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Counting and Probability Theory

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Counting and Probability Theory

Why do we need to learn Counting Principle ?

What are the principals of Counting ?

- Multiplication Principle
- Subtraction Principle

What are sequences and how do you count them ?

What are collections and how do you count them ?

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Need for Probability Theory

Goal: Is to study a large collection of people or Objects

Challenges: Infeasible, Expensive and time consuming

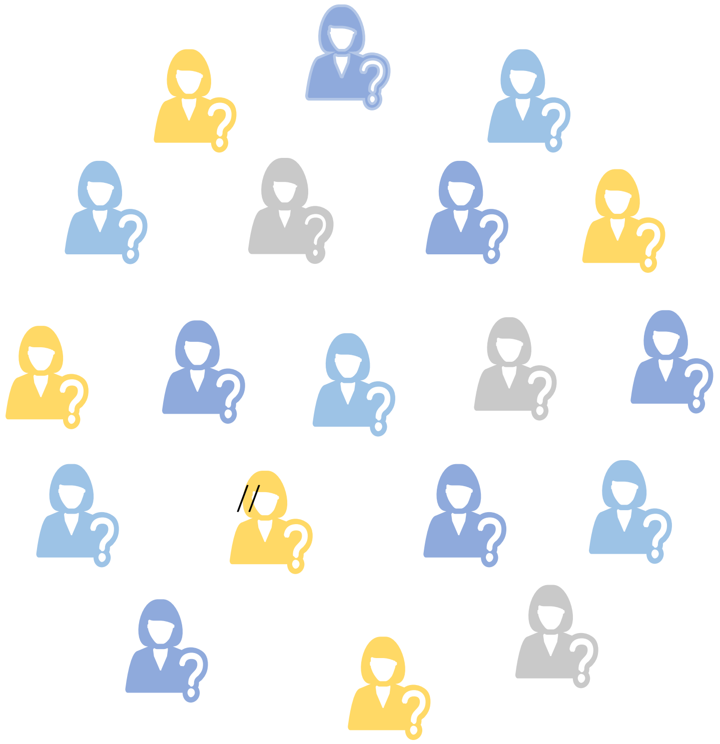
Solution: Survey only a few elements and draw inference about all elements from this smaller group

Population: Total collection of Objects that we want to study

Sample: It's a subgroup of the population that we study to draw inference about the population

What is the probability that a statistic computed from a sample is closed to that computed from a population?

Statistics: Proportion, Mean, Median, standard deviation, variance when computed from a sample is called a statistic

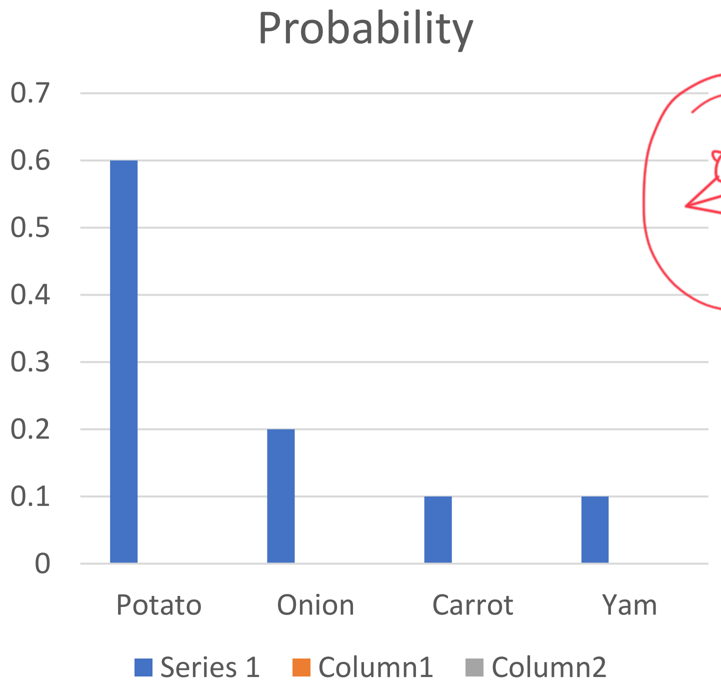


Need for Probability Theory

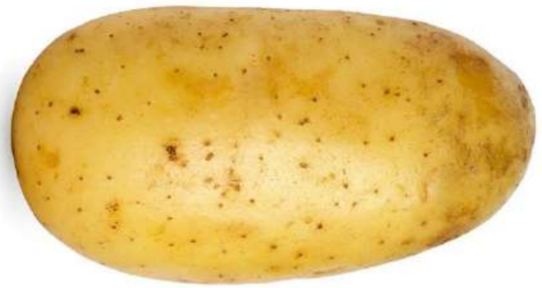
Machine Learning

Classification → ~~Probability estimate problems~~

P (label = potato | image) ?



$\frac{0.6}{1} = 60\%$



Is potato, Carrat, Onion,…..

And is it

Russet, Red, Yellow, White, Purple,

Fingerling, Petite ?

Predict a distribution over classes

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Need for Counting ?

