
- What are some simple ways of defining a probability function?
- What are some important theorems:
- Multiplication rule, total probability, theorem and Bayes theorem ?
- What are independent events ?

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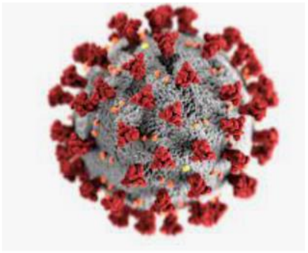
The Elements of Chance (Nothing in life is certain)

The element of chance

(Nothing in life is certain)

The Randomness everywhere !



What is the chance that he would get infected if he went to the super market ?

Due to the random nature of the world around us

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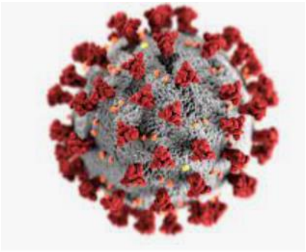
The Elements of Chance (Nothing in life is certain)

The Randomness everywhere !


What is the mode of transport ?

Is private care always more safer then public transport ?

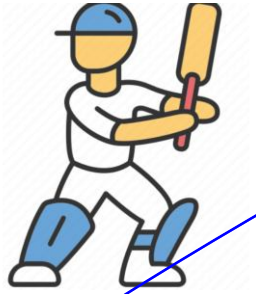
How good is his immune system?

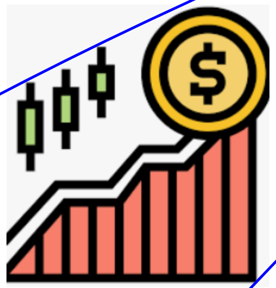
Does he have any co-morbidities?

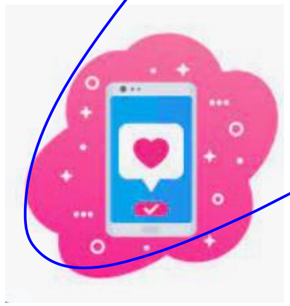
How many infections are there in the neighbourhood?

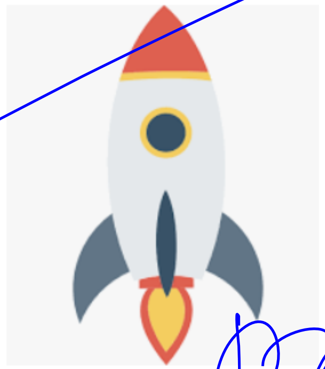
The Elements of Chance (Nothing in life is certain)

The Randomness everywhere !









covered in previous PDFs

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The Elements of Chance (Nothing in life is certain)

The Randomness everywhere !

The study of these chances is the subject matter of Probability Theory !

- Set Theory
- Experiments, sample spaces, events
- Axioms of probability
- Random Variables
- Distributions
- Exceptions

Overview of Set Theory

Set is a collection of elements

S = { a, e, I, o, u}

E = {0, 2, 4 ....94, 96, 98, 100}

E = { x: $0 \leq x \geq 100, x \% 2 = 0$} (Compact notation useful for large data set)

$x \in S$, mean x belongs to set S,

$2 \in E$, $3 \notin E$

Overview of Set Theory

Set is a collection of elements

Subsets and equal sets

I = set of all integers

$S = \{ x : x \in I, x < 0 \}$

Every element of S in contained in I

$S \subset I$ subset

Equal Sets:

$A = B$ if $A \subset B$ and $B \subset A$ equal sets

Overview of Set Theory

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Universal set

Every set of interest is a subset of the universal set

Ω = set of 52 cards

A : set of all aces — $A \subset \Omega$

H : set of all hearts — $H \subset \Omega$

B : set of all black — $B \subset \Omega$

F : set of all face — $F \subset \Omega$

Empty Set:

Set with no elements (null set)

$\emptyset = \{ \}$

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Overview of Set Theory

Set Operations

Complement:

$A^C = \{x : x \in \Omega \ and \ x \notin A\}$

Union (2 Sets)

$\mathrm{A} \cup \mathrm{B} = \{x : x \in A \ and \ x \in \mathrm{B}\}$

Intersection (2 Sets)

$\mathrm{A} \cap \mathrm{B} = \{x : x \in A \ or \ x \in \mathrm{B}\}$

Overview of Set Theory

Properties of Set Operations

Commutativity

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Associativity

$A \cup (B \cup C) = (A \cup B) \cup C$

Distributive laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof: Distributive laws

$X \in A \cap (B \cup C) = x \in A \text{ and } x \in (B \cup C)$

$= x \in A \text{ and } x \in B \text{ or } x \in C$

$= x \in A \text{ and } x \in B \text{ or } x \in A \text{ and } x \in C$

$= x \in (A \cap B) \cup (A \cap C)$

Overview of Set Theory

Properties of Set Operations

De Morgan's Laws

$(A \cup \mathrm{B})^C = A^C \cap B^C$

$(A \cap \mathrm{B})^C = A^C \cup B^C$

Proof: De Morgan's Laws

$\mathrm{X} \in (A \cap \mathrm{B})^C = \mathrm{x} \in A^C \sqcup B^C$

not eq

$= \mathrm{x} \in A^C \text{ or } \mathrm{x} \in B^C$

$\underline{= \mathrm{x} \in A^C \cap B^C}$

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Countable Vs Uncountable

Infinite Sets

R : Set of all real numbers has infinite elements (uncountable)

I : Set of all integers has infinite elements (Countable)

An **infinite set** is said to be **countable** if there is a 1-1 correspondence b/n the elements of this set and the set of positive integers.

Uncountable Infinite sets

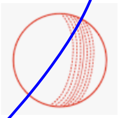
R: set of all real numbers

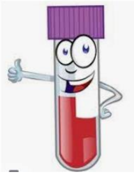
Q = [0,1]

There are infinite set of numbers between 0 to 1 and this infinite set is bugger then the infinite set of integers

Experiments and Sample Spaces

Certainty with in uncertainty





Experiment: Going to the mall

Outcome: infected, Not infected

An Experiment or trail is any procedure that can be repeated infinite times and has a well defined set of outcomes

The set of all possible outcomes of an experiment is called the **sample space**. The elements in a sample space are **mutually exclusive** and **collectively exhaustive**

The outcome in every trail is uncertain but the set of outcomes is certain.

