PRIME INTUIT

#Shorts

Recap
Yesterday Stats Session : Sets Day 2

SWAROOP N C , 06th April of 2022

So Far Learnt : Sets Day 1

- Recap By Akshay
  - **Randomness**
  - **Sets**
  - **Sub Sets , Equal Sets**
  - **Universal Set , Empty Set**
  - **Set Operations**
    - Complement , Union & Intersection
  - **Properties**
    - Commutativity
    - Associativity
    - Distributive laws
    - De Morgan's Laws

Yesterday : Sets Day 2

- **Infinite Sets**
- **Experiments & Sample Spaces**
  - Experiments involving
    - Coin Toss
    - Dice
    - Cards
    - Dart Board
- **Events of an experiment**
  - Union , Intersection and Complement of Events
  - Multiple events
  - Disjoint events
- **Intro to Axioms of Probability**

Infinite Sets

- **sets that are not finite**
- Ex : Real numbers , Integers . Natural numbers
- They are classified as
  - **Countable** Infinite sets
    - If there is 1-1 correspondence bw the elements of this set and set of positive integers
    - Ex : Integers, Natural numbers , positive even numbers
  - **Uncountable** Infinite sets
    - If no correlation
    - Ex : Real numbers

Countable Infinite Sets

Positive even numbers

$\{2, 4, 6, 8, 10, \ldots\ldots\ldots, \infty\}$

$\downarrow \; \downarrow \; \downarrow \; \downarrow \; \downarrow$

$\{1, 2, 3, 4, 5, \ldots\ldots\ldots, \infty\}$

Perfect squares numbers

$\{1, 4, 9, 16, 25, \ldots\ldots\ldots, \infty\}$

$\downarrow \; \downarrow \; \downarrow \; \downarrow \; \downarrow$

$\{1, 2, 3, 4, 5, \ldots\ldots\ldots, \infty\}$

As we can relate every element in set to +Ve integer set , So countable

UnCountable Infinite Sets

{ ....0 .. ∞ ..1 ... ∞... 2 ... ∞... 3 ....}

{ .....0 , ..., 0.1111 ,......., 0.1112 ,......, 0.1113 , ...,,...,0.1,...........0.7.......1.... , ∞ }

0 , 0.1 , 0.01 , 0.001 0.0001. 0.000001 0.0000000000001 ....

{ 0 ... ∞... 1 } > **{ 1,2,3, ... ∞}**

Can not relate

{ 1 , 2 , 3 , 4 , 5 , ... , ∞ }

- there are infinite numbers between 0 and 1 and
- This infinite set itself bigger than the infinite set of integers

And As we CAN NOT relate every element in the set to +Ve integer number set , So Uncountable

Experiment / Trail

- any procedure that can be repeated infinite times and has well defined set of outcomes

Sample Space

- set of all possible outcomes of an experiment

Experiment / Trail & Sample Spaces

Experiment / Trail	Sample Space
Coin Toss	{Head , Tail }
Rolling a dice	{1,2,3,4,5,6}
Outcomes of facing a ball in Cricket	{Wicket , 0 - 6 Runs }

- Sample space are **mutually exclusive** and **collectively exhaustive**
- Outcome in experiment = UnCertain Vs Set of Outcomes = Certain

Experiment : **Coin Toss**

- n = Possible outcomes
- n = 2 (H , T)

# of Coins (k)	**Sample Space**	**# of Outcomes (n^k)**
1	H , T	2
2	HH , HT , TH , TT	4
3	HHH , HHT , HTH , HTT , THH , THT , TTH , TTH , TTT	8
k		**2^k**

Experiment : **Dice**

- n = 6

# of Dice Roll (k)	**Sample Space (Ω)**	**# of Outcomes (n^k)**
1	1,2,3,4,5,6	6
2	1,1 1,2 1,3 1,4 1,5 1,6 2,1 2,2 2,3 2,4 2,5 2,6 3,1 3,2 3,3 3,4 3,5 3,6 4,1 4,2 4,3 4,4 4,5 4,6 5,1 5,2 5,3 5,4 5,5 5,6 6,1 6,2 6,3 6,4 6,5 6,6	36
k		**6^k**