What is the probability of getting an Heads?

1 / 2 = 50%

How did you compute this ?

2 Possible outcomes each equally likely

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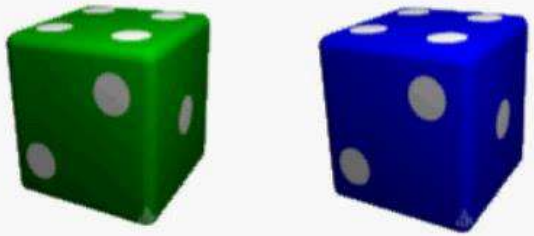
Need for Counting ?

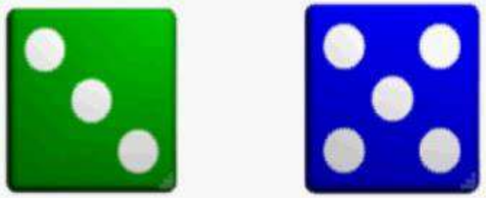
$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

What are the chances of rolling a 3 with the green dice and a 5 with the blue dice?

The probability of rolling a 5 with the blue dice is 1/6.

This is because there are 6 possible outcomes when rolling the blue dice. One of the six outcomes is a 5, therefore 1/6 is the probability.





$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Need for Counting ?

Count the number of outcomes

Number of Outcomes is say “n”

Is there a equal / Fair chance of getting any one value with in n?

Chances of probability = 1 / n

Counting n in this example was easier

Need for Counting ?

52

Here is a table showing all 52 cards in a standard deck.

												Face cards		
Color	**Suit**	Ace	Two	Three	Four	Five	Six	Seven	Eight	Nine	Ten	Jack	Queen	King
Red	Hearts	A♥	2♥	3♥	4♥	5♥	6♥	7♥	8♥	9♥	10♥	J♥	Q♥	K♥
Red	Diamonds	A♦	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	10♦	J♦	Q♦	K♦
Black	Spades	A♠	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠
Black	Clubs	A♣	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣

Red ♡ ◇

Black Spade ♤
Clubs ♧

A → 1
2 to 10 — 9
J, Q, K → 3
13
Face cards
4 = 52

Suppose one card is drawn at random from a standard deck.
Answer each part. Write your answers as fractions.

(a) What is the probability that the card drawn is a face card?

$12/52 = \frac{6}{26} = \frac{3}{13}$

(b) What is the probability that the card drawn is a black card?

$\frac{26}{52} = \frac{1}{2}$

(c) What is the probability that the card drawn is a face card or a black card?

$\frac{6+26}{52} = \frac{32}{52} = \frac{16}{26} = \frac{8}{13}$

× ↶ ?

Need for Counting ?

What is the probability of getting 4 aces

Number of Outcomes is say “n”

If n is the number of outcomes then n is all possible combinations of 4 cards that you can get.

How do you count n ?

- Using principles of counting

1 ace

$\frac{4}{52} = \frac{1}{13}$

4 ac

$1 = \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49}$

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Need for Counting ?

What is the probability of getting 4 aces

Number of Outcomes is say “n”

Without knowing how to count the number of outcomes we will not be able to compute the probability.

4×3×2×1

52×51×50×49

Turns out that there are 270725 ways of selecting 4 cards from 52 cards! (0.00036% chance of getting 4 aces)

Need for Counting ?

Learn how to count the number of outcomes of an experiments

How many numbers are there between 73 and 358 (both inclusive)

k n

$n-(k-1)$

$358-(73-1)$

$358-72 = 286$

Easy !

How many numbers are there between 73 and 358, which are divisable by 7 (both inclusive)

$77, 84, \ldots, 350, 357$

$\div 7$

$11, 12, \ldots, 50, 51$

$n-(k-1) = 51-(11-1)$

$= 51-10 = 41$

A little hard …….

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Very simple counting

Start from absolute basics

How many numbers are there between 1 to 358

Principal 1. Number of numbers between 1 and n is n

How many numbers are there b/n 73 and 358 (both inclusive)

73, 74, 75 ………………………………..356, 357, 358

I know principal 1 , to count from 1 to n, can we use that principal

-72 here ? -72

-72 from the sequence above

1, 2, 3, ……………………………………284, 285, 286

1, 2, 3 286

$$\begin{array}{r} 358 \\ -\ 72 \\ \hline 286 \end{array}$$

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Very simple counting

How many numbers are there between 73 and 358 (both inclusive)

73, 74, 75 …… …….. 356, 357, 358

358 – 72 = 358 – (73 -1) = 358 – 73 + 1 = 286

If k = 73 and n = 358 then above sequence is n – k + 1

Principal 2. Number of numbers between k and n is n – k + 1

$n-(k-1)$ or $(n-k+1)$

Very simple counting

How many numbers are there between 73 and 358 which are

divisable by 7 (both inclusive)

73, 74, 75 …… …….. 356, 357, 358

77, 84, 91, …………………………343, 350, 357

Its not a sequence of consecutive numbers

Divide by 7 =

11, 12, 13, …………………………49, 50, 51

k=4 n=51

51 - (11-1)