Experiments Involving Coin Tosses

Certainty with in uncertainty

1 Coin	{H, T}	2
2 Coin	{HH, HT, TH, TT}	4
3 Coin	{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}	8

Experiments Involving Fair Dice

Certainty with in uncertainty

	Ω	$I\Omega I$
1 Dice	{1, 2, 3, 4, 5, 6}	6
2 Dice		36
N dice		6^n

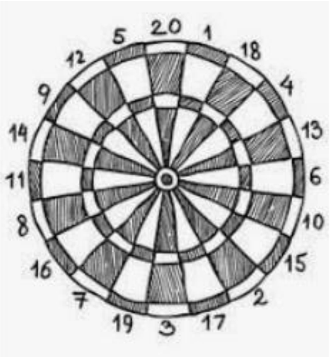
Experiments Involving Cards

Certainty with in uncertainty

With Repetition	Ω	$I\Omega I$
1 card	{52}	52
2 cards		52^2
N Cards		52^n

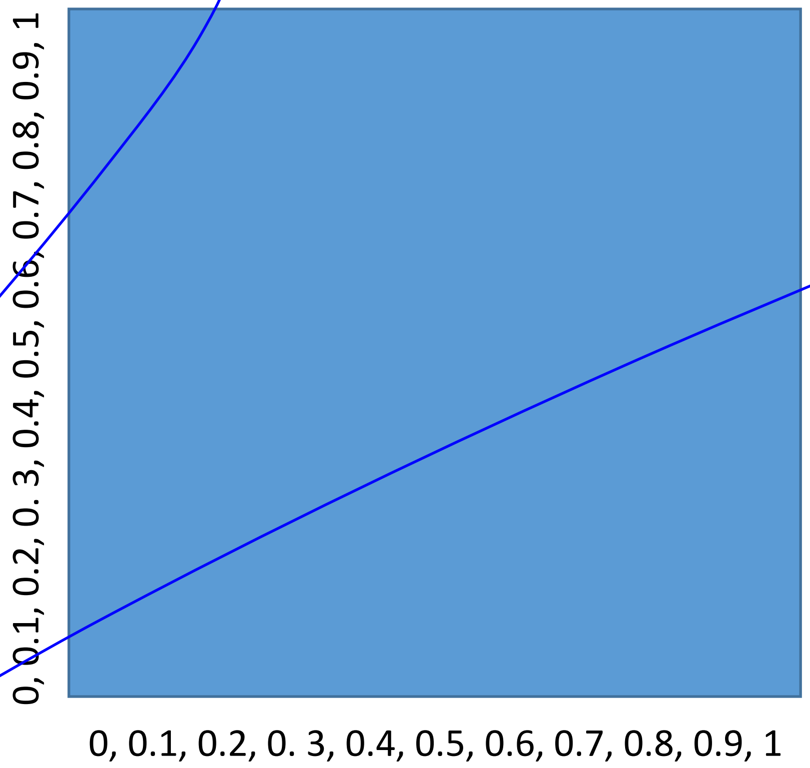
Without Repetition		
1 card	{52}	52
2 cards		52 * 51
N Cards		**52* 51* …..**

Experiments: Continuous outcomes



Certainty with in uncertainty

Dart board of square 1 mts by 1 mts

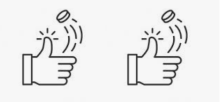


$$\Omega = \{(x, y) s.t\ 0 < x, y < 1\}$$

Events of an Experiment

Certainty with in uncertainty

An event is a set of outcomes of **an** experiment. This set is a subset of sample set.



Ω = { HT, TH, HH, TT, }   A = {HT, HH } A is an even that the first toss results in an head

Event of both tosses resulting in tails   B = {TT }

Event that there are exactly 2 aces in a hand of 3 cards

$$|C| = \binom{4}{2} * \binom{48}{1} = 288$$

We say an even has occurred if the outcome of the experiment lies in the set A.

Events of an Experiment

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Union of events

A = { (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) } Event that the first die shows a 2

B = { (1,4), (2,4), (3,4), (4,4), (5,4), (6,4) } Event that the second die shows a 4

D = A ∩B = { 2,4} Event that first die shows 2 and the second die shows a 4

E = A^C Event that first die does not shows 2

Events of an Experiment

Multiple events

A = The hand contains ace of spades

B = The hand contains ace of Clubs

C = The hand contains ace of hearts

A ∪ B ∪ C   hand contains atleast 1 ace

A ∩ B ∩ C   hand contains all aces

Events of an Experiment

Disjoint events

$2 \ events \ A \ and \ B \ are \ said \ to \ be \ disjoints \ if \ they \ can \ not \ occur \ simultaneously.$

i.e, A ∩ B = ∅

$simple \ example$ = A and A^C

Not necessary that the disjoints events should be a complement always.

A = event of first die showing 1 and B = event of first die showing 2, they can not occur together and hence are disjoint events.

The events A1, A2, A3, ……, An are said to be mutually disjoint or pairwise disjoint, if
$A_i \cap A_j = \emptyset \ \forall$ i, s.t i≠ j

A = {HH}
B = {TT}
C = {HT, TH} here A ∩ B = ∅, B ∩ C = ∅ and A ∩ C = ∅

In addition if A ∪ B ∪ C = **Ω**
Then, they are said to partition the sample space

The events A1, A2, A3, ……, An are mutually Disjoint and A1 ∪ A2 ∪ A3 ∪ An = Ω then A1, A2, A3, ……, An are said to partition the sample space.

Axioms of Probability

Recap

Experiments

Sample spaces

Events

What is the chance of an event?

Goal: Assign a number to each event such that this number reflects the chance the experiment resulting in that event.

Axioms of Probability

The probability function

P(A) = ? **Where: P is Probability function and A is an event.**

What are the conditions that such a probability function must satisfy ?

(Axioms of Probability)

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Axioms of Probability

The Axioms of probability:

Axiom 1 $P(A) \geq 0 \; \forall \; A$
(Non negativity)

Axiom 2 $P(\Omega) = 1$
(Normalisation)

Axiom 3 If the events A1, A2, A3, ……, An are mutually disjoint then $P(A1 \cup A2 \cup A3 \cup An) = \sum_i^n P(A_i)$

(finite additivity)

Axioms of Probability

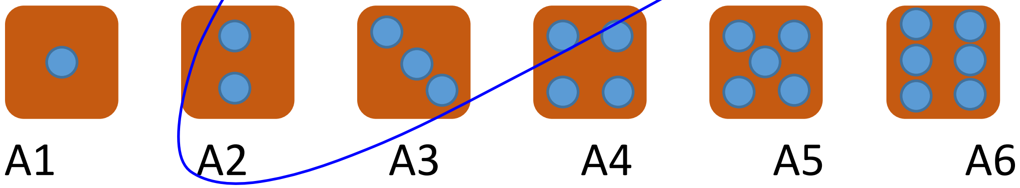
The Axioms of probability:

Axiom 3 **If the events A1, A2, A3, ……, An are mutually disjoint then** $P(A1 \cup A2 \cup A3 \cup An) = \sum_i^n P(A_i)$

(finite additivity)

Compute probabilities of large events from small events

Smallest possible event = one outcome



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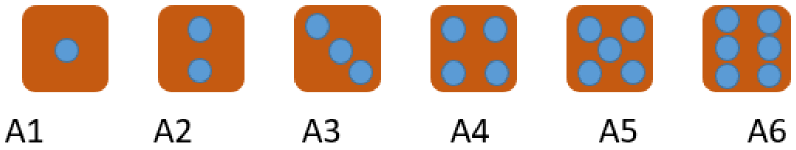
Axioms of Probability

The Axioms of probability:



Given P(A1), P(A2), P(A3), P(A4), P(A5), P(A6)

We can compute other probabilities

B: that event that the outcome is and odd no.
P(B) = P(A1) + P(A3) + P(A5),

C: that event that the outcome is ≥ 5.
P(C) = P(A5) + P(A6)

D: that event that the outcome is $\boldsymbol{multiple\ of\ 3}$.
P(D) = P(A3) + P(A6)

Axioms of Probability

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Some properties of probability:

Property 1:

P(A) = 1 – P(A^C)

$A \cup A^C = \Omega$

$P(\Omega) = 1 = P(A \cup A^C) = P(A) + P(A^C)$

Therefore $P(A) = 1 - P(A^C)$

Axioms of Probability

Some properties of probability:

Property 2:

P(A) $\leq$ 1

$P(A) = 1 - P(A^C)$

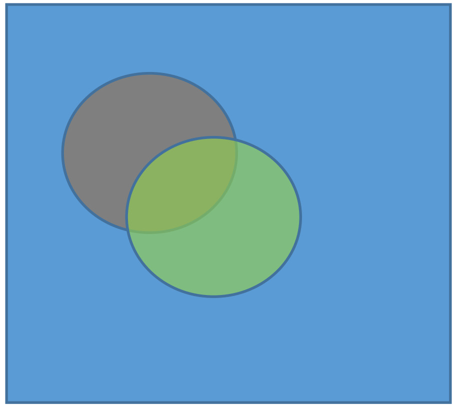
We know that A^C **is always greater then zero**