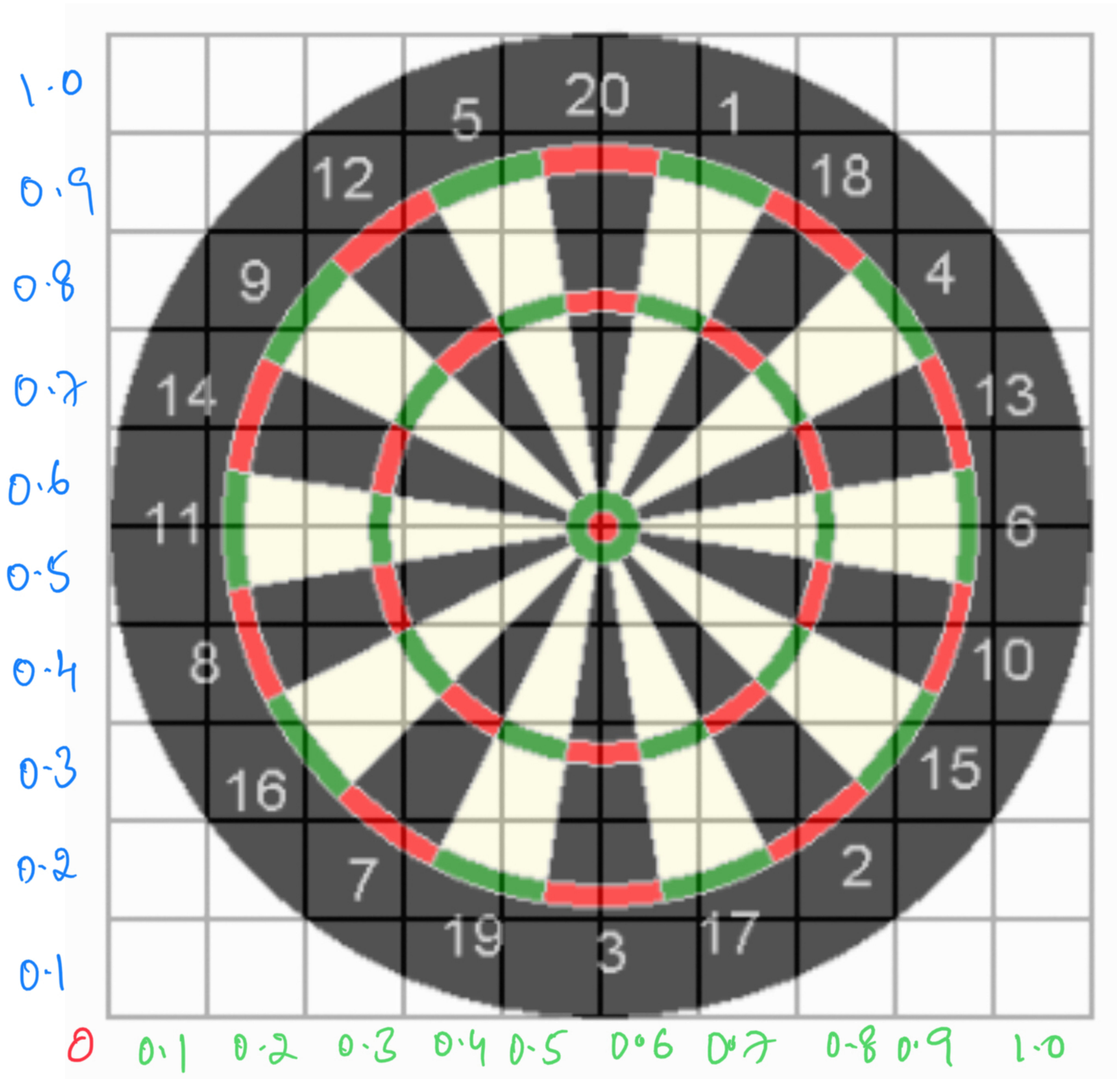
Experiment : **Cards**

- n = 52

# of Cards (k)	**Sample Space (Ω) WITHOUT ~~REPETITION~~**	**# of Outcomes**
1	One of 52 cards	52
2	One of 52 cards, another One of remaining 51 cards	52 * 51
k		**52 * 51 * .... * n-(k-1)**