= 41

Principal 2. Number of numbers between k and n is n – k + 1

K = 11 and n = 51

So, number of elements in the sequence is 51 – 11 + 1 = 41

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Very simple counting

How many numbers are there in the sequence

-21, -17, -13 ……………………………391, 395, 399

Difference of 4 but not divisable by 4

Now if we add 1 to this sequence

-20, -16, -12 ……………………………392, 396, 400

Divide by 4 =

-5, -4, -3, ……………………………98, 99, 100

Principal 2. Number of numbers between k and n is n – k + 1

K = -5 and n = 100

So, number of elements in the sequence is 100 – (-5) + 1 = 106

~~+1 add~~
+1 all
-20, -16, -12 ——— 396, 400
÷ 4
-5 to 100 (k-1)
100 - (-5+1) = ~~100~~ 1
100 - (-4) = 104
100 - (-5-1)
100 - (-6) = 106

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Very simple counting

240
24
4
268

How many numbers are there in this sequence

9 5/12, 9 5/6, 10 1/ 4 ………………..21 ½, 21 11/12, 22 1/3

9 5/12, 9 10/12, 10 3/12 ………… 21 6/12, 21 11/12, 22 4/12

Convert to proper fraction

$\frac{113}{12}, \frac{118}{12}, \frac{123}{12}, \text{-------}, \frac{268}{12}$

(9*12 +5) /12

×12 all sides

113 / 12, 117 / 12, 123 / 12 ………………258 / 12, 263 / 12, 268 / 12

Multiply by 12

+2

115, 120, 125 — 270

113, 118, 123 ……………………..258, 263, 268

÷5

23, 24, — 54

+ 2

115, 120, 125………………………..260, 265, 270

54 − (23 − 1)

32

Divide by 5 = 23, 24, 25 ………………………52, 53, 54

The multiplication Principal

South Indian: Vada, Idly, Dosa, Pongal, uthappam

North Indian: Alo Paratha, Sandwich, Poha

Beverages: Coffee, Tea, Milk

5
x 3
x 3

48

Combo = one South Indian, One North Indian, One Beverages

What are the different combinations possible can I have different breakfast every day of the month

For every combination of South Indian dish
I can make 3 x 3 = 9 Combinations (dicision_tree)
So I can make 5 x 3 x 3 = 45 combinations

Then,
Principal 3: number of ways of making a sequence of independent choices is just the product of the number of choices at each step.

The multiplication Principal

8 Boys ……………………..

12 Girls …………………….

Combination of 8 / B 12 / G

If a students committee consists of 1 boy and 1 girl

How many combinations of boys and girls are possible.

8 | 12

B G

96

Committee = 8 * 12 * …….* ……………..*…………….

Regardless of number steps the rule (Principle) remains the same

5 r, 3 m, 4 sh, 5 sn, 3 tran

5 * 3* 4* 5 * 3

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The multiplication Principal (Special Case 1)

How can you make a sequence of k objects from given n objects

with repetition.

n^k

Example: a fitness enthusiast has 10 different activities to choose from

Walking, running, Arobics, Zomba, Crossfit, Yoga, Squash, badminton,

Swimming, Gym

Monday, Tuesday, Wednesday, Thursday, Friday, Saturday & Sunday

How many weekly exercise plans can you make, if you can repeat

the same exercise more than once.

10 * 10 * 10 * 10 * 10 * 10 * 10
M T W T F S S

$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

Principal 4: The number of sequences of K objects made from given n objects, when any objects in the sequence can be repeated any number of times is N ** k

The multiplication Principal (Special Case 1)

How many 5 letter words can you form using the alphabets of the English language?

26^5

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

5 decisions eg: b I n g o

Applying the principle:

The number of sequences of K objects made from given n objects, when any objects in the sequence can be repeated any number of times is N ** k

The number of 5 letter words that can be created is 26 ** 5

Thank You!

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The multiplication Principal (Special Case 2)

How can you make a sequence of k objects from given n objects **without** repetition.

Example: a fitness enthusiast has 10 different activities to choose from

Walking, running, Arobics, Zomba, Crossfit, Yoga, Squash, badminton, Swimming, Gym

Monday, Tuesday, Wednesday, Thursday, Friday, Saturday & Sunday

How many weekly exercise plans can you make, if you can **not repeat** the exercise more than once per week.

$$\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$$

10 *	9 *	8 *	7 *	6 *	5 *	4
M	T	W	T	F	S	S

$$= \frac{n!}{(n-k)!} \qquad \frac{10!}{3!} = \frac{10!}{(10-7)!}$$

K = 7
N = 10

The multiplication Principal (Special Case 2)

