EET 109 Notes on Fast Decoupled Load Flow

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1. Introduction

The Fast Decoupled Load Flow (FDLF), first proposed by Stott and Alsac [1], offers a computationally efficient approximation that leverages the weak coupling between active/reactive powers and bus voltage angle/magnitude. It lies in between DC and AC Power Flow.

2. Power Flow Equations– You Know It

For an n-bus system, the complex power injection at bus i is

$$S_i = P_i + jQ_i = V_i \sum_{k=1}^{n} V_k Y_{ik}^* e^{j(\theta_i - \theta_k)}, \tag{1}$$

where, V_i and θ_i are the voltage magnitude and angle at bus i, and $Y_{ik} = |Y_{ik}|e^{j\phi_{ik}}$ is the (i, k) element of the bus admittance matrix. Separating into real and imaginary parts, the active and reactive power injections are

$$P_i = \sum_{k=1}^{n} V_i V_k \left(G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)\right), \tag{2}$$

$$Q_i = \sum_{k=1}^{n} V_i V_k \left(G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)\right), \tag{3}$$

where G_{ik} and B_{ik} are the real and imaginary parts of Y_{ik}.

3. Newton–Raphson Formulation

The Newton–Raphson method linearizes the nonlinear power flow equations as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}, \tag{4}$$

where ΔP and ΔQ are the power mismatches, and the Jacobian blocks are

$$J_{11} = \frac{\partial P}{\partial \theta}, \qquad J_{12} = \frac{\partial P}{\partial V}, \tag{5}$$

$$J_{21} = \frac{\partial Q}{\partial \theta}, \qquad J_{22} = \frac{\partial Q}{\partial V}. \tag{6}$$

The NR method updates (θ, V) iteratively until mismatches are within tolerance.

4. Decoupling Assumptions

Empirical observations and physical reasoning suggest two key properties in high-voltage transmission networks [2]:

1. Active power P is strongly coupled with voltage angle θ, but weakly dependent on voltage magnitude V.
2. Reactive power Q is strongly coupled with voltage magnitude V, but weakly dependent on angle θ.

These lead to the following simplifications:

$$J_{12} \approx 0, \qquad J_{21} \approx 0. \tag{7}$$

Thus, the Jacobian becomes approximately block-diagonal:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \approx \begin{bmatrix} J_{11} & 0 \\ 0 & J_{22} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}. \tag{8}$$

5. Further Approximations: Fast Decoupled Method

Stott and Alsac introduced additional approximations [1]:

- Transmission lines have high reactance-to-resistance ratio ($X \gg R$), so $G_{ik} \approx 0$.
- Voltage magnitudes vary slowly around 1 p.u., i.e., $V_i \approx 1$.

Under these assumptions:

$$J_{11} \approx -B', \qquad J_{22} \approx -B'', \tag{9}$$

where B' and B'' are constant matrices derived from the network susceptances:

- B' governs the relation between active power and voltage angle.
- B'' governs the relation between reactive power and voltage magnitude.

6. FDLF Iterations

The FDLF algorithm alternates between solving two linear systems:

$$-B' \, \Delta\theta = \Delta P/V, \tag{10}$$
$$-B'' \, \Delta V = \Delta Q/V, \tag{11}$$

where mismatches are normalized by bus voltages. The updates are

$$\theta^{(k+1)} = \theta^{(k)} + \Delta\theta, \tag{12}$$
$$V^{(k+1)} = V^{(k)} + \Delta V. \tag{13}$$

References

[1] B. Stott and O. Alsac, "Fast Decoupled Load Flow," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-93, no. 3, pp. 859–869, May 1974.

[2] P. Kundur, *Power System Stability and Control*. McGraw-Hill, 1994.

[3] J. J. Grainger and W. D. Stevenson, *Power System Analysis*. McGraw-Hill, 1994.