Experiment : **Dart Board**

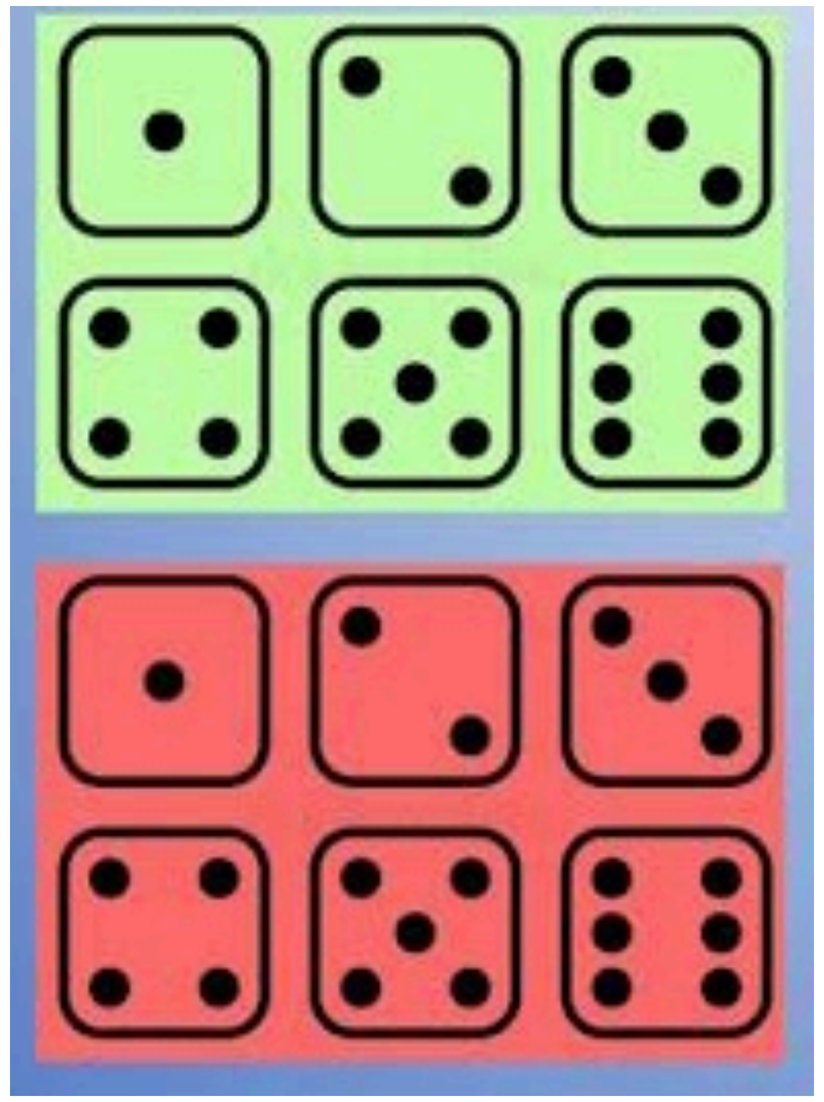
- 1 metre Dartboard
- Divided into 0.1 sq m unit grids
- Possible hit outcomes = 10*10 = 100
- Ω = [(x,y)s.t 0 <x,y<1]
- st = such that
- Ω = [(x,y)∀ 0 <x,y<1]



Events of Experiment

- **Event**
  - Set of outcomes of an experiment
  - This event set is a subset of sample Space of experiment
  - **Experiment Vs Sample Space Vs Event**
    - Rolling a dice = **Experiment**
    - {1,2,3,4,5,6} = **Sample space**
    - All even number {2,4,6} - **Event**
  - Ex : Event A = event that first toss is head .i.e. A = {HH, HT }
  - Ex : Event B = Event of both tosses is Tails , B = { TT }