$10 \times 9 \times 8 \times 7 \times 6 \times 5 \to 4$

$10 - 6$

$(10 - (7 - 1))$

10 *	9 *	8 *	7 *	6 *	5 *	4
M	T	W	T	F	S	S

K = 7
N = 10

10 * (10 - 1) * (10 – 2) * (10 – 3) * (10 – 4) * (10 -5) * (10 -6)

$$\frac{n!}{(n-k)!} = n \times (n-1) \times (n-2) \ldots \times (n-(k-1))$$

Principal 5 : The number of sequences of K objects made from given n objects, such that no objects in the sequence can be repeated is :

n * (n - 1) * (n – 2) * (n – 3) * (n – 4) * (n -5) * (n – (k-1))

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The multiplication Principal (Special Case 2)

with repeat | w/o repeat

9^3 | $\frac{n!}{(n-k)!}$

$\frac{9!}{6!} = 9 \times 8 \times 7$

How many 3 digit numbers

9 Choices **1, 2, 3, 4, 5, 6, 7, 8, 9**

3 Decisions ______ ______ _______

How many weekly exercise plans can you make, if you can not repeat the exercise more than once per week.

10 * 9 * 8 * 7 * 6 * 5 * 4
M T W T F S S

K = 7
N = 10

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The multiplication Principal (Special Case 2)

w/o rep odd

$\boxed{7 \times 8 \times 5}$

Twist How many of the above number are odd ?

9 Choices 1, 2, 3, 4, 5, 6, 7, 8, 9

3 Decisions ___9___ ___8___ ___5___ ?

9 * 8 * ?

I can still apply multiplication principal

How ? (Number of chooses made is independent of previous step)

Start from the last digit

7 * 8 * 5

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The multiplication Principal (Special Case 2)

26 X 25 X 24 X 23 X 22

Similarly if I take another example:

How many 5 letter words can you form using the alphabets of English

language so that no letter is repeated ?

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

5 decisions eg: 26 choices

Applying Principle:

26 * 25 * 24 * 23 * 22

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The multiplication Principal (Special Case 2)

25	24	23	22	21

Twist How many of those words would end with a consonant ?

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

5 decisions eg:

B	c	d	f	?
26	25	24	23	17

All consonant

Now if I change the order in which I make the decision then:

~~7 * 8 * 5~~

5th	4th	3rd	2nd	1st
22	23	24	25	21

Now we can apply the multiplication principal because number of choices at each step is independent of choices made in previous step.

Principal 6: if the problem specifies a constraint or restriction then always start by addressing the restriction first

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The multiplication Principal (Special Case 2)

Example: A different kind of sequence given a class of 15 students, on how many ways can you form a committee comprising of a president, vice president, treasure and secretary

P VP T S

P VP T S

15 14 13 12

9 choices: 1 2 3 4 5 6 7 8 9

Decisions: ________ ___________ _________

26 choices: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, x, y, z

5 decisions: ___ ___ ___ ___ ___

A sequence is some thing in which the order matters, In this case does the order matter ?

$n = 15 \quad k = 4$

$\frac{n!}{n-k} = \frac{15!}{11!} =$

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The multiplication Principal (Special Case 2)

Principal 7: The numbers of ways of filling K named or numbered slots using a collection of n objects is the same as the number of ways of creating a sequence of k elements such that no objects in the sequence can be repeated

n * (n - 1) * (n – 2) * (n – 3) * (n – 4) * (n -5) * (n – (k-1))

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The multiplication Principal (Special Case 2)

How can you make a sequence of n objects from a given n objects ?

Problem: Suppose you have 9 flower pots that you arrange in a line at the entrance of your house, In how many different ways can you arrange these pots ?

$9 \times 8 \times \ldots\ldots)$

N = 9
K = 9
Number of data elements = n (n-1) (n-2) (n-3) ……..(n-k +1)
9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1
= 9!   (Factorial of 9)

9!

$\frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$

Principal 8: The number of sequence of length n that can be formed using n objects, such that no objects in the sequence is repeated is n ! (factorial of n)

The number of ways in which n objects can be arranged amongst themselves is n !

The number of permutations of n objects is n!

The multiplication Principal (Special Case 2)

How counting sequences appears in probability ?

For letters a, b, c. What are the probabilities of combination for getting world “cab”

$\frac{1}{3\times 2\times} = \frac{1}{6}$

3 n objects , sequence of length 3	ABC	BCA
3! = 6	ACB	CAB
Probability is 1 / 6	BAC	CBA

Let us revisit our principal formula

The number of sequences of k objects made from a collection of n objects, such that no objects in the sequence can be repeated is $\frac{n(n-1)(n-2)\ldots\ldots(n-k+1)\,(n-k)(n-k-1)(n-k-2)\ldots.3*2*1}{(n-k)(n-k-1)(n-k-2)\ldots.3*2*1}$

$$\frac{n!}{(n-k)!}$$

This is a more compact representation of the formula

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The multiplication Principal (Special Case 2)

5R 44

5 | 4 | 4 | 3 | 3 | 2 | 2 | 1 | 1

Flower pots problem with twist:

Red	Red	Red	Red	Red	Yellow	Yellow	Yellow	Yellow
1	2	3	4	5	6	7	8	9

5! X 4!

