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Counting and Probability Theory

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Introduction to Descriptive statistics

Descriptive Statistics

- ✓ Different types of data
- ✓ Different types of plots
- ✓ Measure of centrality and Spread

Probability Theory

- ✓ Counting, Sample Specs, events, axioms
- ✓ Discrete and continuous RVs
- ✓ Bernoulli, Uniform, Normal dist
- ✓ Sampling strategies

Inferential Statistics

- ✓ Interval Estimators
- ✓ Hypothesis testing (z-test, t-test)
- ✓ ANOVA, Chi-square test
- ✓ Linear Regression

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Counting and Probability Theory

Why do we need to learn Counting Principle ?

What are the principals of Counting ?

- **Multiplication Principle**
- **Subtraction Principle**

What are sequences and how do you count them ?

What are collections and how do you count them ?

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Need for Probability Theory

Goal: Is to study a large collection of people or Objects

Challenges: Infeasible, Expensive and time consuming

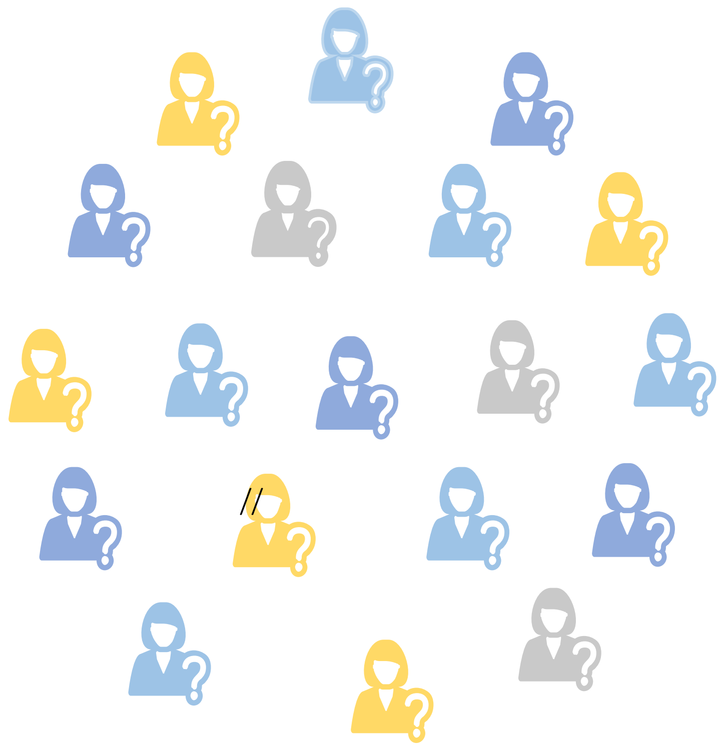
Solution: Survey only a few elements and draw inference about all elements from this smaller group

Population: Total collection of Objects that we want to study

Sample: It's a subgroup of the population that we study to draw inference about the population

What is the probability that a statistic computed from a sample is closed to that computed from a population?

Statistics: Proportion, Mean, Median, standard deviation, variance when computed from a sample is called a statistic

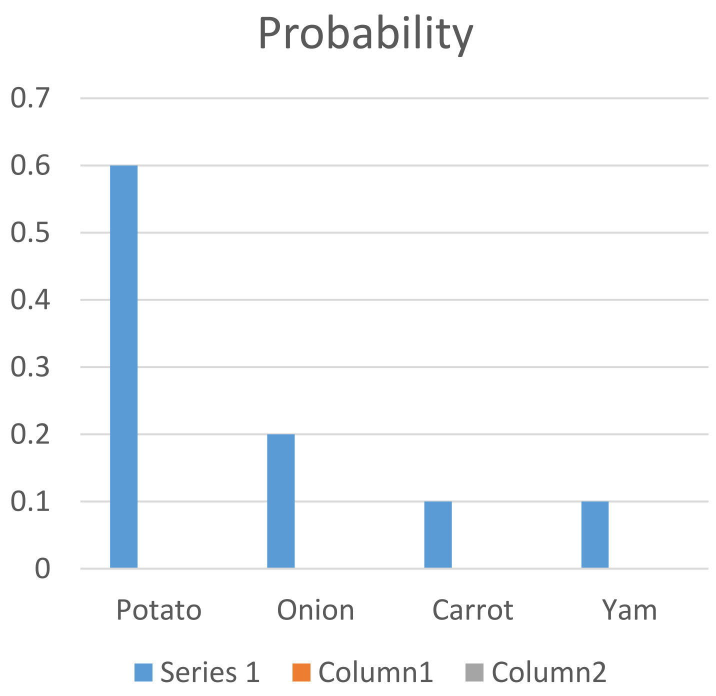


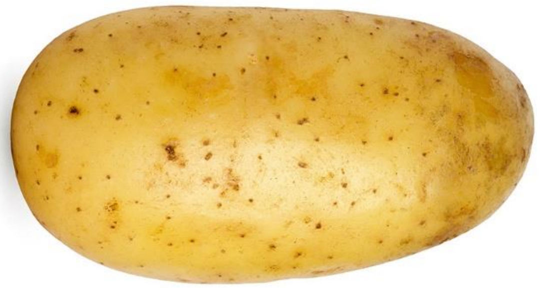
Need for Probability Theory

Machine Learning

Classification → Probability estimate problems

P (label = potato | image) ?





Is potato, Carrat, Onion,…..

And is it

Russet, Red, Yellow, White, Purple,

Fingerling, Petite ?

Predict a distribution over classes

Need for Counting ?

What is the probability of getting an Heads?

1 / 2 = 50%

How did you compute this ?

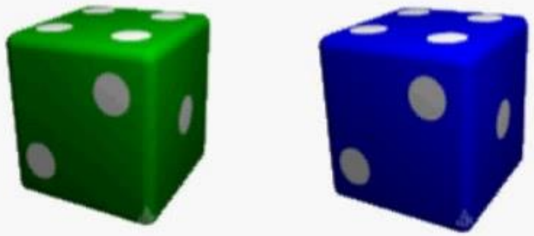
2 Possible outcomes each equally likely

Need for Counting ?

What are the chances of rolling a 3 with the green dice and a 5 with the blue dice?

The probability of rolling a 5 with the blue dice is 1/6.

This is because there are 6 possible outcomes when rolling the blue dice. One of the six outcomes is a 5, therefore 1/6 is the probability.



$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Need for Counting ?

Count the number of outcomes

Number of Outcomes is say “n”

Is there a equal / Fair chance of getting any one value with in n?

Chances of probability = 1 / n

Counting n in this example was easier

Need for Counting ?

Here is a table showing all 52 cards in a standard deck.

												Face cards		
Color	**Suit**	Ace	Two	Three	Four	Five	Six	Seven	Eight	Nine	Ten	Jack	Queen	King
Red	Hearts	A♥	2♥	3♥	4♥	5♥	6♥	7♥	8♥	9♥	10♥	J♥	Q♥	K♥
Red	Diamonds	A♦	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	10♦	J♦	Q♦	K♦
Black	Spades	A♠	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠
Black	Clubs	A♣	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣

Suppose one card is drawn at random from a standard deck.
Answer each part. Write your answers as fractions.

(a) What is the probability that the card drawn is a face card?

(b) What is the probability that the card drawn is a black card?

(c) What is the probability that the card drawn is a face card or a black card?

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Need for Counting ?

What is the probability of getting 4 aces

Number of Outcomes is say "n"

If n is the number of outcomes then n is all possible combinations of 4 cards that you can get.

How do you count n ?

- Using principles of counting

Need for Counting ?

What is the probability of getting 4 aces

Number of Outcomes is say “n”

Without knowing how to count the

number of outcomes we will not be able

to compute the probability.

Turns out that there are 270725 ways of selecting 4 cards from 52

cards! (0.00036% chance of getting 4 aces)

Need for Counting ?

Learn how to count the number of outcomes of an experiments

How many numbers are there between 73 and 358 (both inclusive)

Easy !

How many numbers are there between 73 and 358, which are divisable by 7 (both inclusive)

A little hard …….

Very simple counting

Start from absolute basics

How many numbers are there between 1 to 358

Principal 1. Number of numbers between 1 and n is n

How many numbers are there b/n 73 and 358 (both inclusive)

73, 74, 75 ………………………………..356, 357, 358

I know principal 1 , to count from 1 to n, can we use that principal here ?

-72 from the sequence above

1, 2, 3, ……………………………………284, 285, 286

Very simple counting

How many numbers are there between 73 and 358 (both inclusive)

73, 74, 75 ……   …….. 356, 357, 358

358 – 72 = 358 – (73 -1) = 358 – 73 + 1 = 286

If k = 73 and n = 358 then above sequence is n – k + 1

Principal 2. Number of numbers between k and n is n – k + 1

Very simple counting

How many numbers are there between 73 and 358 which are

divisable by 7 (both inclusive)

73, 74, 75 ……   …….. 356, 357, 358

77, 84, 91, ………………………….343, 350, 357

Its not a sequence of consecutive numbers

Divide by 7 =

11, 12, 13, ………………………….49, 50, 51

Principal 2. Number of numbers between k and n is n – k + 1

K = 11 and n = 51

So, number of elements in the sequence is 51 – 11 + 1 = 41

Very simple counting

How many numbers are there in the sequence

-21, -17, -13 ……………………………391, 395, 399

Difference of 4 but not divisable by 4

Now if we add 1 to this sequence

-20, -16, -12 ……………………………392, 396, 400

Divide by 4 =

-5, -4, -3, ……………………………98, 99, 100

Principal 2. Number of numbers between k and n is n – k + 1

K = -5 and n = 100

So, number of elements in the sequence is 100 – (-5) + 1 = 106

Very simple counting

How many numbers are there in this sequence

9 5/12, 9 5/6, 10 1/ 4 ………………..21 ½, 21 11/12, 22 1/3

9 5/12, 9 10/12, 10 3/12 …………   21 6/12, 21 11/12, 22 4/12

Convert to proper fraction

(9*12 +5) /12

113 / 12, 117 / 12, 123 / 12 ……………….258 / 12, 263 / 12, 268 / 12

Multiply by 12

113, 118, 123 ……………………..258, 263, 268

+ 2
115, 120, 125…………………………260, 265, 270

Divide by 5 = 23, 24, 25 ………………………52, 53, 54

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The multiplication Principal

South Indian: Vada, Idly, Dosa, Pongal, uthappam

North Indian: Alo Paratha, Sandwich, Poha

Beverages: Coffee, Tea, Milk

Combo = one South Indian, One North Indian, One Beverages

What are the different combinations possible can I have different

breakfast every day of the month

For every combination of South Indian dish
I can make 3 x 3 = 9 Combinations (dicision_tree)
So I can make 5 x 3 x 3 = 45 combinations

Then,
Principal 3: number of ways of making a sequence of independent choices is just the product of the number of choices at each step.

The multiplication Principal

8 Boys ……………………..

12 Girls …………………….

Combination of 8 / B   12 / G

If a students committee consists of 1 boy and 1 girl

How many combinations of boys and girls are possible.