Union, Intersection and Complement of Events

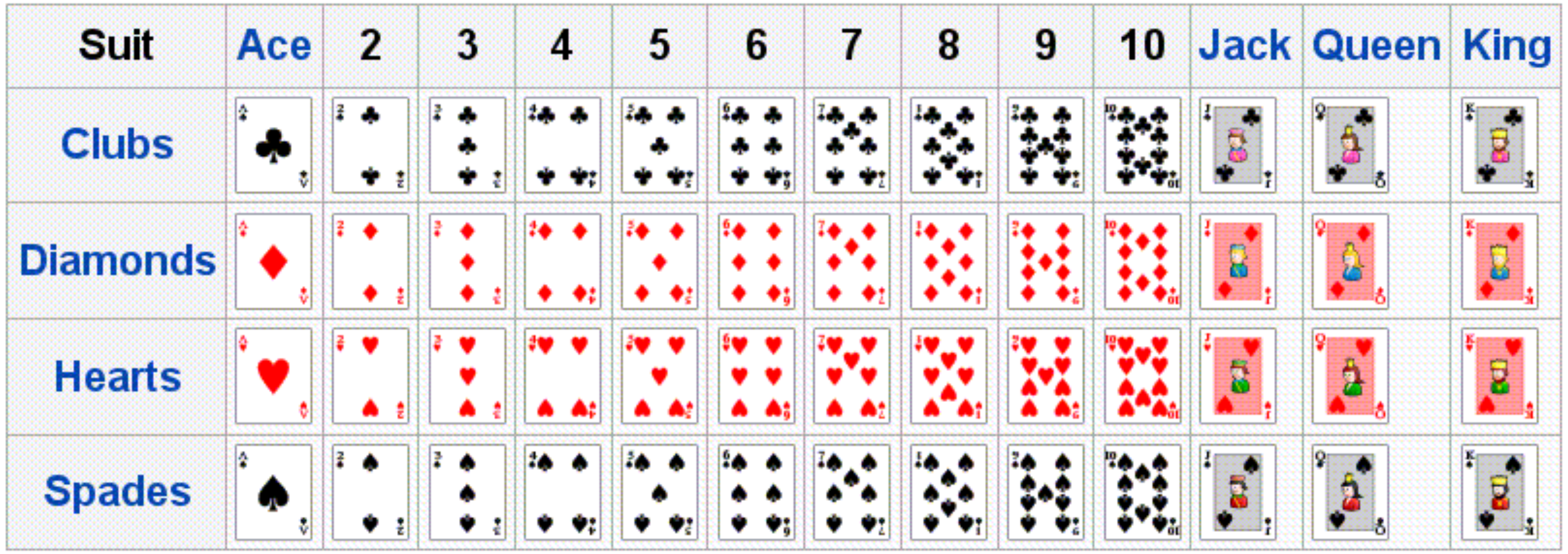


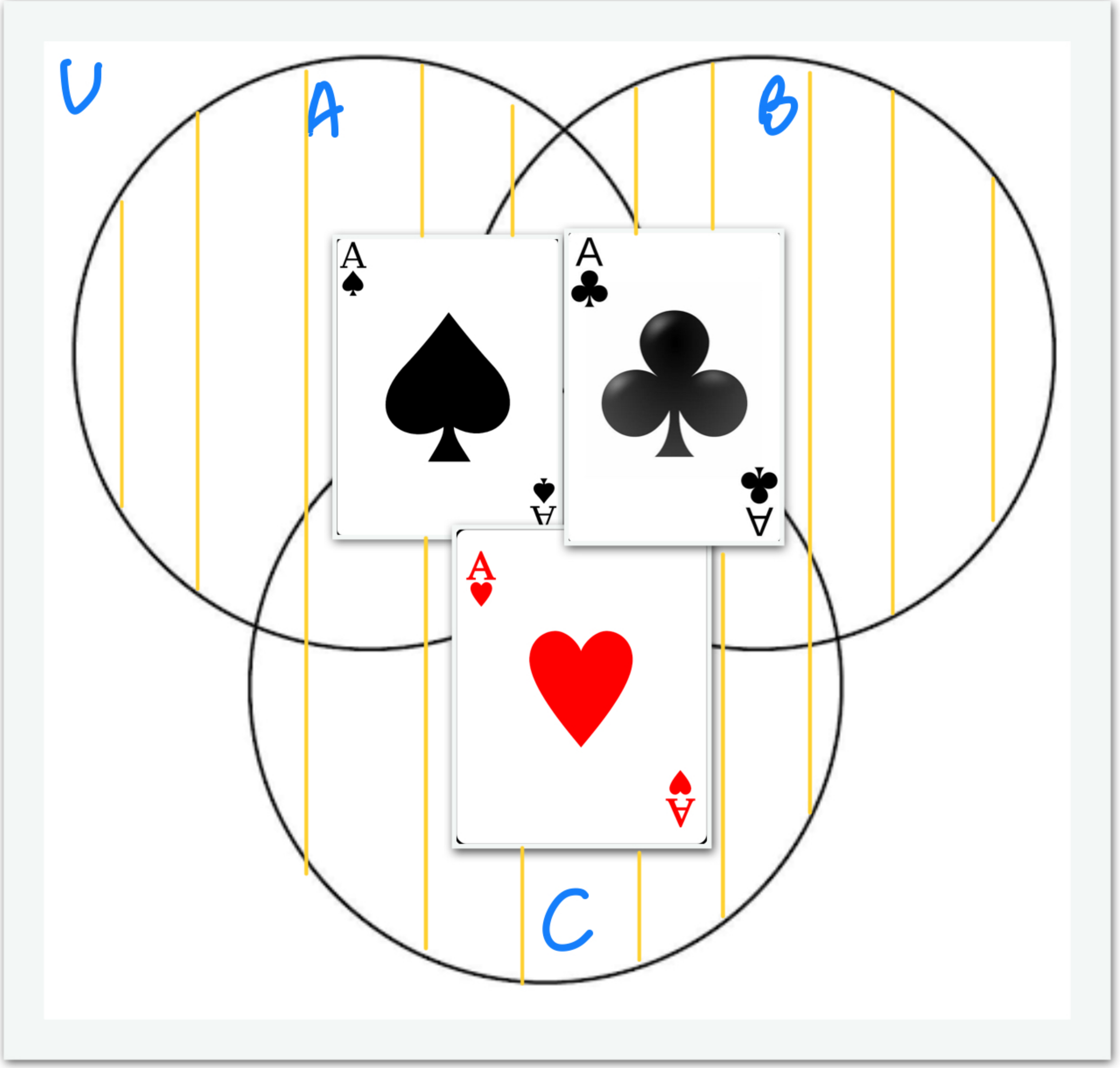
- **Event A : first die shows a 2**
- A = { 2- 1 , 2-2 , 2-3 , **2- 4** , 2- 5 , 2 -6 }
- **Event B : second die shows a 4**
- B = { 1-4 , **2-4** , 3-4 , 4-4 ,5-4 , 6-4 }
- **Event C : either first die is 2 or second die is 4**
- C = A ∪ B = {2- 1 , 2-2 , 2-3 , **2- 4** , 2- 5 , 2 -6 , 1-4 , **2-4** , 3-4 , 4-4 , 5-4 , 6-4}
- **Event D : first Die 2 and second 4**
- D = A ∩ B = **{2-4}**
- **Event E : First Die not shows 2**
- E = A^C