In how many ways can you arrange the pots so that no 2 red pots are adjacent to each other

N = 5 + 4
K = 9

Red ↑ Yellow ↑ Red ↑ Yellow ↑ Red ↑ Yellow ↑ Red ↑ Yellow ↑ Red ↑

5!
4!

Number of possible ways of arranging the pots adjacent to each other is 5! * 4!

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Recap

Principal 1. Number of numbers between 1 and n is n

Principal 2. Number of numbers between k and n is n – k + 1

n – (k – 1)

Multiplication

Principal 3: number of ways of making a sequence of independent choices is just the product of the number of choices at each step.

Principal 4: The number of sequences of K objects made from given n objects, when any objects in the sequence can be repeated any number of times is N ** k

Principal 5 : The number of sequences of K objects made from given n objects, such that no objects in the sequence can be repeated is :

n * (n - 1) * (n – 2) * (n – 3) * (n – 4) * (n -5) * (n – (k-1))

Principal 6: if the problem specifies a constraint or restriction then always start by addressing the restriction first

Principal 7: The numbers of ways of filling K named or numbered slots using a collection of n objects is the same as the number of ways of creating a sequence of k elements such that no objects in the sequence can be repeated

Principal 8: The number of sequence of length n that can be formed using n objects, such that no objects in the sequence is repeated is n ! (factorial of n)

The Subtraction Principal

Recap on rules:

Always address the restriction first

The number of choices at each step should be independent of the choices made at previous steps

What if you can not follow the above rules?

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The Subtraction Principal

How many 3 letter words can you form which contain atleast one vowel and no letter is repeated

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

3 decisions eg: ______ ______ ______

Start with restriction:	3	2	1
	24*	25*	5 (aeiou)
Start with middle letter:	2	1	3
	25*	5*	24

What is happening her is that we are not being able to rearrange the decision making in such a way that we are able to satisfy the original condition in the question.

- 3 letter word, atleast 1 vowel w/o rep

= 3 letter word All − 3 letter consonant

$= (26 \times 25 \times 24) - (21 \times 20 \times 19)$

?

confu

The Subtraction Principal

We can not easily apply multiplication function as the number of choices for the last decision depends on the previous choices

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

3 decisions eg: _______ ________ _________

Subtraction Principal:

The number of objects that satisfy some condition is equal to the total number of objects in the collection minus the ones which do not satisfy this condition

The Subtraction Principal

A = Set of all 3 letter words with no letter repeated

B = Set of all 3 letter words with no letter repeated and atleast one oval

C = Set of all 3 letter words with no letter repeated and no vowels

Then B = A - C

A = 26 * 25 * 24

C = 21 * 20 * 19

B = (26 * 25* 24) – (21 * 20 * 19)

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The Subtraction Principal

Another Example:

How many 5 letter words can you form which contain atleast 2 consecutive letters which are the same ?

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

5 decisions eg: ______ ______ ______ ______ ______

repetition allowed

Atleast 2 consecut letter Same

= All comb

– No 2 cons same

$26 \times 26 \times 26 \times 26 \times 26$

$= 26 \times \cancel{25 \times 24 \times 23 \times 22}$

– (26 | 25 | 25 | 25 | 25)

+1

- ✓ **Apple, Sheep, Utter, Atta, Loop**
- X **Bears, Rusty, Doduo**

The Subtraction Principal

A = Set of all 5 letter words with no letter repeated

B = Set of all 5 letter words containing atleast 2 consecutive letters which are same

C = Set of all 5 letter words with no consecutive letters which are the same

Then B = A - C

A = 26 * *5

C = 26 * 25**4

B = (26 ** 5) - (26 * 25 **4)

Collection

Collections

Recap on sequence

In sequence the order matters

Cat $\neq$ act

Even though both have the same set of letters: (t, c, a)

In collection order does not matter

Cat = act = tac = tca = atc = cta (all factorial combinations)

All the 6 words have the same letters: a, c, t

Collections

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Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

3 decisions eg: ______ ______ ______

How many sequences of 3 letters can you form (no repetition) ?

$$\frac{n!}{(n-k)!}$$

$26 \times 25 \times 24$

How many collections of 3 letters can be formed (no repetition) ?

We don't know that

$\cancel{\frac{26 \times 25 \times 24}{3 \times 2 \times 1}}$

$\left(\frac{n!}{(n-k)!}\right)\frac{1}{k!} = \frac{26, 25, 24}{3!}$

But, we now how to count the sequences. Can we reuse that knowledge ??

Collections

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

3 decisions eg: ______ ______ ______

Breaking down the Sequences :

Step 1: select the 3 letters to be put in the word

Making a collection

Step 2: re-arrange the 3 letters in 3! Ways

Re arranging elements in the Sequence

not do in ~~comb~~

In collection I am only concerned about the step 1

Collections

Alphabets: a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

26 Choices

3 decisions eg: ______ ______ ______

Making a collection

N = Number of ways of selecting k elements

Re arranging elements in the collections

k! = number of ways of re arranging the k terms

Number of Sequences = N*k! = $\frac{n!}{(n-k)!}$

Therefore $\binom{n}{k} = \cancel{N} = \frac{n!}{(n-k)! * k!}$

Collections

Perm = Comb X K!