Committee = 8 * 12 * …….* ……………..*…………….

Regardless of number steps the rule (Principle) remains the same

5 r, 3 m, 4 sh, 5 sn, 3 tran

5 * 3* 4* 5 * 3

The multiplication Principal (Special Case 1)

How can you make a sequence of k objects from given n objects

with repetition.

Example: a fitness enthusiast has 10 different activities to choose from

Walking, running, Arobics, Zomba, Crossfit, Yoga, Squash, badminton,

Swimming, Gym

Monday, Tuesday, Wednesday, Thursday, Friday, Saturday & Sunday

How many weekly exercise plans can you make, if you can repeat

the same exercise more than once.

10 *	10 *	10 *	10 *	10 *	10 *	10
M	T	W	T	F	S	S

Principal 4: The number of sequences of K objects made from given n objects, when any objects in the sequence can be repeated any number of times is N ** k

The multiplication Principal (Special Case 1)

How many 5 letter words can you form using the alphabets of the

English language?

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

5 decisions eg: b I n g o

Applying the principle:

The number of sequences of K objects made from given n objects, when any objects in the sequence can be repeated any number of times is N ** k

The number of 5 letter words that can be created is 26 ** 5

Thank You!

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The multiplication Principal (Special Case 2)

How can you make a sequence of k objects from given n objects without repetition.

Example: a fitness enthusiast has 10 different activities to choose from

Walking, running, Arobics, Zomba, Crossfit, Yoga, Squash, badminton, Swimming, Gym

Monday, Tuesday, Wednesday, Thursday, Friday, Saturday & Sunday

How many weekly exercise plans can you make, if you can not repeat the exercise more than once per week.

10	*	9	*	8	*	7	*	6	*	5	*	4
M		T		W		T		F		S		S

K = 7
N = 10

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The multiplication Principal (Special Case 2)

10 *	9 *	8 *	7 *	6 *	5 *	4	K = 7
M	T	W	T	F	S	S	N = 10

10 * (10 - 1) * (10 – 2) * (10 – 3) * (10 – 4) * (10 -5) * (10 -6)

Principal 5 : The number of sequences of K objects made from given n objects, such that no objects in the sequence can be repeated is :

n * (n - 1) * (n – 2) * (n – 3) * (n – 4) * (n -5) * (n – (k-1))

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The multiplication Principal (Special Case 2)

How many 3 digit numbers

9 Choices 1, 2, 3, 4, 5, 6, 7, 8, 9

3 Decisions _____ _____ ______

How many weekly exercise plans can you make, if you can not repeat the exercise more than once per week.

10 *	9 *	8 *	7 *	6 *	5 *	4
M	T	W	T	F	S	S

K = 7
N = 10

The multiplication Principal (Special Case 2)

Twist How many of the above number are odd ?

9 Choices 1, 2, 3, 4, 5, 6, 7, 8, 9

3 Decisions ___9___ ___8___ ___5___ ?

9 * 8 * ?

I can still apply multiplication principal

How ? (Number of chooses made is independent of previous step)

Start from the last digit

7 * 8 * 5

The multiplication Principal (Special Case 2)

Similarly if I take another example:

How many 5 letter words can you form using the alphabets of English language so that no letter is repeated ?

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

5 decisions eg: 26 choices

Applying Principle:

26 * 25 * 24 * 23 * 22

The multiplication Principal (Special Case 2)

Twist How many of those words would end with a consonant ?

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

5 decisions eg:

B	c	d	f	?
26	25	24	23	17

All consonant

Now if I change the order in which I make the decision then:

7 * 8 * 5

5^{th}	4^{th}	3^{rd}	2^{nd}	1st
22	23	24	25	21

Now we can apply the multiplication principal because number of choices

at each step is independent of choices made in previous step.

Principal 6: if the problem specifies a constraint or restriction then always start by addressing the restriction first

The multiplication Principal (Special Case 2)

Example: A different kind of sequence given a class of 15 students, on how many ways can you form a committee comprising of a president, vice president, treasure and secretary

P VP T S

9 choices: 1 2 3 4 5 6 7 8 9
Decisions: __________ ______________ ___________

26 choices: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, x, y, z
5 decisions: ___ ___ ___ ___ ___

A sequence is some thing in which the order matters, In this case does the order matter ?

The multiplication Principal (Special Case 2)

Principal 7: The numbers of ways of filling K named or numbered slots using a collection of n objects is the same as the number of ways of creating a sequence of k elements such that no objects in the sequence can be repeated

n * (n - 1) * (n – 2) * (n – 3) * (n – 4) * (n -5) * (n – (k-1))

The multiplication Principal (Special Case 2)

How can you make a sequence of n objects from a given n objects ?

Problem: Suppose you have 9 flower pots that you arrange in a line at the entrance of your house, In how many different ways can you arrange these pots ?

N = 9
K = 9
Number of data elements = n (n-1) (n-2) (n-3) ……..(n-k +1)
9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1
= 9!   (Factorial of 9)

Principal 8: The number of sequence of length n that can be formed using n objects, such that no objects in the sequence is repeated is n ! (factorial of n)

The number of ways in which n objects can be arranged amongst themselves is n !

The number of permutations of n objects is n!

The multiplication Principal (Special Case 2)

How counting sequences appears in probability ?

For letters a, b, c. What are the probabilities of combination for getting

world “cab”

3 n objects , sequence of length 3	ABC	BCA
3! = 6	ACB	CAB
Probability is 1 / 6	BAC	CBA

Let us revisit our principal formula

The number of sequences of k objects made from a collection of n objects, such that no objects in the sequence can be repeated is

$$\frac{n(n-1)(n-2)\ldots\ldots(n-k+1)\,(n-k)\,(n-k-1)(n-k-2)\ldots.3*2*1}{(n-k)\,(n-k-1)(n-k-2)\ldots.3*2*1}$$

$$\frac{n!}{(n-k)!}$$

This is a more compact representation of the formula

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The multiplication Principal (Special Case 2)

Flower pots problem with twist:

Red	Red	Red	Red	Red	Yellow	Yellow	Yellow	Yellow
1	2	3	4	5	6	7	8	9

In how many ways can you arrange the pots so that no 2 red pots are adjacent to each other

N = 5 + 4
K = 9

Red	Yellow	Red	Yellow	Red	Yellow	Red	Yellow	Red	5!
↑	↑	↑	↑	↑	↑	↑	↑	↑	4!

Number of possible ways of arranging the pots adjacent to each other is 5! * 4!

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Recap

Principal 1. Number of numbers between 1 and n is n

Principal 2. Number of numbers between k and n is n – k + 1

Multiplication

Principal 3: number of ways of making a sequence of independent choices is just the product of the number of choices at each step.

Principal 4: The number of sequences of K objects made from given n objects, when any objects in the sequence can be repeated any number of times is N ** k

Principal 5 : The number of sequences of K objects made from given n objects, such that no objects in the sequence can be repeated is :

n * (n - 1) * (n – 2) * (n – 3) * (n – 4) * (n -5) * (n – (k-1))

Principal 6: if the problem specifies a constraint or restriction then always start by addressing the restriction first

Principal 7: The numbers of ways of filling K named or numbered slots using a collection of n objects is the same as the number of ways of creating a sequence of k elements such that no objects in the sequence can be repeated

Principal 8: The number of sequence of length n that can be formed using n objects, such that no objects in the sequence is repeated is n ! (factorial of n)

The Subtraction Principal

Recap on rules:

Always address the restriction first

The number of choices at each step should be independent of the choices made at previous steps

What if you can not follow the above rules?

The Subtraction Principal

How many 3 letter words can you form which contain atleast one vowel and no letter is repeated

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

3 decisions eg: ______ ______ ______

Start with restriction:	3	2	1
	24*	25*	5 (aeiou)
Start with middle letter:	2	1	3
	25*	5*	24

What is happening her is that we are not being able to rearrange the decision making in such a way that we are able to satisfy the original condition in the question.

The Subtraction Principal

We can not easily apply multiplication function as the number of choices for the last decision depends on the previous choices

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

3 decisions eg: ______ ______ ______

Subtraction Principal:

The number of objects that satisfy some condition is equal to the total number of objects in the collection minus the ones which do not satisfy this condition

The Subtraction Principal

A = Set of all 3 letter words with no letter repeated

B = Set of all 3 letter words with no letter repeated and atleast one oval

C = Set of all 3 letter words with no letter repeated and no vowels

Then B = A - C

A = 26 * 25 * 24

C = 21 * 20 * 19

B = (26 * 25* 24) – (21 * 20 * 19)

The Subtraction Principal

Another Example:

How many 5 letter words can you form which contain atleast 2 consecutive letters which are the same ?

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

5 decisions eg: ______ ______ _______ ______ _______

✓ **Apple, Sheep, Utter, Atta, Loop**

X **Bears, Rusty, Doduo**

The Subtraction Principal

A = Set of all 5 letter words with no letter repeated

B = Set of all 5 letter words containing atleast 2 consecutive letters which are same

C = Set of all 5 letter words with no consecutive letters which are the same

Then B = A - C

A = 26 * *5

C = 26 * 25**4

B = (26 ** 5)   - (26 * 25 **4)

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Collection

Collections

Recap on sequence

In sequence the order matters

Cat ≠ act

Even though both have the same set of letters: (t, c, a)

In collection order does not matter

Cat = act = tac = tca = atc = cta (all factorial combinations)

All the 6 words have the same letters: a, c, t

Collections

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

3 decisions eg: _______ ________ _________

How many sequences of 3 letters can you form (no repetition) ?

$$\frac{n!}{(n-k)!}$$

How many collections of 3 letters can be formed (no repetition) ?

We don’t know that

But, we now how to count the sequences. Can we reuse that knowledge ??

Collections

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

3 decisions eg: _______ ________ _________

Breaking down the Sequences :

Step 1: select the 3 letters to be put in the word

Making a collection

Step 2: re-arrange the 3 letters in 3! Ways

Re arranging elements in the Sequence

In collection I am only concerned about the step 1

Collections

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

3 decisions eg: ________ ________ ________

Making a collection

N = Number of ways of selecting k elements

Re arranging elements in the collections

k! = number of ways of re arranging the k terms

Number of Sequences = N*k! = $\frac{n!}{(n-k)!}$

Therefore $\binom{n}{k} = N = \frac{n!}{(n-k)! * k!}$

Collections

What is the number of ways of choosing 3 Vowels form 5 Vowels ?

a e i o u

Collections:	Sequences:
(a, e, i)	{(a, e, i), (a, I, e), (e, a, i), (e, I, a), (I, a, e), (I, e, a)}
(a, e, o)	(a, e, o)
(a, e, u)	(a, e, u)
(a, i, o)	(a, i, o)
(a, I, u)	(a, I, u)
(a, o, u)	(a, o, u)
(e, I, o)	(e, I, o)
(e, I, u)	(e, I, u)
(e, o, u)	(e, o, u)
(I, o, u)	(I, o, u)

Number of collections possible by selecting 3 letters from given 5 letters is 10

Number of Sequences possible is : number of collections N * k ! = 10 * 3 ! = 60

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Collections

Given a class of 15 students in how many ways can you form a committee of 4 members?