Therefore $P(A) = 1 - P(A^C)$ *Because* $P(A^C)$ *can not be zero*

$P(A) \leq 1$

Axioms of Probability

Some properties of probability:



Property 3:

$$\mathbf{P}(A \cup B) = \boldsymbol{P}(\boldsymbol{A}) + \boldsymbol{P}(\boldsymbol{B}) - \boldsymbol{P}(A \cap B)$$

$$\mathbf{P}(A \cup B) = P\left(A \cup (B \cap A^C)\right)$$

$$= \boldsymbol{P}(\boldsymbol{A}) + P(B \cap A^C)$$

$$= \boldsymbol{P}(\boldsymbol{A}) + \mathbf{P}(\mathbf{B}) - P(B \cap \boldsymbol{A})$$

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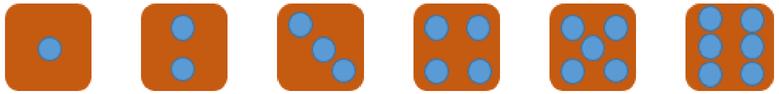
Axioms of Probability

Some properties of probability:



A1 A2 A3 A4 A5 A6

Property 4:

The sum of the probability of all outcomes is equal to 1

$$P(\Omega) = P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6)$$

$$= \sum_{i=1}^{n} P(A_i) = \mathbf{1}$$

Axioms of Probability

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Some properties of probability:

Property 5:

$\mathbf{P}(\phi) = 0$

$\mathbf{P}(\Omega) = \boldsymbol{P}(\Omega \cup \phi) = \boldsymbol{P}(\Omega) + \boldsymbol{P}(\phi) = \mathbf{1}$

$\boldsymbol{P}(\phi) = \mathbf{1} - \boldsymbol{P}(\Omega) = \mathbf{0}$

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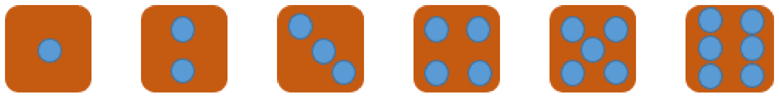
Axioms of Probability

The Axioms of probability:



A1 A2 A3 A4 A5 A6

Given P(A1), P(A2), P(A3), P(A4), P(A5), P(A6)

We can compute other probabilities

Axioms of Probability

The Axioms of probability:

Given P(A1), P(A2), P(A3), P(A4), P(A5), P(A6)

We can compute other probabilities

B: that event that the outcome is and odd no.
P(B) = P(A1) + P(A3) + P(A5),

C: that event that the outcome is $\geq$ 5.
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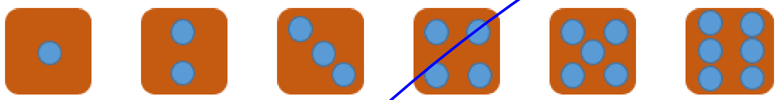
D: that event that the outcome is $multiple\ of\ 3$.
P(D) = P(A3) + P(A6)

Axioms of Probability

The Axioms of probability:



A1 A2 A3 A4 A5 A6

Given P(A1), P(A2), P(A3), P(A4), P(A5), P(A6)

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D: that event that the outcome is $multiple\ of\ 3$.
P(D) = P(A3) + P(A6)

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Designing Probability Functions

Probability as Relative frequency:

Goal: Assign a number to the event such that this number reflects the chance of the experiment resulting in that event

Required: The probability function should satisfy the axioms of probability

We can think of probability of an event as fraction of the times the event occurs when an experiment is repeated a large number of times

Head

P(H) = 12012 / 24000 = 0.5005

$$P(A_i) = \frac{Number\ of\ times\ the\ event\ is\ in\ A_i}{total\ number\ of\ times\ the\ experiment\ was\ repeated}$$

But does such a P() satisfy the axioms of probability?

Axioms of Probability

Probability as relative frequency:

Does P() satisfy the axioms?

P(A_1 U A_2) = P(A_1) + P(A_2) ?

$|\Omega| = \boldsymbol{n} = 2^n\ \boldsymbol{subsets} = 2^n\ \boldsymbol{events}$

(axioms are about events)

P(A_1 U A_2) = $\frac{k_1+k_2}{k} = \frac{k_1}{k} + \frac{k_2}{k} =$ **P(A_1) + P(A_2)**

P(A_i) $= \dfrac{\boldsymbol{Number\ of\ times\ the\ event\ is\ in\ A_i}}{\boldsymbol{total\ number\ of\ times\ the\ experiment\ was\ repeated}} = \mathbf{1}$

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Designing Probability Functions

Example :

A dataset contains images of beaches (60000), mountains (25000) and forests(15000)

What is the probability that a randomly picked image would be a forest?

Experiment: Select an image

Number of trials: 100000

Frequency of the event "forest": 15000

$$P(\text{forest}) = \frac{15000}{100000} = 0.15$$

P

$$\frac{15k}{\cancel{(60+25+15)k}}$$

$$\frac{15k}{100k} = \boxed{0.15}$$

1L F

$$\frac{15k}{(60+25+15)k} \times 1L \text{ trails} = \approx 15k$$

Designing Probability Functions

Example :

A country tests 20 million randomly selected people and finds that 1 million are infected

What is the probability that a randomly picked person would be infected?

Experiment: perform a test

Number of trials: 20 million

$\frac{1}{20} = 0.05$

Frequency of the event "infected": 1 million

P(infected) $= \frac{1000000}{20000000} = 0.05$

Designing Probability Functions

Example :

A subtle point: the sample from which the probabilities were estimated should be drawn from the same population on which we are interested in making inferences.

By May-10-2020, India had tested 1673688 samples of which 67176 were found to be positive. Does this mean the probability that a randomly selected person being infected is 0.04 ?

No be

$\frac{67176}{1673688} \approx 0.04$

No: Testing in India was not random but only for people with flu-like symptoms

Designing a probability function(Equally likely outcomes)

Equally Likely Outcomes:

$|\Omega| = \{H, T\}$

P(H) = P(T) = k

Ω = H ∪ T

P(Ω) = P (H ∪ T)

= P(H) + P(T) = 2k = 1

Therefore: P(H) = P(T) = k = ∪ 1/2

We can now compute the probability of 4 subsets of Ω

∅, {H}, {T}, {H,T}

0

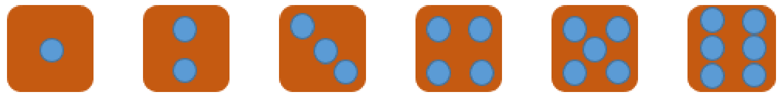
$\frac{1}{2}$

1/2

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Equally likely outcomes

The Axioms of probability:



A1 A2 A3 A4 A5 A6

$|\Omega| = \{1, 2, 3, 4, 5, 6\}$

A_i: *Events that the outcome is i*

$A_1, A_2, A_3, A_4, A_5, A_6$ *partition* Ω

$P(A_1) = P(A_2) = P(A_3) = P(A_4) = P(A_5) = P(A_6) = k$

$P(\Omega) = \Sigma_{i=1}^{6} P(A_i) = 6k = 1$

Hence, $P(A_i) = 1/6$

We can now compute the probability of all subsets of Ω

Equally likely outcomes

$$P(X) = \frac{\text{Number of outcomes in } X}{\text{number of outcomes in } \Omega}$$

Are the axioms of probability satisfied?

$P(A) \geq 0 \ \forall \ A$? : Ratio of 2 positive numbers

$P(\Omega) = 1$? : Contains all outcomes

$P(A1 \cup A2) = P(A1) + P(A2) = \Sigma_{i=1}^{k} \frac{1}{n} = \frac{k}{n}$

$P(A1 \cup A2) = \frac{k1+k2}{n} = \frac{k1}{n} + \frac{k2}{n} = P(A1) + P(A2)$

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Equally likely outcomes

Examples:

What is the probability of getting a black card?