What is the number of ways of choosing 3 Vowels form 5 Vowels ?

a e i o u

Collections:	Sequences:
(a, e, i)	{(a, e, i), (a, I, e), (e, a, i), (e, I, a), (I, a, e), (I, e, a)}
(a, e, o)	(a, e, o) eao oae
(a, e, u)	(a, e, u)
(a, i, o)	(a, i, o)
(a, I, u)	(a, I, u)
(a, o, u)	(a, o, u)
(e, I, o)	(e, I, o)
(e, I, u)	(e, I, u)
(e, o, u)	(e, o, u)
(I, o, u)	(I, o, u)

Colle

$\frac{5 \times 4 \times 3}{3 \times 2 \times 1}$

10

Seq

$5 \times 4 \times 3$

= 60

Number of collections possible by selecting 3 letters from given 5 letters is 10 Coll

Number of Sequences possible is : number of collections $\boxed{N * k ! = 10 * 3 ! = 60}$

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Collections

Given a class of 15 students in how many ways can you form a committee of 4 members?

Are we creating a collection or Sequence

Sequences:

ABCD
ABDC
ACBD
ADBC
ADCB
BACD
….
…..
……. 4!
24

perm → $\boxed{15 \mid 14 \mid 13 \mid 12}$

comb = $\dfrac{\qquad}{4!}$

$$\frac{15!}{(15 - 4)!\ *\ 4!}$$

$$\frac{n!}{(n - k)!\ *\ k!}$$

$^{8}C_{6} = {}^{8}C_{2} =$

$^{n}C_{k} = {}^{n}C_{n-k}$

$\dfrac{n!}{(n-r)!\,r!} =$

Collections

Collections Principal: The number of ways of selecting k objects from a given n objects is

$$\frac{n!}{(n-k)! * k!}$$

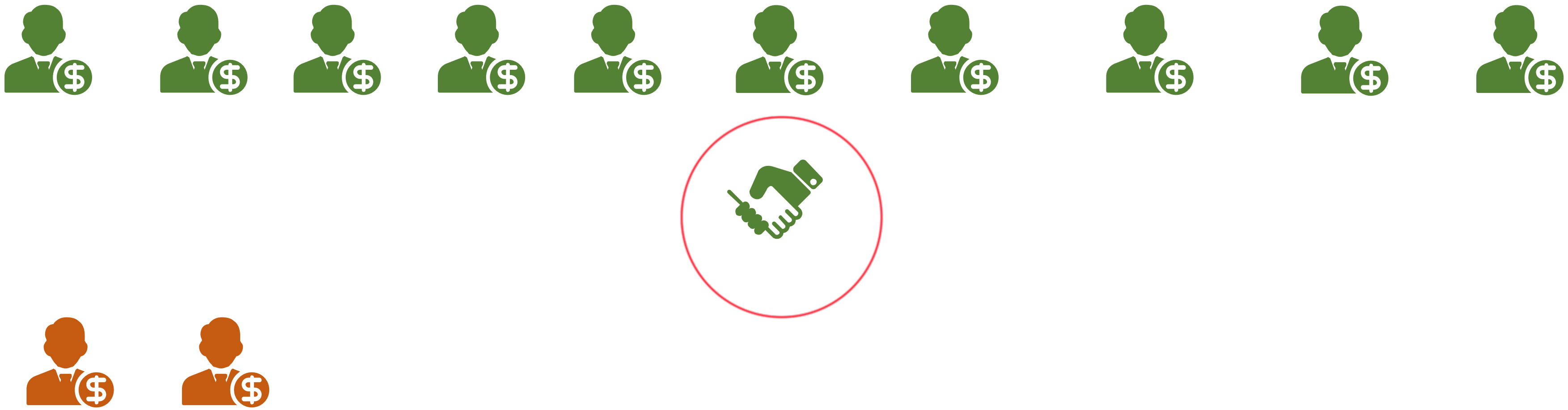
And is denoted as $\binom{n}{k}$

Collections, Some examples

Consider 10 people in a meeting room, each person shakes hands with every other person in the meeting room what is the total number of handshakes?

$^{10}C_2$

$\frac{10\times 9}{2} = 5\times 9 = 45$



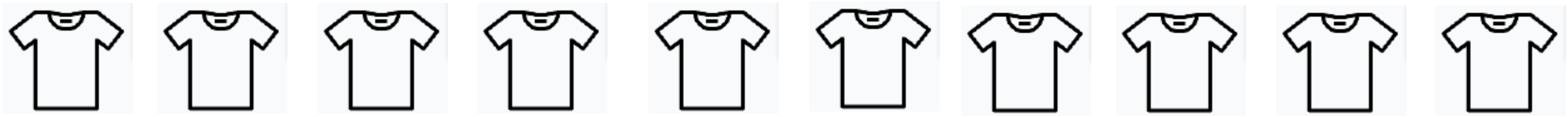
$$\frac{n!}{(n-k)!*k!}$$

$$\frac{10!}{(10-2)!*2!}$$

$$\binom{10}{2} = 45$$

Collections, Some examples

You are going on a vacation and your suitcase has space for only 3 shirts, in how many ways can you fill the suitcase ?