Are we creating a collection or Sequence

Sequences:

ABCD
ABDC
ACBD
ADBC
ADCB
BACD
….
…..
……. 4!
24

$$\frac{15!}{(15 - 4)! * 4!}$$

$$\frac{n!}{(n - k)! * k!}$$

Collections

Collections Principal: The number of ways of selecting k objects from a given n objects is

$$\frac{n!}{(n-k)!\,*\,k!}$$

And is denoted as $\binom{n}{k}$

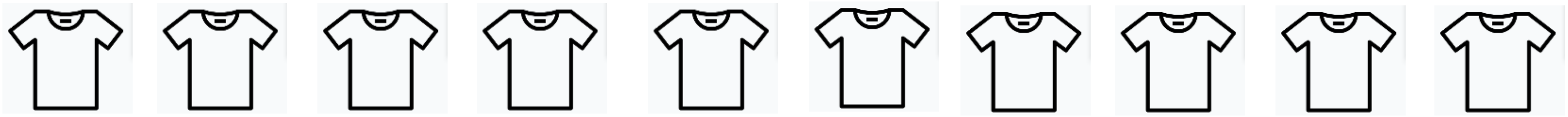
Collections, Some examples

Consider 10 people in a meeting room, each person shakes hands with every other person in the meeting room what is the total number of handshakes?

$$\frac{n!}{(n-k)!*k!} \qquad \frac{10!}{(10-2)!*2!} \qquad \binom{10}{2} = 45$$

Collections, Some examples

You are going on a vacation and your suitcase has space for only 3 shirts, in how many ways can you fill the suitcase ?



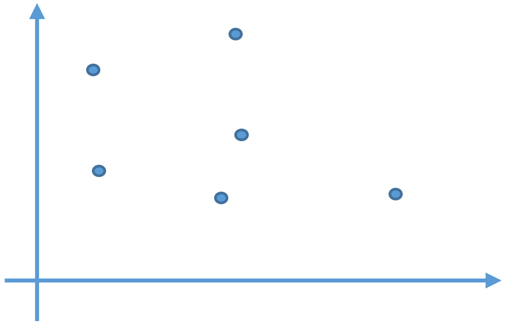
$$\frac{n!}{(n-k)!*k!}$$

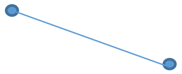
$$\frac{10!}{(10-3)!*3!}$$

$$\binom{10}{3} =$$

Collections, Some examples

There are six points on a two dimensional plane such that no three points are co-llinear. How many segments can you draw using these 6 points?





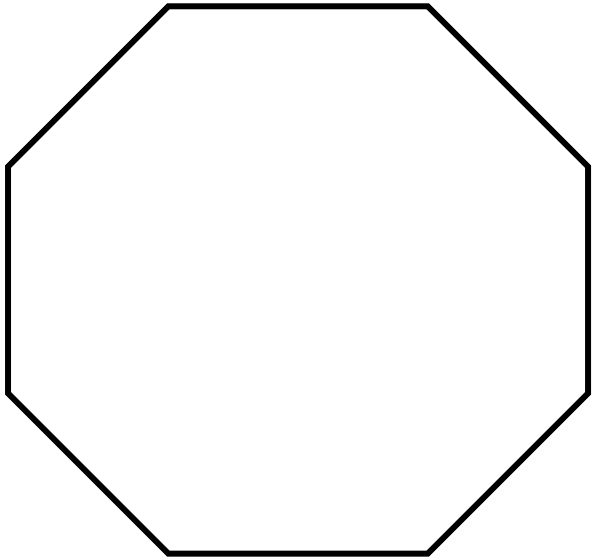
$$\frac{n!}{(n-k)!\,*\,k!}$$

$$\frac{6!}{(6-2)!\,*\,2!}$$

$$\binom{6}{2} =$$

Collections, Some examples

How many triangles can be formed from the vertices of a polygon of sides n, n =8?



$$\frac{n!}{(n-k)!\,*\,k!}$$

$$\frac{8!}{(8-3)!\,*\,3!}$$

$$\binom{8}{3} =$$

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Collections with Repetitions

Recap

Sequences

Without repetitions $$\frac{n!}{(n-k)!}$$

With repetitions **n**k**

Collections

Without repetitions $$\frac{n!}{(n-k)! * k!}$$

With repetitions $$\binom{n+k-1}{k}$$

Collections with Repetitions, Examples:

How many breakfast combos containing 5 items can you form if you are allowed to have multiple servings of the same dish ?

Items: ~~1D~~ ~~2I~~ 3P ~~4V~~ 5U 6P 7S 8PH ~~9C~~ 10T

Magic Counter: ~~1D~~ 2D 3I 4V

Combo: 5 items 1D 1D 2I 4V 9C

Without repetitions $\frac{\mathbf{n1}}{\mathbf{(n - k)\,!\ *\ k!}}$ **n = 10 + k-1**

With repetitions $\binom{n+k-1}{(n-K)!*k!}$ $\binom{10+(5-1)}{(\mathbf{14}-\mathbf{5})!*5!}$ $\binom{\mathbf{14}}{5}$

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Collections

Collections Principal: The number of ways of selecting k objects from a given n objects with repetitions is $\binom{n+k-1}{k}$

Collections Recap

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Recap

Sequences

Without repetitions $$\frac{n!}{(n - k)!}$$

With repetitions **n**k**

Collections

Without repetitions $$\frac{n!}{(n - k)! * k!}$$

With repetitions $$\binom{n+k-1}{k}$$

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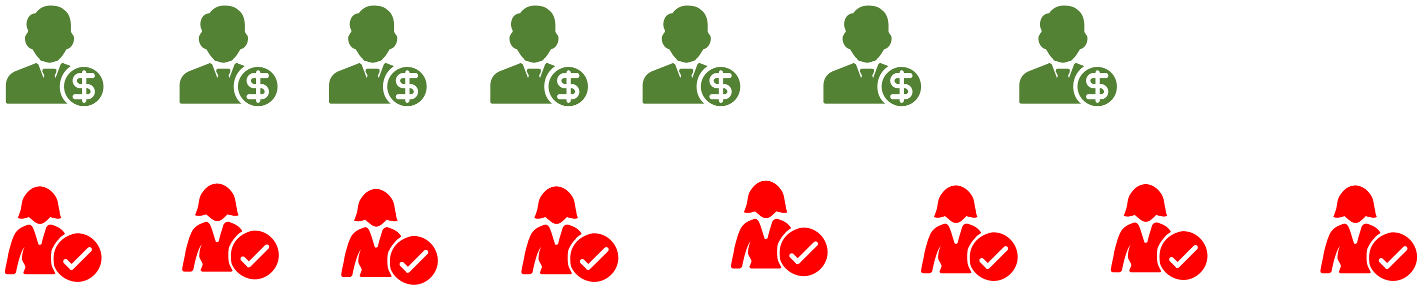
Collections with multiplication principal

Given a class of 7 boys and 8 girls, in how many ways can you form a committee of 4 members with 2 boys and 2 girls ?

Boys

Girls



Break the problem into 2 :

Number of ways of selecting 2 boys from 7 = $\binom{7}{2}$

Number of ways girls from 8 = $\binom{8}{2}$

Number of ways of combining section of bays and girls is the product of individual collections = $\binom{7}{2} * \binom{8}{2}$

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Collections with multiplication principal

Different ways of forming a cricket team using the below available players?

Available		Select
7	Batsmen	5
2	Keepers	1
4	**Pacers**	**3**
3	Spinners	2

Total $= \binom{7}{5} * \binom{2}{1} * \binom{4}{3} * \binom{3}{2}$

Collections with multiplication principal

Given: n items of l different types

M1 + m2+ m3+ ……….+mi = n

Form: Colloction of k items

K1 + k2 + k3 …..Ki = K

Available		Select
7 m1	Batsmen	5 k1
2 m2	Keepers	1 k2
4 m3	**Pacers**	**3 k3**
3 mi	Spinners	2 ki
N=16	l = 4	k = 11

$$N = \binom{n}{k} = \binom{m1}{k1} * \binom{m2}{k2} * \binom{m3}{k3} * \text{..........} * \binom{mi}{ki}$$

Collections with subtraction principal

How many different ways can we form a 4 members committee containing atleast one gynecologist ?

3 Cardiologists

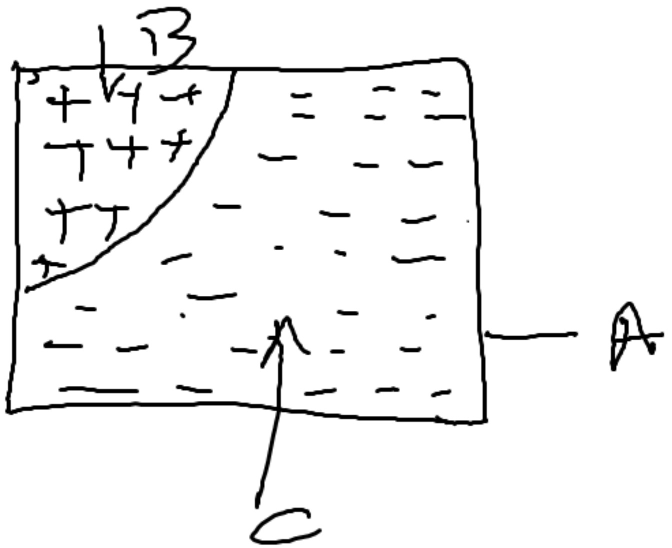
2 Neurologists

4 dialectologists

5 gynecologists

7 general physicians

Total 21 Doctors, form a committee of 4 doctors ,



N = $\binom{n}{k}$ = $\binom{21}{4}$

A = all possible combination of 4 members / Commitees

B = all possible combinations of committees containing atleast 1 Gyno

C = all possible combinations of committees not containing Gyno

B = A - C

Collections with Subtraction principal

Count (A) = $\binom{21}{4}$

Count (C) = $\binom{16}{4}$

Count (B) = $\binom{21}{4}$ - $\binom{16}{4}$

Sample spaces and Events

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Introduction to Descriptive statistics

Descriptive Statistics

- ✓ Different types of data
- ✓ Different types of plots
- ✓ Measure of centrality and Spread

Probability Theory

- ✓ Counting, Sample Specs, events, axioms
- ✓ Discrete and continuous RVs
- ✓ Bernoulli, Uniform, Normal dist
- ✓ Sampling strategies

Inferential Statistics

- ✓ Interval Estimators
- ✓ Hypothesis testing (z-test, t-test)
- ✓ ANOVA, Chi-square test
- ✓ Linear Regression

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Counting and Probability Theory

- What are sets and some of their properties
- What are experiments, sample spaces, outcomes and events ?
- What are the axioms of probability
- What are some simple ways of defining a probability function?
- What are some important theorems:
- Multiplication rule, total probability, theorem and Bayes theorem ?
- What are independent events ?