P(B) = $\frac{26}{52}$ $1/2$ ✓

Total Aces = 3

What is the probability of getting 3 aces?

$$\binom{52}{3} = 22100 \quad \text{and} \quad \binom{4}{3} = 4$$

$$P(A) = \frac{4}{22100} = \frac{\binom{4}{3}}{\binom{52}{3}} = \frac{{}^{4}C_{3}}{{}^{52}C_{3}} = \frac{4}{22100}$$

THANK YOU VERY MUCH!!!!!!

Conditional Probabilities

Change in belief

Setting Context Example 1:

Assume fair play conditions & equally good teams

Before the start of the play: What is the chance of India wining ? 0.5

India scores 395 batting first: What is the chance of India winning ? > 0.5

What has happened here ?

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Change in belief

Setting Context Example 1:

(Assume fair play conditions & equally good teams }

What exactly happened here ?

A: event that India will win

B: India scored 395 runs

P(A) changes once we know that event B has occurred

P of A given B

$$\boxed{P(A \mid B) \neq P(A)}$$

given

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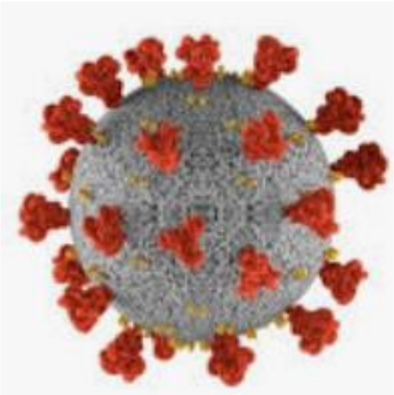
Change in belief



Setting Context Example 2:

10% of the population is infected

What is the probability that a randomly selected person is healthy or infected ?

healthy

infected

$\boxed{P(\bar{A}) = 0.1}$

A: event that a person is healthy P(A) = 0.9

$B: event\ that\ a\ person\ has\ Covid\ 19\ symptoms$

P(A | B) $\neq$ P(A)

Definition: P(A | B) is called the conditional probability of the event A given the event B

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Conditional Probability

The definition of P(A | B)

A: Sum is 8
B: first dice shows a 4

(1, 2)	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(1, 5)**	**(1, 6)**
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

What is the probability that the sum is 8 ?

sum=8

P(A) = $\frac{5}{36}$

What is the probability that the sum is 8 given that the first dice shows a 4 ?

$P(A|B)$ = to find?

first dice = 4

P(B) = $\frac{1}{6}$

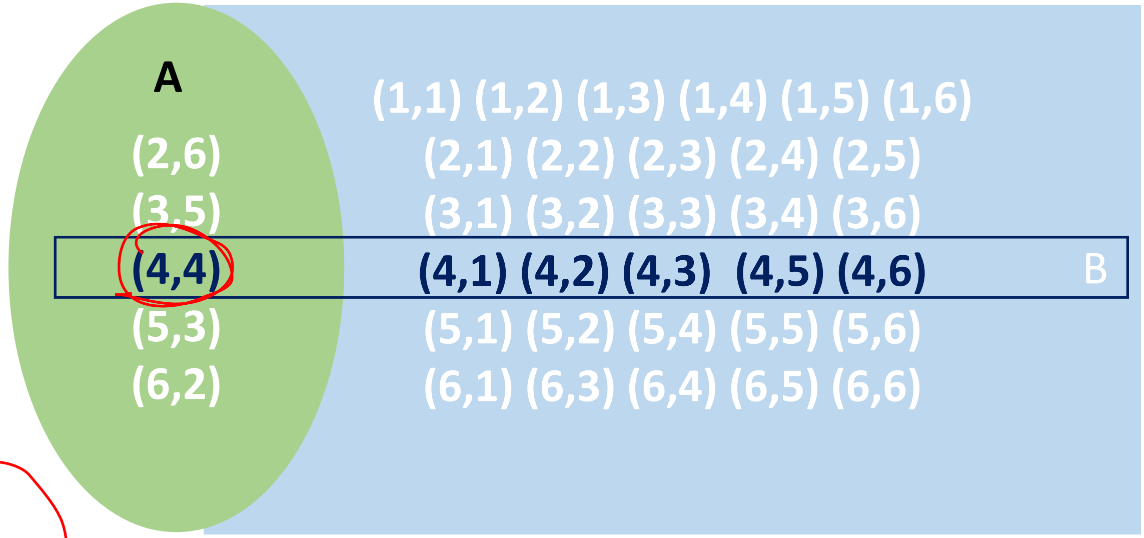
Conditional Probability

The definition of P(A | B)

A: Sum is 8
B: first dice shows a 4

$(A \cap B)$

sum also 8 & first dice also 4?



What is the probability that the probability of P(A|B)

$$P(B) = \frac{1}{6}$$

$$P(A \cap B) = \frac{1}{36}$$

only one pos

$$P(A|B) = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{6}$$

Conditional Probability

The definition of P(A | B)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Probability **Normal Probability**

$\boldsymbol{P(A|B)\ is\ called\ the\ conditional\ probability\ of\ the\ event\ A\ given\ the\ event\ B}$

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Conditional Probability

Examples:

Think of a 2 digit number, If I tell you that atleast 1 on the number Is even what is the probability that both the numbers are even

10 , 11, 12, 13, ……..96, 97, 98, 99 $= 90 = P(\Omega)$

~~P(A) = Atleast 1 on the no. is even~~

All are equally likely

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{20}{90}$$

$$P(A) = \frac{Number\ of\ outcomes\ in\ x}{Number\ of\ outcomes\ in\ \Omega}$$

$$= \frac{\frac{20}{90}}{\frac{65}{90}} = \frac{4}{13}$$

30%

$P(B) =$ atleast 1 is even $= 90 - P(\bar{B})$

$90 - 20$

A event that both the digits are even
B event that at least one digit is even

(20, 22, 24, 26, 28) = 5 × 4 (20, 40, 60, 80)

5×4

11, 33, 55, 77, 99

65

$$P(A) = \frac{20}{90} = \frac{2}{9}$$

But, we are interested in P(A|B) to find

$P(A \cap B) = P(A) \cap P(B)$

$20 \cap (20 + x) = \frac{20}{90}$

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Conditional Probability

Examples:

60 % of students in a class opt for ML. 20% of the students opt for both ML and DL. Given that the students has opted for ML what is the probability that she has also opted for DL?

B   A

A event that student has opted for DL
B event that student has opted for ML

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.20}{.60} = \frac{1}{3}$$

$P(A) = DL =$

$P(B) = ML = \cancel{60/100} = 0.6$

$P(A \cap B) = 0.2$

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}} =$$

Axioms of Probability

Does Conditional probability satisfy the axioms of probability:

How ?

$P(A|B) = \frac{P(A \cap B)}{P(B)} \geq 0$: Ratio of 2 probabilities, Hence its always > 0

$P(\Omega|B) = \frac{P(\Omega \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$

$P(A_1 \cup A_2 \cancel{\cap A_3}|B) = \frac{P(A_1 \cup A_2) \cap B)}{P(B)} = \frac{P((A_1 \cap B) \cup (A_2 \cap B))}{P(B)} = \boxed{\frac{P(A_1 \cap B)}{P(B)}} + \boxed{\frac{P(A_2 \cap B)}{P(B)}} = P(A_1|B) + P(A_2|B)$

?