$$\frac{10 \times 9 \times 8}{3 \times 2}$$

$$\frac{n!}{(n-k)!\,*\,k!}$$

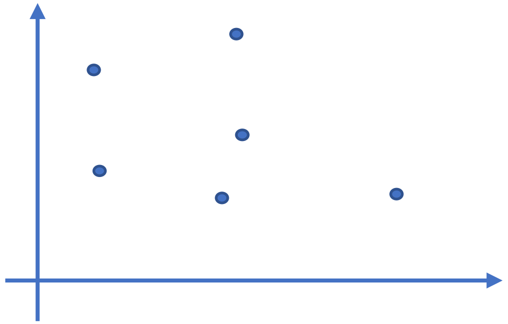
$$\frac{10!}{(10-3)!\,*\,3!}$$

$$\binom{10}{3} =$$

$$\binom{10}{3} =$$

Collections, Some examples

There are six points on a two dimensional plane such that no three points are co-llinear. How many segments can you draw using these 6 points?



$$^6C_2 = \frac{6 \times 5}{2 \times 1} = 15$$

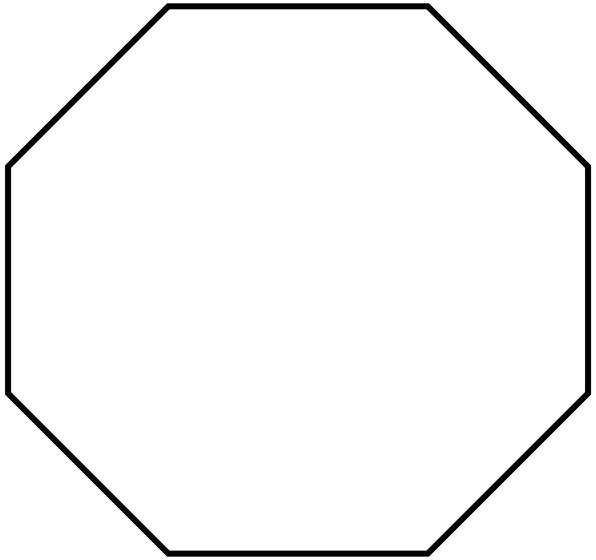
$$\frac{n!}{(n-k)!*k!}$$

$$\frac{6!}{(6-2)!*2!}$$

$$\binom{6}{2} =$$

Collections, Some examples

How many triangles can be formed from the vertices of a polygon of sides n, n =8?



${}^{n}C_3$ ${}^{8}C_3 - 3$

${}^{8}C_3$

$$\frac{n!}{(n-k)!\, *\, k!}$$

$$\frac{8!}{(8-3)!\, *\, 3!}$$

$$\binom{8}{3} =$$

Collections with Repetitions

Recap

Sequences

Without repetitions $\dfrac{n!}{(n-k)!}$

With repetitions $n^{**}k$

Collections

Without repetitions ${}^{n}C_{k}$ $\dfrac{n!}{(n-k)! * k!}$ $\binom{n}{k}$ ${}^{n}C_{k}$

With repetitions ${}^{n+k-1}C_{k}$ $\binom{n+k-1}{k}$

$n - \cancel{k} + 1 + \cancel{k}$

$(n+1) - k$

$C_{(n+1)}$

$n-(k-1)$
k

$n-(k-1)\,C_{k} = C_{n+1}$

$n-k+1$

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Collections with Repetitions,

Examples:

How many breakfast combos containing 5 items can you form if you are allowed to have multiple servings of the same dish ?

Items: ~~1D~~ ~~2I~~ 3P ~~4V~~ 5U 6P 7S 8PH ~~9C~~ 10T

Magic Counter: ~~1D~~ 2D 3I 4V

Combo: 5 items 1D 1D 2I 4V 9C

$$\frac{10 \times 10 \times 10 \times 10 \times 10}{5 \times 4 \times 3 \times 2 \times 1}$$

$^{(n+1)-k}C_{n+1}$ $\quad {}^{11-5}C_{11}$ $\quad {}^{11}C_6$

Without repetitions $\dfrac{n!}{(n-k)! * k!}$ ~~n = 10 + k-1~~

$4 = 5 - 1$

With repetitions $\binom{n+k-1}{(n-K)! * k!}$ total item $\dfrac{\binom{10+(5-1)}{(14-5)! * 5!}}{}$ 9 $\binom{14}{5}$ $\quad {}^{n+k-1}C_{k}$

Collections

Collections Principal: The number of ways of selecting k objects from a given n objects with repetitions is $\binom{n+k-1}{k}$