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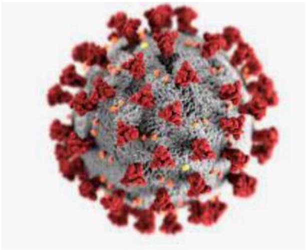
The Elements of Chance (Nothing in life is certain)

The element of chance

(Nothing in life is certain)

The Randomness everywhere !

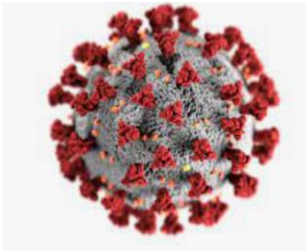


What is the chance that he would get infected if he went to the super market ?

Due to the random nature of the world around us

The Elements of Chance (Nothing in life is certain)

The Randomness everywhere !



What is the mode of transport ?

Is private care always more safer then public transport ?

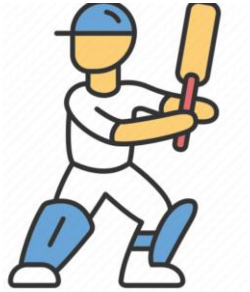
How good is his immune system?

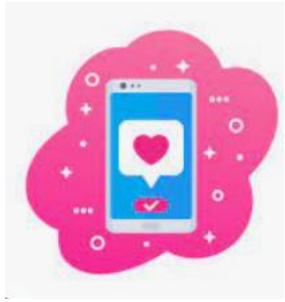
Does he have any co-morbidities?

How many infections are there in the neighbourhood?

The Elements of Chance (Nothing in life is certain)

The Randomness everywhere !

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The Elements of Chance (Nothing in life is certain)

The Randomness everywhere !

The study of these chances is the subject matter of Probability Theory !

Set Theory

Experiments, sample spaces, events

Axioms of probability

Random Variables

Distributions

Exceptions

Overview of Set Theory

Set is a collection of elements

S = { a, e, I, o, u}

E = {0, 2, 4 ………94, 96, 98, 100}

E = { x: $0 \leq x \geq 100, x \% 2 = 0$} (Compact notation useful for large data set)

$x \in S$, mean x belongs to set S,

$2 \in E$, $3 \notin E$

Overview of Set Theory

Set is a collection of elements

Subsets and equal sets

I = set of all integers

$S = \{ x : x \in I, x < 0 \}$

Every element of S in contained in I

$S \subset I$ subset

Equal Sets:

$A = B$ if $A \subset B$ and $B \subset A$ equal sets

Overview of Set Theory

Universal set

Every set of interest is a subset of the universal set

Ω = set of 52 cards

A : set of all aces $A \subset \Omega$

H : set of all hearts $H \subset \Omega$

B : set of all black $B \subset \Omega$

F : set of all face $F \subset \Omega$

Empty Set:

Set with no elements (null set)

$\emptyset = \{ \}$

Overview of Set Theory

Set Operations

Complement:

$$A^C = \{\boldsymbol{x} : \boldsymbol{x} \in \boldsymbol{\Omega}\ \boldsymbol{and}\ \boldsymbol{x} \notin \boldsymbol{A}\}$$

Union (2 Sets)

$$\mathbf{A} \cup \mathbf{B} = \{\boldsymbol{x} : \boldsymbol{x} \in \boldsymbol{A}\ \boldsymbol{and}\ \boldsymbol{x} \in \mathbf{B}\}$$

Intersection (2 Sets)

$$\mathbf{A} \cap \mathbf{B} = \{\boldsymbol{x} : \boldsymbol{x} \in \boldsymbol{A}\ \boldsymbol{or}\ \boldsymbol{x} \in \mathbf{B}\}$$

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Overview of Set Theory

Properties of Set Operations

Commutativity

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Associativity

$A \cup (B \cup C) = (A \cup B) \cup C$

Distributive laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof: Distributive laws

$X \in A \cap (B \cup C) = x \in A \text{ and } x \in (B \cup C)$

$= x \in A \text{ and } x \in B \text{ or } x \in C$

$= x \in A \text{ and } x \in B \text{ or } x \in A \text{ and } x \in C$

$\underline{= x \in (A \cap B) \cup (A \cap C)}$

Overview of Set Theory

Properties of Set Operations

De Morgan's Laws

$$(A \cup \mathrm{B})^C = A^C \cap B^C$$

$$(A \cap \mathrm{B})^C = A^C \cup B^C$$

Proof: De Morgan's Laws

$$\mathrm{X} \in (A \cap \mathrm{B})^C = \mathrm{x} \in A^C \sqcup B^C$$

not eq

$$= \mathrm{x} \in A^C \text{ or } \mathrm{x} \in B^C$$

$$\underline{= \mathrm{x} \in A^C \cap B^C}$$

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Countable Vs Uncountable

Infinite Sets

R : Set of all real numbers has infinite elements (uncountable)

I : Set of all integers has infinite elements (Countable)

An **infinite set** is said to be **countable** if there is a 1-1 correspondence b/n the elements of this set and the set of positive integers.

Uncountable Infinite sets

R: set of all real numbers

$Q = [0,1]$

There are infinite set of numbers between 0 to 1 and this infinite set is bugger then the infinite set of integers

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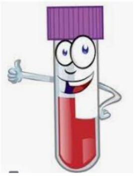
Experiments and Sample Spaces

Certainty with in uncertainty



Experiment: Going to the mall

Outcome: infected, Not infected

An Experiment or trail is any procedure that can be repeated infinite times and has a well defined set of outcomes

The set of all possible outcomes of an experiment is called the **sample space**. The elements in a sample space are **mutually exclusive** and **collectively exhaustive**

The outcome in every trail is uncertain but the set of outcomes is certain.

Experiments Involving Coin Tosses

Certainty with in uncertainty

1 Coin	{H, T}	2
2 Coin	{HH, HT, TH, TT}	4
3 Coin	{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}	8

Experiments Involving Fair Dice

Certainty with in uncertainty

	Ω	$I\Omega I$
1 Dice	{1, 2, 3, 4, 5, 6}	6
2 Dice		36
N dice		6^n

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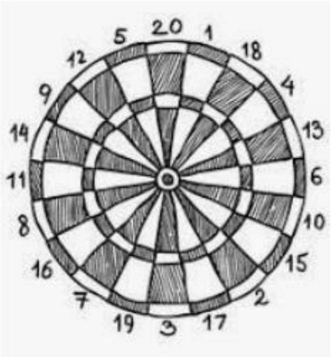
Experiments Involving Cards

Certainty with in uncertainty

	Ω	$I\Omega I$
With Repetition		
1 card	{52}	52
2 cards		52^2
N Cards		52^n

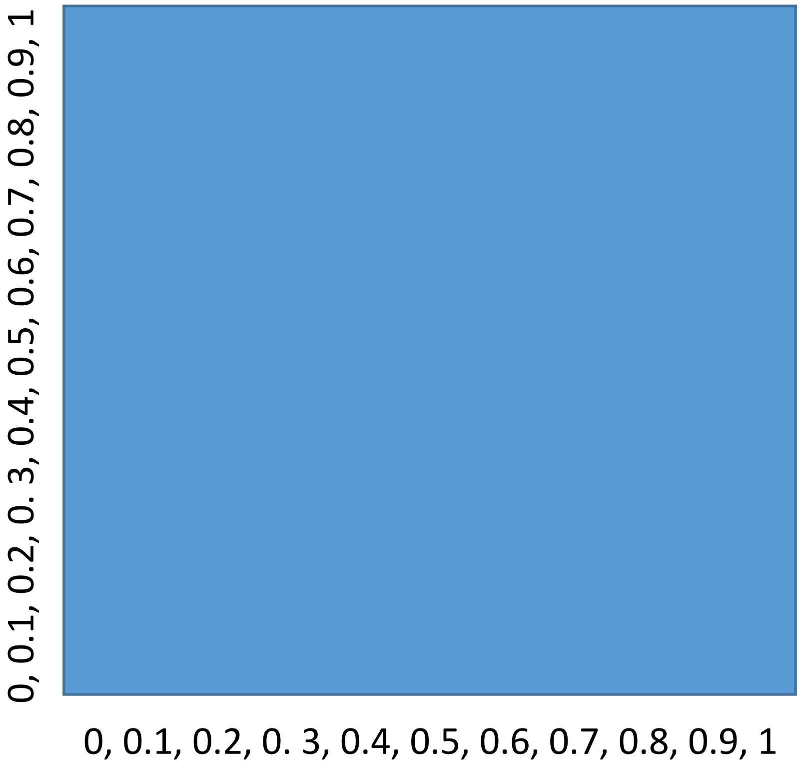
Without Repetition		
1 card	{52}	52
2 cards		52 * 51
N Cards		**52* 51* …..**

Experiments: Continuous outcomes



Certainty with in uncertainty

Dart board of square 1 mts by 1 mts

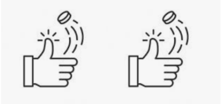


$$\Omega = \{(x, y) s.t\ 0 < x, y < 1\}$$

Events of an Experiment

Certainty with in uncertainty

An event is a set of outcomes of **an** experiment. This set is a subset of sample set.



Ω = { HT, TH, HH, TT, }

A = {HT, HH } A is an even that the first toss results in an head

Event of both tosses resulting in tails B = {TT }

Event that there are exactly 2 aces in a hand of 3 cards

$$|C| = \binom{4}{2} * \binom{48}{1} = 288$$

We say an even has occurred if the outcome of the experiment lies in the set A.

Events of an Experiment

Union of events

A = { (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) } Event that the first die shows a 2

B = { (1,4), (2,4), (3,4), (4,4), (5,4), (6,4) } Event that the second die shows a 4

D = A ∩B = { 2,4} Event that first die shows 2 and the second die shows a 4

E = A^C Event that first die does not shows 2

Events of an Experiment

Multiple events

A = The hand contains ace of spades

B = The hand contains ace of Clubs

C = The hand contains ace of hearts

$A \cup B \cup C$ hand contains atleast 1 ace

$A \cap B \cap C$ hand contains all aces

Events of an Experiment

Disjoint events

$2\ events\ A\ and\ B\ are\ said\ to\ be\ disjoints\ if\ they\ can\ not\ occur\ simultaneously.$

i.e, $A \cap B = \emptyset$

$simple\ example$ = A and A^C

Not necessary that the disjoints events should be a complement always.

A = event of first die showing 1 and B = event of first die showing 2, they can not occur together and hence are disjoint events.

The events A1, A2, A3, ……, An are said to be mutually disjoint or pairwise disjoint, if

$A_i \cap A_j = \emptyset\ \forall$ i, s.t i$\neq$ j

A = {HH}

B = {TT}

C = {HT, TH} here $A \cap B = \emptyset$, $B \cap C = \emptyset$ and $A \cap C = \emptyset$

In addition if $A \cup B \cup C = \boldsymbol{\Omega}$

Then, they are said to partition the sample space

The events A1, A2, A3, ……, An are mutually Disjoint and A1 ∪ A2 ∪ A3 ∪ An = Ω then A1, A2, A3, ……, An are said to partition the sample space.

Axioms of Probability

Recap

Experiments

Sample spaces

Events

What is the chance of an event?