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Conditional Probability

Chain Rule of probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\textit{therfore} \quad P\langle A \cap B\rangle = P(A|\boldsymbol{B}).P(B)$$

$$P(\boldsymbol{B}|\boldsymbol{A}) = \frac{P(\mathbf{B} \cap \boldsymbol{A})}{P(\boldsymbol{A})}$$

$$\textit{therfore}\ P\langle \mathbf{B} \cap \boldsymbol{A}\rangle = P(\boldsymbol{B}|\boldsymbol{A}).P(\boldsymbol{A})$$

Therefore $P\langle A \cap B\rangle = P(A|\boldsymbol{B}).P(B)$ **=** $P(\boldsymbol{B}|\boldsymbol{A}).P(\boldsymbol{A})$

Conditional Probability

$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B \cap A)}{0.1} = 0.99$

$\boxed{P(B \cap A) = P(A \cap B) = 0.099}$

$P(B^c|A) = \frac{P(B^c \cap A)}{P(A)}$

$0.01 = \frac{P(B^c \cap A)}{0.1}$

$\boxed{P(B^c \cap A) = 0.001}$

Chain Rule of probability

Facts:

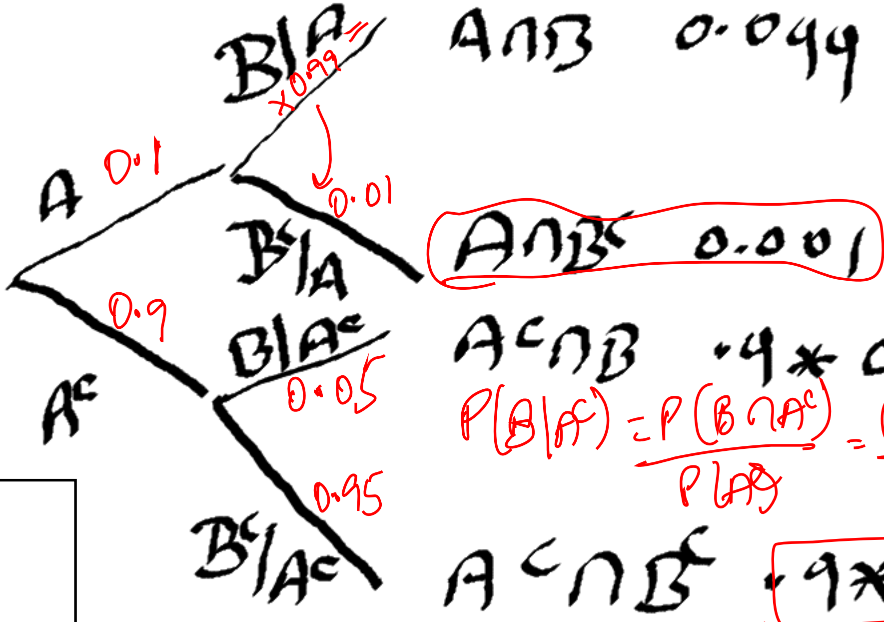
infection **P(A) = 0.1** → $P(\bar{A}) = 0.9$

not positive | infected **P(B^C| A) = 0.01**

+ve | infect **P(B|A) = 0.99**

+ve | not infected **P(B|A^C) = 0.05**

not +ve | not infect **P(B^C|A^C) = 0.95**

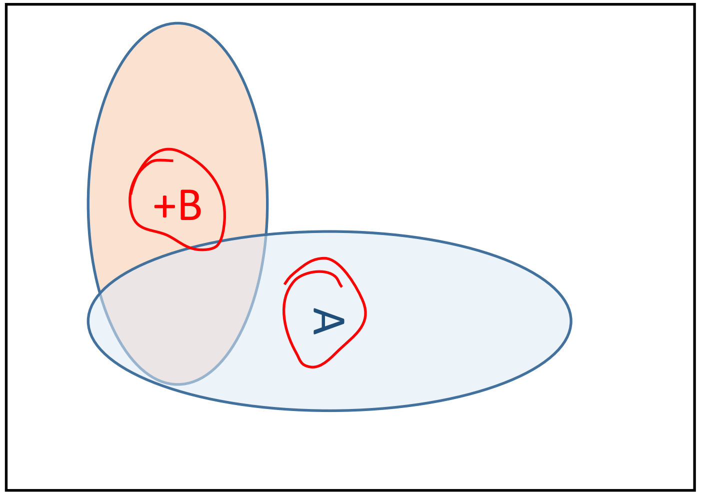


$P(B|A^c) = \frac{P(B \cap A^c)}{P(A^c)} = \frac{(\quad)}{0.9} \times 0.05$

$P(B^c|A^c) = \frac{P(B^c \cap A^c)}{P(A^c)} = \frac{(\quad)}{0.9} = 0.95$

A : Infected

B: Tested Positive



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Conditional Probability

Chain Rule of probability

$P\langle A \cap B \cap \boldsymbol{C}\rangle = P((A \cap B) \cap \boldsymbol{C})$

Let $\langle A \cap B\rangle = \boldsymbol{X}$

therefore $P\langle A \cap B \cap \boldsymbol{C}\rangle = P(\boldsymbol{X} \cap \boldsymbol{C})$

therefore $P\langle A \cap B \cap \boldsymbol{C}\rangle = P(\boldsymbol{X}).\, P(\boldsymbol{C}|\boldsymbol{X})$

therefore $P\langle A \cap B \cap \boldsymbol{C}\rangle = P\langle A \cap B\rangle.\, P(\boldsymbol{C}|A \cap B)$

therefore $P\langle A \cap B \cap \boldsymbol{C}\rangle = P(\boldsymbol{A}).\, P(\boldsymbol{B}|\boldsymbol{A}).\, P(\boldsymbol{C}|A \cap B)$

replac
$A \to A_1$ | $B \to A_2$ | $C \to A_3$

$\boxed{P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2)}$

$\because P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}$

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Conditional Probability

Chain Rule of probability

$$P\langle A \cap B \cap \boldsymbol{C} \cap \boldsymbol{D}\rangle =$$

$$P\langle A \cap B \cap \boldsymbol{C} \cap \boldsymbol{D}\rangle = P(\boldsymbol{A}).\,P(\boldsymbol{B}|\boldsymbol{A}).\,P(\boldsymbol{C}|A \cap B).\,P(\boldsymbol{D}|A \cap B \cap \boldsymbol{C})$$

$$P\langle A_1 \cap A_2 \cap A_3 \cap A_4\rangle = P(A_1)\mathbf{P}(A_2|A_1)P(A_3|A_2 \cap A_1)P(A_4|A_1 \cap A_2 \cap A_3)$$

$$P\langle A_1 \cap A_2 \cap A_3 \cap A_4\rangle = P(A_1)\prod_{i=2}^{4}\boldsymbol{P}(A_i|\boldsymbol{A_1} \cap A_2 \dots .\,A_{i-\mathbf{1}})$$

$$P\langle A_1 \cap A_2 \cap A_3 \cap A_4 \dots \dots A_{\mathbf{n}}\rangle = P(A_1)\prod_{i=2}^{\mathbf{n}}\boldsymbol{P}(A_i|\boldsymbol{A_1} \cap A_2 \dots .\,A_{i-\mathbf{1}})$$

Conditional Probability

Chain Rule of probability

Suppose you draw 3 cards one by one with out ~~replacement~~.

what is the probability that all the 3 cards are aces

$$\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50}$$

Using counting principles:

$$\approx \quad P \cong \frac{\binom{4}{3}}{\binom{52}{3}} = \frac{\frac{4!}{1!,3!}}{\frac{52!}{49!.3!}} = \frac{4*3*2}{52*51*50}$$

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Conditional Probability

Chain Rule of probability

Suppose you draw 3 cards one by one with out replacement.

what is the probability that all the 3 cards are aces

$P(A_1) = \frac{4}{52}$

$P(A_2|A_1) = \frac{3}{51}$

$P(A_3|A_1 \cap A_2) = \frac{2}{50}$

Using chain rule:

A_i: the event that the i-th card is an ace

$$P(A_1 \cap A_2 \cap A_3) = \underset{\frac{4}{52}}{P(A_1)}\,\underset{\frac{3}{51}}{P(A_2|A_1)}\,\underset{\frac{2}{50}}{P(A_3|A_2 \cap A_1)}$$

$$P = \frac{4 * 3 * 2}{52 * 51 * 50}$$

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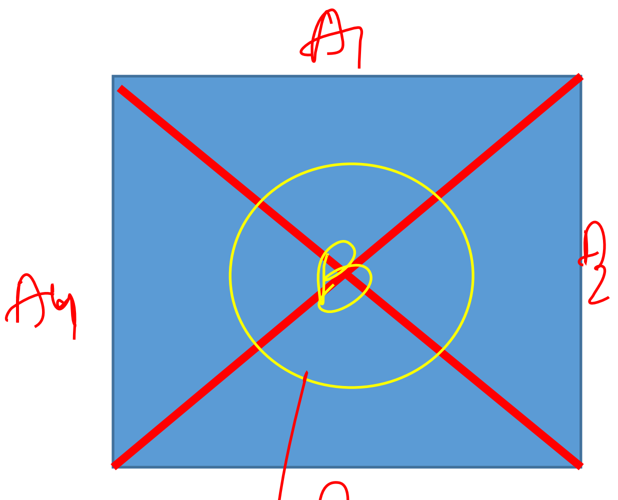
Conditional Probability



Total Probability Theorem

$A_1, A_2, A_3 \dots \dots . A_n$ $\boldsymbol{Partition}$ $\Omega.$

$A_1 \cup A_2 \cup A_3 \dots \dots . A_n = \Omega.$

$A_1 \cap A_j = \phi \forall i \neq j.$

4 Aces

our event B

3 aces

B = $\langle \boldsymbol{B} \cap \boldsymbol{A_1}) \cup (\boldsymbol{B} \cap \boldsymbol{A_2}) \cup \cdots \dots . . \cup (\boldsymbol{B} \cap \boldsymbol{A_2} \rangle$

P(B) = $P \langle \boldsymbol{B} \cap \boldsymbol{A_1}) + \boldsymbol{P}(\boldsymbol{B} \cap \boldsymbol{A_2}) + \cdots \dots . . + (\boldsymbol{B} \cap \boldsymbol{A_4} \rangle$

$P \; \frac{5}{52} \times P(B|A_2)$

$P(B$

Total Probability Is:

P(B) = $P(A_1) . P(\boldsymbol{B}|A_1) + P(A_2) . P(\boldsymbol{B}|A_2) + \cdots \dots \dots \dots + P(A_n) . P(\boldsymbol{B}|A_n)$

$$P = \frac{4 * 3 * 2}{52 * 51 * 50}$$

$= P(A_1) \cdot P(B|A_1) + P(A_2) \cdot P(B|A_2) + P(A_3) + P(B|A_3)$

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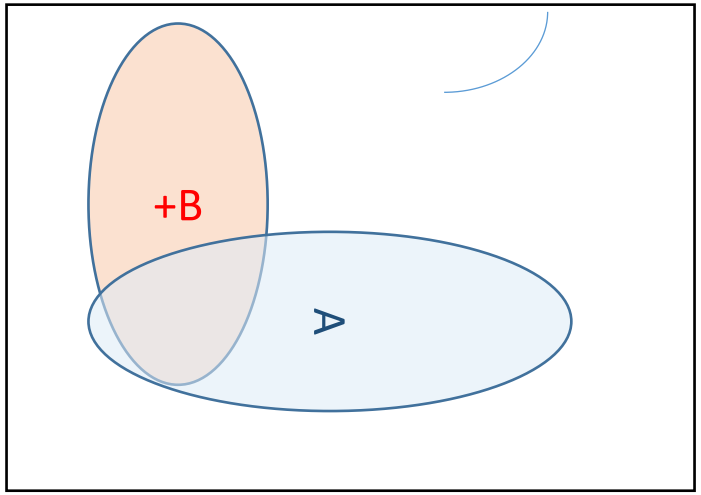
Conditional Probability

Total Probability Theorem

$P(A^C) = 0.9$

Infected → A

Positive → B



Facts:

P(A) = 0.1

P(B^C| A) = 0.01

P(B|A) = 0.99

P(B|A^C) = 0.05

P(B^C|A^C) = 0.95

Using Total Probability Theorem:

$$\mathbf{P(B)} = P(\boldsymbol{A}).\,P(\boldsymbol{B}|\boldsymbol{A}) + P(A^C).\,P(\boldsymbol{B}|A^C)$$

$$\mathbf{P(B)} = 0.1 * 0.99 + 0.9 * 0.05 = \boxed{0.144}$$

14%

P(positive)

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Conditional Probability

Facts:

Total Probability Theorem

P(B|A_1) = 0.3

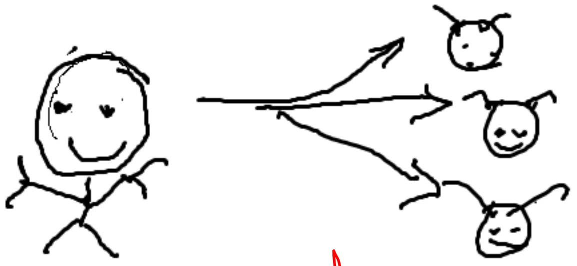
$P(B^c|A_1) = 0.7$

P(B|A_2) = 0.6

$P(B^c|A_2) = 0.4$

P(B|A_3) = 0.75

$P(B^c|A_3) = 0.25$



monster not encounter

$P(B^c) =$

$P(A_1).P(B^c|A_1)$
$+P(A_2).P(B^c|A_2)$
$+P(A_3).P(B^c|A_3)$

A **= i-th path taken**

B: monster encountered

P(A_1) = P($A_{\cancel{1}\,2}$) = P($A_{\cancel{1}\,3}$) = 1/3

Using Total Probability Theorem:

$$\mathbf{P}(B^C) = P(A_1).P(B^C|A_1) + P(A_2).P\left((B^C|A_2) + P(A_3).P(B^C|A_3)\right)$$

P(B) = 1/3 * 0.7 + 1/3 * 0.4 + 1/3 * 0.25 = 0.45

Conditional Probability

Facts:

$P(B|A_1) = 0.3$

$P(B|A_2) = 0.6$

$P(B|A_3) = 0.75$

A_i = i-th path taken

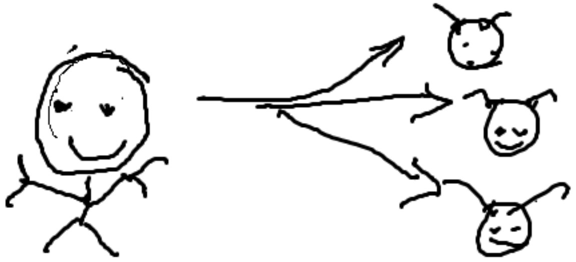
B: monster encountered

$P(A_1) = P(A_1) = P(A_1) = 1/3$

Probability:

$P(A_1|\, B) = ?$

Bayes' Theorem



If he does not come out alive what is the probability that he took path A1 ?