Collections Recap

Recap

Sequences

Without repetitions $$\frac{n!}{(n-k)!}$$

With repetitions **n**k**

Collections

Without repetitions $$\frac{n!}{(n-k)! * k!}$$

With repetitions $$\binom{n+k-1}{k}$$

n

k

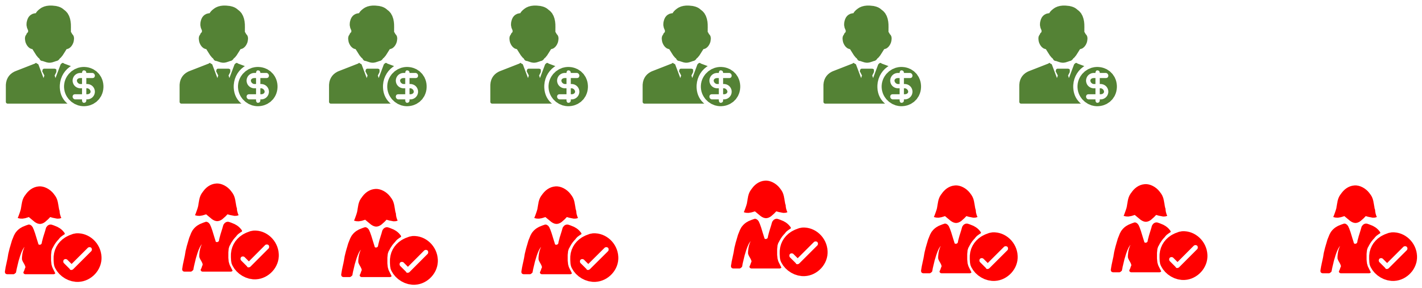
Collections with multiplication principal

Given a class of 7 boys and 8 girls, in how many ways can you form a committee of 4 members with 2 boys and 2 girls ?

$^7C_2 \times {}^8C_2$

Boys

Girls



Break the problem into 2 :

Number of ways of selecting 2 boys from 7 = $\binom{7}{2}$

Number of ways girls from 8 = $\binom{8}{2}$

Number of ways of combining section of bays and girls is the product of individual collections = $\binom{7}{2} * \binom{8}{2}$

Collections with multiplication principal

Different ways of forming a cricket team using the below available players?

$^7C_5 \times {^2C_1} \times {^4C_3} \times {^3C_2}$

Available		Select
7	Batsmen	5
2	Keepers	1
4	**Pacers**	**3**
3	Spinners	2

Total $= \binom{7}{5} * \binom{2}{1} * \binom{4}{3} * \binom{3}{2}$

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Collections with multiplication principal

Generic version

Derivation

Given: n items of l different types

M1 + m2+ m3+ ……….+mi = n

Form: Colloction of k items

K1 + k2 + k3 …..Ki = K

Available		Select
7 m1	Batsmen	5 k1
2 m2	Keepers	1 k2
4 m3	**Pacers**	**3 k3**
3 mi	Spinners	2 ki
N=16	l = 4	k = 11

$$N = \binom{n}{k} = \binom{m1}{k1} * \binom{m2}{k2} * \binom{m3}{k3} * \text{……….} * \binom{mi}{ki}$$

$^{7}C_{5}$ $\quad$ $^{2}C_{1}$

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Collections with subtraction principal

How many different ways can we form a 4 members committee containing atleast one gynecologist ?

3 Cardiologists

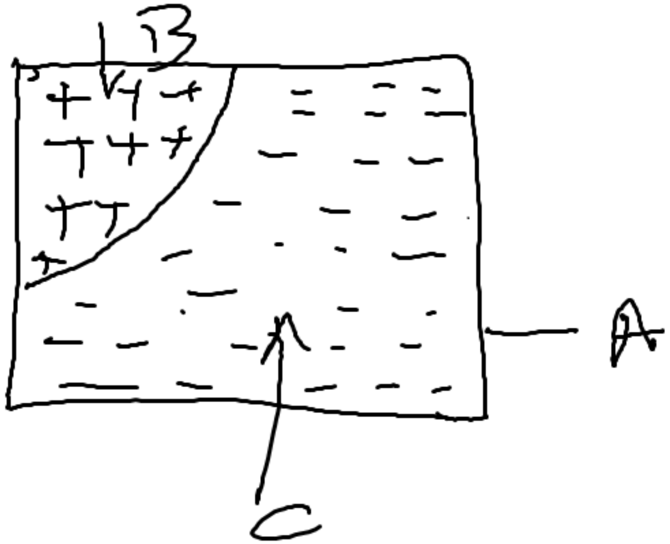
2 Neurologists

4 dialectologists

5 gynecologists

7 general physicians

Total 21 Doctors, form a committee of 4 doctors ,



N = $\binom{n}{k}$ = $\binom{21}{4}$

A = all possible combination of 4 members / Commitees

B = all possible combinations of committees containing atleast 1 Gyno

C = all possible combinations of committees not containing Gyno

B = A - C

$^{21}C_4 - {}^{16}C_4$

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Collections with Subtraction principal

Count (A) = $\binom{21}{4}$

Count (C) = $\binom{16}{4}$

Count (B) = $\binom{21}{4}$ - $\binom{16}{4}$