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Events of Experiment : **Multiple Events**

Example set of 52 poker playing cards





- **A = hand that contain ace of spades**
- **B = hand that contain ace of Clubs**
- **C = hand that contain ace of Hearts**
- **$A \cup B \cup C$ = event that contains at-least 1 ace**
- **$A \cap B \cap C$ = event that contains All 3 aces**

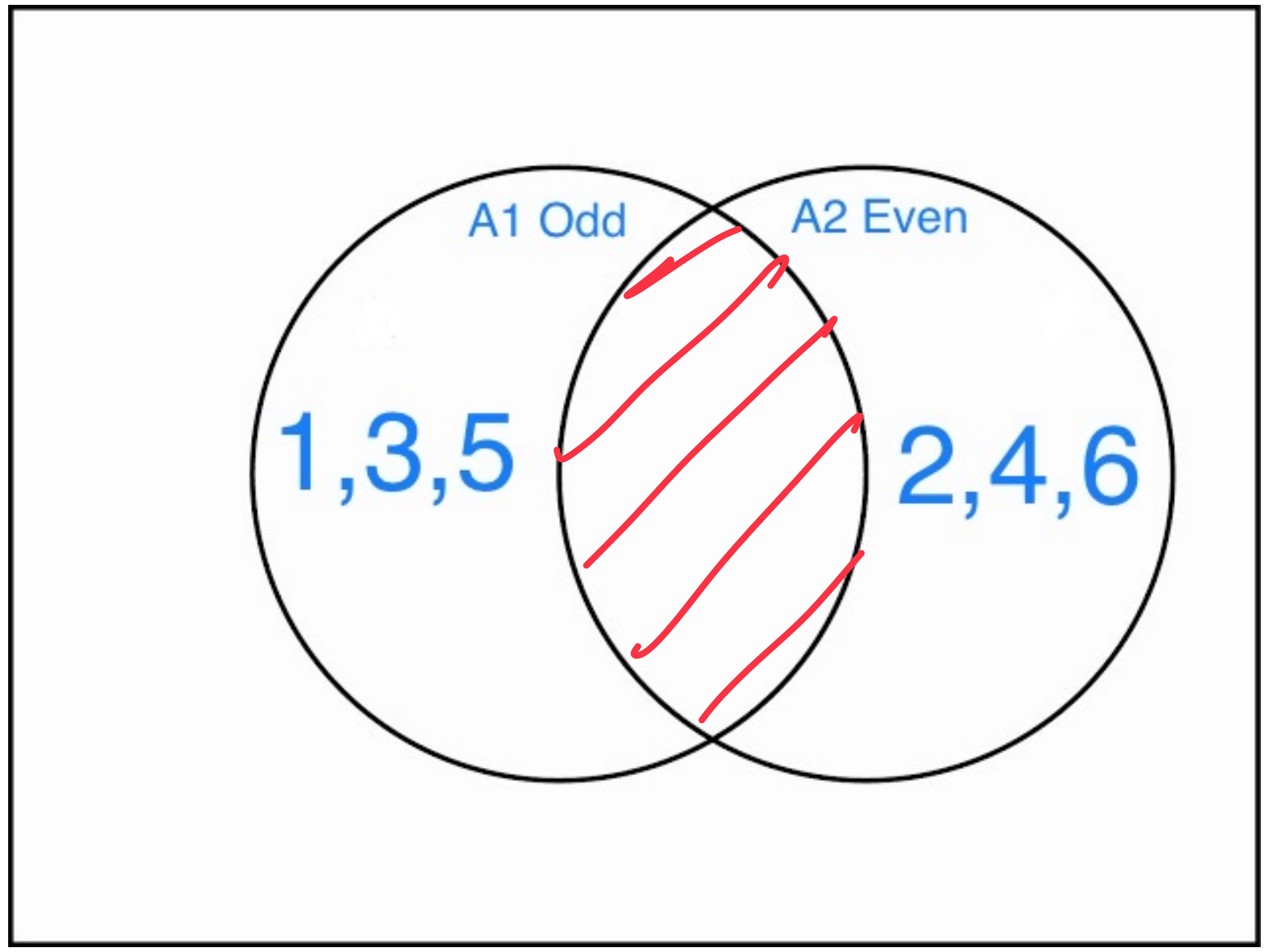
Events of Experiment : **Disjoint Events**

- **2 Events A and B are disjoints if they can not occur simultaneously i.e, $A \cap B = \varnothing$**
- **Simple example 1 : A and A^c**
  - **Consider rolling 2 dice**
    - **Event A : first die shows a 2**
    - A = { 2- 1 , 2-2 , 2-3 , 2- 4 , 2- 5 , 2 -6 }
    - **Event E : First Die not shows 2**
    - $E = A^C$
- **Not All Disjoint sets are complement always**
- **Ex 2 : A = event of First die showing 1 ,**
  - **B = event of first die showing 2**
  - **Can't occur together , Mutually exclusive**

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Mutually / Pairwise Disjoint Events & Partition of Sample Space

- **The Events A1 , A2 , A3 .... An , are mutually disjoint or pairwise disjoint ,**
  - **If $A_i \cap A_j = \varnothing$, $\forall$ i,j where $i \neq j$**
- Consider rolling a dice Ω = {1,2,3,4,5,6}
- A1 = odd number in rolling dice {1,3,5}
- A2 = Even number in rolling dice { 2,4,5}



- **In Addition, if A1 U A2 U A3 = Ω , they are said to partition the sample Space**
- **The events A1 A2 A3 .... An are mutually disjoint and A1 U A2 U A3 ...U An = Ω ,**
  - **Then , A1 A2 A3 .... An are said to partition the sample space**

Intro to Axioms of Probability

- **Probability**
  - **What is the chance of an event ?**
  - **Goal : Assign a number to each event such that it reflects the chance the experiment resulting in that event**
  - Ex : Event A : Odd Number when rolling a dice
  - Chance = 3/6 = 1/2 or 50%
- **Probability Function**
  - **P(A) = ?   ; P = Probability function and A is event**
- **Axioms of probability**
  - are conditions that probability function must satisfy

Intro to Axioms of Probability

Axiom 1
(non-negativity)

$$P(A) \geq 0 \; \forall A$$

Axiom 2
(normalisation)

$$P(\Omega) = 1$$

Axiom 3
(finite additivity)

If the events $A_1, A_2, \ldots, A_n$ are mutually disjoint then

$$P(A_1 \cup A_2 \cup \cdots \cup A_n)$$

$$= \sum_{i=1}^{n} P(A_i)$$

To Summarise

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Thank You