Goal: Assign a number to each event such that this number reflects the chance the experiment resulting in that event.

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Axioms of Probability

The probability function

$P(A) = ?$ **Where: P is Probability function and A is an event.**

What are the conditions that such a probability function must satisfy ?

(Axioms of Probability)

Axioms of Probability

The Axioms of probability:

Axiom 1 **(Non negativity)** $P(A) \geq 0 \; \forall \; A$

Axiom 2 **(Normalisation)** $P(\Omega) = 1$

Axiom 3 If the events A1, A2, A3, ……, An are mutually disjoint then $P(A1 \cup A2 \cup A3 \cup An) = \sum_i^n P(A_i)$

(finite additivity)

Axioms of Probability

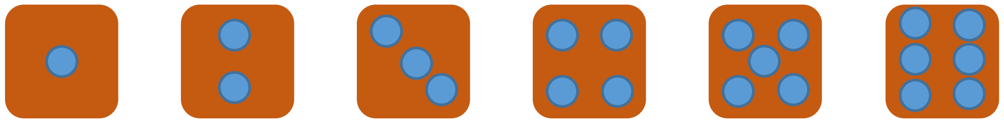
The Axioms of probability:

Axiom 3 If the events A1, A2, A3, ……, An are mutually disjoint then $P(A1 \cup A2 \cup A3 \cup An) = \sum_i^n P(A_i)$

(finite additivity)

Compute probabilities of large events from small events

Smallest possible event = one outcome



A1 A2 A3 A4 A5 A6

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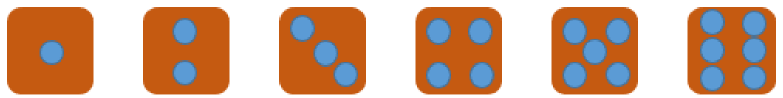
Axioms of Probability

The Axioms of probability:



A1 A2 A3 A4 A5 A6

Given P(A1), P(A2), P(A3), P(A4), P(A5), P(A6)

We can compute other probabilities

B: that event that the outcome is and odd no.
P(B) = P(A1) + P(A3) + P(A5),

C: that event that the outcome is $\geq$ 5.
P(C) = P(A5) + P(A6)

D: that event that the outcome is $\boldsymbol{multiple\ of\ 3}$.
P(D) = P(A3) + P(A6)

Axioms of Probability

Some properties of probability:

Property 1:

P(A) = 1 – P(A^C)

$A \cup A^C = \Omega$

$P(\Omega) = 1 = P(A \cup A^C) = P(A) + P(A^C)$

Therefore $P(A) = 1 - P(A^C)$

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Axioms of Probability

Some properties of probability:

Property 2:

$P(A) \leq 1$

$P(A) = 1 - P(A^C)$

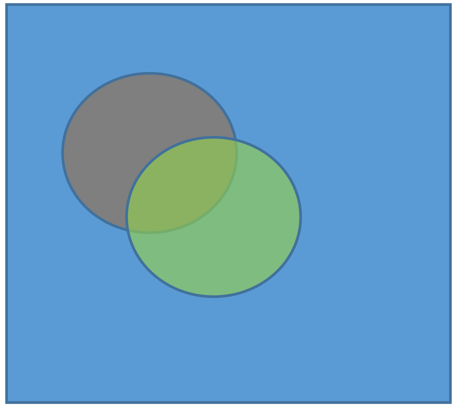
$We\ know\ that\ A^C$ is always greater then zero

Therefore $P(A) = 1 - P(A^C)$ $Because\ P(A^C)\ can\ not\ be\ zero$

$P(A) \leq 1$

Axioms of Probability

Some properties of probability:



Property 3:

$$\mathbf{P}(A \cup B) = \boldsymbol{P}(\boldsymbol{A}) + \boldsymbol{P}(\boldsymbol{B}) - \boldsymbol{P}(A \cap B)$$

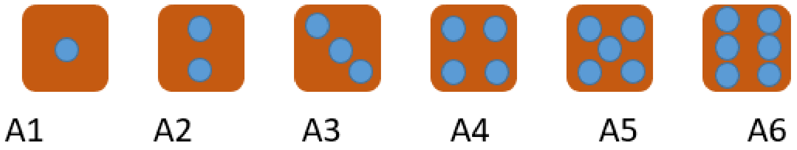
$$\mathbf{P}(A \cup B) = P\left(A \cup (B \cap A^C)\right)$$

$$= \boldsymbol{P}(\boldsymbol{A}) + P(B \cap A^C)$$

$$= \boldsymbol{P}(\boldsymbol{A}) + \mathbf{P}(\mathbf{B}) - P(B \cap \boldsymbol{A})$$

Axioms of Probability

Some properties of probability:



Property 4:

The sum of the probability of all outcomes is equal to 1

$$P(\Omega) = P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6)$$

$$= \sum_{i=1}^{n} P(A_i) = 1$$

Axioms of Probability

Some properties of probability:

Property 5:

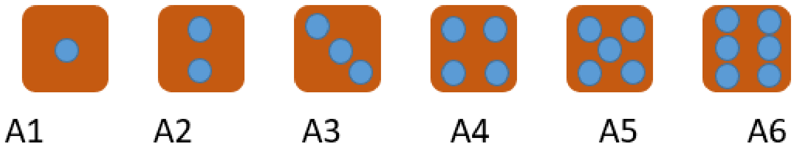
$\mathbf{P}(\phi) = \mathbf{0}$

$\mathbf{P}(\Omega) = \boldsymbol{P}(\Omega \cup \phi) = \boldsymbol{P}(\Omega) + \boldsymbol{P}(\phi) = \mathbf{1}$

$\boldsymbol{P}(\phi) = \mathbf{1} - \boldsymbol{P}(\Omega) = \mathbf{0}$

Axioms of Probability

The Axioms of probability:



Given P(A1), P(A2), P(A3), P(A4), P(A5), P(A6)

We can compute other probabilities

Axioms of Probability

The Axioms of probability:

Given P(A1), P(A2), P(A3), P(A4), P(A5), P(A6)

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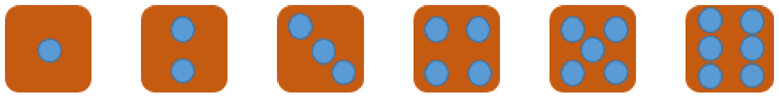
Axioms of Probability

The Axioms of probability:



A1 A2 A3 A4 A5 A6

Given P(A1), P(A2), P(A3), P(A4), P(A5), P(A6)

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P(C) = P(A5) + P(A6)

D: that event that the outcome is $multiple\ of\ 3$.
P(D) = P(A3) + P(A6)

Designing Probability Functions

Probability as Relative frequency:

Goal: Assign a number to the event such that this number reflects the chance of the experiment resulting in that event

Required: The probability function should satisfy the axioms of probability

We can think of probability of an event as fraction of the times the event occurs when an experiment is repeated a large number of times

P(H) = 12012 / 24000 = 0.5005

$$P(A_i) = \frac{Number\ of\ times\ the\ event\ is\ in\ A_i}{total\ number\ of\ times\ the\ experiment\ was\ repeated}$$

But does such a P() satisfy the axioms of probability?

Axioms of Probability

Probability as relative frequency:

Does P() satisfy the axioms?

P(A_1 U A_2) = P(A_1) + P(A_2) ?

$|\Omega| = n = 2^n\ subsets = 2^n\ events$

(axioms are about events)

P(A_1 U A_2) = $\frac{k_1+k_2}{k} = \frac{k_1}{k} + \frac{k_2}{k}$ **= P(A_1) + P(A_2)**

$$P(A_i) = \frac{Number\ of\ times\ the\ event\ is\ in\ A_i}{total\ number\ of\ times\ the\ experiment\ was\ repeated} = 1$$

Designing Probability Functions

Example :

A dataset contains images of beaches (60000), mountains (25000) and forests(15000)

What is the probability that a randomly picked image would be a forest?

Experiment: Select an image

Number of trials: 100000

Frequency of the event “forest”: 15000

$$P(\text{forest}) = \frac{15000}{100000} = 0.15$$

Designing Probability Functions

Example :

A country tests 20 million randomly selected people and finds that 1 million are infected

What is the probability that a randomly picked person would be infected?

Experiment: perform a test

Number of trials: 20 million

Frequency of the event "infected": 1 million

P(infected) $= \frac{1000000}{20000000} = 0.05$

Designing Probability Functions

Example :

A subtle point: the sample from which the probabilities were estimated should be drawn from the same population on which we are interested in making inferences.

By May-10-2020. India had tested 1673688 samples of which 67176 were found to be positive. Does this mean the probability that a randomly selected person being infected is 0.04

No: Testing in India was not random but only for people with flu-like symptoms

Designing a probability function(Equally likely outcomes)

Equally Likely Outcomes:

$|\Omega| = \{H, T\}$

P(H) = P(T) = k

Ω = H ∪ T

P(Ω) = P (H ∪ T)

= P(H) + P(T) = 2k = 1

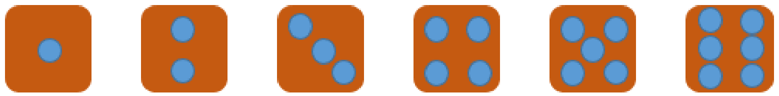
Therefore: P(H) = P(T) = k = ∪ 1/2

We can now compute the probability of 4 subsets of Ω

$\emptyset$, {H}, {T}, {H,T}

Equally likely outcomes

The Axioms of probability:



A1 A2 A3 A4 A5 A6

$$|\Omega| = \{1, 2, 3, 4, 5, 6\}$$

A_i: ***Events that the outcome is i***

$A_1,\ A_2, A_3,\ \ A_4,\ A_5,\ A_6$ ***partition*** Ω

P(A_1) = P(A_2) = P(A_3) = P(A_4) = P(A_5) = P(A_6) = k

P(Ω) = $\Sigma_{i=1}^{6}$P(A_i) = 6k = 1

Hence, P(A_i) = 1/6

We can now compute the probability of all subsets of Ω

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Equally likely outcomes

$$P(X) = \frac{Number\ of\ outcomes\ in\ X}{number\ of\ outcomes\ in\ \Omega}$$

Are the axioms of probability satisfied?

$P(A) \geq 0\ \forall\ A\ ?$: **Ratio of 2 positive numbers**

$P(\Omega) = 1$? : **Contains all outcomes**

$$P(A1 \cup A2) = P(A1) + P(A2) = \Sigma_{i=1}^{k} \frac{1}{n} = \frac{k}{n}$$

$$P(A1 \cup A2) = \frac{k1+k2}{n} = \frac{k1}{n} + \frac{k2}{n} = P(A1) + P(A2)$$

Equally likely outcomes

Examples:

What is the probability of getting a black card?

P(B) = $\frac{26}{52}$

What is the probability of getting 3 aces?

$$\binom{52}{3} = 22100 \quad \text{and} \quad \binom{4}{3} = 4$$

P(A) = $\frac{4}{22100}$

THANK YOU VERY MUCH!!!!!!