$P(A_1|\, B) = ?$

$$\boxed{P(\boldsymbol{A_1}|\, \boldsymbol{B}) = \frac{P(\boldsymbol{A_1} \cap \boldsymbol{B})}{P(\boldsymbol{B})}}$$

Applying total probability theorm:

$$\mathbf{P(B)} = P(A_1).\, P(\mathbf{B}|A_1) + P(A_2).\, P\big((\mathbf{B}|A_2) + P(A_3).\, P(\mathbf{B}A_3)\big)$$

$$P\langle \boldsymbol{A_1} \cap B\rangle = P(\boldsymbol{A_1}|\boldsymbol{B}).\, P(B) = P(\boldsymbol{B}|\boldsymbol{A_1}).\, P(\boldsymbol{A_1})$$

$1/3 \times 0.3$

$$P(\boldsymbol{A_1}|\, \boldsymbol{B}) = \frac{P(\boldsymbol{A_1} \cap \boldsymbol{B}) = P(A_1).\, P(B|A_1)}{P(A_1).\, P(\mathbf{B}|A_1) + P(A_2).\, P\big((\mathbf{B}|A_2) + P(A_3).\, P(\mathbf{B}A_3)\big)} = \mathbf{0.182}$$

$(0.4\ldots)$

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Conditional Probability

Facts:

P(B|A_1) = 0.3

P(B|A_2) = 0.6

P(B|A_3) = 0.75

A_i = i-th path taken

B: monster encountered

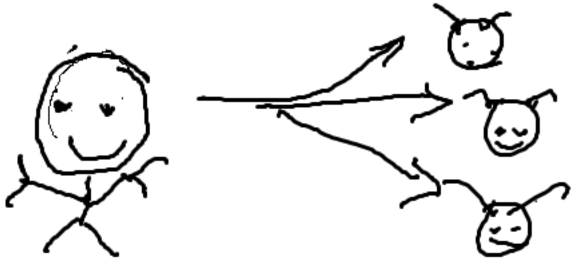
P(A_1) = P(A_1) = P(A_1) = 1/3

Probability:

P(A_3| B) = ?

Bayes' Theorem

If he does not come out alive what is the probability that he took path A_3 ?



P(A_3| B) = ?

$$P(A_3|\,\boldsymbol{B}) = \frac{P(A_3 \cap \boldsymbol{B})}{P(\boldsymbol{B})}$$

Applying total probability theorem:

P(B) = $P(A_1).\,P(\mathbf{B}|A_1) + P(A_2).\,P\big((\mathbf{B}|A_2) + P(A_3).\,P(\mathbf{B}A_3)\big)$ 0.75

1/3

$= P(A_3) \cdot P(B|A_3)$

$P\langle A_3 \cap B\rangle = P(A_3|\boldsymbol{B}).\,P(B) = P(\boldsymbol{B}|A_3).\,P(A_3)$

$$P(A_3|\,\boldsymbol{B}) = \frac{P(A_3 \cap \boldsymbol{B})}{P(A_1).\,P(\mathbf{B}|A_1) + P(A_2).\,P\big((\mathbf{B}|A_2) + P(A_3).\,P(\mathbf{B}A_3)\big)} = \mathbf{0.45}$$

0.45 ≠ 0.55

Conditional Probability

Breaking down Bayes Theorem

Exploit the Multiplication Rule:

$$P(A_1).P(B|A_1) = P(B).P(A_1|B)$$

Exploit the Total Probability Theorem:

$$P(B) = P(A_1).P(B|A_1) + P(A_2).P((B|A_2) + P(A_3).P(B|A_3))$$

Exploiting the known probabilities:

$$P(A_1|B) = \frac{P(A_1).P(B|A_1)}{\Sigma_{i=1}^{n} P(A_i).P(B|A_i)}$$ **Bayes Theorem**

$$\frac{P(A_1) \cdot P(B|A_1)}{P(B)} \quad \Sigma$$

Conditional Probability

Facts:

P(A) = 0.01

$P(\bar{A}) = 0.99$

P(B|A) = 0.95

P(B|A^C) = 0.05

A = Ship 1 sends a signal 1

B = Ship 2 receives a signal 1

Bayes' Theorem

P(A | *B*) = ?

$$\frac{P(A \cap B)}{P(B)} \rightarrow$$

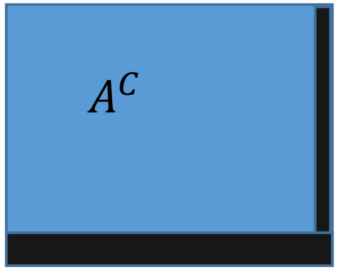
0.01×0.95

0.95

$= P(A).P(B|A)$

$$P(\boldsymbol{A} \mid \boldsymbol{B}) = \frac{P(\mathbf{A} \cap \boldsymbol{B})}{P(\boldsymbol{A}).P(\mathbf{B}|\boldsymbol{A}) + P(A^C).P(\mathbf{B}|A^C)} = \mathbf{0.18}$$

$0.01 \quad 0.95 \quad 0.99 \cdot 0.05$



A

10000

9900

495

100

95

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Conditional Probability

Bayes' Theorem

Facts:

$P(\bar{A}) = 0.9$

P(A) = 0.1

P(B^C | A) = 0.01

=> P(B | A) = 0.99

P(B | A^C) = 0.05

=> P(B^C | A^C) = 0.95

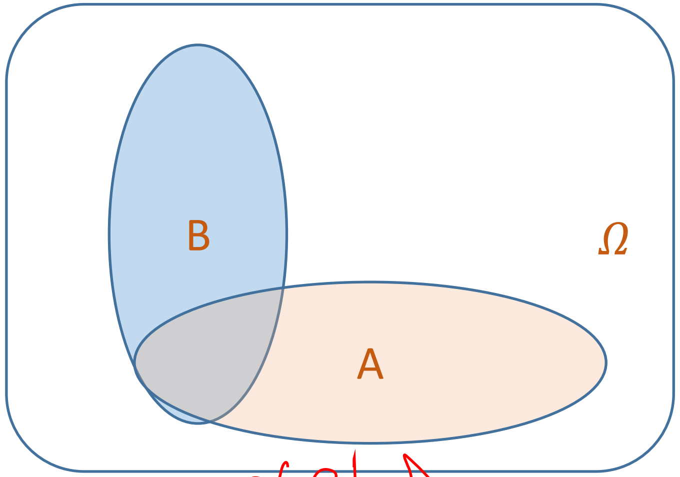
A = Person is infected

B = Tested positive

What is the chance that a person is actually infected, if the results of the test are Positive

A

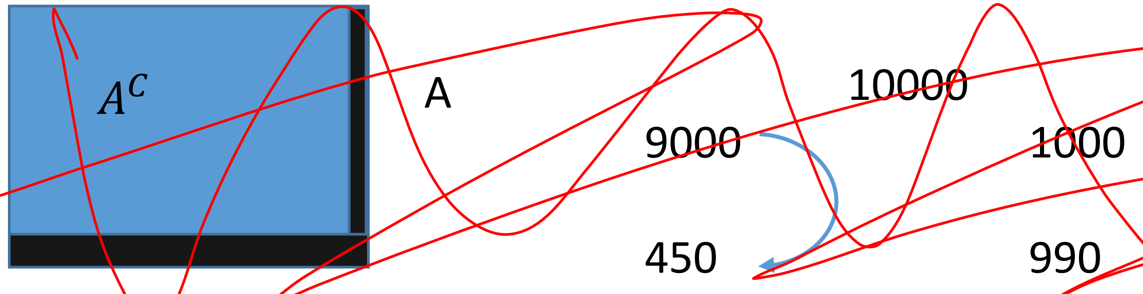
B



P(A | B) = ?

$\rightarrow P(A) \cdot P(B|A)$

$$P(A \mid B) = \frac{P(\boldsymbol{A} \cap \boldsymbol{B})}{P(\boldsymbol{A}).\ P(\mathbf{B}|\boldsymbol{A}) + P(A^C).\ P(\mathbf{B}|A^C)} = \mathbf{0.67}$$



Independent Events

Consider the below 2 Events:

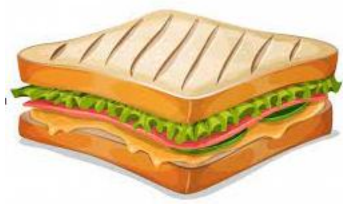
A: I had a sandwich for breakfast

B: It will rain today

If A occurs will you update your belief about A ?