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Conditional Probabilities

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Change in belief

Setting Context Example 1:

Assume fair play conditions & equally good teams

Before the start of the play: What is the chance of India wining ? 0.5

India scores 395 batting first: What is the chance of India winning ? > 0.5

What has happened here ?

Change in belief

Setting Context Example 1:

(Assume fair play conditions & equally good teams }

What exactly happened here ?

A: event that India will win

B: India scored 395 runs

P(A) changes once we know that event B has occurred

$P(A \mid B) \neq P(A)$

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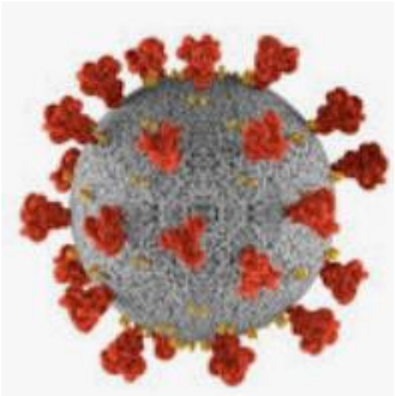
Change in belief



Setting Context Example 2:

10% of the population is infected

What is the probability that a randomly selected person is healthy or infected ?

Definition: P(A | B) is called the conditional probability of the event A given the event B

A: event that a person is healthy P(A) = 0.9

$B: event\ that\ a\ person\ has\ Covid\ 19\ symptoms$

P(A | B) $\neq$ P(A)

Conditional Probability

The definition of P(A | B)

A: Sum is 8
B: first dice shows a 4

(1, 2)	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(1, 5)**	**(1, 6)**
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

What is the probability that the sum is 8 ?

P(A) = $\frac{5}{36}$

What is the probability that the sum is 8 given that the first dice shows a 4 ?

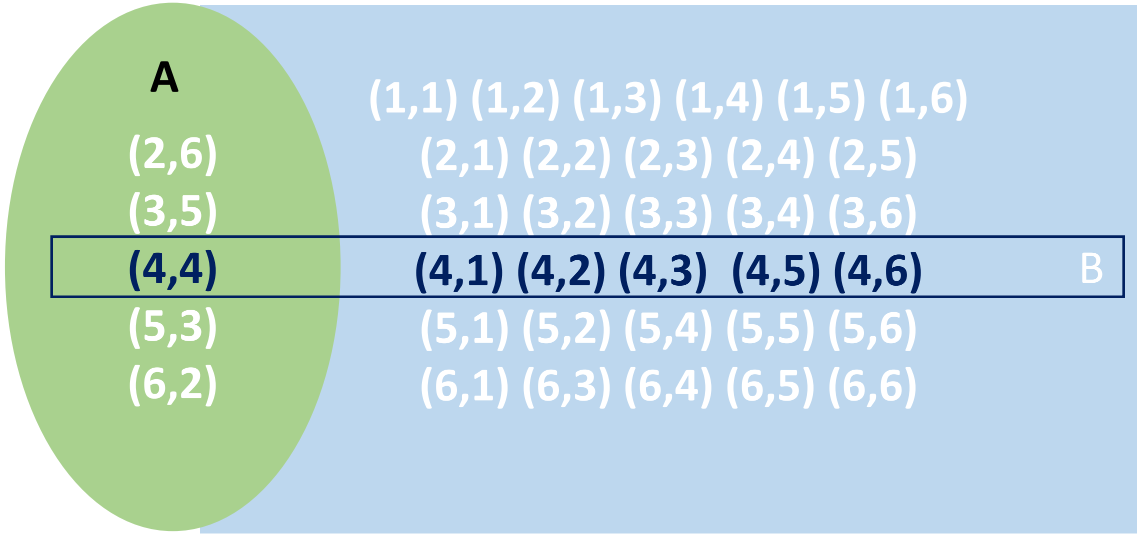
P(B) = $\frac{1}{6}$

Conditional Probability

The definition of P(A | B)

A: Sum is 8
B: first dice shows a 4

$(A \cap B)$



What is the probability that the probability of P(A|B)

$$P(B) = \frac{1}{6}$$

$$P(A \cap B) = \frac{1}{36}$$

$$P(A|B) = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{6}$$

Conditional Probability

The definition of P(A | B)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Probability **Normal Probability**

$\boldsymbol{P(A|B)}$ ***is called the conditional probability of the event A given the event B***

Conditional Probability

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Examples:

$$\textit{Think of a 2 digit number, If I tell you that atleast 1 on the number}$$
$$\textit{Is even what is the probability that both the numbers are even}$$

10 , 11, 12, 13, ……..96, 97, 98, 99

All are equally likely

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{20}{90}}{\frac{65}{90}} = \frac{4}{13}$$

$$P(A) = \frac{Number\ of\ outcomes\ in\ x}{Number\ of\ outcomes\ in\ \Omega}$$

A event that both the digits are even
B event that at least one digit is even

$$P(A) = \frac{20}{90} = \frac{2}{9}$$

But, we are interested in P(A|B)

Conditional Probability

Examples:

$60\ \%\ of\ students\ in\ a\ class\ opt\ for\ ML.20\%$
of the students opt for both ML and DL . Given that the students has opted for ML what is the probability that she has also opted for DL?

A event that student has opted for DL
B event that student has opted for ML

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.20}{.60} = \frac{1}{3}$$

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Axioms of Probability

Does Conditional probability satisfy the axioms of probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \geq 0 \ \ \text{: Ratio of 2 probabilities, Hence its always} > 0$$

$$P(\Omega|B) = \frac{P(\Omega \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

$$P(A_1 \cup A_2 \cap A_3|B) = \frac{P(A_1 \cup A_2) \cap B)}{P(B)} = \frac{P((A_1 \cap B) \cup (A_2 \cap B))}{P(B)} = \frac{P(A_1 \cap B)}{P(B)} + \frac{P(A_2 \cap B)}{P(B)} = \mathrm{P}(A_1|\mathrm{B}) + \mathrm{P}(A_2|\mathrm{B})$$

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Conditional Probability

Chain Rule of probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\boldsymbol{therfore} \quad P\langle A \cap B\rangle = P(A|\mathbf{B}).P(B)$$

$$P(\boldsymbol{B}|\boldsymbol{A}) = \frac{P(\mathbf{B} \cap \boldsymbol{A})}{P(\boldsymbol{A})}$$

$$\boldsymbol{therfore}\, P\langle \mathbf{B} \cap \boldsymbol{A}\rangle = P(\boldsymbol{B}|\boldsymbol{A}).P(\boldsymbol{A})$$

Therefore $P\langle A \cap B\rangle = P(A|\mathbf{B}).P(B)$ **=** $P(\boldsymbol{B}|\boldsymbol{A}).P(\boldsymbol{A})$

Conditional Probability

Facts:

P(A) = 0.1

P(B^C| A) = 0.01

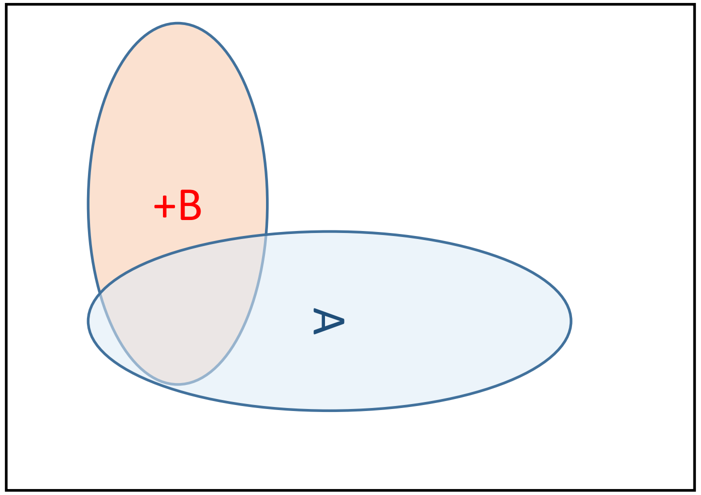
P(B|A) = 0.99

P(B|A^C) = 0.05

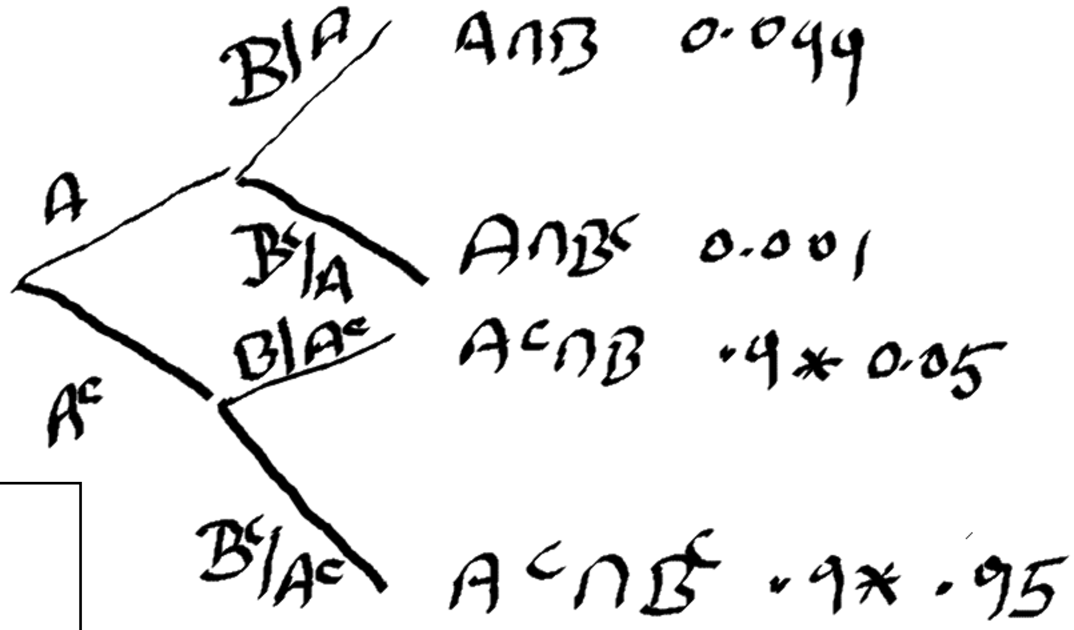
P(B^C|A^C) = 0.95

A : Infected

B: Tested Positive



Chain Rule of probability



Conditional Probability

Chain Rule of probability

$$P\langle A \cap B \cap \boldsymbol{C} \rangle = P((A \cap B) \cap \boldsymbol{C})$$

Let $\langle A \cap B \rangle = \boldsymbol{X}$

therefore $P\langle A \cap B \cap \boldsymbol{C} \rangle = P(\boldsymbol{X} \cap \boldsymbol{C})$

therefore $P\langle A \cap B \cap \boldsymbol{C} \rangle = P(\boldsymbol{X}).\, P(\boldsymbol{C}|\boldsymbol{X})$

therefore $P\langle A \cap B \cap \boldsymbol{C} \rangle = P\langle A \cap B \rangle. P(\boldsymbol{C}|A \cap B)$

therefore $P\langle A \cap B \cap \boldsymbol{C} \rangle = P(\boldsymbol{A}).\, P(\boldsymbol{B}|\boldsymbol{A}). P(\boldsymbol{C}|A \cap B)$

Conditional Probability

Chain Rule of probability

$$P\langle A \cap B \cap \boldsymbol{C} \cap \boldsymbol{D}\rangle =$$

$$P\langle A \cap B \cap \boldsymbol{C} \cap \boldsymbol{D}\rangle = P(\boldsymbol{A}).\, P(\boldsymbol{B}|\boldsymbol{A}).\, P(\boldsymbol{C}|A \cap B).\, P(\boldsymbol{D}|A \cap B \cap \boldsymbol{C})$$

$$P\langle A_1 \cap A_2 \cap A_3 \cap A_4\rangle = P(A_1)\mathbf{P}(A_2|A_1)P(A_3|A_2 \cap A_1)P(A_4|A_1 \cap A_2 \cap A_3)$$

$$P\langle A_1 \cap A_2 \cap A_3 \cap A_4\rangle = P(A_1)\prod_{i=2}^{4} \boldsymbol{P}(A_i|A_{\mathbf{1}} \cap A_2 \ldots .\, A_{i-\mathbf{1}})$$

$$P\langle A_1 \cap A_2 \cap A_3 \cap A_4 \ldots \ldots A_{\mathbf{n}}\rangle = P(A_1)\prod_{i=2}^{\mathbf{n}} \boldsymbol{P}(A_i|A_{\mathbf{1}} \cap A_2 \ldots .\, A_{i-\mathbf{1}})$$

Conditional Probability

Chain Rule of probability

$Suppose\ you\ draw\ 3\ cards\ one\ by\ one\ with\ out\ replacement.$

$what\ is\ \ the\ probability\ that\ all\ the\ 3\ cards\ are\ aces$

Using counting principles:

$$P \cong \frac{\binom{4}{3}}{\binom{52}{3}} = \frac{\frac{4!}{1!,3!}}{\frac{52!}{49!.3!}} = \frac{4*3*\mathbf{2}}{52*51*\mathbf{50}}$$

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Conditional Probability

Chain Rule of probability

Suppose you draw 3 cards one by one with out replacement.

what is the probability that all the 3 cards are aces

$P(A_1) = \frac{4}{52}$

$P(A_2|A_1) = \frac{3}{51}$

$P(A_3|A_1 \cap A_2) = \frac{2}{50}$

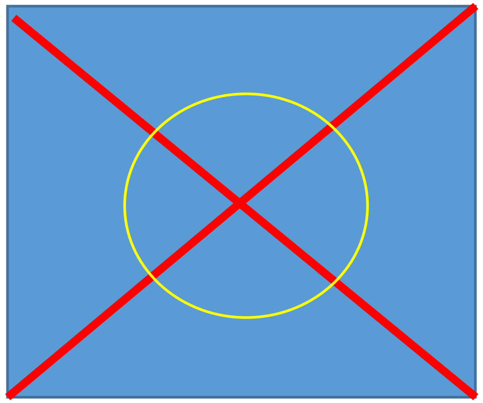
Using chain rule:

A_i: the event that the i-th card is an ace

$$P\langle A_1 \cap A_2 \cap A_3 \rangle = P(A_1)\mathbf{P}(A_2|A_1)P(A_3|A_2 \cap A_1)$$

$$P = \frac{4 * 3 * \mathbf{2}}{52 * 51 * \mathbf{50}}$$

Conditional Probability



Total Probability Theorem

$A_1, A_2, A_3 \dots \dots . A_{\mathrm{n}}$ $\boldsymbol{Partition}\ \Omega.$

$A_1 \cup A_2 \cup A_3 \dots \dots . A_{\mathrm{n}} = \Omega.$

$A_1 \cap A_j = \phi \forall i \neq j.$

B = $\langle \boldsymbol{B} \cap A_1) \cup (\boldsymbol{B} \cap A_2) \cup \cdots \dots . . \cup (\boldsymbol{B} \cap A_2 \rangle$

P(B) = $P\langle \boldsymbol{B} \cap A_1) + \boldsymbol{P}(\boldsymbol{B} \cap A_2) + \cdots \dots . . + (\boldsymbol{B} \cap A_4 \rangle$

Total Probability Is:

P(B) = $P(A_1). P(\boldsymbol{B}|A_1) + P(A_2). P(\boldsymbol{B}|A_2) + \cdots \dots \dots \dots + P(A_n). P(\boldsymbol{B}|A_n)$

$$P = \frac{4 * 3 * \mathbf{2}}{52 * 51 * \mathbf{50}}$$

Conditional Probability

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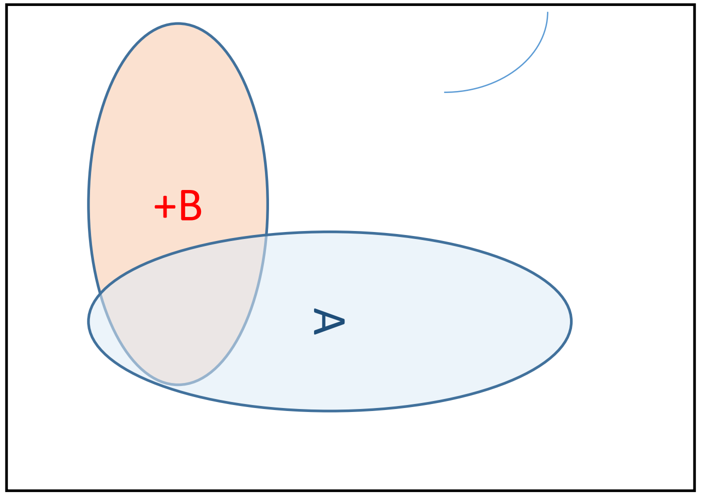
Total Probability Theorem



Facts:

P(A) = 0.1

P(B^C| A) = 0.01

P(B|A) = 0.99

P(B|A^C) = 0.05

P(B^C|A^C) = 0.95

Using Total Probability Theorem:

P(B) = $P(\boldsymbol{A}).\ P(\boldsymbol{B}|\boldsymbol{A}) + P(A^C).\ P(\boldsymbol{B}|A^C)$

P(B) = 0.1*0.99 + 0.9 * 0.05 = 0.144

Conditional Probability

Facts:

Total Probability Theorem

P(B|A_1) = 0.3

P(B|A_2) = 0.6

P(B|A_3) = 0.75

= i-th path taken

B: monster encountered

P(A_1) = P(A_1) = P(A_1) = 1/3

$$P(B^c) = P(A_1).P(B^c|A_1) + P(A_2).P(B^c|A_2) + P(A_3).P(B^c|A_3)$$

Using Total Probability Theorem:

P(B^C) = $P(A_1).\,P(B^C|A_1) + P(A_2).\,P\left((B^C|A_2) + P(A_3).\,P(B^C|A_3)\right)$

P(B) = 1/3 * 0.7 + 1/3 * 0.4 + 1/3 * 0.25 = 0.45

Conditional Probability

Facts:

P(B|A_1) = 0.3

P(B|A_2) = 0.6

P(B|A_3) = 0.75

A_i = i-th path taken

B: monster encountered

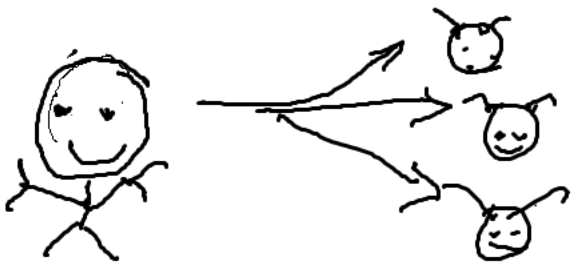
P(A_1) = P(A_1) = P(A_1) = 1/3

Probability:

P(A_1| B) = ?

Bayes' Theorem

If he does not come out alive what is the probability that he took path A1 ?



P(A_1| B) = ?

$$P(\boldsymbol{A_1}|\ \boldsymbol{B}) = \frac{P(\boldsymbol{A_1} \cap \boldsymbol{B})}{P(\boldsymbol{B})}$$

Applying total probability theorm:

$$\mathbf{P(B)} = P(A_1).\ P(\mathbf{B}|A_1) + P(A_2).\ P\big((\mathbf{B}|A_2) + P(A_3).\ P(\mathbf{B}A_3)\big)$$

$$P\langle \boldsymbol{A_1} \cap B\rangle = P(\boldsymbol{A_1}|\boldsymbol{B}).P(B) = P(\boldsymbol{B}|\boldsymbol{A_1}).P(\boldsymbol{A_1})$$

$$P(\boldsymbol{A_1}|\ \boldsymbol{B}) = \frac{P(\boldsymbol{A_1} \cap \boldsymbol{B})}{P(A_1).\ P(\mathbf{B}|A_1) + P(A_2).\ P\big((\mathbf{B}|A_2) + P(A_3).\ P(\mathbf{B}A_3)\big)} = \mathbf{0.182}$$

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Conditional Probability

Facts:

P(B|A_1) = 0.3

P(B|A_2) = 0.6

P(B|A_3) = 0.75

A_i = i-th path taken

B: monster encountered

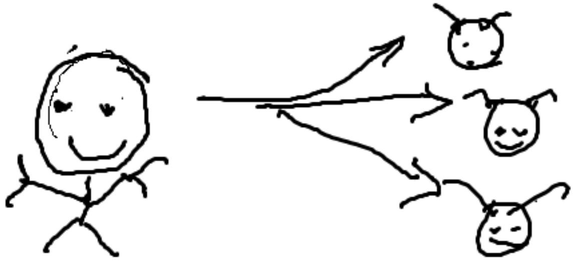
P(A_1) = P(A_1) = P(A_1) = 1/3

Probability:

P(A_3| B) = ?

Bayes' Theorem

If he does not come out alive what is the probability that he took path A_3 ?