What do we call such events ?

Independent Events



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Independent Events

Example: 50 girls and 70 boys in a class, of these, 35 girls and 49 boys are good at maths. If I tell you that a student is very good at maths what is the probability that she is a girl?

A:Student is a Girl

B: Student is good at Maths

there is no correlation but calculated

P(A | B) = ?

$P(A \cap B) = P(A) \cdot P(B|A)$

$P(A \mid B) = \frac{P(\mathbf{B}|A).P(A)}{P(B)} = (\frac{7}{10} * \frac{5}{12}) / 7/10 = 5/12$

P(B) = $P(A).\ P(\mathbf{B}|A) + P(A^C).\ P(\mathbf{B}|A^C)$ = 7/10

Facts:

Student: Girl, Boy

P(A) = 50 / (50+70) = 5/12

P(A^C) = 7/12

Good math: Girl, Boy

=> P(B | A) = 35/50 = 7/10

P(B | A^C) = 49/70 = 7/10

P(A|B) = 5/12 = P(A)

Knowing about B does not change my belief about A

Similarly, P(B|A) = 7/10 = P(B|A^C) = 7/10

in this case

So, Knowing about A does not change by belief about B

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Independent Events

Two events A & B are independent if $P(A|B) = P(B)$ or $P(B|A) = P(B)$

More Robust way of saying the above:

Two events are A& B are independent if

$$P(A \cap B) = P(A) * P(A|B)$$
$$= \boxed{P(A) * P(B)}$$

$$P(A \cap B) = P(A) * P(B|A)$$
$$= \boxed{P(A) * P(B)}$$

Independent Events

Take an example of tossing 3 coins simultaneously

A:First toss results is an Head

B: Exactly 2 tosses results in heads

Are A and B independent ?

Coin 1	Coin 2	Coin 3	A	B	A ∩ B
H	H	H	*		
H	H	T	*	*	*
H	T	H	*	*	*
H	T	T	*		
T	H	H		*	
T	H	T			
T	T	H			
T	T	T			

Facts:

P(A) = 4/8

P(B) = 3/8

P(A ∩ B) = 2/8 = 1/4

P(A ∩ B) = P(A) * P(B)

$\frac{4}{8} \times \frac{3}{8_2} = \frac{3}{16}$

ideally should match

if i add this one

4/16 → 1/4 eq

Independent Events

Example: I am rolling 2 dice, What is the probability that the sum of both the dice is 7 and second dice shows an even number.

A: Sum of the dice is 7

B: Second dice shows an even number

Facts:

~~P(A) = 4/8~~ 6/36

~~P(B) = 3/8~~

~~P(A ∩ B) = 2/8~~

~~P(A ∩ B) = P(A) * P(B)~~

Are A and B independent ?

Event A {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)} 6

Event B { (1,2), (1,4), (1,6), (2,2), (2,4), (2,6)
(3,2), (3,4), (3,6), (4,2), (4,4), (4,6) 18
(5,2), (5,4), (5,6), (6,2), (6,4), (6,6)}

A ∩ B = {(1,6), (3,4), (5,2)}

P(A) = 6/36 = 1/6

P(B) = 18/36 = 18/36 = 1/2

P(A ∩ B) = 3/36 = 1/12 , Therefore P(A ∩ B) = P(A)*P(B)

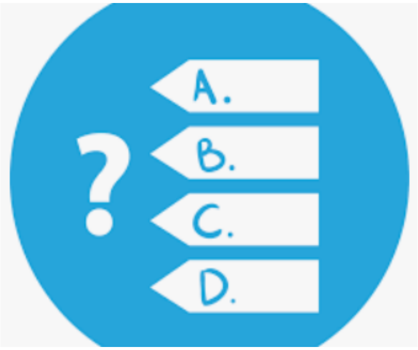
$\frac{1}{12} = \frac{1}{6} \times \frac{1}{2}$

Independent Events

Example: A Quiz has 2 multiple choice questions. The first question has 4 choices of which I is correct and the second question has 3 choices of which one is correct. If a student randomly guesses the answers, what is the probability that he will answer both questions correctly.

A: First answer is correct

B: Second answer is correct both events are independent



Facts:

P(A) = 1/4

P(B) = 1/3

$\boxed{P(A \cap B) = P(A) * P(B)}$ 1/12

$P(A \cap B) = 1/4 * 1/3 = 1/12$

Independent Events: n events

We say that events a1, a2, a3, ……an. are pair wise independent if

P($A_i \cap A_j$) = P(A_i) * P(A_j) $\forall$ i $\neq j$

We say that events A1, A2, A3…..An are mutually independent or independent of all subsets if

I $\subset$ {1, 2, 3, 4,…..n}

P ($\bigcap \boldsymbol{i} \in I\ A_i = \pi_{\mathrm{i=1}}^{n}\ \boldsymbol{P}(A_i)$)

Eg: for n = 3 {1,2,3}

{1,2}, {1,3}, {2,3}, {1,2, 3}

P($A_1 \cap A_2$) = P(A_1)* P(A_2)

P($A_1 \cap A_3$) = P(A_1)* P(A_3)

P($A_2 \cap A_3$) = P(A_2)* P(A_3)

P($A_1 \cap A_2 \cap A_3$) = P(A_1)* P(A_2) * P(A_3)

Summary

Set Theory

Finite, infinite Countable, infinite uncountable

Intersection, Union, Complement

Properties of set operations

Associativity

$A \cup (B \cup C) = (A \cup B) \cup C$

Commutativity

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Distributive laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Demorgon's Law

$(A \cup B)^C = A^C \cap B^C$

$(A \cap B)^C = A^C \cup B^C$

Disjoint sets

$P(A_i \cap A_j) = P(A_i) * P(A_j) \; \forall \, i \neq j$

Summary

Axioms:

Axiom 1 P(A) $\geq \mathbf{0}\ \forall\ \mathbf{A}$
(Non negativity)

Axiom 2 **P(Ω) = 1**
(Normalisation)

Axiom 3 **If the events A1, A2, A3, ……, An are mutually disjoint then P(A1 ∪ A2 ∪ A3 ∪ An) =**

$$\sum_i^n P(A_i)$$

(finite additivity)

Independent events:

We say that events a1, a2, a3, ……an. are pair wise independent if

P($A_i \cap A_j$) = P(A_i) * P(A_j) $\forall$ $\mathbf{i} \neq j$

We say that events A1, A2, A3…..An are mutually independent or independent of all subsets if

I ⊂ {1, 2, 3, 4,…..n} **P** $(\cap\, \boldsymbol{i} \in I\ A_i = \pi_{\mathrm{i}=1}^{n}\ \boldsymbol{P}(A_i)$**)**

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Chain rule of probability

$$P\langle A \cap B\rangle = P(A|\boldsymbol{B}).P(B) = P(\boldsymbol{B}|\boldsymbol{A}).P(\boldsymbol{A})$$

Total probability theorem

P(B) = $P(A_1).P(\boldsymbol{B}|A_1) + P(A_2).P(\boldsymbol{B}|A_2) + \cdots \ldots \ldots \ldots + P(A_n).P(\boldsymbol{B}|A_n)$

Bayes theorem

$$P(A_1|\,\boldsymbol{B}) = \frac{P(A_1).\,P(\boldsymbol{B}|A_1)}{\Sigma_{\mathrm{i}=1}^{n} P(A_\mathrm{i}).\,P(\boldsymbol{B}|A_\mathrm{i})}$$