P(A_3| B) = ?

$$P(A_3|\,\boldsymbol{B}) = \frac{P(A_3 \cap \boldsymbol{B})}{P(\boldsymbol{B})}$$

Applying total probability theorem:

P(B) = $P(A_1).\,P(\mathbf{B}|A_1) + P(A_2).\,P\big((\mathbf{B}|A_2) + P(A_3).\,P(\mathbf{B}A_3)\big)$

$$P\langle A_3 \cap B\rangle = P(A_3|\boldsymbol{B}).\,P(B) = P(\boldsymbol{B}|A_3).\,P(A_3)$$

$$P(A_3|\,\boldsymbol{B}) = \frac{P(A_3 \cap \boldsymbol{B})}{P(A_1).\,P(\mathbf{B}|A_1) + P(A_2).\,P\big((\mathbf{B}|A_2) + P(A_3).\,P(\mathbf{B}A_3)\big)} = \mathbf{0.45}$$

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Conditional Probability

Breaking down Bayes Theorem

Exploit the Multiplication Rule:

$$P(\boldsymbol{A_1}).P(\boldsymbol{B}|\boldsymbol{A_1}) = P(B).P(\boldsymbol{A_1}|\boldsymbol{B})$$

Exploit the Total Probability Theorem:

$$\mathbf{P(B)} = P(A_1).P(\mathrm{B}|A_1) + P(A_2).P\big((\mathrm{B}|A_2) + P(A_3).P(\mathrm{B}A_3)\big)$$

Exploiting the known probabilities:

$$P(A_1|\,\boldsymbol{B}) = \frac{P(A_1).\,P(\boldsymbol{B}|A_1)}{\Sigma_{i=1}^{n} P(A_i).\,P(\boldsymbol{B}|A_i)}$$ **Bayes Theorem**

Conditional Probability

Facts:

P(A) = 0.01

P(B|A) = 0.95

P(B|A^C) = 0.05

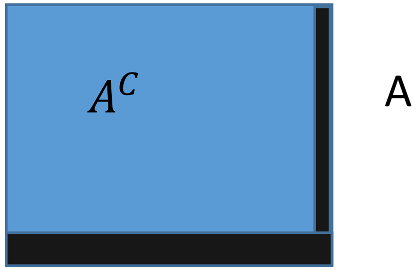
A = Ship 1 sends a signal 1

B = Ship 2 receives a signal 1

Bayes' Theorem

P(A | B) = ?

$$P(\boldsymbol{A} \mid \boldsymbol{B}) = \frac{P(\mathbf{A} \cap \boldsymbol{B})}{P(\boldsymbol{A}).\, P(\mathbf{B}|\boldsymbol{A}) + P(A^C).\, P(\mathbf{B}|A^C)} = \mathbf{0.18}$$



10000

9900 → 495

100 → 95

Conditional Probability

Bayes' Theorem

Facts:

P(A) = 0.1

P(B^C | A) = 0.01

=> P(B | A) = 0.99

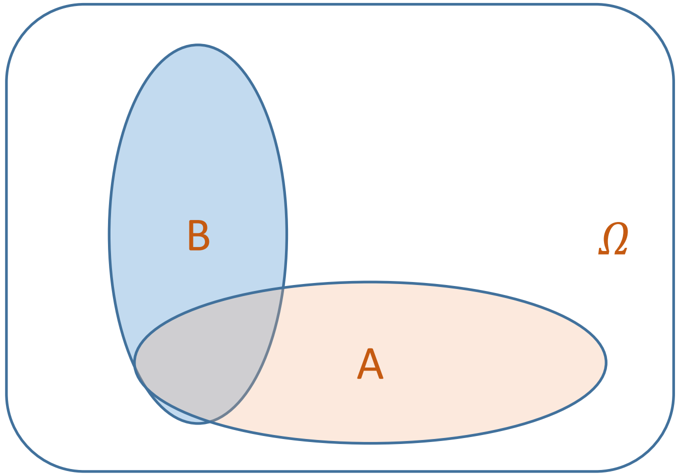
P(B | A^C) = 0.05

=> P(B^C | A^C) = 0.95

A = Person is infected

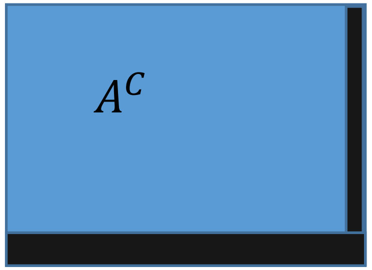
B = Tested positive

What is the chance that a person is actually infected , if the results of the test are Positive



P(A | B) = ?

$$P(\boldsymbol{A} \mid \boldsymbol{B}) = \frac{P(\boldsymbol{A} \cap \boldsymbol{B})}{P(\boldsymbol{A}).\,P(\mathbf{B}|\boldsymbol{A}) + P(A^C).\,P(\mathbf{B}|A^C)} = \mathbf{0.67}$$



A

10000

9000 → 450

1000 → 990

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Independent Events

Consider the below 2 Events:

A: I had a sandwich for breakfast

B: It will rain today

If A occurs will you update your belief

about A ?

What do we call such events ?

Independent Events

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Independent Events

Example: 50 girls and 70 boys in a class, of these, 35 girls and 49 boys are good at maths. If I tell you that a student is very good at maths what is the probability that she is a girl?

A:Student is a Girl

B: Student is good at Maths

$P(A \mid B)$ = ?

$$P(A \mid B) = \frac{P(\mathbf{B}|A).P(A)}{P(B)} = \left(\frac{7}{10} * \frac{5}{12}\right) / 7/10 = 5/12$$

Facts:

P(A) = 50 / (50+70) = 5/12

P(A^C) = 7/12

=> P(B | A) = 35/50 = 7/10

P(B | A^C) = 49/70 = 7/10

P(B) = $P(A).\,P(\mathbf{B}|A) + P(A^C).\,P(\mathbf{B}|A^C)$ = 7/10

P(A|B) = 5/12 = P(A)

Knowing about B does not change my belief about A

Similarly, P(B|A) = 7/10 = P(B|A^C) = 7/10

So, Knowing about A does not change by belief about B

Independent Events

Two events A & B are independent if P(A|B) = P(B) or P(B|A) = P(B)

More Robust way of saying the above:

Two events are A& B are independent if

P$(A \cap B)$ = P(A) * P(A|B)

= P(A) * P(B)

P$(A \cap B)$ = P(A) * P(B|A)

= P(A) * P(B)

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Independent Events

Take an example of tossing 3 coins simultaneously

A:First toss results is an Head

B: Exactly 2 tosses results in heads

Facts:

$P(A) = 4/8$

$P(B) = 3/8$

$P(A \cap B) = 2/8$

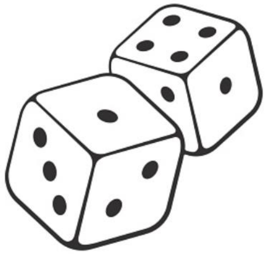
$P(A \cap B) = P(A) * P(B)$

Are A and B independent ?

Coin 1	Coin 2	Coin 3	A	B	A ∩ B
H	H	H	*		
H	H	T	*	*	*
H	T	H	*	*	*
H	T	T	*		
T	H	H		*	
T	H	T			
T	T	H			
T	T	T			

Independent Events

Example: I am rolling 2 dice, What is the probability that the sum of both the dice is 7 and second dice shows an even number.



A: Sum of the dice is 7

B: Second dice shows an even number

Are A and B independent ?

Event A {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

Event B { (1,2), (1,4), (1,6), (2,2), (2,4), (2,6)
(3,2), (3,4), (3,6), (4,2), (4,4), (4,6)
(5,2), (5,4), (5,6), (6,2), (6,4), (6,6)}

Facts:

$P(A) = 4/8$

$P(B) = 3/8$

$P(A \cap B) = 2/8$

$P(A \cap B) = P(A) * P(B)$

$A \cap B = \{(1,6), (3,4), (5,2)\}$

$P(A) = 6/36 = 1/6$

$P(B) = 18/36 = 18/36 = 1/2$

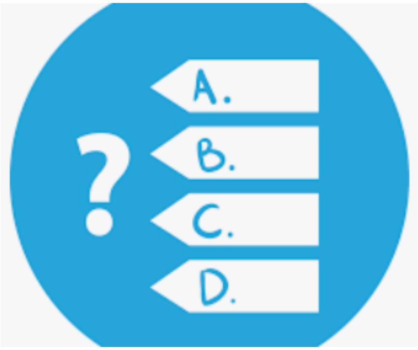
$P(A \cap B) = 3/36 = 1/12$, Therefore $P(A \cap B) = P(A)*P(B)$

Independent Events

Example: A Quiz has 2 multiple choice questions. The first question has 4 choices of which I is correct and the second question has 3 choices of which one is correct. If a student randomly guesses the answers, what is the probability that he will answer both questions correctly.

A: First answer is correct

B: Second answer is correct both events are independent



Facts:

$P(A) = 1/4$

$P(B) = 1/3$

$P(A \cap B) = P(A) * P(B)$

$P(A \cap B) = 1/4 * 1/3 = 1/12$

Independent Events: n events

We say that events a1, a2, a3, ……an. are pair wise independent if

P($A_i \cap A_j$) = P(A_i) * P(A_j) $\forall$ i $\neq j$

We say that events A1, A2, A3…..An are mutually independent or independent of all subsets if

I $\subset$ {1, 2, 3, 4,…..n}

P ($\bigcap \boldsymbol{i} \in I\ A_i = \pi_{\mathrm{i=1}}^{n}\ \boldsymbol{P}(A_i)$)

Eg: for n = 3 {1,2,3}

{1,2}, {1,3}, {2,3}, {1,2, 3}

P($A_1 \cap A_2$) = P(A_1)* P(A_2)

P($A_1 \cap A_3$) = P(A_1)* P(A_3)

P($A_2 \cap A_3$) = P(A_2)* P(A_3)

P($A_1 \cap A_2 \cap A_3$) = P(A_1)* P(A_2) * P(A_3)

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Summary

Set Theory

Finite, infinite Countable, infinite uncountable

Intersection, Union, Complement

Properties of set operations

Associativity

$A \cup (B \cup C) = (A \cup B) \cup C$

Commutativity

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Distributive laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Demorgon's Law

$(A \cup B)^C = A^C \cap B^C$

$(A \cap B)^C = A^C \cup B^C$

Disjoint sets

$P(A_i \cap A_j) = P(A_i) * P(A_j) \; \forall \, i \neq j$

Summary

Axioms:

Axiom 1 P(A) $\geq \mathbf{0}\ \forall\ \mathbf{A}$

(Non negativity)

Axiom 2 **P(Ω) $= \mathbf{1}$**

(Normalisation)

Axiom 3 **If the events A1, A2, A3, ……, An are mutually disjoint then P(A1 ∪ A2 ∪ A3 ∪ An) =**

$$\sum_i^n P(A_i)$$

(finite additivity)

Independent events:

We say that events a1, a2, a3, ……an. are pair wise independent if

P($A_i \cap A_j$) = P(A_i) * P(A_j) $\forall$ i $\neq j$

We say that events A1, A2, A3…..An are mutually independent or independent of all subsets if

I $\subset$ {1, 2, 3, 4,…..n} P ($\cap\, \boldsymbol{i} \in I\ A_i = \pi_{\mathbf{i=1}}^{n}\ \boldsymbol{P}(A_i)$)

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Chain rule of probability

$$P\langle A \cap B\rangle = P(A|\boldsymbol{B}).P(B) = P(\boldsymbol{B}|\boldsymbol{A}).P(\boldsymbol{A})$$

Total probability theorem

P(B) = $P(A_1).P(\boldsymbol{B}|A_1) + P(A_2).P(\boldsymbol{B}|A_2) + \cdots \ldots \ldots \ldots + P(A_{\boldsymbol{n}}).P(\boldsymbol{B}|A_{\boldsymbol{n}})$

Bayes theorem

$$P(A_{\mathbf{1}}|\ \boldsymbol{B}) = \frac{P(A_1).\ P(\boldsymbol{B}|A_1)}{\Sigma_{i=\mathbf{1}}^{n} P(A_{\mathbf{i}}).\ P(\boldsymbol{B}|A_{\mathbf{i}})}$$